Revisit:
Passive elements & Filters

Impedance:

Passive elements' - Resistive, inductive & capacitive

Ohm's law:

The voltage across a conducting material is directly proportional to the current flowing through the material.

$$\boldsymbol{V = IZ}$$

Resistor

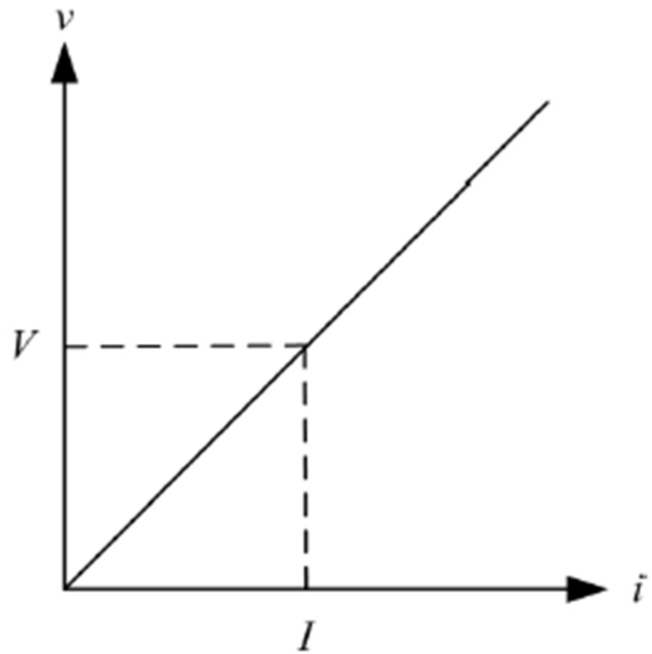
$$V = IR$$

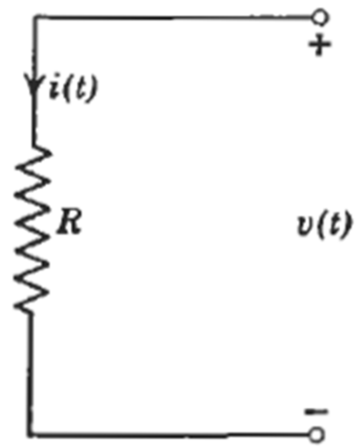
$$R = \frac{V}{I} \rightarrow resistance$$

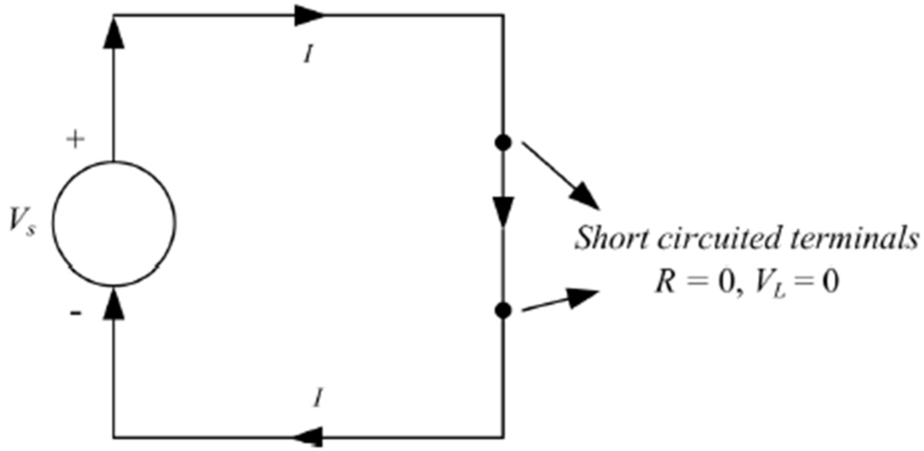
$$G = 1/R \rightarrow conductance$$

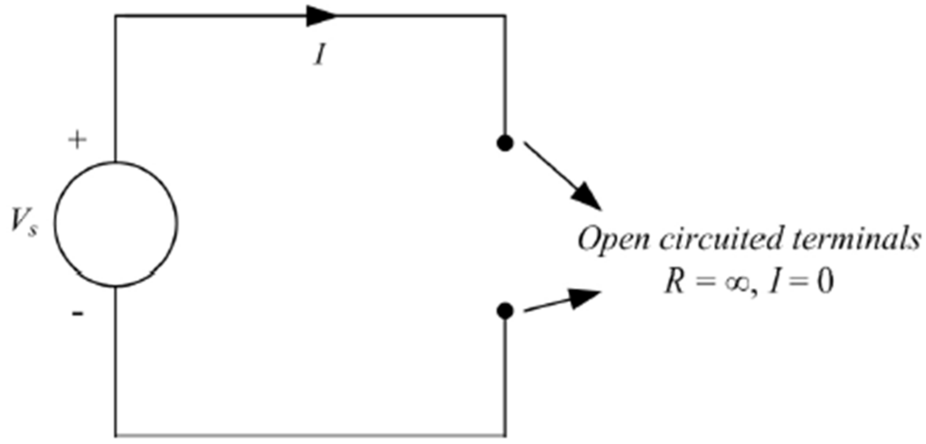
$$R = R_1 + R_2 \qquad \text{In series}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{In parallel}$$



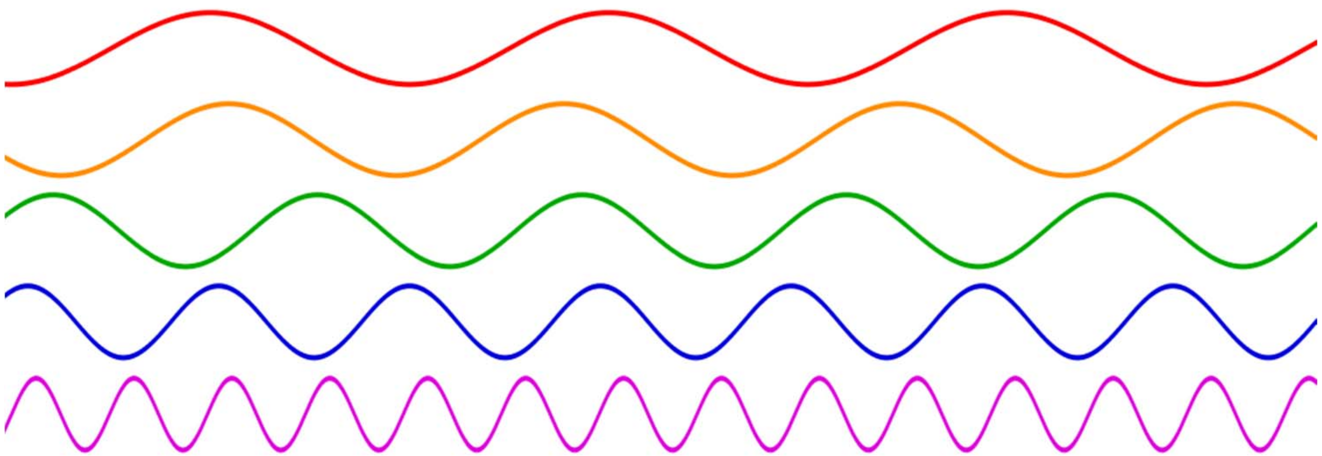






Voltage source: Direct & Alternating

Different frequencies:



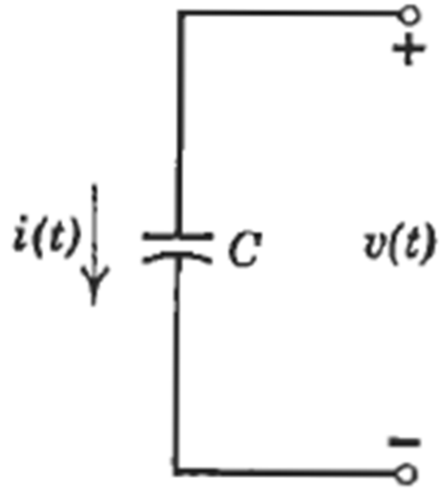
$$\boldsymbol{\omega = 2\pi f}$$

$$\boldsymbol{V sin(\omega t)}$$

$$\boldsymbol{I sin(\omega t)}$$

Capacitor

$$I = C\frac{dV}{dt}, C \rightarrow capacitance$$



$$V = I\frac{1}{j\omega C}$$

$$Impedance: \frac{1}{j\omega C}, capacitive\ reactance \rightarrow X_C = \frac{1}{\omega C}$$

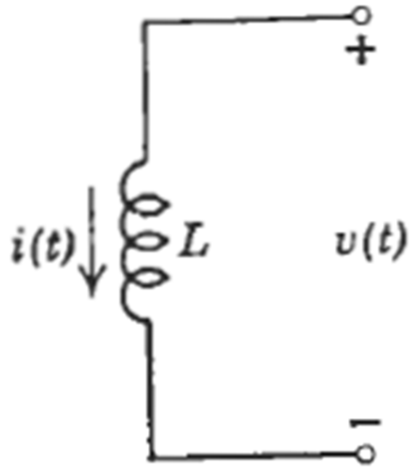
Magnitude

$$X_C \rightarrow \infty\ as\ \omega \rightarrow 0\ (DC)$$

Capacitor is a open circuit for DC alone!

Inductor

$$V = L\frac{dI}{dt}, C \rightarrow inductance$$



$$V = j\omega LI$$

$$Impedance: j\omega L,\ inductive\ reactance \rightarrow X_L = \omega L$$

Magnitude

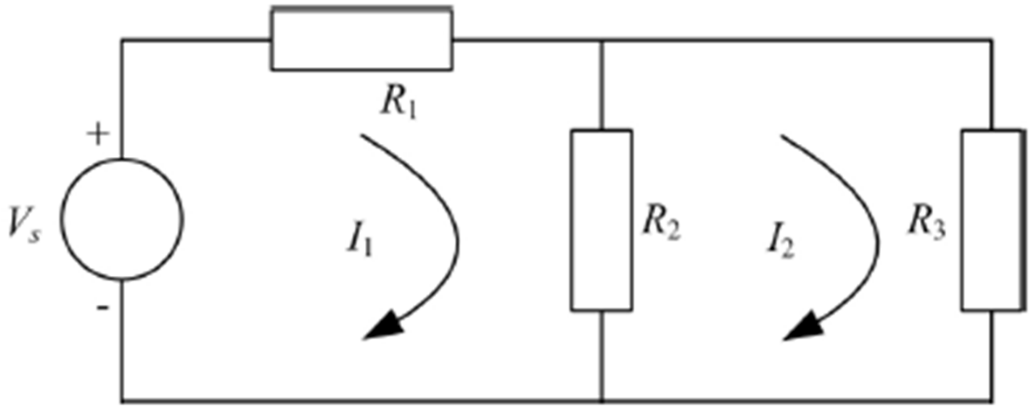
$$X_L \rightarrow 0\ as\ \omega \rightarrow 0\ (DC)$$

Inductor is a short circuit for DC alone!

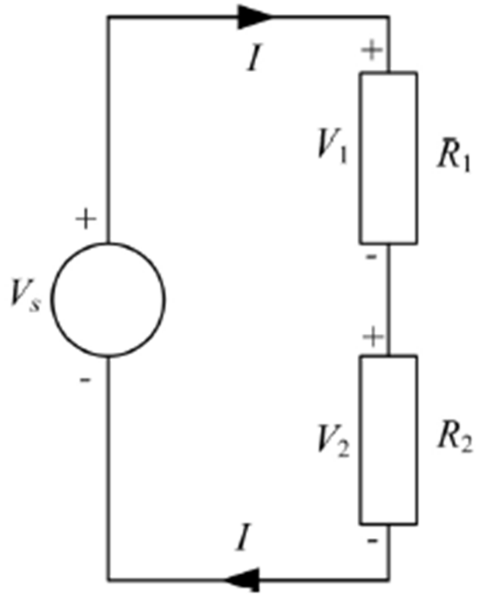
Kirchoff's Laws:

Voltage law:

The sum of all the voltages (rises and drops) around a closed loop is equal to zero.



$$V_s = I_1R_1 + (I_1 - I_2)R_2 \qquad\qquad 0 = I_2R_3 - (I_1 - I_2)R_2$$



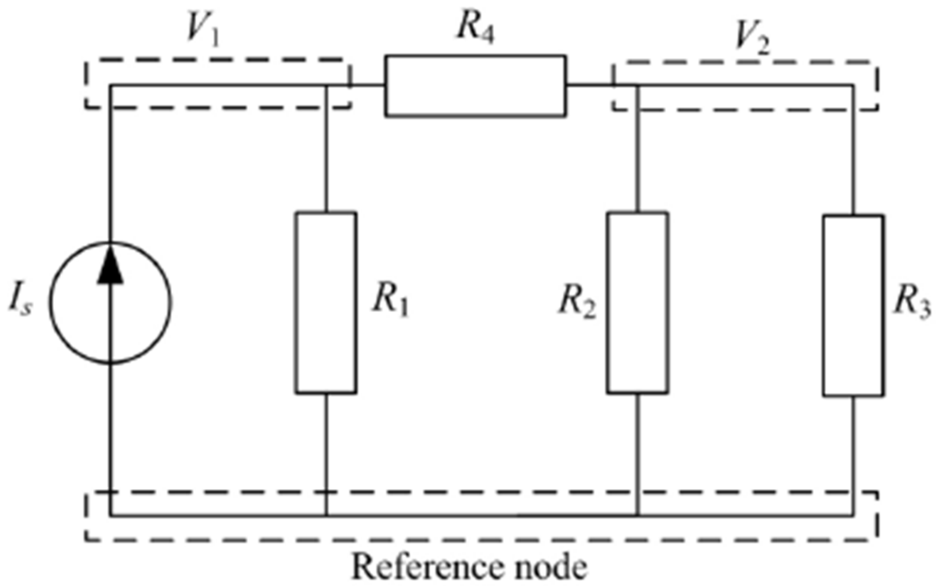
$$KVL\text{: } V_s = V_1 + V_2$$

Voltage divider rule

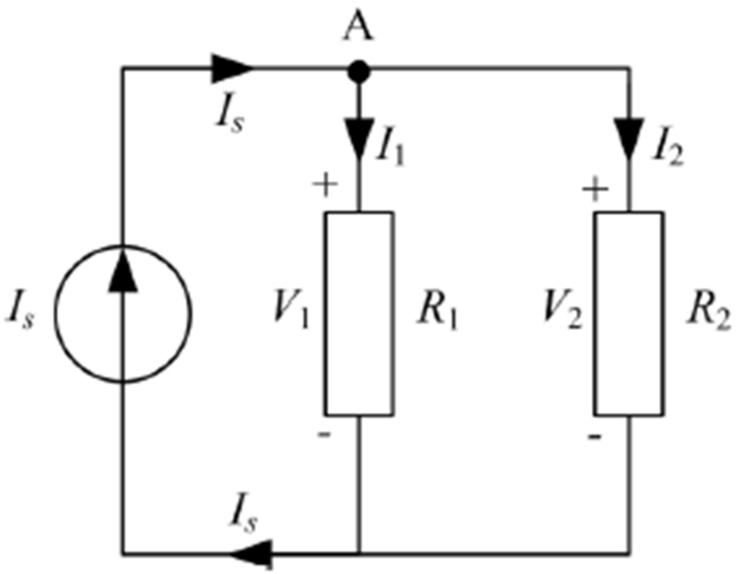
$$V_1 = \frac{R_1}{R_1 + R_2} V_s, V_2 = \frac{R_2}{R_1 + R_2} V_s$$

Current law:

The algebraic sum of all the currents entering or leaving a node in an electric circuit is equal to zero



$$I_s = V_1/R_1 + (V_1 - V_2)/R_4 \qquad 0 = -\frac{V_1 - V_2}{R_4} + \frac{V_2}{R_2} + \frac{V_2}{R_3}$$



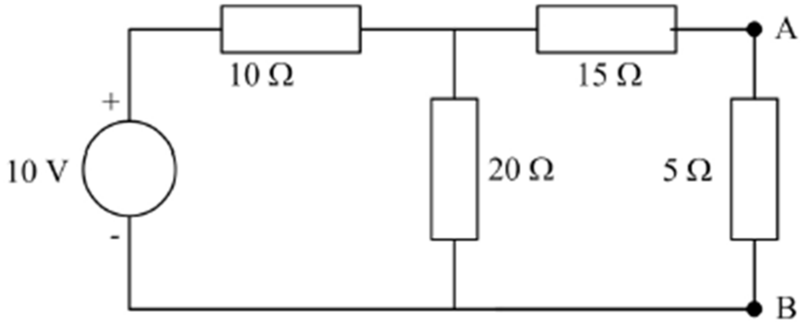
$$KCL\text{: } I_s = V_1/R_1 + V_2/R_2$$

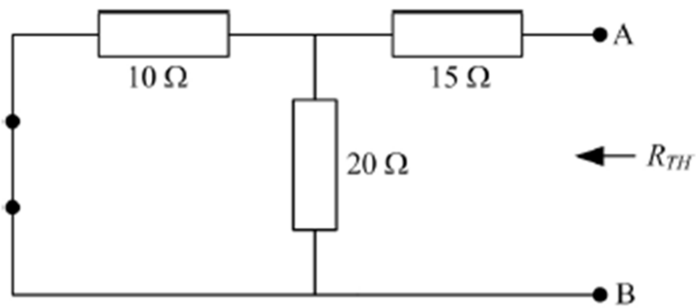
Current divider rule

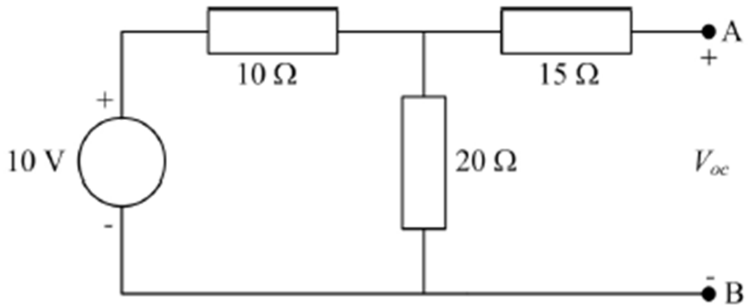
$$I_1 = \frac{R_2}{R_1 + R_2} I_s, I_2 = \frac{R_1}{R_1 + R_2} I_s$$

Thevenin's Theorem:

Any two-terminal linear circuit of resistor and sources can be replaced by an equivalent circuit with a single voltage source in series with a resistor.





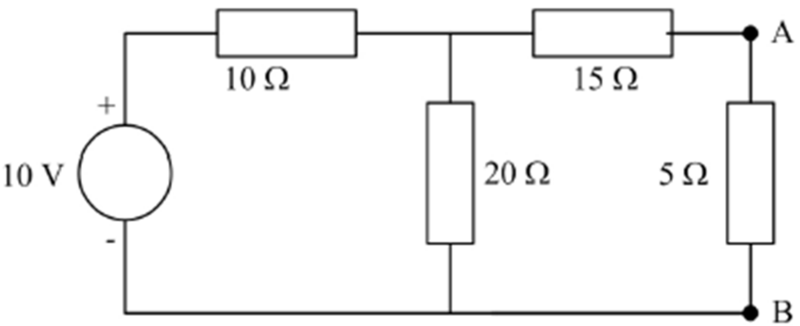


$$R_{TH} = (10||20) + 15 = \frac{10 \times 20}{10 + 20} + 15 = 21.67\ \Omega$$

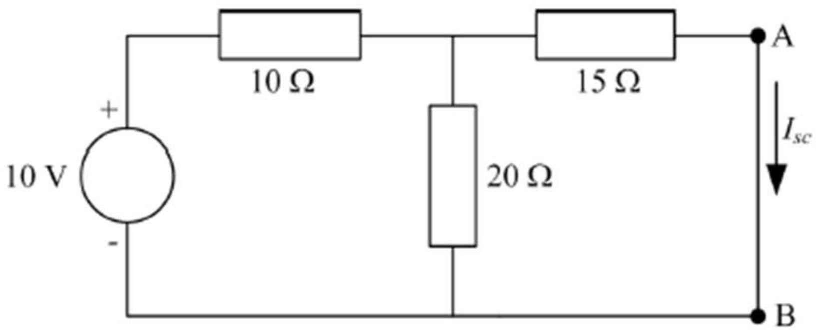
$$V_{TH} = V_{oc} = 10\frac{20}{10 + 20} = 6.67\ \text{V}$$

Norton's Theorem:

Any two-terminal linear circuit of resistor and sources can be replaced by an equivalent circuit with a single current source in parallel with a resistor.



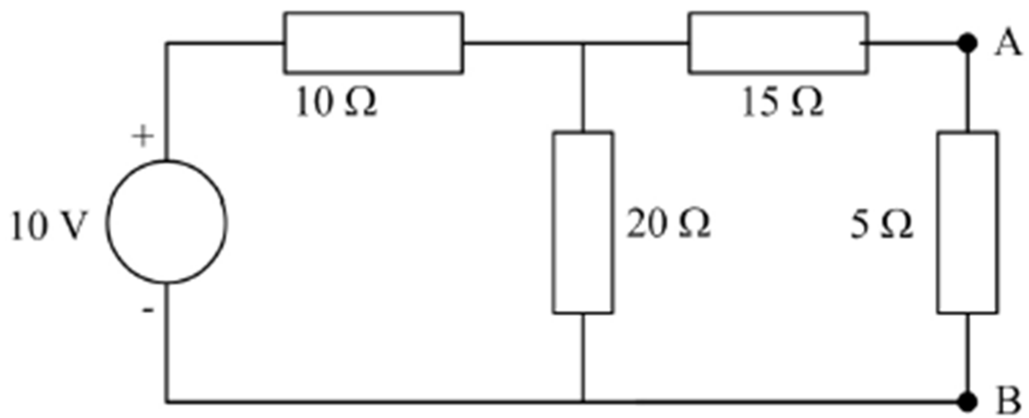
$$R_N = 21.67\ \Omega$$



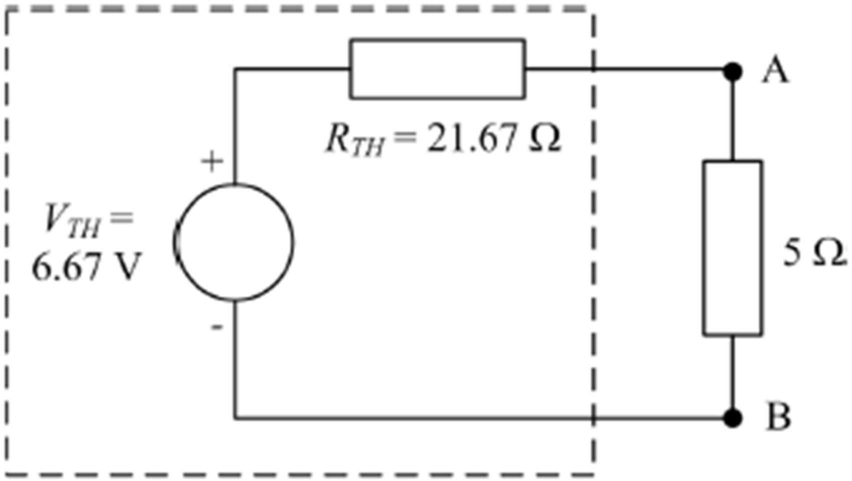
$$R_{eq} = (15||20) + 10 = \frac{15 \times 20}{15 + 20} + 10 = 18.57\ \Omega$$

$$I_s = \frac{V_s}{R_{eq}} = \frac{10}{18.57} = 0.54\ \text{A}$$

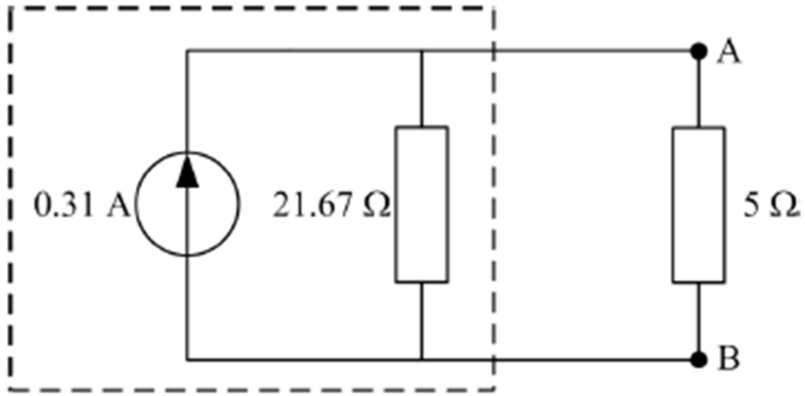
$$\boldsymbol{I_N} = I_{sc} = I_s \frac{20}{15 + 20} = 0.54 \frac{20}{35} = 0.31\ \text{A}$$



$$V_{TH} = I_N R_{TH}$$



Thévenin's Equivalent Circuit



Norton's Equivalent Circuit

A current source unlike a voltage source is not a physical reality

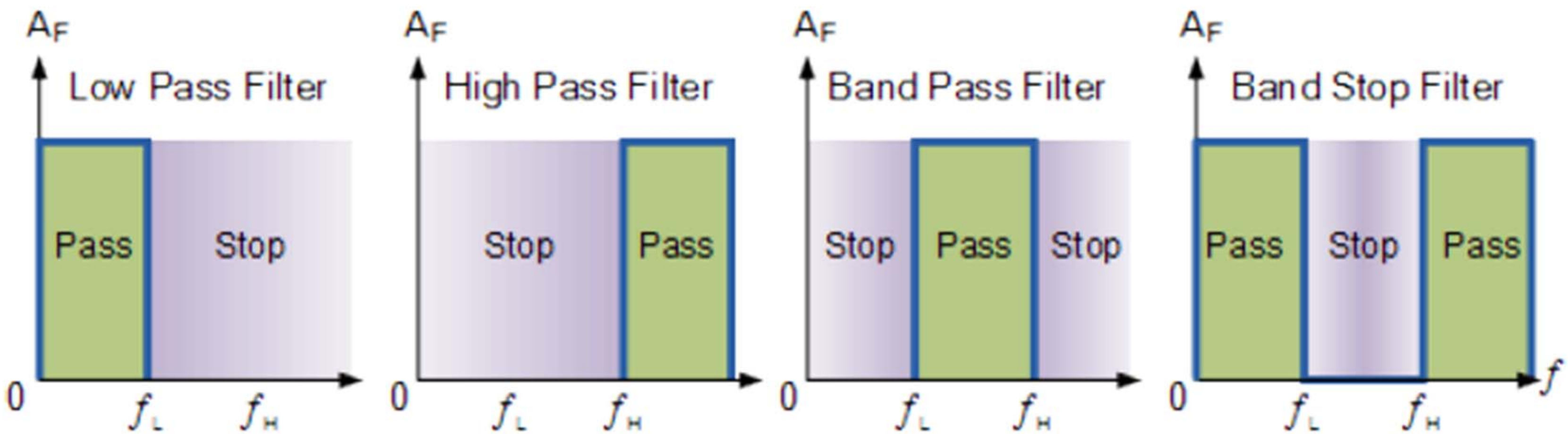
Filters:

To modify the parts (frequencies) of a signal

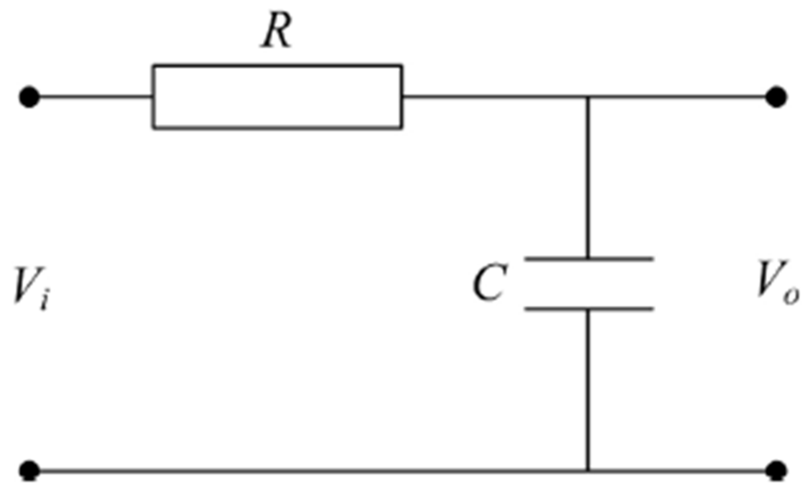
- Active (amplifiers)
- Passive (only reduction / removal of certain freqs)

Passive : Combinations of R, L & C

Types:



LPF

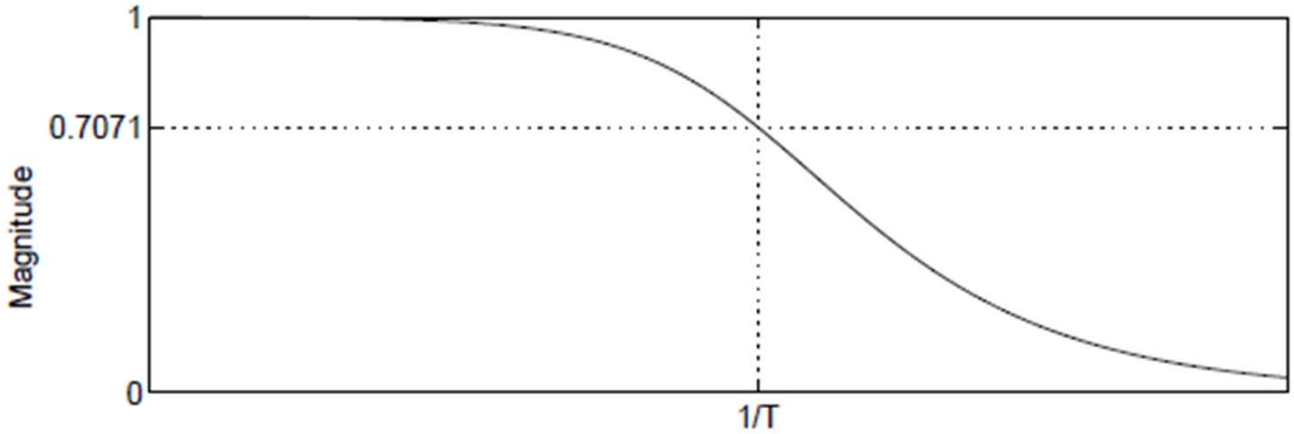


$$V_o = V_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

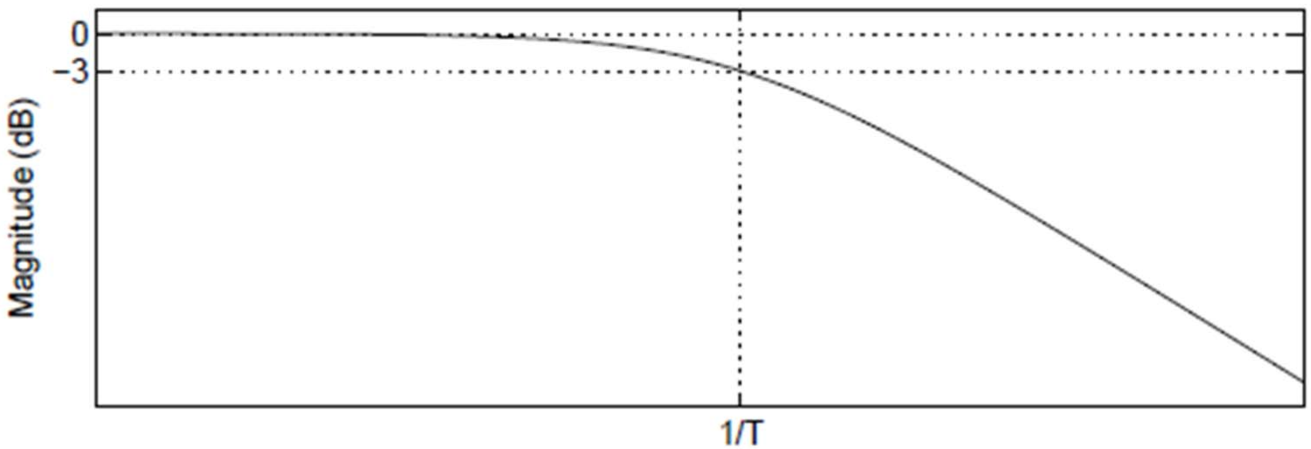
$$V_o = V_i \frac{1}{1 + j\omega RC}$$

$$H(j\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega T} \qquad T = RC$$

$$A = |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega T)^2}} \qquad A_{dB} = 20 \log_{10} A$$



$$A \geq \frac{1}{\sqrt{2}} \rightarrow \boldsymbol{\omega} = \text{Bandwidth} = \frac{1}{T} = \frac{1}{RC}$$



$$\boldsymbol{A} \rightarrow |\boldsymbol{V_O}| = |\boldsymbol{V_i}| \frac{\boldsymbol{X_C}}{\sqrt{\boldsymbol{R}^2 + \boldsymbol{X_C^2}}}$$

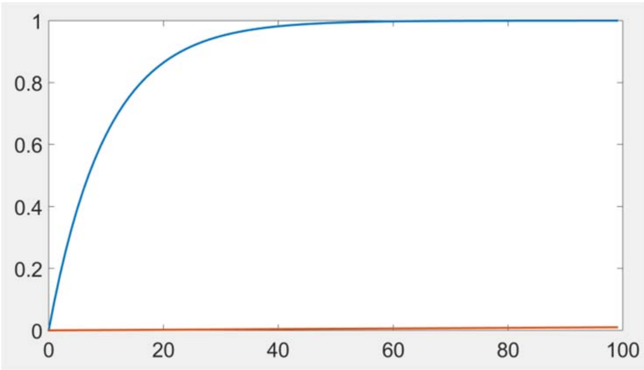
Charging - discharging

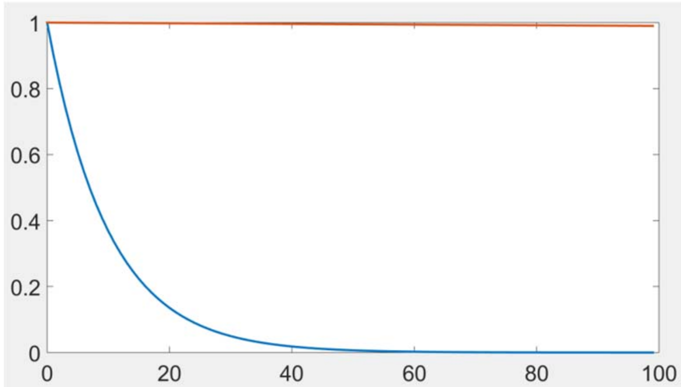
Charging $V(t) = V_0(1 - e^{-t/\tau})$

Discharging $V(t) = V_0(e^{-t/\tau})$

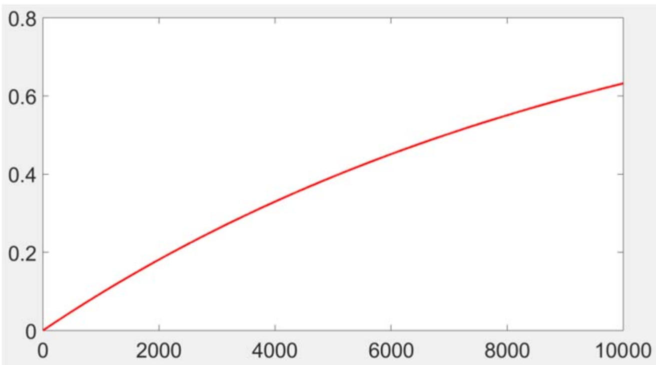
$$\tau = RC$$

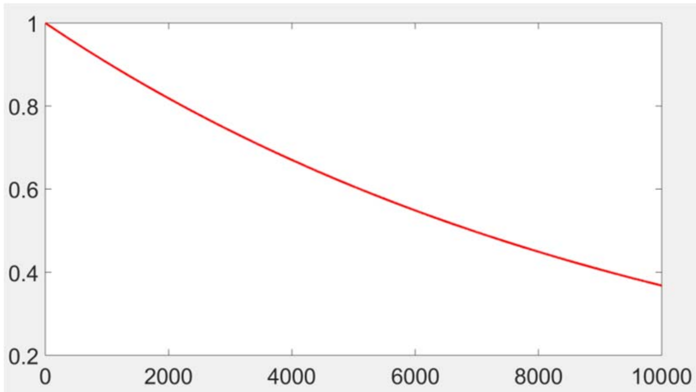
RC small & large

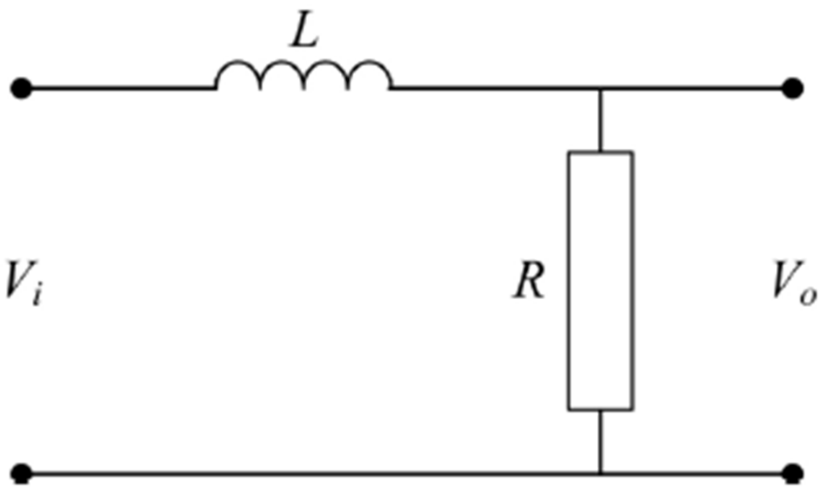




RC large



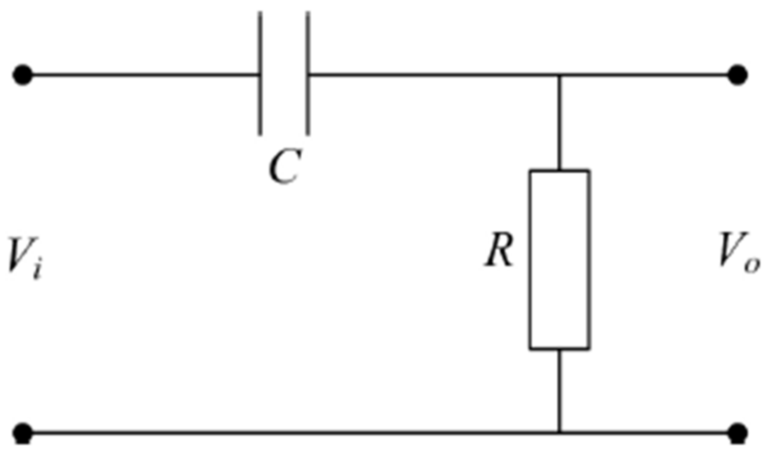




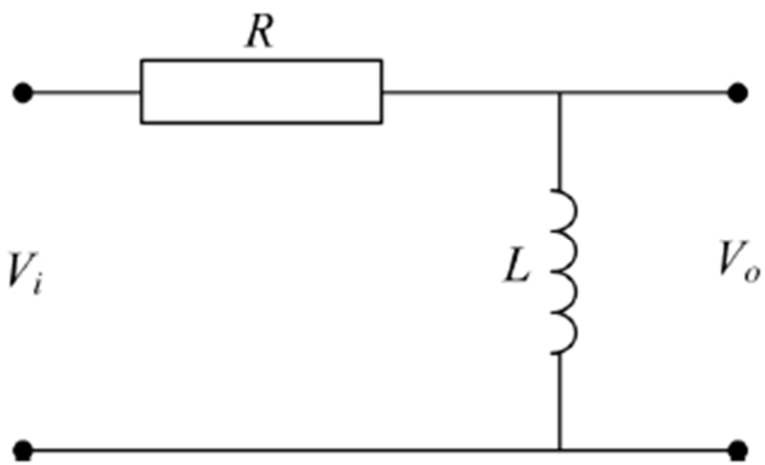
$$V_O = V_i \frac{R}{R + j\omega L}$$

$$T = \frac{L}{R} \qquad |V_O| = |V_i| \frac{R}{\sqrt{R^2 + X_L^2}}$$

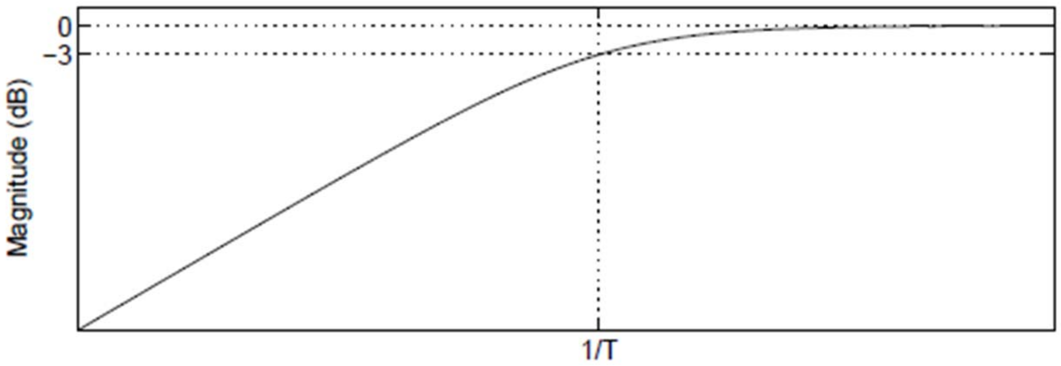
HPF



$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{1}{j\omega RC}}$$



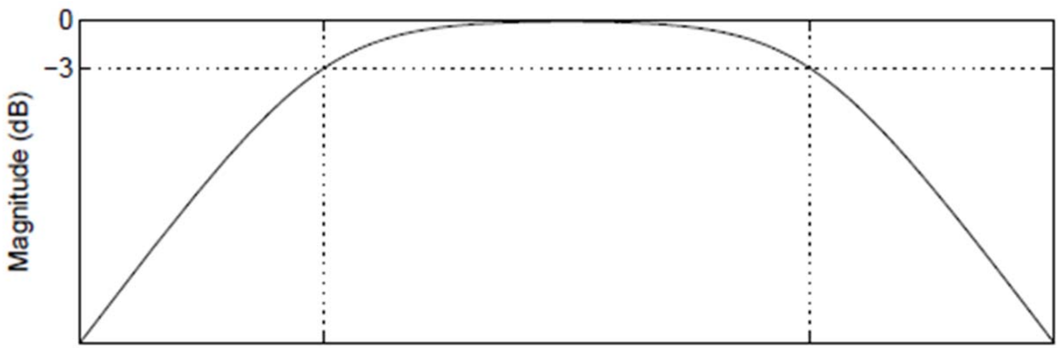
$$\frac{V_O}{V_i} = \frac{1}{1 + \frac{R}{j\omega L}}$$



$\omega = 1/T$

Cut-off

BPF →LPF (small RC) – LPF (large RC)
or HPF (large RC) – LPF (small RC)



BSF →LPF (large RC) +HPF (small RC)

Pulse response: