Radiative Corrections

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For further info see:

- M. Vanderhaeghen et al., (2000) Phys. Rev. C 62 (2000) 025501.
- D. Lhuillier, These Univ. Caen, DAPNIA-SPhN-97-01T (1997)
- D. Marchand, These Univ. Blaise Pascal, DAPNIA-SphN-98-04T (1998)
- P.Janssens et al, Nucl. Instr. Meth. A566 (2006) 675-686.

$$\sigma_{exp} = (1 + \delta_{tot})\sigma_{th}. \tag{7}$$

The correction term δ_{tot} is negative and depends on the cut in the radiative tail accompanying the scattering process. The internal radiative corrections to VCS are discussed in great detail in [12]. Written in first order, one gets

$$\delta_{tot}^{(1)} = \delta_{vac} + \delta_{ver} + \delta_{rad}, \tag{8}$$

δ_{vac} accounts for vacuum polarisation diagrams, δ_{ver} is the vertex correction and δ_{rad} is the correction for radiation in the one additional photon approximation. One can approximately take into account higher order radiative corrections by writing [12]:

$$\sigma_{exp} = \frac{e^{\delta_{ver}+\delta_{rad}}}{(1-\delta_{vac}/2)^2}\sigma_{th}. \tag{9}$$

For $Q^2 >> m^2$, one can write:

Functions of :
- Q^2
- Beam Energy
- e^- out momentum

ΔE^c_{cm} depends on MM cut

$$\delta_{rad} \approx \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E^c_{cm})^2}{E_{cm}E'_{cm}}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] - \frac{1}{2}\ln^2\left(\frac{E_{cm}}{E'_{cm}}\right) + \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right) - \frac{\pi^2}{3} + Sp\left(\cos^2\frac{\theta_{e',cm}}{2}\right)\right\}, \tag{10}$$

$$\delta_{ver} \approx \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{Q^2}{m^2}\right) - 2 - \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right) + \frac{\pi^2}{6}\right\}, \tag{11}$$

$$\delta_{vac} \approx \frac{2\alpha}{3\pi}\left\{-\frac{5}{3} + \ln\left(\frac{Q^2}{m^2}\right)\right\}, \tag{12}$$

Cut dependent variable

been used in the last line ($E_e'^{el}$ denotes the elastic scattered electron *lab* energy, to distinguish it from E_e'). From Eq. (A48), one determines then ΔE_s in terms of *lab* quantities from the scattered electron spectrum through

$$\Delta E_s = \eta \left(E_e'^{el} - E_e' \right) \ , \tag{A49}$$

where the recoil factor η is given by $\eta = E_e / E_e'^{el}$.

$$E_{e'}^{\text{elastic}} = \frac{E_e}{1 + \frac{E_e}{M_{Tg}(1-\cos\theta)}} .$$

$$\Delta E_{cm}^{c} = \frac{\sqrt{M_X^2}}{2} .$$

ΔE_s determined by resolution of the detector,

Catches radiative tail corrections,

Determined by cuts (shown for E' in elastic scattering, MM^2 for our exp)

QED radiative corrections

Values of radiative corrections for the first VCS experiment at MAMI.

MAMI VCS kinematics	
Kinematics	q_{cm}=0.6 GeV/c, $\epsilon = 0.61$, q'_{cm} = 111 MeV/c E=0.843 GeV, E'=0.528 GeV, $\theta_e = 53.14°$ $Q^2 = 0.356$ GeV^2
T_{anal} δ_{cont} δ_V	+22.0 % -1.2 % (*) -15.6 %
F_{rad}	(+94.9 ± 2.0) %

(*) = the value of $(\delta_1 + \delta_2^{(0)})$ averaged over the two MAMI kinematics of Table 1 in ref. [4].

$$\delta_V = \delta_{vertex} + \delta_{vacuum} + \delta_{self\ energy}$$

A partir de la formule (2.96) du chapitre 2, δ_R s'écrit sous la forme :

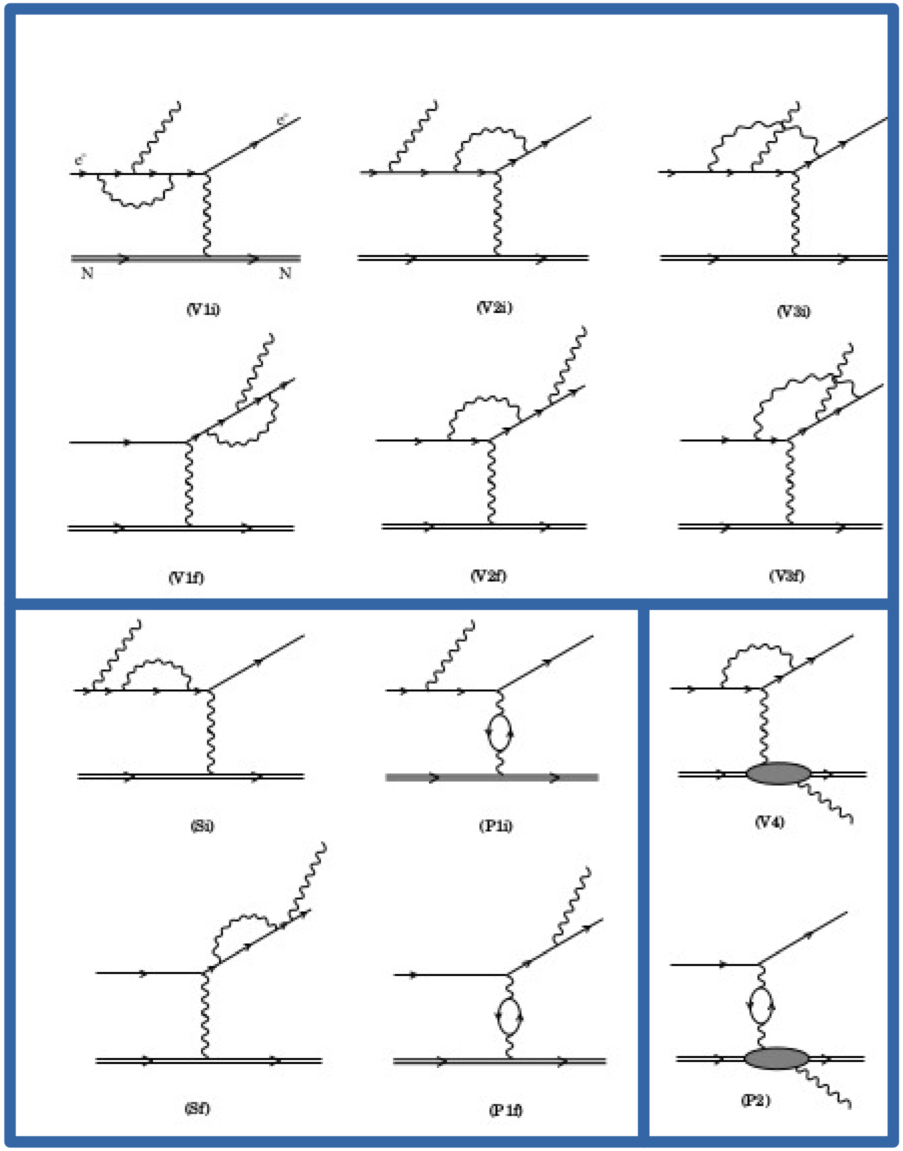
$$\delta_R = T_{anal} + f(\Delta E_{cut}) \quad (3.55)$$

T_{anal} est un terme analytique fonction uniquement de la cinématique et $f(\Delta E_{cut})$ est dépendant de la coupure expérimentale ΔE_{cut}.

Internal Corrections

Vertex

δ_{vert}



Self energy and vacuum

δ_{vac}

Non-Born (VCS)

δ_1, δ_2

External Corrections

Soft photon,
Radiative tail

$\delta_R = \delta_{rad}$

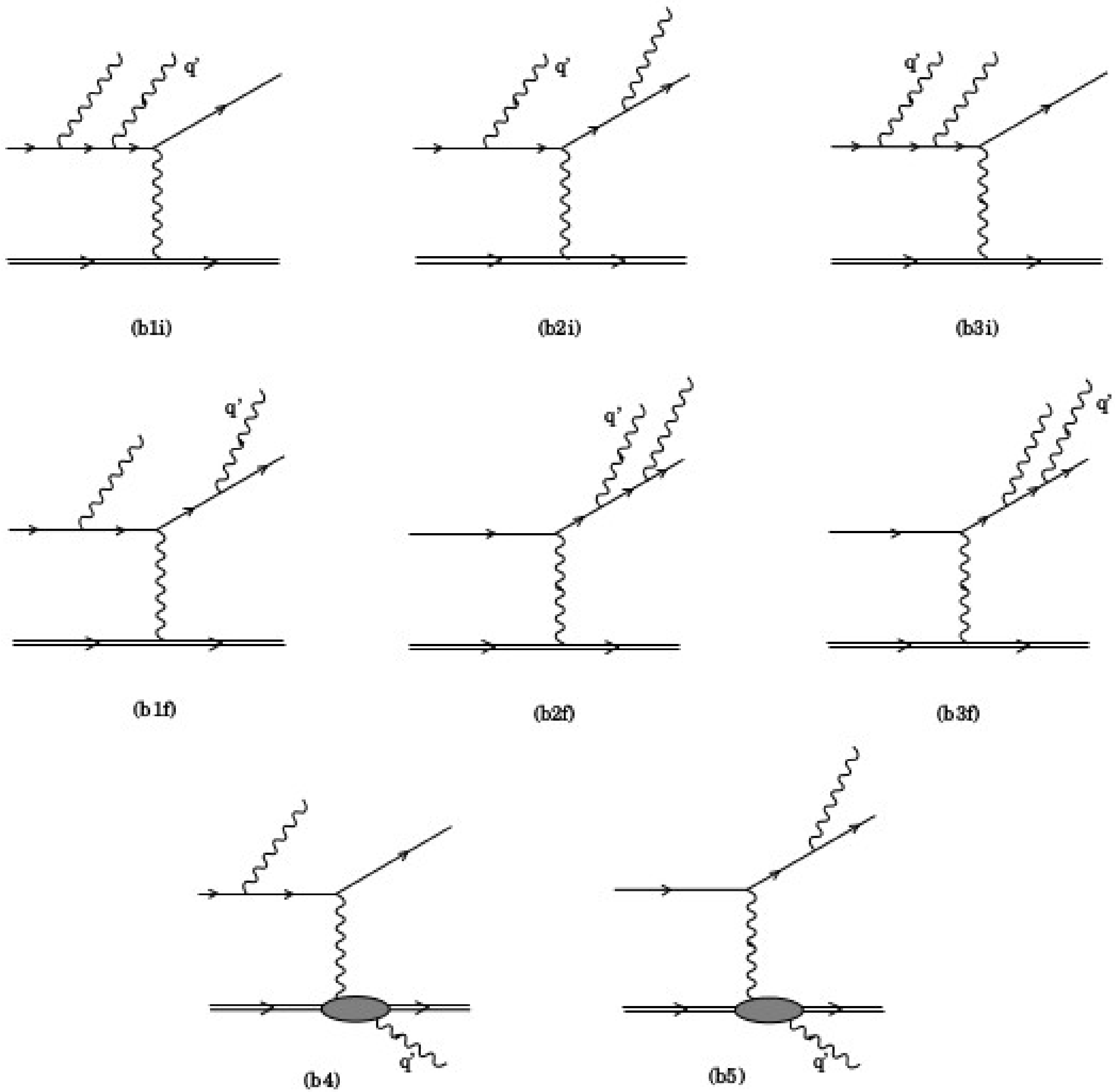


FIG. 3. First order soft photon emission contributions to the $ep \rightarrow ep\gamma$ reaction.

Hadron Corrections

$$\left(\frac{d\sigma}{d\Omega'_e}\right)_{TOTAL} = \left(\frac{d\sigma}{d\Omega'_e}\right)_{BORN} \left(1 + \delta_{vac} + \delta_{vertex} + \delta_R + Z\,\delta_1 + Z^2\,(\delta_2^{(0)} + \delta_2^{(1)}\right) , \quad \text{(A74)}$$

where δ_{vac}, δ_{vertex} and δ_R are given as above (Eq.(A72)). The terms in Eq. (A74) proportional to Z (hadron charge) and Z^2 contain the corrections from the proton side. The correction δ_1, proportional to Z, was calculated in Ref. [24] as

$$\delta_1 = \frac{2\alpha_{em}}{\pi} \left\{ \ln\left(\frac{4(\Delta E_s)^2}{Q^2\,x}\right) \ln\eta + Sp\left(1 - \frac{\eta}{x}\right) - Sp\left(1 - \frac{1}{\eta\,x}\right)\right\}, \quad \text{(A75)}$$

where ΔE_s and η are given as in Eq. (A49) and where the variable x is defined by

$\delta_{cont} = \delta_1 + \delta_2$

$$x = \frac{(Q+\rho)^2}{4M_N^2}\,, \qquad \rho^2 = Q^2 + 4M_N^2\,, \quad \text{(A76)}$$

The correction proportional to Z^2 was split into two parts in Ref. [24]. The contribution $\delta_2^{(0)}$, independent of the nucleon form factors was calculated in Ref. [24] as :

$$\begin{aligned}\delta_2^{(0)} = \frac{\alpha_{em}}{\pi} &\left\{ \ln\left(\frac{4(\Delta E_s)^2}{M_N^2}\right)\left(\frac{E'_N}{|\vec{p}\,'_N|}\ln x - 1\right) + 1 \right.\\ + &\left. \frac{E'_N}{|\vec{p}\,'_N|}\left(-\frac{1}{2}\ln^2 x - \ln x\,\ln\left(\frac{\rho^2}{M_N^2}\right) + \ln x - Sp\left(1 - \frac{1}{x^2}\right) + 2Sp\left(-\frac{1}{x}\right) + \frac{\pi^2}{6}\right)\right\}, \quad \text{(A77)}\end{aligned}$$

where ρ is defined as in Eq. (A76), and where E'_N ($|\vec{p}\,'_N|$) are the *lab* energy (momentum) of the recoiling nucleon. For the lengthier expression of $\delta_2^{(1)}$, which depends on the nucleon form factors, we refer to Ref. [24].

Cross Checks*

Correction	Helene's Paper	My calculation
T_{anal}	22.0	21.8
$f(\Delta E_{cut})$ (acct in tail)	-17.3	-16.2
Δ_R (sum above)	4.7	5.6
δ_V	-15.6	-15.9
δ_{cont}	-1.2	-1.1

Vanderhaeghen, et al

E_e	θ_e	Q^2	δ_{vertex}	δ_{vacpol}	δ_R	δ_1	$\delta_2^{(0)}$	δ_{tot}	EXP
0.705	40.66	0.203	-0.1673	0.0208	-0.0453	-0.0067	-0.0018	-0.2003	-0.2025
0.855	52.18	0.418	-0.1881	0.0228	-0.0245	-0.0123	-0.0034	-0.2054	-0.2087

0.705	40.66	0.203	-0.1672	0.0184	-0.0475	-0.0063	-0.0018	-0.1989	-0.1953
0.855	52.18	0.418	-0.1881	0.0195	-0.0273	-0.0118	-0.0035	-0.2111	-0.2059

* Helene and Marc mention simulations. It is not clear if any values are averages across the phase space

Simul++ RadCorr

Simul++ has T_{anal}, and δ_V built in to both PS and Yield_Sim (cancels?)

Hadron effects not considered ~ 1% effect, can split like T_{anal} & f(ΔE)

Exponentiation differences from Marc's paper

$$\left(\frac{d\sigma}{d\Omega'_e}\right)_{VIRTUAL\,\gamma} + \left(\frac{d\sigma}{d\Omega'_e}\right)_{REAL\,SOFT\,\gamma} = \left(\frac{d\sigma}{d\Omega'_e}\right)_{BORN} (1 + \delta_{vac} + \delta_{vertex} + \delta_R)\ , \quad \text{(A71)}$$

where δ_{vac}, δ_{vertex} and δ_R are given by Eqs. (A69), (A68), and (A64)-(A66) respectively. Bringing the three contributions together, leads to the expression (in the $Q^2 >> m^2$ limit)

$$\delta_{vac} + \delta_{vertex} + \delta_R = \frac{\alpha_{em}}{\pi}\left\{ \ln\left(\frac{(\Delta E_s)^2}{E_e E'_e}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] + \frac{13}{6}\ln\left(\frac{Q^2}{m^2}\right) - \frac{28}{9} - \frac{1}{2}\ln^2\left(\frac{E_e}{E'_e}\right) - \frac{\pi^2}{6} + Sp\left(\cos^2\frac{\theta_e}{2}\right)\right\}, \quad \text{(A72)}$$

where ΔE_s, which is the maximum soft-photon energy in the c.m. system of (recoiling proton + soft-photon), is determined as in Eq. (A49), when applying this formula to the

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[3] Note that an incorrect expression is used in [50] for the vacuum polarization contribution due to $\mu^+\mu^-$ pairs (Eq. (A5) in their paper).

Exponentiation

$$\delta_{tot}^{(1)} = \delta_{vac} + \delta_{ver} + \delta_{rad}, \tag{8}$$

δ_{vac} accounts for vacuum polarisation diagrams, δ_{ver} is the vertex correction and δ_{rad} is the correction for radiation in the one additional photon approximation. One can approximately take into account higher order radiative corrections by writing [12]:

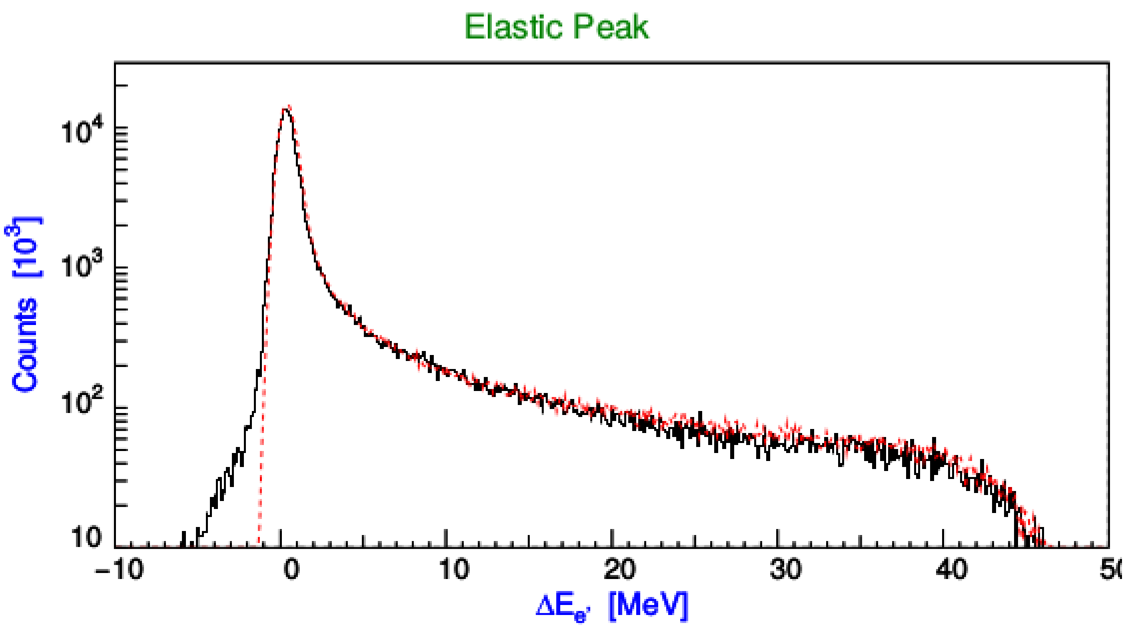
$$\sigma_{exp} = \frac{e^{\delta_{ver}+\delta_{rad}}}{(1-\delta_{vac}/2)^2}\sigma_{th}. \tag{9}$$

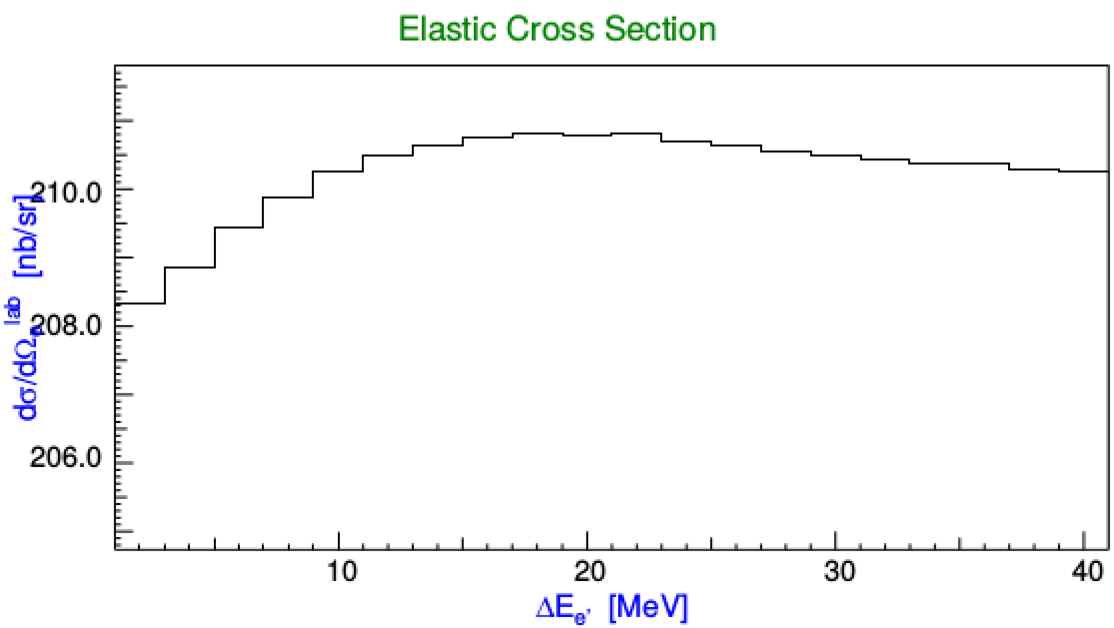
Simul++ applies $\sigma_{exp} = e^{\delta tot}\ \sigma_{th}$

$(1-x/2)^{-2}$	e^x	Difference
$1+x+\frac{3x^2}{4}+\frac{x^3}{2}+\frac{5x^4}{16}+\frac{3x^5}{16}+O(x^6)$ (Taylor series) (converges when $\lvert x\rvert < 2$)	$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+O(x^6)$ (Taylor series) (converges everywhere)	$x^2/4 + O(x^3)$

Exponentiating δ_{vac} ~ 2%, difference of 0.04% is negligible

$f(\Delta E_{cut})$


Elastic Peak
Counts [10^3]
10^4
10^3
10^2
10
−10
0
10
20
30
40
50
ΔE_e' [MeV]


Elastic Cross Section
dσ/dΩ_e^lab [nb/sr]
210.0
208.0
206.0
10
20
30
40
ΔE_e' [MeV]

Corrections for our Kinematics

Correction	KinI	KinII
Tanal *	20.6	20.6
Δ_V *	-14.8	-14.8
δ_{cont}	-1.8	-1.8

* - in Simul++

The correction factors are constant since the electron spectrometer was fixed.
The change is below uncertainty since the proton spectrometer changed very little.

Cross Section Results