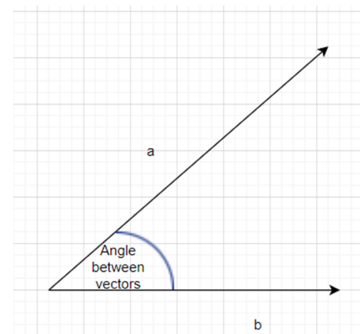
Q) What are dot products and cross products? Explain use cases where dot products are used and cross products are used in graphics environment.

Ans:

A quantity has both magnitude and direction. A vector can be multiplied in two ways which are

1. Dot Product (Scalar Product)
2. Cross Product(Vector Product)

Dot Product :



The dot product is calculated by multiplying the magnitude of the vectors and the cosine of the angle between them. A · B, is a scalar quantity that represents the projection of A onto B.

The formula is given as below:

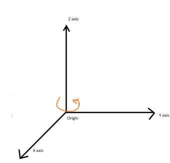
A . B = |A| |B| cos α

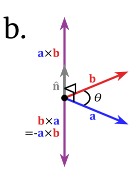
Use cases :

1. **Lighting:** To create realistic lighting effects, determine how much light is reflected from a surface in the direction of the spectator.
2. **Angle Calculation** Calculating the angle between two vectors is useful for applications like collision detection and camera orientation.
3. **Projection:**Projecting points onto planes or other surfaces in order to apply shadows or clip objects.

Cross Product

The cross Product is defined as the vector that is perpendicular to both vectors a and b. It captures the "rotational" relationship between them.





It is mathematically represented by the following equation:

A X B = |A| |B| sin (theta)

Use cases in Graphics :

- **Normal Calculation:** Calculating the normal vector to a surface, crucial for shading and lighting calculations.

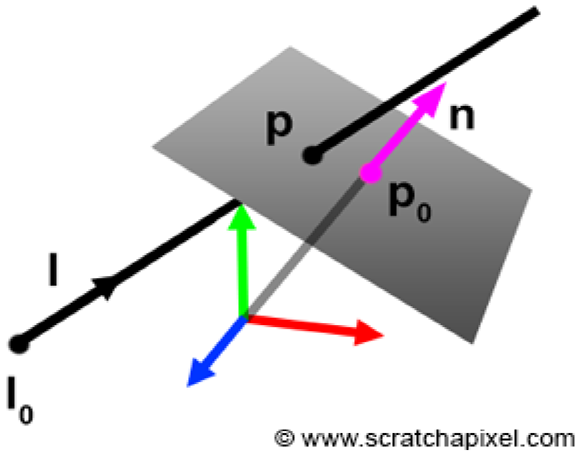
- **Physics Simulation:** Determining the torque and angular velocity of rigid bodies for realistic movement and interaction.

- **Intersection Finding:** Finding the intersection point of two lines or planes for clipping objects or implementing shadows.

BONUS QUESTION

Calculate the intersection Between the Ray and a plane

ANS :



Let us define a plane by the equation: Ax + By + Cz + D = 0, The vector could be written as [A B C D].

If A2 + B2 + C2 = 1 then the unit normal vector Nv = [A B C].

The distance between the unit normal vector Nv and the origin to the plane is given by D

Let the ray be defined as R0 = [X0, Y0, Z0]

Rd = [Xd, Yd, Zd]

so R(t) = R0 + t * Rd , t > 0

To determine if there is an intersection with the plane, substitute for R(t) into the plane equation and get:

$A(X_0 + X_d * t) + B(Y_0 + Y_d * t) + (Z_0 + Z_d * t) + D = 0$

which yields:

$t = -(AX_0 + BY_0 + CZ_0 + D) / (AX_d + BY_d + CZ_d)$

$= -(N_v \cdot R_0 + D) / (N_v \cdot R_d)$

References :

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