Solution

(a) The potential energy $U = mgh$ = $2 \times 10 \times 5 = 100$ J

Here the positive sign implies that the energy is stored on the mass.

(b) This potential energy is transferred from external agency which applies the force on the mass.

(c) The external applied force $\vec{F}_a$ which takes the object to the height 5 m is $\vec{F}_a = -\vec{F}_g$

$$\vec{F}_a = -\left(-mg\,\hat{j}\right) = mg\,\hat{j}$$

where, $\hat{j}$ represents unit vector along vertical upward direction.

(d) From the definition of potential energy, the object must be moved at constant velocity. So the net force acting on the object is zero.

$$\vec{F}_g + \vec{F}_a = 0$$

4.2.6 Elastic Potential Energy

When a spring is elongated, it develops a restoring force. *The potential energy possessed by a spring due to a deforming force which stretches or compresses the spring is termed as elastic potential energy*. The work done by the applied force against the restoring force of the spring is stored as the elastic potential energy in the spring.

Consider a spring-mass system. Let us assume a mass, m lying on a smooth horizontal table as shown in Figure 4.9. Here, $x = 0$ is the equilibrium position. One end of the spring is attached to a rigid wall and the other end to the mass.

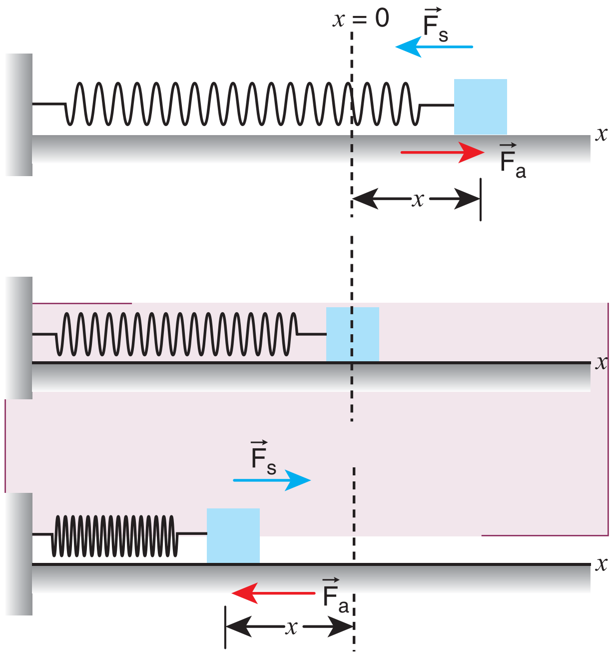


Figure 4.9 Potential energy of the spring (elastic potential energy)

As long as the spring remains in equilibrium position, its potential energy is zero. Now an external force $\vec{F}_a$ is applied so that it is stretched by a distance (x) in the direction of the force.

There is a restoring force called spring force $\vec{F}_s$ developed in the spring which tries to bring the mass back to its original position. This applied force and the spring force are equal in magnitude but opposite in direction i.e., $\vec{F}_a = -\vec{F}_s$. According Hooke's law, the restoring force developed in the spring is

$$\vec{F}_s = -k\vec{x} \qquad (4.20)$$

The negative sign in the above expression implies that the spring force is always opposite to that of displacement $\vec{x}$ and k

is the force constant. Therefore applied force is $\vec{F}_a = +k\vec{x}$. The positive sign implies that the applied force is in the direction of displacement $\vec{x}$. The spring force is an example of variable force as it depends on the displacement $\vec{x}$. Let the spring be stretched to a small distance $d\vec{x}$. The work done by the applied force on the spring to stretch it by a displacement $\vec{x}$ is stored as elastic potential energy.

$$U = \int \vec{F}_a . d\vec{r} = \int_0^x |\vec{F}_a| \, |d\vec{r}| \cos\theta$$

$$= \int_0^x F_a \, dx \cos\theta \qquad (4.21)$$

The applied force $\vec{F}_a$ and the displacement $d\vec{r}$ (i.e., here dx) are in the same direction. As, the initial position is taken as the equilibrium position or mean position, x=0 is the lower limit of integration.

$$U = \int_0^x kx\,dx \qquad (4.22)$$

$$U = k\left[\frac{x^2}{2}\right]_0^x \qquad (4.23)$$

$$U = \frac{1}{2}kx^2 \qquad (4.24)$$

If the initial position is not zero, and if the mass is changed from position x_i to x_f, then the elastic potential energy is

$$U = \frac{1}{2}k\left(x_f^2 - x_i^2\right) \qquad (4.25)$$

From equations (4.24) and (4.25), we observe that the potential energy of the stretched spring depends on the force constant k and elongation or compression x.

Note The potential energy stored in the spring does not depend on the mass that is attached to the spring.

Force-displacement graph for a spring

Since the restoring spring force and displacement are linearly related as $F = -kx$, and are opposite in direction, the graph between F and x is a straight line with dwelling only in the second and fourth quadrant as shown in Figure 4.10. The elastic potential energy can be easily calculated by drawing a F - x graph. The shaded area (triangle) is the work done by the spring force.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2} \times (x) \times (kx).$$

$$= \frac{1}{2}kx^2$$

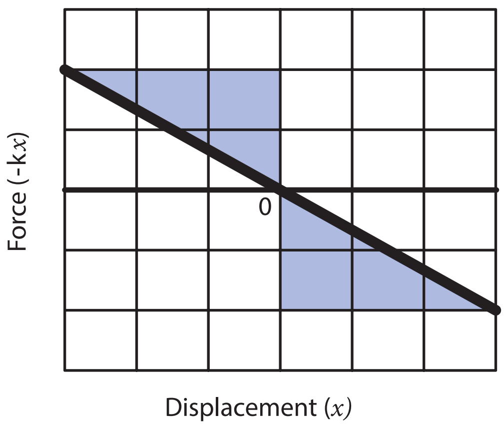


Figure 4.10 Force–displacement graph for a spring

Potential energy-displacement graph for a spring

A compressed or extended spring will transfer its stored potential energy into kinetic energy of the mass attached to the spring. The potential energy-displacement graph is shown in Figure 4.11.

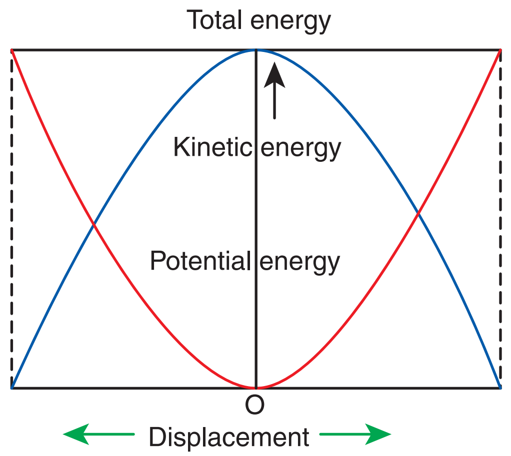


Figure 4.11 Potential energy–displacement graph for a spring-mass system

In a frictionless environment, the energy gets transferred from kinetic to potential and potential to kinetic repeatedly such that the total energy of the system remains constant. At the mean position,

$$\Delta KE = \Delta U \qquad ([illegible])$$

EXAMPLE 4.9

Let the two springs A and B be such that $k_A > k_B$. On which spring will more work has to be done if they are stretched by the same force?

Solution

$$F = k_A x_A = k_B x_B$$

$$x_A = \frac{F}{k_A}; \quad x_B = \frac{F}{k_B}$$

The work done on the springs are stored as potential energy in the springs.

$$U_A = \frac{1}{2} k_A x_A^2 \; ; \quad U_B = \frac{1}{2} k_B x_B^2$$

$$\frac{U_A}{U_B} = \frac{k_A x_A^2}{k_B x_B^2} = \frac{k_A \left(\frac{F}{k_A}\right)^2}{k_B \left(\frac{F}{k_B}\right)^2} = \frac{\frac{1}{k_A}}{\frac{1}{k_B}}$$

$$\frac{U_A}{U_B} = \frac{k_B}{k_A}$$

$k_A > k_B$ implies that $U_B > U_A$. Thus, more work is done on B than A.

EXAMPLE 4.10

A body of mass m is attached to the spring which is elongated to 25 cm by an applied force from its equilibrium position.

(a) Calculate the potential energy stored in the spring-mass system?

(b) What is the work done by the spring force in this elongation?

(c) Suppose the spring is compressed to the same 25 cm, calculate the potential energy stored and also the work done by the spring force during compression. (The spring constant, $k = 0.1 \text{ N m}^{-1}$).

Solution

The spring constant, $k = 0.1 \text{ N m}^{-1}$

The displacement, $x = 25 \text{ cm} = 0.25 \text{ m}$

(a) The potential energy stored in the spring is given by

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \times 0.1 \times (0.25)^2 = 0.0031 \text{J}$$

(b) The work done W_s by the spring force $\vec{F}_s$ is given by,

$$W_s = \int_0^x \vec{F}_s \cdot d\vec{r} = \int_0^x \left(-kx\hat{i}\right) \cdot (dx\,\hat{i})$$

The spring force $\vec{F}_s$ acts in the negative x direction while elongation acts in the positive x direction.

$$W_s = \int_0^x (-kx)\,dx = -\frac{1}{2}kx^2$$

$$W_s = -\frac{1}{2} \times 0.1 \times (0.25)^2 = -0.0031\,\mathrm{J}$$

Note that the potential energy is defined through the work done by the external agency. The positive sign in the potential energy implies that the energy is transferred from the agency to the object. But the work done by the restoring force in this case is negative since restoring force is in the opposite direction to the displacement direction.

(c) During compression also the potential energy stored in the object is the same.

$$U = \frac{1}{2}kx^2 = 0.0031\,J.$$

Work done by the restoring spring force during compression is given by

$$W_s = \int_0^x \vec{F}_s . d\vec{r} = \int_0^x \left(kx\hat{i}\right).\left(-dx\,\hat{i}\right)$$

In the case of compression, the restoring spring force acts towards positive x-axis and displacement is along negative x direction.

$$W_s = \int_0^x (-kx)\,dx = -\frac{1}{2}kx^2 = -0.0031\,\mathrm{J}$$

4.2.7 Conservative and non-conservative forces

Conservative force

A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.

Let us consider an object at point A on the Earth. It can be taken to another point B at a height h above the surface of the Earth by three paths as shown in Figure 4.12.

Whatever may be the path, the work done against the gravitational force is the same as long as the initial and final positions are the same. This is the reason why gravitational force is a conservative force. Conservative force is equal to the negative gradient of the potential energy. In one dimensional case,

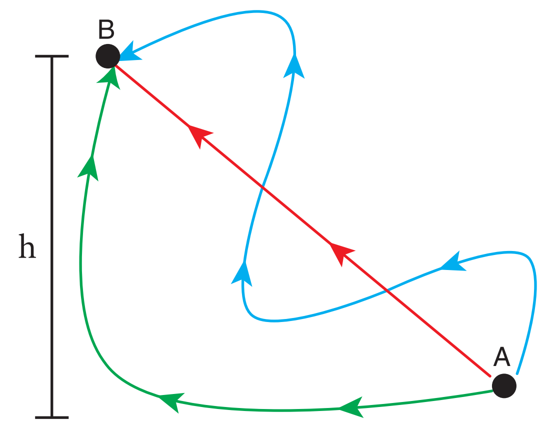


Figure 4.12 Conservative force

$$F_x = -\frac{dU}{dx} \qquad (47)$$

Examples for conservative forces are elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

Table 4.3 Comparison of conservative and non-conservative forces

S.No	Conservative forces	Non-conservative forces
1.	Work done is independent of the path	Work done depends upon the path
2.	Work done in a round trip is zero	Work done in a round trip is not zero
3	Total energy remains constant	Energy is dissipated as heat energy
4	Work done is completely recoverable	Work done is not completely recoverable.
5	Force is the negative gradient of potential energy	No such relation exists.

Non-conservative force

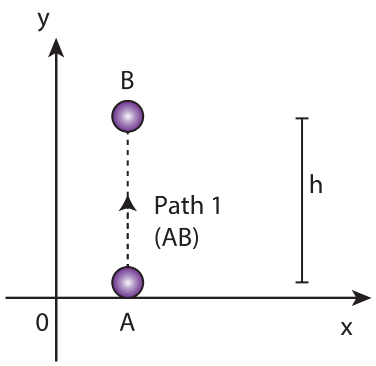
A force is said to be non-conservative if the work done by or against the force in moving a body depends upon the path between the initial and final positions. This means that the value of work done is different in different paths.

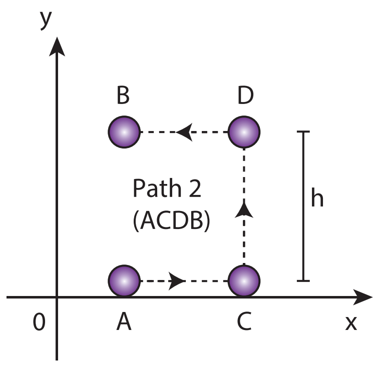
1. Frictional forces are non-conservative forces as the work done against friction depends on the length of the path moved by the body.
2. The force due to air resistance, viscous force are also non-conservative forces as the work done by or against these forces depends upon the velocity of motion.

The properties of conservative and non-conservative forces are summarized in the Table 4.3.

EXAMPLE 4.11

Compute the work done by the gravitational force for the following cases





Solution

Force $\vec{F} = mg\left(-\hat{j}\right) = -mg\hat{j}$

Displacement vector $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

(As the displacement is in two dimension; unit vectors $\hat{i}$ and $\hat{j}$ are used)

(a) Since the motion is only vertical, horizontal displacement component dx is zero. Hence, work done by the force along path 1 (of distance h).

$$W_{path\ 1} = \int_A^B \vec{F}.d\vec{r} = \int_A^B \left(-mg\hat{j}\right).\left(dy\,\hat{j}\right)$$

$$= -mg\int_0^h dy = -mgh$$

Total work done for path 2 is

$$W_{path\ 2} = \int_A^B \vec{F}.d\vec{r} = \int_A^C \vec{F}.d\vec{r} + \int_C^D \vec{F}.d\vec{r} + \int_D^B \vec{F}.d\vec{r}$$

But

$$\int_A^C \vec{F}.d\vec{r} = \int_A^C (-mg\hat{j}).(dx\hat{i}) = 0$$

$$\int_C^D \vec{F}.d\vec{r} = \int_C^D (-mg\hat{j}).(dy\hat{j})$$

$$= -mg\int_0^h dy = -mgh$$

$$\int_D^B \vec{F}.d\vec{r} = \int_D^B (-mg\hat{j}).(-dx\hat{i}) = 0$$

Therefore, the total work done by the force along the path 2 is

$$W_{\text{path 2}} = \int_A^B \vec{F}.d\vec{r} = -mgh$$

Note that the work done by the conservative force is independent of the path.

EXAMPLE 4.12

Consider an object of mass 2 kg moved by an external force 20 N in a surface having coefficient of kinetic friction 0.9 to a distance 10 m. What is the work done by the external force and kinetic friction ? Comment on the result. (Assume $g = 10\ ms^{-2}$)

Solution

$m = 2$ kg, $d = 10$ m, $F_{ext} = 20$ N, $\mu_k = 0.9$.
When an object is in motion on the horizontal surface, it experiences two forces.

(a) External force, $F_{ext} = 20$ N

(b) Kinetic friction,
$f_k = \mu_k mg = 0.9 \times (2) \times 10 = 18\,\text{N}$.

The work done by the external force $W_{ext} = F\,d = 20 \times 10 = 200\,\text{J}$

The work done by the force of kinetic friction $W_k = f_k d = (-18) \times 10 = -180$ J. Here the negative sign implies that the force of kinetic friction is opposite to the direction of displacement.

The total work done on the object $W_{total} = W_{ext} + W_k = 200\ \text{J} - 180\ \text{J} = 20\ \text{J}$.

Since the friction is a non-conservative force, out of 200 J given by the external force, the 180 J is lost and it can not be recovered.

4.2.8 Law of conservation of energy

When an object is thrown upwards its kinetic energy goes on decreasing and consequently its potential energy keeps increasing (neglecting air resistance). When it reaches the highest point its energy is completely potential. Similarly, when the object falls back from a height its kinetic energy increases whereas its potential energy decreases. When it touches the ground its energy is completely kinetic. At the intermediate points the energy is both kinetic and potential as shown in Figure 4.13. When the body reaches the ground the kinetic energy is completely dissipated into some other form of energy like sound, heat, light and deformation of the body etc.

In this example the energy transformation takes place at every point. The sum of kinetic energy and potential energy i.e., the total mechanical energy always remains constant, implying that the total energy is conserved. This is stated as the law of conservation of energy.

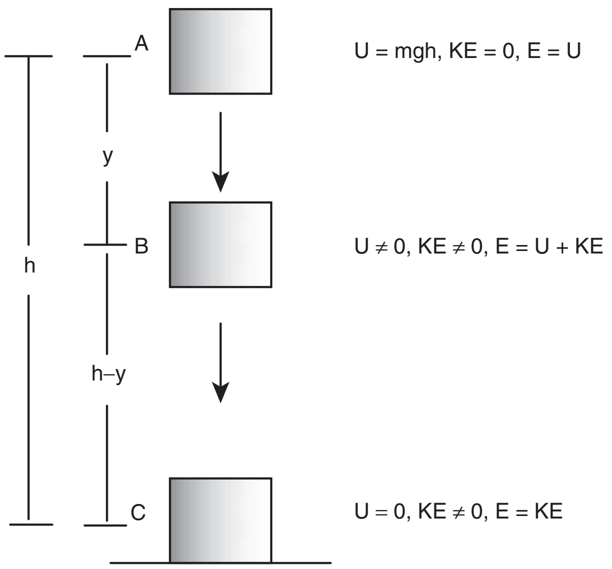


Figure 4.13 Conservation of energy

The law of conservation of energy states that *energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.*

Figure 4.13 illustrates that, if an object starts from rest at height *h*, the total energy is purely potential energy (U=mgh) and the kinetic energy (KE) is zero at *h*. When the object falls at some distance *y*, the potential energy and the kinetic energy are not zero whereas, the total energy remains same as measured at height *h*. When the object is about to touch the ground, the potential energy is zero and total energy is purely kinetic.

EXAMPLE 4.13

An object of mass 1 kg is falling from the height h = 10 m. Calculate

(a) The total energy of an object at $h = 10$ m
(b) Potential energy of the object when it is at $h = 4$ m
(c) Kinetic energy of the object when it is at $h = 4$ m
(d) What will be the speed of the object when it hits the ground?

(Assume $g = 10$ m s^{-2})

Solution

(a) The gravitational force is a conservative force. So the total energy remains constant throughout the motion. At $h = 10$ m, the total energy E is entirely potential energy.

$$E = U = mgh = 1 \times 10 \times 10 = 100 \text{ J}$$

(b) The potential energy of the object at $h = 4$ m is

$$U = mgh = 1 \times 10 \times 4 = 40 \text{ J}$$

(c) Since the total energy is constant throughout the motion, the kinetic energy at $h = 4$ m must be $KE = E - U = 100 - 40 = 60$ J

Alternatively, the kinetic energy could also be found from velocity of the object at 4 m. At the height 4 m, the object has fallen through a height of 6 m.

The velocity after falling 6 m is calculated from the equation of motion,

$$\text{v} = \sqrt{2gh} = \sqrt{2 \times 10 \times 6} = \sqrt{120} \text{ m s}^{-1};$$
$$\text{v}^2 = 120$$

The kinetic energy is $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 120 = 60$ J

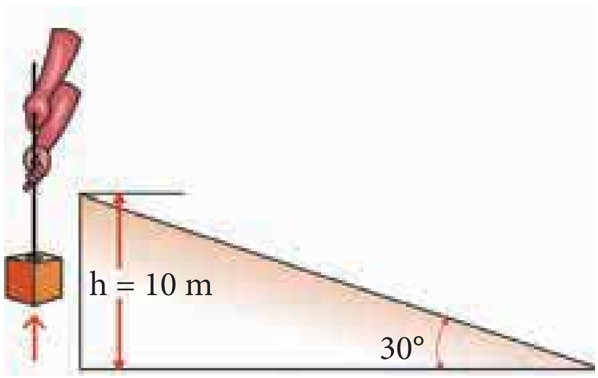
(d) When the object is just about to hit the ground, the total energy is completely kinetic and the potential energy, $U = 0$.

$$E = KE = \frac{1}{2}mv^2 = 100 \text{ J}$$

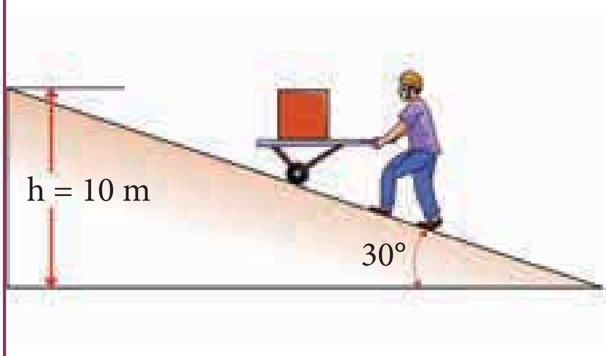
$$v = \sqrt{\frac{2}{m}KE} = \sqrt{\frac{2}{1} \times 100} = \sqrt{200} \approx 14.12 \text{ m s}^{-1}$$

EXAMPLE 4.14

A body of mass 100 kg is lifted to a height 10 m from the ground in two different ways as shown in the figure. What is the work done by the gravity in both the cases? Why is it easier to take the object through a ramp?



Path (1) straight up



Path (2) along the ramp

Solution

m = 100 kg, h = 10 m

Along path (1):

The minimum force F_1 required to move the object to the height of 10 m should be equal to the gravitational force, $F_1 = mg = 100 \times 10 = 1000$ N

The distance moved along path (1) is, $h = 10$ m

The work done on the object along path (1) is

$$W = F\ h = 1000 \times 10 = 10{,}000 \text{ J}$$

Along path (2):

In the case of the ramp, the minimum force F_2 that we apply on the object to take it up is not equal to mg, it is rather equal to $mg\sin\theta$. $\left(\text{mg}\sin\theta < mg\right)$.

Here, angle θ = 30°

Therefore, $F_2 = mg \sin\theta = 100 \times 10 \times \sin30° = 100 \times 10 \times 0.5 = 500$ N

Hence, $\left(\text{mg}\sin\theta < mg\right)$

The path covered along the ramp is,

$$l = \frac{h}{\sin 30} = \frac{10}{0.5} = 20\,m$$

The work done on the object along path (2) is, $W = F_2 l = 500 \times 20 = 10{,}000$ J

Since the gravitational force is a conservative force, the work done by gravity on the object is independent of the path taken.

In both the paths the work done by the gravitational force is 10,000 J

Along path (1): more force needs to be applied against gravity to cover lesser distance .

Along path (2): lesser force needs to be applied against the gravity to cover more distance.

As the force needs to be applied along the ramp is less, it is easier to move the object along the ramp.

EXAMPLE 4.15

An object of mass m is projected from the ground with initial speed v_0.

Find the speed at height h.

Solution

Since the gravitational force is conservative; the total energy is conserved throughout the motion.

	Initial	Final
Kinetic energy	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv^2$
Potential energy	0	mgh
Total energy	$\frac{1}{2}mv_0^2+0=\frac{1}{2}mv_0^2$	$\frac{1}{2}mv^2+mgh$

Final values of potential energy, kinetic energy and total energy are measured at the height *h*.

By law of conservation of energy, the initial and final total energies are the same.

$$\frac{1}{2}mv_0^2=\frac{1}{2}mv^2+mgh$$

$$v_0^2=v^2+2gh$$

$$v=\sqrt{v_0^2-2gh}$$

Note that in section (2.11.2) similar result is obtained using kinematic equation based on calculus method. However, calculation through energy conservation method is much easier than calculus method.

EXAMPLE 4.16

An object of mass 2 kg attached to a spring is moved to a distance $x=10$ m from its equilibrium position. The spring constant $k=1$ N m^{-1} and assume that the surface is frictionless.

(a) When the mass crosses the equilibrium position, what is the speed of the mass?

(b) What is the force that acts on the object when the mass crosses the equilibrium position and extremum position $x=\pm 10$ m.

Solution

(a) Since the spring force is a conservative force, the total energy is constant. At $x=10$ m, the total energy is purely potential.

$$E=U=\frac{1}{2}kx^2=\frac{1}{2}\times(1)\times(10)^2=50\ \mathrm{J}$$

When the mass crosses the equilibrium position $(x=0)$, the potential energy

$$U=\frac{1}{2}\times 1\times(0)=0\ \mathrm{J}$$

The entire energy is purely kinetic energy at this position.

$$E=KE=\frac{1}{2}mv^2=50\,\mathrm{J}$$

The speed

$$v=\sqrt{\frac{2KE}{m}}=\sqrt{\frac{2\times 50}{2}}=\sqrt{50}\,ms^{-1}\approx 7.07\,ms^{-1}$$

(b) Since the restoring spring force is F = - kx, when the object crosses the equilibrium position, it experiences no force. Note that at equilibrium position, the object moves very fast. When the object is at x = +10 m (elongation), the force F = – k x

F = – (1) (10) = – 10 N. Here the negative sign implies that the force is towards equilibrium i.e., towards negative x-axis and when the object is at $x = -10m$ (compression), it experiences a forces F = – (1) (– 10) = +10 N. Here the positive sign implies that the force points towards positive x-axis.

The object comes to momentary rest at $x = \pm 10\,m$ even though it experiences a maximum force at both these points.

4.2.9 Motion in a vertical circle

Imagine that a body of mass (m) attached to one end of a massless and inextensible string executes circular motion in a vertical plane with the other end of the string fixed. The length of the string becomes the radius ($\vec{r}$) of the circular path (Figure 4.14).

Let us discuss the motion of the body by taking the free body diagram (FBD) at a position where the position vector ($\vec{r}$) makes an angle θ with the vertically downward direction and the instantaneous velocity is as shown in Figure 4.14.

There are two forces acting on the mass.

1. Gravitational force which acts downward
2. Tension along the string.

Applying Newton's second law on the mass,

In the tangential direction,

$$mg\sin\theta = m a_t$$
$$mg\sin\theta = -m\left(\frac{dv}{dt}\right) \tag{4.28}$$

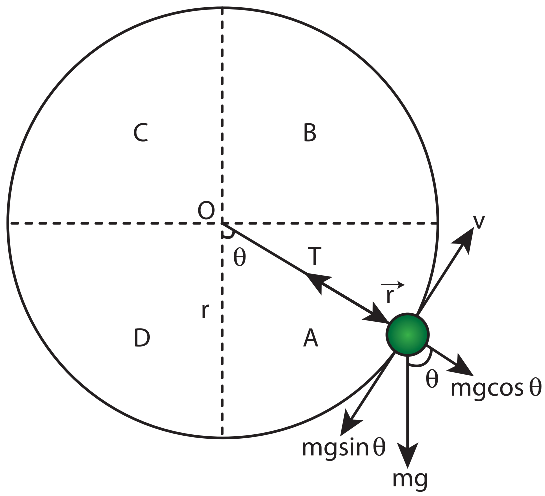


Figure 4.14 Motion in vertical circle

where, $a_t = -\frac{dv}{dt}$ is tangential retardation

In the radial direction,

$$T - mg\cos\theta = m a_r$$
$$T - mg\cos\theta = \frac{mv^2}{r} \tag{4.29}$$

where, $a_r = \frac{v^2}{r}$ is the centripetal acceleration.

The circle can be divided into four sections A, B, C, D for better understanding of the motion. The four important facts to be understood from the two equations are as follows:

(i) The mass is having tangential acceleration (g sin θ) for all values of θ (except θ = 0°), it is clear that this vertical cirular motion is not a uniform circular motion.

(ii) From the equations (4.28) and (4.29) it is understood that as the magnitude of velocity is not a constant in the course of motion, the tension in the string is also not constant.

(iii) The equation (4.29), $T = mg\cos\theta + \frac{mv^2}{r}$ highlights that in sections A and D of the circle, $\left(\text{for } -\frac{\pi}{2} < \theta < \frac{\pi}{2};\ \cos\theta \text{ is positive}\right)$, the term $mg\cos\theta$ is always greater than zero. Hence the tension cannot vanish even when the velocity vanishes.

(iv) The equation (4.29), $\frac{mv^2}{r} = T - mg\cos\theta$; further highlights that in sections B and C of the circle, $\left(\text{for } \frac{\pi}{2} < \theta < \frac{3\pi}{2};\ \cos\theta \text{ is negative}\right)$, the second term is always greater than zero. Hence velocity cannot vanish, even when the tension vanishes.

These points are to be kept in mind while solving problems related to motion in vertical circle.

To start with let us consider only two positions, say the lowest point 1 and the highest point 2 as shown in Figure 4.15 for further analysis. Let the velocity of the body at the lowest point 1 be $\vec{v}_1$, at the highest point 2 be $\vec{v}_2$ and $\vec{v}$ at any other point. The direction of velocity is tangential to the circular path at all points. Let $\vec{T}_1$ be the tension in the string at the lowest point and $\vec{T}_2$ be the tension at the highest point and $\vec{T}$ be the tension at any other point. Tension at each point acts towards the center. The tensions and velocities at these two points can be found by applying the law of conservation of energy.

For the lowest point (1)

When the body is at the lowest point 1, the gravitational force $m\vec{g}$ which acts on the body (vertically downwards) and another one is the tension $\vec{T}_1$ acting vertically upwards, i.e. towards the center. From the equation (4.29), we get

$$T_1 - mg = \frac{mv_1^2}{r} \qquad (4.30)$$

$$T_1 = \frac{mv_1^2}{r} + mg \qquad (4.31)$$

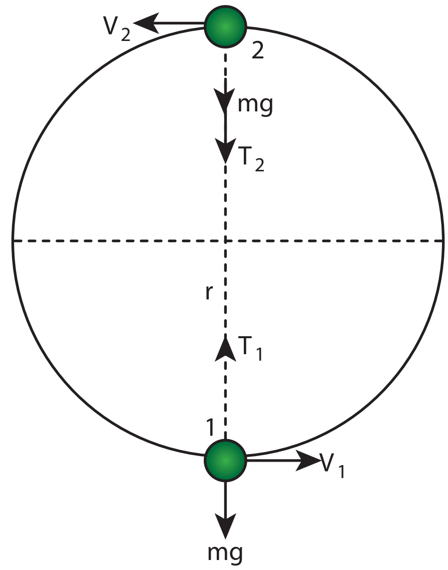


Figure 4.15 Motion in vertical circle shown for lowest and highest points

For the highest point (2)

At the highest point 2, both the gravitational force $m\vec{g}$ on the body and the tension $\vec{T}_2$ act downwards, i.e. towards the center again.

$$T_2 + mg = \frac{mv_2^2}{r} \qquad (4.32)$$

$$T_2 = \frac{mv_2^2}{r} - mg \qquad (4.33)$$

From equations (4.31) and (4.33), it is understood that $T_1 > T_2$. The difference in tension $T_1 - T_2$ is obtained by subtracting equation (4.33) from equation (4.31).

$$T_1 - T_2 = \frac{mv_1^2}{r} + mg - \left(\frac{mv_2^2}{r} - mg\right)$$

$$= \frac{mv_1^2}{r} + mg - \frac{mv_2^2}{r} + mg$$

$$T_1 - T_2 = \frac{m}{r}\left[v_1^2 - v_2^2\right] + 2mg \qquad (4.34)$$

The term $\left[v_1^2 - v_2^2\right]$ can be found easily by applying law of conservation of energy at point 1 and also at point 2.

Note The tension will not do any work on the mass as the tension and the direction of motion is always perpendicular.

The gravitational force is doing work on the mass, as it is a conservative force the total energy of the mass is conserved throughout the motion.

Total Energy at point 1 (E_1) is same as the total energy at a point 2 (E_2)

$$E_1 = E_2 \qquad (4.35)$$

Potential Energy at point 1, $U_1 = 0$ (by taking reference as point 1)

Kinetic Energy at point 1, $KE_1 = \frac{1}{2}mv_1^2$

Total Energy at point 1, $E_1 = U_1 + KE_1 =$

$= 0 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_1^2$

Similarly, Potential Energy at point 2, $U_2 = mg\ (2r)$ (h is 2r from point 1)

Kinetic Energy at point 2, $KE_2 = \frac{1}{2}mv_2^2$

Total Energy at point 2, $E_2 = U_2 + KE_2 =$

$2\ mg\ r + \frac{1}{2}mv_2^2$

From the law of conservation of energy given in equation (4.35), we get

$$\frac{1}{2}mv_1^2 = 2mgr + \frac{1}{2}mv_2^2$$

After rearranging,

$$\frac{1}{2}m\left(v_1^2 - v_2^2\right) = 2mgr$$

$$v_1^2 - v_2^2 = 4gr \qquad (4.36)$$

Substituting equation (4.36) in equation (4.34) we get,

$$T_1 - T_2 = \frac{m}{r}\left[4gr\right] + 2mg$$

Therefore, the difference in tension is

$$T_1 - T_2 = 6\ mg \qquad (4.37)$$

Minimum speed at the highest point (2)

The body must have a minimum speed at point 2 otherwise, the string will slack before reaching point 2 and the body will not loop the circle. To find this minimum speed let us take the tension $T_2 = 0$ in equation (4.33).

$$0 = \frac{mv_2^2}{r} - mg$$

$$\frac{mv_2^2}{r} = mg$$

$$v_2^{\,2} = rg$$

$$v_2 = \sqrt{gr} \qquad (4.38)$$

The body must have a speed at point 2, $v_2 \geq \sqrt{gr}$ to stay in the circular path.

Minimum speed at the lowest point 1

To have this minimum speed $\left(v_2 = \sqrt{gr}\right)$ at point 2, the body must have minimum speed also at point 1.

By making use of equation (4.36) we can find the minimum speed at point 1.

$$v_1^2 - v_2^2 = 4gr$$

Substituting equation (4.38) in (4.36),

$$v_1^2 - gr = 4gr$$

$$v_1^2 = 5gr$$

$$v_1 = \sqrt{5gr} \qquad (4.39)$$

The body must have a speed at point 1, $v_1 \geq \sqrt{5gr}$ to stay in the circular path.

From equations (4.38) and (4.39), it is clear that the minimum speed at the lowest point 1 should be $\sqrt{5}$ times more than the minimum speed at the highest point 2, so that the body loops without leaving the circle.

EXAMPLE 4.17

Water in a bucket tied with rope is whirled around in a vertical circle of radius 0.5 m. Calculate the minimum velocity at the lowest point so that the water does not spill from it in the course of motion. ($g = 10$ ms^{-2})

Solution

Radius of circle r = 0.5 m

The required speed at the highest point $v_2 = \sqrt{gr} = \sqrt{10 \times 0.5} = \sqrt{5}$ ms^{-1}. The speed at lowest point $v_1 = \sqrt{5gr} = \sqrt{5} \times \sqrt{gr} = \sqrt{5} \times \sqrt{5} = 5$ ms^{-1}

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4.3 POWER

4.3.1 Definition of power

Power is a measure of how fast or slow a work is done. *Power is defined as the rate of work done or energy delivered.*

$$\text{Power } (P) = \frac{\text{work done}(W)}{\text{time taken}(t)}$$

$$P = \frac{W}{t}$$

Average power

The average power (P_{av}) is defined as the ratio of the total work done to the total time taken.

$$P_{av} = \frac{\text{total work done}}{\text{total time taken}}$$

Instantaneous power

The instantaneous power (P_{inst}) is defined as the power delivered at an instant (as time interval approaches zero),

$$P_{inst} = \frac{dW}{dt}$$

4.3.2 Unit of power

Power is a scalar quantity. Its dimension is $[ML^2T^{-3}]$. The SI unit of power is watt (W), named after the inventor of the steam engine James Watt. *One watt is defined as* the power *when one joule of work is done in one second*, ($1\ W = 1\ J\ s^{-1}$).

The higher units are kilowatt(kW), megawatt(MW), and Gigawatt(GW).

$$1kW = 1000\ W = 10^3\ watt$$
$$1MW = 10^6\ watt$$
$$1GW = 10^9\ watt$$

For motors, engines and some automobiles an old unit of power still commercially in use which is called as the horse-power (hp). We have a conversion for horse-power (hp) into watt (W) which is,

$$1\ hp = 746\ W$$

All electrical goods come with a definite power rating in watt printed on them. A 100 watt bulb consumes 100 joule of electrical energy in one second. The energy measured in joule in terms of power in watt and time in second is written as, 1 J =1 W s. When electrical appliances are put in use for long hours, they consume a large amount of energy. Measuring the electrical energy in a small unit watt. second (W s) leads to handling large numerical values. Hence, electrical energy is measured in the unit called kilowatt hour (kWh).

$$1\ \text{electrical unit} = 1\ kWh = 1 \times (10^3\ W) \times (3600\ s)$$
$$1\ \text{electrical unit} = 3600\times10^3\ W\ s$$
$$1\ \text{electrical unit} = 3.6\times10^6\ J$$
$$1\ kWh = 3.6\times10^6\ J$$

Electricity bills are generated in units of kWh for electrical energy consumption. 1 unit of electrical energy is 1 kWh. (**Note**: kWh is unit of energy and not of power.)

EXAMPLE 4.18

Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).

Solution

Power, P = 75 W

Time of usage, t = 8 hour × 30 days = 240 hours

Electrical energy consumed is the product of power and time of usage.

Electrical energy = power × time of usage = P × t

$$= 75 \text{ watt} \times 240 \text{ hour}$$
$$= 18000 \text{ watt hour}$$
$$= 18 \text{ kilowatt hour} = 18\,\text{kWh}$$
$$1 \text{ electrical unit} = 1 \text{ kWh}$$
$$\text{Electrical energy} = 18 \text{ unit}$$

DO YOU KNOW?

Incandescent lamps glow for 1000 hours. CFL lamps glow for 6000 hours. But LED lamps glow for 50000 hrs (almost 25 years at 5.5 hour per day).

4.3.3 Relation between power and velocity

The work done by a force $\vec{F}$ for a displacement $d\vec{r}$ is

$$W = \int \vec{F} \cdot d\vec{r} \qquad (4.40)$$

Left hand side of the equation (4.40) can be written as

$$W = \int dW = \int \frac{dW}{dt} dt$$

(multiplied and divided by dt)

(4.41)

Since, velocity is $\vec{v} = \frac{d\vec{r}}{dt}$; $d\vec{r} = \vec{v}\,dt$. Right hand side of the equation (4.40) can be written as

$$\int \vec{F} \cdot d\vec{r} = \int \left(\vec{F} \cdot \frac{d\vec{r}}{dt} \right) dt = \int \left(\vec{F} \cdot \vec{v} \right) dt \qquad \left[\vec{v} = \frac{d\vec{r}}{dt} \right]$$

(4.42)

Substituting equation (4.41) and equation (4.42) in equation (4.40), we get

$$\int \frac{dW}{dt} dt = \int \left(\vec{F} \cdot \vec{v} \right) dt$$
$$\int \left(\frac{dW}{dt} - \vec{F} \cdot \vec{v} \right) dt = 0$$

This relation is true for any arbitrary value of dt. This implies that the term within the bracket must be equal to zero, i.e.,

$$\frac{dW}{dt} - \vec{F} \cdot \vec{v} = 0$$

Or

$$\frac{dW}{dt} = \vec{F} \cdot \vec{v} \qquad (4.43)$$

EXAMPLE 4.19

A vehicle of mass 1250 kg is driven with an acceleration 0.2 m s^{-2} along a straight level road against an external resistive force 500 N. Calculate the power delivered by the vehicle's engine if the velocity of the vehicle is 30 $\text{m} s^{-1}$.

Solution

The vehicle's engine has to do work against resistive force and make vechile to move with an acceleration. Therefore, power delivered by the vehicle engine is

P = (resistive force + mass × acceleration) (velocity)

$$P = \vec{F}_{-tot}\vec{v} = (F_{resistive} + F)\vec{v}$$
$$P = \vec{F}_{tot} \cdot \vec{v} = (F_{resistive} + ma)\vec{v}$$
$$= (500\text{ N} + (1250\text{ kg}) \times (0.2\ ms^{-2}))\,(30\text{ ms}^{-1}) = 22.5\text{ kW}$$

4.4 COLLISIONS

Collision is a common phenomenon that happens around us every now and then. For example, carom, billiards, marbles, etc.,. Collisions can happen between two bodies with or without physical contacts.

Linear momentum is conserved in all collision processes. When two bodies collide, the mutual impulsive forces acting between them during the collision time (Δt) produces a change in their respective momenta. That is, the first body exerts a force $\vec{F}_{12}$ on the second body. From Newton's third law, the second body exerts a force $\vec{F}_{21}$ on the first body. This causes a change in momentum $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$ of the first body and second body respectively. Now, the relations could be written as,

$$\Delta\vec{p}_1 = \vec{F}_{12}\Delta t \qquad (4.44)$$

$$\Delta\vec{p}_2 = \vec{F}_{21}\Delta t \qquad (4.45)$$

Adding equation (4.44) and equation (4.45), we get

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{F}_{12}\Delta t + \vec{F}_{21}\Delta t = \left(\vec{F}_{12} + \vec{F}_{21}\right)\Delta t$$

According to Newton's third law, $\vec{F}_{12} = -\vec{F}_{21}$

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$$\Delta\left(\vec{p}_1 + \vec{p}_2\right) = 0$$

Dividing both sides by Δt and taking limit $\Delta t \to 0$, we get

$$\lim_{\Delta t \to 0} \frac{\Delta\left(\vec{p}_1 + \vec{p}_2\right)}{\Delta t} = \frac{d\left(\vec{p}_1 + \vec{p}_2\right)}{dt} = 0$$

The above expression implies that the total linear momentum is a conserved quantity.

Note: The momentum is a vector quantity. Hence, vector addition has to be followed to find the total momentum of the individual bodies in collision.

4.4.1 Types of Collisions

In any collision process, the total linear momentum and total energy are always conserved whereas the total kinetic energy need not be conserved always. Some part of the initial kinetic energy is transformed to other forms of energy. This is because, the impact of collisions and deformation occurring due to collisions may in general, produce heat, sound, light etc. By taking these effects into account, we classify the types of collisions as follows:

(a) Elastic collision

(b) Inelastic collision

(a) Elastic collision

In a collision, the total initial kinetic energy of the bodies (before collision) is equal to the total final kinetic energy of the bodies (after collision) then, it is called as elastic collision. i.e.,

Total kinetic energy before collision = Total kinetic energy after collision

(b) Inelastic collision

In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e.,

Total kinetic energy before collision ≠ Total kinetic energy after collision

$$\begin{pmatrix} \textit{Total kinetic energy} \\ \textit{after collision} \end{pmatrix} - \begin{pmatrix} \textit{Total kinetic energy} \\ \textit{before collision} \end{pmatrix} = \begin{pmatrix} \textit{loss in energy} \\ \textit{during collision} \end{pmatrix} = \Delta Q$$

Even though kinetic energy is not conserved but the total energy is conserved. This is because the total energy contains the kinetic energy term and also a term ΔQ, which includes all the losses that take place during collision. Note that loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc. Further, if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision. Such a collision is found very often. For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

4.4.2 Elastic collisions in one dimension

Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positive x direction) on a frictionless horizontal surface as shown in Figure 4.16.

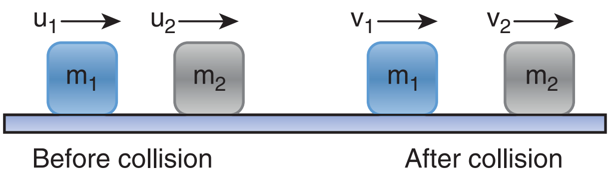


Figure 4.16 Elastic collision in one dimension

Mass	Initial velocity	Final velocity
Mass m_1	u_1	v_1
Mass m_2	u_2	v_2

In order to have collision, we assume that the mass m_1 moves faster than mass m_2 i.e., $u_1 > u_2$. For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

Table 4.4 Comparison between elastic and inelastic collisions

S.No.	Elastic Collision	Inelastic Collision
1.	Total momentum is conserved	Total momentum is conserved
2.	Total kinetic energy is conserved	Total kinetic energy is not conserved
3.	Forces involved are conservative forces	Forces involved are non-conservative forces
4.	Mechanical energy is not dissipated.	Mechanical energy is dissipated into heat, light, sound etc.

	Momentum of mass m_1	**Momentum of mass m_2**	**Total linear momentum**
Before collision	$p_{i1} = m_1 u_1$	$p_{i2} = m_2 u_2$	$p_i = p_{i1} + p_{i2}$ $p_i = m_1 u_1 + m_2 u_2$
After collision	$p_{f1} = m_1 v_1$	$p_{f2} = m_2 v_2$	$p_f = p_{f1} + p_{f2}$ $p_f = m_1 v_1 + m_2 v_2$

From the law of conservation of linear momentum,

Total momentum before collision (p_i) = Total momentum after collision (p_f)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \qquad (4.46)$$

$$\text{Or} \quad m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \qquad (4.47)$$

Further,

	Kinetic energy of mass m_1	**Kinetic energy of mass m_2**	**Total kinetic energy**
Before collision	$KE_{i1} = \frac{1}{2} m_1 u_1^2$	$KE_{i2} = \frac{1}{2} m_2 u_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$
After collision	$KE_{f1} = \frac{1}{2} m_1 v_1^2$	$KE_{f2} = \frac{1}{2} m_2 v_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad (4.48)$$

After simplifying and rearranging the terms,

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2)$$

Using the formula $a^2 - b^2 = (a+b)(a-b)$, we can rewrite the above equation as

$$m_1 (u_1 + v_1)(u_1 - v_1) = m_2 (v_2 + u_2)(v_2 - u_2) \qquad (4.49)$$

Dividing equation (4.49) by (4.47) gives,

$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)}=\frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$$u_1+v_1=v_2+u_2$$

Rearranging, (4.50)

$$u_1-u_2=v_2-v_1$$

Equation (4.50) can be rewritten as

$$u_1-u_2=-(v_1-v_2)$$

This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2,

$$v_1=v_2+u_2-u_1 \qquad (4.51)$$

Or

$$v_2=u_1+v_1-u_2 \qquad (4.52)$$

To find the final velocities v_1 and v_2:

Substituting equation (4.52) in equation (4.47) gives the velocity of m_1 as

$$m_1(u_1-v_1)=m_2(u_1+v_1-u_2-u_2)$$

$$m_1(u_1-v_1)=m_2(u_1+v_1-2u_2)$$

$$m_1u_1-m_1v_1=m_2u_1+m_2v_1-2m_2u_2$$

$$m_1u_1-m_2u_1+2m_2u_2=m_1v_1+m_2v_1$$

$$(m_1-m_2)u_1+2m_2u_2=(m_1+m_2)v_1$$

$$\text{or } v_1=\left(\frac{m_1-m_2}{m_1+m_2}\right)u_1+\left(\frac{2m_2}{m_1+m_2}\right)u_2 \qquad (4.53)$$

Similarly, by substituting (4.51) in equation (4.47) or substituting equation (4.53) in equation (4.52), we get the final velocity of m_2 as

$$v_2=\left(\frac{2m_1}{m_1+m_2}\right)u_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)u_2 \qquad (4.54)$$

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

$$\text{equation (4.53)} \Rightarrow v_1=(0)\,u_1+\left(\frac{2m_2}{2m_2}\right)u_2$$

$$v_1=u_2 \qquad (4.55)$$

$$\text{equation (4.54)} \Rightarrow v_2=\left(\frac{2m_1}{2m_1}\right)u_1+(0)u_2$$

$$v_2=u_1 \qquad (4.56)$$

The equations (4.55) and (4.56) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$ and second body (usually called target) is at rest ($u_2 = 0$),

By substituting $m_1 = m_2$ and $u_2 = 0$ in equations (4.53) and equations (4.54) we get,

$$\text{from equation (4.53)} \Rightarrow v_1=0 \qquad (4.57)$$

$$\text{from equation (4.54)} \Rightarrow v_2=u_1 \qquad (4.58)$$

Equations (4.57) and (4.58) show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3:

The first body is very much lighter than the second body

$\left(m_1 \ll m_2, \frac{m_1}{m_2} \ll 1\right)$ th n th ratio $\frac{m_1}{m_2} \approx 0$

ad lsoi ft h t arg ti sa tr est $\left(u_2 = 0\right)$

Dividing numerator and denominator of equation (4.53) by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right)u_1 + \left(\frac{2}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_1 = \left(\frac{0-1}{0+1}\right)u_1$$

$$v_1 = -u_1 \qquad (4.59)$$

Similarly,

Dividing numerator and denominator of equation (4.54) by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = (0)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = 0 \qquad (4.60)$$

The equation (4.59) implies that the first body which is lighter returns back (rebounds) in the opposite direction with the same initial velocity as it has a negative sign. The equation (4.60) implies that the second body which is heavier in mass continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4:

The second body is very much lighter than the first body

$\left(m_2 \ll m_1, \frac{m_2}{m_1} \ll 1\right)$ th n th ratio $\frac{m_2}{m_1} \approx 0$

ad lsoi ft h t arg ti sa tr est $\left(u_2 = 0\right)$

Dividing numerator and denominator of equation (4.53) by m_1, we get

$$v_1 = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)(0)$$

$$v_1 = \left(\frac{1-0}{1+0}\right)u_1 + \left(\frac{0}{1+0}\right)(0)$$

$$v_1 = u_1 \qquad (4$$

Similarly,

Dividing numerator and denominator of equation (4.58) by m_1, we get

$$v_2 = \left(\frac{2}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{\frac{m_2}{m_1} - 1}{1+\frac{m_2}{m_1}}\right)(0)$$

$$v_2 = \left(\frac{2}{1+0}\right)u_1$$

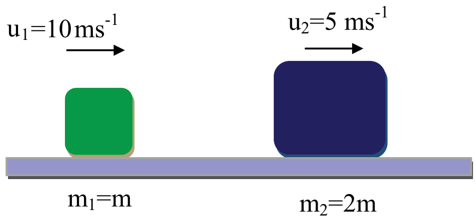
$$v_2 = 2u_1 \qquad (4.62)$$

The equation (4.61) implies that the first body which is heavier continues to move with the same initial velocity. The equation (4.62) suggests that the second body which is lighter will move with twice the initial velocity of the first body. It means that the lighter body is thrown away from the point of collision.

EXAMPLE 4.20

A lighter particle moving with a speed of 10 m s^{-1} collides with an object of double its mass moving in the same direction with half its speed. Assume that the collision is a one dimensional elastic collision. What will be the speed of both particles after the collision?

Solution



Let the mass of the first body be m which moves with an initial velocity, $u_1 = 10$ m s^{-1}. Therefore, the mass of second body is $2m$ and its initial velocity is $u_2 = \frac{1}{2}\ u_1 = \frac{1}{2}(10 \text{ m s}^{-1})$,

Then, the final velocities of the bodies can be calculated from the equation (4.53) and equation (4.54)

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = \left(\frac{m - 2m}{m + 2m}\right)10 + \left(\frac{2 \times 2m}{m + 2m}\right)5$$

$$v_1 = -\left(\frac{1}{3}\right)10 + \left(\frac{4}{3}\right)5 = \frac{-10 + 20}{3} = \frac{10}{3}$$

$$v_1 = 3.33 \text{ ms}^{-1}$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m}{m + 2m}\right)10 + \left(\frac{2m - m}{m + 2m}\right)5$$

$$v_2 = \left(\frac{2}{3}\right)10 + \left(\frac{1}{3}\right)5 = \frac{20 + 5}{3} = \frac{25}{3}$$

$$v_2 = 8.33 \text{ms}^{-1}$$

As the two speeds v_1 and v_2 are positive, they move in the same direction with the velocities, 3.33 m s^{-1} and 8.33 m s^{-1} respectively.

4.4.3 Perfect inelastic collision

In a perfectly inelastic or completely inelastic collision, the objects stick together permanently after collision such that they move with common velocity. Let the two bodies with masses m_1 and m_2 move with initial velocities u_1 and u_2 respectively before collision. After perfect inelastic collision both the objects move together with a common velocity v as shown in Figure (4.17).

Since, the linear momentum is conserved during collisions,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\ v$$

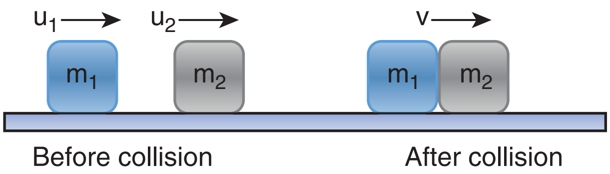


Figure 4.17 Perfect inelastic collision in one dimension

	Velocity		Linear momentum	
	Initial	Final	Initial	Final
Mass m_1	u_1	v	m_1u_1	m_1v
Mass m_2	u_2	v	m_2u_2	m_2v
Total			$m_1u_1 + m_2u_2$	$(m_1 + m_2)$ v

The common velocity can be computed by

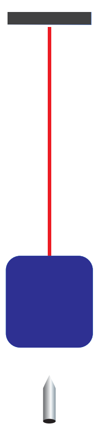
$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \qquad (4.63)$$

EXAMPLE 4.21

A bullet of mass 50 g is fired from below into a suspended object of mass 450 g. The object rises through a height of 1.8 m with bullet remaining inside the object. Find the speed of the bullet. Take g = 10 ms^{-2}.

Solution

m_1 = 50 g = 0.05 kg; m_2 = 450 g = 0.45 kg



The speed of the bullet is u_1. The second body is at rest $(u_2 = 0)$. Let the common velocity of the bullet and the object after the bullet is embedded into the object is v.

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

$$v = \frac{0.05u_1 + (0.45 \times 0)}{(0.05 + 0.45)} = \frac{0.05}{0.50}u_1$$

The combined velocity is the initial velocity for the vertical upward motion of the combined bullet and the object. From second equation of motion,

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 10 \times 1.8} = \sqrt{36}$$

$$v = 6\,ms^{-1}$$

Substituting this in the above equation, the value of u_1 is

$$6 = \frac{0.05}{0.50}u_1 \quad \text{or} \quad u_1 = \frac{0.50}{0.05} \times 6 = 10 \times 6$$

$$u_1 = 60\,ms^{-1}$$

4.4.4 Loss of kinetic energy in perfect inelastic collision

In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc. Let KE_i be the total kinetic energy before collision and KE_f be the total kinetic energy after collision.

Total kinetic energy before collision,

$$KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \quad \text{[illegible]}$$

Total kinetic energy after collision,

$$KE_f = \frac{1}{2}(m_1 + m_2)v^2 \quad \text{[illegible]}$$

Then the loss of kinetic energy is
Loss of KE, $\Delta Q = KE_i - KE_f$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2 \quad (4.66)$$

Substituting equation (4.63) in equation (4.66), and on simplifying (expand v by using the algebra $(a+b)^2 = a^2 + b^2 + 2ab$, we get

$$\text{Loss of KE, } \Delta Q = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2 \quad (4.67)$$

4.4.5 Coefficient of restitution (e)

Suppose we drop a rubber ball and a plastic ball on the same floor. The rubber ball will bounce back higher than the plastic ball. This is because the loss of kinetic energy for an elastic ball is much lesser than the loss of kinetic energy for a plastic ball. The amount of kinetic energy after the collision of two bodies, in general, can be measured through a dimensionless number called the coefficient of restitution (COR).

It is defined as the ratio of velocity of separation (relative velocity) after collision to the velocity of approach (relative velocity) before collision, i.e.,

$$e = \frac{\text{velocity of separation (after collision)}}{\text{velocity of approach (before collision)}}$$
$$= \frac{(v_2 - v_1)}{(u_1 - u_2)} \quad (4.68)$$

In an elastic collision, we have obtained the velocity of separation is equal to the velocity of approach i.e.,

$$(u_1 - u_2) = (v_2 - v_1) \rightarrow \frac{(v_2 - v_1)}{(u_1 - u_2)} = 1 = e$$

This implies that, coefficient of restitution for an elastic collision, $e = 1$. Physically, it means that there is no loss of kinetic energy after the collision. So, the body bounces back with the same kinetic energy which is usually called as perfect elastic.

In any real collision problems, there will be some losses in kinetic energy due to collision, which means e is not always equal to unity. If the ball is perfectly plastic, it will never bounce back and therefore their separation of velocity is zero after the collision. Hence, the value of coefficient of restitution, $e = 0$.

In general, the coefficient of restitution for a material lies between $0 < e < 1$.

EXAMPLE 4.22

Show that the ratio of velocities of equal masses in an inelastic collision when one of the masses is stationary is $\frac{v_1}{v_2} = \frac{1-e}{1+e}$.

Solution

$$e = \frac{\text{velocity of separation (after collision)}}{\text{velocity of approach (before collision)}}$$

$$= \frac{(v_2 - v_1)}{(u_1 - u_2)} = \frac{(v_2 - v_1)}{(u_1 - 0)} = \frac{(v_2 - v_1)}{u_1}$$

$$\Rightarrow v_2 - v_1 = e u_1 \qquad (1)$$

From the law of conservation of linear momentum,

$$m u_1 = m v_1 + m v_2 \Rightarrow u_1 = v_1 + v_2 \ (2)$$

Using the equation (2) for u_1 in (1), we get

$$v_2 - v_1 = e(v_1 + v_2)$$

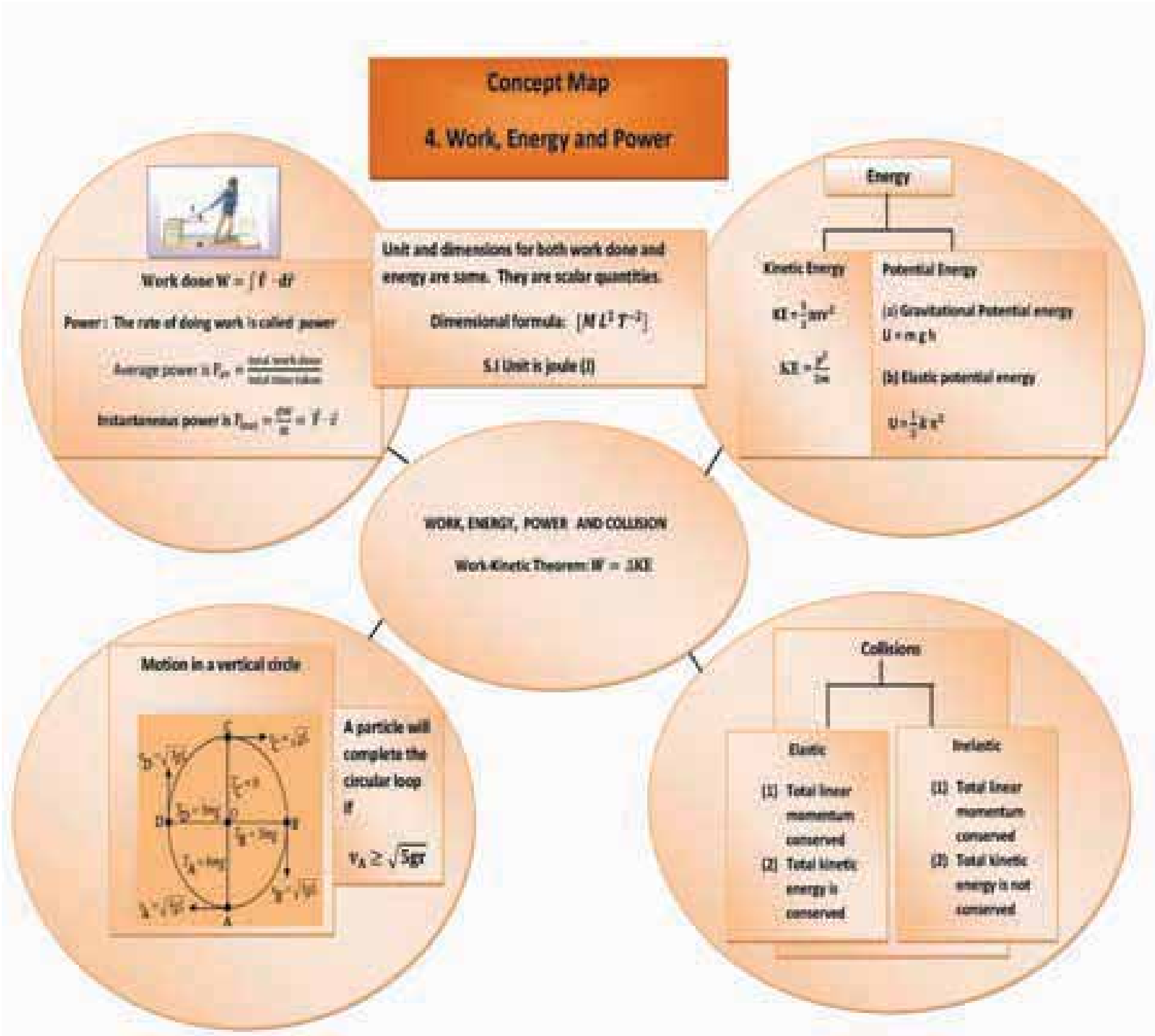
On simplification, we get

$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

SUMMARY

- When a force $\vec{F}$ acting on an object displaces it by $d\vec{r}$, then the work done(W) by the force is $W = \vec{F}.d\vec{r} = Fdr\ cos\theta$.
- The work done by the variable force is defined by $\int_i^f \vec{F}.d\vec{r}$
- Work-kinetic energy theorem: The work done by a force on the object is equal to the change in its kinetic energy.
- The kinetic energy can also be defined in terms of momentum which is given by $K.E = \frac{p^2}{2m}$.
- The potential energy at a point P is defined as the amount of work required to move the object from some reference point O to the point P with constant velocity. It is given by $U = \int_O^P \vec{F}_{ext}.d\vec{r}$. The reference point can be taken as zero potential energy.
- The gravitational potential energy at a height h is given by $U = mgh$. When the elongation or compression is x, the spring potential energy is given by $U = \frac{1}{2}kx^2$. Here k is spring constant.
- The work done by a conservative force around the closed path is zero and for a non-conservative force it is not zero.
- The gravitational force, spring force and Coulomb force are all conservative but frictional force is non-conservative.
- In the conservative force field, the total energy of the object is conserved.
- In the vertical circular motion, the minimum speed required by the mass to complete the circle is $\sqrt{5gr}$. Where r, is the radius of the circle.
- Power is defined as the rate of work done or energy delivered. It is equal to $P = \frac{W}{t} = \vec{F}.\vec{v}$
- The total linear momentum of the system is always conserved for both the elastic and inelastic collisions.
- The kinetic energy of the system is conserved in elastic collisions.
- The coefficient of restitution = $\frac{velocity\ of\ separation(after\ collision)}{velocity\ of\ approach(before\ collision)}$

CONCEPT MAP


Concept Map
4. Work, Energy and Power
Work done $W = \int \vec{F} \cdot d\vec{r}$
Power : The rate of doing work is called power
Average power is P_{av} = $\frac{\text{total work done}}{\text{total time taken}}$
Instantaneous power is $P_{inst} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$
Unit and dimensions for both work done and energy are same. They are scalar quantities.
Dimensional formula: $[M L^2 T^{-2}]$
S.I Unit is joule (J)
Energy
Kinetic Energy
$KE = \frac{1}{2}mv^2$
$KE = \frac{p^2}{2m}$
Potential Energy
(a) Gravitational Potential energy
U = m g h
(b) Elastic potential energy
$U = \frac{1}{2}kx^2$
WORK, ENERGY, POWER AND COLLISION
Work-Kinetic Theorem: $W = \Delta KE$
Motion in a vertical circle
A particle will complete the circular loop if
$v_A \geq \sqrt{5gr}$
Collisions
Elastic
(1) Total linear momentum conserved
(2) Total kinetic energy is conserved
Inelastic
(1) Total linear momentum conserved
(2) Total kinetic energy is not conserved

EXERCISE

I. Multiple Choice Questions

1. A uniform force of $(2\hat{i}+\hat{j})$ N acts on a particle of mass 1 kg. The particle displaces from position $(3\hat{j}+\hat{k})$ m to $(5\hat{i}+3\hat{j})$ m. The work done by the force on the particle is

(AIPMT model 2013)

(a) 9 J (b) 6 J

(c) 10 J (d) 12 J

2. A ball of mass 1 kg and another of mass 2 kg are dropped from a tall building whose height is 80 m. After, a fall of 40 m each towards Earth, their respective kinetic energies will be in the ratio of

(AIPMT model 2004)

(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$

(c) 2 : 1 (d) 1 : 2

3. A body of mass 1 kg is thrown upwards with a velocity 20 m s^{-1}. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction?.

(Take $g=10\,m\,s^{-2}$) (AIPMT 2009)

(a) 20 J (b) 30 J

(c) 40 J (d) 10 J

4. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water of the jet. What is the rate at which kinetic energy is imparted to water ?.

(AIPMT 2009)

(a) $\frac{1}{2}mv^2$ (b) mv^3

(c) $\frac{3}{2}mv^2$ (d) $\frac{5}{2}mv^2$

5. A body of mass 4 m is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass m move perpendicular to each other with equal speed v. The total kinetic energy generated due to explosion is

(AIPMT 2014)

(a) mv^2 (b) $\frac{3}{2}mv^2$

(c) $2mv^2$ (d) $4mv^2$

6. The potential energy of a system increases, if work is done

(a) by the system against a conservative force

(b) by the system against a non-conservative force

(c) upon the system by a conservative force

(d) upon the system by a non-conservative force

7. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?.

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$

(c) $\sqrt{5gR}$ (d) $\sqrt{gR}$

8. The work done by the conservative force for a closed path is

(a) always negative

(b) zero

(c) always positive

(d) not defined

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9. If the linear momentum of the object is increased by 0.1%, then the kinetic energy is increased by

(a) 0.1 % (b) 0.2%

(c) 0.4% (d) 0.01%

10. If the potential energy of the particle is $\alpha - \frac{\beta}{2}x^2$, then force experienced by the particle is

(a) $F = \frac{\beta}{2}x^2$ (b) $F = \beta x$

(c) $F = -\beta x$ (d) $F = -\frac{\beta}{2}x^2$

11. A wind-powered generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2

(c) v^3 (d) v^4

12. Two equal masses m_1 and m_2 are moving along the same straight line with velocities $5\,ms^{-1}$ and $-9\,ms^{-1}$ respectively. If the collision is elastic, then calculate the velocities after the collision of m_1 and m_2, respectively

(a) $-4\,ms^{-1}$ and $10\,ms^{-1}$

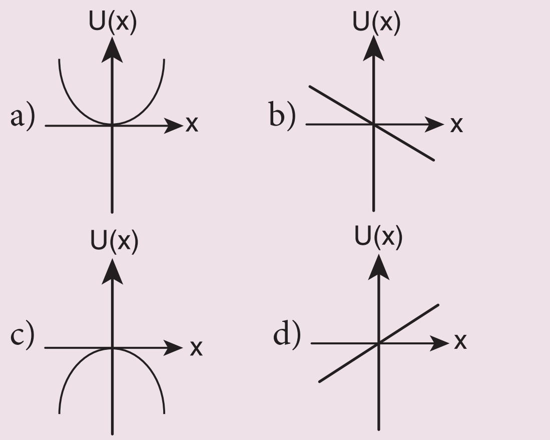
(b) $10\,ms^{-1}$ and $0\,ms^{-1}$

(c) $-9\,ms^{-1}$ and $5\,ms^{-1}$

(d) $5\,ms^{-1}$ and $1\,ms^{-1}$

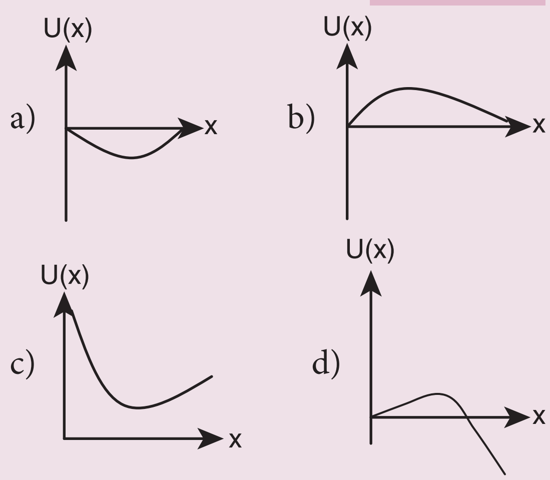
13. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function)

(IIT 2004)



14. A particle which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here, k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is

(IIT 2002)



15. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$

(c) $3k$ (d) $6k$

Answers

1) c	**2)** d	**3)** a	**4)** a	**5)** b
6) a	**7)** c	**8)** b	**9)** b	**10)** c
11) c	**12)** c	**13)** c	**14)** d	**15)** b

II. Short Answer Questions

1. Explain how the definition of work in physics is different from general perception.
2. Write the various types of potential energy. Explain the formulae.
3. Write the differences between conservative and Non-conservative forces. Give two examples each.
4. Explain the characteristics of elastic and inelastic collision.
5. Define the following
   a) Coefficient of restitution
   b) Power
   c) Law of conservation of energy
   d) loss of kinetic energy in inelastic collision.

III. Long Answer Questions

1. Explain with graphs the difference between work done by a constant force and by a variable force.
2. State and explain work energy principle. Mention any three examples for it.
3. Arrive at an expression for power and velocity. Give some examples for the same.
4. Arrive at an expression for elastic collision in one dimension and discuss various cases.
5. What is inelastic collision? In which way it is different from elastic collision. Mention few examples in day to day life for inelastic collision.

IV. Numerical Problems

1. Calculate the work done by a force of 30 N in lifting a load of 2kg to a height of 10m ($g = 10ms^{-2}$)

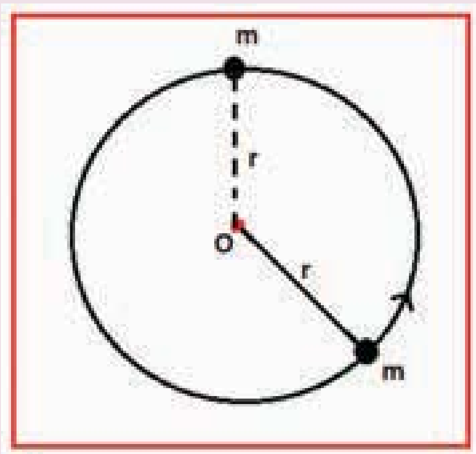
   Ans: 300J

2. A ball with a velocity of 5 ms^{-1} impinges at angle of 60° with the vertical on a smooth horizontal plane. If the coefficient of restitution is 0.5, find the velocity and direction after the impact.

   Ans: $v = 0.3 \text{ m s}^{-1}$

3. A bob of mass m is attached to one end of the rod of negligible mass and length r, the other end of which is pivoted freely at a fixed center O as shown in the figure. What initial speed must be given to the object to reach the top of the circle? (Hint: Use law of conservation of energy). Is this speed less or greater than speed obtained in the section 4.2.9?

   Ans: $v = \sqrt{4gr} \text{ m s}^{-1}$



4. Two different unknown masses A and B collide. A is initially at rest when B has a speed v. After collision B has a speed v/2 and moves at right angles to its original

direction of motion. Find the direction in which A moves after collision.

Ans: $\theta = 26° 33'$

5. A bullet of mass 20 g strikes a pendulum of mass 5 kg. The centre of mass of pendulum rises a vertical distance of 10 cm. If the bullet gets embedded into the pendulum, calculate its initial speed.

Ans: $v = 351.4 ms^{-1}$

V. Conceptual Questions

1. A spring which in initially in un-stretched condition, is first stretched by a length x and again by a further length x. The work done in the first case W_1 is one third of the work done in second case W_2. True or false?
2. Which is conserved in inelastic collision? Total energy (or) Kinetic energy?
3. Is there any net work done by external forces on a car moving with a constant speed along a straight road?
4. A car starts from rest and moves on a surface with uniform acceleration. Draw the graph of kinetic energy versus displacement. What information you can get from that graph?
5. A charged particle moves towards another charged particle. Under what conditions the total momentum and the total energy of the system conserved?

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UNIT 6 GRAVITATION

"The most remarkable discovery in all of astronomy is that the stars are made up of atoms of same kind as those in the Earth" – Richard Feynman

Learning Objectives

In this unit, the student is exposed to

- Kepler's laws for planetary motion
- Newton's law of gravitation
- connection between Kepler's laws and law of gravitation
- calculation of gravitational field and potential
- calculation of variation of acceleration due to gravity
- calculation of escape speed and energy of satellites
- concept of weightlessness
- advantage of heliocentric system over geocentric system
- measurement of the radius of Earth using Eratosthenes method
- recent developments in gravitation and astrophysics

6.1 INTRODUCTION

We are amazed looking at the glittering sky; we wonder how the Sun rises in the East and sets in the West, why there are comets or why stars twinkle. The sky has been an object of curiosity for human beings from time immemorial. We have always wondered about the motion of stars, the Moon, and the planets. From Aristotle to Stephen Hawking, great minds have tried to understand the movement of celestial objects in space and what causes their motion.

The 'Theory of Gravitation' was developed by Newton in the late 17th century to explain the motion of celestial objects and terrestrial objects and answer most of the queries raised. In spite of the study of gravitation and its effect on celestial objects, spanning last three centuries, "gravitation" is still one of the active areas of research in physics today. In 2017, the Nobel Prize in Physics was given for the detection of 'Gravitational waves' which was theoretically predicted by Albert Einstein in the year 1915. Understanding planetary motion, the formation of stars and galaxies, and recently massive objects like black holes and their life cycle have remained the focus of study for the past few centuries in physics.

Geocentric Model of Solar System

In the second century, Claudius Ptolemy, a famous Greco-Roman astronomer, developed a theory to explain the motion of celestial objects like the Sun, the Moon, Mars, Jupiter etc. This theory was called the geocentric model. According to the geocentric model, the Earth is at the center of the universe and all celestial objects including the Sun, the Moon, and other planets orbit the Earth. Ptolemy's model closely matched with the observations of the sky with our naked eye. But later, astronomers found that even though Ptolemy's model successfully explained the motion of the Sun and the Moon up to a certain level, the motion of Mars and Jupiter could not be explained effectively.

Heliocentric Model of Nicholas Copernicus

In the 15th century, a Polish astronomer, Nicholas Copernicus (1473-1543) proposed a new model called the 'Heliocentric model' in which the Sun was considered to be at the center of the solar system and all planets including the Earth orbited the Sun in circular orbits. This model successfully explained the motion of all celestial objects.

Around the same time, Galileo, a famous Italian physicist discovered that all objects close to Earth were accelerated towards the Earth at the same rate. Meanwhile, a noble man called Tycho Brahe (1546-1601) spent his entire lifetime in recording the observations of the stellar and planetary positions with his naked eye. The data that he compiled were analyzed later by his assistant Johannes Kepler (1571–1630) and eventually the analysis led to the deduction of the laws of the planetary motion. These laws are termed as 'Kepler's laws of planetary motion'.

6.1.1 Kepler's Laws of Planetary Motion

Kepler's laws are stated as follows:

1. **Law of orbits:**
   Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci.

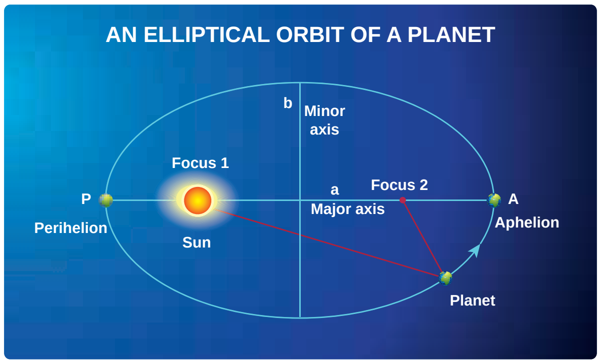


Figure 6.1 An ellipse traced out by a planet around the Sun.

The closest point of approach of the planet to the Sun 'P' is called perihelion and the farthest point 'A' is called aphelion (Figure 6.1). The semi-major axis is 'a' and semi-minor axis is 'b'. In fact, both Copernicus and Ptolemy considered planetary orbits to be circular, but Kepler discovered that the actual orbits of the planets are elliptical.

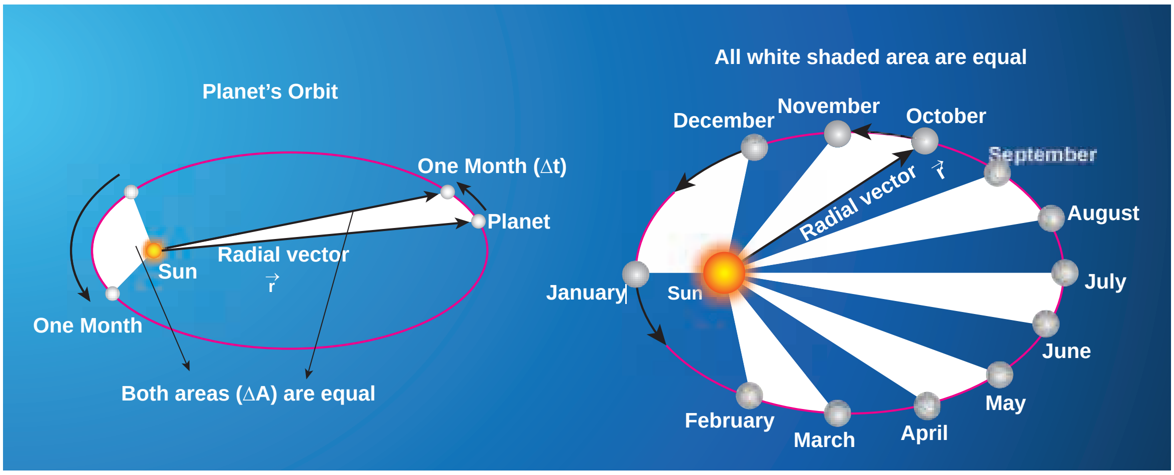


Figure 6.2 Motion of a planet around the Sun depicting 'law of area'.

2. **Law of area:**

The radial vector (line joining the Sun to a planet) sweeps equal areas in equal intervals of time.

In Figure 6.2, the white shaded portion is the area ΔA swept in a small interval of time Δt, by a planet around the Sun. Since the Sun is not at the center of the ellipse, *the planets travel faster when they are nearer to the Sun and slower when they are farther from it, to cover equal area in equal intervals of time.* Kepler discovered the law of area by carefully noting the variation in the speed of planets.

3. **Law of period:**

The square of the time period of revolution of a planet around the Sun in its elliptical orbit is directly proportional to the cube of the semi-major axis of the ellipse. It can be written as:

$$T^2 \propto a^3 \quad (6.1)$$

$$\frac{T^2}{a^3} = constant \quad (6.2)$$

where, T is the time period of revolution for a planet and a is the semi-major axis. Physically this law implies that as the distance of the planet from the Sun increases, the time period also increases but not at the same rate.

In Table 6.1, the time period of revolution of planets around the Sun along with their semi-major axes are given. From column four, we can realize that $\frac{T^2}{a^3}$ is nearly a constant endorsing Kepler's third law.

Table 6.1 The time period of revolution of the planets revolving around the Sun and their semi-major axes.

Planet	a (10^{10} m)	T (years)	$\frac{T^2}{a^3}$
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99

Points to Contemplate

DATA			PROBLEM
Planet	**a**	**T**	What is the law connecting *a* and T?
A	1	3	
B	2	6	
C	4	18	

Comment on the relation between *a* and T for these imaginary planets

6.1.2 Universal Law of Gravitation

Even though Kepler's laws were able to explain the planetary motion, they failed to explain the forces responsible for it. It was Isaac Newton who analyzed Kepler's laws, Galileo's observations and deduced the law of gravitation.

Newton's law of gravitation states that a particle of mass M_1 attracts any other particle of mass M_2 in the universe with an attractive force. The strength of this force of attraction was found to be directly proportional to the product of their masses and is inversely proportional to the square of the distance between them. In mathematical form, it can be written as:

$$\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r} \qquad (6.3)$$

where $\hat{r}$ is the unit vector from M_1 towards M_2 as shown in Figure 6.3, and G is the Gravitational constant that has the value of $6.626\times10^{-11}\,N\,m^2kg^{-2}$, and r is the distance between the two masses M_1 and M_2. In Figure 6.3, the vector $\vec{F}$ denotes the gravitational force experienced by M_2 due to M_1. Here the negative sign indicates that the gravitational force is always attractive in nature and the direction of the force is along the line joining the two masses.

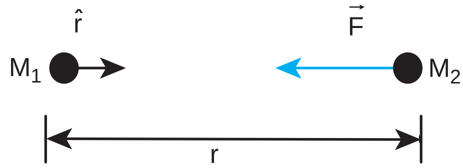
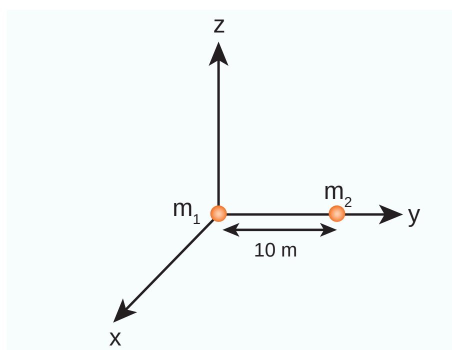


Figure 6.3 Attraction of two masses towards each other.

In cartesian coordinates, the square of the distance is expressed as $r^2 = \left(x^2 + y^2 + z^2\right)$. This is dealt in unit 2.

EXAMPLE 6.1

Consider two point masses m_1 and m_2 which are separated by a distance of 10 meter as shown in the following figure. Calculate the force of attraction between them and draw the directions of forces on each of them. Take m_1= 1 kg and m_2 = 2 kg



Solution

The force of attraction is given by

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

From the figure, r =10 m.

First, we can calculate the magnitude of the force

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{100}$$
$$= 13.34\times10^{-13}\,N.$$

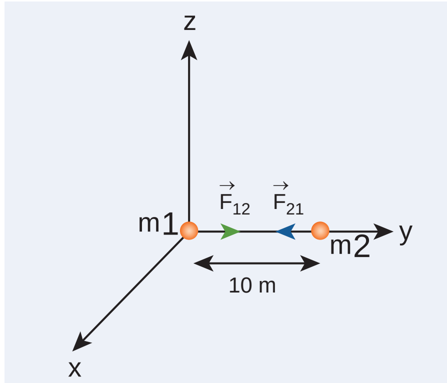
It is to be noted that this force is very small. This is the reason we do not feel the gravitational force of attraction between each other. The small value of G plays a very crucial role in deciding the strength of the force.

The force of attraction ($\vec{F}_{21}$) experienced by the mass m_2 due to m_1 is in the negative 'y' direction ie., $\hat{r} = -\hat{j}$. According to Newton's third law, the mass m_2 also exerts equal and opposite force on m_1. So the force of attraction ($\vec{F}_{12}$) experienced by m_1 due to m_2 is in the direction of positive 'y' axis ie., $\hat{r} = \hat{j}$.

$$\vec{F}_{21} = -13.34\times10^{-13}\,\hat{j}$$

$$\vec{F}_{12} = 13.34\times10^{-13}\,\hat{j}$$

The direction of the force is shown in the figure,



Gravitational force of attraction between m_1 and m_2

$\vec{F}_{12} = -\vec{F}_{21}$ which confirms Newton's third law.

Important features of gravitational force:

- As the distance between two masses increases, the strength of the force tends to decrease because of inverse dependence on r^2. Physically it implies that the planet Uranus experiences less gravitational force from the Sun than the Earth since Uranus is at larger distance from the Sun compared to the Earth.

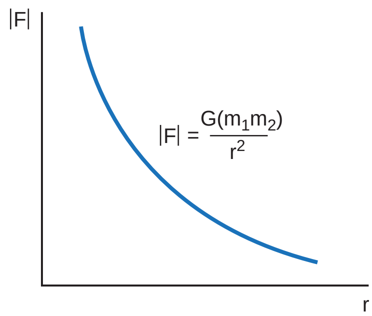


Figure 6.4 Variation of gravitational force with distance

- The gravitational forces between two particles always constitute an action-reaction pair. It implies that the gravitational force exerted by the Sun on the Earth is always towards the Sun. The reaction-force is exerted by the Earth on the Sun. The direction of this reaction force is towards Earth.

- The torque experienced by the Earth due to the gravitational force of the Sun is given by

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left(-\frac{GM_S M_E}{r^2}\hat{r} \right) = 0$$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$

So $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$. It implies that angular momentum $\vec{L}$ is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion. It is true for all the planets. In fact, this constancy of angular momentum leads to the Kepler's second law.

- The expression $\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r}$ has one inherent assumption that both M_1 and M_2 are treated as point masses. When it is said that Earth orbits around the Sun due to Sun's gravitational force, we assumed Earth and Sun to be point masses. This assumption is a good approximation because the distance between the two bodies is very much larger than their diameters. For some irregular and extended objects separated by a small distance, we cannot directly use the equation (6.3). Instead, we have to invoke separate mathematical treatment which will be brought forth in higher classes.
- However, this assumption about point masses holds even for small distance for one special case. To calculate force of attraction between a hollow sphere of mass M with uniform density and point mass m kept outside the hollow sphere, we can replace the hollow sphere of mass M as equivalent to a point mass M located at the center of the hollow sphere. The force of attraction between the hollow sphere of mass M and point mass m can be calculated by treating the hollow sphere also as another point mass. Essentially the entire mass of the hollow sphere appears to be concentrated at the center of the hollow sphere. It is shown in the Figure 6.5(a).
- There is also another interesting result. Consider a hollow sphere of mass M. If we place another object of mass 'm' inside this hollow sphere as in Figure 6.5(b), the force experienced by this mass 'm' will be zero. This calculation will be dealt with in higher classes.

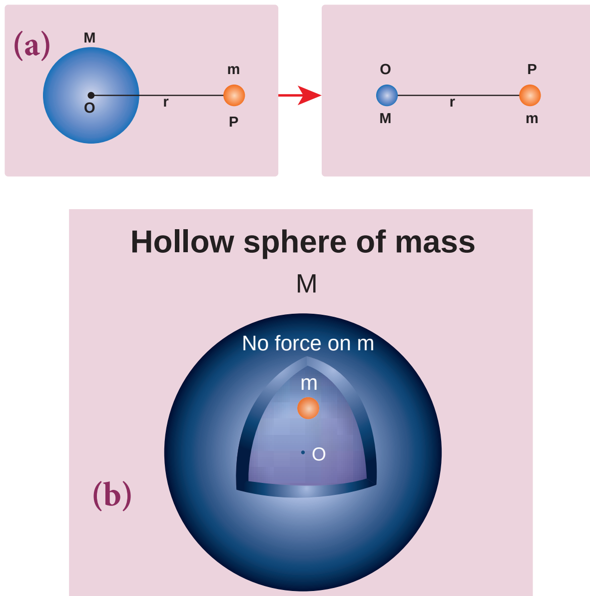


Figure 6.5 A mass placed in a hollow sphere.

- The triumph of the law of gravitation is that it concludes that the mango that is falling down and the Moon orbiting the Earth are due to the same gravitational force.

Newton's inverse square Law:

Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the center is

$$a = -\frac{v^2}{r} \quad (6.4)$$

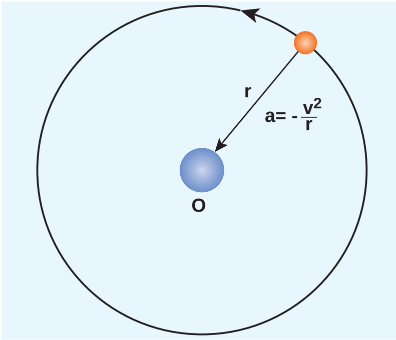


Figure 6.6 Point mass orbiting in a circular orbit.

Here v is the velocity and r, the distance of the planet from the center of the orbit (Figure 6.6).

The velocity in terms of known quantities r and T, is

$$v = \frac{2\pi r}{T} \tag{6.5}$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (6.4) we get,

$$a = -\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = -\frac{4\pi^2 r}{T^2} \tag{6.6}$$

Substituting the value of 'a' from (6.6) in Newton's second law, $F = ma$, where 'm' is the mass of the planet.

$$F = -\frac{4\pi^2 mr}{T^2} \tag{6.7}$$

From Kepler's third law,

$$\frac{r^3}{T^2} = k(constant) \tag{6.8}$$

$$\frac{r}{T^2} = \frac{k}{r^2} \tag{6.9}$$

By substituting equation 6.9 in the force expression, we can arrive at the law of gravitation.

$$F = -\frac{4\pi^2 mk}{r^2} \tag{6.10}$$

Here negative sign implies that the force is attractive and it acts towards the center. In equation (6.10), mass of the planet 'm' comes explicitly. But Newton strongly felt that according to his third law, if Earth is attracted by the Sun, then the Sun must also be attracted by the Earth with the same magnitude of force. So he felt that the Sun's mass (M) should also occur explicitly in the expression for force (6.10). From this insight, he equated the constant $4\pi^2 k$ *to* GM which turned out to be the law of gravitation

$$F = -\frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

In the above discussion we assumed that the orbit of the planet to be circular which is not true as the orbit of the planet around the Sun is elliptical. But this circular orbit assumption is justifiable because planet's orbit is very close to being circular and there is only a very small deviation from the circular shape.

Points to Contemplate

If Kepler's third law was "r^3T^2 = constant" instead of "$\frac{r^3}{T^2}$ = constant" what would be the new law of gravitation? Would it still be an inverse square law? How would the gravitational force change with distance? In this new law of gravitation, will Neptune experience greater gravitational force or lesser gravitational force when compared to the Earth?

EXAMPLE 6.2

Moon and an apple are accelerated by the same gravitational force due to Earth. Compare the acceleration of the two.

The gravitational force experienced by the apple due to Earth

$$F = -\frac{GM_EM_A}{R^2}$$

Here M_A – Mass of the apple, M_E – Mass of the Earth and R – Radius of the Earth.

Equating the above equation with Newton's second law,

$$M_Aa_A = -\frac{GM_EM_A}{R^2}.$$

Simplifying the above equation we get,

$$a_A = -\frac{GM_E}{R^2}$$

Here a_A is the acceleration of apple that is equal to 'g'.

Similarly the force experienced by Moon due to Earth is given by

$$F = -\frac{GM_EM_m}{R_m^2}.$$

Here R_m - distance of the Moon from the Earth, M_m – Mass of the Moon

The acceleration experienced by the Moon is given by

$$a_m = -\frac{GM_E}{R_m^2}.$$

The ratio between the apple's acceleration to Moon's acceleration is given by

$$\frac{a_A}{a_m} = \frac{R_m^2}{R^2}.$$

From the Hipparchrus measurement, the distance to the Moon is 60 times that of Earth radius. R_m = 60R.

$$a_A / a_m = \frac{(60R)^2}{R^2} = 3600.$$

The apple's acceleration is 3600 times the acceleration of the Moon.

The same result was obtained by Newton using his gravitational formula. The apple's acceleration is measured easily and it is 9.8 m s^{-2}. Moon orbits the Earth once in 27.3 days and by using the centripetal acceleration formula, (Refer unit 3).

$$\frac{a_A}{a_m} = \frac{9.8}{0.00272} = 3600$$

which is exactly what he got through his law of gravitation.

Note The above calculation depends on knowing the distance between the Earth and the Moon and the radius of the Earth. The radius of the Earth was measured by Greek librarian Eratosthenes and distance between the Earth and the Moon was measured by Greek astronomer Hipparchrus 2400 years ago. It is very interesting to note that in order to measure these distances he used only high school geometry and trigonometry. These details are discussed in the astronomy section (6.5).

6.1.3 Gravitational Constant

In the law of gravitation, the value of gravitational constant G plays a very important role. The value of G explains why the gravitational force between the Earth and the Sun is so great while the same force between two small objects (for example between two human beings) is negligible.

The force experienced by a mass 'm' which is on the surface of the Earth (Figure 6.7) is given by

$$F = -\frac{GM_E m}{R_E^{\ 2}} \qquad (6.11)$$

M_E-mass of the Earth, m - mass of the object, R_E- radius of the Earth.

Equating Newton's second law, $F = mg$, to equation (6.11) we get,

$$mg = -\frac{GM_E m}{R_E^{\ 2}}$$

$$g = -\frac{GM_E}{R_E^{\ 2}} \qquad (6.12)$$

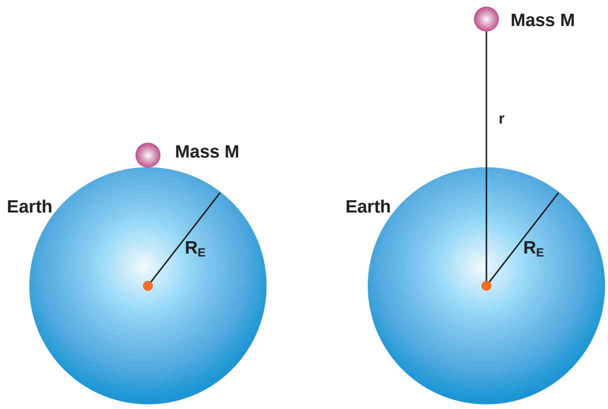


Figure 6.7 Force experienced by a mass on the (i) surface of the Earth (ii) at a distance from the centre of the Earth

Now the force experienced by some other object of mass M at a distance r from the center of the Earth is given by,

$$F = -\frac{GM_E M}{r^2}$$

Using the value of g in equation (6.12), the force F will be,

$$F = -gM\frac{R_E^{\ 2}}{r^2} \qquad (6.13)$$

From this it is clear that the force can be calculated simply by knowing the value of g. It is to be noted that in the above calculation G is not required.

DO YOU KNOW? In the year 1798, Henry Cavendish experimentally determined the value of gravitational constant 'G' by using a torsion balance. He calculated the value of 'G' to be equal to $6.75 \times 10^{-11}\, N\, m^2 kg^{-2}$. Using modern techniques a more accurate value of G could be measured. The currently accepted value of G is $6.67259 \times 10^{-11}\, N\, m^2 kg^{-2}$.

6.2 GRAVITATIONAL FIELD AND GRAVITATIONAL POTENTIAL

6.2.1 Gravitational field

Force is basically due to the interaction between two particles. Depending upon the type of interaction we can have two kinds of forces: Contact forces and Non-contact forces (Figure 6.8).

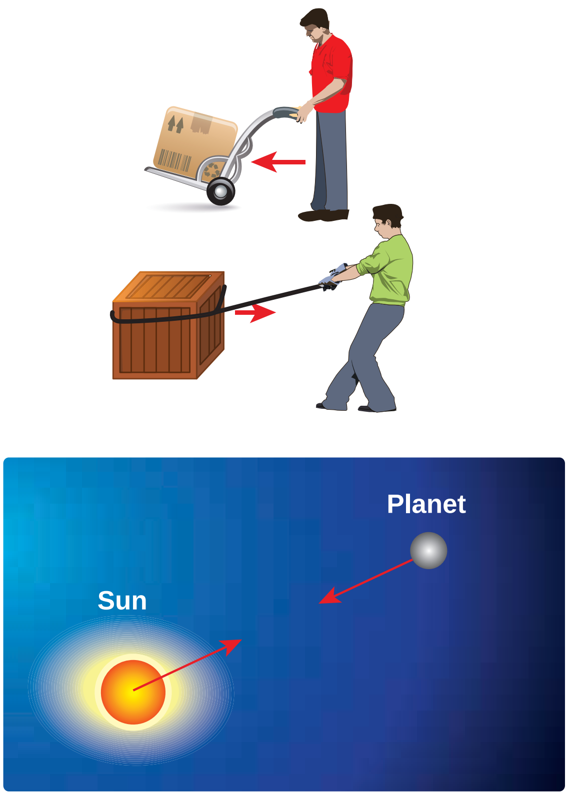


Figure 6.8 Depiction of contact and non-contact forces

Contact forces are the forces applied where one object is in physical contact with the other. The movement of the object is caused by the physical force exerted through the contact between the object and the agent which exerts force.

Consider the case of Earth orbiting around the Sun. Though the Sun and the Earth are not physically in contact with each other, there exists an interaction between them. This is because of the fact that the Earth experiences the gravitational force of the Sun. This gravitational force is a non-contact force.

It sounds mysterious that the Sun attracts the Earth despite being very far from it and without touching it. For contact forces like push or pull, we can calculate the strength of the force since we can feel or see. But how do we calculate the strength of non-contact force at different distances? To understand and calculate the strength of non-contact forces, the concept of 'field' is introduced.

The gravitational force on a particle of mass 'm_2' due to a particle of mass 'm_1' is

$$\vec{F}_{21} = -\frac{Gm_1m_2}{r^2}\hat{r} \qquad (6.14)$$

where $\hat{r}$ is a unit vector that points from m_1 to m_2 along the line joining the masses m_1 and m_2.

The gravitational field intensity $\vec{E}_1$ (here after called as gravitational field) at a point which is at a distance r from m_1 is defined as the gravitational force experienced by unit mass placed at that point. It is given by the ratio $\frac{\vec{F}_{21}}{m_2}$ (where m_2 is the mass of the object on which $\vec{F}_{21}$ acts)

Using $\vec{E}_1 = \frac{\vec{F}_{21}}{m_2}$ in equation (6.14) we get,

$$\vec{E}_1 = -\frac{Gm_1}{r^2}\hat{r} \qquad (6.15)$$

$\vec{E}_1$ is a vector quantity that points towards the mass m_1 and is independent of mass m_2, Here m_2 is taken to be of unit magnitude . The unit $\hat{r}$ is along the line between m_1 and the point in question. The field $\vec{E}_1$ is due to the mass m_1.

In general, the gravitational field intensity due to a mass M at a distance r is given by

$$\vec{E} = -\frac{GM}{r^2}\hat{r} \qquad (6.16)$$

Now in the region of this gravitational field, a mass 'm' is placed at a point P (Figure 6.9). Mass 'm' interacts with the field $\vec{E}$ and experiences an attractive force due to M as shown in Figure 6.9. The gravitational force experienced by 'm' due to 'M' is given by

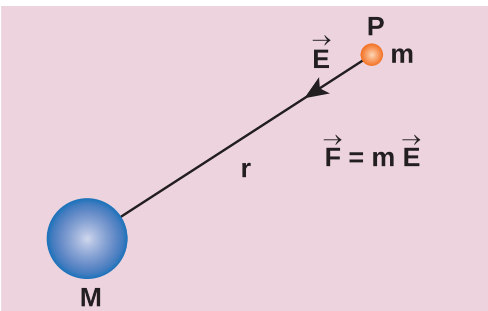


Figure 6.9 Gravitational Field intensity measured with an object of unit mass

$$\vec{F}_m = m\vec{E} \qquad (6.17)$$

Now we can equate this with Newton's second law $\vec{F} = m\vec{a}$

$$m\vec{a} = m\vec{E} \qquad (6.18)$$

$$\vec{a} = \vec{E} \qquad (6.19)$$

In other words, equation (6.18) implies that the gravitational field at a point is equivalent to the acceleration experienced by a particle at that point. However, it is to be noted that $\vec{a}$ and $\vec{E}$ are separate physical quantities that have the same magnitude and direction. The gravitational field $\vec{E}$ is the property of the source and acceleration $\vec{a}$ is the effect experienced by the test mass (unit mass) which is placed in the gravitational field $\vec{E}$. The non-contact interaction between two masses can now be explained using the concept of "Gravitational field".

Points to be noted:

i) The strength of the gravitational field decreases as we move away from the mass M as depicted in the Figure 6.10. The magnitude of $\vec{E}$ decreases as the distance r increases.

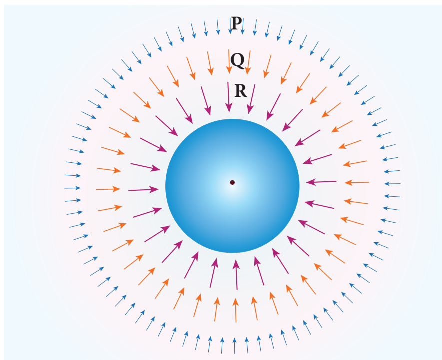


Figure 6.10 Strength of the Gravitational field lines decreases with distance

Figure 6.10 shows that the strength of the gravitational field at points P, Q, and R is given by $|\vec{E}_P| < |\vec{E}_Q| < |\vec{E}_R|$. It can be understood by comparing the length of the vectors at points P, Q, and R.

ii) The "field" concept was introduced as a mathematical tool to calculate gravitational interaction. Later it was found that field is a real physical quantity and it carries energy and momentum in space. The concept of field is inevitable in understanding the behavior of charges.

iii) The unit of gravitational field is Newton per kilogram (N/kg) or m s^{-2}.

6.2.2 Superposition principle for Gravitational field

Consider 'n' particles of masses $m_1, m_2, \ldots m_n$, distributed in space at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots$ etc, with respect to point P. The total gravitational field at a point P due to all the masses is given by the vector sum of the gravitational field due to the individual masses (Figure 6.11). This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\hat{r}_1 - \frac{Gm_2}{r_2^2}\hat{r}_2 - \ldots - \frac{Gm_n}{r_n^2}\hat{r}_n$$

$$= -\sum_{i=1}^{n} \frac{Gm_i}{r_i^2}\hat{r}_i. \quad (6.20)$$

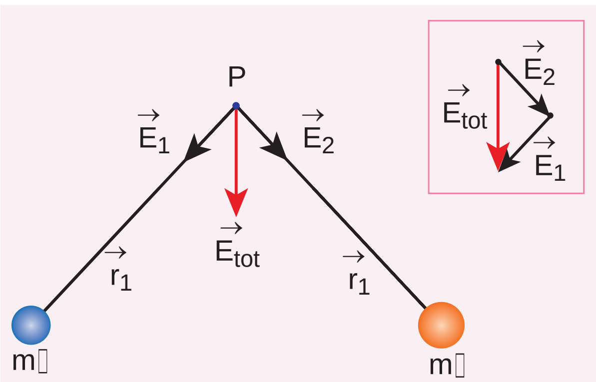
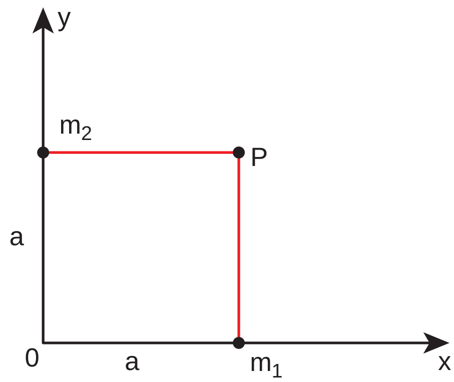


Figure 6.11 Superposition of two gravitational field intensities giving resultant field.

Instead of discrete masses, if we have continuous distribution of a total mass M, then the gravitational field at a point P is calculated using the method of integration.

EXAMPLE 6.3

(a) Two particles of masses m_1 *and* m_2 are placed along the x and y axes respectively at a distance 'a' from the origin. Calculate the gravitational field at a point P shown in figure below.



Solution

Gravitational field due to m_1 at a point P is given by,

$$\vec{E}_1 = -\frac{Gm_1}{a^2}\hat{j}$$

Gravitational field due to m_2 at the point p is given by,

$$\vec{E}_2 = -\frac{Gm_2}{a^2}\hat{i}$$

$$\vec{E}_{total} = -\frac{Gm_1}{a^2}\hat{j} - \frac{Gm_2}{a^2}\hat{i}$$

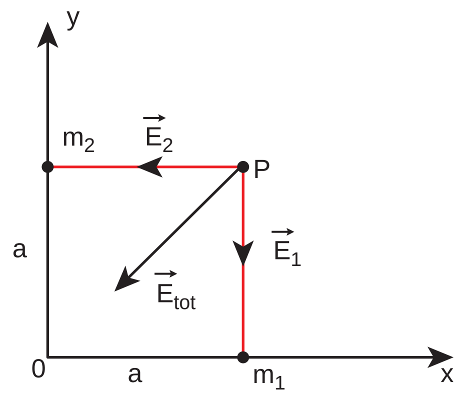
$$= -\frac{G}{a^2}\left(m_1\hat{j} + m_2\hat{i}\right)$$

The direction of the total gravitational field is determined by the relative value of m_1 and m_2.

When $m_1 = m_2 = m$

$$\vec{E}_{total} = -\frac{Gm}{a^2}(\hat{i}+\hat{j})$$

$(\hat{i}+\hat{j}=\hat{j}+\hat{i}$ as vectors obeys commutation law).

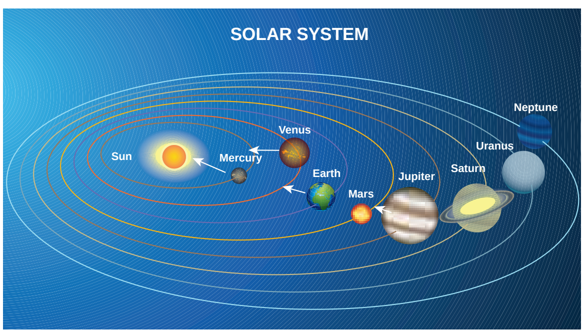


$\vec{E}_{total}$ points towards the origin of the co-ordinate system and the magnitude of $\vec{E}_{total}$ is $\frac{Gm}{a^2}$.

EXAMPLE 6.4

Qualitatively indicate the gravitational field of Sun on Mercury, Earth, and Jupiter shown in figure.

Since the gravitational field decreases as distance increases, Jupiter experiences a weak gravitational field due to the Sun. Since Mercury is the nearest to the Sun, it experiences the strongest gravitational field.



Solar System

6.2.3 Gravitational Potential Energy

The concept of potential energy and its physical meaning were dealt in unit 4. The gravitational force is a conservative force and hence we can define a gravitational potential energy associated with this conservative force field.

Two masses m_1 and m_2 are initially separated by a distance r'. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r as shown in Figure 6.12(a)

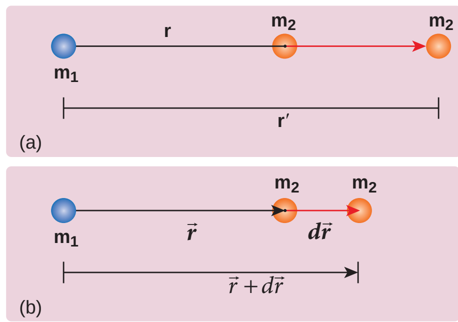


Figure 6.12 Two distant masses changing the linear distance

To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from $\vec{r}$ *to* $\vec{r}+d\vec{r}$ (shown in the Figure 6.12(b)), work has to be done externally. This infinitesimal work is given by

$$dW = \vec{F}_{ext}.d\vec{r} \qquad (6.21)$$

The work is done against the gravitational force, therefore,

$$\left|\vec{F}_{ext}\right| = \left|\vec{F}_G\right| = \frac{Gm_1m_2}{r^2} \qquad (6.22)$$

Substituting Equation (6.22) in 6.21, we get

$$dW = \frac{Gm_1m_2}{r^2}\hat{r}.d\vec{r} \qquad (6.23)$$

Also we know,

$$d\vec{r} = dr\,\hat{r} \qquad (6.24)$$

$$\Rightarrow dW = \frac{Gm_1m_2}{r^2}\hat{r}.\left(dr\,\hat{r}\right) \qquad (6.25)$$

$$\hat{r}.\hat{r} = 1\left(since\ both\ are\ unit\ vectors\right)$$

$$\therefore \quad dW = \frac{Gm_1m_2}{r^2}dr \qquad (6.26)$$

Thus the total work done for displacing the particle from r' to r is

$$W = \int_{r'}^{r} dW = \int_{r'}^{r} \frac{Gm_1m_2}{r^2}dr \qquad (6.27)$$

$$W = -\left(\frac{Gm_1m_2}{r}\right)_{r'}^{r}$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'} \qquad (6.28)$$

$$W = U(r) - U(r')$$

where $U(r) = \frac{-Gm_1m_2}{r}$

This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the separation between them are r and r' respectively.

Case 1: If $r < r'$

Since gravitational force is attractive, m_2 is attracted by m_1.Then m_2 can move from r to r' without any external work (Figure 6.13). Here work is done by the system spending its internal energy and hence the work done is said to be negative.

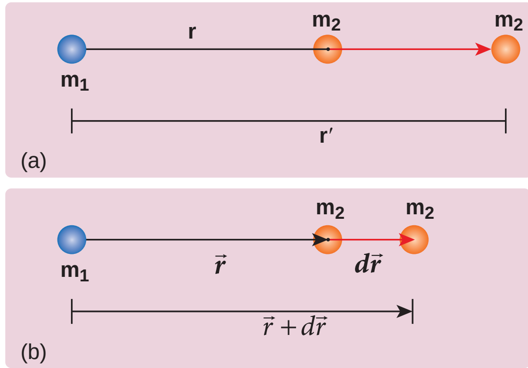


Figure 6.13 Cases for calculation of work done by gravity

Case 2: If $r > r'$

Work has to be done against gravity to move the object from r' to r. Therefore work is done on the body by external force and hence work done is positive.

It is to be noted that only potential energy difference has physical significance. Now gravitational potential energy can be discussed by choosing one point as the reference point.

Let us choose $r' = \infty$. Then the second term in the equation (6.28) becomes zero.

$$W = -\frac{Gm_1m_2}{r} + 0 \qquad (6.29)$$

Now we can define gravitational potential energy of a system of two masses m_1 and m_2 separated by a distance r as the amount of work done to bring the mass m_2 from infinity to a distance r assuming m_1 to be fixed in its position and is written as $U(r) = -\frac{Gm_1m_2}{r}$. It is to be noted that the gravitational potential energy of the system consisting of two masses m_1 and m_2 separated by a distance r, is the gravitational potential energy difference of the system when the masses are separated by an infinite distance and by distance r. $U(r) = U(r) - U(\infty)$. Here we choose $U(\infty)$ = 0 as the reference point. The gravitational potential energy $U(r)$ is always negative because when two masses come together slowly from infinity, work is done by the system.

The unit of gravitational potential energy $U(r)$ is Joule and it is a scalar quantity. The gravitational potential energy depends upon the two masses and the distance between them.

6.2.4 Gravitational potential energy near the surface of the Earth

It is already discussed in chapter 4 that when an object of mass m is raised to a height h, the potential energy stored in the object is mgh (Figure 6.14). This can be derived using the general expression for gravitational potential energy.

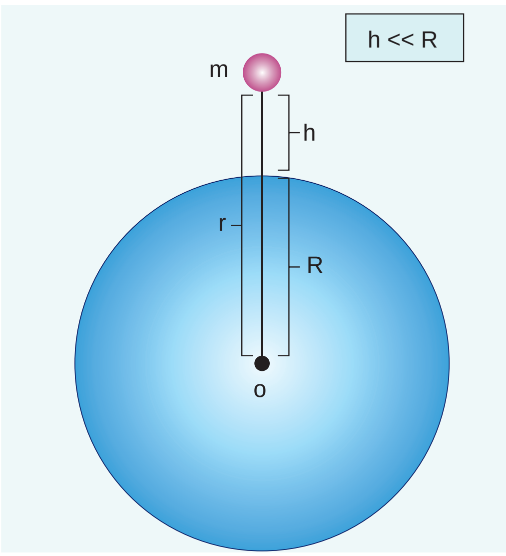


Figure 6.14 Mass placed at a distance r from the center of the Earth

Consider the Earth and mass system, with *r*, the distance between the mass m and the Earth's centre. Then the gravitational potential energy,

$$U = -\frac{GM_e m}{r} \quad (6.30)$$

Here r = R_e+h, where R_e is the radius of the Earth. h is the height above the Earth's surface

$$U = -G\frac{M_e m}{(R_e + h)} \quad (6.31)$$

If h << R_e, equation (6.31) can be modified as

$$U = -G\frac{M_e m}{R_e(1 + h/R_e)}$$

$$U = -G\frac{M_e m}{R_e}(1 + h/R_e)^{-1} \quad (6.32)$$

By using Binomial expansion and neglecting the higher order terms, we get

$$U = -G\frac{M_e m}{R_e}\left(1 - \frac{h}{R_e}\right). \quad (6.33)$$

We know that, for a mass m on the Earth's surface,

$$G\frac{M_e m}{R_e} = mgR_e \quad (6.34)$$

Substituting equation (6.34) in (6.33) we get,

$$U = -mgR_e + mgh \quad (6.35)$$

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from height h_1 to h_2 then the potential energy at h_1 is

$$U(h_1) = -mgR_e + mgh_1 \quad (6.36)$$

and the potential energy at h_2 is

$$U(h_2) = -mgR_e + mgh_2 \quad (6.37)$$

The potential energy difference between h_1 and h_2 is

$$U(h_2) - U(h_1) = mg(h_1 - h_2). \quad (6.38)$$

The term mgR_e in equations (6.36) and (6.37) plays no role in the result. Hence in the equation (6.35) the first term can be omitted or taken to zero. Thus it can be stated that The gravitational potential energy stored in the particle of mass m at a height h from the surface of the Earth is $U = mgh$. On the surface of the Earth, $U = 0$, since h is zero.

It is to be noted that mgh is the work done on the particle when we take the mass m from the surface of the Earth to a height h. This work done is stored as a gravitational potential energy in the mass m. Even though mgh is gravitational potential energy of the system (Earth and mass m), we can take mgh as the gravitational potential energy of the mass m since Earth is stationary when the mass moves to height h.

6.2.5 Gravitational potential V(r)

It is explained in the previous sections that the gravitational field $\vec{E}$ depends only on the source mass which creates the field. It is a vector quantity. We can also define a scalar quantity called "gravitational potential" which depends only on the source mass.

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r and it is denoted as V(r). In other words, the gravitational potential at distance r is equivalent to gravitational potential energy per unit mass at the same distance r. It is a scalar quantity and its unit is J kg^{-1}

We can determine gravitational potential from gravitational potential energy. Consider two masses m_1 and m_2 separated by a distance r which has gravitational potential energy $U(r)$ (Figure 6.15). The gravitational potential due to mass m_1 at a point P which is at a distance r from m_1 is obtained by making m_2 equal to unity (m_2 = 1kg). Thus the gravitational potential $V(r)$ due to mass m_1 at a distance r is

$$V(r) = -\frac{Gm_1}{r} \quad (6.39)$$

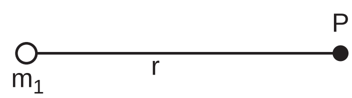


Figure 6.15 Point mass placed at a distance

Gravitational field and gravitational force are vector quantities whereas the gravitational potential and gravitational potential energy are scalar quantities. The motion of particles can be easily analyzed using scalar quantities than vector quantities. Consider the example of a falling apple:

Figure 6.16 shows an apple which falls on Earth due to Earth's gravitational force. This can be explained using the concept of gravitational potential $V(r)$ as follows.

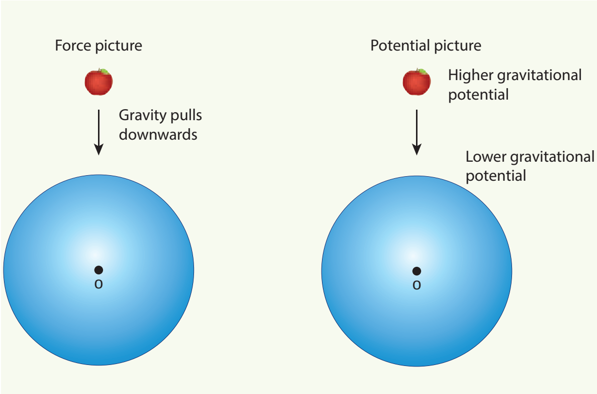


Figure 6.16 Apple falling freely under gravity

The gravitational potential $V(r)$ at a point of height h from the surface of the Earth is given by,

$$V(r=R+h)=-\frac{GM_e}{(R+h)} \quad (6.40)$$

The gravitational potential $V(r)$ on the surface of Earth is given by,

$$V(r=R)=-\frac{GM_e}{R} \quad (6.41)$$

Thus we see that

$$V(r=R)<V(r=R+h). \quad (6.42)$$

It is already discussed in the previous section that the gravitational potential energy near the surface of the Earth at height h is mgh. The gravitational potential at this point is simply $V(h)=U(h)/m$ = gh. In fact, the gravitational potential on the surface of the Earth is zero since h is zero. So the apple falls from a region of a higher gravitational potential to a region of lower gravitational potential. In general, the mass will move from a region of higher gravitational potential to a region of lower gravitational potential.

EXAMPLE 6.5

Water falls from the top of a hill to the ground. Why?

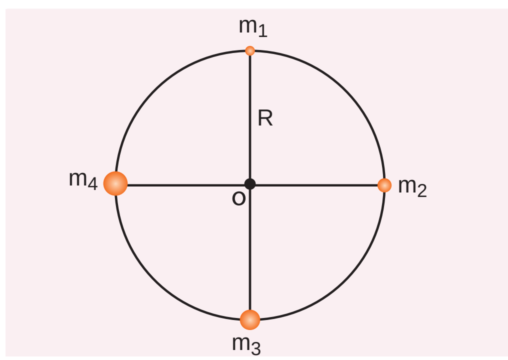
This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth i.e. $V_{hill} > V_{ground}$

Water falling from hill top

The motion of particles can be analyzed more easily using scalars like $U(r)$ *or* $V(r)$ than vector quantities like $\vec{F}$ *or* $\vec{E}$. In modern theories of physics, the concept of potential plays a vital role.

EXAMPLE 6.6

Consider four masses m_1, m_2, m_3, and m_4 arranged on the circumference of a circle as shown in figure below



Calculate

(a) The gravitational potential energy of the system of 4 masses shown in figure.

(b) The gravitational potential at the point O due to all the 4 masses.

Solution

The gravitational potential energy $U(r)$ can be calculated by finding the sum of gravitational potential energy of each pair of particles.

$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_1m_4}{r_{14}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_2m_4}{r_{24}} - \frac{Gm_3m_4}{r_{34}}$$

Here r_{12}, r_{13} ... are distance between pair of particles

$$r_{14}{}^2 = R^2 + R^2 = 2R^2$$

$$r_{14} = \sqrt{2}R = r_{12} = r_{23} = r_{34}$$

$$r_{13} = r_{24} = 2R$$

$$U = -\frac{Gm_1m_2}{\sqrt{2}R} - \frac{Gm_1m_3}{2R} - \frac{Gm_1m_4}{\sqrt{2}R} - \frac{Gm_2m_3}{\sqrt{2}R} - \frac{Gm_2m_4}{2R} - \frac{Gm_3m_4}{\sqrt{2}R}$$

$$U = -\frac{G}{R}\left[\frac{m_1m_2}{\sqrt{2}} + \frac{m_1m_3}{2} + \frac{m_1m_4}{\sqrt{2}} + \frac{m_2m_3}{\sqrt{2}} + \frac{m_2m_4}{2} + \frac{m_3m_4}{\sqrt{2}}\right]$$

If all the masses are equal, then $m_1 = m_2 = m_3 = m_4 = M$

$$U = -\frac{GM^2}{R}\left[\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}}\right]$$

$$U = -\frac{GM^2}{R}\left[1 + \frac{4}{\sqrt{2}}\right]$$

$$U = -\frac{GM^2}{R}\left[1 + 2\sqrt{2}\right]$$

The gravitational potential V(r) at a point O is equal to the sum of the gravitational potentials due to individual mass. Since potential is a scalar, the net potential at point O is the algebraic sum of potentials due to each mass.

$$V_O(r) = -\frac{Gm_1}{R} - \frac{Gm_2}{R} - \frac{Gm_3}{R} - \frac{Gm_4}{R}$$

If $m_1 = m_2 = m_3 = m_4 = M$

$$V_O(r) = -\frac{4GM}{R}$$

6.3 ACCELERATION DUE TO GRAVITY OF THE EARTH

When objects fall on the Earth, the acceleration of the object is towards the Earth. From Newton's second law, an object is accelerated only under the action of a force. In the case of Earth, this force is the gravitational pull of Earth. This force produces a constant acceleration near the Earth's surface in all bodies, irrespective of their masses. The gravitational force exerted by Earth on the mass m near the surface of the Earth is given by

$$\vec{F} = -\frac{GmM_e}{R_e^{\,2}}\hat{r}$$

Now equating Gravitational force to Newton's second law,

$$m\vec{a} = -\frac{GmM_e}{R_e^2}\hat{r}$$

hence, acceleration is,

$$\vec{a} = -\frac{GM_e}{R_e^2}\hat{r} \qquad (6.43)$$

The acceleration experienced by the object near the surface of the Earth due to its gravity is called acceleration due to gravity. It is denoted by the symbol g. The magnitude of acceleration due to gravity is

$$|g| = \frac{GM_e}{R_e^2}. \qquad (6.44)$$

It is to be noted that the acceleration experienced by any object is independent of its mass. The value of g depends only on the mass and radius of the Earth. Infact, Galileo arrived at the same conclusion 400 years ago that *all objects fall towards the Earth with the same acceleration* through various quantitative experiments. The acceleration due to gravity g is found to be 9.8 m s^{-2} on the surface of the Earth near the equator.

6.3.1 Variation of g with altitude, depth and latitude

Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is

$$g' = \frac{GM}{(R_e + h)^2} \qquad (6.45).$$

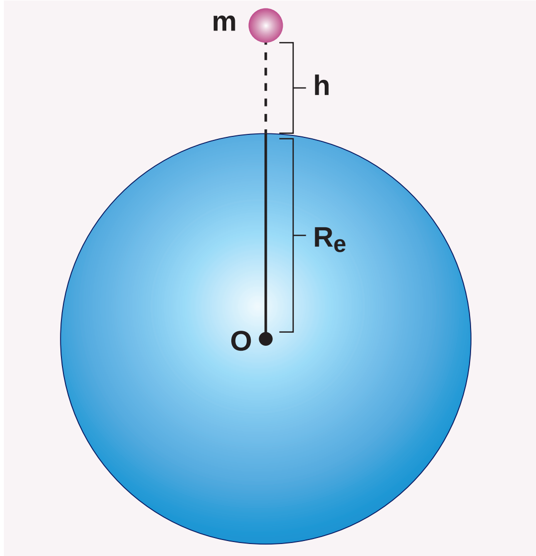


Figure 6.17(a) Mass at a height h from the center of the Earth

$$g' = \frac{GM}{R_e^2\left(1 + \frac{h}{R_e}\right)^2}$$

$$g' = \frac{GM}{R_e^2}\left(1 + \frac{h}{R_e}\right)^{-2}$$

If $h \ll R_e$

We can use Binomial expansion. Taking the terms upto first order

$$g' = \frac{GM}{R_e^2}\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = g\left(1 - 2\frac{h}{R_e}\right) \qquad (6.46)$$

We find that $g' < g$. This means that as altitude h increases the acceleration due to gravity g decreases.

EXAMPLE 6.7

1. Calculate the value of g in the following two cases:
   (a) If a mango of mass ½ kg falls from a tree from a height of 15 meters, what is the acceleration due to gravity when it begins to fall?

Solution

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = 9.8\left(1 - \frac{2\times15}{6400\times10^3}\right)$$

$$g' = 9.8\left(1 - 0.469\times10^{-5}\right)$$

But $\quad 1 - 0.00000469 \cong 1$

Therefore $g' = g$

   (b) Consider a satellite orbiting the Earth in a circular orbit of radius 1600 km above the surface of the Earth. What is the acceleration experienced by the satellite due to Earth's gravitational force?

Solution

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = g\left(1 - \frac{2\times1600\times10^3}{6400\times10^3}\right)$$

$$g' = g\left(1 - \frac{2}{4}\right)$$

$$g' = g\left(1 - \frac{1}{2}\right) = g/2$$

The above two examples show that the acceleration due to gravity is a constant near the surface of the Earth.

Note Can we substitute h = R_e in the equation 6.46? No. To get equation 6.46 we assumed that h << R_e. However h = R_e can be substituted in equation 6.45.

Variation of g with depth:

Consider a particle of mass m which is in a deep mine on the Earth. (Example: coal mines in Neyveli). Assume the depth of the mine as d. To calculate g' at a depth d, consider the following points.

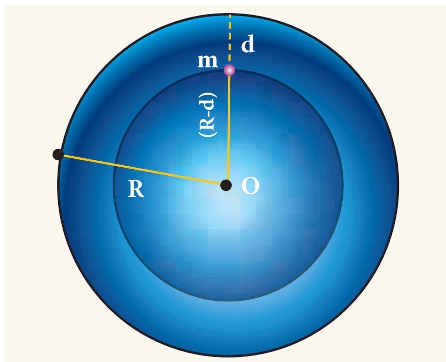


Figure 6.17(b) Particle in a mine

The part of the Earth which is above the radius $(R_e - d)$ do not contribute to the acceleration. The result is proved earlier and is given as

$$g' = \frac{GM'}{(R_e - d)^2} \qquad (6.47)$$

Here M' is the mass of the Earth of radius $(R_e - d)$

Assuming the density of Earth ρ to be constant,

$$\rho = \frac{M}{V} \qquad (6.48)$$

where M is the mass of the Earth and V its volume, Thus,

$$\rho = \frac{M'}{V'}$$

$$\frac{M'}{V'} = \frac{M}{V} \text{ and } M' = \frac{M}{V}V'$$

$$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^3}\right)\left(\frac{4}{3}\pi(R_e - d)^3\right)$$

$$M' = \frac{M}{R_e^3}(R_e - d)^3 \quad (6.49)$$

$$g' = G\frac{M}{R_e^3}(R_e - d)^3 . \frac{1}{(R_e - d)^2}$$

$$g' = GM\frac{R_e\left(1 - \frac{d}{R_e}\right)}{R_e^3}$$

$$g' = GM\frac{\left(1 - \frac{d}{R_e}\right)}{R_e^2}$$

Thus

$$g' = g\left(1 - \frac{d}{R_e}\right) \quad (6.50)$$

Here also $g' < g$. As depth increases, g' decreases. It is very interesting to know that acceleration due to gravity is maximum on the surface of the Earth but decreases when we go either upward or downward.

Variation of g with latitude:

Whenever we analyze the motion of objects in rotating frames [explained in chapter 3] we must take into account the centrifugal force. Even though we treat the Earth as an inertial frame, it is not exactly correct because the Earth spins about its own axis. So when an object is on the surface of the Earth, it experiences a centrifugal force that depends on the latitude of the object on Earth. If the Earth were not spinning, the force on the object would have been mg. However, the object experiences an additional centrifugal force due to spinning of the Earth.

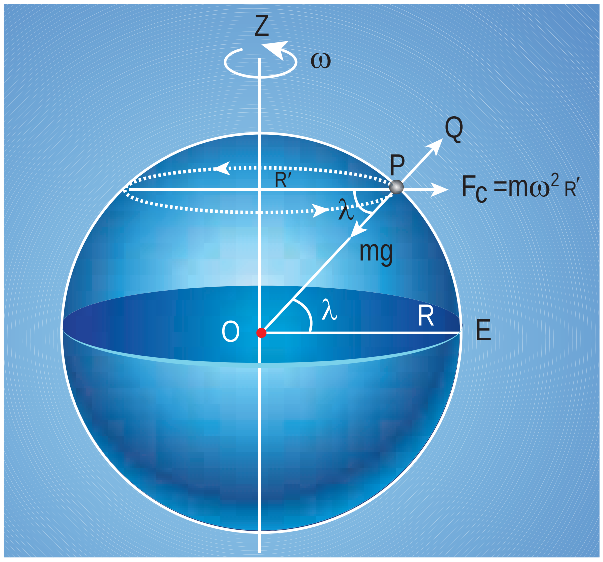


Figure 6.18 Variation of g with latitude

This centrifugal force is given by $m\omega^2 R'$.

$$R' = R\cos\lambda \quad (6.51)$$

where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is

$$a_{PQ} = \omega^2 R' \cos\lambda = \omega^2 R \cos^2\lambda$$

$$since\ R' = R\cos\lambda$$

Therefore,

$$g' = g - \omega^2 R \cos^2 \lambda \qquad (6.52)$$

From the expression (6.52), we can infer that at equator, $\lambda = 0$; $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles $\lambda = 90$; $g' = g$, it is maximum. At the equator, g' is minimum.

EXAMPLE 6.8

Find out the value of g' in your school laboratory?

Solution

Calculate the latitude of the city or village where the school is located. The information is available in Google search. For example, the latitude of Chennai is approximately 13 degree.

$$g' = g - \omega^2 R \cos^2 \lambda$$

Here $\omega^2 R = (2\text{x}3.14/86400)^2 \text{x} (6400\text{x}10^3) = 3.4\text{x}10^{-2}\ \text{m s}^{-2}$.

It is to be noted that the value of λ should be in radian and not in degree. 13 degree is equivalent to 0.2268 rad.

$$g' = 9.8 - \left(3.4 \times 10^{-2}\right) \times \left(\cos 0.2268\right)^2$$

$$g' = 9.7677\ \text{m s}^{-2}$$

Points to Contemplate

Suppose you move towards east-west along the same latitude. Will the value of g' change?

6.4 ESCAPE SPEED AND ORBITAL SPEED

Hydrogen and helium are the most abundant elements in the universe but Earth's atmosphere consists mainly of nitrogen and oxygen. The following discussion brings forth the reason why hydrogen and helium are not found in abundance on the Earth's atmosphere. When an object is thrown up with some initial speed it will reach a certain height after which it will fall back to Earth. If the same object is thrown again with a higher speed, it reaches a greater height than the previous one and falls back to Earth. This leads to the question of what should be the speed of an object thrown vertically up such that it escapes the Earth's gravity and would never come back.

Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i, the initial total energy of the object is

$$E_i = \frac{1}{2} M v_i^2 - \frac{GMM_E}{R_E} \qquad (6.53)$$

where, M_E is the mass of the Earth and R_E- the radius of the Earth. The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero $[U(\infty) = 0]$ and the kinetic energy becomes zero as well. Therefore the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be nonzero.

$$E_f = 0$$

According to the law of energy conservation,

$$E_i = E_f \quad (6.54)$$

Substituting (6.53) in (6.54) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \quad (6.55)$$

Consider the escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace v_i *with* v_e. i.e,

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E}.\frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

Using $g = \frac{GM_E}{R_e^2}$,

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \quad (6.56)$$

From equation (6.56) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object. By substituting the values of g (9.8 $m\,s^{-2}$) and $R_e = 6400\,km$, the escape speed of the Earth is $v_e = 11.2\,km\,s^{-1}$. The escape speed is independent of the direction in which the object is thrown. Irrespective of whether the object is thrown vertically up, radially outwards or tangentially it requires the same initial speed to escape Earth's gravity. It is shown in Figure 6.19

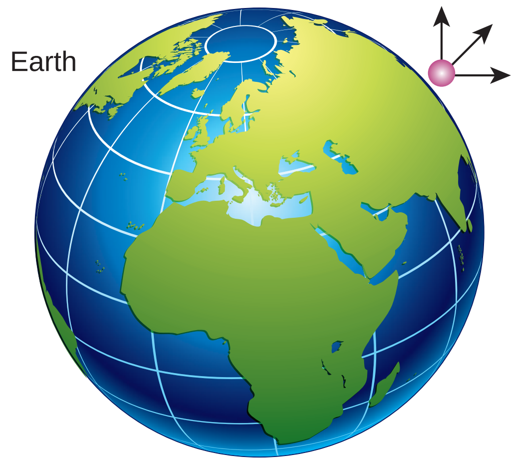


Figure 6.19 Escape speed independent of angle

Lighter molecules such as hydrogen and helium have enough speed to escape from the Earth, unlike the heavier ones such as nitrogen and oxygen. (The average speed of hydrogen and helium atoms compaired with the escape speed of the Earth,is presented in the kinetic theory of gases, unit 9).

6.4.1 Satellites, orbital speed and time period

We are living in a modern world with sophisticated technological gadgets and are able to communicate to any place on Earth. This advancement was made possible because of our understanding of solar system. Communication mainly depends on the satellites that orbit the Earth (Figure 6.20). Satellites revolve around the Earth just like the planets revolve around the Sun. Kepler's laws are applicable to man-made satellites also.

For a satellite of mass M to move in a circular orbit, centripetal force must be acting on the satellite. This centripetal force is provided by the Earth's gravitational force.

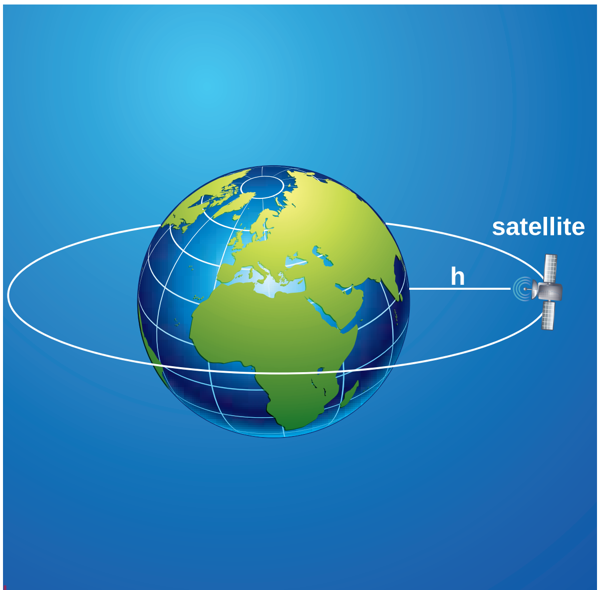


Figure 6.20 Satellite revolving around the Earth.

$$\frac{Mv^2}{(R_E+h)} = \frac{GMM_E}{(R_e+h)^2} \quad (6.57)$$

$$v^2 = \frac{GM_E}{(R_E+h)}$$

$$v = \sqrt{\frac{GM_E}{(R_E+h)}} \quad (6.58)$$

As h increases, the speed of the satellite decreases.

Time period of the satellite:

The distance covered by the satellite during one rotation in its orbit is equal to $2\pi(R_E+h)$ and time taken for it is the time period, T. Then

$$Speed\ v = \frac{Distance\ travelled}{Time\ taken} = \frac{2\pi(R_E+h)}{T}$$

From equation (6.58)

$$\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi(R_E+h)}{T} \quad (6.59)$$

$$T = \frac{2\pi}{\sqrt{GM_E}}(R_E+h)^{3/2} \quad (6.60)$$

Squaring both sides of the equation (6.60), we get

$$T^2 = \frac{4\pi^2}{GM_E}(R_E+h)^3$$

$$\frac{4\pi^2}{GM_E} = constant\ say\ c$$

$$T^2 = c(R_E+h)^3 \quad (6.61)$$

Equation (6.61) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E. Then,

$$T^2 = \frac{4\pi^2}{GM_E}R_E^3$$

$$T^2 = \frac{4\pi^2}{GM_E/R_E^2}R_E$$

$$T^2 = \frac{4\pi^2}{g}R_E$$

$$since\ GM_E/R_E^2 = g$$

$$T = 2\pi\sqrt{\frac{R_E}{g}} \quad (6.62)$$

By substituting the values of $R_E = 6.4 \times 10^6 m$ and g = 9.8 m s^{-2}, the orbital time period is obtained as T $\cong$ 85 minutes.

EXAMPLE 6.9

Moon is the natural satellite of Earth and it takes 27 days to go once around its orbit. Calculate the distance of the Moon from the surface of the Earth assuming the orbit of the Moon as circular.

Solution

We can use Kepler's third law,

$$T^2 = c\left(R_E + h\right)^3$$

$$T^{2/3} = c^{1/3}\left(R_E + h\right)$$

$$\left(\frac{T^2}{c}\right)^{1/3} = \left(R_E + h\right)$$

$$\left(\frac{T^2 GM_E}{4\pi^2}\right)^{\frac{1}{3}} = \left(R_E + h\right);$$

$$c = \frac{4\pi^2}{GM_E}$$

$$h = \left(\frac{T^2 GM_E}{4\pi^2}\right)^{1/3} - R_E$$

Here h is the distance of the Moon from the surface of the Earth. Here,

R_E – radius of the Earth = $6.4 \times 10^6 m$

M_E – mass of the Earth = $6.02 \times 10^{24} kg$

G – Universal gravitational constant = $6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

By substituting these values, the distance to the Moon from the surface of the Earth is calculated to be $3.77 \times 10^5 km$.

6.4.2 Energy of an Orbiting Satellite

The total energy of a satellite orbiting the Earth at a distance h from the surface of Earth is calculated as follows; The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy. The potential energy of the satellite is,

$$U = -\frac{GM_s M_E}{\left(R_E + h\right)} \qquad (6.63)$$

Here M_s - mass of the satellite, M_E - mass of the Earth, R_E - radius of the Earth.

The Kinetic energy of the satellite is

$$K.E = \frac{1}{2} M_s v^2 \qquad (6.64)$$

Here v is the orbital speed of the satellite and is equal to

$$v = \sqrt{\frac{GM_E}{\left(R_E + h\right)}} \qquad (6.65)$$

Substituting the value of v in (6.64), the kinetic energy of the satellite becomes,

$$K.E = \frac{1}{2} \frac{GM_E M_s}{\left(R_E + h\right)}$$

Therefore the total energy of the satellite is

$$E = \frac{1}{2}\frac{GM_E M_s}{(R_E + h)} - \frac{GM_s M_E}{(R_E + h)}$$

$$E = -\frac{GM_s M_E}{2(R_E + h)} \qquad (6.66)$$

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

As h approaches ∞, the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distances.

EXAMPLE 6.10

Calculate the energy of the (i) Moon orbiting the Earth and (ii) Earth orbiting the Sun.

Solution

Assuming the orbit of the Moon to be circular, the energy of Moon is given by,

$$E_m = -\frac{GM_E M_m}{2R_m}$$

where M_E is the mass of Earth 6.02×10^{24} kg; M_m is the mass of Moon 7.35×10^{22} kg; and R_m is the distance between the Moon and the center of the Earth $3.84 \times 10^5\,km$

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}.$$

$$E_m = -\frac{6.67\times10^{-11}\times6.02\times10^{24}\times7.35\times10^{22}}{2\times3.84\times10^5\times10^3}$$

$$E_m = -38.42\times10^{-19}\times10^{46}$$

$$E_m = -38.42\times10^{46}\ Joule$$

The negative energy implies that the Moon is bound to the Earth.

Same method can be used to prove that the energy of the Earth is also negative.

6.4.3 Geo-stationary and polar satellite

The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Can we calculate the orbital radius of a satellite if its time period is 24 hours?

Kepler's third law is used to find the radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$(R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$R_E + h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

Substituting for the time period (24 hrs = 86400 seconds), mass, and radius of the Earth, h turns out to be 36,000 km. Such satellites are called "geo-stationary satellites", since they appear to be stationary when seen from Earth.

India uses the INSAT group of satellites that are basically geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance

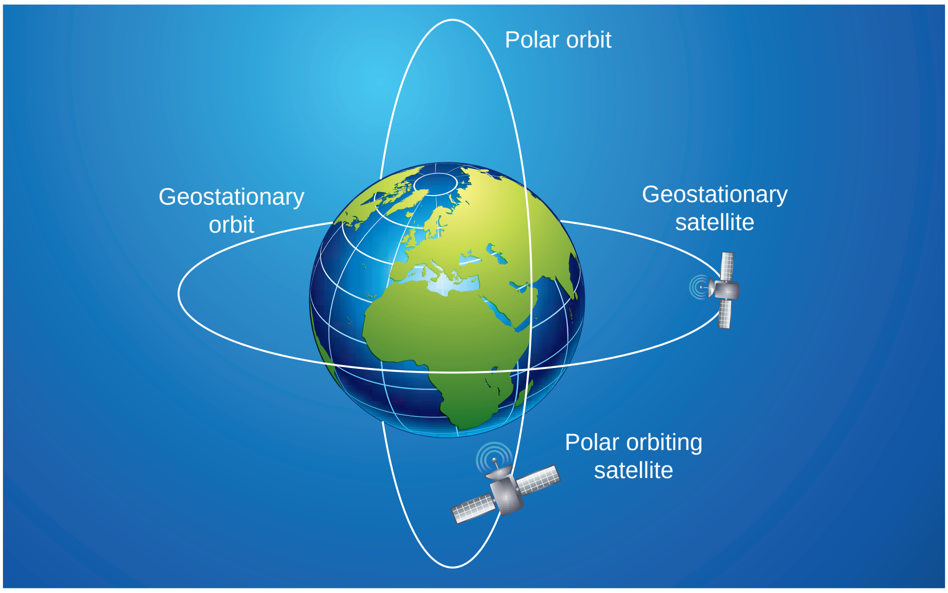


Figure 6.21 Polar orbit and geostationary satellite

of 500 to 800 km from the surface of the Earth orbits the Earth from north to south direction. This type of satellite that orbits Earth from North Pole to South Pole is called a polar satellite. The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day. A Polar satellite covers a small strip of area from pole to pole during one revolution. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

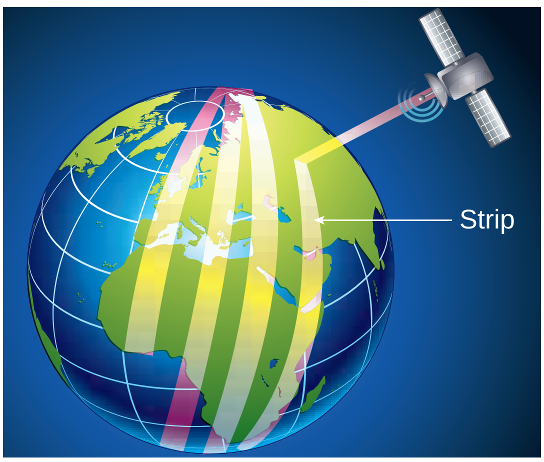


Figure 6.22 Strip of communication region, covered by a polar satellite.

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6.4.4 Weightlessness

Weight of an object

Objects on Earth experience the gravitational force of Earth. The gravitational force acting on an object of mass m is mg. This force always acts downwards towards the center of the Earth. When we stand on the floor, there are two forces acting on us. One is the gravitational force, acting downwards and the other is the normal force exerted by the floor upwards on us to keep us at rest. The weight of an object $\vec{W}$ is defined as the downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth. The direction of weight is in the direction of gravitational force. So the magnitude of weight of an object is denoted as, W=N=mg. Note that even though magnitude of weight is equal to mg, it is not same as gravitational force acting on the object.

Apparent weight in elevators

Everyone who used an elevator would have felt a jerk when the elevator takes off or stops. Why does it happen? Understanding the concept of weight is crucial for explaining this effect. Let us consider a man inside an elevator in the following scenarios.

When a man is standing in the elevator, there are two forces acting on him.

1. Gravitational force which acts downward. If we take the vertical direction as positive y direction, the gravitational force acting on the man is $\vec{F}_G = -mg\hat{j}$
2. The normal force exerted by floor on the man which acts vertically upward, $\vec{N} = N\check{j}$

Case (i) When the elevator is at rest

The acceleration of the man is zero. Therefore the net force acting on the man is zero. With respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = 0$$
$$-mg\hat{j} + N\hat{j} = 0$$

By comparing the components, we can write

$$N - mg = 0 \text{ (or) } N = mg \qquad (6.67)$$

Since weight, W =N, the apparent weight of the man is equal to his actual weight.

Case (ii) When the elevator is moving uniformly in the upward or downward direction

In uniform motion (constant velocity), the net force acting on the man is still zero. Hence, in this case also the apparent weight of the man is equal to his actual weight. It is shown in Figure 6.23(a)

Case (iii) When the elevator is accelerating upwards

If an elevator is moving with upward acceleration $(\vec{a} = a\vec{j})$ with respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = ma\hat{j}$$

By comparing the components,

$$N = m(g + a) \qquad (6.68)$$

Therefore, apparent weight of the man is greater than his actual weight. It is shown in Figure 6.23(b)

Case (iv) When the elevator is accelerating downwards

If the elevator is moving with downward acceleration $(\vec{a} = -a\hat{j})$, by applying Newton's second law on the man, we can write

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = -ma\hat{j}$$

By comparing the components,

$$N = m(g - a) \qquad (6.69)$$

Therefore, apparent weight W = N = m(g-a) of the man is lesser than his actual weight. It is shown in Figure 6.23(c)

Weightlessness of freely falling bodies

Freely falling objects experience only gravitational force. As they fall freely, they are not in contact with any surface (by neglecting air friction). The normal force acting on the object is zero. The downward acceleration is equal to the acceleration due to the gravity of the Earth. i.e (a = g). From equation (6.69) we get.

$$a = g \quad \therefore N = m\,(g - g) = 0.$$

This is called the state of weightlessness. When the lift falls (when the lift wire cuts) with downward acceleration a = g, the person inside the elevator is in the state of weightlessness or free fall. It is shown in Figure 6.23(d)

When the apple was falling from the tree it was weightless.As soon as it hit Newton's head, it gained weight! and Newton gained physics!

Weightlessness in satellites:

There is a wrong notion that the astronauts in satellites experience no gravitational force because they are far away from the Earth. Actually the Earth satellites that orbit very close to Earth experience only gravitational force. The astronauts inside the satellite also experience the same gravitational force. Because of this, they cannot exert any force on the floor of the satellite. Thus, the floor of the satellite also cannot exert any normal force on the astronaut. Therefore, the astronauts inside a satellite are in the state of weightlessness. Not only the astronauts, but all the objects in the satellite will be in the state of weightlessness which is similar to that of a free fall. It is shown in the Figure 6.24.

(a)


(b)


(c)


(d)


Figure 6.23 Apparent weight in the lift

Figure 6.24 The well known scientist Stephen Hawking in the state of weightlessness.
https://www.youtube.com/watch?v=OCsuHvv_D0s

6.5 ELEMENTARY IDEAS OF ASTRONOMY

Astronomy is one of the oldest sciences in the history of mankind. In the olden days, astronomy was an inseparable part of physical science. It contributed a lot to the development of physics in the 16th century. In fact Kepler's laws and Newton's theory of gravitation were formulated and verified using astronomical observations and data accumulated over the centuries by famous astronomers like Hippachrus, Aristachrus, Ptolemy, Copernicus and Tycho Brahe. Without Tycho Brahe's astronomical observations, Kepler's laws would not have emerged. Without Kepler's laws, Newton's theory of gravitation would not have been formulated.

It was mentioned in the beginning of this chapter that Ptolemy's geocentric model was replaced by Copernicus' heliocentric model. It is important to analyze and explain the shortcoming of the geocentric model over heliocentric model.

6.5.1 Heliocentric system over geocentric system

When the motion of the planets are observed in the night sky by naked eyes over a period of a few months, it can be seen that the planets move eastwards and reverse their motion for a while and return to eastward motion again. This is called "retrograde motion" of planets.

Figure 6.25 shows the retrograde motion of the planet Mars. Careful observation for a period of a year clearly shows that Mars initially moves eastwards (February to June), then reverses its path and moves backwards (July, August, September). It changes its direction of motion once again and continues its forward motion (October onwards). In olden days, astronomers recorded the retrograde motion of all

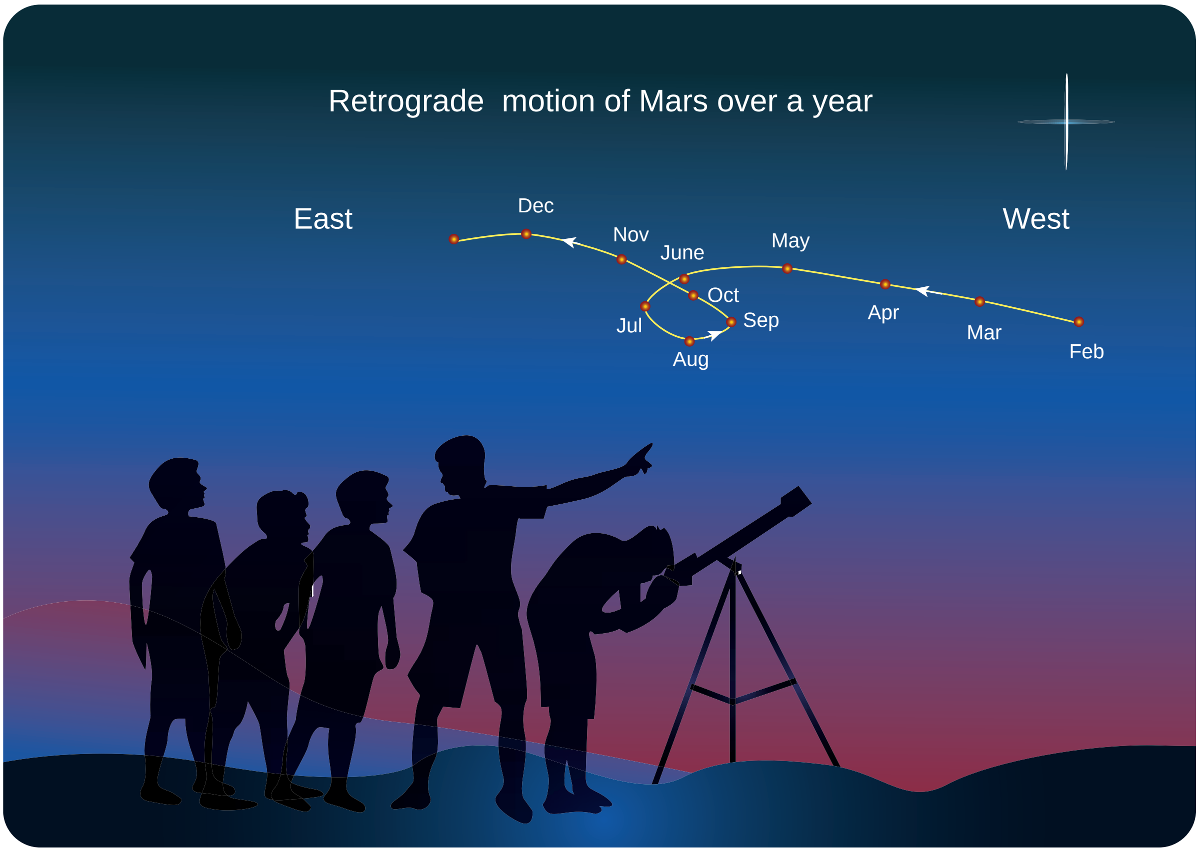


Figure 6.25 Retrograde motion of planets

visible planets and tried to explain the motion. According to Aristotle, the other planets and the Sun move around the Earth in the circular orbits. If it was really a circular orbit it was not known how the planet could reverse its motion for a brief interval. To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle". A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth (Figure 6.26). Essentially Ptolemy retained the Earth centric idea of Aristotle and added the epicycle motion to it.

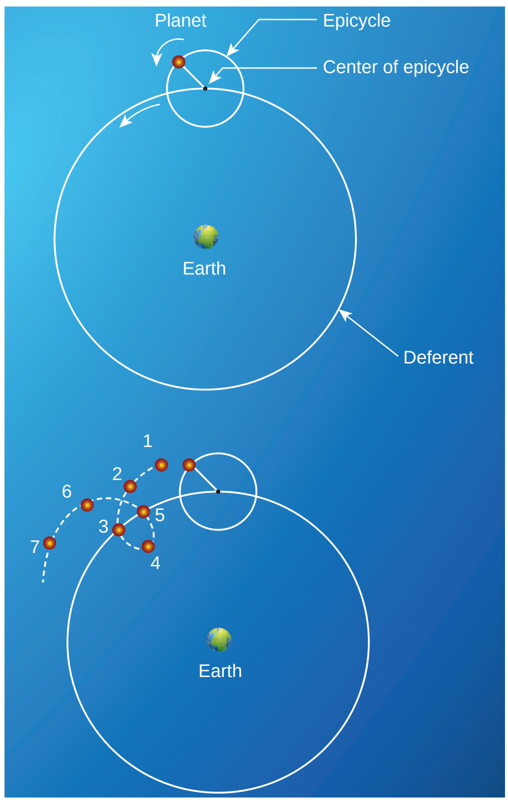


Figure 6.26 "Epicycle" motion of planetary objects around Earth, depicted with respect to months of observation.

But Ptolemy's model became more and more complex as every planet was found to undergo retrograde motion. In the 15th century, the Polish astronomer Copernicus proposed the heliocentric model to explain this problem in a simpler manner. According to this model, the Sun is at the center of the solar system and all planets orbited the Sun. The retrograde motion of planets with respect to Earth is because of the relative motion of the planet with respect to Earth. The retrograde motion from the heliocentric point of view is shown in Figure 6.27.

Figure 6.27 shows that the Earth orbits around the Sun faster than Mars. Because of the relative motion between Mars and Earth, Mars appears to move backwards from July to October. In the same way the retrograde motion of all other planets was explained successfully by the Copernicus model. It was because of its simplicity, the heliocentric model slowly replaced the geocentric model. Historically, if any natural phenomenon has one or more explanations, the simplest one is usually accepted. Though this was not the only reason to disqualify the geocentric model, a detailed discussion on correctness of the Copernicus model over to Ptolemy's model can be found in astronomy books.

BLFQD9

Activity

Students are encouraged to observe the motion of the planet Mars by naked eye and identify its retrograde motion. As mentioned above, to observe the retrograde motion six to seven months are required. So students may start their observation of Mars from the month of June and continue till April next year. Mars is the little bright planet with reddish color. The position of the planet Mars in the sky can be easily taken from 'Google'.

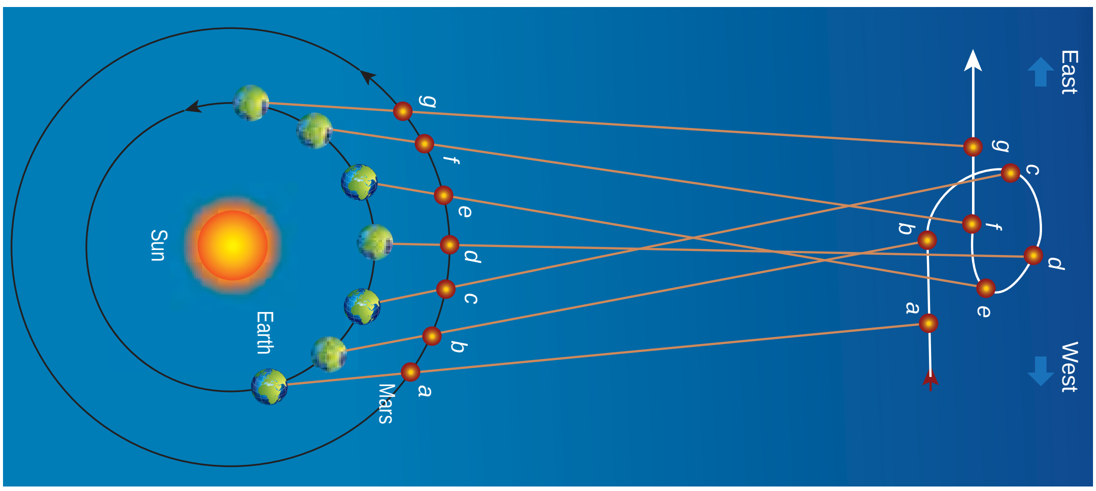


Figure 6.27 'Retrograde motion' in heliocentric model

6.5.2 Kepler's Third Law and The Astronomical Distance

When Kepler derived his three laws, he strongly relied on Tycho Brahe's astronomical observation. In his third law, he formulated the relation between the distance of a planet from the Sun to the time period of revolution of the planet. Astronomers cleverly used geometry and trigonometry to calculate the distance of a planet from the Sun in terms of the distance between Earth and Sun. Here we can see how the distance of Mercury and Venus from the Sun were measured. The Venus and Mercury, being inner planets with respect to Earth, the maximum angular distance they can subtend at a point on Earth with respect to the Sun is 46 degree for Venus and 22.5 degree for Mercury. It is shown in the Figure 6.28

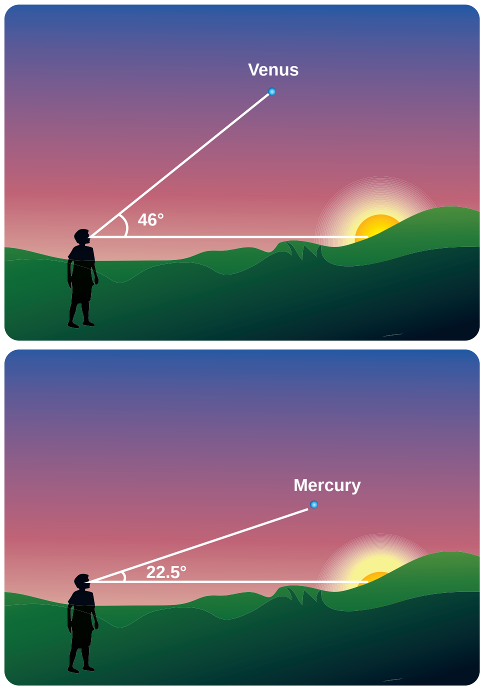


Figure 6.28 Angle of elevation for Venus and Mercury from horizon

Figure 6.29 shows that when Venus is at maximum elongation (i.e., 46 degree) with respect to Earth, Venus makes 90 degree to Sun. This allows us to find the distance between Venus and Sun. The distance between Earth and Sun is taken as one Astronomical unit (1 AU).

The trigonometric relation satisfied by this right angled triangle is shown in Figure 6.29.

$$\sin\theta = \frac{r}{R}$$

where R = 1 AU.

$$r = R\sin\theta = (1AU)(\sin 46°)$$

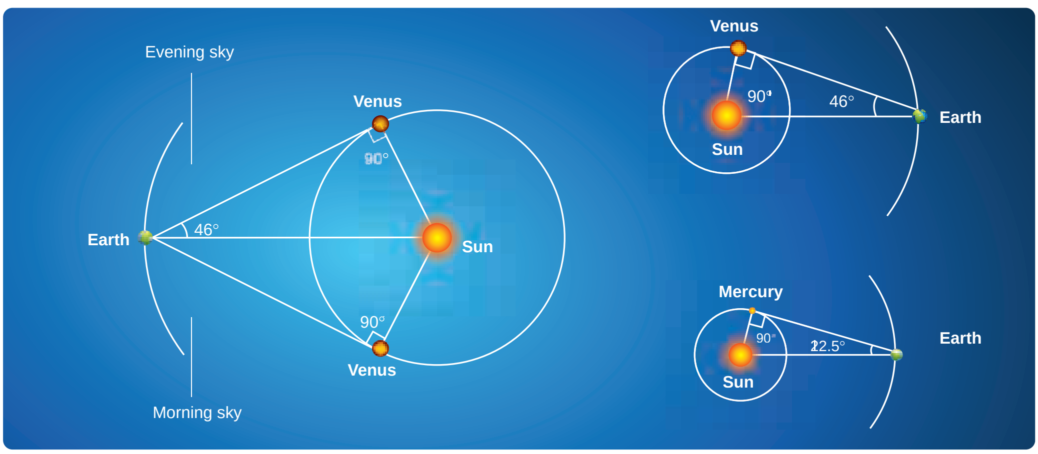


Figure 6.29 Angle of elevation for Mercury from horizon

Here $\sin 46° = 0.77$. Hence Venus is at a distance of 0.77 AU from Sun. Similarly, the distance between Mercury (θ is 22.5 degree) and Sun is calculated as 0.38 AU. To find the distance of exterior planets like Mars and Jupiter, a slightly different method is used. The distances of planets from the Sun is given in the table below.

Table 6.2 a^3/T^2 **for different planets**

Planet	semi major axis of the orbit(a)	Period T (years)	a^3/T^2
Mercury	0.389 AU	87.77	7.64
Venus	0.724 AU	224.70	7.52
Earth	1.000 AU	365.25	7.50
Mars	1.524 AU	686.98	7.50
Jupiter	5.200 AU	4332.62	7.49
Saturn	9.510 AU	10,759.20	7.40

It is to be noted that to verify the Kepler's law we need only high school level geometry and trigonometry.

BLKYQZ

ACTIVITY

Venus can be observed with the naked eye. We can see Venus during sunrise or sunset. Students are encouraged to observe the motion of Venus and verify that the maximum elevation is at 46 degree and calculate the distance of Venus from the Sun. As pointed out already Google or Stellarium will be helpful in locating the position of Venus in the sky.

6.5.3 Measurement of radius of the Earth

Around 225 B.C a Greek librarian "Eratosthenes" who lived at Alexandria measured the radius of the Earth with a small error when compared with results using modern measurements. The technique he used involves lower school geometry and

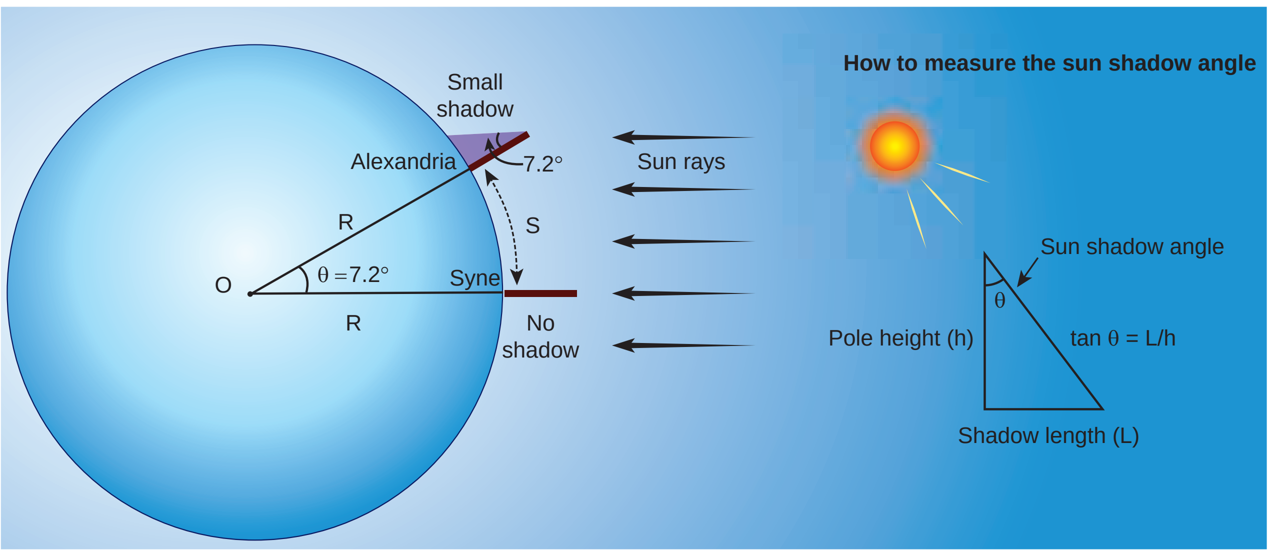


Figure 6.30 Measuring radius of The Earth

brilliant insight. He observed that during noon time of summer solstice the Sun's rays cast no shadow in the city Syne which was located 500 miles away from Alexandria. At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical as shown in the Figure 6.30. He realized that this difference of 7.2 degree was due to the curvature of the Earth.

The angle 7.2 degree is equivalent to $\frac{1}{8}$ *radian*. So $\theta = \frac{1}{8} rad$;

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then

$$S = R\theta = 500\, miles,$$

so radius of the Earth

$$R = \frac{500}{\theta} \text{ miles}$$

$$R = 500 \frac{miles}{\frac{1}{8}}$$

R = 4000 miles

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

The distance of the Moon from Earth was measured by a famous Greek astronomer Hipparchus in the 3rd century BC.

BLPLEH

ACTIVITY

To measure the radius of the Earth, choose two different places (schools) that are separated by at least 500 km. It is important to note that these two places have to be along the same longitude of the Earth (For example Hosur and Kanyakumari lie along the same longitude of 77.82° E). Take poles of known length (h) and fix them vertically in the ground (it may be in the school playgrounds) at both the places. At exactly noon in both the places the length of the shadow (L) cast by each pole has to be noted down. Draw the picture like in Figure 6.30. By using the equation $\tan\theta = \frac{L}{h}$, the angle in radian can be found at each place. The difference in angle (θ') is due to the curvature of the Earth. Now the distance between the two schools can be obtained from 'Google maps'. Divide the distance with the angle (θ' in radians) which will give the radius of the Earth.

6.5.4 Interesting Astronomical Facts

1. **Lunar eclipse and measurement of shadow of Earth**
   On January 31, 2018 there was a total lunar eclipse which was observed from various places including Tamil Nadu. It is possible to measure the radius of shadow of the Earth at the point where the Moon crosses. Figure 6.31 illustrates this.

   When the Moon is inside the umbra shadow, it appears red in color. As soon as the Moon exits from the umbra

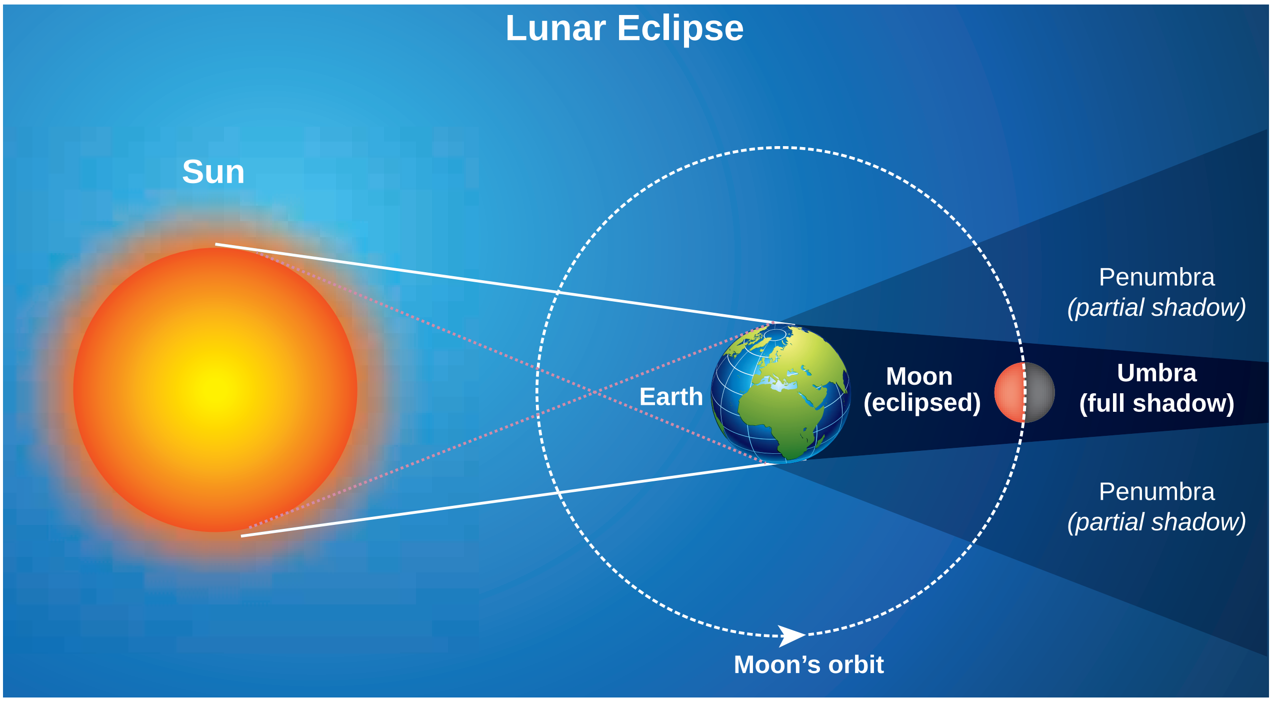


Figure 6.31 Total lunar eclipse

shadow, it appears in crescent shape. Figure 6.32 is the photograph taken by digital camera during Moon's exit from the umbra shadow.

Figure 6.32 Image of the Moon when it exits from umbra shadow

By finding the apparent radii of the Earth's umbra shadow and the Moon, the ratio of the these radii can be calculated. This is shown in Figures 6.33 and 6.34.

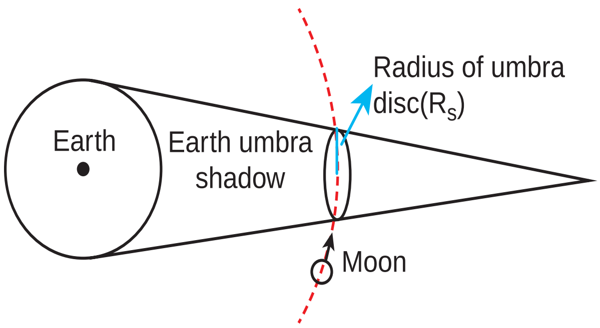


Figure 6.33 Schematic diagram of umbra disk radius

The apparent radius of Earth's umbra shadow = R_s = 13.2 cm

The apparent radius of the Moon = R_m = 5.15 cm

The ratio $\frac{R_s}{R_m} \approx 2.56$

The radius of the Earth's umbra shadow is $R_s = 2.56 \times R_m$

The radius of Moon $R_m = 1737\,km$

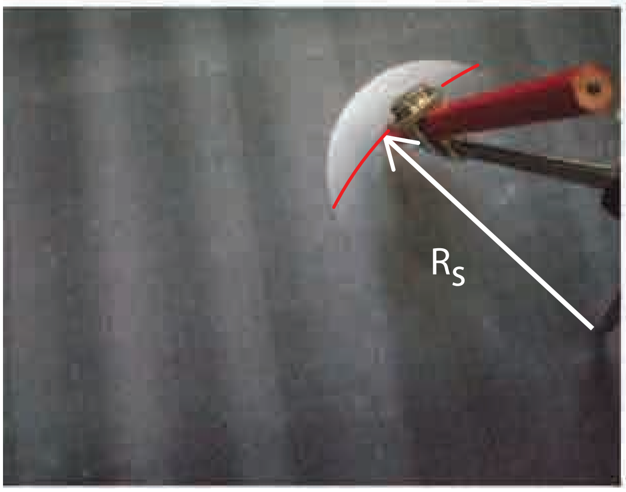


Figure 6.34 Calculation of umbra radius

The radius of the Earth's umbra shadow is $R_s = 2.56 \times 1737 km \cong 4446 km$.

The correct radius is 4610 km.

The percentage of error in the calculation $= \frac{4610 - 4446}{4610} \times 100 = 3.5\%$.

The error will reduce if the pictures taken using a high quality telescope are used. It is to be noted that this calculation is done using very simple mathematics.

Early astronomers proved that Earth is spherical in shape by looking at the shape of the shadow cast by Earth on the Moon during lunar eclipse.

2. **Why there are no lunar eclipse and solar eclipse every month?**
If the orbits of the Moon and Earth lie on the same plane, during full Moon of every month, we can observe lunar eclipse. If this is so during new Moon we can observe solar eclipse. But Moon's orbit is tilted 5° with respect to Earth's orbit. Due to this 5° tilt, only during certain periods of the year, the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment. This is shown in Figure 6.35

3. **Why do we have seasons on Earth?**
The common misconception is that 'Earth revolves around the Sun, so when the Earth is very far away, it is winter and when the Earth is nearer, it is summer'. Actually, the seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. This is shown in Figure 6.36

Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

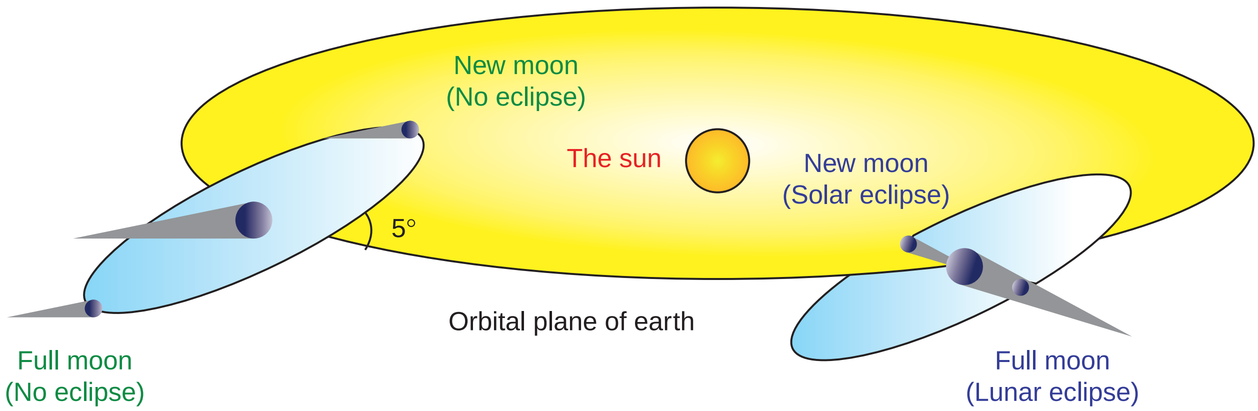


Figure 6.35 Orbital tilt of the Moon

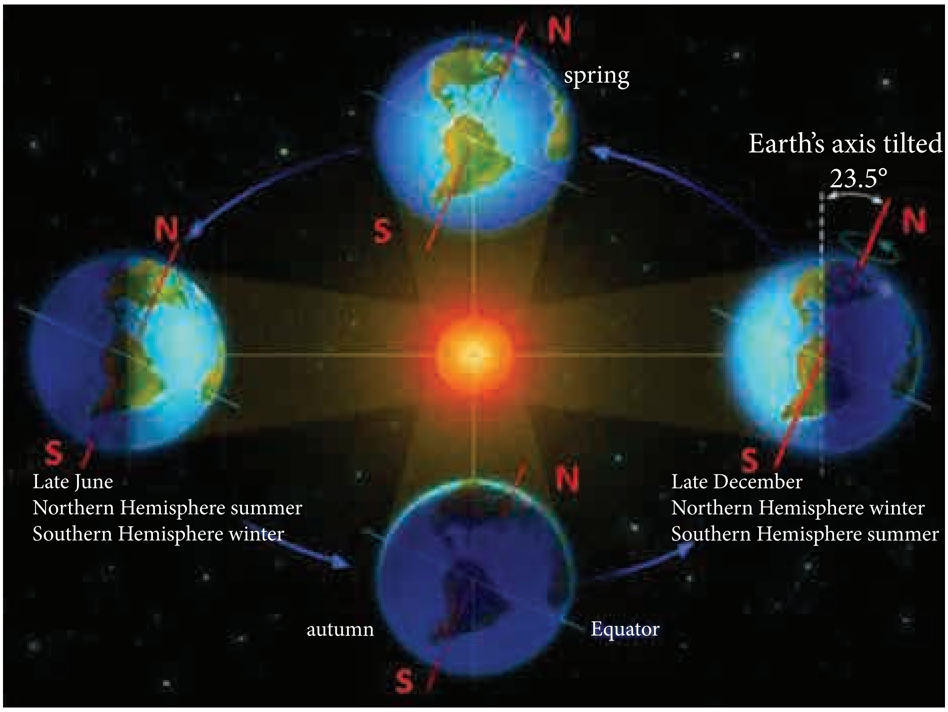


Figure 6.36 Seasons on Earth

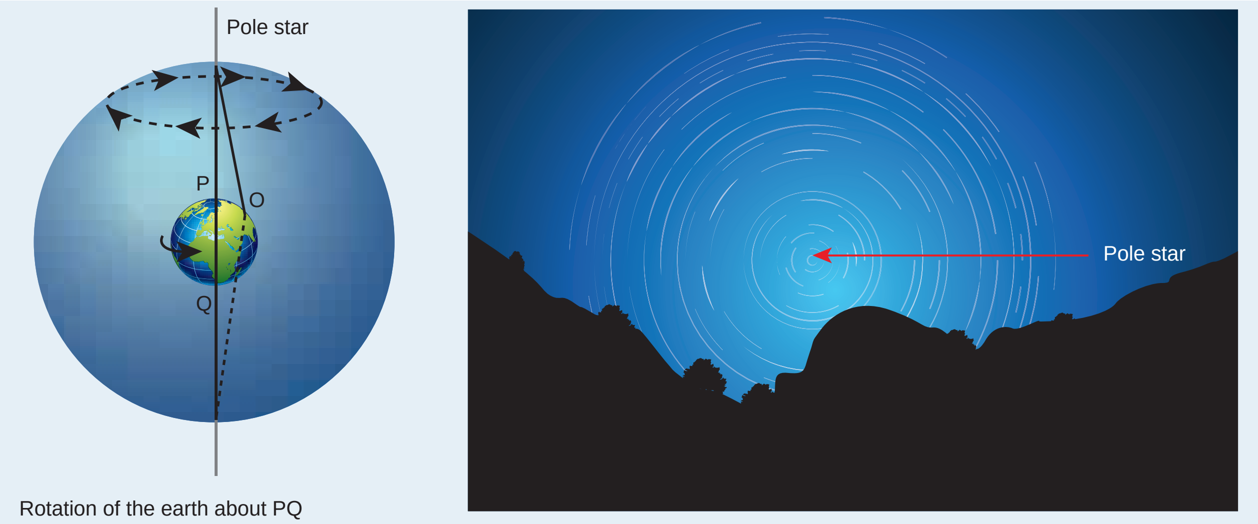
Note Pole star is a star located exactly above the Earth's axis of rotation, hence it appears to be stationary. The Star Polaris is our pole star.

4. **Star's apparent motion and spinning of the Earth**

The Earth's spinning motion can be proved by observing star's position over a night. Due to Earth's spinning motion, the stars in sky appear to move in circular motion about the pole star as shown in Figure 6.37

Point to ponder

Using Sun rays and shadows, How will you prove that the Earth's tilt is 23.5° ?



Rotation of the earth about PQ explains the apparent rotation of stars (in opposite direction) about PQ.

If we observe the stars, it slowly changes its position and appears to move in a circle over the night.

Figure 6.37 Star's apparent circular motion due to Earth's rotation.

6.5.5 Recent developments of astronomy and gravitation

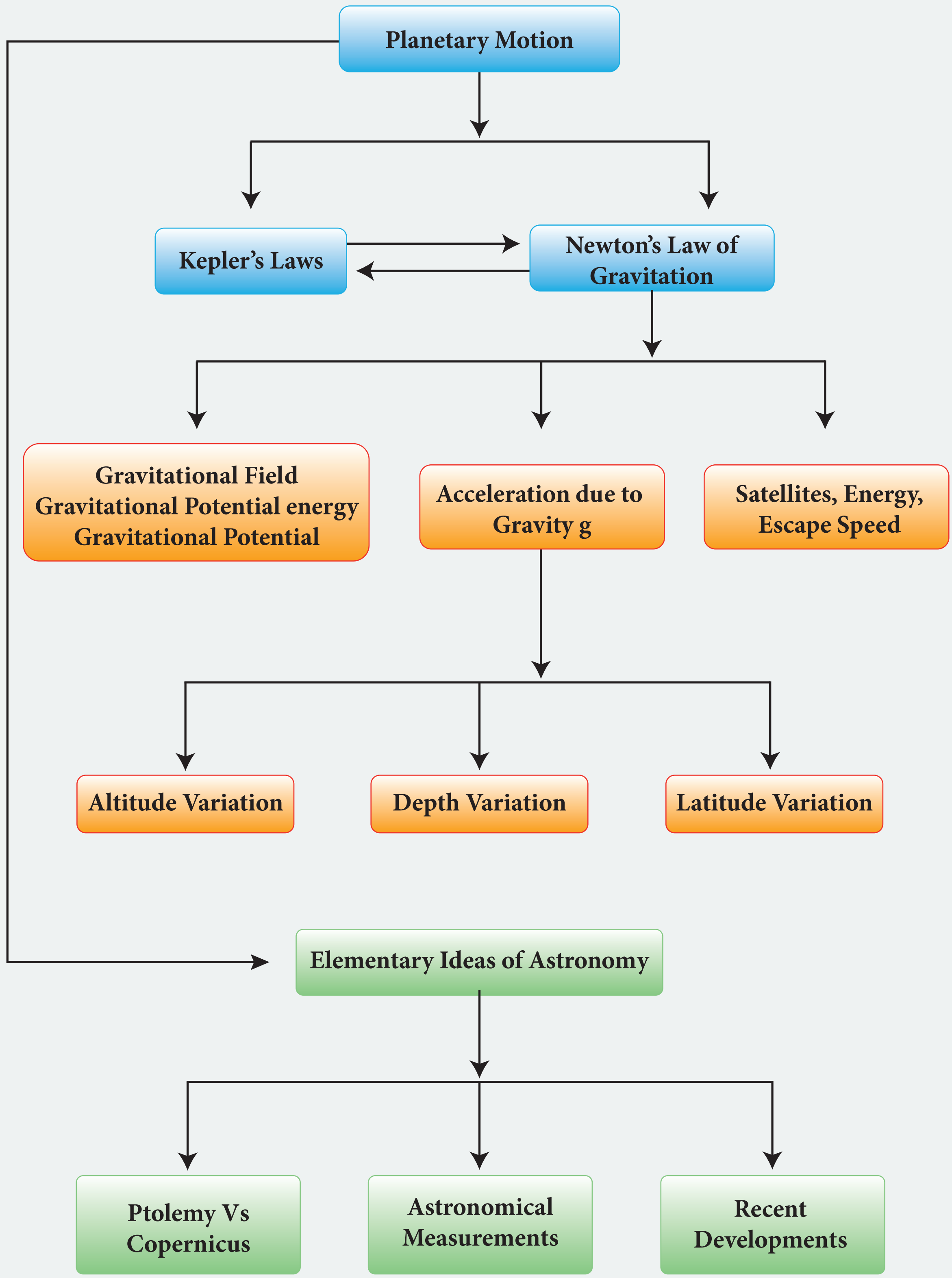
Till the 19th century astronomy mainly depended upon either observation with the naked eye or telescopic observation. After the discovery of the electromagnetic spectrum at the end of the 19th century, our understanding of the universe increased enormously. Because of this development in the late 19th century it was found that Newton's law of gravitation could not explain certain phenomena and showed some discrepancies. Albert Einstein formulated his 'General theory of relativity' which was one of the most successful theories of 20th century in the field of gravitation.

In the twentieth century both astronomy and gravitation merged together and have grown in manifold. The birth and death of stars were more clearly understood. Many Indian physicists made very important contributions to the field of astrophysics and gravitation. Subramanian Chandrasekar formulated the theory of black holes and explained the life of stars. These studies brought him the Nobel prize in the year 1983. Another very notable Indian astrophysicist Meghnad Saha discovered the ionization formula which was useful in classifying stars. This formula is now known as "Saha ionization formula". In the field of gravitation Amal Kumar Raychaudhuri solved an equation now known as "Raychaudhuri equation" which was a very important contribution. Another notable Indian Astrophysicist Jayant V Narlikar made pioneering contribution in the field of astrophysics and has written interesting books on astronomy and astrophysics. IUCAA (Inter University Center for Astronomy and Astrophysics) is one of the important Indian research institutes where active research in astrophysics and gravitation are conducted. The institute was founded by Prof. J.V. Narlikar. Students are encouraged to read more about the recent developments in these fields.

SUMMARY

- The motion of planets can be explained using Kepler's laws.
- **Kepler's first law**: All the planets in the solar system orbit the Sun in elliptical orbits with the Sun at one of the foci.
- **Kepler's second law:** The radial vector line joining the Sun to a planet sweeps equal areas in equal intervals of time.
- **Kepler's third law:** The ratio of the square of the time period of planet to the cubic power of semi major axis is constant for all the planets in the solar system.
- **Newton's law of gravitation** states that the gravitational force between two masses is directly proportional to product of masses and inversely proportional to square of the distance between the masses. In vector form it is given by $\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$
- Gravitational force is a central force.
- Kepler's laws can be derived from Newton's law of gravitation.
- The gravitational field due to a mass m at a point which is at a distance r from mass m is given by $\vec{E} = -\frac{Gm}{r^2}\hat{r}$. It is a vector quantity.
- The gravitational potential energy of two masses is given by $U = -\frac{Gm_1m_2}{r}$. It is a scalar quantity.
- The gravitational potential at a point which is at a distance r from mass m is given by $V = -\frac{Gm}{r}$. It is a scalar quantity.
- The acceleration due to Earth's gravity decreases as altitude increases and as depth increases.
- Due to rotation of the Earth, the acceleration due to gravity is maximum at poles and minimum at Earth's equator.
- The (escape) speed of any object required to escape from the Earth's gravitational field is $v_e = \sqrt{2gR_e}$. It is independent of mass of the object.
- The energy of the satellite is negative. It implies that the satellite is bound to Earth's gravitational force.
- Copernicus model explained that retrograde motion is due to relative motion between planets. This explanation is simpler than Ptolemy's epicycle explanation which is complicated
- Copernicus and Kepler measured the distance between a planet and the Sun using simple geometry and trigonometry.
- 2400 years ago, Eratosthenes measured the radius of the Earth using simple geometry and trigonometry.

CONCEPT MAP


Planetary Motion
Kepler's Laws
Newton's Law of Gravitation
Gravitational Field
Gravitational Potential energy
Gravitational Potential
Acceleration due to Gravity g
Satellites, Energy, Escape Speed
Altitude Variation
Depth Variation
Latitude Variation
Elementary Ideas of Astronomy
Ptolemy Vs Copernicus
Astronomical Measurements
Recent Developments

EVALUATION

I. Multiple Choice Questions

1. The linear momentum and position vector of the planet is perpendicular to each other at
   (a) perihelion and aphelion
   (b) at all points
   (c) only at perihelion
   (d) no point

2. If the masses of the Earth and Sun suddenly double, the gravitational force between them will
   (a) remain the same
   (b) increase 2 times
   (c) increase 4 times
   (d) decrease 2 times

3. A planet moving along an elliptical orbit is closest to the Sun at distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are linear speeds at these points respectively. Then the ratio $\frac{v_1}{v_2}$ is

   (NEET 2016)

   (a) $\frac{r_2}{r_1}$ (b) $\left(\frac{r_2}{r_1}\right)^2$

   (c) $\frac{r_1}{r_2}$ (d) $\left(\frac{r_1}{r_2}\right)^2$

4. The time period of a satellite orbiting Earth in a cirular orbit is independent of .
   (a) Radius of the orbit
   (b) The mass of the satellite
   (c) Both the mass and radius of the orbit
   (d) Neither the mass nor the radius of its orbit

5. If the distance between the Earth and Sun were to be doubled from its present value, the number of days in a year would be
   (a) 64.5 (b) 1032
   (c) 182.5 (d) 730

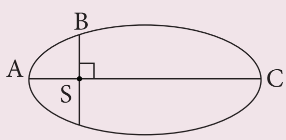
6. According to Kepler's second law, the radial vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of
   (a) conservation of linear momentum
   (b) conservation of angular momentum
   (c) conservation of energy
   (d) conservation of kinetic energy

7. The gravitational potential energy of the Moon with respect to Earth is
   (a) always positive
   (b) always negative
   (c) can be positive or negative
   (d) always zero

8. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then

   (NEET 2018)

   (a) $K_A > K_B > K_C$
   (b) $K_B < K_A < K_C$
   (c) $K_A < K_B < K_C$
   (d) $K_B > K_A > K_C$



9. The work done by the Sun's gravitational force on the Earth is
   (a) always zero
   (b) always positive
   (c) can be positive or negative
   (d) always negative

10. If the mass and radius of the Earth are both doubled, then the acceleration due to gravity g'
   (a) remains same (b) $\frac{g}{2}$
   (c) 2 g (d) 4 g

11. The magnitude of the Sun's gravitational field as experienced by Earth is
   (a) same over the year
   (b) decreases in the month of January and increases in the month of July
   (c) decreases in the month of July and increases in the month of January
   (d) increases during day time and decreases during night time.

12. If a person moves from Chennai to Trichy, his weight
   (a) increases
   (b) decreases
   (c) remains same
   (d) increases and then decreases

BLUUSH

13. An object of mass 10 kg is hanging on a spring scale which is attached to the roof of a lift. If the lift is in free fall, the reading in the spring scale is
   (a) 98 N (b) zero
   (c) 49 N (d) 9.8 N

14. If the acceleration due to gravity becomes 4 times its original value, then escape speed
   (a) remains same
   (b) 2 times of original value
   (c) becomes halved
   (d) 4 times of original value

15. The kinetic energy of the satellite orbiting around the Earth is
   (a) equal to potential energy
   (b) less than potential energy
   (c) greater than kinetic energy
   (e) zero

Answers

1) a	**2)** c	**3)** a	**4)** b	**5)** b
6) b	**7)** b	**8)** a	**9)** c	**10)** b
11) c	**12)** a	**13)** b	**14)** b	**15)** b

II. Short Answer Questions

1. State Kepler's three laws.
2. State Newton's Universal law of gravitation.
3. Will the angular momentum of a planet be conserved? Justify your answer.
4. Define the gravitational field. Give its unit.
5. What is meant by superposition of gravitational field?
6. Define gravitational potential energy.
7. Is potential energy the property of a single object? Justify.
8. Define gravitational potential.
9. What is the difference between gravitational potential and gravitational potential energy?
10. What is meant by escape speed in the case of the Earth?
11. Why is the energy of a satellite (or any other planet) negative?
12. What are geostationary and polar satellites?
13. Define weight

14. Why is there no lunar eclipse and solar eclipse every month?
15. How will you prove that Earth itself is spinning?

III. Long Answer Questions

1. Discuss the important features of the law of gravitation.
2. Explain how Newton arrived at his law of gravitation from Kepler's third law.
3. Explain how Newton verified his law of gravitation.
4. Derive the expression for gravitational potential energy.
5. Prove that at points near the surface of the Earth, the gravitational potential energy of the object is $U = mgh$
6. Explain in detail the idea of weightlessness using lift as an example.
7. Derive an expression for escape speed.
8. Explain the variation of g with lattitude.
9. Explain the variation of g with altitude.
10. Explain the variation of g with depth from the Earth's surface.
11. Derive the time period of satellite orbiting the Earth.
12. Derive an expression for energy of satellite.
13. Explain in detail the geostationary and polar satellites.
14. Explain how geocentric theory is replaced by heliocentric theory using the idea of retrograde motion of planets.
15. Explain in detail the Eratosthenes method of finding the radius of Earth.
16. Describe the measurement of Earth's shadow (umbra) radius during total lunar eclipse

IV. Conceptual Questions

1. In the following, what are the quantities which that are conserved?
   a) Linear momentum of planet
   b) Angular momentum of planet
   c) Total energy of planet
   d) Potential energy of a planet
2. The work done by Sun on Earth in one year will be
   a) Zero
   b) Non zero
   c) positive
   d) negative
3. The work done by Sun on Earth at any finite interval of time is
   a) positive, negative or zero
   b) Strictly positive
   c) Strictly negative
   d) It is always zero
4. If a comet suddenly hits the Moon and imparts energy which is more than the total energy of the Moon, what will happen?
5. If the Earth's pull on the Moon suddenly disappears, what will happen to the Moon?
6. If the Earth has no tilt, what happens to the seasons of the Earth?
7. A student was asked a question 'why are there summer and winter for us? He replied as 'since Earth is orbiting in an elliptical orbit, when the Earth is very far away from the Sun(aphelion) there will be winter, when the Earth is nearer to the Sun(perihelion) there will be winter'. Is this answer correct? If not,

what is the correct explanation for the occurrence of summer and winter?

8. The following photographs are taken from the recent lunar eclipse which occurred on January 31, 2018. Is it possible to prove that Earth is a sphere from these photographs?

V. Numerical Problems

1. An unknown planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is T_1, what is the time period of this unknown planet?

Ans: $T_2 = 2\sqrt{2}T_1$

2. Assume that you are in another solar system and provided with the set of data given below consisting of the planets' semi major axes and time periods. Can you infer the relation connecting semi major axis and time period?

Planet (imaginary)	Time period(T) (in year)	Semi major axis (a) (in AU)
Kurinji	2	8
Mullai	3	18
Marutham	4	32
Neithal	5	50
Paalai	6	72

Ans: $a \propto 2T^2$

3. If the masses and mutual distance between the two objects are doubled, what is the change in the gravitational force between them?

Ans: No change

4. Two bodies of masses m and 4m are placed at a distance r. Calculate the gravitational potential at a point on the line joining them where the gravitional field is zero.

Ans: $V = -\frac{9Gm}{r}$

5. If the ratio of the orbital distance of two planets $\frac{d_1}{d_2}=2$, what is the ratio of gravitational field experienced by these two planets?

Ans: $E_2 = 4\ E_1$

6. The Moon Io orbits Jupiter once in 1.769 days. The orbital radius of the Moon Io is 421700 km. Calculate the mass of Jupiter?

Ans: 1.898×10^{27} kg

7. If the angular momentum of a planet is given by $\vec{L} = 5t^2\hat{i} - 6t\hat{j} + 3\hat{k}$. What is the torque experienced by the planet? Will the torque be in the same direction as that of the angular momentum?

Ans: $\vec{\tau} = 10t\hat{i} - 6\hat{j}$

8. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. Calculate the speed of each particle

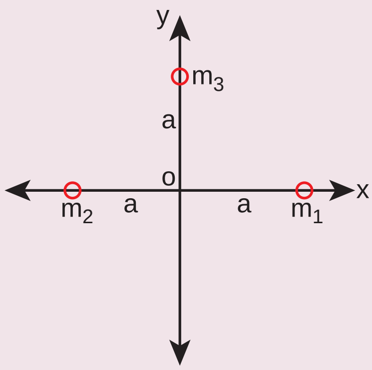
Ans: $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

9. Suppose unknowingly you wrote the universal gravitational constant value

as $G = 6.67 \times 10^{11}$ instead of the correct value $G = 6.67 \times 10^{-11}$, what is the acceleration due to gravity g' for this incorrect G? According to this new acceleration due to gravity, what will be your weight W'?

Ans: $g' = 10^{22} g$, $W' = 10^{22} W$

10. Calculate the gravitational field at point O due to three masses m_1,m_2 and m_3 whose positions are given by the following figure. If the masses m_1 and m_2 are equal what is the change in gravitational field at the point O?



Ans: $\vec{E} = \frac{GM}{a^2}\left[(m_1 - m_2)\hat{i} + m_3\hat{j}\right]$

if $m_1 = m_2$, $\vec{E} = \frac{GM}{a^2}\left[m_3\hat{j}\right]$

11. What is the gravitational potential energy of the Earth and Sun? The Earth to Sun distance is around 150 million km. The mass of the Earth is 5.9×10^{24} kg and mass of the Sun is 1.9×10^{30} kg.

Ans: V= -49.84×10^{32} Joule

12. Earth revolves around the Sun at 30 km s^{-1}. Calculate the kinetic energy of the Earth. In the previous example you calculated the potential energy of the Earth. What is the total energy of the Earth in that case? Is the total energy positive? Give reasons.

Ans: K.E = 26.5×10^{32} J

E = -23.29×10^{32} J

(−) ve implies that Earth is bounded with Sun

13. An object is thrown from Earth in such a way that it reaches a point at infinity with non-zero kinetic energy $\left[K.E(r=\infty) = \frac{1}{2}Mv_\infty^{\,2}\right]$, with what velocity should the object be thrown from Earth?

Ans: $v_e = \sqrt{v_\infty^2 + 2gR_E}$

14. Suppose we go 200 km above and below the surface of the Earth, what are the g values at these two points? In which case, is the value of g small?

Ans: $g_{down} = 0.96$ g

$g_{up} = 0.94$ g

15. Calculate the change in g value in your district of Tamil nadu. (Hint: Get the latitude of your district of Tamil nadu from the Google). What is the difference in g values at Chennai and Kanyakumari?

Ans: $g_{chennai} = 9.767$ m s^{-2}

$g_{Kanyakumari} = 9.798$ m s^{-2}

$\Delta g = 0.031$ m s^{-2}

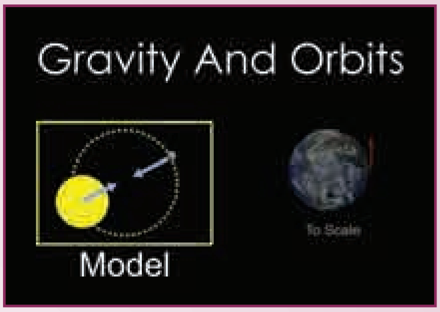
BOOKS FOR REFERENCE

1. Mechanics by Charles Kittel, Walter Knight, Malvin Ruderman, Carl Helmholtz and Moyer
2. Newtonian Mechanics by A.P. French
3. Introduction to Mechanics by Daniel Kepler and Robert Kolenkow
4. Mechanics by Somnath Datta
5. Concepts of Physics volume 1 and Volume 2 by H.C. Verma
6. Physics for Scientist and Engineers with Modern physics by Serway and Jewett
7. Physics for Scientist and Engineers by Paul Tipler and Gene Mosca
8. Physics for the Inquiring Mind by Eric Rogers
9. Fundamental laws of Mechanics by Irodov.
10. Question and Problems in School Physics by Tarasov and Tarasova

ICT CORNER

Gravitation

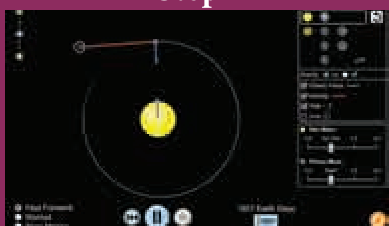
Through this activity you will be able to learn about the gravitational force and orbital paths.



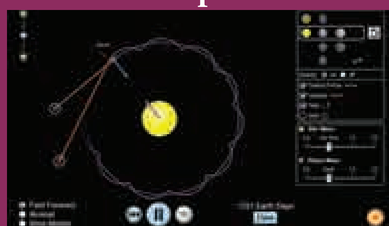
STEPS:

- Click the URL or scan the QR code to launch the activity page. Click on the box labelled "Model" to start the activity.
- In the activity window, a diagram of sun and earth is given. Click the play icon to see the motion of earth.
- We can change the objects by selecting objects from the table given in the right side window.
- The path of the gravity, velocity and the object in motion can be viewed. Check on the relevant boxes given in the table.

Step1

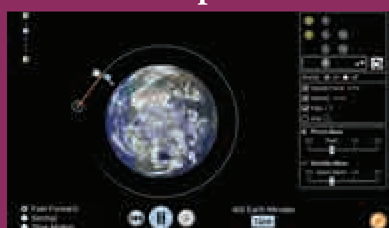


Step2



Step3

Step4



URL:

https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

UNIT 7

PROPERTIES OF MATTER

Many of the greatest advances that have been made from the beginning of the world to the present time have been made in the earnest desire to turn the knowledge of the properties of matter to some purpose useful to mankind– Lord Kelvin

LEARNING OBJECTIVES

In this unit, the student is exposed to

- inter atomic or intermolecular forces in matter
- stress, strain and elastic modulus
- surface tension
- viscosity
- properties of fluids and their applications

BM4QUA

7.1 INTRODUCTION

One of the oldest dams in the world is Kallanai (கல்லணை) located at Trichy. Kallanai was built across river Kaveri for the purpose of irrigation. During heavy floods, the velocity of water is generally very high in river Kaveri. The stability and utility of Kallanai dam reveal the intuitive scientific understanding of Tamils who designed this dam as early as 2nd century AD. The other example known for insightful constructions of the past is the pyramids in Egypt. The flyovers and over bridges are common worldwide today. Heavy vehicles ply over and hence, the bridges are always under stress. Without effective design using suitable materials, the bridges and flyovers will not


Kallanai (கல்லணை)

be stable. The growth of human civilization is due to the understanding of various forms of matter (solid, liquid, and gas).

The study of properties of matter is very essential in selecting any material for a specific application. For example, in technology, the materials used for space applications should be of lightweight but should be strong. Materials used for artificial human organ replacements should be biocompatible. Artificial body fluids are used as tissue substitute for radiotherapy analysis in medicine. Fluids used as lubricants or fuel should possess certain properties. Such salient macroscopic properties are decided by the microscopic phenomena within matter. This unit deals with the properties of solids and fluids and the laws governing the behaviour of matter.

7.2 MICROSCOPIC UNDERSTANDING OF VARIOUS STATES OF MATTER

Even though various forms of matter such as solid food, liquids like water, and the air that we breathe are familiar in the day – to – day lifestyle for the past several thousand years, the microscopic understanding of solids, liquids, and gases was established only in the 20th century. In the universe, everything is made up of atoms. If so, why the same materials exist in three states? For example, water exists in three forms as solid ice, liquid water, and gaseous steam. Interestingly ice, water, and steam are made up of same types of molecules; two hydrogen atoms and one oxygen atom form a water molecule. Physics helps us to explore this beauty of nature at the microscopic level. The distance between the atoms or molecules determines whether it exist in the solid, liquid or gaseous state.

Solids

In solids, atoms or molecules are tightly fixed. In the solid formation, atoms get bound together through various types of bonding. Due to the interaction between the atoms, they position themselves at a particular interatomic distance. This position of atoms in this bound condition is called their mean positions.

Liquids

When the solid is not given any external energy such as heat, it will remain as a solid due to the bonding between atoms. When heated, atoms of the solid receive thermal energy and vibrate about their mean positions. When the solid is heated above its melting point, the heat energy will break the bonding between atoms and eventually the atoms receive enough energy and wander around. Here also the intermolecular (or interatomic) forces are important, but the molecules will have enough energy to move around, which makes the structure mobile.

Gases

When a liquid is heated at constant pressure to its boiling point or when the pressure is reduced at a constant temperature it will convert to a gas. This process of a liquid changing to a gas is called evaporation. The gas molecules have either very weak bonds or no bonds at all. This enables them to move freely and quickly. Hence, the gas will conform to the shape of its container and also will expand to fill the container. The transition from solid to liquid to gaseous states with the variation in external energy is schematically shown in Figure 7.1.

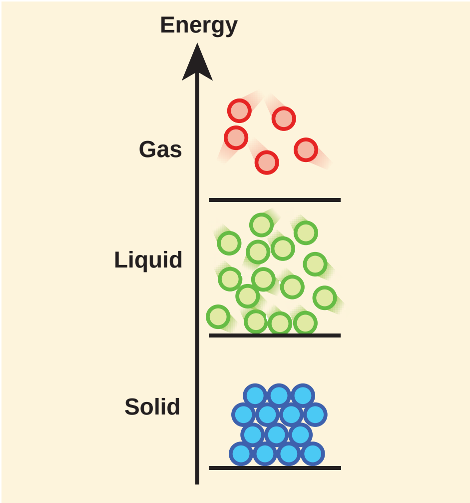
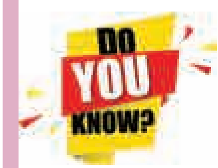


Figure 7.1 Schematic representations of the transition from solid to liquid to gaseous states with a change in external energy



In addition to the three physical states of matter (solid, liquid, and gas), in extreme environments, matter can exist in other states such as plasma, Bose-Einstein condensates. Additional states, such as quark-gluon plasmas are also believed to be possible. A major part of the atomic matter of the universe is hot plasma in the form of rarefied interstellar medium and dense stars.

In the study of Newtonian mechanics (Volume 1), we assumed the objects to be either as point masses or perfect rigid bodies (collection of point masses). Both these are idealized models. In rigid bodies, changes in the shape of the bodies are so small that they are neglected. In real materials, when a force is applied on the objects, there could be some deformations due to the applied force. It is very important to know how materials behave when a deforming force is applied.

7.2.1 Elastic behaviour of materials

In a solid, interatomic forces bind two or more atoms together and the atoms occupy the positions of stable equilibrium. When a deforming force is applied on a body, its atoms are pulled apart or pushed closer. When the deforming force is removed, interatomic forces of attraction or repulsion restore the atoms to their equilibrium positions. If a body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity. The force which changes the size or shape of a body is called a deforming force.

Examples: Rubber, metals, steel ropes.

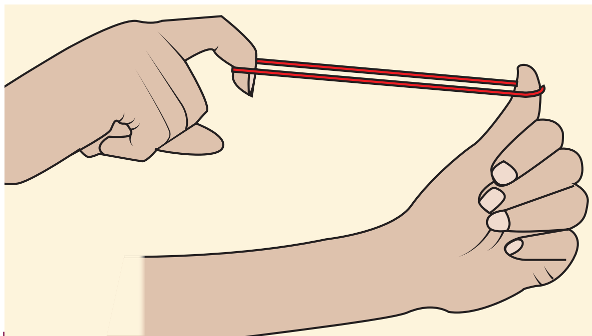


Figure 7.2 Elasticity

Plasticity:

If a body does not regain its original shape and size after removal of the deforming force, it is said to be a plastic body and the property is called plasticity.

Example: Glass

7.2.2 Stress and strain

(a) Stress:

When a force is applied, the size or shape or both may change due to the change in relative positions of atoms or molecules. This deformation may not be noticeable to our naked eyes but it exists in the material itself. When a body is subjected to such a deforming force, internal force is developed in it, called as restoring force. The force per unit area is called as stress.

$$\text{Stress, } \sigma = \frac{Force}{Area} = \frac{F}{A} \quad (7.1)$$

The SI unit of stress is N m^{-2} or pascal (Pa) and its dimension is [ML^{-1}T^{-2}]. Stress is a tensor.

(i) Longitudinal stress and shearing stress:

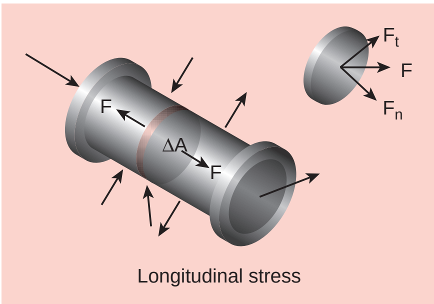


Figure 7.3 Longitudinal stress

Let us consider a body as shown in Figure 7.3. When many forces act on the system (body), the center of mass (defined in unit 5) remains at rest. However, the body gets deformed due to these forces and so the internal forces appear. Let ΔA be the cross sectional area of the body. The parts of the body on the two sides of ΔA exert internal forces $\vec{F}$ and $-\vec{F}$ on each other which is due to deformation. The force can be resolved in two components, F_n normal to the surface ΔA (perpendicular to the surface) and F_t tangential to the surface ΔA (tangent to the surface). The normal stress or longitudinal stress (σ_n) over the area is defined as

$$\sigma_n = \frac{F_n}{\Delta A}$$

Similarly, the tangential stress or shearing stress σ_t over the area is defined as

$$\sigma_t = \frac{F_t}{\Delta A}$$

Longitudinal stress can be classified into two types, tensile stress and compressive stress.

Tensile stress

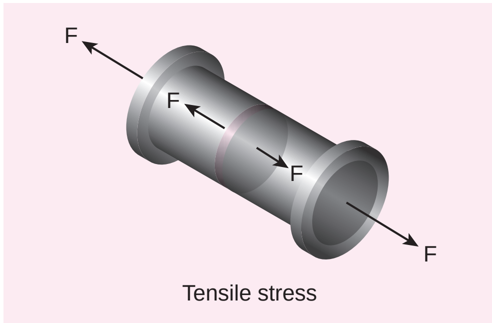


Figure 7.4 Tensile stress

Internal forces on the two sides of ΔA may pull each other, i.e., it is stretched by equal and opposite forces. Then, the longitudinal stress is called tensile stress.

Compressive stress

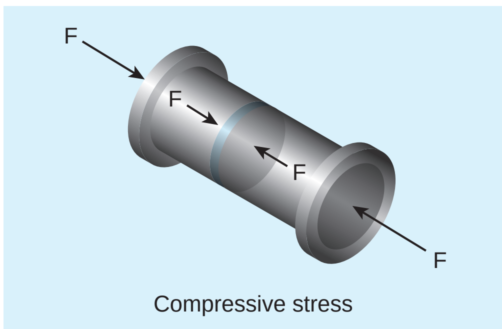


Figure 7.5 Compressive stress

When forces acting on the two sides of ΔA push each other, ΔA is pushed by equal and opposite forces at the two ends. In this case, ΔA is said to be under compression. Then, the longitudinal stress is called compressive stress.

(ii) Volume stress

This happens when a body is acted by forces everywhere on the surface such that the force at any point is normal to the surface and the magnitude of the force on a small surface area is proportional to the area. For instance, when a solid is immersed in a fluid, the pressure at the location of the solid is P, the force on any area ΔA is

$$F = P\,\Delta A$$

Where, F is perpendicular to the area. Thus, force per unit area is called volume stress.

$$\sigma_v = \frac{F}{A}$$

which is the same as the pressure.

(b) Strain:

Strain measures how much an object is stretched or deformed when a force is applied. Strain deals with the fractional change in the size of the object, in other words, strain measures the degree of deformation. As an example, in one dimension, consider a rod of length l when it stretches to a new length Δl then

$$\text{Strain, } \varepsilon = \frac{\textit{change in size}}{\textit{original size}} = \frac{\Delta l}{l} \quad (7.2)$$

ε is a dimensionless quantity and has no unit. Strain is classified into three types.

(1) Longitudinal strain

When a rod of length l is pulled by equal and opposite forces, the longitudinal strain is defined as

$$\varepsilon_l = \frac{\textit{increase in length of the rod}}{\textit{original or natural length of the rod}} = \frac{\Delta l}{l} \quad (7.3)$$

Longitudinal strain can be classified into two types

(i) Tensile strain: If the length is increased from its natural length then it is known as tensile strain.

(ii) Compressive strain: If the length is decreased from its natural length then it is known as compressive strain.

(2) Shearing strain

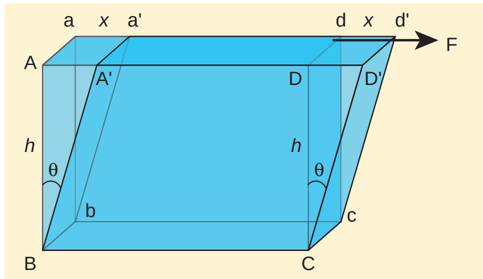


Figure 7.6 Shearing strain

Consider a cuboid as shown in Figure 7.6. Let us assume that the body remains in translational and rotational equilibrium. Let us apply the tangential force F along AD such that the cuboid deforms as shown in Figure 7.6. Hence, shearing strain or shear is (ε_s)

$$\varepsilon_s = \frac{AA'}{BA} = \frac{x}{h} = \tan\theta \quad (7.4)$$

For small angle, $\tan\theta \approx \theta$

Therefore, shearing strain or shear,

$$\varepsilon_s = \frac{x}{h} = \theta = \text{Angle of shear}$$

(3) Volume strain

If the body is subjected to a volume stress, the volume will change. Let V be the original volume of the body before stress and $V + \Delta V$ be the change in volume due to stress. The volume strain which measures the fractional change in volume is

$$\text{Volume strain, } \varepsilon_v = \frac{\Delta V}{V} \quad (7.5)$$

Elastic Limit

The maximum stress within which the body regains its original size and shape after the removal of deforming force is called the elastic limit.

If the deforming force exceeds the elastic limit, the body acquires a permanent deformation. For example, rubber band loses its elasticity if pulled apart too much. It changes its size and becomes misfit to be used again.

7.2.3 Hooke's law and its experimental verification

Hooke's law is for a small deformation, when the stress and strain are proportional to each other. It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed point O. A pan and a pointer are attached at the free end of the wire as shown in Figure 7.7 (a). The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL. It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis. This graph is a straight line passing through the origin as shown in Figure 7.7 (b).

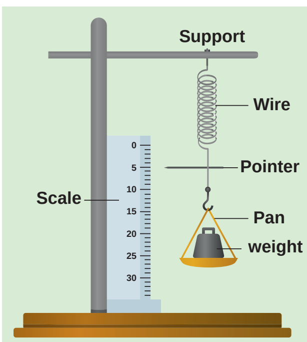


Figure 7.7 (a) Experimental verification of Hooke's law

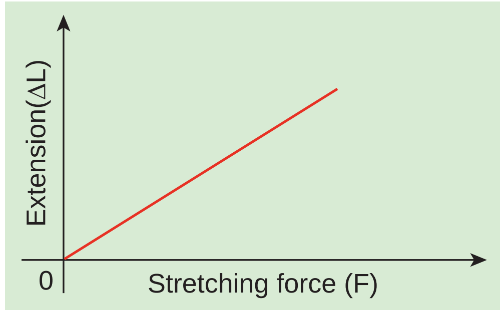


Figure 7.7 (b) Variation of ΔL with F

Therefore,

$$\Delta L = (\text{slope})F$$

Multiplying and dividing by volume,

$$V = A\,L,$$

$$F\,(\text{slope}) = \frac{AL}{AL}\Delta L$$

Rearranging, we get

$$\frac{F}{A} = \left(\frac{L}{A(slope)}\right)\frac{\Delta L}{L}$$

Therefore, $$\frac{F}{A} \propto \left(\frac{\Delta L}{L}\right)$$

Comparing with equation (7.1) and equation (7.2), we get equation (7.5) as

$$\sigma \propto \varepsilon$$

i.e., the stress is proportional to the strain in the elastic limit.

Stress – Strain profile curve:

The stress versus strain profile is a plot in which stress and strain are noted for each load and a graph is drawn taking strain along the X-axis and stress along the Y-axis. The elastic characteristics of the materials can be analyzed from the stress-strain profile.

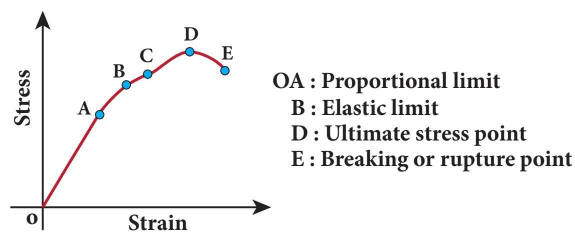


Figure 7.8 Stress-Strain profile

(a) Portion OA:

In this region, stress is very small such that stress is proportional to strain, which means Hooke's law is valid. The point A is called *limit of proportionality* because above this point Hooke's law is not valid. The slope of the line OA gives the Young's modulus of the wire.

(b) Portion AB:

This region is reached if the stress is increased by a very small amount. In this region, stress is not proportional to the strain. But once the stretching force is removed, the wire will regain its original length. This behaviour ends at point B and hence, the point B is known as *yield point* (elastic limit). The elastic behaviour of the material (here wire) in stress-strain curve is OAB.

(c) Portion BC:

If the wire is stretched beyond the point B (elastic limit), stress increases and the wire will not regain its original length after the removal of stretching force.

(d) Portion CD:

With further increase in stress (beyond the point C), the strain increases rapidly and reaches the point D. Beyond D, the strain increases even when the load is removed and breaks (ruptures) at the point E. Therefore, the maximum stress (here D) beyond which the wire breaks is called *breaking stress* or *tensile strength*. The corresponding point D is known as *fracture point*. The region BCDE represents the plastic behaviour of the material of the wire.

7.2.4 Moduli of elasticity

From Hooke's law, the stress in a body is proportional to the corresponding strain, provided the deformation is very small. In this section, we shall define the elastic modulus of a given material. There are three types of elastic modulus.

(a) Young's modulus

(b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.

Young modulus of a material

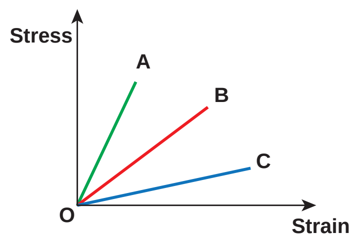
$$= \frac{\textit{Tensile stress or compressive stress}}{\textit{Tensile strain or compressive strain}}$$

$$Y = \frac{\sigma_t}{\varepsilon_t} \quad \text{or} \quad Y = \frac{\sigma_c}{\varepsilon_c} \qquad (7.6)$$

The unit for Young modulus has the same unit of stress because, strain has no unit. So, S.I. unit of Young modulus is N m^{-2} or pascal.

EXAMPLE 7.1

Within the elastic limit, the stretching strain produced in wires A, B, and C due to stress is shown in the figure. Assume the load applied are the same and discuss the elastic property of the material.



Write down the elastic modulus in ascending order.

Solution

Here, the elastic modulus is Young modulus and due to stretching, stress is tensile stress and strain is tensile strain.

Within the elastic limit, stress is proportional to strain (obey Hooke's law). Therefore, it shows a straight line behavior. So, the modulus of elasticity (here, Young modulus) can be computed by taking slope from this straight line. Hence, calculating the slope for the straight line, we get

Slope of A > Slope of B > Slope of C

Which implies,

Young modulus of C < Young modulus of B < Young modulus of A

Notice that larger the slope, lesser the strain (fractional change in length). So, the material is much stiffer. Hence, the elasticity of wire A is greater than wire B which is greater than C. From this example, we have understood that Young's modulus measures the resistance of solid to a change in its length.

EXAMPLE 7.2

A wire 10 m long has a cross-sectional area 1.25 x 10^{-4} m^2. It is subjected to a load of 5 kg. If Young's modulus of the material is 4 x 10^{10} N m^{-2}, calculate the elongation produced in the wire.

Take g = 10 ms^{-2}.

Solution

We know that $\frac{F}{A} = Y \times \frac{\Delta L}{L}$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

$$= \left(\frac{50}{1.25\times10^{-4}}\right)\left(\frac{10}{4\times10^{10}}\right) = 10^{-4}\,\text{m}$$

Bulk modulus:

Bulk modulus is defined as the ratio of volume stress to the volume strain.

Bulk modulus, K =

$$\frac{Normal\,(Perpendicular)\,stress\,or\,Pressure}{Volume\,strain}$$

The normal stress or pressure is

$$\sigma_n = \frac{F_n}{\Delta A} = \Delta p$$

The volume strain is

$$\varepsilon_v = \frac{\Delta V}{V}$$

Therefore, Bulk modulus is

$$K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta P}{\frac{\Delta V}{V}} \qquad (7.7)$$

The negative sign in the equation (7.7) means that when pressure is applied on the body, its volume decreases. Further, the equation (7.7) implies that a material can be easily compressed if it has a small value of bulk modulus. In other words, bulk modulus measures the resistance of solids to change in their volume. For an example, we know that gases can be easily compressed than solids, which means, gas has a small value of bulk modulus compared to solids. The S.I. unit of K is the same as that of pressure i.e., N m^{-2} or Pa (pascal).

Compressibility

The reciprocal of the bulk modulus is called compressibility. It is defined as the fractional change in volume per unit increase in pressure.

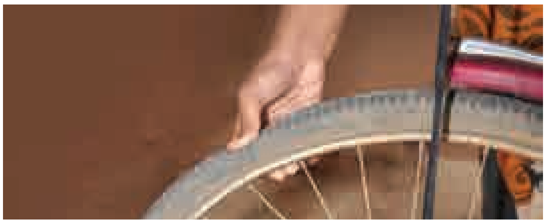
From the equation (7.7) we can say that the compressibility

$$C = \frac{1}{K} = -\frac{\varepsilon_v}{\sigma_n} = -\frac{\frac{\Delta V}{V}}{\Delta P} \qquad (7.8)$$

Since, gases have small value of bulk modulus than solids, their, values of compressibility is very high.

DO YOU KNOW?

After pumping the air in the cycle tyre, usually we press the cycle tyre to check whether it has enough air. What is checked here is essentially the compressibility of air. The tyre should be less compressible for its easy rolling



In fact the rear tyre is less compressible than front tyre for a smooth ride

EXAMPLE 7.3

A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 10^6 pascal. If the volume changes by 1.5×10^{-5} m^3, calculate the bulk modulus of the material.

Solution

By definition, $$K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\frac{V}{\Delta V}$$

$$K = \frac{10^6 \times 1}{1.5 \times 10^{-5}} = 6.67 \times 10^{10}\, N\, m^{-2}$$

The rigidity modulus or shear modulus: The rigidity modulus is defined as

Rigidity modulus or Shear modulus,

$$\eta_R = \frac{\textit{shearing stress}}{\textit{angle of shear or shearing strain}}$$

The shearing stress is

$$\sigma_s = \frac{\text{tangential force}}{\textit{area over which it is applied}} = \frac{F_t}{\Delta A}$$

The angle of shear or shearing strain

$$\varepsilon_s = \frac{x}{h} = \theta$$

Therefore, Rigidity modulus is

$$\eta_R = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{\frac{F_t}{\Delta A}}{\theta} \quad (7.9)$$

Further, the equation (7.9) implies, that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire, when it is twisted through an angle θ, a restoring torque is developed, that is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since, rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small. The S.I. unit of η_R is the same as that of pressure i.e., N m^{-2} or pascal. For the best understanding, the elastic coefficients of some of the important materials are listed in Table 7.1.

Table 7.1 Elastic coefficients of some of the materials in N m^{-2}

Material	**Young's modulus (Y) (10^{10}N m^{-2})**	**Bulk modulus (K) (10^{10} N m^{-2})**	**Shear modulus (η_R) (10^{10} N m^{-2})**
Steel	20.0	15.8	8.0
Aluminium	7.0	7.0	2.5
Copper	12.0	12.0	4.0
Iron	19.0	8.0	5.0
Glass	7.0	3.6	3.0

EXAMPLE 7.4

A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20 m, F = 4000 N, x = 0.50 cm = 0.005 m and Area A = L^2 = 0.04 m^2

Therefore,Shear modulus

$$\eta_R = \frac{F}{A} \times \frac{L}{x} = \frac{4000}{0.04} \times \frac{0.20}{0.005} = 4 \times 10^6 \, N\, m^{-2}$$

7.2.5 Poisson's ratio

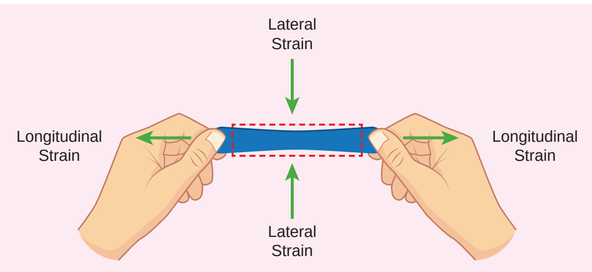


Figure 7.9 Lateral strain verses longitudinal strain

Suppose we stretch a wire, its length increases (elongation) but its diameter decreases (contraction). Similarly, when we stretch a rubber band (elongation), it becomes noticeably thinner (contraction). That is, deformation of the material in one direction produces deformation in another direction. To quantify this, French Physicist S.D. Poisson proposed a ratio, known as Poisson's ratio, which is defined as the ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

$$poisson's\, ratio, \mu = \frac{lateral\, strain}{longitudinal\, strain} \quad (7.10)$$

For a wire of length L with diameter D, due to applied force, wire stretches and hence, increase in length be l and decrease in diameter be d, then

$$\text{Poisson's ratio, } \mu = -\frac{\frac{d}{D}}{\frac{l}{L}} = -\frac{L}{l} \times \frac{d}{D}$$

Negative sign indicates the elongation along longitudinal and contraction along lateral dimension. Further, notice that it is the ratio between quantities of the same dimension. So, Poisson's ratio has no unit and no dimension (dimensionless number). The Poisson's ratio values of some of the materials are listed in Table 7.2.

Table 7.2 The Poisson's ratio of some of the materials

Material	Poisson's ratio
Rubber	0.4999
Gold	0.42 -0.44
Copper	0.33
Stainless steel	0.30-0.31
Steel	0.27-0.30
Cast iron	0.21-0.26
Concrete	0.1-0.2
Glass	0.18-0.3
Foam	0.10-0.50
Cork	0.0

7.2.6 Elastic energy

When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy. Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension *l* and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by d*l*, dW = F d*l*

The total work done in stretching the wire from 0 to *l* is

$$W = \int_0^l F\,dl \qquad (7.11)$$

From Young's modulus of elasticity,

$$Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L} \qquad (7.12)$$

Substituting equation (7.12) in equation (7.11), we get

$$W = \int_0^l \frac{YAl}{L} dl$$

Since, l *is* the dummy variable in the integration, we can change l *to* l'(not in limits), therefore

$$W = \int_0^l \frac{YAl'}{L} dl' = \frac{YA}{L}\left(\frac{l'^2}{2}\right)\Bigg|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right)l = \frac{1}{2}Fl$$

$$W = \frac{1}{2}Fl = Elastic\ potential\ energy$$

Energy per unit volume is called energy density, $u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2}Fl}{AL}$

$$\frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}\ (\text{Stress} \times \text{Strain}) \qquad (7.13)$$

EXAMPLE 7.5

A wire of length 2 m with the area of cross-section $10^{-6} m^2$ is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: $Y = 12 \times 10^{10} N\,m^{-2}$.

Solution

(i) stress $= \frac{F}{A} = \frac{980}{10^{-6}} = 98 \times 10^7\ N\,m^{-2}$

(ii) $strain = \frac{stress}{Y} = \frac{98 \times 10^7}{12 \times 10^{10}} = 8.17 \times 10^{-3}$ (no unit)

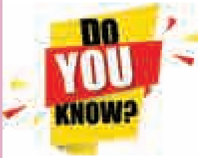
(iii) Since, volume = $2 \times 10^{-6} m^3$

$$Energy = \frac{1}{2}(stress \times strain) \times volume \Rightarrow$$

$$\frac{1}{2}(98 \times 10^7) \times (8.17 \times 10^{-3}) \times 2 \times 10^{-6} = 8\,joule$$

7.2.7 Applications of elasticity

The mechanical properties of materials play a very vital role in everyday life. The elastic behavior is one such property which especially decides the structural design of the columns and beams of a building. As far as the structural engineering is concerned, the amount of stress that the design could withstand is a primary safety factor. A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight. The elastic behavior or in other words the bending of beams is a major concern over the stability of the buildings or bridges. For an example, to reduce the bending of a beam for a given load, one should use the material with a higher Young's modulus of elasticity (Y). It is obvious from Table 7.1 that the Young's modulus of steel is greater than aluminium or copper. Iron comes next to steel. This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.



There is common misconception that rubber is more elastic. Which one is more elastic? Rubber or steel? Steel is more elastic than rubber. If an equal stress is applied to both steel and rubber, the steel produces less strain. So the Young's modulus is higher for steel than rubber. The object which has higher young's modulus is more elastic.

7.3 FLUIDS

7.3.1 Introduction

Fluids are found everywhere in the world. Earth has about two-thirds of water and one-third of land. Fluids are different from solids. Unlike solid, fluid has no defined shape of its own. As far as fluid is concerned, liquid has fixed volume whereas gas fills the entire volume of the container.

Pressure of a fluid:

Fluid is a substance which begins to flow when an external force is applied on it. It offers a very small resistance to the applied force. If the force acts on a smaller area, then the impact will be more and vice versa. This particular idea decides yet another quantity called '*pressure*'. Assume that an object is submerged in a fluid (say water) at rest. In this case, the fluid will exert a force on the surface of the object. This force is always normal to the surface of the object. If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.

$$P = \frac{F}{A} \qquad (7.14)$$

Pressure is a scalar quantity. It's S.I. unit and dimensions are $N\,m^{-2}$ or pascal (Pa) and $[ML^{-1}T^{-2}]$, respectively. Another common unit of pressure is atmosphere, which is abbreviated as 'atm'. It is defined as the pressure exerted by the atmosphere at sea level. i.e., 1 atm = 1.013×10^5 Pa or $N\,m^{-2}$. Apart from pressure, there are two more parameters such as density and relative density (or specific gravity) which are used to describe the nature of fluids.

Density of a fluid:

The density of a fluid is defined as its mass per unit volume. For a fluid of mass 'm' occupying volume 'V', the density $\rho = \frac{m}{V}$. The dimensions and S.I unit of ρ are $[ML^{-3}]$ and $kg\,m^{-3}$, respectively. It is a positive scalar quantity. Mostly, a liquid is largely incompressible and hence its density is nearly constant at ambient pressure (i.e. at 1 atm. pressure). In the case of gases, there are variations in densities with reference to pressure.

Relative density or specific gravity:

The relative density of a substance is defined as the ratio of the density of a substance to the density of water at 4 °C. It is a dimensionless positive scalar quantity. For example, the density of mercury is $13.6 \times 10^3 kg\,m^{-3}$. Its relative density is equal to $\frac{13.6 \times 10^3 kg\,m^{-3}}{1.0 \times 10^3 kg\,m^{-3}} = 13.6$.

EXAMPLE 7.6

A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

Solution

Radius of the sphere R = 1.5 cm

mass m = 0.038 kg

Volume of the sphere $V = \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times (3.14) \times (1.5\times10^{-2})^3 = 1.413 \times 10^{-5} m^3$

Therefore, density

$$\rho = \frac{m}{V} = \frac{0.038 kg}{1.413\times10^{-5} m^3} = 2690\, kg\, m^{-3}$$

Hence, the specific gravity of the sphere

$$= \frac{2690}{1000} = 2.69$$

7.3.2 Pressure due to fluid column at rest

A mountaineer climbing the mountain is able to experience a decrease in pressure with altitude. A person jumping into the swimming pool always realizes an increase in pressure with depth below the water surface. In both the cases, the pressure encountered by the mountaineer and diver is usually due to the hydrostatic pressure, because they are due to fluids that are static. In order to understand the increase in pressure with depth below the water surface, consider a water sample of cross sectional area in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder, respectively as shown in Figure 7.10(a). Let F_1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2, such that, $F_1 = P_1 A$ and $F_2 = P_2 A$ Let us assume the mass of the sample to be *m* and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force (F_1 + mg), in other words, the gravitational force will act downward which is being exactly balanced by the difference between the force $F_2 - F_1$

$$F_2 - F_1 = mg = F_G \qquad (7.15)$$

where *m* is the mass of the water available in the sample element. Let ρ be the density of the water then, the mass of water available in the sample element is

$$m = \rho V = \rho A (h_2 - h_1)$$
$$V = A (h_2 - h_1)$$

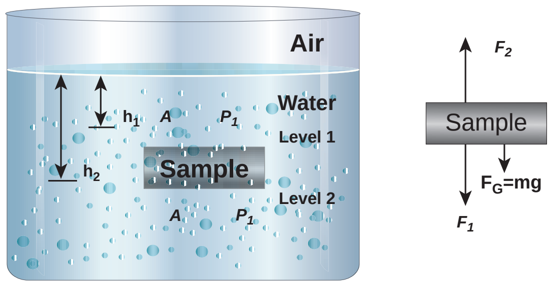


Figure 7.10 (a) A sample of water with base area A in a static fluid with its forces in equilibrium

Hence, gravitational force,

$$F_G = \rho A (h_2 - h_1) g$$

On substituting the value of W in equation (7.15)

$$F_2 = F_1 + m g \Rightarrow P_2 A = P_1 A + \rho A (h_2 - h_1) g$$

Cancelling out A on both sides,

$$P_2 = P_1 + \rho (h_2 - h_1) g \qquad (7.16)$$

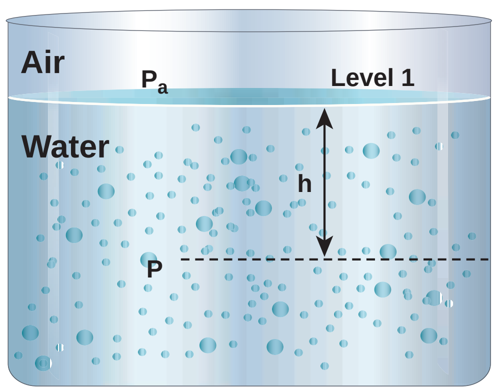


Figure 7.10 (b) The pressure (P) at a depth (h) below the water surface

If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface (as shown in Figure 7.10(b)), then the value of h_1 becomes zero ($h_1 = 0$) and in turn P_1 assumes the value of atmospheric pressure (say P_a). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation (7.16), we get

$$P = P_a + \rho g h \qquad (7.17)$$

which means, the pressure at a depth h is greater than the pressure on the surface of the liquid, where P_a is the atmospheric pressure which is equal to 1.013×10^5 Pa.

If the atmospheric pressure is neglected or ignored then

$$P = \rho g h \qquad (7.18)$$

For a given liquid, ρ is fixed and g is also constant, then the pressure due to the fluid column is directly proportional to vertical distance or height of the fluid column. This implies, the height of the fluid column is more important to decide the pressure and not the cross sectional or base area or even the shape of the container.

When we talk about liquid at rest, the liquid pressure is the same at all points at the same horizontal level (or same depth). This statement can be demonstrated by an experiment called 'hydrostatic paradox'. Let us consider three vessels of different shapes A, B, and C as shown in Figure 7.11. These vessels are connected at the bottom by a horizontal pipe. When they are filled with a liquid (say water), it occupies the same level even though the vessels hold different amounts of water. It is true because the liquid at the bottom of each section of the vessel experiences the same pressure.

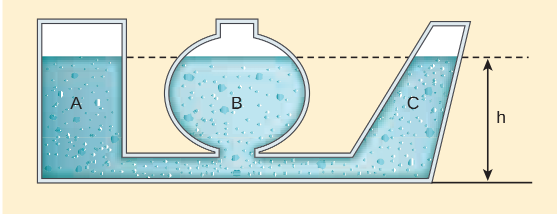
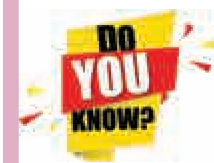


Figure 7.11 Illustration of hydrostatic paradox



The atmospheric pressure at a place is the gravitational force exerted by air above that place per unit surface area. It changes with height and weather conditions (i.e. density of air). In fact, the atmospheric pressure decreases with increasing elevation.

The decrease of atmospheric pressure with altitude has an unwelcome consequence in daily life. For example, it takes longer time to cook at higher altitudes. Nose bleeding is another common experience at higher altitude because of larger difference in atmospheric pressure and blood pressure.

Its value on the surface of the Earth at sea level is 1atm.

Activity

Take a metal container with an opening. Connect a vacuum pump to the opening. Evacuate the air from inside the container. Why the shape of the metal container gets crumbled?

Inference:

Due to the force of atmospheric pressure acting on its outer surface, the shape of the container crumbles.

ACTIVITY

Take a glass tumbler. Fill it with water to the brim. Slide a cardboard on its rim so that no air remains in between the cardboard and the tumbler. Invert the tumbler gently. The water does not fall down.

Inference:

This is due to the fact that the weight of water in the tumbler is balanced by the upward thrust caused due to the atmospheric pressure acting on the lower surface of the cardboard that is exposed to air.

7.3.3 Pascal's law and its applications

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. Statement of Pascal's law is *If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.*

<u>**Application of Pascal's law**</u>

Hydraulic lift

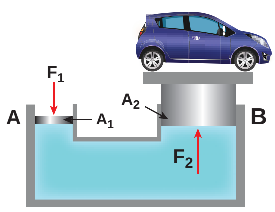


Figure 7.12 Hydraulic lift

A practical application of Pascal's law is the hydraulic lift which is used to lift a heavy load with a small force. It is a force multiplier. It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure 7.12). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to $P\left(where, P = \frac{F_1}{A_1}\right)$. But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2 \Rightarrow F_2 = \frac{A_2}{A_1} \times F_1 \quad (7.19)$$

Therefore by changing the force on the smaller piston A, the force on the piston B has been increased by the factor $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

EXAMPLE 7.7

Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?

Solution

Since, the diameter of the pistons are given, we can calculate the radius of the piston

$$r = \frac{D}{2}$$

Area of smaller piston, $A_1 = \pi\left(\frac{5}{2}\right)^2 = \pi(2.5)^2$

Area of larger piston, $A_2 = \pi\left(\frac{60}{2}\right)^2 = \pi(30)^2$

$$F_2 = \frac{A_2}{A_1} \times F_1 = (50N) \times \left(\frac{30}{2.5}\right)^2 = 7200\,N$$

This means, with the force of 50 N, the force of 7200 N can be lifted.

7.3.4 Buoyancy

When a body is partially or fully immersed in a fluid, it displaces a certain amount of fluid. The displaced fluid exerts an upward force on the body. The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called *upthrust* or *buoyant force* and the phenomenon is called *buoyancy*.

Archimedes principle:

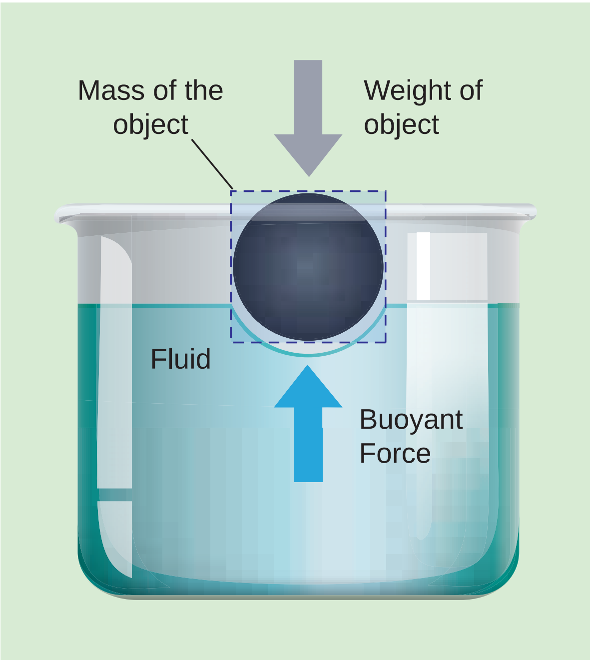


Figure 7.13 Archimedes principle

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

upthrust or buoyant force = weight of liquid displaced.

Law of floatation

It is well-known that boats, ships, and some wooden objects move on the upper part of the water, we say they float. Floatation can be defined as the tendency of an object to rise up to the upper levels of the fluid or to stay on the surface of the fluid.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body. For example, a wooden object weighs 300 kg (about 3000 N) floats in water displaces 300 kg (about 3000 N) of water.

Note If an object floats, the volume of fluid displaced is equal to the volume of the object submerged and the percentage of the volume of the object submerged is equal to the relative density of an object with respect to the density of the fluid in which it floats.

For example, if an ice cube of density 0.9 g cm^{-3} floats in the fresh water of density 1.0 g cm^{-3} then the percentage volume of an object submerged in fresh water is. $\frac{0.9\,g\,cm^{-3}}{1.0\,g\,cm^{-3}} \times 100\% = 90\%$.

Conversely, if the same ice cube floats in sea water of density 1.3 g cm^{-3}, then the percentage volume of the object submerged in seawater would be $\frac{0.9\,g\,cm^{-3}}{1.3\,g\,cm^{-3}} \times 100\% = 69.23\%$ only.

EXAMPLE 7.8

A cube of wood floating in water supports a 300 g mass at the centre of its top face. When the mass is removed, the cube rises by 3 cm. Determine the volume of the cube.

Solution

Let each side of the cube be l. The volume occupied by 3 cm depth of cube,

$$V=(3cm)\times l^2 = 3l^2 cm$$

According to the principle of floatation, we have

$$V\rho g = mg \Rightarrow V\rho = m$$

ρ is density of water = 1000 $kg\ m^{-3}$

$$\Rightarrow (3l^2\times 10^{-2}m)\times(1000\ kgm^{-3}) = 300\times 10^{-3}kg$$

$$l^2 = \frac{300\times10^{-3}}{3\times10^{-2}\times1000}m^2 \Rightarrow l^2 = 100\times10^{-4}m^2$$

$$l = 10\times10^{-2}m = 10\ cm$$

Therefore, volume of cube V = l^3 = 1000 cm^3

Submarines can sink or rise in the depth of water by controlling its buoyancy. To achieve this, the submarines have ballast tanks that can be filled with water or air, alternatively, when the ballast tanks are filled with air, the overall density of the submarine becomes lesser than the surrounding water, and it surfaces (positive buoyancy). If the tanks are flooded with water replacing air, the overall density becomes greater than the surrounding water and submarine begins to sink (negative buoyancy). To keep the submarine at any depth, tanks are filled with air and water (neutral buoyancy).

Examples of floating bodies:

i) A person can swim in sea water more easily than in river water.

ii) Ice floats on water.

iii) The ship is made of steel but its interior is made hollow by giving it a concave shape.

7.4 VISCOSITY

7.4.1 Introduction

In section 7.3, the behavior of fluids at rest is discussed. Successive discussions will bring out the influence of fluid motion on different properties. A fluid in motion is a complex phenomenon, as it possesses potential, kinetic, and gravitational energy besides causing friction viscous forces to come into play. Therefore, it is necessary to consider the case of an ideal liquid to simplify the task. An ideal liquid is incompressible (i.e., bulk modulus is infinity) and in which no shearing forces can be maintained (i.e., the coefficient of viscosity is zero).

Most of the fluids offer resistance towards motion. A frictional force acts at the contact surface when a fluid moves relative to a solid or when two fluids move relative to each other. This resistance to fluid motion is similar to the friction produced when a solid moves on a surface. The internal friction existing between the layers of a moving fluid is viscosity. So, viscosity is defined as 'the property of a fluid to oppose the relative motion between its layers'.

ACTIVITY

Consider three steel balls of the same size, dropped simultaneously in three tall jars each filled with air, water, and oil respectively. It moves easily in air, but not so easily in water. Moving in oil would be even more difficult. There is a relative motion produced between the different layers of the liquid by the falling ball, which causes a viscous force. This frictional force depends on the density of the liquid. This property of a moving fluid to oppose the relative motion between its layers is called viscosity.

Cause of Viscosity:

Consider a liquid flowing through a horizontal surface with two neighboring layers. The upper layer tends to accelerate the lower layer and in turn, the lower layer tends to retard the upper layer. As a result, a backward tangential force is set-up. This tends to destroy the relative motion. This accounts for the viscous behavior of fluids.

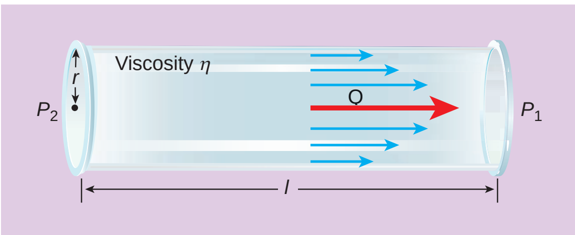


Figure 7.14 Viscosity

Coefficient of Viscosity:

Consider a liquid flowing steadily over a horizontal fixed layer (Figure 7.15). The velocities of the layers increase uniformly as we move away from the fixed layer. Consider any two parallel layers A and B. Let v and $v + dv$ be the velocities of the neighboring layers at distances x and $x + dx$ respectively from the fixed layer.

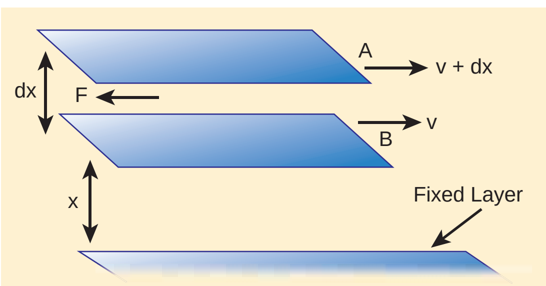


Figure 7.15 Flow of liquid over the horizontal layers

The force of viscosity F acting tangentially between two layers is given by Newton's First law. This force is proportional to (i) area A of the liquid and (ii) the velocity gradient $\frac{dv}{dx}$

$$F \propto A \, and \, F \propto \frac{dv}{dx}$$

$$\Rightarrow F = -\eta A \frac{dv}{dx} \qquad (7.20)$$

Where the constant of proportionality η is called the coefficient of viscosity of the liquid and the negative sign implies that the force is frictional and it opposes the relative motion. The dimensional formula for coefficient of viscosity is $\left[ML^{-1}T^{-1}\right]$

Note Viscosity is similar to friction. The kinetic energy of the substance is dissipated as heat energy.

EXAMPLE 7.9

A metal plate of area $2.5\times10^{-4} m^2$ is placed on a $0.25\times10^{-3} m$ thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity $3\times10^{-2} m\,s^{-1}$, calculate the coefficient of viscosity of castor oil.

Given: $A=2.5\times10^{-4}\ m^2$, $dx = 0.25\times10^{-3} m$, $F=2.5N$ and $dv = 3\times10^{-2}\ m\,s^{-1}$

Solution

$$F = -\eta A \frac{dv}{dx}$$

In magnitude, $\eta = \frac{F}{A}\frac{dx}{dv}$

$$= \frac{(2.5N)}{(2.5\times10^{-4}m^2)} \frac{(0.25\times10^{-3}m)}{(3\times10^{-2}m s^{-1})}$$

$= 0.083\times10^3\ Nm^{-2}s$

7.4.2 Streamlined flow

The flow of fluids occurs in different ways. It can be a steady or streamlined flow, unsteady or turbulent flow, compressible or incompressible flow or even viscous or non-viscous flow. For example, consider a calm flow of water through a river. Careful observation reveals that the velocity of water at different locations of the river is quite different. It is almost faster at the center and slowest near the banks. However, the velocity of the particle at any point is constant. For better understanding, assume that the velocity of the particle is about 4 meter per second at the center of the river. Hence it will be of the same value for all other particles crossing through this point. In a similar way, if the velocity of the particle flowing near the bank of the river is 0.5 meter per second, then the succeeding particles flowing through it will have the same value.

When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a *streamlined flow*. It is also referred to as steady or laminar flow. The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point as shown in Figure 7.16. It is named so because the flow looks like the flow of a stream or river under ideal conditions.

If we assume a bundle of streamlines having the same velocity over any cross section perpendicular to the direction of flow then such bundle is called a '*tube of flow*'. Thus, it is important to note that any particle in a tube of flow always remains in the tube throughout its motion and cannot mix with liquid in another tube. Always the axis of the tube of flow gives the streamline. The streamlines always represent the trajectories of the fluid particles. The flow of fluid is streamlined up to a certain velocity called critical velocity. This means a steady flow can be achieved at low flow speeds below the critical speed.

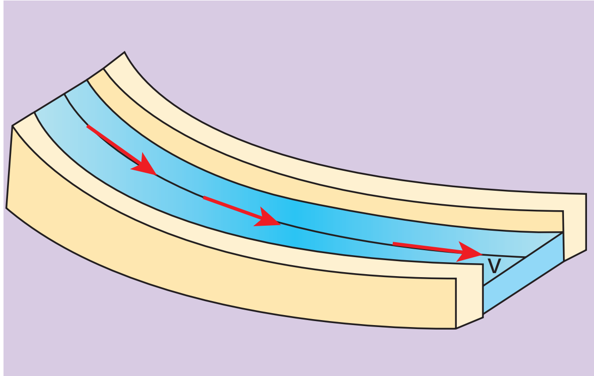


Figure 7.16 Flow is steady velocity at any point in the liquid remains constant

7.4.3 Turbulent flow

When the speed of the moving fluid exceeds the critical speed, v_c the motion becomes turbulent. In this case, the velocity changes both in magnitude and direction from particle to particle and hence the individual particles do not move in a streamlined path. Hence, the path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies (Figure 7.17 (a) and (b)). The flow of water just behind a boat or a ship and the air flow behind a moving bus are a few examples of turbulent flow.

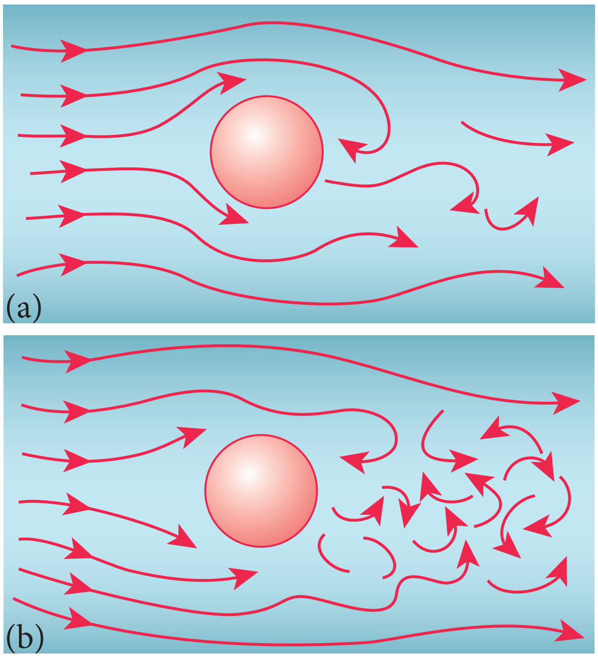


Figure 7.17 (a) turbulent flow around a sphere (when $v = v_c$) (b) turbulent flow around a sphere (when $v > v_c$)

The distinction between the two types of motion can be easily demonstrated by injecting a jet of ink axially in a wide tube through which water flows. When the velocity of the fluid is small the ink will move in a straight line path. Conversely, when the velocity is increased beyond a certain value, the ink will spread out showing the disorderliness and hence the motion becomes turbulent. The zig-zag motion results in the formation of eddy currents and as a consequence, much energy is dissipated.

7.4.4 Reynold's number

We have learnt that the flow of a fluid becomes steady or laminar when the velocity of flow is less than the critical velocity v_c otherwise, the flow becomes turbulent. Osborne Reynolds (1842-1912) formulated an equation to find out the nature of the flow of fluid, whether it is streamlined or turbulent.

$$R_c = \frac{\rho v D}{\eta} \qquad (7.21)$$

It is a dimensionless number called '*Reynold's number*'. It is denoted by the symbol R_c or K. In the equation, ρ denotes the density of the fluid, v the velocity of the flowing fluid, D is the diameter of the pipe in which the fluid flow, and η is the coefficient of viscosity of the fluid. The value of R_c remains the same in any system of units.

Table 7.3 To understand the flow of liquid, Reynold has estimated the value of R_c as follows

S. No.	Reynold's number	Flow
1	Rc < 1000	Streamline
2	1000 < Rc < 2000	Unsteady
3	Rc > 2000	Turbulent

Hence, Reynold's number R_c is a critical variable, which decides whether the flow of a fluid through a cylindrical pipe is streamlined or turbulent. In fact, the critical value of R_c at which the turbulent sets is found to be the same for geometrically similar flows. For example, when two liquids (say oil and water) of different densities and viscosities flow in pipes of same shapes and sizes, the turbulence sets in at almost the same value of R_c. The above fact leads to the *Law of similarity* which states that when there are two geometrically similar flows, both are essentially equal to each other, as long as they embrace the same Reynold's number. The *Law of similarity* plays a very important role in technological applications.

The shape of ships, submarines, racing cars, and airplanes are designed in such a way that their speed can be maximized.

7.4.5 Terminal velocity

To understand terminal velocity, consider a small metallic sphere falling freely from rest through a large column of a viscous fluid. The forces acting on the sphere are (i) gravitational force of the sphere acting vertically downwards, (ii) upthrust U due to buoyancy and (iii) viscous drag acting upwards (viscous force always acts in a direction opposite to the motion of the sphere).

Initially, the sphere is accelerated in the downward direction so that the upward force is less than the downward force. As the velocity of the sphere increases, the velocity of the viscous force also increases. A stage is reached when the net downward force balances the upward force and hence the resultant force on the sphere becomes zero. It now moves down with a constant velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity V_T. In the Figure 7.18, a graph is drawn with velocity along y- axis and time along x- axis. It is evident from the graph that the sphere is accelerated initially and in course of time it becomes constant, and attains terminal velocity (V_T).

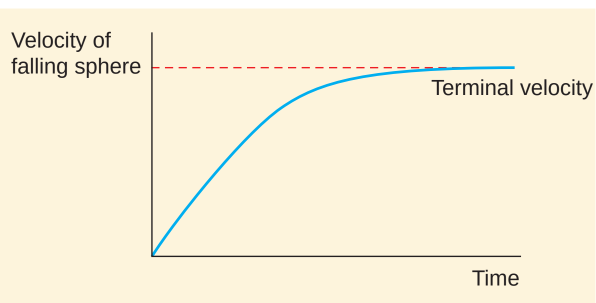


Figure 7.18 Velocity verses time graph

Expression for terminal velocity:

Consider a sphere of radius *r* which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

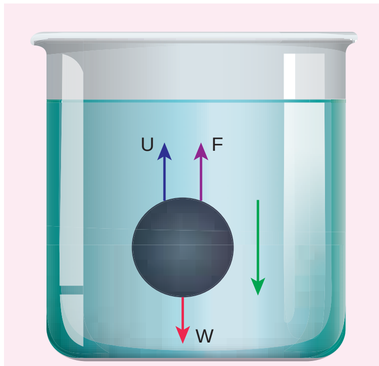


Figure 7.19 Forces acting on the sphere when it falls in a viscous liquid

Gravitational force acting on the sphere,

$$F_G = mg = \frac{4}{3}\pi r^3 \rho g \text{ (downward force)}$$

Up thrust, $U = \frac{4}{3}\pi r^3 \sigma g$ (upward force)

viscous force $F = 6\pi\eta r v_t$

At terminal velocity v_t.

downward force = upward force

$$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v_t$$

$$v_t = \frac{2}{9} \times \frac{r^2(\rho - \sigma)}{\eta} g \Rightarrow v_t \propto r^2 \quad (7.22)$$

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius. If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity. That is why air bubbles rise up through water or any fluid. This is also the reason for the clouds in the sky to move in the upward direction.

Point to ponder

i) The terminal speed of a sphere is directly proportional to the square of the radius of the sphere. Hence, larger raindrops fall with greater speed as compared to the smaller raindrops.
ii) If the density of the material of the sphere is less than the density of the medium, then the sphere shall attain terminal velocity in the upward direction. That is why gas bubbles rise up in soda water.

7.4.6 Stoke's law and its applications

When a body falls through a viscous medium, it drags the layer of the fluid immediately in contact with it. This produces a relative motion between the different layers of the liquid. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force F acting on a spherical body of radius r depends directly on

i) radius (r) of the sphere
ii) velocity (v) of the sphere and
iii) coefficient of viscosity η of the liquid

Therefore $F \propto \eta^x r^y v^z \Rightarrow F = k\eta^x r^y v^z$, where k is a dimensionless constant.

Using dimensions, the above equation can be written as

$$[MLT^{-2}] = k[ML^{-1}T^{-1}]^x \times [L]^y \times [LT^{-1}]^z$$

On solving, we get x=1, y=1, and z=1

Therefore, F=kη rv

Experimentally, Stoke found that the value of k = 6π

$$F = 6\pi\eta\, rv \qquad (7.23)$$

This relation is known as Stoke's law

Practical applications of Stoke's law

Since the raindrops are smaller in size and their terminal velocities are small, remain suspended in air in the form of clouds. As they grow up in size, their terminal velocities increase and they start falling in the form of rain.

This law explains the following:

a) Floatation of clouds
b) Larger raindrops hurt us more than the smaller ones
c) A man coming down with the help of a parachute acquires constant terminal velocity.

7.4.7 Poiseuille's equation

Poiseuille analyzed the steady flow of liquid through a capillary tube. He derived an expression for the volume of the liquid flowing per second through the capillary tube.

As per the theory, the following conditions must be retained while deriving the equation.

- The flow of liquid through the tube is streamlined.
- The tube is horizontal so that gravity does not influence the flow
- The layer in contact with the wall of the tube is at rest
- The pressure is uniform over any cross section of the tube

We can derive Poiseuille's equation using dimensional analysis. Consider a liquid flowing steadily through a horizontal capillary tube. Let $v = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient $\left(\frac{P}{l}\right)$. Then,

$$v \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$v = k\eta^a r^b \left(\frac{P}{l}\right)^c \qquad (7.24)$$

where, k is a dimensionless constant. Therefore,

$$[\upsilon] = \frac{volume}{time} = [L^3T^{-1}], \left[\frac{dP}{dx}\right] = \frac{Pressure}{distance} =$$

$$[ML^{-2}T^{-2}], [\eta] = [ML^{-1}T^{-1}] \, and \, [r] = [L]$$

Substituting in equation (7.24)

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

$$M^0L^3T^{-1} = M^{a+b}L^{-a+b-2c}T^{-a-2c}$$

So, equating the powers of M, L, and T on both sides, we get

$$a + c = 0, -a + b - 2c = 3, \text{ and } -a - 2c = -1$$

We have three unknowns a, b, and c. We have three equations, on solving, we get

$$a = -1, b = 4, \text{ and } c = 1$$

Therefore, equation (7.24) becomes,

$$v = k\eta^{-1}r^4 \left(\frac{P}{l}\right)^1$$

Experimentally, the value of k is shown to be $\frac{\pi}{8}$, we have

$$v = \frac{\pi r^4 P}{8\eta l} \qquad (7.25)$$

The above equation is known as *Poiseuille's equation* for the flow of liquid through a narrow tube or a capillary tube. This relation holds good for the fluids whose velocities are lesser than the critical velocity (v_c).

7.4.8 Applications of viscosity

The importance of viscosity can be understood from the following examples.

(1) The oil used as a lubricant for heavy machinery parts should have a high viscous coefficient. To select a suitable lubricant, we should know its viscosity and how it varies with temperature [Note: As temperature increases, the viscosity of the liquid decreases]. Also, it helps to choose oils with low viscosity used in car engines (light machinery).

(2) The highly viscous liquid is used to damp the motion of some instruments and is used as brake oil in hydraulic brakes.

(3) Blood circulation through arteries and veins depends upon the viscosity of fluids.

(4) Millikan conducted the oil drop experiment to determine the charge of an electron. He used the knowledge of viscosity to determine the charge.

7.5 SURFACE TENSION

7.5.1 Intermolecular forces

Different liquids do not mix together due to their physical properties such as density, surface tension force, etc. For example, water and kerosene do not mix together. Mercury does not wet the glass but water sticks to