

it. Water rises up to the leaves through the stem. They are mostly related to the free surfaces of liquids. Liquids have no definite shape but have a definite volume. Hence they acquire a free surface when poured into a container. Therefore, the surfaces have some additional energy, called as surface energy. The phenomenon behind the above fact is called surface tension. Laplace and Gauss developed the theory of surface and motion of a liquid under various situations.

The molecules of a liquid are not rigidly fixed like in a solid. They are free to move about. The force between the like molecules which holds the liquid together is called '*cohesive force*'. When the liquid is in contact with a solid, the molecules of the these solid and liquid will experience an attractive force which is called '*adhesive force*'.

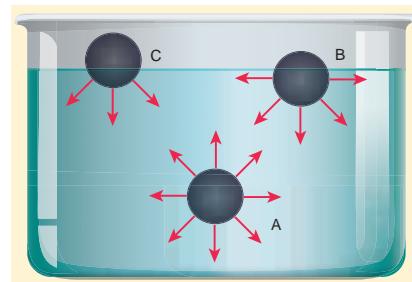
These molecular forces are effective only when the distance between the molecules is very small about  $10^{-9}$  m (i.e.,  $10\text{ \AA}$ ). The distance through which the influence of these molecular forces can be felt in all directions constitute a range and is called *sphere of influence*. The forces outside this range are rather negligible.

Consider three different molecules A, B, and C in a given liquid as shown in Figure 7.20. Let a molecule 'A' be considered well inside the liquid within the sphere of influence. Since this molecule interacts with all other molecules in all directions, the net force experienced by A is zero. Now consider a molecule 'B' in which three-fourth lies below the liquid surface and one-fourth on the air. Since B has more molecules towards its lower side than the upper side, it experiences a net force in the downward direction. In a similar way, if another molecule 'C' is chosen on the liquid surface (i.e, upper half in air and lower half in liquid),

it experiences a maximum downward force due to the availability of more number of liquid molecules on the lower part. Hence it is obvious that all molecules of the liquid that falls within the molecular range inside the liquid interact with the molecule and hence experience a downward force.



BMMHXX



**Figure 7.20** Molecules at different levels of liquid

When any molecule is brought towards the surface from the interior of the liquid, work is done against the cohesive force among the molecules of the surface. This work is stored as potential energy in molecules. So the molecules on the surface will have greater potential energy than that of molecules in the interior of the liquid. But for a system to be under stable equilibrium, its potential energy (or surface energy) must be a minimum. Therefore, in order to maintain stable equilibrium, a liquid always tends to have a minimum number of molecules. In other words, the liquid tends to occupy a minimum surface area. This behaviour of the liquid gives rise to surface tension.

### Examples for surface tension.

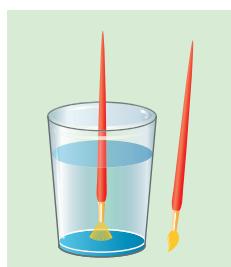
Water bugs and water striders walk on the surface of water (Figure 7.21). The water molecules are pulled inwards and the surface of water acts like a springy or stretched membrane. This balance the weight of water bugs and enables them to walk on the

surface of water. We call this phenomenon as surface tension.



**Figure 7.21** Water stiders can walk on water because of the surface tension of water

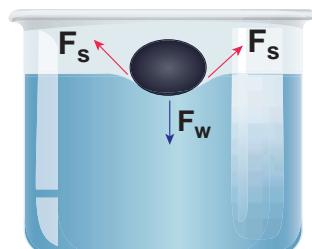
The hairs of the painting brush cling together when taken out of water. This is because the water films formed on them tends to contract to a minimum area (Figure 7.22).



**Figure 7.22** Painting brush hairs cling together due to surface tension

#### Activity 1: Needle floats on water surface

Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating.

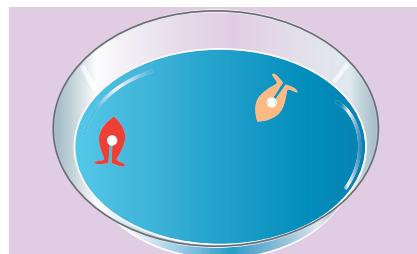


**Figure 7.23** Floating needle

The floating needle causes a little depression; the forces  $F$ , due to the surface tension of the curved surface are inclined as shown in Figure 7.23. The vertical components of these two forces support the weight of the needle. Now add liquid soap to the water and stir it. We find that the needle sinks.

#### Activity 2:

Take a plastic sheet and cut out a piece in the shape of a boat (Figure 7.24). A tapering and smooth front with a notch at the back is suggested. Put a piece of camphor into the notch of the boat. Gently release the boat on the surface of the water and we find that the boat is propelled forward when the camphor dissolves. The surface tension is lowered, as the camphor dissolves and produces a difference in surface tension in the water nearby the notch. This causes the water to flow away from the back of the boat, which moves the boat forward.



**Figure 7.24** Camphor boat

### 7.5.2 Factors affecting the surface tension of a liquid

Surface tension for a given liquid varies in following situations

- (1) *The presence of any contamination or impurities* considerably affects the force of surface tension depending upon the degree of contamination.
- (2) *The presence of dissolved substances* can also affect the value of surface

tension. For example, a highly soluble substance like sodium chloride ( $NaCl$ ) when dissolved in water ( $H_2O$ ) increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water.

- (3) **Electrification** affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases.
- (4) **Temperature** plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature. For a small range of temperature, the surface tension at  $T_t$  at  $t^{\circ}C$  is  $T_t = T_0 (1 - \alpha t)$

Where,  $T_0$  is the surface tension at temperature  $0^{\circ}C$  and  $\alpha$  is the temperature coefficient of surface tension. It is to be noted that at the critical temperature, the surface tension is zero as the interface between liquid and vapour disappear. For example, the critical temperature of water is  $374^{\circ}C$ . Therefore, the surface tension of water is zero at that temperature. van der Wall suggested the important relation between the surface tension and the critical temperature as

$$T_t = T_0 \left(1 - \frac{t}{t_c}\right)^{\frac{3}{2}}$$

Generalizing the above relation, we get

$$T_t = T_0 \left(1 - \frac{t}{t_c}\right)^n$$

which gives more accurate value. Here  $n$ , varies for different liquids and  $t$  and  $t_c$  denotes

the temperature and critical temperature in absolute scale (Kelvin scale), respectively.

### 7.5.3 Surface energy (S.E.) and surface tension (S.T.)

#### Surface Energy

Consider a sample of liquid in a container. A molecule inside the liquid is being pulled in all direction by other molecules that surround it. However, near the surface, a molecule is pulled down only by the molecules below them and there is a net downward force. As a result, the entire surface of the liquid is being pulled inward. The liquid surface thus tends to have the least surface area. To increase the surface area, some molecules are brought from the interior to the surface. For this reason, work has to be done against the forces of attraction. The amount of work done is stored as potential energy. Thus, the molecules lying on the surface possess greater potential energy than other molecules. This excess energy per unit area of the free surface of the liquid is called '*surface energy*'. In other words, the work done in increasing the surface area per unit area of the liquid against the surface tension force is called the surface energy of the liquid.

$$\text{Surface energy} = \frac{\text{work done in increasing the surface area}}{\text{increase in surface area}} \\ = \frac{W}{\Delta A} \quad (7.26)$$

It is expressed in  $J m^{-2}$  or  $N m^{-1}$ .

#### Surface tension

The surface tension of a liquid is defined as the energy per unit area of the surface of a liquid

$$T = \frac{F}{l} \quad (7.27)$$

The SI unit and dimensions of T are  $N\text{m}^{-1}$  and  $\text{M T}^{-2}$ , respectively.

### Relation between surface tension and surface energy:

Consider a rectangular frame of wire ABCD in a soap solution (Figure 7.25). Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inward due to surface tension. Let  $F$  be the force due to surface tension, then

$$F = (2T)l$$

here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance  $\Delta x$  to new a position A'B'. Since the area increases, some work has to be done against the inward force due to surface tension.

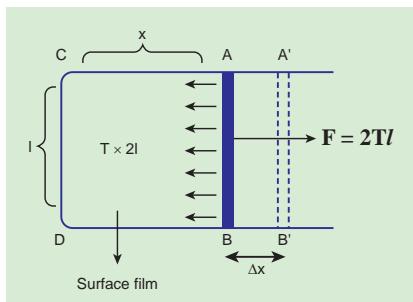
Work done = Force  $\times$  distance =  $(2T l) (\Delta x)$

Increase in area of the film  $\Delta A = (2l)(\Delta x) = 2l \Delta x$

Therefore,

$$\begin{aligned} \text{surface energy} &= \frac{\text{work done}}{\text{increase in surface area}} \\ &= \frac{2Tl \Delta x}{2l \Delta x} = T \end{aligned} \quad (7.28)$$

Hence, the surface energy per unit area of a surface is numerically equal to the surface tension.



**Figure 7.25** A horizontal soap film on a rectangular frame of wire ABCD



It should be remembered that a liquid drop has only one free surface. Therefore, the surface area of a spherical drop of radius  $r$  is equal to  $4\pi r^2$ , whereas, a bubble has two free surfaces and hence the surface area of a spherical bubble is equal to  $2 \times 4\pi r^2$ .

### EXAMPLE 7.10

Let  $2.4 \times 10^{-4}\text{ J}$  of work is done to increase the area of a film of soap bubble from  $50\text{ cm}^2$  to  $100\text{ cm}^2$ . Calculate the value of surface tension of soap solution.

#### Solution:

A soap bubble has two free surfaces, therefore increase in surface area  $\Delta A = A_2 - A_1 = 2(100-50) \times 10^{-4}\text{ m}^2 = 100 \times 10^{-4}\text{ m}^2$ .

Since, work done  $W = T \times \Delta A \Rightarrow T =$

$$\frac{W}{\Delta A} = \frac{2.4 \times 10^{-4}\text{ J}}{100 \times 10^{-4}\text{ m}^2} = 2.4 \times 10^{-2}\text{ N m}^{-1}$$

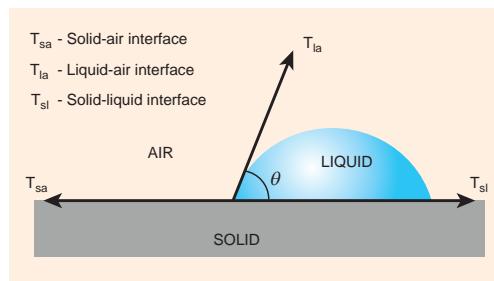
### 7.5.4 Angle of contact

When the free surface of a liquid comes in contact with a solid, then the surface of the liquid becomes curved at the point of contact. Whenever the liquid surface becomes a curve, then the angle between the two medium (solid-liquid interface) comes in the picture. For an example, when a glass plate is dipped in water with sides vertical as shown in figure, we can observe that the water is drawn up to the plate. In the same manner, instead of water the glass plate is dipped in mercury, the surface is curved but now the curve is depressed as shown in Figure 7.29

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is known as the *angle of contact between the solid and the liquid*. It is denoted by  $\theta$  (Read it as “theta” which is Greek alphabet small letter).

Its value is different at interfaces of different pairs of solids and liquids. In fact, it is the factor which decides whether a liquid will spread on the surface of a chosen solid or it will form droplets on it.

Let us consider three interfaces such as liquid-air, solid-air and solid-liquid with reference to the point of contact ‘O’ and the interfacial surface tension forces  $T_{sa}$ ,  $T_{sl}$  and  $T_{la}$  on the respective interfaces as shown in Figure 7.26.



**Figure 7.26** Angle of contact of a liquid

Since the liquid is stable under equilibrium, the surface tension forces between the three interfaces must also be in equilibrium. Therefore,

$$T_{sa} = T_{la} \cos\theta + T_{sl} \Rightarrow \cos\theta = \frac{T_{sa} - T_{sl}}{T_{la}} \quad (7.29)$$

From the above equation, there are three different possibilities which can be discussed as follows.

- (i) If  $T_{sa} > T_{sl}$  and  $T_{sa} - T_{sl} > 0$  (water-plastic interface) then the angle of contact  $\theta$  is acute angle ( $\theta$  less than  $90^\circ$ ) as  $\cos\theta$  is positive.
- (ii) If  $T_{sa} < T_{sl}$  and  $T_{sa} - T_{sl} < 0$  (water-leaf interface) then the angle of contact is

obtuse angle ( $\theta$  less than  $180^\circ$ ) and as  $\cos\theta$  is negative.

- (iii) If  $T_{sa} > T_{la} + T_{sl}$  then there will be no equilibrium and liquid will spread over the solid.

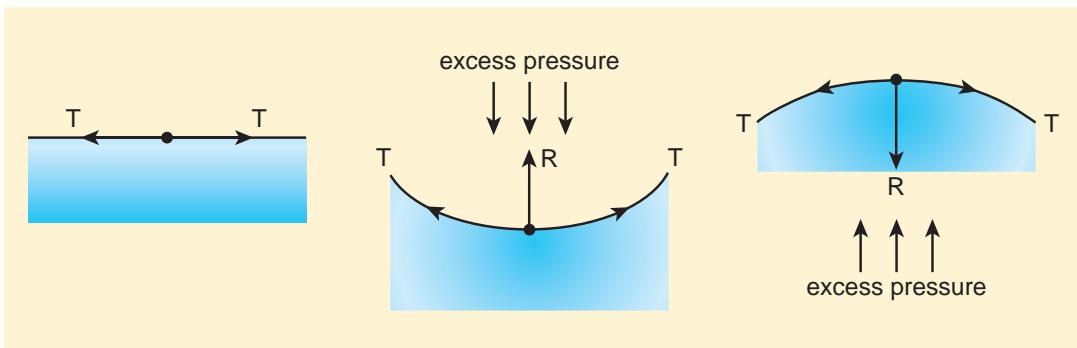
Therefore, the concept of *angle of contact* between the solid-liquid interface leads to some practical applications in real life. For example, soaps and detergents are wetting agents. When they are added to an aqueous solution, they will try to minimize the angle of contact and in turn penetrate well in the cloths and remove the dirt. On the other hand, water proofing paints are coated on the outer side of the building so that it will enhance the angle of contact between the water and the painted surface during the rainfall.

### 7.5.5 Excess of pressure inside a liquid drop, a soap bubble, and an air bubble

As it is discussed earlier, the free surface of a liquid becomes curved when it has contact with a solid. Depending upon the nature of liquid-air or liquid-gas interface, the magnitude of interfacial surface tension varies. In other words, as a consequence of surface tension, the above such interfaces have energy and for a given volume, the surface will have a minimum energy with least area. Due to this reason, the liquid drop becomes spherical (for a smaller radius).

When the free surface of the liquid is curved, there is a difference in pressure between the inner and outer the side of the surface (Figure 7.27).

- i) When the liquid surface is plane, the forces due to surface tension ( $T$ ,  $T$ ) act tangentially to the liquid surface in



**Figure 7.27** Excess of pressure across a liquid surface

opposite directions. Hence, the resultant force on the molecule is zero. Therefore, in the case of a plane liquid surface, the pressure on the liquid side is equal to the pressure on the vapour side.

- ii) When the liquid surface is curved, every molecule on the liquid surface experiences forces ( $F_r, F_T$ ) due to surface tension along the tangent to the surface. Resolving these forces into rectangular components, we find that horizontal components cancel out each other while vertical components get added up. Therefore, the resultant force normal to the surface acts on the curved surface of the liquid. Similarly, for a convex surface, the resultant force is directed inwards towards the centre of curvature, whereas the resultant force is directed outwards from the centre of curvature for a concave surface. Thus, for a curved liquid surface in equilibrium, the pressure on its concave side is greater than the pressure on its convex side.

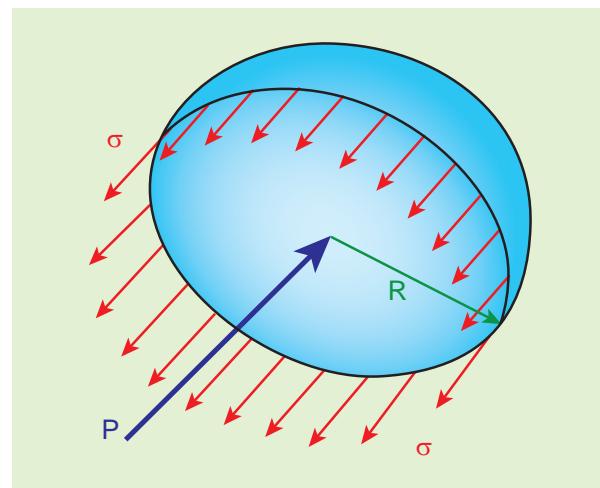
#### Excess of pressure inside a bubble and a liquid drop:

The small bubbles and liquid drops are spherical because of the forces of surface tension. The fact that a bubble or a liquid drop does not collapse due to the combined effect

indicates that the pressure inside a bubble or a drop is greater than that outside it.

#### 1) Excess of pressure inside air bubble in a liquid.

Consider an air bubble of radius  $R$  inside a liquid having surface tension  $T$  as shown in Figure 7.28 (a). Let  $P_1$  and  $P_2$  be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is  $\Delta P = P_1 - P_2$ .



**Figure 7.28.** (a) Air bubble

In order to find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble. For the hemispherical portion of the bubble, considering the forces acting on it, we get,

- The force due to surface tension acting towards right around the rim of length  $2\pi R$  is  $F_T = 2\pi RT$
- The force due to outside pressure  $P_1$  is to the right acting across a cross sectional area of  $\pi R^2$  is  $F_{P_1} = P_1 \pi R^2$
- The force due to pressure  $P_2$  inside the bubble, acting to the left is  $F_{P_2} = P_2 \pi R^2$ .

As the air bubble is in equilibrium under the action of these forces,  $F_{P_2} = F_T + F_{P_1}$

$$P_2 \pi R^2 = 2\pi RT + P_1 \pi R^2$$

$$\Rightarrow (P_2 - P_1) \pi R^2 = 2\pi RT$$

Excess pressure is  $\Delta P = P_2 - P_1 = \frac{2T}{R}$  (7.30)

## 2) Excess pressure inside a soap bubble

Consider a soap bubble of radius  $R$  and the surface tension of the soap bubble be  $T$  as shown in Figure 7.28 (b). A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is  $2 \times 2\pi RT$ . The various forces acting on the soap bubble are,

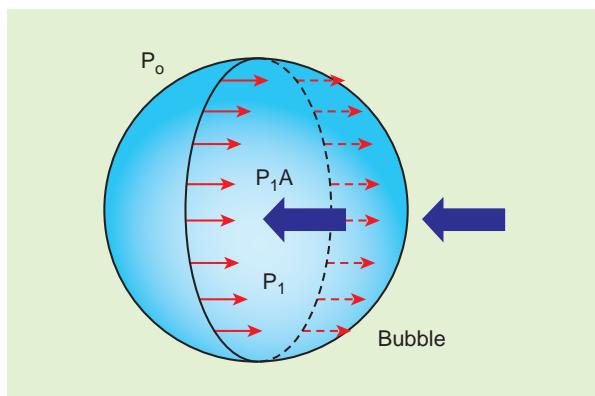
- Force due to surface tension  $F_T = 4\pi RT$  towards right
- Force due to outside pressure,  $F_{P_1} = P_1 \pi R^2$  towards right
- Force due to inside pressure,  $F_{P_2} = P_2 \pi R^2$  towards left

As the bubble is in equilibrium,  $F_{P_2} = F_T + F_{P_1}$

$$P_2 \pi R^2 = 4\pi RT + P_1 \pi R^2$$

$$\Rightarrow (P_2 - P_1) \pi R^2 = 4\pi RT$$

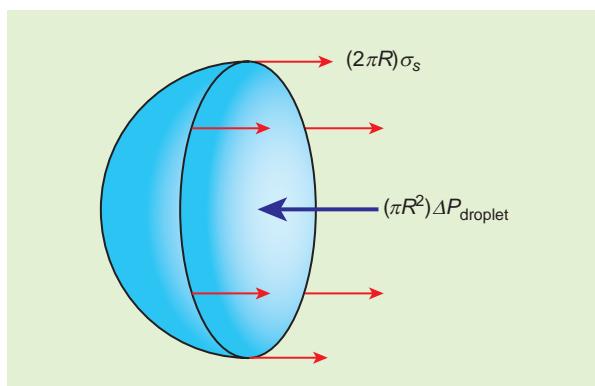
Excess pressure is  $\Delta P = P_2 - P_1 = \frac{4T}{R}$  (7.31)



**Figure 7.28 (b) Soap bubble**

## 3) Excess pressure inside the liquid drop

Consider a liquid drop of radius  $R$  and the surface tension of the liquid is  $T$  as shown in Figure 7.28 (c).



**Figure 7.28 (c) Liquid drop**

The various forces acting on the liquid drop are,

- Force due to surface tension  $F_T = 2\pi RT$  towards right
- Force due to outside pressure,  $F_{P_1} = P_1 \pi R^2$  towards right
- Force due to inside pressure,  $F_{P_2} = P_2 \pi R^2$  towards left

As the drop is in equilibrium,  $F_{P_2} = F_T + F_{P_1}$

$$P_2 \pi R^2 = 2\pi RT + P_1 \pi R^2$$

$$\Rightarrow (P_2 - P_1) \pi R^2 = 2\pi RT$$

Excess pressure is  $\Delta P = P_2 - P_1 = \frac{2T}{R}$  (7.32)



The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside, the tiny fog droplets are rigid enough to behave like solids.

When an ice-skater skate over the surface of the ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates, the tiny droplets of water act as rigid ball-bearings and help the skaters to run along smoothly.

### EXAMPLE 7.11

If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 mm high, where  $R = 2.0\text{ cm}$ , find the surface tension of the soap bubble.

#### Solution

The excess of pressure inside the soap bubble is  $\Delta P = P_2 - P_1 = \frac{4T}{R}$

$$\text{But } \Delta P = P_2 - P_1 = \rho gh \Rightarrow \rho gh = \frac{4T}{R}$$

$\Rightarrow$  Surface tension,

$$T = \frac{\rho gh R}{4} = \frac{(800)(9.8)(4 \times 10^{-3})(2 \times 10^{-2})}{4} =$$

$$T = 15.68 \times 10^{-2} \text{ N m}^{-1}$$

### 7.5.6 Capillarity

The word ‘capilla’ means hair in Latin. If the tubes were hair thin, then the rise would be very large. It means that the tube having a very small diameter is called a ‘capillary tube’. When a glass capillary tube open at both ends is dipped vertically in water, the water in the tube will rise above the level of water in the vessel. In case of mercury, the liquid is depressed in the tube below the level of mercury in the vessel (shown in Figure 7.29). In a liquid whose angle of contact with solid is less than  $90^\circ$ , suffers capillary rise. On the other hand, in a liquid whose angle of contact is greater than  $90^\circ$ , suffers capillary fall (Table 7.4). The rise or fall of a liquid in a narrow tube is called capillarity or capillary action. Depending on the diameter of the capillary tube, liquid rises or falls to different heights.

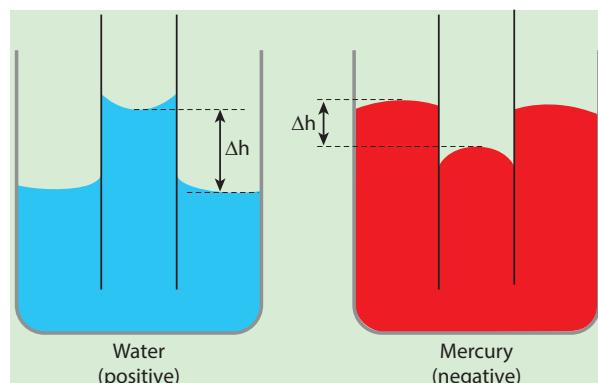
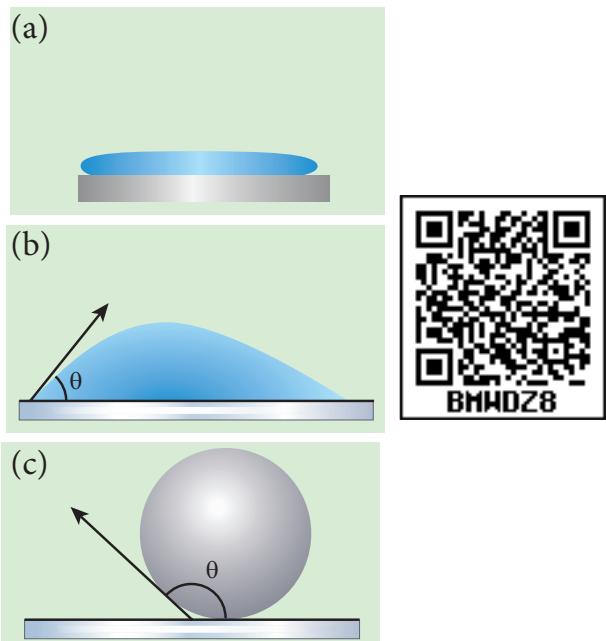


Figure 7.29 Capillary rise or fall

Table 7.4 Capillary rise and fall

Contact angle	Strength of		Degree of wetting	Miniscus	Rise or fall of liquid in the capillary tube
	Cohesive force	Adhesive force			
$\theta=0$ (A)	Weak	Strong	Perfect Wetting	Plane	Neither rises nor is depressed
$\theta<90$ (B)	Weak	Strong	High	Concave	Rise of liquid
$\theta>90$ (C)	Strong	Weak	Low	Convex	Fall of liquid



**Figure 7.30** (a) water on silver surface  
(b) glass plate on water (c) glass on mercury

### Practical applications of capillarity

- Due to capillary action, oil rises in the cotton within an earthen lamp. Likewise, sap rises from the roots of a plant to its leaves and branches.
- Absorption of ink by a blotting paper
- Capillary action is also essential for the tear fluid from the eye to drain constantly.
- Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for sweat.

#### 7.5.7 Surface Tension by capillary rise method

The pressure difference across a curved liquid-air interface is the basic factor behind the rising up of water in a narrow tube (influence of gravity is ignored). The capillary rise is more dominant in the case of very fine tubes. But this phenomenon is the outcome of the force of surface tension. In order to arrive at a relation between the capillary rise ( $h$ ) and surface tension ( $T$ ),

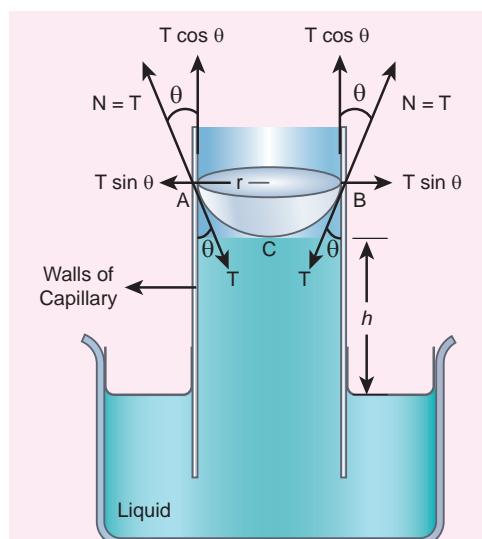
consider a capillary tube which is held vertically in a beaker containing water; the water rises in the capillary tube to a height  $h$  due to surface tension (Figure 7.31).

The surface tension force  $F_T$ , acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension  $T$ , is resolved into two components i) Horizontal component  $T \sin\theta$  and ii) Vertical component  $T \cos\theta$  acting upwards, all along the whole circumference of the meniscus.

$$\begin{aligned} \text{Total upward force} \\ = (T \cos\theta) (2\pi r) = 2\pi r T \cos\theta \end{aligned}$$

where  $\theta$  is the angle of contact,  $r$  is the radius of the tube. Let  $\rho$  be the density of water and  $h$  be the height to which the liquid rises inside the tube. Then,

$$\begin{aligned} \left( \begin{array}{l} \text{the volume of} \\ \text{liquid column in} \\ \text{the tube, } V \end{array} \right) &= \left( \begin{array}{l} \text{volume of the} \\ \text{liquid column of radius } r \\ \text{height } h \end{array} \right) \\ + \left( \begin{array}{l} \text{volume of liquid of radius } r \\ \text{and height } r - \text{Volume of the} \\ \text{hemisphere of radius } r \end{array} \right) \end{aligned}$$



**Figure 7.31** Capillary rise by surface tension

$$V = \pi r^2 h + \left( \pi r^2 \times r - \frac{2}{3} \pi r^3 \right) \Rightarrow V = \pi r^2 h + \frac{1}{3} \pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore,

$$2\pi r T \cos\theta = \pi r^2 \left( h + \frac{1}{3} r \right) \rho g \Rightarrow T = \frac{r \left( h + \frac{1}{3} r \right) \rho g}{2 \cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then  $\frac{r}{3}$  can be neglected when it is compared to the height  $h$ . Therefore,

$$T = \frac{r \rho g h}{2 \cos\theta} \quad (7.33)$$

Liquid rises through a height  $h$

$$h = \frac{2T \cos\theta}{r \rho g} \Rightarrow h \propto \frac{1}{r} \quad (7.34)$$

This implies that the capillary rise ( $h$ ) is inversely proportional to the radius ( $r$ ) of the tube. i.e, the smaller the radius of the tube greater will be the capillarity.

### EXAMPLE 7.12

Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?

#### Solution

From equation (7.34), we have

$$h \propto \frac{1}{r} \Rightarrow hr = \text{constant}$$

Consider two capillary tubes with radius  $r_1$  and  $r_2$  which on placing in a liquid, capillary rises to height  $h_1$  and  $h_2$ , respectively. Then,

$$h_1 r_1 = h_2 r_2 = \text{constant}$$

$$\Rightarrow h_2 = \frac{h_1 r_1}{r_2} = \frac{(2 \times 10^{-2} m) \times r}{\frac{r}{3}} = h_2 = 6 \times 10^{-2} m$$

### EXAMPLE 7.13

Mercury has an angle of contact equal to  $140^\circ$  with soda lime glass. A narrow tube of radius  $2 mm$ , made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury  $T=0.456 N m^{-1}$ ; Density of mercury  $\rho = 13.6 \times 10^3 kg m^{-3}$

#### Solution

Capillary descent,

$$h = \frac{2T \cos\theta}{r \rho g} = \frac{2 \times (0.465 N m^{-1}) (\cos 140^\circ)}{(2 \times 10^{-3} m) (13.6 \times 10^3) (9.8 m s^{-2})}$$

$$\Rightarrow h = -6.89 \times 10^{-4} m$$

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

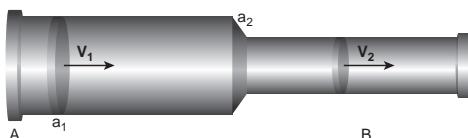
### 7.5.8 Applications of surface tension

- Mosquitoes lay their eggs on the surface of water. To reduce the surface tension of water, a small amount of oil is poured. This breaks the elastic film of water surface and eggs are killed by drowning.
- Chemical engineers must finely adjust the surface tension of the liquid, so it forms droplets of designed size and so it adheres to the surface without smearing. This is used in desktop printing, to paint automobiles and decorative items.
- Specks of dirt get removed when detergents are added to hot water while washing clothes because surface tension is reduced.
- A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact.

**7.6****BERNOULLI'S THEOREM****7.6.1 Equation of continuity**

In order to discuss the mass flow rate through a pipe, it is necessary to assume that the flow of fluid is steady, the flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant with respect to time. Under this condition, the path taken by the fluid particle is a streamline.

Consider a pipe AB of varying cross sectional area  $a_1$  and  $a_2$  such that  $a_1 > a_2$ . A non-viscous and incompressible liquid flows steadily through the pipe, with velocities  $v_1$  and  $v_2$  in area  $a_1$  and  $a_2$ , respectively as shown in Figure 7.32.



**Fig 7.32** A streamlined flow of fluid through a pipe of varying cross sectional area

Let  $m_1$  be the mass of fluid flowing through section A in time  $\Delta t$ ,  $m_1 = (a_1 v_1 \Delta t) \rho$

Let  $m_2$  be the mass of fluid flowing through section B in time  $\Delta t$ ,  $m_2 = (a_2 v_2 \Delta t) \rho$

For an incompressible liquid, mass is conserved  $m_1 = m_2$

$$a_1 v_1 \Delta t \rho = a_2 v_2 \Delta t \rho$$

$$a_1 v_1 = a_2 v_2 \Rightarrow a v = \text{constant} \quad (7.35)$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general,  $a v = \text{constant}$ , which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

**EXAMPLE 7.14**

In a normal adult, the average speed of the blood through the aorta (radius  $r = 0.8 \text{ cm}$ ) is  $0.33 \text{ ms}^{-1}$ . From the aorta, the blood goes into major arteries, which are 30 in number, each of radius  $0.4 \text{ cm}$ . Calculate the speed of the blood through the arteries.

**Solution:**

$$a_1 v_1 = 30 a_2 v_2 \Rightarrow \pi r_1^2 v_1 = 30 \pi r_2^2 v_2$$

$$v_2 = \frac{1}{30} \left( \frac{r_1}{r_2} \right)^2 v_1 \Rightarrow v_2 = \frac{1}{30} \times \left( \frac{0.8 \times 10^{-2} \text{ m}}{0.4 \times 10^{-2} \text{ m}} \right)^2$$

$$\times (0.33 \text{ ms}^{-1})$$

$$v_2 = 0.044 \text{ m s}^{-1}$$

**7.6.2 Pressure, kinetic and potential energy of liquids**

A liquid in a steady flow can possess three kinds of energy. They are (1) Kinetic energy, (2) Potential energy, and (3) Pressure energy, respectively.

- Kinetic energy:** The kinetic energy of a liquid of mass  $m$  moving with a velocity  $v$  is given by

$$KE = \frac{1}{2} mv^2$$

The kinetic energy per unit mass =

$$\frac{KE}{m} = \frac{\frac{1}{2} mv^2}{m} = \frac{1}{2} v^2$$



Similarly, the kinetic energy per unit volume

$$= \frac{KE}{\text{volume}} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2} \left( \frac{m}{V} \right) v^2 = \frac{1}{2} \rho v^2$$

ii) **Potential energy:** The potential energy of a liquid of mass  $m$  at a height  $h$  above the ground level is given by

$$PE = mgh$$

The potential energy per unit mass

$$= \frac{PE}{m} = \frac{mgh}{m} = gh$$

Similarly, the potential energy per unit

$$\text{volume} = \frac{PE}{V} = \frac{mgh}{V} = \left( \frac{m}{V} \right) gh = \rho gh$$

iii) **Pressure energy:** The energy acquired by a fluid by applying pressure on the fluid. We know that

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \Rightarrow \text{Force} = \text{Pressure} \times \text{Area}$$

$$F \times d = (P A) \times d = P (A \times d)$$

$$\Rightarrow F \times d = W = PV = \text{pressure energy}$$

$$\text{Therefore, pressure energy, } E_p = PV$$

The pressure energy per unit mass =

$$\frac{E_p}{m} = \frac{PV}{m} = \frac{P}{\frac{m}{V}} = \frac{P}{\rho}$$

Similarly, the potential energy per unit

$$\text{volume} = \frac{E_p}{V} = \frac{PV}{V} = P$$

### 7.6.3 Bernoulli's theorem and its applications

In 1738, the Swiss scientist Daniel Bernoulli developed a relationship for the flow of fluid through a pipe of varying cross section. He proposed a theorem for the streamline flow of a liquid based on the law of conservation of energy.

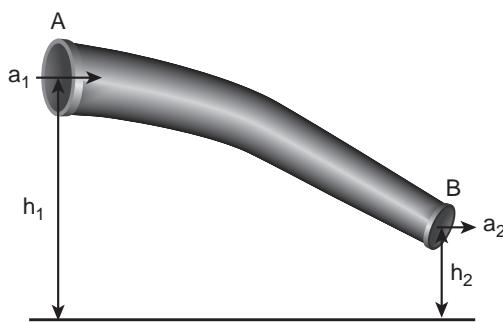
### Bernoulli's theorem

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant. Mathematically,

$$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant} \quad (7.36)$$

This is known as Bernoulli's equation.

**Proof:**



**Figure 7.33** Flow of liquid through a pipe AB

Let us consider a flow of liquid through a pipe AB as shown in Figure 7.33. Let V be the volume of the liquid when it enters A in a time  $t$  which is equal to the volume of the liquid leaving B in the same time. Let  $a_A$ ,  $v_A$  and  $P_A$  be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at A respectively.

Let the force exerted by the liquid at A is

$$F_A = P_A a_A$$

Distance travelled by the liquid in time  $t$  is

$$d = v_A t$$

Therefore, the work done is

$$W = F_A d = P_A a_A v_A t$$

But  $a_A v_A t = a_A d = V$ , volume of the liquid entering at A.

Thus, the work done is the pressure energy (at A),  $W = F_A d = P_A V$

Pressure energy per unit volume at

$$A = \frac{\text{Pressure energy}}{\text{volume}} = \frac{P_A V}{V} = P_A$$

Pressure energy per unit mass at

$$A = \frac{\text{Pressure energy}}{\text{mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$$

Since  $m$  is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is

$$E_{PA} = P_A V = P_A V \times \left( \frac{m}{m} \right) = m \frac{P_A}{\rho}$$

Potential energy of the liquid at A,

$$PE_A = mg h_A,$$

Due to the flow of liquid, the kinetic energy of the liquid at A,

$$KE_A = \frac{1}{2} m v_A^2$$

Therefore, the total energy due to the flow of liquid at A,  $E_A = EP_A + KE_A + PE_A$

$$E_A = m \frac{P_A}{\rho} + \frac{1}{2} mv_A^2 + mg h_A$$

Similarly, let  $a_B$ ,  $v_B$ , and  $P_B$  be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B. Calculating the total energy at EB, we get

$$E_B = m \frac{P_B}{\rho} + \frac{1}{2} mv_B^2 + mg h_B$$

From the law of conservation of energy,

$$EA = EB$$

$$m \frac{P_A}{\rho} + \frac{1}{2} mv_A^2 + mg h_A = m \frac{P_B}{\rho} + \frac{1}{2} mv_B^2 + mg h_B$$

$$\frac{P_A}{\rho} + \frac{1}{2} v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2} v_B^2 + gh_B = \text{constant}$$

Thus, the above equation can be written as

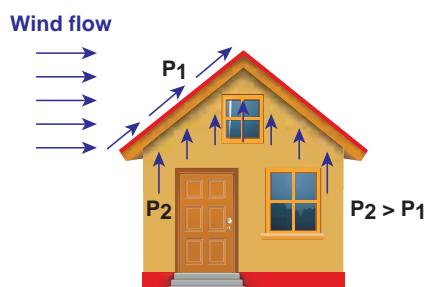
$$\frac{P}{\rho g} + \frac{1}{2} \frac{v^2}{g} + h = \text{constant}$$

The above equation is the consequence of the conservation of energy which is true until there is no loss of energy due to friction. But in practice, some energy is lost due to friction. This arises due to the fact that in a fluid flow, the layers flowing with different velocities exert frictional forces on each other. This loss of energy is generally converted into heat energy. Therefore, Bernoulli's relation is strictly valid for fluids with zero viscosity or non-viscous liquids. Notice that when the liquid flows through a horizontal pipe, then  $h = 0 \Rightarrow \frac{P}{\rho g} + \frac{1}{2} \frac{v^2}{g} = \text{constant}$ .

### Applications of Bernoulli's Theorem

#### (a) Blowing off roofs during wind storm

In olden days, the roofs of the huts or houses were designed with a slope as shown in Figure 7.34. One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.



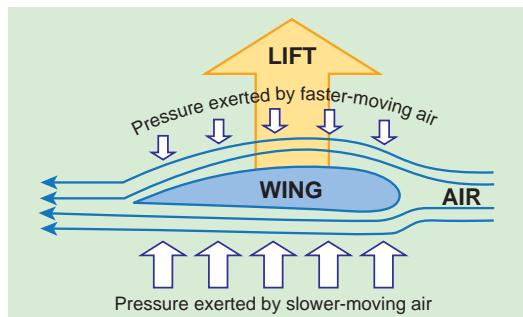
**Figure 7.34** Roofs of the huts or houses

During cyclonic condition, the roof is blown off without damaging the other parts of the house. In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure  $P_1$ . The pressure under the roof  $P_2$  is greater. Therefore, this pressure difference ( $P_2 - P_1$ ) creates an up thrust and the roof is blown off.

### (b) Aerofoil lift

The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the rear edge. As the aircraft moves, the air moves faster above the aerofoil than at the bottom as shown in Figure 7.35.

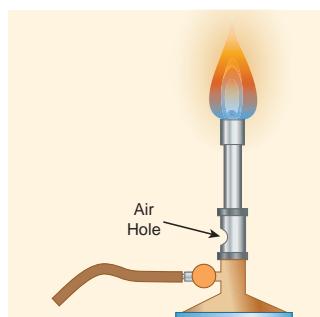
According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an upthrust called the dynamic lift to the aircraft.



**Figure 7.35** Aerofoil lift

### (c) Bunsen burner

In this, the gas comes out of the nozzle with high velocity, hence the pressure in the stem decreases. So outside air reaches into the burner through an air vent and the mixture of air and gas gives a blue flame as shown in Figure 7.36.

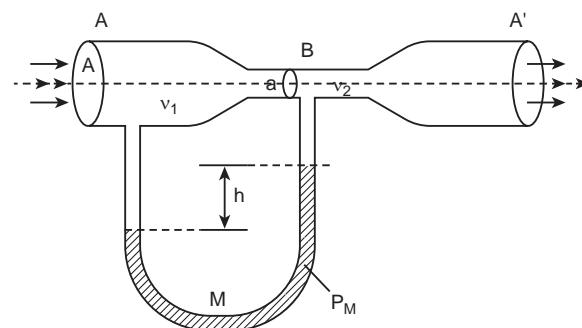


**Figure 7.36** Bunsen burner

### (d) Venturimeter

This device is used to measure the rate of flow (or say flow speed) of the incompressible

fluid flowing through a pipe. It works on the principle of Bernoulli's theorem. It consists of two wider tubes A and A' (with cross sectional area A) connected by a narrow tube B (with cross sectional area a). A manometer in the form of U-tube is also attached between the wide and narrow tubes as shown in Figure 7.37. The manometer contains a liquid of density ' $\rho_m$ '.



**Figure 7.37** A schematic diagram of venturimeter

Let  $P_1$  be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density ' $\rho$ ' flows from the pipe with speed ' $v_1$ ' and into the narrow region, its speed increases to ' $v_2$ '. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure  $P_2$  at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ( $\Delta P = P_1 - P_2$ ) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that  $Av_1 = av_2$  which means that

$$v_2 = \frac{A}{a} v_1$$

Using Bernoulli's equation,

$$P_1 + \rho \frac{v_1^2}{2} = P_2 + \rho \frac{v_2^2}{2} = P_2 + \rho \frac{1}{2} \left( \frac{A}{a} v_1 \right)^2$$

From the above equation, the pressure difference

$$\Delta P = P_1 - P_2 = \rho \frac{v_1^2}{2} \frac{(A^2 - a^2)}{a^2}$$

Thus, the speed of flow of fluid at the wide end of the tube A

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)} \Rightarrow v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$$

The volume of the liquid flowing out per second is

$$V = Av_1 = A \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}} = aA \sqrt{\frac{2(\Delta P)}{\rho(A^2 - a^2)}}$$

#### (e) Other applications

This Bernoulli's concept is mainly used in the design of carburetor of automobiles, filter pumps, atomizers, and sprayers. For

example, the carburetor has a very fine channel called nozzle through which the air is allowed to flow in larger speed. In this case, the pressure is lowered at the narrow neck and in turn, the required fuel or petrol is sucked into the chamber so as to provide the correct mixture of air and fuel necessary for ignition process.

#### ACTIVITY

A bottle is filled with thermocol balls. One end of a flexible tube is kept inside the bottle immersed inside the balls. The free end is rotated and we find the balls sprayed all around. This explains the working of an atomizer or sprayer.



A spider web is much stronger than what we think. A single strand of spider silk can stop flying insects which are tens and thousands times its mass. The young's modulus of the spider web is approximately  $4.5 \times 10^9 \text{ N m}^{-2}$ . Compare this value with Young's modulus of wood.



## SUMMARY

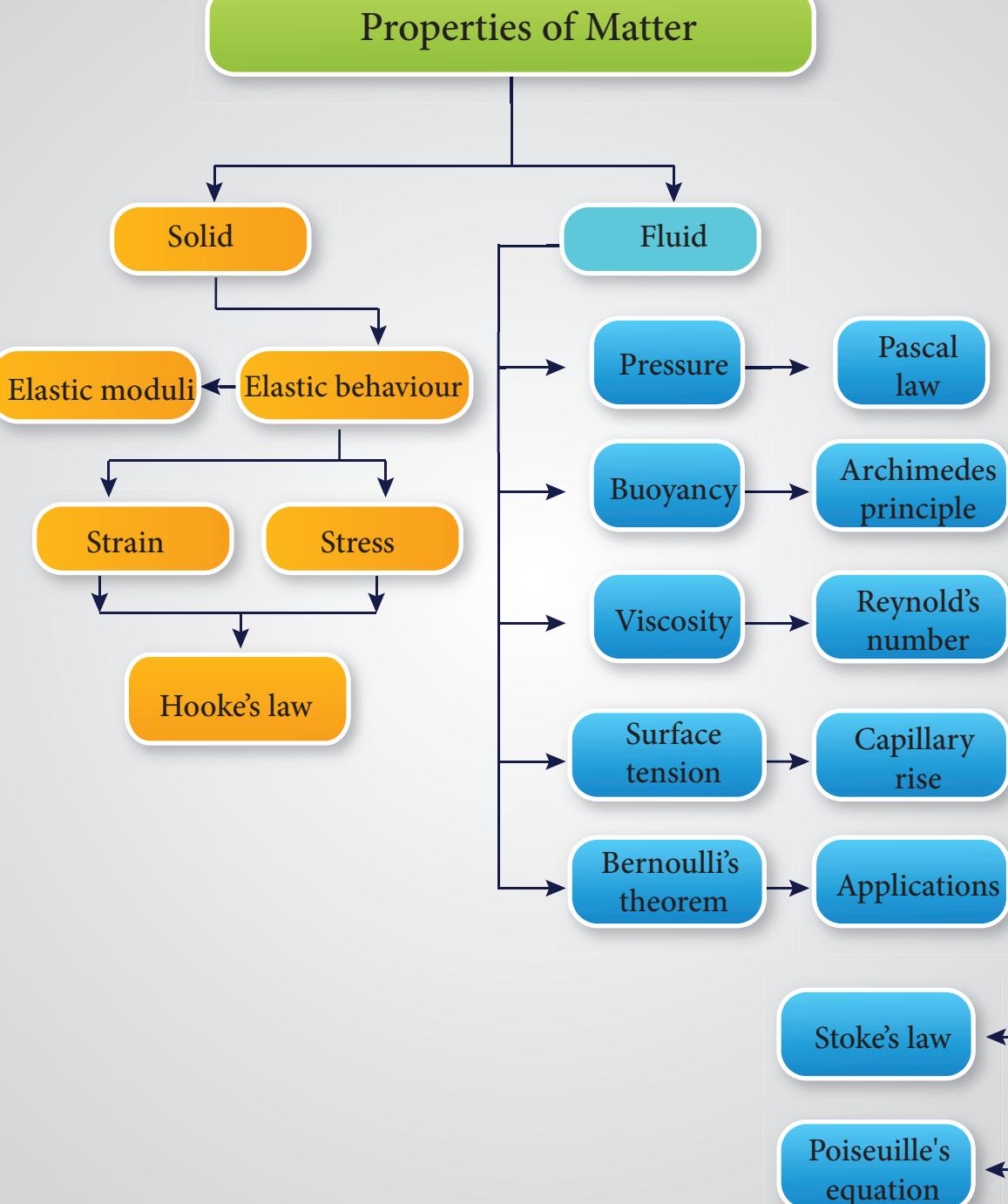
- The force between the atoms of an element is called inter-atomic force whereas the force between the molecules of a compound is called inter-molecular force.
- *Hooke's law*: within the elastic limit, the stress is directly proportional to strain.
- The force per unit area is known as *stress*. If F is the force applied and A is the area of cross section of the body then the magnitude of *stress* is equal to  $F/A$ . Tensile or compressional stress can be expressed using a single term called *longitudinal stress*.
- The ratio of change in length to the original length of a cylinder is  $\Delta L/L$ , which is known as *longitudinal strain*
- Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called the Young's modulus of the material of the wire.
- Within the elastic limit, the ratio of volume stress to the volume strain is called the *bulk modulus*.
- Within the elastic limit, the ratio of shear stress to the shear strain is called the *rigidity modulus*.
- Poisson's ratio = lateral strain/longitudinal strain
- The elastic potential energy stored in the wire per unit volume is  

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$$
, where Y denotes Young's modulus of the material.
- If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.
- The total pressure at a depth h below the liquid surface is  $P = P_a + \rho gh$ , where  $P_a$  is the atmospheric pressure which is equal to  $1.013 \times 10^5$  Pa.
- Pascal's law states that the pressure in a fluid at rest is the same at all points if they are at the same height.
- The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.
- The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.
- When a liquid flows such that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor then the flow of liquid is said to be streamlined flow.
- During the flow of fluid, when the critical velocity is exceeded by the moving fluid, the motion becomes *turbulent*.
- Reynold's number has a significance as it decides which decides whether the flow of fluid through a cylindrical pipe is streamlined or turbulent.

## SUMMARY (cont.)

- Stokes formula  $F = 6\pi\eta av$ , where  $F$  is the viscous force acting on a sphere of radius  $a$  and  $v$  is the terminal velocity of the sphere.
- The surface tension of a liquid is defined as the force of tension acting on a unit length of an imaginary line drawn on the free surface of the liquid, the direction of the force being perpendicular to the line so drawn and acting parallel to the surface.
- The angle between tangents drawn at the point of contact to the liquid surface and solid surface inside the liquid is called the *angle of contact* for a pair of solid and liquid.
- The flow of a fluid is said to be steady if, at any given point, the velocity of each passing fluid particle remains constant with respect to time.
- The equation  $a_1 v_1 = a_2 v_2$  is called the equation of continuity for a flow of fluid through a tube and it is due to the conservation of mass in the flow of fluids. It states that the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains constant. i.e.,  $P/\rho + v^2/2 + gh = \text{constant}$ .

## CONCEPT MAP





## I. Multiple Choice Questions

1. Consider two wires X and Y. The radius of wire X is 3 times the radius of Y. If they are stretched by the same load then the stress on Y is

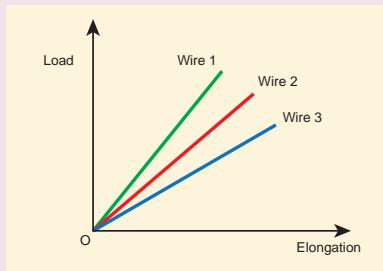
- (a) equal to that on X
- (b) thrice that on X
- (c) nine times that on X
- (d) Half that on X



2. If a wire is stretched to double of its original length, then the strain in the wire is

- |       |       |
|-------|-------|
| (a) 1 | (b) 2 |
| (c) 3 | (d) 4 |

3. The load – elongation graph of three wires of the same material are shown in figure. Which of the following wire is the thickest?



- (a) wire 1
- (b) wire 2
- (c) wire 3
- (d) all of them have same thickness

4. For a given material, the rigidity modulus is  $\left(\frac{1}{3}\right)^{rd}$  of Young's modulus. Its Poisson's ratio is

- |         |          |
|---------|----------|
| (a) 0   | (b) 0.25 |
| (c) 0.3 | (d) 0.5  |

5. A small sphere of radius 2cm falls from rest in a viscous liquid. Heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity is proportional to

(NEET model 2018)

- |           |           |
|-----------|-----------|
| (a) $2^2$ | (b) $2^3$ |
| (c) $2^4$ | (d) $2^5$ |

6. Two wires are made of the same material and have the same volume. The area of cross sections of the first and the second wires are A and 2A respectively. If the length of the first wire is increased by  $\Delta l$  on applying a force F, how much force is needed to stretch the second wire by the same amount?

(NEET model 2018)

- |       |        |
|-------|--------|
| (a) 2 | (b) 4  |
| (c) 8 | (d) 16 |

7. With an increase in temperature, the viscosity of liquid and gas, respectively will

- (a) increase and increase
- (b) increase and decrease
- (c) decrease and increase
- (d) decrease and decrease

8. The Young's modulus for a perfect rigid body is

- |         |              |
|---------|--------------|
| (a) 0   | (b) 1        |
| (c) 0.5 | (d) infinity |

9. Which of the following is not a scalar?
- viscosity
  - surface tension
  - pressure
  - stress
10. If the temperature of the wire is increased, then the Young's modulus will
- remain the same
  - decrease
  - increase rapidly
  - increase by very a small amount
11. Copper of fixed volume  $V$  is drawn into a wire of length  $l$ . When this wire is subjected to a constant force  $F$ , the extension produced in the wire is  $\Delta l$ . If  $Y$  represents the Young's modulus, then which of the following graphs is a straight line?
- (NEET 2014 model)
- $\Delta l$  verses  $V$
  - $\Delta l$  verses  $Y$
  - $\Delta l$  verses  $F$
  - $\Delta l$  verses  $\frac{1}{l}$
12. A certain number of spherical drops of a liquid of radius  $R$  coalesce to form a single drop of radius  $R$  and volume  $V$ . If  $T$  is the surface tension of the liquid, then
- energy =  $4 V T \left( \frac{1}{r} - \frac{1}{R} \right)$  is released
  - energy =  $3 V T \left( \frac{1}{r} + \frac{1}{R} \right)$  is absorbed
  - energy =  $3 V T \left( \frac{1}{r} - \frac{1}{R} \right)$  is released
  - energy is neither released nor absorbed
13. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?
- length = 200 cm, diameter = 0.5 mm
  - length = 200 cm, diameter = 1 mm
  - length = 200 cm, diameter = 2 mm
  - length = 200 cm, diameter = 3 m
14. The wettability of a surface by a liquid depends primarily on
- viscosity
  - surface tension
  - density
  - angle of contact between the surface and the liquid
15. In a horizontal pipe of non-uniform cross section, water flows with a velocity of  $1 \text{ m s}^{-1}$  at a point where the diameter of the pipe is 20 cm. The velocity of water ( $\text{m s}^{-1}$ ) at a point where the diameter of the pipe is
- 8
  - 16
  - 24
  - 32

**Answers:**

- |       |       |       |       |
|-------|-------|-------|-------|
| 1) c  | 2) a  | 3) a  | 4) d  |
| 5) d  | 6) b  | 7) c  | 8) d  |
| 9) d  | 10) b | 11) c | 12) c |
| 13) a | 14) d | 15) b |       |

**II. Short Answer Questions**

- Define stress and strain.
- State Hooke's law of elasticity.
- Define Poisson's ratio.
- Explain elasticity using intermolecular forces.
- Which one of these is more elastic, steel or rubber? Why?

6. A spring balance shows wrong readings after using for a long time. Why?
7. What is the effect of temperature on elasticity?
8. Write down the expression for the elastic potential energy of a stretched wire.
9. State Pascal's law in fluids.
10. State Archimedes principle.
11. What do you mean by upthrust or buoyancy?
12. State the law of floatation.
13. Define coefficient of viscosity of a liquid.
14. Distinguish between streamlined flow and turbulent flow.
15. What is Reynold's number? Give its significance.
16. Define terminal velocity.
17. Write down the expression for the Stoke's force and explain the symbols involved in it.
18. State Bernoulli's theorem.
19. What are the energies possessed by a liquid? Write down their equations.
20. Two streamlines cannot cross each other. Why?
21. Define surface tension of a liquid. Mention its S.I unit and dimension.
22. How is surface tension related to surface energy?
23. Define angle of contact for a given pair of solid and liquid.
24. Distinguish between cohesive and adhesive forces.
25. What are the factors affecting the surface tension of a liquid?
26. What happens to the pressure inside a soap bubble when air is blown into it?
27. What do you mean by capillarity or capillary action?
28. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?
29. State the principle and usage of Venturimeter.

### III. Long Answer Questions

1. State Hooke's law and verify it with the help of an experiment.
2. Explain the different types of modulus of elasticity.
3. Derive an expression for the elastic energy stored per unit volume of a wire.
4. Derive an equation for the total pressure at a depth 'h' below the liquid surface.
5. State and prove Pascal's law in fluids.
6. State and prove Archimedes principle.
7. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.
8. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.
9. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.
10. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.
11. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

12. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.
13. Describe the construction and working of venturimeter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

#### IV. Numerical Problems

1. A capillary of diameter  $d_{mm}$  is dipped in water such that the water rises to a height of  $30\text{ mm}$ . If the radius of the capillary is made  $\left(\frac{2}{3}\right)$  of its previous value, then compute the height up to which water will rise in the new capillary?

(Answer: 45 mm)

2. A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of  $4 \times 10^5 \text{ N}$  is applied at the other end. If the rigidity modulus of the cylinder is  $6 \times 10^{10} \text{ N m}^{-2}$  then, calculate the twist produced in the cylinder.

(Answer: 45.60)

3. A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside

the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

4. A block of Ag of mass  $x \text{ kg}$  hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block. (Answer:  $x = 4 \text{ kg}$ )
5. The reading of pressure meter attached with a closed pipe is  $5 \times 10^5 \text{ N m}^{-2}$ . On opening the valve of the pipe, the reading of the pressure meter is  $4.5 \times 10^5 \text{ N m}^{-2}$ . Calculate the speed of the water flowing in the pipe.

(Answer:  $10 \text{ ms}^{-1}$ )

#### V. Conceptual questions

1. Why coffee runs up into a sugar lump (a small cube of sugar) when one corner of the sugar lump is held in the liquid?
2. Why two holes are made to empty an oil tin?
3. We can cut vegetables easily with a sharp knife as compared to a blunt knife. Why?
4. Why the passengers are advised to remove the ink from their pens while going up in an aeroplane?
5. We use straw to suck soft drinks, why?

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3. H.C.Verma, Concepts of physics volume 1 and Volume 2, Bharati Bhawan Publishers



## ICT CORNER

# Properties of Matter

Through this activity you will be able to learn about the Viscosity.



### STEPS:

- Use the URL or scan the QR code to open ‘Viscosity’ activity page.
- In the activity window , ‘Select Viscosity’ by dragging the ball in the meter in the side.
- Select the ‘Start’ button , sphere falls from top to bottom of the beaker. We can see the changes in Distance and Time.
- Spheres can be reset to the top of each beaker by clicking the ‘Reset’ button.

**Step1**



**Step2**



**Step3**



**Step4**



### URL:

<http://www.geo.cornell.edu/hawaii/220/PRI/viscosity.html>

\* Pictures are indicative only.

\* If browser requires, allow Flash Player or Java Script to load the page.



B163\_11\_Phys\_EM

# UNIT 8

## HEAT AND THERMODYNAMICS



*Classical thermodynamics.... is the only physical theory of universal content which I am convinced... will never be overthrown.* – Albert Einstein



### LEARNING OBJECTIVES

In this unit, a student is exposed to

- meaning of heat, work and temperature
- ideal gas laws
- concept of specific heat capacity
- thermal expansion of solids, liquids and gases
- various states of matter
- Newton's law of cooling
- Stefan's law and Wien's law
- meaning of thermodynamic equilibrium
- meaning of internal energy
- zeroth and first laws of thermodynamics
- various thermodynamic processes
- work done in various thermodynamic processes
- second law of thermodynamics
- working of carnot engine and refrigerator



### 8.1

## HEAT AND TEMPERATURE

### 8.1.1 Introduction

Temperature and heat play very important role in everyday life. All species can function properly only if its body is maintained at a particular temperature. In fact life on Earth is possible because the Sun maintains its temperature. Understanding the meaning of temperature and heat are very crucial to understand the nature. Thermodynamics

is a branch of physics which explains the phenomena of temperature, heat etc. The concepts presented in this chapter will help us to understand the terms 'hot' and 'cold' and also differentiate heat from temperature. In thermodynamics, heat and temperature are two different but closely related parameters.

### 8.1.2 Meaning of heat

When an object at higher temperature is placed in contact with another object at lower temperature, there will be a spontaneous flow of energy from the object

at higher temperature to the one at lower temperature. This energy is called heat. This process of energy transfer from higher temperature object to lower temperature object is called heating. Due to flow of heat sometimes the temperature of the body will increase or sometimes it may not increase.

**Note**

There is a misconception that heat is a quantity of energy. People often talk ‘this water has more heat or less heat’. These words are meaningless. Heat is not a quantity. Heat is an energy in transit which flows from higher temperature object to lower temperature object. Once the heating process is stopped we cannot use the word heat. When we use the word ‘heat’, it is the energy in transit but not energy stored in the body.

### **EXAMPLE 8.1**

- ‘A lake has more rain’.
  - ‘A hot cup of coffee has more heat’.
- What is wrong in these two statements?

#### **Solution**

- When it rains, lake receives water from the cloud. Once the rain stops, the lake will have more water than before raining. Here ‘raining’ is a process which brings water from the cloud. Rain is not a quantity rather it is water in transit. So the statement ‘lake has more rain’ is wrong, instead the ‘lake has more water’ will be appropriate.

- When heated, a cup of coffee receives heat from the stove. Once the coffee is taken from the stove, the cup of coffee has more internal energy than before. ‘Heat’ is the energy in transit and which flows from an object at higher temperature to an object at lower temperature. Heat is not a quantity. So the statement ‘A hot cup of coffee has more heat’ is wrong, instead ‘coffee is hot’ will be appropriate.

### **8.1.3 Meaning of work**

When you rub your hands against each other the temperature of the hands increases. You have done some work on your hands by rubbing. The temperature of the hands increases due to this work. Now if you place your hands on the chin, the temperature of the chin increases. This is because the hands are at higher temperature than the chin. In the above example, the temperature of hands is increased due to work and temperature of the chin is increased due to heat transfer from the hands to the chin. It is shown in the Figure 8.1

By doing work on the system, the temperature in the system will increase and sometimes may not. Like heat, work is also



**Figure 8.1** Difference between work and heat

not a quantity and through the work energy is transferred to the system . So we cannot use the word ‘the object contains more work’ or ‘less work’.

Either the system can transfer energy to the surrounding by doing work on surrounding or the surrounding may transfer energy to the system by doing work on the system. For the transfer of energy from one body to another body through the process of work, they need not be at different temperatures.

#### 8.1.4 Meaning of temperature

Temperature is the degree of hotness or coolness of a body. Hotter the body higher is its temperature. The temperature will determine the direction of heat flow when two bodies are in thermal contact.

The SI unit of temperature is ***kelvin (K)***.

In our day to day applications, ***Celsius*** ( $^{\circ}\text{C}$ ) and ***Fahrenheit*** ( $^{\circ}\text{F}$ ) scales are used.

Temperature is measured with a thermometer.

The conversion of temperature from one scale to other scale is given in Table 8.1

**Table 8.1** Temperature conversion

Scale	To Kelvin	From Kelvin
Celsius	$K = ^{\circ}\text{C} + 273.15$	$^{\circ}\text{C} = K - 273.15$
Fahrenheit	$K = (^{\circ}\text{F} + 459.67) \div 1.8$	$^{\circ}\text{F} = (K \times 1.8) - 459.67$
Scale	To Fahrenheit	From Fahrenheit
Celsius	$^{\circ}\text{F} = (1.8 \times ^{\circ}\text{C}) + 32$	$^{\circ}\text{C} = (^{\circ}\text{F} - 32) \div 1.8$
Scale	To Celsius	From Celsius
Fahrenheit	$^{\circ}\text{C} = (^{\circ}\text{F} - 32) \div 1.8$	$^{\circ}\text{F} = (1.8 \times ^{\circ}\text{C}) + 32$

## 8.2

### THERMAL PROPERTIES OF MATTER

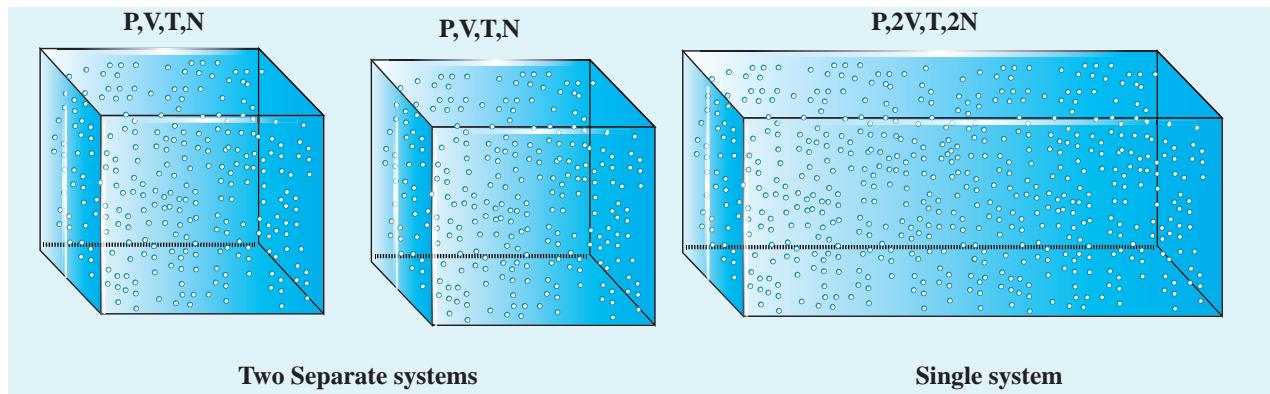
#### 8.2.1 Boyle's law, Charles' law and ideal gas law

For a given gas at low pressure (density) kept in a container of volume V, experiments revealed the following information.

- When the gas is kept at constant temperature, the pressure of the gas is inversely proportional to the volume.  
 $P \propto \frac{1}{V}$ . It was discovered by Robert Boyle (1627-1691) and is known as Boyle's law.
- When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature.  
 $V \propto T$ . It was discovered by Jacques Charles (1743-1823) and is known as Charles' law.
- By combining these two equations we have

$$PV = CT. \text{ Here } C \text{ is a positive constant.}$$

We can infer that C is proportional to the number of particles in the gas container by considering the following argument. If we take two containers of same type of gas with same volume V, same pressure P and same temperature T, then the gas in each container obeys the above equation.  $PV = CT$ . If the two containers of gas is considered as a single system, then the pressure and temperature of this combined system will be same but volume will be twice and number of particles will also be double as shown in Figure 8.2



**Figure 8.2** Ideal gas law

For this combined system,  $V$  becomes  $2V$ , so  $C$  should also double to match with the ideal gas equation  $\frac{P(2V)}{T} = 2C$ . It implies that  $C$  must depend on the number of particles in the gas and also should have the dimension of  $\left[\frac{PV}{T}\right] = JK^{-1}$ . So we can write the constant  $C$  as  $k$  times the number of particles  $N$ .

Here  $k$  is the Boltzmann constant ( $1.381 \times 10^{-23} \text{ JK}^{-1}$ ) and it is found to be a universal constant.

So the ideal gas law can be stated as follows

$$PV = NkT \quad (8.1)$$

The equation (8.1) can also be expressed in terms of mole.

Mole is the practical unit to express the amount of gas. One mole of any substance is the amount of that substance which contains Avogadro number ( $N_A$ ) of particles (such as atoms or molecules). The Avogadro's number  $N_A$  is defined as the number of carbon atoms contained in exactly 12 g of  $^{12}\text{C}$ .

Suppose if a gas contains  $\mu$  mole of particles then the total number of particles can be written as

$$N = \mu N_A \quad (8.2)$$

where  $N_A$  is Avogadro number ( $6.023 \times 10^{23} mol^{-1}$ )

Substituting for  $N$  from equation (8.2), the equation (8.1) becomes

$PV = \mu N_A kT$ . Here  $N_A k = R$  called universal gas constant and its value is 8.314 J/mol.K.

So the ideal gas law can be written for  $\mu$  mole of gas as

$$PV = \mu RT \quad (8.3)$$

This is called the equation of state for an ideal gas. It relates the pressure, volume and temperature of thermodynamic system at equilibrium.

## EXAMPLE 8.2

A student comes to school by a bicycle whose tire is filled with air at a pressure 240 kPa at 27°C. She travels 8 km to reach the school and the temperature of the bicycle tire increases to 39°C. What is the change in pressure in the tire when the student reaches school?



### Solution

We can take air molecules in the tire as an ideal gas. The number of molecules and the volume of tire remain constant. So the air molecules at  $27^\circ\text{C}$  satisfies the ideal gas equation  $P_1 V_1 = NkT_1$  and at  $39^\circ\text{C}$  it satisfies  $P_2 V_2 = NkT_2$

But we know

$$V_1 = V_2 = V$$

$$\frac{P_1 V}{P_2 V} = \frac{NkT_1}{NkT_2}$$

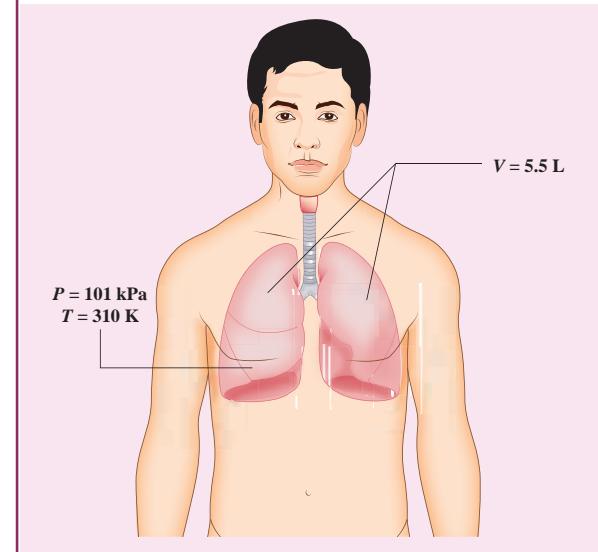
$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$P_2 = \frac{T_2}{T_1} P_1$$

$$P_2 = \frac{312K}{300K} \cdot 240 \cdot 10^3 \text{ Pa} = 249.6 \text{ kPa}$$

### EXAMPLE 8.3

When a person breathes, his lungs can hold up to 5.5 Litre of air at body temperature  $37^\circ\text{C}$  and atmospheric pressure (1 atm = 101 kPa). This Air contains 21% oxygen. Calculate the number of oxygen molecules in the lungs.



### Solution

We can treat the air inside the lungs as an ideal gas. To find the number of molecules, we can use the ideal gas law.

$$PV = NkT$$

Here volume is given in the Litre. 1 Litre is volume occupied by a cube of side 10 cm.

$$1 \text{ Litre} = 10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} = 10^{-3} \text{ m}^3$$

$$N = \frac{PV}{kT} = \frac{1.01 \times 10^5 \text{ Pa} \times 5.5 \times 10^{-3} \text{ m}^3}{1.38 \times 10^{-23} \text{ J K}^{-1} \times 310 \text{ K}}$$

$$= 1.29 \times 10^{23} \text{ Molecules}$$

Only 21% of N are oxygen. The total number of oxygen molecules

$$= 1.29 \times 10^{23} \times \frac{21}{100}$$

Number of oxygen molecules

$$= 2.7 \times 10^{22} \text{ molecules}$$

### EXAMPLE 8.4

Calculate the volume of one mole of any gas at STP and at room temperature (300K) with the same pressure 1 atm.

**Solution:**

Here STP means standard temperature ( $T=273\text{K}$  or  $0^\circ\text{C}$ ) and Pressure ( $P=1\text{ atm}$  or  $101.3\text{ kPa}$ )

We can use ideal gas equation  $V = \frac{\mu RT}{P}$ .

Here  $\mu = 1\text{ mol}$  and  $R = 8.314\text{ J/mol.K}$ .

By substituting the values

$$V = \frac{(1\text{mol})(8.314 \frac{\text{J}}{\text{mol}} \text{K})(273\text{K})}{1.013 \times 10^5 \text{Nm}^{-2}}$$

$$= 22.4 \times 10^{-3} \text{m}^3$$

We know that  $1\text{ Litre (L)} = 10^{-3}\text{m}^3$ . So we can conclude that  $1\text{ mole}$  of any ideal gas has volume  $22.4\text{ L}$ .

By multiplying  $22.4\text{L}$  by  $\frac{300\text{K}}{273\text{K}}$  we get the volume of one mole of gas at room temperature. It is  $24.6\text{ L}$ .

**EXAMPLE 8.5**

Estimate the mass of air in your class room at NTP. Here NTP implies normal temperature (room temperature) and  $1\text{ atmospheric pressure}$ .

**Solution**

The average size of a class is  $6\text{m length}$ ,  $5\text{ m breadth}$  and  $4\text{ m height}$ . The volume of the room  $V = 6 \times 5 \times 4 = 120\text{m}^3$ . We can

determine the number of mole. At room temperature  $300\text{K}$ , the volume of a gas occupied by any gas is equal to  $24.6\text{L}$ .

The number of mole  $\mu = \frac{120\text{m}^3}{24.6 \times 10^{-3} \text{m}^3}$   
 $\approx 4878\text{ mol}$ .

Air is the mixture of about  $20\%$  oxygen,  $79\%$  nitrogen and remaining one percent are argon, hydrogen, helium, and xenon. The molar mass of air is  $29\text{ gmol}^{-1}$ .

So the total mass of air in the room  $m = 4878 \times 29 = 141.4\text{kg}$ .

**8.2.2 Heat capacity and specific heat capacity**

Take equal amount of water and oil at temperature  $27^\circ\text{C}$  and heat both of them till they reach the temperature  $50^\circ\text{C}$ . Note down the time taken by the water and oil to reach the temperature  $50^\circ\text{C}$ . Obviously these times are not same. We can see that water takes more time to reach  $50^\circ\text{C}$  than oil. It implies that water requires more heat energy to raise its temperature than oil. Now take twice the amount of water at  $27^\circ\text{C}$  and heat it up to  $50^\circ\text{C}$ , note the time taken for this rise in temperature. The time taken by the water is now twice compared to the previous case.

We can define 'heat capacity' as the amount of heat energy required to raise the temperature of the given body from  $T$  to  $T + \Delta T$ .

$$\text{Heat capacity } S = \frac{\Delta Q}{\Delta T}$$

*Specific heat capacity of a substance is defined as the amount of heat energy required to raise the temperature of  $1\text{kg}$  of a substance by  $1\text{ Kelvin}$  or  $1^\circ\text{C}$*

$$\Delta Q = m s \Delta T$$

Therefore,

$$s = \frac{1}{m} \left( \frac{\Delta Q}{\Delta T} \right)$$



Where  $s$  is known as *specific heat capacity* of a substance and its value depends only on the nature of the substance not amount of substance

$\Delta Q$  = Amount of heat energy

$\Delta T$  = Change in temperature

$m$  = Mass of the substance

The SI unit for specific heat capacity is  $J \text{ kg}^{-1} \text{ K}^{-1}$ . Heat capacity and specific heat capacity are always positive quantities.

**Table 8.2** Specific heat capacity of some common substances at 1 atm (20°C)

Material	Specific heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ )
Air	1005
Lead	130
Copper	390
Iron (steel)	450
Glass	840
Aluminium	900
Human body	3470
Water	4186

From the table it is clear that water has the highest value of specific heat capacity. For this reason it is used as a coolant in power stations and reactors.



The term heat capacity or specific heat capacity does not mean that object contains a certain amount of heat. Heat is energy transfer from the object at higher temperature to the object at lower temperature. The correct usage is ‘internal energy capacity’. But for historical reason the term ‘heat capacity’ or ‘specific heat capacity’ are retained.



When two objects of same mass are heated at equal rates, the object with *smaller specific heat capacity* will have a *faster temperature increase*.

When two objects of same mass are *left to cool down*, the temperature of the object with *smaller specific heat capacity* will *drop faster*.

When we study properties of gases, it is more practical to use molar specific heat capacity. Molar specific heat capacity is defined as heat energy required to increase the temperature of one mole of substance by 1K or 1°C. It can be written as follows

$$C = \frac{1}{\mu} \left( \frac{\Delta Q}{\Delta T} \right)$$

Here  $C$  is known as *molar specific heat capacity* of a substance and  $\mu$  is number of moles in the substance.

The SI unit for molar specific heat capacity is  $J \text{ mol}^{-1} \text{ K}^{-1}$ . It is also a positive quantity.

### 8.2.3 Thermal expansion of solids, liquids and gases

*Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.*

All three states of matter (solid, liquid and gas) expand when heated. When a solid is heated, its atoms vibrate with higher amplitude about their fixed points. The relative change in the size of solids is small. Railway tracks are given small gaps so that in the summer, the tracks expand and do not buckle. Railroad tracks and bridges have expansion joints to allow them to expand and contract freely with temperature changes. It is shown in Figure 8.3



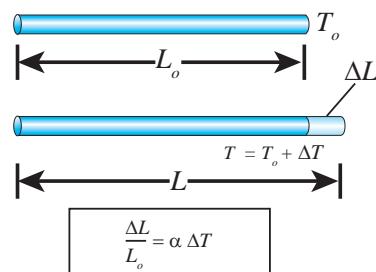
Figure 8.3 Expansion joints for safety

**Liquids**, have less intermolecular forces than solids and hence they expand more than solids. This is the principle behind the mercury thermometers.

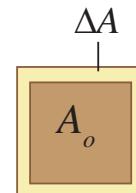
In the case of **gas** molecules, the intermolecular forces are almost negligible and hence they expand much more than solids. For example in hot air balloons when gas particles get heated, they expand and take up more space.

The increase in dimension of a body due to the increase in its temperature is called thermal expansion.

The expansion in length is called **linear expansion**. Similarly the expansion in area is termed as **area expansion** and the

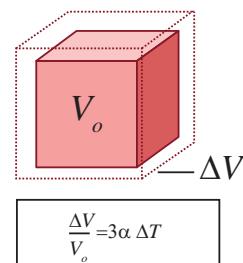


(a) Linear expansion



$$\frac{\Delta A}{A_o} = 2\alpha \Delta T$$

(b) Area expansion



$$\frac{\Delta V}{V_o} = 3\alpha \Delta T$$

(c) Volume expansion

Figure 8.4 Thermal expansions

expansion in volume is termed as **volume expansion**. It is shown in Figure 8.4

### Linear Expansion

In solids, for a small change in temperature  $\Delta T$ , the fractional change in length  $\left(\frac{\Delta L}{L}\right)$  is directly proportional to  $\Delta T$ .

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

Therefore,  $\alpha_L = \frac{\Delta L}{L \Delta T}$

Where,  $\alpha_L$  = coefficient of linear expansion.

$\Delta L$  = Change in length

$L$  = Original length

$\Delta T$  = Change in temperature.



### Solution

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

$$\Delta L = \alpha_L L \Delta T$$

$$\Delta L = 10 \times 10^{-6} \times 300 \times 23 = 0.69 \text{ m} = 69 \text{ cm}$$



- When the lid of a glass bottle is tight, keep the lid near the hot water which makes it easier to open. It is because the lid has higher thermal expansion than glass.
- When the hot boiled egg is dropped in cold water, the egg shell can be removed easily. It is because of the different thermal expansions of the shell and egg.

### EXAMPLE 8.6

Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is  $2^\circ\text{C}$  and in hot summer its average temperature  $25^\circ\text{C}$ . Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron  $\alpha = 10 \times 10^{-6}$  per  $^\circ\text{C}$

### Area Expansion

For a small change in temperature  $\Delta T$  the fractional change in area  $\left(\frac{\Delta A}{A}\right)$  of a substance is directly proportional to  $\Delta T$  and it can be written as

$$\frac{\Delta A}{A} = \alpha_A \Delta T$$

Therefore,  $\alpha_A = \frac{\Delta A}{A \Delta T}$

Where,  $\alpha_A$  = coefficient of area expansion.

$\Delta A$  = Change in area

$A$  = Original area

$\Delta T$  = Change in temperature

### Volume Expansion

For a small change in temperature  $\Delta T$  the fractional change in volume  $\left(\frac{\Delta V}{V}\right)$  of a substance is directly proportional to  $\Delta T$ .

$$\frac{\Delta V}{V} = \alpha_V \Delta T$$

Therefore,  $\alpha_V = \frac{\Delta V}{V \Delta T}$

Where,  $\alpha_v$  = coefficient of volume expansion.

$\Delta V$  = Change in volume

$V$  = Original volume

$\Delta T$  = Change in temperature

Unit of coefficient of linear, area and volumetric expansion of solids is  $^{\circ}\text{C}^{-1}$  or  $\text{K}^{-1}$

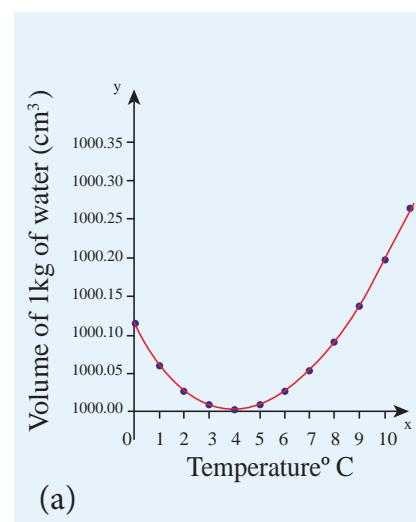


For a given specimen,

$$\frac{\Delta L}{L} = \alpha_L \Delta T \text{ (Linear expansion)}$$

$$\frac{\Delta A}{A} \approx 2 \alpha_L \Delta T \text{ (Area expansion} \approx 2 \times \text{Linear expansion)}$$

$$\frac{\Delta V}{V} \approx 3 \alpha_L \Delta T \text{ (Volume expansion} \approx 3 \times \text{Linear expansion)}$$



(a)

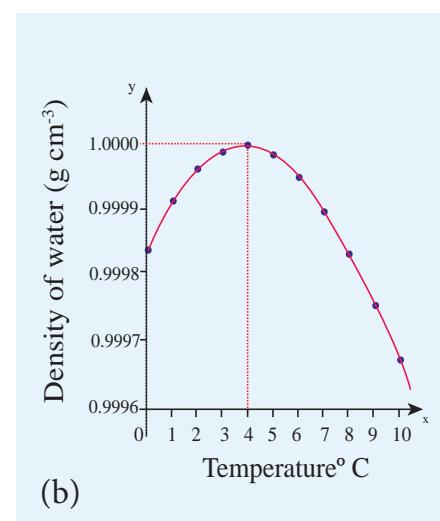


Figure 8.5 Anomalous Expansion of water

#### 8.2.4 Anomalous expansion of water

Liquids expand on heating and contract on cooling at moderate temperatures. But water exhibits an anomalous behavior. It contracts on heating between  $0^{\circ}\text{C}$  and  $4^{\circ}\text{C}$ . The volume of the given amount of water decreases as it is cooled from room temperature, until it reaches  $4^{\circ}\text{C}$ . Below  $4^{\circ}\text{C}$  the volume increases and so the density decreases. This means that the water has a maximum density at  $4^{\circ}\text{C}$ . This behavior of water is called anomalous expansion of water. It is shown in the Figure 8.5

In cold countries during the winter season, the surface of the lakes will be at lower temperature than the bottom as shown in the Figure 8.6. Since the solid water (ice) has lower density than its liquid form, below  $4^{\circ}\text{C}$ , the frozen water will be

on the top surface above the liquid water (ice floats). This is due to the anomalous expansion of water. As the water in lakes and ponds freeze only at the top the species living in the lakes will be safe at the bottom.

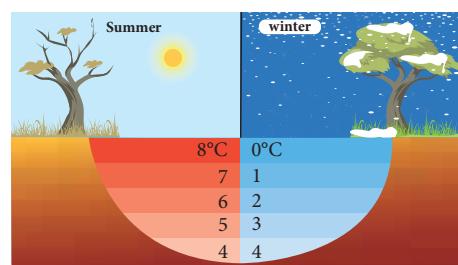


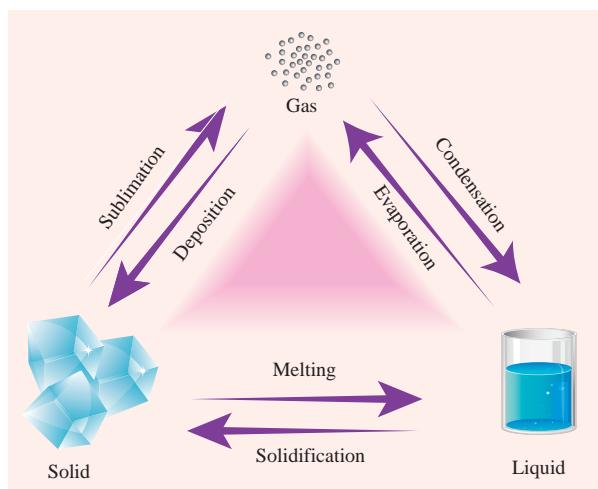
Figure 8.6 Anomalous expansion of water in lakes

### 8.2.5 Change of state

All matter exists normally in three states as solids, liquids or gases. Matter can be changed from one state to another either by heating or cooling.

Examples:

1. Melting (solid to liquid)
2. Evaporation (liquid to gas)
3. Sublimation (solid to gas)
4. Freezing / Solidification (liquid to solid)
5. Condensation (gas to liquid)



**Figure 8.7** Change of states of matter

#### Latent heat capacity:

While boiling a pot of water, the temperature of the water increases until it reaches 100 °C which is the boiling point of water, and then the temperature remains constant until all the water changes from liquid to gas. During this process heat is continuously added to the water. But the temperature of water does not increase above its boiling point. This is the concept of latent heat capacity.

*Latent heat capacity of a substance is defined as the amount of heat energy required to change the state of a unit mass of the material.*

$$Q = m \times L$$

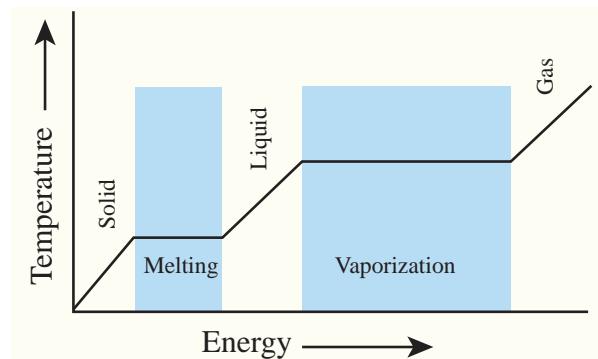
$$\text{Therefore, } L = \frac{Q}{m}$$

Where  $L$  = Latent heat capacity of the substance

$$Q = \text{Amount of heat}$$

$$m = \text{mass of the substance}$$

The SI unit for Latent heat capacity is  $J \text{ kg}^{-1}$ .



**Figure 8.8** Temperature versus heat for water



When heat is added or removed during a change of state, the temperature remains constant.

- The latent heat for a solid - liquid state change is called the latent heat of fusion ( $L_f$ )
- The latent heat for a liquid - gas state change is called the latent heat of vaporization ( $L_v$ )
- The latent heat for a solid - gas state change is called the latent heat of sublimation ( $L_s$ )

#### Triple point

*the triple point of a substance is the temperature and pressure at which the three phases (gas, liquid and solid) of that substance coexist in thermodynamic equilibrium.*

The triple point of water is at 273.1 K and a partial vapour pressure of 611.657 Pascal.

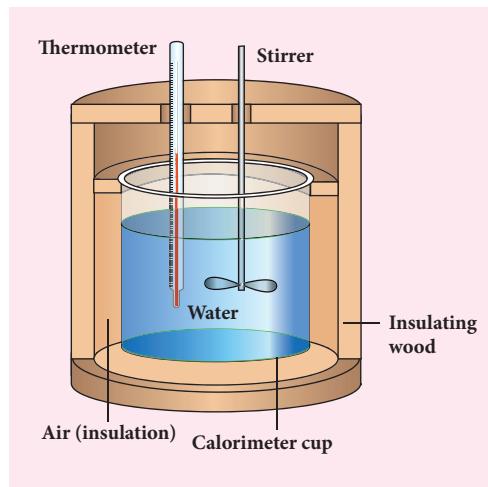
## 8.2.6 Calorimetry

Calorimetry means the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the cold body. No heat is allowed to escape to the surroundings. It can be mathematically expressed as

$$Q_{\text{gain}} = -Q_{\text{lost}}$$

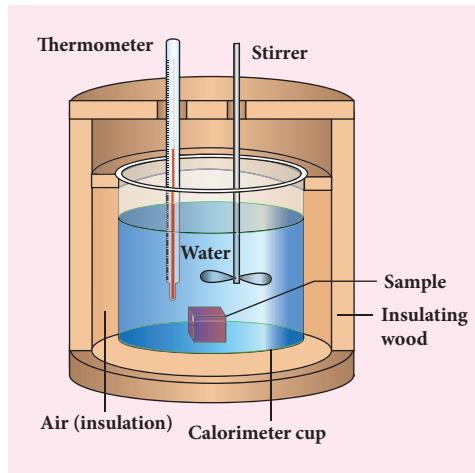
$$Q_{\text{gain}} + Q_{\text{lost}} = 0$$

Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water as shown in Figure 8.9.



**Figure 8.9** Calorimeter

A sample is heated at high temperature ( $T_1$ ) and immersed into water at room temperature ( $T_2$ ) in the calorimeter. After some time both sample and water reach a final equilibrium temperature  $T_f$ . Since the calorimeter is insulated, heat given by the hot sample is equal to heat gained by the water. It is shown in the Figure 8.10



**Figure 8.10** Calorimeter with sample of block

$$Q_{\text{gain}} = -Q_{\text{lost}}$$

Note the sign convention. The heat lost is denoted by negative sign and heat gained is denoted as positive.

From the definition of specific heat capacity

$$Q_{\text{gain}} = m_2 s_2 (T_f - T_2)$$

$$Q_{\text{lost}} = m_1 s_1 (T_f - T_1)$$

Here  $s_1$  and  $s_2$  specific heat capacity of hot sample and water respectively.

So we can write

$$m_2 s_2 (T_f - T_2) = -m_1 s_1 (T_f - T_1)$$

$$m_2 s_2 T_f - m_2 s_2 T_2 = -m_1 s_1 T_f + m_1 s_1 T_1$$

$$m_2 s_2 T_f + m_1 s_1 T_f = m_2 s_2 T_2 + m_1 s_1 T_1$$

The final temperature

$$T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

### EXAMPLE 8.7

If 5 L of water at 50°C is mixed with 4L of water at 30°C, what will be the final temperature of water? Take the specific heat capacity of water as  $4184 \text{ J kg}^{-1} \text{ K}^{-1}$ .

**Solution**

We can use the equation

$$T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

$m_1 = 5L = 5\text{kg}$  and  $m_2 = 4L = 4\text{kg}$ ,  $s_1 = s_2$  and  $T_1 = 50^\circ\text{C} = 323\text{K}$  and  $T_2 = 30^\circ\text{C} = 303\text{K}$ .

So

$$T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = \frac{5 \times 323 + 4 \times 303}{5 + 4} = 314.11\text{ K}$$

$$T_f = 314.11\text{ K} - 273\text{K} \approx 41^\circ\text{C}.$$

Suppose if we mix equal amount of water ( $m_1 = m_2$ ) with  $50^\circ\text{C}$  and  $30^\circ\text{C}$ , then the final temperature is average of two temperatures.

$$T_f = \frac{T_1 + T_2}{2} = \frac{323 + 303}{2} = 313\text{K} = 40^\circ\text{C}$$

Suppose if both the water are at  $30^\circ\text{C}$  then the final temperature will also  $30^\circ\text{C}$ . It implies that they are at equilibrium and no heat exchange takes place between each other.



It is important to note that the final equilibrium temperature of mixing of gas or liquid depends on mass of the substances, their specific heat capacities and their temperatures. Only if we mix the same substances at equal amount, the final temperature will be an average of the individual temperatures.

**8.2.7 Heat transfer**

As we have seen already heat is a energy in transit which is transferred from one body to another body due to temperature difference. There are three modes of heat transfer: Conduction, Convection and Radiation.

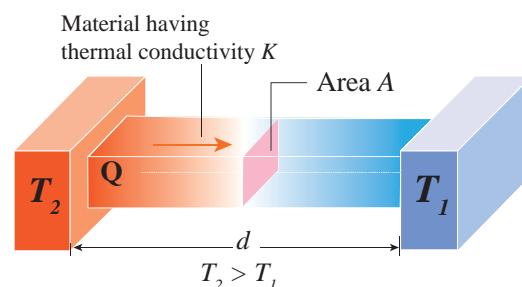
**Conduction**

Conduction is the process of direct transfer of heat through matter due to temperature difference. When two objects are in direct contact with one another, heat will be transferred from the hotter object to the colder one. The objects which allow heat to travel easily through them are called conductors.

**Thermal conductivity**

Thermal conductivity is the ability to conduct heat.

*The quantity of heat transferred through a unit length of a material in a direction normal to unit surface area due to a unit temperature difference under steady state conditions is known as thermal conductivity of a material.*



**Figure 8.11** Steady state heat flow by conduction.

In steady state, the rate of flow of heat  $Q$  is proportional to the temperature difference  $\Delta T$  and the area of cross section  $A$  and is inversely proportional to the length  $L$ . So the rate of flow of heat is written as

$$\frac{Q}{t} = \frac{KA\Delta T}{L}$$

Where,  $K$  is known as the **coefficient of thermal conductivity**.

(Not to be confused with Kelvin represented by upper case  $K$ )

The SI unit of thermal conductivity is  $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$  or  $\text{W m}^{-1} \text{K}^{-1}$ .

**Table 8.3:** Thermal conductivities (in  $\text{W m}^{-1} \text{K}^{-1}$ ) of some materials at 1 atm

Material	Thermal Conductivity	Material	Thermal Conductivity
Diamond	2300	Water	0.56
Silver	420	Human tissue	0.2
Copper	380	Wood	0.17
Aluminum	200	Helium	0.152
Steel	40	Cork	0.042
Glass	0.84	Air	0.023
Brick	0.84		
Ice	2		



Steady state:

The state at which temperature attains constant value everywhere and there is no further transfer of heat anywhere is called steady state.

Thermal conductivity depends on the nature of the material. For example silver and

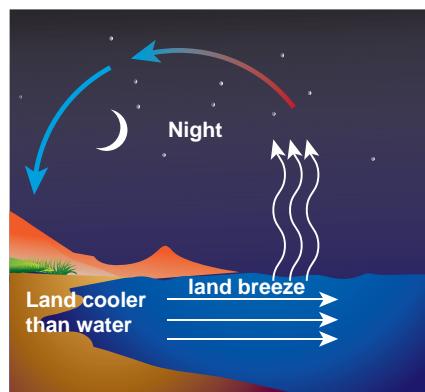
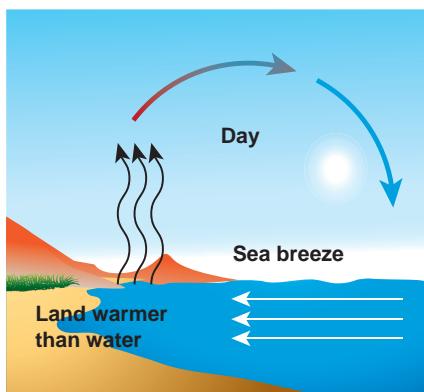
aluminum have high thermal conductivities. So they are used to make cooking vessels.

### Convection

Convection is the process in which heat transfer is by actual movement of molecules in fluids such as liquids and gases. In convection, molecules move freely from one place to another. It happens naturally or forcefully.



During the day, sun rays warm up the land more quickly than sea water. It is because land has less specific heat capacity than water. As a result the air above the land becomes less dense and rises. At the same time the cooler air above the sea flows to land and it is called 'sea breeze'. During the night time the land gets cooled faster than sea due to the same reason (specific heat). The air molecules above sea are warmer than air molecules above the land. So air molecules above the sea are replaced by cooler air molecules from the land. It is called 'land breeze'.



Boiling water in a cooking pot is an example of convection. Water at the bottom of the pot receives more heat. Due to heating, the water expands and the density of water decreases at the bottom. Due to this decrease in density, molecules rise to the top. At the same time the molecules at the top receive less heat and become denser and come to the bottom of the pot. This process goes on continuously. The back and forth movement of molecules is called convection current.

To keep the room warm, we use room heater. The air molecules near the heater will heat up and expand. As they expand, the density of air molecules will decrease and rise up while the higher density cold air will come down. This circulation of air molecules is called convection current.

### Radiation:

When we keep our hands near the hot stove we feel the heat even though our hands are not touching the hot stove. Here heat transferred from the hot stove to our hands is in the form of radiation. We receive energy from the sun in the form of radiations. These radiations travel through vacuum and reach the Earth. It is the peculiar character of radiation which requires no medium to transfer energy from one object to another. The conduction or convection requires medium to transfer the heat.

Radiation is a form of energy transfer from one body to another by electromagnetic waves.

### Example:

1. Solar energy from the Sun.
2. Radiation from room heater.



The parameter temperature is generally thought to be associated with matter (solid, liquid and gas). But radiation is also considered as a thermodynamic system which has well defined temperature and pressure. The visible radiation coming from the Sun is at the temperature of 5700 K and the Earth re emits the radiation in the infrared range into space which is at a temperature of around 300K.

### 8.2.8 Newton's law of cooling

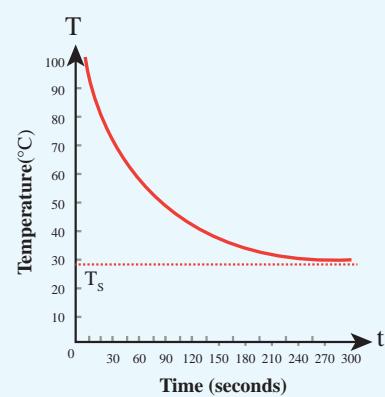
*Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in the temperature between that body and its surroundings .*

$$\frac{dQ}{dt} \propto -(T - T_s) \quad (8.4)$$

The negative sign indicates that the quantity of heat lost by liquid goes on decreasing with time. Where,

$T$  = Temperature of the object

$T_s$  = Temperature of the surrounding



**Figure 8.12** Cooling of hot water with time

From the graph in Figure 8.12 it is clear that the rate of cooling is high initially and decreases with falling temperature.

Let us consider an object of mass  $m$  and specific heat capacity  $s$  at temperature  $T$ . Let  $T_s$  be the temperature of the surroundings. If the temperature falls by a small amount  $dT$  in time  $dt$ , then the amount of heat lost is,

$$dQ = msdT \quad (8.5)$$

Dividing both sides of equation (8.5) by  $dt$

$$\frac{dQ}{dt} = \frac{msdT}{dt} \quad (8.6)$$

From Newton's law of cooling

$$\begin{aligned} \frac{dQ}{dt} &\propto -(T - T_s) \\ \frac{dQ}{dt} &= -a(T - T_s) \end{aligned} \quad (8.7)$$

Where  $a$  is some positive constant.

From equation (8.6) and (8.7)

$$\begin{aligned} -a(T - T_s) &= ms \frac{dT}{dt} \\ \frac{dT}{T - T_s} &= -\frac{a}{ms} dt \end{aligned} \quad (8.8)$$

Integrating equation (8.8) on both sides,

$$\begin{aligned} \int_0^\infty \frac{dT}{T - T_s} &= - \int_0^t \frac{a}{ms} dt \\ \ln(T - T_s) &= -\frac{a}{ms} t + b_1 \end{aligned}$$

Where  $b_1$  is the constant of integration. taking exponential both sides, we get

$$T = T_s + b_2 e^{-\frac{a}{ms} t} \quad (8.9)$$

here  $b_2 = e^{b_1} = \text{constant}$

### EXAMPLE 8.8

A hot water cools from  $92^\circ\text{C}$  to  $84^\circ\text{C}$  in 3 minutes when the room temperature is  $27^\circ\text{C}$ . How long will it take for it to cool from  $65^\circ\text{C}$  to  $60^\circ\text{C}$ ?

The hot water cools  $8^\circ\text{C}$  in 3 minutes. The average temperature of  $92^\circ\text{C}$  and  $84^\circ\text{C}$  is  $88^\circ\text{C}$ . This average temperature is  $61^\circ\text{C}$

above room temperature. Using equation (8.8)

$$\frac{dT}{T - T_s} = -\frac{a}{ms} dt \text{ or } \frac{dT}{dt} = -\frac{a}{ms}(T - T_s)$$

$$\frac{8^\circ\text{C}}{3\text{min}} = -\frac{a}{ms}(61^\circ\text{C})$$

Similarly the average temperature of  $65^\circ\text{C}$  and  $60^\circ\text{C}$  is  $62.5^\circ\text{C}$ . The average temperature is  $35.5^\circ\text{C}$  above the room temperature. Then we can write

$$\frac{5^\circ\text{C}}{dt} = -\frac{a}{ms}(35.5^\circ\text{C})$$

By diving both the equation, we get

$$\begin{aligned} \frac{8^\circ\text{C}}{3\text{min}} &= -\frac{a}{ms}(61^\circ\text{C}) \\ \frac{5^\circ\text{C}}{dt} &= -\frac{a}{ms}(35.5^\circ\text{C}) \\ \frac{8 \times dt}{3 \times 5} &= \frac{61}{35.5} \\ dt &= \frac{61 \times 15}{35.5 \times 8} = \frac{915}{284} = 3.22 \text{ min} \end{aligned}$$

## 8.3

### LAWS OF HEAT TRANSFER

#### 8.3.1 Prevost theory of heat exchange

Every object emits heat radiations at all finite temperatures (except 0 K) as well as it absorbs radiations from the surroundings. For example, if you touch someone, they might feel your skin as either hot or cold.

A body at high temperature radiates more heat to the surroundings than it receives from it. Similarly, a body at a lower temperature receives more heat from the surroundings than it loses to it.

Prevost applied the idea of ‘thermal equilibrium’ to radiation. He suggested that all bodies radiate energy but hot bodies radiate more heat than the cooler bodies. At one point of time the rate of exchange of heat from both the bodies will become the same. Now the bodies are said to be in ‘thermal equilibrium’.

Only at absolute zero temperature a body will stop emitting. Therefore *Prevost theory states that all bodies emit thermal radiation at all temperatures above absolute zero irrespective of the nature of the surroundings.*

### 8.3.2 Stefan Boltzmann law

Stefan Boltzmann law states that, *the total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.*

$$E \propto T^4 \text{ or } E = \sigma T^4 \quad (8.10)$$

Where,  $\sigma$  is known as Stefan’s constant. Its value is  $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$



If a body is not a perfect black body, then

$$E = e \sigma T^4 \quad (8.11)$$

Where ‘e’ is emissivity of surface.

*Emissivity is defined as the ratio of the energy radiated from a material’s surface to that radiated from a perfectly black body at the same temperature and wavelength.*

### 8.3.3 Wien’s displacement law

In the universe every object emits radiation. The wavelengths of these radiations depend on the object’s absolute temperature. These

radiations have different wavelengths and all the emitted wavelengths will not have equal intensity.

Wien’s law states that, *the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body.*

$$\lambda_m \propto \frac{1}{T} \text{ (or) } \lambda_m = \frac{b}{T} \quad (8.12)$$

Where, b is known as Wien’s constant. Its value is  $2.898 \times 10^{-3} \text{ m K}$

It implies that if temperature of the body increases, maximal intensity wavelength ( $\lambda_m$ ) shifts towards lower wavelength (higher frequency) of electromagnetic spectrum. It is shown in Figure 8.13

#### Graphical representation

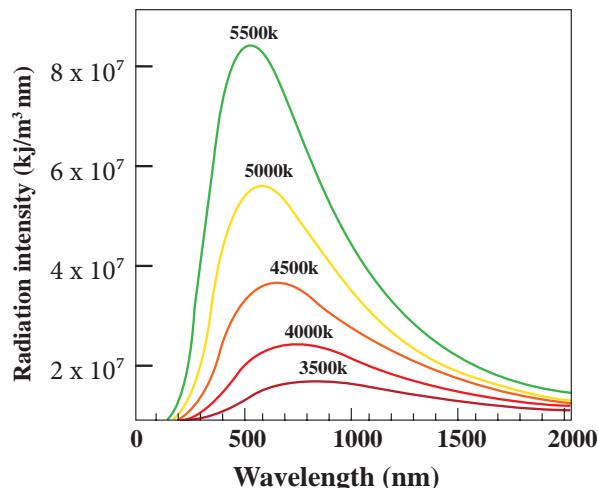


Figure 8.13 Wien’s displacement law

From the graph it is clear that the peak of the wavelengths is inversely proportional to temperature. The curve is known as ‘black body radiation curve’.

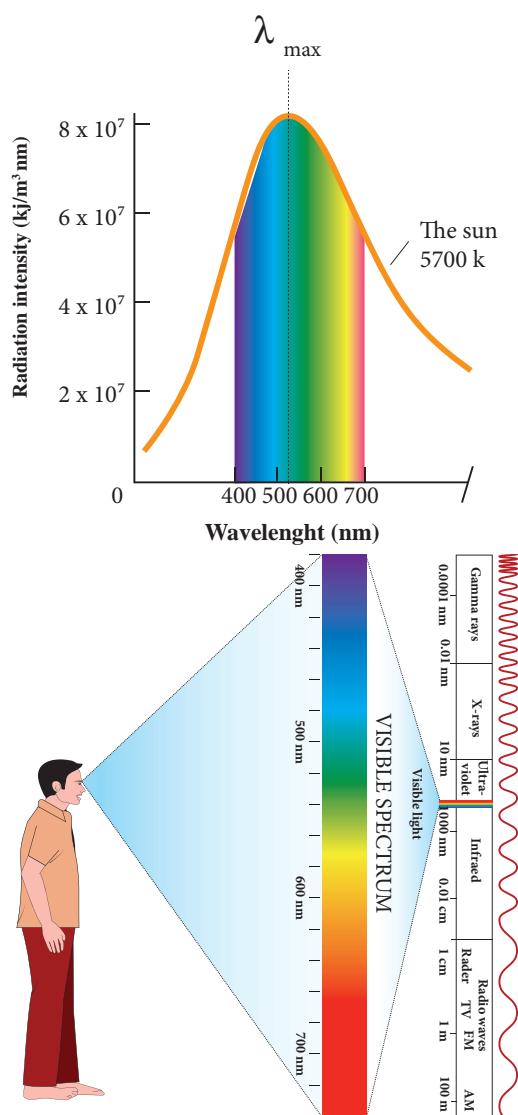
#### Wien’s law and Vision:

Why our eye is sensitive to only visible wavelength (in the range 400 nm to 700nm)?

The Sun is approximately taken as a black body. Since any object above 0 K will emit radiation, Sun also emits radiation. Its surface temperature is about 5700 K. By substituting this value in the equation (8.12),

$$\lambda_m = \frac{b}{T} = \frac{2.898 \times 10^{-8}}{5700} \approx 508 \text{ nm}$$

It is the wavelength at which maximum intensity is 508 nm. Since the Sun's temperature is around 5700K, the spectrum of radiations emitted by Sun lie between 400 nm to 700 nm which is the visible part of the spectrum. It is shown in Figure 8.14



**Figure 8.14** Wien's law and Human's vision

The humans evolved under the Sun by receiving its radiations. The human eye is sensitive only in the visible not in infrared or X-ray ranges in the spectrum.

Suppose if humans had evolved in a planet near the star Sirius (9940K), then they would have had the ability to see the Ultraviolet rays!

### EXAMPLE 8.9

The power radiated by a black body A is  $E_A$  and the maximum energy radiated was at the wavelength  $\lambda_A$ . The power radiated by another black body B is  $E_B = N E_A$  and the radiated energy was at the maximum wavelength,  $\frac{1}{2} \lambda_A$ . What is the value of N?

According to Wien's displacement law

$\lambda_{\max} T = \text{constant}$  for both object A and B

$$\lambda_A T_A = \lambda_B T_B. \text{ Here } \lambda_B = \frac{1}{2} \lambda_A$$

$$\frac{T_B}{T_A} = \frac{\lambda_A}{\lambda_B} = \frac{1}{\frac{1}{2}} = 2$$

$$T_B = 2T_A$$

From Stefan-Boltzmann law

$$\frac{E_B}{E_A} = \left( \frac{T_B}{T_A} \right)^4 = (2)^4 = 16 = N$$

Object B has emitted at lower wavelength compared to A. So the object B would have emitted more energetic radiation than A.

## 8.4

### THERMODYNAMICS:

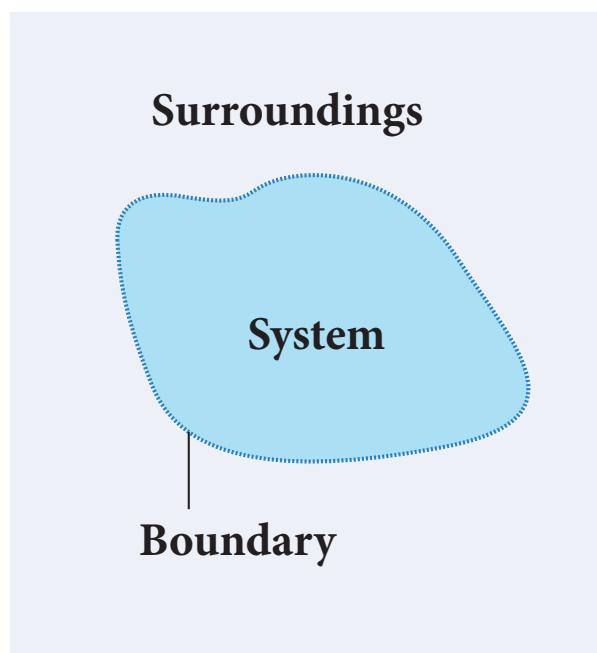
#### 8.4.1 Introduction

In the previous sections we have studied about the heat, temperature and thermal properties of matter. Thermodynamics is

a branch of physics which describes the laws governing the process of conversion of work into heat and conversion of heat into work. The laws of thermodynamics are formulated over three centuries of experimental works of Boyle, Charles, Bernoulli, Joule, Clausius, Kelvin, Carnot and Helmholtz. In our day to day life, the functioning of everything around us and even our body is governed by the laws of thermodynamics. Therefore thermodynamics is one of the essential branches of physics.

### Thermodynamic system:

A thermodynamic system is a finite part of the universe. It is a collection of large number of particles (atoms and molecules) specified by certain parameters called pressure (P), Volume (V) and Temperature (T). The remaining part of the universe is called surrounding. Both are separated by a boundary. It is shown in Figure 8.15



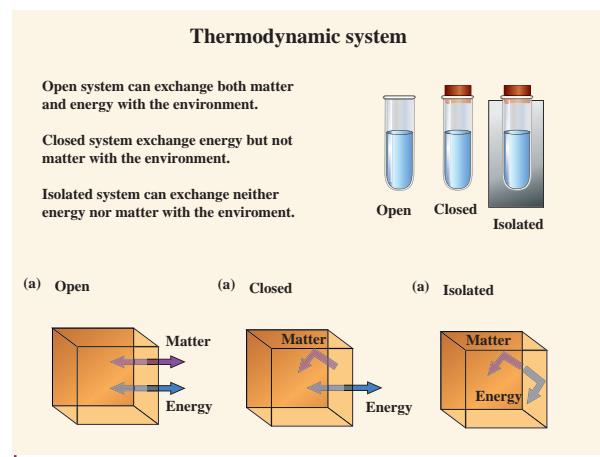
**Figure 8.15** Thermodynamic system

Examples:

A thermodynamic system can be liquid, solid, gas and radiation.

Thermodynamic system	Surrounding
Bucket of water	Open atmosphere
Air molecules in the room	Outside air
Human body	Open atmosphere
Fish in the sea	Sea of water

We can classify thermodynamics system into three types: It is given in Figure 8.16



**Figure 8.16** Different types of thermodynamic systems

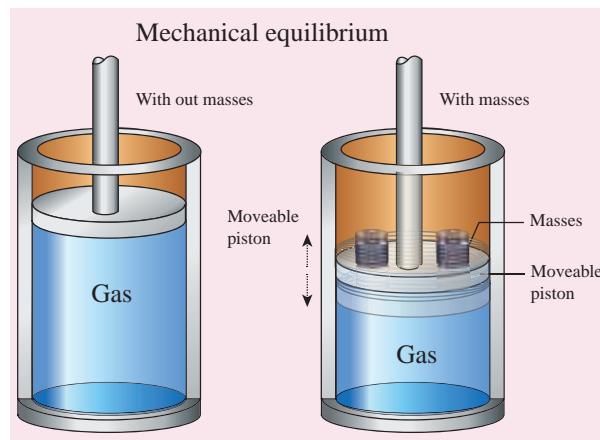
### 8.4.2 Thermal equilibrium

When a hot cup of coffee is kept in the room, heat flows from coffee to the surrounding air. After some time the coffee reaches the same temperature as the surrounding air and there will be no heat flow from coffee to air or air to coffee. It implies that the coffee and surrounding air are in thermal equilibrium with each other.

Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which will not change with time.

### Mechanical equilibrium:

Consider a gas container with piston as shown in Figure 8.17. When some mass is placed on the piston, it will move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position. When the downward gravitational force given by the piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermodynamic system or on the surrounding by thermodynamic system.



**Figure 8.17** Mechanical equilibrium

### Chemical equilibrium:

If there is no net chemical reaction between two thermodynamic systems in contact with each other then it is said to be in chemical equilibrium.

### Thermodynamic equilibrium:

If two systems are set to be in thermodynamic equilibrium, then the systems are at thermal, mechanical and chemical equilibrium with

each other. In a state of thermodynamic equilibrium the macroscopic variables such as pressure, volume and temperature will have fixed values and do not change with time.

### 8.4.3 Thermodynamic state variables

In mechanics velocity, momentum and acceleration are used to explain the state of any moving object (which you would have realized in Volume 1). In thermodynamics, the state of a thermodynamic system is represented by a set of variables called thermodynamic variables.

Examples: Pressure, temperature, volume and internal energy etc.

The values of these variables completely describe the equilibrium state of a thermodynamic system. Heat and work are not state variables rather they are process variables.

There are two types of thermodynamic variables: Extensive and Intensive

Extensive variable depends on the size or mass of the system.

**Example:** Volume, total mass, entropy, internal energy, heat capacity etc.

Intensive variables do not depend on the size or mass of the system.

**Example:** Temperature, pressure, specific heat capacity, density etc.

### Equation of state:

The equation which connects the state variables in a specific manner is called equation of state. A thermodynamic equilibrium is completely specified by these state variables by the equation of state. If the system is not in thermodynamic equilibrium then these equations cannot specify the state of the system.

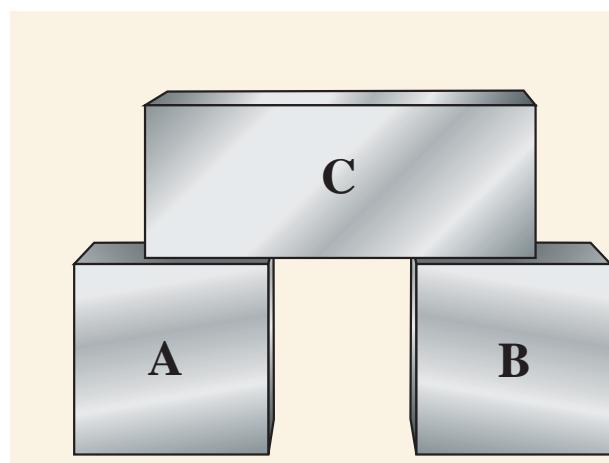
An ideal gas obeys the equation  $PV = NkT$  at thermodynamic equilibrium. Since all four macroscopic variables (P,V,T and N) are connected by this equation, we cannot change one variable alone. For example, if we push the piston of a gas container, the volume of the gas will decrease but pressure will increase or if heat is supplied to the gas, its temperature will increase, pressure and volume of the gas may also increase.

There is another example of equation of state called van der Waals equation. Real gases obey this equation at thermodynamic equilibrium. The air molecules in the room truly obey van der Waals equation of state. But at room temperature with low density we can approximate it into an ideal gas.

## 8.5

### ZEROTH LAW OF THERMODYNAMICS

The zeroth law of thermodynamics states that if two systems, A and B, are in thermal equilibrium with a third system, C,



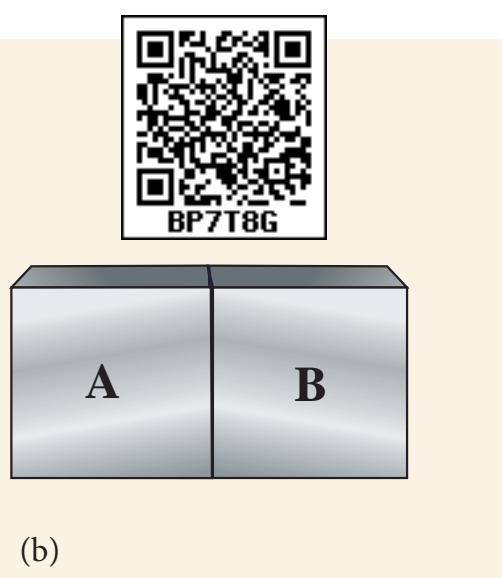
(a)

then A and B are in thermal equilibrium with each other.

Consider three systems A, B and C which are initially at different temperatures. Assume that A and B are not in thermal contact with each other as shown in Figure 8.18 (a) but each of them is in thermal contact with a third system C. After a lapse of time, system A will be in thermal equilibrium with C and B also will be in thermal equilibrium with C. In this condition, if the systems A and B are kept in thermal contact as shown in the Figure 8.18 (b), there is no flow of heat between the systems A and B. It implies that the system A and B are also in thermal equilibrium with each other. Once the three systems are at thermal equilibrium, there will be no heat flow between them as they are at the same temperature. This can be mathematically expressed as

If  $T_A = T_C$  and  $T_B = T_C$ , it implies that  $T_A = T_B$ , where  $T_A$ ,  $T_B$  and  $T_C$  are the temperatures of the systems A, B, and C respectively.

Temperature is the property which determines whether the system is in



(b)

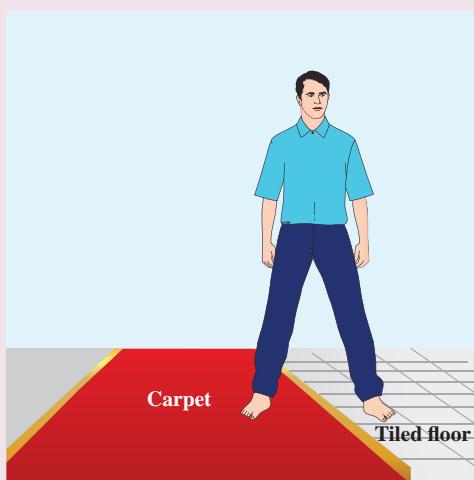
**Figure 8.18** (a) Two systems A and B in thermal contact with object C separately (b) If systems A and B are in thermal contact, they are also in thermal equilibrium with each other.

thermal equilibrium with other systems or not. Zeroth law enables us to determine the temperature. For example, when a thermometer is kept in contact with a human body, it reaches thermal equilibrium with the body. At this condition, the temperature of the thermometer will be same as the human body. This principle is used in finding the body temperature.

### ACTIVITY

We often associate the temperature as a measure of how hot or cold an object is while touching it. Can we use our sensory organs to determine the temperature of an object?

When you stand bare feet with one foot on the carpet and the other on the tiled floor, your foot on tiled floor feels cooler than the foot on the carpet even though both the tiled floor and carpet are at the same room temperature. It is because the tiled floor transfers the heat energy to your skin at higher rate than the carpet. So the skin is not measuring the actual temperature of the object; instead it measures the rate of heat energy transfer. But if we place a thermometer on the tiled floor or carpet it will show the same temperature.



## 8.6

### INTERNAL ENERGY (U)

The internal energy of a thermodynamic system is the sum of kinetic and potential energies of all the molecules of the system with respect to the center of mass of the system.

The energy due to molecular motion including translational, rotational and vibrational motion is called internal kinetic energy ( $E_K$ )

The energy due to molecular interaction is called internal potential energy ( $E_p$ ). Example: Bond energy.

$$U = E_K + E_p$$

- Since ideal gas molecules are assumed to have no interaction with each other the internal energy consists of only kinetic energy part ( $E_K$ ) which depends on the temperature, number of particles and is independent of volume. However this is not true for real gases like Van der Waals gases.
- Internal energy is a state variable. It depends only on the initial and final states of the thermodynamic system. For example, if the temperature of water is raised from 30°C to 40°C either by heating or by stirring, the final internal energy depends only on the final temperature 40°C and not the way it is arrived at.



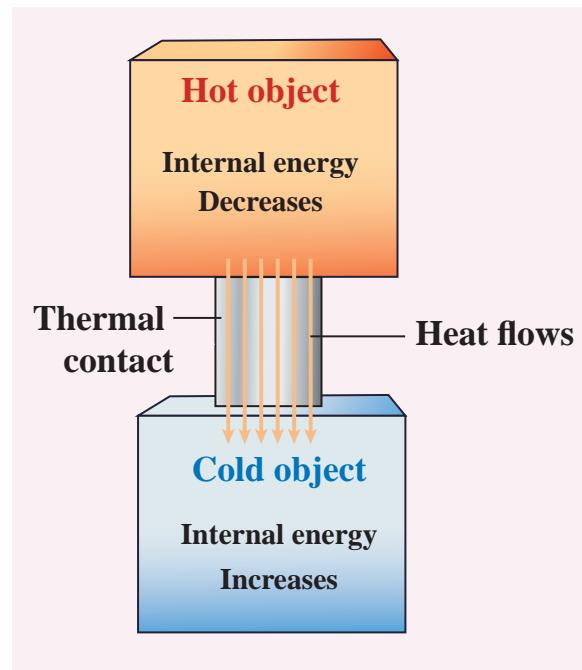
#### Note

It is very important to note that the internal energy of a thermodynamic system is associated with only the kinetic energy of the individual molecule due to its random motion and the potential energy of molecules which depends on their chemical nature. The bulk kinetic energy of the entire system or gravitational potential energy of the system should not be mistaken as a part of internal energy. For example



(a) Consider two gas containers at the same temperature having the same internal energy, one is kept at rest on the ground and the other is kept in a moving train. Even though the gas container in the train is moving with the speed of the train, the internal energy of the gas in it will not increase.

(b) Consider two gas containers at the same temperature having the same internal energy, one is kept on the ground and the other is kept at some height  $h$ . Even though the container at height  $h$  is having higher gravitational potential energy, this has no influence on internal energy of the gas molecules.



### EXAMPLE 8.10

When you mix a tumbler of hot water with one bucket of normal water, what will be the direction of heat flow? Justify.

The water in the tumbler is at a higher temperature than the bucket of normal water. But the bucket of normal water has larger internal energy than the hot water in the tumbler. This is because the internal energy is an extensive variable and it depends on the size or mass of the system.

Even though the bucket of normal water has larger internal energy than the tumbler of hot water, heat will flow from water in the tumbler to the water in the bucket. This is because heat flows from a body at higher temperature to the one at lower temperature and is independent of internal energy of the system.

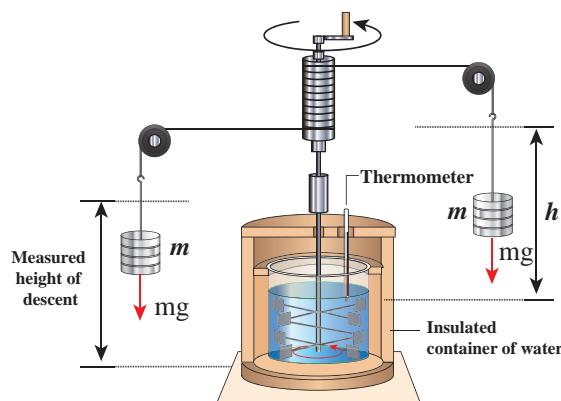
Once the heat is transferred to an object it becomes internal energy of the object. The right way to say is 'object has certain amount of internal energy'. Heat is one of the ways to increase the internal energy of a system as shown in the Figure.



It is to be noted that heat does not always increases the internal energy. Later we shall see that in ideal gases during isothermal process the internal energy will not increase even though heat flows in to the system.

#### 8.6.1 Joule's mechanical equivalent of heat

The temperature of an object can be increased by heating it or by doing some work on it. In the eighteenth century, James Prescott Joule showed that mechanical energy can be converted into internal energy and vice versa. In his experiment, two masses were attached with a rope and a paddle wheel as shown in Figure 8.19. When these masses fall through a distance  $h$  due to gravity, both the masses lose potential energy equal to  $2mgh$ . When the masses fall, the paddle wheel turns. Due to the turning of wheel inside water, frictional force comes in between the water and the paddle wheel. This causes



**Figure 8.19** Joule's experiment for determining the mechanical equivalent of heat energy.

a rise in temperature of the water. This implies that gravitational potential energy is converted to internal energy of water. The temperature of water increases due to the work done by the masses. In fact, Joule was able to show that the mechanical work has the same effect as giving heat. He found that to raise 1 g of an object by  $1^{\circ}\text{C}$ , 4.186 J of energy is required. In earlier days the heat was measured in calorie.

$$1 \text{ cal} = 4.186 \text{ J}$$

This is called Joule's mechanical equivalent of heat.



**Note**

Before James Prescott Joule, people thought that heat was a kind of fluid called caloric fluid which flows from an object at high temperature to that in low temperature. According to caloric fluid idea, the hot object contains more caloric fluid and the cold object contains less caloric fluid since heat was treated as a quantity. Now we understand that heat is not a quantity but it is an energy in transit. So the word 'mechanical equivalent of heat' is wrong terminology. Because mechanical energy is a quantity and any object can contain more or less

mechanical energy, but this is not so with heat as it is not a quantity. However this terminology is retained for historical reasons. A correct name would be 'Joule's mechanical equivalence of internal energy'. Joule essentially converted mechanical energy to internal energy. In his experiment potential energy is converted to rotational kinetic energy of paddle wheel and this rotational kinetic energy is converted to internal energy of water.

### EXAMPLE 8.11

A student had a breakfast of 200 food calories. He thinks of burning this energy by drawing water from the well and watering the trees in his school. Depth of the well is about 25 m. The pot can hold 25 L of water and each tree requires one pot of water. How many trees can he water? (Neglect the mass of the pot and the energy spent by walking. Take  $g = 10 \text{ m s}^{-2}$ )



### Solution

To draw 25 L of water from the well, the student has to do work against gravity by burning his energy.

$$\text{Mass of the water} = 25 \text{ L} = 25 \text{ kg} \quad (1 \text{ L} = 1 \text{ kg})$$

The work required to draw 25 kg of water = gravitational potential energy gained by water.

$$W = mgh = 25 \times 10 \times 25 = 6250 \text{ J}$$

The total energy gained from the food = 200 food cal = 200 kcal.

$$= 200 \times 10^3 \times 4.186 \text{ J} = 8.37 \times 10^5 \text{ J}$$

If we assume that by using this energy the student can draw 'n' pots of water from the well, the total energy spent by him =  $8.37 \times 10^5 \text{ J} = nmgh$

$$n = \frac{8.37 \times 10^5 \text{ J}}{6250 \text{ J}} \approx 134.$$

This n is also equal to the number of trees that he can water.

Is it possible to draw 134 pots of water from the well just by having breakfast? No. Actually the human body does not convert entire food energy into work. It is only approximately 20% efficient. It implies that only 20% of 200 food calories is used to draw water from the well. So 20 % of the 134 is only 26 pots of water. It is quite meaningful. So he can water only 26 trees.

The remaining energy is used for blood circulation and other functions of the body. It is to be noted that some energy is always 'wasted'. Why is it that the body cannot have 100% efficiency? You will find the answer in section 8.9

### 8.6.2 First law of thermodynamics

The first law of thermodynamics is a statement of the law of conservation of energy. In Newtonian mechanics conservation of energy involves kinetic and potential energies of bulk objects. But the first law of thermodynamics includes heat also. This law states that '*Change in internal energy ( $\Delta U$ ) of the system is equal to heat supplied to the system (Q) minus the work done by the system (W) on the surroundings*'. Mathematically it is written as

$$\Delta U = Q - W \quad (8.13)$$

The internal energy of a thermodynamic system can be changed either by heating or by work as shown below.

Heat flows into the system	Internal energy increases
----------------------------	---------------------------

Heat flows out of the system	Internal energy decreases
------------------------------	---------------------------

Work is done on the system	Internal energy increases
----------------------------	---------------------------

Work is done by the system	Internal energy decreases
----------------------------	---------------------------

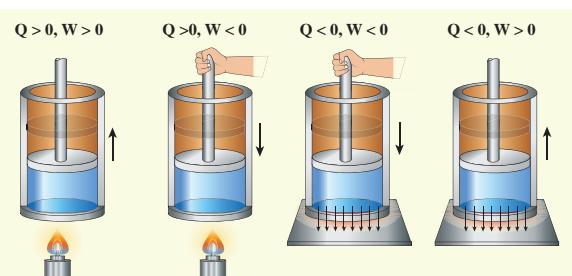
Based on the above table the sign convention is introduced to use first law of thermodynamics appropriately. It is shown in the following table and the Figure 8.20.

System gains heat	$Q$ is positive
-------------------	-----------------

System loses heat	$Q$ is negative
-------------------	-----------------

Work done on the system	$W$ is negative
-------------------------	-----------------

Work done by the system	$W$ is positive
-------------------------	-----------------



**Figure 8.20** The Sign convention for heat and work

Even though we often explain first law of thermodynamics using gases, this law is universal and applies to liquids and solids also.



**Note** Some book presents the first law of thermodynamics as  $\Delta U = Q + W$ . Here the work done by the system is taken as negative and work done on the system is positive. However both the conventions are correct and we can follow any one of the convention.

### EXAMPLE 8.12

A person does 30 kJ work on 2 kg of water by stirring using a paddle wheel. While stirring, around 5 kcal of heat is released from water through its container to the surface and surroundings by thermal conduction and radiation. What is the change in internal energy of the system?

#### Solution

Work done on the system (by the person while stirring),  $W = -30 \text{ kJ} = -30,000 \text{ J}$

Heat flowing out of the system,  $Q = -5 \text{ kcal} = 5 \times 4184 \text{ J} = -20920 \text{ J}$

Using First law of thermodynamics

$$\Delta U = Q - W$$

$$\Delta U = -20920 \text{ J} - (-30,000) \text{ J}$$

$$\Delta U = -20920 \text{ J} + 30,000 \text{ J} = 9080 \text{ J}$$

Here, the heat lost is less than the work done on the system, so the change in internal energy is positive.

### Solution



Work done by the system (body),

$$W = +500 \text{ kJ}$$

Heat released from the system (body),

$$Q = -230 \text{ kJ}$$

The change in internal energy of a body  
 $= \Delta U = -230 \text{ kJ} - 500 \text{ kJ} = -730 \text{ kJ}$

### 8.6.3 Quasi-static process

Consider a system of an ideal gas kept in a cylinder of volume V at pressure P and temperature T. When the piston attached to the cylinder moves outward the volume of the gas will change. As a result the temperature and pressure will also change because all three variables P, T and V are related by the equation of state  $PV = NkT$ . If a block of some mass is kept on the piston, it will suddenly push the piston downward. The pressure near the piston will be larger than other parts of the system. It implies that the gas is in non-equilibrium state. We cannot determine pressure, temperature or internal energy of the system until it reaches another equilibrium state. But if the piston is pushed very slowly such that at every stage it is still in equilibrium with surroundings, we can use the equation of state to calculate the internal energy, pressure or temperature. This kind of process is called quasi-static process.

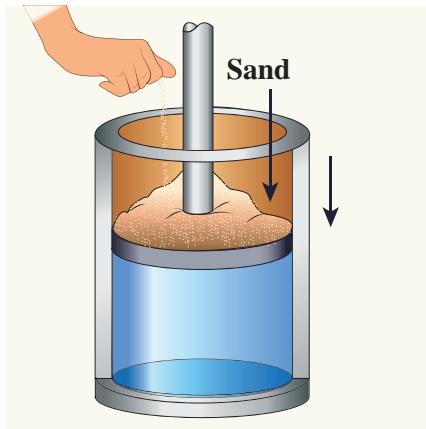
### EXAMPLE 8.13

Jogging every day is good for health. Assume that when you jog a work of 500 kJ is done and 230 kJ of heat is given off. What is the change in internal energy of your body?

A quasi-static process is an infinitely slow process in which the system changes its variables ( $P, V, T$ ) so slowly such that it remains in thermal, mechanical and chemical equilibrium with its surroundings throughout. By this infinite slow variation, the system is always almost close to equilibrium state.

### EXAMPLE 8.14

Give an example of a quasi-static process. Consider a container of gas with volume  $V$ , pressure  $P$  and temperature  $T$ . If we add sand particles one by one slowly on the top of the piston, the piston will move inward very slowly. This can be taken as almost a quasi-static process. It is shown in the figure



Sand particles added slowly- quasi-static process

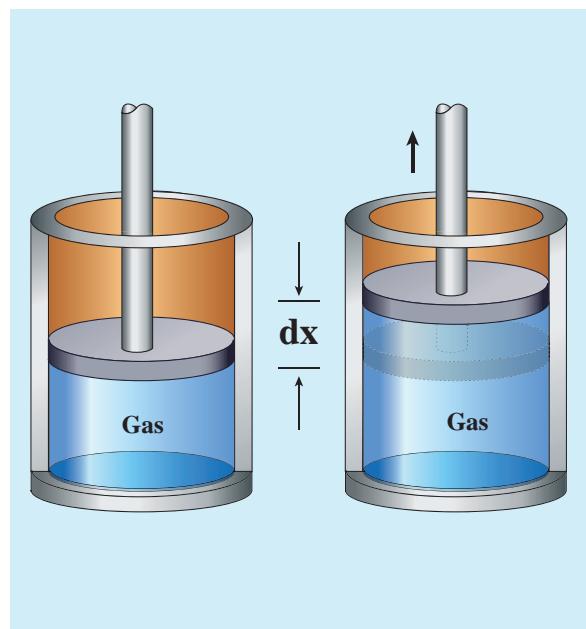
### 8.6.4 Work done in volume changes

Consider a gas contained in the cylinder fitted with a movable piston. Suppose the gas is expanded quasi-statically by pushing the piston by a small distance  $dx$  as shown in Figure 8.21. Since the expansion occurs quasi-statically the pressure, temperature and internal energy will have unique values at every instant.

The small work done by the gas on the piston

$$dW = Fdx \quad (8.14)$$

The force exerted by the gas on the piston  $F = PA$ . Here  $A$  is area of the piston and  $P$  is pressure exerted by the gas on the piston.



**Figure 8.21** Work done by the gas

Equation (8.14) can be rewritten as

$$dW = PA dx \quad (8.15)$$

But  $Adx = dV =$  change in volume during this expansion process.

So the small work done by the gas during the expansion is given by

$$dW = PdV \quad (8.16)$$

Note here that  $dV$  is positive since the volume is increased. Here,  $dV$  is positive.

In general the work done by the gas by increasing the volume from  $V_i$  to  $V_f$  is given by

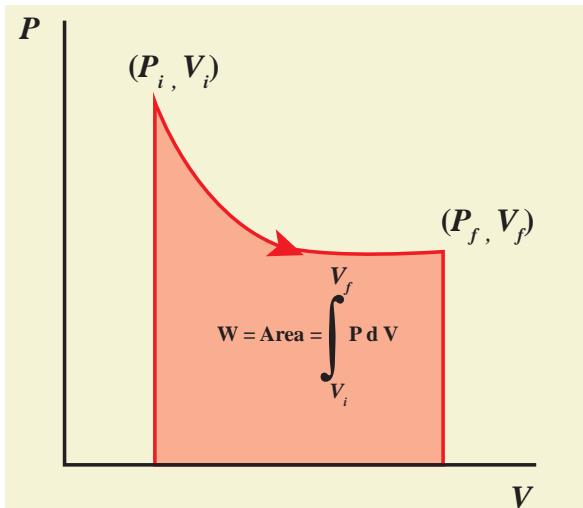
$$W = \int_{V_i}^{V_f} PdV \quad (8.17)$$

Suppose if the work is done on the system, then  $V_i > V_f$ . Then, W is negative.

Note here the pressure P is inside the integral in equation (8.17). It implies that while the system is doing work, the pressure need not be constant. To evaluate the integration we need to first express the pressure as a function of volume and temperature using the equation of state.

### 8.6.5 PV diagram

PV diagram is a graph between pressure P and volume V of the system. The P-V diagram is used to calculate the amount of work done by the gas during expansion or on the gas during compression. In Unit 2, we have seen that the area under the curve will give integration of the function from lower limit to upper limit. The area under the PV diagram will give the work done during expansion or compression which is shown in Figure 8.22



**Figure 8.22** Work done by the gas during expansion

The shape of PV diagram depends on the nature of the thermodynamic process.

### EXAMPLE 8.15

A gas expands from volume  $1\text{m}^3$  to  $2\text{m}^3$  at constant atmospheric pressure.

- Calculate the work done by the gas.
- Represent the work done in PV diagram.

#### Solution

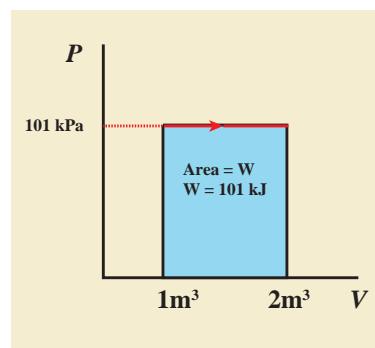
- The pressure  $P = 1 \text{ atm} = 101 \text{ kPa}$ ,  $V_f = 2 \text{ m}^3$  and  $V_i = 1 \text{ m}^3$

From equation (8.17)  $W = \int_{V_i}^{V_f} P dV = P \int_{V_i}^{V_f} dV$

Since P is constant. It is taken out of the integral.

$$W = P(V_f - V_i) = 101 \times 103 \times (2 - 1) = 101 \text{ kJ}$$

- Since the pressure is kept constant, PV diagram is straight line as shown in the figure. The area is equal to work done by the gas.



Note the arrow mark in the curve. Suppose the work is done on the system, then volume will decrease and the arrow will point in the opposite direction.

### 8.7

## SPECIFIC HEAT CAPACITY OF A GAS

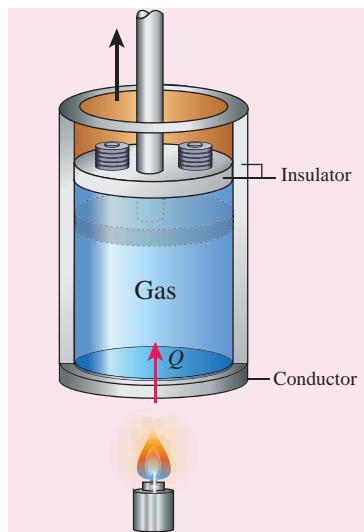
Specific heat capacity of a given system plays a very important role in determining

the structure and molecular nature of the system. Unlike solids and liquids, gases have two specific heats: specific heat capacity at constant pressure ( $s_p$ ) and specific heat capacity at constant volume ( $s_v$ ).

### 8.7.1 Specific heat capacity

**Specific heat capacity at constant pressure ( $s_p$ ):**

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K or 1°C by keeping the pressure constant is called specific heat capacity of at constant pressure. When the heat energy is supplied to the gas, it expands to keep the pressure constant as shown in Figure 8.23



**Figure 8.23** Specific heat capacity at constant pressure

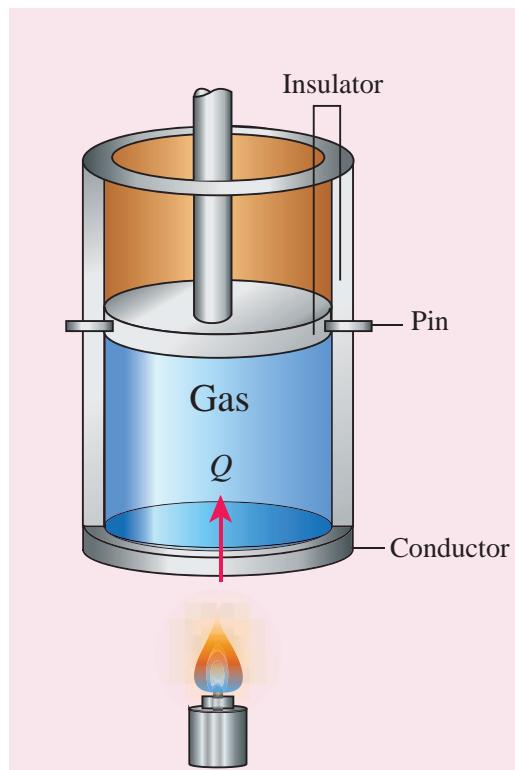
In this process a part of the heat energy is used for doing work (expansion) and the remaining part is used to increase the internal energy of the gas.

**Specific heat capacity at constant volume ( $s_v$ ):**

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K or 1°C by keeping the volume constant

is called specific heat capacity at constant volume.

If the volume is kept constant, then the supplied heat is used to increase only the internal energy. No work is done by the gas as shown in Figure 8.24.



**Figure 8.24** Specific heat capacity at constant volume

It implies that to increase the temperature of the gas at constant volume requires less heat than increasing the temperature of the gas at constant pressure. In other words  $s_p$  is always greater than  $s_v$ .

#### Molar Specific heat capacities

Sometimes it is useful to calculate the molar heat capacities  $C_p$  and  $C_v$ . The amount of heat required to raise the temperature of one mole of a substance by 1K or 1°C at constant volume is called molar specific heat capacity at constant volume ( $C_v$ ). If pressure is kept constant, it is called molar specific heat capacity at constant pressure ( $C_p$ ).

If  $Q$  is the heat supplied to mole of a gas at constant volume and if the temperature changes by an amount  $\Delta T$ , we have

$$Q = \mu C_v \Delta T. \quad (8.18)$$

By applying the first law of thermodynamics for this constant volume process ( $W=0$ , since  $dV=0$ ), we have

$$Q = \Delta U - 0 \quad (8.19)$$

By comparing the equations (8.18) and (8.19),

$$\Delta U = \mu C_v \Delta T \text{ or } C_v = \frac{1}{\mu} \frac{\Delta U}{\Delta T}$$

If the limit  $\Delta T$  goes to zero, we can write

$$C_v = \frac{1}{\mu} \frac{dU}{dT} \quad (8.20)$$

Since the temperature and internal energy are state variables, the above relation holds true for any process.

### 8.7.2 Meyer's relation

Consider  $\mu$  mole of an ideal gas in a container with volume  $V$ , pressure  $P$  and temperature  $T$ .

When the gas is heated at constant volume the temperature increases by  $dT$ . As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be  $dU$ .

If  $C_v$  is the molar specific heat capacity at constant volume, from equation (8.20)

$$dU = \mu C_v dT \quad (8.21)$$

Suppose the gas is heated at constant pressure so that the temperature increases by  $dT$ . If ' $Q$ ' is the heat supplied in this process and ' $dV$ ' the change in volume of the gas.

$$Q = \mu C_p dT \quad (8.22)$$

If  $W$  is the workdone by the gas in this process, then

$$W = PdV \quad (8.23)$$

But from the first law of thermodynamics,

$$Q = dU + W \quad (8.24)$$

Substituting equations (8.21), (8.22) and (8.23) in (8.24), we get,

$$\mu C_p dT = \mu C_v dT + PdV \quad (8.25)$$

For mole of ideal gas, the equation of state is given by

$$PV = \mu RT \Rightarrow PdV + VdP = \mu R dT \quad (8.26)$$

Since the pressure is constant,  $dP=0$

$$\therefore C_p dT = C_v dT + R dT$$

$$\therefore C_p = C_v + R \quad (\text{or}) \quad C_p - C_v = R \quad (8.27)$$

This relation is called Meyer's relation

It implies that the molar specific heat capacity of an ideal gas at constant pressure is greater than molar specific heat capacity at constant volume.

The relation shows that specific heat at constant pressure ( $s_p$ ) is always greater than specific heat at constant volume ( $s_v$ ).

## 8.8

### THERMODYNAMIC PROCESSES

#### 8.8.1 Isothermal process

It is a process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change. The ideal gas equation is

$$PV = \mu RT$$

Here, T is constant for this process

So the equation of state for isothermal process is given by

$$PV = \text{constant} \quad (8.28)$$

This implies that if the gas goes from one equilibrium state  $(P_1, V_1)$  to another equilibrium state  $(P_2, V_2)$  the following relation holds for this process

$$P_1 V_1 = P_2 V_2 \quad (8.29)$$

Since  $PV = \text{constant}$ , P is inversely proportional to V ( $P \propto \frac{1}{V}$ ). This implies that PV graph is a hyperbola. The pressure-volume graph for constant temperature is also called isotherm.

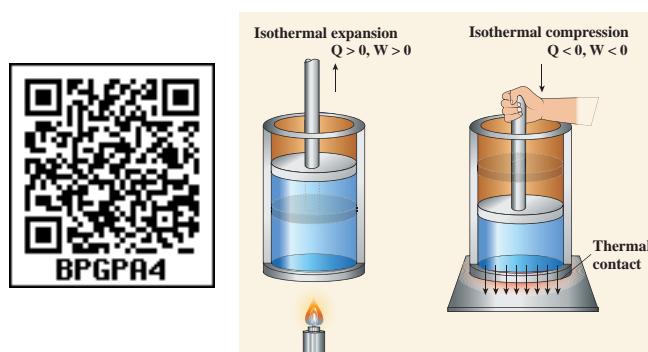
Figure 8.25 shows the PV diagram for quasi-static isothermal expansion and quasi-static isothermal compression.

We know that for an ideal gas the internal energy is a function of temperature only. For an isothermal process since temperature is constant, the internal energy is also constant. This implies that  $dU$  or  $\Delta U = 0$ .

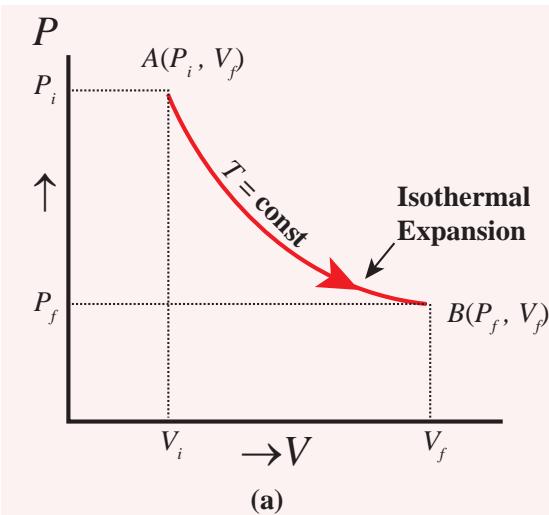
For an isothermal process, the first law of thermodynamics can be written as follows,

$$Q = W \quad (8.30)$$

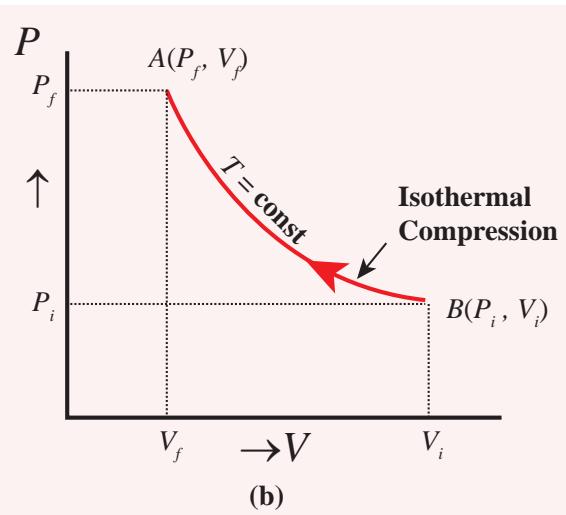
From equation (8.30), we infer that the heat supplied to a gas is used to do only external work. It is a common misconception that when there is flow of heat to the system, the temperature will increase. For isothermal process this is not true. The isothermal compression takes place when the piston of the cylinder is pushed. This will increase the internal energy which will flow out of the system through thermal contact. This is shown in Figure 8.26.



**Figure 8.26** Isothermal expansion and isothermal compression



**Figure 8.25** (a) Quasi-static isothermal expansion (b) Quasi-static isothermal compression



**Examples:**

- (i) When water is heated, at the boiling point, even when heat flows to water, the temperature will not increase unless the water completely evaporates. Similarly, at the freezing point, when the ice melts to water, the temperature of ice will not increase even when heat is supplied to ice.
- (ii) All biological processes occur at constant body temperature ( $37^{\circ}\text{C}$ ).

**Work done in an isothermal process:**

Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state  $(P_i, V_i)$  to the final state  $(P_f, V_f)$ . We can calculate the work done by the gas during this process. From equation (8.17) the work done by the gas,

$$W = \int_{V_i}^{V_f} P dV \quad (8.31)$$

As the process occurs quasi-statically, at every stage the gas is at equilibrium with the surroundings. Since it is in equilibrium at every stage the ideal gas law is valid. Writing pressure in terms of volume and temperature,

$$P = \frac{\mu RT}{V} \quad (8.32)$$

Substituting equation (8.32) in (8.31) we get

$$\begin{aligned} W &= \int_{V_i}^{V_f} \frac{\mu RT}{V} dV \\ W &= \mu RT \int_{V_i}^{V_f} \frac{dV}{V} \end{aligned} \quad (8.33)$$

In equation (8.33), we take  $\mu RT$  out of the integral, since it is constant throughout the isothermal process.

By performing the integration in equation (8.33), we get

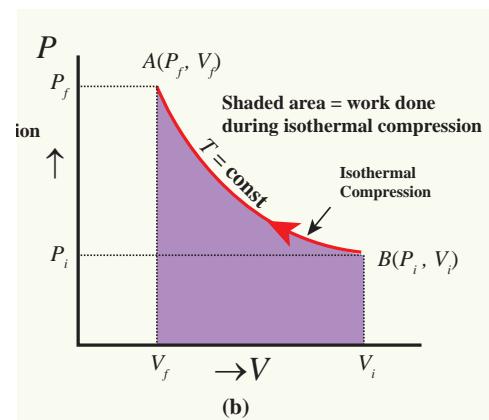
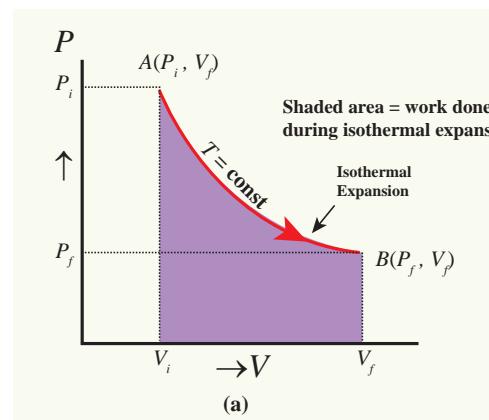
$$W = \mu RT \ln \left( \frac{V_f}{V_i} \right) \quad (8.34)$$

Since we have an isothermal expansion,  $\frac{V_f}{V_i} > 1$ , so  $\ln \left( \frac{V_f}{V_i} \right) > 0$ . As a result the work done by the gas during an isothermal expansion is positive.

The above result in equation (8.34) is true for isothermal compression also. But in an isothermal compression  $\frac{V_f}{V_i} < 1$ , so  $\ln \left( \frac{V_f}{V_i} \right) < 0$ .

As a result the work done on the gas in an isothermal compression is negative.

In the PV diagram the work done during the isothermal expansion is equal to the area under the graph as shown in Figure 8.27



**Figure 8.27** Work done in an isothermal process.

Similarly for an isothermal compression, the area under the PV graph is equal to the

work done on the gas which turns out to be the area with a negative sign.



**Note** To calculate the work done in an isothermal process, we assume that the process is quasi-static. If it is not quasi-static we cannot substitute  $P = \frac{\mu RT}{V}$  in equation (8.31). It is because the ideal gas law is not valid for non equilibrium states. But equation (8.34) is valid even when the isothermal process is not quasi-static. This is because the state variables like pressure and volume depend on initial and final state alone of an ideal gas and do not depend on the way the final state is reached. The assumption of 'quasi-static' requires to do the integration.

### EXAMPLE 8.16

A 0.5 mole of gas at temperature 300 K expands isothermally from an initial volume of 2 L to 6 L

- What is the work done by the gas?
- Estimate the heat added to the gas?
- What is the final pressure of the gas?  
(The value of gas constant,  $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ )

#### Solution

- We know that work done by the gas in an isothermal expansion

Since  $\mu = 0.5$

$$W = 0.5 \text{ mol} \times \frac{8.31 \text{ J}}{\text{mol.K}} \times 300 \text{ K} \ln \left( \frac{6 \text{ L}}{2 \text{ L}} \right)$$

$$W = 1.369 \text{ kJ}$$

Note that  $W$  is positive since the work is done by the gas.

- From the First law of thermodynamics, in an isothermal process the heat supplied is spent to do work.

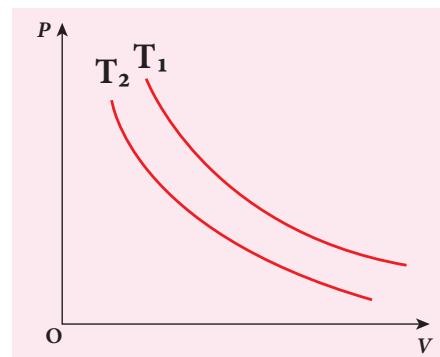
Therefore,  $Q = W = 1.369 \text{ kJ}$ . Thus  $Q$  is also positive which implies that heat flows in to the system.

- For an isothermal process

$$\begin{aligned} P_i V_i &= P_f V_f = \mu R T \\ P_f &= \frac{\mu R T}{V_f} = 0.5 \text{ mol} \times \frac{8.31 \text{ J}}{\text{mol.K}} \times \frac{300 \text{ K}}{6 \times 10^{-3} \text{ m}^3} \\ &= 207.75 \text{ kPa} \end{aligned}$$

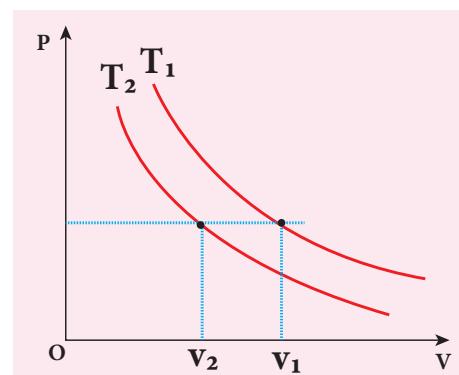
### EXAMPLE 8.17

The following PV curve shows two isothermal processes for two different temperatures and. Identify the higher temperature of these two.



#### Solution

To determine the curve corresponding to higher temperature, draw a horizontal line parallel to x axis as shown in the figure. This is the constant pressure line. The volumes  $V_1$  and  $V_2$  belong to same pressure as the vertical lines from  $V_1$  and  $V_2$  meet the constant pressure line.



At constant pressure, higher the volume of the gas, higher will be the temperature. From the figure, as  $V_1 > V_2$  we conclude  $T_1 > T_2$ . In general the isothermal curve closer to the origin, has lower temperature.

### 8.8.2 Adiabatic process

This is a process in which no heat flows into or out of the system ( $Q=0$ ). But the gas can expand by spending its internal energy or gas can be compressed through some external work. So the pressure, volume and temperature of the system may change in an adiabatic process.

For an adiabatic process, the first law becomes  $\Delta U = W$ .

This implies that the work is done by the gas at the expense of internal energy or work is done on the system which increases its internal energy.

The adiabatic process can be achieved by the following methods

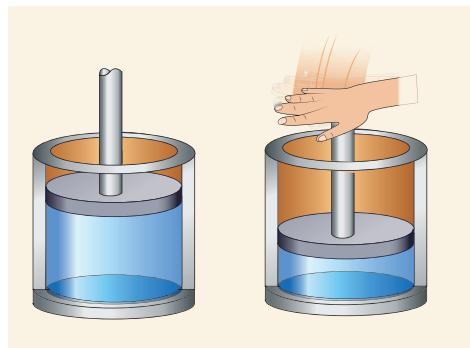
(i) Thermally insulating the system from surroundings so that no heat flows into or out of the system; for example, when thermally insulated cylinder of gas is compressed (adiabatic compression) or expanded (adiabatic expansion) as shown in the Figure 8.28

(ii) If the process occurs so quickly that there is no time to exchange heat with surroundings even though there is no thermal insulation. A few examples are shown in Figure 8.29.

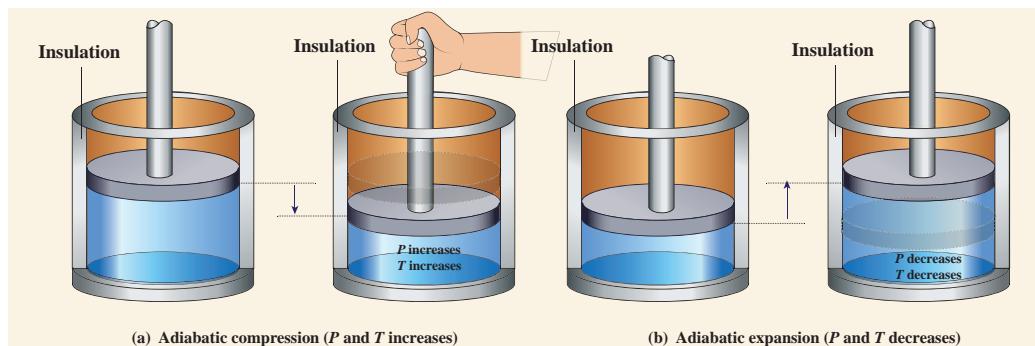
**Examples:**



**Figure 8.29 (a)** When the tyre bursts the air expands so quickly that there is no time to exchange heat with the surroundings.



**Figure 8.29 (b):** When the gas is compressed or expanded so fast, the gas cannot exchange heat with surrounding even though there is no thermal insulation.



**Figure 8.28** Adiabatic compression and expansion



**Figure 8.29 (c):** When the warm air rises from the surface of the Earth, it adiabatically expands. As a result the water vapor cools and condenses into water droplets forming a cloud.

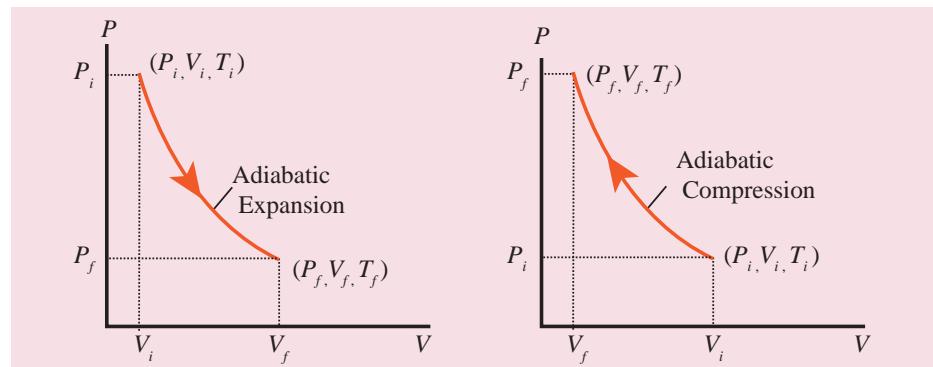
The equation of state for an adiabatic process is given by

$$PV^\gamma = \text{constant} \quad (8.35)$$

Here  $\gamma$  is called adiabatic exponent ( $\gamma = C_p/C_v$ ) which depends on the nature of the gas.

The equation (8.35) implies that if the gas goes from an equilibrium state  $(P_i, V_i)$  to another equilibrium state  $(P_f, V_f)$  adiabatically then it satisfies the relation

$$P_i V_i^\gamma = P_f V_f^\gamma \quad (8.36)$$



**Figure 8.30** PV diagram for adiabatic expansion and adiabatic compression

The PV diagram of an adiabatic expansion and adiabatic compression process are shown in Figure 8.30. The PV diagram for an adiabatic process is also called *adiabat*. Note that the PV diagram for isothermal (Figure 8.25) and adiabatic (Figure 8.30) processes look similar. But actually the adiabatic curve is steeper than isothermal curve.

We can also rewrite the equation (8.35) in terms of T and V. From ideal gas equation,

the pressure  $P = \frac{\mu RT}{V}$ . Substituting this equation in the equation (8.35), we have

$$\frac{\mu RT}{V} V^\gamma = \text{constant} \quad (\text{or}) \quad \frac{T}{V} V^\gamma = \frac{\text{constant}}{\mu R}$$

Note here that is another constant. So it can be written as

$$T V^{\gamma-1} = \text{constant}. \quad (8.37)$$

The equation (8.37) implies that if the gas goes from an initial equilibrium state  $(T_i, V_i)$  to final equilibrium state  $(T_f, V_f)$  adiabatically then it satisfies the relation

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \quad (8.38)$$

The equation of state for adiabatic process can also be written in terms of T and P as

$$T^\gamma P^{1-\gamma} = \text{constant}. \quad (8.39)$$

(The proof of equation (8.39) left as an exercise).

### EXAMPLE 8.18



We often have the experience of pumping air into bicycle tyre using hand pump.

Consider the air inside the pump as a thermodynamic system having volume  $V$  at atmospheric pressure and room temperature,  $27^\circ\text{C}$ . Assume that the nozzle of the tyre is blocked and you push the pump to a volume  $1/4$  of  $V$ . Calculate the final temperature of air in the pump? (For air, since the nozzle is blocked air will not flow into tyre and it can be treated as an adiabatic compression).

#### Solution

Here, the process is adiabatic compression. The volume is given and temperature is to be found. we can use the equation (8.38 )

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}.$$

$$T_i = 300 \text{ K} \quad (273+27^\circ\text{C} = 300 \text{ K})$$

$$V_i = V \text{ & } V_f = \frac{V}{4}$$

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma-1} = 300 \text{ K} \times 4^{1.4-1} = 300 \text{ K} \times 1.741$$

$$T_2 \approx 522 \text{ K or } 249^\circ\text{C}$$

This temperature is higher than the boiling point of water. So it is very dangerous to touch the nozzle of blocked pump when you pump air.

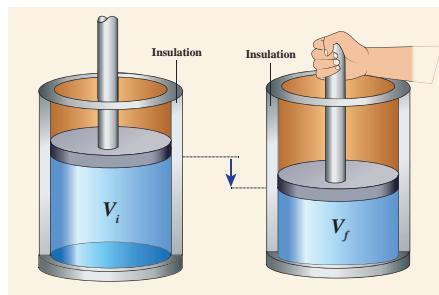


When the piston is compressed so quickly that there is no time to exchange heat to the surrounding, the temperature of the gas increases rapidly. This is shown in the figure. This principle is used in the diesel engine. The air-gasoline mixer is compressed so quickly (adiabatic compression) that the temperature increases enormously, which is enough to produce a spark.



#### Work done in an adiabatic process:

Consider  $\mu$  moles of an ideal gas enclosed in a cylinder having perfectly non conducting walls and base. A frictionless and insulating piston of cross sectional area  $A$  is fitted in the cylinder as shown in Figure 8.31.



**Figure 8.31** Work done in an adiabatic process

Let  $W$  be the work done when the system goes from the initial state  $(P_i, V_i, T_i)$  to the final state  $(P_f, V_f, T_f)$  adiabatically.

$$W = \int_{V_i}^{V_f} P dV \quad (8.40)$$

By assuming that the adiabatic process occurs quasi-statically, at every stage the ideal gas law is valid. Under this condition, the adiabatic equation of state is  $PV^\gamma = \text{constant}$  (or)

$P = \frac{\text{constant}}{V^\gamma}$  can be substituted in the equation (8.40), we get

$$\begin{aligned} \therefore W_{\text{adia}} &= \int_{V_i}^{V_f} \frac{\text{constant}}{V^\gamma} dV \\ &= \text{constant} \int_{V_i}^{V_f} V^{-\gamma} dV \\ &= \text{constant} \left[ \frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_i}^{V_f} \\ &= \text{constant} \left[ \frac{1}{1-\gamma} \left( \frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right) \right] \\ &= \frac{1}{1-\gamma} \left[ \frac{\text{constant}}{V_f^{\gamma-1}} - \frac{\text{constant}}{V_i^{\gamma-1}} \right] \end{aligned}$$

But,  $P_i V_i^\gamma = P_f V_f^\gamma = \text{constant}$ .

$$\begin{aligned} \therefore W_{\text{adia}} &= \frac{1}{1-\gamma} \left[ \frac{P_f V_f^\gamma}{V_f^{\gamma-1}} - \frac{P_i V_i^\gamma}{V_i^{\gamma-1}} \right] \\ W_{\text{adia}} &= \frac{1}{1-\gamma} [P_f V_f - P_i V_i] \end{aligned} \quad (8.41)$$

From ideal gas law,

$$P_f V_f = \mu R T_f \text{ and } P_i V_i = \mu R T_i$$

Substituting in equation (8.41), we get

$$\therefore W_{\text{adia}} = \frac{\mu R}{\gamma-1} [T_i - T_f] \quad (8.42)$$

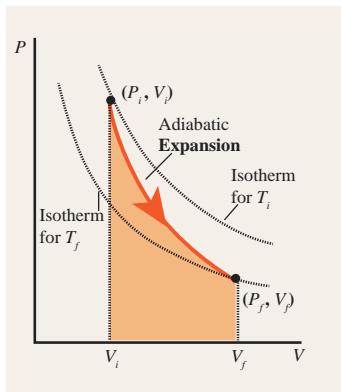
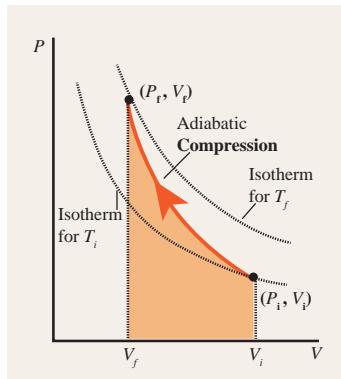
In adiabatic expansion, work is done by the gas. i.e.,  $W_{\text{adia}}$  is positive. As  $T_i > T_f$ , the gas cools during adiabatic expansion.

In adiabatic compression, work is done on the gas. i.e.,  $W_{\text{adia}}$  is negative. As  $T_i < T_f$ , the temperature of the gas increases during adiabatic compression.



Even though we have derived equations (8.41) and (8.42) by assuming that the adiabatic process is quasi-static, both the equations are valid even if the process is not quasi-static. This is because P and V are state variables and are independent of how the state is arrived.

In the adiabatic PV diagram shown in the Figure 8.32, the area under the adiabatic curve from initial state to final state will give the total work done in adiabatic process.



**Figure 8.32** PV diagram -Work done in the adiabatic process

To differentiate between isothermal and adiabatic curves in (Figure 8.32) the adiabatic curve is drawn along with isothermal curve for  $T_f$  and  $T_i$ . Note that adiabatic curve is steeper than isothermal curve. This is because  $\gamma > 1$  always.

### 8.8.3 Isobaric process

This is a thermodynamic process that occurs at constant pressure. Even though pressure is constant in this process, temperature, volume and internal energy are not constant. From the ideal gas equation, we have

$$V = \left( \frac{\mu R}{P} \right) T \quad (8.43)$$

$$\text{Here } \frac{\mu R}{P} = \text{constant}$$

In an isobaric process the temperature is directly proportional to volume.

$$V \propto T \quad (\text{Isobaric process}) \quad (8.44)$$

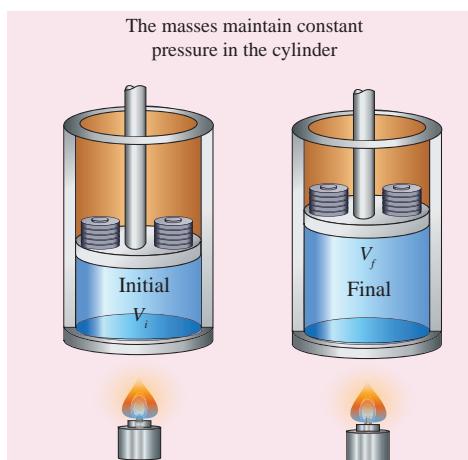
This implies that for a isobaric process, the V-T graph is a straight line passing through the origin.

If a gas goes from a state  $(V_i, T_i)$  to  $(V_f, T_f)$  at constant pressure, then the system satisfies the following equation

$$\frac{T_f}{V_f} = \frac{T_i}{V_i} \quad (8.45)$$

Examples for Isobaric process:

(i) When the gas is heated and pushes the piston so that it exerts a force equivalent to atmospheric pressure plus the force due to gravity then this process is isobaric. This is shown in Figure 8.33



**Figure 8.33** Isobaric process

(ii) Most of the cooking processes in our kitchen are isobaric processes. When the food is cooked in an open vessel, the pressure above the food is always at atmospheric pressure. This is shown in Figure 8.34

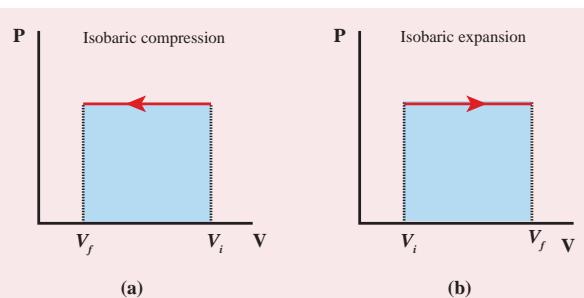


**Figure 8.34** Isobaric process

The PV diagram for an isobaric process is a horizontal line parallel to volume axis as shown in Figure 8.35.

Figure 8.35 (a) represents isobaric process where volume decreases

Figure 8.35 (b) represents isobaric process where volume increases



**Figure 8.35** PV diagram for an isobaric process

The work done in an isobaric process:

Work done by the gas

$$W = \int_{V_i}^{V_f} P dV \quad (8.46)$$

In an isobaric process, the pressure is constant, so  $P$  comes out of the integral,

$$W = P \int_{V_i}^{V_f} dV \quad (8.47)$$

$$W = P[V_f - V_i] = P\Delta V \quad (8.48)$$

Where  $\Delta V$  denotes change in the volume. If  $\Delta V$  is negative,  $W$  is also negative. This implies that the work is done on the gas. If  $\Delta V$  is positive,  $W$  is also positive, implying that work is done by the gas.

The equation (8.48) can also be rewritten using the ideal gas equation.

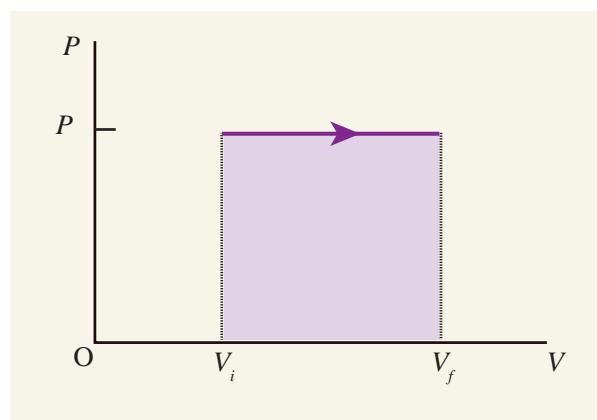
From ideal gas equation

$$PV = \mu RT \text{ and } V = \frac{\mu RT}{P}$$

Substituting this in equation (8.48) we get

$$W = \mu RT_f \left( 1 - \frac{T_i}{T_f} \right) \quad (8.49)$$

In the PV diagram, area under the isobaric curve is equal to the work done in isobaric process. The shaded area in the following Figure 8.36 is equal to the work done by the gas.



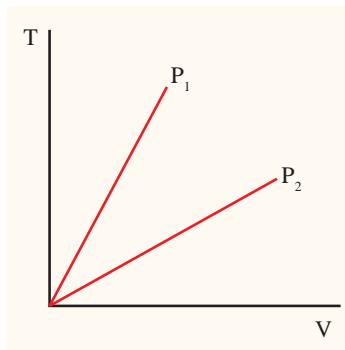
**Figure 8.36** Work done in an isobaric process

The first law of thermodynamics for isobaric process is given by

$$\Delta U = Q - P\Delta V \quad (8.50)$$

### EXAMPLE 8.19

The following graph shows a V-T graph for isobaric processes at two different pressures. Identify which one occurs at higher pressure.



### Solution

From the ideal gas equation,  $V = \left( \frac{\mu R}{P} \right) T$   
V-T graph is a straight line passing the origin.

The slope =  $\frac{\mu R}{P}$

The slope of V-T graph is inversely proportional to the pressure. If the slope is greater, lower is the pressure.

Here  $P_1$  has larger slope than  $P_2$ . So  $P_2 > P_1$ .

Suppose the graph is drawn between T and V (Temperature along the x-axis and Volume along the y-axis) then will we still have  $P_2 > P_1$ ?

### EXAMPLE 8.20

One mole of an ideal gas initially kept in a cylinder at pressure 1 MPa and temperature 27°C is made to expand until its volume is doubled.

- (a) How much work is done if the expansion is (i) adiabatic (ii) isobaric (iii) isothermal?

- (b) Identify the processes in which change in internal energy is least and is maximum.
- (c) Show each process on a PV diagram.
- (d) Name the processes in which the heat transfer is maximum and minimum.

(Take  $\gamma = \frac{5}{3}$  and  $R=8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ )

### Solution

- (a) (i) In an adiabatic process the work done by the system is

$$W_{\text{adia}} = \frac{\mu R}{\gamma - 1} [T_i - T_f]$$

To find the final temperature  $T_f$ , we can use adiabatic equation of state.

$$\begin{aligned} T_f V_f^{\gamma-1} &= T_i V_i^{\gamma-1} \\ T_f &= T_i \left( \frac{V_i}{V_f} \right)^{\gamma-1} = 300 \times \left( \frac{1}{2} \right)^{\frac{2}{3}} \\ &= 0.63 \times 300K = 189.8K \end{aligned}$$

$$W = 1 \times 8.3 \times \frac{3}{2} (300 - 189.8) = 1.37 \text{ kJ}$$

- (ii) In an isobaric process the work done by the system

$$W = P \Delta V = P(V_f - V_i)$$

and  $V_f = 2V_i$  so  $W = 2PV_i$

To find  $V_i$ , we can use the ideal gas law for initial state.  $P_i V_i = RT_i$

$$V_i = \frac{RT_i}{P_i} = 8.3 \times \frac{300}{1} \times 10^{-6} = 24.9 \times 10^{-4} \text{ m}^3$$

The work done during isobaric process,  $W = 2 \times 10^6 \times 24.9 \times 10^{-4} = 4.9 \text{ kJ}$

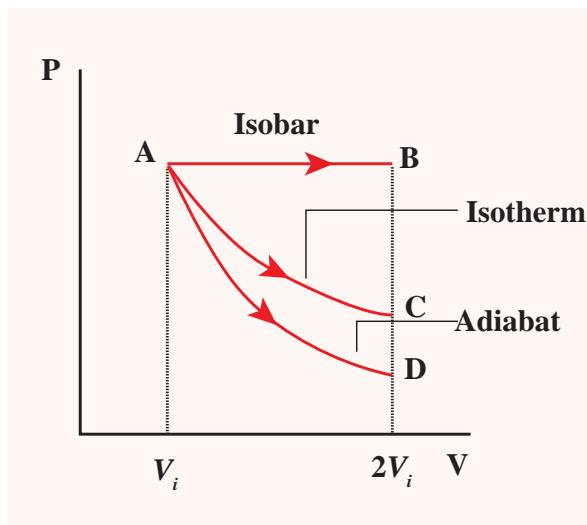
- (iii) In an isothermal process the work done by the system,

$$W = \mu RT \ln \left( \frac{V_f}{V_i} \right)$$

In an isothermal process the initial room temperature is constant.

$$W = 1 \times 8.3 \times 300 \times \ln(2) = 1.7 \text{ kJ}$$

- (b) Comparing all three processes, we see that the work done in the isobaric process is the greatest, and work done in the adiabatic process is the least.
- (c) The PV diagram is shown in the Figure.



The area under the curve AB = Work done during the isobaric process

The area under the curve AC = Work done during the isothermal process

The area under the curve AD = Work done during the adiabatic process

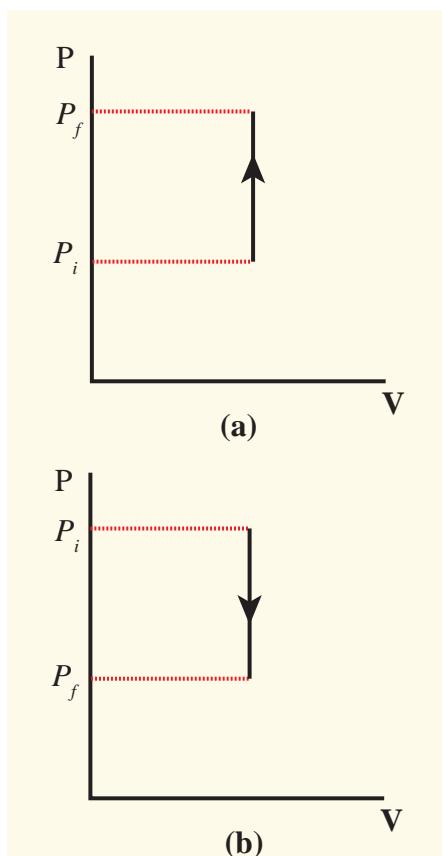
From the PV diagram the area under the curve AB is more, implying that the work done in isobaric process is highest and work done in adiabatic process is least.

- (d) In an adiabatic process no heat enters into the system or leaves from the system. In an isobaric process the work done is more so heat supplied should be more compared to an isothermal process.

### 8.8.4 Isochoric process

This is a thermodynamic process in which the volume of the system is kept constant. But pressure, temperature and internal energy continue to be variables.

The pressure - volume graph for an isochoric process is a vertical line parallel to pressure axis as shown in Figure 8.37.



**Figure 8.37** Isochoric process with (a) increased pressure and (b) decreased pressure

The equation of state for an isochoric process is given by

$$P = \left( \frac{\mu R}{V} \right) T \quad (8.51)$$

where  $\left( \frac{\mu R}{V} \right) = \text{constant}$

We can infer that the pressure is directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin.

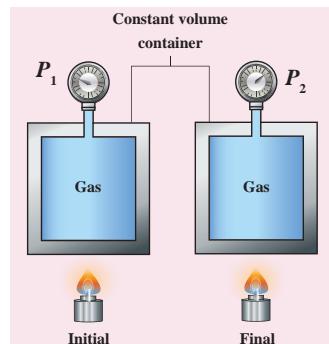
If a gas goes from state  $(P_i, T_i)$  to  $(P_f, T_f)$  at constant volume, then the system satisfies the following equation

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \quad (8.52)$$

For an isochoric processes,  $\Delta V=0$  and  $W=0$ . Then the first law becomes

$$\Delta U = Q \quad (8.53)$$

Implying that the heat supplied is used to increase only the internal energy. As a result the temperature increases and pressure also increases. This is shown in Figure 8.38



**Figure 8.38** Isochoric process

Suppose a system loses heat to the surroundings through conducting walls by keeping the volume constant, then its internal energy decreases. As a result the temperature decreases; the pressure also decreases.

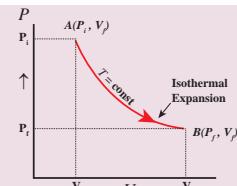
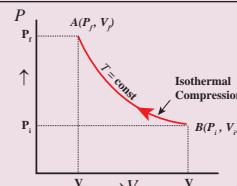
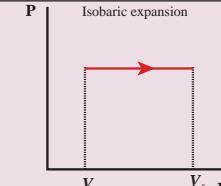
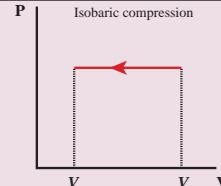
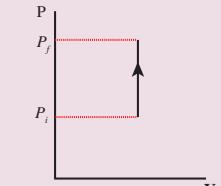
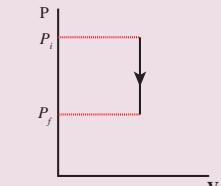
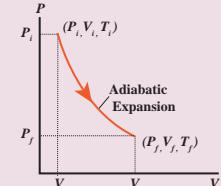
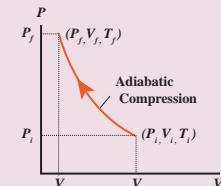
#### Examples:

- When food is cooked by closing with a lid as shown in figure.



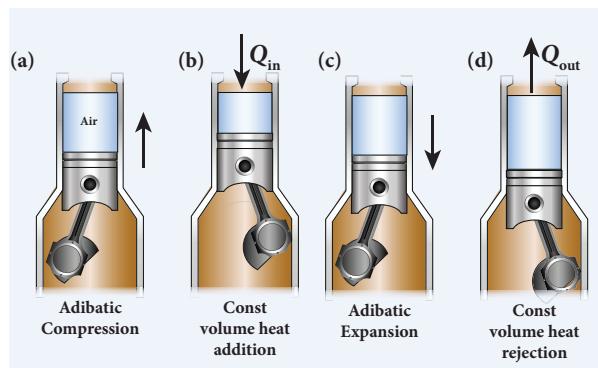
**Table 8.4:** Summary of various thermodynamic processes

S. No.	Process	Heat Q	Temperature & internal energy	Pressure	
1	Isothermal	Expansion	$Q > 0$	Constant	decreases
		Compression	$Q < 0$	Constant	Increases
2	Isobaric	Expansion	$Q > 0$	increases	Constant
		Compression	$Q < 0$	decreases	Constant
3	Isochoric		$Q > 0$	Increases	increases
			$Q < 0$	Decreases	decreases
4	Adiabatic	expansion	$Q = 0$	Decreases	Decreases
		Compression	$Q = 0$	Increases	increases

Volume	Equation of state	Work done (ideal gas)	Indicator diagram (PV diagram)
increases		$W = \mu RT \ln \left( \frac{V_f}{V_i} \right) > 0$	
Decreases	$PV = \text{Constant}$	$W = \mu RT \ln \left( \frac{V_i}{V_f} \right) < 0$	
increases		$W = P [V_f - V_i] = P\Delta V > 0$	
decreases	$\frac{V}{T} = \text{Constant}$	$W = P [V_i - V_f] = P\Delta V < 0$	
Constant	$\frac{P}{T} = \text{Constant}$	Zero	 
increases		$W = \frac{\mu R}{\gamma - 1} (T_i - T_f) > 0$	
Decreases	$PV^\gamma = \text{Constant}$	$W = \frac{\mu R}{\gamma - 1} (T_i - T_f) < 0$	

When food is being cooked in this closed position, after a certain time you can observe the lid is being pushed upwards by the water steam. This is because when the lid is closed, the volume is kept constant. As the heat continuously supplied, the pressure increases and water steam tries to push the lid upwards.

2. In automobiles the petrol engine undergoes four processes. First the piston is adiabatically compressed to some volume as shown in the Figure (a). In the second process (Figure (b)), the volume of the air-fuel mixture is kept constant and heat is being added. As a result the temperature and pressure are increased. This is an isochoric process. For a third stroke (Figure (c)) there will be an adiabatic expansion, and fourth stroke again isochoric process by keeping the piston immovable (Figure (d)).



The summary of various thermodynamic processes is given in the Table 8.4.

### EXAMPLE 8.21

500 g of water is heated from 30°C to 60°C. Ignoring the slight expansion of water, calculate the change in internal energy of the water? (specific heat of water 4184 J/kg.K)

### Solution

When the water is heated from 30°C to 60°C, there is only a slight change in its volume. So we can treat this process as isochoric. In an isochoric process the work done by the system is zero. The given heat supplied is used to increase only the internal energy.

$$\Delta U = Q = m s_v \Delta T$$

The mass of water = 500 g = 0.5 kg

The change in temperature = 30K

The heat  $Q = 0.5 \times 4184 \times 30 = 62.76 \text{ kJ}$

### 8.8.5 Cyclic processes

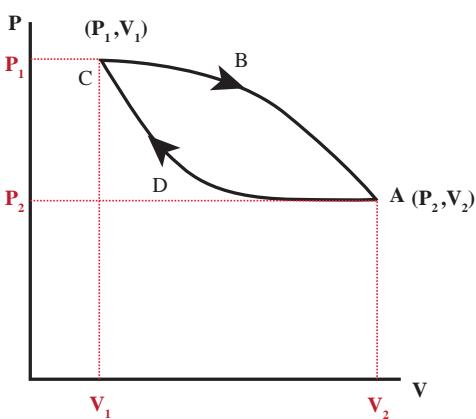
This is a thermodynamic process in which the thermodynamic system returns to its initial state after undergoing a series of changes. Since the system comes back to the initial state, the change in the internal energy is zero. In cyclic process, heat can flow in to system and heat flow out of the system. From the first law of thermodynamics, the net heat transferred to the system is equal to work done by the gas.

$$Q_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = W \quad (\text{for a cyclic process}) \quad (8.54)$$

### 8.8.6 PV diagram for a cyclic process

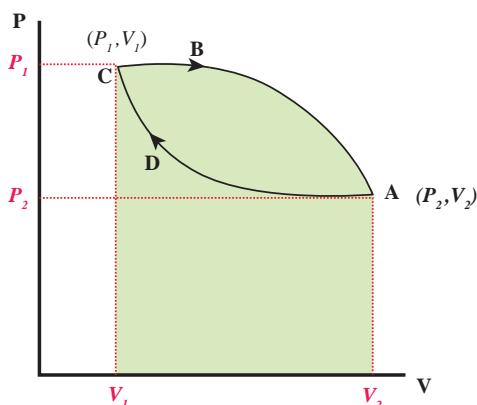
In the PV diagram the cyclic process is represented by a closed curve.

Let the gas undergo a cyclic process in which it returns to the initial stage after an expansion and compression as shown in Figure 8.39

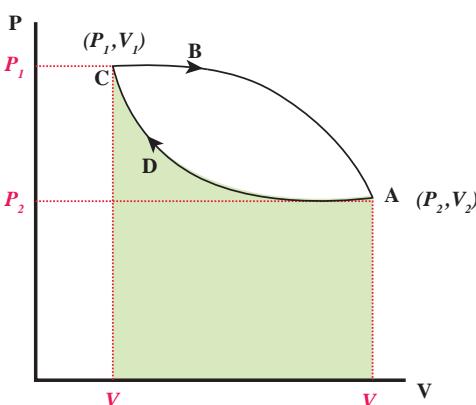


**Figure 8.39** PV diagram for cyclic process

Let  $W_1$  be the work done by the gas during expansion from volume  $V_1$  to volume  $V_2$ . It is equal to area under the graph CBA as shown in Figure 8.40 (a).



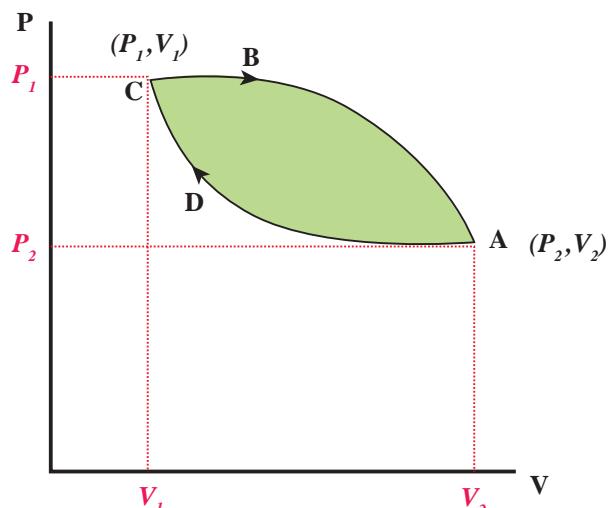
**Figure 8.40 (a)**  $W$  for path CBA



**Figure 8.40 (b)**  $W$  for path ADC

Let  $W_2$  be the work done on the gas during compression from volume  $V_2$  to volume  $V_1$ . It is equal to the area under the graph ADC as shown in Figure 8.40 (b)

The total work done in this cyclic process  
 $= W_1 - W_2 =$  Green shaded area inside the loop, as shown in Figure 8.41.



**Figure 8.41** Net work done in a cyclic process

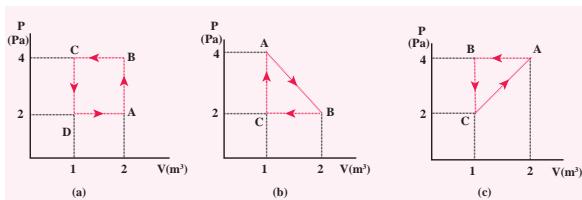
Thus the net work done during the cyclic process shown above is not zero. In general the net work done can be positive or negative. If the net work done is positive, then work done by the system is greater than the work done on the system. If the net work done is negative then the work done by the system is less than the work done on the system.



Further, in a cyclic process the net work done is positive if the process goes clockwise and net work done is negative if the process goes anti-clockwise. In Figure 8.41 the process goes clockwise.

**EXAMPLE 8.22**

The PV diagrams for a thermodynamical system is given in the figure below. Calculate the total work done in each of the cyclic processes shown.

**Solution**

In the case (a) the closed curve is anticlockwise. So the net work done is negative, implying that the work done on the system is greater than the work done by the system. The area under the curve BC will give work done on the gas (isobaric compression) and area under the curve DA (work done by the system) will give the total work done by the system.

$$\text{Area under the curve BC} = \text{Area of rectangle BC12} = 1 \times 4 = -4 \text{ J}$$

$$\text{Area under the curve DA} = 1 \times 2 = +2 \text{ J}$$

$$\begin{aligned}\text{Net work done in cyclic process} \\ = -4 + 2 = -2 \text{ J}\end{aligned}$$

In the case (b) the closed curve is clockwise. So the net work done is positive, implying that the work done on the system is less than the work done by the system. Area under the curve BC will give work done on the gas (isobaric compression) and area under the curve AB will give the total work done by the system.

$$\begin{aligned}\text{Area under the curve AB} = \text{rectangle area} + \\ \text{triangle area} = (1 \times 2) + \frac{1}{2} \times 1 \times 2 = +3 \text{ J}\end{aligned}$$

$$\begin{aligned}\text{Area under the curve BC} &= \text{rectangle area} \\ &= 1 \times 2 = -2 \text{ J}\end{aligned}$$

Network done in the cyclic process = 1 J, which is positive.

In the case (c) the closed curve is anticlockwise. So the net work done is negative, implying that the work done on the system is greater than work done by the system. The area under the curve AB will give the work done on the gas (isobaric compression) and area under the curve CA (work done by the system) will give the total work done by the system.

$$\text{The area under the curve AB} = \text{Rectangle of area} = 4 \times 1 = -4 \text{ J}$$

$$\begin{aligned}\text{The area under the curve CA} &= \text{Rectangle area} + \text{triangle area} = (1 \times 2) + \frac{1}{2} \times 1 \times 2 = +3 \text{ J} \\ \text{The total work in the cyclic process} &= -1 \text{ J. It is negative}\end{aligned}$$

### 8.8.7 Limitations of first law of thermodynamics

The first law of thermodynamics explains well the inter convertibility of heat and work. But it does not indicate the direction of change.

For example,

- When a hot object is in contact with a cold object, heat always flows from the hot object to cold object but not in the reverse direction. According to first law, it is possible for the energy to flow from hot object to cold object or from cold object to hot object. But in nature the direction of heat flow is always from higher temperature to lower temperature.
- When brakes are applied, a car stops due to friction and the work done against friction is converted into heat. But this

heat is not reconverted to the kinetic energy of the car.  
So the first law is not sufficient to explain many of natural phenomena.

### 8.8.8 Reversible process

A thermodynamic process can be considered reversible only if it is possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states as in the initial, direct process.

Example: A quasi-static isothermal expansion of gas, slow compression and expansion of a spring.

Conditions for reversible process:

1. The process should proceed at an extremely slow rate.
2. The system should remain in mechanical, thermal and chemical equilibrium state at all the times with the surroundings, during the process.
3. No dissipative forces such as friction, viscosity, electrical resistance should be present.



All reversible processes are quasi-static but all quasi-static processes need not be reversible. For example when we push the piston very slowly, if there is friction between cylinder wall and piston some amount of energy is lost to surroundings, which cannot be retrieved back.

#### Irreversible process:

All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure, temperature at every stage of the process.

The first law of thermodynamics is the statement about conservation of energy in a thermodynamic process. For example, if a hotter object is placed on a colder object, heat flows from hotter to colder object. Why does heat not flow from the colder object to hotter object? Even if energy flows from colder object to hotter object, the first law of thermodynamics is not violated. For example, if 5 J of heat flows from hotter to colder or from colder to hotter objects the total internal energy of this combined system remains the same. But 5 J of heat never flows from the colder object to hotter object. In nature all such process occur only in one direction but not in the reverse direction, even if the energy is conserved in both the processes. Thus the first law of thermodynamics has no explanation for this irreversibility. When the scientists of the eighteenth century tried to explain this irreversibility, they discovered a new law of nature. This is called the second law of thermodynamics. According to second law of thermodynamics

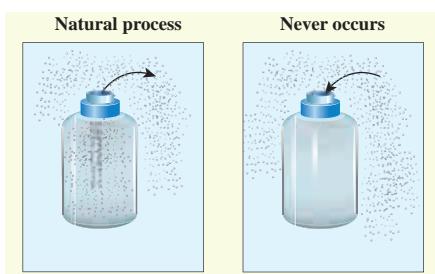
“Heat always flows from hotter object to colder object spontaneously”. This is known as the Clausius form of second law of thermodynamics.

### EXAMPLE 8.23

Give some examples of irreversible processes.

All naturally occurring processes are irreversible. Here we give some interesting examples.

- (a) When we open a gas bottle, the gas molecules slowly spread into the entire room. These gas molecules can never get back in to the bottle.



- (b) Suppose one drop of an ink is dropped in water, the ink droplet slowly spreads in the water. It is impossible to get the ink droplet back.
- (c) When an object falls from some height, as soon as it hits the earth it comes to rest. All the kinetic energy of the object is converted to kinetic energy of molecules of the earth surface, molecules of the object and small amount goes as sound energy. The spreaded kinetic energy to the molecules never collected back and object never goes up by itself.

Note that according to first law of thermodynamics all the above processes are possible in both directions. But second law of thermodynamics forbids the processes to occur in the reverse direction. The second law of thermodynamics is one of the very important laws of nature. It controls the way the natural processes occur.

## 8.9

### HEAT ENGINE

In the modern technological world, the role of automobile engines plays a vital role in for transportation. In motor bikes and cars there are engines which take in petrol or diesel as input, and do work by rotating wheels. Most of these automobile engines have efficiency not greater than 40%. The second law of thermodynamics

puts a fundamental restriction on efficiency of engines. Therefore understanding heat engines is very important.

#### Reservoir:

It is defined as a thermodynamic system which has very large heat capacity. By taking in heat from reservoir or giving heat to reservoir, the reservoir's temperature does not change.

**Example:** Pouring a tumbler of hot water in to lake will not increase the temperature of the lake. Here the lake can be treated as a reservoir.

When a hot cup of coffee attains equilibrium with the open atmosphere, the temperature of the atmosphere will not appreciably change. The atmosphere can be taken as a reservoir.

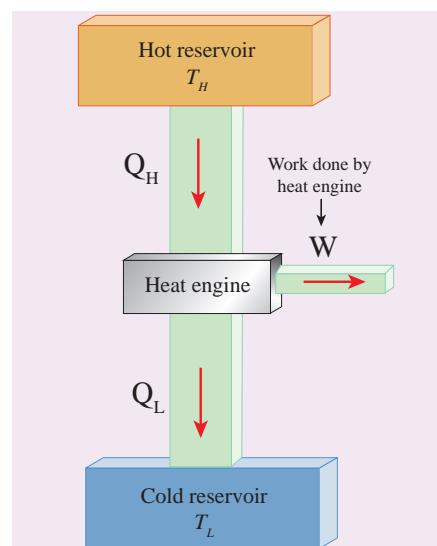
We can define heat engine as follows.

*Heat engine is a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.*

A heat engine has three parts:

- (a) Hot reservoir
- (b) Working substance
- (c) Cold reservoir

A Schematic diagram for heat engine is given below in the figure 8.42.



**Figure 8.42** Heat Engine

1. Hot reservoir (or) Source: It supplies heat to the engine. It is always maintained at a high temperature  $T_H$
2. Working substance: It is a substance like gas or water, which converts the heat supplied into work.

A simple example of a heat engine is a steam engine. In olden days steam engines were used to drive trains. The working substance in these is water which absorbs heat from the burning of coal. The heat converts the water into steam. This steam is does work by rotating the wheels of the train, thus making the train move.

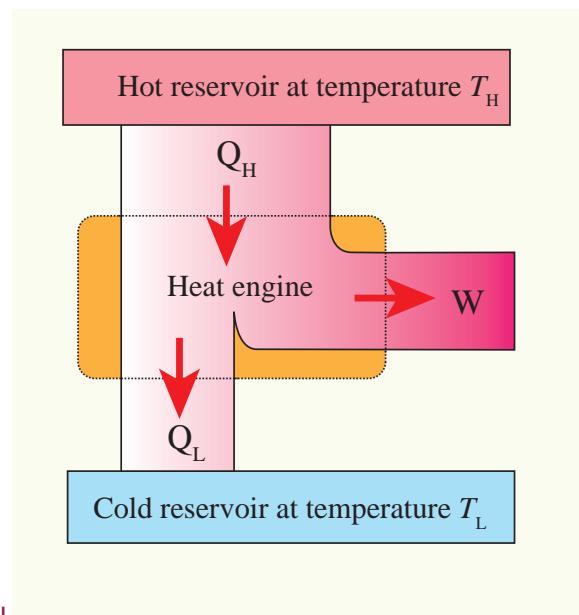
3. Cold reservoir (or) Sink: The heat engine ejects some amount of heat ( $Q_L$ ) in to cold reservoir after it doing work. It is always maintained at a low temperature  $T_L$ .

For example, in the automobile engine, the cold reservoir is the surroundings at room temperature. The automobile ejects heat to these surroundings through a silencer.

The heat engine works in a cyclic process. After a cyclic process it returns to the same state. Since the heat engine returns to the same state after it ejects heat, the change in the internal energy of the heat engine is zero.

The efficiency of the heat engine is defined as the ratio of the work done (output) to the heat absorbed (input) in one cyclic process.

Let the working substance absorb heat  $Q_H$  units from the source and reject  $Q_L$  units to the sink after doing work  $W$  units, as shown in the Figure 8.43.



**Figure 8.43** Heat engine

We can write

$$\text{Input heat} = \text{Work done} + \text{ejected heat}$$

$$Q_H = W + Q_L$$

$$W = Q_H - Q_L$$

Then the efficiency of heat engine

$$\eta = \frac{\text{output}}{\text{input}} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$\eta = \frac{\text{output}}{\text{input}} = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H} \quad (8.55)$$

Note here that  $Q_H$ ,  $Q_L$  and  $W$  all are taken as positive, a sign convention followed in this expression.

Since  $Q_L < Q_H$ , the efficiency ( $\eta$ ) always less than 1. This implies that heat absorbed is not completely converted into work. The second law of thermodynamics placed fundamental restrictions on converting heat completely into work.

We can state the heat engine statement of second law of thermodynamics. This is also called Kelvin-Planck's statement.

### Kelvin-Planck statement:

*It is impossible to construct a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work. This implies that no heat engine in the universe can have 100% efficiency.*

**Note**

According to first law of thermodynamics, in an isothermal process the given heat is completely converted into work ( $Q = W$ ). Is it a violation of the second law of thermodynamics? No. For non-cyclic process like an isothermal expansion, the heat can be completely converted into work. But Second law of thermodynamics implies that 'In a cyclic process only a portion of the heat absorbed is converted into work'. All heat engines operate in a cyclic process.

### EXAMPLE 8.24

During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine?

#### Solution

The efficiency of heat engine is given by

$$\eta = 1 - \frac{Q_L}{Q_H}$$

$$\eta = 1 - \frac{300}{500} = 1 - \frac{3}{5}$$

$$\eta = 1 - 0.6 = 0.4$$

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

### 8.9.1 Carnot's ideal heat engine

In the previous section we have seen that the heat engine cannot have 100% efficiency. What is the maximum possible efficiency can a heat engine have?. In the year 1824 a young French engineer Sadi Carnot proved that a certain reversible engine operated in cycle between hot and cold reservoir can have maximum efficiency. This engine is called Carnot engine.

*A reversible heat engine operating in a cycle between two temperatures in a particular way is called a Carnot Engine.*

The carnot engine has four parts which are given below.

- i Source: It is the source of heat maintained at constant high temperature  $T_H$ . Any amount of heat can be extracted from it, without changing its temperature.
- ii Sink: It is a cold body maintained at a constant low temperature  $T_L$ . It can absorb any amount of heat.
- iii Insulating stand: It is made of perfectly non-conducting material. Heat is not conducted through this stand.
- iv Working substance: It is an ideal gas enclosed in a cylinder with perfectly non-conducting walls and perfectly conducting bottom. A non-conducting and frictionless piston is fitted in it.

The four parts are shown in the following Figure 8.44



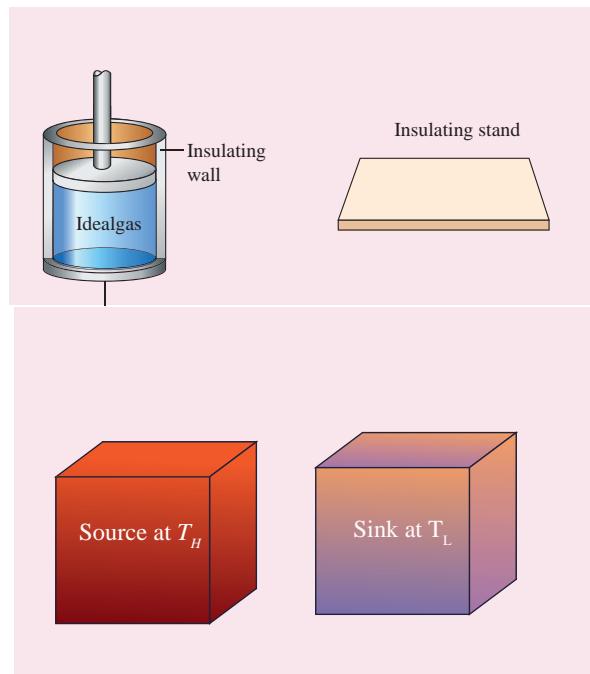


Figure 8.44 Carnot engine

**Carnot's cycle:**

The working substance is subjected to four successive reversible processes forming what is called Carnot's cycle.

Let the initial pressure, volume of the working substance be  $P_1, V_1$ .

**Step A to B:** Quasi-static isothermal expansion from  $(P_1, V_1, T_H)$  to  $(P_2, V_2, T_H)$ :

The cylinder is placed on the source. The heat ( $Q_H$ ) flows from source to the working substance (ideal gas) through the bottom of the cylinder. Since the process is isothermal, the internal energy of the working substance will not change. The input heat increases the volume of the gas. The piston is allowed to move out very slowly(quasi-statically). It is shown in the figure 8.47(a).

$W_1$  is the work done by the gas in expanding from volume  $V_1$  to volume  $V_2$  with a decrease of pressure from  $P_1$  to  $P_2$ . This is represented by the P-V diagram along the path AB as shown in the Figure 8.45.

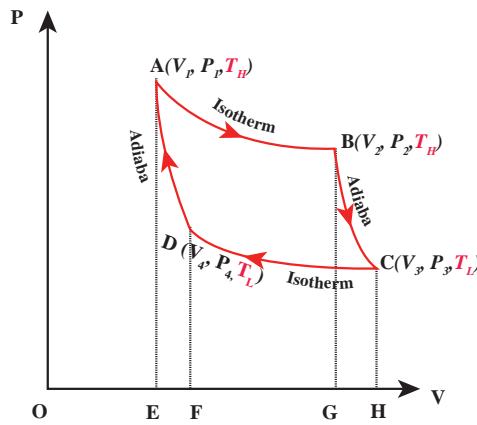


Figure 8.45 PV diagram for Carnot cycle

Then the work done by the gas (working substance) is given by

$$\therefore Q_H = W_{A \rightarrow B} = \int_{V_1}^{V_2} P dV$$

Since the process occurs quasi-statically, the gas is in equilibrium with the source till it reaches the final state. The work done in the isothermal expansion is given by the equation (8.34)

$$W_{A \rightarrow B} = \mu R T_H \ln\left(\frac{V_2}{V_1}\right) = \text{Area under the curve AB}$$
(8.56)

This is shown in Figure 8.46 (a)

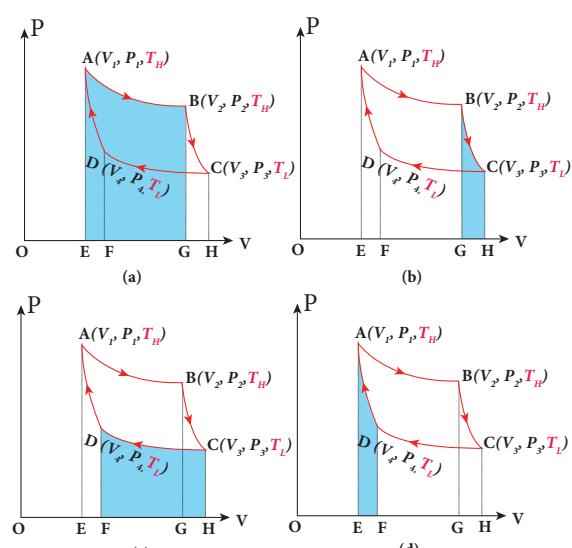
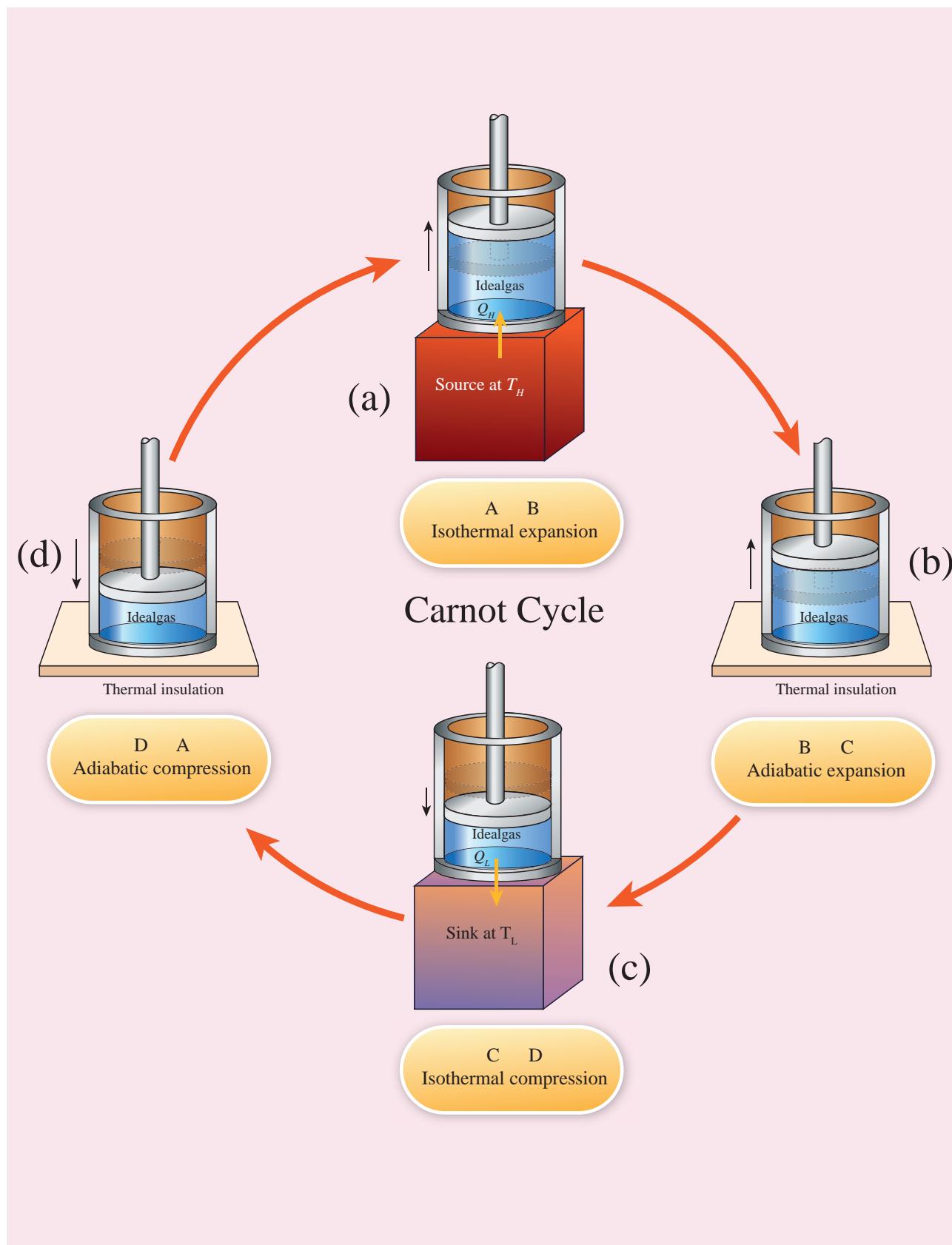


Figure 8.46 Work done in Carnot cycle



**Figure 8.47** Carnot cycle

Step B to C: Quasi-static adiabatic expansion from  $(P_2, V_2, T_H)$  to  $(P_3, V_3, T_L)$

The cylinder is placed on the insulating stand and the piston is allowed to move out. As the gas expands adiabatically from volume  $V_2$  to volume  $V_3$  the pressure falls from  $P_2$  to  $P_3$ . The temperature falls to  $T_L$ . This adiabatic expansion is represented by curve BC in the P-V diagram. This adiabatic process also occurs quasi-statically and implying that this process is reversible and the ideal gas is in equilibrium throughout the process. It is shown in the figure 8.47(b). From the equation (8.42)

The work done by the gas in an adiabatic expansion is given by,

$$W_{B \rightarrow C} = \int_{V_2}^{V_3} P dV = \frac{\mu R}{\gamma - 1} [T_H - T_L] = \\ \text{Area under the curve BC} \quad (8.57)$$

This is shown in Figure 8.46 (b)

Step C → D: Quasi-static isothermal compression from  $(P_3, V_3, T_L)$  to  $(P_4, V_4, T_L)$ : It is shown in the figure 8.47(c)

The cylinder is placed on the sink and the gas is isothermally compressed until the pressure and volume become  $P_4$  and  $V_4$  respectively. This is represented by the curve CD in the PV diagram as shown in Figure 8.45. Let  $W_{C \rightarrow D}$  be the work done on the gas. According to first law of thermodynamics

$$\therefore W_{C \rightarrow D} = \int_{V_3}^{V_4} P dV = \mu R T_L \ln \left( \frac{V_4}{V_3} \right) = -\mu R T_L \ln \left( \frac{V_3}{V_4} \right) \\ . = - \text{Area under the curve CD} \quad (8.58)$$

This is shown in Figure 8.46 (c)

Here  $V_3$  is greater than  $V_4$ . So the work done is negative, implying work is done on the gas.

Step D → A: Quasi-static adiabatic compression from  $(P_4, V_4, T_L)$  to  $(P_1, V_1, T_H)$ : It is shown in the figure 8.47(d)

The cylinder is placed on the insulating stand again and the gas is compressed adiabatically till it attains the initial pressure  $P_1$ , volume  $V_1$  and temperature  $T_H$ . This is shown by the curve DA in the P-V diagram.

$$\therefore W_{D \rightarrow A} = \int_{V_4}^{V_1} P dV = \frac{\mu R}{\gamma - 1} (T_L - T_H) = - \text{Area under the curve DA} \quad (8.59)$$

In the adiabatic compression also work is done on the gas so it is negative, as is shown in Figure 8.46 (d)

Let 'W' be the net work done by the working substance in one cycle

$\therefore W = \text{Work done by the gas} - \text{work done on the gas}$

$$= W_{A \rightarrow B} + W_{B \rightarrow C} - W_{C \rightarrow D} - W_{D \rightarrow A} \\ \text{since } W_{B \rightarrow C} = W_{D \rightarrow A} \\ = W_{A \rightarrow B} - W_{C \rightarrow D}$$

The net work done by the Carnot engine in one cycle  $W = W_{A \rightarrow B} - W_{C \rightarrow D}$  (8.60)

Equation (8.60) shows that the net work done by the working substance in one cycle is equal to the area (enclosed by ABCD) of the P-V diagram (Figure 8.48)

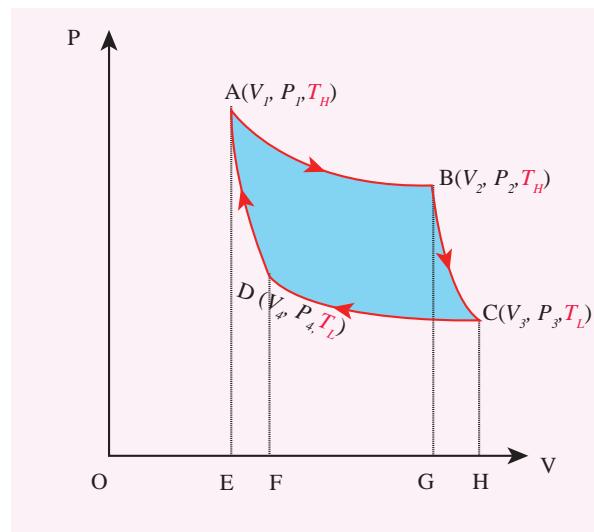


Figure 8.48 Net work done in Carnot cycle

It is very important to note that after one cycle the working substance returns to the initial temperature  $T_H$ . This implies that the change in internal energy of the working substance after one cycle is zero.

### 8.9.2 Efficiency of a Carnot engine

Efficiency is defined as the ratio of work done by the working substance in one cycle to the amount of heat extracted from the source.

$$\eta = \frac{\text{work done}}{\text{Heat extracted}} = \frac{W}{Q_H} \quad (8.61)$$

From the first law of thermodynamics,  
 $W = Q_H - Q_L$

$$\therefore \eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \quad (8.62)$$

Applying isothermal conditions, we get,

$$\begin{aligned} Q_H &= \mu RT_H \ln\left(\frac{V_2}{V_1}\right) \\ Q_L &= \mu RT_L \ln\left(\frac{V_3}{V_4}\right) \end{aligned} \quad (8.63)$$

Here we omit the negative sign. Since we are interested in only the amount of heat ( $Q_L$ ) ejected into the sink, we have

$$\therefore \frac{Q_L}{Q_H} = \frac{T_L \ln\left(\frac{V_3}{V_4}\right)}{T_H \ln\left(\frac{V_2}{V_1}\right)} \quad (8.64)$$

By applying adiabatic conditions, we get,

$$T_H V_2^{\gamma-1} = T_L V_3^{\gamma-1}$$

$$T_H V_1^{\gamma-1} = T_L V_4^{\gamma-1}$$

By dividing the above two equations, we get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

Which implies that  $\frac{V_2}{V_1} = \frac{V_3}{V_4}$  (8.65)

Substituting equation (8.65) in (8.64), we get

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \quad (8.66)$$

$$\therefore \text{The efficiency } \eta = 1 - \frac{T_L}{T_H} \quad (8.67)$$

Note :  $T_L$  and  $T_H$  should be expressed in Kelvin scale.

#### Important results:

- $\eta$  is always less than 1 because  $T_L$  is less than  $T_H$ . This implies the efficiency cannot be 100%. It can be 1 or 100% only when  $T_L = 0K$  (absolute zero of temperature) which is impossible to attain practically.
- The efficiency of the Carnot's engine is independent of the working substance. It depends only on the temperatures of the source and the sink. The greater the difference between the two temperatures, higher the efficiency.
- When  $T_H = T_L$  the efficiency  $\eta = 0$ . No engine can work having source and sink at the same temperature.
- The entire process is reversible in the Carnot engine cycle. So Carnot engine is itself a reversible engine and has maximum efficiency. But all practical heat engines like diesel engine, petrol engine and steam engine have cycles which are not perfectly reversible. So their efficiency is always less than the Carnot efficiency. This can be stated in the form of the Carnot theorem. It is stated as follows '*Between two constant temperatures reservoirs, only Carnot engine can have maximum efficiency. All real heat engines will have efficiency less than the Carnot engine'*

**EXAMPLE 8.25**

- (a) A steam engine boiler is maintained at 250°C and water is converted into steam. This steam is used to do work and heat is ejected to the surrounding air at temperature 300K. Calculate the maximum efficiency it can have?

**Solution**

The steam engine is not a Carnot engine, because all the process involved in the steam engine are not perfectly reversible. But we can calculate the maximum possible efficiency of the steam engine by considering it as a Carnot engine.

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{523\text{ K}} = 0.43$$

The steam engine can have maximum possible 43% of efficiency, implying this steam engine can convert 43% of input heat into useful work and remaining 57% is ejected as heat. In practice the efficiency is even less than 43%.

**EXAMPLE 8.26**

There are two Carnot engines A and B operating in two different temperature regions. For Engine A the temperatures of the two reservoirs are 150°C and 100°C. For engine B the temperatures of the reservoirs are 350°C and 300°C. Which engine has lesser efficiency?

**Solution**

The efficiency for engine A =  $1 - \frac{373}{423} = 0.11$ . Engine A has 11% efficiency

The efficiency for engine B =  $1 - \frac{573}{623} = 0.08$   
Engine B has only 8% efficiency.

Even though the differences between the temperature of hot and cold reservoirs in both engines is same, the efficiency is not same. The efficiency depends on the ratio of the two temperature and not on the difference in the temperature. The engine which operates in lower temperature has highest efficiency.



Diesel engines used in cars and petrol engines used in our motor bikes are all real heat engines.

The efficiency of diesel engines has maximum up to 44% and the efficiency of petrol engines are maximum up to 30%. Since these engines are not ideal heat engines (Carnot engine), their efficiency is limited by the second law of thermodynamics. Now a days typical bikes give a mileage of 50 km per Liter of petrol. This implies only 30% of 1 Liter of petrol is converted into mechanical work and the remaining 70% goes out as wasted heat and ejected into the surrounding atmosphere!

**8.9.3 Entropy and second law of thermodynamics**

We have seen in the equation (8.66) that the quantity  $\frac{Q_H}{T_H}$  is equal to  $\frac{Q_L}{T_L}$ . The quantity  $\frac{Q}{T}$  is called entropy. It is a very important thermodynamic property of a system. It is also a state variable.  $\frac{Q_H}{T_H}$  is the entropy received by the Carnot engine from hot reservoir and  $\frac{Q_L}{T_L}$  is entropy given out by the Carnot engine to the cold reservoir. For reversible engines (Carnot Engine) both entropies should be same, so that the change

in entropy of the Carnot engine in one cycle is zero. This is proved in equation (8.66). But for all practical engines like diesel and petrol engines which are not reversible engines, they satisfy the relation  $\frac{Q_L}{T_L} > \frac{Q_H}{T_H}$ . In fact we can reformulate the second law of thermodynamics as follows

"For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.

We now come back to the question: Why does heat always flows from a state of higher temperature to one of lower temperature and not in the opposite direction? Because entropy increases when heat flows from hot object to cold object. If heat were to flow from a cold to a hot object, entropy will decrease leading to violation of second law thermodynamics.

*Entropy is also called 'measure of disorder'. All natural process occur such that the disorder should always increases.*

Consider a bottle with a gas inside. When the gas molecules are inside the bottle it has less disorder. Once it spreads into the entire room it leads to more disorder. In other words when the gas is inside the bottle the entropy is less and once the gas spreads into entire room, the entropy increases. From the second law of thermodynamics, entropy always increases. If the air molecules go back in to the bottle, the entropy should decrease, which is not allowed by the second law of thermodynamics. The same explanation applies to a drop of ink diffusing into water. Once the drop of ink spreads, its entropy is increased. The diffused ink can never become a drop again. So the natural

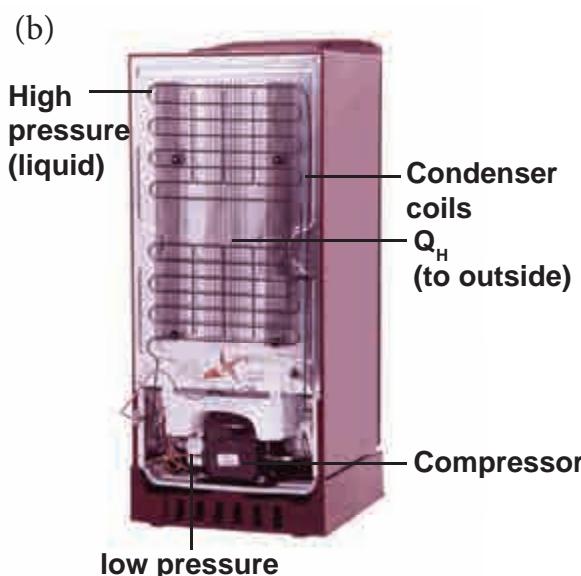
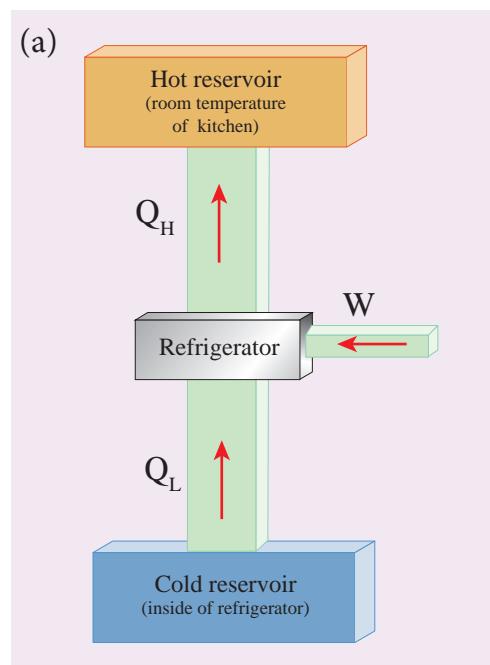
processes occur in such a way that entropy should increase for all irreversible process.

## 8.10

### REFRIGERATOR

A refrigerator is a Carnot's engine working in the reverse order. It is shown in the figure 8.49.

#### Working Principle:



**Fig 8.49** (a) Schematic diagram of a refrigerator (b) Actual refrigerator

The working substance (gas) absorbs a quantity of heat  $Q_L$  from the cold body (sink) at a lower temperature  $T_L$ . A certain amount of work  $W$  is done on the working substance by the compressor and a quantity of heat  $Q_H$  is rejected to the hot body (source) ie, the atmosphere at  $T_H$ . When you stand beneath of refrigerator, you can feel warmth air. From the first law of thermodynamics , we have

$$Q_L + W = Q_H \quad (8.68)$$

As a result the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.

#### Coefficient of performance (COP) ( $\beta$ ):

COP is a measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor  $W$ .

$$\text{COP} = \beta = \frac{Q_L}{W} \quad (8.69)$$

From the equation (8.68)

$$\begin{aligned} \beta &= \frac{Q_L}{Q_H - Q_L} \\ \beta &= \frac{1}{\frac{Q_H}{Q_L} - 1} \end{aligned} \quad (8.70)$$

But we know that  $\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$

Substituting this equation into equation (8.70) we get

$$\beta = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}$$

#### Inferences:

1. The greater the COP, the better is the condition of the refrigerator. A typical refrigerator has COP around 5 to 6.
2. Lesser the difference in the temperatures of the cooling chamber and the atmosphere, higher is the COP of a refrigerator.
3. In the refrigerator the heat is taken from cold object to hot object by doing external work. Without external work heat cannot flow from cold object to hot object. It is not a violation of second law of thermodynamics, because the heat is ejected to surrounding air and total entropy of (refrigerator + surrounding) is always increased.

#### EXAMPLE 8.27

A refrigerator has COP of 3. How much work must be supplied to the refrigerator in order to remove 200 J of heat from its interior?

$$\text{COP} = \beta = \frac{Q_L}{W}$$

$$W = \frac{Qc}{COP} = \frac{200}{3} = 66.67J$$



### Green House Effect

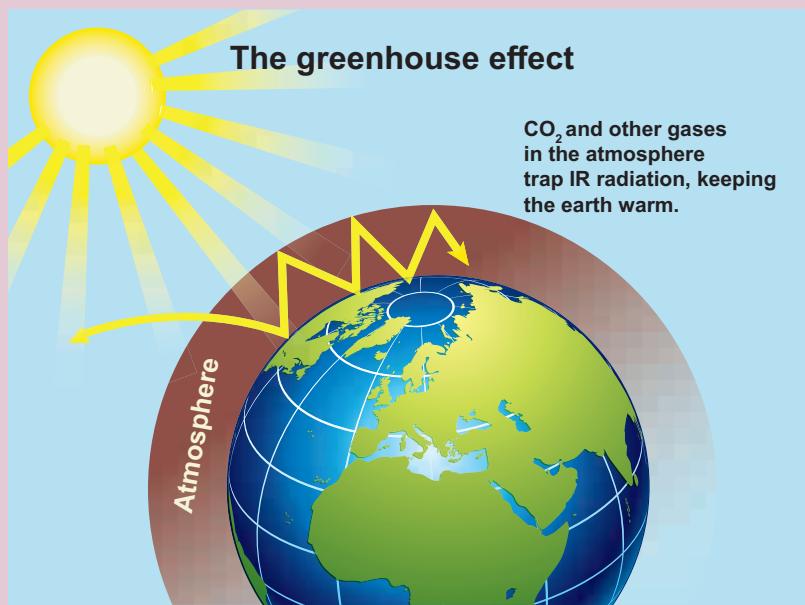
The presence of atmosphere in the earth plays very important role in human lives. Top of the atmosphere is at  $-19^{\circ}\text{C}$  and bottom of the atmosphere is at  $+14^{\circ}\text{C}$ . The increase in  $33^{\circ}\text{C}$  from top to bottom is due to some gases present in the atmosphere. These gases are called Greenhouse gases and this effect is called Greenhouse effect.

The greenhouse gases are mainly  $\text{CO}_2$ , water vapour,  $\text{Ne}$ ,  $\text{He}$ ,  $\text{NO}_2$ ,  $\text{CH}_4$ ,  $\text{Xe}$ ,  $\text{Kr}$ , ozone and  $\text{NH}_3$ . Except  $\text{CO}_2$  and water vapor, all others are present only in very small amount in the atmosphere. The radiation from the Sun is mainly in the visible region of the spectrum. The earth absorbs these radiations and reradiate in the infrared region. Carbon dioxide and water Vapour are good absorbers of infrared radiation since they have more vibrational degree of freedom compared to nitrogen and oxygen (you will learn in unit 9) which keeps earth warmer as shown in Figure.

The amount of  $\text{CO}_2$  present in the atmosphere is increased from 20% to 40% due to human activities since 1900s. The major emission of  $\text{CO}_2$  comes from burning of fossil fuels. The increase in automobile usage worldwide causes this damage. Due to this increase in the  $\text{CO}_2$  content in the atmosphere, the average temperature of the earth increases by  $1^{\circ}\text{C}$ . This effect is called global warming. It has serious influence and alarming effect on ice glaciers on Arctic and Antarctic regions. In addition, the  $\text{CO}_2$  content is also increasing in ocean which is very dangerous to species in the oceans.

In addition to  $\text{CO}_2$ , another very important greenhouse gas is Chloro flouro carbon(CFC) which is used as coolant in refrigerators worldwide. In the human made greenhouse gases  $\text{CO}_2$  is 55%, CFCs are 24%. Nitrogen oxide is 6% and methane is 15%. CFCs also has made huge damage to ozone layer.

Lot of efforts are taken internationally to reduce the emission of  $\text{CO}_2$  and CFCs in various countries. Nowadays a lot of research is going to replace non fossil fuels to replace the fossil-fuels in automobile industry. The major emission of  $\text{CO}_2$  comes from developed countries like USA and European countries. Various treaties are formed between countries to reduce the emission of  $\text{CO}_2$  to considerable level before 2020s. But still global warming is not taken seriously in various countries.





In hot summer, we use earthen pots to drink cold water. The pot reduces the temperature of water inside it. Does the earthen pot act as a refrigerator? No. cyclic process is the basic necessity for heat engine or refrigerator. In earthen pot, the cooling process is not due to any cyclic process. The cooling occurs due to evaporation of water molecules which oozes out through pores of the pot. Once the water molecules evaporate, they never come back to the pot. Even though the heat flows from cold water to open atmosphere, it is not a violation of second law of thermodynamics. The water inside the pot is an open thermodynamic system, so the entropy of water + surrounding always increases.

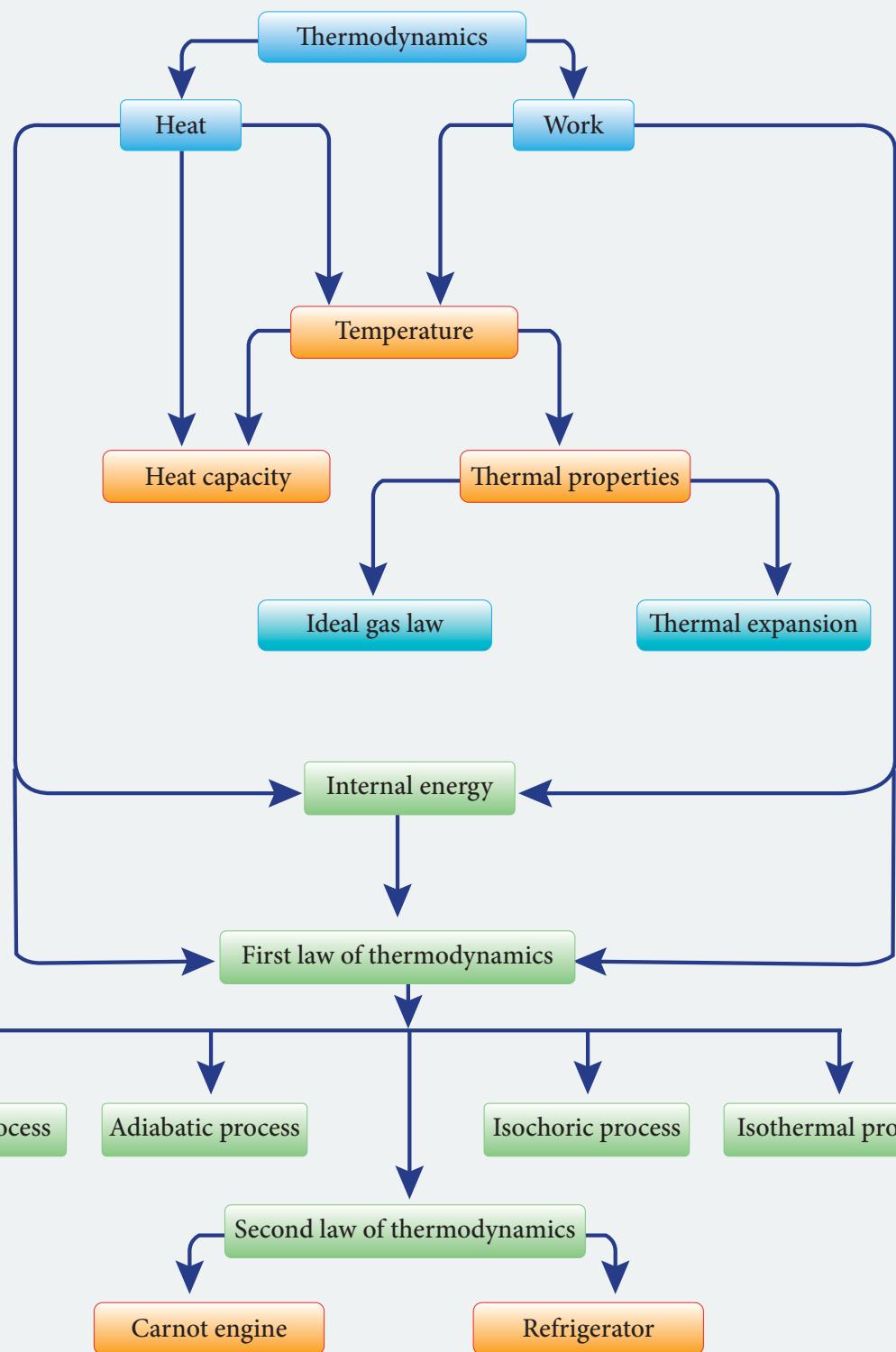


## SUMMARY

- Heat is energy in transit which flows from hot object to cold object. However it is not a quantity.
- Work is a process to transfer energy from one object to another object.
- Temperature is a measure of hotness of the object. It determines the direction of the flow of heat.
- The ideal gas law is  $PV = NkT$  or  $PV = \mu RT$ . The Ideal gas law holds for only at thermodynamic equilibrium. For non-equilibrium process, it is not valid.
- Heat capacity is the amount of heat energy required to increase the object's temperature by  $1^{\circ}\text{C}$  or  $1\text{K}$ . It is denoted by  $S$ .
- Specific heat capacity is the amount of heat energy required to increase the  $1\text{ kg}$  of object's temperature by  $1^{\circ}\text{C}$  or  $1\text{K}$ . It is denoted by  $s$ .
- Molar specific heat capacity is the amount of heat energy requires to increase the  $1$  mole of substance's temperature by  $1^{\circ}\text{C}$  or  $1\text{K}$ . It is denoted by  $C$ .
- Thermal expansion is a tendency of an object to change its shape, area, and volume due to change in temperature.
- Water has an anomalous behavior of expansion.
- Latent heat capacity is the amount of heat energy required to change the phase of the substance.
- Calorimetry is the measurement of the amount of heat energy released or absorbed by a thermodynamic system during the heating process.
- Heat transfers in three different modes: conduction, convection and radiation
- Stefan-Boltmann law:  $E = \sigma T^4$  and Wien's law:  $\lambda_{\max} T = b$
- Thermodynamic equilibrium: thermal, mechanical and chemical equilibrium
- Thermodynamic variables : Pressure, temperature, volume, internal energy and entropy

- Zeroth law of thermodynamics: If two objects are separately in thermal equilibrium with the third object, then these two are in thermal equilibrium. Temperature is a property which is the same for both the systems.
- Internal energy is the sum of kinetic and potential energies of molecules in the thermodynamic system.
- Joule converted mechanical energy to internal energy of the thermodynamic system
- First law of thermodynamics is a statement of conservation of energy. It included heat energy of the thermodynamic system.
- A quasi-static process is an infinitely slow process in which the system is always at equilibrium with the surrounding.
- When the volume of the system changes, the work done  $W = \int P dV$
- The area under the PV diagram gives the work done by the system or work done on the system.
- Specific heat capacity at constant volume is always less than specific heat capacity at constant pressure.
- Isothermal process:  $T = \text{constant}$ , Isobaric process:  $P = \text{constant}$ , Isochoric process:  $V = \text{constant}$ , Adiabatic process  $Q = 0$
- Work done in the isobaric process is most and work done in the adiabatic process is least
- In a cyclic process, change in internal energy is zero.
- The total work done in the cyclic process is given by a closed area in PV diagram
- A reversible process is an ideal process.
- All natural processes are irreversible.
- Heat engine takes input from the hot reservoir, performs work and rejects some amount of heat energy into sink.
- Carnot engine is a reversible engine. It has the highest efficiency. No real heat engine can have the efficiency of that of a Carnot engine.
- A refrigerator is reverse of a Carnot engine. COP (coefficient of performance) of the practical refrigerator is always less than ideal refrigerator.

## CONCEPT MAP



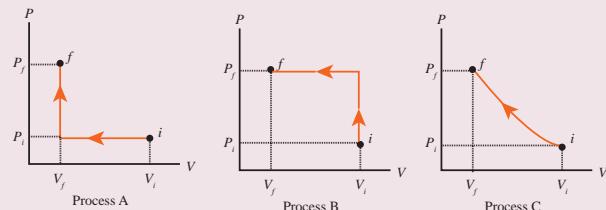


### I. Multiple choice questions:

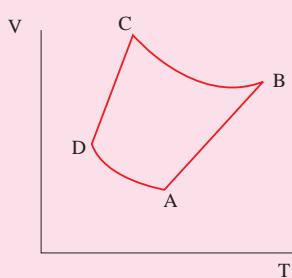
1. In hot summer after a bath, the body's
  - a) internal energy decreases
  - b) internal energy increases
  - c) heat decreases
  - d) no change in internal energy and heat
2. The graph between volume and temperature in Charles' law is
  - a) an ellipse
  - b) a circle
  - c) a straight line
  - d) a parabola
3. When a cycle tyre suddenly bursts, the air inside the tyre expands. This process is
  - a) isothermal
  - b) adiabatic
  - c) isobaric
  - d) isochoric
4. An ideal gas passes from one equilibrium state ( $P_1, V_1, T_1, N$ ) to another equilibrium state ( $2P_1, 3V_1, T_2, N$ ). Then
  - a)  $T_1 = T_2$
  - b)  $T_1 = \frac{T_2}{6}$
  - c)  $T_1 = 6T_2$
  - d)  $T_1 = 3T_2$
5. When a uniform rod is heated, which of the following quantity of the rod will increase
  - a) mass
  - b) weight
  - c) center of mass
  - d) moment of inertia
6. When food is cooked in a vessel by keeping the lid closed, after some time the steam pushes the lid outward. By considering the steam as a thermodynamic system, then in the cooking process

a)  $Q > 0, W > 0$ ,b)  $Q < 0, W > 0$ ,c)  $Q > 0, W < 0$ ,d)  $Q < 0, W < 0$ ,

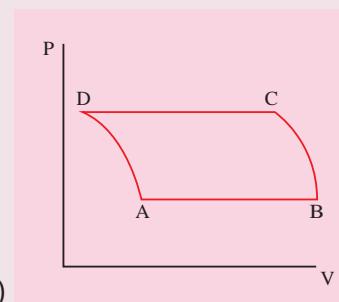
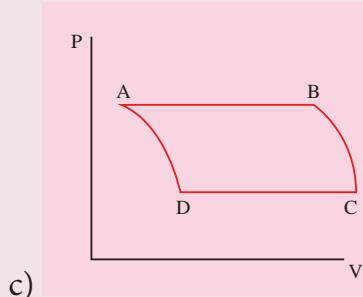
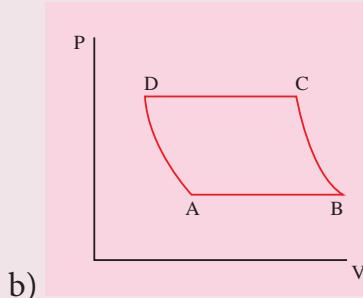
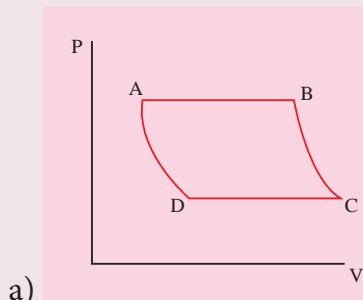
7. When you exercise in the morning, by considering your body as thermodynamic system, which of the following is true?
  - a)  $\Delta U > 0, W > 0$ ,
  - b)  $\Delta U < 0, W > 0$ ,
  - c)  $\Delta U < 0, W < 0$ ,
  - d)  $\Delta U = 0, W > 0$ ,
8. A hot cup of coffee is kept on the table. After some time it attains a thermal equilibrium with the surroundings. By considering the air molecules in the room as a thermodynamic system, which of the following is true
  - a)  $\Delta U > 0, Q = 0$
  - b)  $\Delta U > 0, W < 0$
  - c)  $\Delta U > 0, Q > 0$
  - d)  $\Delta U = 0, Q > 0$
9. An ideal gas is taken from  $(P_i, V_i)$  to  $(P_f, V_f)$  in three different ways. Identify the process in which the work done on the gas the most.



- a) Process A  
 b) Process B  
 c) Process C  
 d) Equal work is done in Process A,B &C
10. The V-T diagram of an ideal gas which goes through a reversible cycle  $A \rightarrow B \rightarrow C \rightarrow D$  is shown below. (Processes  $D \rightarrow A$  and  $B \rightarrow C$  are adiabatic)



The corresponding PV diagram for the process is (all figures are schematic)



11. A distant star emits radiation with maximum intensity at 350 nm. The temperature of the star is  
 a) 8280 K      b) 5000K  
 c) 7260 K      d) 9044 K
12. Identify the state variables given here?  
 a)  $Q, T, W$       b)  $P, T, U$   
 c)  $Q, W$       d)  $P, T, Q$
13. In an isochoric process, we have  
 a)  $W = 0$       b)  $Q = 0$   
 c)  $\Delta U = 0$       d)  $\Delta T = 0$
14. The efficiency of a heat engine working between the freezing point and boiling point of water is  
 (NEET 2018)  
 a) 6.25%      b) 20%  
 c) 26.8%      d) 12.5%
15. An ideal refrigerator has a freezer at temperature  $-12^{\circ}\text{C}$ . The coefficient of performance of the engine is 5. The temperature of the air (to which the heat ejected) is  
 a)  $50^{\circ}\text{C}$       b)  $45.2^{\circ}\text{C}$   
 c)  $40.2^{\circ}\text{C}$       d)  $37.5^{\circ}\text{C}$

### Answers:

- |       |       |       |       |
|-------|-------|-------|-------|
| 1) a  | 2) c  | 3) b  | 4) b  |
| 5) d  | 6) a  | 7) b  | 8) c  |
| 9) b  | 10) b | 11) a | 12) b |
| 13) a | 14) b | 15) c |       |

## II. Short answer questions:

1. 'An object contains more heat'- is it a right statement? If not why?
2. Obtain an ideal gas law from Boyle's and Charles' law.
3. Define one mole.
4. Define specific heat capacity and give its unit.
5. Define molar specific heat capacity.
6. What is a thermal expansion?
7. Give the expressions for linear, area and volume thermal expansions.
8. Define latent heat capacity. Give its unit.
9. State Stefan-Boltzmann law.
10. What is Wien's law?
11. Define thermal conductivity. Give its unit.
12. What is a black body?
13. What is a thermodynamic system? Give examples.
14. What are the different types of thermodynamic systems?
15. What is meant by 'thermal equilibrium'?
16. What is mean by state variable? Give example.
17. What are intensive and extensive variables? Give examples.
18. What is an equation of state? Give an example.
19. State Zeroth law of thermodynamics.
20. Define the internal energy of the system.
21. Are internal energy and heat energy the same? Explain.
22. Define one calorie.
23. Did joule converted mechanical energy to heat energy? Explain.
24. State the first law of thermodynamics.
25. Can we measure the temperature of the object by touching it?
26. Give the sign convention for Q and W.
27. Define the quasi-static process.
28. Give the expression for work done by the gas.
29. What is PV diagram?
30. Explain why the specific heat capacity at constant pressure is greater than the specific heat capacity at constant volume.
31. Give the equation of state for an isothermal process.
32. Give an expression for work done in an isothermal process.
33. Express the change in internal energy in terms of molar specific heat capacity.
34. Apply first law for (a) an isothermal (b) adiabatic (c) isobaric processes.
35. Give the equation of state for an adiabatic process.
36. Give an equation state for an isochoric process.
37. If the piston of a container is pushed fast inward. Will the ideal gas equation be valid in the intermediate stage? If not, why?
38. Draw the PV diagram for
  - a. Isothermal process
  - b. Adiabatic process
  - c. isobaric process
  - d. Isochoric process
39. What is a cyclic process?

40. What is meant by a reversible and irreversible processes?
41. State Clausius form of the second law of thermodynamics
42. State Kelvin-Planck statement of second law of thermodynamics.
43. Define heat engine.
44. What are processes involved in a Carnot engine?
45. Can the given heat energy be completely converted to work in a cyclic process? If not, when can the heat be completely converted to work?
46. State the second law of thermodynamics in terms of entropy.
47. Why does heat flow from a hot object to a cold object?
48. Define the coefficient of performance.
49. Discuss the
  - a. thermal equilibrium
  - b. mechanical equilibrium
  - c. Chemical equilibrium
  - d. thermodynamic equilibrium.
50. Explain Joule's Experiment of the mechanical equivalent of heat.
51. Derive the expression for the work done in a volume change in a thermodynamic system.
52. Derive Mayer's relation for an ideal gas.
53. Explain in detail the isothermal process.
54. Derive the work done in an isothermal process
55. Explain in detail an adiabatic process.
56. Derive the work done in an adiabatic process
57. Explain the isobaric process and derive the work done in this process
58. Explain in detail the isochoric process.
59. What are the limitations of the first law of thermodynamics?
60. Explain the heat engine and obtain its efficiency.
61. Explain in detail Carnot heat engine.
62. Derive the expression for Carnot engine efficiency.
63. Explain the second law of thermodynamics in terms of entropy.
64. Explain in detail the working of a refrigerator.

### III. Long answer Questions:

1. Explain the meaning of heat and work with suitable examples.
2. Discuss the ideal gas laws.
3. Explain in detail the thermal expansion.
4. Describe the anomalous expansion of water. How is it helpful in our lives?
5. Explain Calorimetry and derive an expression for final temperature when two thermodynamic systems are mixed.
6. Discuss various modes of heat transfer.
7. Explain in detail Newton's law of cooling.
8. Explain Wien's law and why our eyes are sensitive only to visible rays?

## IV. Numerical Problems

1. Calculate the number of moles of air is in the inflated balloon at room temperature as shown in the figure.



The radius of the balloon is 10 cm, and pressure inside the balloon is 180 kPa.

**Answer:**  $\mu \approx 0.3 \text{ mol}$

2. In the planet Mars, the average temperature is around  $-53^\circ\text{C}$  and atmospheric pressure is 0.9 kPa. Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

**Answer:**  $\mu_{\text{Mars}} = 0.49 \text{ mol}$

$\mu_{\text{Earth}} \approx 40 \text{ mol}$

3. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume  $V_1$  and contains ideal gas at pressure  $P_1$  and temperature  $T_1$ . The other chamber has volume  $V_2$  and contains ideal gas at pressure  $P_2$  and temperature  $T_2$ . If the partition is removed without doing any work on the gases, calculate the final equilibrium temperature of the container.

$$\text{Answer: } T = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

4. The temperature of a uniform rod of length L having a coefficient of linear expansion  $\alpha_L$  is changed by  $\Delta T$ . Calculate the new moment of inertia of the uniform rod about axis passing through its center and perpendicular to an axis of the rod.

**Answer:**  $I' = I (1 + \alpha_L \Delta T)^2$

5. Draw the TP diagram (P-x axis, T-y axis), VT(T-x axis, V-y axis) diagram for

- Isochoric process
- Isothermal process
- isobaric process

6. A man starts bicycling in the morning at a temperature around  $25^\circ\text{C}$ , he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

**Answer:**  $T = 36.9^\circ\text{C}$

7. Normal human body of the temperature is  $98.6^\circ\text{F}$ . During high fever if the temperature increases to  $104^\circ\text{F}$ , what is the change in peak wavelength that emitted by our body? (Assume human body is a black body)

- Answer:** (a)  $\lambda_{\text{max}} \approx 9348 \text{ nm}$  at  $98.6^\circ\text{F}$   
 (b)  $\lambda_{\text{max}} \approx 9258 \text{ nm}$  at  $104^\circ\text{F}$

8. In an adiabatic expansion of the air, the volume is increased by 4%, what is percentage change in pressure?

(For air  $\gamma = 1.4$ )

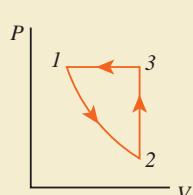
**Answer:** 5.6%

9. In a petrol engine, (internal combustion engine) air at atmospheric pressure and temperature of  $20^{\circ}\text{C}$  is compressed in the cylinder by the piston to  $1/8$  of its original volume. Calculate the temperature of the compressed air.

(For air  $\gamma = 1.4$ )

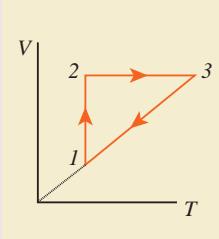
**Answer:**  $T \approx 400^{\circ}\text{C}$

10. Consider the following cyclic process consist of isotherm, isochoric and isobar which is given in the figure.



Draw the same cyclic process qualitatively in the V-T diagram where T is taken along x direction and V is taken along y-direction. Analyze the nature of heat exchange in each process.

**Answer:**  $T = 36.9^{\circ}\text{C}$

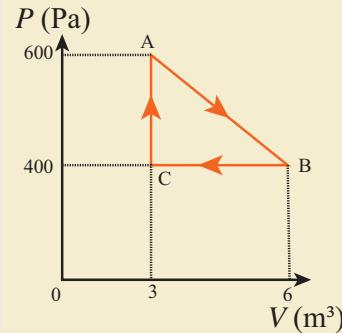


Process 1 to 2 = increase in volume. So heat must be added.

Process 2 to 3 = Volume remains constant. Increase in temperature. The given heat is used to increase the internal energy.

Process 3 to 1 : Pressure remains constant. Volume and Temperature are reduced. Heat flows out of the system. It is an isobaric compression where the work is done on the system.

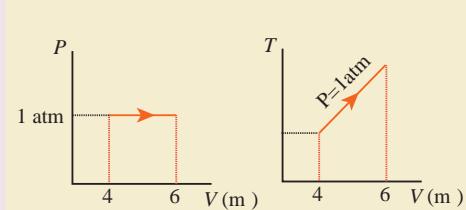
11. An ideal gas is taken in a cyclic process as shown in the figure. Calculate  
 (a) work done by the gas.  
 (b) work done on the gas  
 (c) Net work done in the process



**Answer:** (a)  $W = +1.5\text{ kJ}$   
 (b)  $W = -1.2\text{ kJ}$   
 (c)  $W = +300\text{ J}$ .

12. For a given ideal gas  $6 \times 10^5\text{ J}$  heat energy is supplied and the volume of gas is increased from  $4\text{ m}^3$  to  $6\text{ m}^3$  at atmospheric pressure. Calculate (a) the work done by the gas (b) change in internal energy of the gas (c) graph this process in PV and TV diagram.

**Answer:** (a)  $W = +202.6\text{ kJ}$   
 (b)  $dU = 397.4\text{ kJ}$



- (c)
13. Suppose a person wants to increase the efficiency of the reversible heat engine that is operating between  $100^{\circ}\text{C}$  and  $300^{\circ}\text{C}$ . He had two ways to increase the efficiency. (a) By decreasing the cold reservoir temperature from  $100^{\circ}\text{C}$  to  $50^{\circ}\text{C}$  and keeping the hot reservoir temperature constant (b) by increasing

the temperature of the hot reservoir from  $300^{\circ}\text{C}$  to  $350^{\circ}\text{C}$  by keeping the cold reservoir temperature constant. Which is the suitable method?

**Answer:** Initial efficiency = 44.5%

Efficiency in method (a) = 52 %

Efficiency in method (b) = 48 %

Method (a) is more efficient.

14. A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of  $327^{\circ}\text{C}$ . To have an

engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

**Answer:**  $552^{\circ}\text{C}$

15. An ideal refrigerator keeps its content at  $0^{\circ}\text{C}$  while the room temperature is  $27^{\circ}\text{C}$ . Calculate its coefficient of performance.

**Answer:**  $\beta=10.11$

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# UNIT 1

# ELECTROSTATICS

*Electricity is really just organized lightning*

– George Carlin



## LEARNING OBJECTIVES

In this unit, student is exposed to

- Historical background of electricity and magnetism
- The role of electrostatic force in day – to-day life
- Coulomb's law and superposition principle
- The concept of electric field
- Calculation of electric field for various charge configurations
- Electrostatic potential and electrostatic potential energy
- Electric dipole and dipole moment
- Electric field and electrostatic potential for a dipole
- Electric flux
- Gauss law and its various applications
- Electrostatic properties of conductors and dielectrics
- Polarisation
- Capacitors in series and parallel combinations
- Effect of a dielectric in a capacitor
- Distribution of charges in conductors, corona discharge
- Working of a Van de Graaff generator



### 1.1

## INTRODUCTION

Electromagnetism is one of the most important branches of physics. The technological developments of the modern 21<sup>st</sup> century are primarily due to our understanding of electromagnetism. The forces we experience in everyday life are electromagnetic in nature except gravity.

In standard XI, we studied about the gravitational force, tension, friction, normal force etc. Newton treated them to be independent of each other with each force being a separate natural force. But what is the origin of all these forces? It is now understood that except gravity, all forces which we experience in every day life (tension in the string, normal force from the surface, friction etc.) arise from electromagnetic forces within the atoms. Some examples are



- (i) When an object is pushed, the atoms in our hand interact with the atoms in the object and this interaction is basically electromagnetic in nature.
- (ii) When we stand on Earth's surface, the gravitational force on us acts downwards and the normal force acts upward to counter balance the gravitational force. What is the origin of this normal force?

It arises due to the electromagnetic interaction of atoms on the surface of the Earth with the atoms present in the feet of the person. Though, we are attracted by the gravitational force of the Earth, we stand on Earth only because of electromagnetic force of atoms.

- (iii) When an object is moved on a surface, static friction resists the motion of the object. This static friction arises due to electromagnetic interaction between the atoms present in the object and atoms on the surface. Kinetic friction also has similar origin.

From these examples, it is clear that understanding electromagnetism is very essential to understand the universe in a holistic manner. The basic principles of electromagnetism are dealt in XII physics volume 1. This unit deals with the behaviour and other related phenomena of charges at rest. This **branch of electricity which deals with stationary charges is called Electrostatics.**

### 1.1.1 Historical background of electric charges

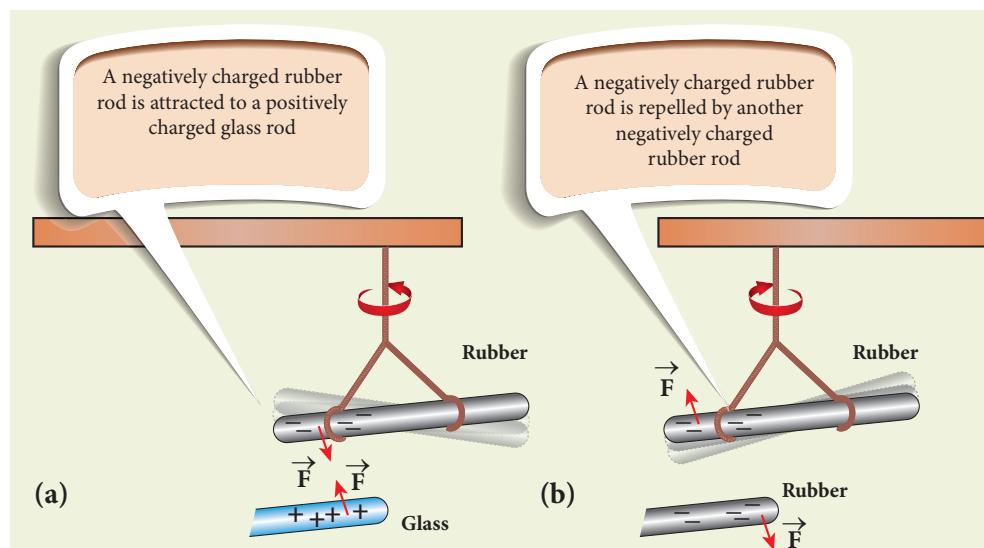
Two millennium ago, Greeks noticed that amber (a solid, translucent material formed from the resin of a fossilized tree)

after rubbing with animal fur attracted small pieces of leaves and dust. The amber possessing this property is said to be 'charged'. It was initially thought that amber has this special property. Later people found that not only amber but even a glass rod rubbed with silk cloth, attracts pieces of papers. So glass rod also becomes 'charged' when rubbed with a suitable material.

Consider a charged rubber rod hanging from a thread as shown in Figure 1.1. Suppose another charged rubber rod is brought near the first rubber rod; the rods repel each other. Now if we bring a charged glass rod close to the charged rubber rod, they attract each other. At the same time, if a charged glass rod is brought near another charged glass rod, both the rods repel each other.

From these observations, the following inferences are made

- (i) The charging of rubber rod and that of glass rod are different from one another.
  - (ii) The charged rubber rod repels another charged rubber rod, which implies that 'like charges repel each other'. We can also arrive at the same inference by observing that a charged glass rod repels another charged glass rod.
  - (iii) The charged amber rod attracts the charged glass rod, implying that the charge in the glass rod is not the same kind of charge present in the rubber. Thus unlike charges attract each other.
- Therefore, two kinds of charges exist in the universe. In the 18<sup>th</sup> century, Benjamin Franklin called one type of charge as positive (+) and another type of charge as negative (-). Based on Franklin's convention, rubber and amber rods are negatively charged while the glass rod is positively charged. **If the net charge is zero in the object, it is said to be electrically neutral.**



**Figure 1.1** (a) Unlike charges attract each other (b) Like charges repel each other

Following the pioneering work of J. J. Thomson and E. Rutherford, in the late 19<sup>th</sup> century and in the beginning of 20<sup>th</sup> century, we now understand that the atom is electrically neutral and is made up of the negatively charged electrons, positively charged protons, and neutrons which have zero charge. The material objects made up of atoms are neutral in general. When an object is rubbed with another object (for example rubber with silk cloth), some amount of charge is transferred from one object to another due to the friction between them and the object is then said to be electrically charged. **Charging the objects through rubbing is called triboelectric charging.**

is another intrinsic and fundamental property of particles. The nature of charges is understood through various experiments performed in the 19<sup>th</sup> and 20<sup>th</sup> century. The SI unit of charge is coulomb.

### (ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged). From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

**'The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical**

## 1.1.2 Basic properties of charges

### (i) Electric charge

Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric charge



process, the net change in charge will always be zero.

### (iii) Quantisation of charges

What is the smallest amount of charge that can be found in nature? Experiments show that the charge on an electron is  $-e$  and the charge on the proton is  $+e$ . Here,  $e$  denotes the fundamental unit of charge. The charge  $q$  on any object is equal to an integral multiple of this fundamental unit of charge  $e$ .

$$q = ne \quad (1.1)$$

Here  $n$  is any integer ( $0, \pm 1, \pm 2, \pm 3, \pm 4, \dots$ ). This is called quantisation of electric charge.

Robert Millikan in his famous experiment found that the value of  $e = 1.6 \times 10^{-19} \text{ C}$ . The charge of an electron is  $-1.6 \times 10^{-19} \text{ C}$  and the charge of the proton is  $+1.6 \times 10^{-19} \text{ C}$ .

When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of  $10^{10}$ . So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

### EXAMPLE 1.1

Calculate the number of electrons in one coulomb of negative charge.

#### Solution

According to the quantisation of charge

$$q = ne$$

Here  $q = 1 \text{ C}$ . So the number of electrons in 1 coulomb of charge is

$$n = \frac{q}{e} = \frac{1 \text{ C}}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons}$$

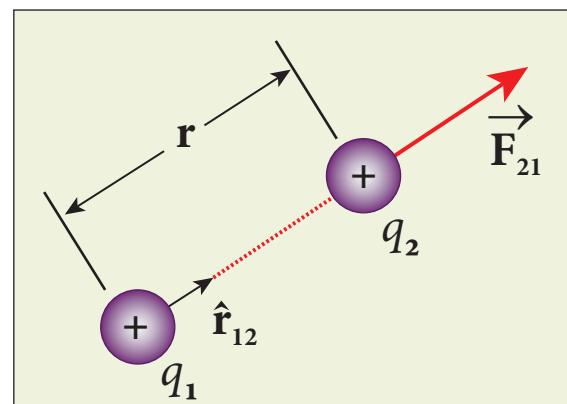
## 1.2

### COULOMB'S LAW

In the year 1786, Coulomb deduced the expression for the force between two stationary point charges in vacuum or free space. Consider two point charges  $q_1$  and  $q_2$  at rest in vacuum, and separated by a distance of  $r$ , as shown in Figure 1.2. According to Coulomb, the force on the point charge  $q_2$  exerted by another point charge  $q_1$  is

$$\vec{F}_{21} = k \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad (1.2)$$

where  $\hat{r}_{12}$  is the unit vector directed from charge  $q_1$  to charge  $q_2$  and  $k$  is the proportionality constant.



**Figure 1.2** Coulomb force between two point charges

#### Important aspects of Coulomb's law

- Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.
- The force on the charge  $q_2$  exerted by the charge  $q_1$  always lies along the line joining the two charges.  $\hat{r}_{12}$  is the unit



vector pointing from charge  $q_1$  to  $q_2$ . It is shown in the Figure 1.2. Likewise, the force on the charge  $q_1$  exerted by  $q_2$  is along  $-\hat{r}_{12}$  (i.e., in the direction opposite to  $\hat{r}_{12}$ ).

(iii) In SI units,  $k = \frac{1}{4\pi\epsilon_0}$  and its value is  $9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ . Here  $\epsilon_0$  is the permittivity of free space or vacuum and the value of  $\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$|F| = \frac{9 \times 10^9 \times 1 \times 1}{1^2} = 9 \times 10^9 \text{ N}$ . This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of  $\mu\text{C}$  (micro coulomb) or  $\text{nC}$  (nano coulomb).

(v) In SI units, Coulomb's law in vacuum takes the form  $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$ . In a medium of permittivity  $\epsilon$ , the force between two point charges is given by  $\vec{F}_{21} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \hat{r}_{12}$ . Since  $\epsilon > \epsilon_0$ , the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as  $\epsilon_r = \frac{\epsilon}{\epsilon_0}$ . For vacuum or air,  $\epsilon_r = 1$  and for all other media  $\epsilon_r > 1$ .

(vi) Coulomb's law has same structure as Newton's law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product

of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses. But there are some important differences between these two laws.

- The gravitational force between two masses is always attractive but Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.
- The value of the gravitational constant  $G = 6.626 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ . The value of the constant  $k$  in Coulomb law is  $k = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ . Since  $k$  is much more greater than  $G$ , the electrostatic force is always greater in magnitude than gravitational force for smaller size objects.
- The gravitational force between two masses is independent of the medium. For example, if 1 kg of two masses are kept in air or inside water, the gravitational force between two masses remains the same. But the electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.
- The gravitational force between two point masses is the same whether two masses are at rest or in motion. If the charges are in motion, yet another force (Lorentz force) comes into play in addition to coulomb force.

(vii) The force on a charge  $q_1$  exerted by a point charge  $q_2$  is given by

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

Here  $\hat{r}_{21}$  is the unit vector from charge  $q_2$  to  $q_1$ .

But  $\hat{r}_{21} = -\hat{r}_{12}$ ,



$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} (-\hat{r}_{12}) = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} (\hat{r}_{12})$$

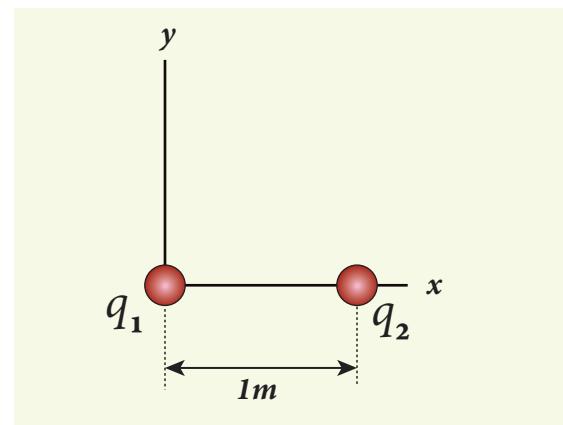
$$(\text{or}) \quad \vec{F}_{12} = -\vec{F}_{21}$$

Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

### EXAMPLE 1.2

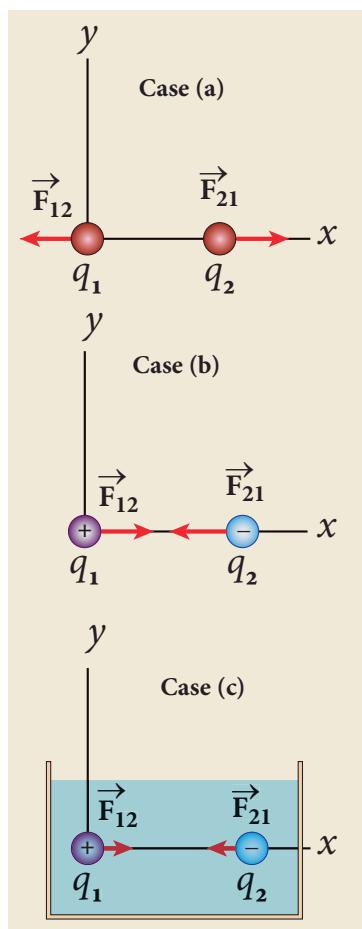
Consider two point charges  $q_1$  and  $q_2$  at rest as shown in the figure.



They are separated by a distance of 1m. Calculate the force experienced by the two charges for the following cases:

- (a)  $q_1 = +2\mu\text{C}$  and  $q_2 = +3\mu\text{C}$
- (b)  $q_1 = +2\mu\text{C}$  and  $q_2 = -3\mu\text{C}$
- (c)  $q_1 = +2\mu\text{C}$  and  $q_2 = -3\mu\text{C}$  kept in water ( $\epsilon_r = 80$ )

### Solution



- (a)  $q_1 = +2\mu\text{C}$ ,  $q_2 = +3\mu\text{C}$ , and  $r = 1\text{m}$ . Both are positive charges. so the force will be repulsive

Force experienced by the charge  $q_2$  due to  $q_1$  is given by

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Here  $\hat{r}_{12}$  is the unit vector from  $q_1$  to  $q_2$ . Since  $q_2$  is located on the right of  $q_1$ , we have

$$\hat{r}_{12} = \hat{i}, \text{ so that}$$

$$\begin{aligned} \vec{F}_{21} &= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{1^2} \hat{i} \left[ \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \right] \\ &= 54 \times 10^{-3} N \hat{i} \end{aligned}$$



According to Newton's third law, the force experienced by the charge  $q_1$  due to  $q_2$  is  $\vec{F}_{12} = -\vec{F}_{21}$

So that  $\vec{F}_{12} = -54 \times 10^{-3} N \hat{i}$ .

The directions of  $\vec{F}_{21}$  and  $\vec{F}_{12}$  are shown in the above figure in case (a)

(b)  $q_1 = +2 \mu C$ ,  $q_2 = -3 \mu C$ , and  $r = 1m$ . They are unlike charges. So the force will be attractive.

Force experienced by the charge  $q_2$  due to  $q_1$  is given by

$$\begin{aligned}\vec{F}_{21} &= \frac{9 \times 10^9 \times (2 \times 10^{-6}) \times (-3 \times 10^{-6})}{1^2} \hat{r}_{12} \\ &= -54 \times 10^{-3} N \hat{i} \text{ (Using } \hat{r}_{12} = \hat{i} \text{ )}\end{aligned}$$

The charge  $q_2$  will experience an attractive force towards  $q_1$  which is in the negative x direction.

According to Newton's third law, the force experienced by the charge  $q_1$  due to  $q_2$  is

$$\vec{F}_{12} = -\vec{F}_{21}$$

so that  $\vec{F}_{12} = 54 \times 10^{-3} N \hat{i}$

The directions of  $\vec{F}_{21}$  and  $\vec{F}_{12}$  are shown in the figure (case (b)).

(c) If these two charges are kept inside the water, then the force experienced by  $q_2$  due to  $q_1$

$$\vec{F}_{21}^W = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

since  $\epsilon = \epsilon_r \epsilon_o$ ,

we have  $\vec{F}_{21}^W = \frac{1}{4\pi\epsilon_r\epsilon_o} \frac{q_1 q_2}{r^2} \hat{r}_{12} = \frac{\vec{F}_{21}}{\epsilon_r}$

Therefore,

$$\vec{F}_{21}^W = -\frac{54 \times 10^{-3} N}{80} \hat{i} = -0.675 \times 10^{-3} N \hat{i}$$



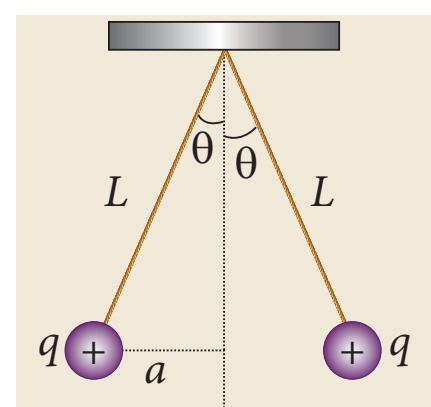
Note that the strength of the force between the two charges in water is reduced by 80 times compared to the force between the same two charges in vacuum.

When common salt (NaCl) is taken in water, the electrostatic force between Na and Cl ions is reduced due to the high relative permittivity of water ( $\epsilon_r = 80$ ). This is the reason water acts as a good solvent.

### EXAMPLE 1.3

Two small-sized identical equally charged spheres, each having mass 1 mg are hanging in equilibrium as shown in the figure. The length of each string is 10 cm and the angle  $\theta$  is  $7^\circ$  with the vertical. Calculate the magnitude of the charge in each sphere.

(Take  $g = 10 \text{ ms}^{-2}$ )

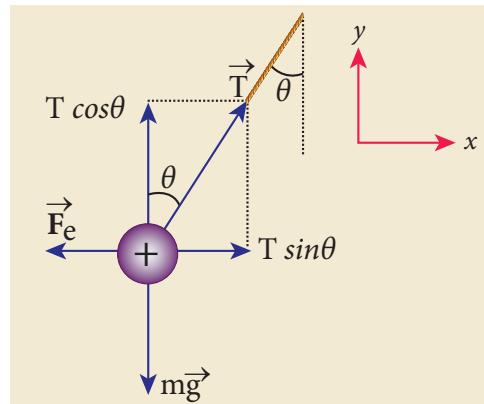


### Solution

If the two spheres are neutral, the angle between them will be  $0^\circ$  when hanged

vertically. Since they are positively charged spheres, there will be a repulsive force between them and they will be at equilibrium with each other at an angle of  $7^\circ$  with the vertical. At equilibrium, each charge experiences zero net force in each direction. We can draw a free body diagram for one of the charged spheres and apply Newton's second law for both vertical and horizontal directions.

The free body diagram is shown below.



In the x-direction, the acceleration of the charged sphere is zero.

Using Newton's second law ( $\vec{F}_{tot} = m\vec{a}$ ), we have

$$\begin{aligned} T \sin \theta \hat{i} - F_e \hat{i} &= 0 \\ T \sin \theta &= F_e \end{aligned} \quad (1)$$

Here  $T$  is the tension acting on the charge due to the string and  $F_e$  is the electrostatic force between the two charges.

In the y-direction also, the net acceleration experienced by the charge is zero.

$$\begin{aligned} T \cos \theta \hat{j} - mg \hat{j} &= 0 \\ \text{Therefore, } T \cos \theta &= mg. \end{aligned} \quad (2)$$

By dividing equation (1) by equation (2),

$$\tan \theta = \frac{F_e}{mg} \quad (3)$$

Since they are equally charged, the magnitude of the electrostatic force is

$$F_e = k \frac{q^2}{r^2} \text{ where } k = \frac{1}{4\pi\epsilon_0}$$

Here  $r = 2a = 2L \sin \theta$ . By substituting these values in equation (3),

$$\tan \theta = k \frac{q^2}{mg(2L \sin \theta)^2} \quad (4)$$

Rearranging the equation (4) to get  $q$

$$\begin{aligned} q &= 2L \sin \theta \sqrt{\frac{mg \tan \theta}{k}} \\ &= 2 \times 0.1 \times \sin 7^\circ \times \sqrt{\frac{10^{-3} \times 10 \times \tan 7^\circ}{9 \times 10^9}} \\ q &= 8.9 \times 10^{-9} \text{ C} = 8.9 \text{ nC} \end{aligned}$$

### EXAMPLE 1.4

Calculate the electrostatic force and gravitational force between the proton and the electron in a hydrogen atom. They are separated by a distance of  $5.3 \times 10^{-11} \text{ m}$ . The magnitude of charges on the electron and proton are  $1.6 \times 10^{-19} \text{ C}$ . Mass of the electron is  $m_e = 9.1 \times 10^{-31} \text{ kg}$  and mass of proton is  $m_p = 1.6 \times 10^{-27} \text{ kg}$ .

#### Solution

The proton and the electron attract each other. The magnitude of the electrostatic force between these two particles is given by

$$\begin{aligned} F_e &= \frac{k e^2}{r^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2} \\ &= \frac{9 \times 2.56}{28.09} \times 10^{-7} = 8.2 \times 10^{-8} \text{ N} \end{aligned}$$



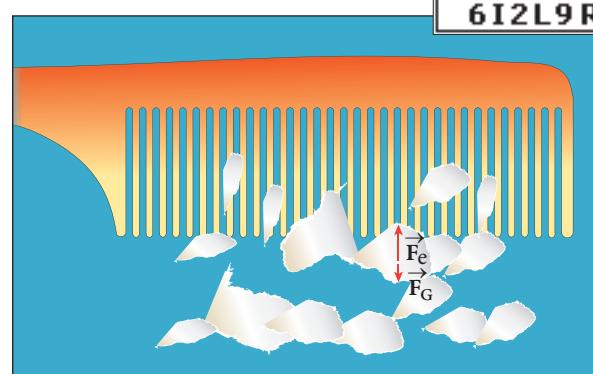
The gravitational force between the proton and the electron is attractive. The magnitude of the gravitational force between these particles is

$$\begin{aligned} F_G &= \frac{Gm_e m_p}{r^2} \\ &= \frac{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-27}}{(5.3 \times 10^{-11})^2} \\ &= \frac{97.11}{28.09} \times 10^{-47} = 3.4 \times 10^{-47} \text{ N} \end{aligned}$$

The ratio of the two forces  $\frac{F_e}{F_G} = \frac{8.2 \times 10^{-8}}{3.4 \times 10^{-47}}$   
 $= 2.41 \times 10^{39}$

Note that  $F_e \approx 10^{39} F_G$

The electrostatic force between a proton and an electron is enormously greater than the gravitational force between them. Thus the gravitational force is negligible when compared with the electrostatic force in many situations such as for small size objects and in the atomic domain. This is the reason why a charged comb attracts an uncharged piece of paper with greater force even though the piece of paper is attracted downward by the Earth. This is shown in Figure 1.3



**Figure 1.3** Electrostatic attraction between a comb and pieces of papers

### 1.2.1 Superposition principle

Coulomb's law explains the interaction between two point charges. If there are more than two charges, the force on one charge due to all the other charges needs to be calculated. Coulomb's law alone does not give the answer. The superposition principle explains the interaction between multiple charges.

According to this superposition principle, **the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.**

Consider a system of  $n$  charges, namely  $q_1, q_2, q_3, \dots, q_n$ . The force on  $q_1$  exerted by the charge  $q_2$

$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Here  $\hat{r}_{21}$  is the unit vector from  $q_2$  to  $q_1$  along the line joining the two charges and  $r_{21}$  is the distance between the charges  $q_1$  and  $q_2$ . The electrostatic force between two charges is not affected by the presence of other charges in the neighbourhood.

The force on  $q_1$  exerted by the charge  $q_3$  is

$$\vec{F}_{13} = k \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31}$$

By continuing this, the total force acting on the charge  $q_1$  due to all other charges is given by

$$\begin{aligned} \vec{F}_1^{tot} &= \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots + \vec{F}_{1n} \\ \vec{F}_1^{tot} &= k \left\{ \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31} + \frac{q_1 q_4}{r_{41}^2} \hat{r}_{41} + \dots \right. \\ &\quad \left. \dots + \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1} \right\} \end{aligned} \quad (1.3)$$