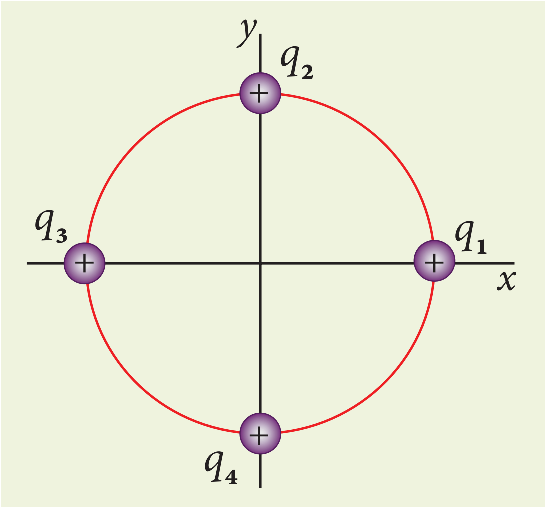
Note

Without the superposition principle, Coulomb's law will be incomplete when applied to more than two charges. Both the superposition principle and Coulomb's law form fundamental principles of electrostatics and explain all the phenomena in electrostatics. But they are not derivable from each other.

EXAMPLE 1.5

Consider four equal charges q_1,q_2, q_3 and q_4 = q = +1μC located at four different points on a circle of radius 1m, as shown in the figure. Calculate the total force acting on the charge q_1 due to all the other charges.

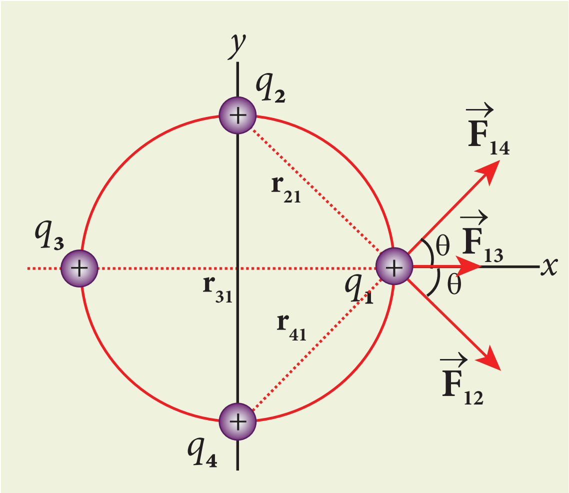


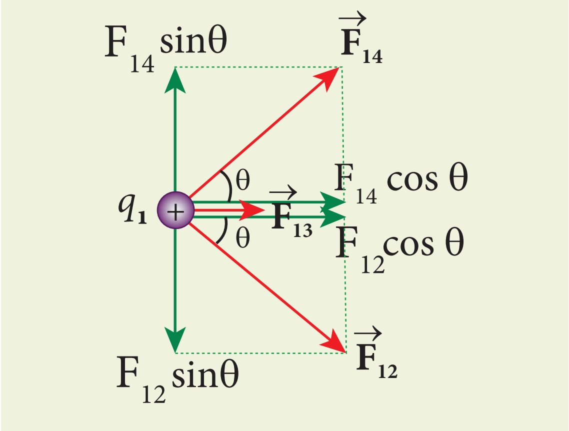
Solution

According to the superposition principle, the total electrostatic force on charge q_1 is the vector sum of the forces due to the other charges,

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

The following diagram shows the direction of each force on the charge q_1.





The charges q_2 and q_4 are equi-distant from q_1. As a result the strengths (magnitude) of the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge q_3 is located farther compared to q_2 and q_4. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F}_{13}$ is lesser than that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$. Hence the vector representing the force $\vec{F}_{13}$ is drawn with smaller length compared to that for forces $\vec{F}_{12}$ and $\vec{F}_{14}$.

From the figure, $r_{21} = \sqrt{2}\,m = r_{41}$ and $r_{31} = 2m$

The magnitudes of the forces are given by

$$F_{13} = \frac{kq^2}{r_{31}^2} = \frac{9\times10^9\times10^{-12}}{4}$$

$F_{13} = 2.25 \times 10^{-3}$ N

$$F_{12} = \frac{kq^2}{r_{21}^2} = F_{14} = \frac{9\times10^9\times10^{-12}}{2}$$

$= 4.5 \times 10^{-3}$ N

From the figure, the angle $\theta = 45^o$. In terms of the components, we have

$$\vec{F}_{12} = F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} - 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{F}_{13} = F_{13}\hat{i} = 2.25\times10^{-3}N\hat{i}$$

$$\vec{F}_{14} = F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} + 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

Then the total force on q_1 is,

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\hat{j}\right) + F_{13}\hat{i} + \left(F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\hat{j}\right)$$

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i} + \left(-F_{12}\sin\theta + F_{14}\sin\theta\right)\hat{j}$$

Since $F_{12} = F_{14}$, the j^{th} component is zero. Hence we have

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i}$$

substituting the values in the above equation,

$$= \left(\frac{4.5}{\sqrt{2}} + 2.25 + \frac{4.5}{\sqrt{2}}\right)\hat{i} = \left(4.5\sqrt{2} + 2.25\right)\hat{i}$$

$$\vec{F}_1^{tot} = 8.61\times10^{-3}N\hat{i}$$

The resultant force is along the positive x axis.

1.3 ELECTRIC FIELD AND ELECTRIC FIELD LINES

1.3.1 Electric Field

The interaction between two charges is determined by Coulomb's law. How does the interaction itself occur? Consider a point charge kept at a point in space. If another point charge is placed at some distance from the first point charge, it experiences either an attractive force or repulsive force. This is called 'action at a distance'. But how does the second charge know about existence of the first charge which is located at some distance away from it? To answer this question, Michael Faraday introduced the concept of **field**.

According to Faraday, every charge in the universe creates an electric field in the surrounding space, and if another charge is brought into its field, it will interact with the electric field at that point and will experience a force. It may be recalled that the interaction of two masses is similarly explained using the concept of gravitational field (Refer unit 6, volume 2, XI physics). Both the electric and gravitational forces are non-contact forces, hence the field concept is required to explain action at a distance.

Consider a source point charge q located at a point in space. Another point charge q_o (test charge) is placed at some point P which is at a distance r from the charge q. The electrostatic force experienced by the charge q_o due to q is given by Coulomb's law.

$$\vec{F} = \frac{kqq_0}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r} \quad \text{where} \quad k = \frac{1}{4\pi\varepsilon_0}$$

The charge q creates an electric field in the surrounding space. The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad (1.4)$$

Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is Newton per Coulomb (NC^{-1}).

Important aspects of Electric field

(i) If the charge q is positive then the electric field points away from the source charge and if q is negative, the electric field points towards the source charge q. This is shown in the Figure 1.4.

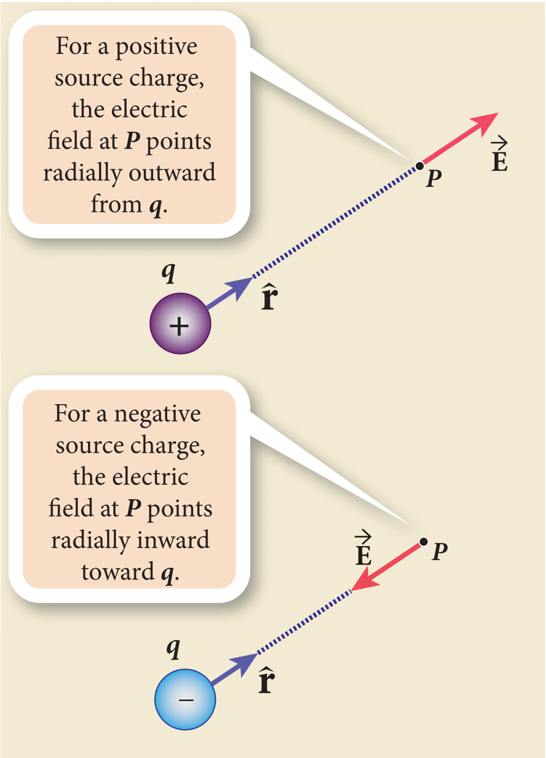


Figure 1.4 Electric field of positive and negative charges

(ii) If the electric field at a point P is $\vec{E}$, then the force experienced by the test charge q_o placed at the point P is

$$\vec{F} = q_0\vec{E} \qquad (1.5)$$

This is Coulomb's law in terms of electric field. This is shown in Figure 1.5

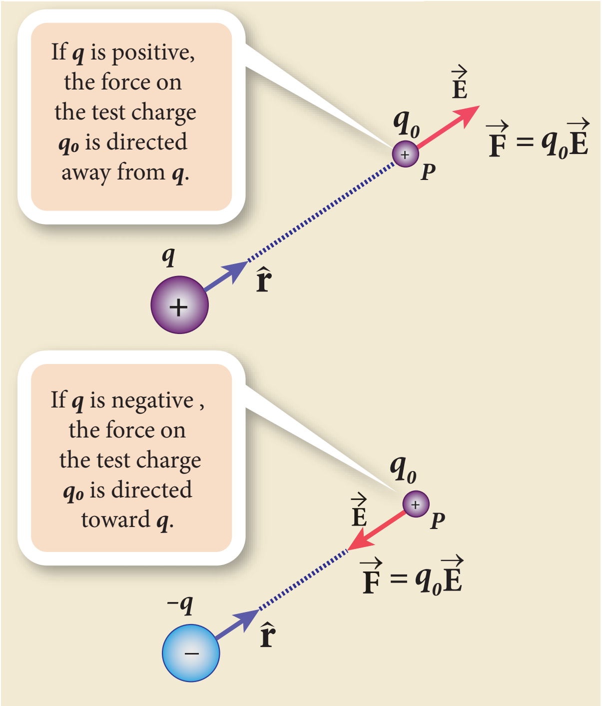


Figure 1.5 Coulomb's law in terms of electric field

(iii) The equation (1.4) implies that the electric field is independent of the test charge q_o and it depends only on the source charge q.

(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures 1.6(a) and (b). From equation (1.4), we can infer that as distance increases, the electric field decreases in magnitude.

Note that in Figures 1.6 (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the points Q and R because the point P is closer to the source charge.

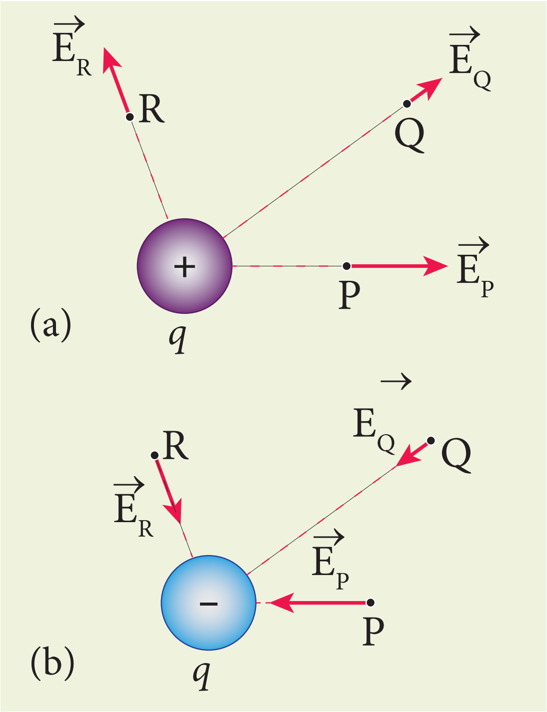


Figure 1.6 (a) Electric field due to positive charge (b) Electric field due to negative charge

(v) In the definition of electric field, it is assumed that the test charge q_0 is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

(vi) The expression (1.4) is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used These will be explained later in the same section. However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge. Note that we similarly treat the Earth as a point mass when we calculate the gravitational field of the Sun on the Earth (refer unit 6, volume 2, XI physics).

(vii) There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases. These are shown in Figure 1.7.

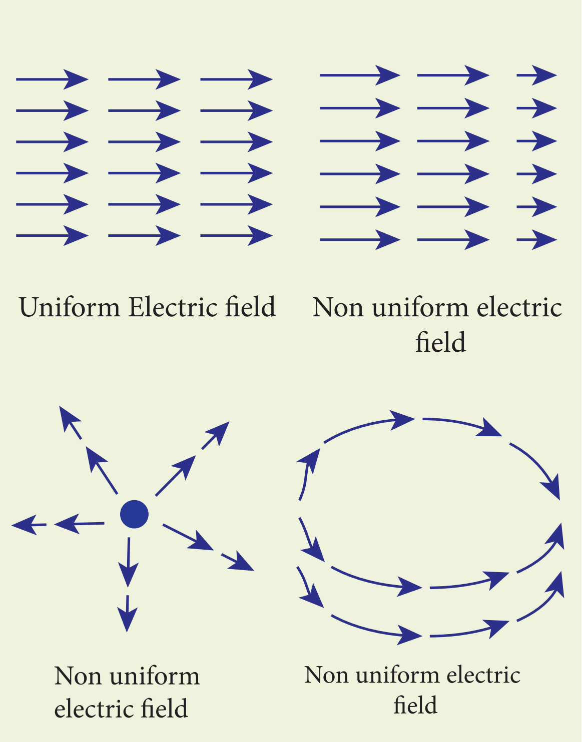


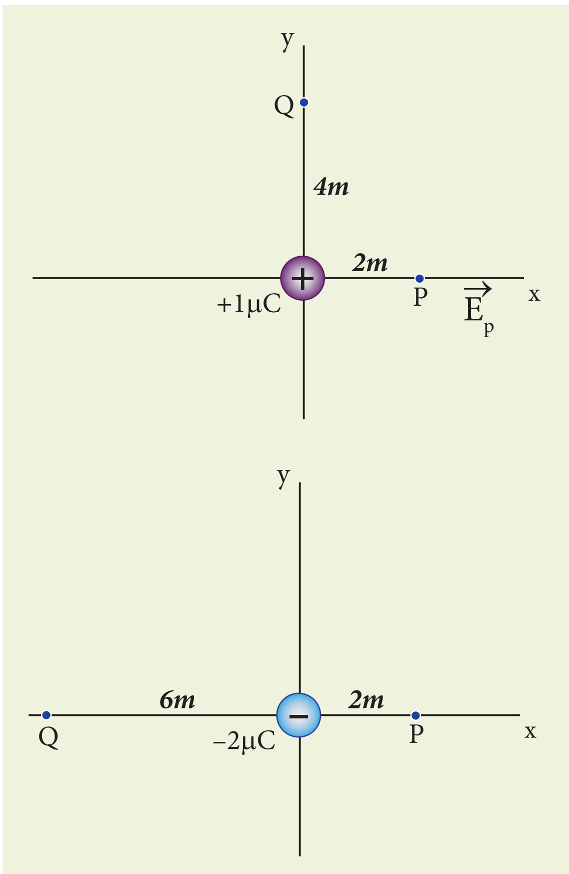
Figure 1.7 Uniform and non-uniform electric field

EXAMPLE 1.6

Calculate the electric field at points P, Q for the following two cases, as shown in the figure.

(a) A positive point charge +1 μC is placed at the origin

(b) A negative point charge -2 μC is placed at the origin



Solution

Case (a)

The magnitude of the electric field at point P is

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times1\times10^{-6}}{4}$$

$$= 2.25\times10^3\, NC^{-1}$$

Since the source charge is positive, the electric field points away from the charge. So the electric field at the point P is given by

$$\vec{E}_P = 2.25\times10^3 N C^{-1}\,\hat{i}$$

For the point Q

$$\left|\vec{E}_Q\right| = \frac{9\times10^9\times1\times10^{-6}}{16} = 0.56\times10^3\, NC^{-1}$$

Hence $\qquad \vec{E}_Q = 0.56\times10^3\,\hat{j}$

Case (b)

The magnitude of the electric field at point P

$$\left|\vec{E}_P\right| = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times2\times10^{-6}}{4}$$

$= 4.5 \times 10^3$ N C^{-1}

Since the source charge is negative, the electric field points towards the charge. So the electric field at the point P is given by

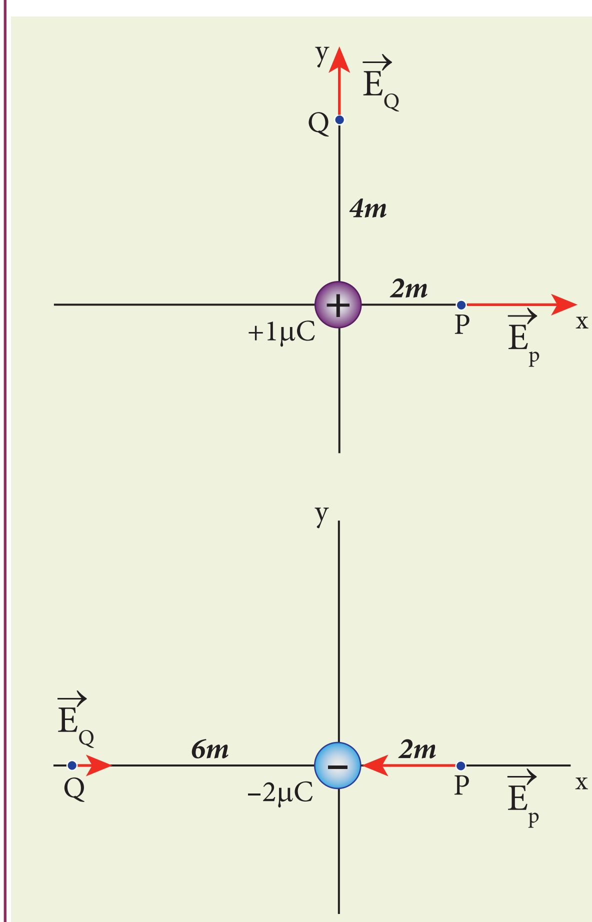
$$\vec{E}_P = -4.5\times10^3\,\hat{i}\; NC^{-1}$$

For the point Q, $\left|\vec{E}_Q\right| = \dfrac{9\times10^9\times2\times10^{-6}}{36}$

$= 0.5 \times 10^3$ N C^{-1}

$$\vec{E}_R = 0.56\times10^3\,\hat{i}\; NC^{-1}$$

At the point Q the electric field is directed along the positive x-axis.



1.3.2 Electric field due to the system of point charges

Suppose a number of point charges are distributed in space. To find the electric field at some point P due to this collection of point charges, superposition principle is used. The electric field at an arbitrary point due to a collection of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields.

Consider a collection of point charges $q_1, q_2, q_3 ....q_n$ located at various points in space. The total electric field at some point P due to all these n charges is given by

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n \quad (1.6)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \frac{q_3}{r_{3P}^2}\hat{r}_{3P} + ... + \frac{q_n}{r_{nP}^2}\hat{r}_{nP}\right\} \quad (1.7)$$

Here $r_{1P}, r_{2P}, r_{3P} ....r_{nP}$ are the distance of the the charges $q_1, q_2, q_3 ....q_n$ from the point P respectively. Also $\hat{r}_{1P}, \hat{r}_{2P}, \hat{r}_{3P} ....\hat{r}_{nP}$ are the corresponding unit vectors directed from $q_1, q_2, q_3 ....q_n$ to P.

Equation (1.7) can be re-written as,

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_\circ}\sum_{i=1}^{n}\left(\frac{q_i}{r_{iP}^2}\hat{r}_{iP}\right) \quad (1.8)$$

For example in Figure 1.8, the resultant electric field due to three point charges q_1, q_2, q_3 at point P is shown.

Note that the relative lengths of the electric field vectors for the charges depend on relative distances of the charges to the point P.

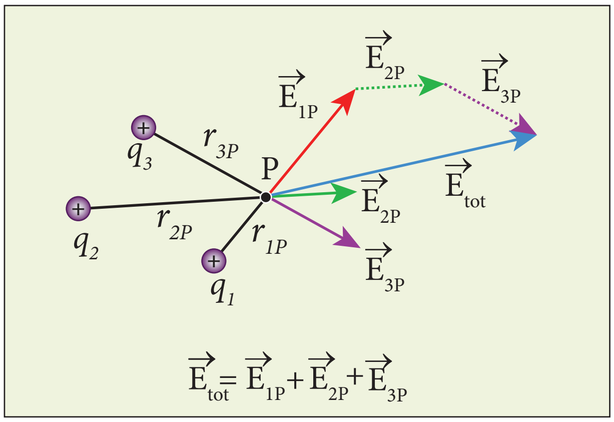
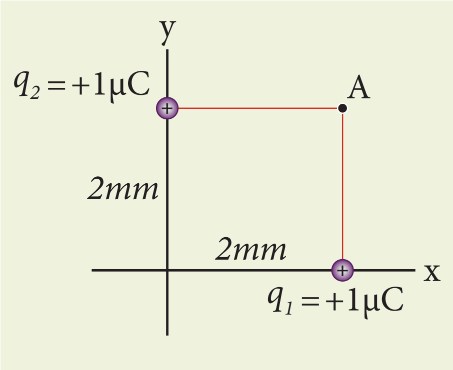


Figure 1.8 Superposition of Electric field

EXAMPLE 1.7

Consider the charge configuration as shown in the figure. Calculate the electric field at point A. If an electron is placed at points A, what is the acceleration experienced by this electron? (mass of the electron = 9.1 × 10^{-31} kg and charge of electron = −1.6 × 10^{-19} C)



Solution

By using superposition principle, the net electric field at point A is

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_{1A}^2}\hat{r}_{1A} + \frac{1}{4\pi\varepsilon_\circ}\frac{q_2}{r_{2A}^2}\hat{r}_{2A},$$

where r_{1A} and r_{2A} are the distances of point A from the two charges respectively.

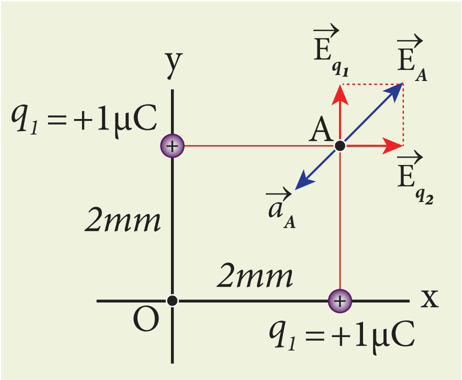
$$\vec{E}_A = \frac{9\times10^9\times1\times10^{-6}}{\left(2\times10^{-3}\right)^2}(\hat{j}) + \frac{9\times10^9\times1\times10^{-6}}{\left(2\times10^{-3}\right)^2}(\hat{i})$$

$= 2.25\times10^9\ \hat{j} + 2.25\times10^9\hat{i} = 2.25\times10^9\left(\hat{i}+\hat{j}\right)$

The magnitude of electric field

$$\left|\vec{E}_A\right| = \sqrt{\left(2.25\times10^9\right)^2 + \left(2.25\times10^9\right)^2}$$
$$= 2.25\times\sqrt{2}\times10^9\ NC^{-1}$$

The direction of $\vec{E}_A$ is given by $\dfrac{\vec{E}_A}{\left|\vec{E}_A\right|} = \dfrac{2.25\times10^9\left(\hat{i}+\hat{j}\right)}{2.25\times\sqrt{2}\times10^9} = \dfrac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$, which is the unit vector along OA as shown in the figure.



The acceleration experienced by an electron placed at point A is

$$\vec{a}_A = \frac{\vec{F}}{m} = \frac{q\vec{E}_A}{m}$$
$$= \frac{\left(-1.6\times10^{-19}\right)\times\left(2.25\times10^9\right)\left(\hat{i}+\hat{j}\right)}{9.1\times10^{-31}}$$
$$= -3.95\times10^{20}\left(\hat{i}+\hat{j}\right)N$$

The electron is accelerated in a direction exactly opposite to $\vec{E}_A$.

1.3.3 Electric field due to continuous charge distribution

The electric charge is quantized microscopically. The expressions (1.2), (1.3), (1.4) are applicable to only point charges. While dealing with the electric field due to a charged sphere or a charged wire etc., it is very difficult to look at individual charges in these charged bodies. Therefore, it is assumed that charge is distributed continuously on the charged bodies and the discrete nature of charges is not considered here. The electric field due to such continuous charge distributions is found by invoking the method of calculus.

Consider the following charged object of irregular shape as shown in Figure 1.9. The entire charged object is divided into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3 ......\Delta q_n$ and each charge element Δq is taken as a point charge.

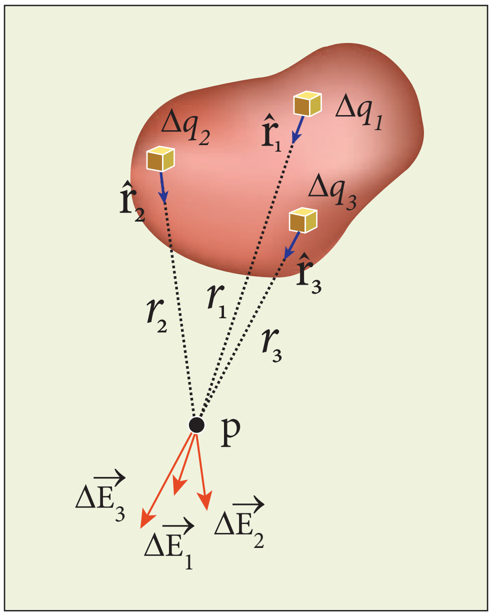


Figure 1.9 Continuous charge distributions

The electric field at a point P due to a charged object is approximately given by the sum of the fields at P due to all such charge elements.

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + ...+\frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right)$$
$$\approx \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP} \qquad (1.9)$$

Here Δq_i is the i[th] charge element, r_{iP} is the distance of the point P from the i[th] charge element and $\hat{r}_{iP}$ is the unit vector from i[th] charge element to the point P.

However the equation (1.9) is only an approximation. To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (1.9) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r} \qquad (1.10)$$

Here r is the distance of the point P from the infinitesimal charge dq and $\hat{r}$ is the unit vector from dq to point P. Even though the electric field for a continuous charge distribution is difficult to evaluate, the force experienced by some test charge q in this electric field is still given by $\vec{F} = q\vec{E}$.

(a) If the charge Q is uniformly distributed along the wire of length L, then linear charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. Its unit is coulomb per meter (Cm^{-1}).

The charge present in the infinitesimal length dl is $dq = \lambda dl$. This is shown in Figure 1.10 (a).

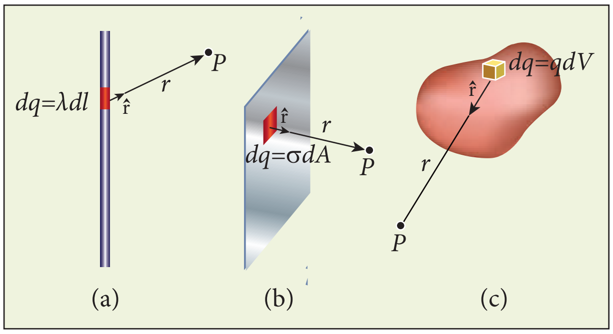


Figure 1.10 Line, surface and volume charge distribution

The electric field due to the line of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl}{r^2}\hat{r} = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{r^2}\hat{r}$$

(b) If the charge Q is uniformly distributed on a surface of area A, then surface charge density (charge per unit area) is $\sigma = \frac{Q}{A}$. Its unit is coulomb per square meter ($C\,m^{-2}$).

The charge present in the infinitesimal area dA is $dq = \sigma dA$. This is shown in the figure 1.10 (b).

The electric field due to a of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\sigma da}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_\circ}\sigma\int \frac{da}{r^2}\hat{r}$$

This is shown in Figure 1.10(b).

(c) If the charge Q is uniformly distributed in a volume V, then volume charge density (charge per unit volume) is given by $\rho = \frac{Q}{V}$. Its unit is coulomb per cubic meter ($C\,m^{-3}$).

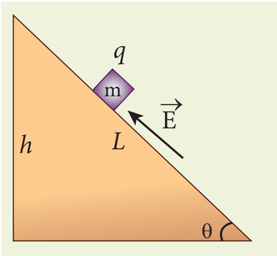
The charge present in the infinitesimal volume element dV is $dq = \rho dV$. This is shown in Figure 1.10(c).

The electric field due to a volume of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\rho dV}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_\circ}\rho\int \frac{dV}{r^2}\hat{r}.$$

EXAMPLE 1.8

A block of mass m and positive charge q is placed on an insulated frictionless inclined plane as shown in the figure. A uniform electric field E is applied parallel to the inclined surface such that the block is at rest. Calculate the magnitude of the electric field E.



Solution

Note: A similar problem is solved in XI[th] Physics volume I, unit 3 section 3.3.2.

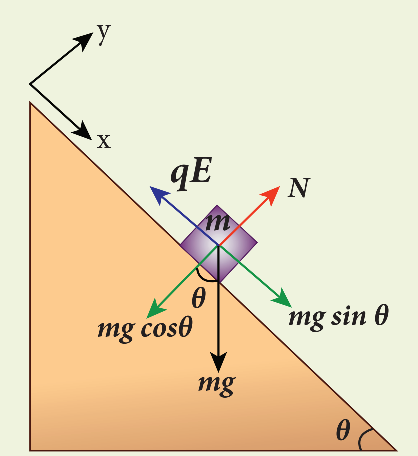
There are three forces that acts on the mass m:

(i) The downward gravitational force exerted by the Earth (mg)

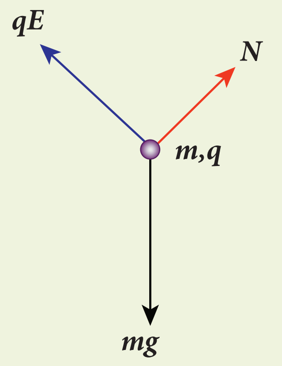
(ii) The normal force exerted by the inclined surface (N)

(iii) The Coulomb force given by uniform electric field (qE)

The free body diagram for the mass m is drawn below.



Forces acting on the mass *m*



Free body diagram

A convenient inertial coordinate system is located in the inclined surface as shown in the figure. The mass m has zero net acceleration both in x and y-direction.

Along x-direction, applying Newton's second law, we have

$$mg\sin\theta\hat{i} - qE\hat{i} = 0$$

$$mg\sin\theta - qE = 0$$

$$\text{or,} \qquad E = \frac{mg\sin\theta}{q}$$

Note that the magnitude of the electric field is directly proportional to the mass m and inversely proportional to the charge q. It implies that, if the mass is increased by keeping the charge constant, then a strong electric field is required to stop the object from sliding. If the charge is increased by keeping the mass constant, then a weak electric field is sufficient to stop the mass from sliding down the plane.

The electric field also can be expressed in terms of height and the length of the inclined surface of the plane.

$$E = \frac{mg\,h}{qL}$$

1.3.4 Electric field lines

Electric field vectors are visualized by the concept of electric field lines. They form a set of continuous lines which are the visual representation of the electric field in some region of space. The following rules are followed while drawing electric field lines for charges.

- The electric field lines start from a positive charge and end at negative charges or at infinity. For a positive point charge the electric field lines point radially outward and for a negative point charge, the electric field lines point radially inward. These are shown in Figure 1.11 (a) and (b).

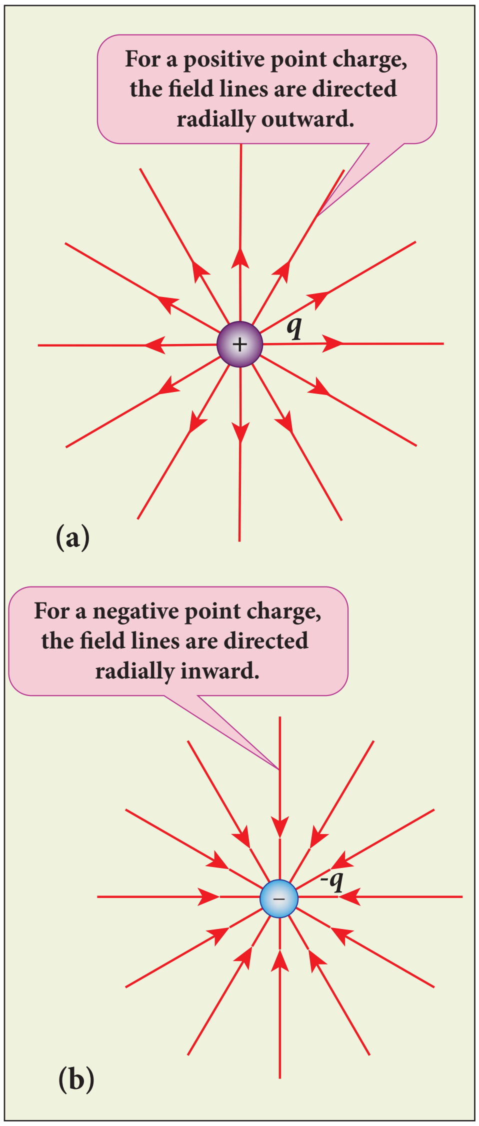


Figure 1.11 Electric field lines for isolated positive and negative charges

Note that for an isolated positive point charge the electric field line starts from the charge and ends only at infinity. For an isolated negative point charge the electric field lines start at infinity and end at the negative charge.

- The electric field vector at a point in space is tangential to the electric field line at that point. This is shown in Figure 1.12

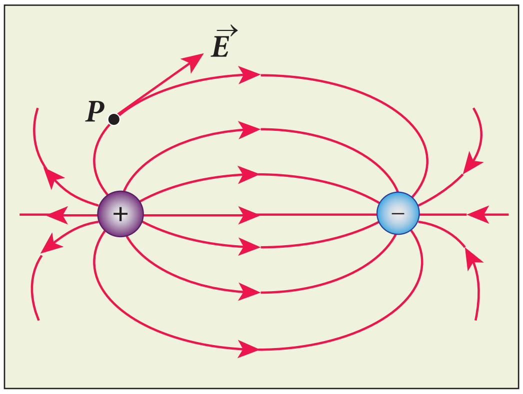


Figure 1.12 Electric field at a point P

- The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region. This is shown in Figure1.13

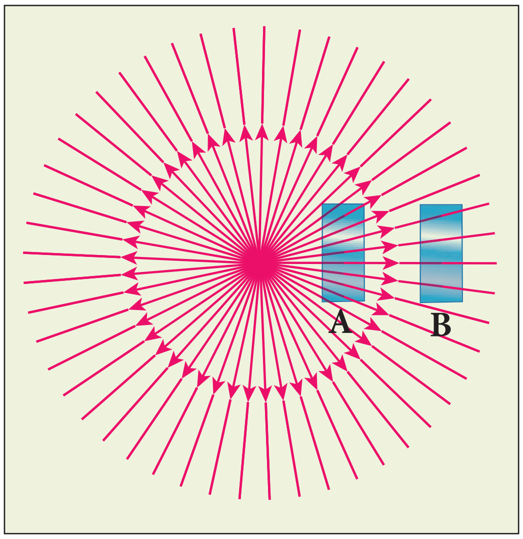


Figure 1.13 Electric field has larger magnitude at surface A than B

Figure 1.13 shows electric field lines from a positive point charge. The magnitude of the electric field for a point charge decreases as the distance increases $\left(\left|\vec{E}\right| \propto \frac{1}{r^2}\right)$. So the electric field has greater magnitude at the surface A than at B. Therefore, the number of lines crossing the surface A is greater than the number of lines crossing the surface B. Note that at surface B the electric field lines are farther apart compared to the electric field lines at the surface A.

- No two electric field lines intersect each other. If two lines cross at a point, then there will be two different electric field vectors at the same point, as shown in Figure 1.14.

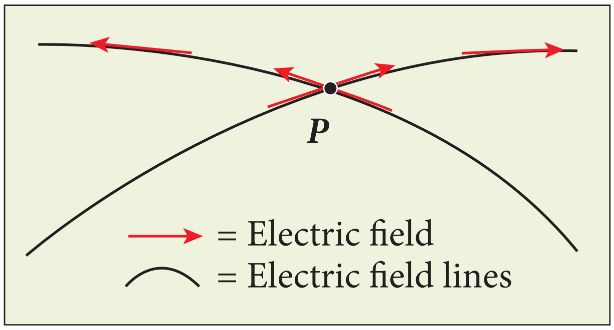


Figure 1.14 Two electric field lines never intersect each other

As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

- The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

For example in the Figure 1.15, the electric field lines are drawn for charges +q and -2q. Note that the number of field lines emanating from +q is 8 and the number of field lines ending at -2q is 16. Since the magnitude of the second charge is twice that

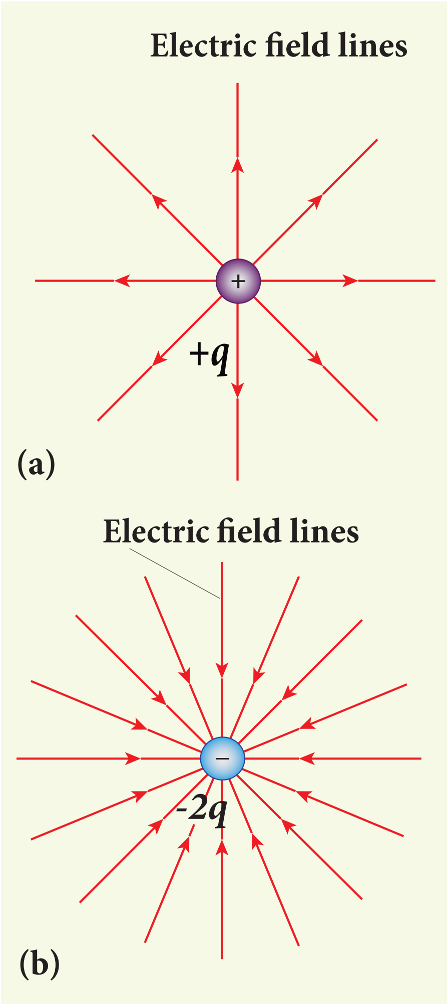
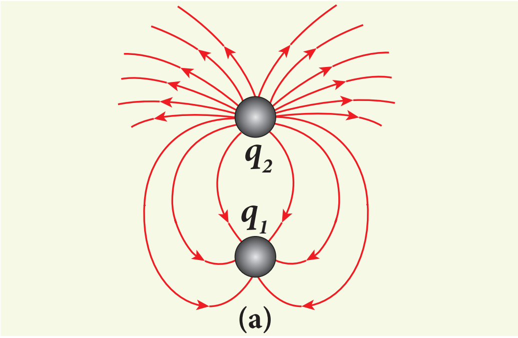


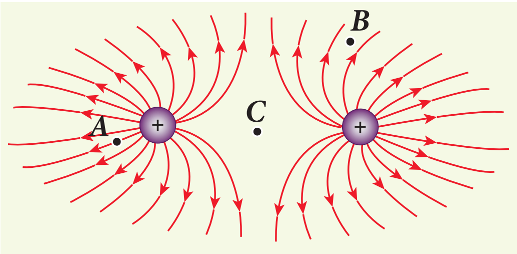
Figure 1.15 Electric field lines and magnitude of the charge

of the first charge, the number of field lines drawn for -2q is twice in number than that for charge +q.

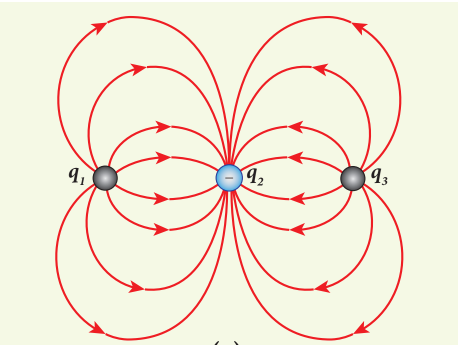
EXAMPLE 1.9

The following pictures depict electric field lines for various charge configurations.





(b)



(c)

(i) In figure (a) identify the signs of two charges and find the ratio $\left|\frac{q_1}{q_2}\right|$

(ii) In figure (b), calculate the ratio of two positive charges and identify the strength of the electric field at three points A, B, and C

(iii) Figure (c) represents the electric field lines for three charges. If q_2 = -20 nC, then calculate the values of q_1 and q_3

Solution

(i) The electric field lines start at q_2 and end at q_1. In figure (a), q_2 is positive and q_1 is negative. The number of lines starting from q_2 is 18 and number of the lines ending at q_1 is 6. So q_2 has greater magnitude. The ratio of $\left|\frac{q_1}{q_2}\right| = \frac{N_1}{N_2} = \frac{6}{18} = \frac{1}{3}$. It implies that $|q_2| = 3|q_1|$

(ii) In figure (b), the number of field lines emanating from both positive charges are equal (N=18). So the charges are equal. At point A, the electric field lines are denser compared to the lines at point B. So the electric field at point A is greater in magnitude compared to the field at point B. Further, no electric field line passes through C, which implies that the resultant electric field at C due to these two charges is zero.

(iii) In the figure (c), the electric field lines start at q_1 and q_3 and end at q_2. This implies that q_1 and q_3 are positive charges. The ratio of the number of field lines is $\left|\frac{q_1}{q_2}\right| = \frac{8}{16} = \left|\frac{q_3}{q_2}\right| = \frac{1}{2}$, implying that q_1 and q_3 are half of the magnitude of q_2. So $q_1 = q_3 = +10$ nC.

1.4 ELECTRIC DIPOLE AND ITS PROPERTIES

1.4.1 Electric dipole

Two equal and opposite charges separated by a small distance constitute an electric dipole. In many molecules, the centers of positive and negative charge do not coincide. Such molecules behave as permanent dipoles. Examples: CO, water, ammonia, HCl etc.

Consider two equal and opposite point charges (+q, -q) that are separated by a distance 2a as shown in Figure 1.16(a).
The electric dipole moment is defined as $\vec{p} = q\vec{r}_+ - q\vec{r}_-$.
Here $\vec{r}_+$ is the position vector of +q from the origin and $\vec{r}_-$ is the position vector of -q from the origin. Then, from Figure 1.16 (a),

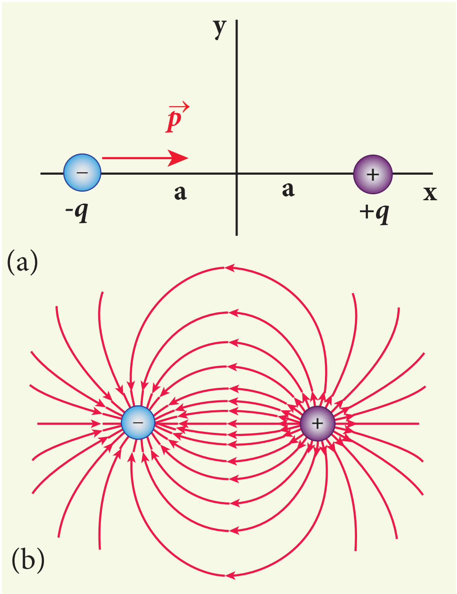


Figure 1.16 (a) Electric dipole (b) Electric field lines for the electric dipole

$$\vec{p} = qa\hat{i} - qa(-\hat{i}) = 2qa\hat{i} \quad (1.11)$$

The electric dipole moment vector lies along the line joining two charges and is directed from –q to +q. The SI unit of dipole moment is coulomb meter (Cm). The electric field lines for an electric dipole are shown in Figure 1.16 (b).

- For simplicity, the two charges are placed on the x-axis. Even if the two charges are placed on y or z-axies, dipole moment will point from –q to +q. The magnitude of the electric dipole moment is equal to the product of the magnitude of one of the charges and the distance between them, $|\vec{p}| = 2qa$
- Though the electric dipole moment for two equal and opposite charges is defined, it is very general. It is possible to define and calculate the electric dipole moment for a single charge, two positive charges, two negative charges and also for more than two charges.

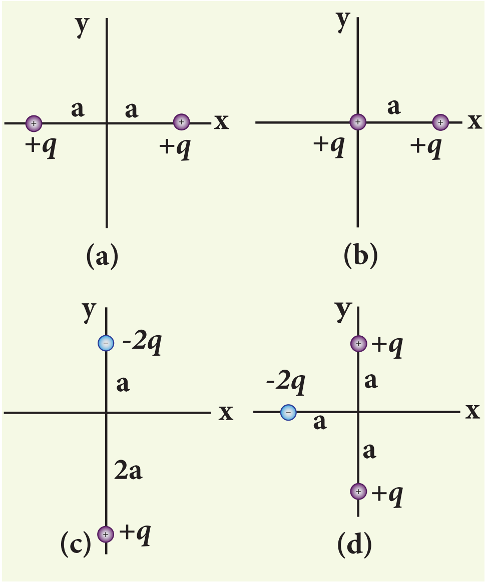
  For a collection of n point charges, the electric dipole moment is defined as follows:

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i \quad (1.12)$$

where $\vec{r}_i$ is the position vector of charge q_i from the origin.

EXAMPLE 1.10

Calculate the electric dipole moment for the following charge configurations.



Solution

Case (a) The position vector for the +q on the positive x-axis is $a\hat{i}$ and position vector for the +q charge the negative x axis is $-a\hat{i}$. So the dipole moment is,

$$\vec{p} = (+q)(a\hat{i}) + (+q)(-a\hat{i}) = 0$$

Case (b) In this case one charge is placed at the origin, so its position vector is zero. Hence only the second charge +q with position vector $a\hat{i}$ contributes to the dipole moment, which is $\vec{p} = qa\hat{i}$.

From both cases (a) and (b), we can infer that in general the electric dipole moment depends on the choice of the origin and charge configuration. But for one special case, the electric dipole moment is independent of the origin. If the total

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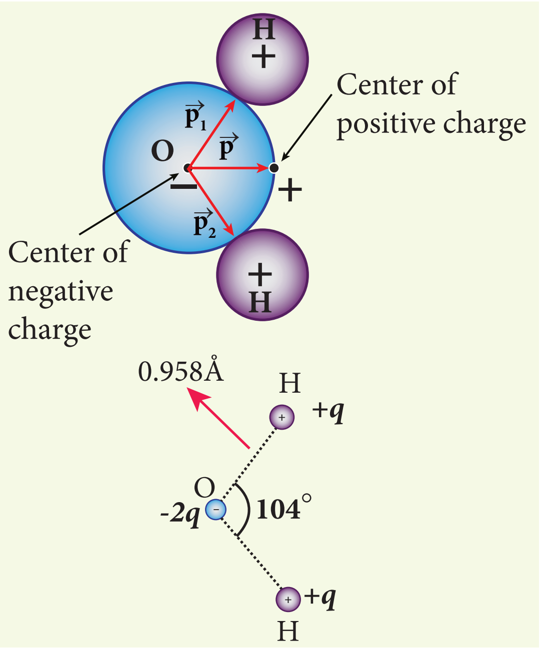
charge is zero, then the electric dipole moment will be the same irrespective of the choice of the origin. It is because of this reason that the electric dipole moment of an electric dipole (total charge is zero) is always directed from –q to +q, independent of the choice of the origin.

Case (c) $\vec{p} = (-2q)a\hat{j} + q(2a)(-\hat{j}) = -4qa\hat{j}$.
Note that in this case $\vec{p}$ is directed from -2q to +q.

Case (d) $\vec{p} = -2qa(-\hat{i}) + qa\hat{j} + qa(-\hat{j})$
$= 2qa\,\hat{i}$

The water molecule (H_2O) has this charge configuration. The water molecule has three atoms (two H atom and one O atom). The centers of positive (H) and negative (O) charges of a water molecule lie at different points, hence it possess permanent dipole moment. The O-H bond length is 0.958 × 10^{-10} m due to which the electric dipole moment of water molecule has the magnitude p = 6.1 x 10^{-30} Cm. The electric dipole moment $\vec{p}$ is directed from center of negative charge to the center of positive charge, as shown in the figure.



1.4.2 Electric field due to a dipole

Case (i) Electric field due to an electric dipole at points on the axial line

Consider an electric dipole placed on the x-axis as shown in Figure 1.17. A point C is located at a distance of *r* from the midpoint O of the dipole along the axial line.

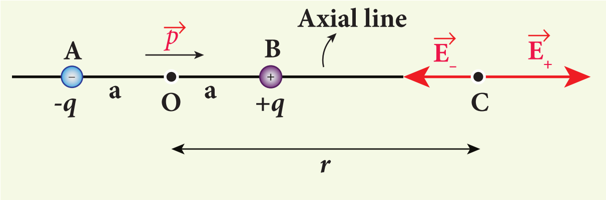


Figure 1.17 Electric field of the dipole along the axial line

The electric field at a point C due to +q is

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ along BC}$$

Since the electric dipole moment vector $\vec{p}$ is from –q to +q and is directed along BC, the above equation is rewritten as

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} \qquad (1.13)$$

where $\hat{p}$ is the electric dipole moment unit vector from –q to +q.

The electric field at a point C due to –q is

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p} \qquad (1.14)$$

Since +q is located closer to the point C than –q, $\vec{E}_+$ is stronger than $\vec{E}_-$. Therefore, the length of the $\vec{E}_+$ vector is drawn larger than that of $\vec{E}_-$ vector.

The total electric field at point C is calculated using the superposition principle of the electric field.

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p} \quad (1.15)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p} \quad (1.16)$$

Note that the total electric field is along $\vec{E}_+$, since $+q$ is closer to C than −q.

The direction of $\vec{E}_{tot}$ is shown in Figure 1.18.

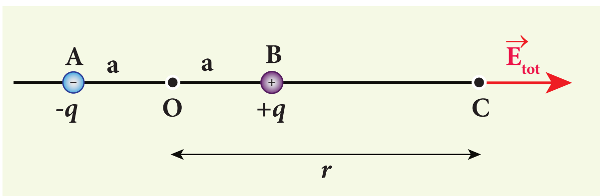


Figure 1.18 Total electric field of the dipole on the axial line

If the point C is very far away from the dipole then (r >> a). Under this limit the term $(r^2-a^2)^2 \approx r^4$. Substituting this into equation (1.16), we get

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p} \quad (r >> a)$$

$$\text{since } 2aq\,\hat{p} = \vec{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} \quad (r >> a) \quad (1.17)$$

If the point C is chosen on the left side of the dipole, the total electric field is still in the direction of $\vec{p}$. We infer this result by examining the electric field lines of the dipole shown in Figure 1.16(b).

Case (ii) Electric field due to an electric dipole at a point on the equatorial plane

Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in Figure 1.19.

Since the point C is equi-distant from +q and −q, the magnitude of the electric fields of +q and −q are the same. The direction of $\vec{E}_+$ is along BC and the direction of $\vec{E}_-$ is along CA. $\vec{E}_+$ and $\vec{E}_-$ are resolved into two components; one component parallel to the dipole axis and the other perpendicular to it. The perpendicular components $|\vec{E}_+|\sin\theta$ and $|\vec{E}_-|\sin\theta$ are oppositely directed and cancel each other. The magnitude of the total electric field at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$ as shown in the Figure 1.19.

$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\,\hat{p} - |\vec{E}_-|\cos\theta\,\hat{p} \quad (1.18)$$

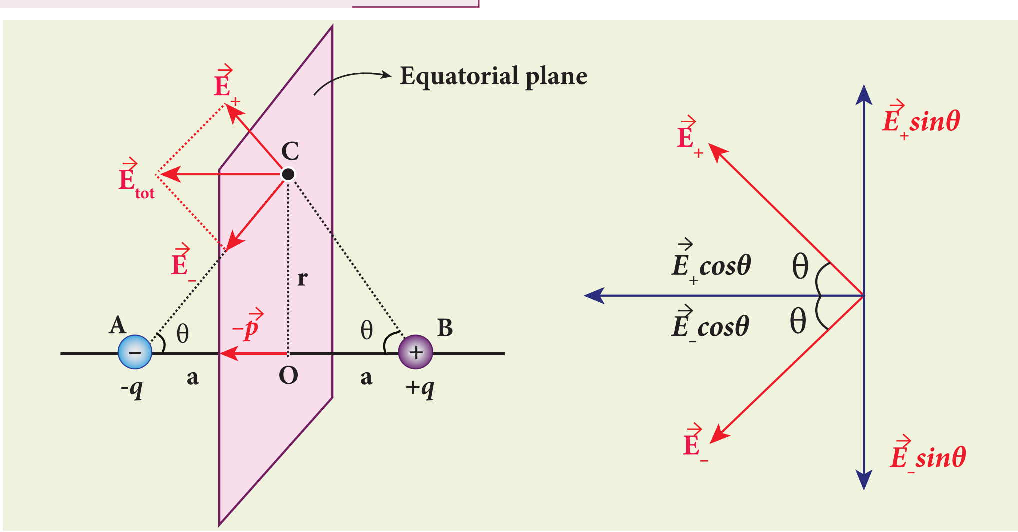


Figure 1.19 Electric field due to a dipole at a point on the equatorial plane

The magnitudes $\vec{E}_{+}$ and $\vec{E}_{-}$ are the same and are given by

$$\left|\vec{E}_{+}\right| = \left|\vec{E}_{-}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2+a^2\right)} \qquad (1.19)$$

By substituting equation (1.19) into equation (1.18), we get

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{2q\cos\theta}{\left(r^2+a^2\right)}\hat{p}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{2qa}{\left(r^2+a^2\right)^{\frac{3}{2}}}\hat{p}$$

$$\text{since } \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{\left(r^2+a^2\right)^{\frac{3}{2}}}$$

$$\text{since } \vec{p} = 2qa\hat{p} \qquad (1.20)$$

At very large distances (r>>a), the equation (1.20) becomes

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}}{r^3} \qquad (r >> a) \qquad (1.21)$$

Important inferences

(i) From equations (1.17) and (1.21), it is inferred that for very large distances, the magnitude of the electric field at points on the dipole axis is twice the magnitude of the electric field at points on the equatorial plane. The direction of the electric field at points on the dipole axis is directed along the direction of dipole moment vector $\vec{p}$ but at points on the equatorial plane it is directed opposite to the dipole moment vector, that is along $-\vec{p}$.

(ii) At very large distances, the electric field due to a dipole varies as $\frac{1}{r^3}$. Note that for a point charge, the electric field varies as $\frac{1}{r^2}$. This implies that the electric field due to a dipole at very large distances goes to zero faster than the electric field due to a point charge. The reason for this behavior is that at very large distance, the two charges appear to be close to each other and neutralize each other.

(iii) The equations (1.17) and (1.21) are valid only at very large distances (r>>a). Suppose the distance 2a approaches zero and q approaches infinity such that the product of $2aq = p$ is finite, then the dipole is called a point dipole. For such point dipoles, equations (1.17) and (1.21) are exact and hold true for any r.

1.4.3 Torque experienced by an electric dipole in the uniform electric field

Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ whose field lines are equally spaced and point in the same direction. The charge +q will experience a force q$\vec{E}$ in the direction of the field and charge –q will experience a force -q$\vec{E}$ in a direction opposite to the field. Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque as shown in Figure 1.20. This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction).

The total torque on the dipole about the point O

$$\vec{\tau} = \overrightarrow{OA}\times\left(-q\vec{E}\right) + \overrightarrow{OB}\times q\vec{E} \qquad (1.22)$$

Using right-hand corkscrew rule (Refer XI, volume 1, unit 2), it is found that total

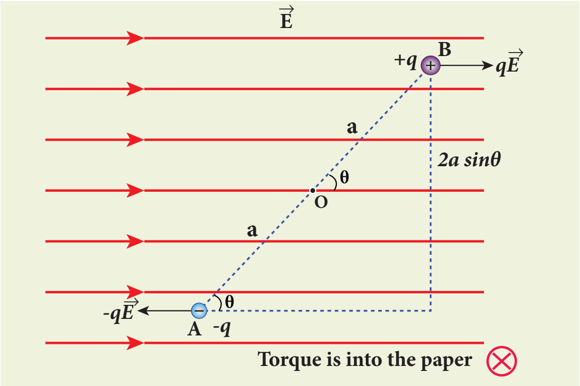


Figure 1.20 Torque on dipole

torque is perpendicular to the plane of the paper and is directed into it.

The magnitude of the total torque $\vec{\tau} = |\overline{OA}||(-q\vec{E})|\sin\theta + |\overline{OB}||q\vec{E}|\sin\theta$

$$\tau = qE \cdot 2a\sin\theta \qquad (1.23)$$

where θ is the angle made by $\vec{p}$ with $\vec{E}$. Since p = 2aq, the torque is written in terms of the vector product as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad (1.24)$$

The magnitude of this torque is $\tau = pE\sin\theta$ and is maximum when $\theta = 90^\circ$.

This torque tends to rotate the dipole and align it with the electric field $\vec{E}$. Once $\vec{p}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero.

If the electric field is not uniform, then the force experienced by +q is different from that experienced by –q. In addition to the torque, there will be net force acting on the dipole. This is shown in Figure 1.21.

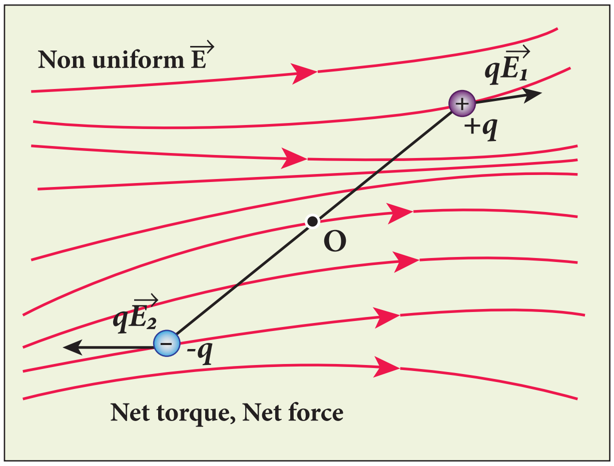


Figure 1.21 The dipole in a non-uniform electric field

EXAMPLE 1.11

A sample of HCl gas is placed in a uniform electric field of magnitude 3 × 10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4 × 10^{-30} Cm. Calculate the maximum torque experienced by each HCl molecule.

Solution

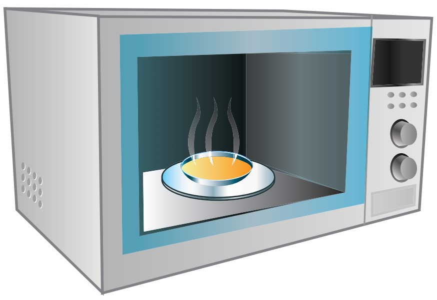
The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

$$\tau_{max} = pE\sin 90^\circ = 3.4 \times 10^{-30} \times 3 \times 10^4\, N\,m$$

$$\tau_{max} = 10.2 \times 10^{-26}\, N\,m$$

DO YOU KNOW?

Microwave oven works on the principle of torque acting on an electric dipole. The food we consume has water molecules which are permanent electric dipoles. Oven produces microwaves that are oscillating electromagnetic fields and produce torque on the water molecules. Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy. Thus, heat generated is used to heat the food.



1.5 ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Introduction

In mechanics, potential energy is defined for conservative forces. Since gravitational force is a conservative force, its gravitational potential energy is defined in XI standard physics (Unit 6). Since Coulomb force is an inverse-square-law force, its also a conservative force like gravitational force. Therefore, we can define potential energy for charge configurations.

1.5.1 Electrostatic Potential energy and Electrostatic potential

Consider a positive charge q kept fixed at the origin which produces an electric field $\vec{E}$ around it. A positive test charge q′ is brought from point R to point P against the repulsive force between q and q′ as shown in Figure 1.22. Work must be done to overcome this repulsion. This work done is stored as potential energy.

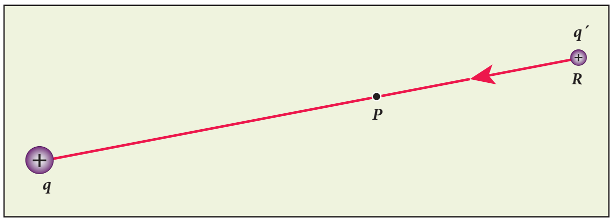


Figure 1.22 Work done is equal to potential energy

The test charge q′ is brought from R to P with constant velocity which means that external force used to bring the test charge q′ from R to P must be equal and opposite to the coulomb force $\left(\vec{F}_{ext} = -\vec{F}_{coloumb}\right)$. The work done is

$$W = \int_R^P \vec{F}_{ext} \cdot d\vec{r} \qquad (1.25)$$

Since coulomb force is conservative, work done is independent of the path and it depends only on the initial and final positions of the test charge. If potential energy associated with q′ at P is U_P and that at R is U_R, then difference in potential energy is defined as the work done to bring a test charge q′ from point R to P and is given as $U_P - U_R = W = \Delta U$

$$\Delta U = \int_R^P \vec{F}_{ext} \cdot d\vec{r} \qquad (1.26)$$

$$\text{Since } \vec{F}_{ext} = -\vec{F}_{coloumb} = -q'\vec{E} \qquad (1.27)$$

$$\Delta U = \int_R^P -\left(q'\vec{E}\right) \cdot d\vec{r} = q' \int_R^P \left(-\vec{E}\right) \cdot d\vec{r} \qquad (1.28)$$

The potential energy difference per unit charge is given by

$$\frac{\Delta U}{q'} = \frac{q' \int_R^P \left(-\vec{E}\right) \cdot d\vec{r}}{q'} = -\int_R^P \vec{E} \cdot d\vec{r} \qquad (1.29)$$

The above equation (1.29) is independent of q′. The quantity $\frac{\Delta U}{q'} = -\int_R^P \vec{E} \cdot d\vec{r}$ is called electric potential difference between P and R and is denoted as $V_P - V_R = \Delta V$.

In otherwords, the electric potential difference is defined as the work done by an external force to bring unit positive charge from point R to point P.

$$V_P - V_R = \Delta V = \int_R^P -\vec{E} \cdot d\vec{r} \qquad (1.30)$$

The electric potential energy difference can be written as ΔU = q′ ΔV. Physically potential difference between two points is a meaningful quantity. The value of the potential itself at one point is not meaningful. Therefore the point R is taken at infinity and its potential is considered as zero $(V_\infty = 0)$.

Then **the electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$**. Mathematically this is written as

$$V_P = -\int_\infty^P \vec{E} \cdot d\vec{r} \qquad (1.31)$$

Important points

1. Electric potential at point P depends only on the electric field which is due to the source charge q and not on the test charge q′. Unit positive charge is brought from infinity to the point P with constant velocity because external agency should not impart any kinetic energy to the test charge.
2. From equation (1.29), the unit of electric potential is Joule per coulomb. The practical unit is volt (V) named after Alessandro Volta (1745-1827) who invented the electrical battery. The potential difference between two points is expressed in terms of voltage.

1.5.2 Electric potential due to a point charge

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q. This is shown in Figure 1.23.

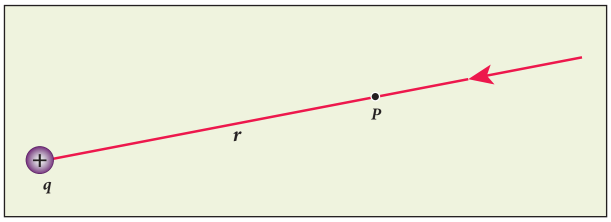


Figure 1.23 Electrostatic potential at a point P

The electric potential at the point P is

$$V = \int_\infty^r \left(-\vec{E}\right) \cdot d\vec{r} = -\int_\infty^r \vec{E} \cdot d\vec{r} \qquad (1.32)$$

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = \frac{-1}{4\pi\varepsilon_0} \int_\infty^r \frac{q}{r^2} \hat{r} . d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr\hat{r}$ and using $\hat{r} . \hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\varepsilon_\circ} \int_\infty^r \frac{q}{r^2} \hat{r} \cdot dr\hat{r} = -\frac{1}{4\pi\varepsilon_\circ} \int_\infty^r \frac{q}{r^2} dr$$

After the integration,

$$V = -\frac{1}{4\pi\varepsilon_\circ} q \left\{ -\frac{1}{r} \right\}_\infty^r = \frac{1}{4\pi\varepsilon_\circ} \frac{q}{r}$$

Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \qquad (1.33)$$

Important points

(i) If the source charge q is positive, V > 0. If q is negative, then V is negative and equal to $V = -\frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

(ii) The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.

(iii) From expression (1.33), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity $(r=\infty)$ electrostatic potential is zero (V = 0).

In the case of gravitational force, mass moves from a point of higher gravitational potential to a point of lower gravitational potential. Similarly a positive charge moves from a point of higher electrostatic potential to lower electrostatic potential. However a negative charge moves from lower electrostatic potential to higher electrostatic potential. This comparison is shown in Figure 1.24.

(iv) The electric potential at a point P due to a collection of charges $q_1, q_2, q_3 \ldots q_n$ is equal to sum of the electric potentials due to individual charges.

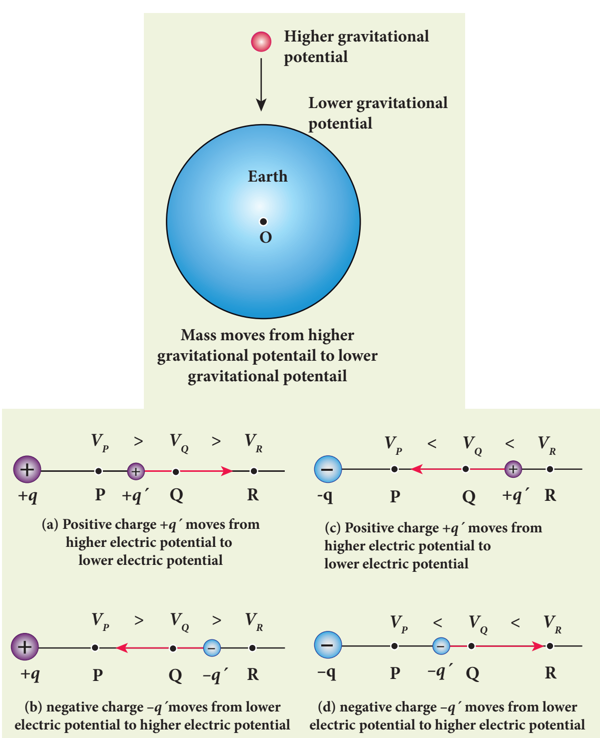


Figure 1.24 Motion of charges in terms of electric potential

$$V_{tot} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \ldots$$
$$\ldots + \frac{kq_n}{r_n} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad (1.34)$$

where $r_1, r_2, r_3 \ldots r_n$ are the distances of $q_1, q_2, q_3 \ldots q_n$ respectively from P (Figure 1.25).

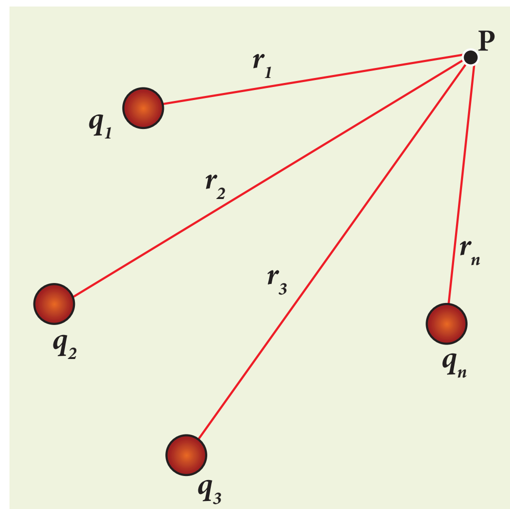
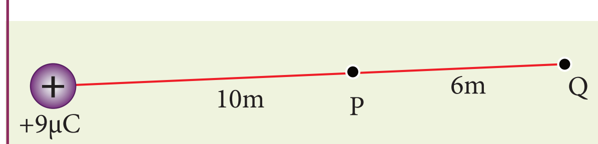


Figure 1.25 Electrostatic potential due to collection of charges

EXAMPLE 1.12

(a) Calculate the electric potential at points P and Q as shown in the figure below.

(b) Suppose the charge +9μC is replaced by -9μC find the electrostatic potentials at points P and Q



(c) Calculate the work done to bring a test charge +2μC from infinity to the point P. Assume the charge +9μC is held fixed at origin and +2μC is brought from infinity to P.

Solution

(a) Electric potential at point P is given by

$$V_P = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_P} = \frac{9\times10^9 \times 9\times10^{-6}}{10} = 8.1\times10^3\,V$$

Electric potential at point Q is given by

$$V_Q = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_Q} = \frac{9\times10^9 \times 9\times10^{-6}}{16} = 5.06\times10^3\,V$$

Note that the electric potential at point Q is less than the electric potential at point P. If we put a positive charge at P, it moves from P to Q. However if we place a negative charge at P it will move towards the charge +9μC.

The potential difference between the points P and Q is given by

$$\Delta V = V_P - V_Q = +3.04\times10^3\,V$$

(b) Suppose we replace the charge +9 μC by -9 μC, then the corresponding potentials at the points P and Q are,

$$V_P = -8.1\times10^3\,V,\; V_Q = -5.06\times10^3\,V$$

Note that in this case electric potential at the point Q is higher than at point P.

The potential difference or voltage between the points P and Q is given by

$$\Delta V = V_P - V_Q = -3.04\times10^3\,V$$

(c) The electric potential V at a point P due to some charge is defined as the work done by an external force to bring a unit positive charge from infinity to P. So to bring the q amount of charge from infinity to the point P, work done is given as follows.

$$W = qV$$

$$W_Q = 2\times10^{-6}\times5.06\times10^3\,J = 10.12\times10^{-3}\,J.$$

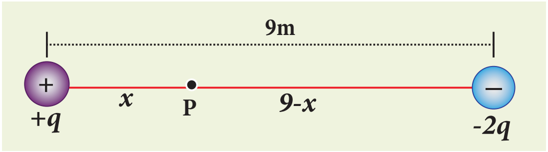
EXAMPLE 1.13

Consider a point charge +q placed at the origin and another point charge -2q placed at a distance of 9 m from the charge +q. Determine the point between the two charges at which electric potential is zero.

Solution

According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point.

Consider the point at which the total potential zero is located at a distance x from the charge +q as shown in the figure.



The total electric potential at P is zero.

$$V_{tot} = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0$$

which gives $\frac{q}{x} = \frac{2q}{(9-x)}$

or $\frac{1}{x} = \frac{2}{(9-x)}$

Hence, $x = 3$ m

1.5.3 Electrostatic potential at a point due to an electric dipole

Consider two equal and opposite charges separated by a small distance 2a as shown in Figure 1.26. The point P is located at a distance r from the midpoint of the dipole. Let θ be the angle between the line OP and dipole axis AB.

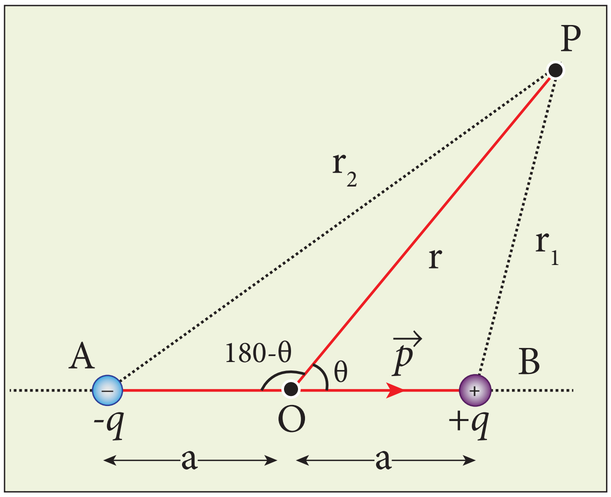


Figure 1.26 Potential due to electric dipole

Let r_1 be the distance of point P from +q and r_2 be the distance of point P from –q.

Potential at P due to charge +q $= \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_1}$

Potential at P due to charge –q $= -\frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_2}$

Total potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_\circ} q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (1.35)$$

Suppose if the point P is far away from the dipole, such that r>>a, then equation (1.35) can be expressed in terms of r.

By the cosine law for triangle BOP,

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$$

Since the point P is very far from dipole, then r>>a. As a result the term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore

$$r_1^2 = r^2\left(1 - 2a\frac{\cos\theta}{r}\right)$$

$$\text{(or)} \qquad r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$

Since $\frac{a}{r} << 1$, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) \qquad (1.36)$$

Similarly applying the cosine law for triangle AOP,

$$r_2^2 = r^2 + a^2 - 2ra\cos(180 - \theta)$$

since $\cos(180 - \theta) = -\cos\theta$ we get

$$r_2^2 = r^2 + a^2 + 2ra\cos\theta$$

Neglecting the term $\frac{a^2}{r^2}$ (because r>>a)

$$r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

$$r_2 = r\left(1 + \frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

Using Binomial theorem, we get

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right) \qquad (1.37)$$

Substituting equation (1.37) and (1.36) in equation (1.35),

$$V = \frac{1}{4\pi\varepsilon_\circ} q\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{q}{4\pi\varepsilon_\circ}\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{2aq}{r^2}\cos\theta$$

But the electric dipole moment $p = 2qa$ and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write p cosθ = $\vec{p}\cdot\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad (r>>a) \qquad (1.38)$$

Equation (1.38) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (1.38) is valid for any distance.

Special cases

Case (i) If the point P lies on the axial line of the dipole on the side of +q, then θ = 0. Then the electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{p}{r^2} \qquad (1.39)$$

Case (ii) If the point P lies on the axial line of the dipole on the side of –q, then θ = 180°, then

$$V = -\frac{1}{4\pi\varepsilon_\circ}\frac{p}{r^2} \qquad (1.40)$$

Case (iii) If the point P lies on the equatorial line of the dipole, then θ = 90°. Hence

$$V = 0 \qquad (1.41)$$

Important points

(i) The potential due to an electric dipole falls as $\frac{1}{r^2}$ and the potential due to a single point charge falls as $\frac{1}{r}$. Thus the potential due to the dipole falls faster than that due to a monopole (point charge). As the distance increases from electric dipole, the effects of positive and negative charges nullify each other.

(ii) The potential due to a point charge is spherically symmetric since it depends only on the distance r. But the potential due to a dipole is not spherically symmetric because the potential depends on the angle between $\vec{p}$ and position vector $\vec{r}$ of the point.
However the dipole potential is axially symmetric. If the position vector $\vec{r}$ is rotated about $\vec{p}$ by keeping θ fixed, then all points on the cone at the same distance r will have the same potential as shown in Figure 1.27. In this figure, all the points located on the blue curve will have the same potential.

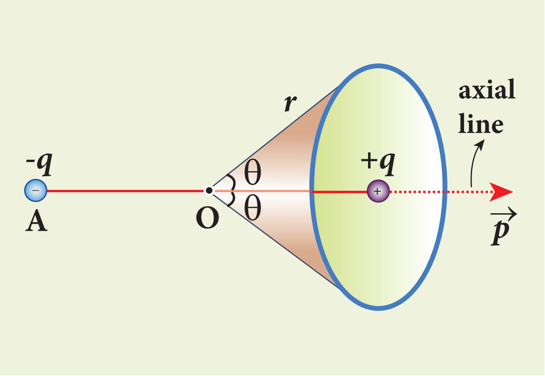


Figure 1.27 Dipole potential is axially symmetric

1.5.4 Equi-potential Surface

Consider a point charge q located at some point in space and an imaginary sphere of radius r is chosen by keeping the charge q at its center (Figure 1.28(a)). The electric potential at all points on the surface of the given sphere is the same. Such a surface is called an equipotential surface.

An equipotential surface is a surface on which all the points are at the same potential.

For a point charge the equipotential surfaces are concentric spherical surfaces as shown in Figure 1.28(b). Each spherical surface is an equipotential surface but the value of the

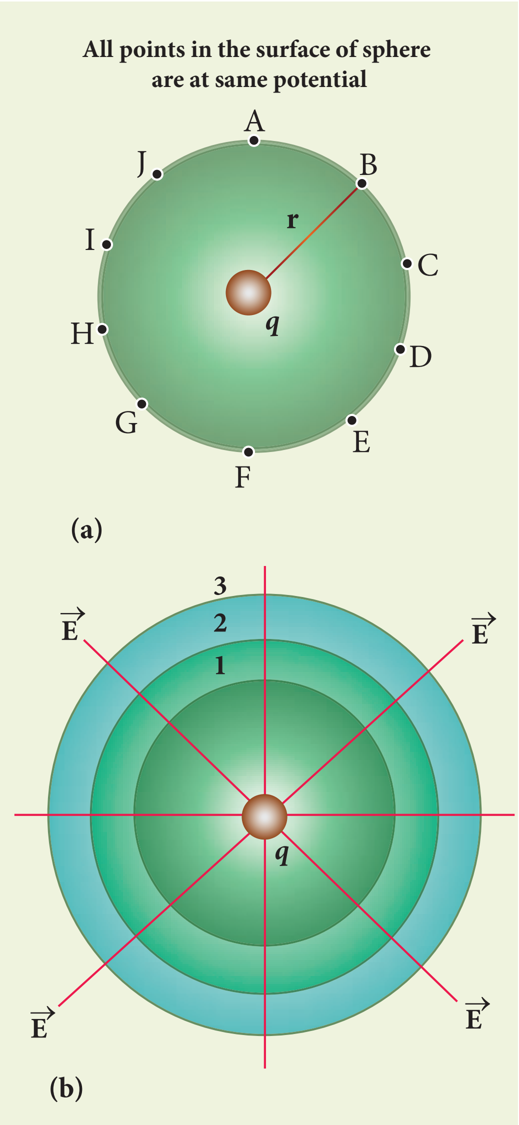


Figure 1.28 Equipotential surface of point Charge

potential is different for different spherical surfaces.

For a uniform electric field, the equipotential surfaces form a set of planes normal to the electric field $\vec{E}$. This is shown in the Figure 1.29.

Properties of equipotential surfaces

(i) The work done to move a charge q between any two points A and B, W = q (V_B – V_A). If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$.

(ii) The electric field is normal to an equipotential surface. If it is not normal, then there is a component of the field parallel to the surface. Then work must be done to move a charge between two points on the same surface. This is a contradiction. Therefore the electric field must always be normal to equipotential surface.

1.5.5 Relation between electric field and potential

Consider a positive charge q kept fixed at the origin. To move a unit positive charge by a small distance dx in the electric field E,

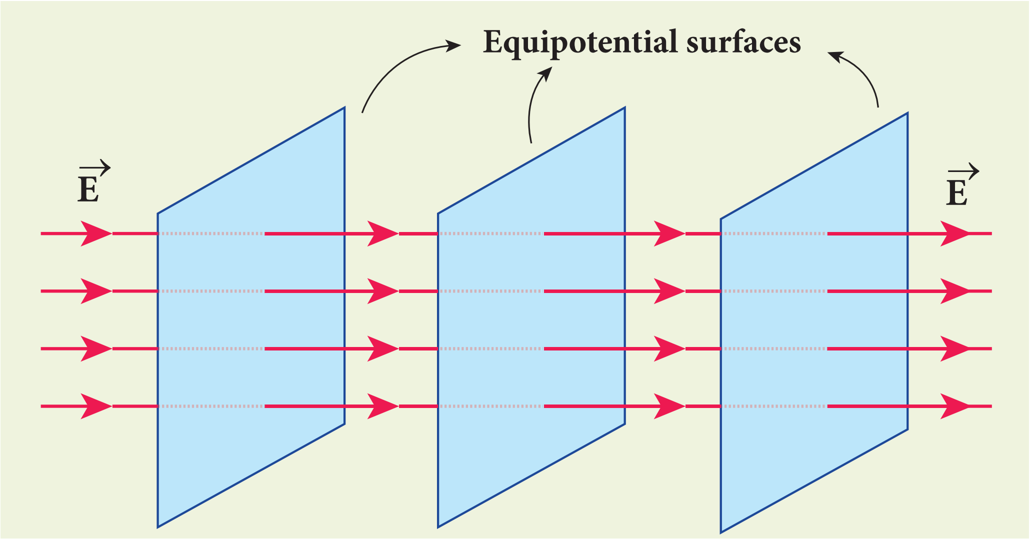


Figure 1.29 Equipotential surface for uniform electric field

the work done is given by $dW = -E\,dx$. The minus sign implies that work is done against the electric field. This work done is equal to electric potential difference. Therefore,

$$dW = dV.$$

$$\text{(or)} \quad dV = -E\,dx \tag{1.42}$$

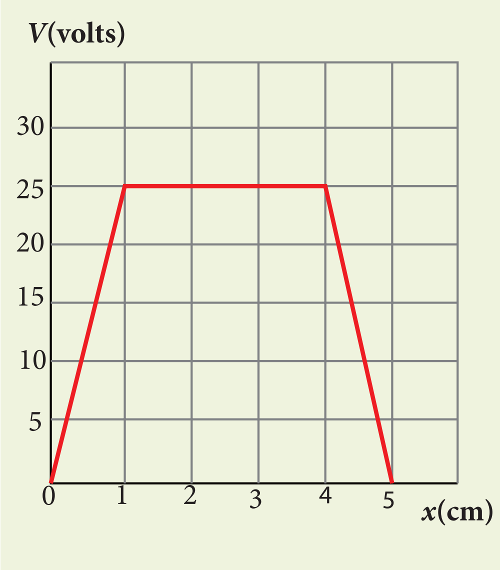
$$\text{Hence } E = -\frac{dV}{dx} \tag{1.43}$$

The electric field is the negative gradient of the electric potential. In general,

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial k}\hat{k}\right) \tag{1.44}$$

EXAMPLE 1.14

The following figure represents the electric potential as a function of x – coordinate. Plot the corresponding electric field as a function of x.



Solution

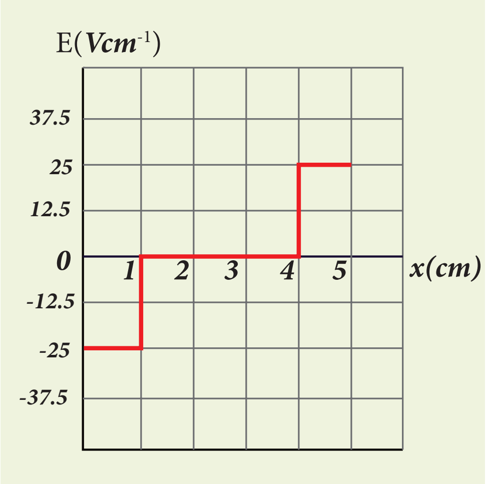
In the given problem, since the potential depends only on x, we can use $\vec{E} = -\frac{dV}{dx}\hat{i}$ (the other two terms $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$ are zero)

From 0 to 1 cm, the slope is constant and so $\frac{dV}{dx} = 25\,V\,cm^{-1}$. So $\vec{E} = -25\,V\,cm^{-1}\hat{i}$

From 1 to 4 cm, the potential is constant, V = 25 V. It implies that $\frac{dV}{dx} = 0$. So $\vec{E} = 0$

From 4 to 5 cm, the slope $\frac{dV}{dx} = -25\,V\,cm^{-1}$. So $\vec{E} = +25\,V\,cm^{-1}\hat{i}$.

The plot of electric field for the various points along the *x* axis is given below.



1.5.6 Electrostatic potential energy for collection of point charges

The electric potential at a point at a distance *r* from point charge q_1 is given by

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r}$$

This potential *V* is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance *r* from q_1, the work done is the product of q_2 and the electric potential at that point. Thus we have

$$W = q_2 V$$

This work done is stored as the electrostatic potential energy U of a system of charges q_1 and q_2 separated by a distance r. Thus we have

$$U = q_2 V = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r} \qquad (1.45)$$

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression (1.45) is derived by assuming that q_1 is fixed and q_2 is brought from infinity. The equation (1.45) holds true when q_2 is fixed and q_1 is brought from infinity or both q_1 and q_2 are simultaneously brought from infinity to a distance r between them.

Three charges are arranged in the following configuration as shown in Figure 1.30.

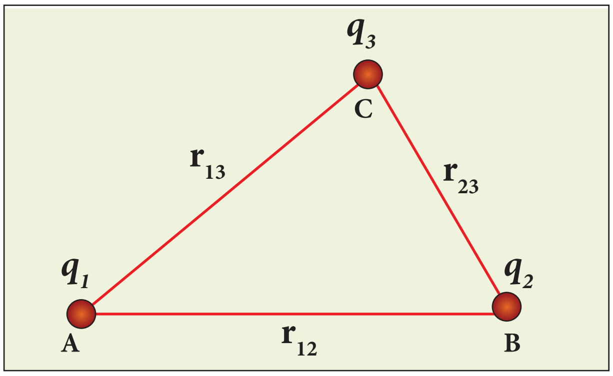


Figure 1.30 Electrostatic potential energy for Collection of point charges

To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown in Figure 1.30.

(i) Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1.

(ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1. So the work done on the charge q_2 is $W = q_2 V_{1B.}$ Here V_{1B} is the electrostatic potential due to the charge q_1 at point B.

$$U = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r_{12}} \qquad (1.46)$$

Note that the expression is same when q_2 is brought first and then q_1 later.

(iii) Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is $= q_3\,(V_{1C} + V_{2C})$. Here V_{1C} is the electrostatic potential due to charge q_1 at point C and V_{2C} is the electrostatic potential due to charge q_2 at point C.

The electrostatic potential is

$$U = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad (1.47)$$

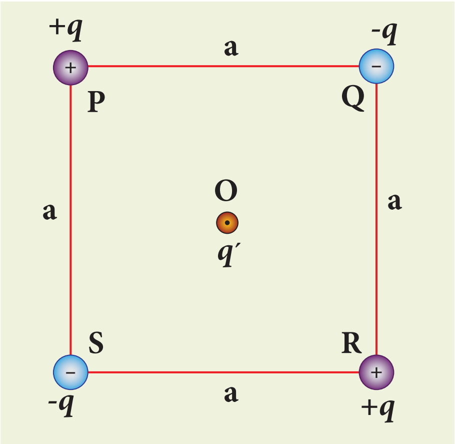
(iv) Adding equations (1.46) and (1.47), the total electrostatic potential energy for the system of three charges $q_{1,}$ q_2 and q_3 is

$$U = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad (1.48)$$

Note that this stored potential energy U is equal to the total external work done to assemble the three charges at the given locations. The expression (1.48) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

EXAMPLE 1.15

Four charges are arranged at the corners of the square *PQRS* of side *a* as shown in the figure.(a) Find the work required to assemble these charges in the given configuration. (b) Suppose a charge q′ is brought to the center of the square, by keeping the four charges fixed at the corners, how much extra work is required for this?



Solution

(a) The work done to arrange the charges in the corners of the square is independent of the way they are arranged. We can follow any order.

(i) First, the charge +q is brought to the corner P. This requires no work since no charge is already present, $W_P = 0$

(ii) Work required to bring the charge –q to the corner Q = (-q) × potential at a point Q due to +q located at a point P.

$$W_Q = -q \times \frac{1}{4\pi\varepsilon_\circ}\frac{q}{a} = -\frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{a}$$

(iii) Work required to bring the charge +q to the corner R= q × potential at the point R due to charges at the point P and Q.

$$W_R = q \times \frac{1}{4\pi\varepsilon_\circ}\left(-\frac{q}{a}+\frac{q}{\sqrt{2}a}\right)$$

$$= \frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{a}\left(-1+\frac{1}{\sqrt{2}}\right)$$

(iv) Work required to bring the fourth charge –q at the position S = q × potential at the point S due the all the three charges at the point P, Q and R

$$W_S = -q \times \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q}{a}+\frac{q}{a}-\frac{q}{\sqrt{2}a}\right)$$

$$W_S = -\frac{1}{4\pi\varepsilon_\circ}\frac{q}{a}\left(2-\frac{1}{\sqrt{2}}\right)$$

(b) Work required to bring the charge q′ to the center of the square = q′ × potential at the center point O due to all the four charges in the four corners

The potential created by the two +*q* charges are canceled by the potential created by the -*q* charges which are located in the opposite corners. Therefore the net electric potential at the center O due to all the charges in the corners is zero.

Hence no work is required to bring any charge to the point O. Physically this implies that if any charge *q*′ when brought close to O, then it moves to the point O without any external force.

1.5.7 Electrostatic potential energy of a dipole in a uniform electric field

Consider a dipole placed in the uniform electric field $\vec{E}$ as shown in the Figure 1.31. A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the

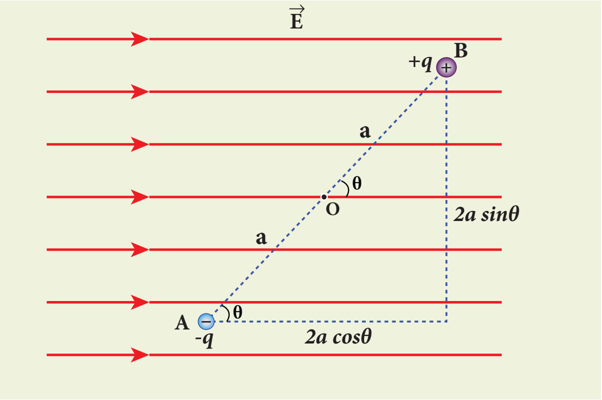


Figure 1.31 The dipole in a uniform electric field

direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle θ' to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

The work done by the external torque to rotate the dipole from angle θ' to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext} d\theta \qquad (1.49)$$

Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have

$$|\vec{\tau}_{ext}| = |\vec{\tau}_E| = |\vec{p} \times \vec{E}| \qquad (1.50)$$

Substituting equation (1.50) in equation (1.49), we get

$$W = \int_{\theta'}^{\theta} pE\sin\theta\, d\theta$$

$$W = pE(\cos\theta' - \cos\theta)$$

This work done is equal to the potential energy difference between the angular positions θ and θ'.

$$U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$$

If the initial angle is $\theta' = 90^\circ$ and is taken as reference point, then $U(\theta') = pE\cos 90^\circ = 0$.

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E} \qquad (1.51)$$

In addition to p and E, the potential energy also depends on the orientation θ of the electric dipole with respect to the external electric field.

The potential energy is maximum when the dipole is aligned anti-parallel ($\theta = \pi$) to the external electric field and minimum when the dipole is aligned parallel ($\theta = 0$) to the external electric field.

EXAMPLE 1.16

A water molecule has an electric dipole moment of 6.3×10^{-30} Cm. A sample contains 10^{22} water molecules, with all the dipole moments aligned parallel to the external electric field of magnitude 3×10^5 N C^{-1}. How much work is required to rotate all the water molecules from $\theta = 0^\circ$ to 90°?

Solution

When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from $\theta = 0^\circ$ to 90° is equal to the potential energy difference between these two configurations.

$$W = \Delta U = U(90^\circ) - U(0^\circ)$$

From the equation (1.51), we write $U = -pE\cos\theta$, Next we calculate the work done to rotate one water molecule from $\theta = 0^\circ$ to 90°.

For one water molecule

$$W = -pE\cos 90^\circ + pE\cos 0^\circ = pE$$

$$W = 6.3 \times 10^{-30} \times 3 \times 10^{5} = 18.9 \times 10^{-25} J$$

For 10^{22} water molecules, the total work done is

$$W_{tot} = 18.9 \times 10^{-25} \times 10^{22} = 18.9 \times 10^{-3} J$$

1.6 GAUSS LAW AND ITS APPLICATIONS

1.6.1 Electric Flux

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter Φ_E and its unit is N m^2 C^{-1}. Electric flux is a scalar quantity and it can be positive or negative. For a simpler understanding of electric flux, the following Figure 1.32 is useful.

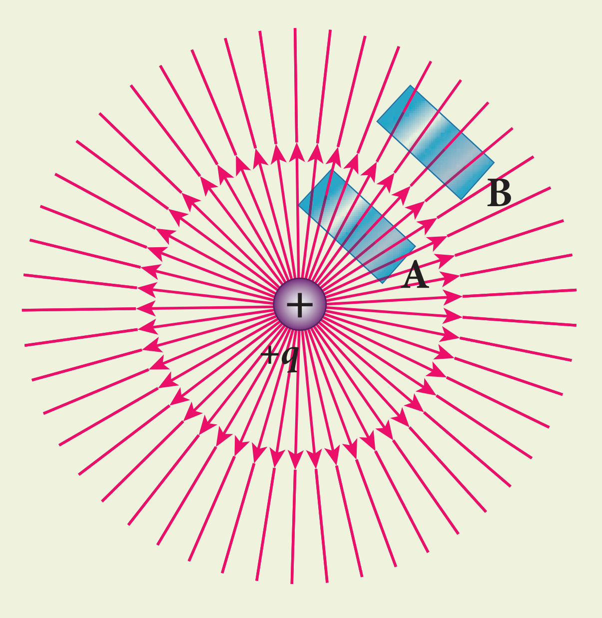


Figure 1.32 Electric flux

The electric field of a point charge is drawn in this figure. Consider two small rectangular area elements placed normal to the field at regions A and B. Even though these elements have the same area, the number of electric field lines crossing the element in region A is more than that crossing the element in region B. Therfore the electric flux in region A is more than that in region B. The electric field strength for a point charge decreases as the distance increases, then for a point charge electric flux also decreases as the distance increases. The above discussion gives a qualitative idea of electric flux. However a precise definition of electric flux is needed.

Electric flux for uniform Electric field

Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown in Figure 1.33 (a). The electric flux for this case is

$$\Phi_E = EA \quad (1.52)$$

Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pierce through the area A , as shown in Figure 1.33(b). The electric flux for this case is zero.

$$\Phi_E = 0 \quad (1.53)$$

If the area is inclined at an angle θ with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. This is shown in Figure 1.33 (c). For this case, the electric flux

$$\Phi_E = (E \cos\theta)\, A \quad (1.54)$$

Further, θ is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is defined as

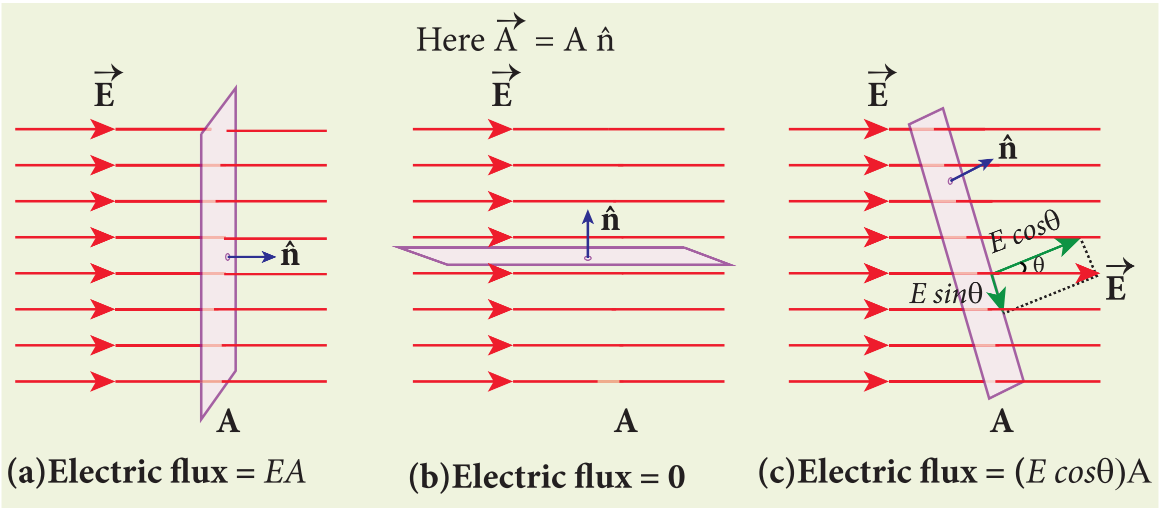


Figure 1.33 The electric flux for Uniform electric field

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta \qquad (1.55)$$

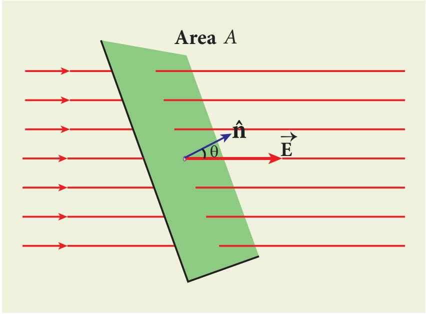
Here, note that $\vec{A}$ is the area vector $\vec{A} = A\hat{n}$. Its magnitude is simply the area A and the direction is along the unit vector $\hat{n}$ perpendicular to the area as shown in Figure 1.33. Using this definition for flux, $\Phi_E = \vec{E}\cdot\vec{A}$, equations (1.53) and (1.54) can be obtained as special cases.

In Figure 1.33 (a), θ = 0° so $\Phi_E = \vec{E}\cdot\vec{A} = EA$

In Figure 1.33 (b), θ = 90° so $\Phi_E = \vec{E}\cdot\vec{A} = 0$

EXAMPLE 1.17

Calculate the electric flux through the rectangle of sides 5 cm and 10 cm kept in the region of a uniform electric field 100 NC^{-1}. The angle θ is 60°. Suppose θ becomes zero, what is the electric flux?



Solution

The electric flux

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta = 100\times5\times10\times10^{-4}\times\cos 60^{\circ}$$

$$\Rightarrow \qquad \Phi_E = 0.25\,N.m^2C^{-1}$$

For θ = 0°,

$$\Phi_E = \vec{E}\cdot\vec{A} = EA = 100\times5\times10\times10^{-4} = 0.5\,N.m^2C^{-1}$$

Electric flux in a non uniform electric field and an arbitrarily shaped area

Suppose the electric field is not uniform and the area A is not flat (Figure 1.34), then the entire area is divided into n small area segments $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3 ....\Delta\vec{A}_n$ such that each area element is almost flat and the electric field over each area element is considered to be uniform.

The electric flux for the entire area A is approximately written as

$$\Phi_E = \vec{E}_1\cdot\Delta\vec{A}_1 + \vec{E}_2\cdot\Delta\vec{A}_2 + \vec{E}_3\cdot\Delta\vec{A}_3 ....\vec{E}_n\cdot\Delta\vec{A}_n$$
$$= \sum_{i=1}^{n}\vec{E}_i\cdot\Delta\vec{A}_i \qquad (1.56)$$

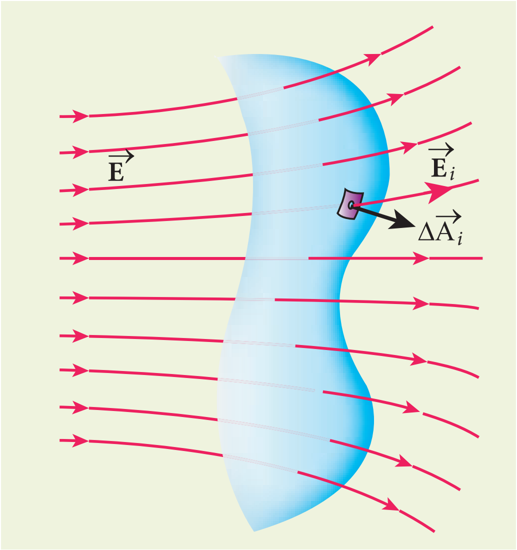


Figure 1.34 Electric flux for non-uniform electric Field

By taking the limit $\Delta\vec{A}_i \rightarrow 0$ (for all i) the summation in equation (1.56) becomes integration. The total electric flux for the entire area is given by

$$\Phi_E = \int \vec{E} \cdot d\vec{A} \qquad (1.57)$$

From Equation (1.57), it is clear that the electric flux for a given surface depends on both the electric field pattern on the surface area and orientation of the surface with respect to the electric field.

1.6.2 Electric flux for closed surfaces

In the previous section, the electric flux for any arbitrary curved surface is discussed. Suppose a closed surface is present in the region of the non-uniform electric field as shown in Figure 1.35 (a).

The total electric flux over this closed surface is written as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \qquad (1.58)$$

Note the difference between equations (1.57) and (1.58). The integration in equation

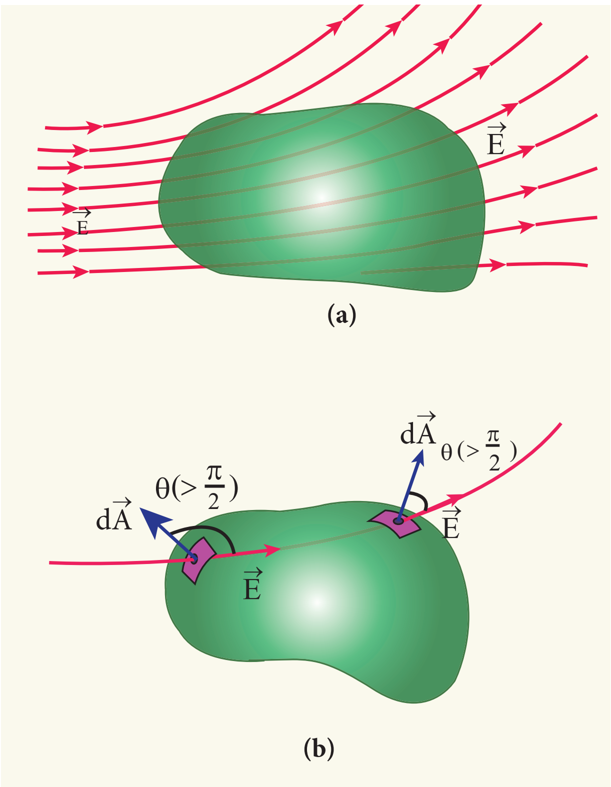


Figure 1.35 Electric flux over a closed surface

(1.58) is a closed surface integration and for each areal element, the outward normal is the direction of $d\vec{A}$ as shown in the Figure 1.35(b).

The total electric flux over a closed surface can be negative, positive or zero. In the Figure 1.35(b), it is shown that in one area element, the angle between $d\vec{A}$ and $\vec{E}$ is less than 90°, then the electric flux is positive and in another areal element, the angle between $d\vec{A}$ and $\vec{E}$ is greater than 90°, then the electric flux is negative.

In general, the electric flux is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

1.6.3 Gauss law

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure 1.36. We can calculate the total electric flux through the closed surface of the sphere using the equation (1.58).

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\,dA\cos\theta$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and $\theta = 0°$.

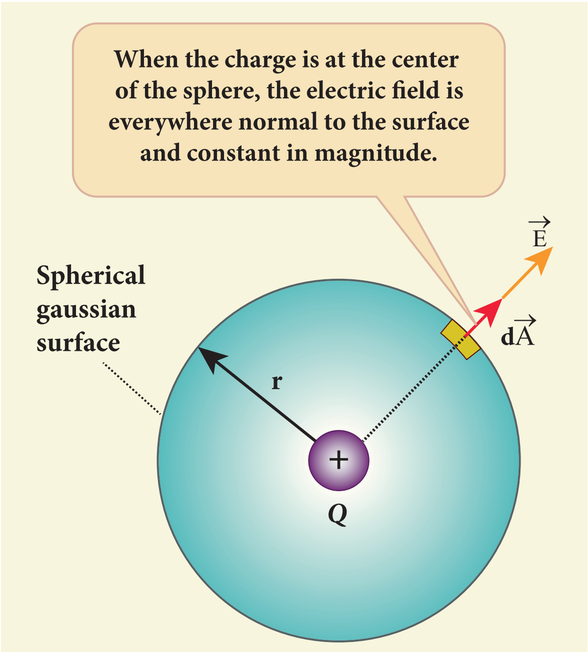


Figure 1.36 Total electric flux of point charge

$$\Phi_E = \oint E\,dA \qquad \text{since } \cos 0^0 = 1 \qquad (1.59)$$

E is uniform on the surface of the sphere,

$$\Phi_E = E\oint dA \qquad (1.60)$$

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ in equation 1.60, we get

$$\Phi_E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi\frac{1}{4\pi\varepsilon_\circ}Q$$

$$\Phi_E = \frac{Q}{\varepsilon_\circ} \qquad (1.61)$$

The equation (1.61) is called as Gauss's law. The remarkable point about this result is that the equation (1.61) is equally true for any arbitrary shaped surface which encloses the charge Q and as shown in the Figure 1.37. It is seen that the total electric flux is the same for closed surfaces A_1, A_2 and A_3 as shown in the Figure 1.37.

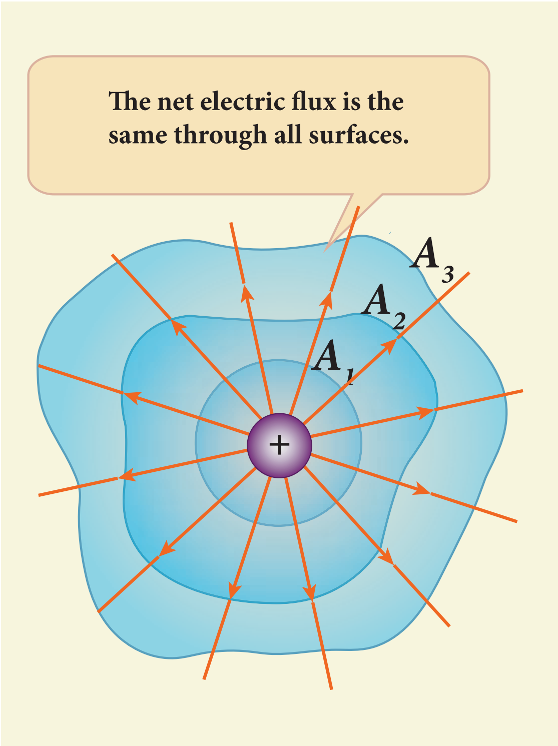


Figure 1.37 Gauss law for arbitrarily shaped surface

Gauss's law states that **if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is**

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_\circ} \qquad (1.62)$$

Here Q_{encl} denotes the charges inside the closed surface.

Discussion of Gauss law

(i) The total electric flux through the closed surface depends only on the charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

(ii) The total electric flux is independent of the location of the charges inside the closed surface.

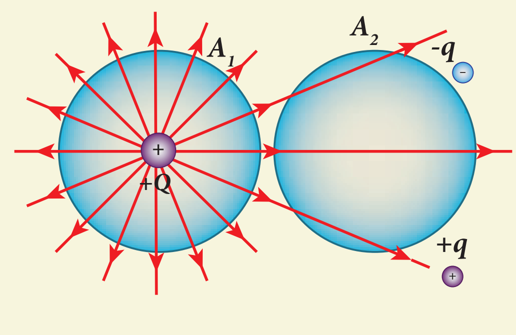
(iii) To arrive at equation (1.62), we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge, therefore spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation (1.62), the electric field $\vec{E}$ is due to charges present inside and outside the Gaussian surface but the charge Q_{encl} denotes the charges which lie only inside the Gaussian surface.

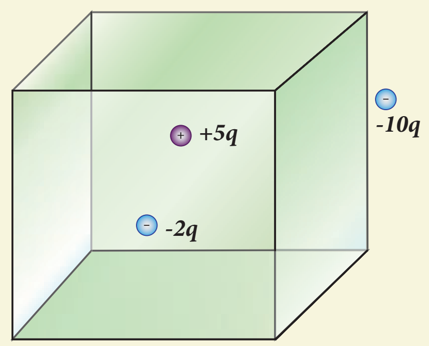
(v) The Gaussian surface cannot pass through any discrete charge but it can pass through continuous charge distributions. It is because, very close to the discrete charges, the electric field is not well defined.

(vi) Gauss law is another form of Coulomb's law and it is also applicable to the charges in motion. Because of this reason, Gauss law is treated as much more general law than Coulomb's law.

EXAMPLE 1.18



(a)



(b)

(i) In figure (a), calculate the electric flux through the closed areas A_1 and A_2.

(ii) In figure (b), calculate the electric flux through the cube

Solution

(i) In figure (a), the area A_1 encloses the charge Q. So electric flux through this closed surface A_1 is $\frac{Q}{\varepsilon_\circ}$. But the closed surface A_2 contains no charges inside, so electric flux through A_2 is zero.

(ii) In figure (b), the net charge inside the cube is 3q and the total electric flux in the cube is therefore $\Phi_E = \frac{3q}{\varepsilon_\circ}$.

Note that the charge -10 q lies outside the cube and it will not contribute the total flux through the surface of the cube.

1.6.4 Applications of Gauss law

Electric field due to any arbitrary charge configuration can be calculated using Coulomb's law or Gauss law. If the charge configuration possesses some kind of symmetry, then Gauss law is a very efficient way to calculate the electric field. It is illustrated in the following cases.

(i) Electric field due to an infinitely long charged wire

Consider an infinitely long straight wire having uniform linear charge density λ. Let P be a point located at a perpendicular distance r from the wire (Figure 1.38(a)).

The electric field at the point P can be found using Gauss law. We choose two small charge elements A_1 and A_2 on the wire which are at equal distances from the point P. The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r. This is shown in the Figure 1.38(b). From this property, we can infer that the charged wire possesses a cylindrical symmetry.

Let us choose a cylindrical Gaussian surface of radius r and length L as shown in the Figure 1.39.

The total electric flux in this closed surface is calculated as follows.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$
$$= \int_{\substack{Curved\\surface}} \vec{E}\cdot d\vec{A} + \int_{\substack{top\\surface}} \vec{E}\cdot d\vec{A} + \int_{\substack{bottom\\surface}} \vec{E}\cdot d\vec{A} \quad (1.63)$$

It is seen from Figure (1.39) that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = E\,dA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = 0$

Substituting these values in the equation (1.63) and applying Gauss law to the cylindrical surface, we have

$$\Phi_E = \int_{\substack{Curved\\surface}} E\,dA = \frac{Q_{encl}}{\varepsilon_\circ} \quad (1.64)$$

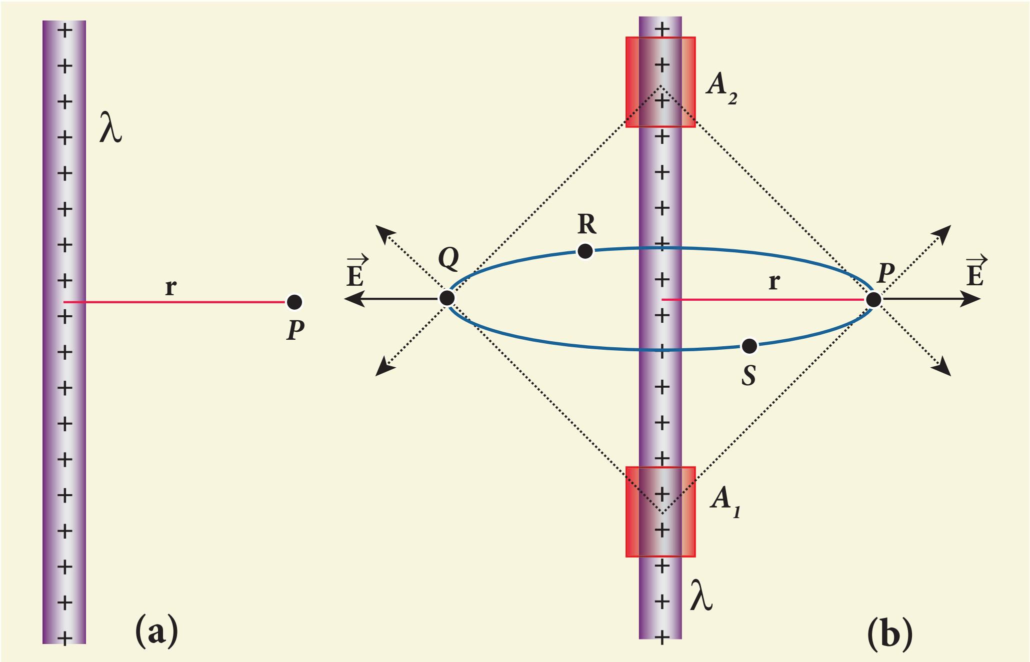


Figure 1.38 Electric field due to infinite long charged wire

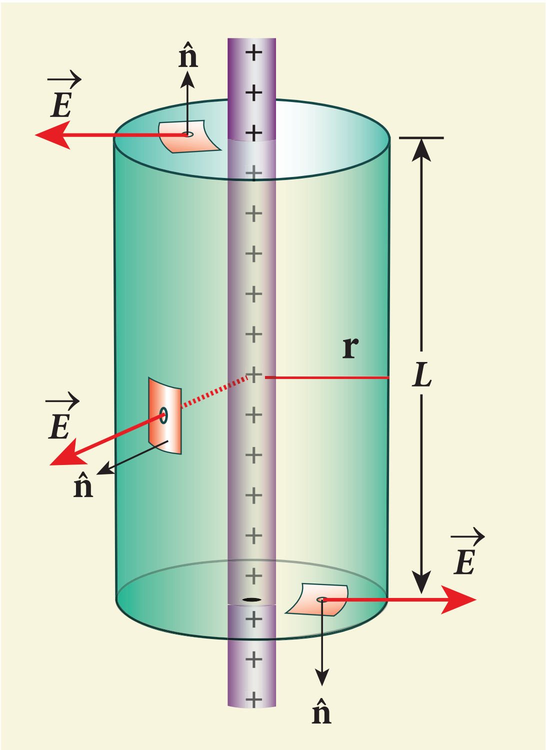


Figure 1.39 Cylindrical Gaussian surface

Since the magnitude of the electric field for the entire curved surface is constant, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \lambda L$.

$$E \int_{\substack{Curved \\ surface}} dA = \frac{\lambda L}{\varepsilon_\circ} \qquad (1.65)$$

Here $\Phi_E = \int_{\substack{Curved \\ surface}} dA =$ total area of the curved surface = 2πrL. Substituting this in equation (1.65), we get

$$E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_\circ}$$

$$E = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r} \qquad (1.66)$$

In vector form $\vec{E} = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r}\hat{r}$ (1.67)

The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ for a point charge.

Equation (1.67) indicates that the electric field is always along the perpendicular direction ($\hat{r}$) to wire. In fact, if λ > 0 then $\vec{E}$ points perpendicular outward ($\hat{r}$) from the wire and if λ < 0, then $\vec{E}$ points perpendicular inward $(-\hat{r})$.

The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

(ii) Electric field due to charged infinite plane sheet

Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet as shown in the Figure 1.40.

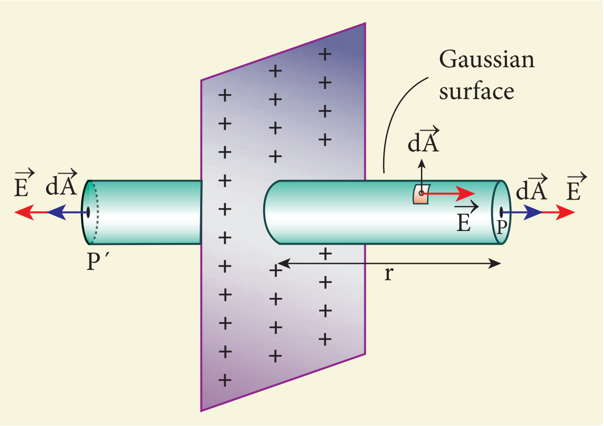


Figure 1.40 Electric field due to charged infinite planar sheet

Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points. A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.

Applying Gauss law for this cylindrical surface,

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$
$$= \int_{\substack{Curved\\surface}} \vec{E}\cdot d\vec{A} + \int_P \vec{E}\cdot d\vec{A} + \int_{P'} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad (1.68)$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P' (Figure 1.40). Then,

$$\Phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0} \quad (1.69)$$

Since the magnitude of the electric field at these two equal surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E\int_P dA = \frac{\sigma A}{\varepsilon_0}$$

The total area of surface either at P or P′

$$\int_P dA = A$$

Hence $2EA = \dfrac{\sigma A}{\varepsilon_0}$ or $E = \dfrac{\sigma}{2\varepsilon_0}$ (1.70)

In vector form, $\vec{E} = \dfrac{\sigma}{2\varepsilon_\circ}\hat{n}$ (1.71)

Here $\hat{n}$ is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance r.

The electric field will be the same at any point farther away from the charged plane. Equation (1.71) implies that if $\sigma > 0$ the electric field at any point P is outward perpendicular $\hat{n}$ to the plane and if $\sigma < 0$ the electric field points inward perpendicularly $(-\hat{n})$ to the plane.

For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

(iii) Electric field due to two parallel charged infinite sheets

Consider two infinitely large charged plane sheets with equal and opposite charge densities $+\sigma$ and $-\sigma$ which are placed parallel to each other as shown in the Figure 1.41.

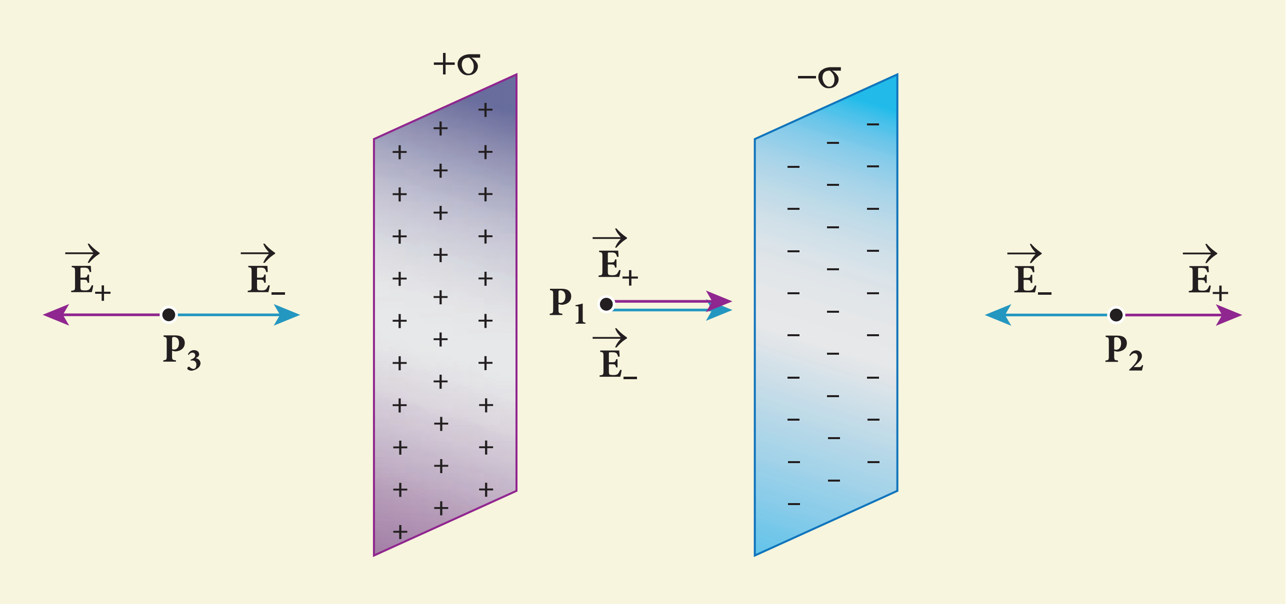


Figure 1.41 Electric field due to two parallel charged sheets

The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_{\circ}}$ and it points perpendicularly outward if $\sigma > 0$ and points inward if $\sigma < 0$.

At the points P_2 and P_3, the electric field due to both plates are equal in magnitude and opposite in direction (Figure 1.41). As a result, electric field at a point outside the plates is zero. But inside the plate, electric fields are in same direction i.e., towards the right, the total electric field at a point P_1

$$E_{inside} = \frac{\sigma}{2\varepsilon_{\circ}} + \frac{\sigma}{2\varepsilon_{\circ}} = \frac{\sigma}{\varepsilon_{\circ}} \tag{1.72}$$

The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plate.

(iv) Electric field due to a uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius R and total charge Q as shown in Figure 1.42. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell (r > R)

Let us choose a point P outside the shell at a distance r from the center as shown in Figure 1.42 (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0. So we choose a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q. Applying Gauss law

$$\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_{\circ}} \tag{1.73}$$

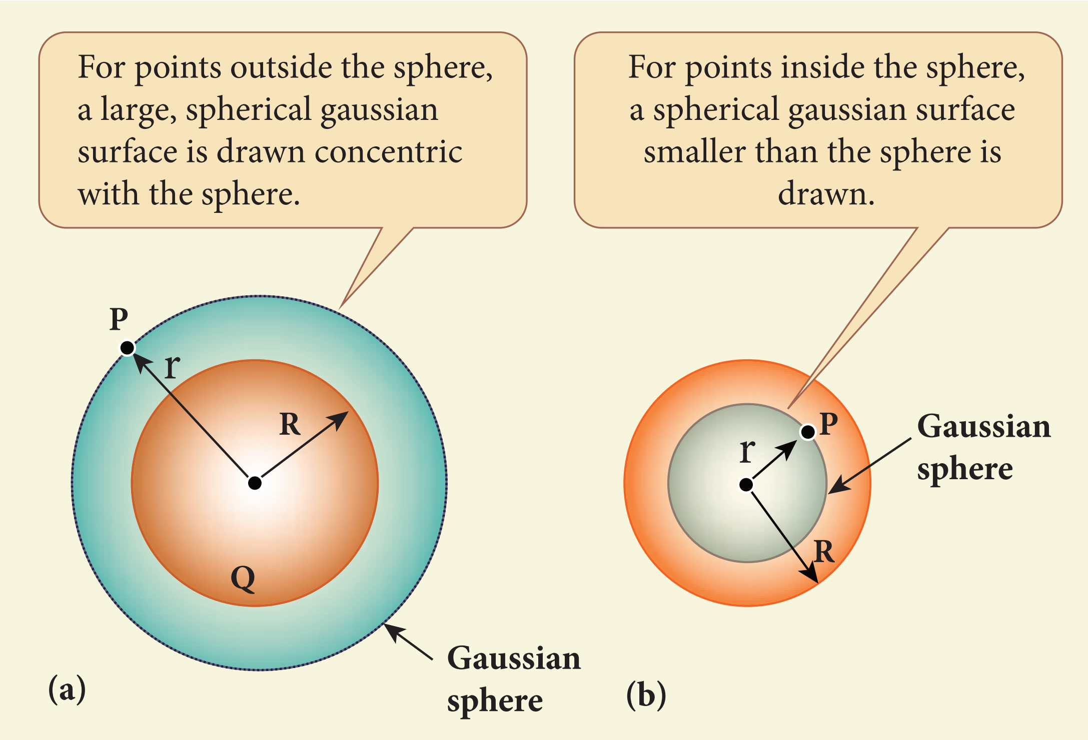


Figure 1.42 The electric field due to a charged spherical shell

The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E \oint_{\text{Gaussian surface}} dA = \frac{Q}{\varepsilon_{\circ}} \quad (1.74)$$

But $\oint_{\text{Gaussian surface}} dA =$ total area of Gaussian surface $= 4\pi r^2$. Substituting this value in equation (1.74)

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_{\circ}}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_{\circ}} \quad \text{(or)} \quad E = \frac{1}{4\pi\varepsilon_{\circ}} \frac{Q}{r^2}$$

In vector form $$\vec{E} = \frac{1}{4\pi\varepsilon_{\circ}} \frac{Q}{r^2}\hat{r} \quad (1.75)$$

The electric field is radially outward if Q > 0 and radially inward if Q < 0. From equation (1.75), we infer that the electric field at a point outside the shell will be same as if the entire charge Q is concentrated at the center of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell (r = R)

The electrical field at points on the spherical shell (r = R) is given by

$$\vec{E} = \frac{Q}{4\pi\varepsilon_{\circ} R^2}\hat{r} \quad (1.76)$$

Case (c) At a point inside the spherical shell (r < R)

Consider a point P inside the shell at a distance r from the center. A Gaussian sphere of radius r is constructed as shown in the Figure 1.42 (b). Applying Gauss law

$$\oint_{\text{Gaussian surface}} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_{\circ}}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_{\circ}} \quad (1.77)$$

Since Gaussian surface encloses no charge, So Q = 0. The equation (1.77) becomes

$$E = 0 \qquad (r < R) \qquad (1.78)$$

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.

A graph is plotted between the electric field and radial distance. This is shown in Figure 1.43.

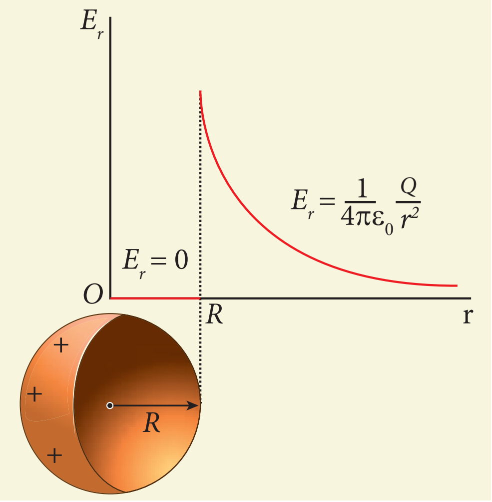


Figure 1.43 Electric field versus distance for a spherical shell of radius R

Note Gauss law is a powerful technique whenever a given charge configuration possesses spherical, cylindrical or planer symmetry, then the electric field due to such a charge configuration can be easily found. If there is no such symmetry, the direct method (Coulomb's law and calculus) can be used. For example, it is difficult to use Gauss law to find the electric field for a dipole since it has no spherical, cylindrical or planar symmetry.

1.7 ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS

1.7.1 Conductors at electrostatic equilibrium

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown in Figure 1.44.

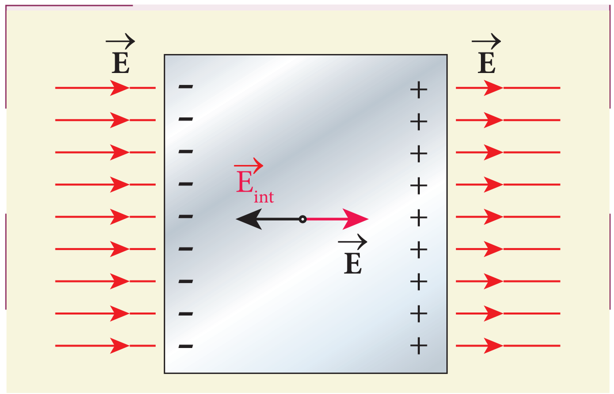


Figure 1.44 Electric field of conductors

Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown in Figure 1.44.

Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of 10^{-16}s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.45.

A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside

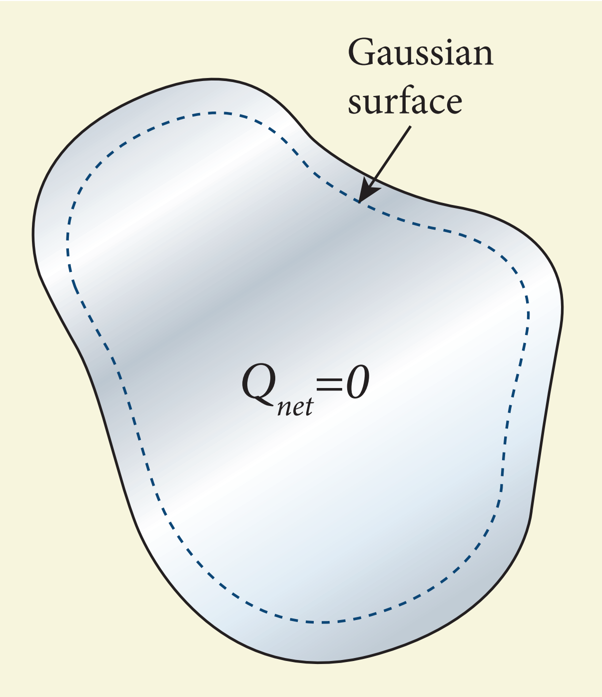


Figure 1.45 No net charge inside the conductor

the conductor, the net electric flux is also zero over this Gaussian surface. From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced inside the conductor, it immediately reaches the surface of the conductor.

(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_\circ}$ where σ is the surface charge density at that point.

If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure 1.46(a)). This means that the conductor is not in equilibrium. Therefore at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure 1.46 (b).

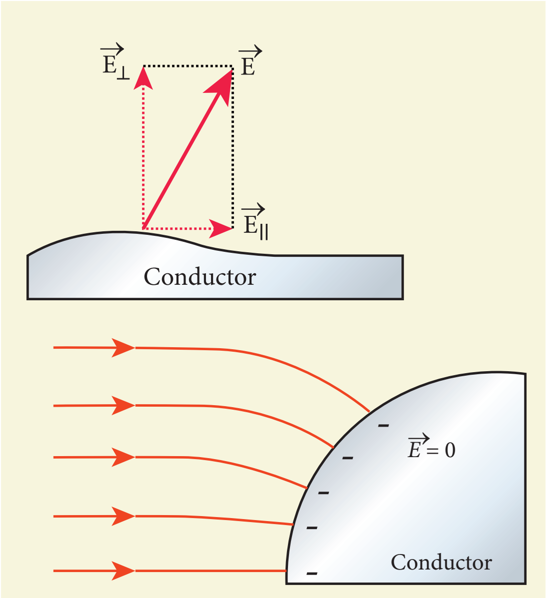


Figure 1.46 (a) Electric field is along the surface (b)Electric field is perpendicular to the surface of the conductor

We now prove that the electric field has magnitude $\frac{\sigma}{\varepsilon_\circ}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the Figure 1.47. One half of this cylinder is embedded inside the conductor.

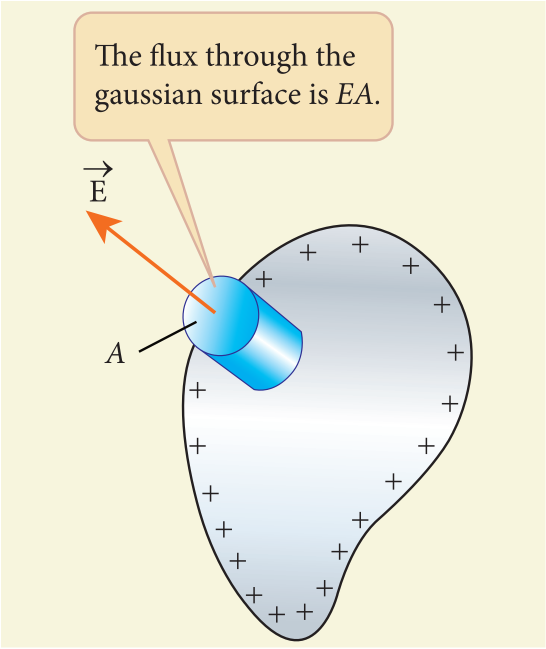


Figure 1.47 The electric field on the surface of the conductor

Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux.

Therefore the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is σA. By applying Gaus's law,

$$EA = \frac{\sigma A}{\varepsilon_\circ}$$

In vector form, $$\vec{E} = \frac{\sigma}{\varepsilon_\circ}\hat{n} \qquad (1.79)$$

Here $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.

Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

1.7.2 Electrostatic shielding

Using Gauss law, we proved that the electric field inside the charged spherical shell is zero, Further, we showed that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.

Consider a cavity inside the conductor as shown in Figure 1.48 (a). Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity. This is called electrostatic shielding.

Faraday cage is an instrument used to demonstrate this effect. It is made up of metal bars configured as shown in Figure1.48 (b). If an artificial lightning jolt is created outside, the person inside is not affected.

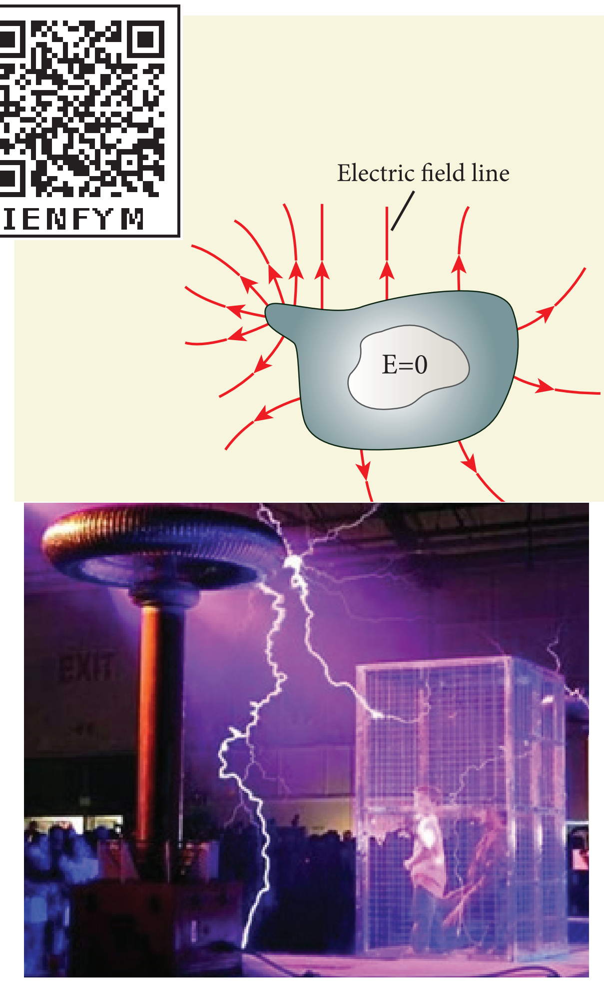


Figure 1.48 (a) Electric field inside the cavity (b) Faraday cage

During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

1.7.3 Electrostatic induction

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of **charging without actual contact is called electrostatic induction.**

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure 1.49(a).

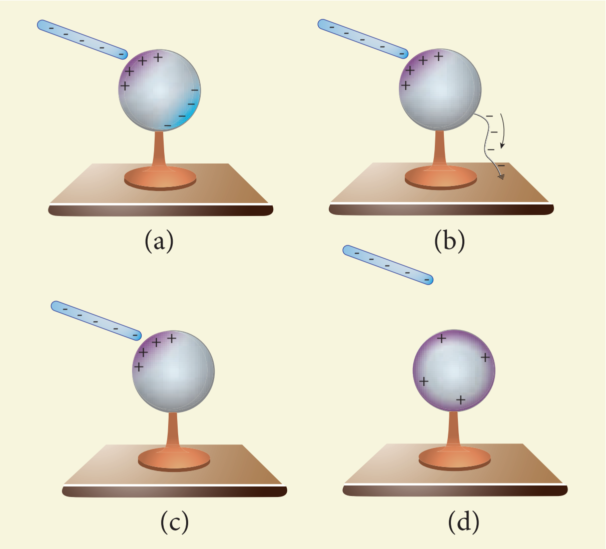


Figure 1.49 Various steps in electrostatic induction

The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side.

Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure 1.49(b)).

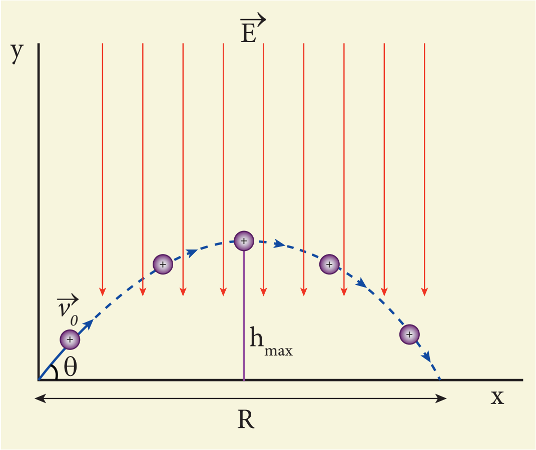
(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure 1.49(c))

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure 1.49 (d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section 1.9

EXAMPLE 1.19

A small ball of conducting material having a charge +q and mass m is thrown upward at an angle θ to horizontal surface with an initial speed v_o as shown in the figure. There exists an uniform electric field E downward along with the gravitational field g. Calculate the range, maximum height and time of flight in the motion of this charged ball. Neglect the effect of air and treat the ball as a point mass.



Solution

If the conductor has no net charge, then its motion is the same as usual projectile motion of a mass m which we studied in Kinematics (unit 2, vol-1 XI physics). Here, in this problem, in addition to downward gravitational force, the charge also will experience a downward uniform electrostatic force.

The acceleration of the charged ball due to gravity = $-g\hat{j}$

The acceleration of the charged ball due to uniform electric field = $-\frac{qE}{m}\hat{j}$

The total acceleration of charged ball in downward direction $\vec{a} = -\left(g + \frac{qE}{m}\right)\hat{j}$

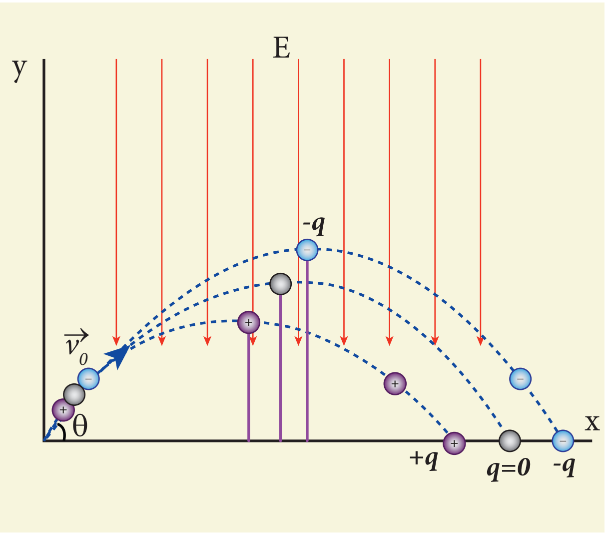
It is important here to note that the acceleration depends on the mass of the object. Galileo's conclusion that all objects fall at the same rate towards the Earth is true only in a uniform gravitational field. When a uniform electric field is included, the acceleration of a charged object depends on both mass and charge.

But still the acceleration $a = \left(g + \frac{qE}{m}\right)$ is constant throughout the motion. Hence we use kinematic equations to calculate the range, maximum height and time of flight. In fact we can simply replace g by $g + \frac{qE}{m}$ in the usual expressions of range, maximum height and time of flight of a projectile.

	Without charge	With the charge +q
Time of flight T	$\frac{2v_o \sin\theta}{g}$	$\frac{2v_o \sin\theta}{\left(g + \frac{qE}{m}\right)}$
Maximum height h_{max}	$\frac{v_o^2 \sin^2\theta}{2g}$	$\frac{v_o^2 \sin^2\theta}{2\left(g + \frac{qE}{m}\right)}$
Range R	$\frac{v_o^2 \sin 2\theta}{g}$	$\frac{v_o^2 \sin 2\theta}{\left(g + \frac{qE}{m}\right)}$

Note that the time of flight, maximum height, range are all inversely proportional to the acceleration of the object. Since $\left(g + \frac{qE}{m}\right) > g$ for charge +q, the quantities T, h_{max}, and R will decrease when compared to the motion of an object of mass m and zero net charge. Suppose the charge is –q, then $\left(g - \frac{qE}{m}\right) < g$, and the quantities T, h_{max} and

R will increase. Interestingly the trajectory is still parabolic as shown in the figure.



1.7.4 Dielectrics or insulators

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms. Ebonite, glass and mica are some examples of dielectrics. When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way. A dielectric is made up of either polar molecules or non-polar molecules.

Non-polar molecules

A non-polar molecule is one in which centers of positive and negative charges coincide. As a result, it has no permanent dipole moment. Examples of non-polar molecules are hydrogen (H_2), oxygen (O_2), and carbon dioxide (CO_2) etc.

When an external electric field is applied, the centers of positive and negative charges are separated by a small distance which induces dipole moment in the direction of the external electric field. Then the dielectric is said to be polarized by an external electric field. This is shown in Figure 1.50.

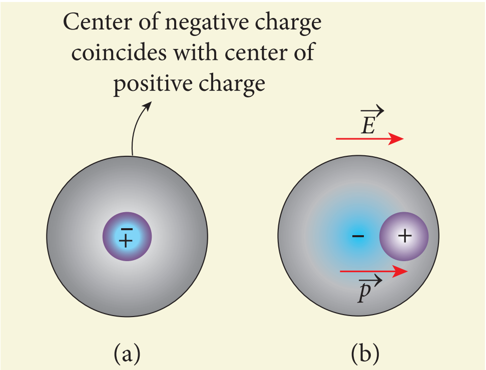


Figure 1.50 (a) Non polar molecules without external field (b) With the external field

Polar molecules

In polar molecules, the centers of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment. Due to thermal motion, the direction of each dipole moment is oriented randomly (Figure 1.51(a)). Hence the net dipole moment is zero in the absence of an external electric field. Examples of polar molecules are H_2O, N_2O, HCl, NH_3.

When an external electric field is applied, the dipoles inside the polar molecule tend to align in the direction of the electric field. Hence a net dipole moment is induced in it. Then the dielectric is said to be polarized by an external electric field (Figure 1.51(b)).

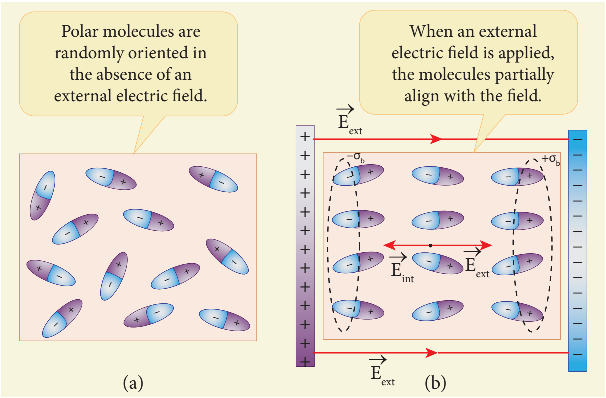


Figure 1.51 (a) Randomly oriented polar molecules (b) Align with the external electric field

Polarisation

In the presence of an external electric field, the dipole moment is induced in the dielectric material. **Polarisation $\vec{P}$ is defined as the total dipole moment per unit volume of the dielectric.** For most dielectrics (linear isotropic), the Polarisation is directly proportional to the strength of the external electric field. This is written as

$$\vec{P} = \chi_e \vec{E}_{ext} \tag{1.80}$$

where χ_e is a constant called the electric susceptibility which is a characteristic of each dielectric.

1.7.5 Induced Electric field inside the dielectric

When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field. Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the Figure 1.52(b).

The uniform electric field between the plates acts as an external electric field $\vec{E}_{ext}$ which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface.

But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the Figure 1.52(b).

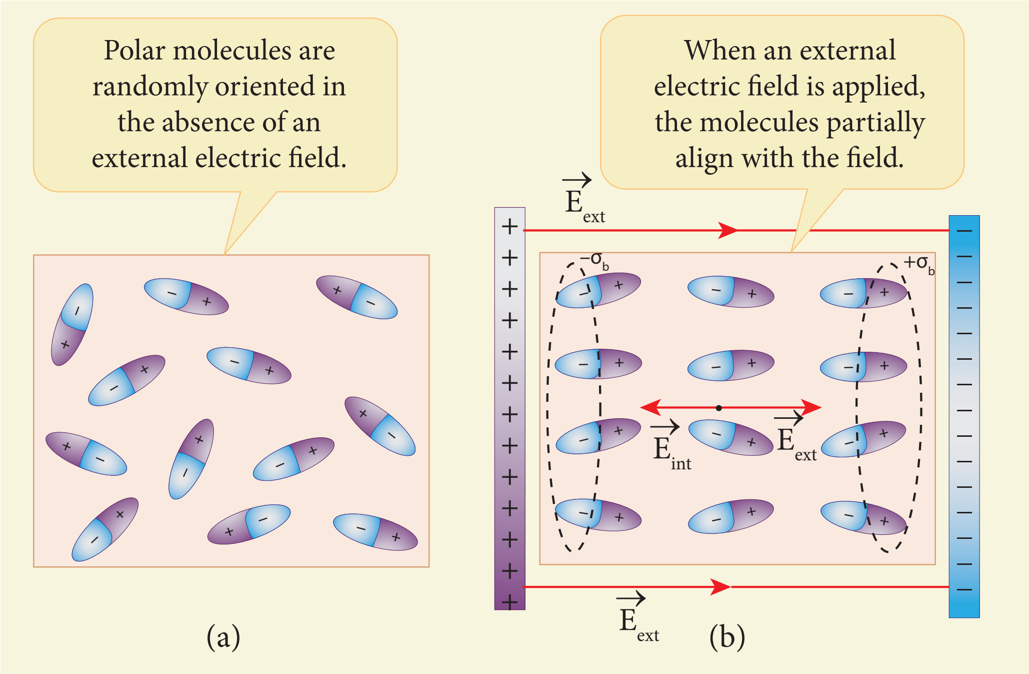


Figure 1.52 Induced electric field lines inside the dielectric

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown in Figure 1.53.

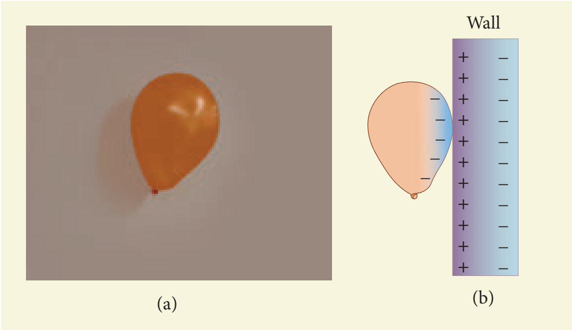


Figure 1.53 (a) Balloon sticks to the wall (b) Polarisation of wall due to the electric field created by the balloon

1.7.6 Dielectric strength

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength. For example, the dielectric strength of air is 3×10^6 V m^{-1}. If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in the Table 1.1.

Table 1.1 Dielectric strength

Substance	Dielectric strength (Vm^{-1})
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Air	3×10^6
Pyrex glass	14×10^6

1.8 CAPACITORS AND CAPACITANCE

1.8.1 Capacitors

Capacitor is a device used to store electric charge and electrical energy. It consists of two conducting objects (usually plates or sheets) separated by some distance. Capacitors are widely used in many electronic circuits and have applications in many areas of science and technology.

A simple capacitor consists of two parallel metal plates separated by a small distance as shown in Figure 1.54 (a).

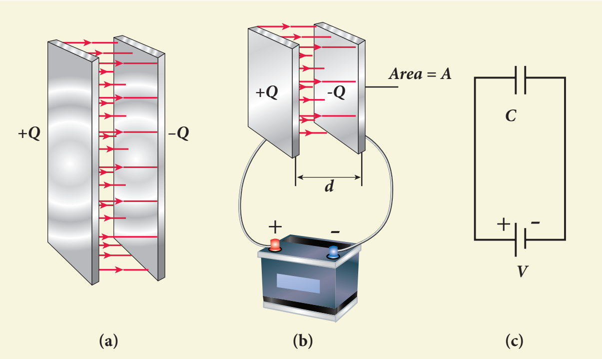


Figure 1.54 (a) Parallel plate capacitor (b) Capacitor connected with a battery (c) Symbolic representation of capacitor.

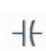
When a capacitor is connected to a battery of potential difference V, the electrons are transferred from one plate to the other plate by battery so that one plate becomes negatively charged with a charge of –Q and the other plate positively charged with +Q. The potential difference between the plates is equivalent to the battery's terminal voltage. This is shown in Figure 1.54(b). If the battery voltage is increased, the amount of charges stored in the plates also increase. In general, the charge stored in the capacitor is proportional to the potential difference between the plates.

$$Q \propto V$$

so that $Q = CV$

where the C is the proportionality constant called capacitance. **The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.**

$$C = \frac{Q}{V} \quad (1.81)$$

The SI unit of capacitance is *coulomb per volt* or *farad (F)* in honor of Michael Faraday. Farad is a very large unit of capacitance. In practice, capacitors are available in the range of microfarad ($1\mu F = 10^{-6}$ F) to picofarad ($1pf = 10^{-12}$ F). A capacitor is represented by the symbol  or  . Note that the total charge stored in the capacitor is zero (Q - Q = 0). When we say the capacitor stores charges, it means the amount of charge that can be stored in any one of the plates.

Nowadays there are capacitors available in various shapes (cylindrical, disk) and types (tantalum, ceramic and electrolytic), as shown in Figure 1.55. These capacitors are extensively used in various kinds of electronic circuits.

Figure 1.55 Various types of capacitors

Capacitance of a parallel plate capacitor

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown in Figure 1.56.

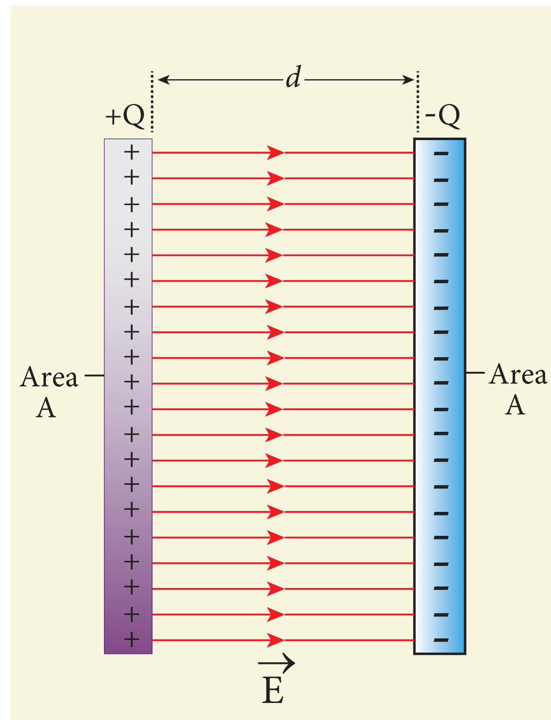


Figure 1.56 Capacitance of a parallel plate capacitor

The electric field between two infinite parallel plates is uniform and is given by $E = \frac{\sigma}{\varepsilon_\circ}$ where σ is the surface charge density on the plates $\left(\sigma = \frac{Q}{A}\right)$. If the separation distance d is very much smaller than the size of the plate ($d^2 << A$), then the above result is used even for finite–sized parallel plate capacitor.

The electric field between the plates is

$$E = \frac{Q}{A\varepsilon_\circ} \quad (1.82)$$

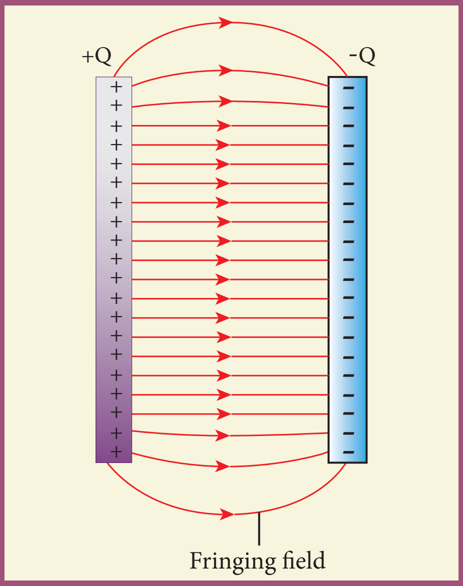
Since the electric field is uniform, the electric potential between the plates having separation d is given by

$$V = Ed = \frac{Qd}{A\varepsilon_0} \quad (1.83)$$

Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d} \quad (1.84)$$

Note While deriving an expression for capacitance of the parallel plate capacitor, the expression of the electric field for infinite plates is used. But for finite sized plates, the electric field is not strictly uniform between the plates. At both edges, the electric field is bent outwards as shown in the Figure.



This is called "fringing field". However under the condition ($d^2 << A$), this effect can be ignored.

From equation (1.84), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance d between the two plates is reduced, the potential difference between the plates ($V = Ed$) decreases with E constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery's terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

EXAMPLE 1.20

A parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm. (a) Calculate the capacitance of this capacitor. (b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates? (The value of $\varepsilon_o = 8.85 \times 10^{-12}\ Nm^2\ C^{-2}$)

Solution

(a) The capacitance of the capacitor is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$$
$$= 221.2\times10^{-13}\,F$$
$$C = 22.12\times10^{-12}\,F = 22.12\,pF$$

(b) The charge stored in any one of the plates is Q = CV, Then

$$Q = 22.12\times10^{-12}\times10 = 221.2\times10^{-12}\,C = 221.2\,pC$$

Sometimes we notice that the ceiling fan does not start rotating as soon as it is switched on. But when we rotate the blades, it starts to rotate as usual. Why it is so? We know that to rotate any object, there must be a torque applied on the object. For the ceiling fan, the initial torque is given by the capacitor widely known as a condenser. If the condenser is faulty, it will not give sufficient initial torque to rotate the blades when the fan is switched on.

1.8.2 Energy stored in the capacitor

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = V\,dQ \qquad (1.85)$$

where $V = \frac{Q}{C}$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C} \qquad (1.86)$$

This work done is stored as electrostatic potential energy (U_E) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (\because Q = CV) \qquad (1.87)$$

where Q = CV is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor. But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the results $C = \frac{\varepsilon_0 A}{d}$ and V = Ed

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 (Ad) E^2 \qquad (1.88)$$

where Ad = volume of the space between the capacitor plates. **The energy stored per unit volume of space is defined as energy density** $\boldsymbol{u_E = \frac{U}{Volume}}$ From equation (1.88), we get

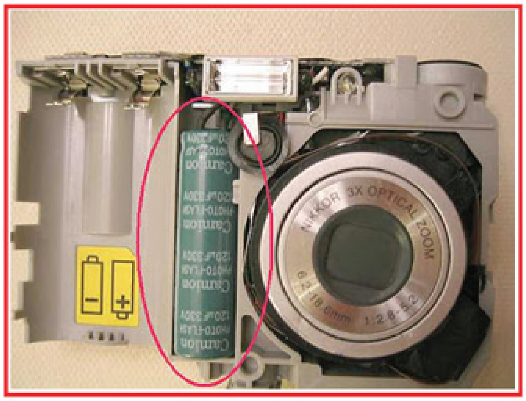
$$u_E = \frac{1}{2}\varepsilon_o E^2 \qquad (1.89)$$

From equation (1.89), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

1.8.3 Applications of capacitors

Capacitors are used in various electronics circuits. A few of the applications.


(a)


(b)

Figure 1.57 (a) Flash capacitor in camera (b) Heart defibrillator

(a) Most people are now familiar with the digital camera. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor (Figure 1.57 (a))

(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of 175 μF charged to a high voltage of around 2000 V. This is shown in Figure 1.57(b).

(c) Capacitors are used in the ignition system of automobile engines to eliminate sparking

(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

However, capacitors have disadvantage as well. Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. For example if the TV is switched off, it is always advisable to not touch the back side of the TV panel.

1.8.4 Effect of dielectrics in capacitors

In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) when the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0} \qquad (1.90)$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in Figure 1.58.

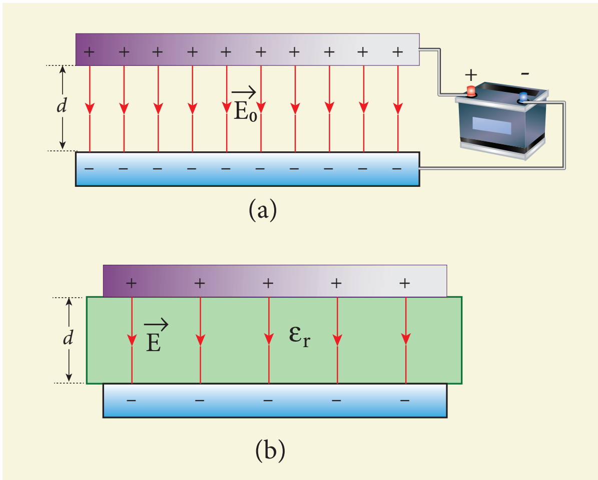


Figure 1.58 (a) Capacitor is charged with a battery (b) Dielectric is inserted after the battery is disconnected

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by

$$E = \frac{E_0}{\varepsilon_r} \quad (1.91)$$

Here E_o is the electric field inside the capacitors when there is no dielectric and ε_r is the relative permeability of the dielectric or simply known as the dielectric constant. Since $\varepsilon_r > 1$, the electric field $E < E_o$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected.

Hence the new potential difference is

$$V = Ed = \frac{E_0}{\varepsilon_r}d = \frac{V_0}{\varepsilon_r} \quad (1.92)$$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases.

Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = \varepsilon_r\frac{Q_0}{V_0} = \varepsilon_r C_0 \quad (1.93)$$

Since $\varepsilon_r > 1$, we have $C > C_o$. Thus insertion of the dielectric constant ε_r increases the capacitance.

Using equation (1.84),

$$C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{\varepsilon A}{d} \quad (1.94)$$

where $\varepsilon = \varepsilon_r \varepsilon_o$ is the permittivity of the dielectric medium.

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0} \quad (1.95)$$

After the dielectric is inserted, the charge Q_0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r} \quad (1.96)$$

Since $\varepsilon_r > 1$ we get $U < U_o$. There is a decrease in energy because,when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage V_0 remains connected to the capacitor when the dielectric is inserted into the capacitor. This is shown in Figure1.59.

The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.

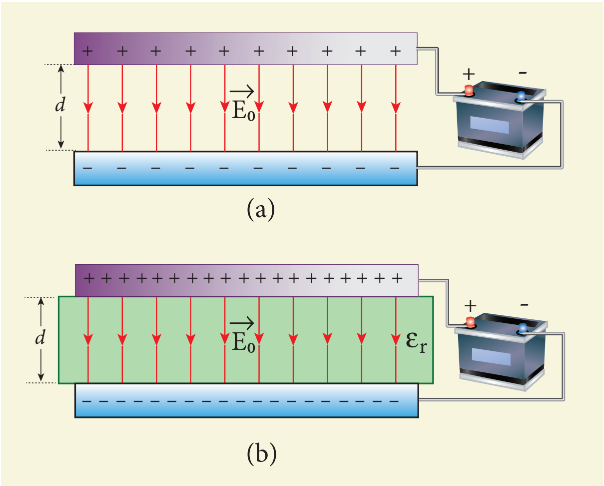


Figure 1.59 (a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

$$Q = \varepsilon_r Q_o \qquad (1.97)$$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \qquad (1.98)$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

$$\text{Now,} \quad C_0 = \frac{\varepsilon_0 A}{d}$$

$$\text{and} \quad C = \frac{\varepsilon A}{d} \qquad (1.99)$$

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2 \qquad (1.100)$$

Note that here we have not used the expression $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ because here, both charge and capacitance are changed, whereas in equation (1.100), V_o remains constant.

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2}\varepsilon_r C_0 V_0^2 = \varepsilon_r U_0 \qquad (1.101)$$

Since $\varepsilon_r > 1$ we have $U > U_o$.

It may be noted here that since voltage between the capacitor V_0 is constant, the electric field between the plates also remains constant.

The energy density is given by

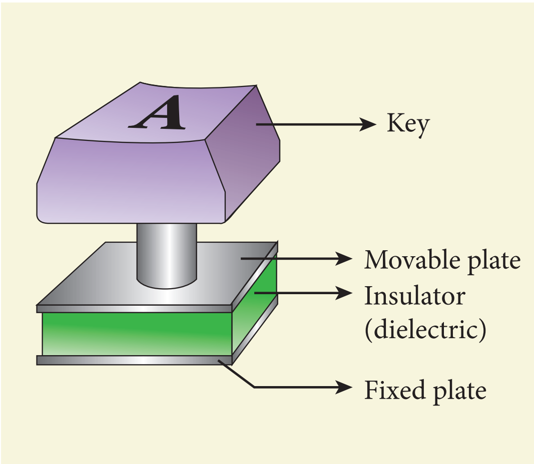
$$u = \frac{1}{2}\varepsilon E_0^2 \qquad (1.102)$$

where ε is the permittivity of the given dielectric material.

The results of the above discussions are summarised in the following Table 1.2

DO YOU KNOW?

Computer keyboard keys are constructed using capacitors with a dielectric as shown in the figure.



When the key is pressed, the separation between the plates decreases leading to an increase in the capacitance. This in turn triggers the electronic circuits in the computer to identify which key is pressed.

Table 1.2

S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

EXAMPLE 1.21

A parallel plate capacitor filled with mica having $\varepsilon_r = 5$ is connected to a 10 V battery. The area of the parallel plate is 6 m^2 and separation distance is 6 mm. (a) Find the capacitance and stored charge.

(b) After the capacitor is fully charged, the battery is disconnected and the dielectric is removed carefully.

Calculate the new values of capacitance, stored energy and charge.

Solution

(a) The capacitance of the capacitor in the presence of dielectric is

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 6}{6 \times 10^{-3}}$$
$$= 44.25 \times 10^{-9} F = 44.25 \text{nF}$$

The stored charge is

$$Q = CV = 44.25 \times 10^{-9} \times 10$$
$$= 442.5 \times 10^{-9} C = 442.5 nC$$

The stored energy is

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 44.25 C \times 10^{-9} \times 100$$
$$= 2.21 \times 10^{-6} J = 2.21\ \mu J$$

(b) After the removal of the dielectric, since the battery is already disconnected the total charge will not change. But the potential difference between the plates increases. As a result, the capacitance is decreased.

New capacitance is

$$C_0 = \frac{C}{\varepsilon_r} = \frac{44.25 \times 10^{-9}}{5}$$
$$= 8.85 \times 10^{-9} F = 8.85 nF$$

The stored charge remains same and 442.5 nC. Hence newly stored energy is

$$U_0 = \frac{Q^2}{2C_0} = \frac{Q^2 \varepsilon_r}{2C} = \varepsilon_r U$$
$$= 5 \times 2.21\ \mu J = 11.05\ \mu J$$

The increased energy is

$$\Delta U = 11.05 \mu J - 2.21 \mu J = 8.84 \mu J$$

When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84 μJ.

1.8.5 Capacitor in series and parallel

(i) Capacitor in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the Figure 1.60 (a).

As soon as the battery is connected to the capacitors in series, the electrons of charge

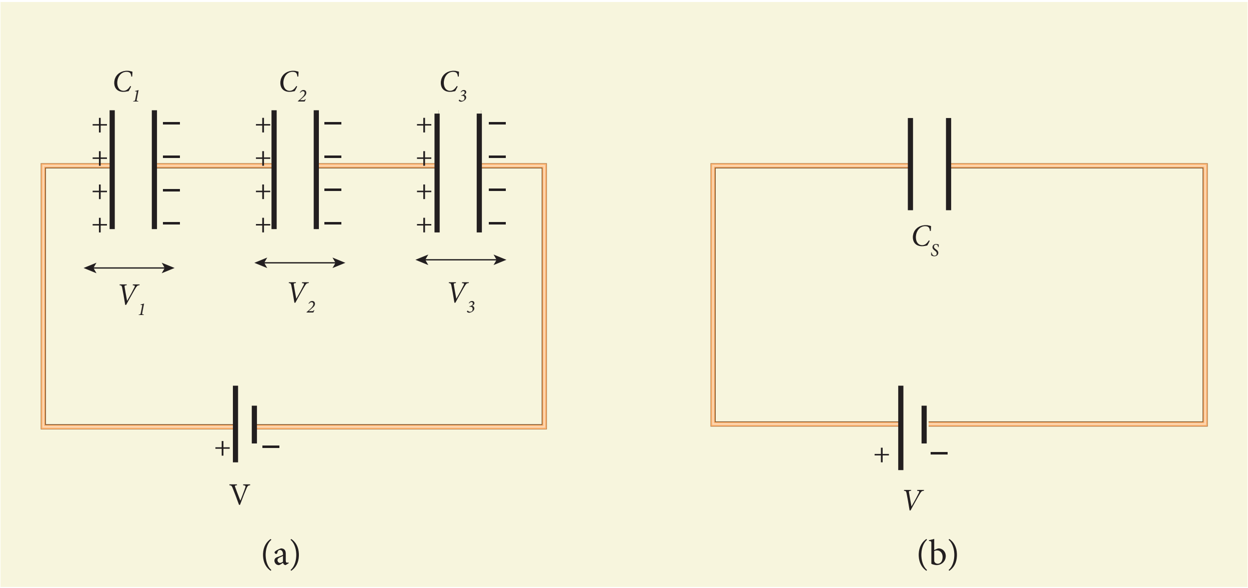


Figure 1.60 (a) Capacitors connected in series (b) Equivalence capacitors C_S

–Q are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount -Q from left plate of C_3 to the right plate of C_2 due to electrostatic induction. Similarly, the left plate of C_2 pushes the charges of –Q to the right plate of C_1 which induces the positive charge +Q on the left plate of C_1. At the same time, electrons of charge –Q are transferred from left plate of C_1 to positive terminal of the battery.

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.

The total voltage across each capacitor must be equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3 \qquad (1.103)$$

Since, $Q = CV$, we have $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad (1.104)$$

If three capacitors in series are considered to form an equivalent single capacitor C_s shown in Figure 1.60(b), then we have $V = \frac{Q}{C_S}$. Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (1.105)$$

Thus, the inverse of the equivalent capacitance C_S of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance C_S is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in Figure 1.61 (a).

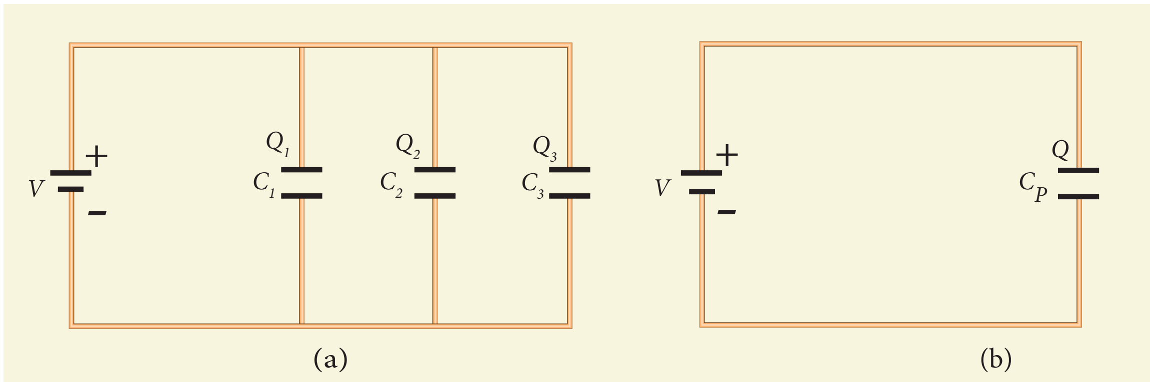


Figure 1.61 (a) capacitors in parallel (b) equivalent capacitance with the same total charge

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2 and Q_3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3 \tag{1.106}$$

Now, since $Q=CV$, we have

$$Q = C_1V + C_2V + C_3V \tag{1.107}$$

If these three capacitors are considered to form a single capacitance C_P which stores the total charge Q as shown in the Figure 1.61(b), then we can write $Q = C_PV$. Substituting this in equation (1.107), we get

$$C_PV = C_1V + C_2V + C_3V$$

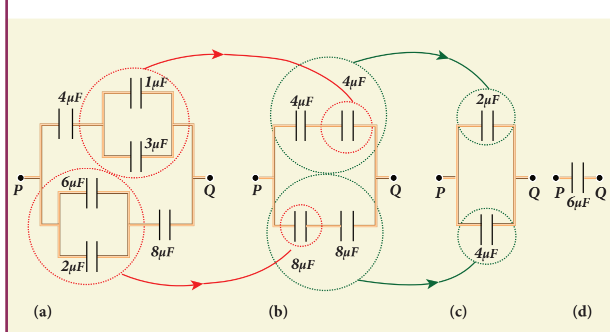
$$C_P = C_1 + C_2 + C_3 \tag{1.108}$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.

The equivalent capacitance C_P in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

EXAMPLE 1.22

Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).



Solution

The capacitors 1 μF and 3μF are connected in parallel and 6μF and 2 μF are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

$$C_{eq} = 1\mu F + 3\mu F = 4\mu F$$

$$C_{eq} = 6\mu F + 2\mu F = 8\mu F$$

From the figure (b), we infer that the two 4 μF capacitors are connected in series and the two 8 μF capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \qquad \Rightarrow C_{eq} = 2\ \mu F$$

and

$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \qquad \Rightarrow C_{eq} = 4\ \mu F$$

From the figure (c), we infer that 2μF and 4μF are connected in parallel. So the equivalent capacitance is given in the figure (d).

$$C_{eq} = 2\mu F + 4\mu F = 6\mu F$$

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 μF.

1.9 DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

1.9.1 Distribution of charges in a conductor

Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in the Figure 1.62. The distance between the spheres is much greater than the radii of either spheres.

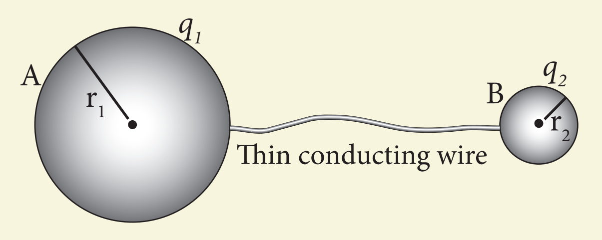


Figure 1.62 Two conductors are connected through conducting wire

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that $Q = q_1 + q_2$. The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\varepsilon_\circ} \frac{q_1}{r_1} \qquad (1.110)$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\varepsilon_\circ} \frac{q_2}{r_2} \qquad (1.111)$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B$$

$$\text{or} \quad \frac{q_1}{r_1} = \frac{q_2}{r_2} \qquad (1.112)$$

Let us take the charge density on the surface of sphere A is σ_1 and charge density on the surface of sphere B is σ_2. This implies that $q_1 = 4\pi r_1^{\,2}\sigma_1$ and

$q_2 = 4\pi r_2^2\sigma_2$. Substituting these values into equation (1.112), we get

$$\sigma_1 r_1 = \sigma_2 r_2 \qquad (1.113)$$

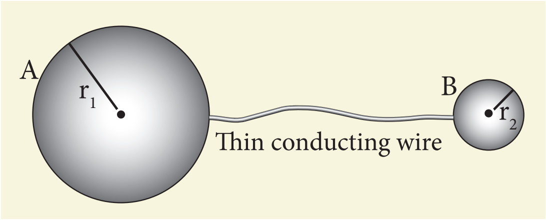
from which we conclude that

$$\sigma r = \text{constant} \qquad (1.114)$$

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

EXAMPLE 1.23

Two conducting spheres of radius r_1 = 8 cm and r_2 = 2 cm are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of Q = +100 nC is placed on one of the spheres. After a fraction of a second, the charge Q is redistributed and both the spheres attain electrostatic equilibrium.



(a) Calculate the charge and surface charge density on each sphere.

(b) Calculate the potential at the surface of each sphere.

Solution

(a) The electrostatic potential on the surface of the sphere A is $V_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_1}$

The electrostatic potential on the surface of the sphere A is $V_B = \frac{1}{4\pi\varepsilon_\circ}\frac{q_2}{r_2}$

Since $V_A = V_B$. We have

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 = \left(\frac{r_1}{r_2}\right)q_2$$

But from the conservation of total charge, Q = $q_1 + q_2$, we get $q_1 = Q - q_2$. By substituting this in the above equation,

$$Q - q_2 = \left(\frac{r_1}{r_2}\right)q_2$$

so that $$q_2 = Q\left(\frac{r_2}{r_1 + r_2}\right)$$

Therefore,

$$q_2 = 100\times10^{-9}\times\left(\frac{2}{10}\right) = 20nC$$

and $q_1 = Q - q_2 = 80\text{nC}$

The electric charge density for sphere A is

$$\sigma_1 = \frac{q_1}{4\pi r_1^2}$$

The electric charge density for sphere B is

$$\sigma_2 = \frac{q_2}{4\pi r_2^2}$$

Therefore,

$$\sigma_1 = \frac{80\times10^{-9}}{4\times64\times10^{-4}} = 0.99\times10^{-6}Cm^{-2}$$

and

$$\sigma_2 = \frac{20\times10^{-9}}{4\pi\times4\times10^{-4}} = 3.9\times10^{-6}Cm^{-2}$$

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere ($\sigma_2 \approx 4\sigma_1$) which confirms the result $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$.

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

$$V_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_1} = \frac{9\times10^9\times80\times10^{-9}}{8\times10^{-2}} = 9kV$$

1.9.2 Action at points or Corona discharge

Consider a charged conductor of irregular shape as shown in Figure 1.63 (a).

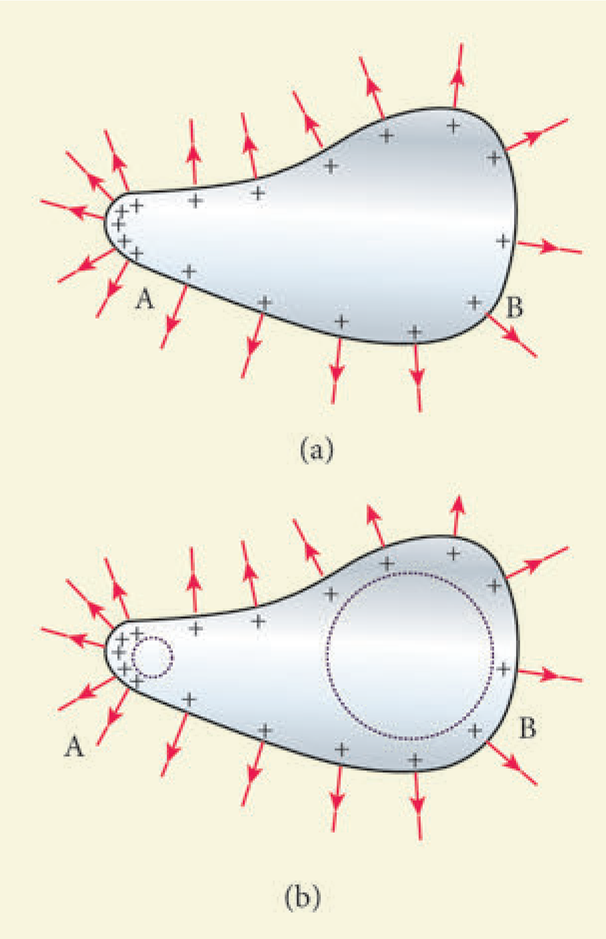


Figure 1.63 Action at a points or corona discharge

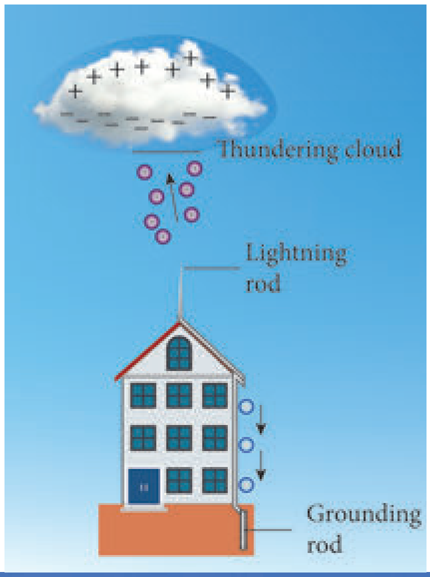
We know that smaller the radius of curvature, the larger is the charge density. The end of the conductor which has larger curvature (smaller radius) has a large charge accumulation as shown in Figure 1.63 (b).

As a result, the electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

1.9.3 Lightning arrester or lightning conductor

This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge.

This device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in Figure 1.64 (a) and (b).



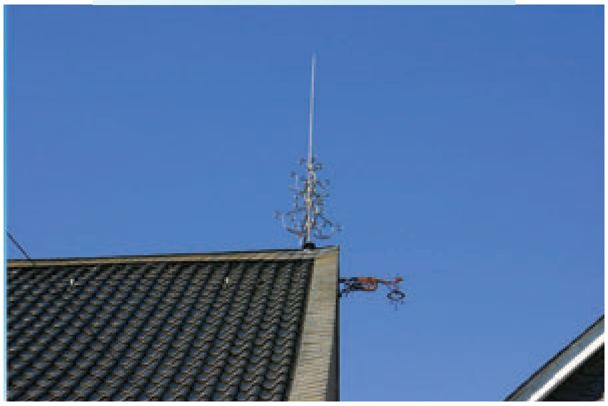


Figure 1.64 (a) Schematic diagram of a lightning arrestor. (b) A house with a lightning arrestor

The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces

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a positive charge on the spike. Since the induced charge density on thin sharp spike is large, it results in a corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod and is safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely.

1.9.4 Van de Graaff Generator

In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V). This Van de Graff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.65. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.

The comb D is maintained at a positive potential of 10^4 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the

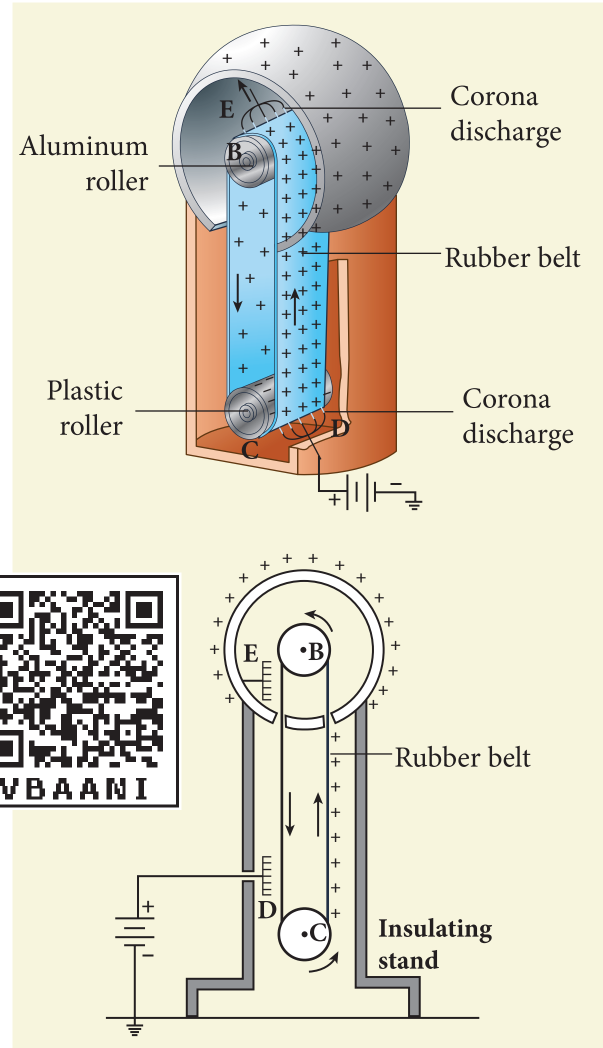

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Figure 1.65 Van de Graaff generator

comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

EXAMPLE 1.24

Dielectric strength of air is 3×10^6 V m^{-1}. Suppose the radius of a hollow sphere in the Van de Graff generator is R = 0.5 m, calculate the maximum potential difference created by this Van de Graaff generator.

The electric field on the surface of the sphere(by Gauss law) is given by

$$E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R^2}$$

The potential on the surface of the hollow metallic sphere is given by

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R} = ER$$

with $V_{max} = E_{max}R$

Here $E_{max} = 3\times10^6 \frac{V}{m}$. So the maximum potential difference created is given by

$V_{max} = 3 \times 10^6 \times 0.5$

$= 1.5 \times 10^6$ V (or) 1.5 million volt

SUMMARY

- Like charges repel and unlike charges attract
- The total charge in the universe is conserved
- Charge is quantized. Total charge in an object q = ne where n = 0,1,2,3… and e is electron charge.
- Coulomb's law in vector form: $\vec{F} = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r^2}\hat{r}$ ($\hat{r}$ is unit vector along joining q_1, q_2)
- For continuous charge distributions, integration methods can be used.
- Electrostatic force obeys the superposition principle.
- Electric field at a distance r from a point charge: $\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r^2}\hat{r}$
- Electric field lines starts at a positive charge and end at a negative charge or at infinity
- Electric field due to electric dipole at points on the axial line : $\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{2\vec{p}}{r^3}\right)$
- Electric field due to electric dipole at points on the equatorial line: $\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_\circ}\left(\frac{\vec{p}}{r^3}\right)$
- Torque experienced by a dipole in a uniform electric field: $\vec{\tau} = \vec{p}\times\vec{E}$
- Electrostatic potential at a distance r from the point charge: $V = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r}$
- Electrostatic potential due to an electric dipole: $V = \frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2}$
- The electrostatic potential is the same at all points on an equipotential surface.
- The relation between electric field and electrostatic potential:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- Electrostatic potential energy for system of charges is equal to the work done to arrange the charges in the given configuration.
- Electrostatic potential energy stored in a dipole system in a uniform electric field: $U = -\vec{p}\cdot\vec{E}$
- The total electric flux through a closed surface : $\Phi_E = \frac{Q}{\varepsilon_\circ}$ where Q is the net charge enclosed by the surface
- Electric field due to a charged infinite wire : $\vec{E} = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r}\hat{r}$
- Electric field due to a charged infinite plane : $\vec{E} = \frac{\sigma}{\varepsilon_\circ}\hat{n}$ ($\hat{n}$ is normal to the plane)
- Electric field inside a charged spherical shell is zero. For points outside: $\vec{E} = \frac{Q}{4\pi\varepsilon_\circ r^2}\hat{r}$

- Electric field inside a conductor is zero. The electric field at the surface of the conductor is normal to the surface and has magnitude $E = \frac{\sigma}{\varepsilon_\circ}$.
- The surface of the conductor has the same potential, at all points on the surface.
- Conductor can be charged using the process of induction.
- A dielectric or insulator has no free electrons. When an electric field is applied, the dielectric is polarised.
- Capacitance is given by $C = \frac{Q}{V}$.
- Capacitance of a parallel plate capacitor: $C = \frac{\varepsilon_\circ A}{d}$
- Electrostatic energy stored in a capacitor: $U = \frac{1}{2}CV^2$
- The equivalent capacitance for parallel combination is equal to the sum of individual capacitance of capacitors.
- For a series combination: The inverse of equivalent capacitance is equal to sum of inverse of individual capacitances of capacitors.
- The distribution of charges in the conductors depends on the shape of conductor. For sharper edge, the surface charge density is greater. This principle is used in the lightning arrestor
- To create a large potential difference, a Van de Graaff generator is used.

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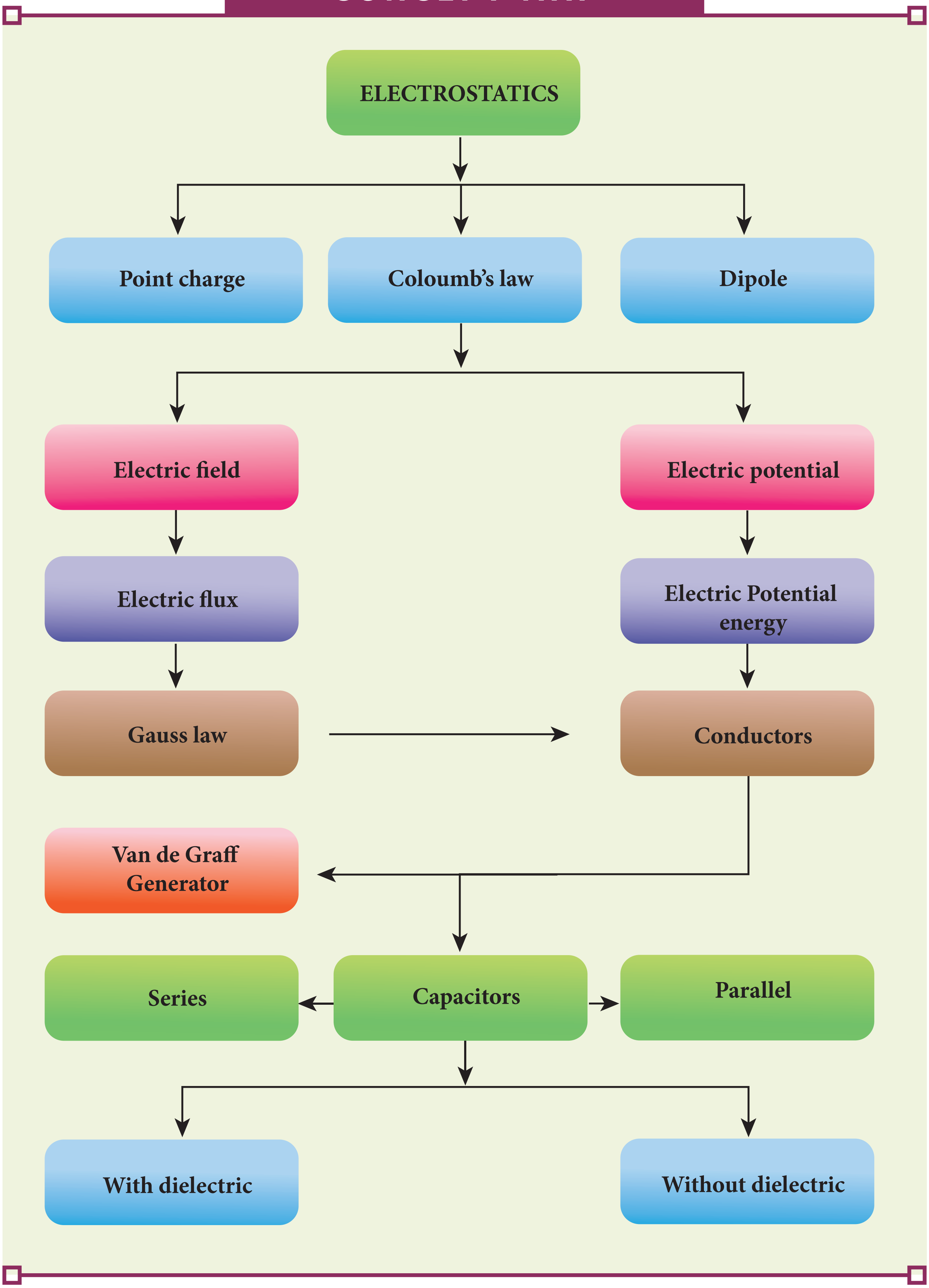
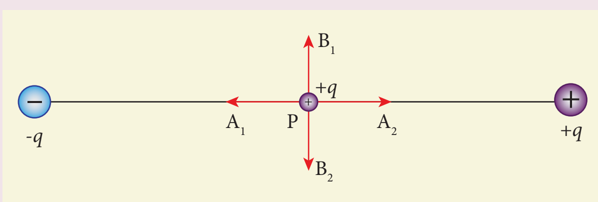
CONCEPT MAP


ELECTROSTATICS
Point charge
Coloumb's law
Dipole
Electric field
Electric potential
Electric flux
Electric Potential energy
Gauss law
Conductors
Van de Graff Generator
Series
Capacitors
Parallel
With dielectric
Without dielectric

EVALUATION

I Multiple choice questions

1. Two identical point charges of magnitude –q are fixed as shown in the figure below. A third charge +q is placed midway between the two charges at the point P. Suppose this charge +q is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will +q be stable with respect to the displacement?



(a) A_1 and A_2 (b) B_1 and B_2

(c) both directions (d) No stable

2. Which charge configuration produces a uniform electric field?

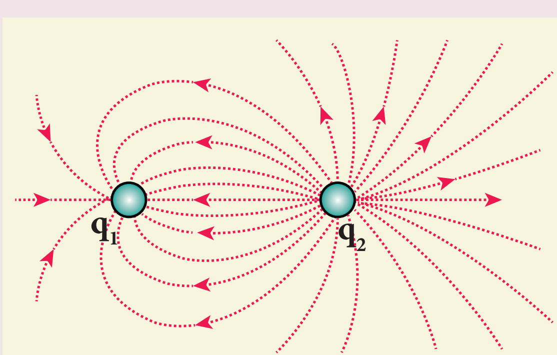
(a) point Charge

(b) infinite uniform line charge

(c) uniformly charged infinite plane

(d) uniformly charged spherical shell

3. What is the ratio of the charges $\left|\frac{q_1}{q_2}\right|$ for the following electric field line pattern?



(a) $\frac{1}{5}$ (b) $\frac{25}{11}$

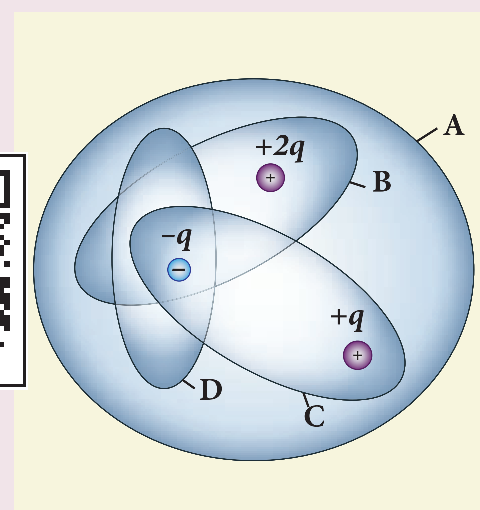
(c) 5 (d) $\frac{11}{25}$

4. An electric dipole is placed at an alignment angle of 30° with an electric field of 2×10^5 N C^{-1}. It experiences a torque equal to 8 N m. The charge on the dipole if the dipole length is 1 cm is

(a) 4 mC (b) 8 mC

(c) 5 mC (d) 7 mC

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.



(a) D < C < B < A

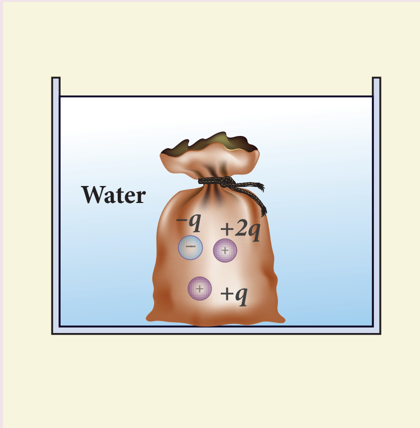
(b) A < B = C < D

(c) C < A = B < D

(d) D > C > B > A

6. The total electric flux for the following closed surface which is kept inside water

87W5DE



(a) $\frac{80q}{\varepsilon_o}$ (b) $\frac{q}{40\varepsilon_o}$

(c) $\frac{q}{80\varepsilon_o}$ (d) $\frac{q}{160\varepsilon_o}$

7. Two identical conducting balls having positive charges q_1 and q_2 are separated by a center to center distance r. If they are made to touch each other and then separated to the same distance, the force between them will be
(NSEP 04-05)

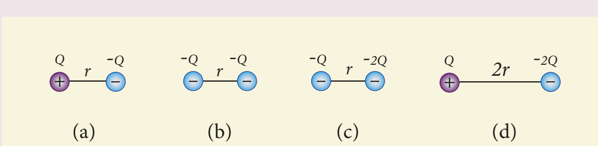
(a) less than before

(b) same as before

(c) more than before

(d) zero

8. Rank the electrostatic potential energies for the given system of charges in increasing order.



(a) 1 = 4 < 2 < 3 (b) 2 = 4 < 3 < 1

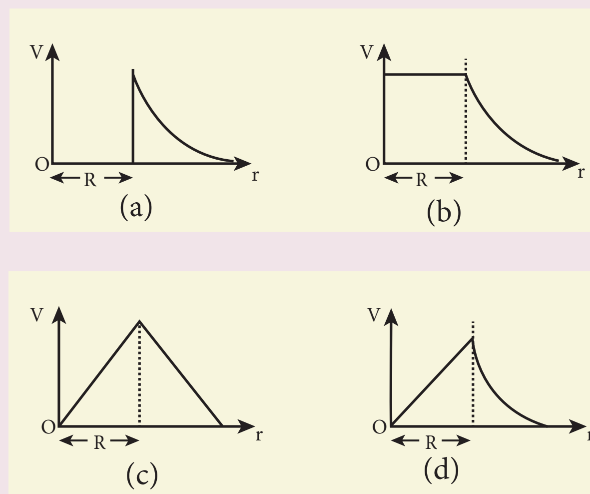
(c) 2 = 3 < 1 < 4 (d) 3 < 1 < 2 < 4

9. An electric field $\vec{E} = 10x\hat{i}$ exists in a certain region of space. Then the potential difference $V = V_o - V_A$, where V_o is the potential at the origin and V_A is the potential at x = 2 m is:

(a) 10 J (b) – 20 J

(c) +20 J (d) -10J

10. A thin conducting spherical shell of radius R has a charge Q which is uniformly distributed on its surface. The correct plot for electrostatic potential due to this spherical shell is



11. Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is

(a) 8.80×10^{-17} J

(b) -8.80×10^{-17} J

(c) 4.40×10^{-17} J

(d) 5.80×10^{-17} J

12. If voltage applied on a capacitor is increased from V to 2V, choose the correct conclusion.

(a) Q remains the same, C is doubled

(b) Q is doubled, C doubled

(c) C remains same, Q doubled

(d) Both Q and C remain same

13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?

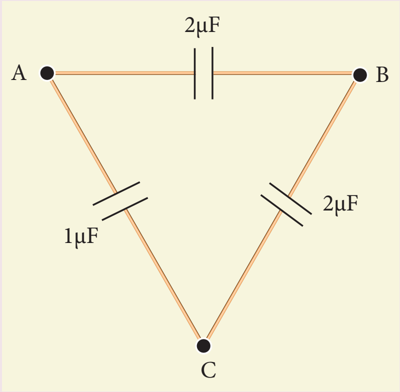
(a) Capacitance

(b) Charge

(c) Voltage

(d) Energy density

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the points A and C is



(a) 1μF (b) 2 μF

(c) 3 μF (d) $\frac{1}{4}\mu F$

15. Two metallic spheres of radii 1 cm and 3 cm are given charges of -1 x 10^{-2} C and 5 x 10^{-2} C respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is (AIIPMT -2012)

(a) 3×10^{-2} C

(b) 4×10^{-2} C

(c) 1×10^{-2} C

(d) 2×10^{-2} C

Answers

1) b	**2)** c	**3)** d	**4)** b	**5)** a
6) b	**7)** c	**8)** a	**9)** b	**10)** b
11) a	**12)** c	**13)** d	**14)** b	**15)** a

II Short Answer Questions

1. What is meant by quantisation of charges?
2. Write down Coulomb's law in vector form and mention what each term represents.
3. What are the differences between Coulomb force and gravitational force?
4. Write a short note on superposition principle.
5. Define 'Electric field'.
6. What is mean by 'Electric field lines'?
7. The electric field lines never intersect. Justify.
8. Define 'Electric dipole'
9. What is the general definition of electric dipole moment?
10. Define 'electrostatic potential".
11. What is an equipotential surface?
12. What are the properties of an equipotential surface?
13. Give the relation between electric field and electric potential.
14. Define 'electrostatic potential energy'.
15. Define 'electric flux'
16. What is meant by electrostatic energy density?
17. Write a short note on 'electrostatic shielding'.
18. What is Polarisation?

19. What is dielectric strength?
20. Define 'capacitance'. Give its unit.
21. What is corona discharge?

III Long Answer questions

1. Discuss the basic properties of electric charges.
2. Explain in detail Coulomb's law and its various aspects.
3. Define 'Electric field' and discuss its various aspects.
4. How do we determine the electric field due to a continuous charge distribution? Explain.
5. Calculate the electric field due to a dipole on its axial line and equatorial plane.
6. Derive an expression for the torque experienced by a dipole due to a uniform electric field.
7. Derive an expression for electrostatic potential due to a point charge.
8. Derive an expression for electrostatic potential due to an electric dipole.
9. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.
10. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.
11. Obtain Gauss law from Coulomb's law.
12. Obtain the expression for electric field due to an infinitely long charged wire.
13. Obtain the expression for electric field due to an charged infinite plane sheet.
14. Obtain the expression for electric field due to an uniformly charged spherical shell.
15. Discuss the various properties of conductors in electrostatic equilibrium.
16. Explain the process of electrostatic induction.
17. Explain dielectrics in detail and how an electric field is induced inside a dielectric.
18. Obtain the expression for capacitance for a parallel plate capacitor.
19. Obtain the expression for energy stored in the parallel plate capacitor.
20. Explain in detail the effect of a dielectric placed in a parallel plate capacitor.
21. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.
22. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.
23. Explain in detail the construction and working of a Van de Graaff generator.

Exercises

1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

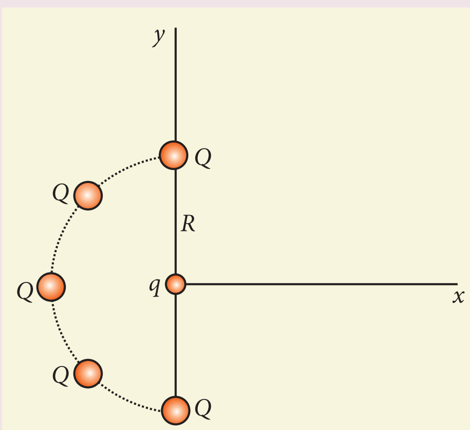
   Ans: 31.25×10^{10} electrons

2. The total number of electrons in the human body is typically in the order of 10^{28}. Suppose, due to some reason, you and your friend lost 1% of this number

of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

Ans: $F_e = 9 \times 10^{61}$ N, W = 588 N

3. Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the center of the circle of radius R. Calculate the electrostatic force experienced by the charge q.



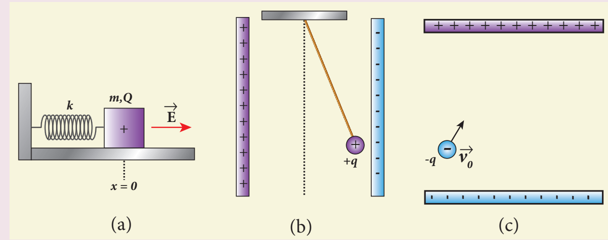
Ans: $\vec{F} = \frac{1}{4\pi\varepsilon_\circ}\frac{qQ}{R^2}\left(1+\sqrt{2}\right)N\hat{i}$

4. Suppose a charge +q on Earth's surface and another +q charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change?

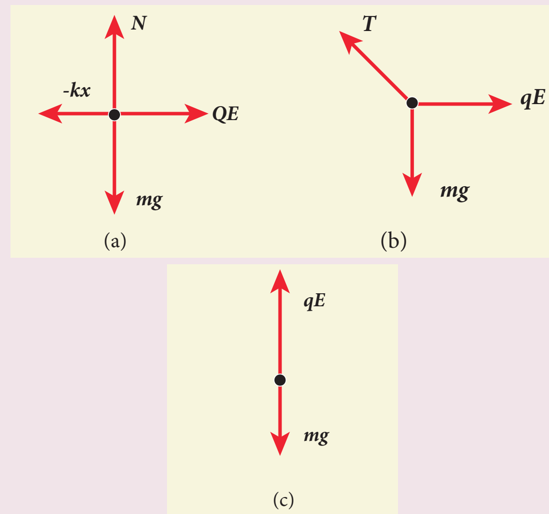
(Take $m_E = 5.9 \times 10^{24}$ kg, $m_M = 7.9 \times 10^{22}$ kg)

Ans: (a) $q \approx +5.64 \times 10^{13}$ C, (b) no change

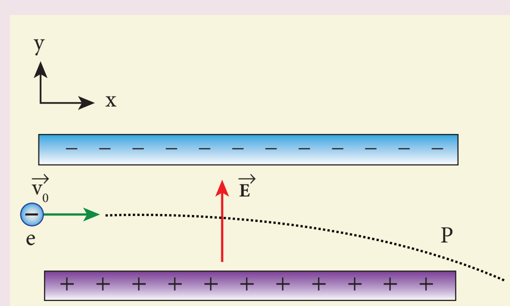
5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).



Ans:



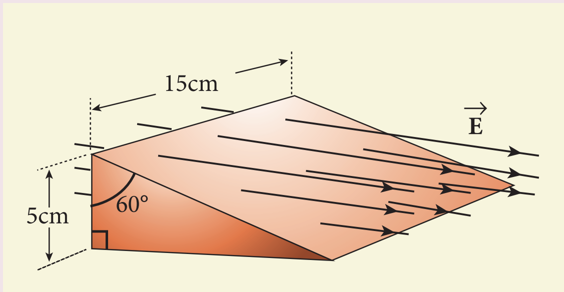
6. Consider an electron travelling with a speed v_o and entering into a uniform electric field $\vec{E}$ which is perpendicular to $\vec{v}_\circ$ as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.



Ans :

$$\vec{a} = -\frac{eE}{m}\hat{j}, \vec{v} = v_\circ\hat{i} - \frac{eE}{m}t\hat{j}, \vec{r} = v_\circ t\hat{i} - \frac{1}{2}\frac{eE}{m}t^2\hat{j}$$

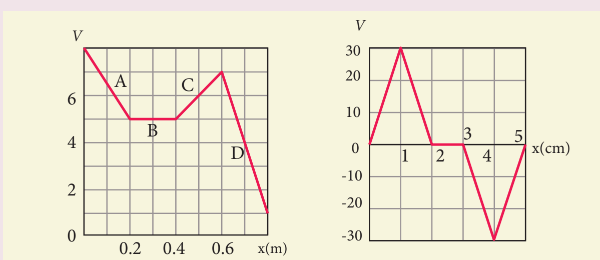
7. A closed triangular box is kept in an electric field of magnitude $E = 2 \times 10^3$ N C^{-1} as shown in the figure.



Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface.

Ans: (a) 15 Nm^2 C^{-1} (b) 15 Nm^2 C^{-1} (c) zero

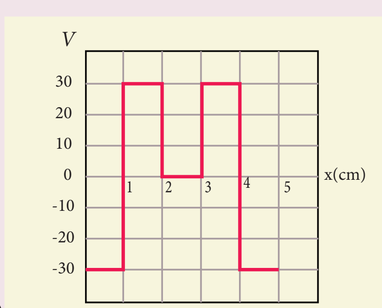
8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).



(a) (b)

Ans: (a) E_X = 15 Vm^{-1} (region A), E_X = -10 Vm^{-1} (region C)

E_X = 0 (region B), E_X = 30 Vm^{-1} (region D)



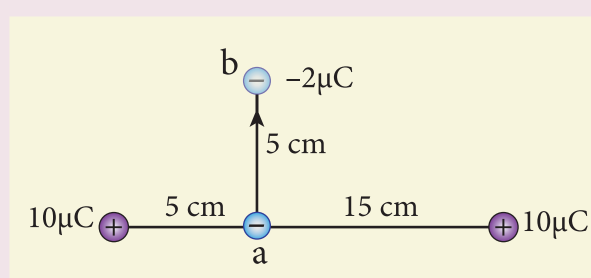
(b)

9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure.

To create the spark, an electric field of magnitude 3 × 10^6 Vm^{-1} is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same? (c) find the potential difference if the gap is 1 mm.

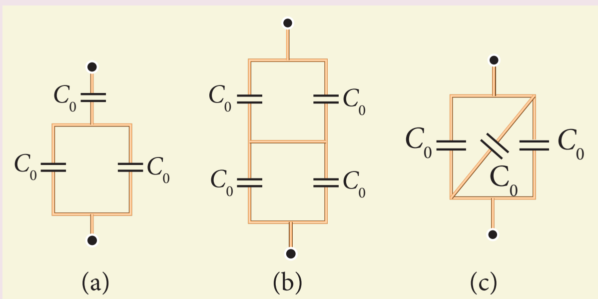
Ans: (a) 1800 V, (b) increases (c) 3000 V

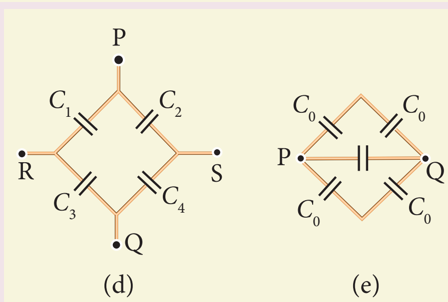
10. A point charge of +10 μC is placed at a distance of 20 cm from another identical point charge of +10 μC. A point charge of -2 μC is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.



Ans: ΔU = -3.246 J, negative sign implies that to move the charge -2μC no external work is required. System spends its stored energy to move the charge from point a to point b.

11. Calculate the resultant capacitances for each of the following combinations of capacitors.





Ans: (a) $\frac{2}{3}C_{\circ}$ (b) $C_{\circ}$ (c) $3C_{\circ}$

(d) across PQ:

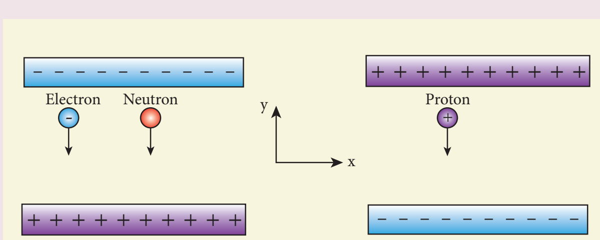
$$\frac{C_1C_2C_3 + C_2C_3C_4 + C_1C_2C_4 + C_1C_3C_4}{(C_1 + C_2)(C_3 + C_4)}$$

across RS:

$$\frac{C_1C_2C_3 + C_2C_3C_4 + C_1C_2C_4 + C_1C_3C_4}{(C_1 + C_2)(C_3 + C_4)}$$

(e) across PQ: 2 C_o

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance h = 1 mm as shown in the figure.



(a) Calculate the time of flight for both electron and proton (b) Suppose if a neutron is allowed to fall, what is the time of flight? (c) Among the three, which one will reach the bottom first? (Take $m_p = 1.6 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg and $g = 10\ m\ s^{-2}$)

Ans:

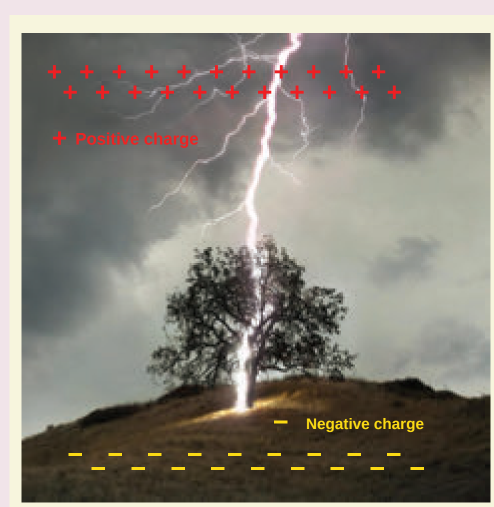
(a) $t_e = \sqrt{\frac{2hm_e}{eE}} \approx 1.5ns$ (ignoring the gravity),

$t_p = \sqrt{\frac{2hm_e}{eE}} \approx 63ns$ (ignoring the gravity)

(b) $t_n = \sqrt{\frac{2h}{g}} \approx 14.1ms$

(c) electron will reach first

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air (3×10^6 Vm^{-1}), lightning will occur.

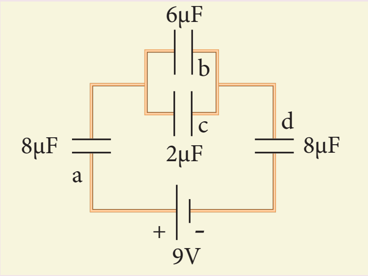


(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Ans: (a) $V = 3 \times 10^9$ V, (b) $U = 75 \times 10^9$ J

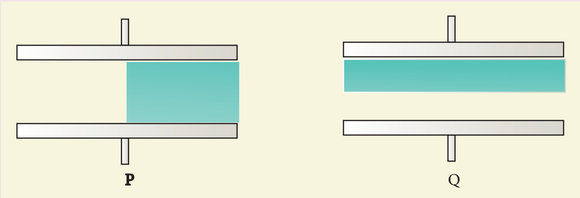
14. For the given capacitor configuration (a) Find the charges on each capacitor (b) potential difference across them (c) energy stored in each capacitor



Ans:

$Q_a = 24$ μC, $Q_b = 18$ μC,
$Q_c = 6$ μC, $Q_d = 24$ μC
$V_a = 3$V, $V_b = 3$V,
$V_c = 3$V, $V_d = 3$V,
$U_a = 36$ μJ, $U_b = 27$ μJ,
$U_c = 9$ μJ, $U_d = 36$ μJ

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ε_r as shown in the figure. Calculate the capacitance of capacitors P and Q.



$$\text{Ans}: C_P = \frac{\varepsilon_\circ A}{2d}(1+\varepsilon_r),\ C_Q = \frac{2\varepsilon_\circ A}{d}\left(\frac{\varepsilon_r}{1+\varepsilon_r}\right)$$

BOOKS FOR REFERENCE

1. Douglas C.Giancoli, , "*Physics for Scientist & Engineers with Modern Physics*", Pearson Prentice Hall, Fourth edition
2. James Walker, "*Physics*", Pearson- Addison Wesley Publishers, Fourth Edition
3. Purcell, Morin, "*Electricity and Magnetism*", "Cambridge University Press, Third Edition.
4. Serway and Jewett, "*Physics for Scientist and Engineers with Modern Physics*", Brook/Coole Publishers, Eighth Edition
5. Tipler, Mosca, "*Physics for scientist and Engineers with Modern Physics*", Freeman and Company, Sixth Edition
6. Tarasov and Tarasova, "*Questions and problems in School Physics*", Mir Publishers
7. H.C.Verma, "*Concepts of Physics: Vol 2*, Bharthi Bhawan Publishers
8. Eric Roger, *Physics for the Inquiring Mind*, Princeton University Press

ICT CORNER

Electrostatics

In this activity you will be able to learn about capacitor and the factors affecting capacitance.

Topic: Capacitor lab

STEPS:

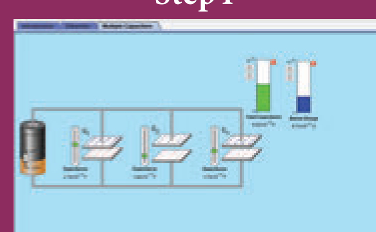
- Open the browser and type "phet.colorado.edu/en/simulation/legacy/capacitor-lab" in the address bar. Go to the tab 'Dielectric'.
- Change the plate area, distance between the plate and dielectric. Identify what you would maximize or minimize to make a capacitor with the greatest capacitance.
- Explore the relationships between charge, voltage, and stored energy for a capacitor. Design a capacitor system to store the greatest energy.
- Charge the capacitor with 1.0 v using the battery. Disconnect the battery. Now insert a dielectric between the plates. Discus how electric field changes in between the plates when dielectric is introduced.
- What is the effect of introducing a dielectric between plates? (Change dielectric materials)

Step1

Step2

Step3

Step4



Connect capacitors parallel and series combination and find the effective capacitance.

Note:

Install Java application if it is not in your system. You can download all the phet simulation and works in off line from https://phet.colorado.edu/en/offline-access .

URL:

https://phet.colorado.edu/en/simulation/legacy/capacitor-lab

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT 2 CURRENT ELECTRICITY

We will make electricity so cheap that only the rich will burn candles
– Thomas A. Edison

Learning Objectives

In this unit, the student is exposed to

- Flow of charges in a metallic conductor
- Ohm's law, electrical resistance, V-I characteristics
- Carbon resistors and combination of resistors
- Kirchhoff's laws - Wheatstone's bridge and its applications
- Electric power and Electric energy
- Heating effect - Joule's law – Experimental verification and applications
- Thermoelectric effects – Seebeck effect – Peltier effect – Thomson effect

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INTRODUCTION

In unit 1, we studied the properties of charges when it is at rest. In reality, the charges are always moving within the materials. For example, the electrons in a copper wire are never at rest and are continuously in random motion. Therefore it is important to analyse the behaviour of charges when it is at motion. The motion of charges is called 'electric current'. Current electricity is the study of flow of electric charges. It owes its origin to Alessandro Volta (1745-1827), who invented the electric battery which produced the first steady flow of electric current. Modern world depends heavily on the use of electricity. It is used to operate machines, communication systems, electronic devices, home appliances etc., In this unit, we will study about the electric current, resistance and related phenomenon in materials.

2.1 ELECTRIC CURRENT

Matter is made up of atoms. Each atom consists of a positively charged nucleus with negatively charged electrons moving around the nucleus. Atoms in metals have one or more electrons which are loosely bound to the nucleus. These electrons are called free electrons and can be easily detached from the atoms. The substances which have an abundance of these free electrons are called conductors. These free electrons move at random throughout the conductor at a given temperature. In general due to this random motion, there is no net transfer of charges from one end of the conductor to other end and hence no current. When a potential difference is applied by the battery across the ends of the conductor, the free electrons drift towards the positive terminal of the battery, producing a net electric current. This is easily understandable from the analogy given in the Figure 2.1.

In the XI Volume 2, unit 6, we studied, that the mass move from higher gravitational potential to lower gravitational potential. Likewise, positive charge flows from higher electric potential to lower electric potential and negative charge flows from lower electric potential to higher electric potential. So battery or electric cell simply creates potential difference across the conductor.

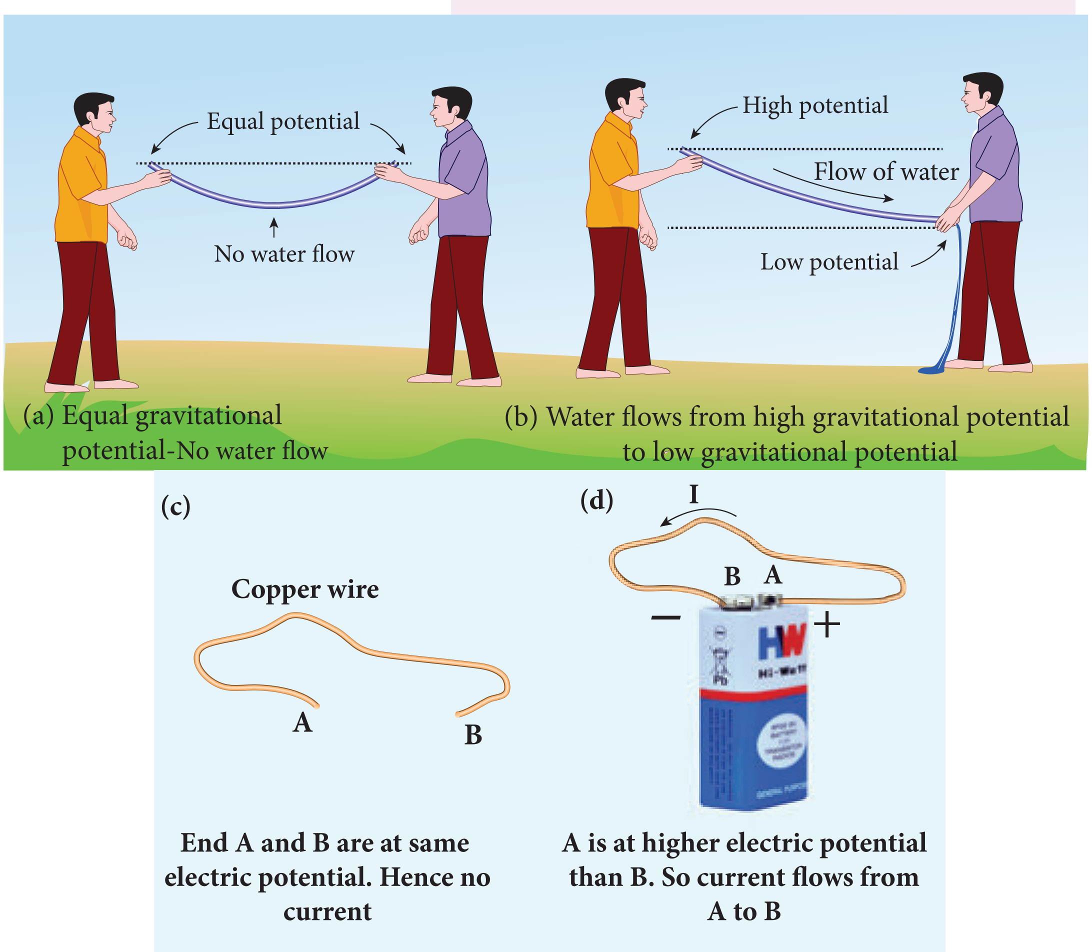

Figure 2.1 Water current and Electric current

The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area A. It is shown in the Figure 2.2.

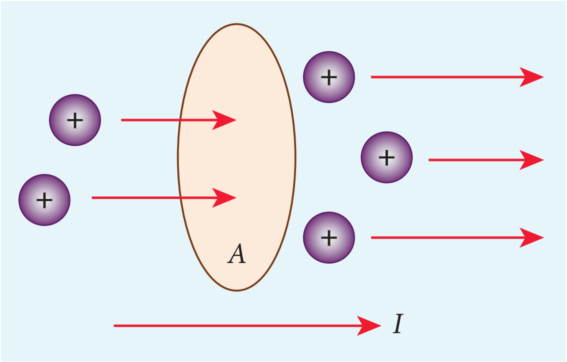

Figure 2.2 Charges flow across the area A

If a net charge Q passes through any cross section of a conductor in time t, then the current is defined as $I = \frac{Q}{t}$. But charge flow is not always constant. Hence current can more generally be defined as

$$I_{avg} = \frac{\Delta Q}{\Delta t} \quad (2.1)$$

Where ΔQ is the amount of charge that passes through the conductor at any cross section during the time interval Δt. If the rate at which charge flows changes in time, the current also changes. The instantaneous current I is defined as the limit of the average current, as $\Delta t \to 0$

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad (2.2)$$

The SI unit of current is the **ampere** (A)

$$1A = \frac{1C}{1s}$$

That is, 1A of current is equivalent to 1 Coulomb of charge passing through a perpendicular cross section in 1second. The electric current is a scalar quantity.

EXAMPLE 2.1

Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

Solution

The current (rate of flow of charge) in the wire is

$$I = \frac{Q}{t} = \frac{120}{60} = 2A$$

2.1.1 Conventional Current

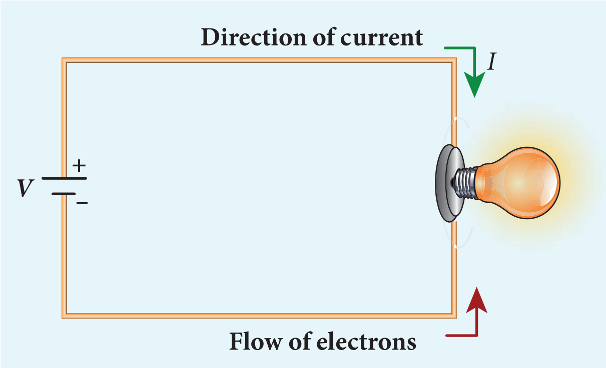

Figure 2.3 Direction of conventional current and electron flow

In an electric circuit, arrow heads are used to indicate the direction of flow of current. By convention, this flow in the circuit should be from the positive terminal of the battery to the negative terminal. This current is called the conventional current or simply current and is in the direction in which a positive test charge would move. In typical circuits the charges that flow are actually electrons, from the negative terminal of the battery to the positive. As a result, the flow of electrons and the direction of conventional current points in opposite direction as shown in Figure 2.3. Mathematically, a transfer of positive charge

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is the same as a transfer of negative charge in the opposite direction.

Electric current is not only produced by batteries. In nature, lightning bolt produces enormous electric current in a short time. During lightning, very high potential difference is created between the clouds and ground so charges flow between the clouds and ground.

2.1.2 Drift velocity

In a conductor the charge carriers are free electrons. These electrons move freely through the conductor and collide repeatedly with the positive ions. If there is no electric field, the electrons move in random directions, so the directions of their velocities are also completely random direction. On an average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. As a result, there is no net flow of electrons in any direction and hence there will not be any current.

Suppose a potential difference is set across the conductor by connecting a battery, an electric field $\vec{E}$ is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons, while ions scatter the electrons and change the direction of motion. Thus, we have zigzag paths of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor in a direction opposite to that of $\vec{E}$ as shown in the Figure 2.4.

Ions

Any material is made up of neutral atoms with equal number of electrons and protons. If the outermost electrons leave the atoms, they become free electrons and are responsible for electric current. The atoms after losing their outer most electrons will have more positive charges and hence are called positive ions. These ions will not move freely within the material like the free electrons. Hence the positive ions will not give rise to current.

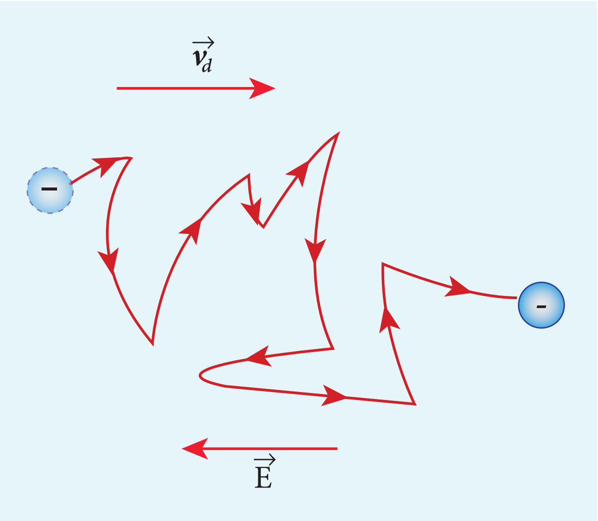


Figure 2.4 Electric current

This velocity is called drift velocity $\vec{v}_d$. The drift velocity is the average velocity acquired by the electrons inside the conductor when it is subjected to an electric field. The average time between successive collisions is called the mean free time denoted by τ. The acceleration $\vec{a}$ experienced by the electron in an electric field $\vec{E}$ is given by

$$\vec{a} = \frac{-e\vec{E}}{m} \qquad \left(\text{since}\,\vec{F} = -e\vec{E}\right) \qquad (2.3)$$

The drift velocity $\vec{v}_d$ is given by

$$\vec{v}_d = \vec{a}\,\tau$$

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E} \qquad (2.4)$$

$$\vec{v}_d = -\mu\vec{E} \qquad (2.5)$$

Here $\mu = \frac{e\tau}{m}$ is the mobility of the electron and it is defined as the magnitude of the drift velocity per unit electric field.

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|} \qquad (2.6)$$

The SI unit of mobility is $\frac{m^2}{V\,s}$.

Note

The typical drift velocity of electrons in the wire is 10^{-4} m s^{-1}. If an electron drifts with this speed, then the electrons leaving the battery will take hours to reach the light bulb. Then how electric bulbs glow as soon as we switch on the battery? When battery is switched on, the electrons begin to move away from the negative terminal of the battery and this electron exerts force on the nearby electrons. This process creates a propagating influence (electric field) that travels through the wire at the speed of light. In other words, the energy is transported from the battery to light bulb at the speed of light through propagating influence (electric field). Due to this reason, the light bulb glows as soon as the battery is switched on.

EXAMPLE 2.2

If an electric field of magnitude 570 N C^{-1}, is applied in the copper wire, find the acceleration experienced by the electron.

Solution:

E = 570 N C^{-1}, e = 1.6 × 10^{-19} C,
m = 9.11 × 10^{-31} kg and a = ?

F = ma = eE

$$a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}$$

$$= \frac{912 \times 10^{-19} \times 10^{31}}{9.11}$$

$$= 1.001 \times 10^{14}\ \text{m s}^{-2}$$

Misconception

(i) There is a common misconception that the battery is the source of electrons. It is not true. When a battery is connected across the given wire, the electrons in the closed circuit resulting the current. Battery sets the potential difference (electrical energy) due to which these electrons in the conducting wire flow in a particular direction. The resulting electrical energy is used by electric bulb, electric fan etc. Similarly the electricity board is supplying the electrical energy to our home.

(ii) We often use the phrases like 'charging the battery in my mobile' and 'my mobile phone battery has no charge' etc. These sentences are not correct.

When we say 'battery has no charge', it means, that the battery has lost ability to provide energy or provide potential difference to the electrons in the circuit. When we say 'mobile is charging', it implies that the battery is receiving energy from AC power supply and not electrons.

2.1.3 Microscopic model of current

Consider a conductor with area of cross section A and an electric field $\vec{E}$ applied from right to left. Suppose there are n electrons per unit volume in the conductor and assume that all the electrons move with the same drift velocity $\vec{v}_d$ as shown in Figure 2.5.

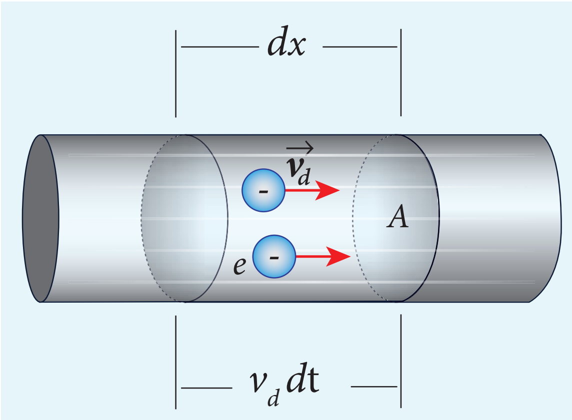


Figure 2.5 Microscopic model of current

The drift velocity of the electrons = v_d

The electrons move through a distance dx within a small interval of dt

$$v_d = \frac{dx}{dt}\,; \qquad dx = v_d\, dt \qquad (2.7)$$

Since A is the area of cross section of the conductor, the electrons available in the volume of length dx is

= volume × number per unit volume

$$= A\,dx \times n \qquad (2.8)$$

Substituting for dx from equation (2.7) in (2.8)

$$= (A\, v_d\, dt)\, n$$

Total charge in volume element dQ = (charge) × (number of electrons in the volume element)

$$dQ = (e)(Av_d dt)n$$

Hence the current $I = \frac{dQ}{dt} = \frac{neAv_d dt}{dt}$

$$I = ne\,Av_d. \qquad (2.9)$$

Current density (J)

The current density (J) is defined as the current per unit area of cross section of the conductor.

$$J = \frac{I}{A}$$

The S.I unit of current density is $\frac{A}{m^2}$ (or) A m^{-2}

$$J = \frac{neAv_d}{A} \text{ (from equation 2.9)}$$

$$J = ne\,v_d \qquad (2.10)$$

The above expression is valid only when the direction of the current is perpendicular to the area A. In general, the current density is a vector quantity and it is given by

$$\vec{J} = ne\vec{v}_d$$

Substituting $\vec{v}_d$ from equation (2.4)

$$\vec{J} = -\frac{n \cdot e^2 \tau}{m}\vec{E} \qquad (2.11)$$

$$\vec{J} = -\sigma\vec{E}$$

But conventionally, we take the direction of (conventional) current density as the direction of electric field. So the above equation becomes

$$\vec{J} = \sigma\vec{E} \qquad (2.12)$$

where $\sigma = \frac{ne^2\tau}{m}$ is called conductivity.

The equation 2.12 is called microscopic form of ohm's law.

The inverse of conductivity is called resistivity (ρ) [Refer section 2.2.1].

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \quad (2.13)$$

EXAMPLE 2.3

A copper wire of cross-sectional area 0.5 mm^2 carries a current of 0.2 A. If the free electron density of copper is 8.4×10^{28} m^{-3} then compute the drift velocity of free electrons.

Solution

The relation between drift velocity of electrons and current in a wire of cross-sectional area A is

$$v_d = \frac{I}{neA} = \frac{0.2}{8.4\times10^{28}\times1.6\times10^{-19}\times0.5\times10^{-6}}$$

$v_d = 0.03 \times 10^{-3}$ m s^{-1}

Note

Why current density is a vector but current is a scalar?

In general, the current I is defined as the scalar product of the current density and area vector in which the charges cross.

$$I = \vec{J}\cdot\vec{A}$$

The current I can be positive or negative depending on the choice of the unit vector normal to the surface area A.

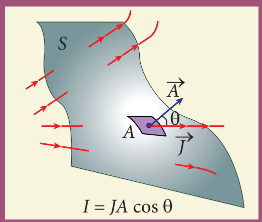


Figure 2.6 Current is a scalar

EXAMPLE 2.4

Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it.

Solution

I = 32 A , t = 1 s

Charge of an electron, e = 1.6×10^{-19} C

The number of electrons flowing per second, n =?

$$I = \frac{q}{t} = \frac{ne}{t}$$

$$n = \frac{It}{e}$$

$$n = \frac{32\times1}{1.6\times10^{-19}C}$$

$n = 20 \times 10^{19} = 2 \times 10^{20}$ electrons

2.2 OHM'S LAW

The ohm's law can be derived from the equation J = σE. Consider a segment of wire of length l and cross sectional area A as shown in Figure 2.7.

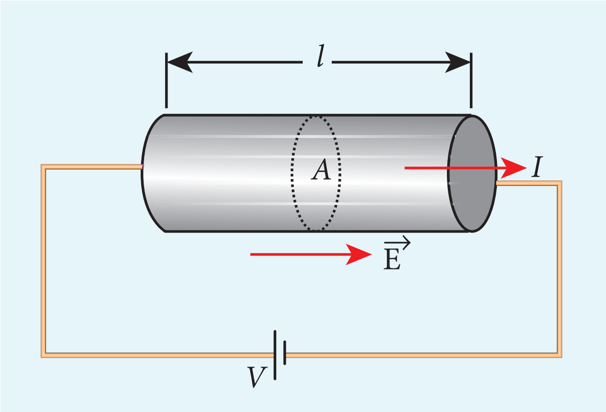


Figure 2.7 Current through the conductor

When a potential difference V is applied across the wire, a net electric field is created in the wire which constitutes the current. For simplicity, we assume that the electric field is uniform in the entire length of the wire, the potential difference (voltage V) can be written as

$$V = El$$

As we know, the magnitude of current density

$$J = \sigma E = \sigma \frac{V}{l} \quad (2.14)$$

But $J = \frac{I}{A}$, so we write the equation (2.14) as

$$\frac{I}{A} = \sigma \frac{V}{l}.$$

By rearranging the above equation, we get

$$V = I\left(\frac{l}{\sigma A}\right) \quad (2.15)$$

The quantity $\frac{l}{\sigma A}$ is called resistance of the conductor and it is denoted as R. Note that the resistance is directly proportional to the length of the conductor and inversely proportional to area of cross section.

Therefore, the macroscopic form of ohm's law can be stated as

$$V = IR \quad (2.16)$$

From the above equation, **the resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.**

$$R = \frac{V}{I} \quad (2.17)$$

The SI unit of resistance is ohm (Ω). From the equation (2.16), we infer that the graph between current versus voltage is straight line with a slope equal to the inverse of resistance R of the conductor. It is shown in the Figure 2.8 (a).

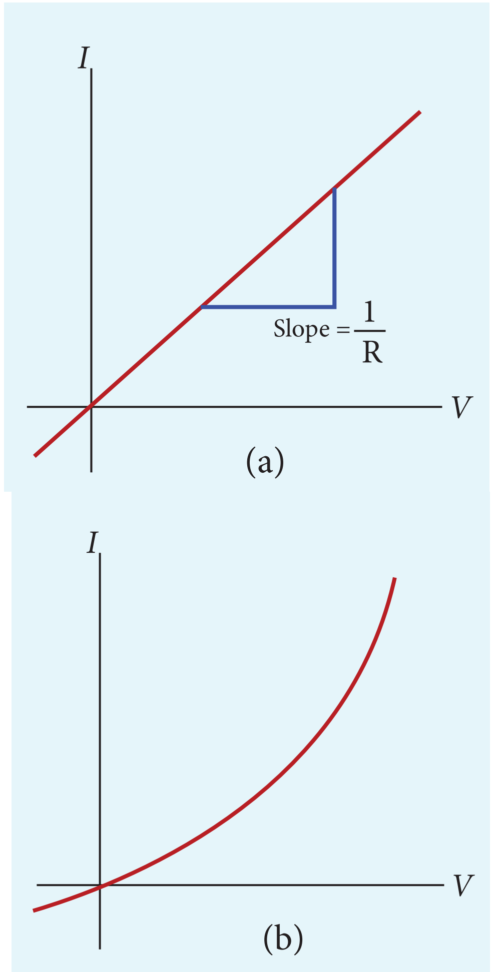


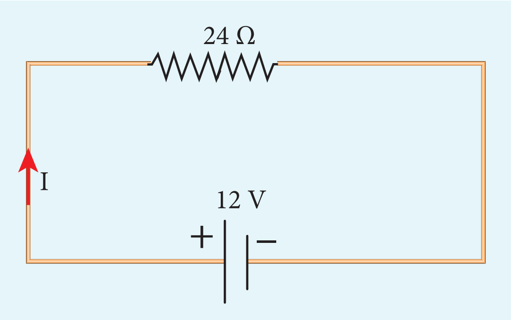
Figure 2.8 Current against voltage for (a) a conductor which obey Ohm's law and (b) for a non-ohmic device (Diode given in XII physics, unit 9 is an example of a non-ohmic device)

Materials for which the current against voltage graph is a straight line through the origin, are said to obey Ohm's law and their behaviour is said to be ohmic as shown in Figure 2.8(a). Materials or devices that do not follow Ohm's law are said to be non-ohmic. These materials have more complex relationships between voltage and current. A plot of I against V for a non-ohmic material is non-linear and they do not have a constant resistance (Figure 2.8(b)).

EXAMPLE 2.5

A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?

Solution



V = 12 V and R = 24 Ω
Current, I = ?
From Ohm's law, $I = \frac{V}{R} = \frac{12}{24} = 0.5\,A$

2.2.1 Resistivity

In the previous section, we have seen that the resistance R of any conductor is given by

$$R = \frac{l}{\sigma A} \quad (2.18)$$

where σ is called the conductivity of the material and it depends only on the type of the material used and not on its dimension.

The resistivity of a material is equal to the reciprocal of its conductivity.

$$\rho = \frac{1}{\sigma} \quad (2.19)$$

Now we can rewrite equation (2.18) using equation (2.19)

$$R = \rho\frac{l}{A} \quad (2.20)$$

The resistance of a material is directly proportional to the length of the conductor and inversely proportional to the area of cross section of the conductor. The proportionality constant ρ is called the resistivity of the material.

If l = 1 m and A = 1 m², then the resistance R = ρ. In other words, the **electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section.** The SI unit of ρ is ohm-metre (Ω m). Based on the resistivity, materials are classified as conductors, insulators and semi-conductors. The conductors have lowest resistivity, insulators have highest resistivity and semiconductors have resistivity greater than conductors but less than insulators. The typical resistivity values of some conductors, insulators and semiconductors are given in the Table 2.1

Table 2.1 Resistivity for various materials

Material	Resistivity, ρ (Ω m) at 20°C
Insulators	
Pure Water	2.5×10^5
Glass	10^{10} - 10^{14}
Hard Rubber	10^{13} - 10^{16}
NaCl	- 10^{14}
Fused Quartz	- 10^{16}
Semiconductors	
Germanium	0.46
Silicon	640
Conductors	
Silver	1.6×10^{-8}
Copper	17×10^{-8}
Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}

EXAMPLE 2.6

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

Solution

$R_1 = 20\ \Omega$, $R_2 = ?$

Let the original length (l_1) be l.

The new length, $l_2 = 8l_1$ (i.,e) $l_2 = 8l$

The original resistance, $R_1 = \rho \frac{l_1}{A_1}$

The new resistance $R_2 = \rho \frac{l_2}{A_2} = \frac{\rho(8l)}{A_2}$

Though the wire is stretched, its volume is unchanged.

Initial volume = Final volume

$A_1 l_1 = A_2 l_2$, $\quad A_1 l = A_2 8l$

$$\frac{A_1}{A_2} = \frac{8l}{l} = 8$$

By dividing equation R_2 by equation R_1, we get

$$\frac{R_2}{R_1} = \frac{\rho(8l)}{A_2} \times \frac{A_1}{\rho l}$$

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8$$

Substituting the value of $\frac{A_1}{A_2}$, we get

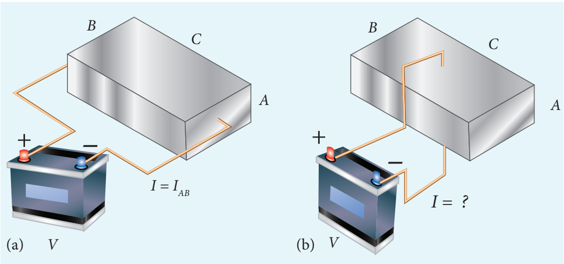
$$\frac{R_2}{R_1} = 8 \times 8 = 64$$

$R_2 = 64 \times 20 = 1280\ \Omega$

Hence, stretching the length of the wire has increased its resistance.

EXAMPLE 2.7

Consider a rectangular block of metal of height A, width B and length C as shown in the figure.



If a potential difference of V is applied between the two faces A and B of the block (figure (a)), the current I_{AB} is observed. Find the current that flows if the same potential difference V is applied between the two faces B and C of the block (figure (b)). Give your answers in terms of I_{AB}.

Solution

In the first case, the resistance of the block

$$R_{AB} = \rho \frac{\text{length}}{\text{Area}} = \rho \frac{C}{AB}$$

$$\text{The current } I_{AB} = \frac{V}{R_{AB}} = \frac{V}{\rho} \cdot \frac{AB}{C} \qquad (1)$$

In the second case, the resistance of the block $R_{BC} = \rho \frac{A}{BC}$

$$\text{The current } I_{BC} = \frac{V}{R_{BC}} = \frac{V}{\rho} \cdot \frac{BC}{A} \qquad (2)$$

To express I_{BC} interms of I_{AB}, we multiply and divide equation (2) by AC, we get

$$I_{BC} = \frac{V}{\rho} \cdot \frac{BC}{A} \frac{AC}{AC} = \left(\frac{V}{\rho} \cdot \frac{AB}{C}\right) \frac{C^2}{A^2} = \frac{C^2}{A^2} \cdot I_{AB}$$

Since C > A, the current $I_{BC} > I_{AB}$

DO YOU KNOW?

The human body contains a large amount of water which has low resistance of around 200 Ω and the dry skin has high resistance of around 500 k Ω. But when the skin is wet, the resistance is reduced to around1000 Ω. This is the reason, repairing the electrical connection with the wet skin is always dangerous.

2.2.2 Resistors in series and parallel

An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors.

Resistors in series

When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices. Figure 2.9 (a) shows three resistors R_1, R_2 and R_3 connected in series.

The amount of charge passing through resistor R_1 must also pass through resistors R_2 and R_3 since the charges cannot accumulate anywhere in the circuit. Due to this reason, the current I passing through all the three resistors is the same. According to Ohm's law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different. Let V_1, V_2 and V_3 be the potential difference (voltage) across each of the resistors R_1, R_2 and R_3 respectively, then we can write $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. But the total voltage V is equal to the sum of voltages across each resistor.

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad (2.21)$$

$$V = I\,(R_1 + R_2 + R_3)$$

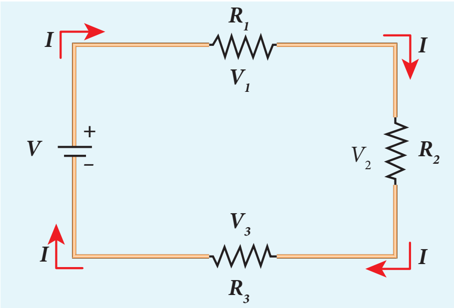
$$V = I.R_S \quad (2.22)$$

where R_S is the equivalent resistance,

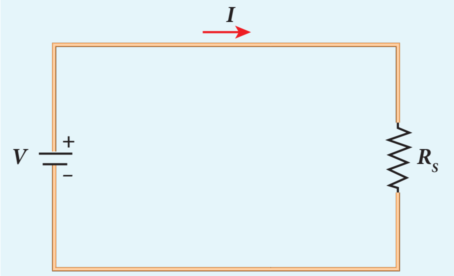
$$R_S = R_1 + R_2 + R_3 \quad (2.23)$$

When several resistances are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the Figure 2.9 (b).

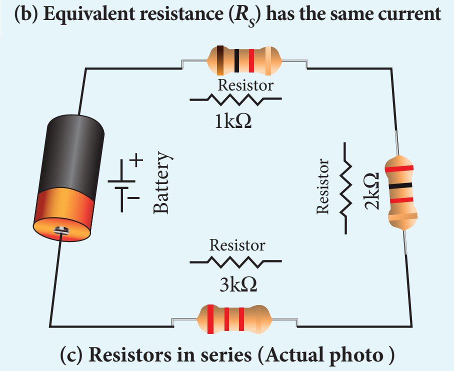
Note: The value of equivalent resistance in series connection will be greater than each individual resistance.



(a) Three resistors in series



(b) Equivalent resistance (R_S) has the same current

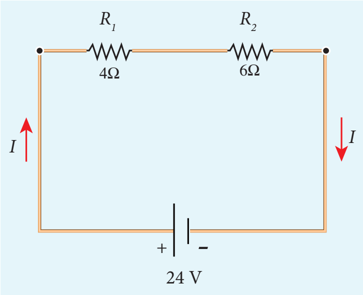


(c) Resistors in series (Actual photo)

Figure 2.9 Resistors in series

EXAMPLE 2.8

Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across 4 Ω and 6 Ω resistors in the circuit.



Solution

Since the resistors are connected in series, the effective resistance in the circuit

$= 4\ \Omega + 6\ \Omega = 10\ \Omega$

The Current I in the circuit $= \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,A$

Voltage across 4Ω resistor

$V_1 = IR_1 = 2.4\,A \times 4\Omega = 9.6\,V$

Voltage across 6 Ω resistor

$V_2 = IR_1 = 2.4\,A \times 6\Omega = 14.4\,V$

Resistors in parallel

Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a).

In this case, the total current I that leaves the battery is split into three separate paths. Let I_1, I_2 and I_3 be the current through the resistors R_1, R_2 and R_3 respectively. Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the three resistors.

$$I = I_1 + I_2 + I_3 \qquad (2.24)$$

Since the voltage across each resistor is the same, applying Ohm's law to each resistor, we have

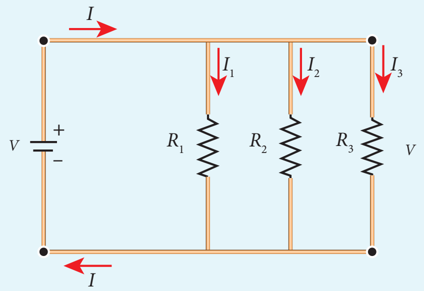
$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \qquad (2.25)$$

Substituting these values in equation (2.24),we get

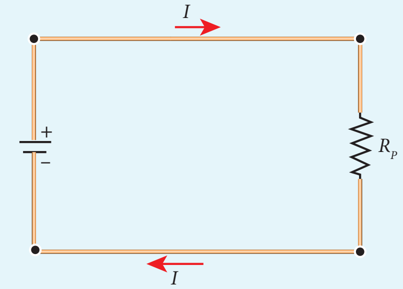
$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$I = \frac{V}{R_P}$$

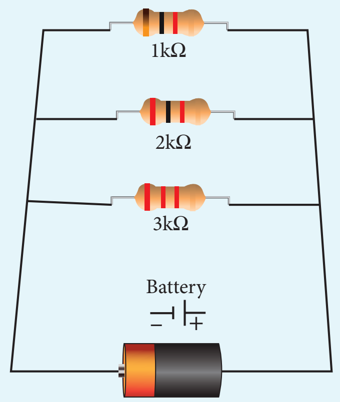
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (2.26)$$



(a) Three resistors in parallel



(b) Equivalent resistance (R_P) has the same current



(c) Resistors in parallel(Actual photo)

Figure 2.10 Resistors in parallel

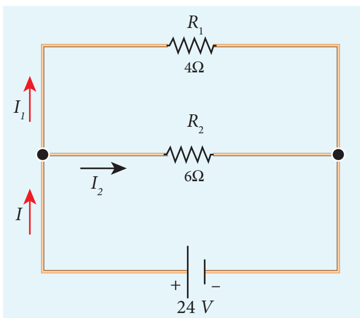
Here R_P is the equivalent resistance of the parallel combination of the resistors. Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the values of resistance of the individual resistor is equal to the reciprocal of the effective resistance of the combination as shown in the Figure 2.10 (b)

Note: The value of equivalent resistance in parallel connection will be lesser than each individual resistance.

House hold appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.

EXAMPLE 2.9

Calculate the equivalent resistance in the following circuit and also find the current I, I_1 and I_2 in the given circuit.



Solution

Since the resistances are connected in parallel, therefore, the equivalent resistance in the circuit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_P} = \frac{5}{12}\Omega \quad \text{or } R_P = \frac{12}{5}\Omega$$

The resistors are connected in parallel, the potential (voltage) across each resistor is the same.

$$I_1 = \frac{V}{R_1} = \frac{24V}{6\Omega} = 6\,A$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4\,A$$

The current I is the total of the currents in the two branches. Then,

$I = I_1 + I_2 = 6$ A + 4 A = 10 A

EXAMPLE 2.10

When two resistances connected in series and parallel their equivalent resistances are 15 Ω and $\frac{56}{15}\Omega$ respectively. Find the individual resistances.

Solution

$R_s = R_1 + R_2 = 15\ \Omega$ (1)

$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{56}{15}\Omega$ (2)

From equation (1) substituting for $R_1 + R_2$ in equation (2)

$$\frac{R_1 R_2}{15} = \frac{56}{15}\Omega$$

$\therefore\ R_1R_2 = 56$

$R_2 = \frac{56}{15}\Omega$ (3)

Substituting for R_2 in equation (1) from equation (3)

$$R_1 + \frac{56}{R_1} = 15$$

$$\text{Then, } \frac{R_1^2 + 56}{R_1} = 15$$

$R_1^{\,2} + 56 = 15\,R_1$

$R_1^{\,2} - 15\,R_1 + 56 = 0$

The above equation can be solved using factorisation.

$R_1^{\,2} - 8\,R_1 - 7\,R_1 + 56 = 0$

$R_1\,(R_1 - 8) - 7\,(R_1 - 8) = 0$

$(R_1 - 8)\,(R_1 - 7) = 0$

If $(R_1 = 8\ \Omega)$

using in equation (1)

$8 + R_2 = 15$

$R_2 = 15 - 8 = 7\ \Omega$,

$R_2 = 7\ \Omega$ i.e , (when $R_1 = 8\ \Omega$; $R_2 = 7\ \Omega$)

If $(R_1 = 7\ \Omega)$

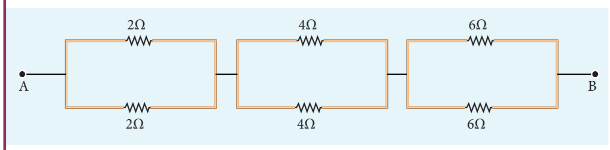
Substituting in equation (1)

$7 + R_2 = 15$

$R_2 = 8\ \Omega$, i.e , (when $R_1 = 8\ \Omega$; $R_2 = 7\ \Omega$)

EXAMPLE 2.11

Calculate the equivalent resistance between A and B in the given circuit.



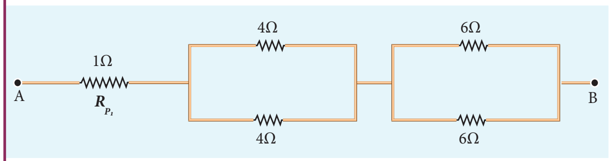
Solution

Parallel connection

Part 1

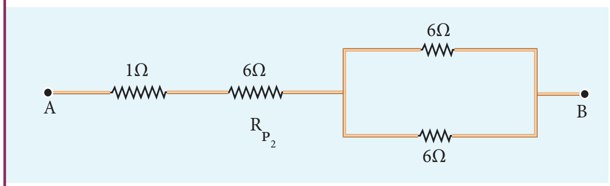
$$\frac{1}{R_{p_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{p_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \qquad R_{p_1} = 1\Omega$$



Part II

$$\frac{1}{R_{p_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}, \quad \frac{1}{R_{p_2}} = \frac{1}{2}, \quad R_{p_2} = 2\Omega$$



Part III

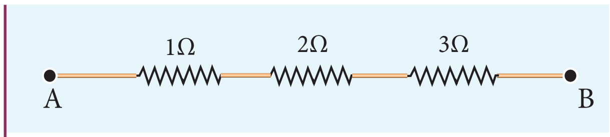
$$\frac{1}{R_{p_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{1}{R_{p_3}} = \frac{1}{3}, \qquad R_{p_3} = 3\Omega$$

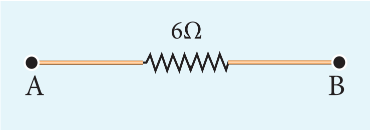
$$R = R_{p_1} + R_{p_2} + R_{p_3}$$

R = 1 + 2 + 3 R = 6 Ω

The circuit became:

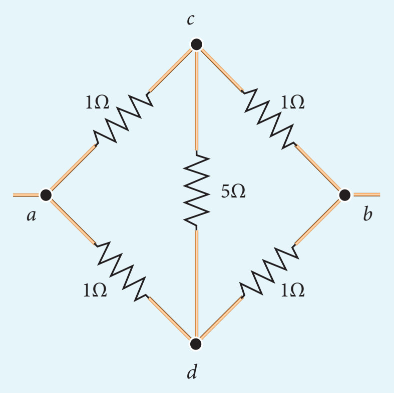


Equivalent resistance between A and B is



EXAMPLE 2.12

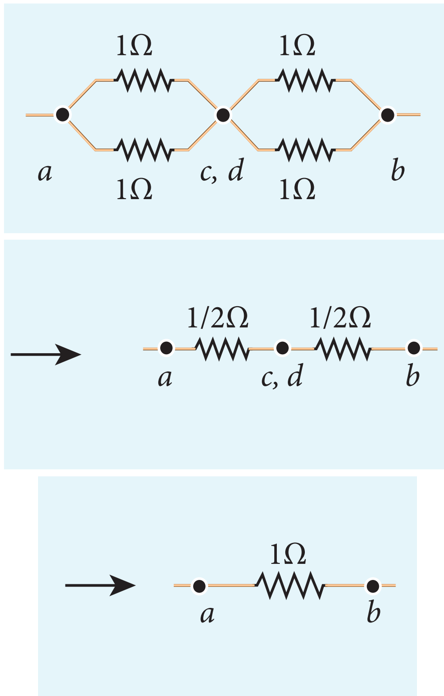
Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.



Solution

Case (a)

To find the equivalent resistance between the points a and b, we assume that current is entering the junction a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. So the electric potential at the point c and d is the same hence no current flows into 5 Ω resistance. It implies that the 5 Ω has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.



The equivalent resistance of the circuit between a and b is $R_{eq} = 1\Omega$

2.2.3 Color code for Carbon resistors

Figure 2.11 Resistance used in our laboratory

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited as shown in Figure 2.11. These resistors are inexpensive, stable and compact in size. Color rings are used to indicate the value of the resistance according to the rules given in the Table 2.2.

Three coloured rings are used to indicate the values of a resistor: the first two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth color, silver or gold, shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12 .If there is no fourth ring, the tolerance is 20%.

For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = 10^3 (orange) and tolerance = 5% (gold). The value of resistance = $56 \times 10^3\ \Omega$ or 56 kΩ with the tolerance value 5%.

Table 2.2 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Sliver		10^{-2}	10%
Colorless			20%

Note While reading the colour code, hold the resistor with colour bands to your left. Resistors never start with a metallic band on the left.

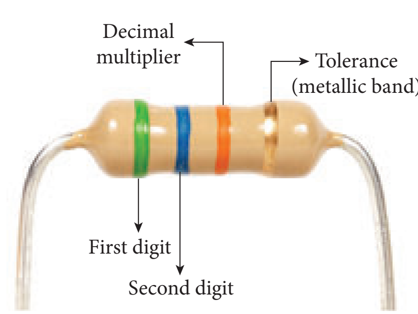


Figure 2.12 Resistor color coding

DO YOU KNOW?

Measuring current

Measuring resistance

Measuring voltage

A multimeter is a very useful electronic instrument used to measure voltage, current, resistance and capacitance. In fact, it can also measure AC voltage and AC current. The circular slider has to be kept in appropriate position to measure each electrical quantity.

2.2.4 Temperature dependence of resistivity

The resistivity of a material is dependent on temperature. It is experimentally found that for a wide range of temperatures, the resistivity of a conductor increases with increase in temperature according to the expression,

$$\rho_T = \rho_\circ\left[1+\alpha\left(T-T_\circ\right)\right] \qquad (2.27)$$

where ρ_T is the resistivity of a conductor at $T\,^\circ C$, ρ_o is the resistivity of the conductor at some reference temperature T_o (usually at 20°C) and α is the temperature coefficient of resistivity. **It is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at T_o.**

From the equation (2.27), we can write

$$\rho_T - \rho_\circ = \alpha\rho_\circ\left(T-T_\circ\right)$$

$$\therefore \alpha = \frac{\rho_T - \rho_\circ}{\rho_\circ\left(T-T_\circ\right)} = \frac{\Delta\rho}{\rho_\circ\Delta T}$$

where $\Delta\rho = \rho_T - \rho_o$ is change in resistivity for a change in temperature $\Delta T = T - T_o$. Its unit is per °C.

α of conductors

For conductors α *is positive.* If the temperature of a conductor increases, the average kinetic energy of electrons in the conductor increases. This results in more frequent collisions and hence the resistivity increases. The graph of the equation (2.27) is shown in Figure 2.13

Even though, the resistivity of conductors like metals varies linearly for wide range of temperatures, there also exists a non-linear region at very low temperatures. The resistivity approaches some finite value as the temperature approaches absolute zero as shown in Figure 2.13(b).

As the resistance is directly proportional to resistivity of the material, we can also write the resistance of a conductor at temperature T °C as

$$R_T = R_\circ\left[1+\alpha\left(T-T_\circ\right)\right] \qquad (2.28)$$

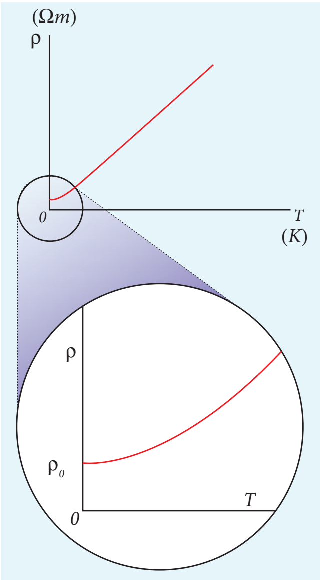


Figure 2.13 (a) Temperature dependence of resistivity for a conductor (b) Non linear region at low temperature

The temperature coefficient can be also be obtained from the equation (2.28),

$$R_T - R_\circ = \alpha R_\circ (T - T_\circ)$$

$$\therefore \alpha = \frac{R_T - R_\circ}{R_\circ (T - T_\circ)} = \frac{1}{R_\circ}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{R_\circ}\frac{\Delta R}{\Delta T} \quad (2.29)$$

where $\Delta R = R_T - R_\circ$ is change in resistance during the change in temperature $\Delta T = T - T_\circ$

α of semiconductors

For semiconductors, the resistivity decreases with increase in temperature. As the temperature increases, more electrons will be liberated from their atoms (Refer unit 9 for conduction in semi conductors). Hence the current increases and therefore the resistivity decreases as shown in Figure 2.14. A semiconductor with a negative temperature coefficient of resistance is called a thermistor.

The typical values of temperature coefficients of various materials are given in table 2.3.

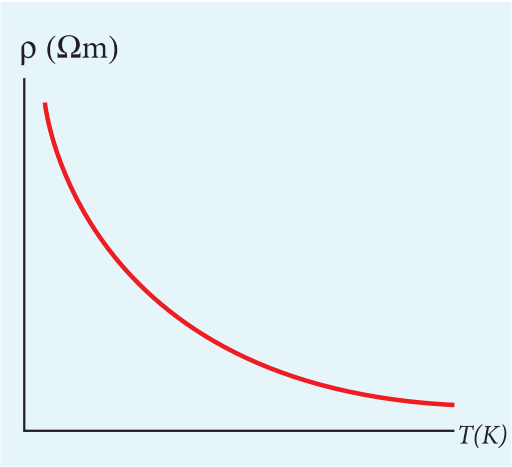


Figure 2.14 Temperature dependence of resistivity for a semiconductor

Table 2.3

Color	Temperature Coefficient α [(°C)$^{-1}$]
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.92×10^{-3}
Lead	3.9×10^{-3}
Nichrome	0.4×10^{-3}
Carbon	-0.5×10^{-3}
Germanium	-48×10^{-3}
Silicon	-75×10^{-3}

We can understand the temperature dependence of resistivity in the following way. In section 2.1.3, we have shown that the electrical conductivity, $\sigma = \frac{ne^2\tau}{m}$. As the resistivity is inverse of σ, it can be written as,

$$\rho = \frac{m}{ne^2\tau} \quad (2.30)$$

The resistivity of materials is

i) inversely proportional to the number density (n) of the electrons

ii) inversely proportional to the average time between the collisions (τ).

In metals, if the temperature increases, the average time between the collision (τ) decreases and n is independent of temperature. In semiconductors when temperature increases, n increases and τ decreases, but increase in n is dominant than decreasing τ, so that overall resistivity decreases.

DO YOU KNOW?

The resistance of certain materials become zero below certain temperature T_c. This temperature is known as critical temperature or transition temperature. The materials which exhibit this property are known as superconductors. This phenomenon was first observed by Kammerlingh Onnes in 1911. He found that mercury exhibits superconductor behaviour at 4.2 K. Since R = 0, current once induced in a superconductor persists without any potential difference.

EXAMPLE 2.13

If the resistance of coil is 3 Ω at 20 ^{0}C and α = 0.004/^{0}C then determine its resistance at 100 ^{0}C.

Solution

$R_0 = 3\ \Omega,\quad T = 100^0\text{C},\quad T_0 = 20^0\text{C}$

$\alpha = 0.004/^0\text{C},\quad R_T = ?$

$R_T = R_0(1 + \alpha(T - T_0))$

$R_{100} = 3(1 + 0.004 \times 80)$

$R_{100} = 3(1 + 0.32)$

$R_{100} = 3(1.32)$

$R_{100} = 3.96\ \Omega$

EXAMPLE 2.14

Resistance of a material at 10^{0}C and 40^{0}C are 45 Ω and 85 Ω respectively. Find its temperature co-efficient of resistance.

Solution

$T_0 = 10\ ^0\text{C},\ T = 40\ ^0\text{C},\ R_0 = 45\ \Omega,\ R = 85\ \Omega$

$$\alpha = \frac{1}{R_\circ}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{45}\left(\frac{85-45}{40-10}\right) = \frac{1}{45}\left(\frac{40}{30}\right)$$

$$\alpha = 0.0296\, per^\circ C$$

2.3 ENERGY AND POWER IN ELECTRICAL CIRCUITS

When a battery is connected between the ends of a conductor, a current is established. The battery is transporting energy to the device which is connected in the circuit. Consider a circuit in which a battery of voltage V is connected to the resistor as shown in Figure 2.15.

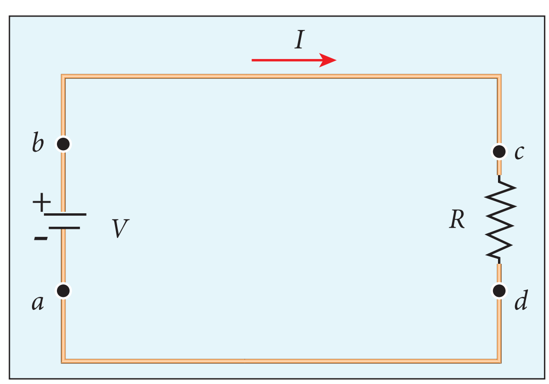


Figure 2.15 Energy given by the battery

Assume that a positive charge of *dQ* moves from point *a* to *b* through the battery and moves from point *c* to *d* through the resistor and back to point a. When the charge moves from point a to b, it gains potential energy $dU = V.dQ$ and the chemical potential energy of the battery decreases by the same amount. When this charge dQ passes through resistor it loses the potential energy $dU = V.dQ$ due to collision with atoms in the resistor and again reaches the point a. This process occurs continuously till the battery is connected in the circuit. The rate at which the charge loses its electrical potential energy in the resistor can be calculated.

The electrical power P is the rate at which the electrical potential energy is delivered,

$$P = \frac{dV}{dt} = \frac{d}{dt}(V.dQ) = V\frac{dQ}{dt} \qquad (2.31)$$

Since the electric current $I = \frac{dQ}{dt}$.

So the equation (2.31) can be rewritten as

$$P = VI \qquad (2.32)$$

This expression gives the power delivered by the battery to any electrical system, where I is the current passing through it and V is the potential difference across it. The SI unit of electrical power is watt ($1W = 1\ J s^{-1}$). Commercially, the electrical bulbs used in houses come with the power and voltage rating of 5W-220V, 30W-220V, 60W-220V etc. (Figure 2.16).

Figure 2.16 Electrical bulbs with power rating

Usually these voltage rating refers AC RMS voltages. For a given bulb, if the voltage drop across the bulb is greater than voltage rating, the bulb will fuse.

Using Ohm's law, power delivered to the resistance R is expressed in other forms

$$P = IV = I\,(IR) = I^2R \qquad (2.33)$$

$$P = IV = \frac{V}{R}V = \frac{V^2}{R} \qquad (2.34)$$

DO YOU KNOW?

The electrical power produced (dissipated) by a resistor is I^2R. It depends on the square of the current. Hence, if current is doubled, the power will increase by four times. Similar explanation holds true for voltage also.

The total energy used by any device is obtained by multiplying the power and duration of the time when it is ON. If the power is in watts and the time is in seconds, the energy will be in joules. In practice, electrical energy is measured in kilowatt hour (kWh). 1 kWh is known as 1 unit of electrical energy.

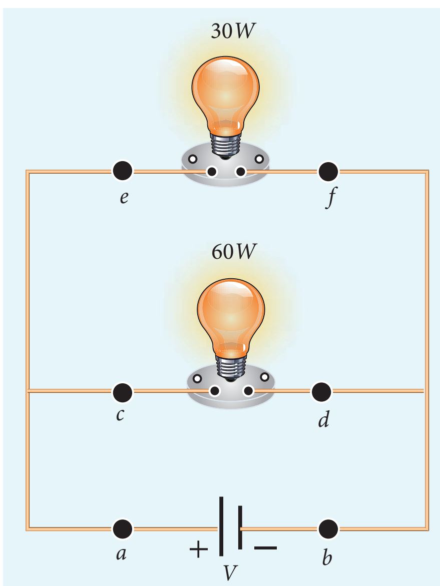
(1 kWh = 1000 Wh = (1000 W) (3600 s) = 3.6 x 10^6 J)

DO YOU KNOW?

The Tamilnadu Electricity Board is charging for the amount of energy you use and not for the power. A current of 1A flowing through a potential difference of 1V produces a power of 1W.

EXAMPLE 2.15

A battery of voltage V is connected to 30 W bulb and 60 W bulb as shown in the figure. (a) Identify brightest bulb (b) which bulb has greater resistance? (c) Suppose the two bulbs are connected in series, which bulb will glow brighter?



Solution

(a) The power delivered by the battery P = VI. Since the bulbs are connected in parallel, the voltage drop across each bulb is the same. If the voltage is kept fixed, then the power is directly proportional to current (P $\propto$ I). So 60 W bulb draws twice as much as current as 30 W and it will glow brighter then others.

(b) To calculate the resistance of the bulbs, we use the relation $P = \frac{V^2}{R}$. In both the bulbs, the voltage drop is the same, so the power is inversely proportional to the resistance or resistance is inversely proportional to the power $\left(R \propto \frac{1}{P}\right)$. It implies that, the 30W has twice as much as resistance as 60 W bulb.

(c) When these two bulbs are connected in series, the current passing through each bulb is the same. It is equivalent to two resistors connected in series. The bulb which has higher resistance has higher voltage drop. So 30W bulb will glow brighter than 60W bulb. So the higher power rating does not always imply more brightness and it depends whether bulbs are connected in series or parallel.

EXAMPLE 2.16

Two electric bulbs marked 20 W – 220 V and 100 W – 220 V are connected in series to 440 V supply. Which bulb will be fused?

Solution

To check which bulb will be fused, the voltage drop across each bulb has to be calculated.

The resistance of a bulb,

$$R = \frac{V^2}{P} = \frac{(\text{Rated voltage})^2}{\text{Rated power}}$$

For 20W-220V bulb,

$$R_1 = \frac{(220)^2}{20}\Omega = 2420\Omega$$

For 100W-220V bulb,

$$R_2 = \frac{(220)^2}{100}\Omega = 484\Omega$$

Both the bulbs are connected in series. So the current which passes through both the bulbs are same. The current that passes through the circuit, $I = \frac{V}{R_{tot}}$.

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$$R_{tot} = (R_1 + R_2)$$

$$R_{tot} = (484 + 2420)\Omega = 2904\Omega$$

$$I = \frac{440V}{2904\Omega} \approx 0.151A$$

The voltage drop across the 20W bulb is

$$V_1 = IR_1 = \frac{440}{2904} \times 2420 \approx 366.6\ V$$

The voltage drop across the 100W bulb is

$$V_2 = IR_2 = \frac{440}{2904} \times 484 \approx 73.3\ V$$

The 20 W bulb will be fused because its voltage rating is only 220 V and 366.6 V is dropped across it.

2.4 ELECTRIC CELLS AND BATTERIES

An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrodes immersed in an electrolyte as shown in Figure 2.17.

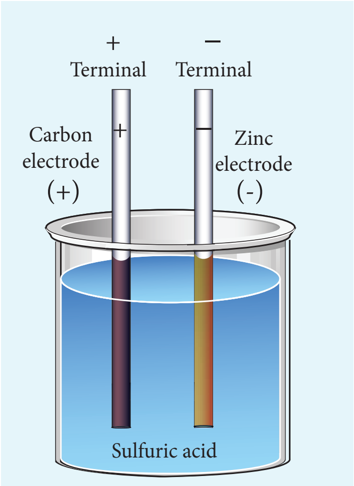


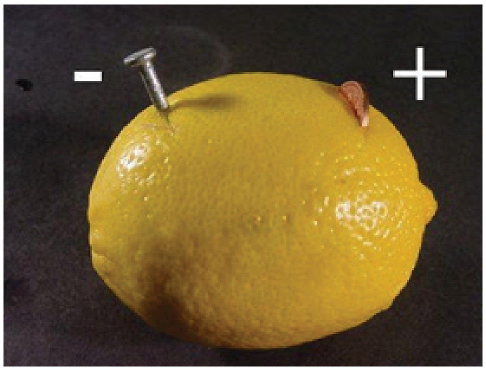
Figure 2.17 Simple electric cell

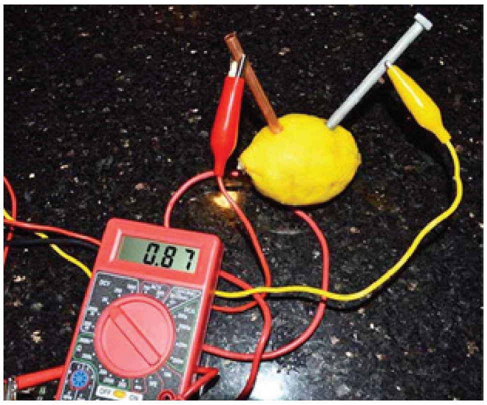
Several electric cells connected together form a battery. When a cell or battery is connected to a circuit, electrons flow from the negative terminal to the positive terminal through the circuit. By using chemical reactions, a battery produces potential difference across its terminals. This potential difference provides the energy to move the electrons through the circuit. Commercially available electric cells and batteries are shown in Figure 2.18

Figure 2.18 Electric cells and Batteries

DO YOU KNOW?

If we connect copper and zinc rod in a lemon, it acts as an electric cell. The citric acid in the lemon acts as an electrolyte. The potential can be measured using a multimeter.





2.4.1 Electromotive force and internal resistance

A battery or cell is called a source of electromotive force (emf). The term 'electromotive force' is a misnomer since it does not really refer to a force but describes a potential difference in volts. The emf of a battery or cell is the voltage provided by the battery when no current flows in the external circuit. It is shown in Figure 2.19.

Figure 2.19 Measuring the emf of a cell

Electromotive force determines the amount of work a battery or cell does to move a certain amount of charge around the circuit. It is denoted by the symbol ξ and to be pronounced as 'xi'. An ideal battery has zero internal resistance and the potential difference (terminal voltage) across the battery equals to its emf. But a real battery is made of electrodes and electrolyte, there is resistance to the flow of charges within the battery. This resistance is called internal resistance r. For a real battery, the terminal voltage is not equal to the emf of the battery. A freshly prepared cell has low internal resistance and it increases with ageing.

2.4.2 Determination of internal resistance

The circuit connections are made as shown in Figure 2.20.

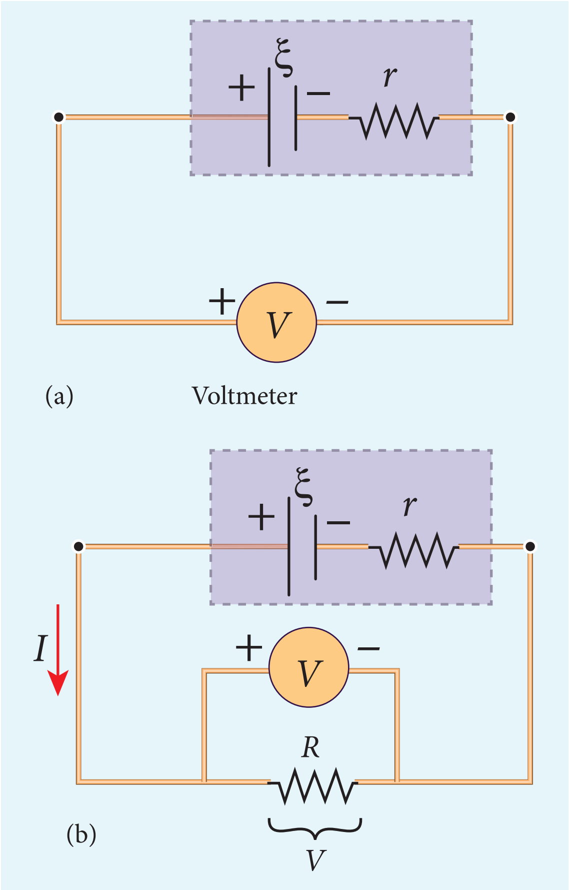


Figure 2.20 Internal resistance of the cell

The emf of cell ξ is measured by connecting a high resistance voltmeter across it without connecting the external resistance R as shown in Figure 2.20(a). Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell. Then, external resistance R is included in the circuit and current I is established in the circuit. The potential difference across R is equal to the potential difference across the cell (V) as shown in Figure 2.20(b).

The potential drop across the resistor R is

$$V = IR \qquad (2.35)$$

Due to internal resistance r of the cell, the voltmeter reads a value V, which is less than the emf of cell ξ. It is because, certain

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amount of voltage (Ir) has dropped across the internal resistance *r*.

Then $V = \xi - Ir$

$$Ir = \xi - V \qquad (2.36)$$

Dividing equation (2.36) by equation (2.35), we get

$$\frac{Ir}{IR} = \frac{\xi - V}{V}$$

$$r = \left[\frac{\xi - V}{V}\right] R \qquad (2.37)$$

Since ξ, V and R are known, internal resistance r can be determined. We can also find the total current that flows in the circuit.

Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery. For a battery of emf ξ, with an internal resistance r, the power delivered to the circuit of resistance R is given by

$P = I\xi = I\,(V + Ir)$ (from equation 2.36)

Here V is the voltage drop across the resistance R and it is equal to IR.

Therefore, P = I (IR +Ir)

$$P = I^2 R + I^2 r \qquad (2.38)$$

Here I^2 r is the power delivered to the internal resistance and I^2 R is the power delivered to the electrical device (here it is the resistance R). For a good battery, the internal resistance r is very small, then $I^2 r << I^2 R$ and almost entire power is delivered to the resistance.

EXAMPLE 2.17

A battery has an emf of 12 V and connected to a resistor of 3 Ω. The current in the circuit is 3.93 A. Calculate (a) terminal voltage and the internal resistance of the battery (b) power delivered by the battery and power delivered to the resistor

Solution

The given values $I = 3.93$ A, $\xi = 12$ V, $R = 3\,\Omega$

(a) The terminal voltage of the battery is equal to voltage drop across the resistor

V = IR = 3.93 × 3 = 11.79 V

The internal resistance of the battery,

$$r = \left[\frac{\xi - V}{V}\right] R = \left[\frac{12 - 11.79}{11.79}\right] \times 3 = 0.05\,\Omega$$

(b) The power delivered by the battery P = Iξ = 3.93 × 12 = 47.1 W

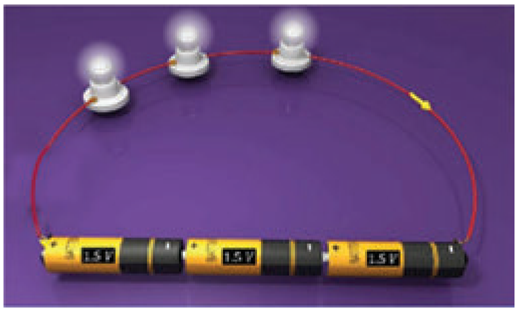
The power delivered to the resistor = I^2 R = 46.3 W

The remaining power = (47.1 – 46.3) P = 0.772 W is delivered to the internal resistance and cannot be used to do useful work. (it is equal to $I^2 r$).

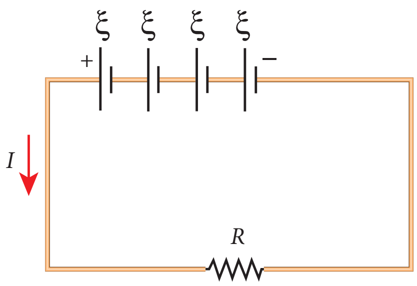
2.4.3 Cells in series

Several cells can be connected to form a battery. In series connection, the negative terminal of one cell is connected to the positive terminal of the second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery.

Suppose n cells, each of emf ξ volts and internal resistance r ohms are connected in series with an external resistance R as shown in Figure 2.21



Cella in series (Schematic diagram)



Cells in series (circuit diagram)

Figure 2.21 cells in series

The total emf of the battery = $n\xi$
The total resistance in the circuit = $nr + R$
By Ohm's law, the current in the circuit is

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n\xi}{nr+R} \qquad (2.39)$$

Case (a) If r << R, then,

$$I = \frac{n\xi}{R} \approx nI_1 \qquad (2.40)$$

where, I_1 is the current due to a single cell

$$\left(I_1 = \frac{\xi}{R}\right)$$

Thus, if r is negligible when compared to R the current supplied by the battery is n times that supplied by a single cell.

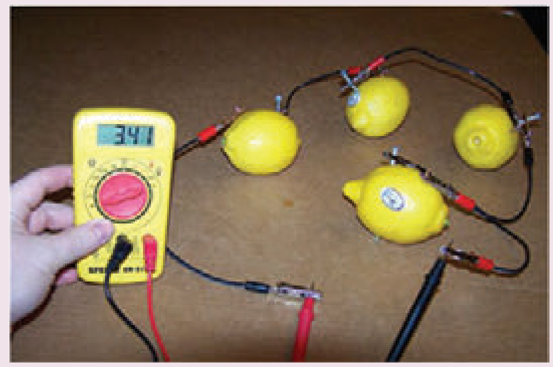
Case (b) If r>>R, $I = \frac{n\xi}{nr} \approx \frac{\xi}{r}$ (2.41)

It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells.

Thus series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with R.

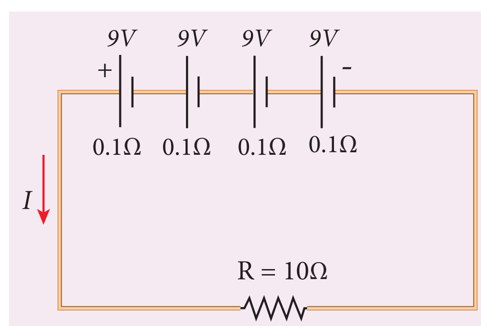
Activity

Construct lemon cells in series and observe the potential of this combination



EXAMPLE 2.18

From the given circuit,



Find

i) Equivalent emf of the combination
ii) Equivalent internal resistance
iii) Total current
iv) Potential difference across external resistance
v) Potential difference across each cell

Solution

i) Equivalent emf of the combination
$\xi_{eq} = n\xi = 4\ 9 = 36$ V

ii) Equivalent internal resistance r_{eq} = nr = 4 × 0.1 = 0.4 Ω

iii) Total current $I = \frac{n\xi}{R+nr}$

$= \frac{4 \times 9}{10+(4 \times 0.1)}$

$= \frac{4 \times 9}{10+0.4} = \frac{36}{10.4}$

I = 3.46 A

iv) Potential difference across external resistance V = IR = 3.46 × 10 = 34.6 V. The remaining 1.4 V is dropped across the internal resistance of cells.

v) Potential difference across each cell $\frac{V}{n} = \frac{34.6}{4} = 8.65V$

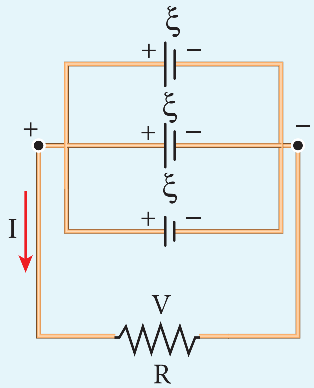
2.4.4 Cells in parallel

In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and negative terminals of the battery.

Let n cells be connected in parallel between the points A and B and a resistance R is connected between the points A and B as shown in Figure 2.22. Let ξ be the emf and r the internal resistance of each cell.

The equivalent internal resistance of the battery is $\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + ... \frac{1}{r}(n terms) = \frac{n}{r}$. So $r_{eq} = \frac{r}{n}$ and the total resistance in the circuit $= R + \frac{r}{n}$. The total emf is the potential difference between the points A and B, which is equal to ξ. The current in the circuit is given by

$$I = \frac{\xi}{\frac{r}{n} + R}$$



cells in parallel (Circuit diagram)

Cells in parallel (Schematic diagram)

Figure 2.22 Cells in parallel

$$I = \frac{n\xi}{r+nR} \quad (2.42)$$

Case (a) If r >> R, $I = \frac{n\xi}{r} = nI_1$ (2.43)

where I_1 is the current due to a single cell and is equal to $\frac{\xi}{r}$ when R is negligible. Thus, the current through the external resistance due to the whole battery is n times the current due to a single cell.

Case (b) If r<<R, $I = \frac{\xi}{R}$ (2.44)

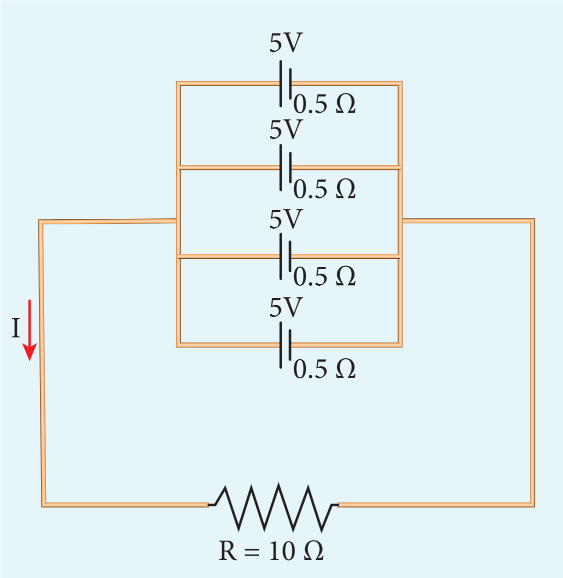
DO YOU KNOW?

When the car engine is started with headlights turned on, they sometimes become dim. This is due to the internal resistance of the car battery.

The above equation implies that current due to the whole battery is the same as that due to a single cell. Hence it is advantageous to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

EXAMPLE 2.19

From the given circuit



Find

i) Equivalent emf
ii) Equivalent internal resistance
iii) Total current (I)
iv) Potential difference across each cell
v) Current from each cell

Solution

i) Equivalent emf $\xi_{eq} = 5$ V

ii) Equivalent internal resistance,

$$R_{eq} = \frac{r}{n} = \frac{0.5}{4} = 0.125\Omega$$

iii) total current, $I = \dfrac{\xi}{R + {r}/{n}}$

$$I = \frac{5}{10 + 0.125} = \frac{5}{10.125}$$

$I \approx 0.5$ A

iv) Potential difference across each cell $V = IR = 0.5 \times 10 = 5$ V

v) Current from each cell, $I' = \dfrac{I}{n}$

$$I' = \frac{0.5}{4} = 0.125\,A$$

2.5 KIRCHHOFF'S RULES

Ohm's law is useful only for simple circuits. For more complex circuits, Kirchhoff's rules can be used to find current and voltage. There are two generalized rules: i) Kirchhoff's current rule ii) Kirchhoff's voltage rule.

2.5.1 Kirchhoff's first rule (Current rule or Junction rule)

It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of conservation of electric charge. All charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction. Current entering the junction is taken as positive and current leaving the junction is taken as negative.

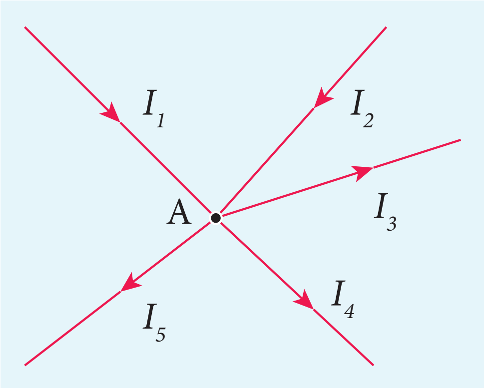


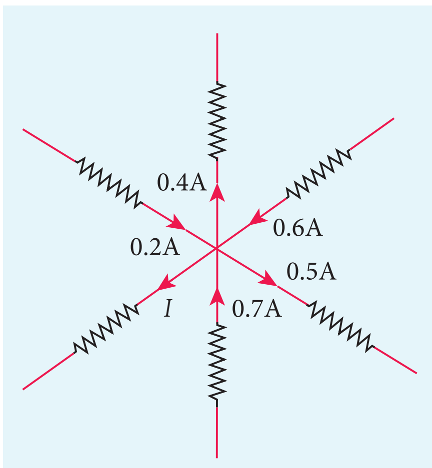
Figure 2.23 Kirchhoff's current rule

Applying this law to the junction A in Figure 2.23

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$
(or)
$$I_1 + I_2 = I_3 + I_4 + I_5$$

EXAMPLE 2.20

From the given circuit find the value of I.



Solution

Applying Kirchoff's rule to the point P in the circuit,

The arrows pointing towards P are positive and away from P are negative.

Therefore, $0.2A - 0.4A + 0.6A - 0.5A + 0.7A - I = 0$

$1.5A - 0.9A - I = 0$

$0.6A - I = 0$

$I = 0.6\ A$

2.5.2 Kirchhoff's Second rule (Voltage rule or Loop rule)

It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors).

The product of current and resistance is taken as positive when the direction of the current is followed. Suppose if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistor is negative. It is shown in Figure 2.24 (a) and (b). The emf is considered positive when proceeding from the negative to the positive terminal of the cell. It is shown in Figure 2.24 (c) and (d).

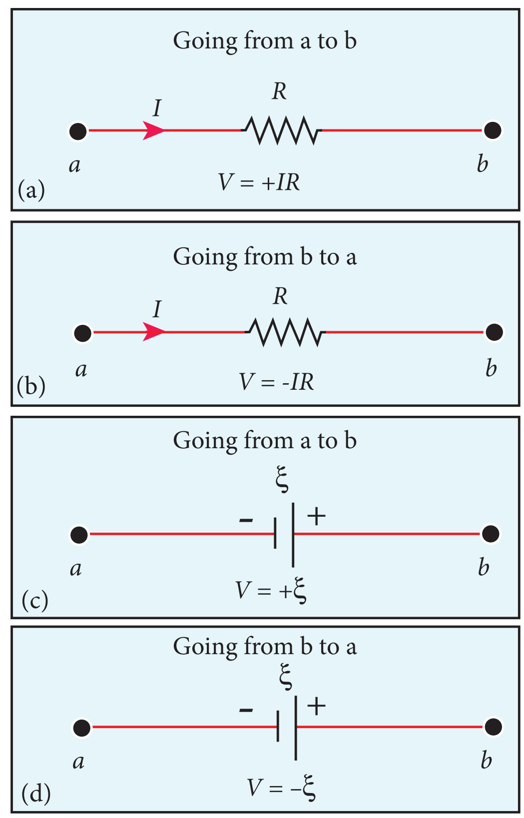
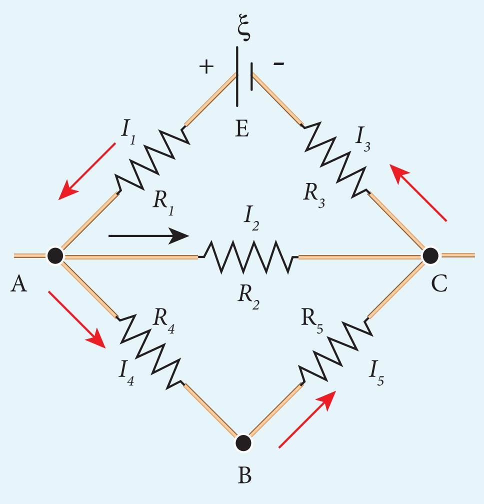


Figure 2.24 Kirchhoff voltage rule

Kirchhoff voltage rule has to be applied only when all currents in the circuit reach a steady state condition (the current in various branches are constant).

EXAMPLE 2.21

The following figure shows a complex network of conductors which can be divided into two closed loops like ACE and ABC. Apply Kirchoff's voltage rule.



Solution

Thus applying Kirchoff's second law to the closed loop EACE

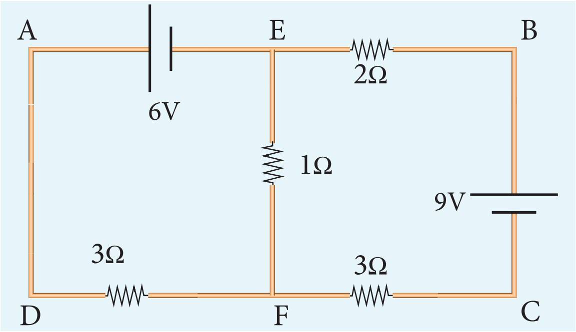
$$I_1R_1 + I_2R_2 + I_3R_3 = \xi$$

and for the closed loop ABCA

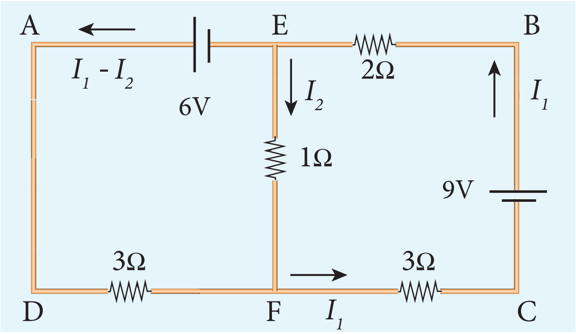
$$I_4R_4 + I_5R_5 - I_2R_2 = 0$$

EXAMPLE 2.22

Calculate the current that flows in the 1 Ω resistor in the following circuit.



Solution



We can denote the current that flows from 9V battery as I_1 and it splits into I_2 and $I_1 - I_2$ in the junction according Kirchoff's current rule (KCR). It is shown below.

Now consider the loop EFCBE and apply KVR, we get

$$1I_2 + 3I_1 + 2I_1 = 9$$

$$5I_1 + I_2 = 9 \qquad (1)$$

Applying KVR to the loop EADFE, we get

$$3\,(I_1 - I_2) - 1I_2 = 6$$

$$3I_1 - 4I_2 = 6 \qquad (2)$$

Solving equation (1) and (2), we get

$$I_1 = 1.83 \text{ A and } I_2 = \text{-}0.13 \text{ A}$$

It implies that the current in the 1 ohm resistor flows from F to E.

2.5.3 Wheatstone's bridge

An important application of Kirchhoff's rules is the Wheatstone's bridge. It is used to compare resistances and also helps in determining the unknown resistance in electrical network. The bridge consists of four resistances P, Q, R and S connected as shown in Figure 2.25. A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current through the galvanometer is I_G and its resistance is G.

Applying Kirchhoff's current rule to junction B

$$I_1 - I_G - I_3 = 0 \qquad (2.45)$$

Applying Kirchhoff's current rule to junction D,

$$I_2 + I_G - I_4 = 0 \qquad (2.46)$$