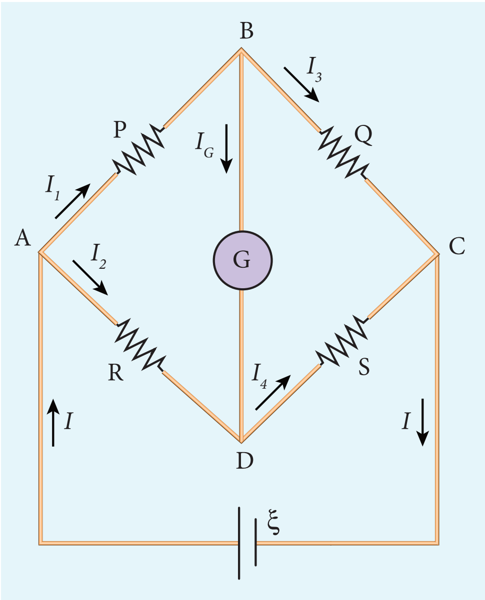


Figure 2.25 Wheatstone's bridge

Applying Kirchhoff's voltage rule to loop ABDA,

$$I_1P + I_GG - I_2R = 0 \qquad (2.47)$$

Applying Kirchhoff's voltage rule to loop ABCDA,

$$I_1P + I_3Q - I_4S - I_2R = 0 \qquad (2.48)$$

When the points B and D are at the same potential, the bridge is said to be balanced. As there is no potential difference between B and D, no current flows through galvanometer ($I_G = 0$). Substituting $I_G = 0$ in equation (2.45), (2.46) and (2.47), we get

$$I_1 = I_3 \qquad (2.49)$$

$$I_2 = I_4 \qquad (2.50)$$

$$I_1P = I_2R \qquad (2.51)$$

Substituting the equation (2.49) and (2.50) in equation (2.48)

$$I_1P + I_1Q - I_2S - I_2R = 0$$

$$I_1(P+Q) = I_2(R+S) \qquad (2.52)$$

Dividing equation (2.52) by equation (2.51), we get

$$\frac{P+Q}{P} = \frac{R+S}{R}$$

$$1+\frac{Q}{P} = 1+\frac{S}{R}$$

$$\frac{Q}{P} = \frac{S}{R}$$

$$\frac{P}{Q} = \frac{R}{S} \qquad (2.53)$$

This is the bridge balance condition. Only under this condition, galvanometer shows null deflection. Suppose we know the values of two adjacent resistances, the other two resistances can be compared. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.

A galvanometer is an instrument used for detecting and measuring even very small electric currents. It is extensively useful to compare the potential difference between various parts of the circuit.

EXAMPLE 2.23

In a Wheatstone's bridge P = 100 Ω, Q = 1000 Ω and R = 40 Ω. If the galvanometer shows zero deflection, determine the value of S.

Solution

$$\frac{P}{Q}=\frac{R}{S}$$

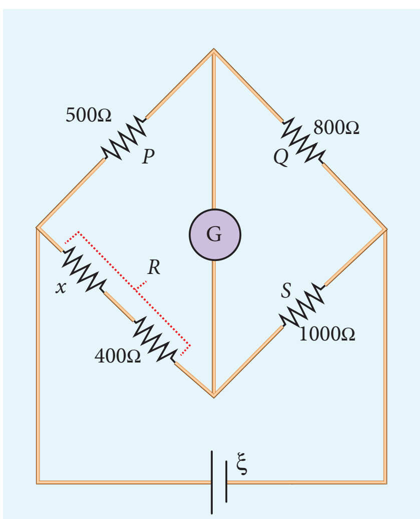
$$S=\frac{Q}{P}\times R$$

$$S=\frac{1000}{100}\times 40 \quad S = 400\ \Omega$$

EXAMPLE 2.24

What is the value of x when the Wheatstone's network is balanced?

P = 500 Ω, Q = 800 Ω, R = x + 400, S = 1000 Ω



Solution

$$\frac{P}{Q}=\frac{R}{S}$$

$$\frac{500}{800}=\frac{x+400}{1000}$$

$$\frac{x+400}{1000}=\frac{500}{800}$$

$$x+400=\frac{500}{800}\times 1000$$

$$x+400=\frac{5}{8}\times 1000$$

$$x + 400 = 0.625 \times 1000$$

$$x + 400 = 625$$

$$x = 625 - 400$$

$$x = 225\ \Omega$$

2.5.4 Meter bridge

The meter bridge is another form of Wheatstone's bridge. It consists of a uniform manganin wire AB of one meter length. This wire is stretched along a meter scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps G_1 and G_2 as shown in Figure 2.26. An unknown resistance P is connected in G_1 and a standard resistance Q is connected in G_2. A jockey (conducting wire) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR). The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected across the ends of the bridge wire.

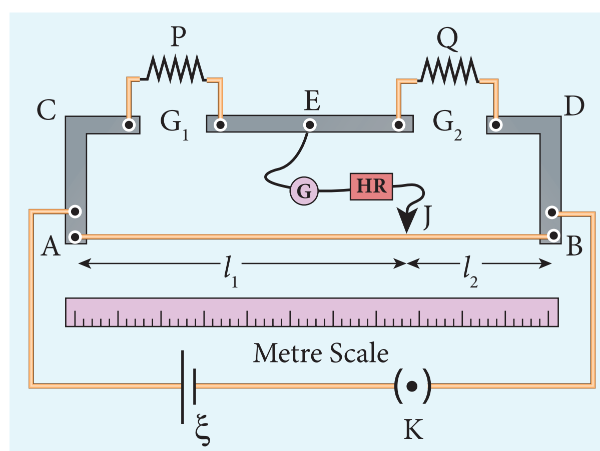


Figure 2.26 Meter bridge

The position of the jockey on the wire is adjusted so that the galvanometer shows zero deflection. Let the point be J. The lengths AJ and JB of the bridge wire now replace the resistance R and S of the Wheatstone's bridge. Then

$$\frac{P}{Q}=\frac{R}{S}=\frac{R'.AJ}{R'.JB} \qquad (2.54)$$

UNIT-2(XII-Physics_Vol-1).indd 111 04-03-2019 11:01:47

where R′ is the resistance per unit length of wire

$$\frac{P}{Q} = \frac{AJ}{JB} = \frac{l_1}{l_2} \quad (2.55)$$

$$P = Q\frac{l_1}{l_2} \quad (2.56)$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistances. This error can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found.

To find the specific resistance of the material of the wire in the coil P, the radius r and length l of the wire is measured. The specific resistance or resistivity ρ can be calculated using the relation

$$\text{Resistance} = \rho\frac{l}{A}$$

By rearranging the above equation, we get

$$\rho = \text{Resistance} \times \frac{A}{l} \quad (2.57)$$

If P is the unknown resistance equation (2.57) becomes,

$$\rho = P\frac{\pi r^2}{l}$$

EXAMPLE 2.25

In a meter bridge with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance.

Solution

Q = 15 Ω, $l_1 : l_2 = 3:2$

$$\frac{l_1}{l_2} = \frac{3}{2}$$

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

$$P = Q\frac{l_1}{l_2}$$

$$P = 15\,\frac{3}{2} = 22.5\ \Omega$$

EXAMPLE 2.26

In a meter bridge, the value of resistance in the resistance box is 10 Ω. The balancing length is l_1 = 55 cm. Find the value of unknown resistance.

Solution

Q = 10 Ω

$$\frac{P}{Q} = \frac{l_1}{100 - l_1} = \frac{l_1}{l_2}$$

$$P = Q \times \frac{l_1}{100 - l_1}$$

$$P = \frac{10 \times 55}{100 - 55}$$

$$P = \frac{550}{45} = 12.2\ \Omega$$

2.5.5 Potentiometer

Potentiometer is used for the accurate measurement of potential differences, current and resistances. It consists of ten meter long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board. The two free ends A and B are brought to the same side and fixed to copper strips with binding screws. A meter scale is fixed parallel to the wire. A jockey is provided for making contact.

The principle of the potentiometer is illustrated in Figure 2.27. A steady current is maintained across the wire CD by a battery

Bt. The battery, key and the potentiometer wire are connected in series forms the primary circuit. The positive terminal of a primary cell of emf ξ is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.

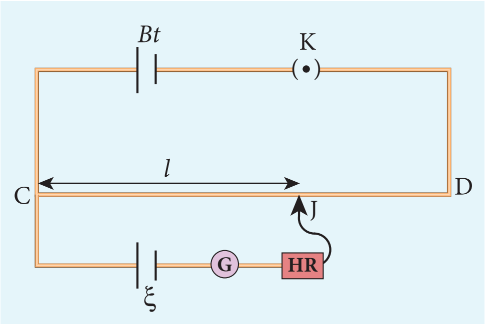


Figure 2.27 Potentiometer

Let contact be made at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell ξ then no current will flow through the galvanometer and it will show zero deflection. CJ is the balancing length l. The potential difference across CJ is equal to Irl where I is the current flowing through the wire and r is the resistance per unit length of the wire.

Hence $\xi = Irl$ (2.58)

Since I and r are constants, $\xi \propto l$. The emf of the cell is directly proportional to the balancing length.

2.5.6 Comparison of emf of two cells with a potentiometer

To compare the emf of two cells, the circuit connections are made as shown in Figure 2.28. Potentiometer wire CD is connected to a battery *Bt* and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR. The cells whose emf ξ_1 and ξ_2 to be compared are connected to the terminals M_1,N_1 and M_2,N_2 of the DPDT switch. The positive terminals of *Bt*, ξ_1 and ξ_2 should be connected to the same end C.

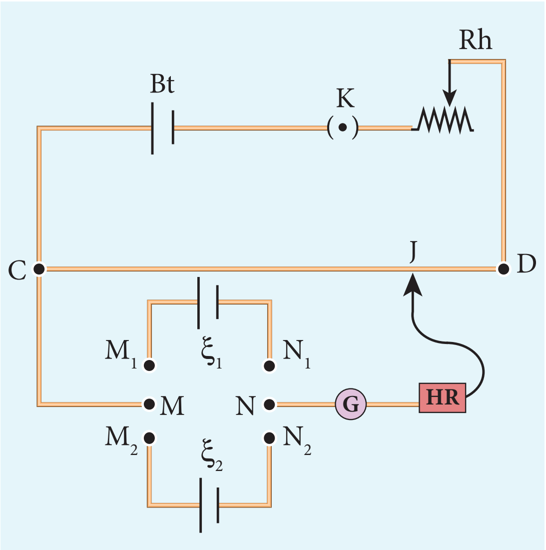


Figure 2.28 Comparison of emf of two cells

The DPDT switch is pressed towards M_1, N_1 so that cell ξ_1 is included in the secondary circuit and the balancing length l_1 is found by adjusting the jockey for zero deflection. Then the second cell ξ_2 is included in the circuit and the balancing length l_2 is determined. Let r be the resistance per unit length of the potentiometer wire and I be the current flowing through the wire.

we have $\xi_1 = Irl_1$ (2.59)

$\xi_2 = Irl_2$ (2.60)

By dividing equation (2.59) by (2.60)

$$\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2} \qquad (2.61)$$

By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it.

2.5.7 Measurement of internal resistance of a cell by potentiometer

To measure the internal resistance of a cell, the circuit connections are made as shown in Figure 2.29. The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K_1. This forms the primary circuit.

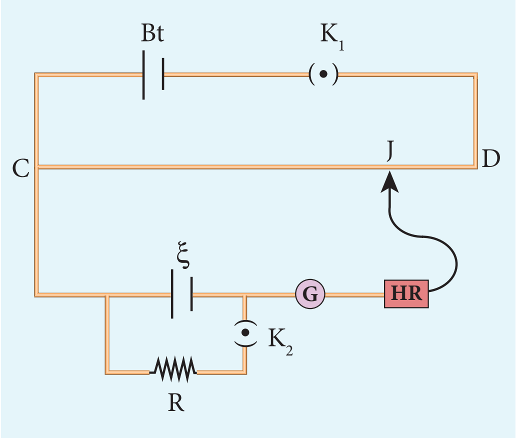


Figure 2.29 measurement of internal resistance

The positive terminal of the cell ξ whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell ξ is connected to a jockey through a galvanometer and a high resistance. A resistance box R and key K_2 are connected across the cell ξ. With K_2 open, the balancing point J is obtained and the balancing length CJ = l_1 is measured. Since the cell is in open circuit, its emf is

$$\xi \propto l_1 \tag{2.62}$$

A suitable resistance (say, 10 Ω) is included in the resistance box and key K_2 is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

$$I = \frac{\xi}{R+r}$$

The potential difference across R is

$$V = \frac{\xi R}{R+r}$$

When this potential difference is balanced on the potentiometer wire, let l_2 be the balancing length.

$$\text{Then } \frac{\xi R}{R+r} \propto l_2 \tag{2.63}$$

From equations (2.62) and (2.63)

$$\frac{R+r}{R} = \frac{l_1}{l_2} \tag{2.64}$$

$$1+\frac{r}{R} = \frac{l_1}{l_2};$$

$$r = R\left[\frac{l_1}{l_2} - 1\right]$$

$$\therefore\ r = R\left(\frac{l_1 - l_2}{l_2}\right) \tag{2.65}$$

Substituting the values of the R, l_1 and l_2, the internal resistance of the cell is determined. The experiment can be repeated for different values of R. It is found that the internal resistance of the cell is not constant but increases with increase of external resistance connected across its terminals.

2.6 HEATING EFFECT OF ELECTRIC CURRENT

When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of

current is known as Joule's heating effect. Just as current produces thermal energy, thermal energy may also be suitably used to produce an electromotive force. This is known as thermoelectric effect.

2.6.1 Joule's law

If a current I flows through a conductor kept across a potential difference V for a time t, the work done or the electric potential energy spent is

$$W = VIt \tag{2.66}$$

In the absence of any other external effect, this energy is spent in heating the conductor. The amount of heat(H) produced is

$$H = VIt \tag{2.67}$$

For a resistance R,

$$H = I^2Rt \tag{2.68}$$

This relation was experimentally verified by Joule and is known as Joule's law of heating. It states that **the heat developed in an electrical circuit due to the flow of current varies directly as**

(i) the square of the current

(ii) the resistance of the circuit and

(iii) the time of flow.

EXAMPLE 2.27

Find the heat energy produced in a resistance of 10 Ω when 5 A current flows through it for 5 minutes.

Solution

R = 10 Ω, I = 5 A, t = 5 minutes = 5 × 60 s

$H = I^2 R t$

$= 5^2 \times 10 \times 5 \times 60$

$= 25 \times 10 \times 300$

$= 25 \times 3000$

= 75000 J (or) 75 kJ

2.6.2 Application of Joule's heating effect

1. Electric heaters

Electric iron, electric heater, electric toaster shown in Figure 2.30 are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.

Figure 2.30 (a) Electric Iron box, (b) electric heater (c) electric Toaster

EXAMPLE 2.28

An electric heater of resistance 10 Ω connected to 220 V power supply is immersed in the water of 1 kg. How long the electrical heater has to be switched on to increase its temperature from 30°C to 60°C. (The specific heat of water is s = 4200 J kg^{-1})

UNIT-2(XII-Physics_Vol-1).indd 115 04-03-2019 11:01:53

Solution

According to Joule's heating law H = I^2 Rt

The current passed through the electrical heater $= \frac{220V}{10\Omega} = 22\,A$

The heat produced in one second by the electrical heater H = I^2 R

The heat produced in one second H = $(22)^2 \times 10$ = 4840 J = 4.84 k J. In fact the power rating of this electrical heater is 4.84 k W.

The amount of energy to increase the temperature of 1kg water from 30°C to 60°C is

Q = ms ΔT (Refer XI physics vol 2, unit 8)

Here m = 1 kg,

s = 4200 J kg^{-1},

ΔT = 30,

so Q = 1 × 4200 x 30 = 126 kJ

The time required to produce this heat energy $t = \frac{Q}{I^2R} = \frac{126\times10^3}{4840} \approx 26.03\ s$

2. Electric fuses

Fuses as shown in Figure 2.31, are connected in series in a circuit to protect the electric devices from the heat developed by the passage of excessive current. It is a short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain value. Lead and copper wire melts and burns out when the current increases above 5 A and 35 A respectively.

The only disadvantage with the above fuses is that once fuse wire is burnt due to excessive current, they need to be replaced. Nowadays in houses, circuit breakers (trippers) are also used instead of fuses.

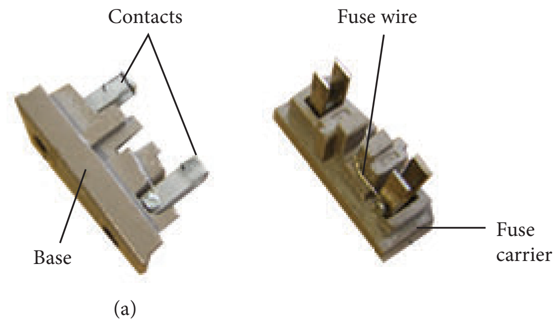


Figure 2.31 Electric Fuse

Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch. It is shown in the Figure 2.32.

Figure 2.32 circuit breakers

3. Electric furnace

Furnaces as shown in Figure 2.33 are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc). To produce temperatures up to 1500°C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000 °C.

Figure 2.33 Electric furnace

4. Electrical lamp

It consists of a tungsten filament (melting point 3380 ^{0}C) kept inside a glass bulb and heated to incandescence by current. In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current as shown in Figure 2.34.

Figure 2.34 Electric bulb, electric arc and electric welding

2.7 THERMOELECTRIC EFFECT

Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect. A thermoelectric device generates voltage when there is a temperature difference on each side. If a voltage is applied, it generates a temperature difference.

2.7.1 Seebeck effect

Seebeck discovered that in a closed circuit consisting of two dissimilar metals, when the junctions are maintained at different temperatures an emf (potential difference) is developed. The current that flows due to the emf developed is called thermoelectric current. The two dissimilar metals connected to form two junctions is known as thermocouple (Figure 2.35).

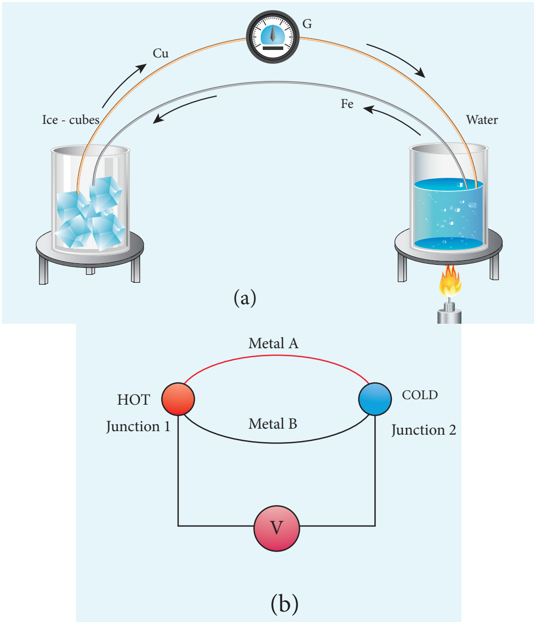


Figure 2.35 Seebeck effect (Thermocouple)

If the hot and cold junctions are interchanged, the direction of current also reverses. Hence the effect is reversible.

The magnitude of the emf developed in a thermocouple depends on (i) the nature of the metals forming the couple and (ii) the temperature difference between the junctions.

Applications of Seebeck effect

1. Seebeck effect is used in thermoelectric generators (Seebeck generators).These thermoelectric generators are used in power plants to convert waste heat into electricity.

2. This effect is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency.
3. Seebeck effect is used in thermocouples and thermopiles to measure the temperature difference between the two objects.

2.7.2 Peltier effect

In 1834, Peltier discovered that **when an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect.**

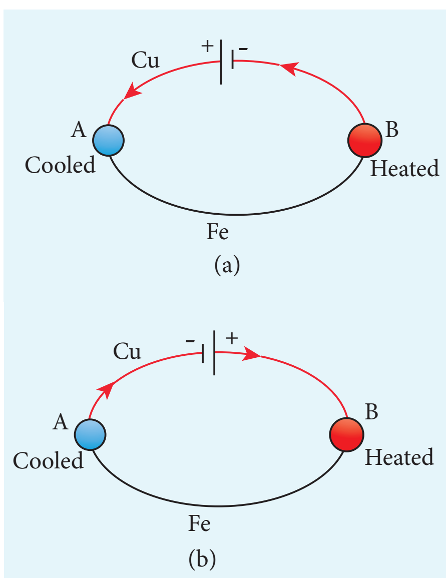


Figure 2.36 Peltier effect: Cu – Fe thermocouple

In the Cu-Fe thermocouple the junctions A and B are maintained at the same temperature. Let a current from a battery flow through the thermocouple (Figure 2.36 (a)). At the junction A, where the current flows from Cu to Fe, heat is absorbed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu heat is liberated and it becomes hot. When the direction of current is reversed, junction A gets heated and junction B gets cooled as shown in the Figure 2.36(b). Hence Peltier effect is reversible.

2.7.3 Thomson effect

Thomson showed that **if two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is also reversible.**

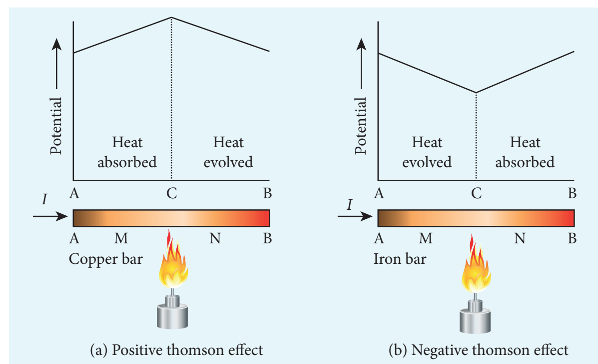


Figure 2.37 (a) Positive Thomson effect (b) Negative Thomson effect

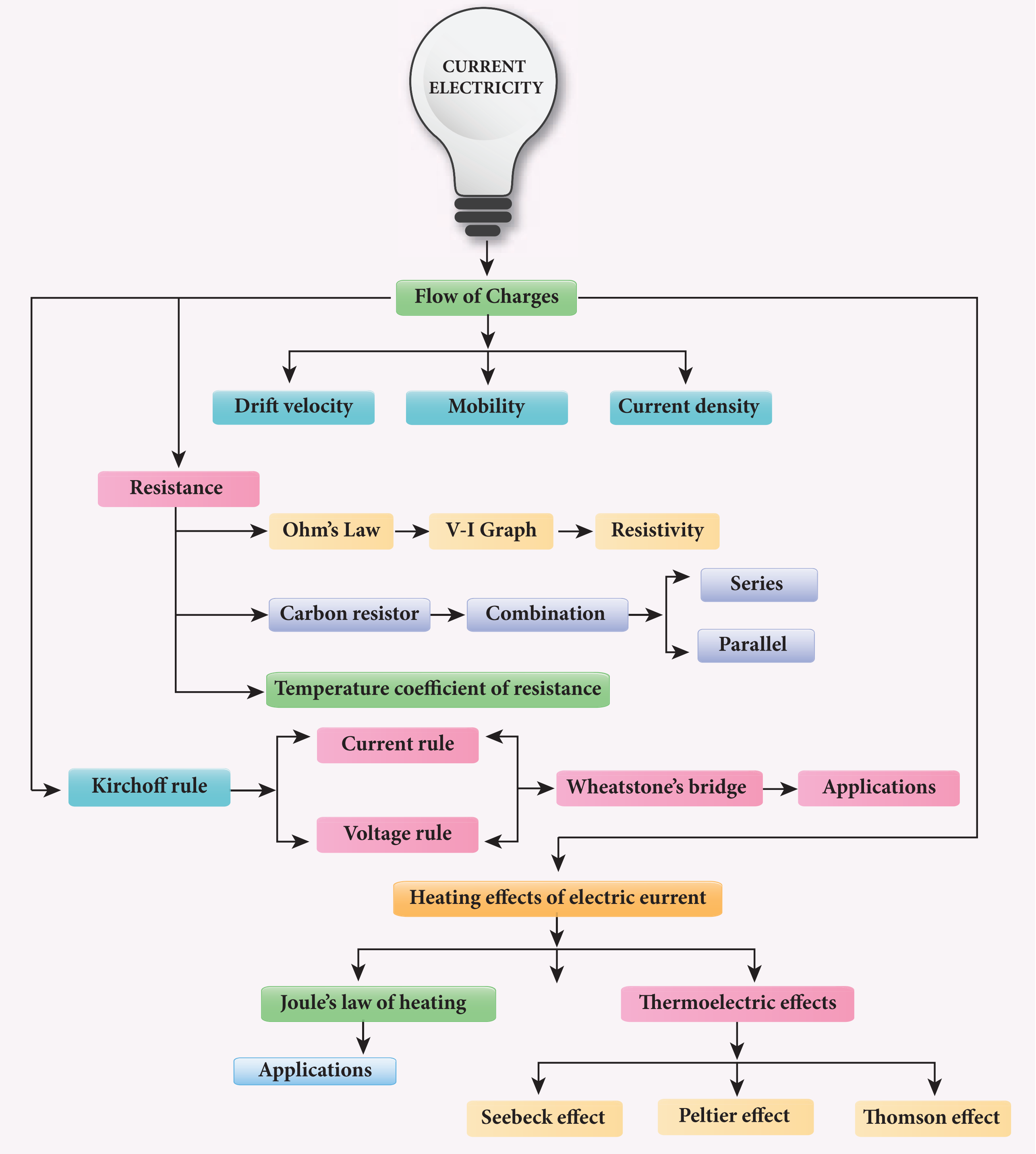
If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is absorbed along AC and evolved along CB of the conductor as shown in Figure 2.37(a). Thus heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Similar effect is observed in metals like silver, zinc, and cadmium.

When the copper bar is replaced by an iron bar, heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect as shown in the Figure 2.37(b). Similar effect is observed in metals like platinum, nickel, cobalt, and mercury.

SUMMARY

- The current, I flowing in a conductor $I = \frac{dQ}{dt}$, where dQ is the charge that flows through a cross-section in a time interval dt. SI unit of current is ampere (A). $1A = 1\ C\ s^{-1}$.
- The current density J in a conductor is the current flowing per unit area. $\left(J = \frac{I}{A}\right)$
- Current is a scalar but current density is a vector.
- The general form of Ohm's law $\vec{J} = \sigma\vec{E}$
- Practical form of Ohm's law states that $V \propto I$, or $V = IR$ where I is the current.
- The resistance R of a conductor is $R = \frac{V}{I}$. SI unit of resistance is ohm (Ω) and $1\ \Omega = \frac{1V}{1A}$
- The resistance of a material $R = \rho\frac{l}{A}$ where l is length of the material and A is the area of cross section.
- The resistivity of a material determines how much resistance it offers to the flow of current.
- The equivalent resistance (R_S) of several resistances (R_1, R_2, R_3........) connected in series combination is $R_S = (R_1 + R_2 + R_3 \ldots\ldots)$
- The equivalent resistance (R_P) of several resistances (R_1, R_2, R_3........) connected in parallel combination is $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots$
- Kirchoff's first rule (Current rule or junction rule): The algebraic sum of the currents at any junction is zero.
- Kirchoff's second rule (Voltage rule or loop rule): In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- Electric power is the rate at which energy is transformed.
- If a current I flows across a potential difference V, the power delivered to the circuit is P = IV.
- In a resistor R, the electrical power converted to heat is $P = I^2R = \frac{V^2}{R}$
- The energy equivalent of one kilowatt-hour (kWh) is $1kWh = 3.6 \times 10^6$ J.
- Metre bridge is one form of Wheatstone's bridge.
- Potentiometer is used to compare potential differences.
- Joule's law of heating is $H = VIt$ (or) $H = I^2Rt$.
- Thermoelectric effect: Conversion of temperature differences into electrical voltage and vice versa.

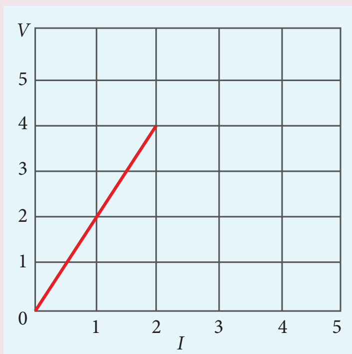
CONCEPT MAP


CURRENT ELECTRICITY
Flow of Charges
Drift velocity
Mobility
Current density
Resistance
Ohm's Law
V-I Graph
Resistivity
Carbon resistor
Combination
Series
Parallel
Temperature coefficient of resistance
Kirchoff rule
Current rule
Voltage rule
Wheatstone's bridge
Applications
Heating effects of electric current
Joule's law of heating
Applications
Thermoelectric effects
Seebeck effect
Peltier effect
Thomson effect

EVALUATION

I Multiple Choice Questions

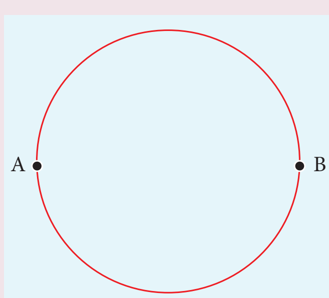
1. The following graph shows current versus voltage values of some unknown conductor. What is the resistance of this conductor?



QMAJDW

(a) 2 ohm (b) 4 ohm
(c) 8 ohm (d) 1 ohm

2. A wire of resistance 2 ohms per meter is bent to form a circle of radius 1m. The equivalent resistance between its two diametrically opposite points, A and B as shown in the figure is



(a) $\pi\ \Omega$ (b) $\frac{\pi}{2}\Omega$
(c) $2\pi\Omega$ (d) $\frac{\pi}{4}\Omega$

3. A toaster operating at 240 V has a resistance of 120 Ω. The power is
a) 400 W b) 2 W
c) 480 W d) 240 W

4. A carbon resistor of (47 ± 4.7) k Ω to be marked with rings of different colours for its identification. The colour code sequence will be
a) Yellow – Green – Violet – Gold
b) Yellow – Violet – Orange – Silver
c) Violet – Yellow – Orange – Silver
d) Green – Orange – Violet - Gold

5. What is the value of resistance of the following resistor?

(a) 100 k Ω (b) 10 k Ω
(c) 1k Ω (d) 1000 k Ω

6. Two wires of A and B with circular cross section made up of the same material with equal lengths. Suppose $R_A = 3\ R_B$, then what is the ratio of radius of wire A to that of B?
(a) 3 (b) $\sqrt{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

7. A wire connected to a power supply of 230 V has power dissipation P_1. Suppose the wire is cut into two equal pieces and connected parallel to the same power supply. In this case power dissipation is P_2. The ratio $\frac{P_2}{P_1}$ is
(a) 1 (b) 2
(c) 3 (d) 4

8. In India electricity is supplied for domestic use at 220 V. It is supplied at 110 V in USA. If the resistance of a 60W bulb for use in India is R, the resistance of a 60W bulb for use in USA will be

(a) R (b) 2R

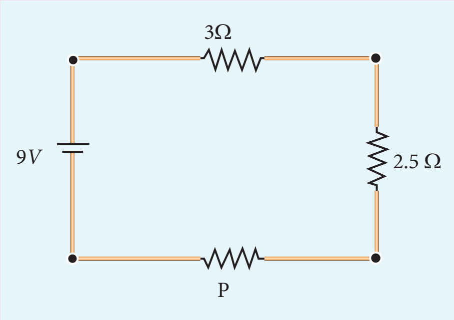
(c) $\frac{R}{4}$ (d) $\frac{R}{2}$

9. In a large building, there are 15 bulbs of 40W, 5 bulbs of 100W, 5 fans of 80W and 1 heater of 1kW are connected. The voltage of electric mains is 220V. The minimum capacity of the main fuse of the building will be (IIT-JEE 2014)

(a) 14 A (b) 8 A

(c) 10 A (d) 12 A

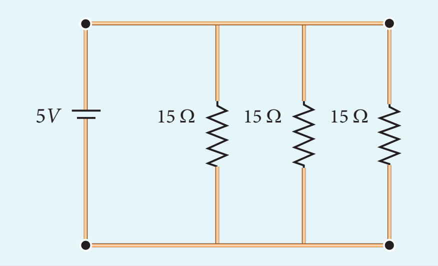
10. There is a current of 1.0 A in the circuit shown below. What is the resistance of P ?



a) 1.5 Ω b) 2.5 Ω

c) 3.5 Ω d) 4.5 Ω

11. What is the current out of the battery?



a) 1A b) 2A

c) 3A d) 4A

12. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K, its resistance is 1 Ω. The resistance of the wire will be 2 Ω at

a) 1154 K b) 1100 K

c) 1400 K d) 1127 K

13. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is

a) 0.2 Ω b) 0.5 Ω

c) 0.8 Ω d) 1.0 Ω

14. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

a) each of them increases

b) each of them decreases

c) copper increases and germanium decreases

d) copper decreases and germanium increases

15. In Joule's heating law, when I and t are constant, if the H is taken along the y axis and I^2 along the x axis, the graph is

a) straight line b) parabola

c) circle d) ellipse

Answers

1) a	**2)** b	**3)** c	**4)** b	**5)** a
6) c	**7)** d	**8)** c	**9)** d	**10)** c
11) a	**12)** d	**13)** b	**14)** d	**15)** a

II Short Answer Questions

1. Why current is a scalar?
2. Distinguish between drift velocity and mobility.
3. State microscopic form of Ohm's law.

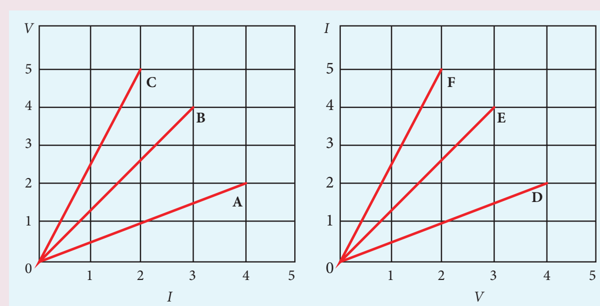
4. State macroscopic form of Ohm's law.
5. What are ohmic and non ohmic devices?
6. Define electrical resistivity.
7. Define temperature coefficient of resistance.
8. What is superconductivity?
9. What is electric power and electric energy?
10. Define current density.
11. Derive the expression for power P=VI in electrical circuit.
12. Write down the various forms of expression for power in electrical circuit.
13. State Kirchhoff's current rule.
14. State Kirchhoff's voltage rule.
15. State the principle of potentiometer.
16. What do you mean by internal resistance of a cell?
17. State Joule's law of heating.
18. What is Seebeck effect?
19. What is Thomson effect?
20. What is Peltier effect?
21. State the applications of Seebeck effect.

III Long Answer Questions

1. Describe the microscopic model of current and obtain general form of Ohm's law
2. Obtain the macroscopic form of Ohm's law from its microscopic form and discuss its limitation.
3. Explain the equivalent resistance of a series and parallel resistor network
4. Explain the determination of the internal resistance of a cell using voltmeter.
5. State and explain Kirchhoff's rules.
6. Obtain the condition for bridge balance in Wheatstone's bridge.
7. Explain the determination of unknown resistance using meter bridge.
8. How the emf of two cells are compared using potentiometer?

IV Numerical problems

1. The following graphs represent the current versus voltage and voltage versus current for the six conductors A,B,C,D,E and F. Which conductor has least resistance and which has maximum resistance?



Ans: Least: $R_F = 0.4\ \Omega$, maximum $R_C = 2.5\ \Omega$

2. Lightning is very good example of natural current. In typical lightning, there is 10^9 J energy transfer across the potential difference of 5×10^7 V during a time interval of 0.2 s.

Using this information, estimate the following quantities (a) total amount of charge transferred between cloud and ground (b) the current in the lightning bolt (c) the power delivered in 0.2 s.

Ans: charge = 20 C, I = 100 A, P = 5 GW

3. A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of electrons per cubic meter is 8×10^{28}, calculate the current density and average drift velocity.

Ans: $J = 2 \times 10^6$ Am^{-2} ; $v_d = 15.6 \times 10^{-5}$ ms^{-1}

4. The resistance of a nichrome wire at 0 ^0C is 10 Ω. If its temperature coefficient of resistance is 0.004/^0C, find its resistance at boiling point of water. Comment on the result.

Ans: $R_T = 14$ Ω.

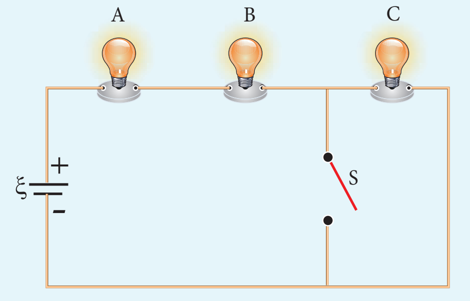
As the temperature increases the resistance of the wire also increases.

5. The rod given in the figure is made up of two different materials.

Both have square cross sections of 3 mm side. The resistivity of the first material is 4 x 10^{-3} Ω.m and it is 25 cm long while second material has resistivity of 5 x 10^{-3} Ω.m and is of 70 cm long. What is the resistivity of rod between its ends?

Ans: 500 Ω

6. Three identical lamps each having a resistance R are connected to the battery of emf as shown in the figure.

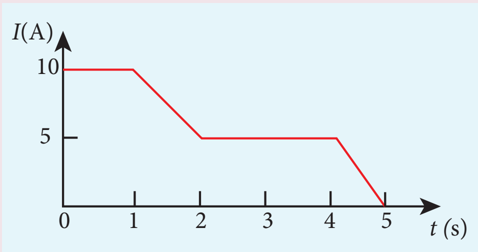


Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A,B and C. (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decreases, increases or remain same?

Ans:

Electrical quantities	Switch S is open	Switch S is closed
Current	$\frac{\xi}{3R}$	$\frac{\xi}{2R}$
Voltage	$V_A = \frac{\xi}{3R}$, $V_B = \frac{\xi}{3R}$, $V_C = \frac{\xi}{3R}$	$V_A = \frac{\xi}{2R}$, $V_B = \frac{\xi}{2R}$, $V_C = 0$
Power	$P_A = \frac{\xi^2}{9R}$, $P_B = \frac{\xi^2}{9R}$, $P_C = \frac{\xi^2}{9R}$	$P_A = \frac{\xi^2}{4R}$, $P_B = \frac{\xi^2}{4R}$, $P_C = 0$ Total power increases
Intensity	All the bulbs glow with equal intensity	The intensities of the bulbs A and B equally increase. Bulb C will not glow since no current pass through it.

7. The current through an element is shown in the figure. Determine the total charge that pass through the element at a) t = 0 s, b) t = 2 s, c) t = 5s



Ans: At t= 0s,dq = 0 C, At t=2 s, dq = 10 C; At t=5 s, dq = 0 C

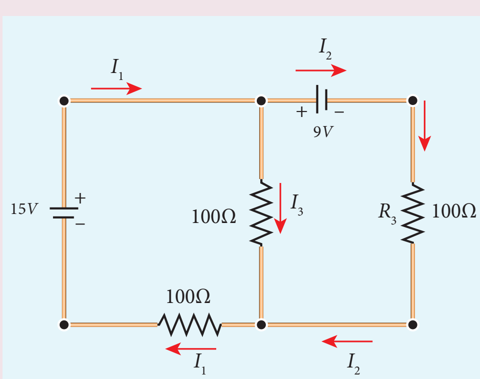
8. An electronics hobbyist is building a radio which requires 150 Ω in her circuit, but she has only 220 Ω, 79 Ω and 92 Ω resistors available. How can she connect the available resistors to get desired value of resistance?

Ans: Parallel combination of 220 Ω and 79 Ω in series with 92 Ω

9. A cell supplies a current of 0.9 A through a 2 Ω resistor and a current of 0.3 A through a 7 Ω resistor. Calculate the internal resistance of the cell.

Ans: 0.5 Ω

10. Calculate the currents in the following circuit.

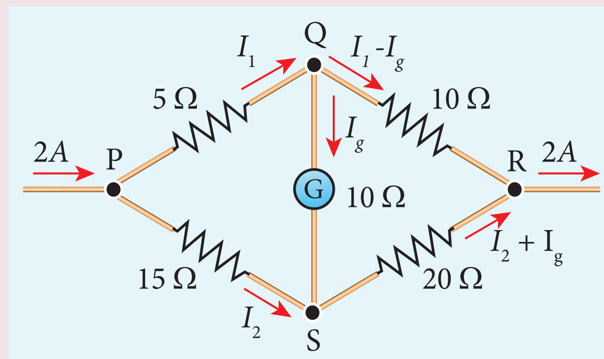


Ans : I_1 = 0.070 A, I_2 = -0.010 A and I_3 = 0.080 A

11. A potentiometer wire has a length of 4 m and resistance of 20 Ω. It is connected in series with resistance of 2980 Ω and a cell of emf 4 V. Calculate the potential along the wire.

Ans: Potential = 0.65×10^{-2} V m^{-1}.

12. Determine the current flowing through the galvanometer (G) as shown in the figure.



Ans: $I_g = \frac{1}{11} A$

13. Two cells each of 5V are connected in series across a 8 Ω resistor and three parallel resistors of 4 Ω, 6 Ω and 12 Ω. Draw a circuit diagram for the above arrangement. Calculate i) the current drawn from the cell (ii) current through each resistor

Ans: The current at 4 Ω ,$I = \frac{2}{4} = 0.5A$,

the current at 6 Ω, $I = \frac{2}{6} = 0.33\,\text{A}$,

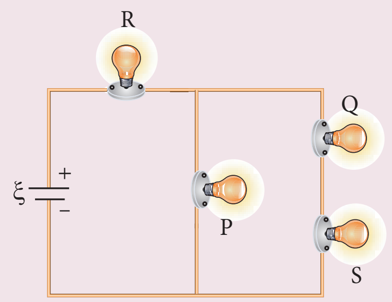
the current at 12 Ω, $I = \frac{2}{12} = 0.17\,\text{A}$

14. Four light bulbs P, Q, R, S are connected in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed.

	P	Q	R	S
P removed	*	on	on	on
Q removed	on	*	on	off
R removed	off	off	*	off
S removed	on	off	on	*

Draw the circuit diagram for these bulbs.

Ans:



15. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?

    Ans: emf of the second cell is 2.25 V

BOOKS FOR REFERENCE:

1. Douglas C.Giancoli, , "*Physics for Scientist &Engineers with Modern Physics*", Pearson Prentice Hall, Fourth edition
2. James Walker, *Physics*, Pearson- Addison Wesley publishers, Fourth edition
3. Tipler, Mosca, "*Physics for scientist and Engineers with Modern Physics*", Freeman and Company, sixth edition
4. Purcell, Morin, *Electricity and magnetism*, Cambridge university press, third edition
5. Serway and Jewett, "*Physics for Scientist and Engineers with Modern Physics*", Brook/Coole publishers, eighth edition
6. Tarasov and Tarasova, "*Questions and problems in School Physics*", Mir Publishers
7. H.C.Verma, "*Concepts of Physics Vol 2*, Bharthi Bhawan publishers
8. Eric Roger, *Physics for the Inquiring Mind*, Princeton University press

UNIT 6

OPTICS

An age is called dark, not because the light fails to shine, but because people refuse to see.
— James Albert Michener

Learning Objectives

In this unit, the students are exposed to,

- The two aspects of treating light as a ray and a wave.
- The behaviour and propagation of light.
- The concepts related to mirrors, lenses, prisms etc.
- The different optical instruments like microscope, telescope etc.
- The terms like magnification and resolving power etc.
- The various phenomena that support the wave nature of light.

X6J5S9

6.1 INTRODUCTION

Light is mystical. Yet, its behaviour is so fascinating. It is difficult to comprehend light to a single entity. In this unit, we learn it in two different scientific aspects called ray optics and wave optics. Ray optics deals with light that is represented as a ray travelling in straight lines. The geometrical constructs get the permanence to understand the various characteristics of light. In wave optics, we study about the phenomenon associated with the propagation of light as a wave. First, let us learn the ray optics followed by the wave optics.

6.1.1 Ray optics

Light travels in a straight line in a medium. Light may deviate in its path only when it encounters another medium or an obstacle. A ray of light gives information about only the direction of light. It does not give information about the other characteristics of light like intensity and colour. However, a ray is a sensible representation of light in ray optics. The path of the light is called a ray of light and a bundle of such rays is called a beam of light. In this chapter, we can explain the phenomena of reflection, refraction, dispersion and scattering of light, using the ray depiction of light.

6.1.2 Reflection

The bouncing back of light into the same medium when it encounters a reflecting surface is called *reflection* of light. Polished surfaces can reflect light. Mirrors which are silver coated at their back can reflect almost 90% of the light falling on them. The angle of incidence i and the angle of reflection r are measured with respect to the normal drawn to the surface at the point of incidence of light. **According to law of reflection,**

(a) The incident ray, reflected ray and normal to the reflecting surface all are coplanar (ie. lie in the same plane).

(b) The angle of incidence i is equal to the angle of reflection r.

$$i = r \qquad (6.1)$$

The law of reflection is shown in Figure 6.1.

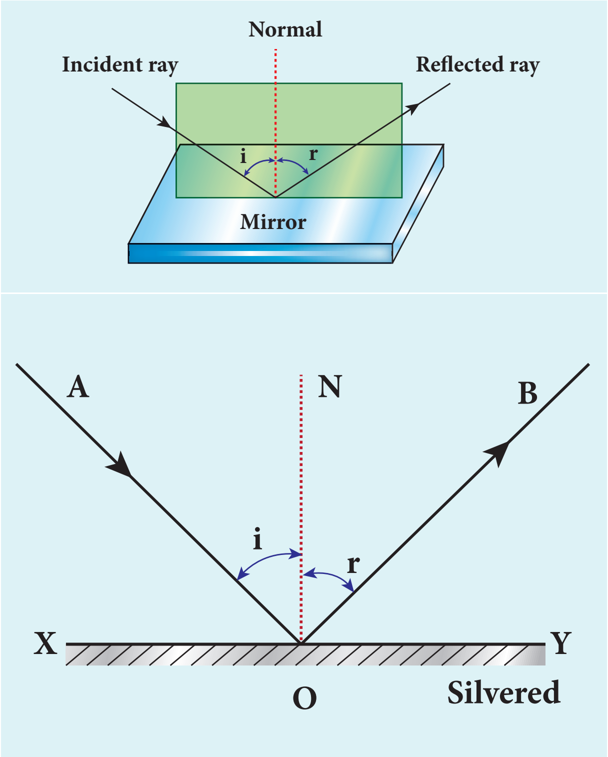


Figure 6.1 Reflection of light

The law of reflection is valid at each point for any reflecting surface whether the surface is plane or curved. If the reflecting surface is flat, then incident parallel rays after reflection come out parallel as per the law of reflection. If the reflecting surface is irregular, then the incident parallel rays after reflection come out irregular (not parallel) rays. Still law of reflection is valid at every point of incidence as shown in Figure 6.2.

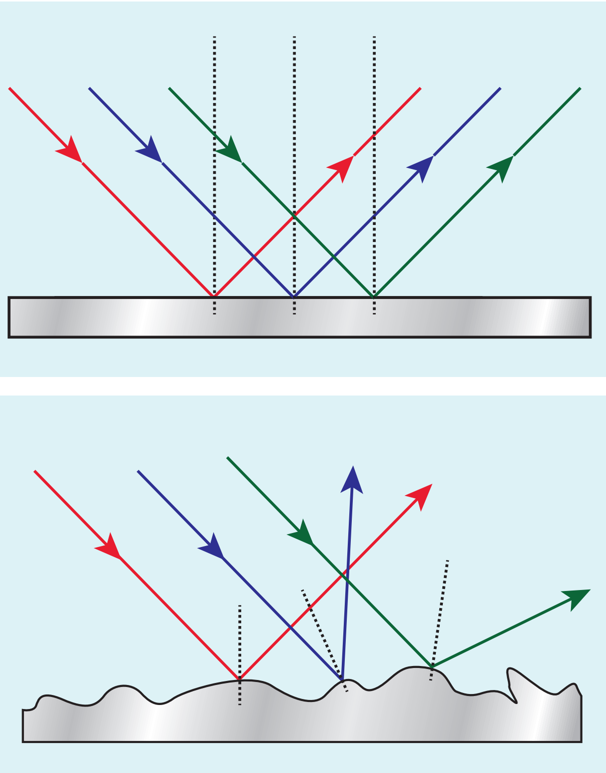


Figure 6.2 Regular and irregular reflections

6.1.3 Angle of deviation due to reflection

The angle between the incident and deviated light ray is called *angle of deviation* of the light ray. In reflection, it is calculated by a simple geometry as shown in Figure 6.3(a). The incident light is AO. The reflected light is OB. The un-deviated light is OC which is the continuation of the incident light. The angle between OB and OC is the angle of deviation d. From the geometry, it is written as, $d = 180 - (i+r)$. As, $i = r$ in reflection, we can write angle of deviation in reflection at plane surface as,

$$d = 180 - 2i \qquad (6.2)$$

The angle of deviation can also be measured in terms of the glancing angle α which is measured between the incident ray AO and the reflecting plane surface XY as shown in Figure 6.3(b). By geometry, the angles $\angle AOX = \alpha$, $\angle BOY = \alpha$ and $\angle YOC = \alpha$ (are all same). The angle of deviation (d) is the angle $\angle BOC$. Therefore,

$$d = 2\alpha \qquad (6.3)$$

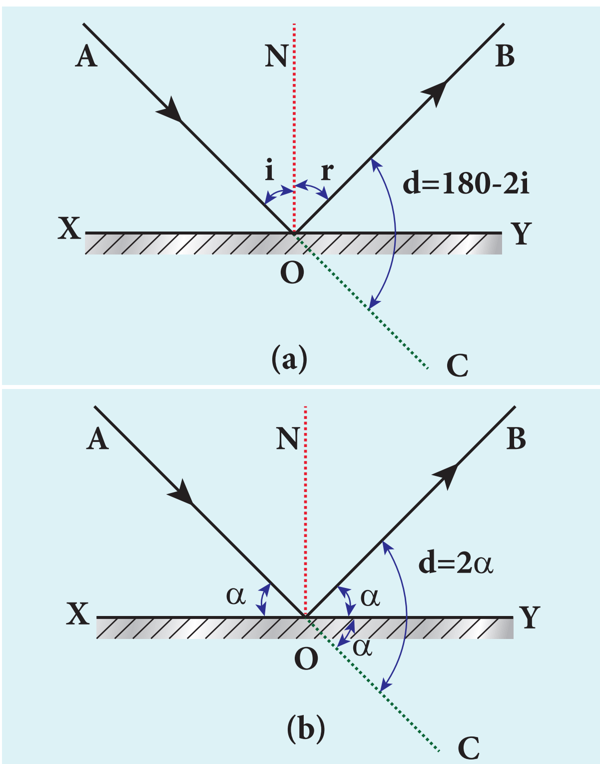


Figure 6.3 Angle of deviation due to reflection

EXAMPLE 6.1

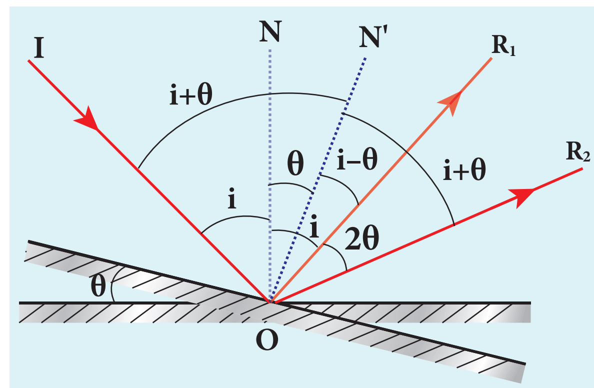
Prove that when a reflecting surface of light is tilted by an angle θ, the reflected light will be tilted by an angle 2θ.

Solution

For the reflecting surface AB, the incident ray IO and the reflected ray OR_1 subtend angle i with the normal N as angle of incidence is equal to angle of reflection as shown in figure. When the surface AB is tilted to $A'B'$ by an angle θ, the normal N' is also is tilted by the same angle θ. Remember the position of incident ray IO remains unaltered. But, in the tilted system the angle of incidence is now $i+\theta$ and the angle of reflection is also $i+\theta$. Now, OR_2 is the reflected ray. The angle between OR_2 and OR_1 is,

$$\angle R_1OR_2 = \angle N'OR_2 - \angle NOR_1$$

$$(i+\theta) - (i-\theta) = 2\theta.$$



6.1.4 Image formation in plane mirror

Let us consider a point object A is placed in front of a plane mirror and the point of incidence is O on the mirror as shown in the Figure. 6.4. A light ray AO from the point object is incident on the mirror and it is reflected along OB. The normal is ON.

The angle of incidence $\angle AON$ = angle of reflection $\angle BON$

Another ray AD incident normally on the mirror at D is reflected back along DA. When BO and AD are extended backwards, they meet at a point A'. Thus, the rays appear to come from a point A' which is behind the plane mirror. The object and its image in a plane mirror are at equal perpendicular distances from the plane mirror which can be shown by the following explanation.

In Figure 6.4, Angle $\angle AON$ = angle $\angle DAO$ [Since they are alternate angles]

Angle $\angle BON$ = angle $\angle OA'D$ [Since they are corresponding angles]

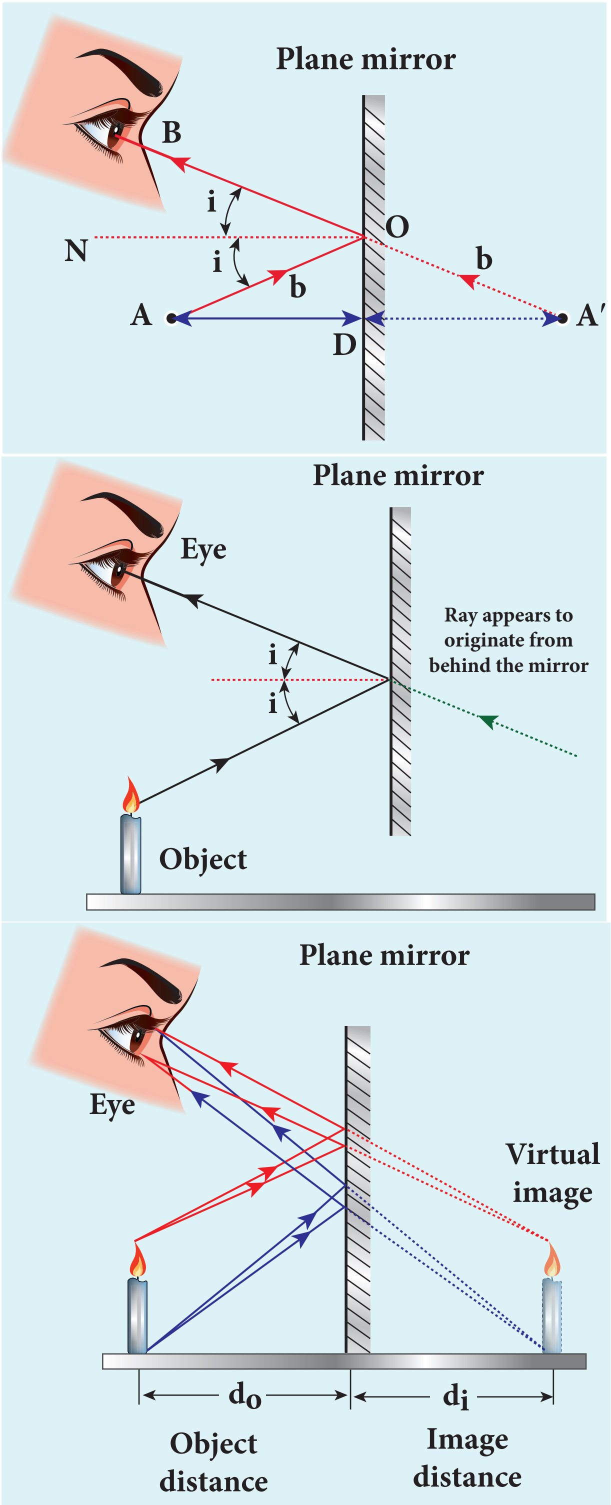


Figure 6.4 Formation of image in plane mirror

Hence, it follows that angle, $\angle DAO = \angle OA'D$

The triangles ΔODA and $\Delta ODA'$ are congruent

$$\therefore AD = A'D$$

This shows that the image distance inside the plane mirror is equal to the object distance in front of the plane mirror.

6.1.5 Characteristics of the image formed by plane mirror

(i) The image formed by a plane mirror is virtual, erect, and laterally inverted.

(ii) The size of the image is equal to the size of the object.

(iii) The image distance far behind the mirror is equal to the object distance in front of it.

(iv) If an object is placed between two plane mirrors inclined at an angle θ, then the number of images n formed is as,

- If $\left(\frac{360}{\theta}\right)$ is even then, $n = \left(\frac{360}{\theta} - 1\right)$ for objects placed symmetrically or unsymmetrically,
- If $\left(\frac{360}{\theta}\right)$ is odd then, $n = \left(\frac{360}{\theta} - 1\right)$ for objects placed symmetrically,
- If $\left(\frac{360}{\theta}\right)$ is odd then, $n = \left(\frac{360}{\theta}\right)$ for objects placed unsymmetrically.

6.1.6 Real and virtual images by a plane mirror

When a real object is placed at a point O in front of a plane mirror it produces divergent rays in all directions as shown in Figure 6.5(a). After reflection from the plane mirror they appear to come out from a point I behind the mirror. This image cannot be formed on a screen as the image is behind the mirror. **This type of image which cannot be formed on the screen but can only be seen with the eyes is called *virtual image*.**

On the other hand, if convergent rays are incident on a plane mirror, the rays after reflection pass through a point I in front of the mirror and form an image as shown in Figure 6.5(b). This image can be formed on a screen as the image is in front of the mirror. **This type of image which canbe formed on a screen and can also be seen with the eyes is called *real image*.**

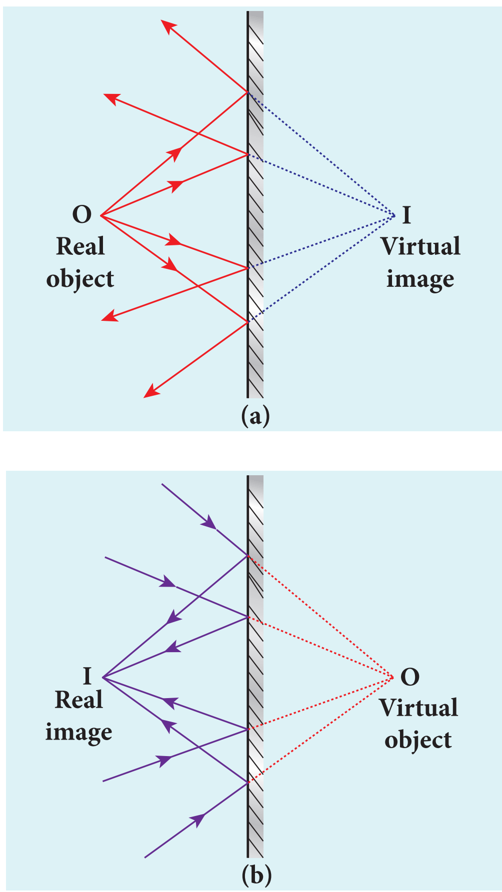


Figure 6.5 Real and virtual images by plane mirror

Note It is generally known that a plane mirror can only form a virtual image. But, now we have understood that a plane mirror can form a real image when converging rays fall on it.

The above discussion is consolidated in Table 6.1. These concepts will be very much useful in deciding about the nature of object and image in ray optics.

Table 6.1 Conditions for nature of objects and images

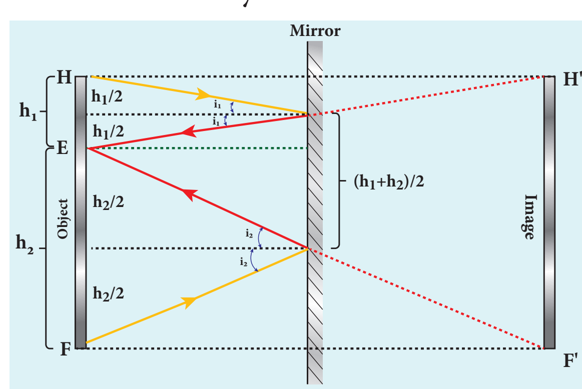
Nature of object/image	Condition
Real Image	Rays actually converge at the image
Virtual Image	Rays appear to diverge from the image
Real Object	Rays actually diverge from the object
Virtual Object	Rays appear to converge at the object

EXAMPLE 6.2

What is the height of the mirror needed to see the image of a person fully on the mirror?

Solution

Let us assume a person of height h is standing in front of a vertical plane mirror. The person could see his/her head when light from the head falls on the mirror and gets reflected to the eyes. Same way, light from the feet falls on the mirror and gets reflected to the eyes.



If the distance between his head H and eye E is h_1 and distance between his feet F and eye E is h_2. The person's total height h is, $h = h_1 + h_2$

By the law of reflection, the angle of incidence and angle of reflection are the same in the two extreme reflections. The normals are now the bisectors of angles between incident and reflected rays in the two reflections. By geometry, the height of the mirror needed is only half of the height of the person. $\frac{h_1 + h_2}{2} = \frac{h}{2}$

Does the height depend on the distance between the person and the mirror?

6.2 SPHERICAL MIRRORS

We shall now study about the reflections that take place in spherical surfaces.

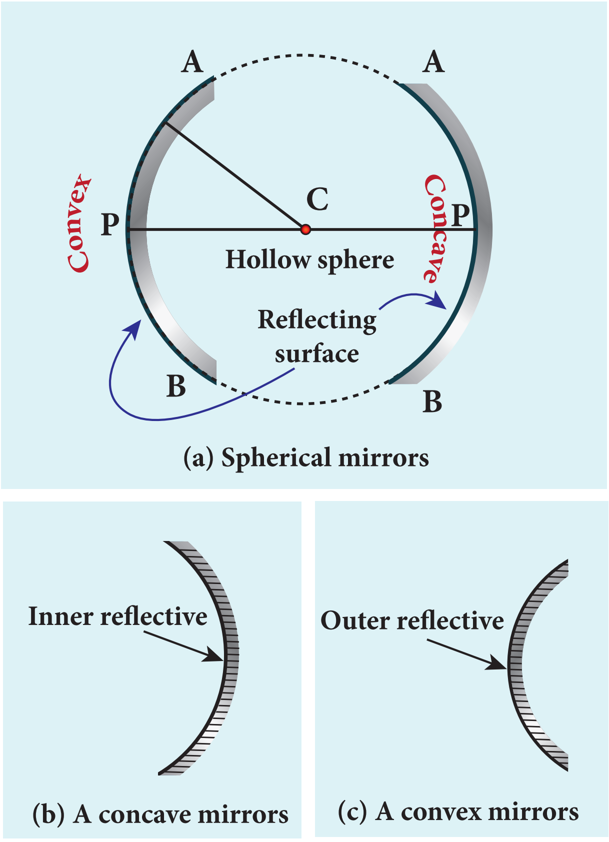


Figure 6.6 Spherical mirrors

A spherical surface is a part cut from a hollow sphere. Spherical mirrors are generally constructed from glass. One surface of the glass is silvered. The reflection takes place at the other polished surface. **If the reflection takes place at the convex surface, it is called a *convex mirror* and if the reflection takes place at the concave surface, it is called a *concave mirror.*** These are shown in Figure 6.6.

We shall now become familiar with some of the terminologies pertaining to spherical mirrors.

Centre of curvature: The centre of the sphere of which the mirror is a part is called the *center of curvature* (C) of the mirror.

Radius of curvature: The radius of the sphere of which the spherical mirror is a part is called the *radius of curvature* (R) of the mirror.

Pole: The middle point on the spherical surface of the mirror (or) the geometrical center of the mirror is called *pole* (P) of the mirror.

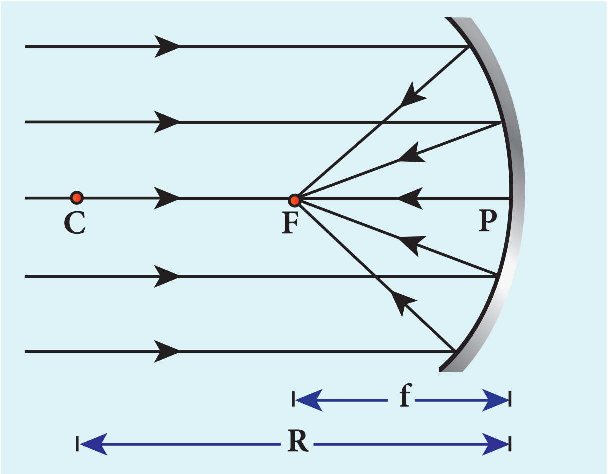
Principal axis: The line joining the pole and the centre of curvature is called the *principal axis* of the mirror. The light ray travelling along the principal axis towards the mirror after reflection travels back along the same principal axis. It is also called optical axis

Focus (or) Focal point: Light rays travelling parallel and close to the principal axis when incident on a spherical mirror, converge at a point for concave mirror or appear to diverge from a point for convex mirror on the principal axis. This point is called the *focus* or *focal point* (F) of the mirror.

Focal length: The distance between the pole and the focus is called the *focal length* (f) of the mirror.

Focal plane: The plane through the focus and perpendicular to the principal axis is called the *focal plane* of the mirror.

All the above mentioned terms are shown in Figure 6.7 for both concave and convex mirrors.



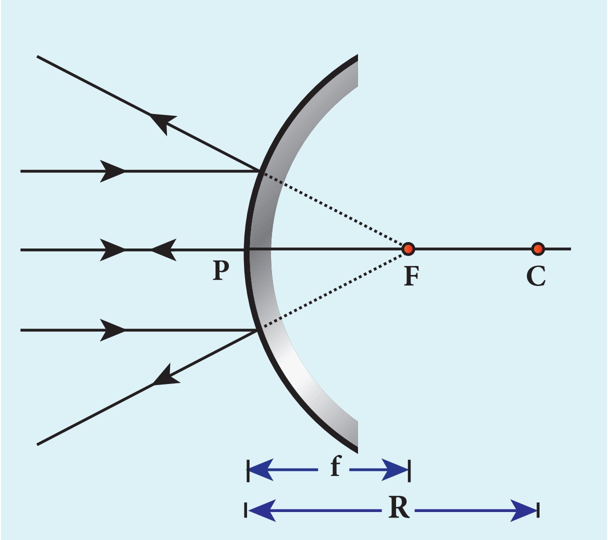


Figure 6.7 Focal length of concave and convex mirrors

6.2.1 Paraxial Rays and Marginal Rays

The rays travelling very close to the principal axis and make small angles with it are called *paraxial rays*. The paraxial rays fall on the mirror very close to the pole of the mirror. On the other hand, **the rays travelling far away from the principal axis and fall on the mirror far away from the pole are called as *marginal rays*.** These two rays behave differently (get focused at different points) as shown in Figure 6.8. In this chapter, we shall restrict our studies only to paraxial rays. As the angles made by the paraxial rays are very small, this helps us to make some approximations with the angles in ray optics.

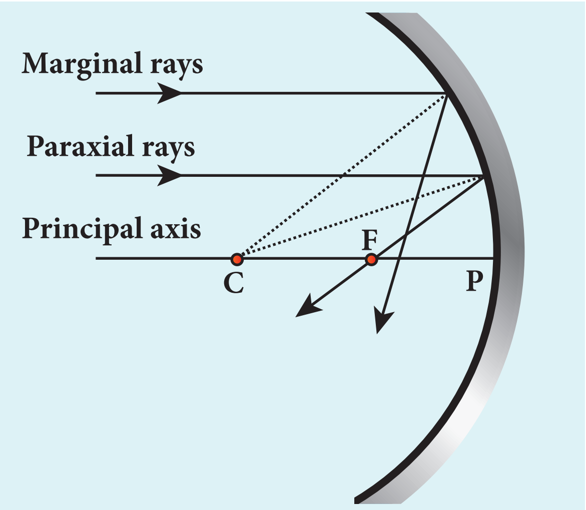


Figure 6.8 Paraxial and marginal rays

6.2.2 Relation between *f* and *R*

Let C be the centre of curvature of the mirror. Consider a light ray parallel to the principal axis is incident on the mirror at M and passes through the principal focus F after reflection. The geometry of reflection of the incident ray is shown in Figure 6.9(a). The line CM is the normal to the mirror at M. Let i be the angle of incidence and the same will be the angle of reflection.

If MP is the perpendicular from M on the principal axis, then from the geometry,

The angles $\angle MCP = i$ and $\angle MFP = 2i$

From right angle triangles ΔMCP and ΔMFP,

$$\tan i = \frac{PM}{PC} \text{ and } \tan 2i = \frac{PM}{PF}$$

As the angles are small, $\tan i \approx i$,

$$i = \frac{PM}{PC} \text{ and } 2i = \frac{PM}{PF}$$

Simplifying further,

$$2\frac{PM}{PC} = \frac{PM}{PF};\ 2PF = PC$$

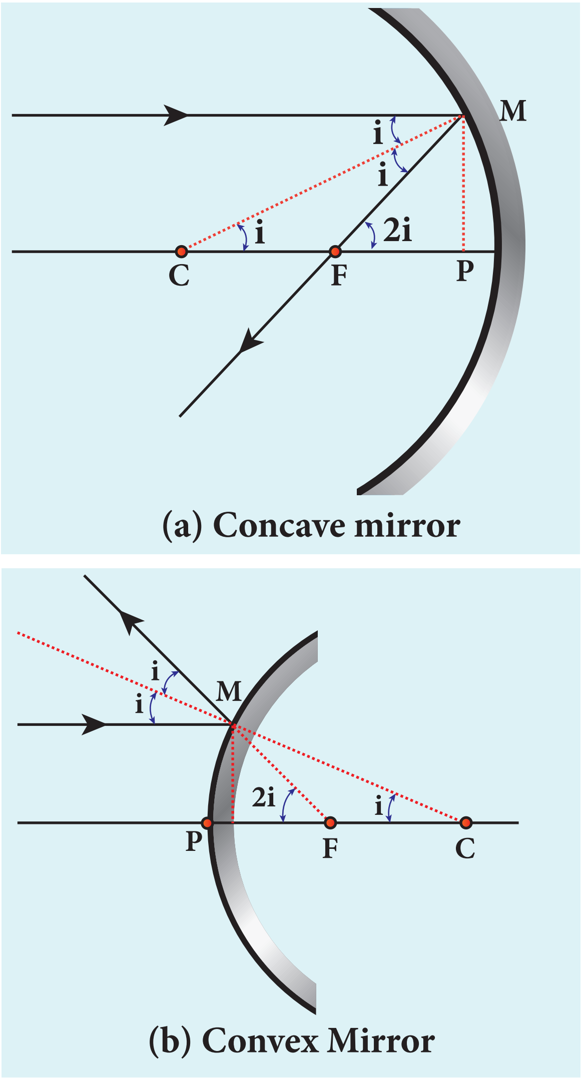


Figure 6.9 relation between R and f

PF is focal length *f* and *PC* is the radius of curvature *R*.

$$2f = R \qquad \text{(or)} \qquad f = \frac{R}{2} \qquad (6.4)$$

Equation (6.4) is the relation between *f* and *R*. The construction is shown for convex mirror in figure 6.9(b)

6.2.3 Image formation in spherical mirrors

The image can be located by graphical construction. To locate the point of an image, a minimum of two rays must meet at that point. We can use at least any two of the following rays to locate the image point as shown in Figure 6.10.

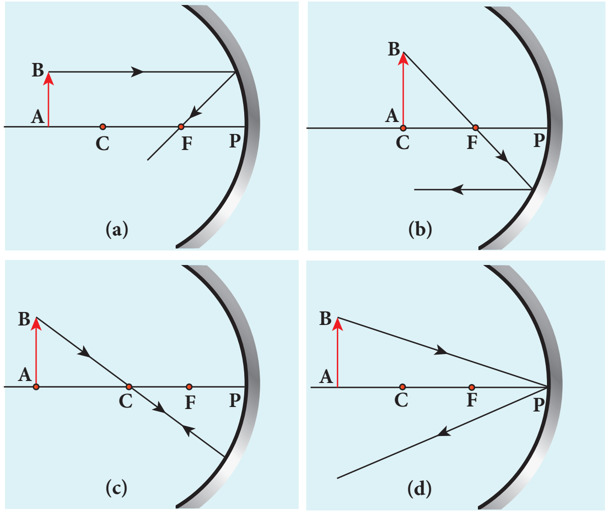


Figure 6.10 Image tracing

(i) A ray parallel to the principal axis after reflection will pass through or appear to pass through the principal focus. (Figure 6.10(a))

(ii) A ray passing through or appear to pass through the principal focus, after reflection will travel parallel to the principal axis. (Figure 6.10(b))

(iii) A ray passing through the centre of curvature retraces its path after reflection as it is a case of normal incidence.(Figure 6.10(c))

(iv) A ray falling on the pole will get reflected as per law of reflection keeping principal axis as the normal. (Figure 6.10(d))

6.2.4 Cartesian sign convention

While tracing the image, we would normally come across the object distance *u*, the image distance *v*, the object height *h*, the image height (h'), the focal length *f* and the radius of curvature *R*. A system of signs for these quantities must be followed so that the relations connecting them are consistent in all types of physical situations. We shall follow the Cartesian sign convention which is now widely used as given below and also shown in Figure 6.11.

(i) The Incident light is taken from left to right (i.e. object on the left of mirror).

(ii) All the distances are measured from the pole of the mirror (pole is taken as origin).

(iii) The distances measured to the right of pole along the principal axis are taken as positive.

(iv) The distances measured to the left of pole along the principal axis are taken as negative.

(v) Heights measured in the upward perpendicular direction to the principal axis are taken as positive.

(vi) Heights measured in the downward perpendicular direction to the principal axis, are taken as negative.

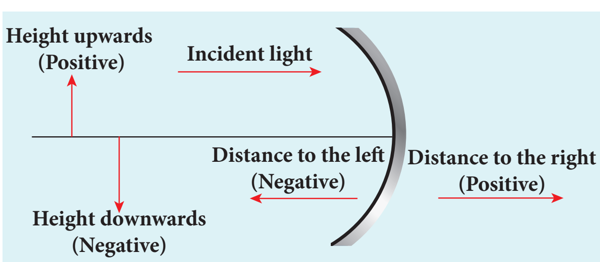


Figure 6.11 Cartesian sign convention

6.2.5 The mirror equation

The mirror equation establishes a relation among object distance u, image distance v and focal length f for a spherical mirror.

An object AB is considered on the principal axis of a concave mirror beyond the center of curvature C. The image formation is shown in the Figure 6.12. Let us consider three paraxial rays from point B on the object. The first paraxial ray BD travelling parallel to principal axis is incident on the concave mirror at D, close to the pole P. After reflection the ray passes through the focus F. The second paraxial ray BP incident at the pole P is reflected along PB'. The third paraxial ray BC passing through centre of curvature C, falls normally on the mirror at E is reflected back along the same path. The three reflected rays intersect at the point B'. A perpendicular drawn as $A'B'$ to the principal axis is the real, inverted image of the object AB.

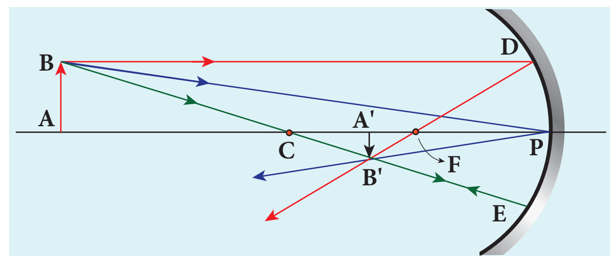


Figure 6.12 Mirror equation

As per law of reflection, the angle of incidence $\angle BPA$ is equal to the angle of reflection $\angle B'PA'$.

The triangles ΔBPA and $\Delta B'PA'$ are similar. Thus, from the rule of similar triangles,

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \quad (6.5)$$

The other set of similar triangles are, ΔDPF and $\Delta B'A'F$. (PD is almost a straight vertical line)

$$\frac{A'B'}{PD} = \frac{A'F}{PF}$$

As, the distances $PD = AB$ the above equation becomes,

$$\frac{A'B'}{AB} = \frac{A'F}{PF} \quad (6.6)$$

From equations (6.5) and (6.6) we can write,

$$\frac{PA'}{PA} = \frac{A'F}{PF}$$

As, $A'F = PA' - PF$, the above equation becomes,

$$\frac{PA'}{PA} = \frac{PA' - PF}{PF} \quad (6.7)$$

We can apply the sign conventions for the various distances in the above equation.

$PA = -u, \qquad PA' = -v, \qquad PF = -f$

All the three distances are negative as per sign convention, because they are measured to the left of the pole. Now, the equation (6.7) becomes,

$$\frac{-v}{-u} = \frac{-v-(-f)}{-f}$$

On further simplification,

$$\frac{v}{u} = \frac{v-f}{f}; \qquad \frac{v}{u} = \frac{v}{f} - 1$$

Dividing either side with v,

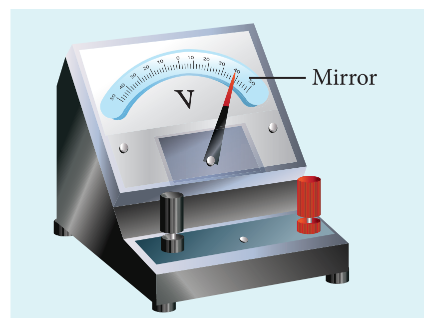
$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

After rearranging,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad (6.8)$$

The above equation (6.8) is called ***mirror equation***. Although this equation is derived for a special situation shown in Figure (6.12), it is also valid for all other situations with any spherical mirror. This is because proper sign convention is followed for u, v and f in equation (6.7).

DO YOU KNOW?



Identify the type of mirror used in each of the application shown above.

6.2.6 Lateral magnification in spherical mirrors

The lateral or transverse magnification is defined as the ratio of the height of the image to the height of the object. The height of the object and image are measured perpendicular to the principal axis.

$$\text{magnification } (m) = \frac{\text{height of the image } (h')}{\text{height of the object } (h)}$$

$$m = \frac{h'}{h} \qquad (6.9)$$

Applying proper sign conventions for equation (6.5),

$$\frac{A'B'}{AB} = \frac{PA'}{PA}$$

$$A'B' = -h, \mathrm{AB} = \mathrm{h}, PA' = -v, PA = -u$$

$$\frac{-h'}{h} = \frac{-v}{-u}$$

On simplifying we get,

$$m = \frac{h'}{h} = -\frac{v}{u} \qquad (6.10)$$

Using mirror equation, we can further write the magnification as,

$$m = \frac{h'}{h} = \frac{f-v}{f} = \frac{f}{f-u} \qquad (6.11)$$

Note The students are advised to refresh themselves with the image tracing for the concave and convex mirrors for various predetermined positions of the object and the position of image, nature of image etc. studied in 9th standard (Science, Unit 6. Light).

EXAMPLE 6.3

An object is placed at a distance of 20.0 cm from a concave mirror of focal length 15.0 cm.

(a) What distance from the mirror a screen should be placed to get a sharp image?

(b) What is the nature of the image?

Solution

Given, $f = -15$ cm, $u = -20$ cm

(a) Mirror equation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Rewriting to find v, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Substituting for f and u, $\frac{1}{v}=\frac{1}{-15}-\frac{1}{-20}$

$$\frac{1}{v}=\frac{(-20)-(-15)}{300}=\frac{-5}{300}=\frac{-1}{60}$$

$$v=-60.0\ cm$$

As the image is formed at 60.0 cm to the left of the concave mirror, the screen is to be placed at distance 60.0 cm to the left of the concave mirror.

(b) Magnification, $m=\frac{h'}{h}=-\frac{v}{u}$

$$m=\frac{h'}{h}=-\frac{(-60)}{(-20)}=-3$$

As the sign of magnification is negative, the image is inverted.

As the magnitude of magnification is 3, the image is enlarged three times.

As the image is formed to the left of the concave mirror, the image is real.

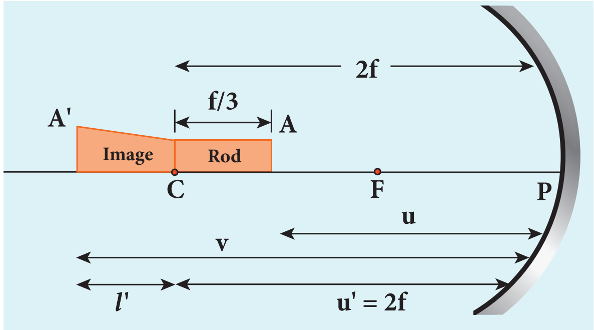
EXAMPLE 6.4

A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the longitudinal magnification.

Solution

$$\text{longitudinal magnification } (m)=\frac{\text{length of image } (l')}{\text{length of object } (l)}$$

Given: length of object, $l=\frac{f}{3}$



Let, l be the length of the image, then,

$$m=\frac{l'}{l}=\frac{l'}{f/3}\ \text{(or)}\ l=\frac{mf}{3}$$

Image of one end coincides with the object. Thus, the coinciding end must be at center of curvature.

Hence, $u' = R = 2f$

$$u'=u+\frac{f}{3}$$

$$u=u'-\frac{f}{3}=2f-\frac{f}{3}=\frac{5f}{3}$$

$$v=u+\frac{f}{3}+\frac{mf}{3}=\frac{5f}{3}+\frac{f}{3}+\frac{mf}{3}=\frac{f(6+m)}{3}$$

Mirror equation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-\left(\frac{f(6+m)}{3}\right)}+\frac{1}{-\left(\frac{5f}{3}\right)}=\frac{1}{-f}$$

After simplifying,

$$\frac{3}{f(6+m)}+\frac{3}{5f}=\frac{1}{f};\frac{3}{(6+m)}=\frac{2}{5}$$

$$6+m=\frac{15}{2};m=\frac{15}{2}-6$$

$$m=\frac{3}{2}=1.5$$

6.3 SPEED OF LIGHT

Light travels with the highest speed in vacuum. The speed of light in vacuum is denoted as c and its value is, $c = 3\times10^8$ m s^{-1}. It is a very high value. Several attempts were made by scientists to determine the speed of light. The earliest attempt was made by a French scientist Hippolyte Fizeau (1819–1896). That paved way for the other scientists too to determine the speed of light.

6.3.1 Fizeau's method to determine speed of light

Apparatus: The apparatus used by Fizeau for determining speed of light in air is shown in Figure 6.13. The light from the source *S* was first allowed to fall on a partially silvered glass plate *G* kept at an angle of 45° to the incident light from the source. The light then was allowed to pass through a rotating toothed-wheel with *N* teeth and *N* cuts of equal widths whose speed of rotation could be varied through an external mechanism (not shown in the Figure). The light passing through one cut in the wheel will get reflected by a mirror *M* kept at a long distance *d*, about 8 *km* from the toothed wheel. If the toothed wheel was not rotating, the reflected light from the mirror would again pass through the same cut and reach the eyes of the observer through the partially silvered glass plate.

Working: The angular speed of rotation of the toothed wheel was increased from zero to a value ω until light passing through one cut would completely be blocked by the adjacent tooth. This is ensured by the disappearance of light while looking through the partially silvered glass plate.

Expression for speed of light: The speed of light in air v is equal to the ratio of the distance the light travelled from the toothed wheel to the mirror and back $2d$ to the time taken t.

$$v=\frac{2d}{t} \qquad (6.12)$$

The distance d is a known value from the arrangement. The time taken t for the light to travel the distance to and fro is calculated from the angular speed ω of the toothed wheel.

The angular speed ω of the toothed wheel when the light disappeared for the first time is,

$$\omega=\frac{\theta}{t} \qquad (6.13)$$

Here, θ is the angle between the tooth and the slot which is rotated by the toothed wheel within that time t.

$$\theta=\frac{\text{total angle of the circle in radian}}{\text{number of teeth+number of cuts}}$$

$$\theta=\frac{2\pi}{2N}=\frac{\pi}{N}$$

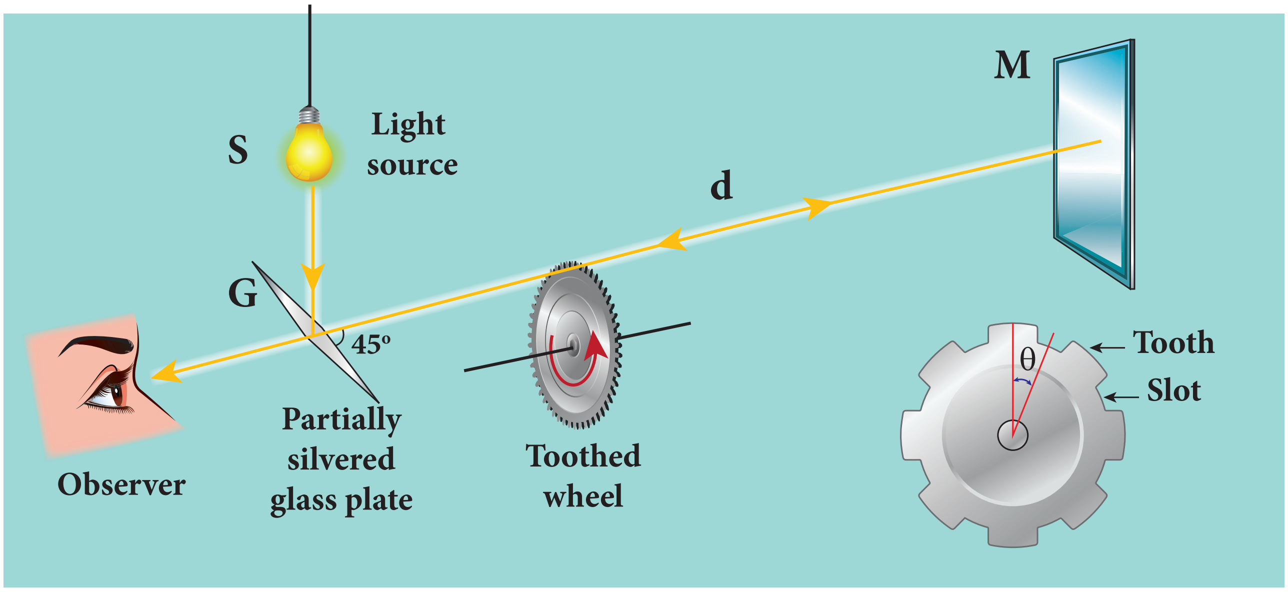


Figure 6.13 Speed of light by Fizeau's method

Substituting for θ in the equation 6.13. for ω,

$$\omega = \frac{\pi / N}{t} = \frac{\pi}{Nt}$$

Rewriting the above equation for t,

$$t = \frac{\pi}{N\omega} \qquad (6.14)$$

Substituting t from equation (6.14) in equation (6.12),

$$v = \frac{2d}{\pi / N\omega}$$

After rearranging,

$$v = \frac{2dN\omega}{\pi} \qquad (6.15)$$

Fizeau had some difficulty to visually estimate the minimum intensity of the light when blocked by the adjacent tooth, and his value for speed of light was very close to the actual value. Later on, with the same idea of Fizeau and with much sophisticated instruments, the speed of light in air was determined as, $v = 2.99792\times10^{8}$ m s^{-1}.

Note After the disappearance of light for the first time while increasing the speed of rotation of the toothed-wheel from zero to ω, on further increase of speed of rotation of the wheel to 2ω, the light would appear again due to the passing of reflected light through the next slot. So, for every odd value of ω, light will disappear (stopped by tooth) and for every even value of ω light will appear (allowed by slot).

6.3.2 Speed of light through different media

Different transparent media like glass, water etc. were introduced in the path of light by scientists like Foucault (1819–1868) and Michelson (1852–1931) to find the speed of light in different media. Even evacuated glass tubes were also introduced in the path of light to find the speed of light in vacuum. It was found that light travels with lesser speed in any medium than its speed in vacuum. The speed of light in vacuum was determined as, $c = 3\times10^8$ m s^{-1}. We could notice that the speed of light in vacuum and in air are almost the same.

6.3.3 Refractive index

Refractive index **of a transparent medium is defined as the ratio of speed of light in vacuum (or air) to the speed of light in that medium.**

$$\left.\begin{array}{r}\text{refractive}\\ \text{index } n \text{ of a}\\ \text{medium}\end{array}\right\} = \frac{\text{speed of light in vacuum } (c)}{\text{speed of light in medium } (v)}$$

$$n = \frac{c}{v} \qquad (6.16)$$

Refractive index of a transparent medium gives an idea about the speed of light in that medium.

EXAMPLE 6.5

One type of transparent glass has refractive index 1.5. What is the speed of light through this glass?

Solution

$$n = \frac{c}{v}; \qquad v = \frac{c}{n}$$

$$v = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m s}^{-1}$$

Light travels with a speed of 2×10^8 m s^{-1} through this glass.

Refractive index does not have unit. The smallest value of refractive index is for vacuum, which is 1. For any other medium refractive index is greater than 1. Refractive index is also called as optical density of the medium. Higher the refractive index of a medium, greater is its optical density and speed of light through the medium is lesser and vice versa. [Note: optical density should not be confused with mass density of the material of the medium. They two are different entities]. The Table 6.2 shows the refractive index of different transparent media.

Table 6.2 Refractive index of different media

Media	Refractive index
Vacuum	1.00
Air	1.0003
Carbon dioxide gas	1.0005
Ice	1.31
Pure water	1.33
Ethyl alcohol	1.36
Quartz	1.46
Vegetable oil	1.47
Olive oil	1.48
Acrylic	1.49
Table salt	1.51
Glass	1.52
Sapphire	1.77
Zircon	1.92
Qubic zirconia	2.16
Diamond	2.42
Gallium phosphide	3.50

6.3.4 Optical path

Optical path **of a medium is defined as the distance d' light travels in vacuum in the same time it travels a distance d in the medium.**

Let us consider a medium of refractive index n and thickness d. Light travels with a speed v through the medium in a time t. Then we can write,

$$v = \frac{d}{t}; \text{ rewritten as, } t = \frac{d}{v}$$

In the same time, light can cover a greater distance d' in vacuum as it travels with greater speed c in vacuum as shown in Figure 6.14. Then we have,

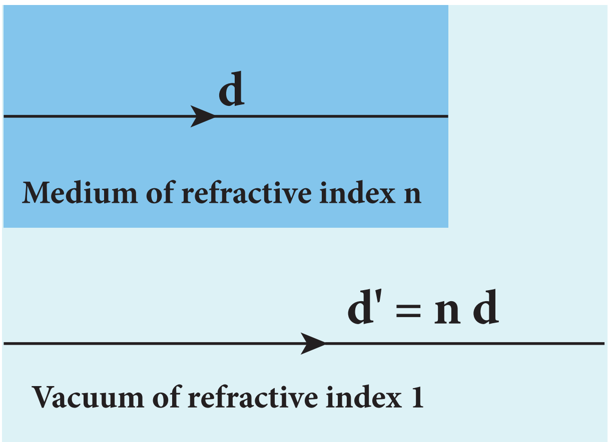


Figure 6.14 Optical path

$$c = \frac{d'}{t}; \text{ rewritten as, } t = \frac{d'}{c}$$

As the time taken in both the cases is the same, we can equate the time t as,

$$\frac{d'}{c} = \frac{d}{v}$$

rewritten for the optical path d' as, $d' = \frac{c}{v}d$

As, $\frac{c}{v} = n$; The optical path d' is,

$$d' = nd \qquad (6.17)$$

As n is always greater than 1, the optical path d' of the medium is always greater than d.

EXAMPLE 6.6

Light travels from air in to glass slab of thickness 50 cm and refractive index 1.5.

(i) What is the speed of light in glass?

(ii) What is the time taken by the light to travel through the glass slab?

(iii) What is the optical path of the glass slab?

Solution

Given, thickness of glass slab, d = 50 cm = 0.5 m, refractive index, n = 1.5

refractive index, $n = \frac{c}{v}$

speed of light in glass is,

$$v = \frac{c}{n} = \frac{3\times10^8}{1.5} = 2\times10^8 m\ s^{-1}$$

Time taken by light to travel through glass slab is,

$$t = \frac{d}{v} = \frac{0.5}{2\times10^8} = 2.5\times10^{-9} s$$

Optical path,

$$d' = nd = 1.5\times0.5 = 0.75\,m = 75\,cm$$

Light would have travelled 25 cm more (75 cm – 50 cm) in vacuum by the same time had there not been a glass slab.

6.4 REFRACTION

Refraction is passing through of light from one optical medium to another optical medium through a boundary. In refraction, the angle of incidence i in one medium and the angle of reflection r in the other medium are measured with respect to the normal drawn to the surface at the point of incidence of light. Law of refraction is called *Snell's law*.

Snell's law states that,

a) **The incident ray, refracted ray and normal to the refracting surface are all coplanar (ie. lie in the same plane).**

b) **The ratio of angle of incident i in the first medium to the angle of reflection r in the second medium is equal to the**

ratio of refractive index of the second medium n_2 to that of the refractive index of the first medium n_1.

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad (6.18)$$

The above equation is in the ratio form. It can also be written in a much useful product form as,

$$n_1 \sin i = n_2 \sin r \quad (6.19)$$

The refraction at a boundary is shown in Figure 6.15.

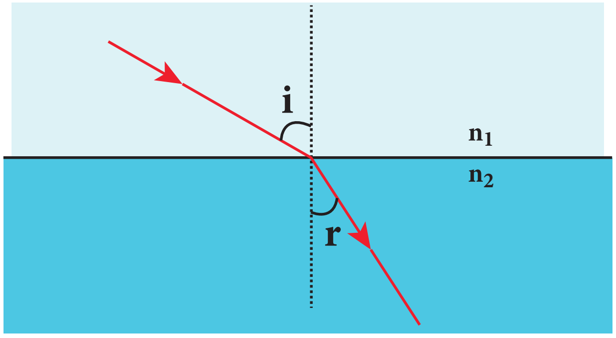


Figure 6.15 Refraction of light

Note For normal incidence of light on a surface, the angle of incidence is zero.

6.4.1 Angle of deviation due to refraction

We know that the angle between the incident and deviated light is called angle of deviation. When light travels from rarer to denser medium it deviates towards normal as shown in Figure 6.16. The angle of deviation in this case is,

$$d = i - r \quad (6.20)$$

On the other hand, if light travels from denser to rarer medium it deviates away from normal as shown in Figure 6.17. The angle of deviation in this case is,

$$d = r - i \quad (6.21)$$

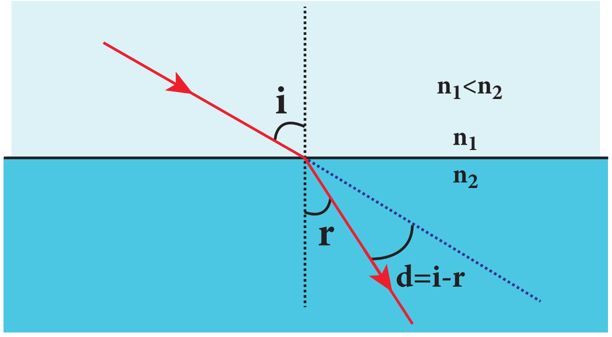


Figure 6.16 Angle of deviation due to refraction from rarer to denser medium

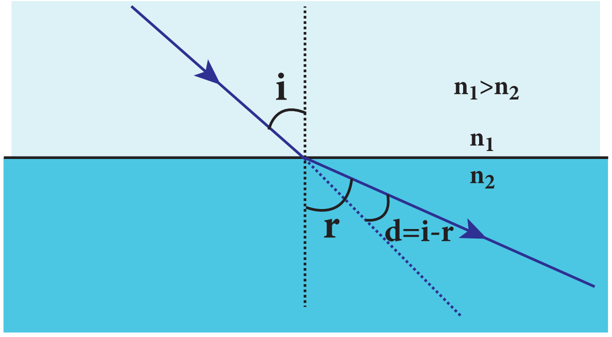


Figure 6.17 Angle of deviation due to refraction from denser to rarer medium

6.4.2 Characteristics of refraction

a) When light passes from rarer medium to denser medium it deviates towards normal in the denser medium.

b) When light passes from denser medium to rarer medium it deviates away from normal in the rarer medium.

c) In any refracting surface there will also be some reflection taking place. Thus, the intensity of refracted light will be lesser than the incident light. **The phenomenon in which a part of light from a source undergoing reflection and the other part of light from the same source undergoing refraction at the same**

UNIT-6(XII-Physics_Vol-2).indd 16 8/3/2019 4:35:09 PM

surface is called *simultaneous reflection* or *simultaneous refraction*. This is shown in Figure 6.18. Such surfaces are available as partially silvered glasses.

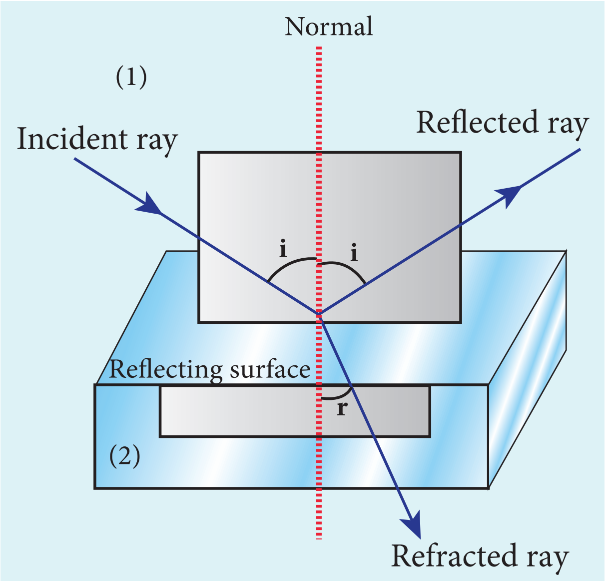


Figure 6.18 Simultaneous reflection and refraction

6.4.3 Principle of reversibility

The *principle of reversibility* states that light will follow exactly the same path if its direction of travel is reversed. This is true for both reflection and refraction as shown in Figure 6.15.

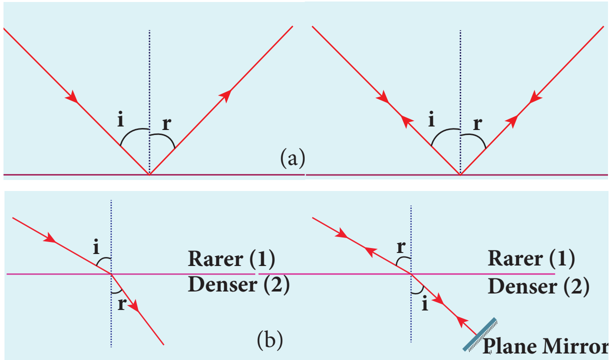
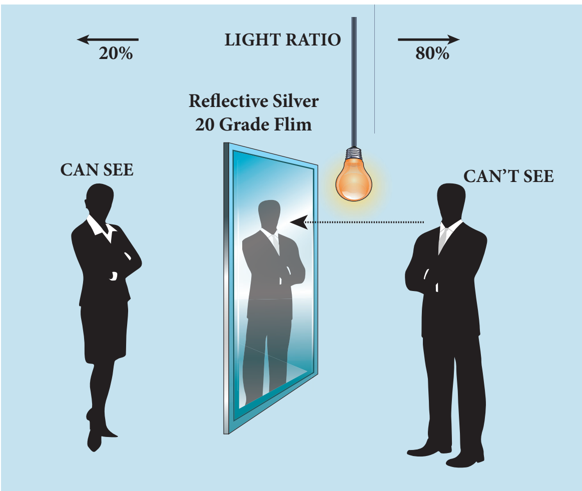
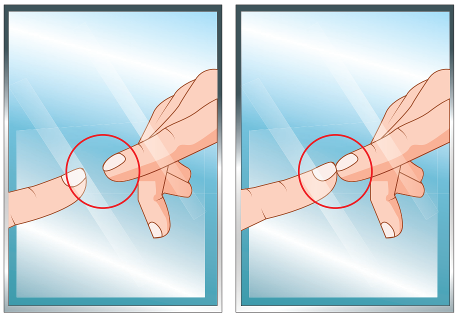


Figure 6.19 Principle of reversibility in (a) reflection and (b) refraction

Production of optical surfaces capable of refracting as well as reflecting is possible by properly coating the surfaces with suitable materials. Thus, a glass can be made partially see through and partially reflecting by varying the amount of coating on its surface. It is commercially called as two way mirror, half-silverd or semi-silvered mirror etc. This gives a perception of regular mirror if the other side is made dark. But, still hidden cameras can be kept behind such mirrors. We need to be cautious when we stand in front of mirrors kept in unknown places. There is a method to test the two way mirror. Place the finger nail against the mirror surface. If there is a gap between nail and its image, then it is a regular mirror. If the fingernail directly touches its image, then it is a two way mirror.





UNIT-6(XII-Physics_Vol-2).indd 17 8/3/2019 4:35:10 PM

6.4.4 Relative refractive index

In the equation for Snell's law, **the term** $\left(\frac{n_2}{n_1}\right)$ **is called *relative refractive index* of second medium with respect to the first medium** which is denoted as (n_{21}).

$$n_{21} = \frac{n_2}{n_1} \qquad (6.22)$$

The concept of relative refractive index gives rise to other useful relation such as,

a) Inverse rule:

$$n_{12} = \frac{1}{n_{21}} \ (or) \ \frac{n_1}{n_2} = \frac{1}{n_2/n_1} \qquad (6.23)$$

b) Chain rule:

$$n_{32} = n_{31} \times n_{12} \ \text{(or)} \ \frac{n_3}{n_2} = \frac{n_3}{n_1} \times \frac{n_1}{n_2} \qquad (6.24)$$

EXAMPLE 6.7

Light travelling through transparent oil enters in to glass of refractive index 1.5. If the refractive index of glass with respect to the oil is 1.25, what is the refractive index of the oil?

Solution

Given, n_{go} =1.25 and n_g =1.5

Refractive index of glass with respect to oil,

$$n_{go} = \frac{n_g}{n_o}$$

Rewriting for refractive index of oil,

$$n_o = \frac{n_g}{n_{go}} = \frac{1.5}{1.25} = 1.2$$

The refractive index of oil is, n_o = 1.2

6.4.5 Apparent depth

It is a common observation that the bottom of a tank filled with water appears raised as shown in Figure 6.19(a). An equation could be derived for the apparent depth for viewing in the near normal direction. The ray diagram is shown in Figure 6.19(b) and (c).

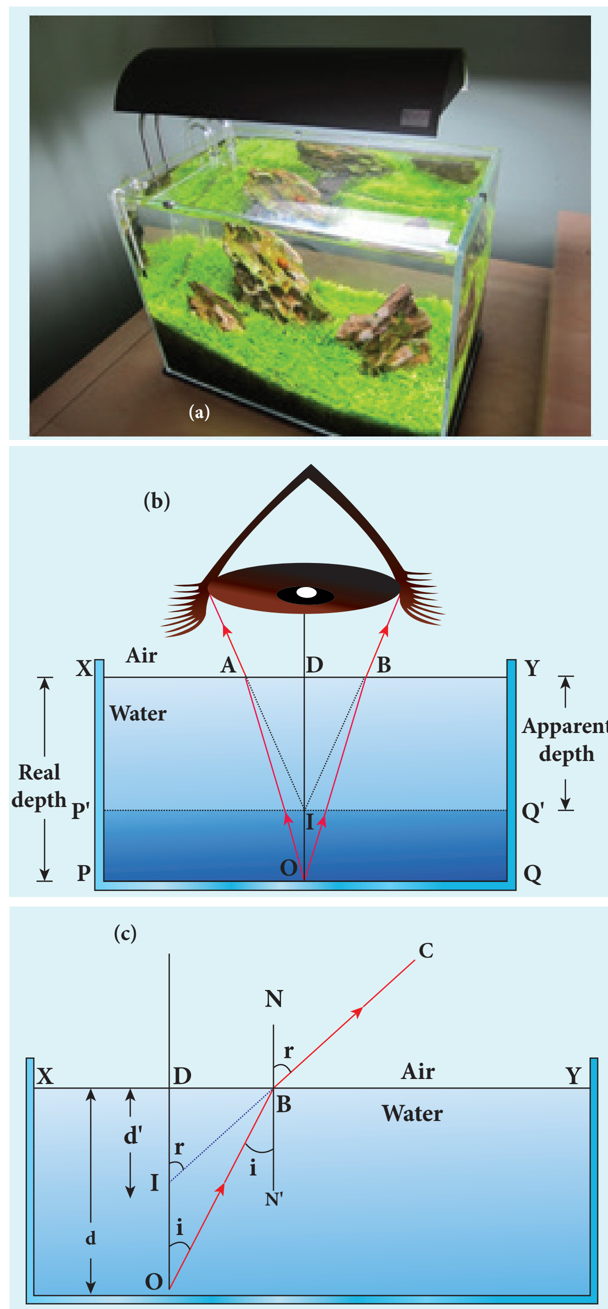


Figure 6.20 Apparent depth

Light from the object *O* at the bottom of the tank passes from denser medium (water) to rarer medium (air) to reach our eyes. It deviates away from the normal in the rarer medium at the point of incidence B. The refractive index of the denser medium is n_1 and rarer medium is n_2. Here, $n_1 > n_2$. The angle of incidence in the denser medium is i and the angle of refraction in the rarer medium is r. The lines NN' and OD are parallel. Thus angle $\angle DIB$ is also r. The angles i and r are very small as the diverging light from O entering the eye is very narrow. The Snell's law in product form for this refraction is,

$$n_1 \sin i = n_2 \sin r \qquad (6.19)$$

As the angles i and r are small, we can approximate, $\sin i \approx \tan i$;

$$n_1 \tan i = n_2 \tan \mathrm{i}$$

In triangles ΔDOB and ΔDIB,

$$\tan(i) = \frac{DB}{DO} \text{ and } \tan(r) = \frac{DB}{DI}$$

$$n_1 \frac{DB}{DO} = n_2 \frac{DB}{DI}$$

DB is cancelled on both sides, DO is the actual depth d and DI is the apparent depth d'.

$$n_1 \frac{1}{d} = n_2 \frac{1}{d'}$$

$$\frac{d'}{d} = \frac{n_2}{n_1} \qquad (6.25)$$

Rearranging the above equation for the apparent depth d',

$$d' = \frac{n_2}{n_1} d \qquad (6.26)$$

As the rarer medium is air and its refractive index n_2 can be taken as 1, (n_2=1). And the refractive index n_1 of denser medium could then be taken as n, (n_1=n). In that case, the equation for apparent depth becomes,

$$d' = \frac{d}{n} \qquad (6.27)$$

The bottom appears to be elevated by d-d',

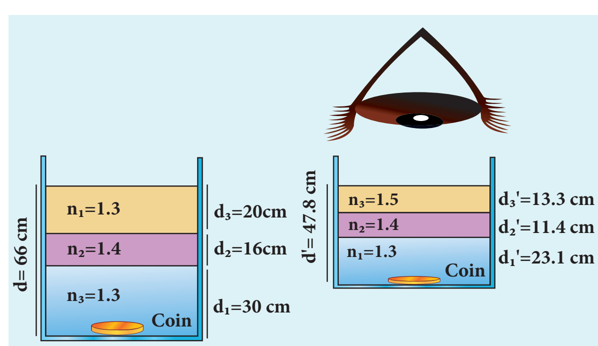
$$d - d' = d - \frac{d}{n} \text{ or } d - d' = d\left(1 - \frac{1}{n}\right) \qquad (6.28)$$

EXAMPLE 6.8

A coin is at the bottom of a trough containing three immiscible liquids of refractive indices 1.3, 1.4 and 1.5 poured one above the other of heights 30 cm, 16 cm, and 20 cm respectively. What is the apparent depth at which the coin appears to be when seen from air medium outside? In which medium the coin will be seen?

Solution

When seen from top, the coin will still appear to be at the bottom with each medium appearing to have shrunk with respect to the air medium outside. This situation is illustrated below.



The equations for apparent depth for each medium is,

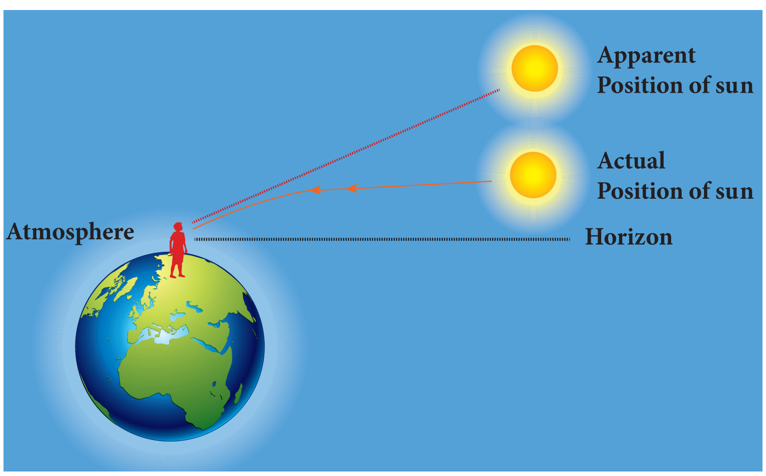
$$d_1' = \frac{d_1}{n_1}; \quad d_2' = \frac{d_2}{n_2}; \qquad d_3' = \frac{d_3}{n_3}$$

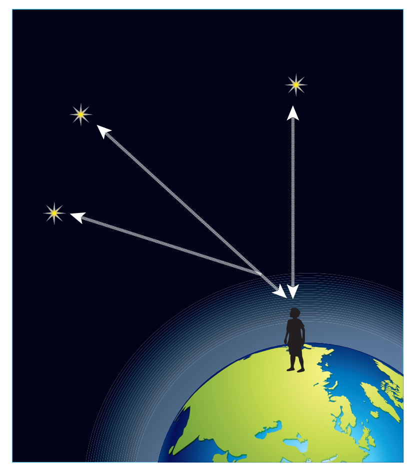
$$d' = d_1' + d_2' + d_3' = \frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3}$$

DO YOU KNOW?

Atmospheric refraction: Due to refraction of light through different layers of atmosphere which vary in refractive index, the path of light deviates continuously when it passes through atmosphere. For example, the Sun is visible a little before the actual sunrise and also until a little after the actual sunset due to refraction of light through the atmosphere. By actual sunrise what we mean is the actual crossing of the sun at the horizon. Figure shows the actual and apparent positions of the sun with respect to the horizon. The figure is highly exaggerated to show the effect. The apparent shift in the direction of the sun is around half a degree and the corresponding time difference between actual and apparent positions is about 2 minutes. Sun appears flattened (oval shaped) during sun rise and sunset due to the same phenomenon.

The same is also applicable for the positions of stars as shown in Figure. The stars actually do not twinkle. They appear twinkling because of the movement of the atmospheric layers with varying refractive indices which is clearly seen in the night sky.





$$d' = \frac{30}{1.3} + \frac{16}{1.4} + \frac{30}{1.5} = 23.1 + 11.4 + 13.3$$

$$d' = 47.8\ cm$$

6.4.6 Critical angle and total internal reflection

When a ray passes from an optically denser medium to an optically rarer medium, it bends away from normal. Because of this, the angle of refraction r on the rarer medium is greater than the corresponding angle of incidence i in the denser medium. As angle of incidence i is gradually increased, r rapidly increases and at a certain stage it becomes 90° or gracing the boundary. **The angle of incidence in the denser medium for which the refracted ray graces the boundary is called *critical angle* i_c.**

If the angle of incidence in the denser medium is increased beyond the critical angle, there is no refraction possible in to the rarer medium. **The entire light is reflected back into the denser medium itself. This phenomenon is called *total internal reflection*.** These are shown in Figure 6.21.

The two conditions for total internal reflection are,

(a) light must travel from denser to rarer medium,

(b) angle of incidence in the denser medium must be greater than critical angle ($i>i_c$).

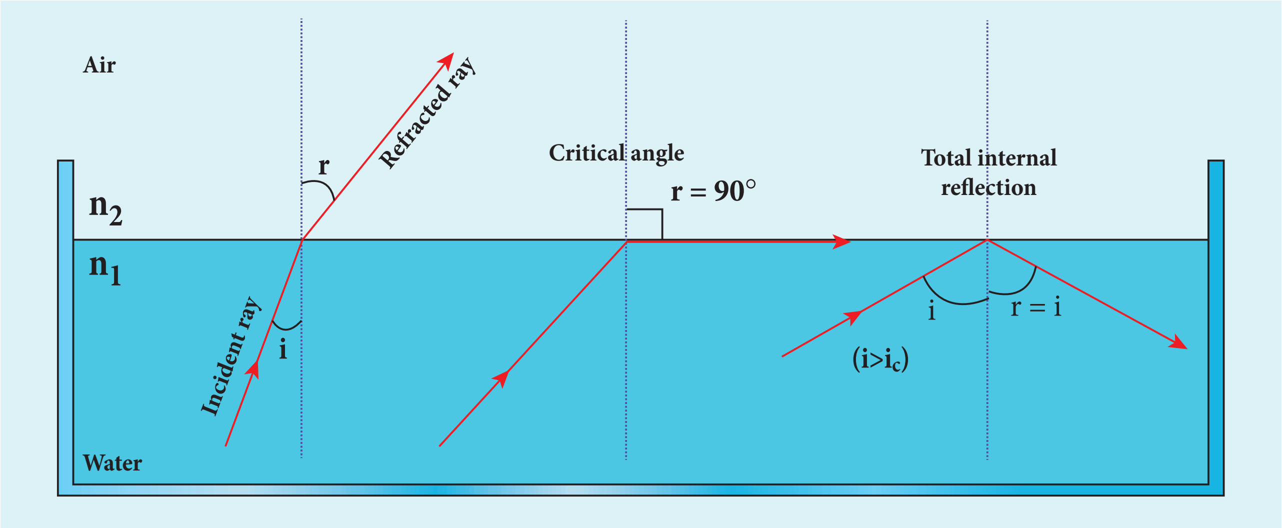


Figure 6.21 Critical angle and total internal reflection

Snell's law in the product form, equation (6.19) for critical angle incidence becomes,

$$n_1 \sin i_c = n_2 \sin 90^\circ \qquad (6.29)$$

$$n_1 \sin i_c = n_2 \qquad \because \sin 90^\circ = 1$$

$$\sin i_c = \frac{n_2}{n_1} \qquad (6.30)$$

Here, $n_1 > n_2$

If the rarer medium is air, then its refractive index is 1 and can be taken as n itself. i.e. (n_2=1) and (n_1=n).

$$\sin i_c = \frac{1}{n} \text{ (or) } i_c = \sin^{-1}\left(\frac{1}{n}\right) \qquad (6.31)$$

For example the refractive index of glass is about 1.5. The critical angle for glass-air interface is, $i_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$.

The critical angle for water-air interface is, $i_c = \sin^{-1}\left(\frac{1}{1.3}\right) = 48.6^\circ$.

The critical angle i_c depends on the refractive index of the medium. Table 6.3 shows the refractive index and the critical angle for different materials.

Table 6.3 Refractive index and critical angle of different media

Material	Refractive index	Critical Angle
Ice	1.310	49.8°
Water	1.333	48.6°
Fused Quartz (SiO_2)	1.458	43.3°
Crown Glass	1.541	40.5°
Flint Glass	1.890	31.9°
Calcite ($CaCO_2$)	1.658	37.0°
Diamond	2.417	24.4°
Strontium Titanate ($SrTiO_3$)	2.417	24.4°
Rutile	2.621	22.4°

6.4.7 Effects due to total internal reflection

6.4.7.1 Glittering of diamond

Diamond appears dazzling because the total internal reflection of light happens inside the diamond. The refractive index of only diamond is about 2.417. It is much larger than that for ordinary glass which is about only 1.5. The critical angle of diamond is about 24.4°. It is much less than that of glass. A skilled diamond cutter makes use of this larger range of angle of incidence (24.4° to 90° inside the diamond), to ensure that light entering the diamond is total internally reflected from the many cut faces before getting out as shown in Figure 6.22. This gives a sparkling effect for diamond.

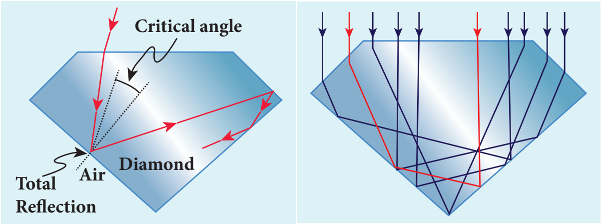
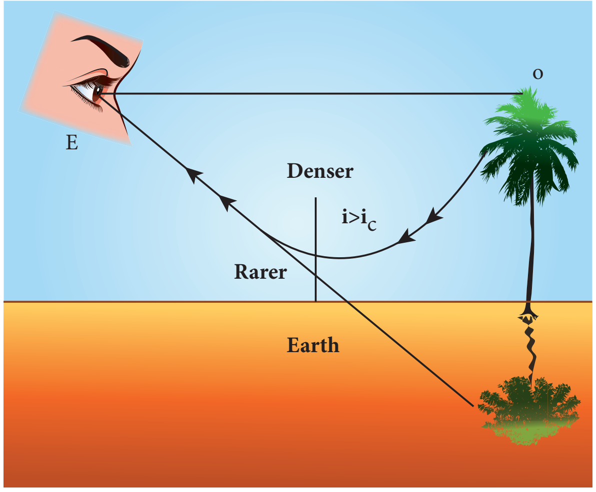


Figure 6.22 Total internal reflection in diamond

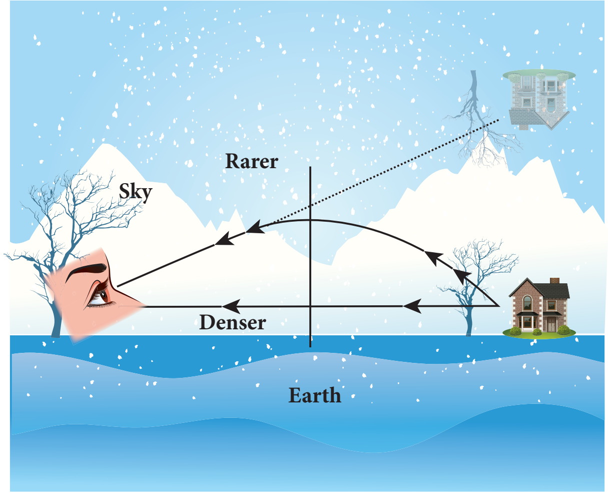
6.4.7.2 Mirage and looming

The refractive index of air increases with its density. In hot places, air near the ground is hotter than air at a height. Hot air is less dense. Hence, in still air the refractive index of air increases with height. Because of this, light from tall objects like a tree, passes through a medium whose refractive index decreases towards the ground. Hence, a ray of light successively deviates away from the normal at different layers of air and undergoes total internal reflection when the angle of incidence near the ground exceeds the critical angle. This gives an illusion as if the light comes from somewhere below the ground. For of the shaky nature of the layers of air, the observer feels as if the object is getting reflected by a pool of water or wet surface beneath the object as shown in Figure 6.23(a). This phenomenon is called ***mirage***.

In the cold places the refractive index increases towards the ground because the temperature of air close to the ground is lesser than the temperature above the surface of earth. Thus, the density and refractive index of air near the ground is greater than at a height. In the cold regions like glaciers and frozen lakes and seas, the reverse effect of mirage will happen. Hence, an inverted image is formed little above the surface as shown in Figure 6.23(b). This phenomenon is called ***looming***.



(a) Mirage



(b)Looming

Figure 6.23 Mirage and looming

6.4.7.3 Prisms making using of total internal reflection

Prisms can be designed to reflect light by 90° or by 180° by making use of total internal reflection as shown in Figure 6.24(a) and (b). In the first two cases, the critical angle i_c for the material of the prism must be less than 45°. We see from Table 6.3 that this is true for both crown glass and flint glass. Prisms are also used to invert images without changing their size as shown in Figure. 6.24(c).

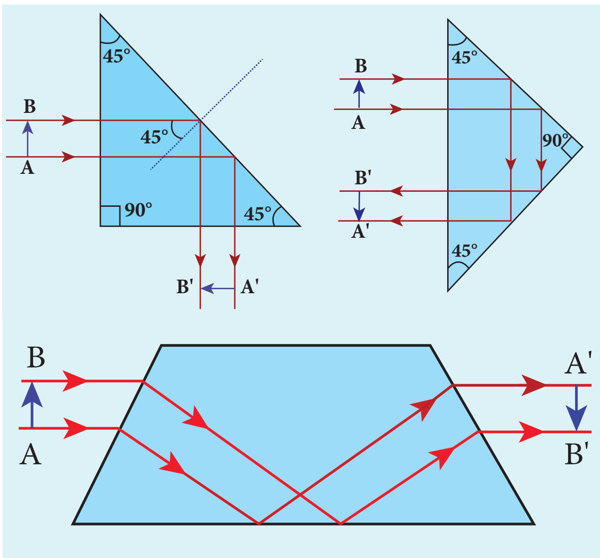


Figure 6.24 Prisms making use of total internal reflection

6.4.7.4 Radius of illumination (Snell's window)

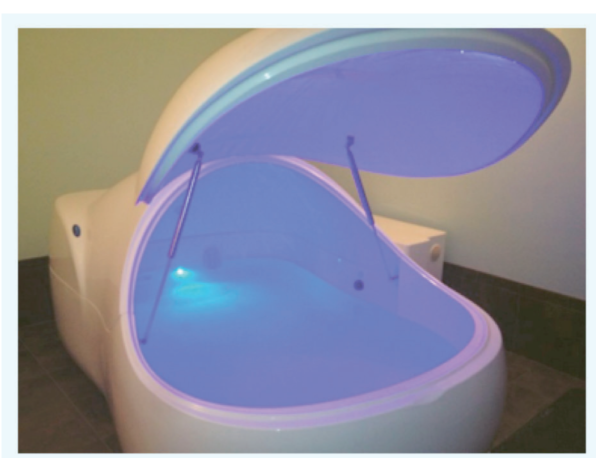


Figure 6.25 Light source inside water tank

When a light source like electric bulb is kept inside a water tank, the light from the source travels in all direction inside the water. The light that is incident on the water surface at an angle less than the critical angle will undergo refraction and emerge out from the water. The light incident at an angle greater than critical angle will undergo total internal reflection.The light falling particularly at critical angle graces the surface.Thus, the entire surface of water appears illuminated when seen from outside as shown in Figure 6.25.

On the other hand, when light entering the water from outside is seen from inside the water, the view is restricted to a particular angle equal to the critical angle i_c. The restricted illuminated circular area is called *Snell's window* as shown in Figure 6.26(a).The Figure 6.26(b) shows the angle of view for water animals.

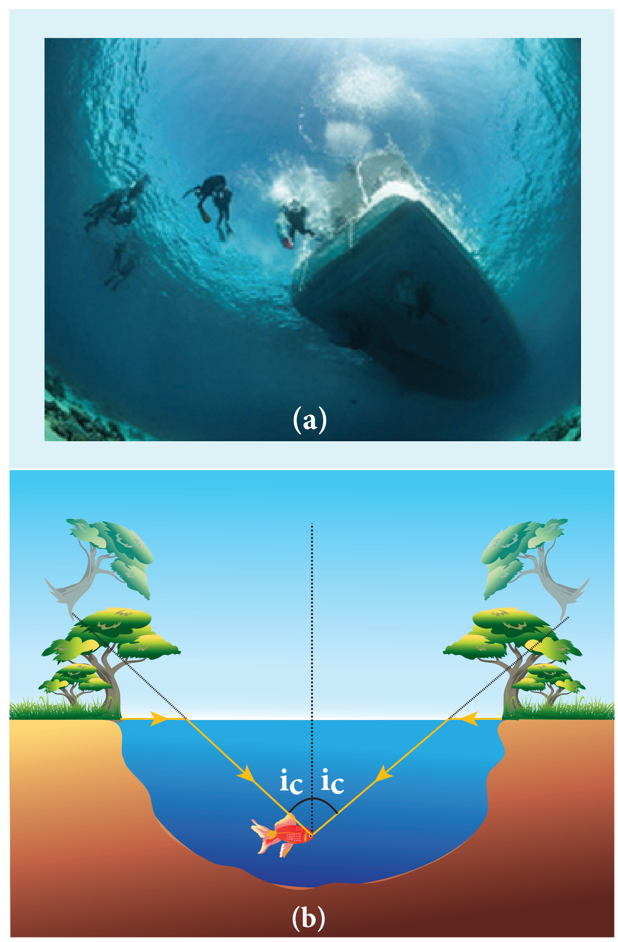


Figure 6.26 (a) Snell's window and (b) angle of view for water animals

The angle of view for water animals is restricted to twice the critical angle $2i_c$. The critical angle for water is 48.6°. Thus the angle of view is 97.2°. The radius R of the circular area depends on the depth d from which it is seen and also the refractive indices of the media. The radius of Snell's window can be deduced with the illustration as shown in Figure 6.27.

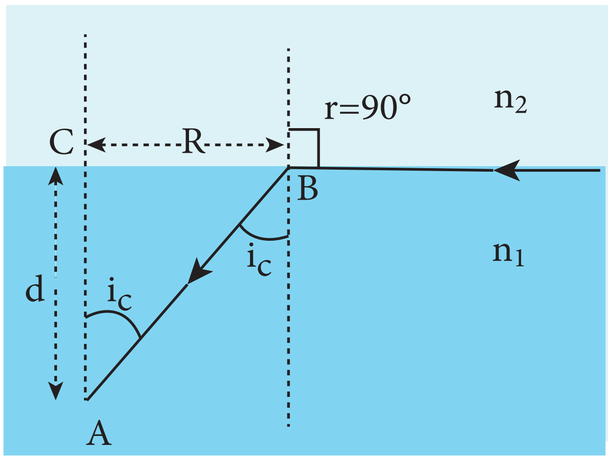


Figure 6.27 Radius of Snell's window

Light is seen from a point *A* at a depth *d*. The Snell's law in product form, equation (6.19) for the refraction happening at the point *B* on the boundary between the two media is,

$$n_1 \sin i_c = n_2 \sin 90^0 \quad (6.32)$$

$$n_1 \sin i_c = n_2 \qquad \because \sin 90^0 = 1$$

$$\sin i_c = \frac{n_2}{n_1} \quad (6.33)$$

From the right angle triangle ΔABC,

$$\sin i_c = \frac{CB}{AB} = \frac{R}{\sqrt{d^2 + R^2}} \quad (6.34)$$

Equating the above two equation 6.34 and equation 6.35, $\frac{R}{\sqrt{d^2+R^2}} = \frac{n_2}{n_1}$

Squaring on both sides, $\frac{R^2}{R^2+d^2} = \left(\frac{n_2}{n_1}\right)^2$

Taking reciprocal, $\frac{R^2+d^2}{R^2} = \left(\frac{n_1}{n_2}\right)^2$

On further simplifying,

$$1 + \frac{d^2}{R^2} = \left(\frac{n_1}{n_2}\right)^2; \qquad \frac{d^2}{R^2} = \left(\frac{n_1}{n_2}\right)^2 - 1;$$

$$\frac{d^2}{R^2} = \frac{n_1^2}{n_2^2} - 1 = \frac{n_1^2 - n_2^2}{n_2^2}$$

Again taking reciprocal and rearranging,

$$\frac{R^2}{d^2} = \frac{n_2^2}{n_1^2 - n_2^2}; \qquad R^2 = d^2\left(\frac{n_2^2}{n_1^2 - n_2^2}\right)$$

The radius of illumination is,

$$R = d\sqrt{\frac{n_2^2}{(n_1^2 - n_2^2)}} \quad (6.35)$$

If the rarer medium outside is air, then, n_2 = 1, and we can take $n_1 = n$

$$R = d\left(\frac{1}{\sqrt{n^2-1}}\right) \text{ (or) } R = \frac{d}{\sqrt{n^2-1}} \quad (6.36)$$

EXAMPLE 6.9

What is the radius of the illumination when seen above from inside a swimming pool from a depth of 10 m on a sunny day? What is the total angle of view? [Given, refractive index of water is 4/3]

Solution

Given, n = 4/3, d = 10 m.

Radius of illumination, $R = \frac{d}{\sqrt{n^2-1}}$

$$R = \frac{10}{\sqrt{(4/3)^2 - 1}} = \frac{10 \times 3}{\sqrt{16-9}}$$

$$R = \frac{30}{\sqrt{7}} = 11.32m$$

To find the angle of the view of the cone,

$$i_c = \sin^{-1}\left(\frac{1}{n}\right)$$

$$i_c = \sin^{-1}\left(\frac{1}{4/3}\right) = \sin^{-1}\left(\frac{3}{4}\right) = 48.6^\circ$$

The total angle of view is,

$$2i_c = 2 \times 48.6^\circ = 97.2^\circ$$

6.4.7.5 Optical fiber

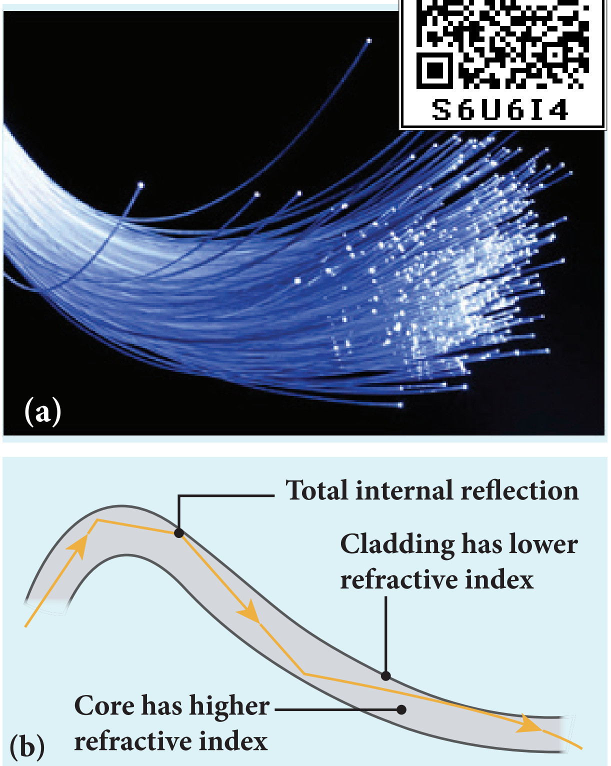


Figure 6.28 Optical fibre

Transmitting signals through optical fibres is possible due to the phenomenon of total internal reflection. **Optical fibres consists of inner part called *core* and outer part called *cladding* (or) *sleeving*.** The refractive index of the material of the core must be higher than that of the cladding for total internal reflection to happen. Signal in the form of light is made to incident inside the core-cladding boundary at an angle greater than the critical angle. Hence, it undergoes repeated total internal reflections along the length of the fibre without undergoing any refraction. The light travels inside the core with no appreciable loss in the intensity of the light as shown in Figure 6.28(a). Even while bending the optic fiber, it is done in such a way that the condition for total internal reflection is ensured at every reflection as shown in Figure 6.28(b).

6.4.7.6 Acceptance angle in optical fibre

To ensure the critical angle incidence in the core-cladding boundary inside the optical fibre, the light should be incident at a certain angle at the end of the optical fiber while entering in to it. This angle is called ***acceptance angle***. It depends on the refractive indices of the core n_1, cladding n_2 and the outer medium n_3. Assume the light is incident at an angle called acceptance angle i_a at the outer medium and core boundary at A.

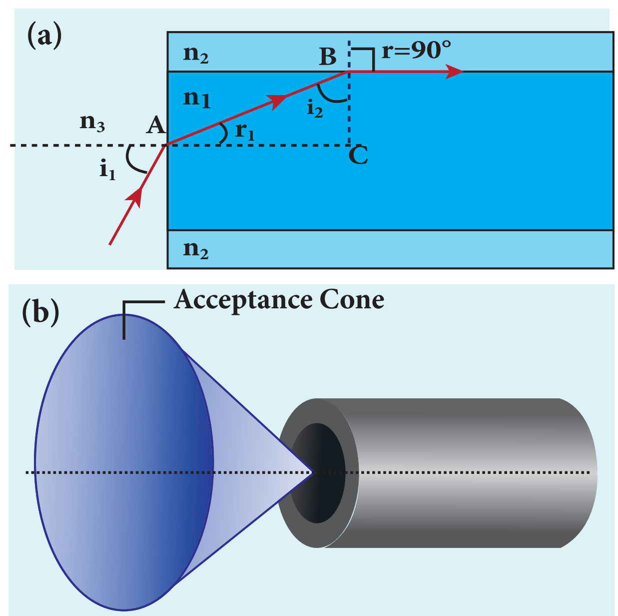


Figure 6.29 (a) acceptance angle and (b) acceptance cone.

The Snell's law in the product form, equation (6.19) for this refraction at the point A is as shown in the Figure 6.29(a),

$$n_3 \sin i_a = n_1 \sin r_a \qquad (6.37)$$

To have the total internal reflection inside optical fibre, the angle of incidence at the core-cladding interface at B should be atleast critical angle i_c. Snell's law in the product form, equation (6.19) for the refraction at point B is,

$$n_1 \sin i_c = n_2 \sin 90^\circ \qquad (6.38)$$

$$n_1 \sin i_c = n_2 \qquad \because \sin 90° = 1$$

$$\therefore \sin i_c = \frac{n_2}{n_1} \qquad (6.39)$$

From the right angle triangle ΔABC,

$$i_c = 90° - r_a$$

Now, equation (6.39) becomes,

$$\sin\left(90^{\circ} - r_a\right) = \frac{n_2}{n_1}$$

Using trigonometry, $\cos r_a = \frac{n_2}{n_1}$ (6.40)

$$\sin r_a = \sqrt{1 - \cos^2 r_a}$$

Substituting for $\cos r_a$

$$\sin r_a = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \qquad (6.41)$$

Substituting this in equation (6.37).

$$n_3 \sin i_a = n_1 \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2} \qquad (6.42)$$

On further simplification,

$$\sin i_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_3} \text{ (or) } \sin i_a = \sqrt{\frac{n_1^2 - n_2^2}{n_3^2}} \qquad (6.43)$$

$$i_a = \sin^{-1}\left(\sqrt{\frac{n_1^2 - n_2^2}{n_3^2}}\right) \qquad (6.44)$$

If outer medium is air, then n_3 = 1. The acceptance angle i_a becomes,

$$i_a = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right) \qquad (6.45)$$

Light can have any angle of incidence from 0 to i_a with the normal at the end of the optical fibre forming a conical shape called ***acceptance cone*** as shown in Figure 6.29(b). In the equation (6.42), **the term ($n_3 \sin i_a$) is called *numerical aperture NA*** of the optical fibre.

$$NA = n_3 \sin i_a = \sqrt{n_1^2 - n_2^2} \qquad (6.46)$$

If outer medium is air, then n_3 = 1. The numerical aperture *NA* becomes,

$$NA = \sin i_a = \sqrt{n_1^2 - n_2^2} \qquad (6.47)$$

EXAMPLE 6.10

A optical fibre is made up of a core material with refractive index 1.68 and a cladding material of refractive index 1.44. What is the acceptance angle of the fibre kept in air medium? What is the answer if there is no cladding?

Solution

Given, n_1 = 1.68, n_2 = 1.44, n_1 = 1

Acceptance angle, $i_a = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$

$$i_a = \sin^{-1}\left(\sqrt{(1.68)^2 - (1.44)^2}\right) = \sin^{-1}(0.865)$$

$$i_a \approx 60°$$

If there is no cladding then, n_2 = 1

Acceptance angle, $i_a = \sin^{-1}\left(\sqrt{n_1^2 - 1}\right)$

$$i_a = \sin^{-1}\left(\sqrt{(1.68)^2 - 1}\right) = \sin^{-1}(1.35)$$

$\sin^{-1}(\text{more than } 1)$ is not possible. But, this includes the range 0º to 90º. Hence, all the rays entering the core from flat surface will undergo total internal reflection.

Note: If there is no cladding then there is a condition on the refractive index (n_1) of the core.

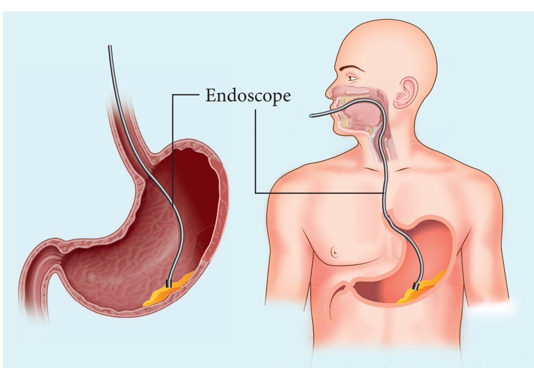
$$i_a = \sin^{-1}\left(\sqrt{n_1^2 - 1}\right)$$

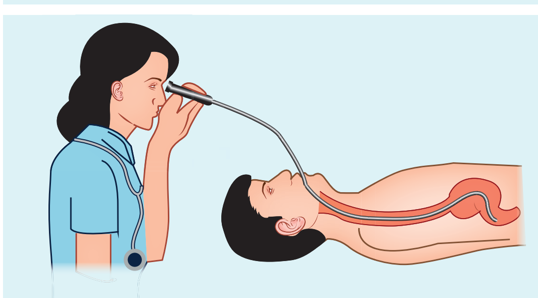
Here, as per mathematical rule, $\left(n_1^2 - 1\right) \le 1$ *or* $\left(n_1^2\right) \le 2$

or $n_1 \le \sqrt{2}$

Hence, in air (no cladding) the refractive index n_1 of the core should be, $n_1 \leq 1.414$

An endoscope is an instrument used by doctors which has a bundle of optical fibres that are used to see inside a patient's body. Endoscopes work on the phenomenon of total internal reflection. The optical fibres are inserted in to the body through mouth, nose or a special hole made in the body. Even operations could be carried out with the endoscope cable which has the necessary instruments attached at their ends.





6.4.8 Refraction in glass slab

When a ray of light passes through a glass slab it refracts at two refracting surfaces. When the light ray enters the slab it travels from rarer medium (air) to denser medium (glass). This results in deviation of ray towards the normal. When the light ray leaves the slab it travels from denser medium to rarer medium resulting in deviation of ray away from the normal. After the two refractions, the emerging ray has the same direction as that of the incident ray on the slab with a lateral displacement or shift L. i.e. There is no change in the direction of ray but the path of the incident ray and refracted ray are different and parallel to each other. To calculate the lateral displacement, a perpendicular is drawn in between the paths of incident ray and refracted ray as shown in Figure 6.30.

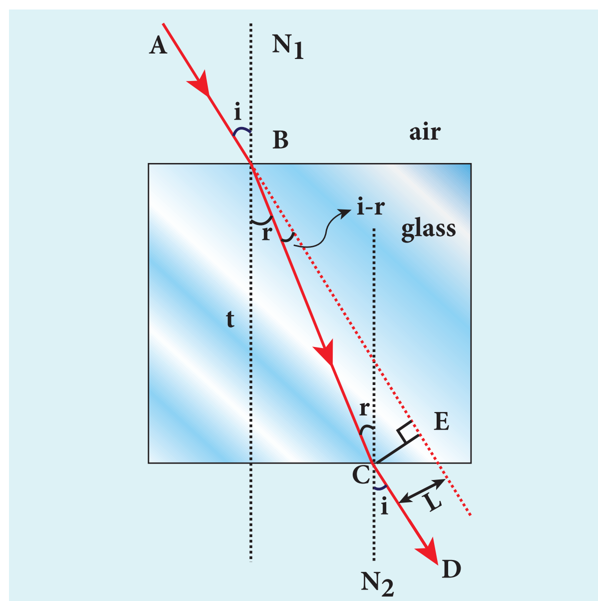


Figure 6.30 Refraction in glass slab

Consider a glass slab of thickness t and refractive index n is kept in air medium. The path of the light is $ABCD$ and the refractions occur at two points B and C in the glass slab. The angles of incidence i and refraction r are measured with respect to the normal N_1 and N_2 at the two points B and C respectively. The lateral displacement L is the perpendicular distance CE drawn between the path of light and the undeviated path of light at point C.

In the right angle triangle ΔBCE,

$$\sin(i-r) = \frac{L}{BC}; BC = \frac{L}{\sin(i-r)} \tag{6.48}$$

In the right angle triangle ΔBCF,

$$\cos(r)=\frac{t}{BC};BC=\frac{t}{\cos(r)} \quad (6.49)$$

Equating equations (6.48) and (6.49),

$$\frac{L}{\sin(i-r)}=\frac{t}{\cos(r)}$$

After rearranging,

$$L=t\left(\frac{\sin(i-r)}{\cos(r)}\right) \quad (6.50)$$

Lateral displacement depends upon the thickness of the slab. Thicker the slab, greater will be the lateral displacement. Greater the angle of incident, larger will be the lateral displacement.

EXAMPLE 6.11

The thickness of a glass slab is 0.25 m. it has a refractive index of 1.5. A ray of light is incident on the surface of the slab at an angle of 60°. Find the lateral displacement of the light when it emerges from the other side of the mirror.

Solution

Given, thickness of the slab, t = 0.25 m, refractive index, n = 1.5, angle of incidence, i = 60°.

Using Snell's law, $1 \times \sin i = n \sin r$

$$\sin r=\frac{\sin \mathrm{i}}{n}=\frac{\sin 60}{1.5}=0.58$$

$$r=\sin^{-1}0.58=35.25^{\circ}$$

Lateral displacement is, $L=t\left(\frac{\sin(i-r)}{\cos(r)}\right)$

$$L=(0.25)\times\left(\frac{\sin(60-35.25)}{\cos(35.25)}\right)=0.1281\,m$$

The lateral displacement is, $L=12.81\,cm$

6.5 REFRACTION AT SINGLE SPHERICAL SURFACE

We have so far studied only the refraction at a plane surfaces. The refractions also do take place at spherical surface between two transparent media. The laws of refraction hold good at every point on the spherical surface. The normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point. Therefore, the normal always passes through its center of curvature. The study of refraction at single spherical surface paves way to the understanding of thin lenses which consist of two surfaces of which one or both must be spherical.

The following assumptions are made while considering refraction at spherical surfaces.

(a) The incident light is assumed to be monochromatic (single colour)

(b) The incident ray of light is very close to the principal axis (paraxial rays).

The sign conventions are similar to that of the spherical mirrors.

6.5.1 Equation for refraction at single spherical surface

Let us consider two transparent media having refractive indices n_1 and n_2 are separated by a spherical surface as shown in Figure 6.31. Let C be the centre of curvature of the spherical surface. Let a point object O be in the medium n_1. The line OC cuts the spherical surface at the pole P of the surface. As the rays considered are paraxial rays, the perpendicular dropped for the point of incidence to the principal axis is very close to the pole or passes through the pole itself.

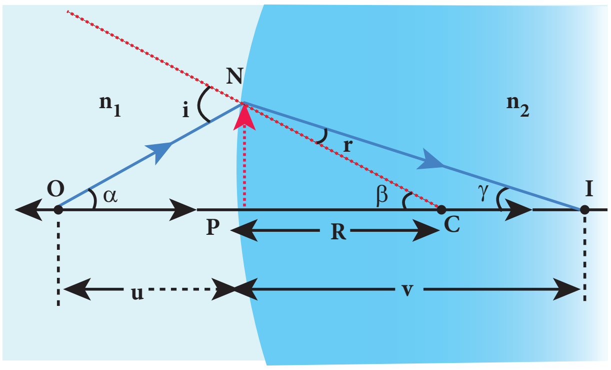


Figure 6.31 Refraction at single spherical surface

Light from O falls on the refracting surface at N. The normal drawn at the point of incidence passes through the centre of curvature C. As $n_2 > n_1$, light in the denser medium deviates towards the normal and meets the principal axis at I where the image is formed.

Snell's law in product form for the refraction at the point N could be written as,

$$n_1 \sin i = n_2 \sin r \tag{6.19}$$

As the angles are small, sine of the angle could be approximated to the angle itself.

$$n_1 i = n_2 r \tag{6.51}$$

Let the angles,

$\angle NOP = \alpha$, $\angle NCP = \beta$, $\angle NIP = \gamma$

$$\tan\alpha = \frac{PN}{PO}; \quad \tan\beta = \frac{PN}{PC}; \quad \tan\gamma = \frac{PN}{PI}$$

As these angles are small, tan of the angle could be approximated to the angle itself.

$$\alpha = \frac{PN}{PO}; \ \beta = \frac{PN}{PC}; \ \gamma = \frac{PN}{PI} \tag{6.52}$$

For the triangle, ΔONC,

$$i = \alpha + \beta \tag{6.53}$$

For the triangle, ΔINC,

$$\beta = r + \gamma \text{ (or) } r = \beta - \gamma \tag{6.54}$$

Substituting for i and r from equations (6.53) and (6.54) in the equation (6.51).

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

Rearranging,

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

Substituting for α, β and γ from equation (6.52),

$$n_1\left(\frac{PN}{PO}\right) + n_2\left(\frac{PN}{PI}\right) = (n_2 - n_1)\left(\frac{PN}{PC}\right)$$

Further simplifying by cancelling PN,

$$\frac{n_1}{PO} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC} \tag{6.55}$$

Following sign conventions, $PO = -u$, $PI = +v$ and $PC = +R$ in equation (6.58),

$$\frac{n_1}{-u} + \frac{n_2}{v} = \frac{(n_2 - n_1)}{R}$$

After rearranging, finally we get,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R} \tag{6.56}$$

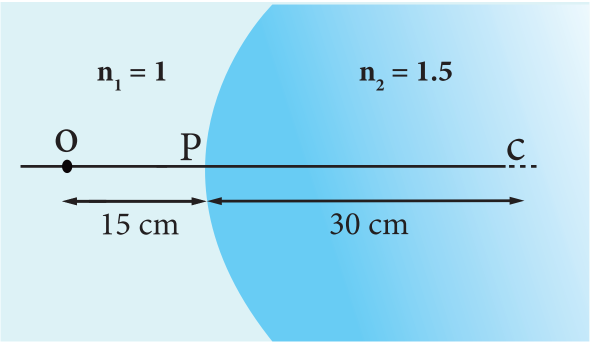
Equation (6.56) gives the relation among the object distance u, image distance v, refractive indices of the two media (n_1 and n_2) and the radius of curvature R of the spherical surface. It holds for any spherical surface.

If the first medium is air then, $n_1 = 1$ and the second medium is taken just as $n_2 = n$, then the equation is reduced to,

$$\frac{n}{v} - \frac{1}{u} = \frac{(n-1)}{R} \tag{6.57}$$

EXAMPLE 6.12

Locate the image of the point object O in the situation shown. The point C denotes the centre of curvature of the separating surface.



Solution

Given, u = –15 cm, R = 30 cm, n_1 = 1 and n_2 = 1.5

Equation for single spherical surface is,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

Substituting the values,

$$\frac{1.5}{v} - \frac{1}{-15} = \frac{(1.5-1)}{30}; \qquad \frac{1.5}{v} + \frac{1}{15} = \frac{(0.5)}{30}$$

$$\frac{1.5}{v} + \frac{1}{15} = \frac{1}{60}; \qquad \frac{1.5}{v} = \frac{1}{60} - \frac{1}{15};$$

$$\frac{1.5}{v} = \frac{1-4}{60} = \frac{-3}{60}; = -\frac{1}{20}$$

v = –30 cm

The image is a virtual image formed 30 cm to the left of the spherical surface.

6.5.2 Lateral magnification in single spherical surface

Let us, consider an extended object OO' is kept perpendicular to the principal axis to the left of the single spherical surface as shown in Figure 6.32. The image formed on the other side of the surface is II'. Consider a ray from O' in the first medium towards C in the second medium. As this ray is incident normal to the spherical surface, it goes undeviated in the second medium.The position of image may be located using the Equation (6.60).

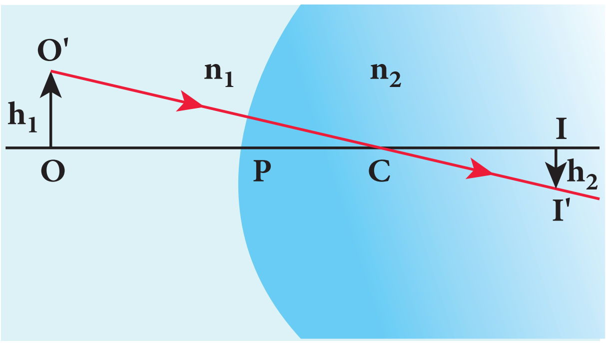


Figure 6.32 Lateral magnification in single spherical surface

The lateral or transverse magnification m is defined as the ratio of height of the image to the height of the object.

$$m = \frac{II'}{OO'} \qquad (6.58)$$

From the two similar triangles $\Delta COO'$ and $\Delta CII'$, we can write,

$$\frac{II'}{OO'} = \frac{CI}{CO}$$

From the geometry,

$$\frac{CI}{CO} = \frac{PI - PC}{PC + PO}$$

Hence,

$$m = \frac{II'}{OO'} = \frac{PI - PC}{PC + PO} \qquad (6.59)$$

Applying sign conventions in the above equation (6.59),

$$II' = -h_2, \quad OO' = \mathrm{h}_1, PI = +v,$$
$$PC = +R, \quad PO = -u$$

Where, h_1 is the height of the object and h_2 is the height of the image.

$$m = \frac{-h_2}{h_1} = \frac{v-R}{R+(-u)}; \ \mathrm{m} = \frac{h_2}{h_1} = -\left(\frac{v-R}{R-u}\right)$$

After rearranging,

$$m = \frac{h_2}{h_1} = \frac{R - v}{R - u} \tag{6.60}$$

We can also arrive at an equation for lateral magnification involving the refractive indices of the two media.

Let us consider the equation for single spherical surface as,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

Further simplifying, $\dfrac{n_2 u - n_1 v}{vu} = \dfrac{(n_2 - n_1)}{R}$

Rewriting for R, $\quad R = \dfrac{(n_2 - n_1)vu}{n_2 u - n_1 v}$

Rearranging,

$$R - u = \frac{n_2 u(v - u)}{n_2 u - n_1 v} \tag{6.61}$$

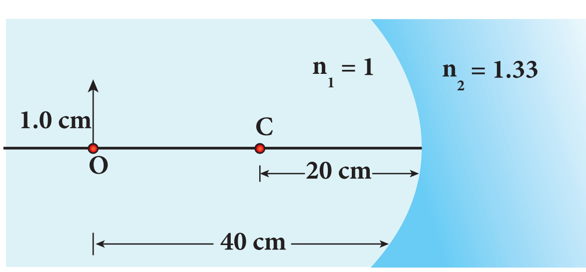
$$R - v = \frac{n_1 v(v - u)}{n_2 u - n_1 v} \tag{6.62}$$

Substituting equations (6.61) and (6.62) in equation (6.60) we get the equation for lateral magnification as,

$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} \tag{6.63}$$

EXAMPLE 6.13

Find the size of the image formed in the given figure.



Solution

Given, $u = -40$ cm, $R = -20$ cm, $n_1 = 1$ and $n_2 = 1.33$

Equation for single spherical surface is,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

Substituting the values,

$$\frac{1.33}{v} - \frac{1}{-40} = \frac{(1.33 - 1)}{-20}; \qquad \frac{1.33}{v} + \frac{1}{40} = \frac{(0.33)}{-20}$$

$$\frac{1.33}{v} = -\frac{(0.33)}{20} - \frac{1}{40};$$

$$\frac{1.33}{v} = \frac{-0.66 - 1}{40} = -\frac{1.66}{40}$$

$$v = -40 \times \frac{1.33}{1.66} = -32.0\ cm$$

Equation for magnification is, $m = \dfrac{h_2}{h_1} = \dfrac{n_1 v}{n_2 u}$

$$\frac{h_2}{1.0} = \frac{(1.0) \times (-32)}{(1.33) \times (-40)} = 0.6 cm \quad \text{(or)} \quad h_2 = 0.6 cm$$

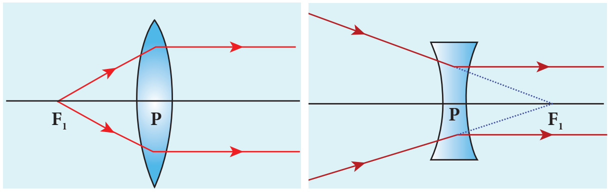
The erect virtual image of height 0.6 cm is formed at 32.0 cm to the left of the single spherical surface.

6.6 THIN LENS

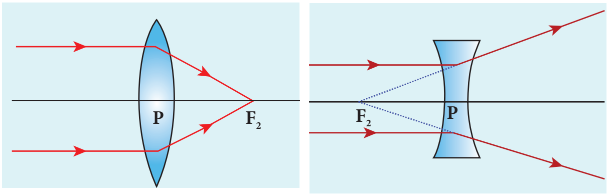
A lens is formed by a transparent material bounded between two spherical surfaces or one plane and another spherical surface. In a thin lens, the distance between the surfaces is very small. If there are two spherical surfaces, then there will be two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2. A plane surface has its center of curvature C at infinity and its radius of curvature R is infinity ($R = \infty$). The terminologies of spherical mirrors also hold good very much for thin lens except for focal length.

6.6.1 Primary and secondary focal points

As the thin lens is formed by two spherical surfaces, the lens may separate two different media. i.e. the media to the left and right of the lens may be different. Hence, we have two focal lengths.



(a) Primary focal point



(b) Secondary focal point

Figure 6.33 Focal length of convex and concave lenses

The *primary focus* F_1 is defined as a point where an object should be placed to give parallel emergent rays to the principal axis as shown in Figure 6.33(a). For a convergent lens, such an object is a real object and for a divergent lens, it is a virtual object. The distance PF_1 is the *primary focal length* f_1.

The *secondary focus* F_2 is defined as a point where all the parallel rays travelling close to the principal axis converge to form an image on the principal axis as shown in Figure 6.33(b). For a convergent lens, such an image is a real image and for a divergent lens, it is a virtual image. The distance PF_2 is the secondary focal length f_2.

If the media on the two sides of a thin lens have same refractive index, then the two focal lengths are equal. We will mostly be using the secondary focus F_2 in our further discussions.

6.6.2 Sign conventions for lens on focal length

The sign conventions for thin lenses differ only in the signs followed for focal lengths.

(a) The sign of focal length is *not decided* on the direction of measurement of the focal length from the pole of the lens as they have two focal lengths, one to the left and another to the right (primary and secondary focal lengths on either side of the lens).

(b) The focal length of the thin lens is taken as positive for a converging lens and negative for a diverging lens.

The other sign conventions for object distance, image distance, radius of curvature, object height and image height (except for the focal lengths as mentioned above) remain the same for thin lenses as that of spherical mirrors.

6.6.3 Lens maker's formula and lens equation

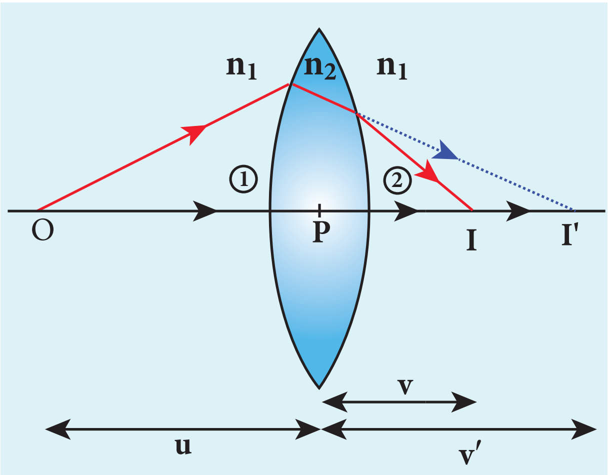


Figure 6.34 Refraction through thin lens

Let us consider a thin lens made up of a medium of refractive index n_2 is placed in a medium of refractive index n_1. Let R_1 and R_2 be the radii of curvature of two spherical surfaces ① and ② respectively and P be the pole as shown in figure 6.34. Consider a point object O on the principal axis. The ray which falls very close to P, after refraction at the surface ① forms image at I'. Before it does so, it is again refracted by the surface ②. Therefore the final image is formed at I.

The general equation for the refraction at a spherical surface is given from Equation (6.59),

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

For the refracting surface ①, the light goes from n_1 to n_2.

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R_1} \quad (6.64)$$

For the refracting surface ②, the light goes from medium n_2 to n_1.

$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{(n_1 - n_2)}{R_2} \quad (6.65)$$

Adding the above two equations (6.64) and (6.65)

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Further simplifying and rearranging,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (6.66)$$

If the object is at infinity, the image is formed at the focus of the lens. Thus, for $u = \infty$, $v = f$. Then the equation becomes.

$$\frac{1}{f} - \frac{1}{\infty} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (6.67)$$

If the refractive index of the lens is n_2 and it is placed in air, then $n_2 = n$ and $n_1 = 1$. So the equation (6.67) becomes,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (6.68)$$

The above equation is called the ***lens maker's formula***, because it tells the lens manufactures what curvature is needed to make a lens of desired focal length with a material of particular refractive index. This formula holds good also for a concave lens. By comparing the equations (6.66) and (6.67) we can write,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (6.69)$$

This equation is known as ***lens equation*** which relates the object distance u and image distance v with the focal length f of the lens. This formula holds good for a any type of lens.

6.6.4 Lateral magnification in thins lens

Let us consider an object OO' of height h_1 placed on the principal axis with its height perpendicular to the principal axis as shown in Figure 6.35. The ray OP passing through the pole of the lens goes undeviated. The inverted real image II' formed has a height h_2.

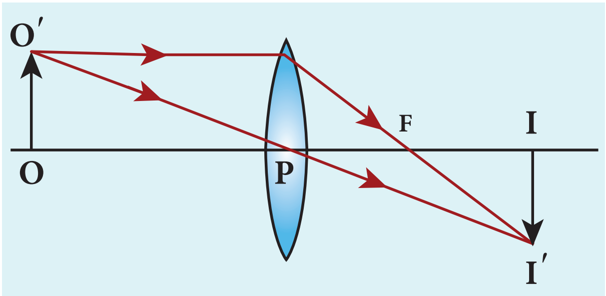


Figure 6.35 Lateral magnification in thin lens

The lateral or transverse magnification m is defined as the ratio of the height of the image to that of the object.

$$m = \frac{II'}{OO'} \qquad (6.70)$$

From the two similar triangles $\Delta\, POO'$ and $\Delta\, PII'$, we can write,

$$\frac{II'}{OO'} = \frac{PI}{PO} \qquad (6.71)$$

Applying sign convention,

$$\frac{-h_2}{h_1} = \frac{v}{-u}$$

Substituting this in the equation (6.70) for magnification,

$$m = \frac{-h_2}{h_1} = \frac{v}{-u}$$

After rearranging,

$$m = \frac{h_2}{h_1} = \frac{v}{u} \qquad (6.72)$$

The magnification is negative for real image and positive for virtual image. In the case of a concave lens, the magnification is always positive and less than one.

We can also have the equations for magnification by combining the lens equation with the formula for magnification as,

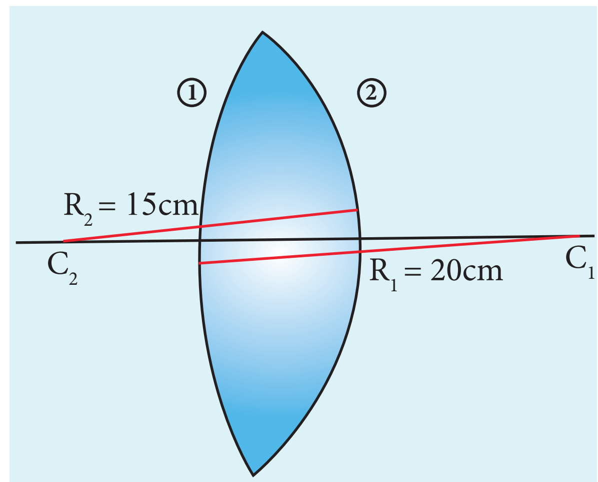
$$m = \frac{h_2}{h_1} = \frac{f}{f+u} \text{ (or) } m = \frac{h_2}{h_1} = \frac{f-v}{f} \qquad (6.73)$$

EXAMPLE 6.14

A biconvex lens has radii of curvature 20 cm and 15 cm each. The refractive index of the material of the lens is 1.5. What is its focal length? Will the focal length change if the lens is flipped by the side?

Solution

For a biconvex lens, radius of curvature of the first surface is positive and that of the second surface is negative side as shown in the figure.



Given, $n = 1.5$, R_1=20 cm and R_2= –15 cm

Lens maker's formula, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Substituting the values,

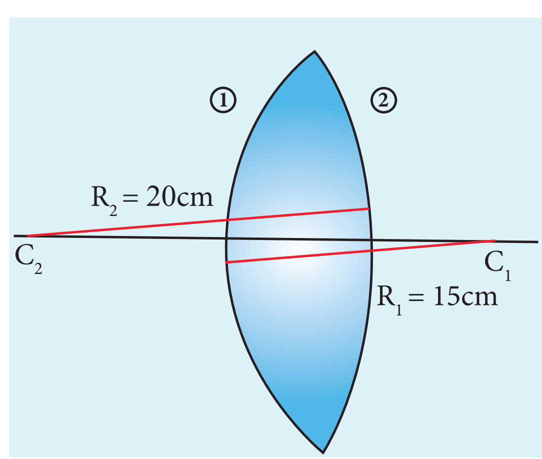
$$\frac{1}{f} = (1.5-1)\left(\frac{1}{20} - \frac{1}{-15}\right)$$

$$\frac{1}{f} = (0.5)\left(\frac{1}{20} + \frac{1}{15}\right) = (0.5)\left(\frac{3+4}{60}\right) = \left(\frac{1}{2} \times \frac{7}{60}\right) = \frac{7}{120}$$

$$f = \frac{120}{7} = 17.14\,cm$$

As the focal length is positive the lens is a converging lens.

If the lens is flipped back to front,



Now, $R_1 = 15$ cm and $R_2 = -20$ cm, $n = 1.5$

Substituting the values,

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{15} - \frac{1}{-20}\right)$$

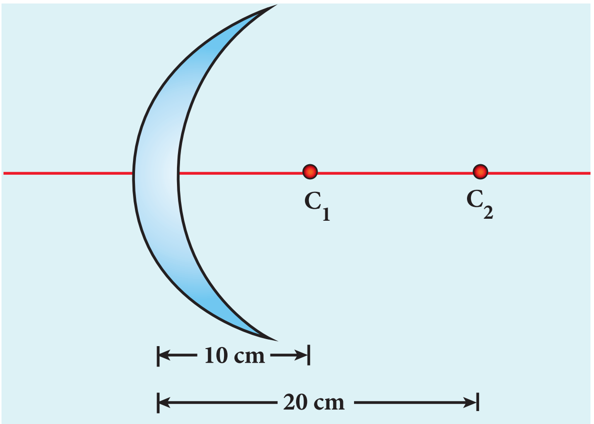
$$\frac{1}{f} = (1.5-1)\left(\frac{1}{15} + \frac{1}{20}\right)$$

This will also result in, $f = 17.14\ cm$

Thus, it is concluded that the focal length of the lens will not change if it is flipped side wise. This is true for any lens. Students can verify this for any lens.

EXAMPLE 6.15

Determine the focal length of the lens made up of a material of refractive index 1.52 as shown in the diagram. (Points C_1 and C_2 are the centers of curvature of the first and second surface.)



Solution

This lens is called convexo-concave lens

Given, $n = 1.52$, $R_1 = 10$ cm and $R_2 = 20$ cm

Lens makers formula, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Substituting the values,

$$\frac{1}{f} = (1.52-1)\left(\frac{1}{10} - \frac{1}{20}\right)$$

$$\frac{1}{f} = (0.52)\left(\frac{2-1}{20}\right) = (0.52)\left(\frac{1}{20}\right) = \frac{0.52}{20}$$

$$f = \frac{20}{0.52} = 38.46\,\text{cm}$$

As the focal length is positive, the lens is a converging lens.

6.6.5 Power of a lens

Power of lens is the measurement of deviating strength of a lens i.e. when a ray is incident on a lens then the degree with which the lens deviates the ray is determined by the power of the lens. Power of the lens is inversely proportional to focal length i.e. greater the power of lens, greater will be the deviation of ray and smaller will be the focal length. In Figure 6.36, the lens (b) has greater deviating strength than that of (a). As (b) has greater deviating strength, its focal length is less and vice versa.

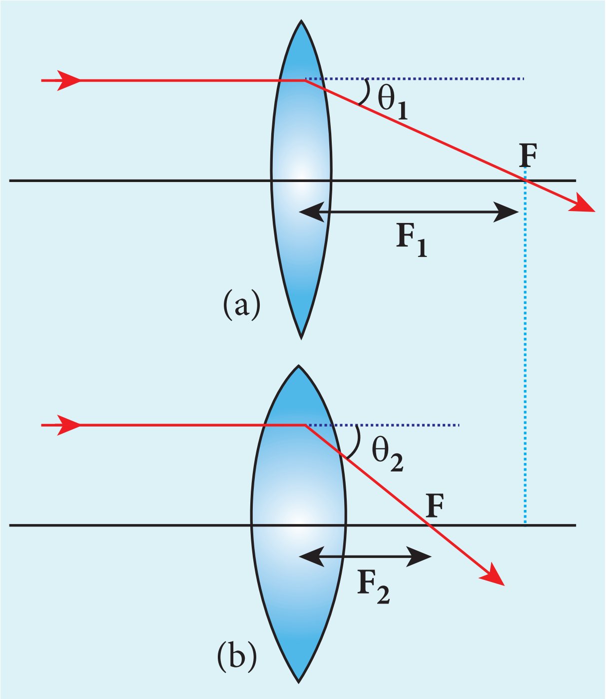


Figure 6.36 Power of lens

In other words, the *power* of a lens is a measure of the degree of convergence or divergence of light falling on it. The power of a lens P is defined as the reciprocal of its focal length.

$$P = \frac{1}{f} \tag{6.74}$$

The unit of power is diopter D. 1 D = 1 m^{-1}. Power is positive for converging lens and negative for diverging lens.

From the lens makers formula, equation (6.68), the equation for power can be written as,

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{6.75}$$

The outcome of this equation of power is that larger the value of refractive index, greater is the power of lens and vice versa. Also for lenses with small radius of curvature (bulky) the power is large and for lenses with large the radius of curvature (skinny), the power is small.

EXAMPLE 6.16

If the focal length is 150 *cm* for a glass lens, what is the power of the lens?

Solution

Given, focal length, f = 150 cm (or) f = 1.5 *m*

Equation for power of lens is, $P = \frac{1}{f}$

Substituting the values,

$$P = \frac{1}{1.5} = 0.67 \text{diopter}$$

As the power is positive, it is a converging lens.

6.6.6 Focal length of lenses in contact

Let us consider two lenses ① and ② of focal length f_1 and f_2 are placed coaxially in contact with each other so that they have a common principal axis. For an object placed at O beyond the focus of the first lens ① on the principal axis, an image is formed by it at I'. This image I' acts as an object for the second lens ② and the final image is formed at I as shown in Figure. 6.37. As these two lenses are thin, the measurements are done with respect to the common optical centre P in the middle of the two lenses.

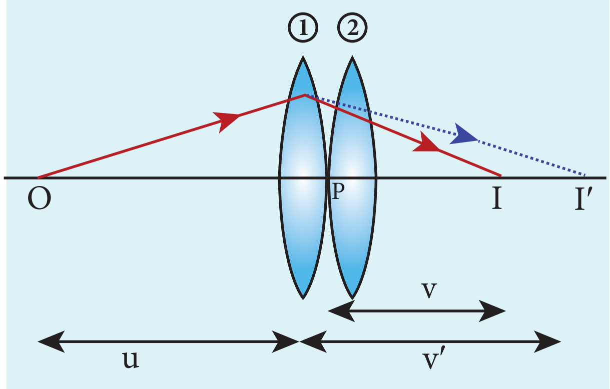


Figure. 6.37 Lenses in contact

Let, PO be object distance u and PI' be the image distance (v') for the first lens ① and object distance for the second lens ② and $PI = v$ be the image distance for the second lens ② .

Writing the lens equation for first lens ①,

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \tag{6.76}$$

Writing the lens equation for second lens ②,

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \tag{6.77}$$

Adding the above two equations (6.76) and (6.77),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \tag{6.78}$$

If the combination acts as a single lens of focal length f so that for an object at the position O it forms the image at I then,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{F} \qquad (6.79)$$

Comparing equations (6.78) and (6.79) we can write,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \qquad (6.80)$$

The above equation can be extended for any number of lenses in contact as,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}+\text{.........} \qquad (6.81)$$

The above equation can be written as power of the lenses as,

$$P=P_1+P_2+P_3+P_4+\text{.........} \qquad (6.82)$$

Where, P is the net power of the lens combination of lenses in contact. One should note that the sum in equation (6.82) is an algebraic sum. The powers of individual lenses may be positive (for convex lenses) or negative (for concave lenses). Combination of lenses helps to obtain diverging or converging lenses of desired magnification. Also, combination of lenses enhances the sharpness of the images. As the image formed by the first lens becomes the object for the second and so on, the total magnification m of the combination is a product of magnification of individual lenses. We can write, $m = m_1 \times m_2 \times m_3 \ldots.$ (without proof).

EXAMPLE 6.17

What is the focal length of the combination if a lens of focal length –70 cm is brought in contact with a lens of focal length 150 cm? What is the power of the combination?

Solution

Given, focal length of first lens, f_1 = –70 cm, focal length of second lens, f_2 = 150 cm.

Equation for focal length of lenses in contact, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$

Substituting the values,

$$\frac{1}{F}=\frac{1}{-70}+\frac{1}{150}=-\frac{1}{70}+\frac{1}{150}$$

$$\frac{1}{F}=\frac{-150+70}{70\times150}=\frac{-80}{70\times150}=-\frac{80}{10500}$$

$$F=\frac{-1050}{8}=-131.25\,cm$$

As the focal length is negative, the combination of two lenses is a diverging system of lenses.

The power of combination is,

$$P=\frac{1}{F}=\frac{1}{-1.3125\,m}=-0.76\,\text{diopter}$$

6.6.7 Focal length of lenses in out of contact

When two thin lenses are separated by a distance d common optical center cannot be chosen for them. Hence, they cannot be treated as a single thin lens. Actually, such a combination should be treated as a thick lens for which the theory is more involved (beyond the scope of present study). However, as a special case, only when the object is placed at infinity, the combination can be replaced by a single thin lens. The focal length and position of the equivalent lens can be derived by considering the concept of angle of deviation.

Let O be a point object on the principal axis of a lens as shown in Figure 6.38. OA is the incident ray on the lens at a point A at a height h above the optical centre. The ray is deviated through an angle δ and forms the image at I on the principal axis.

The incident and refracted rays subtend the angles, $\angle AOP = \alpha$ and $\angle AIP = \beta$ with the principal axis respectively.

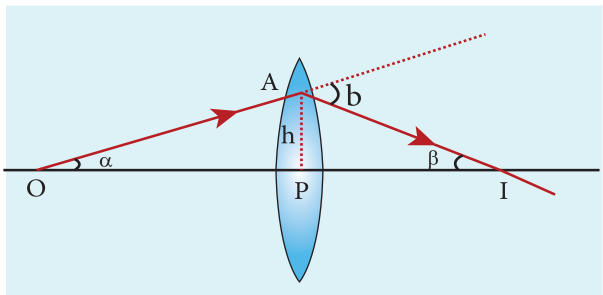


Figure 6.38 Angle of deviation in lens

In the triangle ΔOAI, the angle of deviation δ can be written as,

$$\delta = \alpha + \beta \qquad (6.83)$$

If the height h is small as compared to *PO* and *PI*, the angles *α*, *β* and *δ* are also small. Then,

$$\alpha \approx \tan\alpha = \frac{PA}{PO}; \text{ and } \beta \approx \tan\beta = \frac{PA}{PI} \qquad (6.84)$$

$$\text{Then, } \delta = \frac{PA}{PO} + \frac{PA}{PI} \qquad (6.85)$$

Here, $PA = h$, $PO = -u$ and $PI = v$

$$\delta = \frac{h}{-u} + \frac{h}{v} = h\left(\frac{1}{-u} + \frac{1}{v}\right) \qquad (6.86)$$

After rearranging

$$\delta = h\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{h}{f}$$

$$\delta = \frac{h}{f} \qquad (6.87)$$

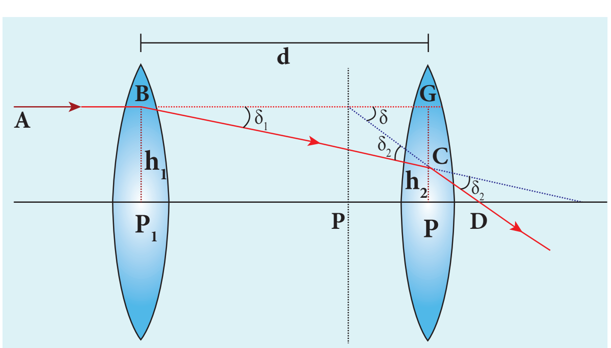


Figure 6.39 Lens in out of contact

The above equation tells that the angle of deviation is the ratio of height to the focal length. Now, the case of two lenses of focal length f_1 and f_2 arranged coaxially but separated by a distance d can be considered as shown in Figure 6.39.

For a parallel ray that falls on the arrangement, the two lenses produce deviations δ_1 and δ_2 respectively and The net deviation δ is.

$$\delta = \delta_1 + \delta_2 \qquad (6.88)$$

From Equation (6.87),

$$\delta_1 = \frac{h_1}{f_1}; \; \delta_2 = \frac{h_2}{f_2} \text{ and } \delta = \frac{h_1}{f} \qquad (6.89)$$

The equation (6.88) becomes,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \qquad (6.90)$$

From the geometry,

$$h_2 - h_1 = P_2G - P_2C = CG$$

$$h_2 - h_1 = BG \tan\, \delta_1 \approx BG\delta_1$$

$$h_2 - h_1 = d\frac{h_1}{f_1}$$

$$h_2 = h_1 + d\frac{h_1}{f_1} \qquad (6.91)$$

Substituting the above equation in Equation (6.90)

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2} + \frac{h_1 d}{f_1 f_2}$$

On further simplification,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2} \qquad (6.92)$$

The above equation could be used to find the equivalent focal length. To find the position of the equivalent lens, we can further write from the geometry,

$$PP_2 = EG = \frac{GC}{\tan\delta}$$

$$PP_2 = EG = \frac{h_1 - h_2}{\tan\delta} = \frac{h_1 - h_2}{\delta}$$

From equations (6.89) and (6.91)

$$h_2 - h_1 = d\frac{h_1}{f_1} \quad \text{and} \quad \delta = \frac{h_1}{f}$$

$$PP_2 = \left(d\frac{h_1}{f_1}\right) \times \left(\frac{f}{h_1}\right)$$

$$PP_2 = \left(d\frac{f}{f_1}\right) \qquad (6.93)$$

POINTS TO PONDER

System of combination of lenses is commonly used in designing lenses for cameras, microscopes, telescopes and other optical instruments. They produce better magnification as well as sharpness of the images.

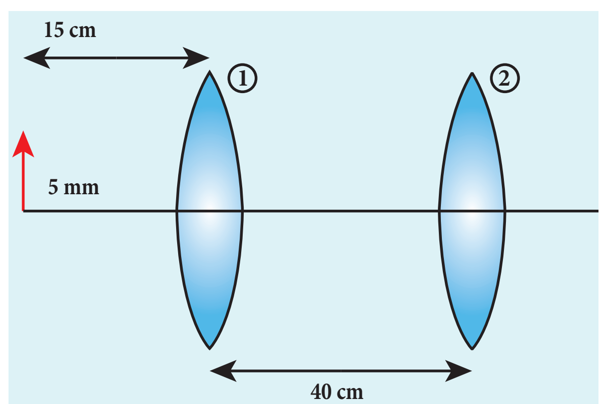
The above equation (6.93) is the position of the equivalent single lens from the second lens. Its position from the first lens is,

$$PP_1 = d - \left(d\frac{f}{f_1}\right)$$

$$PP_1 = d\left(1 - \frac{f}{f_1}\right) \qquad (6.94)$$

Note The Equation (6.92), (6.93) and (6.94) hold good only for the special case of parallel incident rays or object at infinity. We cannot use these equations if the object is at a finite distance. For finite distance of the object, the image positions must be calculated separately using the lens equation for the two lenses.

EXAMPLE 6.18



An object of 5 mm height is placed at a distance of 15 cm from a convex lens of focal length 10 cm. A second lens of focal length 5 cm is placed 40 cm from the first lens and 55 cm from the object. Find (a) the position of the final image, (b) its nature and (c) its size.

Solution

Given, h_1 = 5 mm = 0.5 cm, u_1 = –15 cm, f_1 = 10 cm, f_2 = 5 cm, d = 40 cm

For the first lens, the lens equation is,

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}$$

Substituting the values,

$$\frac{1}{v_1}-\frac{1}{-15}=\frac{1}{10}; \qquad \frac{1}{v_1}+\frac{1}{15}=\frac{1}{10}$$

$$\frac{1}{v_1}=\frac{1}{10}-\frac{1}{15}=\frac{15-10}{150}=\frac{5}{150}=\frac{1}{30}$$

v_1= 30 cm

First lens forms image 30 cm to the right of first lens.

Let us find the height of this image.

Equation for magnification is, $m=\frac{h_2}{h_1}=\frac{v}{u}$

Substituting the values, $\frac{h_2}{0.5}=\frac{30}{-15}$

$$h_2=0.5\times\frac{30}{-15}=-1\ cm$$

As the height of the lens is negative, the image is inverted, real image.

Object is at 10 cm to the left of the second lens (40–30=10 cm). Hence, u_2 = –10 cm

For the second, the lens equation is,

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

Substituting the values,

$$\frac{1}{v_2}-\frac{1}{-10}=\frac{1}{5}; \ \frac{1}{v_2}+\frac{1}{10}=\frac{1}{5}$$

$$\frac{1}{v_2}=\frac{1}{5}-\frac{1}{10}=\frac{10-5}{50}=\frac{5}{50}=\frac{1}{10}$$

$$v_2=10\ cm$$

The image is formed 10 cm to the right of the second lens.

Let us find the height of the final image. Assume, the final height of the image formed by the second lens is h_2' and the height of the object for the second lens h_1' is the image height of the first lens, $h_1'=h_2'$

Equation for magnification is, $m=\frac{h_2'}{h_1'}=\frac{v_2}{u_2}$

Substituting the values, $\frac{h_2'}{-1}=\frac{10}{-10}$

$$h_2'=(-1)\times\left(\frac{10}{-10}\right)=1\ cm=10\ mm$$

As the height of the image is positive, the image is erect, and it is real.

6.7 PRISM

A prism is a triangular block of glass or plastic. It is bounded by the three plane faces not parallel to each other. Its one face is grounded which is called base of the prism. The other two faces are polished which are called refracting faces of the prism. The angle between the two refracting faces is called angle of prism (or) refracting angle (or) apex angle of the prism represented as *A* as shown in Figure 6.40.

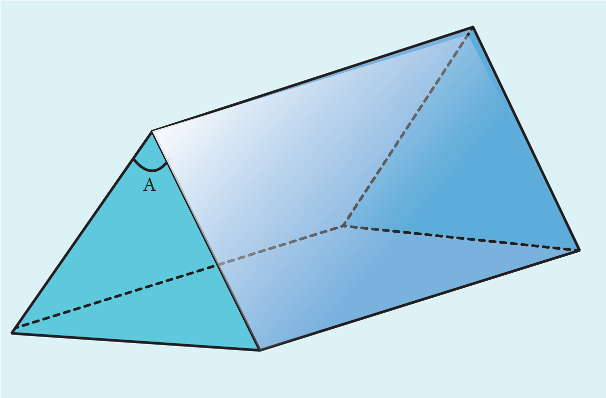


Figure 6.40 Prism

6.7.1 Angle of deviation produced by prism

Let light ray *PQ* is incident on one of the refracting faces of the prism as shown in Figure 6.41. The angles of incidence and refraction at the first face *AB* are i_1 and r_1. The path of the light inside the prism is *QR*. The angle of incidence and refraction at the second face *AC* is r_2 and i_2 respectively. RS is the ray emerging from the second face. Angle i_2 is also called angle of emergence. **The angle between the direction of the incident ray *PQ* and the emergent ray *RS* is called the *angle of deviation d*.** The two normals drawn at the point of incidence *Q* and emergence *R* are *QN* and *RN*. They meet at point *N*. The incident ray and the emergent ray meet at a point *M*.

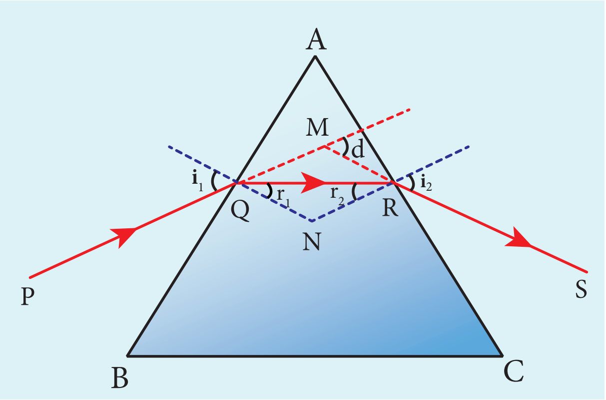


Figure 6.41 Refraction through prism

The deviation d_1 at the surface *AB* is,

$$angle\ \angle RQM = d_1 = i_1 - r_1 \qquad (6.95)$$

The deviation d_2 at the surface *AC* is,

$$angle\ \angle QRM = d_2 = i_2 - r_2 \qquad (6.96)$$

Total angle of deviation *d* produced is,

$$d = d_1 + d_2 \qquad (6.97)$$

Substituting for d_1 and d_2,

$$d = (i_1 - r_1) + (i_2 - r_2)$$

After rearranging,

$$d = (i_1 - r_1) + (i_2 - r_2) \qquad (6.98)$$

In the quadrilateral *AQNR*, two of the angles (at the vertices *Q* and *R*) are right angles. Therefore, the sum of the other angles of the quadrilateral is 180°.

$$\angle A + \angle QNR = 180^{o} \qquad (6.99)$$

From the triangle Δ*QNR*,

$$r_1 + r_2 + \angle QNR = 180^{o} \qquad (6.100)$$

Comparing these two equations (6.99) and (6.100) we get,

$$r_1 + r_2 = A \qquad (6.101)$$

Substituting this in equation (6.98) for angle of deviation,

$$d = i_1 + i_2 - A \qquad (6.102)$$

Thus, the angle of deviation depends on the angle of incidence angle of emergence and the angle for the prism. For a given angle of incidence the angle of emergence is decided by the refractive index of the material of the prism. Hence, the angle of deviation depends on these following factors.

(i) the angle of incidence
(ii) the angle of the prism
(iii) the material of the prism
(iv) the wave length of the light

EXAMPLE 6.19

A monochromatic light is incident on an equilateral prism at an angle 30° and emerges at an angle of 75°. What is the angle of deviation produced by the prism?

Solution

Given, as the prism is equilateral,

$A = 60°;\ i_1 = 30°;\ i_2 = 75°$

Equation for angle of deviation, $d = i_1 + i_2 - A$

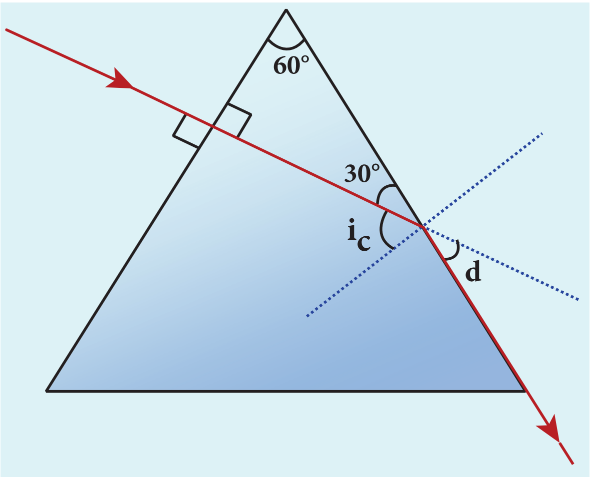
Substituting the values, $d = 30°+75°-60°=45°$

The angle of deviation produced is, $d = 45°$

EXAMPLE 6.20

Light ray falls at normal incidence on the first face of an equilateral prism and emerges gracing the second face. What is the angle of deviation? What is the refractive index of the material of the prism?

Solution



The given situation is shown in the figure.

Here, $A = 60°$; $i_1 = 0°$; $i_2 = 90°$

Equation for angle of deviation,

$$d = i_1 + i_2 - A$$

Substituting the values,

$$d = 0° + 90° - 60° = 30°$$

The angle of deviation produced is, $d = 30°$

The light inside the prism must be falling on the second face at critical angle as it graces the boundary.

Equation for critical angle is, $\sin i_c = \frac{1}{n}$

$$n = \frac{1}{\sin i_c}; \quad n = \frac{1}{\sin 30^\circ} = \frac{1}{1/2} = 2$$

The refractive index of the material of the prism is, $n = 2$

6.7.2 Angle of minimum deviation

A graph plotted between the angle of incidence and angle of deviation is shown in Figure 6.42. One could observe that the angle of deviation decreases with increase in angle of incidence and reaches a minimum value and then continues to increase.

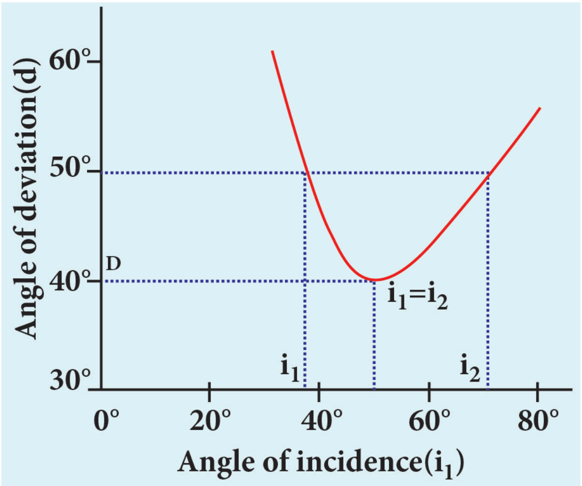


Figure 6.42 Graph between i and d

The minimum value of angle of deviation is called angle of *minimum deviation D*. At minimum deviation,

(a) the angle of incidence is equal to the angle of emergence, $i_1 = i_2$.

(b) the angle of refraction at the face one and face two are equal, $r_1 = r_2$).

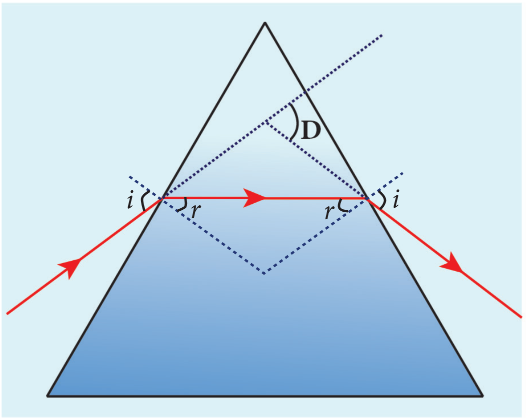


Figure 6.43 Angle of minimum deviation

(c) the incident ray and emergent ray are symmetrical with respect to the prism.

(d) the refracted ray inside the prism is parallel to its base of the prism.

The case of angle of minimum deviation is shown in Figure 6.43.

6.7.3 Refractive index of the material of the prism

At minimum deviation, $i_1=i_2=i$ and $r_1=r_2=r$

Now, the equation (6.102) becomes,

$$D = i_1 + i_2 - A = 2i - A \quad \text{(or)} \quad i = \frac{(A+D)}{2}$$

The equation (6.101) becomes,

$$r_1 + r_2 = A \Rightarrow 2r = A \quad \text{(or)} \quad r = \frac{A}{2}$$

Substituting i and r in Snell's law,

$$n = \frac{\sin i}{\sin r}$$

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (6.103)$$

The above equation is used to find the refractive index of the material of the prism. The angles A and D can be measured experimentally.

EXAMPLE 6.21

The angle of minimum deviation for a prism is 37°. If the angle of prism is 60°, find the refractive index of the material of the prism.

Solution

Given, A=60°; D=37°

Equation for refractive index is,

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Substituting the values,

$$n = \frac{\sin\left(\frac{60^0+37^0}{2}\right)}{\sin\left(\frac{60^0}{2}\right)} = \frac{\sin\left(48.5^0\right)}{\sin\left(30^0\right)} = \frac{0.75}{0.5} = 1.5$$

The refractive index of the material of the prism is, $n = 1.5$

6.7.4 Dispersion of white light through prism

So far the angle of deviation produced by a prism is discussed for monochromatic light (i.e. light of single colour). When white light enter in to a prism, the effect called *dispersion* takes place. **Dispersion is splitting of white light into its constituent colours. This band of colours of light is called its *spectrum*.** When a narrow beam of parallel rays of white light is incident on the face of a prism and the refracted beam is received on a white screen, a band of colours is obtained in the order, recollected by the word: VIBGYOR i.e., Violet, Indigo, Blue, Green, Yellow, Orange and Red. Violet is the most deviated and red is the least deviated colour as shown in Figure 6.44.

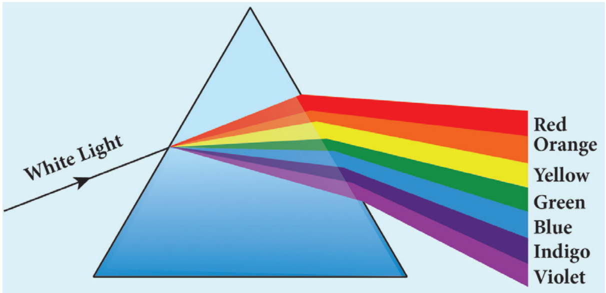


Figure 6.44 Dispersion of white light in to its constituent colours

The colours obtained in a spectrum depend on the nature of the source of the light used. Each colour of light is associated with a definite wavelength. Red light is at the longer wavelength end (700 nm) while the violet light is at the shorter wavelength end (400 nm). Therefore the violet ray travels with a smaller velocity in glass prism than red ray.

Points to Ponder

Sir Isaac Newton has demonstrated through a classic experiment to produce white light when all the colours of VIBGYOR are recombined. He used a prism to produce dispersion and made all the colours to incident on another inverted prism to combine all the colours to get white light as shown in figure.

Dispersion takes place because light of different wave lengths travel with different speeds inside the prism. In other words, the refractive index of the material of the prism is different for different colours. For violet, the refractive index is high and for red the refractive index is the low. In Vacuum, all the colours travel with the same speed. The refractive index of two different glasses for different colours is shown in Table 6.4.

The speed of light is independent of wavelength in vacuum. Therefore, vacuum is a non-dispersive medium in which all colours travel with the same speed.

Table 6.4 Refractive indices for different wavelengths

Colour	Wavelength (nm)	Crown glass	Flint glass
Violet	396.9	1.533	1.663
Blue	486.1	1.523	1.639
Yellow	589.3	1.517	1.627
Red	656.3	1.515	1.622

6.7.5 Dispersive Power

Consider a beam of white light passes through a prism; it gets dispersed into its constituent colours as shown in Figure 6.45. Let δ_V, δ_R are the angles of deviation for violet and red light. Let n_V and n_R are the refractive indices for the violet and red light respectively.

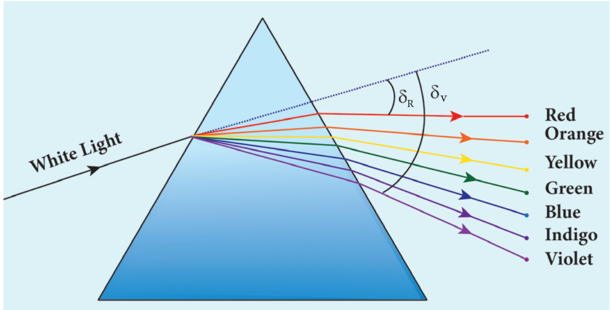


Figure 6.45 Angle of deviation for different colours

The refractive index of the material of a prism is given by the equation (6.103),

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here A is the angle of the prism and D is the angle of minimum deviation. If the angle of prism is small of the order of 10°, the prism is said to be a small angle prism. When rays of light pass through such prisms, the angle of deviation also becomes small. If A be the angle of a small angle prism and

δ the angle of deviation then the prism formula becomes.

$$n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad (6.104)$$

For small angles of A and δ_m,

$$\sin\left(\frac{A+\delta}{2}\right) \approx \left(\frac{A+\delta}{2}\right) \quad (6.105)$$

$$\sin\left(\frac{A}{2}\right) \approx \left(\frac{A}{2}\right) \quad (6.106)$$

$$\therefore n = \frac{\left(\frac{A+\delta}{2}\right)}{\left(\frac{A}{2}\right)} = \frac{A+\delta}{A} = 1 + \frac{\delta}{A}$$

Further simplifying, $\frac{\delta}{A} = n - 1$

$$\delta = (n-1)A \quad (6.107)$$

When white light enters the prism, the deviation is different for different colours. Thus, the refractive index is also different for different colours.

For Violet light, $\delta_V = (n_V - 1)A$ (6.108)

For Red light, $\delta_R = (n_R - 1)A$ (6.109)

As, angle of deviation for violet colour δ_V is greater than angle of deviation for red colour δ_R, the refractive index for violet colour n_V is greater than the refractive index for red colour n_R.

Subtracting δ_V from δ_R we get,

$$\delta_V - \delta_R = (n_V - n_R)A \quad (6.110)$$

The term (δ_V–δ_R) is the angular separation between the two extreme colours (violet and red) in the spectrum is called the *angular dispersion*. Clearly, the angular dispersion produced by a prism depends upon.

(i) Angle of the prism

(ii) Nature of the material of the prism.

If we take δ is the angle of deviation for any middle ray (green or yellow) and n the corresponding refractive index. Then,

$$\delta = (n - 1)A \quad (6.111)$$

Dispersive power (ω) is the ability of the material of the prism to cause dispersion. It is defined as the ratio of the angular dispersion for the extreme colours to the deviation for any mean colour.

Dispersive power (ω),

$$\omega = \frac{\text{angular dispersion}}{\text{mean deviation}} = \frac{\delta_V - \delta_R}{\delta} \quad (6.112)$$

Substituting for $(\delta_V - \delta_R)$ and (δ),

$$\omega = \frac{(n_V - n_R)}{(n-1)} \quad (6.113)$$

Dispersive power is a dimensionless quality. It has no unit. Dispersive power is always positive. The dispersive power of a prism depends only on the nature of material of the prism and it is independent of the angle of the prism.

EXAMPLE 6.22

Find the dispersive power of flint glass if the refractive indices of flint glass for red, green and violet light are 1.613, 1.620 and 1.632 respectively.

Solution

Given, n_V=1.632; n_R=1.613; n_G=1.620

Equation for dispersive power is,

$$\omega = \frac{(n_V - n_R)}{(n_G - 1)}$$

Substituting the values,

$$\omega = \frac{1.632 - 1.613}{1.620 - 1} = \frac{0.019}{0.620} = 0.0306$$

The dispersive power of flint glass is, $\omega = 0.0306$

6.7.6 Scattering of sunlight

When sunlight enters the atmosphere of the earth, the atmospheric particles present in the atmosphere change the direction of the light. This process is known as scattering of light.

H9D1Z9

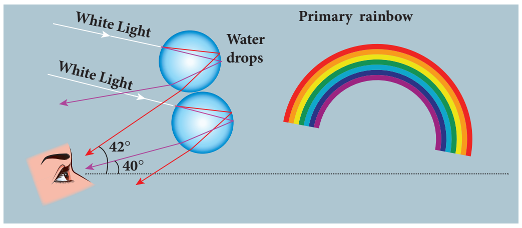
If the scattering of light is by atoms and molecules which have size *a* very less than that of the wave length λ of light $a << \lambda$, the scattering is called *Rayleigh's scattering*. The intensity of Rayleigh's scattering is inversely proportional to fourth power of wavelength.

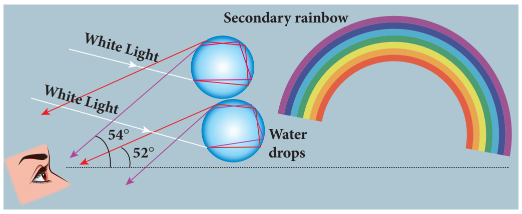
$$I \propto \frac{1}{\lambda^4} \qquad (6.114)$$

According to equation 6.114, violet colour which has the shortest wavelength gets much scattered during day time. The next scattered colour is blue. As our eyes are more sensitive to blue colour than violet colour the sky appears blue during day time as shown in Figure 6.46(a). But, during sunrise and sunset, the light from sun travels a greater distance through the atmosphere. Hence, the blue light which has shorter wavelength is scattered away and the less-scattered red light of longer wavelength manages to reach

DO YOU KNOW?

Rainbow is an example of dispersion of sunlight through droplets of water during rainy days. Rainbow is observed during a rainfall or after the rainfall or when we look at a water fountain provided the sun is at the back of the observer. When sunlight falls on the water drop suspended in air, it splits (or dispersed) into its constituent seven colours. Thus, water drop suspended in air behaves as a glass prism. Primary rainbow is formed when light entering the drop undergoes one total internal reflection inside the drop before coming out from the drop as shown in figure. The angle of view for violet to red in primary rainbow is 40° to 42°. A secondary rainbow appears outside of a primary rainbow and develops when light entering a raindrop undergoes two internal reflections. The angle of view for red to violet in a secondary rainbow is, 52° to 54°.





our eye. This is the reason for the reddish appearance of sky during sunrise and sunset as shown in Figure 6.46(b).

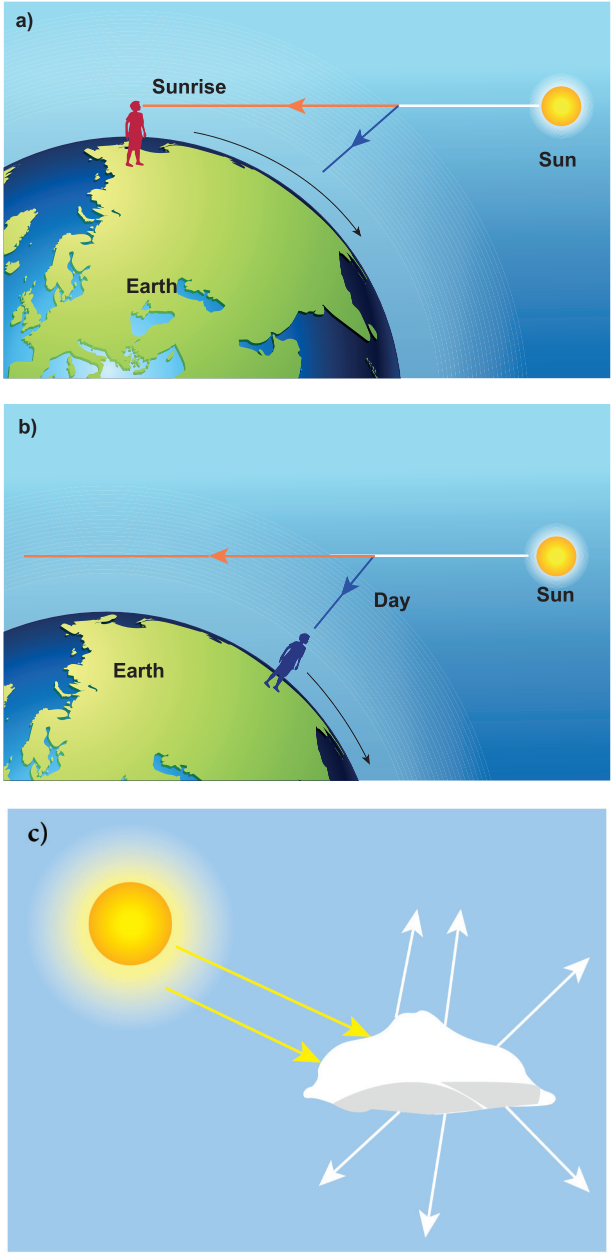


Figure 6.46. Scattering of different types

If light is scattered by large particles like dust and water droplets present in the atmosphere which have size a greater than the wavelength λ of light, $a >> \lambda$, the intensity of scattering is equal for all the wavelengths. It is happening in clouds which contains large amount of dust and water droplets. Thus, in clouds all the colours get equally scattered irrespective of wavelength. This is the reason for the whitish appearance of cloud as shown in Figure 6.46(c). But, the rain clouds appear dark because of the condensation of water droplets on dust particles that makes the cloud become opaque.

If earth has no atmosphere there would not have been any scattering and the sky would appear dark. That is why sky appears dark for the astronauts who could see the sky from above the atmosphere.

6.8 THEORIES ON LIGHT

Light is a form of energy that is transferred from one place to another. A glance at the evolution of various theories of light put forward by scientists will give not only an over view of the nature of light but also its propagation and some phenomenon demonstrated by it.

6.8.1 Corpuscular theory

Sir Isaac Newton (1672) gave the corpuscular theory of light which was also suggested earlier by Descartes (1637) to explain the laws of reflection and refraction. According this theory, light is emitted as tiny, massless (negligibly small mass) and perfectly elastic particles called corpuscles. As the corpuscles are very small, the source of light does not suffer appreciable loss of mass even if it emits light for a long time. On account of high speed, they are unaffected by the force of gravity and their path is a straight line in a medium of uniform refractive index. The energy of light is the kinetic energy of these corpuscles. When these corpuscles impinge on the retina of the eye, the vision is produced. The different size of the corpuscles is the reason for different colours of light. When the corpuscles approach a surface between two media, they are either attracted or repelled. The reflection of light is due to the repulsion of the corpuscles by the medium and refraction of light is due to the attraction of the corpuscles by the medium.

This theory could not explain the reason why the speed of light is lesser in denser medium than in rarer medium and also the phenomena like interference, diffraction and polarisation.

6.8.2 Wave theory

Christian Huygens (1678) proposed the wave theory to explain the propagation of light through a medium. According to him, light is a disturbance from a source that travels as longitudinal mechanical waves through the ether medium that was presumed to pervade all space as mechanical wave requires medium for its propagation. The wave theory could successfully explain phenomena of reflection, refraction, interference and diffraction of light.

Later, the existence of ether in all space was proved to be wrong. Hence, this theory could not explain the propagation of light through vacuum. The phenomenon of polarisation could not be explained by this theory as it is the property of only transverse waves.

6.8.3 Electromagnetic wave theory

Maxwell (1864) proved that light is an electromagnetic wave which is transverse in nature carrying electromagnetic energy. He could also show that no medium is necessary for the propagation of electromagnetic waves. All the phenomenon of light could be successfully explained by this theory.

Nevertheless, the interaction phenomenon of light with matter like photoelectric effect, Compton effect could not be explained by this theory.

6.8.4 Quantum theory

Albert Einstein (1905), endorsing the views of Max Plank (1900), was able to explain photoelectric effect (discussed in Unit 7) in which light interacts with matter as photons to eject the electrons. A *photon* is a discrete packet of energy. Each photon has energy *E* of,

$$E = h\nu \qquad (6.115)$$

Where, h is Plank's constant ($h = 6.625 \times 10^{-34}$ J s) and ν is frequency of electromagnetic wave.

As light has both wave as well as particle nature it is said to have dual nature. Thus, it is concluded that light propagates as a wave and interacts with matter as a particle.

6.9 WAVE NATURE OF LIGHT

Light is a transverse, electromagnetic wave. The wave nature of light was first illustrated through experiments on interference and diffraction. Like all electromagnetic waves, light can travel through vacuum. The transverse nature of light is demonstrated in polarization.

6.9.1 Wave optics

Wave optics deals with the wave characteristics of light. With the help of wave optics, we are going to learn in details the phenomena of interference, diffraction and polarization. Even the law of reflection and refraction are proved only with the help of wave optics. Though light propagates as a wave, its direction of propagation is still represented as a ray.

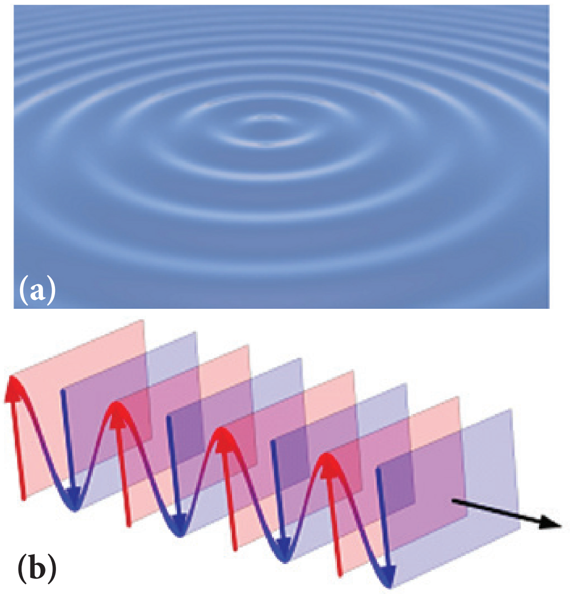


Figure 6.47 (a) Ripples on water surface (b) Wavefront and ray

An example for wave propagation is the spreading of circular ripples on the surface of still water from a point at which a stone is dropped. The molecules or particles of water are moving only up and down (oscillate) when a ripple passes out that part. All these particles on the circular ripple are in the same phase of vibration as they are all at the same distance from the center. The ripple represents a wavefront as shown in Figure 6.47(a). **A *wavefront* is the locus of points which are in the same state or phase of vibration.** When a wave propagates it is treated as the propagation of wavefront. The wavefront is always perpendicular to the direction of the propagation of the wave. As the direction of ray is in the direction of propagation of the wave, the wavefront is always perpendicular to the ray as shown in Figure 6.47(b).

The shape of a wavefront observed at a point depends on the shape of the source and also the distance at which the source is located. A point source located at a finite distance gives spherical wavefronts. An extended (or) line source at finite distance gives cylindrical wavefronts. The plane wavefronts are received from any source that is located at infinity as shown in Figure 6.48.

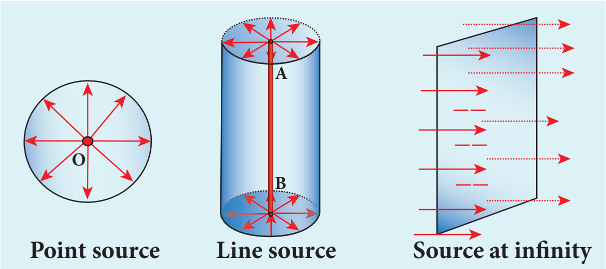


Figure 6.48 Wavefronts

6.9.2 Huygens' Principle

Huygens principle is a geometrical construction which gives the shape of the wavefront at any time if we know its shape at $t = 0$. **According to Huygens principle, each point of the wavefront is the source of secondary wavelets emanating from these points spreading out in all directions with the speed of the wave. These are called as secondary wavelets**. The common tangent, in other words the envelope to all these wavelets gives the position and shape of the new wavefront at a later time. Thus, Huygens' principle explains the propagation of a wavefront.

The propagation of a spherical and plane wavefront is explained in using Huygens' principle. Let, AB be the wavefront at a time, $t = 0$. According to Huygens' principle, every point on AB acts as a source of secondary wavelet which travels with the speed of the wave (speed of light c). To find the position of the wavefront after a time t, circles of radius equal to ct are drawn with points P, Q, R ... etc., as centers on AB. The tangent or forward envelope $A'B'$ of the small circles is the new wavefront at that instant. The wavefront $A'B'$ will be a spherical wavefront from a point object which is at a finite distance as shown in Figure 49(a) and it is a plane wavefront if the source of light is at a large distance (infinity) as shown in Figure 6.49(b).

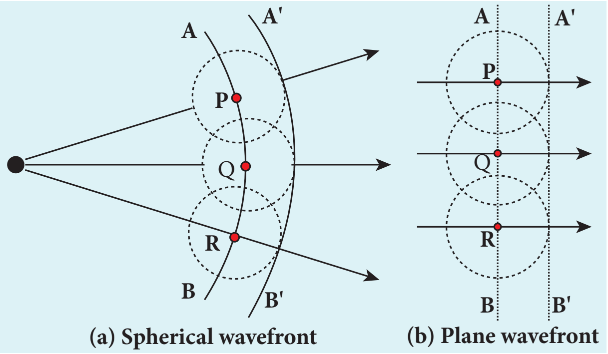


Figure 6.49 Huygens' Principle

There is one shortcoming in the above Huygens' construction for propagation of a wavefront. It could not explain the absence of backwave which also arises in the above construction. According to electromagnetic wave theory, the backwave is ruled out inherently. However, Huygens' construction diagrammatically explains the propagation of the wavefront.

6.9.3 Proof for laws of reflection using Huygens' Principle

Let us consider a parallel beam of light, incident on a reflecting plane surface such as a plane mirror XY as shown in Figure 6.50. The incident wavefront is AB and the reflected wavefront is $A'B'$ in the same medium. These wavefronts are perpendicular to the incident rays L, M and reflected rays L', M' respectively. By the time point A of the incident wavefront touches the reflecting surface, the point B is yet to travel a distance BB' to touch the reflecting surface at B'. When the point B falls on the reflecting surface at B', the point A would have reached A'. This is applicable to all the points on the wavefront. Thus, the reflected wavefront $A'B'$ emanates as a plane wavefront. The two normals N and N' are considered at the points where the rays L and M fall on the reflecting surface. As reflection happens in the same medium, the speed of light is same before and after the reflection. Hence, the time taken for the ray to travel from B to B' is the same as the time taken for the ray to travel from A to A'. Thus, the distance BB' is equal to the distance AA'; $(AA' = BB')$.

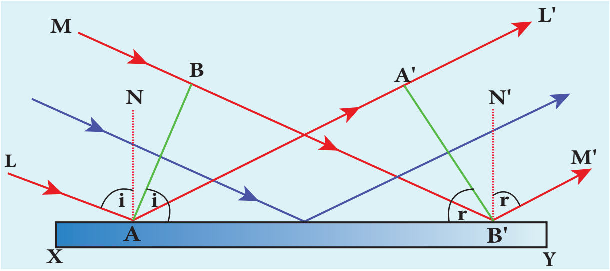


Figure 6.50 Laws of reflection

(i) The incident rays, the reflected rays and the normal are in the same plane.

(ii) Angle of incidence,

$\angle i = \angle NAL = 90^{o} - \angle NAB = \angle BAB'$

Angle of reflection,

$\angle r = \angle N'B'M' = 90^{o} - \angle N'B'A' = \angle A'B'A$

For the two right angle triangles, $\Delta ABB'$ and $\Delta B'A'A$, the right angles, $\angle B$ and $\angle A'$ are equal, ($\angle B$ and $\angle A' = 90^{o}$); the two sides, AA' and BB' are equal, $(AA' = BB')$; the side AB' is the common. Thus, the two triangles are congruent. As per the property of congruency, the two angles, $\angle BAB'$ and $\angle A'B'A$ must also be equal.

$$i = r \qquad (6.1)$$

Hence, the laws of reflection are proved.

6.9.4 Proof for laws of refraction using Huygens' Principle

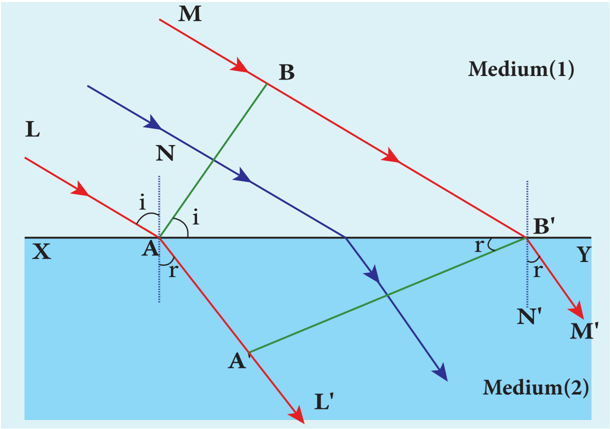


Figure 6.51 Law of refraction

Let us consider a parallel beam of light is incident on a refracting plane surface *XY* such as a glass surface as shown in Figure 6.51. The incident wavefront *AB* is in rarer medium (1) and the refracted wavefront $A'B'$ is in denser medium (2). These wavefronts are perpendicular to the incident rays *L*, *M* and refracted rays L', M' respectively. By the time the point *A* of the incident wavefront touches the refracting surface, the point *B* is yet to travel a distance BB' to touch the refracting surface at B'. When the point *B* falls on the refracting surface at B', the point *A* would have reached A' in the other medium. This is applicable to all the points on the wavefront. Thus, the refracted wavefront $A'B'$ emanates as a plane wavefront. The two normals *N* and N' are considered at the points where the rays *L* and *M* fall on the refracting surface. As refraction happens from rarer medium (1) to denser medium (2), the speed of light is v_1 and v_2 before and after refraction and v_1 is greater than v_2 ($v_1 > v_2$). But, the time taken *t* for the ray to travel from *B* to B' is the same as the time taken for the ray to travel from *A* to A'.

$$t = \frac{BB'}{v_1} = \frac{AA'}{v_2} \text{ (or) } \frac{BB'}{AA'} = \frac{v_1}{v_2}$$

(i) The incident rays, the refracted rays and the normal are in the same plane.

(ii) Angle of incidence,

$i = \angle NAL = 90^{o} - \angle NAB = \angle BAB'$

Angle of refraction,

$r = \angle N'B'M' = 90^{o} - \angle N'B'A' = \angle A'B'A$

For the two right angle triangles $\Delta ABB'$ and $\Delta B'A'A$,

$$\frac{\sin i}{\sin r} = \frac{BB'/AB'}{AA'/AB'} = \frac{BB'}{AA'} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1}$$

Here, *c* is speed of light in vacuum. The ratio c/v is the constant, called refractive index of the medium. The refractive index of medium (1) is, $c/v_1 = n_1$ and that of medium (2) is, $c/v_2 = n_2$.

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \qquad (6.18)$$

In product form,

$$n_1 \sin i = n_2 \sin r \qquad (6.19)$$

Hence, the laws of refraction are proved.

In the same way the laws of refraction can also be proved for wavefront travelling from denser to rarer medium.

Light travels with greater speed in rarer medium and lesser speed in denser medium. Hence, the wavelength of the light is longer in rarer medium and shorter in denser medium.

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \qquad (6.116)$$

Note If light of a particular frequency travels through different media, then, its frequency remains unchanged in all the media. Only the wavelength changes according to speed of light in that medium.

EXAMPLE 6.23

The wavelength of light from sodium source in vacuum is 5893Å.What are its (a) wavelength, (b) speed and (c) frequency when this light travels in water which has a refractive index of 1.33.

Solution

The refractive index of vacuum, $n_1 = 1$

The wavelength in vacuum, $\lambda_1 = 5893$ Å.

The speed in vacuum, $c = 3 \times 10^8$ m s^{-1}

The refractive index of water, $n_2 = 1.33$

The wavelength of light in water, λ_2

The speed of light in water, v_2

(a) The equation relating the wavelength and refractive index is,

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

Rewriting, $\lambda_2 = \frac{n_1}{n_2} \times \lambda_1$

Substituting the values,

$$\lambda_2 = \frac{1}{1.33} \times 5893\,\text{Å} = 4431\,\text{Å}$$

$$\lambda_2 = 4431\,\text{Å}$$

(b) The equation relating the speed and refractive index is,

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Rewriting, $v_2 = \frac{n_1}{n_2} \times v_1$

Substituting the values,

$$v_2 = \frac{1}{1.33} \times 3 \times 10^8 = 2.256 \times 10^8$$

$$v_2 = 2.256 \times 10^8\, m s^{-1}$$

(c) Frequency of light in vacuum is,

$$\nu_1 = \frac{c}{\lambda_1}$$

Substituting the values,

$$\nu_1 = \frac{3 \times 10^8}{5893 \times 10^{-10}} = 5.091 \times 10^{14}\, Hz$$

Frequency of light in water is, $\nu_2 = \frac{v}{\lambda_2}$

Substituting the values,

$$\nu_2 = \frac{2.256 \times 10^8\, m s^{-1}}{4431 \times 10^{-10}} = 5.091 \times 10^{14}\, Hz$$

The results show that the frequency remains same in all media.

6.10 INTERFERENCE

The phenomenon of addition or superposition of two light waves which produces increase in intensity at some points and decrease in intensity at some other points is called *interference* of light.

Superposition of waves refers to addition of waves. The concept of superposition of mechanical waves is studied in (XI Physics 11.7). When two waves simultaneously pass through a particle in a medium, the resultant displacement of that particle is the vector addition of the displacements due to the individual waves. The resultant displacement will be maximum or minimum depending upon the phase difference between the two superimposing waves. These concepts hold good for light as well.

Let us consider two light waves from the two sources S_1 and S_2 meeting at a point P as shown in Figure 6.52.

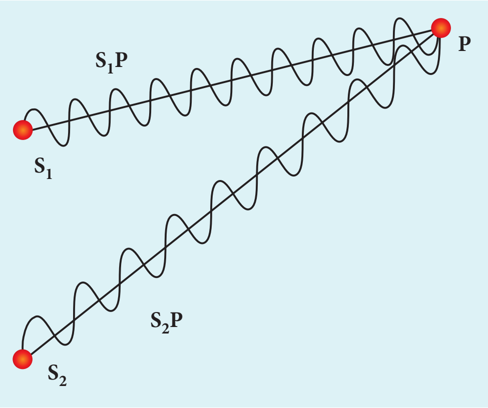


Figure 6.52 Superposition principle

The wave from S_1 at an instant t at P is,

$$y_1 = a_1 \sin \omega t \qquad (6.117)$$

The wave form S_2 at an instant t at P is,

$$y_2 = a_2 \sin (\omega t + \phi) \qquad (6.118)$$

The two waves have different amplitudes a_1 and a_2, same angular frequency ω, and a phase difference of ϕ between them. The resultant displacement will be given by,

$$y = y_1 + y_2 = a_1 \sin \omega t + a_1 \sin_2 (\omega t + \phi) \qquad (6.119)$$

The simplification of the above equation by using trigonometric identities as done in (XI Physics 11.7) gives the equation,

$$y = A \sin (\omega t + \theta) \qquad (6.120)$$

$$\text{Where, } A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi} \qquad (6.121)$$

$$\theta = \tan^{-1} \frac{a_2 \sin\phi}{a_1 + a_2 \cos\phi} \qquad (6.122)$$

The resultant amplitude is maximum,

$$A_{max} = \sqrt{(a_1 + a_2)^2}; \text{when } \phi = 0, \pm 2\pi, \pm 4\pi \ldots, \qquad (6.123)$$

The resultant amplitude is minimum,

$$A_{min} = \sqrt{(a_1 - a_2)^2}; \text{when } \phi = \pm\pi, \pm 3\pi, \pm 5\pi \ldots, \qquad (6.124)$$

The intensity of light is proportional to square of amplitude,

$$I \propto A^2 \qquad (6.125)$$

Now, equation 6.121 becomes,

$$I \propto I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi \qquad (6.126)$$

In equation 6.126 if the phase difference, $\phi = 0, \pm 2\pi, \pm 4\pi \ldots$, it corresponds to the condition for maximum intensity of light called as ***constructive interference***.

The resultant maximum intensity is,

$$I_{max} \propto (a_1 + a_2)^2 \propto I_1 + I_2 + 2\sqrt{I_1 I_2} \qquad (6.127)$$

In equation 6.126 if the phase difference, $\phi = \pm\pi, \pm 3\pi, \pm 5\pi \ldots$, it corresponds to the condition for minimum intensity of light called ***destructive interference***.

The resultant minimum intensity is,

$$I_{min} \propto (a_1 - a_2)^2 \propto I_1 + I_2 - 2\sqrt{I_1 I_2} \qquad (6.128)$$

As a special case, if $a_1 = a_2 = a$, then equation 6.121. becomes,

$$A = \sqrt{2a^2 + 2a^2 \cos\phi} = \sqrt{2a^2(1 + \cos\phi)}$$
$$= \sqrt{2a^2 2\cos^2(\phi/2)}$$

$$A = 2a \cos(\phi/2) \qquad (6.129)$$

$$I \propto 4a^2 \cos^2(\phi/2) \quad [\because I \propto A^2] \qquad (6.130)$$

$$I = 4\ I_0 \cos^2(\phi/2) \quad [\because I_0 \propto a^2] \qquad (6.131)$$

$$I_{max} = 4I_0 \text{ when, } \phi = 0, \pm 2\pi, 4\pi \ldots, \qquad (6.132)$$

$$I_{\min} = 0 \text{ when, } \phi = \pm\pi, \pm 3\pi, \pm 5\pi ... \quad (6.133)$$

We conclude that the phase difference ϕ, between the two waves decides the intensity of light at that point where the two waves meet.

EXAMPLE 6.24

Two light sources with amplitudes 5 units and 3 units respectively interfere with each other. Calculate the ratio of maximum and minimum intensities.

Solution

Amplitudes, $a_1 = 5$, $a_2 = 3$

Resultant amplitude,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

Resultant amplitude is maximum when,

$$\phi = 0, \cos 0 = 1, A_{\max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$$

$$A_{\max} = \sqrt{(a_1 + a_2)^2} = \sqrt{(5+3)^2} = \sqrt{(8)^2}$$
$$= 8 \text{ units}$$

Resultant amplitude is minimum when,

$$\phi = \pi, \cos\pi = -1, A_{\max} = \sqrt{a_1^2 + a_2^2 - 2a_1a_2}$$

$$A_{\min} = \sqrt{(a_1 - a_2)^2} = \sqrt{(5-3)^2} = \sqrt{(2)^2}$$
$$= 2 \text{ units}$$

$I \propto A^2$

$$\frac{I_{\max}}{I_{\min}} = \frac{(A_{\max})^2}{(A_{\min})^2}$$

Substituting,

$$\frac{I_{\max}}{I_{\min}} = \frac{(8)^2}{(2)^2} = \frac{64}{4} = 16 \text{ (or)}$$

$$I_{\max} : I_{\min} = 16 : 1$$

EXAMPLE 6.25

Two light sources of equal amplitudes interfere with each other. Calculate the ratio of maximum and minimum intensities.

Solution

Let the amplitude be a.

The intensity is, $I \propto 4a^2\cos^2(\phi/2)$

or $I = 4I_0\cos^2(\phi/2)$

Resultant intensity is maximum when,

$\phi = 0, \cos 0 = 1, \mathrm{I}_{\max} \propto 4a^2$

Resultant amplitude is minimum when,

$\phi = \pi, \cos(\pi/2) = 0, \mathrm{I}_{\min} = 0$

$\mathrm{I}_{\max} : I_{\min} = 4a^2 : 0$

EXAMPLE 6.26

Two light sources have intensity of light as I_0. What is the resultant intensity at a point where the two light waves have a phase difference of $\pi/3$?

Solution

Let the intensities be I_0.

The resultant intensity is, $I = 4\, I_0\cos^2(\phi/2)$

Resultant intensity when, $\phi = \pi/3$, is $I = 4I_0\cos^2(\pi/6)$

$$I = 4I_0\left(\sqrt{3}/2\right)^2 = 3I_0$$

6.10.1 Phase difference and path difference

Phase is the angular position of a vibration. As a wave is progressing, there is a relation between the phase of the vibration and the path travelled by the wave. One can express the phase in terms of path and vice versa. In the path of the wave, one wavelength λ corresponds to a phase of 2π as shown in Figure 6.53. A path difference δ corresponds to a phase difference ϕ as given by the equation,

$$\delta = \frac{\lambda}{2\pi} \times \phi \text{ (or) } \phi = \frac{2\pi}{\lambda} \times \delta \qquad (6.134)$$

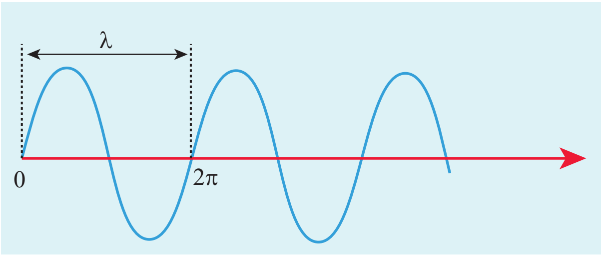


Figure 6.53 Path difference and phase difference

For constructive interference, the phase difference should be, $\phi = 0, 2\pi, 4\pi \ldots$ Hence, the path difference must be, $\delta = 0, \lambda, 2\lambda \ldots$ In general, the integral multiples of λ.

$$\delta = n\lambda \text{ where, } n = 0, 1, 2, 3 \ldots \qquad (6.135)$$

For destructive interference, phase difference should be, $\phi = \pi, 3\pi, 5\pi \ldots$ Hence, the path difference must be, $\delta = \frac{\lambda}{2}, \frac{3\lambda}{2} \ldots\ldots$

In general, the half integral multiples of λ.

$$\delta = (2n-1)\frac{\lambda}{2} \text{ where, } n = 1, 2, 3 \ldots (6.136)$$

EXAMPLE 6.27

The wavelength of a light is 450 nm. How much phase it will differ for a path of 3 mm?

Solution

The wavelength is, $\lambda = 450\ nm = 450 \times 10^{-9}\ m$

Path difference is, $\delta = 3\ mm = 3 \times 10^{-3}\ m$

Relation between phase difference and path difference is, $\phi = \frac{2\pi}{\lambda} \times \delta$

Substituting,

$$\phi = \frac{2\pi}{450 \times 10^{-9}} \times 3 \times 10^{-3} = \frac{\pi}{75} \times 10^{6}$$

$$\phi = \frac{\pi}{75} \times 10^{6}\ rad$$

6.10.2 Coherent sources

Two light sources are said to be coherent if they produce waves which have same phase or constant phase difference, same frequency or wavelength (monochromatic), same waveform and preferably same amplitude. Coherence is a property of waves that enables to obtain stationary interference patterns.

Two independent monochromatic sources can never be coherent, because they may emit waves of same frequency and same amplitude, but not with same phase. This is because, atoms while emitting light, produce change in phase due to thermal vibrations. Hence, these sources are said to be incoherent sources.

To obtain coherent light waves, we have three techniques. They are,

(i) Intensity or amplitude division
(ii) wavefront division
(iii) source and images.

(i) Intensity or amplitude division: If we allow light to pass through a partially silvered mirror (beam splitter), both reflection and refraction take place simultaneously. As the two light beams are obtained from the same light source, the two divided light beams will be coherent beams. They will be either in-phase or at constant phase difference as shown in Figure 6.54. Instruments like

Michelson's interferometer, Fabray-Perrot etalon work on this principle.

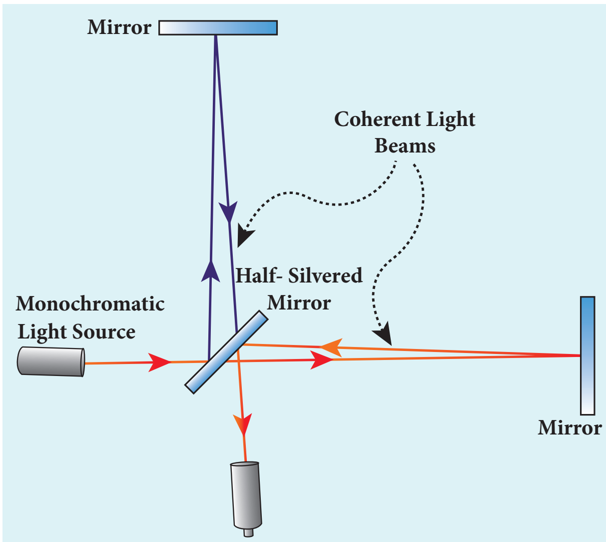


Figure 6.54 Intensity or amplitude division

(ii) Wavefront division: This is the most commonly used method for producing two coherent sources. We know a point source produces spherical wavefronts. All the points on the wavefront are at the same phase. If two points are chosen on the wavefront by using a double slit, the two points will act as coherent sources as shown in Figure 6.55.

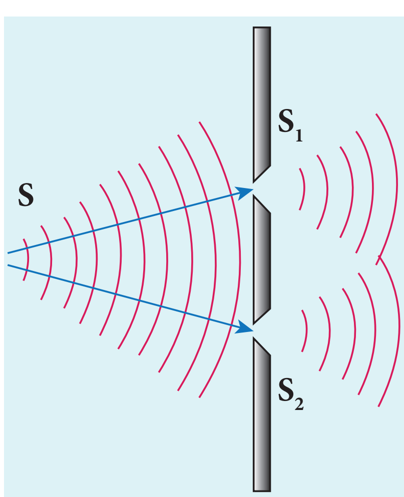


Figure 6.55 Wavefront division

(iii) Source and images: In this method a source and its image will act as a set of coherent source, because the source and its image will have waves in-phase or constant phase difference as shown in Figure 6.56. The Instrument, Fresnel's biprism uses two virtual sources as two coherent sources and the instrument, Lloyd's mirror uses a source and its virtual image as two coherent sources.

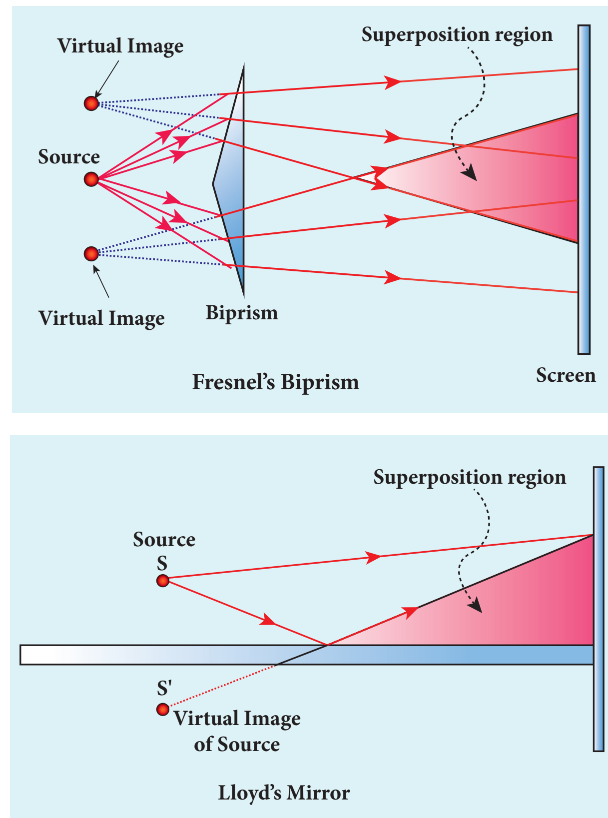


Figure 6.56 Using virtual and real images of a source as coherent sources

6.10.3 Double slit as coherent sources

Double slit uses the principle of wavefront division. Two slits S_1 and S_2 illuminated by a single monochromatic source S act as a set of coherent sources. The waves from these two coherent sources travel in the same medium and superpose. The constructive and destructive interference are shown in Figure 6.57(a).The crests of the waves are shown by thick continuous lines and troughs are shown by broken lines in Figure 6.57(b).

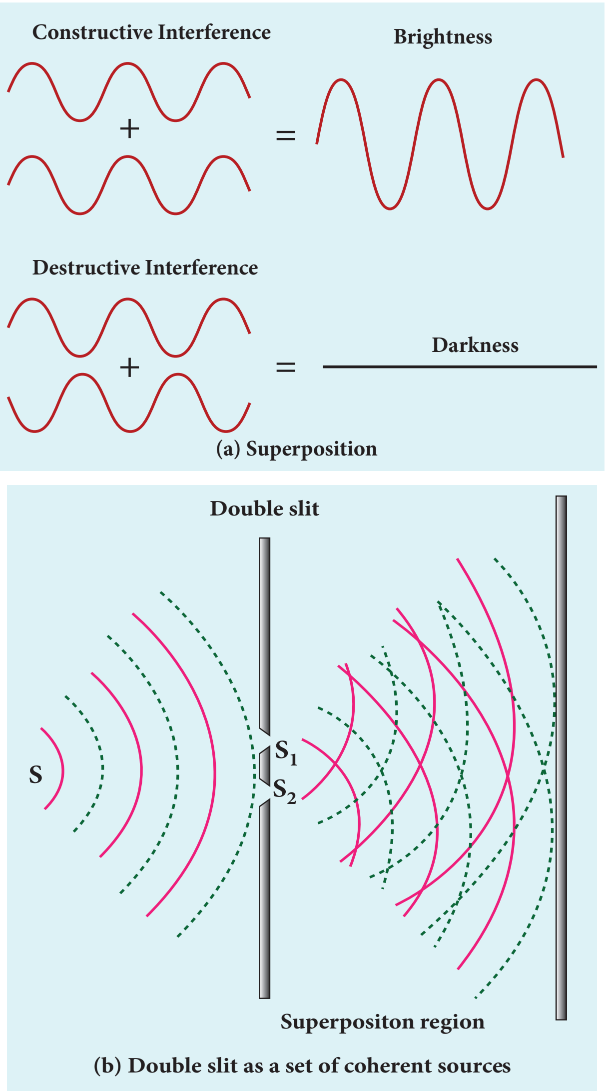


Figure 6.57 Interference due to double slit

At points where the crest of one wave meets the crest of the other wave or the trough of one wave meets the trough of the other wave, the waves are in-phase. Hence, the displacement is maximum and these points appear bright. This type of interference is said to be ***constructive interference***.

At points where the crest of one wave meets the trough of the other wave and vice versa, the waves are out-of-phase. Hence, the displacement is minimum and these points appear dark. This type of interference is said to be ***destructive interference***.

On a screen the intensity of light will be alternatively maximum and minimum i.e. bright and dark bands which are referred as interference fringes.

6.10.4 Young's double slit experiment

Thomas Young, a British Physicist used an opaque screen with two small openings called double slit S_1 and S_2 kept equidistance from a source S as shown in Figure 6.62. The width of each slit is about 0.03 mm and they are separated by a distance of about 0.3 mm. As S_1 and S_2 are equidistant from S, the light waves from S reach S_1 and S_2 in-phase. So, S_1 and S_2 act as coherent sources which are the requirement of obtaining interference pattern.

Experimental setup

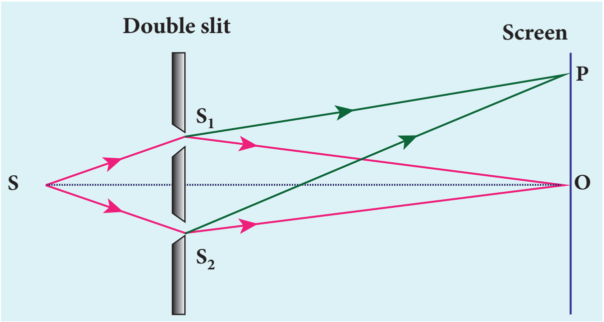


Figure 6.58 Young's double slit experiment

Wavefronts from S_1 and S_2 spread out and overlapping takes place to the right side of double slit. When a screen is placed at a distance of about 1 meter from the slits, alternate bright and dark fringes which are equally spaced appear on the screen. These are called interference fringes or bands. Using an eyepiece the fringes can be seen directly. At the center point O on the screen, waves from S_1 and S_2 travel equal distances and arrive in-phase as shown in Figure 6.58. These two waves constructively interfere and bright fringe is observed at O. This is called central bright fringe. The fringes disappear and there is uniform illumination on the screen when one of the slits is covered. This shows clearly that the bands are due to interference.

Equation for path difference

The schematic diagram of the experimental set up is shown in Figure 6.59. The Let d be the distance between the double slits S_1 and S_2 which act as coherent sources of wavelength λ. A screen is placed parallel to the double slit at a distance D from it. The mid-point of S_1 and S_2 is C and the mid-point of the screen O is equidistant from S_1 and S_2. P is any point at a distance y from O. The waves from S_1 and S_2 meet at P either in-phase or out-of-phase depending upon the path difference between the two waves.

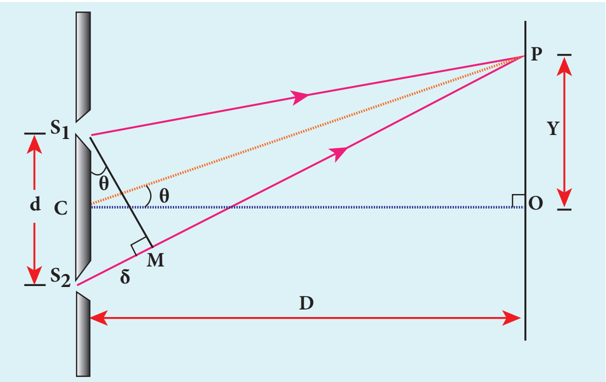


Figure 6.59 Young's double slit experimental setup

The path difference δ between the light waves from S_1 and S_2 to the point P is, $\delta = S_2P - S_1P$

A perpendicular is dropped from the point S_1 to the line S_2P at M to find the path difference more precisely.

$$\delta = S_2P - MP = S_2M \tag{6.137}$$

The angular position of the point P from C is θ. $\angle OCP = \theta$.

From the geometry, the angles $\angle OCP$ and $\angle S_2S_1M$ are equal.

$\angle OCP = \angle S_2S_1M = \theta$.

In right angle triangle ΔS_1S_2M, the path difference, $S_2M = d \sin\theta$

$$\delta = d \sin\theta \tag{6.138}$$

If the angle θ is small, $\sin\theta \approx \tan\theta \approx \theta$

From the right angle triangle ΔOCP,

$\tan\theta = \frac{y}{D}$

$$\text{The path difference, } \delta = \frac{d\,y}{D} \tag{6.139}$$

Based on the condition on the path difference, the point P may have a bright or dark fringe.

Condition for bright fringe (or) maxima

The condition for the constructive interference or the point P to be have a bright fringe is,

Path difference, $\delta = n\lambda$

where, $n = 0, 1, 2, \ldots$

$\therefore \frac{d\,y}{D} = n\lambda$

$$y = n\frac{\lambda D}{d} \quad \text{(or)} \quad y_n = n\frac{\lambda D}{d} \tag{6.140}$$

This is the condition for the point P to be a bright fringe. The distance is the distance of the n^{th} bright fringe from the point O.

Condition for dark fringe (or) minima

The condition for the destructive interference or the point P to be have a dark fringe is,

Path difference, $\delta = (2n-1)\frac{\lambda}{2}$

where, $n = 1, 2, 3 \ldots$

$\therefore \frac{d\,y}{D} = (2n-1)\frac{\lambda}{2}$

$$y = \frac{(2n-1)}{2}\frac{\lambda D}{d} \text{ (or) } y_n = \frac{(2n-1)}{2}\frac{\lambda D}{d} \tag{6.141}$$

This is the condition for the point P to be a dark fringe. The distance y_n is the distance of the n^{th} dark fringe from the point O.

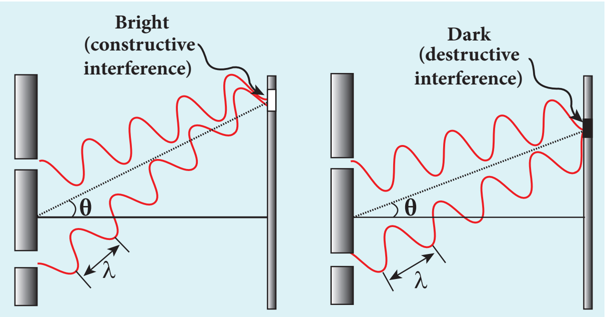


Figure 6.60 Formation of bright and dark fringes

The formation of bright and dark fringes is shown in Figure 6.60.

This shows that on the screen, alternate bright and dark bands are seen on either side of the central bright fringe. The central bright is referred as 0^{th} bright followed by 1^{st} dark and 1^{st} bright and then 2^{nd} dark and 2^{nd} bright and so on, on either side of O successively as shown in Figure 6.61.

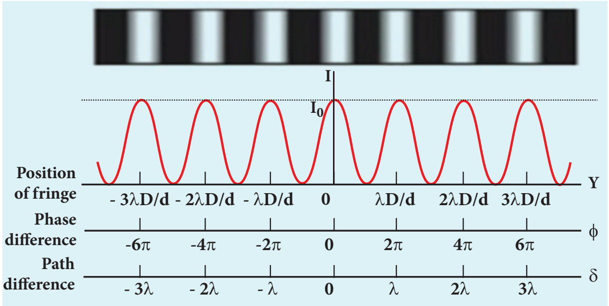


Figure 6.61 Interference fringe pattern

Equation for bandwidth

The *bandwidth* (β) is defined as the distance between any two consecutive bright or dark fringes.

The distance between $(n+1)^{th}$ and n^{th} consecutive bright fringes from O is given by,

$$\beta = y_{(n+1)} - y_n = \left((n+1)\frac{\lambda D}{d}\right) - \left(n\frac{\lambda D}{d}\right)$$

$$\beta = \frac{\lambda D}{d} \qquad (6.142)$$

Similarly, the distance between $(n+1)^{th}$ and n^{th} consecutive dark fringes from O is given by,

$$\beta = y_{(n+1)} - y_n = \left(\frac{(2(n+1)-1)}{2}\frac{\lambda D}{d}\right) - \left(\frac{(2n-1)}{2}\frac{\lambda D}{d}\right)$$

$$\beta = \frac{\lambda D}{d} \qquad (6.142)$$

Equations (6.142) show that the bright and dark fringes are of same width equally spaced on either side of central bright fringe.

Conditions for obtaining clear and broad interference bands

(i) The screen should be as far away from the source as possible.

(ii) The wavelength of light used must be larger.

(iii) The two coherent sources (here S_1 and S_2) must be as close as possible.

EXAMPLE 6.28

In Young's double slit experiment, the two slits are 0.15 mm apart. The light source has a wavelength of 450 nm. The screen is 2 m away from the slits.

(i) Find the distance of the second bright fringe and also third dark fringe from the central maximum.

(ii) Find the fringe width.

(iii) How will the fringe pattern change if the screen is moved away from the slits?

(iv) What will happen to the fringe width if the whole setup is immersed in water of refractive index 4/3.

UNIT-6(XII-Physics_Vol-2).indd 59
8/3/2019 4:36:38 PM

Solution

d = 0.15 mm = 0.15× 10^{-3} m; D = 2 m; λ = 450 nm = 450 × 10^{-9} m; n = 4/3

(i) Equation for n^{th} bright fringe is,

$$y_n = n\frac{\lambda D}{d}$$

Distance of 2^{nd} bright fringe is,

$$y_2 = 2\times\frac{450\times10^{-9}\times2}{0.15\times10^{-3}}$$

$$y_2 = 12\times10^{-3}m = 12mm$$

Equation for n^{th} dark fringe is,

$$y_n = \frac{(2n-1)}{2}\frac{\lambda D}{d}$$

Distance of 3^{rd} dark fringe is,

$$y_3 = \frac{5}{2}\times\frac{450\times10^{-9}\times2}{0.15\times10^{-3}}$$

$$y_2 = 15\times10^{-3}m = 15\,mm$$

(ii) Equation for fringe width is, $\beta = \frac{\lambda D}{d}$

Substituting, $\beta = \frac{450\times10^{-9}\times2}{0.15\times10^{-3}}$

$$\beta = 6\times10^{-3}m = 6\,mm$$

(iii) The fringe width will increase as D is increased, $\beta = \frac{\lambda D}{d}$ (or) $\beta \propto D$

(iv) The fringe width will decrease as the setup is immersed in water of refractive index 4/3

$$\beta = \frac{\lambda D}{d} \quad \text{(or)} \quad \beta \propto \lambda$$

The wavelength will decrease refractive index n times. Hence, $\beta \propto \lambda$ and $\beta' \propto \lambda'$

We know that, $\lambda' = \frac{\lambda}{n}$

$$\frac{\beta'}{\beta} = \frac{\lambda'}{\lambda} = \frac{\lambda/n}{\lambda} = \frac{1}{n} \quad \text{(or)} \quad \beta' = \frac{\beta}{n} = \frac{6\times10^{-3}}{4/3}$$

$$\beta' = 4.5\times10^{-3}m = 4.5\,mm$$

6.10.5 Interference with polychromatic light

When a polychromatic light (white light) is used in interference experiment, coloured fringes of varied thickness will be formed on the screen. This is because, different colours have different wavelengths. However, the central fringe or 0^{th} fringe will always be bright and white in colour, because for all the colours falling at the point O will have no path difference. Hence, only constructive interference is possible at O for all the colours.

EXAMPLE 6.29

Two lights of wavelengths 560 nm and 420 nm are used in Young's double slit experiment. Find the least distance from the central fringe where the bright fringe of the two wavelengths coincides. Given D = 1 m and d = 3 mm.

Solution

$$\lambda_1 = 560\,nm = 560\times10^{-9}m;$$

$$\lambda_2 = 420\,nm = 420\times10^{-9}m;$$

$$D = 1\,m; d = 3\,mm = 3\times10^{-3}\,m$$

For a given y, n and λ are inversely proportional.

Let n^{th} order bright fringe of λ_1 coincides with $(n+1)^{th}$ order bright fringe of λ_2.

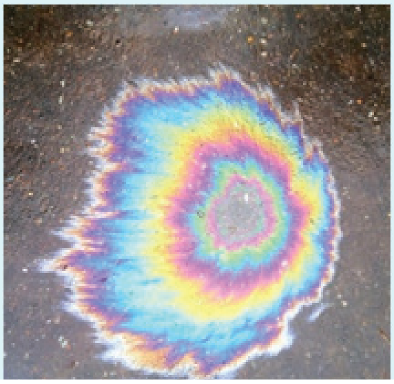
Equation for n^{th} bright fringe is, $y_n = n\frac{\lambda D}{d}$

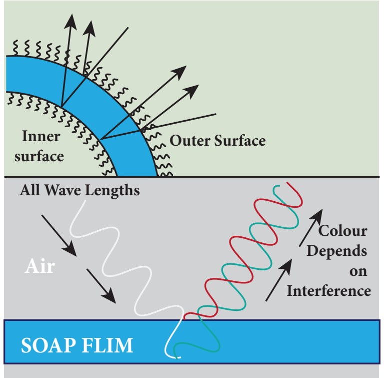
Here, $n\frac{\lambda_1 D}{d} = (n+1)\frac{\lambda_2 D}{d}$ (as $\lambda_1 > \lambda_2$)

$$n\lambda_1 = (n+1)\lambda_2 \quad \text{(or)} \quad \frac{\lambda_1}{\lambda_2} = \frac{(n+1)}{n}$$

$$1+\frac{1}{n} = \frac{560\times10^{-9}}{420\times10^{-9}} \quad \text{(or)} \quad 1+\frac{1}{n} = \frac{4}{3}$$

Dazzling colours are exhibited by thin films of oil spread on the surface of water and also by soap bubbles as shown in the figure. These colours are due to interference of white light undergoing multiple reflections from the top and the bottom surfaces of thin films. The colour depends upon the thickness of the film, refractive index of the film and also the angle of incidence of the light.





$$\frac{1}{n} = \frac{1}{3} \text{ (or) } n = 3$$

Thus, the 3^{rd} bright fringe of λ_1 and 4^{th} bright fringe of λ_2 coincide at the least distance y.

The least distance from the central fringe where the bright fringes of the two wavelengths coincides is, $y_n = n\frac{\lambda D}{d}$

$$y_n = 3 \times \frac{560 \times 10^{-9} \times 1}{3 \times 10^{-3}} = 560 \times 10^{-6} m$$

$$y_n = 0.560 \times 10^{-3} \ m = 0.560 \ mm$$

6.10.6 Interference in thin films

Let us consider a thin film of transparent material of refractive index μ (not to confuse with order of fringe n) and thickness d. A parallel beam of light is incident on the film at an angle i as shown in Figure 6.62. The wave is divided into two parts at the upper surface, one is reflected and the other is refracted. The refracted part, which enters into the film, again gets divided at the lower surface into two parts; one is transmitted out of the film and the other is reflected back in to the film. Reflected as well as refracted waves are sent by the film as multiple reflections take place inside the film. The interference is produced by both the reflected and transmitted light.

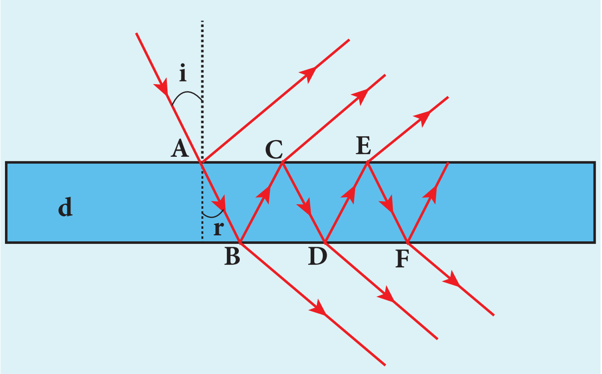


Figure 6.62 Interference in thin films

For transmitted light

The light transmitted may interfere to produce a resultant intensity. Let us consider the path difference between the two light waves transmitted from B and D. The two waves moved together and remained in phase up to B where splitting occurred. The extra path travelled by the wave transmitted from D is the path inside the film, $BC + CD$. If we approximate the incidence to be nearly normal ($i = 0$), then the points B and D are very close to each other. The extra distance travelled by the wave is approximately twice thickness of the film, $BC + CD = 2d$. As this extra path is traversed in a medium of refractive index μ, the optical path difference is, $\delta = 2\mu d$.

The condition for constructive interference in transmitted ray is,

$$2\mu d = n\lambda \qquad (6.143)$$

Similarly, the condition for destructive interference in transmitted ray is,

$$2\mu d = (2n-1)\frac{\lambda}{2} \qquad (6.144)$$

For reflected light

It is experimentally and theoretically proved that a wave while travelling in a rarer medium and getting reflected by a denser medium, undergoes a phase change of π. Hence, an additional path difference of $\lambda/2$ should be considered.

Let us consider the path difference between the light waves reflected by the upper surface at A and the other wave coming out at C after passing through the film. The additional path travelled by wave coming out from C is the path inside the film, $AB + BC$. For nearly normal incidence this distance could be approximated as, $AB + BC = 2d$. As this extra path is travelled in the medium of refractive index μ, the optical path difference is, $\delta = 2\mu d$.

The condition for constructive interference for reflected ray is,

$$2\mu d + \frac{\lambda}{2} = n\lambda \text{ (or) } 2\mu d = (2n-1)\frac{\lambda}{2} \qquad (6.145)$$

The additional path difference $\lambda/2$ is due to the phase change of π in rarer to denser reflection taking place at A.

The condition for destructive interference for reflected ray is,

$$2\mu d + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \text{ (or) } 2\mu d = n\lambda \qquad (6.146)$$

Note If the incidence is not nearly normal, but at an angle of incidence i which has an angle of refraction r, then the expression for path difference $2\mu d$ on the left hand side of the above equations are to be replaced with the expression, $2\mu d \cos r$.

EXAMPLE 6.30

Find the minimum thickness of a film of refractive index 1.25, which will strongly reflect the light of wavelength 589 nm. Also find the minimum thickness of the film to be anti-reflecting.

Solution

$\lambda = 589 \text{ nm} = 589 \times 10^{-9} m$

For the film to have strong reflection, the reflected waves should interfere constructively. The least optical path difference introduced by the film should be $\lambda/2$. The optical path difference between the waves reflected from the two surfaces of the film is $2\mu d$. Thus, for strong reflection, $2\mu d = \lambda/2$ [As given in equation 6.145. with $n = 1$]

Rewriting, $d = \dfrac{\lambda}{4\mu}$

Substituting, $d = \dfrac{589 \times 10^{9}}{4 \times 1.25} = 117.8 \times 10^{-9}$

$d = 117.8 \times 10^{-9} = 117.8\ nm$

For the film to be anti-reflecting, the reflected rays should interfere destructively. The least optical path difference introduced by the film should be λ. The optical path difference between the waves reflected from the two surfaces of the film is $2\mu d$. For strong reflection, $2\mu d = \lambda$ [As given in equation 6.146. with $n = 1$].

Rewriting, $d = \dfrac{\lambda}{2\mu}$

Substituting, $d = \dfrac{589\times10^{9}}{2\times1.25} = 235.6\times10^{-9}$

$d = 235.6\times10^{-9} = 235.6\,nm$

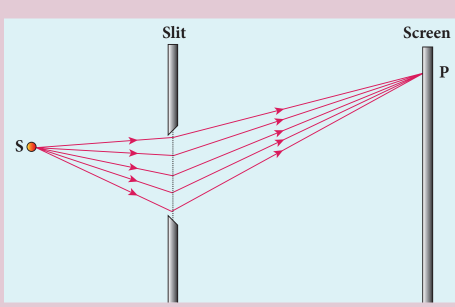
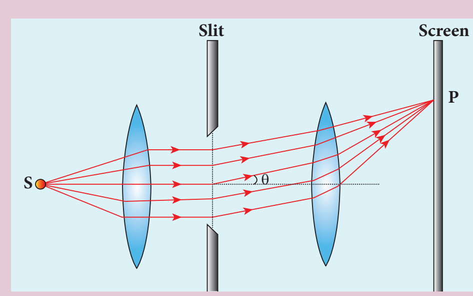
6.11 DIFFRACTION

Diffraction is a general characteristic of all types of waves, be it sound wave, light wave, water wave etc. ***Diffraction* is bending of waves around sharp edges into the geometrically shadowed region.** This is a violation to the rectilinear propagation of light, we have studied in ray optics, which says light should travel in straight line in a medium without bending. But, the diffraction is prominent only when the size of the obstacle is comparable to the wavelength of light. This is the reason why sound waves get diffracted prominently by obstacles like doors, windows, buildings etc. The wavelength of sound wave is large and comparable to the geometry of these obstacles. But the diffraction in light is more pronounced when the obstacle size is of the order of wavelength of light.

6.11.1 Fresnel and Fraunhofer diffractions

Based on the type of wavefront which undergoes diffraction, the diffraction could be classified as Fresnel and Fraunhofer diffractions. The differences between Fresnel and Fraunhofer diffractions are shown in Table 6.4.

Table 6.4 Difference between Fresnel and Fraunhofer diffractions

S.No.	Fresnel diffraction	Fraunhofer diffraction
1	Spherical or cylindrical wavefront undergoes diffraction	Plane wavefront undergoes diffraction
2	Light wave is from a source at finite distance	Light wave is from a source at infinity
3	For laboratory conditions, convex lenses need not be used	In laboratory conditions, convex lenses are to be used
4	Difficult to observe and analyse	Easy to observe and analyse
5		

As Fraunhofer diffraction is easy to observe and analyse, let us take it up for further discussions.

6.11.2. Diffraction at single slit

Let a parallel beam of light fall normally on a single slit AB of width a as shown in Figure 6.63. The diffracted beam falls on a screen kept at a distance. The center of the slit is *C*. A straight line through *C* perpendicular to the plane of slit meets the center of the screen at *O*. We would like to find the intensity at any point *P* on the screen. The lines joining *P* to the different points on the slit can be treated as parallel lines, making an angle θ with the normal *CO*.

All the waves start parallel to each other from different points of the slit and interfere at point *P* and other points to give the resultant intensities. The point *P* is in the geometrically shadowed region, up to which the central maximum is spread due to diffraction as shown Figure 6.63. We need to give the condition for the point *P* to be of various minima.

The basic idea is to divide the slit into much smaller even number of parts. Then, add their contributions at *P* with the proper path difference to show that destructive interference takes place at that point to make it minimum. To explain maximum, the slit is divided into odd number of parts.

Condition for P to be first minimum

Let us divide the slit *AB* into two half's *AC* and *CB*. Now the width of *AC* is (a/2). We have different points on the slit which are separated by the same width (here a/2) called *corresponding points* as shown in Figure 6.64.

The path difference of light waves from different corresponding points meeting at point P and interfere destructively to make it first minimum. The path difference δ between waves from these corresponding points is, $\delta = \frac{a}{2}\sin\theta$

The condition for P to be first minimum,

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$$a \sin\theta = \lambda \text{ (first minimum)} \quad (6.147)$$

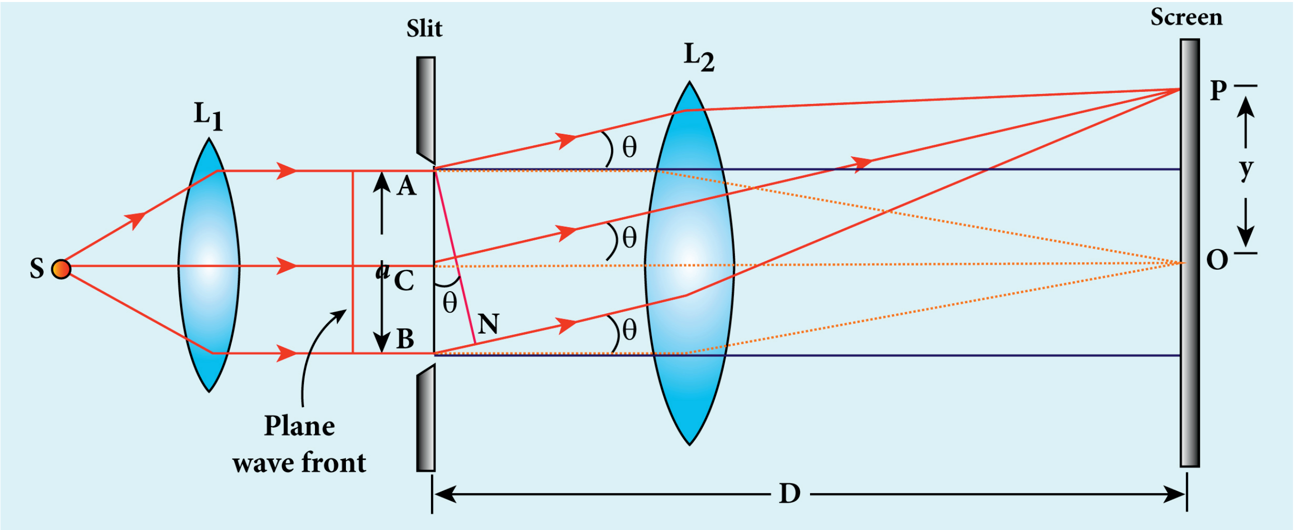


Figure 6.63 Diffraction at single slit

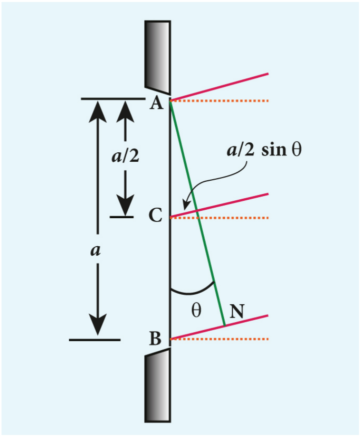


Figure 6.64 Corresponding points

Condition for P to be second minimum

Let us divide the slit AB into four equal parts. Now, the width of each part is $a/4$. We have several corresponding points on the slit which are separated by the same width $a/4$. The path difference δ between waves from these corresponding points is, $\delta = \frac{a}{4}\sin\theta$.

The condition for P to be second minimum, $\frac{a}{4}\sin\theta = \frac{\lambda}{2}$

$$a \sin\theta = 2\lambda \text{ (second minimum)} \qquad (6.148)$$

Condition for P to be third order minimum

The same way the slit is divided in to six equal parts to explain the condition for P to be third minimum is, $\frac{a}{6}\sin\theta = \frac{\lambda}{2}$

$$a \sin\theta = 3\lambda \text{ (third minimum)} \qquad (6.149)$$

Condition for P to be n^{th} order minimum

Dividing the slit into $2n$ number of (even number of) equal parts makes the light produced by one of the corresponding points to be cancelled by its counterpart. Thus, the condition for n^{th} order minimum is, $\frac{a}{2n}\sin\theta = \frac{\lambda}{2}$

$$a \sin\theta = n\lambda \quad (n^{th} \text{ minimum}) \qquad (6.150)$$

Condition for maxima

For points of maxima, the slit is to be divided in to odd number of equal parts so that one part remains un-cancelled making the point P appear bright.

The condition for first maximum is,

$$\frac{a}{3}\sin\theta = \frac{\lambda}{2} \text{ (or) } a\sin\theta = \frac{3\lambda}{2} \qquad (6.151)$$

The condition for second maximum is,

$$\frac{a}{5}\sin\theta = \frac{\lambda}{2} \text{ (or) } a\sin\theta = \frac{5\lambda}{2} \qquad (6.152)$$

The condition for third maximum is,

$$\frac{a}{7}\sin\theta = \frac{\lambda}{2} \text{ (or) } a\sin\theta = \frac{7\lambda}{2} \qquad (6.153)$$

In the same way, condition for n^{th} maximum is,

$$a\sin\theta = (2n+1)\frac{\lambda}{2} \; (n^{th} \text{ maximum}) \qquad (6.154)$$

where, n = 0, 1, 2, 3, . . . , is the order of diffraction maximum.

The central maximum is called 0^{th} order maximum. The points of the maximum intensity lie nearly midway between the successive minima.

Note Here, $\sin\theta$ gives the angular spread of the diffraction. The position of the minimum or maximum in terms of y may be expressed by replacing $\sin\theta$ approximated by $\tan\theta$, as θ is small, $\sin\theta = \tan\theta \frac{y}{D}$

Where, y is the position of the minimum from the center of the screen and D is the distance between single slit and the screen.

EXAMPLE 6.31

Light of wavelength 500 nm passes through a slit of 0.2 mm wide. The diffraction pattern is formed on a screen 60 cm away. Determine the,

(i) angular spread of central maximum

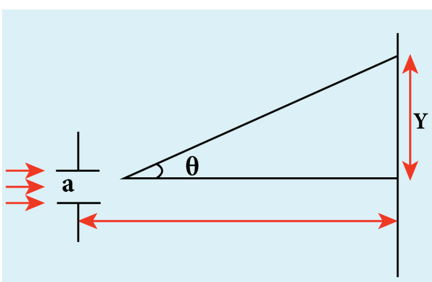
(ii) the distance between the central maximum and the second minimum.

Solution

λ = 500 nm = 500×10^{-9} m; a = 0.2 mm = 0.2×10^{-3} m; D = 60 cm = 60×10^{-2} m

(i) Equation for diffraction minimum is, $a \sin\theta = n\lambda$

The central maximum is spread up to the first minimum. Hence, $n = 1$



Rewriting, $\sin\theta = \dfrac{\lambda}{a}$ (or) $\theta = \sin^{-1}\left(\dfrac{\lambda}{a}\right)$

Substituting,

$$\theta = \sin^{-1}\left(\frac{500\times10^{-9}}{0.2\times10^{-3}}\right) = \sin^{-1}\left(2.5\times10^{-3}\right)$$

$$\theta = 0.0025\ rad$$

(ii) To find the value of y_1 for central maximum, which is spread up to first minimum with ($n = 1$) is, $a \sin\theta = \lambda$

As θ is very small, $\sin\theta \approx \tan\theta = \dfrac{y_1}{D}$

$$a\frac{y_1}{D} = \lambda \qquad \text{rewriting, } y_1 = \frac{\lambda D}{a}$$

Substituting,

$$y_1 = \frac{500\times10^{-9}\times60\times10^{-2}}{0.2\times10^{-3}} = 1.5\times10^{-3} = 1.5\ mm$$

To find the value of y_2 for second minimum with ($n = 2$) is, $a \sin\theta = 2\lambda$

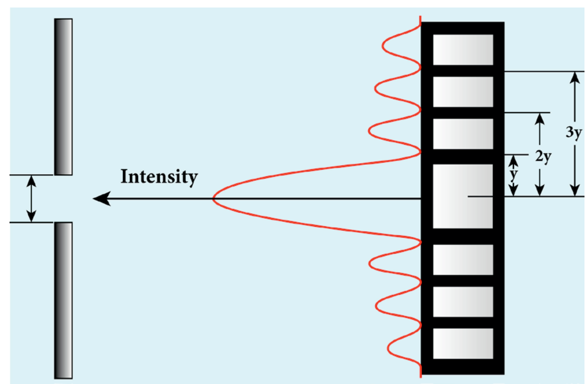
$$a\frac{y_2}{D} = 2\lambda \qquad \text{rewriting, } y_2 = \frac{2\lambda D}{a}$$

Substituting,

$$y_2 = \frac{2\times500\times10^{-9}\times60\times10^{-2}}{0.2\times10^{-3}} = 3\times10^{-3} = 3\text{ mm}$$

The distance between the central maximum and second minimum is, $y_2 - y_1$

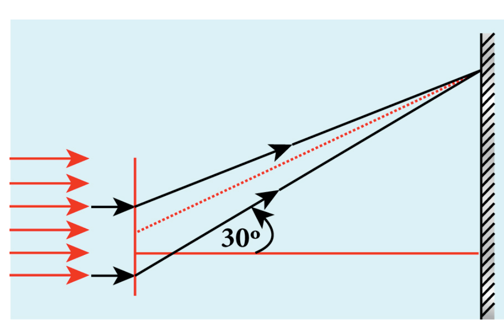
$y_2 - y_1$ = 3 mm – 1.5 mm = 1.5 mm



Note: The above calculation shows that in the diffraction pattern caused by single slit, the width of each maximum is equal with central maximum as the double that of others. But the bright and dark fringes are not of equal width.

EXAMPLE 6.32

A monochromatic light of wavelength 5000 Å passes through a single slit producing diffraction pattern for the central maximum as shown in the figure. Determine the width of the slit.



Solution

λ = 5000 Å = 5000×10^{-10} m; sin 30° = 0.5; n = 1; a =?

Equation for diffraction minimum is, $a\sin\theta = n\lambda$

The central maximum is spread up to the first minimum. Hence, n = 1

Rewriting, $a = \dfrac{\lambda}{\sin\theta}$

Substituting, $a = \dfrac{5000\times10^{-10}}{0.5}$

$a = 1\times10^{-6}m = 0.001\times10^{-3}m = 0.001\text{mm}$

6.11.3 Discussion on first minimum

Let us consider the condition for first minimum with (n = 1). $a\sin\theta = \lambda$

The first minimum has an angular spread of, $\sin\theta = \dfrac{\lambda}{a}$

Now, we have special cases to discuss on the above condition.

(i) When $a < \lambda$, the diffraction is not possible, because $\sin\theta$ can never be greater than 1.

(ii) When $a \geq \lambda$, the diffraction is possible.

- For $a = \lambda$, $\sin\theta = 1$ i.e, $\theta = 90^0$. That means the first minimum is at 90°. Hence, the central maximum spreads fully in to the geometrically shadowed region leading to bending of the diffracted light to 90°.
- For $a >> \lambda$, $\sin\theta << 1$ i.e, the first minimum will fall within the width of the slit itself. The diffraction will not be noticed at all.

(iii) When $a > \lambda$ and also comparable, say $a = 2\lambda$, $\sin\theta = \dfrac{\lambda}{a} = \dfrac{\lambda}{2\lambda} = \dfrac{1}{2}$; then $\theta = 30^0$. These are practical cases where diffraction could be observed effectively.

6.11.4 Fresnel's distance

Fresnel's distance is the distance up to which the ray optics is valid in terms of rectilinear propagation of light. As there is bending of light in diffraction, the rectilinear propagation of light is violated. But, this bending is not significant till the diffracted ray crosses the central maximum at a distance z as shown in Figure 6.65. Hence, **Fresnel's distance is the distance upto which ray optics is obeyed and beyond which ray optics is not obeyed but, wave optics becomes significant.**

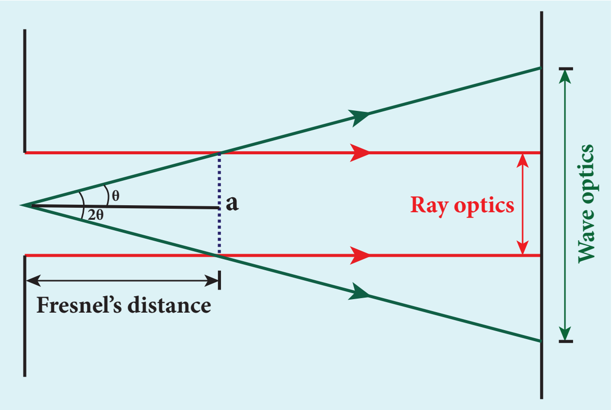


Figure 6.65 Fresnel's distance

From the diffraction equation for first minimum, $\sin\theta = \dfrac{\lambda}{a}$; $\theta = \dfrac{\lambda}{a}$

From the definition of Fresnel's distance,

$\sin 2\theta = \dfrac{a}{z}$; $2\theta = \dfrac{a}{z}$

Equating the above two equation gives,

$\dfrac{\lambda}{a} = \dfrac{a}{2z}$

After rearranging, we get Fresnel's distance z as,

$$z = \frac{a^2}{2\lambda} \qquad (6.155)$$

EXAMPLE 6.33

Calculate the distance for which ray optics is good approximation for an aperture of 5 mm and wavelength 500 nm.

Solution

$a = 5 \text{ mm} = 5 \times 10^{-3}\ m;$

$\lambda = 500nm = 500 \times 10^{-9} m;\ z = ?$

Equation for Fresnel's distance, $z = \frac{a^2}{2\lambda}$

Substituting,

$$z = \frac{\left(5 \times 10^{-3}\right)^2}{2 \times 500 \times 10^{-9}} = \frac{25 \times 10^{-6}}{1 \times 10^{-6}} = 25 \text{ m}$$

$z = 25$ m

6.11.5 Difference between interference and diffraction

It is very difficult to distinguish between interference and diffraction as they both exhibit the wave nature of light. In both the phenomena the light reaches the geometrically shadowed regions and also interferes to produce maxima and minima. Nevertheless, based on the appearance, the differences are given in Table 6.5.

6.11.6 Diffraction in grating

Grating has multiple slits with equal widths of size comparable to the wavelength of diffracting light. Grating is a plane sheet of transparent material on which opaque rulings are made with a fine diamond pointer. The modern commercial grating contains about 6000 lines per centimetre. The rulings act as obstacles having a definite width b and the transparent space between the rulings act as slit of width a. **The combined width of a ruling and a slit is called *grating element* ($e = a + b$). Points on successive slits separated by a distance equal to the grating element are called *corresponding points*.**

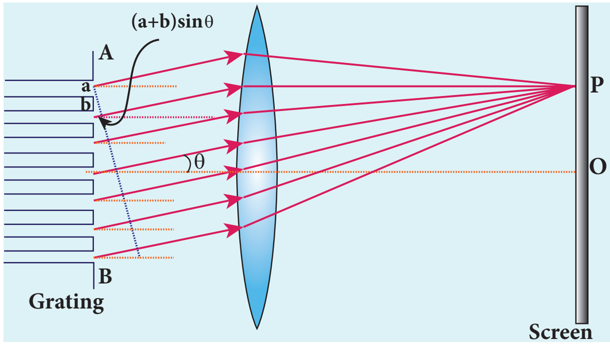


Figure 6.66 Diffraction grating experiment

A plane transmission grating is represented by AB in Figure 6.66. Let a

Table 6.5 Difference between interference and diffraction

S.No.	Interference	Diffraction
1	Superposition of two waves	Bending of waves around edges
2	Superposition of waves from two coherent sources.	Superposition wavefronts emitted from various points of the same wavefront.
3	Equally spaced fringes.	Unequally spaced fringes
4	Intensity of all the bright fringes is almost same	Intensity falls rapidly for higher orders
5	Large number of fringes are obtained	Less number of fringes are obtained

plane wavefront of monochromatic light with wave length λ be incident normally on the grating. As the slits size is comparable to that of wavelength, the incident light diffracts at the grating.

A diffraction pattern is obtained on the screen when the diffracted waves are focused on a screen using a convex lens. Let us consider a point P at an angle θ with the normal drawn from the center of the grating to the screen. The path difference δ between the diffracted waves from one pair of corresponding points is,

$$\delta = (a + b)\sin\theta \tag{6.156}$$

This path difference is the same for any pair of corresponding points. The point P will be bright, when

$$\delta = m\lambda \text{ where } m = 0, 1, 2, 3 \tag{6.157}$$

Combining the above two equations, we get,

$$(a + b)\sin\theta = m\lambda \tag{6.158}$$

Here, m is called order of diffraction.

Condition for zero order maximum, m = 0

For $(a + b)\sin\theta = 0$, the position, $\theta = 0$. $\sin\theta = 0$ and $m = 0$. This is called zero order diffraction or central maximum.

Condition for first order maximum, m = 1

If $(a + b)\sin\theta_1 = \lambda$, the diffracted light meet at an angle θ_1 to the incident direction and the first order maximum is obtained.

Condition for second order maximum, m = 2

Similarly, $(a + b)\sin\theta_2 = 2\lambda$ forms the second order maximum at the angular position θ_2.

Condition for higher order maximum

On either side of central maxima different higher orders of diffraction maxima are formed at different angular positions.

If we take,

$$N = \frac{1}{a+b} \tag{6.159}$$

Then, N gives the number of grating elements or rulings drawn per unit width of the grating. Normally, this number N is specified on the grating itself. Now, the equation becomes,

$$\frac{1}{N}\sin\theta = m\lambda \text{ (or) } \sin\theta = Nm\lambda \tag{6.160}$$

Note The students should remember that in a single slit experiment the formula, $a\sin\theta = n\lambda$ is condition for minimum with n as order of minimum. But, the formula in diffraction grating, $\sin\theta = Nm\lambda$ is condition for maxima with m as the order of diffraction.

EXAMPLE 6.34

A diffraction grating consisting of 4000 slits per centimeter is illuminated with a monochromatic light that produces the second order diffraction at an angle of 30°. What is the wavelength of the light used?

Solution

Number of lines per cm = 4000; $m = 2$;

$\theta = 30°$; $\lambda = ?$

Number of lines per unit length,

$$N = \frac{4000}{1\times10^{-2}} = 4\times10^{5}$$

Equation for diffraction maximum in grating is, $\sin\theta = Nm\lambda$

Rewriting, $\lambda = \dfrac{\sin\theta}{Nm}$

Substituting,

$$\lambda = \frac{\sin 30^\circ}{4\times10^5\times2} = \frac{0.5}{4\times10^5\times2}$$
$$= \frac{1}{2\times4\times10^5\times2} = \frac{1}{16\times10^5}$$

$$\lambda = 6250\times10^{-10} m = 6250\ \text{Å}$$

EXAMPLE 6.35

A monochromatic light of wavelength of 500 nm strikes a grating and produces fourth order bright line at an angle of 30°. Find the number of slits per centimeter.

Solution

λ = 500 nm = 500×10^{-9} m; m = 4;

θ = 30°; number of lines per cm = ?

Equation for diffraction maximum in grating is, $\sin\theta = Nm\lambda$

Rewriting, $N = \frac{\sin\theta}{m\lambda}$

Substituting,

$$N = \frac{0.5}{4\times500\times10^{-9}} = \frac{1}{2\times4\times500\times10^{-9}}$$
$$= 2.5\times10^5 \text{ lines per meter}$$

number of lines per centimeter =

$2.5\times10^5\times10^{-2} = 2500$ lines per centimetre

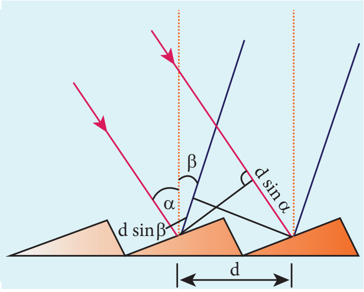
6.11.7 Experiment to determine the wavelength of monochromatic light

The wavelength of a spectral line can be very accurately determined with the help of a diffraction grating and a spectrometer. Initially all the preliminary adjustments of the spectrometer are made. The slit of collimator is illuminated by a monochromatic light, whose wavelength is to be determined. The telescope is brought in line with collimator to view the image of the slit. The given plane transmission grating is then mounted on the prism table with its plane perpendicular to the incident beam of light coming from the collimator. The telescope is turned to one side until the first order diffraction image of the slit coincides with the vertical cross wire of the eye piece. The reading of the position of the telescope is noted.

Similarly the first order diffraction image on the other side is made to coincide with

DO YOU KNOW?

You would have noticed the colourful appearance of the compact disc. On the read/writable side which is polished, there are many narrow circular tracks with widths comparable to the wavelength of visible light. Hence, the diffraction takes place after reflection for incident white light to give colourful appearance. The tracks act as reflecting grating.



the vertical cross wire and corresponding reading is noted. The difference between two positions gives 2θ. Half of its value gives θ, the diffraction angle for first order maximum as shown in Figure 6.67. The wavelength of light is calculated from the equation,

$$\lambda = \frac{\sin\theta}{Nm} \qquad (6.161)$$

Here, N is the number of rulings per metre in the grating and m is the order of the diffraction image.

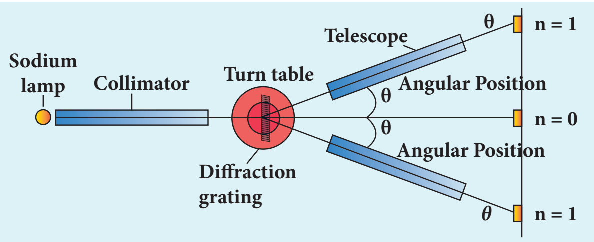


Figure 6.67 Determination of wavelength using grating and spectrometer

6.11.8 Determination of wavelength of different colours

When white light is used, the diffraction pattern consists of a white central maximum and on both sides continuous coloured diffraction patters are formed. The central maximum is white as all the colours meet here constructively with no path difference. As θ increases, the path difference, $(a+b)\sin\theta$, passes through condition for maxima of diffraction of different orders for all colours from violet to red. It produces a spectrum of diffraction pattern from violet to red on either side of central maximum as shown in Figure 6.68. By measuring the angle at which these colours appear for various orders of diffraction, the wavelength of different colours could be calculated using the formula,

$$\lambda = \frac{\sin\theta}{Nm} \qquad (6.161)$$

Here, N is the number of rulings per metre in the grating and m is the order of the diffraction image.

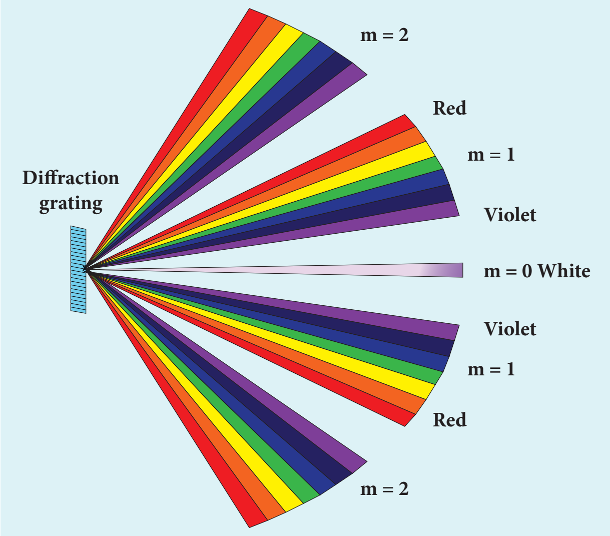


Figure 6.68 Diffraction with white light

6.11.9 Resolution

The effect of diffraction has an adverse impact in the image formation by the optical instruments such as microscope and telescope. For a single rectangular slit, the half angle θ subtended by the spread of central maximum (or position of first minimum) is given by the relation,

$$a \sin\theta = \lambda \qquad (6.162)$$

Similar to a rectangular slit, when a circular aperture or opening (like a lens or the iris of our eye) forms an image of a point object, the image formed will not be a point but a diffraction pattern of concentric circles that becomes fainter while moving away from the center as shown in Figure 6.69. These are known as Airy's discs. The circle of central maximum has the half angular spread given by the equation,

$$a \sin\theta = 1.22\,\lambda \quad (6.163)$$

Here, the numerical value 1.22 comes for central maximum formed by circular apertures. This involves higher level mathematics which is avoided in this discussion.

W9N9D3

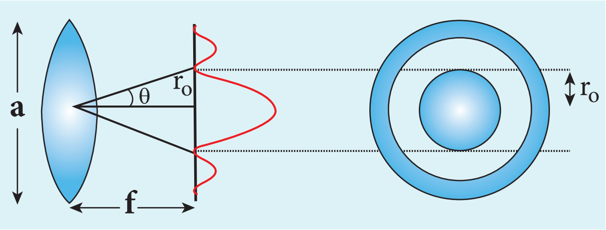


Figure 6.69 Airy's discs

For small angles, $\sin\theta \approx \theta$

$$a\,\theta = 1.22\,\lambda \quad (6.164)$$

Rewriting further,

$$\theta = \frac{1.22\lambda}{a} \text{ and } \frac{r_0}{f} = \frac{1.22\lambda}{a}$$

$$r_0 = \frac{1.22\lambda f}{a} \quad (6.165)$$

When two point sources close to each another form image on the screen, the diffraction pattern of one point source can overlap with another and produce a blurred image as shown in Figure 6.70(a). To obtain a good image of the two sources, the two point sources must be resolved i.e., the point sources must be imaged in such a way that their images are sufficiently far apart that their diffraction patterns do not overlap. According to *Rayleigh's criterion*, for two point objects to be just resolved, the minimum distance between their diffraction images must be in such a way that the central maximum of one coincides with the first minimum of the other and vice versa as shown in Figure 6.70(b). Such an image is said to be just resolved image of the object. The Rayleigh's criterion is said to be limit of resolution.

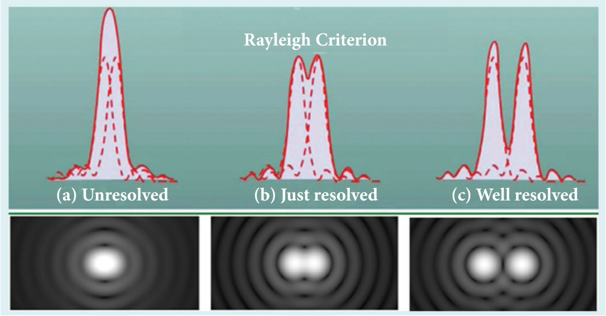


Figure 6.70 Rayleigh's criterion

According to Rayleigh's criterion the two point sources are said to be just resolved when the distance between the two maxima is at least r_o. The ***angular resolution*** has a unit in radian (rad) and it is given by the equation,

$$\theta = \frac{1.22\lambda}{a} \quad (6.166)$$

It shows that the first order diffraction angle must be as small as possible for greater resolution. This further shows that for better resolution, the wavelength of light used must be as small as possible and the size of the aperture of the instrument used must be as large as possible. The Equation 6.165 is used to calculate ***spacial resolution***.

The inverse of resolution is called resolving power. This implies, smaller the resolution, greater is the resolving power of the instrument. **The ability of an optical instrument to separate or distinguish small or closely adjacent objects through the image formation is said to be *resolving power* of the instrument.** In general, the term resolution is pertaining to the quality of the image formed and the term resolving power is associated with the ability of the optical instrument.

EXAMPLE 6.36

The optical telescope in the Vainu Bappu observatory at Kavalur has an objective lens of diameter 2.3 m. What is its angular resolution if the wavelength of light used is 589 nm?

Solution

$a = 2.3$ m; $\lambda = 589$ nm $= 589\times10^{-9}$ m; $\theta = ?$

The equation for angular resolution is,

$$\theta = \frac{1.22\lambda}{a}$$

Substituting,

$$\theta = \frac{1.22\times589\times10^{-9}}{2.3} = 321.4\times10^{-9}$$

$$\theta = 3.214\times10^{-7}\,rad \approx 0.0011'$$

<u>Note</u>: The angular resolution of human eye is approximately, $3\times10^{-4}\,rad \approx 1.03'$.

6.12 POLARISATION

The phenomena of interference and diffraction demonstrated that light is propagated in the form of waves. They did not specify whether the light waves are transverse or longitudinal. The phenomena of interference and diffraction are possible in both transverse and longitudinal waves. The phenomenon of polarization distinctly proves that light waves are only transverse in nature. Light is propagated in the form of electromagnetic waves. **The phenomenon of restricting the vibrations of light (electric or magnetic field vector) to a particular direction perpendicular to the direction of propagation of wave is called *polarization* of light**. In this lesson only the electric field is considered for discussion.

6.12.1 Plane polarised light

A transverse wave which has vibrations in all directions in a plane perpendicular to the direction of propagation of wave is said to be *unpolarised light* as shown in Figure 6.71(a). All these vibrations could be resolved into parallel and perpendicular components as shown in Figure 6.71(b) which represents unpolarised light. **If the vibrations of a wave are present in only one direction in a plane perpendicular to the direction of propagation of wave is said to be *polarised or plane polarised light*** as shown in Figure 6.71(c) and 6.71(d).

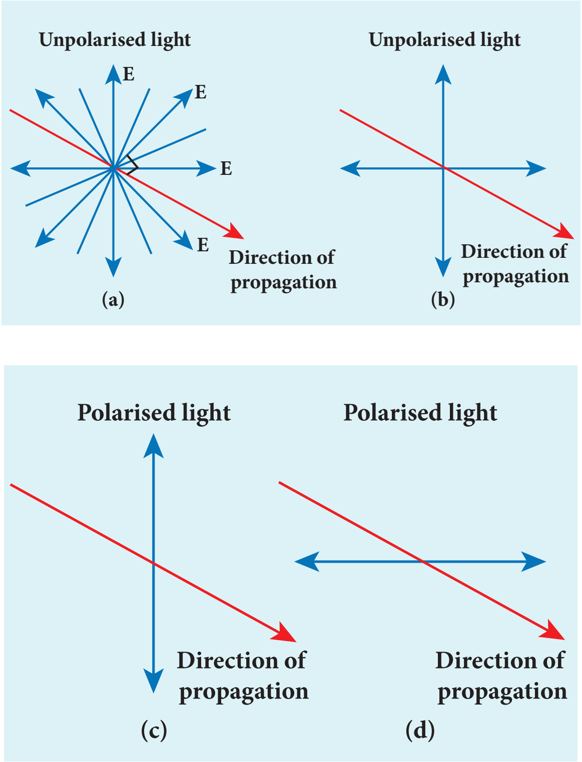


Figure 6.71 Unpolarised and polarised light

The plane containing the vibrations of the electric field vector is known as the ***plane of vibration*** *ABCD* as shown in Figure 6.72. The plane perpendicular to the plane of vibration and containing the ray of light is known as the ***plane of polarisation*** *EFGH*.

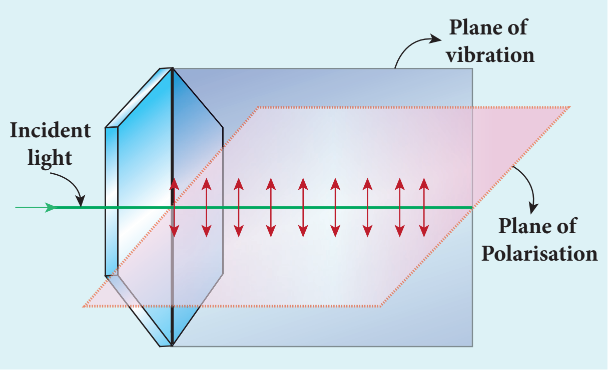


Figure 6.72 Plane of vibration and plane of polarisation

The Table 6.6 consolidates few characteristics of polarised and unpolarised light.

6.12.2 Polarisation Techniques

The unpolarised light can be polarised by several techniques. Here, we are discussing the following four methods,

(i) polarisation by selective absorption
(ii) polarisation by reflection
(iii) polarisation by double refraction
(iv) polarisation by scattering.

6.12.3 Polarisation by selective absorption

Selective absorption is the property of a material which transmits waves whose electric fields vibrate in a plane parallel to a certain direction of orientation and absorbs all other waves. The *polaroids* or *polarisers* are thin commercial sheets which make use of the property of selective absorption to produce an intense beam of plane polarised light. Selective absorption is also called as *dichroism*.

In 1932, an American scientist Edwin Land developed polarisers in the form of sheets. Tourmaline is a natural polarising material. Polaroids are also made artificially. It was discovered that small needle shaped crystals of quinine iodosulphate have the property of polarising light. A number of these crystals with their axes parallel to one another packed in between two transparent plastic sheets serve as a good polaroid. Recently new types of polaroids are prepared in which thin film of polyvinyl alcohol is used. These are colourless crystals which transmit more light, and give better polarisation. Polaroids have many applications as the one shown in Figure 6.73.

Table 6.6 Characteristics of polarised light and unpolarised light

S.No	Polarised light	Unpolarised light
1	Consists of waves having their electric field vibrations in a single plane normal to the direction of ray.	Consists of waves having their electric field vibrations equally distributed in all directions normal to the direction of ray.
2	Asymmetrical about the ray direction	Symmetrical about the ray direction
3	It is obtained from unpolarised light with the help of polarisers	Produced by conventional light sources.

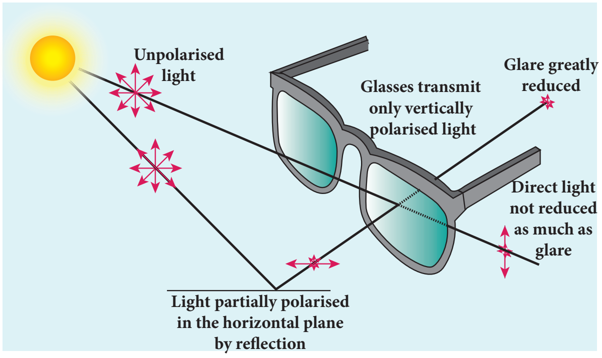


Figure 6.73 Polaroid sun glasses

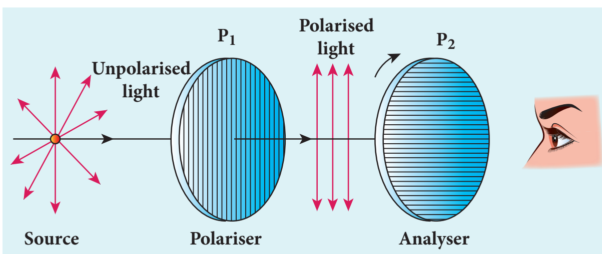


Figure 6.74 Polariser and analyser

6.12.3.1 Polariser and analyser

Let us consider an unpolarised beam of light. The vibrations can be in all possible directions all of them being perpendicular to the direction of propagation as shown in Figure 6.74. When this light passes through polaroid P_1 the vibrations are restricted to only one plane. The emergent beam can be further passed through another polaroid P_2. If the polaroid P_2 is rotated about the ray of light as axis, for a particular position of P_2 the intensity is maximum. When the polaroid P_2 is rotated from this position the intensity starts decreasing. There is complete extinction of the light when P_2 is rotated through 90°. On further rotation of P_2 the light reappears and the intensity increases and becomes a maximum for a further rotation through 90°. The light coming out from polaroid P_1 is said to be plane polarised. The Polaroid (here P_1) which plane polarises the unpolarised light passing through it is called a *polariser*. The polaroid (here P_2) which is used to examine whether a beam of light is polarised or not is called an *analyser*.

If the intensity of the unpolarised light is I then the intensity of plane polarised light will be $\left(\frac{I}{2}\right)$. The other half of intensity is restricted by the polariser.

6.12.3.2 Plane and partially polarised light

In *plane polarised* light the intensity varies from maximum to zero for every rotation of 90° of the analyser as shown in Figure 6.75(a). This is because the vibrations are allowed in one axis and completely restricted in the perpendicular axis. On the other hand, **if the intensity of light varies between maximum and minimum for every rotation of 90° of the analyser, the light is said to be *partially polarised* light** as shown in Figure 6.75(b). This is because the light is not fully restricted in that particular axis which shows a minimum intensity.

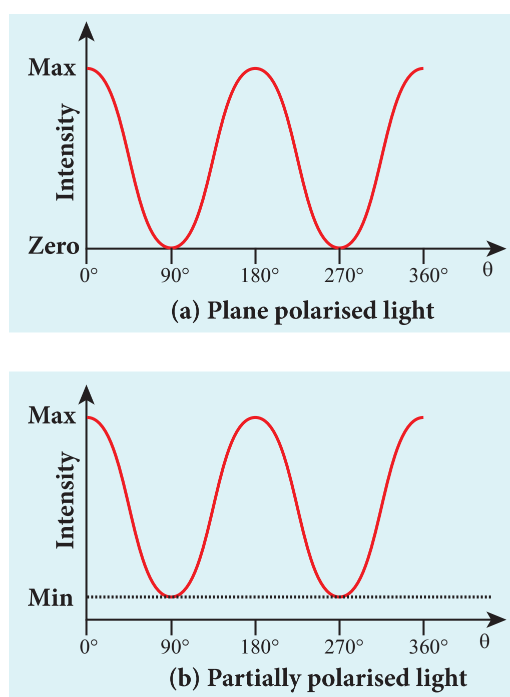


Figure 6.75 Intensity variation in plane and partially polarised light

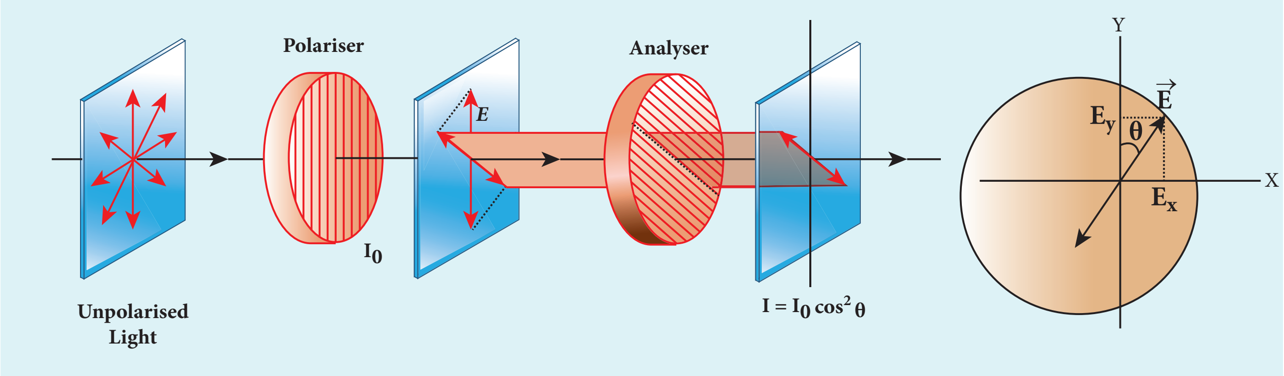


Figure 6.76 Malus's law

6.12.3.3 Malus' law

When a plane polarised light is seen through an analyser, the intensity of transmitted light varies as the analyser is rotated through an angle perpendicular to the incident direction. In 1809, French Physicist E.N Malus discovered that when a beam of plane polarised light of intensity I_0 is incident on an analyser, the light transmitted of intensity I from the analyser varies directly as the square of the cosine of the angle θ between the transmission axis of polariser and analyser as shown in Figure 6.76. This is known as Malus' law.

$$I = I_0 \cos^2\theta \qquad (6.167)$$

The proof of Malus's law is as follows. Let us consider the plane of polariser and analyser are inclined to each other at an angle θ is as shown in Figure 6.77. Let I_0 be the intensity and a be the amplitude of the electric vector transmitted by the polariser. The amplitude a of the incident light has two rectangular components, $a\cos\theta$ and $a\sin\theta$ which are the parallel and perpendicular components to the axis of transmission of the analyser.

Only the component $a\cos\theta$ will be transmitted by the analyser. The intensity of light transmitted from the analyser is proportional to the square of the component of the amplitude transmitted by the analyser.

$$I \propto (a\cos\theta)^2$$

$$I = k(a\cos\theta)^2$$

Where k is constant of proportionality.

$$I = ka^2\cos^2\theta$$

$$I = I_0\cos^2\theta$$

Where, $I_0 = ka^2$ is the maximum intensity of light transmitted from the analyser.

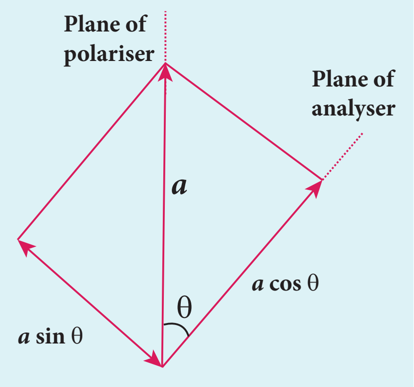


Figure 6.77 Malus' law

The following are few special cases.

Case (i) When $\theta = 0^o$, $\cos 0^o = 1$, $I = I_0$

When the transmission axis of polariser is along that of the analyser, the intensity of light transmitted from the analyser is equal to the incident light that falls on it from the polariser.

Case (ii) When $\theta = 90^o$, $\cos 90^o = 0$, $I = 0$

When the transmission axes of polariser and analyser are perpendicular to each other, the intensity of light transmitted from the analyser is zero.

EXAMPLE 6.37

Two polaroids are kept with their transmission axes inclined at 30^o. Unpolarised light of intensity I falls on the first polaroid. Find out the intensity of light emerging from the second polaroid.

Solution

As the intensity of the unpolarised light falling on the first polaroid is I, the intensity of polarized light emerging will be, $I_0 = \left(\frac{I}{2}\right)$.

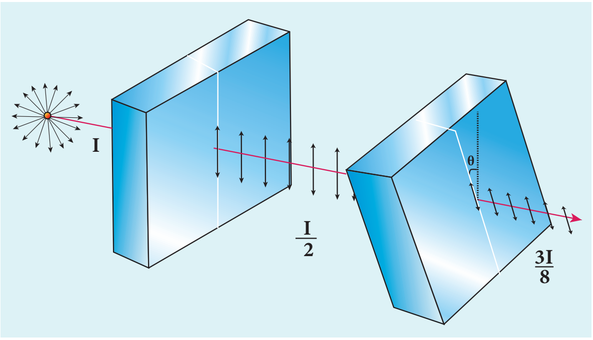
Let I' be the intensity of light emerging from the second polaroid.

Malus' law, $I' = I_0 \cos^2\theta$

Substituting,

$$I' = \left(\frac{I}{2}\right)\cos^2(30^\circ) = \left(\frac{I}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2 = I\frac{3}{8}$$

$$I' = \left(\frac{3}{8}\right)I$$



EXAMPLE 6.38

Two polaroids are kept crossed (transmission axes at 90^o) to each other.

(i) What will be the intensity of the light coming out from the second polaroid when an unpolarised light of intensity I falls on the first polaroid?

(ii) What will be the intensity of light coming out from the second polaroid if a third polaroid is kept at 45^o inclination to both of them.

Solution

(i) As the intensity of the unpolarised light falling on the first polaroid is I, the intensity of polarized light emerging from it will be $I_0 = \left(\frac{I}{2}\right)$. Let I' be the intensity of light emerging from the second polaroid.

Malus' law, $I' = I_0 \cos^2\theta$

Here θ is 90^o as the transmission axes are perpendicular to each other.

Substituting,

$$I' = \left(\frac{I}{2}\right)\cos^2\left(90^\circ\right) = 0 \quad \left[\because \cos\left(90^\circ\right) = 0\right]$$

No light comes out from the second polaroid.

(ii) Let the first polaroid be P_1 and the second polaroid be P_2. They are oriented at 90^o. The third polaroid P_3 is introduced between them at 45^o. Let I' be the intensity of light emerging from P_3.

Angle between P_1 and P_3 is 45^o. The intensity of light coming out from P_3 is, $I' = I_0 \cos^2\theta$

Substituting,

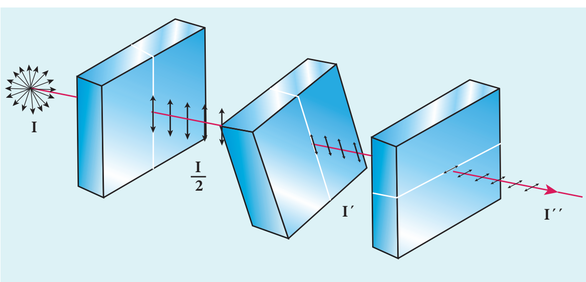
$$I' = \left(\frac{I}{2}\right)\cos^2(45^\circ) = \left(\frac{I}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{4'};\ I' = \frac{I}{4}$$

Angle between P_3 and P_2 is 45°. Let I'' is the intensity of light coming out from P_2
$I'' = I' \cos^2 \theta$

Here, the intensity of polarized light existing between P_3 and P_2 is $\frac{I}{4}$.
Substituting,

$$I'' = \left(\frac{I}{4}\right)\cos^2(45^\circ) = \left(\frac{I}{4}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{8}$$

$$I'' = \frac{I}{8}$$



6.12.3.4. Uses of polaroids

1. Polaroids are used in goggles and cameras to avoid glare of light.
2. Polaroids are useful in three dimensional motion pictures i.e., in holography.
3. Polaroids are used to improve contrast in old oil paintings.
4. Polaroids are used in optical stress analysis.
5. Polaroids are used as window glasses to control the intensity of incoming light.
6. Polarised laser beam acts as needle to read/write in compact discs (CDs).
7. Polaroids produce polarised lights to be used in liquid crystal display (LCD).

6.12.4 Polarisation by reflection

The simplest method of producing plane polarised light is by reflection. Consider a beam of unpolarised light *AB* is incident at any angle on the reflecting glass surface *XY*. Vibrations in *AB* which are parallel to the plane of the diagram are shown by arrows. The vibrations which are perpendicular to the plane of the diagram and parallel to the reflecting surface are shown by dots in Figure 6.78. A part of the light is reflected along *BC*, and the rest is refracted along *BD*. On examining the reflected beam *BC* with an analyser, it is found that the ray is partially plane polarised. When the light is allowed to be incident at a particular angle the reflected beam is found to be plane polarised. **The angle of incidence at which the reflected beam is plane polarised is called *polarising angle* i_p.**

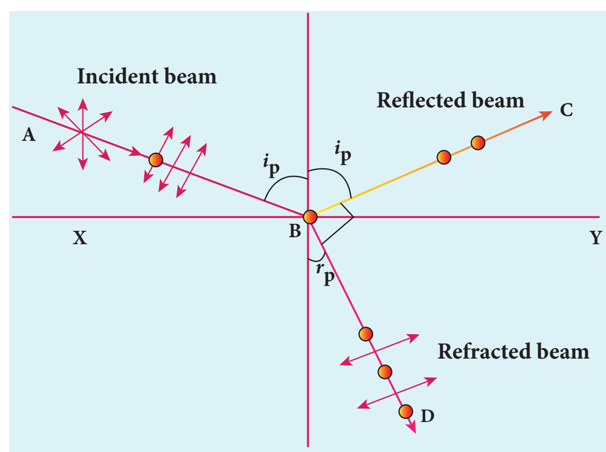


Figure 6.78 Polarisation by reflection

6.12.4.1 Brewster's Law

In 1808, Malus discovered that when ordinary light is incident on the surface of a transparent medium, the reflected light is partially plane polarised. The extent of polarisation depends on the angle of incidence. For a particular angle of incidence, the reflected light is found to be plane polarised. The angle of incidence at which a beam of unpolarised light falling on a transparent surface is reflected as a beam of plane polarised light is called *polarising angle* or *Brewster's angle*. It is denoted by i_p

Further, the British Physicist, Sir. David Brewster found that at the incidence of polarising angle, the reflected and transmitted rays are perpendicular to each other. Suppose, i_p is the polarising angle and r_p is the corresponding angle of refraction. Then from Figure 6.82,

$$i_p + 90^0 + r_p = 180^0 \tag{6.168}$$

$$r_p = 90^0 - i_p \tag{6.169}$$

From Snell's law, the refractive index of the transparent medium is,

$$\frac{\sin i_p}{\sin r_p} = n \tag{6.170}$$

where n is the refractive index of the medium with respect to air.

Substituting the value of r_p from Equation 6.163, we get,

$$\frac{\sin i_p}{\sin(90^\circ - i_p)} = \frac{\sin i_p}{\cos i_p} = n$$

$$\tan i_p = n \tag{6.171}$$

This relation is known as *Brewster's law*. The law states that the tangent of the polarising angle for a transparent medium is equal to its refractive index. The value of Brewster's angle depends on the nature of the transparent refracting medium and the wavelength of light used.

EXAMPLE 6.39

Find the polarizing angles for (i) glass of refractive index 1.5 and (ii) water of refractive index 1.33.

Solution

Brewster's law, $\tan i_p = n$

For glass, $\tan i_p = 1.5$; $i_p = \tan^{-1} 1.5$; $i_p = 56.3^\circ$

For water, $\tan i_p = 1.33$; $i_p = \tan^{-1} 1.33$; $i_p = 53.1^\circ$

6.12.4.2 Pile of plates

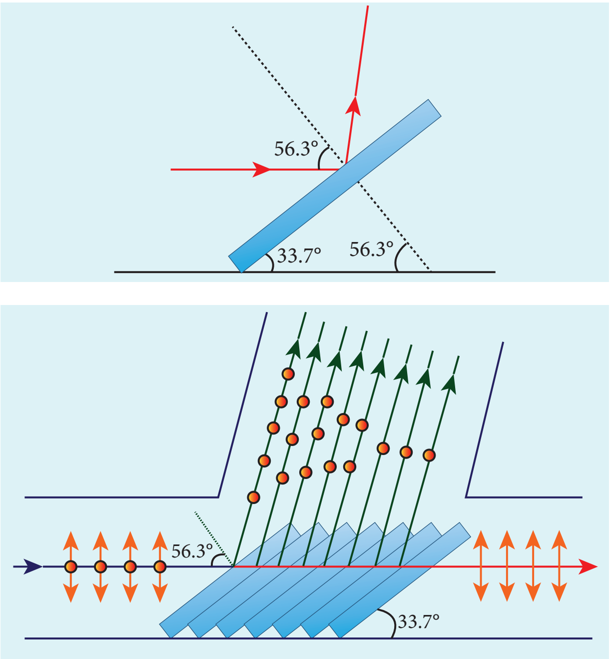


Figure 6.79 Pile of plates

The phenomenon of polarisation by reflection is used in the construction of pile of plates. It consists of a number of glass plates placed one over the other in a tube as shown in Figure 6.79. The plates are inclined at an angle of $33.7^\circ (90^\circ - 56.3^\circ)$ to the axis of the tube. A beam of unpolarised light is allowed to fall on the pile of plates along the axis of the tube. So, the angle of incidence of light will be at 56.3° which is the polarising angle for glass. The vibrations perpendicular to the plane of incidence are reflected at each surface and those parallel to it are transmitted. The larger the number of surfaces, the greater is the intensity of the reflected plane polarised light. The pile of plates is used as a polarizer and also as an analyser.

EXAMPLE 6.40

What is the angle at which a glass plate of refractive index 1.65 is to be kept with respect to the horizontal surface so that an unpolarised light travelling horizontal after reflection from the glass plate is found to be plane polarised?

Solution

$n = 1.65$

Brewster's law, $\tan i_p = n$

$\tan i_p = 1.65;\ i_p = \tan^{-1} 1.65;\ i_p = 58.8^0$

The inclination with the horizontal surface is, $(90^0 - 58.8^0) = 31.2^0$

6.12.5 Polarisation by double refraction

Erasmus Bartholinus, a Danish physicist discovered that **when a ray of unpolarised light is incident on a calcite crystal, two refracted rays are produced. Hence, two images of a single object are formed. This phenomenon is called *double refraction*** as shown in Figure 6.80. **Double refraction is also called *birefringence*.** This phenomenon is also exhibited by several other crystals like quartz, mica etc.

When an ink dot on a sheet of paper is viewed through a calcite crystal, two images will be seen. On rotating the crystal, one image remains stationary, while the other rotates around the first. The stationary image is known as the ordinary image *O*, produced by the refracted rays which obey the laws of refraction, called as ordinary rays. The other image is extraordinary image *E*, produced by the refracted rays which do not obey the laws of refraction, called as extraordinary rays. The extraordinary ray is found to be plane polarised. Inside a double refracting crystal the ordinary ray travels with same velocity in all directions and the extra ordinary ray travels with different velocities along different directions. A point source inside a refracting crystal produces spherical wavefront corresponding to ordinary ray and elliptical wavefront corresponding to extraordinary ray. Inside the crystal, there is a particular direction in which both the rays travel with same velocity. This direction is called *optic axis*. Along the optic axis, the refractive index is same for both the rays and there is no double refraction along this direction.

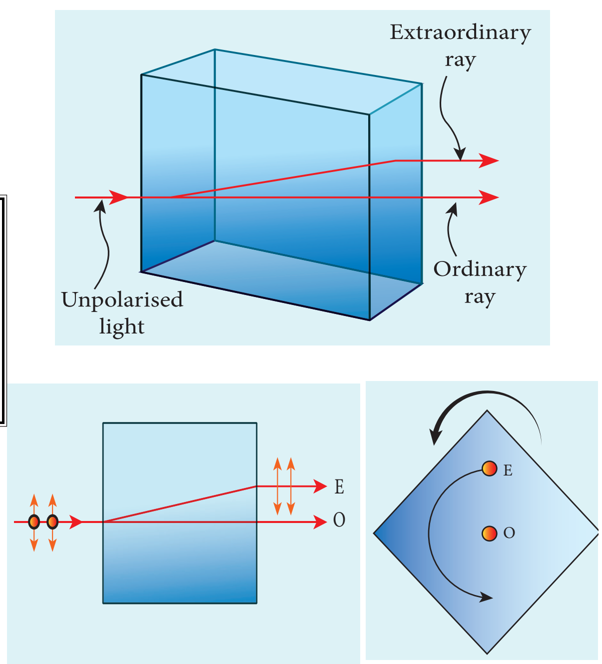


Figure 6.80 Double refraction

6.12.6 Types of optically active crystals

Crystals like calcite, quartz, tourmaline and ice having only one optic axis are called uniaxial crystals.

Crystals like mica, topaz, selenite and aragonite having two optic axes are called biaxial crystals.

6.12.7 Nicol prism

Nicol prism is an optical device incorporated in optical instruments both for producing and analysing plane polarised light. The construction of a Nicol prism is based on the phenomenon of Double refraction and was designed by William Nicol in 1828.

One of the most common forms of the Nicol prism is made by taking a calcite crystal which is a double refracting crystal with its length three times its breadth. As shown in Figure 6.81, *ABCD* represents the principal section of a calcite crystal. It is cut into two halves along the diagonal so that their face angles are 72° and 108°. The two halves are joined together by a layer of *canada balsam*, a transparent cement.

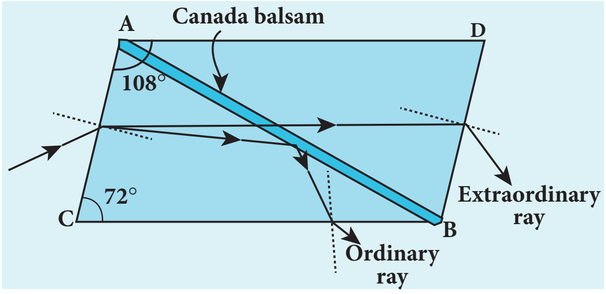


Figure 6.81 Nicol Prism

Let us consider a ray of unpolarised light from monochromatic source such as a sodium vapour lamp is incident on the face *AC* of the Nicol prism. Double refraction takes place and the ray is split into ordinary and extraordinary rays. They travel with different velocities. The refractive index of the crystal for the ordinary ray (monochromatic sodium light) is 1.658 and for extraordinary ray is 1.486. The refractive index of canada balsam is 1.523. Canada balsam does not polarise light.

The ordinary ray is total internally reflected at the layer of canada balsam and is prevented from emerging from the other face. The extraordinary ray alone is transmitted through the crystal which is plane polarised.

Uses of Nicol prism

(i) It produces plane polarised light and functions as a polariser

(ii) It can also be used to analyse the plane polarised light i.e used at an analyser.

Drawbacks of Nicol prism

(i) Its cost is very high due to scarity of large and flawless calcite crystals

(ii) Due to extraordinary ray passing obliquely through it, the emergent ray is always displaced a little to one side.

(iii) The effective field of view is quite limited

(iv) Light emerging out of it is not uniformly plane polarised.

6.12.8 Polarisation by scattering

The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polaroid which is rotated. This is because of sunlight, which has changed its direction (having been scattered) on encountering the molecules of the earth's atmosphere. As Figure 6.81 shows, the incident sunlight is unpolarised. The electric field of light interact with the electrons present in the air molecules. Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions. We have shown an observer looking at 90° to the direction of the sun. Clearly, charges accelerating parallel do not radiate energy towards this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore polarized perpendicular to the plane of the Figure 6.82. This explains the reason for polarisation of sunlight by scattering.

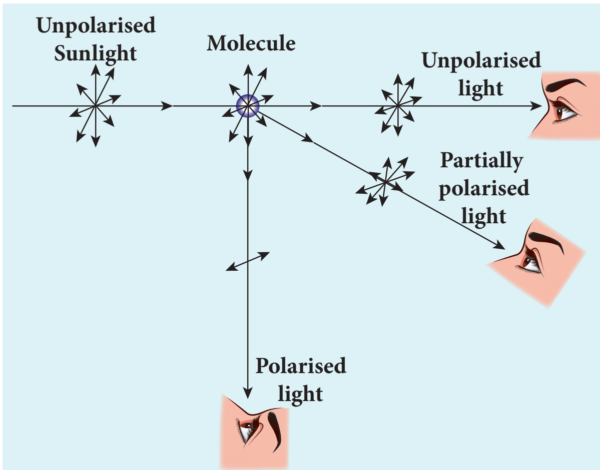


Figure 6.82 Polarisation by scattering

6.13 OPTICAL INSTRUMENTS

There are plenty of optical instruments we used in our daily life. We shall discuss here about microscopes, telescopes, spectrometer and of course human eye.

6.13.1 Simple microscope

A simple microscope is a single magnifying (converging) lens of small focal length. The idea is to get an erect, magnified and virtual image of the object. For this the object is placed between F and P on one side of the lens and viewed from other side of the lens. There are two magnifications to be discussed for two kinds of focussing.

(1) *Near point focusing* – The image is formed at near point, i.e. 25 cm for normal eye. This distance is also called as *least distance D* of distinct vision. In this position, the eye feels comfortable but there is little strain on the eye. This is shown in Figure 6.83

(2) *Normal focusing* – The image is formed at infinity. In this position the eye is most relaxed to view the image. This is shown in Figure 6.84(b).

6.13.1.1 Magnification in near point focusing

The near point focusing is shown in Figure 6.83. Object distance u is less than f. The image distance is the near point D. The magnification m is given by the relation,

$$m=\frac{v}{u} \qquad (6.172)$$

With the help of lens equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ the magnification can further be written as,

$$m=1-\frac{v}{f} \qquad (6.173)$$

Substituting for v with sign convention, $v = -D$

$$m=1+\frac{D}{f} \qquad (6.174)$$

This is the magnification for near point focusing.

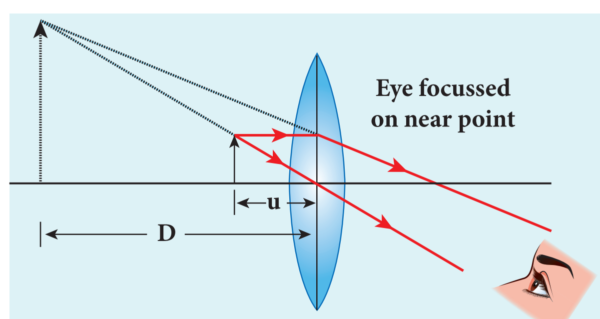


Figure 6.83 Near point focusing

6.13.1.2 Magnification in normal focusing (angular magnification)

The normal focusing is shown in Figure 6.84(b). We will now find the magnification for the image formed at infinity. If we take the ratio of height of image to height of

UNIT-6(XII-Physics_Vol-2).indd 82 8/3/2019 4:37:11 PM

object $\left(m=\frac{h'}{h}\right)$ to find the magnification, we will not get a practical relation, as the image will also be of infinite size when the image is formed at infinity. Hence, we can practically use the angular magnification. The angular magnification is defined as the ratio of angle θ_i subtended by the image with aided eye to the angle θ_0 subtended by the object with unaided eye.

$$m=\frac{\theta_i}{\theta_0} \qquad (6.175)$$

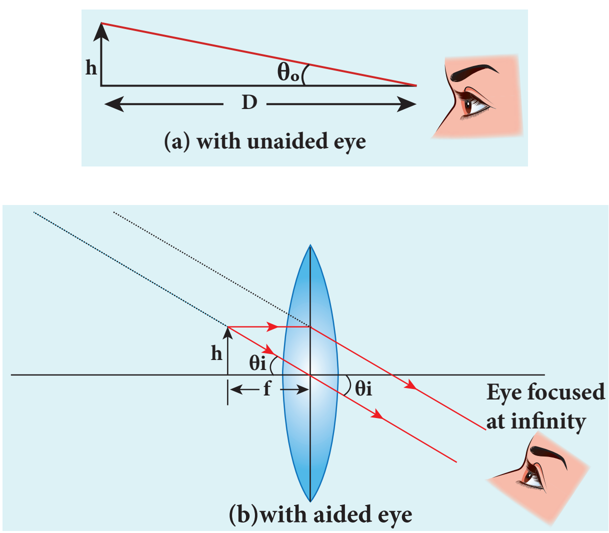


Figure 6.84 Normal focusing

For unaided eye shown in Figure 6.84(a),

$$\tan\theta_0 \approx \theta_0 = \frac{h}{D} \qquad (6.176)$$

For aided eye shown in Figure 6.83(b),

$$\tan\theta_i \approx \theta_i = \frac{h}{f} \qquad (6.177)$$

The angular magnification is,

$$m=\frac{\theta_i}{\theta_0}=\frac{h/f}{h/D}$$

$$m=\frac{D}{f} \qquad (6.178)$$

This is the magnification for normal focusing.

The magnification for normal focusing is one less than that for near point focusing. But, the viewing is more comfortable in normal focusing than near point focusing. For large values of D/f, the difference in magnification is usually small. In subsequent discussions, we shall only consider the normal focusing.

EXAMPLE 6.41

A man with a near point of 25 cm reads a book with small print using a magnifying glass, a convex lens of focal length 5 cm. (a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass? (b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Solution

D = 25 cm; f = 5 cm;

For closest object distance, u; the image distance, v is, –25 cm. (near point focusing)

For farthest object distance, u'; the corresponding image distance, v' is, $v'=\infty$ (normal focusing)

(a) To find closest image distance, lens equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Rewriting for closest object distance, $\frac{1}{u}=\frac{1}{v}-\frac{1}{f}$

Substituting,

$$\frac{1}{u}=\frac{1}{-25}-\frac{1}{5}=\frac{1}{25}-\frac{1}{5}=\left(\frac{-1-5}{25}\right)=-\frac{6}{25}$$