DO YOU KNOW?

Acoustical wonder of Golconda fort (Hyderabad, Telangana)

The Clapping portico in Golconda Fort is a series of arches on one side, each smaller than the preceding one. So, a sound wave generated under the dome would get compressed and then bounce back amplified sufficiently to reach a considerable distance.

5.2.2 Reflection at the boundary of a denser medium

54TNGQ

A longitudinal wave travels in a medium in the form of compressions and rarefactions. Suppose a compression travelling in air from left to right reaches a rigid wall. The compression exerts a force F on the rigid wall. In turn, the wall exerts an equal and opposite reaction R = – F on the air molecules. This results in a compression near the rigid wall. Thus, a compression travelling towards the rigid wall is reflected back as a compression. That is the direction of compression is reversed (Figure 5.5).

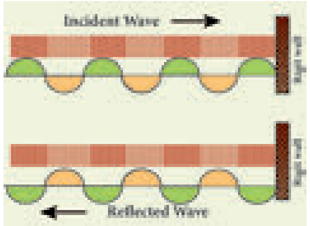
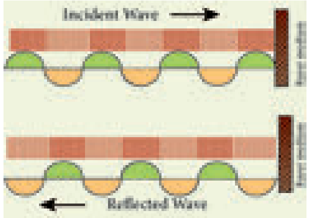


Figure 5.5 Reflection of sound at a denser medium

5.2.3 Reflection at the boundary of a rarer medium

78X6K2

Consider a wave travelling in a solid medium striking on the interface between the solid and the air. The compression exerts a force F on the surface of the rarer medium. As a rarer medium has smaller resistance for any deformation, the surface of separation is pushed backwards (Figure 5.6). As the particles of the rarer medium are free to move, a rarefaction is produced at the interface. Thus, a compression is reflected as a rarefaction and a rarefaction travels from right to left.



5.6 Reflection of sound at a rarer medium

More to know:

What is meant by rarer and denser medium?
The medium in which the velocity of sound increases compared to other medium is called rarer medium. (Water is rarer compared to air for sound).
The medium in which the velocity of sound decreases compared to other medium is called denser medium. (Air is denser compared to water for sound)

5.2.4 Reflection of sound in plane and curved surfaces

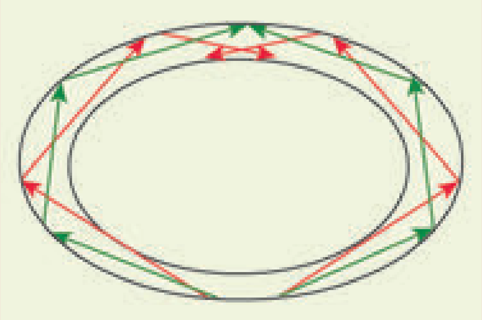
When sound waves are reflected from a plane surface, the reflected waves travel in a direction, according to the law of reflection. The intensity of the reflected wave is neither decreased nor increased. But, when the sound waves are reflected from the curved surfaces, the intensity of the reflected waves is changed. When reflected from a convex surface, the reflected waves are diverged out and the intensity is decreased. When sound is reflected from a concave surface, the reflected waves are converged and focused at a point. So the intensity of reflected waves is concentrated at a point. Parabolic surfaces are used when it is required to focus the sound at a particular point. Hence, many halls are designed with parabolic reflecting surfaces. In elliptical surfaces, sound from one focus will always be reflected to the other focus, no matter where it strikes the wall.

This principle is used in designing whispering halls. In a whispering hall, the speech of a person standing in one focus can be heard clearly by a listener standing at the other focus.

DO YOU KNOW?

Whispering Gallery

One of the famous whispering galleries is in St. Paul's cathedral church in London. It is built with elliptically shaped walls. When a person is talking at one focus, his voice can be heard distinctly at the other focus. It is due to the multiple reflections of sound waves from the curved walls.



5.3 ECHOES

An echo is the sound reproduced due to the reflection of the original sound from various rigid surfaces such as walls, ceilings, surfaces of mountains, etc.

If you shout or clap near a mountain or near a reflecting surface, like a building you can hear the same sound again. The sound, which you hear is called an echo. It is due to the reflection of sound. One does not experience any echo sound in a small room. This does not mean that sound is not reflected in a small room. This is because smaller rooms do not satisfy the basic conditions for hearing an echo.

5.3.1 Conditions necessary for hearing echo

1. The persistence of hearing for human ears is 0.1 second. This means that you can hear two sound waves clearly, if the time interval between the two sounds is at least 0.1 s. Thus, the minimum time gap between the original sound and an echo must be 0.1 s.

2. The above criterion can be satisfied only when the distance between the source of sound and the reflecting surface would satisfy the following equation:

$$\text{Velocity} = \frac{\text{distance travelled by sound}}{\text{time taken}}$$

$$v = \frac{2d}{t}$$

$$d = \frac{vt}{2}$$

Since, t = 0.1 second, then $d = \frac{v \times 0.1}{2} = \frac{v}{20}$

Thus the minimum distance required to hear an echo is 1/20th part of the magnitude of the velocity of sound in air. If you consider the velocity of sound as 344 m s^{-1}, the minimum distance required to hear an echo is 17.2 m.

5.3.2 Applications of echo

- Some animals communicate with each other over long distances and also locate objects by sending the sound signals and receiving the echo as reflected from the targets.
- The principle of echo is used in obstetric ultrasonography, which is used to create real-time visual images of the developing embryo or fetus in the mother's uterus. This is a safe testing tool, as it does not use any harmful radiations.
- Echo is used to determine the velocity of sound waves in any medium.

5.3.3 Measuring velocity of sound by echo method

Apparatus required:

A source of sound pulses, a measuring tape, a sound receiver, and a stop watch.

Procedure:

1. Measure the distance 'd' between the source of sound pulse and the reflecting surface using the measuring tape.
2. The receiver is also placed adjacent to the source. A sound pulse is emitted by the source.
3. The stopwatch is used to note the time interval between the instant at which the sound pulse is sent and the instant at which the echo is received by the receiver. Note the time interval as 't'.
4. Repeat the experiment for three or four times. The average time taken for the given number of pulses is calculated.

Calculation of speed of sound:

The sound pulse emitted by the source travels a total distance of 2d while travelling from the source to the wall and then back to the receiver. The time taken for this has been observed to be 't'. Hence, the speed of sound wave is given by:

$$\text{Speed of sound} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2d}{t}$$

5.4 APPLICATIONS REFLECTION OF SOUND

5.4.1 Sound board

These are basically curved surfaces (concave), which are used in auditoria and halls to improve the quality of sound. This board is placed such that the speaker is at the focus of the concave surface. The sound of the speaker is reflected towards the audience thus improving the quality of sound heard by the audience.

5.4.2 Ear trumpet

Ear trumpet is a hearing aid, which is useful by people who have difficulty in hearing. In this device, one end is wide and the other end is narrow. The sound from the sources fall into the wide end and are reflected by its walls into the narrow part of the device. This helps in concentrating the sound and the sound enters the ear drum with more intensity. This enables a person to hear the sound better.

5.4.3 Mega phone

A megaphone is a horn-shaped device used to address a small gathering of people. Its one end is wide and the other end is narrow. When a person speaks at the narrow end, the sound of his speech is concentrated by the multiple reflections from the walls of the tube. Thus, his voice can be heard loudly over a long distance.

5.5 DOPPLER EFFECT

The whistle of a fast moving train appears to increase in pitch as it approaches a stationary listener and it appears to decrease as the train moves away from the listener. This apparent change in frequency was first observed and explained by Christian Doppler (1803-1853), an Austrian Mathematician and Physicist. He observed that the frequency of the sound as received by a listener is different from the original frequency produced by the source whenever there is a relative motion between the source and the listener. This is known as Doppler effect This relative motion could be due to various possibilities as follows:

(i) The listener moves towards or away from a stationary source

(ii) The source moves towards or away from a stationary listener

(iii) Both source and listener move towards or away from one other

(iv) The medium moves when both source and listener are at rest

DEFINITION

When ever there is a relative motion between a source and a listener, the frequency of the sound heard by the listener is different from the original frequency of sound emitted by the source. This is known as "Doppler effect".

For simplicity of calculation, it is assumed that the medium is at rest. That is the velocity of the medium is zero.

Let S and L be the source and the listener moving with velocities v_S and v_L respectively. Consider the case of source and listener moving towards each other (Figure 5.7). As the distance between them decreases, the apparent frequency will be more than the actual source frequency.

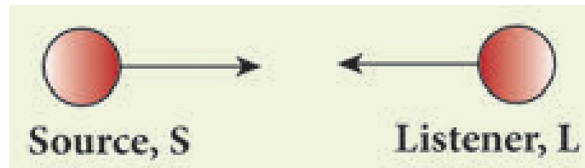


Figure 5.7 Source and listener moving towards each other

Let n and n' be the frequency of the sound produced by the source and the sound observed by the listener respectively. Then, the expression for the apparent frequency n' is

$$n' = \left(\frac{v + v_L}{v - v_s}\right)n$$

Here, v is the velocity of sound waves in the given medium. Let us consider different possibilities of motions of the source and the listener. In all such cases, the expression for the apparent frequency is given in table 5.2.

Table 5.2 Expression for apparent frequency due to Doppler effect

Case No.	Position of source and listener	Note	Expression for apparent frequency
1	❖ Both source and listener move ❖ They move towards each other	a) Distance between source and listener decreases. b) Apparent frequency is more than actual frequency.	$n' = \left(\frac{v + v_L}{v - v_s}\right)n$
2	❖ Both source and listener move ❖ They move away from each other	a) Distance between source and listener increases. b) Apparent frequency is less than actual frequency. c) v_S and v_L become opposite to that in case-1.	$n' = \left(\frac{v - v_L}{v + v_s}\right)n$
3	❖ Both source and listener move ❖ They move one behind the other ❖ Source follows the listener	a) Apparent frequency depends on the velocities of the source and the listener. b) v_S becomes opposite to that in case-2.	$n' = \left(\frac{v - v_L}{v - v_s}\right)n$

4	❖ Both source and listener move ❖ They move one behind the other ❖ Listener follows the source	a) Apparent frequency depends on the velocities of the source and the listener. b) v_S and v_L become opposite to that in case-3.	$n' = \left(\frac{v + v_L}{v + v_s}\right) n$
5	❖ Source at rest ❖ Listener moves towards the source	a) Distance between source and listener decreases. b) Apparent frequency is more than actual frequency. c) $v_S = 0$ in case-1.	$n' = \left(\frac{v + v_L}{v}\right) n$
6	❖ Source at rest ❖ Listener moves away from the source	a) Distance between source and listener increases. b) Apparent frequency is less than actual frequency. c) $v_S = 0$ in case-2.	$n' = \left(\frac{v - v_L}{v}\right) n$
7	❖ Listener at rest ❖ Source moves towards the listener	a) Distance between source and listener decreases. b) Apparent frequency is more than actual frequency. c) $v_L = 0$ in case-1.	$n' = \left(\frac{v}{v - v_s}\right) n$
8	❖ Listener at rest ❖ Source moves away from the listener	a) Distance between source and listener increases. b) Apparent frequency is less than actual frequency. c) $v_L = 0$ in case-2.	$n' = \left(\frac{v}{v + v_s}\right) n$

Suppose the medium (say wind) is moving with a velocity W in the direction of the propagation of sound. For this case, the velocity of sound, 'v' should be replaced with (v + W). If the medium moves in a direction opposite to the propagation of sound, then 'v' should be replaced with (v – W).

Solved problems

1. A source producing a sound of frequency 90 Hz is approaching a stationary listener with a speed equal to (1/10) of the speed of sound. What will be the frequency heard by the listener?

Solution: When the source is moving towards the stationary listener, the expression for apparent frequency is

$$n' = \left(\frac{v}{v - v_s}\right) n$$

$$= \left(\frac{v}{v - \left(\frac{1}{10}\right) v}\right) n = \left(\frac{10}{9}\right) n$$

$$= \left(\frac{10}{9}\right) \times 90 = 100 \text{ Hz}$$

2. A source producing a sound of frequency 500 Hz is moving towards a listener with a velocity of 30 m s^{-1}. The speed of the sound is 330 m s^{-1}. What will be the frequency heard by listener?

Solution: When the source is moving towards the stationary listener, the expression for apparent frequency is

$$n' = \left(\frac{v}{v - v_s}\right) n$$

$$n' = \left(\frac{330}{330 - 30}\right) \times 500$$

$$= \mathbf{550\ Hz}$$

3. A source of sound is moving with a velocity of 50 m s^{-1} towards a stationary listener. The listener measures the frequency of the source as 1000 Hz. what will be the apparent frequency of the source when it is moving away from the listener after crossing him? (velocity of sound in the medium is 330 m s^{-1})

Solution: When the source is moving towards the stationary listener, the expression for apparent frequency is

$$n' = \left(\frac{v}{v - v_s}\right) n$$

$$1000 = \left(\frac{330}{330 - 50}\right) n$$

$$n = \left(\frac{1000 \times 280}{330}\right)$$

n = 848.48 Hz.

The actual frequency of the sound is 848.48 Hz. When the source is moving away from the stationary listener, the expression for apparent frequency is

$$n' = \left(\frac{v}{v + v_s}\right) n$$

$$= \left(\frac{330}{330 + 50}\right) \times 848.48$$

$$= \mathbf{736.84\ Hz}$$

4. A source and listener are both moving towards each other with a speed v/10 where v is the speed of sound. If the frequency of the note emitted by the source is f, what will be the frequency heard by the listener?

Solution: When source and listener are both moving towards each other, the apparent frequency is

$$n' = \left(\frac{v + v_l}{v - v_s}\right).n$$

$$n' = \left(\frac{v + \frac{v}{10}}{v - \frac{v}{10}}\right).n$$

$$n' = \frac{11}{9}.f$$

$$= \mathbf{1.22\ f}$$

5. At what speed should a source of sound move away from a stationary observer so that observer finds the apparent frequency equal to half of the original frequency?

Solution: When the source is moving away from the stationary listener, the expression for the apparent frequency is

$$n' = \left(\frac{v}{v + v_s}\right).n$$

$$\frac{n}{2} = \left(\frac{v}{v + v_s}\right).n$$

$$V_s = V$$

5.5.1 Conditions for no Doppler effect

Under the following circumstances, there will be no Doppler effect and the apparent frequency as heard by the listener will be the same as the source frequency.

(i) When source (S) and listener (L) both are at rest.

(ii) When S and L move in such a way that distance between them remains constant.

(iii) When source S and L are moving in mutually perpendicular directions.

(iv) If the source is situated at the center of the circle along which the listener is moving.

5.5.2 Applications of Doppler effect

(a) To measure the speed of an automobile

An electromagnetic wave is emitted by a source attached to a police car. The wave is reflected by a moving vehicle, which acts as a moving source. There is a shift in the frequency of the reflected wave. From the frequency shift, the speed of the car can be determined. This helps to track the over speeding vehicles.

(b) Tracking a satellite

The frequency of radio waves emitted by a satellite decreases as the satellite passes away from the Earth. By measuring the change in the frequency of the radio waves, the location of the satellites is studied.

(c) RADAR (RAdio Detection And Ranging)

In RADAR, radio waves are sent, and the reflected waves are detected by the receiver of the RADAR station. From the frequency change, the speed and location of the aeroplanes and aircrafts are tracked.

(d) SONAR

In SONAR, by measuring the change in the frequency between the sent signal and received signal, the speed of marine animals and submarines can be determined.

Points to Remember

- Wave velocity is the velocity with which the wave travels through the medium.
- Velocity of a sound wave is maximum in solids because they are more elastic in nature than liquids and gases. Since gases are least elastic in nature.
- Infrasonic waves are sound wave with a frequency below 20 Hz. A human ear cannot hear these waves.
- Ultrasonic waves are sound waves with frequency greater than 20 kHz, A human ear cannot detect these waves.
- Reflection of sound waves obey the laws of reflection.
- when a compression hits the boundary of a rarer medium, it is reflected as a rarefaction.
- An echo is the sound reproduced due to the reflection of the original sound wave.
- The minimum distance between the source and the reflecting surface should be 17.2 m to hear an echo clearly.
- “The apparent frequency” is the frequency of the sound as heard by the listener.

TEXTBOOK EVALUATION

I. Choose the correct answer

1. When a sound wave travels through air, the air particles
   a) vibrate along the direction of the wave motion
   b) vibrate but not in any fixed direction
   c) vibrate perpendicular to the direction of the wave motion
   d) do not vibrate

2. Velocity of sound in a gaseous medium is 330 m s^{-1}. If the pressure is increased by 4 times without causing a change in the temperature, the velocity of sound in the gas is
   a) 330 m s^{-1} b) 660 m s^{1}
   c) 156 m s^{-1} d) 990 m s^{-1}

3. The frequency, which is audible to the human ear is
   a) 50 kHz b) 20 kHz
   c) 15000 kHz d) 10000 kHz

4. The velocity of sound in air at a particular temperature is 330 m s^{-1}. What will be its value when temperature is doubled and the pressure is halved?
   a) 330 m s^{-1} b) 165 m s^{-1}
   c) $330 \times \sqrt{2}$ m s^{-1} d) $320 / \sqrt{2}$ m s^{-1}

5. If a sound wave travels with a frequency of 1.25×10^4 Hz at 344 m s^{-1}, the wavelength will be
   a) 27.52 m b) 275.2 m
   c) 0.02752 m d) 2.752 m

6. The sound waves are reflected from an obstacle into the same medium from which they were incident. Which of the following changes?
   a) speed b) frequency
   c) wavelength d) none of these

7. Velocity of sound in the atmosphere of a planet is 500 m s^{-1}. The minimum distance between the sources of sound and the obstacle to hear the echo, should be
   a) 17 m b) 20 m c) 25 m d) 50 m

II. Fill up the blanks

1. Rapid back and forth motion of a particle about its mean position is called ______
2. If the energy in a longitudinal wave travels from south to north, the particles of the medium would be vibrating in ______
3. A whistle giving out a sound of frequency 450 Hz, approaches a stationary observer at a speed of 33 m s^{-1}. The frequency heard by the observer is (speed of sound = 330 m s^{-1}) ______.
4. A source of sound is travelling with a velocity 40 km/h towards an observer and emits a sound of frequency 2000 Hz. If the velocity of sound is 1220 km/h, then the apparent frequency heard by the observer is ____________.

III. True or false:- (If false give the reason)

1. Sound can travel through solids, gases, liquids and even vacuum.
2. Waves created by Earth Quake are Infrasonic.
3. The velocity of sound is independent of temperature.
4. The Velocity of sound is high in gases than liquids.

IV. Match the following

1. Infrasonic	-	(a) Compressions
2. Echo	-	(b) 22 kHz
3. Ultrasonic	-	(c) 10 Hz
4. High pressure region	-	(d) Ultrasonography

V. Assertion and Reason Questions

Mark the correct choice as

a. If both the assertion and the reason are true and the reason is the correct explanation of the assertion.

b. If both the assertion and the reason are true but the reason is not the correct explanation of the assertion.

c. Assertion is true, but the reason is false.

d. Assertion is false, but the reason is true.

1) Assertion: The change in air pressure affects the speed of sound.

Reason: The speed of sound in a gas is proportional to the square of the pressure

2) Assertion: Sound travels faster in solids than in gases.

Reason: Solid posses a greater density than that of gases.

VI. Answer very briefly

1. What is a longitudinal wave?
2. What is the audible range of frequency?
3. What is the minimum distance needed for an echo?
4. What will be the frequency sound having 0.20 m as its wavelength, when it travels with a speed of 331 m s^{-1}?
5. Name three animals, which can hear ultrasonic vibrations.

VII. Answer briefly

1. Why does sound travel faster on a rainy day than on a dry day?
2. Why does an empty vessel produce more sound than a filled one?
3. Air temperature in the Rajasthan desert can reach 46°C. What is the velocity of sound in air at that temperature? (V_0 = 331 m s^{-1})
4. Explain why, the ceilings of concert halls are curved.
5. Mention two cases in which there is no Doppler effect in sound?

VIII. Problem Corner

1. A sound wave has a frequency of 200 Hz and a speed of 400 m s^{-1} in a medium. Find the wavelength of the sound wave.
2. The thunder of cloud is heard 9.8 seconds later than the flash of lightning. If the speed of sound in air is 330 m s^{-1}, what will be the height of the cloud?
3. A person who is sitting at a distance of 400 m from a source of sound is listening to a sound of 600 Hz. Find the time period between successive compressions from the source?
4. An ultrasonic wave is sent from a ship towards the bottom of the sea. It is found that the time interval between the transmission and reception of the wave is 1.6 seconds. What is the depth of the sea, if the velocity of sound in the seawater is 1400 m s^{-1}?
5. A man is standing between two vertical walls 680 m apart. He claps his hands and hears two distinct echoes after 0.9 seconds and 1.1 second respectively. What is the speed of sound in the air?
6. Two observers are stationed in two boats 4.5 km apart. A sound signal sent by one, under water, reaches the other after 3 seconds. What is the speed of sound in the water?
7. A strong sound signal is sent from a ship towards the bottom of the sea. It is received back after 1s.What is the depth of sea given that the speed of sound in water 1450 m s^{-1}?

IX. Answer in Detail

1. What are the factors that affect the speed of sound in gases?

2. What is mean by reflection of sound? Explain:

   a) reflection at the boundary of a rarer medium

   b) reflection at the boundary of a denser medium

   c) Reflection at curved surfaces

3. a) What do you understand by the term 'ultrasonic vibration'?

   b) State three uses of ultrasonic vibrations.

   c) Name three animals which can hear ultrasonic vibrations.

4. What is an echo?

   a) State two conditions necessary for hearing an echo.

   b) What are the medical applications of echo?

   c) How can you calculate the speed of sound using echo?

X. HOT Questions

1. Suppose that a sound wave and a light wave have the same frequency, then which one has a longer wavelength?

   a) Sound b) Light

   c) both a and b d) data not sufficient

2. When sound is reflected from a distant object, an echo is produced. Let the distance between the reflecting surface and the source of sound remain the same. Do you hear an echo sound on a hotter day? Justify your answer.

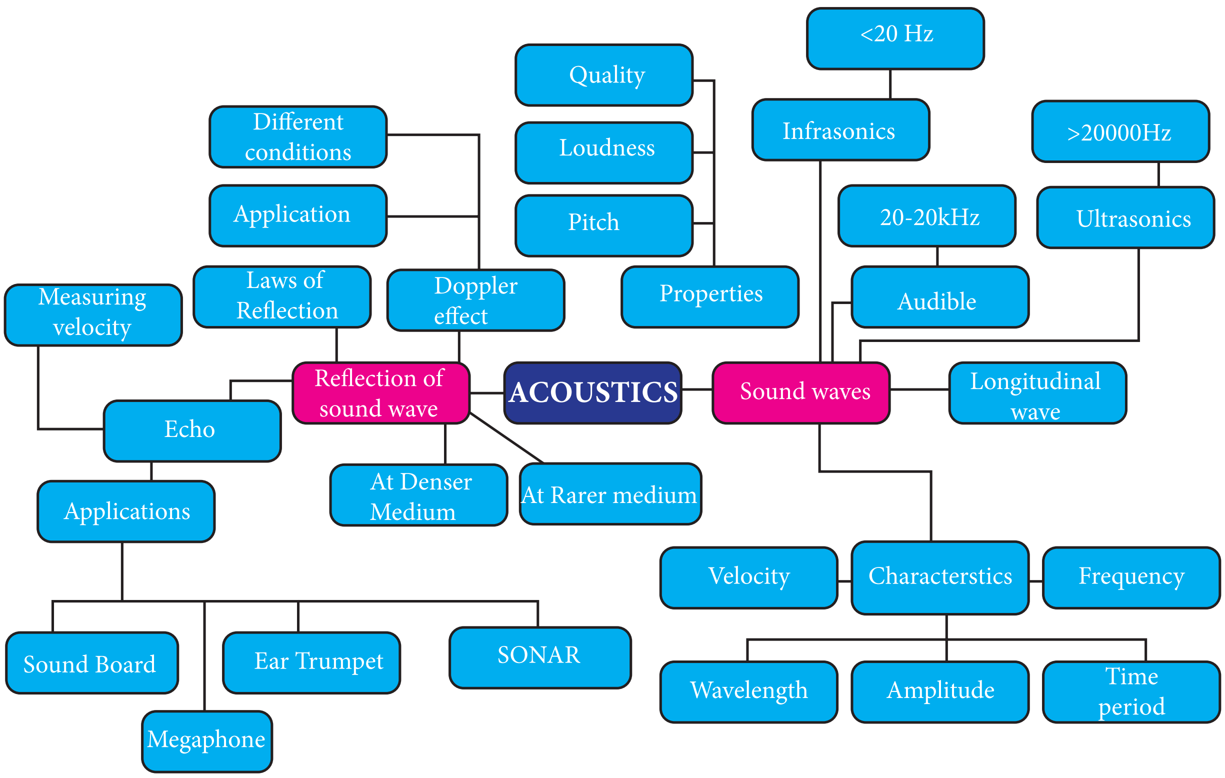
REFERENCE BOOKS

1. Fundamental Physics by K.L. Gomber and K.L. Gogia
2. Fundamentals of sound and vibration by Franky Fahy and David Thombson
3. The theory of sound by Rayleigh and John William Strutt

INTERNET RESOURCE

1. http://people.bath.ac.uk/ensmjc/Notes/acoustics.pdf

Concept Map



ICT CORNER

Doppler effect

In this activity you will be able to learn how the observed frequencies of a sound changes with the velocities of the source and the observer (Doppler effect).

Steps

- Open the browser and type “vlab.amrita.edu” in the address bar. Click ‘Physical Sciences’ and then click ‘Harmonic Motion and Waves Virtual Lab’. Click ‘Doppler Effect’ and Go to “simulator” tab to do the experiment. sign up one time with your e-mail
- Select medium of travel, detector direction and source direction by clicking the drop down menu.
- Change relative motion between source and observer by adjusting the velocity of the source and observer using the slider.
- Discuss how apparent frequency is changes with respect to actual frequency by changing position of source and listener. Also try for different source frequencies.

Link

http://vlab.amrita.edu/?sub=1&brch=201&sim=368&cnt=4

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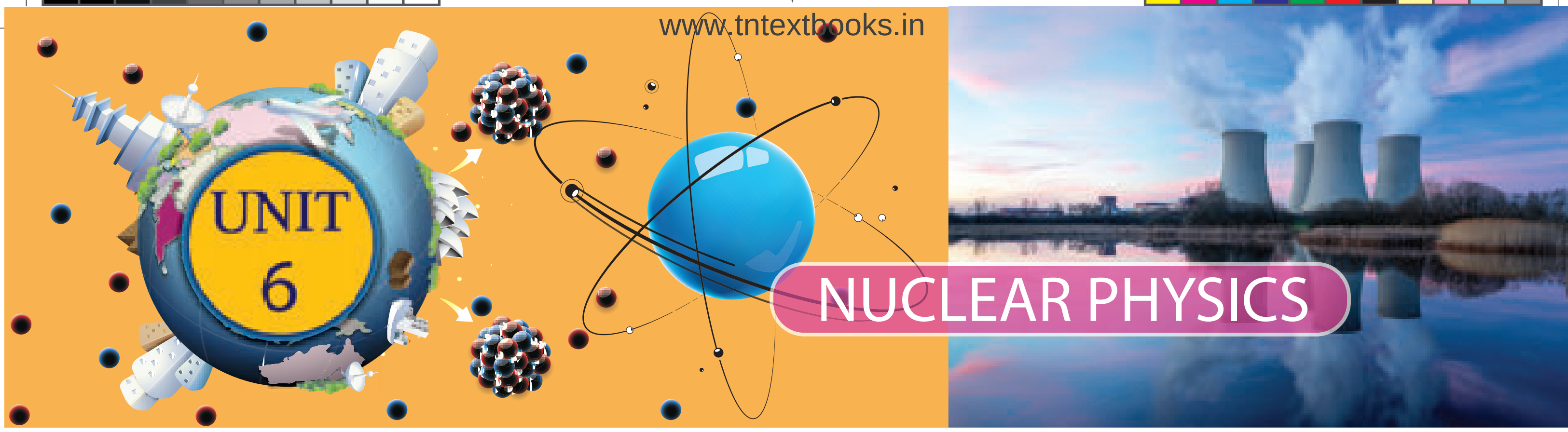

A83DR1

Learning Objectives

After learning this unit, students will be able to

- Define radio activity.
- Distingush between natural and artificial radio activity.
- Relate the properties of alpha, beta and gamma rays.
- State Soddy and Fajan's displacement law of nuclear disintegration.
- Understand the concept of nuclear fission and nuclear fusion.
- Identify fissionable materials.
- Analyze controlled and uncontrolled chain reactions.
- Explain the principle of atom bomb and hydrogen bomb.
- List the uses of radio activity.
- Understand the components of a nuclear reactor.
- Identify the precautionary measures while handling a radioactive material.

INTRODUCTION

Humans are very much interested in knowing about atoms. Things around us are made up of atoms. A Greek Philosopher 'Democritus' in 400 BC believed that matter is made up of tiny indestructible units called atoms. Later, in 1803, John Dalton considered that elements consist of atoms, which are identical in nature. J J Thomson discovered cathode rays, known as electrons, experimentally and Goldstein discovered positive rays, which were named as protons by Rutherford. In 1932, James Chadwick discovered the chargeless particles called neutrons. Presently, a large number of elementary particles like photon, meson, positron and nutrino have been discovered. In 1911, the British scientist, Ernest **Rutherford** explained that the mass of an atom is concentrated in its central part called **Nucleus**. You have already learnt about the atomic structure in the earlier classes.

6.1 RADIOACTIVITY

6.1.1 Discovery of radioactivity

In 1896, French physicist **Henri Becquerel** finished his research for the week and stored a certain amount of uranium compound away in a drawer for the week end. By chance, an unexposed photographic plate was also stored in the same drawer. After a week he returned and noticed that the film had been exposed to some radiation. He discovered that he could reproduce the effect whenever he placed uranium near a photographic film. Apparently, uranium radiated something that could affect a photographic plate. This phenomenon was called as **Radioactivity**. Uranium was identified to be a radioactive element.

Two years later, the Polish physicist **Marie Curie** and her husband **Pierre Curie** detected radioactivity in 'Pitchblende', a tiny black substance. They were not surprised at the radioactivity of pitchblende, which is known as an ore of uranium. Later, they discovered that the radiation was more intense from pure uranium. Also, it was found that the pitchblende had less concentration of uranium. They concluded that **some other substance** was present in pitchblende. After separating this new substance, they discovered that it had unknown chemical properties and it also emitted radiations spontaneously like uranium. They named this new substance as **'Radium'**.The radioactive elements emit harmful radioactive radiations like alpha rays or beta rays or gamma rays.

6.1.2 Definition of radioactivity

The nucleus of some elements is unstable. Such nuclei undergo nuclear decay and get converted into more stable nuclei. During this nuclear reaction, these nuclei emit certain harmful radiations and elementary particles. The phenomenon of nuclear decay of certain elements with the emission of radiations like alpha, beta, and gamma rays is called 'radioactivity' and the elements, which undergo this phenomenon are called 'radioactive elements'.

6.1.3 Natural Radioactivity

The elements such as uranium and radium undergo radioactivity and emit the radiations on their own without any human intervention. This phenomenon of spontaneous emission of radiation from certain elements on their own is called 'natural radioactivity'.

The elements whose atomic number is more than 83 undergo spontaneous radioactivity. Eg: uranium, radium, etc. There are only two elements, which have been identified as radioactive substances with atomic number less than 83. They are technetium (Tc) with atomic number 43 and promethium (Pm) with atomic number 61.

DO YOU KNOW?

There have been 29 radioactive substances discovered so far. Most of them are rare earth metals and transition metals.

6.1.4 Artificial Radioactivity (or) Induced Radioactivity

The phenomenon by which even light elements are made radioactive, by artificial or induced methods, is called 'artificial radioactivity' or 'man-made radioactivity'.

This kind of radioactivity was discovered by Irene Curie and F.Joliot in 1934. Artificial radioactivity is induced in certain lighter elements like boron, aluminium etc., by bombarding them with radiations such as 'alpha particles' emitted during the natural radioactivity of uranium. This also results in the emission of invisible radiations and elementary particles. During such a disintegration, the nucleus which undergoes disintegration is called 'parent nucleus' and that which is produced after the disintegration is called a 'daughter nucleus'. The particle, which

Table 6.1 Comparison between Natural and Artificial Radioactivity

S.No.	Natural radioactivity	Artificial radioactivity
1	Emission of radiation due to self-disintegration of a nucleus.	Emission of radiation due to disintegration of a nucleus through induced process.
2	Alpha, beta and gamma radiations are emitted.	Mostly elementary particles such as neutron, positron, etc. are emitted.
3	It is a spontaneous process.	It is an induced process.
4	Exhibited by elements with atomic number more than 83.	Exhibited by elements with atomic number less than 83.
5	This cannot be controlled.	This can be controlled.

is used to induce the artificial disintegration is termed as projectile and the particle which is produced after the disintegration is termed as ejected particle. When the projectile hits the parent nucleus, it is converted into an unstable nucleus, which in turn decays spontaneously emitting the daughter nucleus along with an ejected particle.

Activity 6.1

Using the periodic table, list out the radioactive elements. Also identify the name of the groups in which they are present.

If you denote the parent and daughter nuclei as X and Y respectively, then the nuclear disintegration is represented as follows: X (P,E) Y. Here, P and E represent the projectile particle and ejected particle respectively.

Example:

$${}_4Be^9 + {}_2He^4 \longrightarrow {}_6C^{13*}$$

$${}_6C^{13*} \longrightarrow {}_6C^{12} + {}_0n^1$$

In the above nuclear reaction, ${}_6C^{13*}$ is unstable and is radioactive. This reaction can be represented as ${}_4Be^9$ (α, n) ${}_6C^{12}$

$${}_4Be^9 + {}_2He^4 \longrightarrow {}_6C^{12} + {}_0n^1$$

6.1.5 Units of Radioactivity

AML8SC

Curie: It is the traditional unit of radioactivity. It is defined as the quantity of a radioactive substance which undergoes 3.7×10^{10} disintegrations in one second. This is actually close to the activity of 1 g of radium 226.

1 curie = 3.7×10^{10} disintegrations per second.

Rutherford (Rd): It is another unit of radioactivity. It is defined as the quantity of a radioactive substance, which produces 10^6 disintegrations in one second.

1 Rd = 10^6 disintegrations per second.

Becquerel (Bq) : It is The SI unit of radioactivity is becquerel. It is defined as the quantity of one disintegration per second.

Roentgen (R): It is The radiation exposure of γ and x-rays is measured by another unit called roentgen. One roentgen is defined as the quantity of radioactive substance which produces a charge of 2.58×10^{-4} coulomb in 1 kg of air under standard conditions of pressure, temperature and humidity.

6.2 ALPHA, BETA AND GAMMA RAYS

When a radioactive nucleus undergoes radioactivity, it emits harmful radiations. These radiations are usually comprised of any of the three types of particles. They are **alpha(α), beta (β) and gamma(γ)** rays.

DO YOU KNOW?

Uranium, named after the planet Uranus, was discovered by Martin Klaproth, a German chemist in a mineral called pitchblende.

6.2.1 Properties of Alpha, Beta and Gamma rays

These three particles possess certain similarities and dissimilarities in their properties as listed below in Table 6.2.

6.2.2 Radioactive displacement law

In 1913, Soddy and Fajan framed the displacement laws governing the daughter nucleus produced during an alpha and beta decay. They are stated below:

(i) When a radioactive element emits an alpha particle, a daughter nucleus is formed whose mass number is less by 4 units and the atomic number is less by 2 units, than the mass number and atomic number of the parent nucleus.

(ii) When a radioactive element emits a beta particle, a daughter nucleus is formed whose mass number is the same and the atomic number is more by 1 unit, than the atomic number of the parent nucleus.

Table 6.2 Properties of alpha, beta and gamma rays

Properties	α rays	β rays	γ rays
What are they?	Helium nucleus ($_2He^4$) consisting of two protons and two neutrons.	They are electrons ($_{-1}e^0$), basic elementary particle in all atoms.	They are electromagnetic waves consisting of photons.
Charge	Positively charged particles. Charge of each alpha particle = +2e	Negatively charged particles. Charge of each beta particle = –e	Neutral particles. Charge of each gamma particle = zero
Ionising power	100 time greater than β rays and 10,000 times greater than γ rays	Comparatively low	Very less ionization power
Penetrating power	Low penetrating power (even stopped by a thick paper)	Penetrating power is greater than that of α rays. They can penetrate through a thin metal foil.	They have a very high penetrating power greater than that of β rays. They can penetrate through thick metal blocks.
Effect of electric and magnetic field	Deflected by both the fields. (in accordance with Fleming's left hand rule)	Deflected by both the fields; but the direction of deflection is opposite to that for alpha rays. (in accordance with Fleming's left hand rule)	They are not deflected by both the fields.
Speed	Their speed ranges from 1/10 to 1/20 times the speed of light.	Their speed can go up to 9/10 times the speed of light.	They travel with the speed of light.

6.2.3 Alpha decay

A nuclear reaction in which an unstable parent nucleus emits an alpha particle and forms a stable daughter nucleus, is called 'alpha decay'.

E.g.: Decay of uranium (U^{238}) to thorium (Th^{234}) with the emission of an alpha particle.

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} \quad (\alpha \text{ - decay})$$

In α - decay, the parent nucleus emits an α particle and so it is clear that for the daughter nucleus, the mass number decreases by four and the atomic number decreases by two as illustrated in Figure 6.1

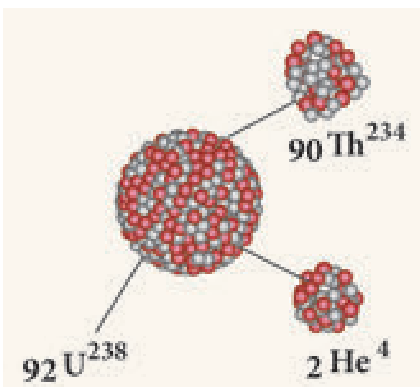

Figure 6.1 Alpha decay

6.2.4 Beta decay

A nuclear reaction, in which an unstable parent nucleus emits a beta particle and forms a stable daughter nucleus, is called 'beta decay'.

E.g.: Beta decay of phosphorous.

$$_{15}P^{32} \longrightarrow {}_{16}S^{32} + {}_{-1}e^{0} \quad (\beta \text{ - decay})$$

In β - decay there is no change in the mass number of the daughter nucleus but the atomic number increases by one.

Note: In a nuclear reaction, the element formed as the product nucleus is identified by the atomic number of the resulting nucleus and not by its mass number.

6.2.5 Gamma decay

In a γ - decay, only the energy level of the nucleus changes. The atomic number and mass number of the radioactive nucleus remain the same.

6.3 NUCLEAR FISSION

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6.3.1 Definition

In 1939, German Scientist Otto Hahn and F.Strassman discovered that when a uranium nucleus is bombarded with a neutron, it breaks up into two smaller nuclei of comparable mass along with the emission of a few neutrons and energy. This process of breaking (splitting) up of a heavier nucleus into two smaller nuclei with the release of a large amount of energy and a few neutrons is called 'nuclear fission'.

E.g.: Nuclear fission of a uranium nucleus (U^{235})

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + Q \text{ (energy)}$$

The average energy released in each fission process is about 3.2×10^{-11} J. Nuclear fission is pictorially represented in Figure 6.2.

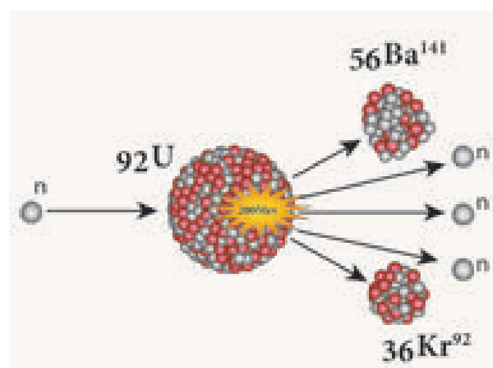

Figure 6.2 Nuclear fission

6.3.2 Fissionable materials

A fissionable material is a radioactive element, which undergoes fission in a sustained manner when it absorbs a neutron. It is also termed as 'fissile material'.

E.g.: U^{235}, plutonium (Pu^{239} and Pu^{241})

All isotopes of uranium do not undergo nuclear fission when they absorb a neutron. For example, natural uranium consists of 99.28 % of $_{92}U^{238}$ and 0.72 % of $_{92}U^{235}$. Of these two, U^{238} does not undergo fission

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whereas U^{235} undergoes fission. Hence, U^{235} is a fissionable material and U^{238} is non-fissionable.

There are some radioactive elements, which can be converted into fissionable material. They are called as **fertile materials**.

E.g.: Uranium-238, Thorium-232, Plutonium-240.

6.3.3 Chain Reaction

A uranium nucleus (U-235) when bombarded with a neutron undergoes fission producing three neutrons. These three neutrons in turn can cause fission in three other uranium nuclei present in the sample, thus producing nine neutrons. These nine neutrons in turn may produce twenty seven neutrons and so on. This is known as 'chain reaction'. A chain reaction is a self-propagating process in which the number of neutrons goes on multiplying rapidly almost in a geometrical progression.

Two kinds of chain reactions are possible. They are: (i) controlled chain reaction and (ii)uncontrolled chain reaction.

(a) Controlled chain reaction

In the controlled chain reaction the number of neutrons released is maintained to be one. This is achieved by absorbing the extra neutrons with a neutron absorber leaving only one neutron to produce further fission. Thus, the reaction is sustained in a controlled manner. The energy released due to a controlled chain reaction can be utilized for constructive purposes. Controlled chain reaction is used in a nuclear reactor to produce energy in a sustained and controlled manner.

(b) Uncontrolled chain reaction

In the uncontrolled chain reaction the number of neutrons multiplies indefinitely and causes fission in a large amount of the fissile material. This results in the release of a huge amount of energy within a fraction of a second. This kind of chain reaction is used in the atom bomb to produce an explosion. Figure 6.3 represents an uncontrolled chain reaction.

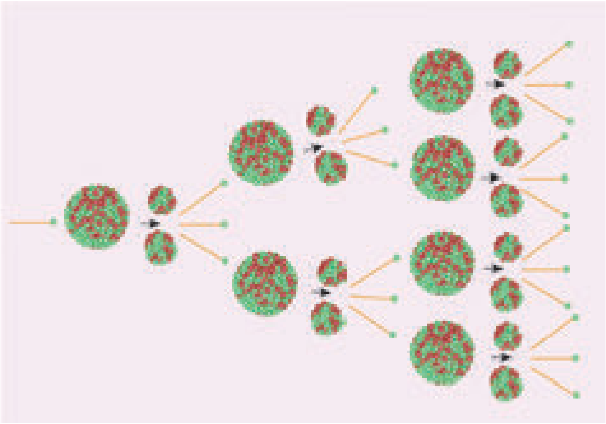

Figure 6.3 Uncontrolled chain reaction

6.3.4 Critical Mass

During a nuclear fission process, about 2 to 3 neutrons are released. But, all these neutrons may not be available to produce further fission. Some of them may escape from the system, which is termed as 'leakage of neutrons' and some may be absorbed by the non-fissionable materials present in the system. These two factors lead to the loss of neutrons. To sustain the chain reaction, the rate of production of neutrons due to nuclear fission must be more than the rate of its loss. This can be achieved only when the size (i.e., mass) of the fissionable material is equal to a certain optimum value. This is known as 'critical mass'.

The minimum mass of a fissile material necessary to sustain the chain reaction is called 'critical mass (m_c)'. It depends on the nature, density and the size of the fissile material.

If the mass of the fissile material is less than the critical mass, it is termed as 'subcritical'. If the mass of the fissile material is more than the critical mass, it is termed as 'supercritical'.

Activity 6.2

Using beads make a chain reaction model

6.3.5 Atom bomb

The atom bomb is based on the principle of uncontrolled chain reaction. In an uncontrolled chain reaction, the number of neutrons and the number of fission reactions multiply almost in a geometrical progression. This releases a huge amount of energy in a very small time interval and leads to an explosion.

Structure:

An atom bomb consists of a piece of fissile material whose mass is subcritical. This piece has a cylindrical void. It has a cylindrical fissile material which can fit into this void and its mass is also subcritical. When the bomb has to be exploded, this cylinder is injected into the void using a conventional explosive. Now, the two pieces of fissile material join to form the supercritical mass, which leads to an explosion. The structure of an atom bomb is shown in Figure 6.4

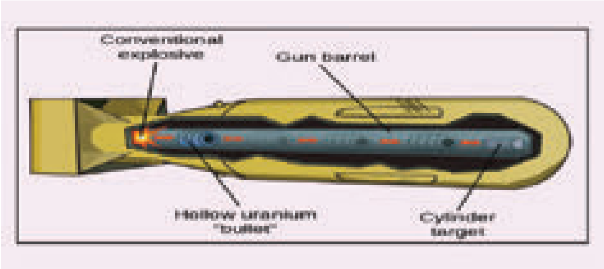

Figure 6.4 Atom bomb

During this explosion tremendous amount of energy in the form of heat, light and radiation is released. A region of very high temperature and pressure is formed in a fraction of a second along with the emission of hazardous radiation like γ rays, which adversely affect the living creatures. This type of atom bombs were exploded in 1945 at Hiroshima and Nagasaki in Japan during the World War II.

DO YOU KNOW?

Electron Volt (eV) is the unit used in nuclear physics to measure the energy of small particles. It is nothing but the energy of one electron when it is accelerated using an electric potential of one volt.

$1 eV = 1.602 \times 10^{-19}$ joule.

1 million electron volt = 1 MeV = 10^6 eV (mega electron volt)

The energy released in a nuclear fission process is about 200 MeV.

6.4 NUCLEAR FUSION

You have learnt that energy can be produced when a heavy nucleus is split up into two smaller nuclei. Similarly, energy can be produced when two lighter nuclei combine to form a heavier nucleus. This phenomenon is known as nuclear fusion.

BHE2TC

6.4.1 Definition

The process in which two lighter nuclei combine to form a heavier nucleus is termed as 'nuclear fusion'.

E.g.: ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + Q$ (Energy)

Here, ${}_1H^2$ represents an isotope of hydrogen known as 'deuterium'. The average energy released in each fusion reaction is about 3.84×10^{-12} J. Figure 6.5 represents this.

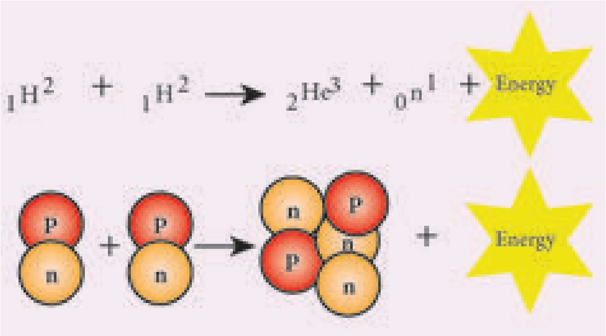

Figure 6.5 Nuclear fusion

The mass of the daughter nucleus formed during a nuclear reaction (fission and fusion) is lesser than the sum of the masses of the two parent nuclei. This difference in mass is called mass defect. This mass is converted into energy, according to the mass-energy equivalence. This concept of mass-energy equivalence was proposed by Einstein in 1905. It stated that mass can be converted into energy and vice versa. The relation between mass and energy proposed by Einstein is **$E = mc^2$** where c is the velocity of light in vacuum and is equal to 3×10^8 ms^{-1}.

> **DO YOU KNOW?** The nuclear bomb that was dropped in Hiroshima during World War II was called as 'Little boy'. It was a gun-type bomb which used a uranium core. The bomb, which was subsequently dropped over Nagasaki was called as 'Fat man'. It was an explosion type bomb, which used a plutonium core.

6.4.2 Conditions necessary for nuclear fusion

Earth's atmosphere contains a small trace of hydrogen. If nuclear fusion is a spontaneous process at normal temperature and pressure, then a number of fusion processes would happen in the atmosphere which may lead to explosions. But, we do not encounter any such explosions. Can you explain why?

The answer is that nuclear fusion can take place only under certain conditions.

Nuclear fusion is possible only at an extremly high temperature of the order of 10^7 to 10^9 K and a high pressure to push the hydrogen nuclei closer to fuse with each other. Hence, it is named as 'Thermonuclear reaction'.

> **DO YOU KNOW?** Nuclear fusion is the combination of two lighter nuclei. The charge of both nuclei is positive. According to electrostatic theory, when they come closer they tend to repel each other. This repulsive force will be overcome by the kinetic energy of the nuclei at higher temperature of the order of 10^7 to 10^9 K.

6.4.3 Stellar Energy

The stars like our Sun emit a large amount of energy in the form of light and heat. This energy is termed as the stellar energy. Where does this high energy come from? All stars contain a large amount of hydrogen. The surface temperature of the stars is very high which is sufficient to induce fusion of the hydrogen nuclei.

Fusion reaction that takes place in the cores of the Sun and other stars results in an enormous amount of energy, which is called as 'stellar energy. Thus, nuclear fusion or thermonuclear reaction is the source of light and heat energy in the Sun and other stars.

6.4.4 Hydrogen Bomb

Hydrogen bomb is based on the principle of nuclear fusion. A hydrogen bomb is always designed to have an inbuilt atom bomb which creates the high temperature and pressure required for fusion when it explodes. Then, fusion takes place in the hydrogen core and leads to the release of a very large amount of energy in an uncontrolled manner. The energy released in a hydrogen bomb (or fusion bomb) is much higher than that released in an atom bomb (or fission bomb).

Table 6.3 Features of Nuclear fission and nuclear fusion

S.No.	NUCLEAR FISSION	NUCLEAR FUSION
1	The process of breaking up (splitting) of a heavy nucleus into two smaller nuclei is called '**nuclear fission**'.	Nuclear fusion is the combination of two lighter nuclei to form a heavier nucleus.
2	Can be performed at room temperature.	Extremely high temperature and pressure is needed.
3	Alpha, beta and gamma radiations are emitted.	Alpha rays, positrons, and neutrinos are emitted.
4	Fission leads to emission of gamma radiation. This triggers the mutation in the human gene and causes genetic transform diseases.	Only light and heat energy is emitted.

DO YOU KNOW?

Sun fuses about 620 million metric tons of hydrogen each second and radiates about 3.8×10^{26} joule of energy per second. When this energy is radiated towards the Earth, it decreases in its intensity. When it reaches the Earth its value is about 1.4 kilo joule per unit area in unit time.

6.5 USES OF RADIOACTIVITY

Many radio isotopes can be obtained from radioactivity. These radio isotopes have found wide variety of applications in the fields of medicine, agriculture, industry and archeological research.

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6.5.1 Agriculture

The radio isotope of phosphorous (P-32) helps to increase the productivity of crops. The radiations from the radio isotopes can be used to kill the insects and parasites and prevent the wastage of agricultural products. Certain perishable cereals exposed to radiations remain fresh beyond their normal life, enhancing the storage time. Very small doses of radiation prevent sprouting and spoilage of onions, potatoes and gram.

6.5.2 Medicine

Medical applications of radio isotopes can be divided into two parts:

i) Diagnosis ii) Therapy

Radio isotopes are used as tracers to diagnose the nature of circulatory disorders of blood, defects of bone metabolism, to locate tumors, etc. Some of the radio isotopes which are used as tracers are: hydrogen, carbon, nitrogen, sulphur, etc.

- Radio sodium (Na^{24}) is used for the effective functioning of heart.
- Radio – Iodine (I^{131}) is used to cure goiter.
- Radio-iron is (Fe^{59}) is used to diagnose anaemia and also to provide treatment for the same.
- Radio phosphorous (P^{32}) is used in the treatment of skin diseases.

- Radio cobalt (Co^{60}) and radio-gold (Au^{198}) are used in the treatment of skin cancer.
- Radiations are used to sterilize the surgical devices as they can kill the germs and microbes.

6.5.3 Industries

In industries, radioactive isotopes are used as tracers to detect any manufacturing defects such as cracks and leaks. Packaging faults can also be identified through radio activity. Gauges, which have radioactive sources are used in many industries to check the level of gases, liquids and solids.

- An isotope of californium (Cf^{252}) is used in the airlines to detect the explosives in the luggage.
- An isotope of Americium (Am^{241}) is used in many industries as a smoke detector.

6.5.4. Archeological research

Using the technique of radio carbon dating, the age of the Earth, fossils, old paintings and monuments can be determined. In radio carbon dating, the existing amount of radio carbon is determined and this gives an estimate about the age of these things.

6.6 SAFETY MEASURES

In day to day life, you do receive some natural radiation from the Sun. The radioactive elements present in the soil and rocks, the house hold appliances like television, microwave ovens, cell phones and the X-rays used in hospitals. These radiations do not produce any severe effects as they are very low in intensity.

The second source of radiation exposure is man-made. These are due to nuclear reactors and during the testing of the nuclear devices in the atmosphere or in the ground.

Improper and careless handling of radioactive materials release harmful radiations in our environment. These radiations are very harmful to the human body. A person who is exposed to radiations very closely or for a longer duration, is at a greater health risk and can be affected genetically.

DO YOU KNOW?

How old is our mother Earth? Any guess?? It is nearly 4.54×10^9 years (around 45 Crore 40 lakh years). Wow!!

6.6.1 Permitted range

The International Commission on Radiological Protection (ICRP) has recommended certain maximum permissible exposure limits to radiation that is believed to be safe without producing any appreciable injury to a person. Safe limit of overall exposure to radiation is given as 20 milli sievert per year. In terms of roentgen, the safe limit of receiving the radiation is about 100 mR per week. If the exposure is 100 R, it may cause fatal diseases like leukemia (death of red blood corpuscle in the blood) or cancer. When the body is exposed to about 600 R, it leads to death.

DO YOU KNOW?

*Dosimeter is a device used to detect the levels of exposure to an ionizing radiation. It is frequently used in the environments where exposure to radiation may occur such as nuclear power plants and medical imaging facilities. Pocket dosimeter is used to provide the wearer with an immediate reading of his/her exposure to X-rays and γ rays.

6.6.2 Preventive measures

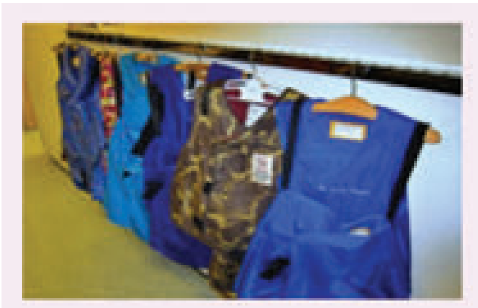

Figure. 6.6 Lead coated aprons model.

- Radioactive materials should be kept in a thick walled lead container.
- Lead coated aprons and lead gloves should be used while working with hazardous radioactive materials.
- You should avoid eating while handling radioactive materials.
- The radioactive materials should be handled only by tongs or by a remote control device.
- Dosimeters should be worn by the users to check the level of radiation.

6.7 NUCLEAR REACTOR

A Nuclear reactor is a device in which the nuclear fission reaction takes place in a self-sustained and controlled manner to produce electricity. The first nuclear reactor was built in 1942 at Chicago, USA.

6.7.1 Types of nuclear reactors

Breeder reactor, fast breeder reactor, pressurized water reactor, pressurized heavy water reactor, boiling water reactor, water-cooled reactor, gas-cooled reactor, fusion reactor and thermal reactor are some types of nuclear reactors, which are used in different places world-wide.

6.7.2 Components of a nuclear reactors

The essential components of a nuclear reactor are (i) fuel, (ii) moderator, (iii) control rod, (iv) coolant and (v) protection wall.

i. **Fuel:** A fissile material is used as the fuel. The commonly used fuel material is uranium.

ii. **Moderator:** A moderator is used to slow down the high energy neutrons to provide slow neutrons. Graphite and heavy water are the commonly used moderators.

iii. **Control rod:** Control rods are used to control the number of neutrons in order to have sustained chain reaction. Mostly boron or cadmium rods are used as control rods. They absorb the neutrons.

iv. **Coolant:** A coolant is used to remove the heat produced in the reactor core, to produce steam. This steam is used to run a turbine in order to produce electricity. Water, air and helium are some of the coolants.

v. **Protection wall:** A thick concrete lead wall is built around the nuclear reactor in order to prevent the harmful radiations from escaping into the environment.

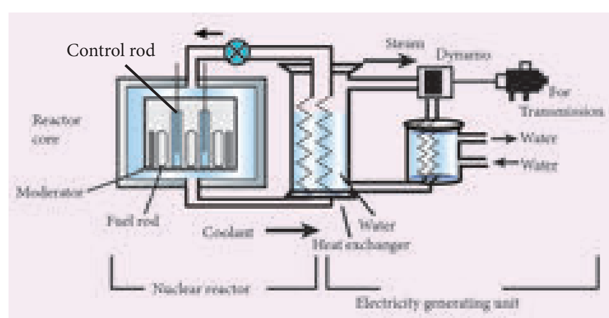

Figure 6.7 Schematic diagram of a nuclear reactor

6.7.3 Uses of a nuclear reactor

- Nuclear reactors are widely used in power generation.
- They are also used to produce radio isotopes, which are used in a variety of applications.
- Some reactors help us to do research in the field of nuclear physics.
- Breeder reactors are used to convert non-fissionable materials into fissionable materials.

6.7.4 Nuclear power plants in India

Indian Atomic Energy Commission (AEC) was established in August 1948 by the

Department of Indian Scientific Research committee at Bombay (now Mumbai) in Maharashtra. It is the nodal agency for all the research done in the field of atomic energy. Dr. Homi Jahangir Bhaba was the first chairman of Indian Atomic Energy Commission. Now, it is known as Bhaba Atomic Research Centre (BARC).

Nuclear power is the fifth largest source of power in India. Tarapur Atomic Power Station is India's first nuclear power station. Now, there are a total of seven power stations, one each in Maharashtra, Rajasthan, Gujarat, Uttar Pradesh and two in Tamilnadu. In Tamilnadu, we have nuclear power stations in Kalpakkam and Kudankulam. Apsara was the first nuclear reactor built in India and Asia. Now, there are 22 nuclear reactors which are operating in India. Some other operating reactors are

- Cirus
- Dhuruva
- Purnima

Solved problem 6.1

Identify A, B, C, and D from the following nuclear reactions.

(i) $_{13}Al^{27} + A$ --------> $_{15}P^{30} + B$

(ii) $_{12}Mg^{24} + B$ --------> $_{11}Na^{24} + C$

(iii) $_{92}U^{238} + B$ --------> $_{93}Np^{239} + D$

Solution:

(i) $_{13}Al^{27} + {}_{2}He^{4}$ --------> $_{15}P^{30} + {}_{0}n^{1}$

(ii) $_{12}Mg^{24} + {}_{0}n^{1}$ --------> $_{11}Na^{24} + {}_{1}H^{1}$

(iii) $_{92}U^{238} + {}_{0}n^{1}$ --------> $_{93}Np^{239} + {}_{-1}e^{0}$

A is alpha particle, B is neutron, C is proton, and D is electron.

Solved problem 6.2

A radon specimen emits radiation of 3.7×10^{3} GBq per second. Convert this disintegration in terms of curie. (one curie = 3.7×10^{10} disintegration per second)

$$1 \text{ Bq} = \text{one disintegration per second}$$

$$\text{one curie} = 3.7 \times 10^{10} \text{ Bq}$$

$$1 \text{ Bq} = \frac{1}{3.7 \times 10^{10}} \text{ curie}$$

$$\therefore 3.7 \times 10^{3} \text{ G Bq} = 3.7 \times 10^{3} \times 10^{9} \times \frac{1}{3.7 \times 10^{10}}$$

$$= 100 \text{ curie}$$

Solved problem 6.3

$_{92}U^{235}$ experiences one **α** - decay and one **β** - decay. Find number of neutrons in the final daughter nucleus that is formed.

Solution:

Let X and Y be the resulting nucleus after the emission of the alpha and beta particles respectively.

$$_{92}U^{235} \xrightarrow{\alpha \text{ decay}} {}_{90}X^{231} + {}_{2}He^{4}$$

$$_{90}X^{231} \xrightarrow{\beta \text{ decay}} {}_{91}Y^{231} + {}_{-1}e^{0}$$

Number of neutrons = Mass number – Atomic number

= 231 – 91 = 140

Solved problem 6.4

Calculate the amount of energy released when a radioactive substance undergoes fusion and results in a mass defect of 2 kg.

Solution:

Mass defect in the reaction (m) = 2 kg

Velocity of light (c) = 3×10^{8} m s^{-1}

By Einstein's equation,

Energy released $E = mc^2$

So $E = 2 \times (3 \times 10^{8})^2$

$= 1.8 \times 10^{17}$ J

Points to Remember

- This phenomenon of spontaneous emission of radiation from certain elements on its own is called 'natural radioactivity'.
- **Curie** is defined as the quantity of a radioactive substance, which undergoes 3.7×10^{10} disintegrations in one second. This is actually close to the activity of 1 g of radium-226.

- **Rutherford (Rd)** is defined as the quantity of a radioactive substance which produces 10^6 disintegrations in one second.
  1 Rd = 10^6 disintegrations per second.

CCFRVZ

- The SI unit of radioactivity is **becquerel.** It is defined as the quantity of one disintegration per second.
- Helium nucleus ($_2He^4$) consisting of two protons and two neutrons is known as alpha particle.
- Beta particles are electrons ($_{-1}e^0$), which are the basic elementary particles present in all atoms.
- Gamma rays are electromagnetic waves consisting of photons.
- A nuclear reaction in which an unstable parent nucleus emits an alpha particle and forms a stable daughter nucleus is called as 'alpha decay'.
- A nuclear reaction in which an unstable parent nucleus emits a beta particle and forms a stable daughter nucleus is called as 'beta decay'.
- The process of breaking (splitting) up of a heavier nucleus into two smaller nuclei with the release of a large amount of energy is called **'nuclear fission'.**
- The energy released in a nuclear fission process is about 200 MeV.
- There are some radioactive elements which can be converted into a fissionable material. They are called as '**fertile materials**'. e.g. Uranium-238, Thorium-232, Plutonium-240.
- Controlled chain reaction is used in a nuclear reactor to produce energy in a sustained and controlled manner.
- The process in which two lighter nuclei combine to form a heavier nucleus is termed as **'nuclear fusion'.**
- Nuclear fusion or thermonuclear reaction is the source of light and heat energy in the Sun and other stars.
- The safe limit of receiving the radiation is about 100 mR per week.

TEXTBOOK EVALUATION

CRPQVN

I. Choose the correct answer

1. Man-made radioactivity is also known as ____________

   a. Induced radioactivity

   b. Spontaneous radioactivity

   c. Artificial radioactivity

   d. a & c

2. Unit of radioactivity is ____________

   a. roentgen   b. curie

   c. becquerel   d. all the above

3. Artificial radioactivity was discovered by ____________

   a. Bequerel   b. Irene Curie

   c. Roentgen   d. Neils Bohr

4. In which of the following, no change in mass number of the daughter nuclei takes place

   i) α decay   ii) β decay

   iii) γ decay   iv) neutron decay

   a. (i) is correct

   b (ii) and (iii) are correct

   c (i) & (iv) are correct

   d (ii) & (iv) are correct

5. ____________ isotope is used for the treatment of cancer.

a. Radio Iodine b. Radio Cobalt

c. Radio Carbon d. Radio Nickel

6. Gamma radiations are dangerous because

a. it affects eyes & bones

b. it affects tissues

c. it produces genetic disorder

d. it produces enormous amount of heat

7. ____________ aprons are used to protect us from gamma radiations

a. Lead oxide b. Iron

c. Lead d. Aluminium

8. Which of the following statements is/are correct?

i. α particles are photons

ii. Penetrating power of γ radiation is very low

iii. Ionization power is maximum for α rays

iv. Penetrating power of γ radiation is very high

a. (i) & (ii) are correct

b. (ii) & (iii) are correct

c. (iv) only correct

d. (iii) & (iv) are correct

9. Proton - Proton chain reaction is an example of ______________

a. Nuclear fission b. α - decay

c. Nuclear fusion d. β - decay

10. In the nuclear reaction ${}_{6}X^{12} \xrightarrow{\alpha\ decay} {}_{Z}Y^{A}$, the value of A & Z.

a. 8, 6 b. 8, 4

c. 4, 8 d. cannot be determined with the given data

11. Kamini reactor is located at __________

a. Kalpakkam b. Koodankulam

c. Mumbai d. Rajasthan

12. Which of the following is/are correct?

i. Chain reaction takes place in a nuclear reactor and an atomic bomb.

ii. The chain reaction in a nuclear reactor is controlled

iii. The chain reaction in a nuclear reactor is not controlled

iv. No chain reaction takes place in an atom bomb

a. (i) only correct b. (i) & (ii) are correct

c. (iv) only correct d. (iii) & (iv) are correct

II. Fill in the blanks

1. One roentgen is equal to __________ disintegrations per second
2. Positron is an__________.
3. Anemia can be cured by __________ isotope
4. Abbreviation of ICRP______
5. _________is used to measure exposure rate of radiation in humans.
6. ________ has the greatest penetration power.
7. ${}_{Z}Y^{A} \rightarrow {}_{Z+1}Y^{A} + X$; Then, X is __________
8. ${}_{Z}X^{A} \rightarrow {}_{Z}Y^{A}$ This reaction is possible in ____________ decay.
9. The average energy released in each fusion reaction is about ________ J.
10. Nuclear fusion is possible only at an extremely high temperature of the order of ____ K.
11. The radio isotope of ________ helps to increase the productivity of crops.
12. If the radiation exposure is 100 R, it may cause ____________.

III State whether the following statements are true or false: If false, correct the statement

1. Plutonium -239 is a fissionable material.
2. Elements having atomic number greater than 83 can undergo nuclear fusion.
3. Nuclear fusion is more dangerous than nuclear fission.
4. Natural uranium U-238 is the core fuel used in a nuclear reactor.
5. If a moderator is not present, then a nuclear reactor will behave as an atom bomb.
6. During one nuclear fission on an average, 2 to 3 neutrons are produced.
7. Einstein's theory of mass energy equivalence is used in nuclear fission and fusion.

IV. Match the following

Match: I

a. BARC	Kalpakkam
b. India's first atomic power station	Apsara
c. IGCAR	Mumbai
d. First nuclear reactor in India	Tarapur

Match: II

a. Fuel	lead
b. Moderator	heavy water
c. Coolant	cadmium rods
d. Shield	uranium

Match: III

a. Soddy Fajan	Natural radioactivity
b. Irene Curie	Displacement law
c. Henry Bequerel	Mass energy equivalence
d. Albert Einstein	Artificial Radioactivity

Match: IV

a. Uncontrolled fission reaction	Hydrogen Bomb
b. Fertile material	Nuclear Reactor
c. Controlled fission reaction	Breeder reactor
d. Fusion reaction	Atom bomb

Match: V

a. Co - 60	Age of fossil
b. I - 131	Function of Heart
c. Na - 24	Leukemia
d. C - 14	Thyroid disease

V. Arrange the following in the correct sequence:

1. **Arrange in descending order, on the basis of their penetration power**

   Alpha rays, beta rays, gamma rays, cosmic rays

2. **Arrange the following in the chronological order of discovery**

   Nuclear reactor, radioactivity, artificial radioactivity, discovery of radium.

VI. Use the analogy to fill in the blank

1. Spontaneous process : Natural Radioactivity, Induced process : ________
2. Nuclear Fusion : Extreme temperature, Nuclear Fission : ____________
3. Increasing crops : Radio phosphorous, Effective functioning of heart : _____
4. Deflected by electric field : α ray, Null Deflection : ___________

VII. Numerical problems:

1. $_{88}\mathbf{Ra}^{226}$ experiences three **α** - decay. Find the number of neutrons in the daughter element.

2. A cobalt specimen emits induced radiation of 75.6 millicurie per second. Convert this disintegration in to becquerel (one curie = 3.7×10^{10} Bq)

VIII. Assertion and reason type questions:

Mark the correct choice as

(a) If both the assertion and the reason are true and the reason is the correct explanation of the assertion.

(b) If both the assertion and the reason are true, but the reason is not the correct explanation of the assertion.

(c) Assertion is true, but the reason is false.

(d) Assertion is false, but the reason is true.

1. **Assertion:** A neutron impinging on U^{235}, splits it to produce Barium and Krypton.
   Reason: U - 235 is a fissile material.
2. **Assertion**: In a β - decay, the neutron number decreases by one.
   Reason: In β - decay atomic number increases by one.
3. **Assertion**: Extreme temperature is necessary to execute nuclear fusion.
   Reason: In a nuclear fusion, the nuclei of the reactants combine releasing high energy.

4. **Assertion**: Control rods are known as 'neutron seeking rods'
   Reason: Control rods are used to perform sustained nuclear fission reaction

IX. Answer in one or two word (VSA)

1. Who discovered natural radioactivity?
2. Which radioactive material is present in the ore of pitchblende?
3. Write any two elements which are used for inducing radioactivity?
4. Write the name of the electromagnetic radiation which is emitted during a natural radioactivity.
5. If A is a radioactive element which emits an α - particle and produces $_{104}Rf^{259}$. Write the atomic number and mass number of the element A.
6. What is the average energy released from a single fission process?
7. Which hazardous radiation is the cause for the genetic disease?
8. What is the amount of radiation that may cause death of a person when exposed to it?
9. When and where was the first nuclear reactor built?
10. Give the SI unit of radioactivity.
11. Which material protects us from radiation?

X. Answer the following questions in few sentences.

1. Write any three features of natural and artificial radioactivity.
2. Define critical mass.
3. Define one roentgen.
4. State Soddy and Fajan's displacement law.
5. Give the function of control rods in a nuclear reactor.
6. In Japan, some of the new born children are having congenital diseases. Why?
7. Mr. Ramu is working as an X - ray technician in a hospital. But, he does not wear the lead aprons. What suggestion will you give to Mr. Ramu?
8. What is stellar energy?
9. Give any two uses of radio isotopes in the field of agriculture?

XI. Answer the following questions in detail.

1. Explain the process of controlled and uncontrolled chain reactions.
2. Compare the properties of alpha, beta and gamma radiations.
3. What is a nuclear reactor? Explain its essential parts with their functions.

XII. HOT Questions:

1. Mass number of a radioactive element is 232 and its atomic number is 90. When this element undergoes certain nuclear reactions, it transforms into an isotope of lead with a mass number 208 and an atomic number 82. Determine the number of alpha and beta decay that can occur.
2. 'X – rays should not be taken often'. Give the reason.
3. Cell phone towers should be placed far away from the residential area – why?

REFERENCE BOOKS

1. Physics concepts and connections – by Art Hobson Edition: Pearson education
2. Modern Physics – by Dr. R Murugesan & Er. Kiruthiga Sivaprasath – S. Chand publications

INTERNET RESOURCES

1. https://physics.columbia.edu/research/nuclear-physics
2. http://www.newworldencyclopedia.org/entry/Nuclear_physics

UNIT 3

LAWS OF MOTION

"In the beginning there was a mechanics" – Von Laue

Learning Objectives

In this unit, the student is exposed to

- Newton's laws
- logical connection between laws of Newton
- free body diagram and related problems
- law of conservation of momentum
- role of frictional forces
- centripetal and centrifugal forces
- origin of centrifugal force

2XMV5I

3.1 INTRODUCTION

Each and every object in the universe interacts with every other object. The cool breeze interacts with the tree. The tree interacts with the Earth. In fact, all species interact with nature. But, what is the difference between a human's interaction with nature and that of an animal's. Human's interaction has one extra quality. We not only interact with nature but also try to understand and explain natural phenomena scientifically.

In the history of mankind, the most curiosity driven scientific question asked was about motion of objects–'How things move?' and 'Why things move?' Surprisingly, these simple questions have paved the way for development from early civilization to the modern technological era of the 21st century.

Objects move because something pushes or pulls them. For example, if a book is at rest, it will not move unless a force is applied on it. In other words, to move an object a force must be applied on it. About 2500 years ago, the famous philosopher, Aristotle, said that '*Force causes motion*'. This statement is based on common sense. But any scientific answer cannot be based on common sense. It must be endorsed with quantitative experimental proof.

In the 15th century, Galileo challenged Aristotle's idea by doing a series of experiments. He said force is not required to maintain motion.

Galileo demonstrated his own idea using the following simple experiment. When a ball rolls from the top of an inclined plane to its bottom, after reaching the ground it moves some distance and continues to move on to another inclined plane of same angle of inclination as shown in the Figure 3.1(a). By increasing the smoothness of both the inclined planes, the ball reach almost the same height(h) from where it was released (L1) in the second plane (L2) (Figure 3.1(b)). The motion of the ball is then observed by varying the angle of inclination of the second plane keeping the same smoothness. If the angle of inclination is reduced, the ball travels longer distance in the second plane to reach the same height (Figure 3.1 (c)). When the angle of inclination is made zero, the ball moves forever in the horizontal direction (Figure 3.1(d)). If the Aristotelian idea were true, the ball would not have moved in the second plane even if its smoothness is made maximum since no

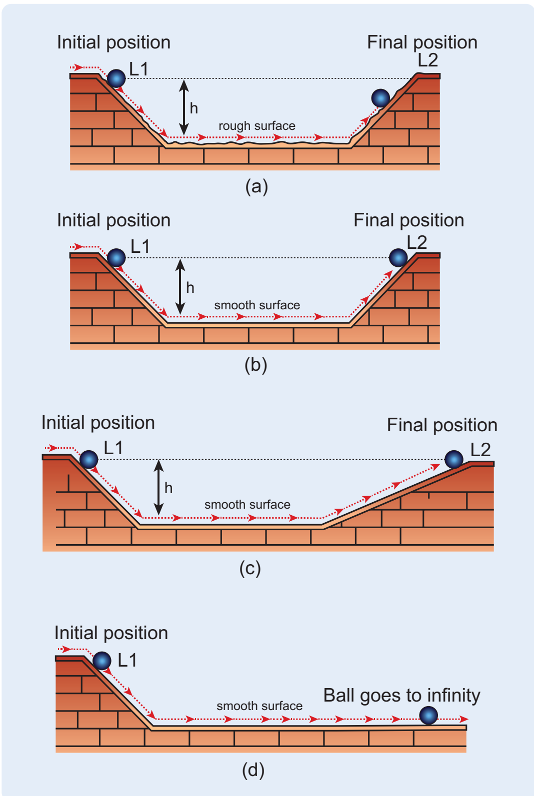

Figure 3.1 Galileo's experiment with the second plane (a) at same inclination angle as the first (b) with increased smoothness (c) with reduced angle of inclination (d) with zero angle of inclination

force acted on it in the horizontal direction. From this simple experiment, Galileo proved that force is not required to maintain motion. An object can be in motion even without a force acting on it.

In essence, Aristotle coupled the motion with force while Galileo decoupled the motion and force.

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3.2 NEWTON'S LAWS

Newton analysed the views of Galileo, and other scientist like Kepler and Copernicus on motion and provided much deeper insights in the form of three laws.

3.2.1 Newton's First Law

Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it.

This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state. Depending on the circumstances, there can be three types of inertia.

1. **Inertia of rest:** When a stationary bus starts to move, the passengers experience a sudden backward push. Due to inertia, the body (of a passenger) will try to

Figure 3.2 Passengers experience a backward push due to inertia of rest

continue in the state of rest, while the bus moves forward. This appears as a backward push as shown in Figure 3.2. *The inability of an object to change its state of rest is called inertia of rest.*

2. **Inertia of motion:** When the bus is in motion, and if the brake is applied suddenly, passengers move forward and hit against the front seat. In this case, the bus comes to a stop, while the body (of a passenger) continues to move forward due to the property of inertia as shown in Figure 3.3. *The inability of an object to change its state of uniform speed (constant speed) on its own is called inertia of motion.*

3. **Inertia of direction:** When a stone attached to a string is in whirling motion, and if the string is cut suddenly, the stone will not continue to move in circular motion but moves tangential to the circle as illustrated in Figure 3.4. This is because the body cannot change its direction of motion without any force acting on it. *The inability of an object to change its direction of motion on its own is called inertia of direction.*

Figure 3.3 Passengers experience a forward push due to inertia of motion

When we say that an object is at rest or in motion with constant velocity, it has a meaning only if it is specified with respect to some reference frames. In physics, any motion has to be stated with respect to a reference frame. It is to be noted that Newton's first law is valid only in certain special reference frames called inertial frames. In fact, Newton's first law defines an inertial frame.

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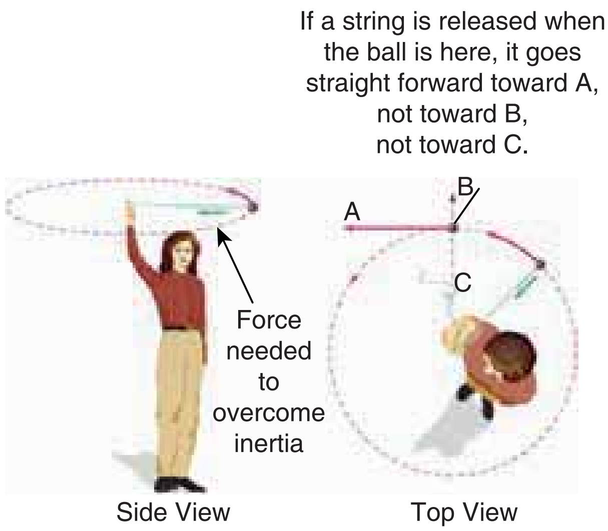


Figure 3.4 A stone moves tangential to circle due to inertia of direction

Inertial Frames

If an object is free from all forces, then it moves with constant velocity or remains at rest when seen from inertial frames. Thus, there exists some special set of frames in which if an object experiences no force it moves with constant velocity or remains at rest. But how do we know whether an object is experiencing a force or not? All the objects in the Earth experience Earth's gravitational force. In the ideal case, if an object is in deep space (very far away from any other object), then Newton's first law will be certainly valid. Such deep space can be treated as an inertial frame. But practically it is not possible to reach such deep space and verify Newton's first law.

For all practical purposes, we can treat Earth as an inertial frame because an object on the table in the laboratory appears to be at rest always. This object never picks up acceleration in the horizontal direction since no force acts on it in the horizontal direction. So the laboratory can be taken as an inertial frame for all physics experiments and calculations. For making these conclusions, we analyse only the horizontal motion of the object as there is no horizontal force that acts on it. We should not analyse the motion in vertical direction as the two forces (gravitational force in the downward direction and normal force in upward direction) that act on it makes the net force is zero in vertical direction. Newton's first law deals with the motion of objects in the absence of any force and not the motion under zero net force. Suppose a train is moving with constant velocity with respect to an inertial frame, then an object at rest in the inertial frame (outside the train) appears to move with constant velocity with respect to the train (viewed from within the train). So the train can be treated as an inertial frame. All inertial frames are moving with constant velocity relative to each other. If an object appears to be at rest in one inertial frame, it may appear to move with constant velocity with respect to another inertial frame. For example, in Figure 3.5, the car is moving with uniform velocity v with respect to a person standing (at rest) on the ground. As the car is moving with constant velocity with respect to ground to the person is at rest on the ground, both frames (with respect to the car and to the ground) are inertial frames.

Figure 3.5 The person and vehicle are inertial frames

Suppose an object remains at rest on a smooth table kept inside the train, and if the train suddenly accelerates (which we may not sense), the object appears to accelerate backwards even without any force acting on it. It is a clear violation of Newton's first law as the object gets accelerated without being acted upon by a force. It implies that the train is not an inertial frame when it is accelerated. For example, Figure 3.6 shows that car 2 is a non-inertial frame since it moves with acceleration $\vec{a}$ with respect to the ground.

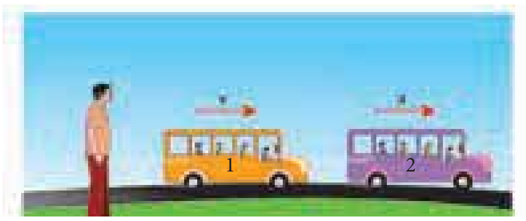


Figure 3.6 Car 2 is a non-inertial frame

These kinds of accelerated frames are called non-inertial frames. A rotating frame is also a non inertial frame since rotation requires acceleration. In this sense, Earth is not really an inertial frame since it has self-rotation and orbital motion. But these rotational effects of Earth can be ignored for the motion involved in our day-to-day life. For example, when an object is thrown, or the time period of a simple pendulum is measured in the physics laboratory, the Earth's self-rotation has very negligible effect on it. In this sense, Earth can be treated as an inertial frame. But at the same time, to analyse the motion of satellites and wind patterns around the Earth, we cannot treat Earth as an inertial frame since its self-rotation has a strong influence on wind patterns and satellite motion.

3.2.2 Newton's Second Law

This law states that

The force acting on an object is equal to the rate of change of its momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{3.1}$$

In simple words, whenever the momentum of the body changes, there must be a force acting on it. The momentum of the object is defined as $\vec{p} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

$$\vec{F} = m\vec{a}. \tag{3.2}$$

The above equation conveys the fact that if there is an acceleration $\vec{a}$ on the body, then there must be a force acting on it. This implies that if there is a change in velocity, then there must be a force acting on the body. The force and acceleration are always in the same direction. Newton's second law was a paradigm shift from Aristotle's idea of motion. According to Newton, the force need not cause the motion but only a change in motion. It is to be noted that *Newton's second law is valid only in inertial frames*. In non-inertial frames Newton's second law cannot be used in this form. It requires some modification.

In the SI system of units, the unit of force is measured in newtons and it is denoted by symbol 'N'.

One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 m s^{-2} in the direction of the force.

Aristotle vs. Newton's approach on sliding object

Newton's second law gives the correct explanation for the experiment on the inclined plane that was discussed in section 3.1. In normal cases, where friction is not negligible, once the object reaches the bottom of the inclined plane (Figure 3.1), it travels some distance and stops. Note that it stops because there is a frictional force acting in the direction opposite to its velocity. It is this frictional force that reduces the velocity of the object to zero and brings it to rest. As per Aristotle's idea, as soon as the body reaches the bottom of the plane, it can travel only a small distance and stops because there is no force acting on the object. Essentially, he did not consider the frictional force acting on the object.

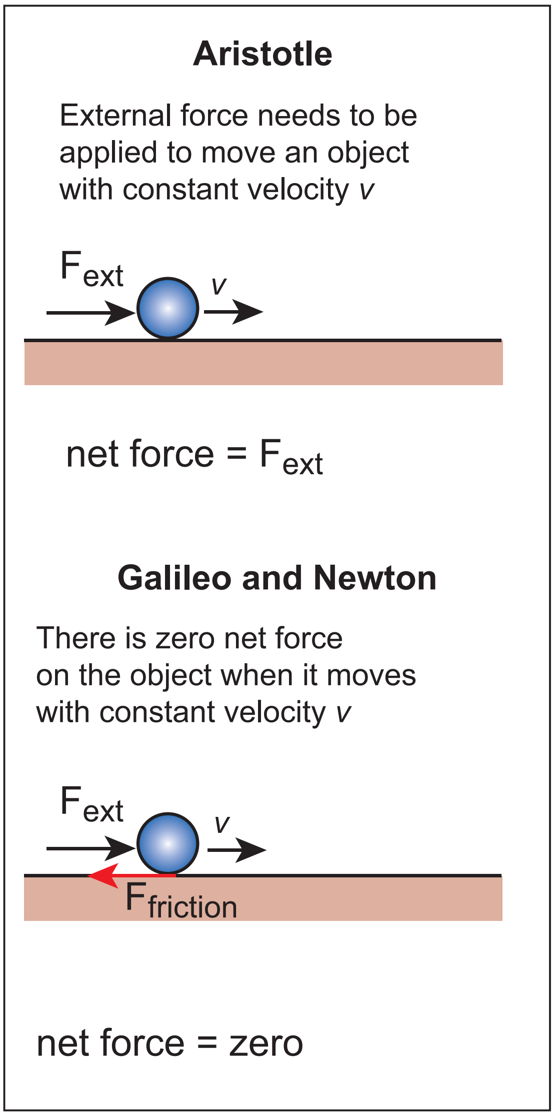


Figure 3.7 Aristotle, Galileo and Newton's approach

3.2.3 Newton's Third Law

Consider Figure 3.8(a) whenever an object 1 exerts a force on the object 2 ($\vec{F}_{21}$), then object 2 must also exert equal and opposite force on the object 1 ($\vec{F}_{12}$). These forces must lie along the line joining the two objects.

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature. *Newton's third law states that for every action there is an equal and opposite reaction*. Here, action and reaction pair of forces do not act on the same body but on two different bodies. Any one of the forces can be called as an action force and the other the reaction force. Newton's third law is valid in both inertial and non-inertial frames.

These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

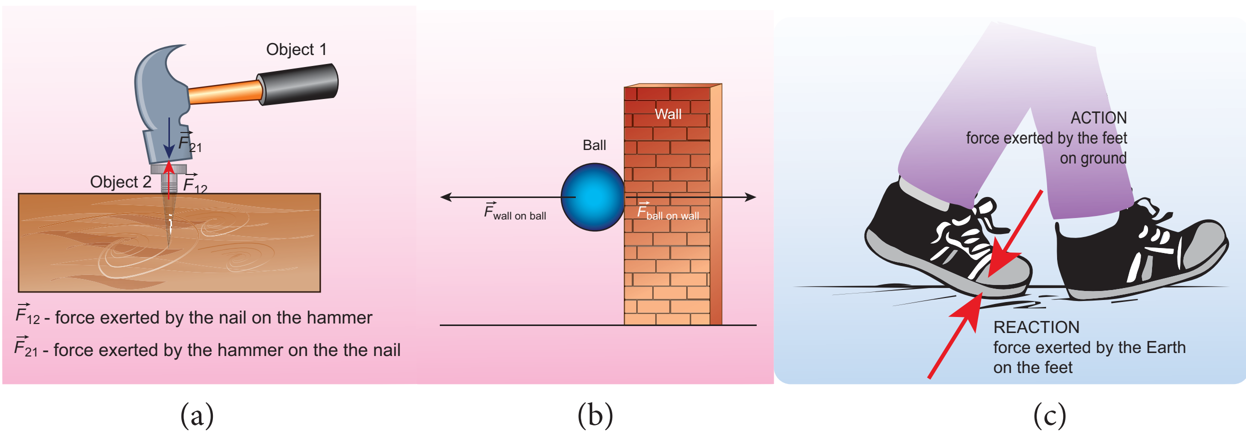


Figure 3.8 Demonstration of Newton's third law (a) Hammer and the nail (b) Ball bouncing off the wall (c) Walking on the floor with friction

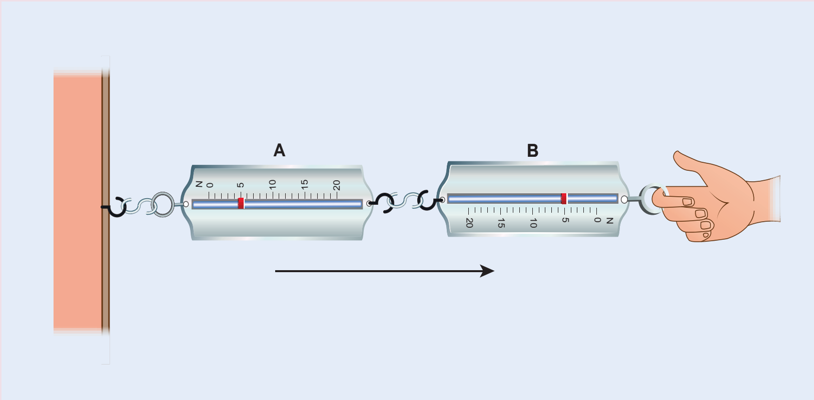
ACTIVITY

Verification of Newton's third law

Attach two spring balances as shown in the figure. Fix one end with rigid support and leave the other end free, which can be pulled with the hand.

Pull one end with some force and note the reading on both the balances.

Repeat the exercise a number of times.



Note The reading in the spring balance A is due to the force given by spring balance B. The reading in the spring balance B is due to the reaction force given by spring balance A. Note that according to Newton's third law, both readings (force) are equal.

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3.2.4 Discussion on Newton's Laws

1. Newton's laws are vector laws. The equation $\vec{F} = m\vec{a}$ is a vector equation and essentially it is equal to three scalar equations. In Cartesian coordinates, this equation can be written as $F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = ma_x\hat{i} + ma_y\hat{j} + ma_z\hat{k}$.

   By comparing both sides, the three scalar equations are

   $F_x = ma_x$ The acceleration along the x direction depends only on the component of force acting along the x-direction.

   $F_y = ma_y$ The acceleration along the y direction depends only on the component of force acting along the y-direction.

   $F_z = ma_z$ The acceleration along the z direction depends only on the component of force acting along the z-direction.

   From the above equations, we can infer that the force acting along y direction cannot alter the acceleration along x direction. In the same way, F_z cannot affect a_y and a_x. This understanding is essential for solving problems.

2. The acceleration experienced by the body at time t depends on the force which acts on the body at that instant of time. It does not depend on the force which acted on the body before the time t. This can be expressed as

$$\vec{F}(t) = m\vec{a}(t)$$

Acceleration of the object does not depend on the previous history of the force. For example, when a spin bowler or a fast bowler throws the ball to the batsman, once the ball leaves the hand of the bowler, it experiences only gravitational force and air frictional force. The acceleration of the ball is independent of how the ball was bowled (with a lower or a higher speed).

3. In general, the direction of a force may be different from the direction of motion. Though in some cases, the object may move in the same direction as the direction of the force, it is not always true. A few examples are given below.

Case 1: Force and motion in the same direction

When an apple falls towards the Earth, the direction of motion (direction of velocity) of the apple and that of force are in the same downward direction as shown in the Figure 3.9 (a).

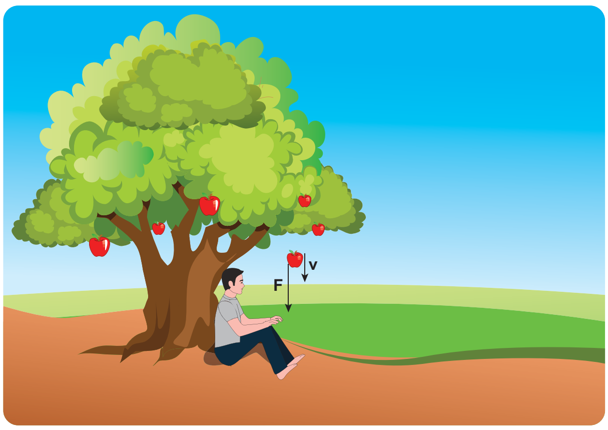


Figure 3.9 (a) Force and motion in the same direction

Case 2: Force and motion not in the same direction

The Moon experiences a force towards the Earth. But it actually moves in elliptical orbit. In this case, the direction of the force is different from the direction of motion as shown in Figure 3.9 (b).

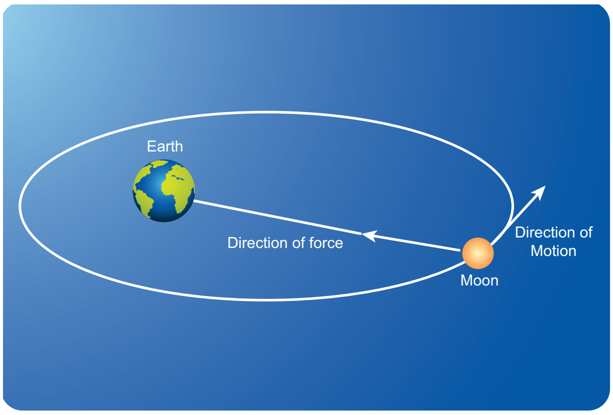


Figure 3.9 (b) Moon orbiting in elliptical orbit around the Earth

Case 3: Force and motion in opposite direction

If an object is thrown vertically upward, the direction of motion is upward, but gravitational force is downward as shown in the Figure 3.9 (c).

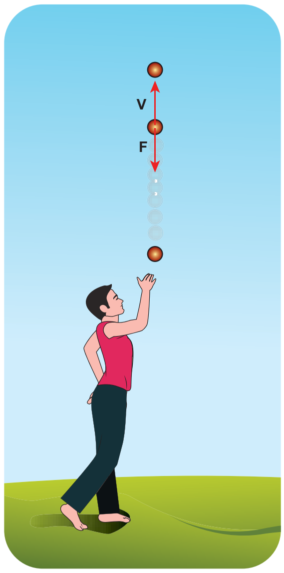


Figure 3.9 (c) Force and direction of motion are in opposite directions

Case 4: Zero net force, but there is motion

When a raindrop gets detached from the cloud it experiences both downward gravitational force and upward air drag force. As it descends towards the Earth, the upward air drag force increases and after a certain time, the upward air drag force cancels the downward gravity. From then on the raindrop moves at constant velocity till it touches the surface of the Earth. Hence the raindrop comes with zero net force, therefore with zero acceleration but with non-zero terminal velocity. It is shown in the Figure 3.9 (d).

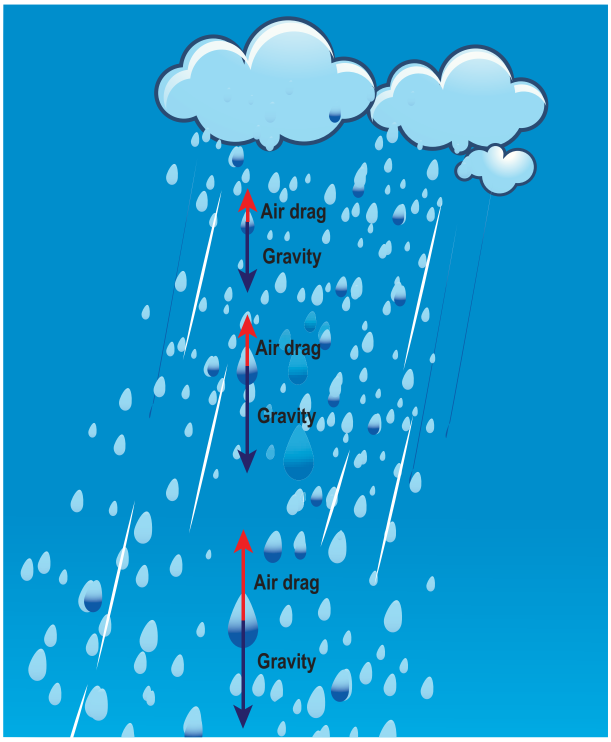


Figure 3.9 (d) Zero net force and non zero velocity

4. If multiple forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots . \vec{F}_n$ act on the same body, then the total force $(\vec{F}_{net})$ is equivalent to the vectorial sum of the individual forces. Their net force provides the acceleration.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

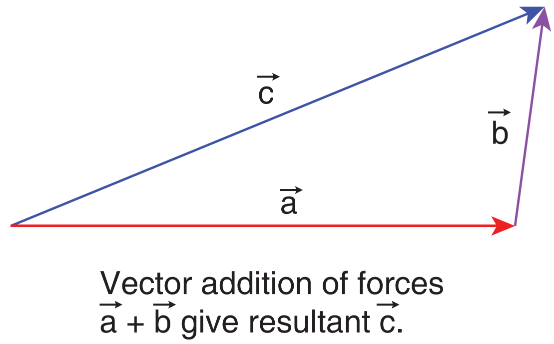


Figure 3.10 Vector addition of forces

Newton's second law for this case is

$$\vec{F}_{net} = m\vec{a}$$

In this case the direction of acceleration is in the direction of net force.

Example

Bow and arrow

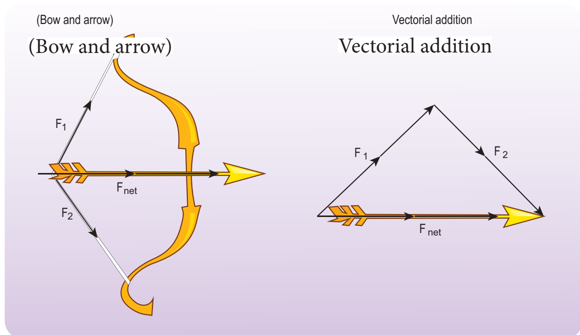


Figure 3.11 Bow and arrow – Net force is on the arrow

5. Newton's second law can also be written in the following form.

Since the acceleration is the second derivative of position vector of the body $\left(\vec{a} = \frac{d^2\vec{r}}{dt^2}\right)$, the force on the body is

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}.$$

From this expression, we can infer that Newton's second law is basically a second order ordinary differential equation and whenever the second derivative of position vector is not zero, there must be a force acting on the body.

6. If no force acts on the body then Newton's second law, $m\frac{d\vec{v}}{dt} = 0$.

   It implies that $\vec{v} = constant$. It is essentially Newton's first law. It implies that the second law is consistent with the first law. However, it should not be thought of as the reduction of second law to the first when no force acts on the object. Newton's first and second laws are independent laws. They can internally be consistent with each other but cannot be derived from each other.

7. Newton's second law is cause and effect relation. Force is the cause and acceleration is the effect. Conventionally, the effect should be written on the left and cause on the right hand side of the equation. So the correct way of writing Newton's second law is $m\vec{a} = \vec{F}$ or $\frac{d\vec{p}}{dt} = \vec{F}$

3.3 APPLICATION OF NEWTON'S LAWS

3.3.1 Free Body Diagram

Free body diagram is a simple tool to analyse the motion of the object using Newton's laws.

The following systematic steps are followed for developing the free body diagram:

1. Identify the forces acting on the object.
2. Represent the object as a point.
3. Draw the vectors representing the forces acting on the object.

When we draw the free body diagram for an object or a system, the forces exerted by the object should not be included in the free body diagram.

EXAMPLE 3.1

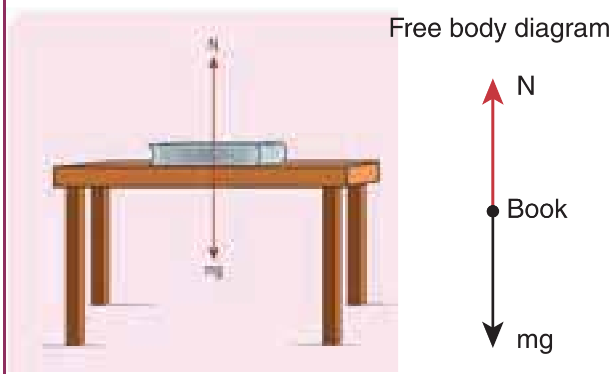
A book of mass m is at rest on the table. (1) What are the forces acting on the book? (2) What are the forces exerted by the book? (3) Draw the free body diagram for the book.

Solution

(1) There are two forces acting on the book.

   (i) Gravitational force (mg) acting downwards on the book

   (ii) Normal contact force (N) exerted by the surface of the table on the book. It acts upwards as shown in the figure.



Note In the free body diagram, as the magnitudes of the normal force and the gravitational force are same, the lengths of both these vectors are also same.

(2) According to Newton's third law, there are two reaction forces exerted by the book.

(i) The book exerts an equal and opposite force (mg) on the Earth which acts upwards.

(ii) The book exerts a force which is equal and opposite to normal force on the surface of the table (N) acting downwards.

Note It is to be emphasized that while applying Newton's third law it is wrong to conclude that the book on the table is at rest due to the downward gravitational force exerted by the Earth and the equal and opposite reacting normal force exerted by the table on the book. Action and reaction forces never act on the same body.

(3) The free body diagram of the book is shown in the figure.

EXAMPLE 3.2

If two objects of masses 2.5 kg and 100 kg experience the same force 5 N, what is the acceleration experienced by each of them?

Solution

From Newton's second law (in magnitude form), F = ma

For the object of mass 2.5 kg, the acceleration is $a = \dfrac{F}{m} = \dfrac{5}{2.5} = 2\ m\ s^{-2}$

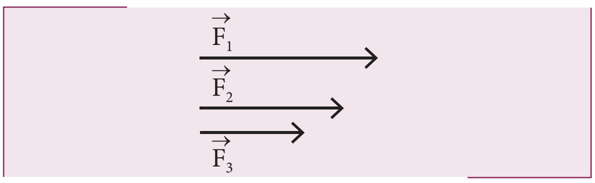
For the object of mass 100 kg, the acceleration is $a = \dfrac{F}{m} = \dfrac{5}{100} = 0.05\ m\ s^{-2}$

Note Even though the force applied on both the objects is the same, acceleration experienced by each object differs. The acceleration is inversely proportional to mass. For the same force, the heavier mass experiences lesser acceleration and the lighter mass experiences greater acceleration.

When an apple falls, it experiences Earth's gravitational force. According to Newton's third law, the apple exerts equal and opposite force on the Earth. Even though both the apple and Earth experience the same force, their acceleration is different. The mass of Earth is enormous compared to that of an apple. So an apple experiences larger acceleration and the Earth experiences almost negligible acceleration. Due to the negligible acceleration, Earth appears to be stationary when an apple falls.

EXAMPLE 3.3

Which is the greatest force among the three force $\vec{F}_1, \vec{F}_2, \vec{F}_3$ shown below



Solution

Force is a vector and magnitude of the vector is represented by the length of the vector. Here $\vec{F}_1$ has greater length compared to other two. So $\vec{F}_1$ is largest of the three.

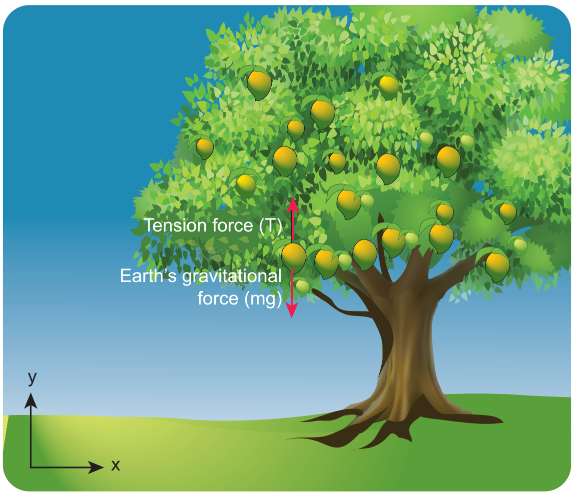
EXAMPLE 3.4

Apply Newton's second law to a mango hanging from a tree. (Mass of the mango is 400 gm)

Solution

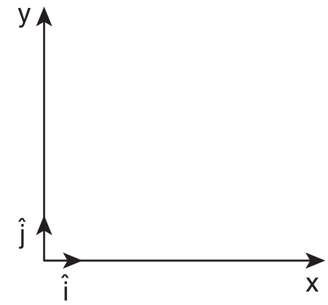
Note: Before applying Newton's laws, the following steps have to be followed:

1) Choose a suitable inertial coordinate system to analyse the problem. For most of the cases we can take Earth as an inertial coordinate system.
2) Identify the system to which Newton's laws need to be applied. The system can be a single object or more than one object.
3) Draw the free body diagram.
4) Once the forces acting on the system are identified, and the free body diagram is drawn, apply Newton's second law. In the left hand side of the equation, write the forces acting on the system in vector notation and equate it to the right hand side of equation which is the product of mass and acceleration. Here, acceleration should also be in vector notation.
5) If acceleration is given, the force can be calculated. If the force is given, acceleration can be calculated.



By following the above steps:

We fix the inertial coordinate system on the ground as shown in the figure.

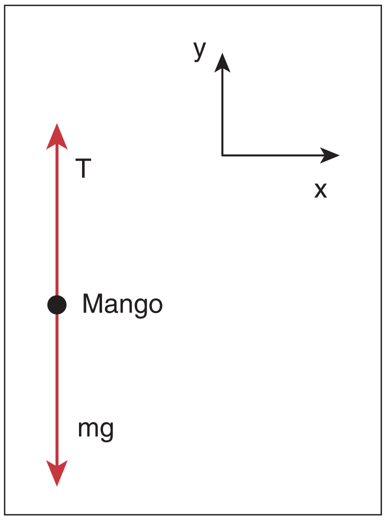


The forces acting on the mango are

i) Gravitational force exerted by the Earth on the mango acting downward along negative y axis

ii) Tension (in the cord attached to the mango) acts upward along positive y axis.

The free body diagram for the mango is shown in the figure



$$\vec{F}_g = mg\left(-\hat{j}\right) = -mg\hat{j}$$

Here, mg is the magnitude of the gravitational force and $\left(-\hat{j}\right)$ represents the unit vector in negative y direction

$$\vec{T} = T\hat{j}$$

Here T is the magnitude of the tension force and $(\hat{j})$ represents the unit vector in positive y direction

$$\vec{F}_{net} = \vec{F}_g + \vec{T} = -mg\hat{j} + T\hat{j} = (T - mg)\hat{j}$$

From Newton's second law $\vec{F}_{net} = m\vec{a}$

Since the mango is at rest with respect to us (inertial coordinate system) the acceleration is zero $(\vec{a} = 0)$.

So $\vec{F}_{net} = m\vec{a} = 0$

$$(T - mg)\hat{j} = 0$$

By comparing the components on both sides of the above equation, we get $T - mg = 0$

So the tension force acting on the mango is given by $T = mg$

Mass of the mango m = 400g and $g = 9.8\ m\ s^{-2}$

Tension acting on the mango is $T = 0.4 \times 9.8 = 3.92$ N

EXAMPLE 3.5

A person rides a bike with a constant velocity $\vec{v}$ with respect to ground and another biker accelerates with acceleration $\vec{a}$ with respect to ground. Who can apply Newton's second law with respect to a stationary observer on the ground?

Solution

Second biker cannot apply Newton's second law, because he is moving with acceleration $\vec{a}$ with respect to Earth (he is not in inertial frame). But the first biker can apply Newton's second law because he is moving at constant velocity with respect to Earth (he is in inertial frame).

EXAMPLE 3.6

The position vector of a particle is given by $\vec{r} = 3t\hat{i} + 5t^2\hat{j} + 7\hat{k}$. Find the direction in which the particle experiences net force?

Solution

Velocity of the particle,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t)\hat{i} + \frac{d}{dt}(5t^2)\hat{j} + \frac{d}{dt}(7)\hat{k}$$

$$\frac{d\vec{r}}{dt} = 3\hat{i} + 10t\hat{j}$$

Acceleration of the particle

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 10\hat{j}$$

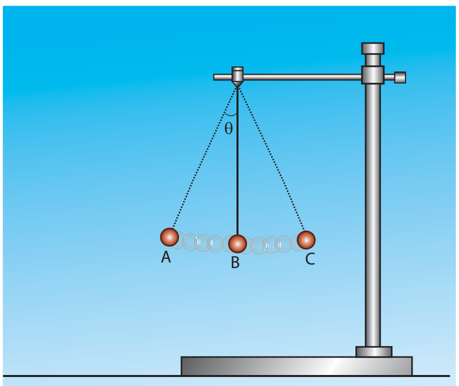
Here, the particle has acceleration only along positive y direction. According to Newton's second law, net force must also act along positive y direction. In addition, the particle has constant velocity in positive x direction and no velocity in z direction. Hence, there are no net force along x or z direction.

EXAMPLE 3.7

Consider a bob attached to a string, hanging from a stand. It oscillates as shown in the figure.

Solution

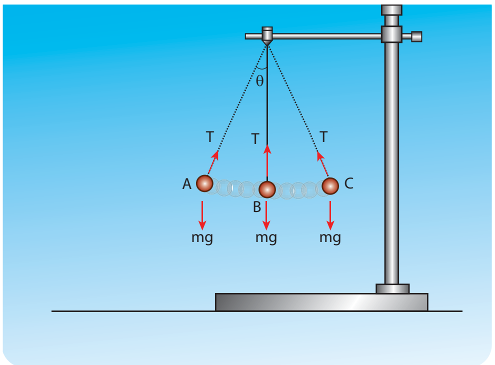
a) Identify the forces that act on the bob?
b) What is the acceleration experienced by the bob?



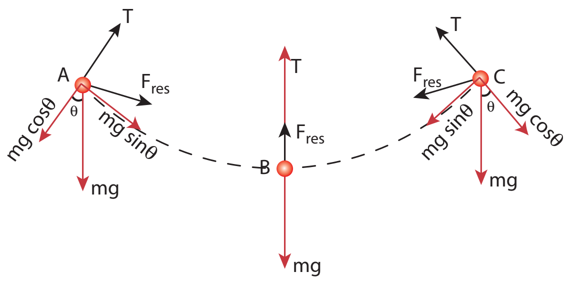
Two forces act on the bob.

(i) Gravitational force (mg) acting downwards

(ii) Tension (T) exerted by the string on the bob, whose position determines the direction of T as shown in figure.



The bob is moving in a circular arc as shown in the above figure. Hence it has centripetal acceleration. At a point A and C, the bob comes to rest momentarily and then its velocity increases when it moves towards point B. Hence, there is a tangential acceleration along the arc. The gravitational force can be resolved into two components (mg cosθ, mg sinθ) as shown below

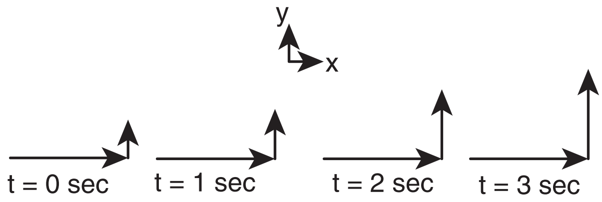


Note Note that the bob does not move in the direction of the resultant force. At the points A and C, tension T = mg cosθ. At all other points, tension T is greater than mg cosθ, since it has non zero centripetal acceleration. At point B, the resultant force acts upward along the string. It is an example of a non uniform circular motion because the bob has both the centripetal and tangential accelerations.

EXAMPLE 3.8

The velocity of a particle moving in a plane is given by the following diagram. Find out the direction of force acting on the particle?

Solution



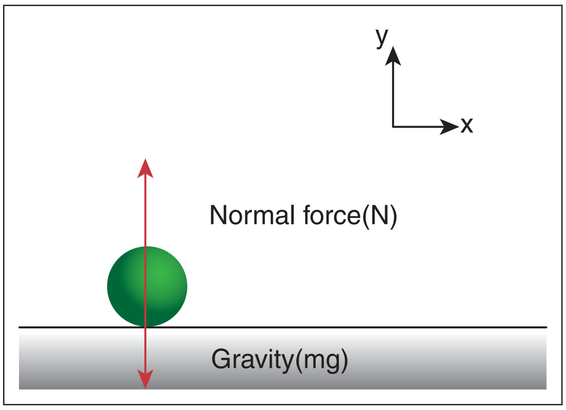
The velocity of the particle is $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$. As shown in the figure, the particle is moving in the xy plane, there is no motion in the z direction. So velocity in the z direction is zero ($v_z = 0$). The velocity of the particle has x component (v_x) and y component (v_y). From figure, as time increases from t = 0 sec to t = 3 sec, the length of the vector in y direction is changing (increasing). It means y component of velocity $\left(v_y\right)$ is increasing with respect to time. According to Newton's second law, if velocity changes with respect to time then there must be acceleration. In this case, the particle has acceleration in the y direction since the y component of velocity changes. So the particle experiences force in the y direction. The length of the vector in x direction does not change. It means that the particle has constant velocity in the x direction. So no force or zero net force acts in the x direction.

EXAMPLE 3.9

Apply Newton's second law for an object at rest on Earth and analyse the result.

Solution

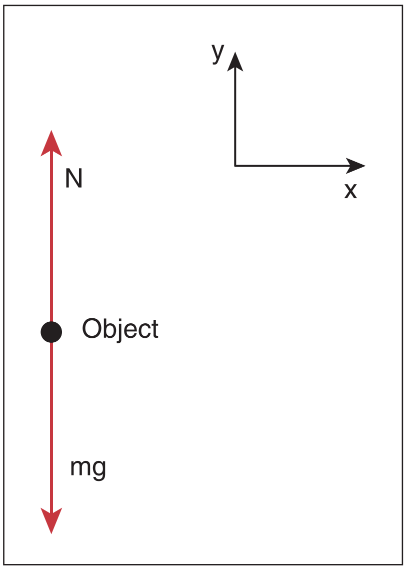
The object is at rest with respect to Earth (inertial coordinate system). There are two forces that act on the object.



i) Gravity acting downward (negative y-direction)

ii) Normal force by the surface of the Earth acting upward (positive y-direction)

The free body diagram for this object is



$$\vec{F}_g = -mg\hat{j}$$
$$\vec{N} = N\hat{j}$$

Net force $\vec{F}_{net} = -mg\hat{j} + N\hat{j}$

But there is no acceleration on the ball. So $\vec{a} = 0$. By applying Newton's second law $(\vec{F}_{net} = m\vec{a})$

Since $\vec{a} = 0, \vec{F}_{net} = -mg\hat{j} + N\hat{j}$

$$(-mg + N)\hat{j} = 0$$

By comparing the components on both sides of the equation, we get

$$-mg + N = 0$$
$$N = mg$$

We can conclude that if the object is at rest, the magnitude of normal force is exactly equal to the magnitude of gravity.

EXAMPLE 3.10

A particle of mass 2 kg experiences two forces, $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} + 3\hat{k}$. What is the acceleration of the particle?

Solution

We use Newton's second law, $\vec{F}_{net} = m\vec{a}$ where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. From the above equations the acceleration is $\vec{a} = \frac{\vec{F}_{net}}{m}$, where

$$\vec{F}_{net} = (5+3)\hat{i} + (8-4)\hat{j} + (7+3)\hat{k}$$
$$\vec{F}_{net} = 8\hat{i} + 4\hat{j} + 10\hat{k}$$
$$\vec{a} = \left(\frac{8}{2}\right)\hat{i} + \left(\frac{4}{2}\right)\hat{j} + \left(\frac{10}{2}\right)\hat{k}$$
$$\vec{a} = 4\hat{i} + 2\hat{j} + 5\hat{k}$$

EXAMPLE 3.11

Identify the forces acting on blocks A, B and C shown in the figure.

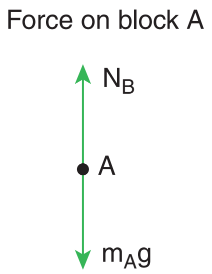
Solution

Forces on block A:

(i) Downward gravitational force exerted by the Earth ($m_A g$)

(ii) Upward normal force (N_B) exerted by block B (N_B)

The free body diagram for block A is as shown in the following picture.

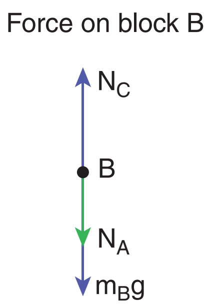


Forces on block B :

(i) Downward gravitational force exerted by Earth ($m_B g$)

(ii) Downward force exerted by block A (N_A)

(iii) Upward normal force exerted by block C (N_C)

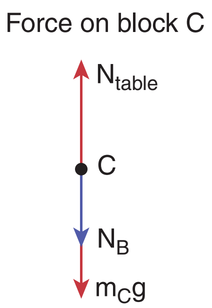


Forces onblock C:

(i) Downward gravitational force exerted by Earth ($m_C g$)

(ii) Downward force exerted by block B (N_B)

(iii) Upward force exerted by the table (N_{table})



EXAMPLE 3.12

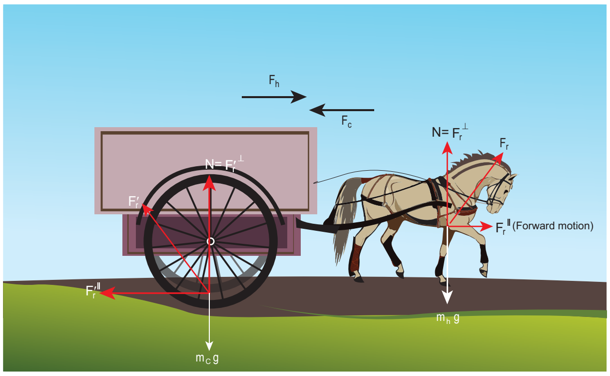
Consider a horse attached to the cart which is initially at rest. If the horse starts walking forward, the cart also accelerates in the forward direction. If the horse pulls the cart with force F_h in forward direction, then according to Newton's third law, the cart also pulls the horse by equivalent opposite force $F_c = F_h$ in backward direction. Then total force on 'cart+horse' is zero. Why is it then the 'cart+horse' accelerates and moves forward?

Solution

This paradox arises due to wrong application of Newton's second and third laws. Before applying Newton's laws, we should decide 'what is the system?'. Once we identify the 'system', then it is possible to identify all the forces acting on the system. We should not consider the force exerted by the system. If there is an unbalanced force acting on the system, then it should have acceleration in the direction of the resultant force. By following these steps we will analyse the horse and cart motion.

If we decide on the cart+horse as a 'system', then we should not consider the force exerted by the horse on the cart or the force exerted by cart on the horse. Both are internal forces acting on each other. According to Newton's third law, total internal force acting on the system is zero and it cannot accelerate the system. The acceleration of the system is caused by some external force. In this case, the force exerted by the road on the system is the external force acting on the system. It is wrong to conclude that the total force acting on the system (cart+horse) is zero without including all the forces acting on the system. The road is pushing the horse and cart forward with acceleration. As there is an external force acting on the system, Newton's second law has to be applied and not Newton's third law.

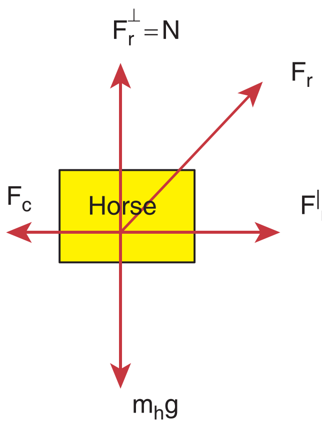
The following figures illustrates this.



If we consider the horse as the 'system', then there are three forces acting on the horse.

(i) Downward gravitational force ($m_h g$)
(ii) Force exerted by the road (F_r)
(iii) Backward force exerted by the cart (F_c)

It is shown in the following figure.



F_r – Force exerted by the road on the horse

F_c – Force exerted by the cart on the horse

$F_r^{\perp}$ – Perpendicular component of F_r=N

$F_r^{\parallel}$ – Parallel component of F_r which is reason for forward movement

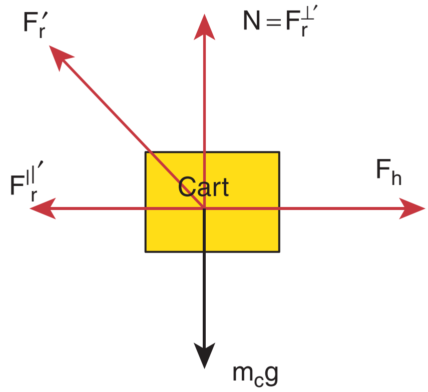
The force exerted by the road can be resolved into parallel and perpendicular components. The perpendicular component balances the downward gravitational force. There is parallel component along the forward direction. It is greater than the backward force (F_c). So there is net force along the forward direction which causes the forward movement of the horse.

If we take the cart as the system, then there are three forces acting on the cart.

(i) Downward gravitational force ($m_c g$)
(ii) Force exerted by the road ($F_r^{'}$)
(iii) Force exerted by the horse (F_h)

It is shown in the figure

Force on the cart



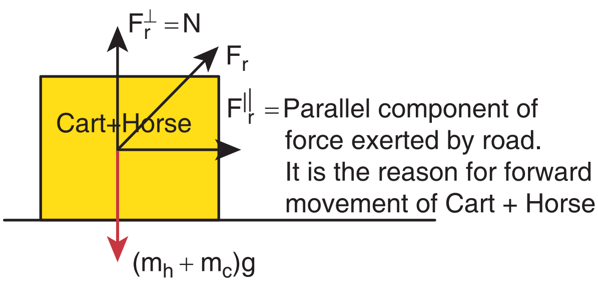
The force exerted by the road ($\vec{F_r^{'}}$) can be resolved into parallel and perpendicular components. The perpendicular component cancels the downward gravity ($m_c g$). Parallel component acts backwards and the force exerted by the horse ($\vec{F_h}$) acts forward. Force ($\vec{F_h}$) is greater than the parallel component acting in the opposite direction. So there is an overall unbalanced force in the forward direction which causes the cart to accelerate forward.

If we take the cart+horse as a system, then there are two forces acting on the system.

(i) Downward gravitational force $(m_h + m_c)$g

(ii) The force exerted by the road (F_r) on the system.

It is shown in the following figure.



(iii) In this case the force exerted by the road (F_r) on the system (cart+horse) is resolved in to parallel and perpendicular components. The perpendicular component is the normal force which cancels the downward gravitational force $(m_h + m_c)$g. The parallel component of the force is not balanced, hence the system (cart+horse) accelerates and moves forward due to this force.

EXAMPLE 3.13

The position of the particle is represented by $y = ut - \frac{1}{2}gt^2$.

a) What is the force acting on the particle?
b) What is the momentum of the particle?

Solution

To find the force, we need to find the acceleration experienced by the particle.

The acceleration is given by $a = \frac{d^2y}{dt^2}$

$$\text{(or)} \quad a = \frac{dv}{dt}$$

Here

v =velocity of the particle in y direction

$$v = \frac{dy}{dt} = u - gt$$

The momentum of the particle = mv = m (u-gt).

$$a = \frac{dv}{dt} = -g$$

The force acting on the object is given by $F = ma = -mg$

The negative sign implies that the force is acting on the negative y direction. This is exactly the force that acts on the object in projectile motion.

3.3.2 Particle Moving in an Inclined Plane

When an object of mass m slides on a frictionless surface inclined at an angle θ as shown in the Figure 3.12, the forces acting on it decides the

a) acceleration of the object
b) speed of the object when it reaches the bottom

The force acting on the object is

(i) Downward gravitational force (mg)
(ii) Normal force perpendicular to inclined surface (N)

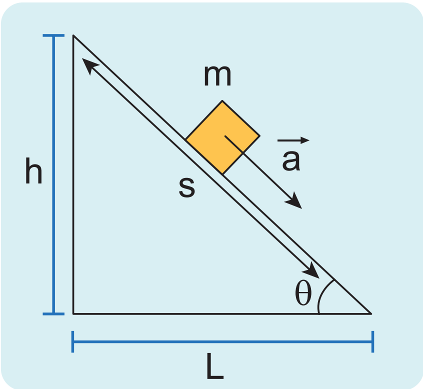


Figure 3.12 Object moving in an inclined plane

To draw the free body diagram, the block is assumed to be a point mass (Figure 3.13 (a)). Since the motion is on the inclined surface, we have to choose the coordinate system parallel to the inclined surface as shown in Figure 3.13 (b).

The gravitational force mg is resolved in to parallel component $mg\sin\theta$ along the inclined plane and perpendicular component $mg\cos\theta$ perpendicular to the inclined surface (Figure 3.13 (b)).

Note that the angle made by the gravitational force (mg) with the perpendicular to the surface is equal to the angle of inclination θ as shown in Figure 3.13 (c).

There is no motion(acceleration) along the y axis. Applying Newton's second law in the y direction

$$-mg\cos\theta\hat{j} + N\hat{j} = 0(No\ acceleration)$$

By comparing the components on both sides, $N - mg\cos\theta = 0$

$$N = mg\cos\theta$$

The magnitude of normal force (N) exerted by the surface is equivalent to $mg\ cos\theta$.

The object slides (with an acceleration) along the x direction. Applying Newton's second law in the x direction

$$mg\sin\theta\hat{i} = ma\hat{i}$$

By comparing the components on both sides, we can equate

$$mg\sin\theta = ma$$

The acceleration of the sliding object is

$$a = g\ sin\theta$$

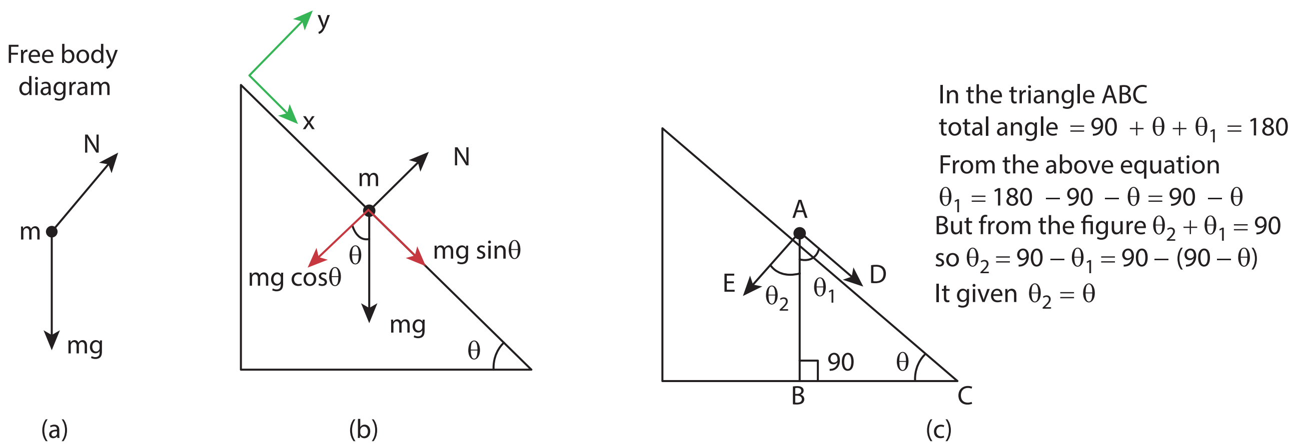


Figure 3.13 (a) Free body diagram, (b) mg resolved into parallel and perpendicular components (c) The angle θ_2 is equal to θ

Note that the acceleration depends on the angle of inclination θ. If the angle θ is 90 degree, the block will move vertically with acceleration $a = g$.

Newton's kinematic equation is used to find the speed of the object when it reaches the bottom. The acceleration is constant throughout the motion.

$$v^2 = u^2 + 2as \text{ along the x direction} \quad (3.3)$$

The acceleration a is equal to g sinθ. The initial speed (u) is equal to zero as it starts from rest. Here s is the length of the inclined surface.

The speed (v) when it reaches the bottom is (using equation (3.3))

$$v = \sqrt{2sg\ sin\theta} \quad (3.4)$$

Note Here we choose the coordinate system along the inclined plane. Even if we choose the coordinate system parallel to the horizontal surface, we will get the same result. But the mathematics will be quite complicated. Choosing a suitable inertial coordinate system for the given problem is very important.

3.3.3 Two Bodies in Contact on a Horizontal Surface

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface as shown in Figure 3.14.

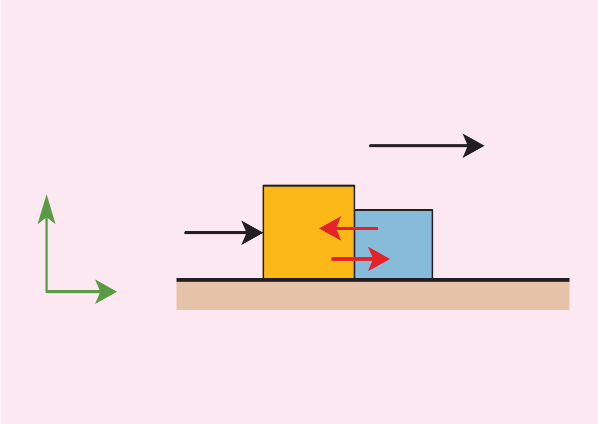


Figure 3.14 (a) Two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface

By the application of a horizontal force F, both the blocks are set into motion with acceleration 'a' simultaneously in the direction of the force F.

To find the acceleration $\vec{a}$, Newton's second law has to be applied to the system (combined mass $m = m_1 + m_2$)

$$\vec{F} = m\vec{a}$$

If we choose the motion of the two masses along the positive x direction,

$$F\hat{i} = ma\hat{i}$$

By comparing components on both sides of the above equation

$$F = ma \quad \text{where } m = m_1 + m_2$$

The acceleration of the system is given by

$$\therefore a = \frac{F}{m_1 + m_2} \quad (3.5)$$

The force exerted by the block m_1 on m_2 due to its motion is called force of contact ($\vec{f}_{21}$). According to Newton's third law, the block m_2 will exert an equivalent opposite reaction force ($\vec{f}_{12}$) on block m_1.

Figure 3.14 (b) shows the free body diagram of block m_1.

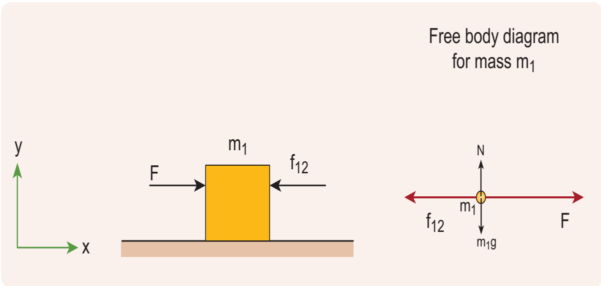


Figure 3.14 (b) Free body diagram of block of mass m_1

$$\therefore F\hat{i} - f_{12}\hat{i} = m_1 a\hat{i}$$

By comparing the components on both sides of the above equation, we get

$$F - f_{12} = m_1 a$$

$$f_{12} = F - m_1 a \qquad (3.6)$$

Substituting the value of acceleration from equation (3.5) in (3.6) we get

$$f_{12} = F - m_1\left(\frac{F}{m_1 + m_2}\right)$$

$$f_{12} = F\left[1 - \frac{m_1}{m_1 + m_2}\right]$$

$$f_{12} = \frac{Fm_2}{m_1 + m_2} \qquad (3.7)$$

Equation (3.7) shows that the magnitude of contact force depends on mass m_2 which provides the reaction force. Note that this force is acting along the negative x direction.

In vector notation, the reaction force on mass m_1 is given by $\vec{f}_{12} = -\frac{Fm_2}{m_1 + m_2}\hat{i}$

For mass m_2 there is only one force acting on it in the x direction and it is denoted by $\vec{f}_{21}$. This force is exerted by mass m_1. The free body diagram for mass m_2 is shown in Figure 3.14 (c).

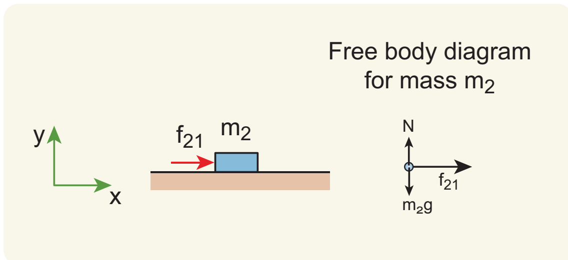


Figure 3.14 (c) Free body diagram of block of mass m_2

Applying Newton's second law for mass m_2

$$f_{21}\hat{i} = m_2 a\hat{i}$$

By comparing the components on both sides of the above equation

$$f_{21} = m_2 a \qquad (3.8)$$

Substituting for acceleration from equation (3.5) in equation (3.8), we get $f_{21} = \frac{Fm_2}{m_1 + m_2}$

In this case the magnitude of the contact force is

$f_{21} = \frac{Fm_2}{m_1 + m_2}$ The direction of this force is along the positive x direction.

In vector notation, the force acting on mass m_2 exerted by mass m_1 is $\vec{f}_{21} = \frac{Fm_2}{m_1 + m_2}\hat{}$

Note $\vec{f}_{12} = -\vec{f}_{21}$ which confirms Newton's third law.

3.3.4 Motion of Connected Bodies

When objects are connected by strings and a force F is applied either vertically or horizontally or along an inclined plane, it produces a tension T in the string, which affects the acceleration to an extent. Let us discuss various cases for the same.

Case 1: Vertical motion

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley as shown in Figure 3.15.

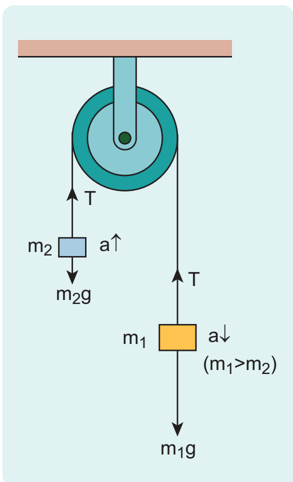


Figure 3.15 Two blocks connected by a string over a pulley

Let the tension in the string be T and acceleration a. When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration a. The gravitational force m_1g on mass m_1 is used in lifting the mass m_2.

The upward direction is chosen as y direction. The free body diagrams of both masses are shown in Figure 3.16.

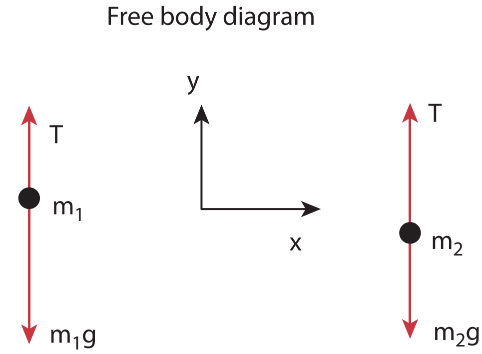


Figure 3.16 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for mass m_2

$$T\hat{j} - m_2g\hat{j} = m_2a\hat{j}$$

The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get

$$T - m_2g = m_2a \qquad (3.9)$$

Similarly, applying Newton's second law for mass m_1

$$T\hat{j} - m_1g\hat{j} = -m_1a\hat{j}$$

As mass m_1 moves downward $(-\hat{j})$, its acceleration is along $(-\hat{j})$

By comparing the components on both sides, we get

$$T - m_1 g = -m_1 a$$

$$m_1 g - T = m_1 a \qquad (3.10)$$

Adding equations (3.9) and (3.10), we get

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$(m_1 - m_2) g = (m_1 + m_2) a \qquad (3.11)$$

From equation (3.11), the acceleration of both the masses is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad (3.12)$$

If both the masses are equal ($m_1 = m_2$), from equation (3.12)

$$a = 0$$

This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest.

To find the tension acting on the string, substitute the acceleration from the equation (3.12) into the equation (3.9).

$$T - m_2 g = m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g + m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad (3.13)$$

By taking $m_2 g$ common in the RHS of equation (3.13)

$$T = m_2 g \left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g \left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

Equation (3.12) gives only magnitude of acceleration.

For mass m_1, the acceleration vector is given by $\vec{a} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\hat{j}$

For mass m_2, the acceleration vector is given by $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\hat{j}$

Case 2: Horizontal motion

In this case, mass m_2 is kept on a horizontal table and mass m_1 is hanging through a small pulley as shown in Figure 3.17. Assume that there is no friction on the surface.

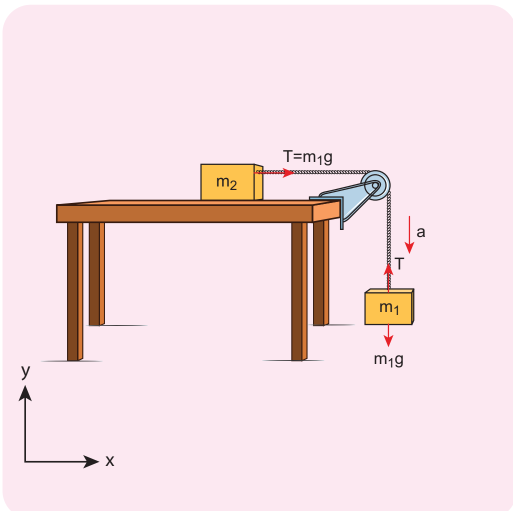


Figure 3.17 Blocks in horizontal motion

As both the blocks are connected to the unstretchable string, if m_1 moves with an acceleration a downward then m_2 also moves with the same acceleration a horizontally.

The forces acting on mass m_2 are

(i) Downward gravitational force (m_2g)
(ii) Upward normal force (N) exerted by the surface
(iii) Horizontal tension (T) exerted by the string

The forces acting on mass m_1 are

(i) Downward gravitational force (m_1g)
(ii) Tension (T) acting upwards

The free body diagrams for both the masses is shown in Figure 3.18.

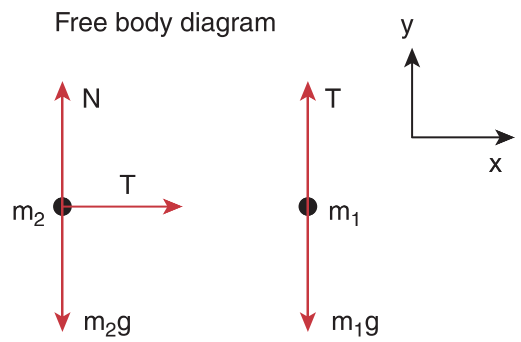


Figure 3.18 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for m_1

$$T\hat{j} - m_1g\hat{j} = -m_1a\hat{j} \text{ (along y direction)}$$

By comparing the components on both sides of the above equation,

$$T - m_1g = -m_1a \qquad (3.14)$$

Applying Newton's second law for m_2

$$T\hat{i} = m_2a\hat{i} \text{ (along x direction)}$$

By comparing the components on both sides of above equation,

$$T = m_2a \qquad (3.15)$$

There is no acceleration along y direction for m_2.

$$N\hat{j} - m_2g\hat{j} = 0$$

By comparing the components on both sides of the above equation

$$N - m_2g = 0$$
$$N = m_2g \qquad (3.16)$$

By substituting equation (3.15) in equation (3.14), we can find the tension T

$$m_2a - m_1g = -m_1a$$
$$m_2a + m_1a = m_1g$$
$$a = \frac{m_1}{m_1 + m_2}g \qquad (3.17)$$

Tension in the string can be obtained by substituting equation (3.17) in equation (3.15)

$$T = \frac{m_1m_2}{m_1 + m_2}g \qquad (3.18)$$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.

This result has an important application in industries. The ropes used in conveyor belts (horizontal motion) work for longer duration than those of cranes and lifts (vertical motion).

3.3.5 Concurrent Forces and Lami's Theorem

A collection of forces is said to be concurrent, if the lines of forces act at a common point. Figure 3.19 illustrates concurrent forces.

Concurrent forces need not be in the same plane. If they are in the same plane, they are concurrent as well as coplanar forces.

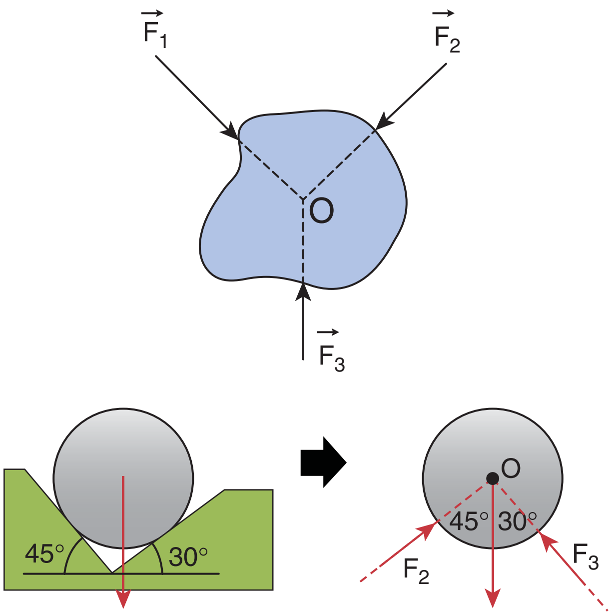


Figure 3.19 Concurrent forces

3.4 LAMI'S THEOREM

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

Let us consider three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ which act at a common point O as shown in Figure 3.20. If the point is at equilibrium, then according to Lami's theorem

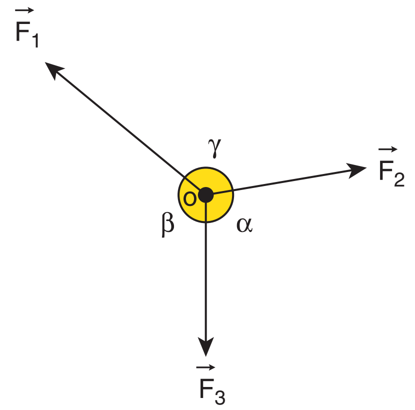


Figure 3.20 Three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ acting at O

$$\left|\vec{F}_1\right| \propto \sin\alpha$$

$$\left|\vec{F}_2\right| \propto \sin\beta$$

$$\left|\vec{F}_3\right| \propto \sin\gamma$$

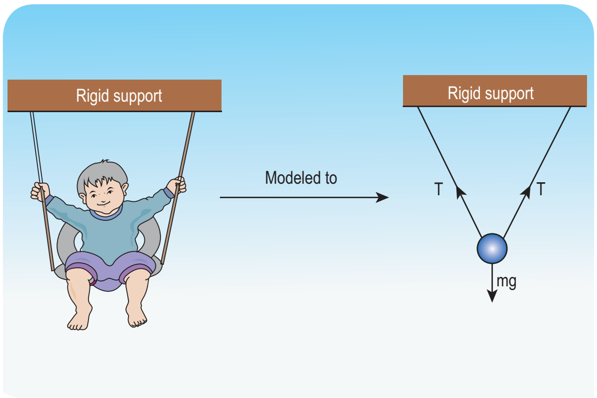
Therefore, $$\frac{\left|\vec{F}_1\right|}{\sin\alpha} = \frac{\left|\vec{F}_2\right|}{\sin\beta} = \frac{\left|\vec{F}_3\right|}{\sin\gamma} \quad (3.19)$$

Lami's theorem is useful to analyse the forces acting on objects which are in static equilibrium.

Application of Lami's Theorem

EXAMPLE 3.14

A baby is playing in a swing which is hanging with the help of two identical chains is at rest. Identify the forces acting on the baby. Apply Lami's theorem and find out the tension acting on the chain.



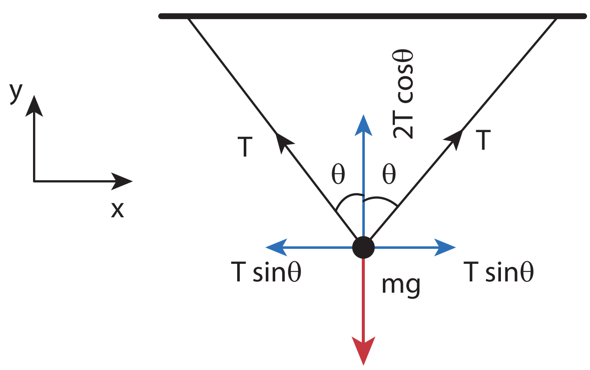
Solution

The baby and the chains are modeled as a particle hung by two strings as shown in the figure. There are three forces acting on the baby.

i) Downward gravitational force along negative y direction (mg)

ii) Tension (T) along the two strings

These three forces are coplanar as well as concurrent as shown in the following figure.



By using Lami's theorem

$$\frac{T}{\sin(180-\theta)} = \frac{T}{\sin(180-\theta)} = \frac{mg}{\sin(2\theta)}$$

Since $\sin(180-\theta) = \sin\theta$ and $\sin(2\theta) = 2\sin\theta\cos\theta$

We get $$\frac{T}{\sin\theta} = \frac{mg}{2\sin\theta\cos\theta}$$

From this, the tension on each string is $T = \dfrac{mg}{2\cos\theta}$.

Note When $\theta = 0°$, the strings are vertical and the tension on each string is $T = \dfrac{mg}{2}$

3.5 LAW OF CONSERVATION OF TOTAL LINEAR MOMENTUM

In nature, conservation laws play a very important role. The dynamics of motion of bodies can be analysed very effectively using conservation laws. There are three conservation laws in mechanics. Conservation of total energy, conservation of total linear momentum, and conservation of angular momentum. By combining Newton's second and third laws, we can derive the law of conservation of total linear momentum.

When two particles interact with each other, they exert equal and opposite forces on each other. The particle 1 exerts force $\vec{F}_{21}$ on particle 2 and particle 2 exerts an exactly equal and opposite force $\vec{F}_{12}$ on particle 1, according to Newton's third law.

$$\vec{F}_{21} = -\vec{F}_{12} \tag{3.20}$$

In terms of momentum of particles, the force on each particle (Newton's second law) can be written as

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \quad \text{and} \quad \vec{F}_{21} = \frac{d\vec{p}_2}{dt}. \tag{3.21}$$

Here $\vec{p}_1$ is the momentum of particle 1 which changes due to the force $\vec{F}_{12}$ exerted by particle 2. Further $\vec{p}_2$ is the momentum of particle 2. This changes due to $\vec{F}_{21}$ exerted by particle 1.

Substitute equation (3.21) in equation (3.20)

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \tag{3.22}$$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \tag{3.23}$$

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0$$

It implies that $\vec{p}_1 + \vec{p}_2 =$ constant vector (always).

$\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system. From this result, the law of conservation of linear momentum can be stated as follows.

If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector. In otherwords, the total linear momentum of the system is conserved in time. Here the word 'conserve' means that $\vec{p}_1$ and $\vec{p}_2$ can vary,in such a way that $\vec{p}_1 + \vec{p}_2$ is a constant vector.

The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are called the internal forces of the system, because they act only between the two particles. There is no external force acting on the two particles from outside. In such a case the total linear momentum of the system is a constant vector or is conserved.

EXAMPLE 3.15

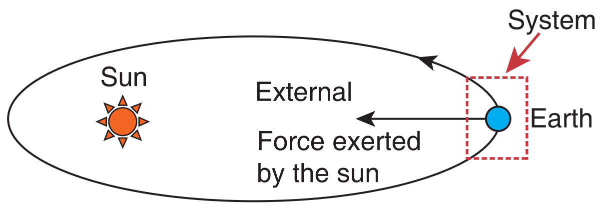
Identify the internal and external forces acting on the following systems.

a) Earth alone as a system
b) Earth and Sun as a system
c) Our body as a system while walking
d) Our body + Earth as a system

Solution

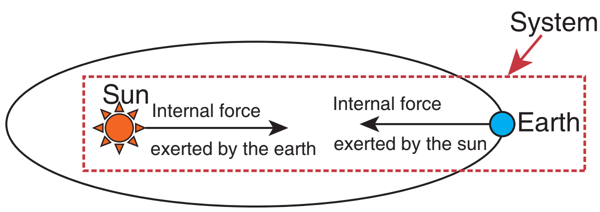
a) Earth alone as a system

Earth orbits the Sun due to gravitational attraction of the Sun. If we consider Earth as a system, then Sun's gravitational force is an external force. If we take the Moon into account, it also exerts an external force on Earth.



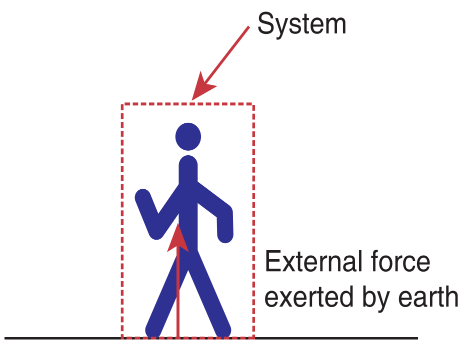
b) (Earth + Sun) as a system

In this case, there are two internal forces which form an action and reaction pair- the gravitational force exerted by the Sun on Earth and gravitational force exerted by the Earth on the Sun.



c) Our body as a system

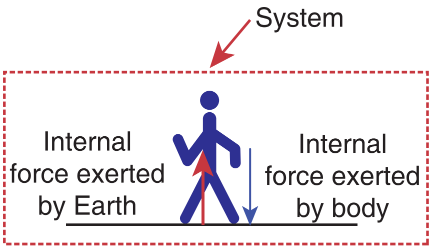
While walking, we exert a force on the Earth and Earth exerts an equal and opposite force on our body. If our body alone is considered as a system, then the force exerted by the Earth on our body is external.



Our body as a system

d) (Our body + Earth) as a system

In this case, there are two internal forces present in the system. One is the force exerted by our body on the Earth and the other is the equal and opposite force exerted by the Earth on our body.

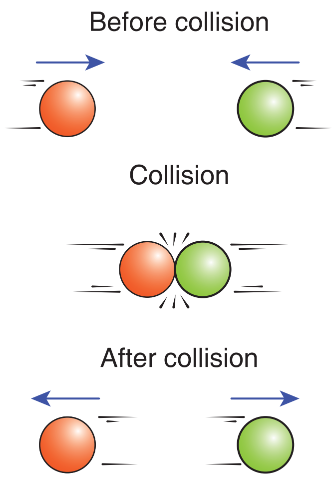


Our body + Earth as a system

Meaning of law of conservation of momentum

1) The Law of conservation of linear momentum is a vector law. It implies that both the magnitude and direction of total linear momentum are constant. In some cases, this total momentum can also be zero.
2) To analyse the motion of a particle, we can either use Newton's second law or the law of conservation of linear momentum. Newton's second law requires us to specify the forces involved in the process. This is difficult to specify in real situations. But conservation of linear momentum does not require any force involved in the process. It is covenient and hence important.

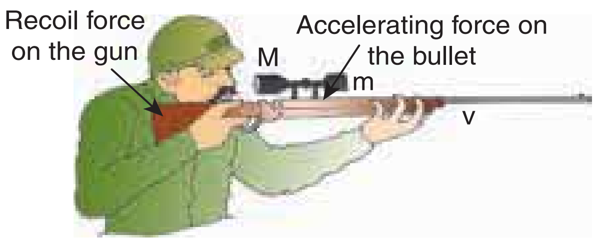
For example, when two particles collide, the forces exerted by these two particles on each other is difficult to specify. But it is easier to apply conservation of linear momentum during the collision process.



Collision of two particles

Examples

- Consider the firing of a gun. Here the system is Gun+bullet. Initially the gun and bullet are at rest, hence the total linear momentum of the system is zero. Let $\vec{p}_1$ be the momentum of the bullet and $\vec{p}_2$ the momentum of the gun before firing. Since initially both are at rest,



$$\vec{p}_1 = 0, \vec{p}_2 = 0.$$

Total momentum before firing the gun is zero, $\vec{p}_1 + \vec{p}_2 = 0$.

According to the law of conservation of linear momentum, total linear momemtum has to be zero after the firing also.

When the gun is fired, a force is exerted by the gun on the bullet in forward direction. Now the momentum of the bullet changes from $\vec{p}_1$ to $\vec{p}_1'$. To conserve the total linear momentum of the system, the momentum of the gun must also change from $\vec{p}_2$ to $\vec{p}_2'$. Due to the conservation of linear momentum, $\vec{p}_1' + \vec{p}_2' = 0$. It implies that $\vec{p}_1' = -\vec{p}_2'$, the momentum of the gun is exactly equal, but in the opposite direction to the momentum of the bullet. This is the reason after firing, the gun suddenly moves backward with the momentum $\left(-\vec{p}_2'\right)$. It is called 'recoil momemtum'. This is an example of conservation of total linear momentum.

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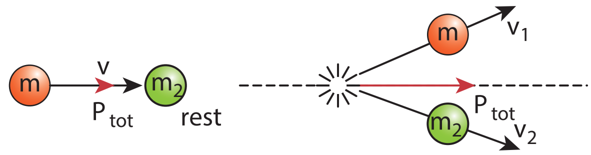
- Consider two particles. One is at rest and the other moves towards the first particle (which is at rest). They collide and after collison move in some arbitrary directions. In this case, before collision, the total linear momentum of the system is equal to the initial linear momentum of the moving particle. According to conservation of momentum, the total linear momentum after collision also has to be in the forward direction. The following figure explains this.



Before collision   After collision

A more accurate calculation is covered in section 4.4. It is to be noted that the total momentum vector before and after collison points in the same direction. This simply means that the total linear momentum is constant before and after the collision. At the time of collision, each particle exerts a force on the other. As the two particles are considered as a system, these forces are only internal, and the total linear momentum cannot be altered by internal forces.

3.5.1 Impulse

If a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.

If a force (F) acts on the object in a very short interval of time (Δt), from Newton's second law in magnitude form

$$F dt = dp$$

Integrating over time from an initial time t_i to a final time t_f, we get

$$\int_i^f dp = \int_{t_i}^{t_f} F\, dt$$

$$p_f - p_i = \int_{t_i}^{t_f} F\, dt$$

p_i = initial momentum of the object at time t_i

p_f = final momentum of the object at time t_f

$p_f - p_i = \Delta p =$ change in momentum of the object during the time interval $t_f - t_i = \Delta t$.

The integral $\int_{t_i}^{t_f} F\,dt = J$ is called the impulse and it is equal to change in momentum of the object.

If the force is constant over the time interval, then

$$\int_{t_i}^{t_f} F\,dt = F\int_{t_i}^{t_f} dt = F\left(t_f - t_i\right) = F\Delta t$$

$$F\,\Delta t = \Delta p \qquad (3.24)$$

Equation (3.24) is called the 'impulse-momentum equation'.

For a constant force, the impulse is denoted as $J = F\Delta t$ and it is also equal to change in momentum (Δp) of the object over the time interval Δt.

Impulse is a vector quantity and its unit is Ns.

The average force acted on the object over the short interval of time is defined by

$$F_{avg} = \frac{\Delta p}{\Delta t} \qquad (3.25)$$

From equation (3.25), the average force that act on the object is greater if Δt is smaller. Whenever the momentum of the body changes very quickly, the average force becomes larger.

The impulse can also be written in terms of the average force. Since Δp is change in momentum of the object and is equal to impulse (J), we have

$$J = F_{avg}\Delta t \qquad (3.26)$$

The graphical representation of constant force impulse and variable force impulse is given in Figure 3.21.

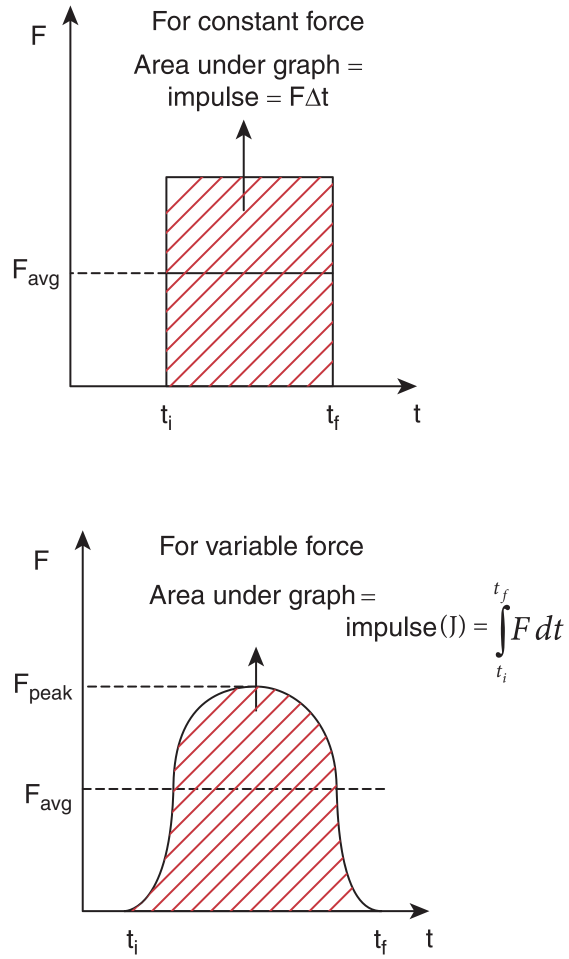


Figure 3.21 Constant force impulse and variable force impulse

Illustration

1. When a cricket player catches the ball, he pulls his hands gradually in the direction of the ball's motion. Why?

   If he stops his hands soon after catching the ball, the ball comes to rest very quickly. It means that the momentum of the ball is brought to rest very quickly. So the average force acting on the body will be very large. Due to this large average force, the hands will get hurt. To avoid getting hurt, the player brings the ball to rest slowly.

2. When a car meets with an accident, its momentum reduces drastically in a very short time. This is very dangerous for the passengers inside the car since they will experience a large force. To prevent this fatal shock, cars are designed with air bags in such a way that when the car meets with an accident, the momentum of the passengers will reduce slowly so that the average force acting on them will be smaller.

3. The shock absorbers in two wheelers play the same role as airbags in the car. When there is a bump on the road, a sudden force is transferred to the vehicle. The shock absorber prolongs the period of transfer of force on to the body of the rider. Vehicles without shock absorbers will harm the body due to this reason.
4. Jumping on a concrete cemented floor is more dangerous than jumping on the sand. Sand brings the body to rest slowly than the concrete floor, so that the average force experienced by the body will be lesser.

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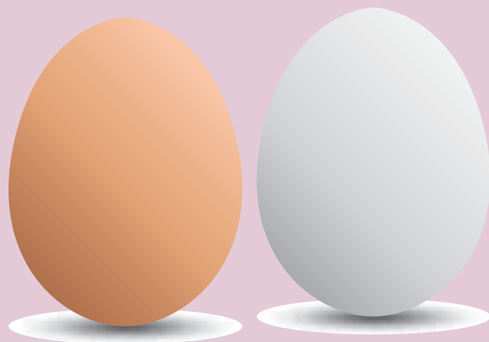
Impulse



If an egg is thrown, can you catch the egg safely without breaking it? How?

EXAMPLE 3.16

An object of mass 10 kg moving with a speed of 15 $m\ s^{-1}$ hits the wall and comes to rest within

a) 0.03 second
b) 10 second

Calculate the impulse and average force acting on the object in both the cases.

Solution

Initial momentum of the object $p_i = 10 \times 15 = 150\ kg m s^{-1}$

Final momentum of the object $p_f = 0$

$$\Delta p = 150 - 0 = 150\ kg\ m s^{-1}$$

(a) Impulse $J = \Delta p = 150\, N\, s.$

(b) Impulse $J = \Delta p = 150\, N\, s$

(a) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{0.03} = 5000\, N$

(b) Average force $F_{avg} = \frac{150}{10} = 15\, N$

We see that, impulse is the same in both cases, but the average force is different.

3.6 FRICTION

3.6.1 Introduction

If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move. It is because of the opposing force exerted by the surface on the object which resists its motion. This force is called the *frictional force which always opposes the relative motion between an object and the surface where it is placed.* If the force applied is increased, the object moves after a certain limit.

Relative motion: when a force parallel to the surface is applied on the object, the force tries to move the object with respect to the surface. This 'relative motion' is opposed by the surface by exerting a frictional force on the object in a direction opposite to applied force. Frictional force always acts on the object parallel to the surface on which the object is placed. There are two kinds of friction namely 1) Static friction and 2) Kinetic friction.

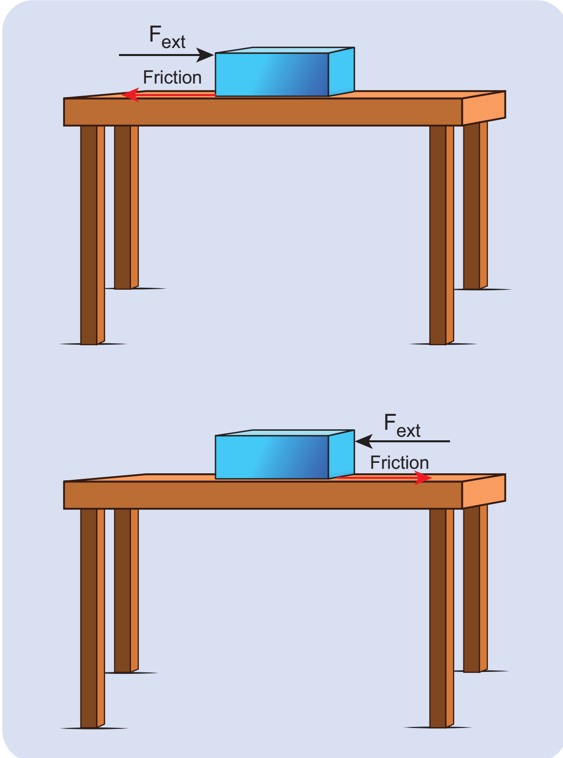


Figure 3.22 Frictional force

3.6.2 Static Friction ($\vec{f}_s$)

Static friction is the force which opposes the initiation of motion of an object on the surface. When the object is at rest on the surface, only two forces act on it. They are the downward gravitational force and upward normal force. The resultant of these two forces on the object is zero. As a result the object is at rest as shown in Figure 3.23.

If some external force F_{ext} is applied on the object parallel to the surface on which the object is at rest, the surface exerts exactly an equal and opposite force on the object to resist its motion and tries to keep the object at rest. It implies that external force and frictional force are exactly equal and opposite. Therefore, no motion parallel to the surface takes place. But if the external force is increased, after a particular limit, the surface cannot provide sufficient opposing frictional force to balance the external force on the object. Then the object starts to slide. This is the maximal static friction that can be exerted by the surface. Experimentally, it is found that the magnitude of static frictional force f_s satisfies the following empirical relation.

$$0 \leq f_s \leq \mu_s N, \qquad (3.27)$$

where μ_s is the coefficient of static friction. It depends on the nature of the surfaces in contact. N is normal force exerted by the surface on the body and sometimes it is equal to mg. But it need not be equal to *mg* always.

Equation (3.27) implies that the force of static friction can take any value from zero to $\mu_s N$.

If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero ($f_s = 0$).

If the object is at rest, and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object ($f_s = F_{ext}$). But still the static friction f_s is less than $\mu_s N$.

When object begins to slide, the static friction (f_s) acting on the object attains maximum,

The static and kinetic frictions (which we discuss later) depend on the normal force acting on the object. If the object is pressed hard on the surface then the normal force acting on the object will increase. As a consequence it is more difficult to move the object. This is shown in Figure 3.23 (a) and (b). The static friction does not depend upon the area of contact.

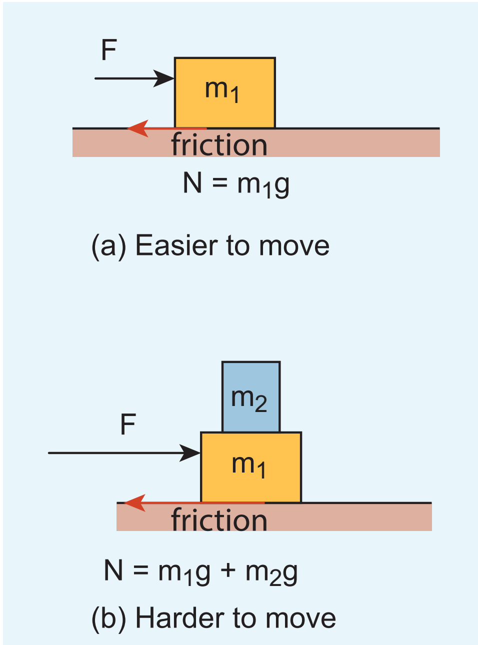


Fig 3.23 Static friction and kinetic friction (a) Easier to move (b) Harder to move

EXAMPLE 3.17

Consider an object of mass 2 kg resting on the floor. The coefficient of static friction between the object and the floor is $\mu_s = 0.8$. What force must be applied on the object to move it?

Solution

Since the object is at rest, the gravitational force experienced by an object is balanced by normal force exerted by floor.

$$N = mg$$

The maximum static frictional force $f_s^{max} = \mu_s N = \mu_s mg$

$$f_s^{max} = 0.8 \times 2 \times 9.8 = 15.68\,N$$

Therefore to move the object the external force should be greater than maximum static friction.

$$F_{ext} > 15.68\,N$$

EXAMPLE 3.18

Consider an object of mass 50 kg at rest on the floor. A Force of 5 N is applied on the object but it does not move. What is the frictional force that acts on the object?

Solution

When the object is at rest, the external force and the static frictional force are equal and opposite.

The magnitudes of these two forces are equal, $f_s = F_{ext}$

Therefore, the static frictional force acting on the object is

$$f_s = 5N.$$

The direction of this frictional force is opposite to the direction of F_{ext}.

EXAMPLE 3.19

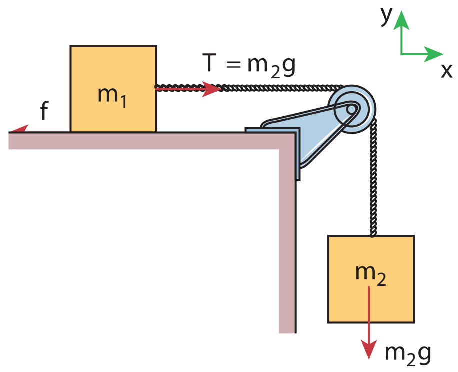
Two bodies of masses 7 kg and 5 kg are connected by a light string passing over a smooth pulley at the edge of the table as shown in the figure. The coefficient of static friction between the surfaces (body and table) is 0.9. Will the mass $m_1 = 7$ kg on the surface move? If not what value of m_2 should be used so that mass 7 kg begins to slide on the table?

Solution

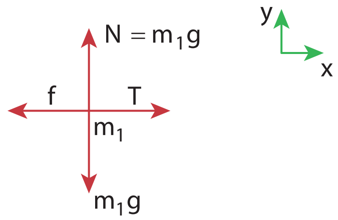
As shown in the figure, there are four forces acting on the mass m_1

a) Downward gravitational force along the negative y-axis (m_1g)
b) Upward normal force along the positive y axis (N)
c) Tension force due to mass m_2 along the positive x axis
d) Frictional force along the negative x axis

Since the mass m_1 has no vertical motion, $m_1g = N$



Free body diagram for mass m_1



To determine whether the mass m_1 moves on the surface, calculate the maximum static friction exerted by the table on the mass m_1. If the tension on the mass m_1 is equal to or greater than this maximum static friction, the object will move.

$$f_s^{max} = \mu_s N = \mu_s m_1 g$$

$$f_s^{max} = 0.9X7X9.8 = 61.74\ N$$

The tension $T = m_2 g = 5X9.8 = 49\ N$

$$T < f_s^{max}$$

The tension acting on the mass m_1 is less than the maximum static friction. So the mass m_1 will not move.

To move the mass m_1, $T > f_s^{max}$ where $T = m_2 g$

$$m_2 = \frac{\mu_s m_1 g}{g} = \mu_s m_1$$

$$m_2 = 0.9X7 = 6.3\ kg$$

If the mass m_2 is greater than 6.3 kg then the mass m_1 will begin to slide. Note that if there is no friction on the surface, the mass m_1 will move for m_2 even for just 1 kg.

The values of coefficient of static friction for pairs of materials are presented in Table 3.1. Note that the ice and ice pair have very low coefficient of static friction. This means a block of ice can move easily over another block of ice.

Table 3.1 Coefficient of Static Friction for a Pair of Materials

Material	Coefficient of Static Friction
Glass and glass	1.0
Ice and ice	0.10
Steel and steel	0.75
Wood and wood	0.35
Rubber tyre and dry concrete road	1.0
Rubber tyre and wet road	0.7

3.6.3 Kinetic Friction

If the external force acting on the object is greater than maximum static friction, the objects begin to slide. When an object slides, the surface exerts a frictional force called **kinetic friction** $\vec{f}_k$ (also called sliding friction or dynamic friction). To move an object at constant velocity we must apply a force which is equal in magnitude and opposite to the direction of kinetic friction.

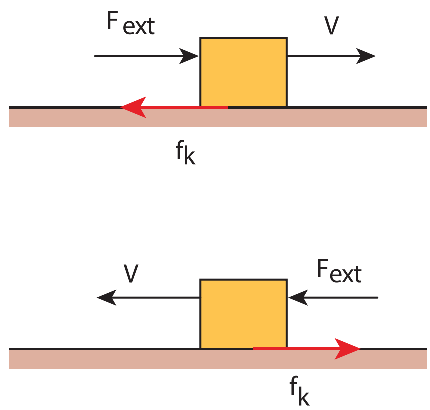


Figure 3.24 Kinetic friction

Experimentally it was found that the magnitude of kinetic friction satisfies the relation

$$f_k = \mu_k N \qquad (3.28)$$

where μ_k is the coefficient of kinetic friction and N the normal force exerted by the surface on the object,

and $$\mu_k < \mu_s$$

This implies that starting of a motion is more difficult than maintaining it. The salient features of static and kinetic friction are given in Table 3.2.

Table 3.2 Salient Features of Static and Kinetic Friction

Static friction	Kinetic friction
It opposes the starting of motion	It opposes the relative motion of the object with respect to the surface
Independent of surface of contact	Independent of surface of contact
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of applied force	Independent of magnitude of applied force
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$ whatever be the speed (true <10 ms^{-1})
$f_s^{max} > f_k$	It is less than maximal value of static friction
$\mu_s > \mu_k$	Coefficient of kinetic friction is less than coefficient of static friction

The variation of both static and kinetic frictional forces with external applied force is graphically shown in Figure 3.25.

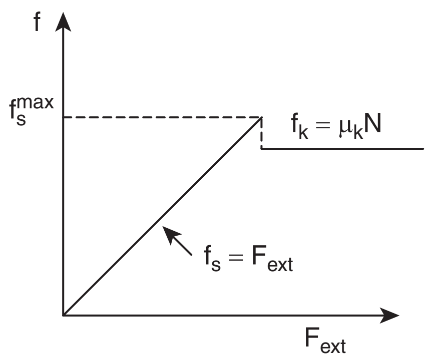


Figure 3.25 Variation of static and kinetic frictional forces with external applied force

The Figure 3.25 shows that static friction increases linearly with external applied force till it reaches the maximum. If the object begins to move then the kinetic friction is slightly lesser than the maximum static friction. Note that the kinetic friction is constant and it is independent of applied force.

Note The relation $f_s = \mu_s N$ is not a vector relation. This is because the normal force N and f_s are not in the same direction even though f_s is equal to μ_s times the normal force. This is also true in the case of kinetic friction.

3.6.4 To Move an Object - Push or pull? Which is easier?

When a body is pushed at an arbitrary angle θ $\left(0 \text{ to } \frac{\pi}{2}\right)$, the applied force F can be resolved into two components as F sinθ parallel to the surface and F cosθ perpendicular to the surface as shown in Figure 3.26. The total downward force acting on the body is mg + Fcosθ. It implies *that* the normal force acting on the body increases. Since there is no acceleration along the vertical direction the normal force N is equal to

$$N_{push} = mg + F\cos\theta \qquad (3.29)$$

As a result the maximal static friction also increases and is equal to

$$f_s^{max} = \mu_s N_{push} = \mu_s\left(mg + F\cos\theta\right) \qquad (3.30)$$

Equation (3.30) shows that a greater force needs to be applied to push the object into motion.

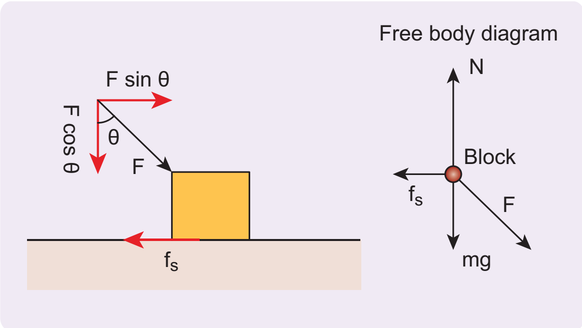


Figure 3.26 An object is pushed at an angle θ

When an object is pulled at an angle θ, the applied force is resolved into two components as shown in Figure 3.27. The total downward force acting on the object is

$$N_{pull} = mg - F\cos\theta \qquad (3.31)$$

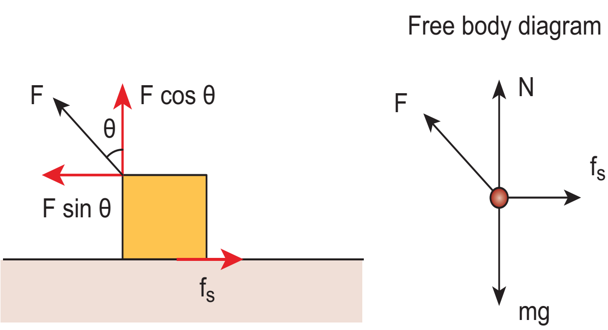


Figure 3.27 An object is pulled at an angle θ

Equation (3.31) shows that the normal force is less than N_{push}. From equations (3.29) and (3.31), it is easier to pull an object than to push to make it move.

3.6.5 Angle of Friction

The angle of friction is defined as the angle between the normal force (N) and the resultant force (R) of normal force and maximum friction force (f_s^{max})

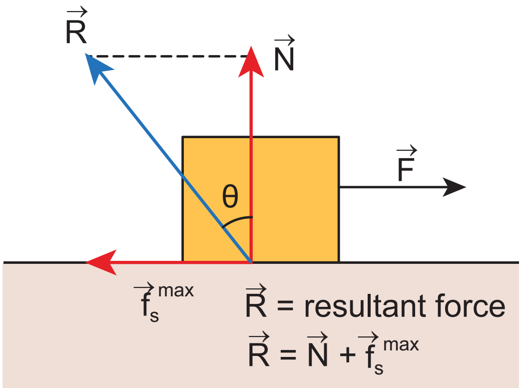


Figure 3.28 Angle of Friction

In Figure 3.28 the resultant force is

$$R = \sqrt{\left(f_s^{max}\right)^2 + N^2}$$

$$\tan\theta = \frac{f_s^{max}}{N} \qquad (3.32)$$

But from the frictional relation, the object begins to slide when $f_s^{max} = \mu_s N$

$$\text{or when } \frac{f_s^{max}}{N} = \mu_s \qquad (3.33)$$

From equations (3.32) and (3.33) the coefficient of static friction is

$$\mu_s = \tan\theta \qquad (3.34)$$

The coefficient of static friction is equal to tangent of the angle of friction

3.6.6 Angle of Repose

Consider an inclined plane on which an object is placed, as shown in Figure 3.30. Let the angle which this plane makes with the horizontal be θ. For small angles of θ, the object may not slide down. As θ is increased, for a particular value of θ, the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

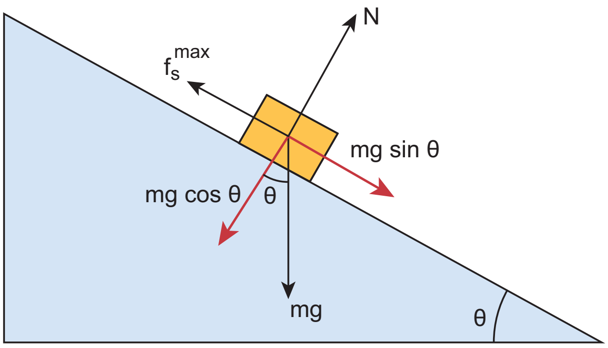


Figure 3.29 Angle of repose

Let us consider the various forces in action here. The gravitational force mg is resolved into components parallel $(mg\sin\theta)$ and perpendicular $(mg\cos\theta)$ to the inclined plane.

The component of force parallel to the inclined plane (mg sinθ) tries to move the object down.

The component of force perpendicular to the inclined plane (mg cosθ) is balanced by the Normal force (N).

$$N = mg\cos\theta$$

When the object just begins to move, the static friction attains its maximum value

$$f_s = f_s^{max} = \mu_s N = \mu_s mg\cos\theta \qquad (3.35)$$

This friction also satisfies the relation

$$f_s^{max} = mg\sin\theta \qquad (3.36)$$

Equating the right hand side of equations (3.35) and (3.36), we get

$$\mu_s = \sin\theta / \cos\theta$$

From the definition of angle of friction, we also know that

$$\tan\theta = \mu_s, \qquad (3.37)$$

in which θ is the angle of friction.

Thus the angle of repose is the same as angle of friction. But the difference is that the angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

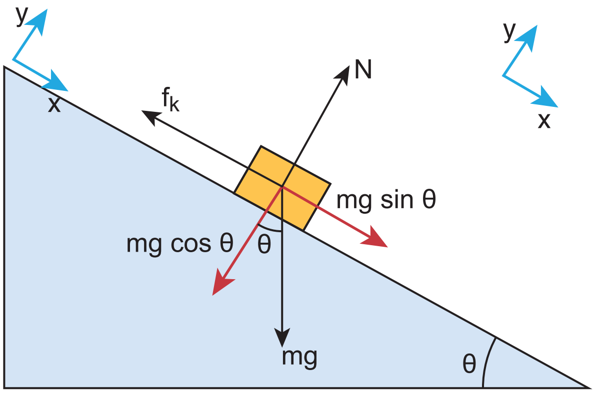
EXAMPLE 3.20

A block of mass m slides down the plane inclined at an angle 60° with an acceleration $\frac{g}{2}$. Find the coefficient of kinetic friction?

Solution

Kinetic friction comes to play as the block is moving on the surface.

The forces acting on the mass are the normal force perpendicular to surface, downward gravitational force and kinetic friction f_k along the surface.



Along the x-direction

$$mg\sin\theta - f_k = \text{ma}$$

But a = g/2

$$mg\sin 60° - f_k = \text{mg/2}$$

$$\frac{\sqrt{3}}{2}\text{mg} - f_k = \text{mg/2}$$

$$f_k = mg\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$f_K = \left(\frac{\sqrt{3}-1}{2}\right)\text{mg}$$

There is no motion along the y-direction as normal force is exactly balanced by the mg $\cos\theta$.

$$\text{mg}\cos\theta = \text{N} = \text{mg/2}$$

$$f_K = \mu_K \text{N} = \mu_K \text{mg/2}$$

$$\mu_K = \frac{\left(\frac{\sqrt{3}-1}{2}\right)mg}{\frac{mg}{2}}$$

$$\mu_K = \sqrt{3} - 1$$

3.6.7 Application of Angle of Repose

1. Antlions make sand traps in such a way that when an insect enters the edge of the trap, it starts to slide towards the bottom where the antilon hide itself. The angle of inclination of sand trap is made to be equal to angle of repose. It is shown in the Figure 3.30.

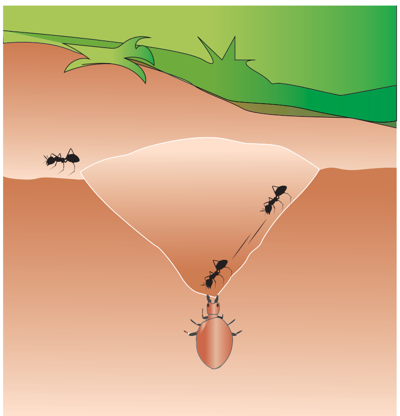


Figure 3.30 Sand trap of antlions

2. Children are fond of playing on sliding board (Figure 3.31). Sliding will be easier

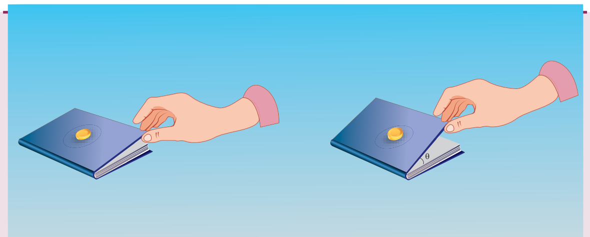
when the angle of inclination of the board is greater than the angle of repose. At the same time if inclination angle is much larger than the angle of repose, the slider will reach the bottom at greater speed and get hurt.

Figure 3.31 Sliding board

ACTIVITY

Measuring the coefficients of friction

Take a hard bound note book and a coin. Keep the coin on the note book. The note book cover has to be in an inclined position as shown in the figure. Slowly increase the angle of inclination of the cover with respect to rest of the pages. When the angle of inclination reaches the angle of repose, the parallel component of gravitational force (mg sinθ) to book surface becomes equal to the frictional force and the coin begins to slide down. Measure the angle of inclination and take the tangent of this angle. It gives the coefficient of static friction between the surface of the cover and coin. The same can be repeated with other objects such as an eraser in



order to observe that the coefficient of static friction differs from case to case.

Note At the point of sliding $\tan\theta_s = \mu_s$

To measure the coefficient of kinetic friction, reduce the inclination of the book after it starts sliding, such that the coin/eraser moves with uniform velocity. Now measure the angle from which coefficient of kinetic friction can be calculated as

$$\mu_K = \tan\theta_k$$

Observe that $\theta_K < \theta_S$

3.6.8 Rolling Friction

The invention of the wheel plays a crucial role in human civilization. One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage. When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion. In rolling motion when a wheel moves on a surface, the point of contact with surface is always at rest. Since the point of contact is at rest, there is no relative motion between the wheel and surface. Hence the frictional force is very less. At the same time if an object moves without a wheel, there is a relative motion between the object and the surface. As a result frictional force is larger. This makes it difficult to move the object. The Figure 3.32 shows the difference between rolling and kinetic friction.

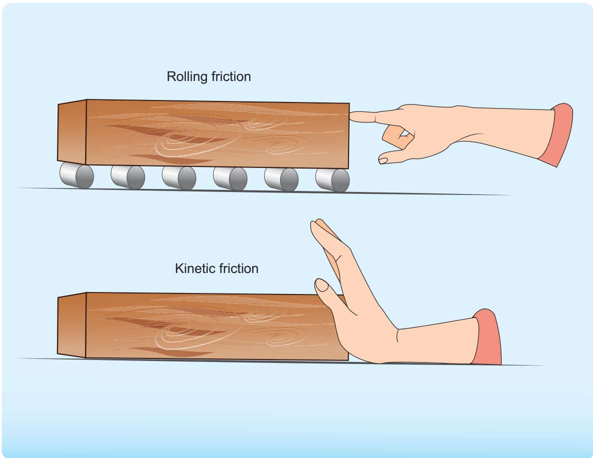


Figure 3.32 Rolling and kinetic friction

Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so. Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface as shown in Figure 3.33. Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'. In fact, 'rolling friction' is much smaller than kinetic friction.

Figure 3.33 Rolling friction

3.6.9 Methods to Reduce Friction

Frictional force has both positive and negative effects. In some cases it is absolutely necessary. Walking is possible because of frictional force. Vehicles (bicycle, car) can move because of the frictional force between the tyre and the road. In the braking system, kinetic friction plays a major role. As we have already seen, the frictional force comes into effect whenever there is relative motion between two surfaces. In big machines used in industries, relative motion between different parts of the machine produce unwanted heat which reduces its efficiency. To reduce this kinetic friction lubricants are used as shown in Figure 3.34.

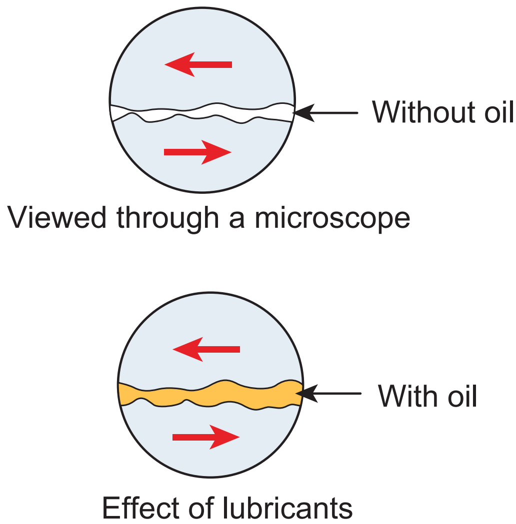
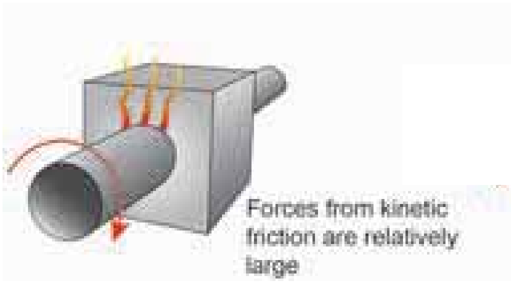


Figure 3.34 Reducing kinetic friction using lubricant

Ball bearings provides another effective way to reduce the kinetic friction (Figure 3.35) in machines. If ball bearings are fixed between two surfaces, during the relative motion only the rolling friction comes to effect and not kinetic friction. As we have seen earlier, the rolling friction is much smaller than kinetic friction; hence the machines are protected from wear and tear over the years.



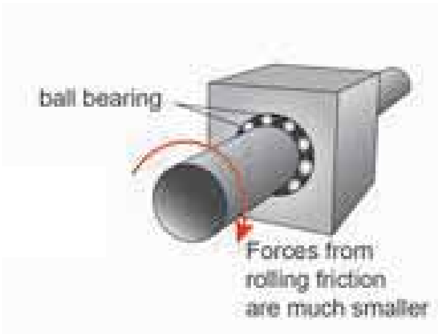


Figure 3.35 Reducing kinetic friction using ball bearing

During the time of Newton and Galileo, frictional force was considered as one of the natural forces like gravitational force. But in the twentieth century, the understanding on atoms, electron and protons has changed the perspective. The frictional force is actually the electromagnetic force between the atoms on the two surfaces. Even well polished surfaces have irregularities on the surface at the microscopic level as seen in the Figure 3.36.

POINTS TO PONDER

When you walk on the tiled floor where water is spilled, you are likely to slip. Why?

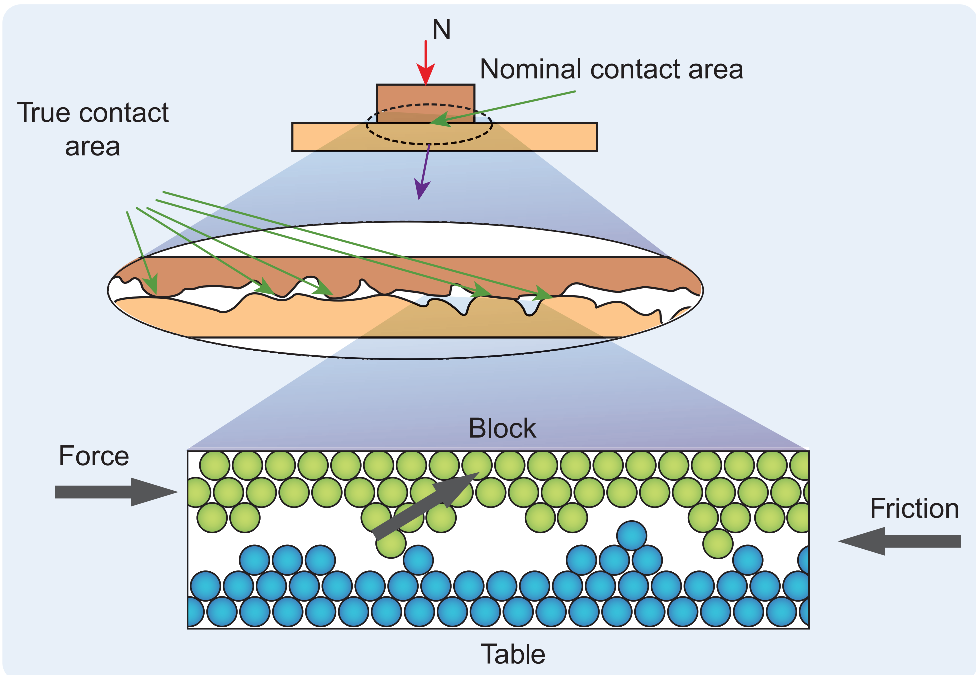
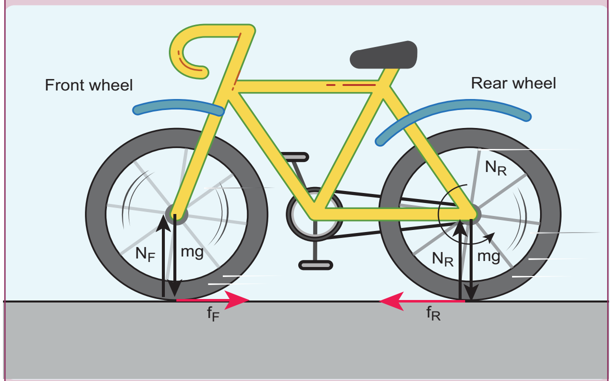


Figure 3.36 Irregularities on the surface at the microscopic level

Frictional force in the motion of a bicycle

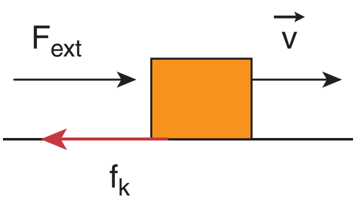
When a bicycle moves in the forward direction, what is the direction of frictional force in the rear and front wheels?



When we pedal a bicycle, we try to push the surface backward and the velocity of point of contact in the rear wheel is backwards. So, the frictional force pushes the rear wheel to move forward. But as the front wheel is connected with a rigid support to the back wheel, the forward motion of back wheel pushes the front wheel in the forward direction. So, the frictional forces act backward. Remember both frictional forces correspond to only static friction and not kinetic friction. If the wheel slips then kinetic friction comes into effect. In addition to static friction, the rolling friction also acts on both wheels in the backward direction.

EXAMPLE 3.21

Consider an object moving on a horizontal surface with a constant velocity. Some external force is applied on the object to keep the object moving with a constant velocity. What is the net force acting on the object?



Solution

If an object moves with constant velocity, then it has no acceleration. According to Newton's second law there is no net force acting on the object. The external force is balanced by the kinetic friction.

Note It is not that 'no force acts on the object'. In fact there are two forces acting on the object. Only the net force acting on the object is zero.

3.7 DYNAMICS OF CIRCULAR MOTION

In the previous sections we have studied how to analyse linear motion using Newton's laws. It is also important to know how to apply Newton's laws to circular motion, since circular motion is one of the very common types of motion that we come across in our daily life. A particle can be in linear motion with or without any external force. But when circular motion occurs there must necessarily be some force acting on the object. There is no Newton's first law for circular motion. In other words without a force, circular motion cannot occur in nature. A force can change the velocity of a particle in three different ways.

1. The magnitude of the velocity can be changed without changing the direction of the velocity. In this case the particle will move in the same direction but with acceleration.

Examples

Particle falling down vertically, bike moving in a straight road with acceleration.

2. The direction of motion alone can be changed without changing the magnitude (speed). If this happens continuously then we call it 'uniform circular motion'.
3. Both the direction and magnitude (speed) of velocity can be changed. If this happens non circular motion occurs. For example oscillation of a swing or simple pendulum, elliptical motion of planets around the Sun.

In this section we will deal with uniform circular motion and non-circular motion.

3.7.1 Centripetal force

If a particle is in uniform circular motion, there must be centripetal acceleration towards the center of the circle. If there is acceleration then there must be some force acting on it with respect to an inertial frame. This force is called centripetal force.

As we have seen in chapter 2, the centripetal acceleration of a particle in the circular motion is given by $a = \frac{v^2}{r}$ and it acts towards center of the circle. According to Newton's second law, the centripetal force is given by

$$F_{cp} = ma_{cp} = \frac{mv^2}{r}$$

The word Centripetal force means center seeking force.

In vector notation $\vec{F}_{cp} = -\frac{mv^2}{r}\hat{r}$

For uniform circular motion $\vec{F}_{cp} = -m\omega^2 r\,\hat{r}$

The direction $-\hat{r}$ points towards the center of the circle which is the direction of centripetal force as shown in Figure 3.38.

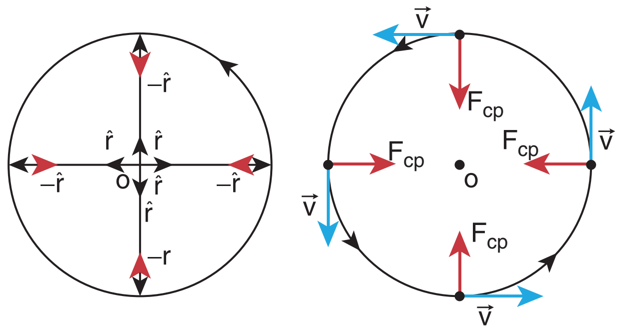


Figure 3.38 Centripetal force

It should be noted that 'centripetal force' is not other forces like gravitational force or spring force. It can be said as 'force towards center'. The origin of the centripetal force can be gravitational force, tension in the string, frictional force, Coulomb force etc. Any of these forces can act as a centripetal force.

1. In the case of whirling motion of a stone tied to a string, the centripetal force on the particle is provided by the tensional force on the string. In circular motion in an amusement park, the centripetal force is provided by the tension in the iron ropes.
2. In motion of satellites around the Earth, the centripetal force is given by Earth's gravitational force on the satellites. Newton's second law for satellite motion is

$$F = \text{earth's gravitational force} = \frac{mv^2}{r}$$

Where r - distance of the planet from the center of the Earth.

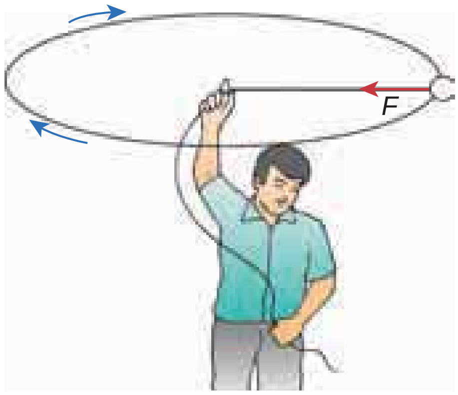


Figure 3.39 Whirling motion of objects

m-mass of the satellite

v-speed of the satellite

3. When a car is moving on a circular track the centripetal force is given by the frictional force between the road and the tyres.

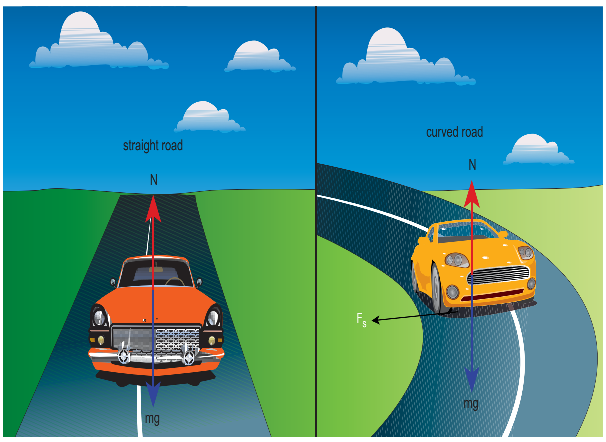


Figure 3.40 Car in the circular track

Newton's second law for this case is

$$\text{Frictional force} = \frac{mv^2}{r}$$

m-mass of the car

v-speed of the car

r-radius of curvature of track

Even when the car moves on a curved track, the car experiences the centripetal force which is provided by frictional force between the surface and the tyre of the car. This is shown in the Figure 3.41.

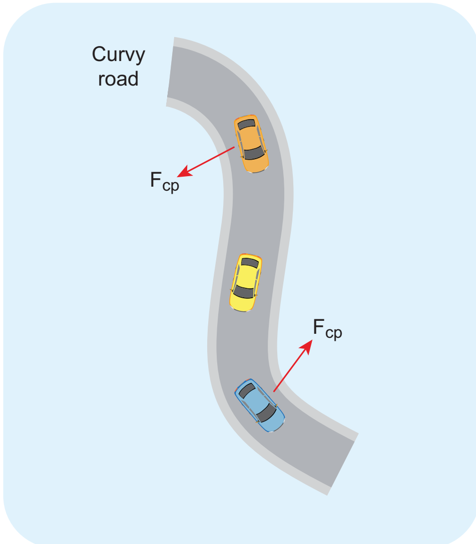


Figure 3.41 Centripetal force due to frictional force between the road and tyre

4. When the planets orbit around the Sun, they experience centripetal force towards the center of the Sun. Here gravitational force of the Sun acts as centripetal force on the planets as shown in Figure 3.42

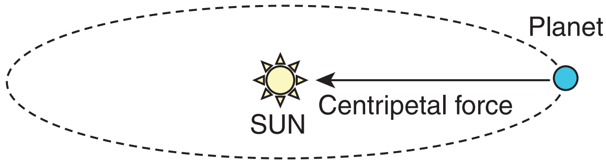


Figure 3.42 Centripetal force on the orbiting planet due Sun's gravity

Newton's second law for this motion

Gravitational force of Sun on the planet $= \frac{mv^2}{r}$

EXAMPLE 3.22

If a stone of mass 0.25 kg tied to a string executes uniform circular motion with a speed of 2 m s^{-1} of radius 3 m, what is the magnitude of tensional force acting on the stone?

Solution

$$F_{cp} = \frac{\frac{1}{4}\times(2)^2}{3} = 0.333 \text{ N.}$$

EXAMPLE 3.23

The Moon orbits the Earth once in 27.3 days in an almost circular orbit. Calculate the centripetal acceleration experienced by the Moon? (Radius of the Earth is 6.4×10^6 m)

Solution

The centripetal acceleration is given by $a = \frac{v^2}{r}$. This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula

$$\omega^2 R_m = a_m$$

a_m is centripetal acceleration of the Moon due to Earth's gravity.

ω is angular velocity.

R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

$$R_m = 60R = 60\times 6.4\times 10^6 = 384\times 10^6\ m$$

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and T = 27.3 days = 27.3 × 24 × 60 × 60 second = 2.358×10^6 sec

By substituting these values in the formula for acceleration

$$a_m = \frac{\left(4\pi^2\right)\left(384\times 10^6\right)}{\left(2.358\times 10^6\right)^2} = 0.00272 \text{ m s}^{-2}$$

The centripetal acceleration of Moon towards the Earth is 0.00272 m s^{-2}

Note: This result was calculated by Newton himself. In unit 6 we will use this result.

3.7.2 Vehicle on a leveled circular road

When a vehicle travels in a curved path, there must be a centripetal force acting on it. This centripetal force is provided by the frictional force between tyre and surface of the road. Consider a vehicle of mass 'm' moving at a speed 'v' in the circular track of radius 'r'. There are three forces acting on the vehicle when it moves as shown in the Figure 3.43

1. Gravitational force (mg) acting downwards
2. Normal force (mg) acting upwards
3. Frictional force (F_s) acting horizontally inwards along the road

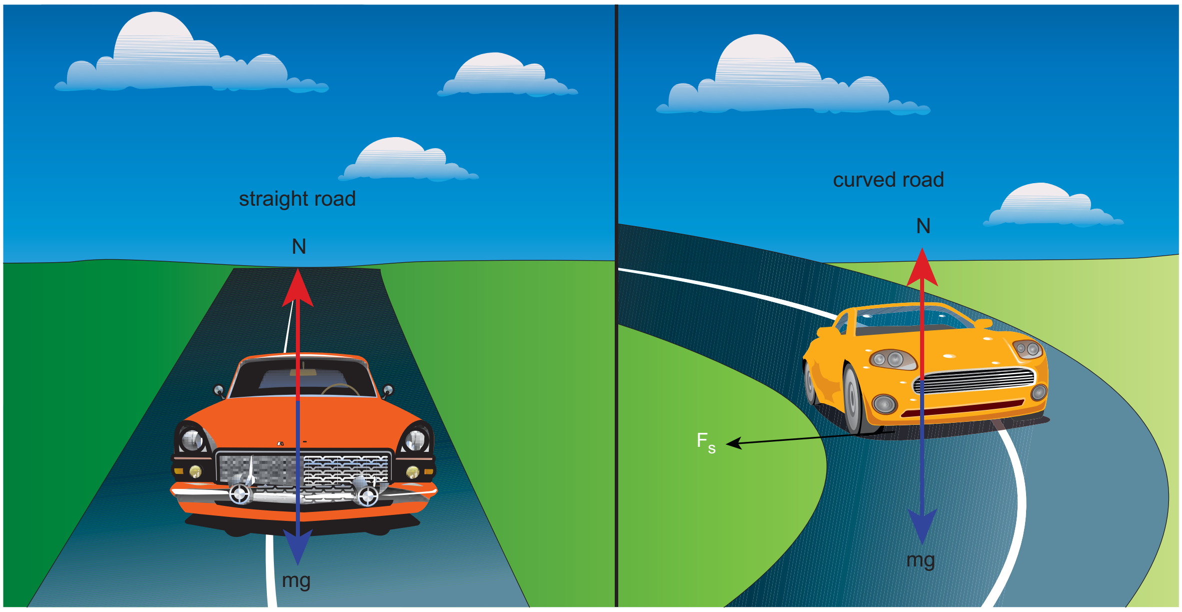


Figure 3.43 Forces acting on the vehicle on a leveled circular road

Suppose the road is horizontal then the normal force and gravitational force are exactly equal and opposite. The centripetal force is provided by the force of static friction F_s between the tyre and surface of the road which acts towards the center of the circular track,

$$\frac{mv^2}{r} = F_s$$

As we have already seen in the previous section, the static friction can increase from zero to a maximum value

$$F_s \le \mu_s mg.$$

There are two conditions possible:

a) If $\frac{mv^2}{r} \le \mu_s mg$, or $\mu_s \ge \frac{v^2}{rg}$ or $\sqrt{\mu_s rg} \ge v$
(Safe turn)

The static friction would be able to provide necessary centripetal force to bend the car on the road. So the coefficient of static friction between the tyre and the surface of the road determines what maximum speed the car can have for safe turn.

b) If $\frac{mv^2}{r} > \mu_s mg$, or $\mu_s < \frac{v^2}{rg}$ (skid)

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid.

EXAMPLE 3.24

Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed 7 m s^{-1}, 8 m s^{-1} and 10 ms^{-1} respectively. Which car will skid when it moves in the circular level road? (g =10 m s^{-2})

Solution

From the safe turn condition the speed of the vehicle (v) must be less than or equal to $\sqrt{\mu_s rg}$

$$v \le \sqrt{\mu_s rg}$$

$$\sqrt{\mu_s rg} = \sqrt{0.81 \times 10 \times 10} = 9\, m s^{-1}$$

For Car C, $\sqrt{\mu_s rg}$ is less than v

The speed of car A, B and C are 7 m s^{-1}, 8 m s^{-1} and 10 m s^{-1} respectively. The cars A and B will have safe turns. But the car C has speed 10 m s^{-1} while it turns which exceeds the safe turning speed. Hence, the car C will skid.

3.7.3 Banking of Tracks

In a leveled circular road, skidding mainly depends on the coefficient of static friction μ_s The coefficient of static friction depends on the nature of the surface which has a maximum limiting value. To avoid this problem, usually the outer edge of the road is slightly raised compared to inner edge as shown in the Figure 3.44. This is called banking of roads or tracks. This introduces an inclination, and the angle is called banking angle.

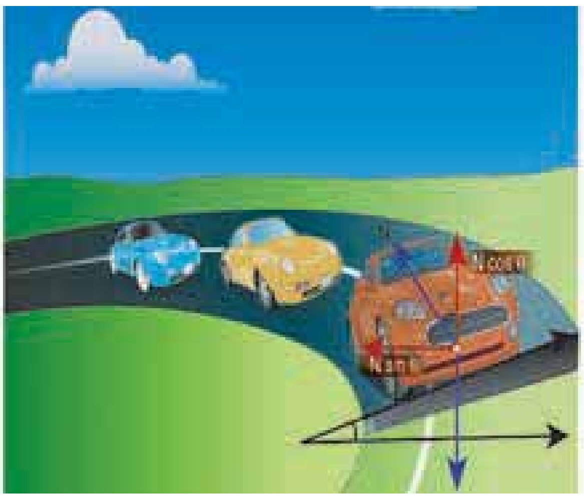


Figure 3.44 Outer edge of the road is slightly raised to avoid skidding

Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical. When the car takes a turn, there are two forces acting on the car:

a) Gravitational force mg (downwards)

b) Normal force N (perpendicular to surface)

We can resolve the normal force into two components. $N\cos\theta$ and $N\sin\theta$ as shown in Figure 3.46. The component $N\cos\theta$ balances the downward gravitational force 'mg' and component $N\sin\theta$ will provide the necessary centripetal acceleration. By using Newton second law

$$N\cos\theta = mg$$

$$N\sin\theta = \frac{mv^2}{r}$$

By dividing the equations we get $\tan\theta = \dfrac{v^2}{rg}$

$$v = \sqrt{rg\tan\theta}$$

The banking angle θ and radius of curvature of the road or track determines the safe speed of the car at the turning. If the speed of car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding. At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding. However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

EXAMPLE 3.25

Consider a circular road of radius 20 meter banked at an angle of 15 degree. With what speed a car has to move on the turn so that it will have safe turn?

Solution

$$v = \sqrt{(rg\tan\theta)} = \sqrt{20 \times 9.8 \times \tan 15^\circ}$$

$$= \sqrt{20 \times 9.8 \times 0.26} = 7.1 \text{ m s}^{-1}$$

The safe speed for the car on this road is 7.1 m s^{-1}

3.7.4 Centrifugal Force

Circular motion can be analysed from two different frames of reference. One is the inertial frame (which is either at rest or in uniform motion) where Newton's laws are obeyed. The other is the rotating frame of reference which is a non-inertial frame of reference as it is accelerating. When we examine the circular motion from these frames of reference the situations are entirely different. To use Newton's first and second laws in the rotational frame of reference, we need to include a pseudo force called 'centrifugal force'. This 'centrifugal force' appears to act on the object with respect to rotating frames. To understand the concept of centrifugal force, we can take a specific case and discuss as done below.

Consider the case of a whirling motion of a stone tied to a string. Assume that the stone has angular velocity ω in the inertial frame (at rest). If the motion of the stone is observed from a frame which is also rotating along with the stone with same angular velocity ω then, the stone appears to be at rest. This implies that in addition to the inward centripetal force $-m\omega^2 r$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$. So the total force acting on the stone in a rotating frame is equal to zero ($-m\omega^2 r + m\omega^2 r = 0$). This outward force $+m\omega^2 r$ is called the centrifugal force. The word 'centrifugal' means 'flee from center'. Note that the 'centrifugal force' appears to act on the particle, only when we analyse the motion from a rotating frame. With respect to an inertial frame there is only centripetal force which is given by the tension in the string. For this reason centrifugal force is called as a 'pseudo force'. A pseudo force has no origin. It arises due to the non inertial nature of the frame considered. When circular motion problems are solved from a rotating frame of reference, while drawing free body diagram of a particle, the centrifugal force should necessarily be included as shown in the Figure 3.45.

3.7.5 Effects of Centrifugal Force

Although centrifugal force is a pseudo force, its effects are real. When a car takes a turn in a curved road, person inside the car feels an outward force which pushes the person away. This outward force is also called centrifugal force. If there is sufficient friction between the person and the seat, it will prevent the person from moving outwards. When a car moving in a straight line suddenly takes a turn, the objects not fixed to the car try to continue in linear motion due to their inertia of direction. While observing this motion from an inertial frame, it appears as a straight line as shown in Figure 3.46. But, when it is observed from the rotating frame it appears to move outwards.

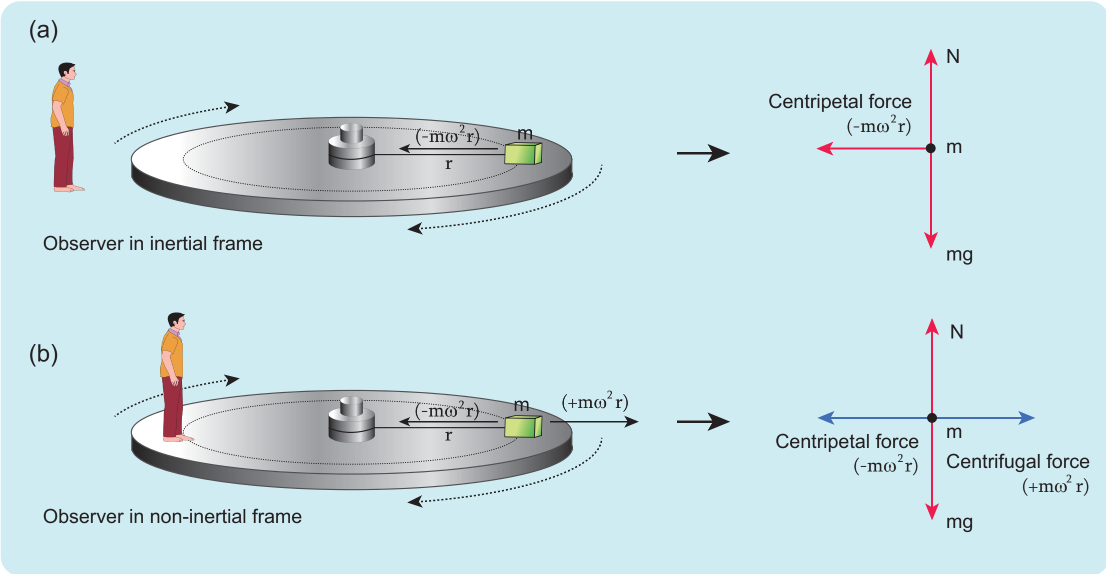


Figure 3.45 Free body diagram of a particle including the centrifugal force

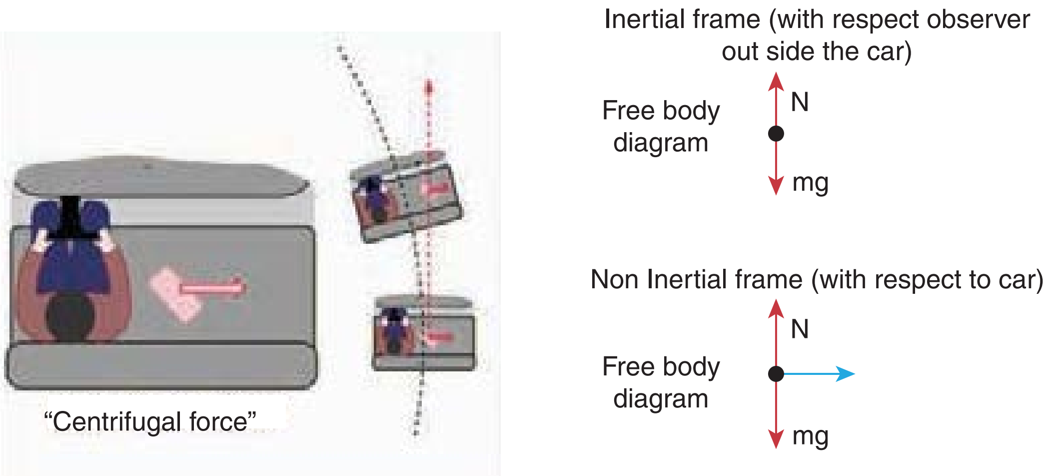


Figure 3.46 Effects of centrifugal force

A person standing on a rotating platform feels an outward centrifugal force and is likely to be pushed away from the platform. Many a time the frictional force between the platform and the person is not sufficient to overcome outward push. To avoid this, usually the outer edge of the platform is little inclined upwards which exerts a normal force on the person which prevents the person from falling as illustrated in Figures 3.47.

Caution!

It is dangerous to stand near the open door (or) steps while travelling in the bus. When the bus takes a sudden turn in a curved road, due to centrifugal force the person is pushed away from the bus. Even though centrifugal force is a pseudo force, its effects are real.

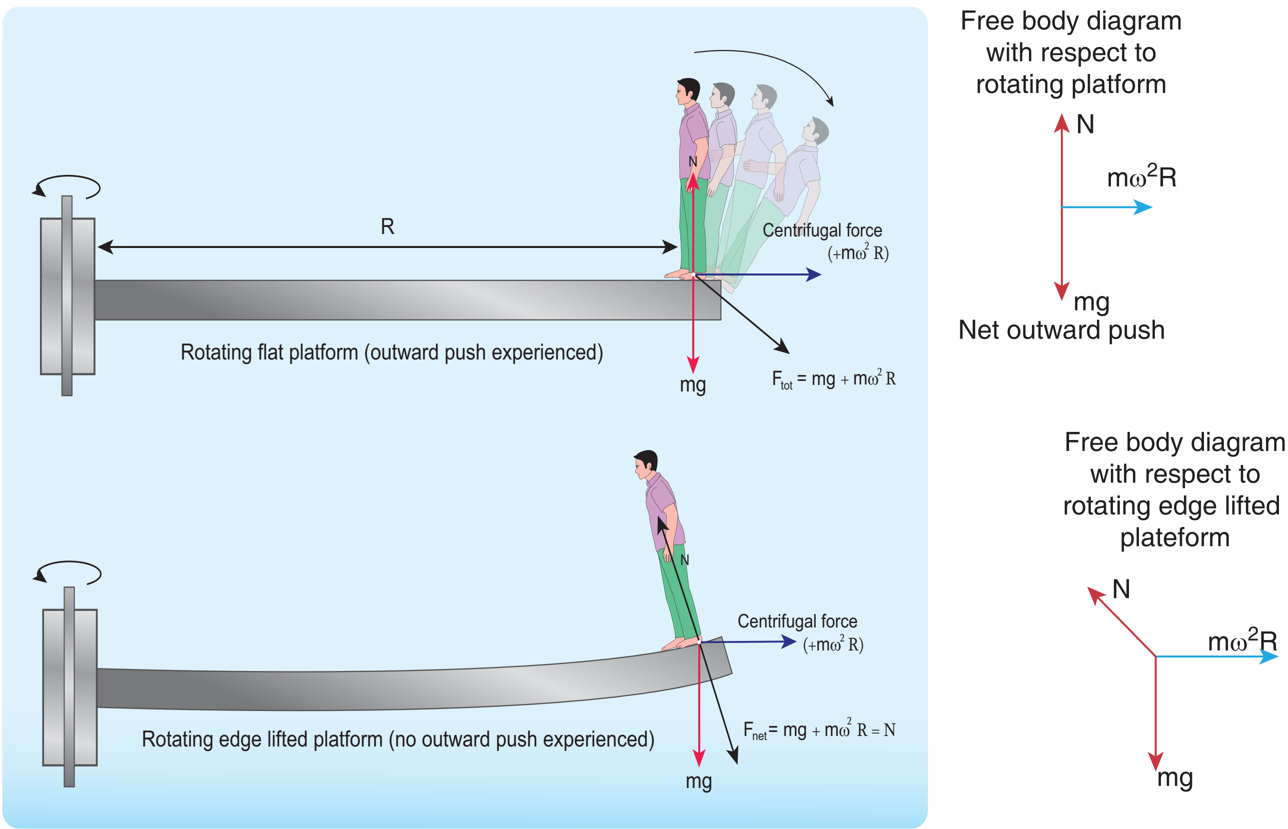


Figure 3.47 Outward centrifugal force in rotating platform

3.7.6 Centrifugal Force due to Rotation of the Earth

Even though Earth is treated as an inertial frame, it is actually not so. Earth spins about its own axis with an angular velocity ω. Any object on the surface of Earth (rotational frame) experiences a centrifugal force. The centrifugal force appears to act exactly in opposite direction from the axis of rotation. It is shown in the Figure 3.48.

The centrifugal force on a man standing on the surface of the Earth is $F_c = m\omega^2 r$

where r is perpendicular distance of the man from the axis of rotation. By using right angle triangle as shown in the Figure 3.48, the distance $r = R\cos\theta$

Here R = radius of the Earth

and θ = latitude of the Earth where the man is standing.

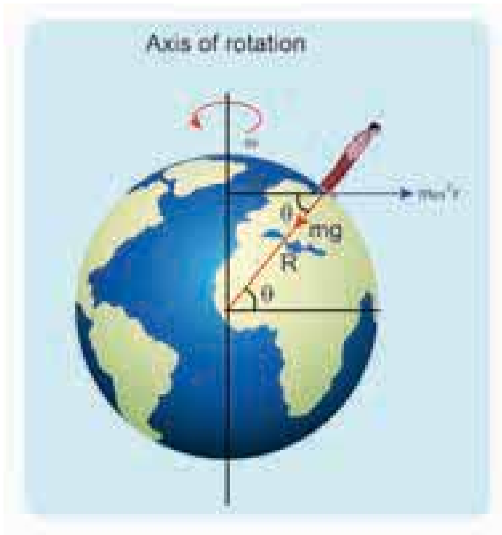


Figure 3. 48 Centrifugal force acting on a man on the surface of Earth

EXAMPLE 3.26

Calculate the centrifugal force experienced by a man of 60 kg standing at Chennai? (Given: Latitude of Chennai is $13°$

Solution

The centrifugal force is given by $F_c = m\omega^2 R\cos\theta$

The angular velocity (ω) of Earth $= \frac{2\pi}{T}$, where T is time period of the Earth (24 hours)

$$\omega = \frac{2\pi}{24\times60\times60} = \frac{2\pi}{86400}$$
$$= 7.268\times10^{-5}\, rad sec^{-1}$$

The radius of the Earth R = 6400 Km = 6400×10^3 m

Latitude of Chennai $= 13°$

$$F_{cf} = 60\times\left(7.268\times10^{-5}\right)^2\times6400\times10^3$$
$$\times\cos\left(13°\right) = 1.9678\,N$$

A 60 kg man experiences centrifugal force of approximately 2 Newton. But due to Earth's gravity a man of 60 kg experiences a force = mg = 60 × 9.8 = 588N. This force is very much larger than the centrifugal force.

3.7.7 Centripetal Force Versus Centrifugal Force

Salient features of centripetal and centrifugal forces are compared in Table 3.4.

Table 3.4 Salient Features of Centripetal and Centrifugal Forces

Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or center of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the center of the circular motion
$\left\|F_{cp}\right\| = m\omega^2 r = \frac{mv^2}{r}$	$\left\|F_{cf}\right\| = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects.	Origin of centrifugal force is inertia. It does not arise from interaction. In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

SUMMARY

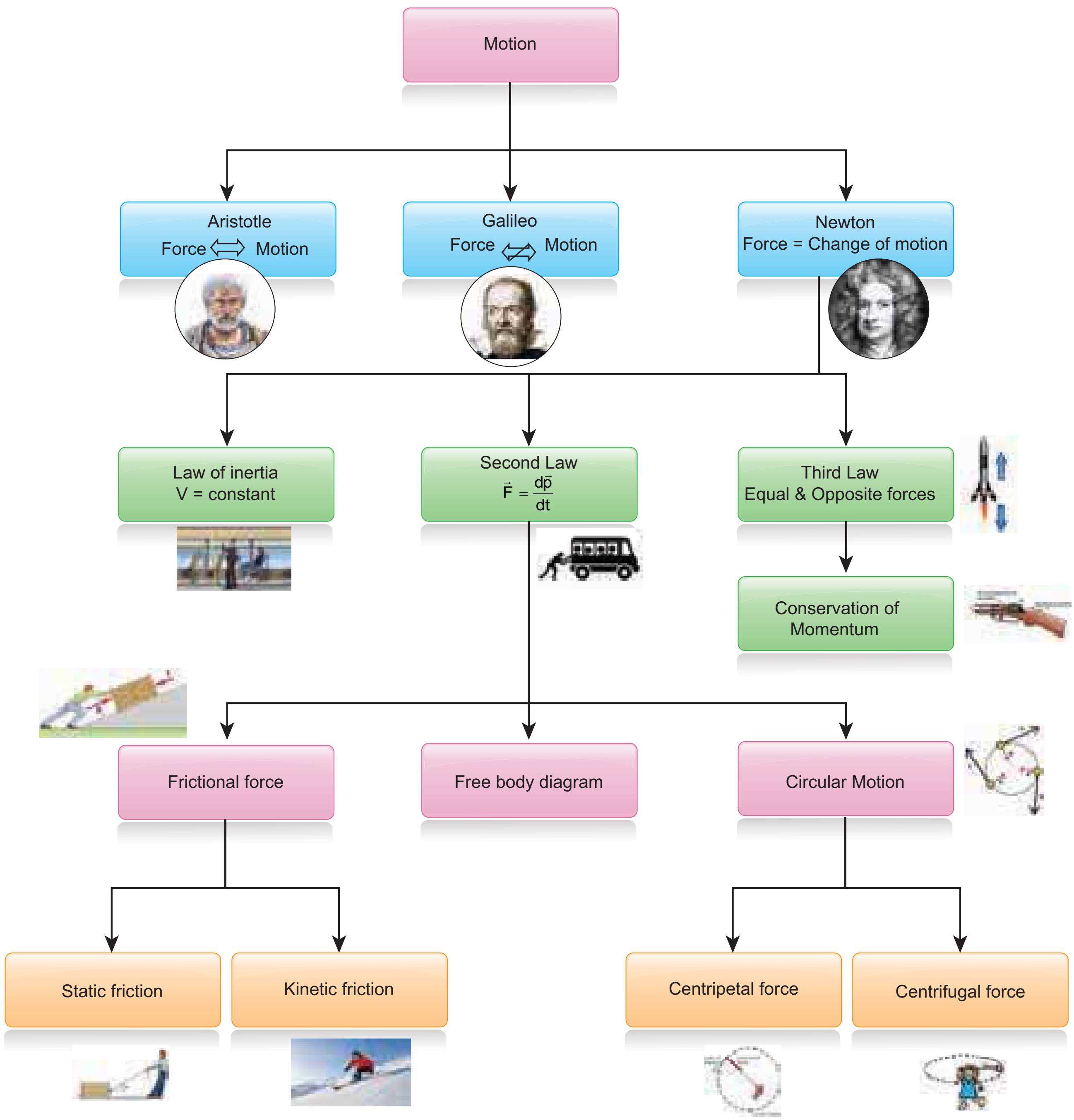
- Aristotle's idea of motion: To maintain motion, a force is required
- Galileo's idea of motion: To maintain motion, a force is not required
- Mass is a measure of inertia of the body
- Newton's first law states that under no external force, the object continues its state of motion
- Newton's second law states that to change the momentum of the body, external force is required

  Mathematically it is defined as $\vec{F} = \frac{d\vec{p}}{dt}$
- Both Newton's first and second laws are valid only in inertial frames
- Inertial frame is the one in which if there is no force on the object, the object moves at constant velocity.
- Newton's third law states that for every force there is an equivalent and opposite force and such a pair of forces is called action and reaction pair.
- To draw a free body diagram for an object,
  - Isolate the object from other objects and identify the forces acting on it
  - The force exerted by that object should not be taken into account
  - Draw the direction of each force with relative magnitude
  - Apply Newton's second law in each direction
- If no net external force acts on a collection of particles (system), then the total momentum of the collection of particles (system) is a constant vector.
- Internal forces acting in the system cannot change the total momentum of the system.
- Lami's theorem states that if an object is in equilibrium under the concurrent forces, then the ratio of each force with the sine of corresponding opposite angle is same.
- An impulse acting on a body is equal to the change in momentum of the body. Whenever a force acts on the object for a very short time, it is difficult to calculate the force. But impulse can be calculated.
- Static friction is the force which always opposes the movement of the object from rest. It can take values from zero to $\mu_s N$. If an external force is greater than $\mu_s N$ then object begins to move.
- If the object begins to move, kinetic friction comes into effect. To move an object with constant velocity, the external force must be applied to overcome the kinetic friction. The kinetic friction is $\mu_k N$.
- Rolling friction is much smaller than static and kinetic friction. This is the reason that to move an object roller coaster is fixed in the bottom of the object. Example: Rolling suitcase

SUMMARY (*cont*)

- The origin of friction is electromagnetic interaction between the atoms of two surfaces which are touching each other.
- Whenever there is a motion along a curve, there must be a centripetal force that acts towards the center of the curve. In uniform circular motion the centripetal force acts at the center of the circle.
- The centripetal force is not a separate natural force. Any natural force can behave as centripetal force. In planetary motion, Sun's gravitational force acts as centripetal force. In the whirling motion of a stone attached to a string, the centripetal force is given by the string. When Moon orbits the Earth, it experiences Earth's gravitational force as centripetal force.
- Centrifugal force arises whenever the motion is analysed from rotating frame. It is a pseudo force. The inertial motion of the object appears as centrifugal force in the rotating frame.
- The magnitude of centrifugal and centripetal force is $m\omega^2 r$. But centripetal force acts towards center of the circular motion and centrifugal force appears to acts in the opposite direction to centripetal force.

CONCEPT MAP


Motion
Aristotle
Force ⟺ Motion
Galileo
Force ⟺ Motion
Newton
Force = Change of motion
Law of inertia
V = constant
Second Law
$\vec{F} = \frac{d\vec{p}}{dt}$
Third Law
Equal & Opposite forces
Conservation of Momentum
Frictional force
Free body diagram
Circular Motion
Static friction
Kinetic friction
Centripetal force
Centrifugal force

EXERCISE

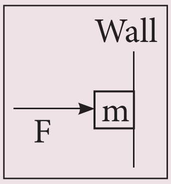
I. Multiple Choice Questions

1. When a car takes a sudden left turn in the curved road, passengers are pushed towards the right due to
   (a) inertia of direction
   (b) inertia of motion
   (c) inertia of rest
   (d) absence of inertia

2. An object of mass m held against a vertical wall by applying horizontal force F as shown in the figure. The minimum value of the force F is

   (IIT JEE 1994)

   (a) Less than mg
   (b) Equal to mg
   (c) Greater than mg
   (d) Cannot determine

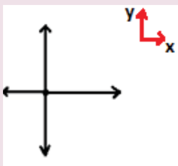


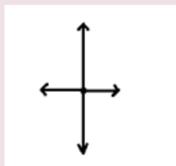
3. A vehicle is moving along the positive x direction, if sudden brake is applied, then
   (a) frictional force acting on the vehicle is along negative x direction
   (b) frictional force acting on the vehicle is along positive x direction
   (c) no frictional force acts on the vehicle
   (d) frictional force acts in downward direction

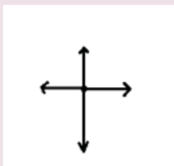
4. A book is at rest on the table which exerts a normal force on the book. If this force is considered as reaction force, what is the action force according to Newton's third law?
   (a) Gravitational force exerted by Earth on the book
   (b) Gravitational force exerted by the book on Earth
   (c) Normal force exerted by the book on the table
   (d) None of the above

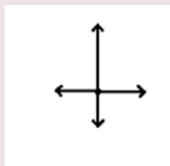
5. Two masses m_1 and m_2 are experiencing the same force where $m_1 < m_2$. The ratio of their acceleration $\frac{a_1}{a_2}$ is
   (a) 1
   (b) less than 1
   (c) greater than 1
   (d) all the three cases

6. Choose appropriate free body diagram for the particle experiencing net acceleration along negative y direction. (Each arrow mark represents the force acting on the system).

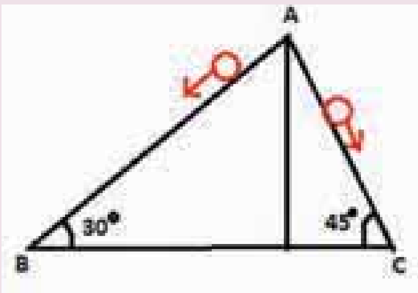
a) 

b) 

c) 

d) 

7. A particle of mass m sliding on the smooth double inclined plane (shown in figure) will experience



(a) greater acceleration along the path AB

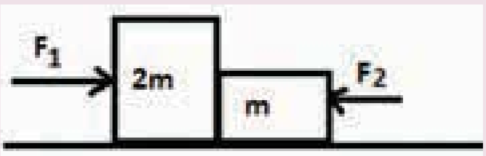
(b) greater acceleration along the path AC

(c) same acceleration in both the paths

(d) no acceleration in both the paths.

8. Two blocks of masses m and 2m are placed on a smooth horizontal surface as shown. In the first case only a force F_1 is applied from the left. Later only a force F_2 is applied from the right. If the force acting at the interface of the two blocks in the two cases is same, then $F_1 : F_2$ is

(Physics Olympiad 2016)



(a) 1:1 (b) 1:2

(c) 2:1 (d) 1:3

9. Force acting on the particle moving with constant speed is

(a) always zero

(b) need not be zero

(c) always non zero

(d) cannot be concluded

10. An object of mass m begins to move on the plane inclined at an angle θ. The coefficient of static friction of inclined surface is μ_s. The maximum static friction experienced by the mass is

(a) mg

(b) $\mu_s$mg

(c) $\mu_s$mg sinθ

(d) μ_s mg cos θ

11. When the object is moving at constant velocity on the rough surface,

(a) net force on the object is zero

(b) no force acts on the object

(c) only external force acts on the object

(d) only kinetic friction acts on the object

12. When an object is at rest on the inclined rough surface,

(a) static and kinetic frictions acting on the object is zero

(b) static friction is zero but kinetic friction is not zero

(c) static friction is not zero and kinetic friction is zero

(d) static and kinetic frictions are not zero

13. The centrifugal force appears to exist

(a) only in inertial frames

(b) only in rotating frames

(c) in any accelerated frame

(d) both in inertial and non-inertial frames

14. Choose the correct statement from the following
    (a) Centrifugal and centripetal forces are action reaction pairs
    (b) Centripetal forces is a natural force
    (c) Centrifugal force arises from gravitational force
    (d) Centripetal force acts towards the center and centrifugal force appears to act away from the center in a circular motion

15. If a person moving from pole to equator, the centrifugal force acting on him
    (a) increases
    (b) decreases
    (c) remains the same
    (d) increases and then decreases

Answers

1) a	**2)** c	**3)** a	**4)** c	**5)** c
6) c	**7)** b	**8)** c	**9)** b	**10)** d
11) a	**12)** c	**13)** b	**14)** d	**15)** a

II. Short Answer Questions

1. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.
2. State Newton's second law.
3. Define one newton.
4. Show that impulse is the change of momentum.
5. Using free body diagram, show that it is easy to pull an object than to push it.
6. Explain various types of friction. Suggest a few methods to reduce friction.
7. What is the meaning by 'pseudo force'?
8. State the empirical laws of static and kinetic friction.
9. State Newton's third law.
10. What are inertial frames?
11. Under what condition will a car skid on a leveled circular road?

III. Long Answer Questions

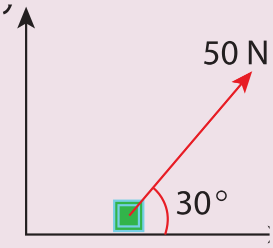
1. Prove the law of conservation of linear momentum. Use it to find the recoil velocity of a gun when a bullet is fired from it.
2. What are concurrent forces? State Lami's theorem.
3. Explain the motion of blocks connected by a string in i) Vertical motion ii) Horizontal motion.
4. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.
5. State Newton's three laws and discuss their significance.
6. Explain the similarities and differences of centripetal and centrifugal forces.
7. Briefly explain 'centrifugal force' with suitable examples.
8. Briefly explain 'rolling friction'.
9. Describe the method of measuring angle of repose.
10. Explain the need for banking of tracks.
11. Calculate the centripetal acceleration of Moon towards the Earth.

IV. Conceptual Questions

1. Why it is not possible to push a car from inside?
2. There is a limit beyond which the polishing of a surface increases frictional resistance rather than decreasing it why?
3. Can a single isolated force exist in nature? Explain your answer.
4. Why does a parachute descend slowly?
5. When walking on ice one should take short steps. Why?
6. When a person walks on a surface, the frictional force exerted by the surface on the person is opposite to the direction of motion. True or false?
7. Can the coefficient of friction be more than one?
8. Can we predict the direction of motion of a body from the direction of force on it?
9. The momentum of a system of particles is always conserved. True or false?

V. Numerical Problems

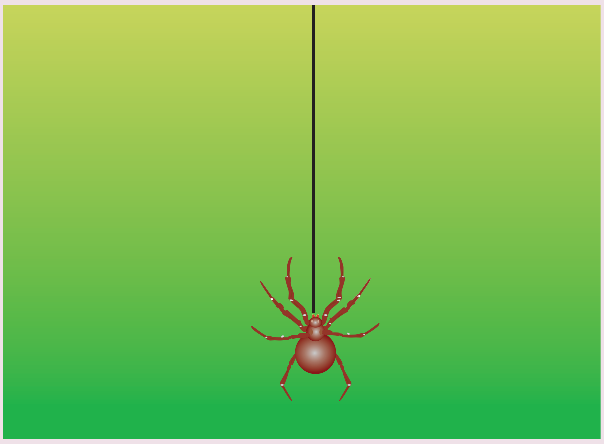
1. A force of 50N act on the object of mass 20 kg. shown in the figure. Calculate the acceleration of the object in x and y directions.



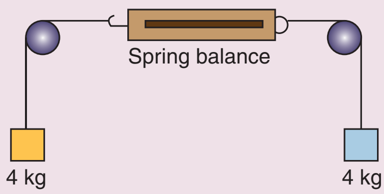
Ans: a_x=2.165 ms^{-2}; a_y=1.25 ms^{-2}

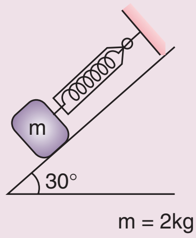
2. A spider of mass 50 g is hanging on a string of a cob web as shown in the figure. What is the tension in the string?

Ans: T=0.49N



3. What is the reading shown in spring balance?



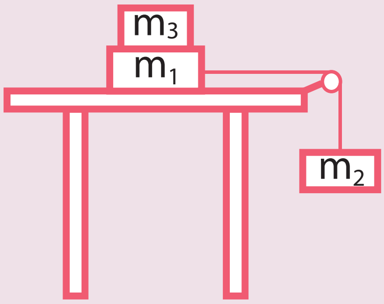


Ans: Zero, 9.8 N

4. The physics books are stacked on each other in the sequence: +1 volumes 1 and 2; +2 volumes 1 and 2 on a table.
   a) Identify the forces acting on each book and draw the free body diagram.
   b) Identify the forces exerted by each book on the other.
5. A bob attached to the string oscillates back and forth. Resolve the forces acting on the bob in to components. What is the acceleration experience by the bob at an angle θ.

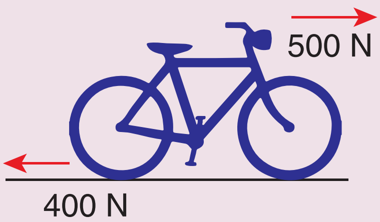
Ans: Tangential acceleration = g sinθ ; centripetal acceleration = $\frac{(T-mg\cos\theta)}{m}$.

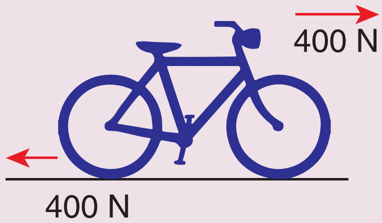
6. Two masses m_1 and m_2 are connected with a string passing over a frictionless pulley fixed at the corner of the table as shown in the figure. The coefficient of static friction of mass m_1 with the table is μ_s. Calculate the minimum mass m_3 that may be placed on m1to prevent it from sliding. Check if m1=15 kg, m2=10 kg, m3=25 and μ_s =0.2



Ans: $m_3 = \frac{m_2}{\mu_s} - m_1$, the combined masses m_1+m_3 will slide.

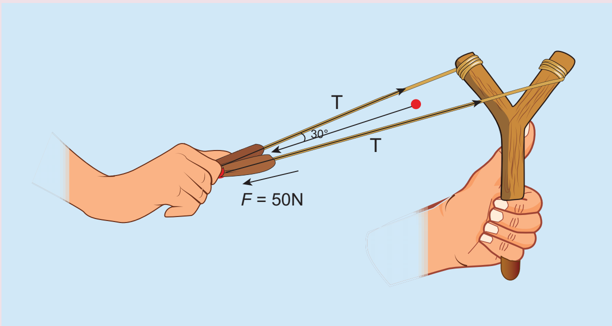
7. Calculate the acceleration of the bicycle of mass 25 kg as shown in Figures 1 and 2.





Ans: a=4 ms^{-2}, zero

8. Apply Lami's theorem on sling shot and calculate the tension in each string ?



Ans: T= 28.868N.

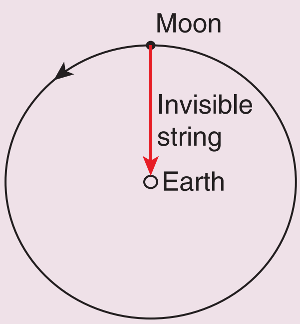
9. A football player kicks a 0.8 kg ball and imparts it a velocity 12 ms^{-1}. The contact between the foot and ball is only for one- sixtieth of a second. Find the average kicking force.

Ans: 576N.

10. A stone of mass 2 kg is attached to a string of length 1 meter. The string can withstand maximum tension 200 N. What is the maximum speed that stone can have during the whirling motion?

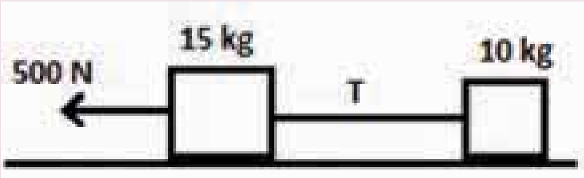
Ans: v_{max}=10ms^{-1}

11. Imagine that the gravitational force between Earth and Moon is provided by an invisible string that exists between the Moon and Earth. What is the tension that exists in this invisible string due to Earth's centripetal force? (Mass of the Moon = 7.34×10^{22} kg, Distance between Moon and Earth = 3.84×10^{8} m)



Ans: $T \approx 2 \times 10^{20} N$.

12. Two bodies of masses 15 kg and 10 kg are connected with light string kept on a smooth surface. A horizontal force F=500 N is applied to a 15 kg as shown in the figure. Calculate the tension acting in the string



Ans: $T = 200N$.

13. People often say "For every action there is an equivalent opposite reaction". Here they meant 'action of a human'. Is it correct to apply Newton's third law to human actions? What is mean by 'action' in Newton third law? Give your arguments based on Newton's laws.

Ans: Newton's third law is applicable to only human's actions which involves physical force. Third law is not applicable to human's psychological actions or thoughts

14. A car takes a turn with velocity 50 ms^{-1} on the circular road of radius of curvature 10 m. calculate the centrifugal force experienced by a person of mass 60kg inside the car?

Ans: 15,000 N

15. A long stick rests on the surface. A person standing 10 m away from the stick. With what minimum speed an object of mass 0.5 kg should he thrown so that it hits the stick. (Assume the coefficient of kinetic friction is 0.7).

Ans: 11.71 ms^{-1}

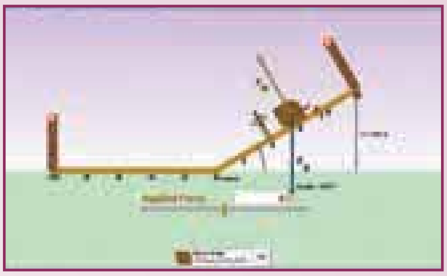
BOOKS FOR REFERENCE

1. Charles Kittel, Walter Knight, Malvin Ruderman, Carl Helmholtz and Moyer, *Mechanics,* 2nd edition, Mc Graw Hill Pvt Ltd,
2. A.P.French, *Newtonian Mechanics,* Viva-Norton Student edition
3. SomnathDatta, *Mechanics,* Pearson Publication
4. H.C.Verma, *Concepts of physics* volume 1 and Volume 2, Bharati Bhawan Publishers
5. Serway and Jewett, *Physics for scientist and Engineers with modern physics*, Brook/Coole publishers, Eighth edition
6. Halliday, Resnick & Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition

ICT CORNER

Force and motion

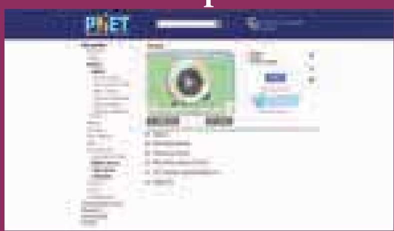
Through this activity you will understand the Force and motion



STEPS:

- Open the browser and type the given URL to open the PhET simulation on force and motion. Click OK to open the java applet.
- Select the values of the applied force to observe the change.
- Observe the change of the ramp angle by changing the position of the object.
- You can also observe the variations in force and ramp angle by changing the weights.

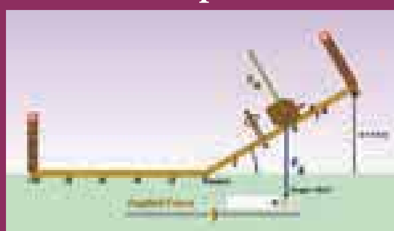
Step1



Step2

Step3

Step4



PhET simulation's URL:

https://phet.colorado.edu/en/simulation/ramp-forces-and-motion

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 4

WORK, ENERGY AND POWER

"Matter is Energy. Energy is Light. We are all Light Beings." – Albert Einstein

LEARNING OBJECTIVES

In this unit, student is exposed to

- definition of work
- work done by a constant and a variable force
- various types of energy
- law of conservation of energy
- vertical circular motion
- definition of power
- various types of collisions

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4.1 INTRODUCTION

The term *work* is used in diverse contexts in daily life. It refers to both physical as well as mental work. In fact, any activity can generally be called as work. But in Physics, the term work is treated as a physical quantity with a precise definition. Work is said to be done by the force when the force applied on a body displaces it. To do work, *energy* is required. In simple words, *energy is defined as the ability to do work*. Hence, work and energy are equivalents and have same dimension. Energy, in Physics exists in different forms such as mechanical, electrical, thermal, nuclear and so on. Many machines consume one form of energy and deliver energy in a different form. In this chapter we deal mainly with mechanical energy and its two types namely kinetic energy and potential energy. The next quantity in this sequence of discussion is the rate of work done or the rate of energy delivered. *The rate of work done is called power*. A powerful strike in cricket refers to a hit on the ball at a fast rate. This chapter aims at developing a good understanding of these three physical quantities namely work, energy and power and their physical significance.

4.1.1 WORK

Let us consider a force ($\vec{F}$), acting on a body which moves it by a displacement in some direction ($d\vec{r}$) as shown in Figure 4.1

The expression for work done (w) by the force on the body is mathematically written as,

$$W = \vec{F} \cdot d\vec{r} \qquad (4.1)$$

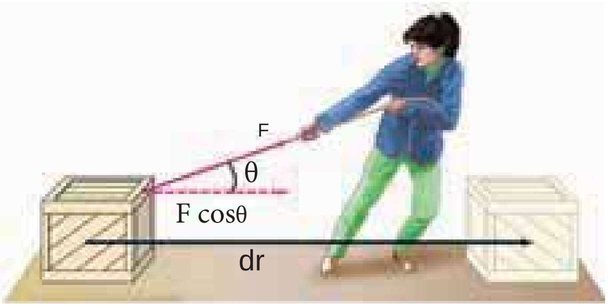

Figure 4.1 Work done by a force

The work done by the force depends on the force (F), displacement (dr) and the angle (θ) between them.

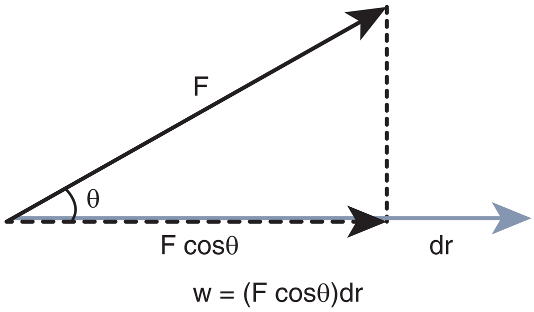

Figure 4.2 Calculating work done.

Here, the product $\vec{F}\cdot d\vec{r}$ is a *scalar product (or dot product)*. The scalar product of two vectors is a scalar (refer section 2.5.1). Thus, work done is a scalar quantity. It has only magnitude and no direction. In SI system, unit of work done is N m (or) joule (J). Its dimensional formula is $[ML^2T^{-2}]$.

The equation (4.1) is,

$$W = F\,dr\cos\theta \quad (4.2)$$

which can be realised using Figure 4.2 (as $\vec{a}\cdot\vec{b} = ab\cos\theta$) where, θ is the angle between applied force and the displacement of the body.

Work done is zero in the following cases.

(i) **When the force is zero (F = 0).** For example, a body moving on a horizontal smooth frictionless surface will continue to do so as no force (not even friction) is acting along the plane. (This is an ideal situation.)

(ii) **When the displacement is zero (dr = 0).** For example, when force is applied on a rigid wall it does not produce any displacement. Hence, the work done is zero as shown in Figure. 4.3(a).

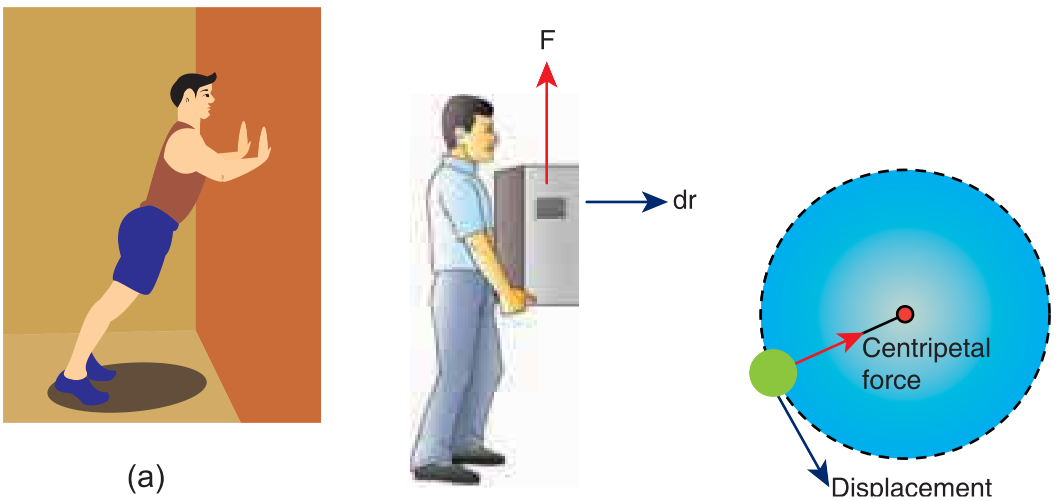

Figure 4.3 Different cases of zero work done

(iii) **When the force and displacement are perpendicular ($\theta = 90^\circ$) to each other.** when a body moves on a horizontal direction, the gravitational force (mg) does no work on the body, since it acts at right angles to the displacement as shown in Figure 4.3(b). In circular motion the centripetal force does not do work on the object moving on a circle as it is always perpendicular to the displacement as shown in Figure 4.3(c).

For a given force (F) and displacement (dr), the angle (θ) between them decides the value of work done as consolidated in Table 4.1.

There are many examples for the negative work done by a force. In a football game, the goalkeeper catches the ball coming towards him by applying a force such that the force is applied in a direction opposite to that of the motion of the ball till it comes to rest in his hands. During the time of applying the force, he does a negative work on the ball as shown in Figure 4.4. We will discuss many more situations of negative work further in this unit.

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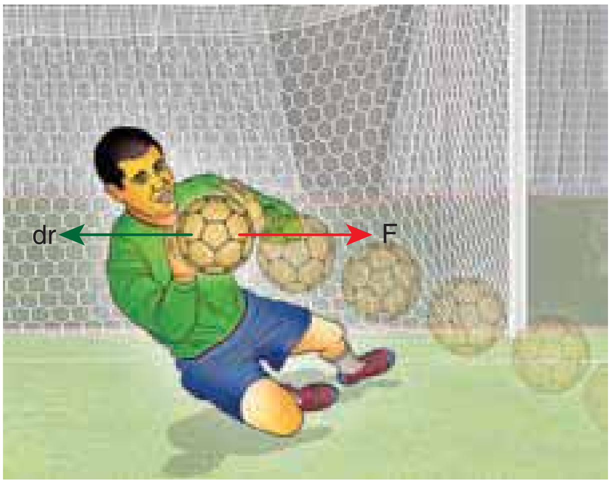
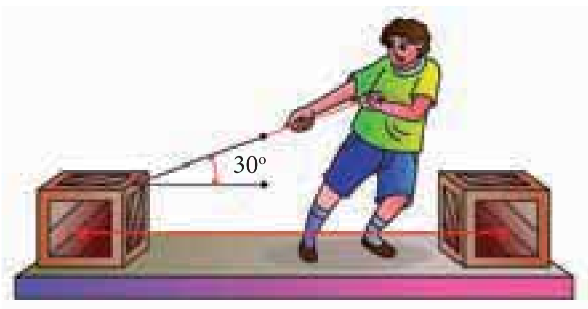


Figure 4.4 Negative work done

EXAMPLE 4.1

A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30°, find the work done by the force.



Solution

Force, F = 25 N

Displacement, dr = 15 m

Angle between F and dr, $\theta = 30^\circ$

Table 4.1 Angle (θ) and the nature of work

Angle (θ)	$\cos\theta$	Work
$\theta = 0^\circ$	1	Positive, Maximum
$0 < \theta < 90^\circ$ (acute)	$0 < \cos\theta < 1$	Positive
$\theta = 90^\circ$ (right angle)	0	Zero
$90^\circ < \theta < 180^\circ$	$-1 < \cos\theta < 0$	Negative
$\theta = 180^\circ$	-1	Negative, Maximum

Work done, $W = F\,dr\cos\theta$

$$W = 25 \times 15 \times \cos 30 = 25 \times 15 \times \frac{\sqrt{3}}{2}$$

$$W = 324.76\ \text{J}$$

4.1.2 Work done by a constant force

When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation,

$$dW = (F\cos\theta)\,dr \qquad (4.3)$$

The total work done in producing a displacement from initial position $\mathbf{r}_i$ to final position $\mathbf{r}_f$ is,

$$W = \int_{r_i}^{r_f} dW \qquad (4.4)$$

$$W = \int_{r_i}^{r_f} (F\cos\theta)\,dr = (F\cos\theta)\int_{r_i}^{r_f} dr =$$

$$(F\cos\theta)(r_f - r_i) \qquad (4.5)$$

The graphical representation of the work done by a constant force is shown in Figure 4.5. The area under the graph shows the work done by the constant force.

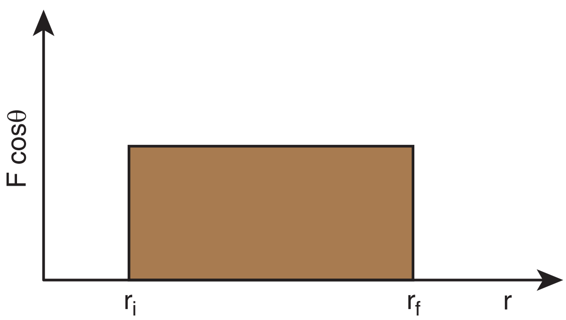
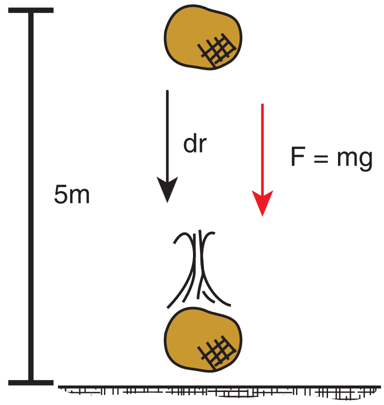


Figure 4.5 Work done by the constant force

EXAMPLE 4.2

An object of mass 2 kg falls from a height of 5 m to the ground. What is the work done by the gravitational force on the object? (Neglect air resistance; Take $g = 10\ \text{m s}^{-2}$)



Solution

In this case the force acting on the object is downward gravitational force $m\vec{g}$. This is a constant force.

Work done by gravitational force is

$$W = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$

$$W = (F\cos\theta)\int_{r_i}^{r_f} dr = (mg\cos\theta)\,(r_f - r_i).$$

The object also moves downward which is in the direction of gravitational force ($\vec{F} = m\vec{g}$) as shown in figure. Hence, the angle between them is $\theta = 0°$; $\cos 0° = 1$ and the displacement, $(r_f - r_i) = 5\,\text{m}$

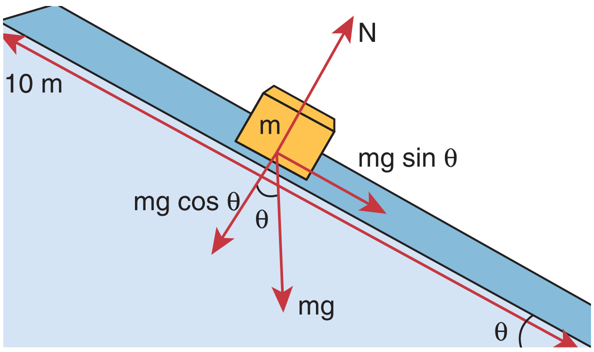
$$W = mg(r_f - r_i)$$

$$W = 2 \times 10 \times 5 = 100$$

The work done by the gravitational force on the object is positive.

EXAMPLE 4.3

An object of mass $m=1$ kg is sliding from top to bottom in the frictionless inclined plane of inclination angle $\theta = 30°$ and the length of inclined plane is 10 m as shown in the figure. Calculate the work done by gravitational force and normal force on the object. Assume acceleration due to gravity, $g = 10\ m\ s^{-2}$



Solution

We calculated in the previous chapter that the acceleration experienced by the object in the inclined plane as $g\sin\theta$.

According to Newton's second law, the force acting on the mass along the inclined plane $F = mg\sin\theta$. Note that this force is constant throughout the motion of the mass.

The work done by the parallel component of gravitational force $(mg\ \sin\theta)$ is given by

$$W = \vec{F}.d\vec{r} = Fdr\cos\phi$$

where ϕ is the angle between the force (mg $\sin\theta$) and the direction of motion (dr). In this case, force (mg $\sin\theta$) and the displacement $(d\vec{r})$ are in the same direction. Hence $\phi = 0$ and $\cos\phi = 1$

$$W = F\,dr = (mg\ \sin\theta)(dr)$$

(dr = length of the inclined place)

$$W = 1 \times 10 \times \sin(30°) \times 10 = 100 \times \frac{1}{2} = 50\ J$$

The component mg $\cos\theta$ and the normal force N are perpendicular to the direction of motion of the object, so they do not perform any work.

EXAMPLE 4.4

If an object of mass 2 kg is thrown up from the ground reaches a height of 5 m and falls back to the Earth (neglect the air resistance). Calculate

(a) The work done by gravity when the object reaches 5 m height

(b) The work done by gravity when the object comes back to Earth

(c) Total work done by gravity both in upward and downward motion and mention the physical significance of the result.

Solution

When the object goes up, the displacement points in the upward direction whereas the gravitational force acting on the object points in downward direction. Therefore, the angle between gravitational force and displacement of the object is 180°.

(a) The work done by gravitational force in the upward motion.

Given that $\Delta r = 5m$ and $F = mg$

$$W_{up} = F\Delta r\ \cos\theta = mg\Delta r\ \ \cos 180°$$

$$W_{up} = 2 \times 10 \times 5 \times (-1) = -100 \text{ joule.}$$

$$[\cos 180° = -1]$$

(b) When the object falls back, both the gravitational force and displacement of the object are in the same direction. This implies that the angle between gravitational force and displacement of the object is 0°.

$$W_{down} = F\Delta r \cos 0°$$
$$W_{down} = 2 \times 10 \times 5 \times (1) = 100 \text{ joule}$$
$$[\cos 0° = 1]$$

(c) The total work done by gravity in the entire trip (upward and downward motion)

$$W_{total} = W_{up} + W_{down}$$
$$= -100 \text{ joule} + 100 \text{ joule} = 0$$

It implies that the gravity does not transfer any energy to the object. When the object is thrown upwards, the energy is transferred to the object by the external agency, which means that the object gains some energy. As soon as it comes back and hits the Earth, the energy gained by the object is transferred to the surface of the Earth (i.e., dissipated to the Earth).

EXAMPLE 4.5

A weight lifter lifts a mass of 250 kg with a force 5000 N to the height of 5 m.

(a) What is the workdone by the weight lifter?
(b) What is the workdone by the gravity?
(c) What is the net workdone on the object?

Solution

(a) When the weight lifter lifts the mass, force and displacement are in the same direction, which means that the angle between them $\theta = 0^0$. Therefore, the work done by the weight lifter,

$$W_{weight\ lifter} = F_w h \cos\theta = F_w h\left(\cos 0^0\right)$$
$$= 5000 \times 5 \times (1) = 25{,}000 \text{ joule} = 25 \text{ kJ}$$

(b) When the weight lifter lifts the mass, the gravity acts downwards which means that the force and displacement are in opposite direction. Therefore, the angle between them $\theta = 180^0$

$$W_{gravity} = F_g h \cos\theta = mgh\left(\cos 180^0\right)$$
$$= 250 \times 10 \times 5 \times (-1)$$
$$= -12{,}500 \text{ joule} = -12.5 \text{ kJ}$$

(c) The net workdone (or total work done) on the object

$$W_{net} = W_{weight\ lifter} + W_{gravity}$$
$$= 25 \text{ kJ} - 12.5 \text{ kJ} = +12.5 \text{ kJ}$$

4.1.3 Work done by a variable force

When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation

$$dW = (F \cos\theta)\, dr$$

[F cos θ is the component of the variable force F]

where, F and θ are variables. The total work done for a displacement from initial position r_i to final position r_f is given by the relation,

$$W = \int_{r_i}^{r_f} dW = \int_{r_i}^{r_f} F\cos\theta\, dr$$ ([illegible]

A graphical representation of the work done by a variable force is shown in Figure 4.6. The area under the graph is the work done by the variable force.

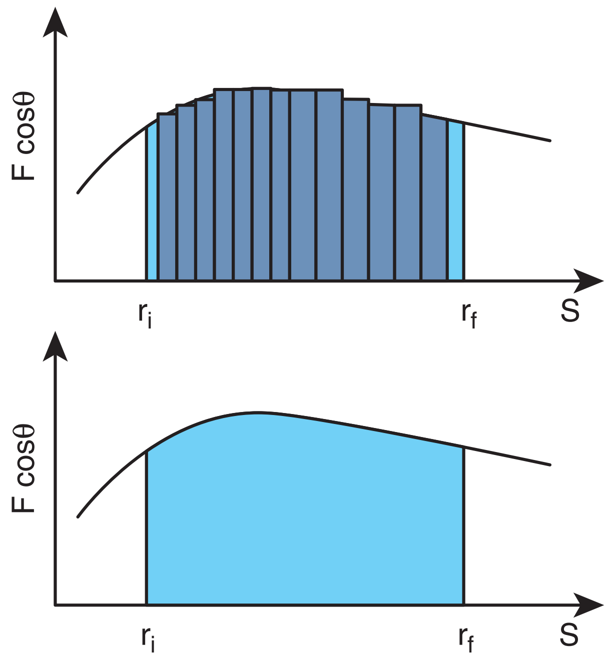


Figure 4.6 Work done by a variable force

EXAMPLE 4.6

A variable force $F = k\,x^2$ acts on a particle which is initially at rest. Calculate the work done by the force during the displacement of the particle from $x = 0$ m to $x = 4$ m. (Assume the constant $k = 1\ \mathrm{N\,m^{-2}}$)

Solution

Work done,

$$W = \int_{x_i}^{x_f} F(x)\,dx = k\int_{0}^{4} x^2 dx = \frac{64}{3}\ \mathrm{N\,m}$$

4.2 ENERGY

Energy is defined as the capacity to do work. In other words, work done is the manifestation of energy. That is why work and energy have the same dimension (ML^2T^{-2})

Work ⇔ Energy

The important aspect of energy is that for an isolated system, the sum of all forms of energy i.e., the total energy remains the same in any process irrespective of whatever internal changes may take place. This means that the energy disappearing in one form reappears in another form. This is known as the law of conservation of energy. In this chapter we shall take up only the mechanical energy for discussion.

In a broader sense, mechanical energy is classified into two types

1. Kinetic energy
2. Potential energy

The energy possessed by a body due to its motion is called kinetic energy. The energy possessed by the body by virtue of its position is called potential energy.

The SI unit of energy is the same as that of work done i.e., N m (or) joule (J). The dimension of energy is also the same as that of work done. It is given by $[ML^2T^{-2}]$. The other units of energy and their SI equivalent values are given in Table 4.2.

Table 4.2 SI equivalent of other units of energy

Unit	Equivalent in joule
1 erg (CGS unit)	10^{-7} J
1 electron volt (eV)	$1.6\mathrm{x}10^{-19}$ J
1 calorie (cal)	4.186 J
1 kilowatt hour (kWh)	$3.6\mathrm{x}10^{6}$ J

4.2.1 Kinetic energy

Kinetic energy is the energy possessed by a body by virtue of its motion. All moving objects have kinetic energy. A body that is in motion has the ability to do work. For example a hammer kept at rest on a nail does not push the nail into the wood. Whereas the same hammer when it strikes the nail, draws the nail into the wood as shown in Figure 4.7. Kinetic energy is measured by the amount of work that the body can perform before it comes to rest. The amount of work done by a moving body depends both on the mass of the body and the magnitude of its velocity. A body which is not in motion does not have kinetic energy.

4.2.2 Work–Kinetic Energy Theorem

Work and energy are equivalents. This is true in the case of kinetic energy also. To prove this, let us consider a body of mass m at rest on a frictionless horizontal surface.

The work (W) done by the constant force (F) for a displacement (s) in the same direction is,

$$W = Fs \qquad (4.7)$$

The constant force is given by the equation,

$$F = ma \qquad (4.8)$$

The third equation of motion (refer section 2.10.3) can be written as,

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

Substituting for a in equation (4.8),

$$F = m\left(\frac{v^2 - u^2}{2s}\right) \qquad (4.9)$$

Substituting equation (4.9) in (4.7),

$$W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad (4.10)$$

The expression for kinetic energy:

The term $\left(\frac{1}{2}mv^2\right)$ in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v).

$$KE = \frac{1}{2}mv^2 \qquad (4.11)$$

Kinetic energy of the body is always positive. From equations (4.10) and (4.11)

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad (4.12)$$

$$\text{Thus, } W = \Delta KE$$

Figure 4.7 Demonstration of kinetic energy

The expression on the right hand side (RHS) of equation (4.12) is the change in kinetic energy (ΔKE) of the body.

This implies that *the work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.*

The work-kinetic energy theorem implies the following.

1. If the work done by the force on the body is positive then its kinetic energy increases.
2. If the work done by the force on the body is negative then its kinetic energy decreases.
3. If there is no work done by the force on the body then there is no change in its kinetic energy, which means that the body has moved at constant speed provided its mass remains constant.

4.2.3 Relation between Momentum and Kinetic Energy

Consider an object of mass m moving with a velocity $\vec{v}$. Then its linear momentum is $\vec{p} = m\vec{v}$ and its kinetic energy, $KE = \frac{1}{2}mv^2$.

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\vec{v}.\vec{v}\right) \quad (4.13)$$

Multiplying both the numerator and denominator of equation (4.13) by mass, m

$$KE = \frac{1}{2}\frac{m^2\left(\vec{v}.\vec{v}\right)}{m}$$

$$= \frac{1}{2}\frac{\left(m\vec{v}\right).\left(m\vec{v}\right)}{m} \quad [\vec{p} = m\vec{v}]$$

$$= \frac{1}{2}\frac{\vec{p}.\vec{p}}{m}$$

$$= \frac{p^2}{2m}$$

$$KE = \frac{p^2}{2m} \quad (4.14)$$

where $|\vec{p}|$ is the magnitude of the momentum. The magnitude of the linear momentum can be obtained by

$$|\vec{p}| = p = \sqrt{2m\ (KE)} \quad (4.15)$$

Note that if kinetic energy and mass are given, only the magnitude of the momentum can be calculated but not the direction of momentum. It is because the kinetic energy and mass are scalars.

EXAMPLE 4.7

Two objects of masses 2 kg and 4 kg are moving with the same momentum of 20 kg m s^{-1}.

(a) Will they have same kinetic energy?
(b) Will they have same speed?

Solution

(a) The kinetic energy of the mass is given by $KE = \frac{p^2}{2m}$

For the object of mass 2 kg, kinetic energy is $KE_1 = \frac{(20)^2}{2\times 2} = \frac{400}{4} = 100\,J$

For the object of mass 4 kg, kinetic energy is $KE_2 = \frac{(20)^2}{2\times4} = \frac{400}{8} = 50\,J$

Note that $KE_1 \neq KE_2$ i.e., even though both are having the same momentum, the kinetic energy of both masses is not the same. The kinetic energy of the heavier object has lesser kinetic energy than smaller mass. It is because the kinetic energy is inversely proportional to the mass ($KE \propto \frac{1}{m}$) for a given momentum.

(b) As the momentum, $p = mv$, the two objects will not have same speed.

4.2.4 Potential Energy

The potential energy of a body is associated with its position and configuration with respect to its surroundings. This is because the various forces acting on the body also depends on position and configuration.

"Potential energy of an object at a point P is defined as the amount of work done by an external force in moving the object at constant velocity from the point O (initial location) to the point P (final location). At initial point O potential energy can be taken as zero.

Mathematically, *potential energy* is defined as $U = \int \vec{F}_a . d\vec{r}$ (4.16)

where the limit of integration ranges from initial location point O to final location point P.

We have various types of potential energies. Each type is associated with a particular force. For example,

(i) The energy possessed by the body due to gravitational force gives rise to gravitational potential energy.

(ii) The energy due to spring force and other similar forces give rise to elastic potential energy.

(iii) The energy due to electrostatic force on charges gives rise to electrostatic potential energy.

We will learn more about conservative forces in the section 4.2.7. Now, we continue to discuss more about gravitational potential energy and elastic potential energy.

4.2.5 Potential energy near the surface of the Earth

The gravitational potential energy (U) at some height h is equal to the amount of work required to take the object from ground to that height h with constant velocity.

Let us consider a body of mass m being moved from ground to the height h against the gravitational force as shown in Figure 4.8.

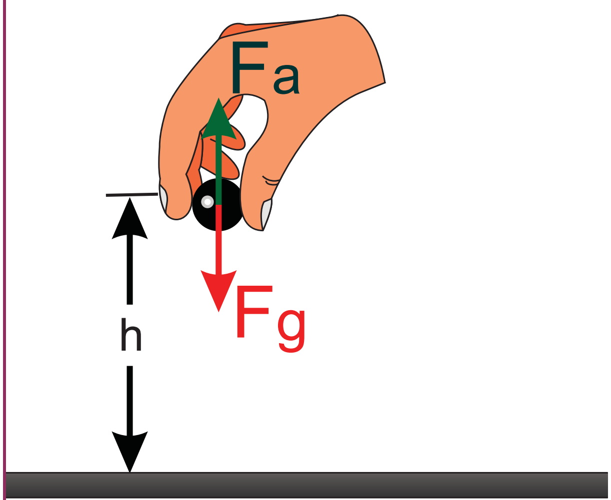


Figure 4.8 Gravitational potential energy

The gravitational force $\vec{F}_g$ acting on the body is, $\vec{F}_g = -mg\,\hat{j}$ (as the force is in y direction, unit vector $\hat{j}$ is used). Here, negative sign implies that the force is acting vertically downwards. In order to move the body without acceleration (or with constant velocity), an external applied force $\vec{F}_a$ equal in magnitude but opposite to that of gravitational force $\vec{F}_g$ has to be applied on the body i.e., $\vec{F}_a = -\vec{F}_g$. This implies that $\vec{F}_a = +mg\,\hat{j}$. The positive sign implies that the applied force is in vertically upward direction. Hence, when the body is lifted up its velocity remains unchanged and thus its kinetic energy also remains constant.

The gravitational potential energy (U) at some height h is equal to the amount of work required to take the object from the ground to that height h.

$$U = \int \vec{F}_a . d\vec{r} = \int_0^h \left|\vec{F}_a\right| \left|d\vec{r}\right| \cos\theta \quad (4.17)$$

Since the displacement and the applied force are in the same upward direction, the angle between them, $\theta = 0°$. Hence, $\cos 0^0 = 1$ and $\left|\vec{F}_a\right| = mg$ and $\left|d\vec{r}\right| = dr$.

$$U = mg\int_0^h dr \quad (4.18)$$

$$U = mg\left[r\right]_0^h = mgh \quad (4.19)$$

Note that the potential energy stored in the object is defined through work done by the external force which is positive. Physically this implies that the agency which is applying the external force is transferring the energy to the object which is then stored as potential energy. If the object is allowed to fall from a height h then the stored potential energy is converted into kinetic energy.

Note

- **How can an object move with zero acceleration (constant velocity) when the external force is acting on the object?**

  It is possible when there is another force which acts exactly opposite to the external applied force. They both cancel each other and the resulting net force becomes zero, hence the object moves with zero acceleration.
- Why should the object be moved at constant velocity when we define potential energy?

  If the object does not move at constant velocity, then it will have different velocities at the initial and final locations. According to work-kinetic energy theorem, the external force will impart some extra kinetic energy. But we associate potential energy to the forces like gravitational force, spring force and coulomb force. So the external agency should not impart any kinetic energy when the object is taken from initial to final location.

EXAMPLE 4.8

An object of mass 2 kg is taken to a height 5 m from the ground $\left(g = 10\,\text{m}\,\text{s}^{-2}\right)$.

(a) Calculate the potential energy stored in the object.

(b) Where does this potential energy come from?

(c) What external force must act to bring the mass to that height?

(d) What is the net force that acts on the object while the object is taken to the height 'h'?