PROGRAMMING AND DATA STRUCTURES

HASH TABLES

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- Hashing and Hash Tables
- Collisions in Hash Tables
- Solutions to the collision problem
 - Open Addressing and Separate Chaining
- Performance of Hash Tables
- Implementing a Hash Table (class HashMap)

STUDENT LEARNING OUTCOMES

At the end of this chapter, your should be able to:

- Describe how hashing and hash tables work
- Apply different solutions to handle collisions
- Implement and test the hash table data structure
- Evaluate the time complexity of the operations on a Hash Table

Search operation

Array List: O(n)

Linked List: O(n)

▶ BST: O(log n) (when kind of balanced)

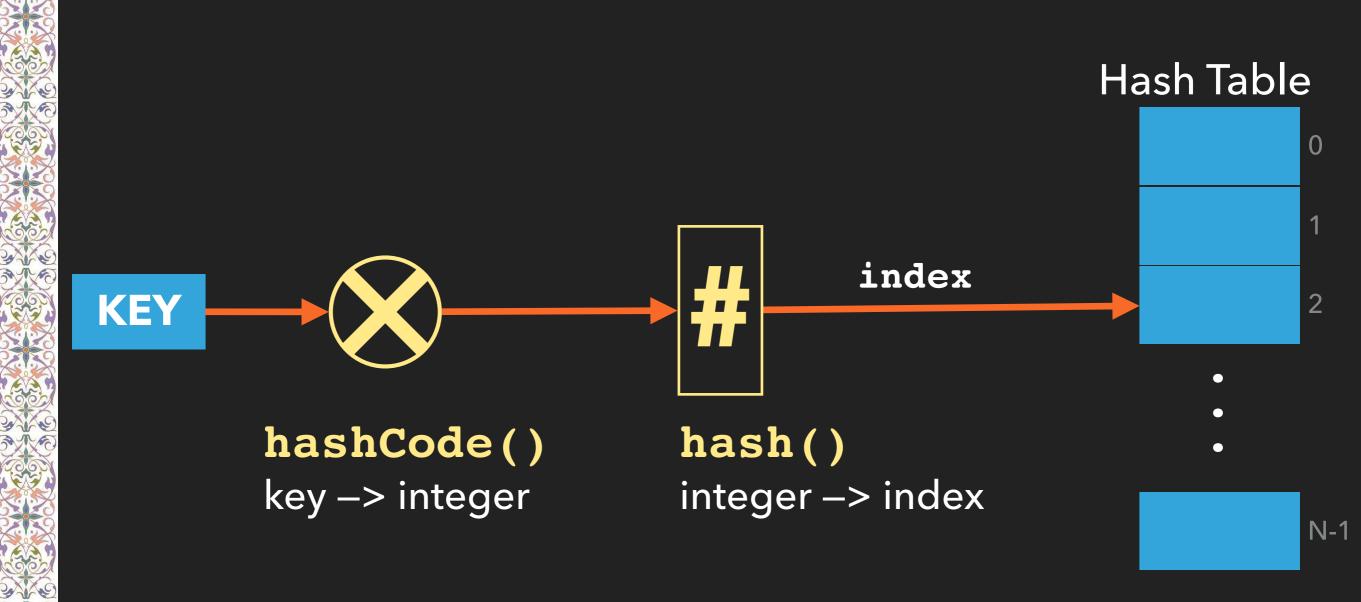
Can we perform search in less time?

- Hash tables allow to perform search operations in O(1)
- Hash tables use associative access to data
- Associative memory: access data using the data itself instead of an address (index)

◆ Store the data in an array - Hash Table (HT)

◆ Access the HT elements using a hash function h() that returns an index in the HT

HASHING



- ◆ If the size of HT is N, then
 - $0 \le hash() \le N-1$ (valid index)
- ◆ Searching for a value key is performed using one comparison with
 - HT[hash(hashCode(key))]
- → How is data added/found in HT using hash()?
- How hashCode() and hash() are defined?

- Adding data to the table
 - Apply hash () to the data to determine the index where the data should be added
- Retrieving data from the table
 - Apply hash () to the data to find the index where it is in the hash table

Example

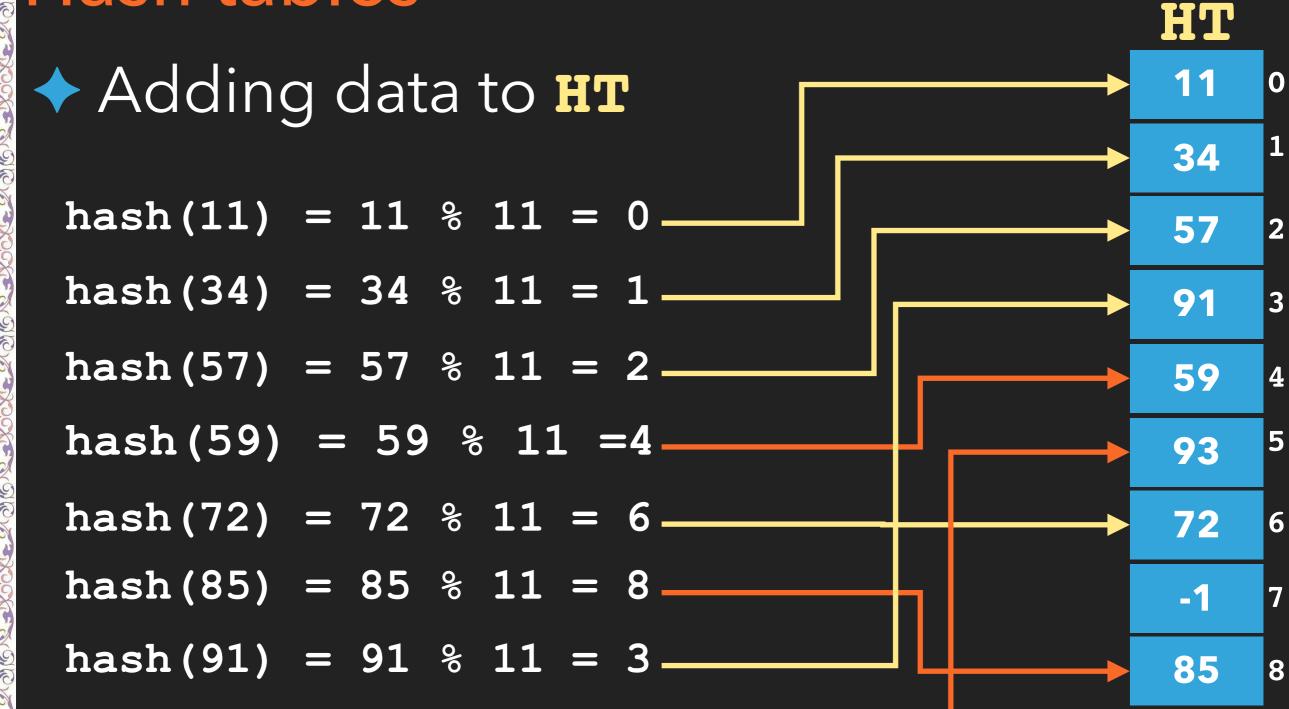
- ♦ Values {11, 34, 57, 59, 72, 85, 91, 93}
 to store in a HT of size 11
- ◆ Each value v is stored at an index i calculated by hash(v) = v%(size of HT) [i = v % 11]
- Searching for a value v: compare v to HT[hash(v)]

HASHING

9

10

Hash tables



hash(93) = 93 % 11 = 5

HASHING

Hash tables

- → Retrieving data from HT
 - ★ key = 91
 hash(key) = 91 % 11 = 3
 key found
 - ★ key = 75
 hash(75) = 75 % 11 = 9
 key not found
 - ♦ Search operation: O (1)

HT

11 0

34 1

57 2

91 3

60

93 5

72 6

-1 7

85 8

1 9

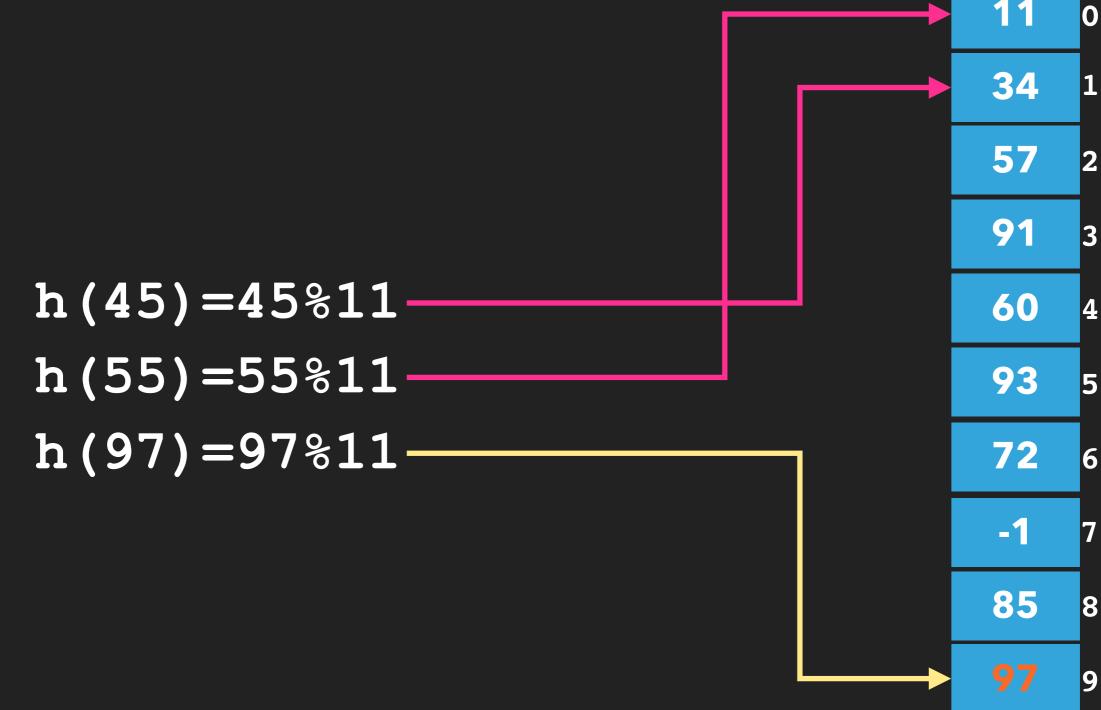
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Collisions

- Issue with the hash method
- ◆ Set of values may be hashed to the same index (23, 34, 56, 78, 89 all have the hash value = 1) for size = 11
- ◆ Collision: two or more values have the same hash function value

HASHING

Collisions



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Collisions

Collisions - two or more values have the same hash function value

- Two solutions for the collision problem
 - Separate Chaining (open hashing)
 - Open Addressing (closed hashing)

Collisions

- Solutions for the collision problem
 - ◆ Separate Chaining collisions are stored outside the hash table in a list (array list or linked list, or even a tree)
 - ◆ Open Addressing collisions are stored in the hash table itself at the next available index

9 h(9)=9%13=9

11

22

33

44

75

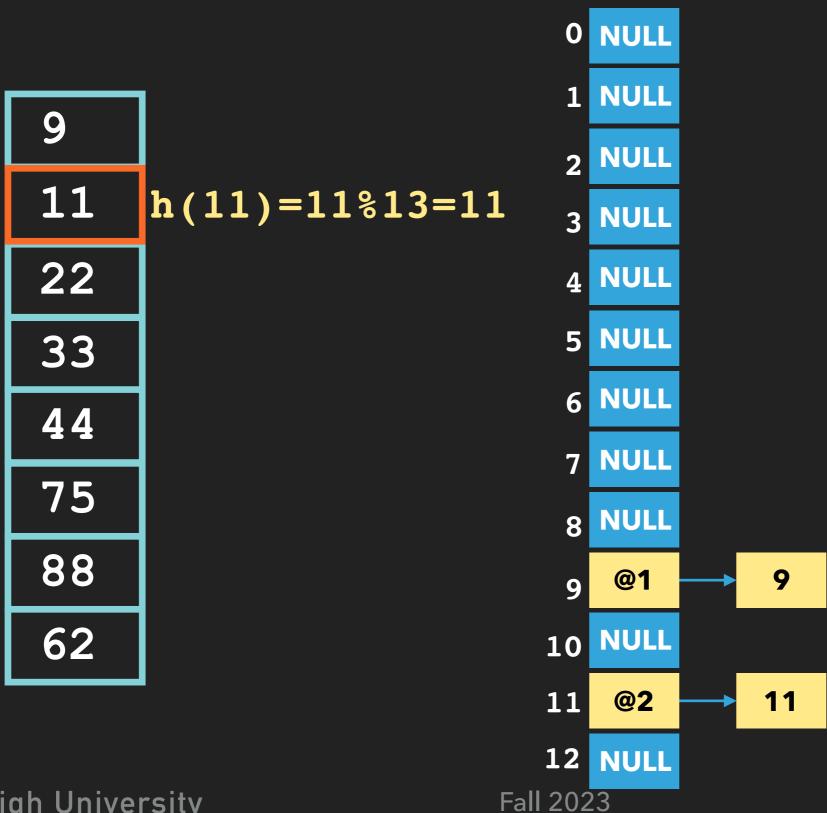
88

62



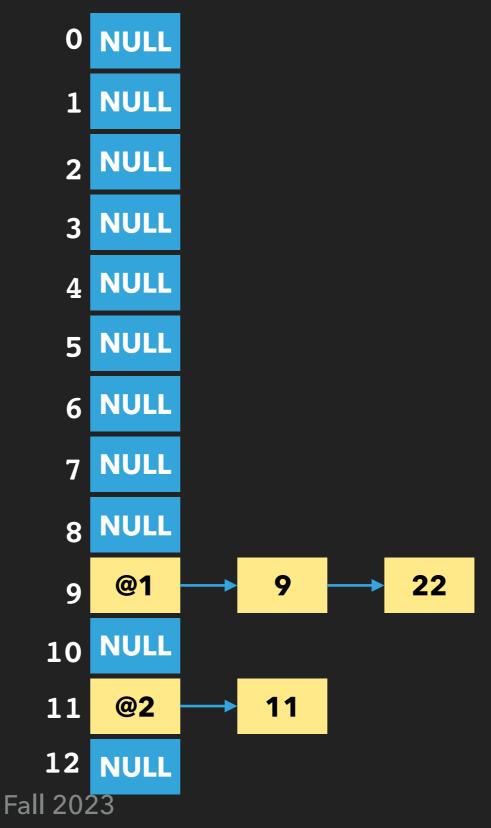
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h(22)=22%13=9



h(33)=33%13=7

9

11

22

33

44

75

88

62



@2

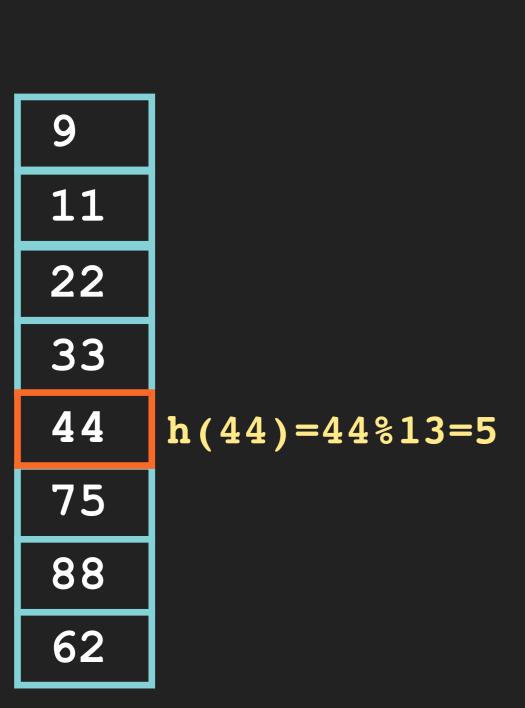
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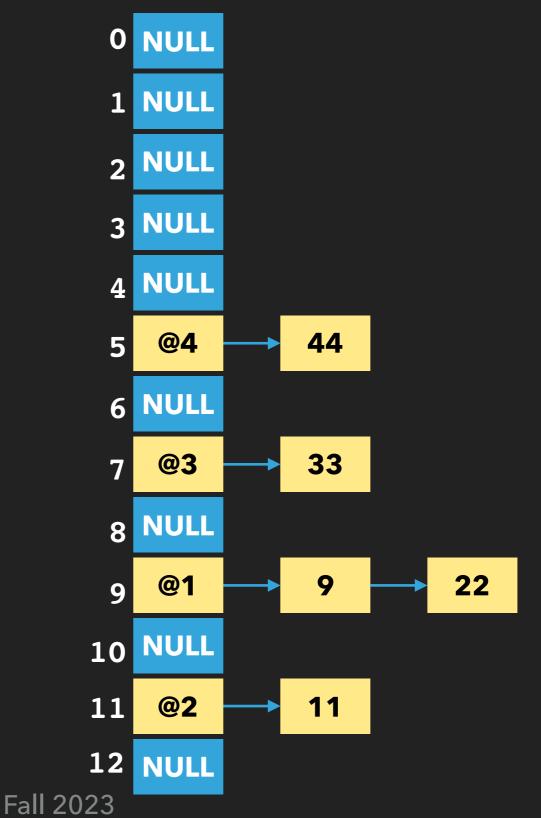
11

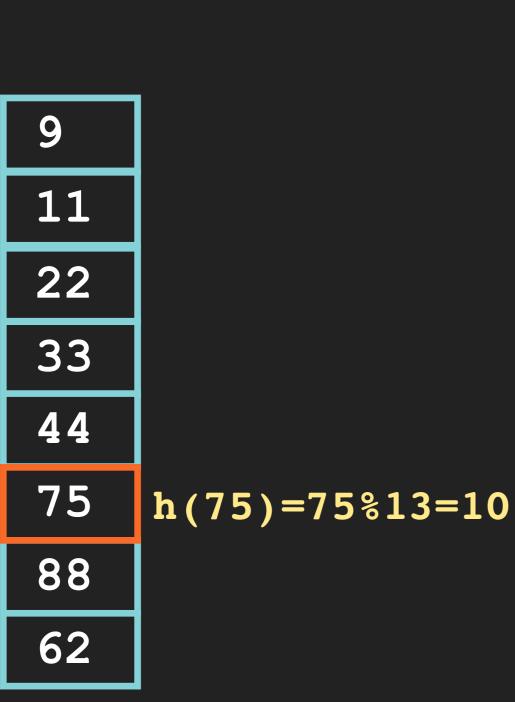
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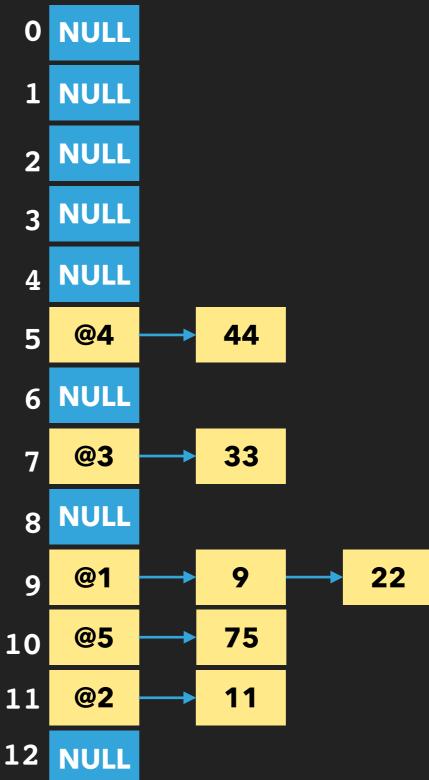
11

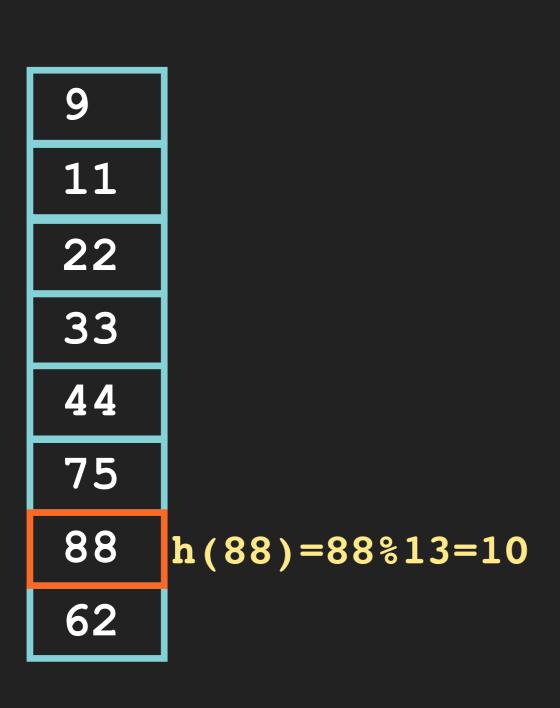
22

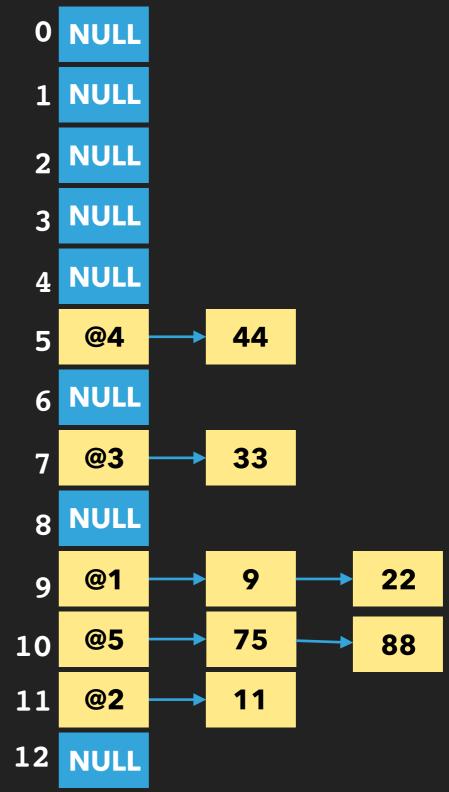


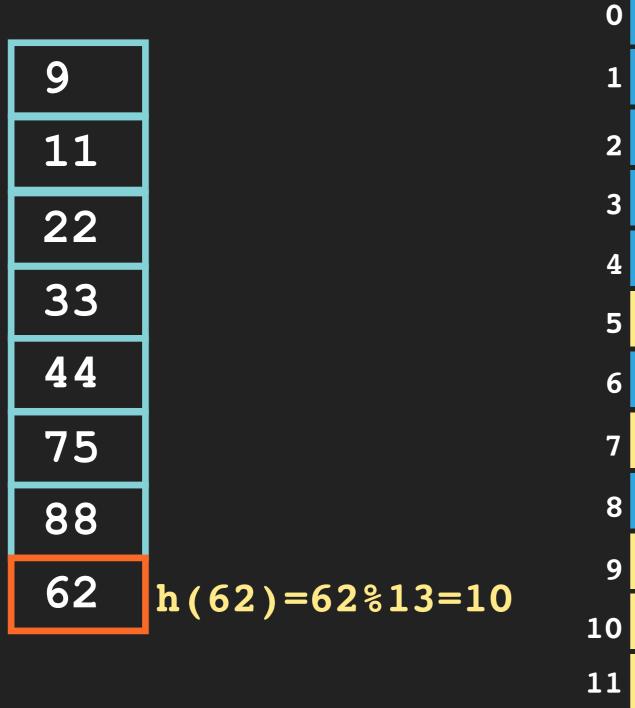


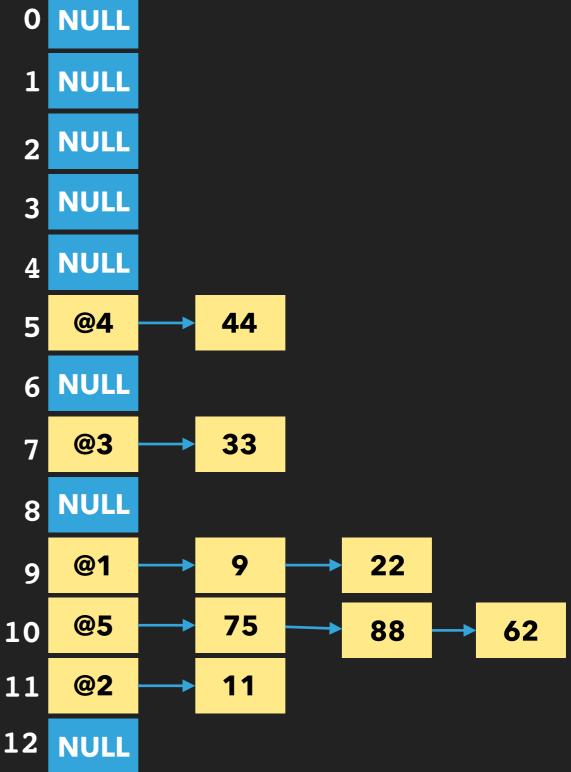












Collisions - Open Addressing

- Linear probing look for the next available slot using a linear function (index+j until an empty element is found)
- Quadratic probing look for the next available slot using a quadratic function (index+j² until an empty element is found)
- ◆ Double hashing look for the next available slot at an index using another hash function (index + hash2(v))



26

Collisions - Linear Probing

HT

u	

11

22

33

44

75

88

62

1	null
_	

Null

2 null

3 null

4 NULL

5 NULL

6 NULL

7 NULL

8 NULL

9 NULL

10 NULL

11 NULL

12 NULL

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9 $h(9) = 9 % 13 =$	9
---------------------	---

11

22

33

44

75

88

62

0	null	null
1	null	null
2	null	null
3	null	null
4	null	null
5	null	null
6	null	null
7	null	null
8	null	null
9	null	9
.0	null	null
.1	null	null

null

nul1



9

11

22

33

44

75

88

62

h(11) = 11 % 13 = 11

4 null

null

null

null

null

5 null

6 null

7 null

8 null

9 9

10 null

11 null

12 null

nul1

null

null

null

null

null

null

null

null

9

null

11

null



h(22)=(9 + 1) % 13

9

11

22

33

44

75

88

62

1	null
2	null
3	null

null

10

5 null

null

6 null

7 null

8 null

9 9

10 null

11 11

12 null

null

null

nul1

null

null

null

null

null

null

9

22

11

null



h(33) = 33 % 13 = 7

9

11

22

33

44

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2	2
3	3
4	ļ.
5	;
6	
7	'
8	3
9	

null 33 NULL NULL NULL 9 9 22 22 10 11 11 11 **12** NULL NULL



h(44) = 44 % 13 = 5

9

11

22

33

44

75

88

62

1	
2	
3	
4	
5	
6	
7	

NULL NULL NULL NULL NULL NULL NULL 33 **NULL** 8 22 **NULL**

10 11

NULL NULL NULL NULL NULL 44 **NULL** 33 **NULL**

9 22

11 **NULL**



h(75)=(10 + 2) % 13 = 12

9

11

22

33

44

75

88

62

•	
0	
1	
2	
3	
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5	
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10

11

NULL

NULL

NULL

NULL

NULL

44

NULL

33

NULL

9

22

NULL

NULL NULL NULL NULL NULL 44 **NULL** 33 **NULL** 9 22 11 **75**



h(88) = (10 + 3) % 13 = 0

11

22

33

44

75

88

62

0
1
2
3
4
5
6
7
8
9

10

11

12

88 **NULL NULL NULL** NULL **NULL NULL** NULL **NULL** NULL 44 44 NULL **NULL** 33 33 NULL **NULL** 9 9 **22 22** 11 **75 75**



9

11

22

33

44

75

88

62

$$h(62) = (10 + 4) % 13 = 1$$

0 88

1 NULL

2 NULL

3 NULL

4 NULL

5 44

6 NULL

7 33

8 NULL

9

10 22

11 11

¹² 75

88

62

NULL

NULL

NULL

44

NULL

33

NULL

9

22

11

75



HT

Collisions - Quadratic Probing

9

11

22

33

44

75

88

62



NULL

3 NULL

4 NULL

5 NULL

6 NULL

7 NULL

8 NULL

9 NULL

10 NULL

11 NULL

12 NULL

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Collisions - Quadratic Probing

9

$$h(9) = 9 % 13 = 9$$

11

22

33

44

75

88

62

NULL

1 NULL

₂ NULL

3 NULL

4 NULL

5 NULL

6 NULL

7 NULL

8 NULL

9 NULL

10 NULL

11 NULL

12 NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

9

NULL

NULL

NULL



h(11) = 11 % 13 = 11

9

11

22

33

44

75

88

62

0 NULL

L NULL

NULL

3 NULL

4 NULL

5 NULL

6 NULL

7 NULL

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0 NULL

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3 NULL

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5 NULL

6 NULL

7 NULL

8 NULL

9

10 NULL

11 11

12 NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

9

22

11



h(33) = 7 % 13 = 7

9

11

22

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44

75

88

62

0	NULL

1 NULL

2 NULL

3 NULL

4 NULL

5 NULL

6 NULL

7 NULL

8 NULL

9

10 22

11 11

12 NULL

NULL

NULL

NULL

NULL

NULL

NULL

NULL

33

NULL

9

22

11



h(44) = 44 % 13 = 5

9

11

22

33

44

75

88

62

0	NULL
1	NULL
2	NULL
3	NULL
4	NULL
5	NULL
6	NULL
7	33
8	NULL
9	9

22

NULL

10

12

```
NULL
NULL
NULL
NULL
NULL
44
NULL
33
NULL
 9
22
11
NULL
```



h(75) = (10 + 4)

9

11

22

33

44

75

88

62

NULL NULL 75 NULL NULL NULL NULL NULL NULL NULL 44 44 **NULL NULL** 33 33 **NULL NULL** 9 9 **22** 22 10 11 11 **NULL NULL**



9										
11										
22										•
33										!
44										(
75										•
88	h(88)	=	(10	+	9)	0/0	13	=	6	
62										10
	[1

0 NULL **NULL 75 75 NULL NULL NULL NULL NULL NULL** 44 44 **NULL** 88 33 33 **NULL NULL** 9 9 22 22 11

NULL



	0	NULL	62
9	1	75	75
	2	NULL	NULL
11	3	NULL	NULL
22	4	NULL	NULL
33	5	44	44
44	6	88	88
	7	33	33
75	8	NULL	NULL
88	9	9	9
62 h(62) = (10 + 1)			22
	11	11	11
	12	NULL	NULL

Collisions

The number of collisions may affect the performance of the search operation

could result in linear search if the number of collisions is high

Practice

◆ We want to add the following data to a hash table of 15 elements

```
57, 8, 19, 30, 20, 34, 37, 42, 35, 21, 27
```

- \rightarrow The hash function is hash(x) = x % 15
- Show the content of the hash table after adding the data using separate chaining, linear probing, and quadratic probing to resolve the collisions (Practice Sheet)

Collisions

- Linear Probing
 - Clusters are formed
 - Clusters can grow in size and slow down the (search, add, and remove) operations
 - Linear probing guarantees to find an empty element if the table is not full

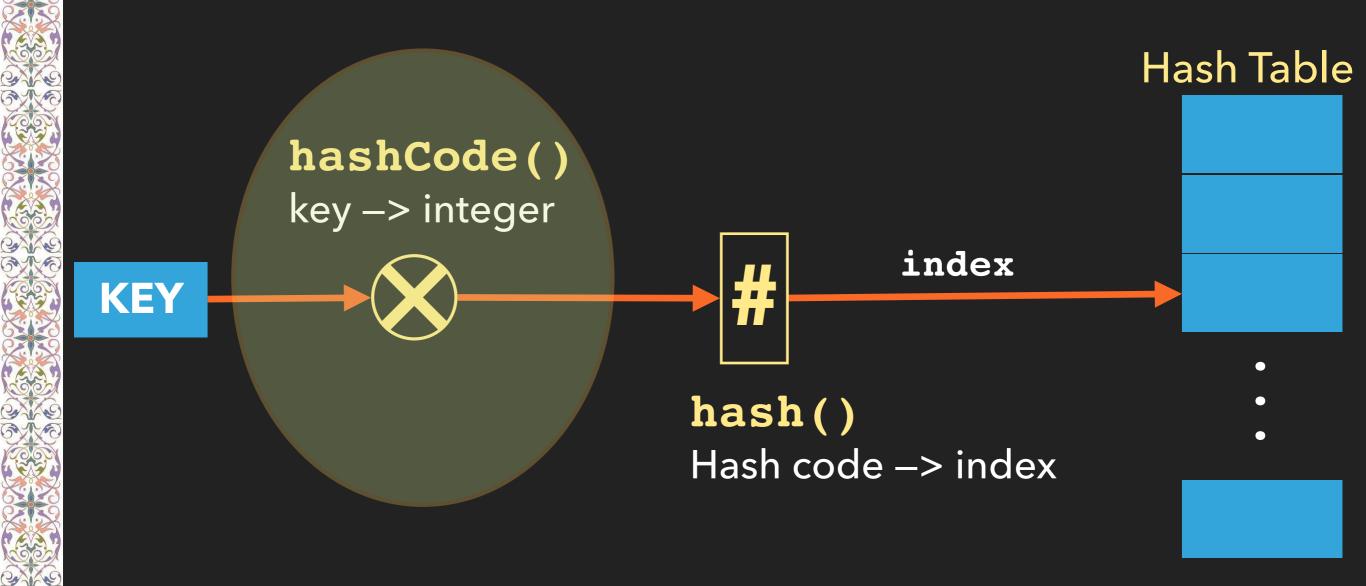
Collisions

- Quadratic Probing
 - Avoids clustering (linear probing)
 - Has its own clustering problem (secondary clustering: keys that collide with an occupied element use the same probing sequence)
 - does not guarantee finding an empty element

Hashtable Performance

- Search, Add, Remove 0(1)
- ◆ Three factors may cause more collisions
 - affect the constant time performance
 - HashCode function
 - Hash function
 - ◆ Size of the hash table

HASHING



- hashCode() is simple for integers return the integer itself
 - hashCode()? for double, strings, etc.
 - Class Object has a method hashCode() that returns the reference to the object
 - Override hashCode() to generate a hash code for the type you are using
 - Wrapper classes and class String override hashCode()

- General guidelines for hashCode()
 - ◆ You should override hashCode() whenever you override equals() to ensure that two equal objects return the same hash code
 - Multiple calls to hashCode return the same integer provided that the object's data did not change
 - Two unequal objects may have the same hash code, but you should implement hashCode() to minimize such cases

- hashCode for primitive types
 - byte (8 bits), short (16 bits), char (16 bits) cast to int (32 bits)
 - ◆ float (32 bits)- covert to int using Float.floatToIntBits(float f) - returns an integer that corresponds to the bit representation of f

floatToIntBits(f) returns $2^{18} + 2^{20} + 2^{21} + 2^{22} + 2^{23} + 2^{24}$

- hashCode for primitive types
 - → long fold 64 bits into 32 bits -

```
(int) (number^(number>>32))
```

```
hashCode(x) =
          10000001111011111101010011000100
```

double - convert to long using doubleToLongBits() and fold into 32 bits



- hashCode for strings
 - ◆ Search keys are often strings good hashCode method for strings
 - ◆ Intuitive approach
 - Sum of the Unicode of all the characters in the string
 - Does not work well if two strings have the same letters

```
hashCode("tod") = hashCode("dot") = 0x0064 + 0x006F + 0x0074
```

- ◆ Better approach
 - ◆ Take the position of the character into consideration
 - Polynomial hashCode

$$hashCode(s) = s_0 \cdot b^{n-1} + s_1 \cdot b^{n-2} + \dots + s_{n-1}, s_i = s \cdot charAt(i)$$

- hashCode for strings
 - Efficient polynomial hashCode evaluation:

```
hashCode(s) = s_{n-1} + b(s_{n-1} + b(\ldots + b(s_2 + b(s_1 + bs_0))) \ldots))
```

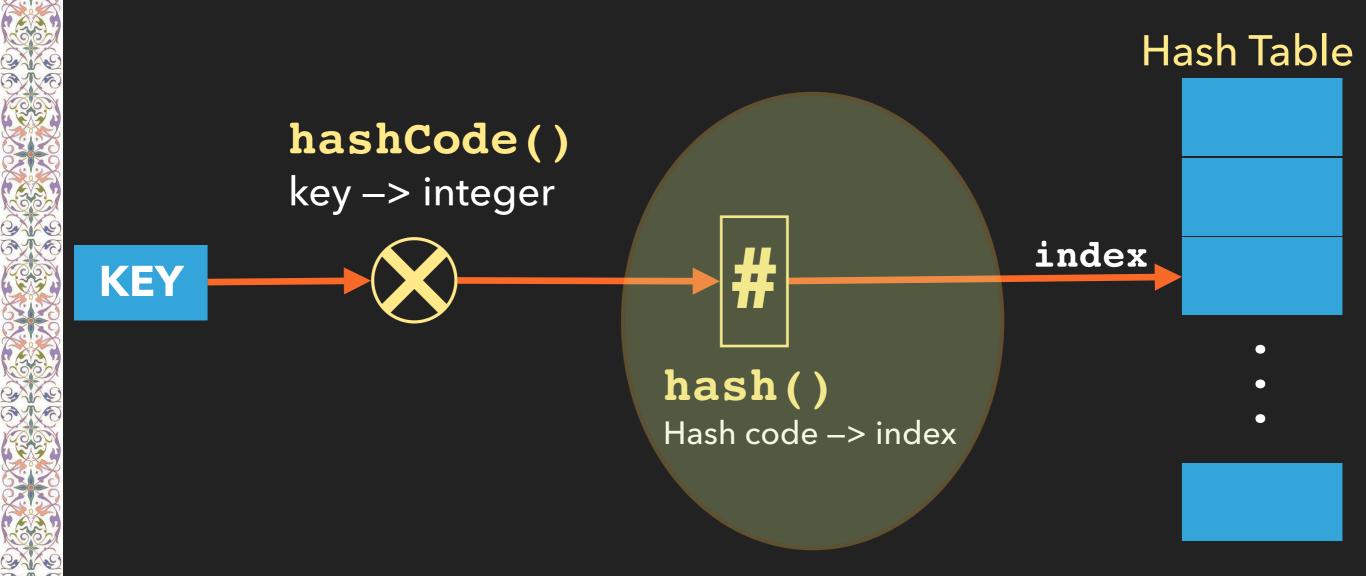
- ◆ The weight b should be selected to minimize similar values for different strings. Experiments show that using b as 31, 33, 37, 39, and 41 minimize similar values
- ◆ Class String overrides hashCode() method to use a polynomial hash code with b=31

```
public static void main(String[] args) {
       Integer i = 55000;
       Long 1 = 99123900555L;
       Double d = 5e200;
       Float fl = 2e-10f;
       Character c = 'B';
       String s = "Lehigh University";
        System.out.printf("%-15s\t%-20d %-10s %d\n", "Integer i = ",
                          i,"hashCode = ",i.hashCode());
        System.out.printf("%-15s\t%-20d %-10s %d\n", "Long l = ",
                          1 ,"hashCode = ", l.hashCode());
        System.out.printf("%-15s\t%-20.0e %-10s %d\n", "Double d = ",
                          d , "hashCode = ", d.hashCode());
        System.out.printf("%-15s\t%-20.0e %-10s %d\n", "Float f = ",
                          f1, "hashCode = ", fl.hashCode());
        System.out.printf("%-15s\t%-20c %-10s %d\n", "Character c = ",
                          c, "hashCode = ", c.hashCode());
        System.out.printf("%-15s\t%-20s %-10s %d\n", "String s = ",
                          s, "hashCode = ", s.hashCode());
```

```
Integer i =
               55000
                                    hashCode = 55000
Long 1 =
               99123900555
                                    hashCode = 339652764
Double d =
               5e+200
                                    hashCode = 1683429934
Float f =
               2e-10
                                    hashCode = 794552063
Character c =
                                    hashCode = 66
               Lehigh University
String s =
                                    hashCode =
                                                1269381427
```



Hash function



Hash Function

- hash() method
 - ◆ Transform return value of hashCode() (can be a large integer) into a valid index in the hash table
 - ◆ For a hash table of size N, the most common hash function is index = hashCode % N (0 to N-1)
 - ♦ N should be prime to spread the indices evenly time consuming to find a large prime number

Hash Function

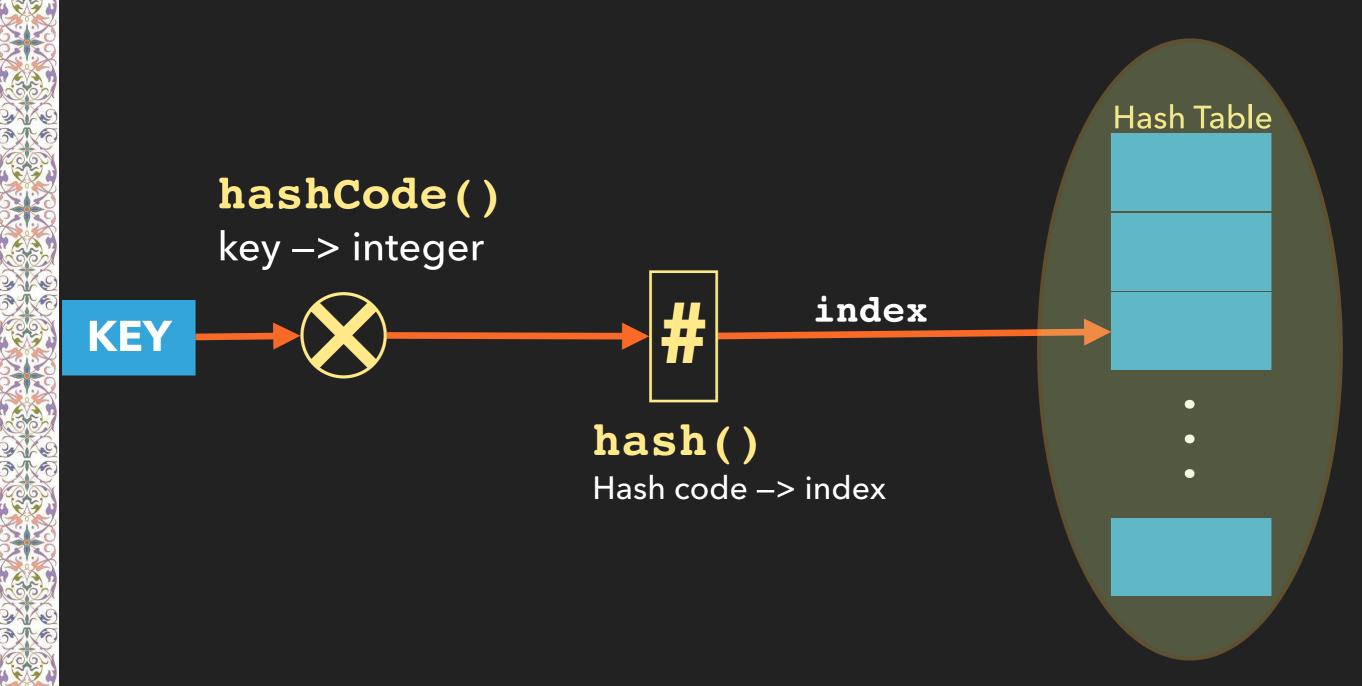
- In practice, the size of the hash table is set to an integer power of 2 to simplify the hash function operation
- java.util.HashMap sets the size of the hash table to a power of 2

```
hash(key) = hashCode(key) & (N-1)
```

◆ Bit-level operations >>, ^, & are faster than arithmetic operations *, /, and %

HASHING

Hash function



Size of the Hash Table

- Choosing the size of the table
 - ◆ A prime number larger than the size of the data set - may take time to find such number
 - Bigger table less collisions waste of memory space
 - Tradeoff space vs. time use power of 2 to simplify calculations

Size of the Hash Table

Load Factor - How full is the hash table?

★ Load Factor = # of added elements / size of the HT

High load factor results in more collisions - requires rehashing

Size of the Hash Table

Rehashing - Increase the size of the table and rehash all the data to add it to the new table

♦ 0.5 < load factor < 0.9</p>
(0.5 for probing and 0.9 for chaining)

Implementation

Hash Table with separate chaining

Array of pointers to linked lists

Each element in the hash table is a reference to a linked list

Implementation

HashMapEntry<K, V>

```
-key: K
```

-value: V

```
+HashMapEntry(K k, V v)
+getKey(): K
+getValue():V
+setKey(K k): void
+setValue(V v): void
+toString(): String
```

HashMap<K, V>

```
-hashTable: LinkedList<HashMapEntry<K,V>>[]
-loadFactor: double
-size: int
+HashMap()
+HashMap(int capacity)
+HashMap(int capacity, double loadFactor)
-trimToPowerOf2(int capacity): int
-hash(int hashCode): int
-rehash(): void
+get(K key): V
+put(K key, V value): V
+remove(K key): void
+containsKey(K key): boolean
+size(): int
+isEmpty(): boolean
+clear(): void
+toString(): String
+toList(): ArrayList<HashMapEntry<K,V>>
```

HashMap (Data members)

```
import java.util.ArrayList;
import java.util.LinkedList;
 * Class HashMap: An implementation of the hash table
 * using separate chaining
public class HashMap <K, V> {
 // data members
private int size;
private double loadFactor;
private LinkedList<HashMapEntry<K,V>>[] hashTable;
```

HashMap (Constructors)

```
* Default constructor
 default capacity: 100
* default load factor: 0.9
public HashMap() {
  this(100, 0.9);
 Constructor with one parameter
 @param c for the capacity
* default load factor: 0.9
public HashMap(int c) {
  this(c, 0.9);
 Constructor with two parameters
 @param c for the capacity
 @param lf for the load factor
public HashMap(int c, double lf) {
  hashTable = new LinkedList[trimToPowerOf2(c)];
  loadFactor = lf;
  size = 0;
```

HashMap (private methods)

```
* Method trimToPowerOf2
* @param c the capacity of the hash table
* @return the closest power of 2 to c
private int trimToPowerOf2(int c) {
  int capacity = 1;
  while (capacity < c)
      capacity = capacity << 1;</pre>
  return capacity;
* Method hash
 @param hashCode
 @return a valid index in the hashtable
private int hash(int hashCode) {
  return hashCode & (hashTable.length-1);
```

HashMap (useful methods)

```
* Method size
* @return the number of pairs (key, value) stored the hashtable
public int size() {
  return size;
* Method clear to clear the hashtable
public void clear() {
  size = 0;
  for(int i=0; i<hashTable.length; i++)</pre>
    if(hashTable[i] != null)
        hashTable[i].clear();
* Method isEmpty
 @return true if the hashtable is empty, false otherwise
public boolean isEmpty() {
  return (size == 0);
```

HashMap (search methods)

```
* Method contains to search for a key in the hashtable
* @param key the value of the key being searched for
* @return true if key was found, false otherwise
public boolean containsKey(K key) {
  if(get(key) != null)
    return true;
  return false;
 Method get to find an entry in the hashtable
 @param key the value of the key being searched for
 @return the value associated with the key if key is found, null otherwise
public V get(K key)
  int HTIndex = hash(key.hashCode());
  if(hashTable[HTIndex] != null) {
    LinkedList<HashMapEntry<K,V>> 11 = hashTable[HTIndex];
    for(HashMapEntry<K, V> entry: 11) {
       if (entry.getKey().equals(key))
         return entry.getValue();
  return null;
```

HashMap (remove method)

```
Method remove to remote an entry from the hashtable
* @param key the key to be removed
* if the key is found, the pair (key and it associated value)
* will be removed from the hashtable
* the hashtable is not modified if key is not found
public void remove(K key) {
  int HTIndex = hash(key.hashCode());
  if (hashTable[HTIndex]!=null) { //key is in the hash map
    LinkedList<HashMapEntry<K,V>> 11 = hashTable[HTIndex];
    for (HashMapEntry<K, V> entry: 11) {
      if(entry.getKey().equals(key)) {
        11.remove(entry);
        size--;
        break;
```

HashMap (put method)

```
* Method put to add a new entry to the hashtable
* @param key the value of the key of the new entry
* @param value the value associated with the key
* @return the old value of the entry if an entry is found for key
         or the new value if a new entry was added to the hashtable
public V put(K key, V value) {
  // check if the key is already in the hashtable
  // modify its associated value if key is found
  if(get(key) != null) {
    int HTIndex = hash(key.hashCode());
     LinkedList<HashMapEntry<K,V>> 11 = hashTable[HTIndex];
     for (HashMapEntry<K, V> entry: 11) {
       if(entry.getKey().equals(key)) {
          V old = entry.getValue();
           entry.setValue(value);
           return old;
  // key was not found. Check if rehashing is needed before adding a new entry
  if(size >= hashTable.length * loadFactor)
     rehash();
  // Add a new entry to the hashtable
  int HTIndex = hash(key.hashCode());
  if(hashTable[HTIndex] == null) {
     hashTable[HTIndex] = new LinkedList<>();
  hashTable[HTIndex].add(new HashMapEntry<>(key, value));
  size++;
  return value;
```



HashMap (rehash method)

```
Method rehash
creates a new hashtable with double capacity
puts all the entries from the old hashtable into the new table
private void rehash() {
  ArrayList<HashMapEntry<K,V>> list = toList();
  hashTable = new LinkedList[hashTable.length << 1];</pre>
  size = 0;
  for (HashMapEntry<K, V> entry: list)
     put(entry.getKey(), entry.getValue());
```

HASHING

HashMap (toList and toString methods)

```
* Method toList
 @return an arraylist with all the entries in the hashtable
public ArrayList<HashMapEntry<K,V>> toList() {
  ArrayList<HashMapEntry<K,V>> list = new ArrayList<>();
   for(int i=0; i< hashTable.length; i++) {</pre>
     if(hashTable[i]!= null) {
       LinkedList<HashMapEntry<K,V>> 11 = hashTable[i];
       for (HashMapEntry<K, V> entry: 11)
          list.add(entry);
   return list;
* Method toString
* @return a formatted string with all the entries in the hashtable
public String toString() {
  String out = "[";
   for(int i=0; i<hashTable.length; i++) {</pre>
     if(hashTable[i]!=null) {
        for(HashMapEntry<K,V> entry: hashTable[i])
           out += entry.toString();
        out += "\n";
   out += "]";
   return out;
```

Testing HashMap

```
import java.util.Scanner;
import java.io.File;
import java.io.FileNotFoundException;
public class TestHashMap{
   public static void main(String[] args) {
        HashMap<String, String> states = new HashMap<>();
        readStates(states, "states.txt");
        System.out.println(states);
   public static void readStates(HashMap<String,String> hm, String filename) {
        try{
            Scanner read = new Scanner(new File(filename));
            while(read.hasNextLine()){
                String line = read.nextLine();
                String[] tokens = line.split("\\|");
                String state = tokens[0];
                String capital = tokens[1];
                hm.put(state, capital);
            read.close();
        catch (FileNotFoundException e) {
            System.out.println("File not found");
```



Performance of the HashMap

Operation	Complexity	Operation	Complexity	
HashMap()	0(1)	isEmpty()	0(1)	
HashMap(int)	O(log n)	containsKey(K)	0(1)	
HashMap(int, double)	O(log n)	get(K)	0(1)	
trimToPowerOf2(int)	O(log n)	put(K, V)	O(1)-O(n)	
hash(int)	0(1)	remove(K)	0(1)	
rehash()	O(n)	toList()	O(n)	
size()	0(1)	toString()	O(n)	



Summary

- → Hash Tables
 - → Efficient search operation (O(1))
 - → Performance affected by:
 - Hash Code function Hash function size of the table (load Factor and rehashing)
 - Collisions
 - ◆ Chaining
 - Probing (linear, quadratic, double hashing)
- → Implementation of the HashMap data structure