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1 Introduction

2 question 1

Equation **??** is a second order linear differential equation. According to theorem 3.6.1 in Boyce [**?**]:
According to Boyce [**?**]:

$$x(t) = c_1 y_1(t) + c_2(t) y_2 + Y(t) \tag{1}$$

$$Y(t) = -y_1(t) \int_{t0}^{t} \frac{y_2(s)F(s)}{W[y_1, y_2](s)} ds + -y_2(t) \int_{t0}^{t} \frac{y_1(s)F(s)}{W[y_1, y_2](s)} ds \tag{2}$$

With $y(t)$ independent solutions solutions for the homogeneous differential equation:

$$\ddot{y} + \frac{\omega}{Q}\dot{y} + \omega^2 y = 0 \tag{3}$$

We first solve $y(t)$ by seeing that, for constant values of ω,m and Q, a solution for equation 1 is $y(t) = e^(r\ t)$. Appliyng to equation 1 yields:

$$(r^2 + \frac{\omega}{Q} r + \omega^2) y(t) = 0 \tag{4}$$

$$r^2 + \frac{\omega}{Q} r + \omega^2 = 0$$

We find two solutions for r:

$$r_1 = \frac{-\frac{\omega}{Q} + \sqrt{\left(\frac{\omega}{Q}\right)^2 - 4\,\omega^2}}{2}$$

$$r_2 = \frac{-\frac{\omega}{Q} + \sqrt{\left(\frac{\omega}{Q}\right)^2 - 4\,\omega^2}}{2}$$

We use this to define:

$$y_1(t) = e^{r_1\ t}$$
$$y_2(t) = e^{r_2\ t}$$

With c_1 and c_2 two constant variables that depend on the boundary values.
For solving x, we will use the method of variation of parameters. Therefore, c_1 and c_2 are changed for time dependent functions $u_1(t)$ and $u_2(t)$;

$$x = u_1(t)\, y_1(t) + u_2(t)\, y_2(t)$$

Differentiating with respect to t yields:

$$\dot{x} = u_1(t)\, r_1\, y_1(t) + u_2(t)\, r_2\, y_2(t) + \dot{u}_1(t)\, y_1(t) + \dot{u}_2(t)\, y_2(t)$$

Since there are two unknown functions $u_1(t)$ and $u_2(t)$ and only one equation to solve those. We may impose an extra condition on $u_1(t)$ and $u_2(t)$. If we require that:

$$\dot{u}_1(t)\, y_1(t) + \dot{u}_2(t)\, y_2(t) = 0 \tag{5}$$

The equation for $\dot{x}$ reduces to:

$$\dot{x} = u_1(t)\, r_1\, y_1(t) + u_2(t)\, r_2\, y_2(t)$$

The second derivative of x with respect to t reads:

$$\ddot{x} = u_1(t)\, r_1^2\, y_1(t) + u_2(t)\, r_2^2\, y_2(t) + \dot{u}_1(t)\, r_1\, y_1(t) + \dot{u}_2(t)\, r_2\, y_2(t)$$

If we now use these results in equation **??** we get the following:

$$m\big[u_1(t)r_1^2y_1(t)+u_2(t)r_2^2y_2(t)+\dot{u}_1(t)r_1y_1(t)+\dot{u}_2(t)r_2y_2(t)\big]+\frac{\omega\,m}{Q}\big[u_1(t)r_1y_1(t)+u_2(t)r_2y_2(t)\big]+m\omega^2\big[u_1(t)y_1(t)+u_2(t)y_2(t)\big]$$

$$u_1(t)\,m\big(\,r_1^2+\frac{\omega\,m}{Q}r_1+\omega^2\big)y_1(t)+u_2(t)\,m\big(\,r_2^2+\frac{\omega\,m}{Q}r_2+\omega^2\big)y_2(t)+\dot{u}_1(t)\,r_1\,y_1(t)+\dot{u}_2(t)\,r_2\,y_1(t)=F(t)$$

Using equation 5 reduces the latter equation to:

$$\dot{u}_1(t)\,r_1\,y_1(t)+\dot{u}_2(t)\,r_2\,y_1(t)=F(t)$$

We can solve this using equation 7:

$$\dot{u}_2(t)=-\dot{u}_1(t)\,\frac{y_1(t)}{y_2(t)}$$

Therefore we find:

$$\dot{u}_1(t)=F(t)$$

3 question 2

4 question 3

5 conclusion