Wolfram|Alpha Step-by-Step Solution

Wolfram|Alpha Input:

STEP 1

Solve the separable equation $\frac{dz(x)}{dx} = \frac{\sqrt{-c^2 + (n0 - \alpha\, z(x))^2}}{c}$:

STEP 2

Divide both sides by $\frac{\sqrt{-c^2 + (n0 - \alpha\, z(x))^2}}{c}$:

$$\frac{c\,\frac{dz(x)}{dx}}{\sqrt{-c^2 + (n0 - \alpha\, z(x))^2}} = 1$$

STEP 3

Integrate both sides with respect to x:

$$\int \frac{c\,\frac{dz(x)}{dx}}{\sqrt{-c^2 + (n0 - \alpha\, z(x))^2}}\,dx = \int 1\,dx$$

STEP 4

Evaluate the integrals:

$$-\frac{c\log\left(n0 + \sqrt{-c^2 + (n0 - \alpha\, z(x))^2} - \alpha\, z(x)\right)}{\alpha}$$

$= x + k_1$, where k_1 is an arbitrary constant.

STEP 5

Solve for $z(x)$:

Answer:

$$z(x) = -\frac{c^2\, e^{(\alpha\,(x+k_1))/c} + e^{-(\alpha\,(x+k_1))/c} - 2\,n0}{2\,\alpha}$$