과제 + 필기

HW 01.

3.6

3.7

4.2

4.5

4.12

due 9/16.

3-6

3.6 Evaluate e^{-5} using two approaches

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \cdots$$

and

$$e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \cdots}$$

and compare with the true value of 6.737947×10^{-3} (0.006737947). Use 20 terms to evaluate each series and compute true and approximate relative errors as terms are added.

① $e^{-5} = 1 - 5 + \frac{5^2}{2} - \frac{5^3}{3!} + \cdots$

term	계산결과	true relative error	approximate relative error
0	1		$\frac{0.006737947 - 1}{0.006737947} \fallingdotseq -147$
1	−4	$\frac{-4-1}{-4} = 1.25$	$\frac{0.006737947 - (-4)}{0.006737947} \fallingdotseq 594.65$
2	8.5	1.47	−1260
⋮	⋮	⋮	⋮
20	0.00674554	0.0058	−0.0011

② $e^{-5} = \frac{1}{e^5} = \frac{1}{1 + 5 + \frac{5^2}{2} + \frac{5^3}{3!} + \cdots}$

term	계산결과	true relative error	approximate relative error
0	1		(−147)
1	0.1666….	$\frac{0.1666\ldots - 1}{0.1666\ldots} = 0.833$	$\frac{0.006737947 - 1.666\ldots}{0.006737947}$ (−23.8)
2	0.0540541	0.696	−7.02
3	0.0254237	0.53	−2.77
⋮	⋮	⋮	⋮
20	0.00673795	0.0000003	−0.00000008

★ ① 보다 ② 가 더 정확하게 근사한다.

★ ① 보다 ② 가 더 빠르게 True 값에 같은 값에 수렴한다.

3-7

3.7 The derivative of $f(x) = 1/(1 - 3x^2)$ is given by

$$\frac{6x}{(1 - 3x^2)^2}$$

Do you expect to have difficulties evaluating this function at $x = 0.577$? Try it using 3- and 4-digit arithmetic with chopping.

chopping?

true value: $\frac{6(0.577)}{(1 - 3(0.577)^2)^2} = 2,352,910.7926$

① 3-digit chopping.

$6x = 6(0.577) = 3.462$

$x^2 = 0.332929$ / $3x^2 = 0.996$ / $1 - 3x^2 = 0.004$

$\therefore \frac{6x}{(1-3x^2)^2} = \frac{3.462}{(0.004)^2} = 216,375$

★ relative error $\left| \frac{2,352,910 - 216,250}{2,352,910} \right| \fallingdotseq 0.9081$

② 4-digit chopping.

$6x = 3.4620$

$x^2 = 0.332929$ / $3x^2 = 0.9987$ / $1 - 3x^2 = 0.0013$

$\frac{6x}{(1-3x^2)^2} = \frac{3.4620}{(0.0013)^2} = 2,048,520.7100$

★ relative error

$\left| \frac{2,352,910 - 2,048,520}{2,352,910} \right| \fallingdotseq 0.1294$

소숫점 몇자리까지 나타내냐에 따라

에러의 퍼센트가 매우 달라지게 나타난다.

90% (3-digits), 12% (4-digits)

4.2

4.2 The Maclaurin series expansion for cos x is

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

Starting with the simplest version, $\cos x = 1$, add terms one at a time to estimate $\cos(\pi/3)$. After each new term is added, compute the true and approximate percent relative errors. Use your pocket calculator to determine the true value. Add terms until the absolute value of the approximate error estimate falls below an error criterion conforming to two significant figures. (0.005%?)

✱ true value of $\cos(\pi/3) = \frac{1}{2} = 0.5$

① $\cos(\pi/3) = 1$

— relative error : $\frac{0.5-1}{0.5} = -1 = -100\%$

② $\cos(\pi/3) \approx 1 - \frac{(\pi/3)^2}{2} = 0.45168864\ldots\ldots$

— relative error : $\frac{0.5 - 0.45168864\ldots}{0.5} = 0.09662272 \approx 9.66\%$

— approximate relative error : $\frac{0.45168864 - 1}{0.45168864} = -1.21391444 \approx -121.39\%$

③ $\cos(\pi/3) \approx 1 - \frac{(\pi/3)^2}{2} + \frac{(\pi/3)^4}{4!} = 0.50179620 \ldots$

— relative error : $\frac{0.5 - 0.50179620}{0.5} = -0.003592 \approx -0.3592\%$

— approximate relative error : $\frac{0.501796 - 0.451689}{0.501796} = 0.09985532 \approx 9.99\%$

④ $\cos(\pi/3) \approx 1 - \frac{(\pi/3)^2}{2} + \frac{(\pi/3)^4}{4!} - \frac{(\pi/3)^6}{6!} = 0.49996457 \ldots$

— relative error : $\frac{0.5 - 0.49996457}{0.5} = 0.00007086$

— approximation relative error : $\frac{0.499965 - 0.501796}{0.499965} = -0.003662 \ldots$

$|-0.003662| < 0.005$

$\therefore \cos(\pi/3) = 1 - \frac{(\pi/3)^2}{2} + \frac{(\pi/3)^4}{4!} - \frac{(\pi/3)^6}{6!} = 0.4999649 \approx 0.5$

4.5

4.5 Use zero- through third-order Taylor series expansions to predict $f(3)$ for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

using a base point at $x = 1$. Compute the true percent relative error ε_t for each approximation.

$f'(x) = 75x^2 - 12x + 7$

$f''(x) = 150x - 12$

$f'''(x) = 150$

True $f(3) = 25\cdot3^3 - 6\cdot3^2 + 7\cdot3 - 88 = 554$

✱ Taylor : $f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(x_i)}{3!}h^3$

① $(n=0)$

$f(3) \approx f(1) = 25 - 6 + 7 - 88 = -62$

$\varepsilon_t = \frac{554-(-62)}{554} \approx 1.111913357$ 111.19%

② $(n=1)$

$f(3) \approx f(1) + f'(1)(3-1) = -62 + (75-12+7)2 = -62 + 140 = 78$

$\varepsilon_t = \frac{554-78}{554} = 0.85920578$ 85.92%

③ $(n=2)$

$f(3) \approx f(1) + 2f'(1) + \frac{f''(1)\times2^2}{2!} = -62 + 140 + \frac{138\times4}{2} = 354$

$\varepsilon_t = \frac{554-354}{554} = 0.36101083$ 36.10%

④ $(n=3)$

$f(3) \approx f(1) + 2f'(1) + \frac{f''(1)\times2^2}{2!} + \frac{f'''(1)\times2^3}{3!} = 354 + \frac{150\times8}{6} = 554$

$\varepsilon_t = \frac{554-554}{554} = 0$.   0.0%

4.12

4.11 Recall that the velocity of the falling parachutist can be computed by [Eq. (1.10)],

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

Use a first-order error analysis to estimate the error of v at $t = 6$, if $g = 9.81$ and $m = 50$ but $c = 12.5 \pm 1.5$.

4.12 Repeat Prob. 4.11 with $g = 9.81$, $t = 6$, $c = 12.5 \pm 1.5$, and $m = 50 \pm 2$.

$$\Delta v(\tilde{c}, \tilde{m}) = \left|\frac{\partial v}{\partial c}\right| \Delta \tilde{c}^{\,1.5} + \left|\frac{\partial v}{\partial m}\right| \Delta \tilde{m}^{\,2}$$

$$= \left| \frac{g\tilde{m}}{\tilde{c}^2}\left(e^{-\frac{\tilde{c}}{\tilde{m}}t} - 1\right) + \frac{gt}{\tilde{c}} e^{-\frac{\tilde{c}}{\tilde{m}}t} \right| \Delta \tilde{c}$$

$$+ \left| \frac{g}{\tilde{c}} - e^{-\frac{\tilde{c}}{\tilde{m}}t}\left(\frac{g}{\tilde{c}} + \frac{gt}{\tilde{m}}\right) \right| \Delta \tilde{m}$$

$$= \left| \frac{9.81 \cdot 50}{(12.5)^2}\left(e^{-\frac{12.5}{50}\cdot 6} - 1\right) + \frac{9.81 \cdot 6}{12.5} e^{-\frac{12.5}{50}\cdot 6} \right| \cdot 1.5$$

$$+ \left| \frac{9.81}{12.5} - e^{-\frac{12.5}{50}\cdot 6}\left(\frac{9.81}{12.5} + \frac{9.81 \cdot 6}{50}\right) \right| \cdot 2$$

$$\approx 2.116$$

$$v \approx \frac{9.81 \cdot 50}{12.5}\left(1 - e^{-\frac{12.5}{50}6}\right)$$

$$\approx 30.484 \pm 2.116$$