HW 09

$$\text{①} \begin{cases} x' = a_1 x + a_2 y + a_3 \\ y' = a_4 x + a_5 y + a_6 \end{cases}$$

$$\text{②} \begin{cases} S(a_1 a_2 a_3 . a_4 a_5 . a_6) = \sum_{i=1}^{N} (e_1^2 + e_2^2) \\ \\ = \sum_{i=1}^{N} \left[\left(x_i' - a_1 x_i - a_2 y_i - a_3 \right)^2 + \left(y_i' - a_4 x_i - a_5 y_i - a_6 \right)^2 \right] \end{cases}$$

$$\text{③} \begin{cases} \dfrac{\partial S}{\partial a_1} = 2 \sum_{i=1}^{N} \left(x_i' - a_1 x_i - a_2 y_i - a_3 \right) \cdot (-x_i), \quad \dfrac{\partial S}{\partial a_4} = 2 \sum_{i=1}^{N} \left(y_i' - a_4 x_i - a_5 y_i - a_6 \right) \cdot (-x_i) \\[2mm] \dfrac{\partial S}{\partial a_2} = 2 \sum_{i=1}^{N} \left(x_i' - a_1 x_i - a_2 y_i - a_3 \right) \cdot (-y_i), \quad \dfrac{\partial S}{\partial a_5} = 2 \sum_{i=1}^{N} \left(y_i' - a_4 x_i - a_5 y_i - a_6 \right) \cdot (-y_i) \\[2mm] \dfrac{\partial S}{\partial a_3} = 2 \sum_{i=1}^{N} \left(x_i' - a_1 x_i - a_2 y_i - a_3 \right) \cdot (-1), \quad \dfrac{\partial S}{\partial a_6} = 2 \sum_{i=1}^{N} \left(y_i' - a_4 x_i - a_5 y_i - a_6 \right) \cdot (-1) \end{cases}$$

$$\text{④} \begin{cases} a_1 \sum x_i^2 + a_2 \sum x_i y_i + a_3 \sum x_i = \sum x_i' x_i \\ a_1 \sum x_i y_i + a_2 \sum y_i^2 + a_3 \sum y_i = \sum x_i' y_i \\ a_1 \sum x_i + a_2 \sum y_i + a_3 \sum 1 = \sum x_i' \end{cases} \Rightarrow \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & \sum 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum x_i' x_i \\ \sum x_i' y_i \\ \sum x_i' \end{bmatrix}$$

$$\begin{cases} a_4 \sum x_i^2 + a_5 \sum x_i y_i + a_6 \sum x_i = \sum y_i' x_i \\ a_4 \sum x_i y_i + a_5 \sum y_i^2 + a_6 \sum y_i = \sum y_i' y_i \\ a_4 \sum x_i + a_5 \sum y_i + a_6 \sum 1 = \sum y_i' \end{cases} \Rightarrow \begin{bmatrix} a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & \sum 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum y_i' x_i \\ \sum y_i' y_i \\ \sum y_i' \end{bmatrix}$$

$$\text{⑤} \quad \begin{bmatrix} a_1 & a_4 \\ a_2 & a_5 \\ a_3 & a_6 \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & \sum 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum x_i' x_i & \sum y_i' x_i \\ \sum x_i' y_i & \sum y_i' y_i \\ \sum x_i' & \sum y_i' \end{bmatrix}$$