Homework #1

Due. 4/6/2020 (Monday Midnight)

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(Goal of Project: Understanding Linear Transformations)

Developing the subsampling/interpolation scheme operated in frequency domain.

From this project, we want to know how we can subsample/interpolate the image in frequency domain (compressed domain), by multiplying the proper frequency version of transformation matrix in frequency domain directly.

(Tools)

1. Subsampling method : Simple averaging method, that is,

$$g(\frac{m}{2},\frac{n}{2}) = \frac{1}{4}\sum_{i=0}^{1}\sum_{j=0}^{1} g(m+i,n+j)$$

2. Interpolation method : Simple linear interpolation method, that is,

$$g(2m+1,2n+1) = \frac{1}{4}(f(m,n)+f(m+1,n)+f(m,n+1)+f(m+1,n+1))$$

$$g(2m+1,2n) = \frac{1}{2}(f(m,n)+f(m+1,n))$$

$$g(2m,2n+1) = \frac{1}{2}(f(m,n)+f(m,n+1))$$

$$g(2m,2n) = f(m,n)$$

for m, n = 0,1,2,3,.....,N

3. For frequency domain transformation, use Discrete Fourier Transform.

(Description of Project)

1. Get a grey level image f which size is N*N. (For example, 256*256, however, N= $8n$), and partition to 8*8 sub images f_8.
2. Apply DFT to these sub images f_8, and get $\boldsymbol{F}_8$, then make fourier transformed image $\boldsymbol{F}$ (N*N) by patching $\boldsymbol{F}_8$s.
3. Derive the proper subsampling function matrix $\boldsymbol{S}$ in spatial domain using upper subsampling equation (tool 1).

$$\underset{(4\times 8)}{S}\ \underset{(8\times 8)}{f_8}\ \underset{(8\times 4)}{S^T} = \underset{(4\times 4)}{f_4}$$

where, f_8 : the original 8*8 image,

f_4 the subsampled 4*4 image,

S : the subsampling matrix in spatial domain

4. Multiplying proper 4*4 DFT matrix D_4 and 8*8 DFT matrix D_8 to matrix S, derive the frequency version of subsampling function matrix, $S_f = D_4 S D_8^H$ in order to be

$$S_f F_8 S_f^H = D_4 S D_8^H F_8 D_8 S^T D_4^H = F_4.$$

5. Derive the proper interpolation matrix I using interpolation equation (tool 2).

$$\underset{(8\times4)}{I}\ \underset{(4\times4)}{f_4}\ \underset{(4\times8)}{I^T} = \underset{(8\times8)}{f_8}$$

where, f_8 : the interpolated image in spatial domain

I : the interpolation matrix in spatial domain.

6. Similarly, derive the frequency version of interpolation matrix I_f by using DFTs and I, as same as before. That is, $I_f = D_8 I D_4^H$

7. Apply subsampling/interpolation function matrix S/I to original image f in spatial domain and apply frequency version of subsampling/interpolation function matrix S_f/I_f to fourier transformed image $\boldsymbol{F}$ (N*N) in frequency domain. Then, compare the final images by taking proper inverse DFT (D_4^H or D_8^H)