Homework 6

Image Restoration

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**Goal of Project** : Understanding on Optimal Filters for Image Restoration.

**Tools :** Inverse Filter, Wiener Filter, and Constrained Matrix Inversion Filter

1. Inverse Filter :

Let’s consider ‘f’ is original image. Image ‘f’ become degradation image ‘g’ by some point spread function ‘h’. Then you can think g = h\*f. In frequency domain, it can be shown as G = HF. Now you know point spread function and degradation image ‘g’. You can get original image’s frequency domain by . To avoid the amplification of noise, apply not use as filter factor , but a windowed version of it, cutting it off at a frequency before H(u,v) becomes too small or before its first zero. So

2. Wiener Filter :

In inverse filter, it never consider error. In this case, let’s consider error. Then g = h\*f + .

Then . You don’t know exact error, so you cannot remove . But it is white gaussian noise, so it can be apply ‘mean’. Let’s assume that identify filter m for f = m\*g.

= E( f(r)g(s) ) = E( ) = ) =

=m(r)\*. Let’s convert to frequency domain then

= M, M = = . When you get , consider = E( f(r)(s) ) = E(f(r))E() = 0.

Now M = = = . Assume for constant because it is not big.

3. Constrained Matrix Inversion Filter :

In this case, we need find that h when ||g - h\*f||2 is minimum. But consider there is error, add normalize factor in error;

|| l\*f ||2. l is Laplacian matrix, for convert f to second derivative for smoothing constraint term.

Also second derivative can reduce difference between data. So find f when

= ||g - h\*f||2 + || l\*f ||2 is minimum. You can find f when error is minimum by = 0.

= .

Before calculation, = + = Af + ATf = (A +AT)f.

= = = 0.

Finally . Now let’s see l.

When you apply Fourier transform in l. How to apply Fourier transform in discussion part.

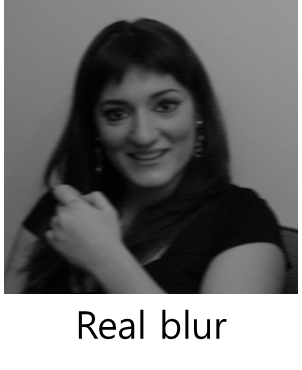
.

’s magnitude =

So F = . Apply inverse Fourier transform for get original image.

**Process :**

< Fig 1 >

There are five pictures in < Fig 1 >; Original image, Cylinder blurred image, Cylinder gaussian noised image, Real blurred image and Real gaussian noised image. They are made from moving 10 pixels in  direction.

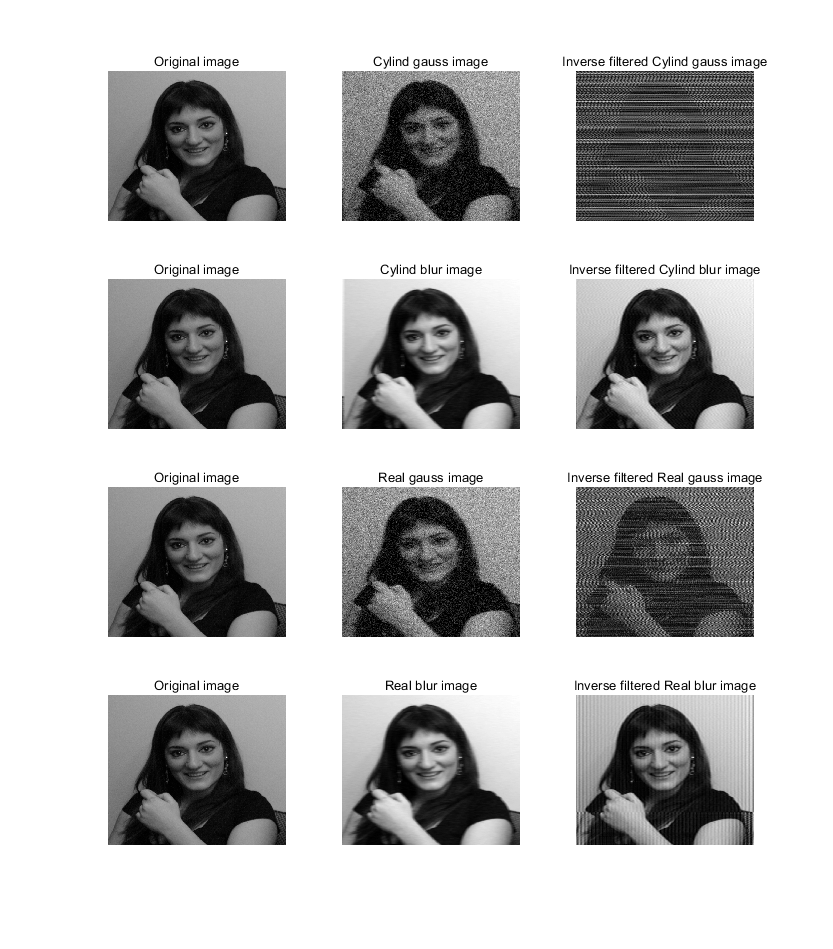
g(x,y) = = . Then G(u,v) = = = = F(u,v) = F(u,v). So impulse response in frequency domain is H(u,v) = .

Image size is 498 x 512 in first row of < Fig 1 > and 498 x 498 in second row of < Fig 2 >

I apply this method by “Matlab” tool.

**Result :**

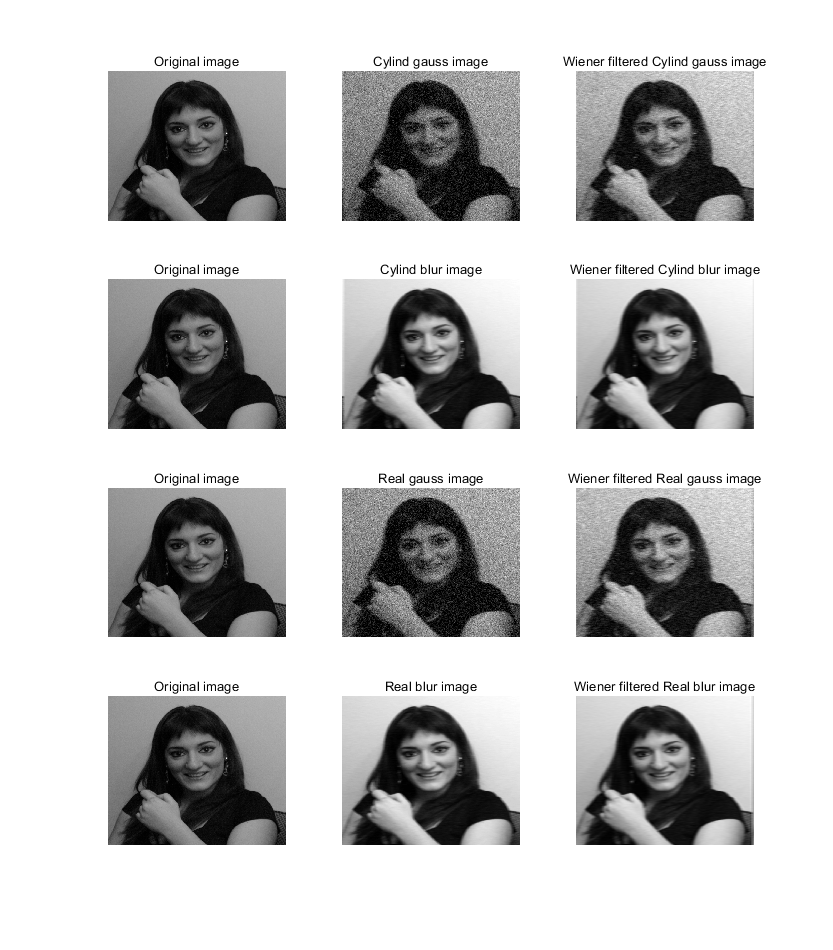
Prob 1. Inverse Filter



< Fig 2 >

It is obvious from this that if we wish to obtain a real image, whatever we do to frequencies lower than m0 that lowest value of H could be 0, we must also do it to the frequencies higher than N −m0. H(u,v) = , that mean m0 50~51. When you see second & fourth row, they seem restore well but still have some specific artifacts. Also in first row and third row, images are deformed.

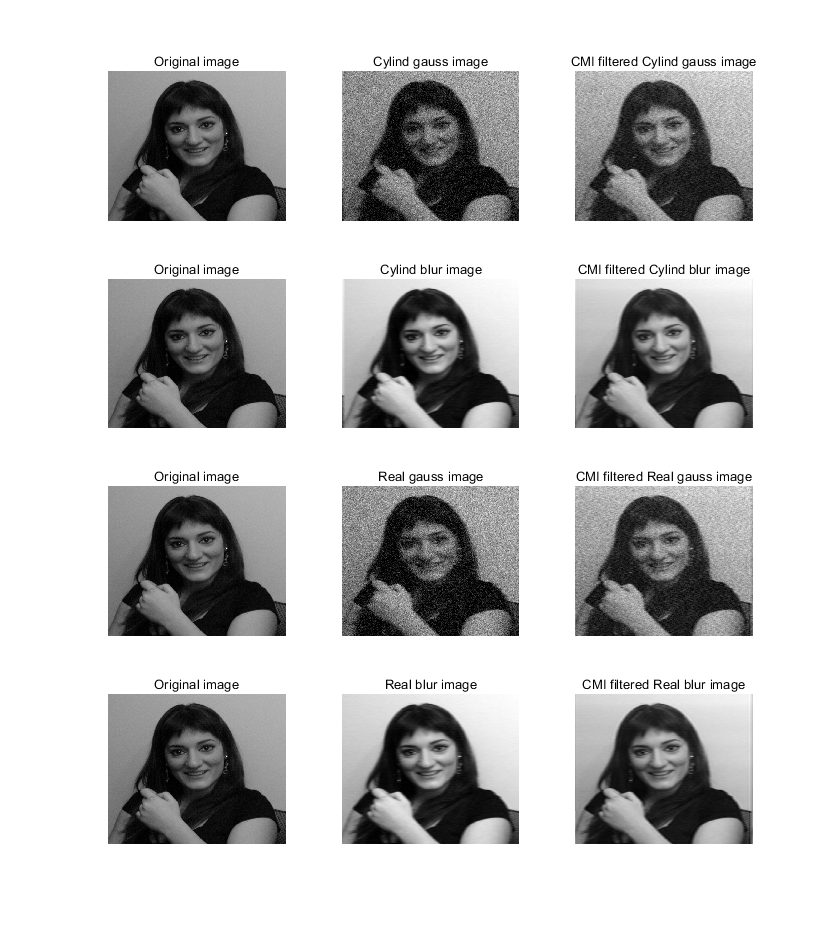
Prob 2. Wiener Filter



< Fig 3 >

Wiener filter is optimized for noise added image. It combine deblurring filter and denosing filter. When you see first row and third row of < Fig 3 >, SNR seems better than previous one.

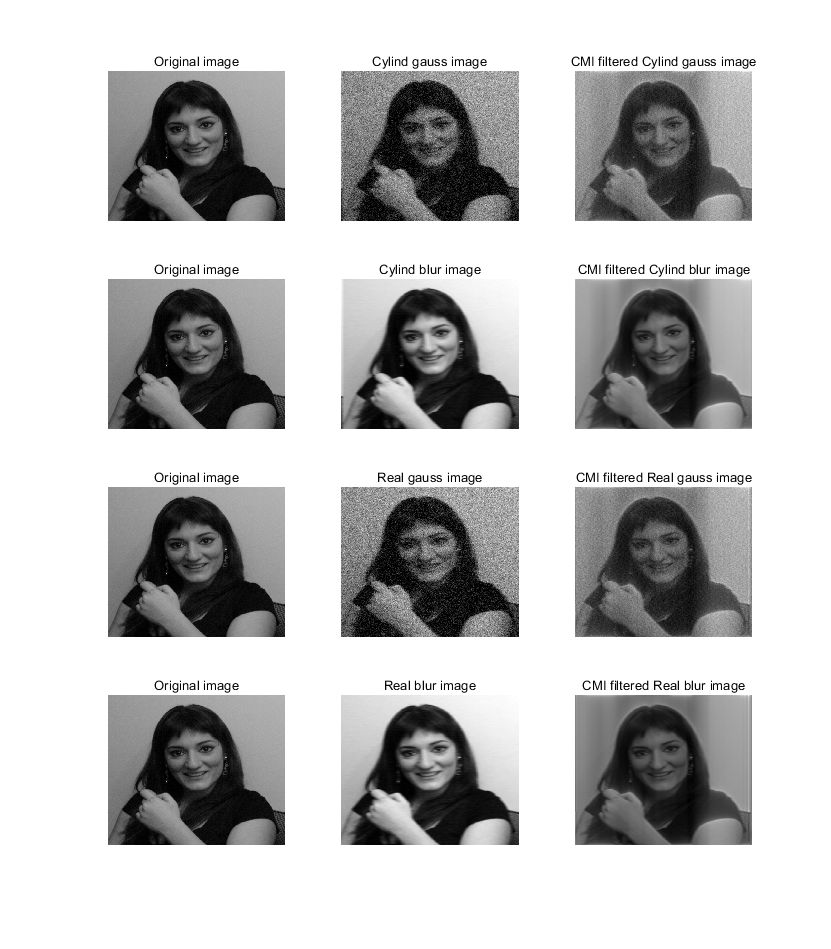
Prob 3. Constrained Matrix Inversion Filter



< Fig 4 >

It seems like Wiener filter. It contains specific terms that matrix linear equation’s constraint term to calculate. That smoothing constraint term acts like low-pass filter. As you can see < Fig 4 > looks like < Fig 3 >.

**Discussion :**



< Fig 5 >

Actually I think there are some error in textbook. Laplace filter is . When it convert to N x N, it is . So when you apply Fourier transform and its magnitude;

However, it has to correct by . So when you apply Fourier transform; -4 + + + + .

< Fig 5 > is apply real “Constrained Matrix Inversion Filter” that I think. When you apply filter, every images become blur. But I think noises are much more removed.

I think textbook is wrong but author may think other things. So I distinguish to 2 ways.

**Reference :**

Chapter5 of TextbookFile