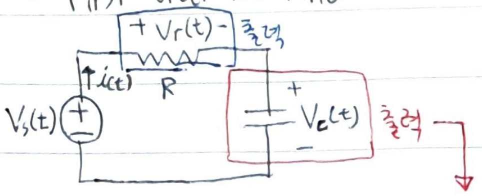
[3.10] 미분방정식으로 표현된 CT Filters의 예

- First-order RC filter



$V_s = R\,i(t) + V_c$

$V_s(t) = RC\frac{dV_c(t)}{dt} + V_c(t)$

$e^{j\omega t} \longrightarrow \boxed{LTI} \longrightarrow H(j\omega)e^{j\omega t}$

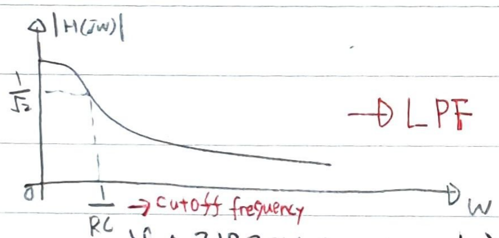
$e^{j\omega t} = RC\frac{d}{dt}[H(j\omega)e^{j\omega t}] + H(j\omega)e^{j\omega t}$

$\cancel{e^{j\omega t}} = RCj\omega H(j\omega)\cancel{e^{j\omega t}} + H(j\omega)\cancel{e^{j\omega t}}$

$\boxed{\therefore H(j\omega) = \frac{1}{1+RCj\omega}}$ 주파수 응답

⇓

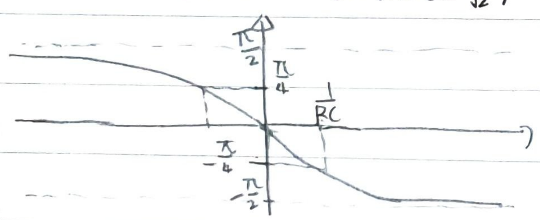
$$\begin{cases} |H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}} \\ \angle H(j\omega) = -\tan^{-1}(RC\omega) \end{cases}$$



→ LPF

$\frac{1}{RC}$ → cutoff frequency

→ 3dB 주파수 (Magnitude $\frac{1}{\sqrt{2}}$)



$V_s(t) = V_r(t) + V_c(t)$

미분 $\boxed{V_r(t) = RC\frac{dV_c(t)}{dt}}$

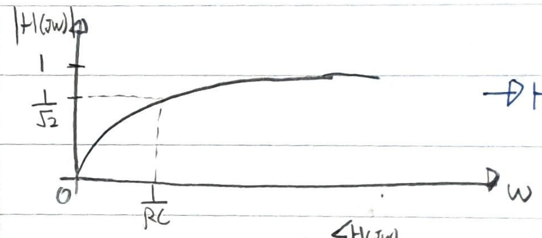
$\frac{dV_s(t)}{dt} = \frac{dV_r(t)}{dt} + \frac{dV_c(t)}{dt}$

$\times RC$: $RC\frac{dV_s(t)}{dt} = RC\frac{dV_r(t)}{dt} + V_r(t)$

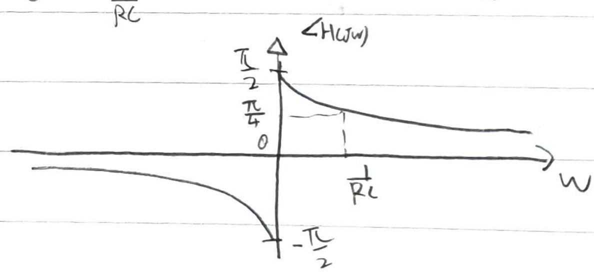
$RCj\omega\cancel{e^{j\omega t}} = RCj\omega H(j\omega)\cancel{e^{j\omega t}} + H(j\omega)\cancel{e^{j\omega t}}$

$\boxed{\therefore H(j\omega) = \frac{j\omega RC}{1+j\omega RC}}$

$$\begin{cases} |H(j\omega)| = \frac{|RC\omega|}{\sqrt{1+(RC\omega)^2}} \\ \angle H(j\omega) = \frac{\pi}{2} - \tan^{-1}(RC\omega) \end{cases}$$



→ HPF



[3.11] 차등방정식으로 표현된 DT filters 의 예

* Recursive DE

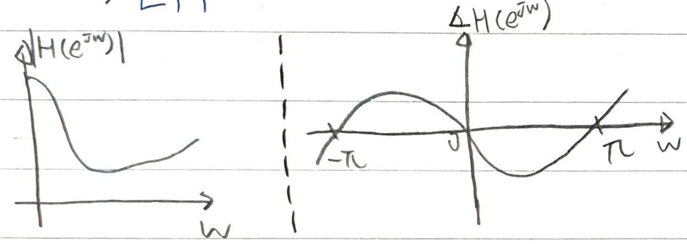
Let, $y[n]-ay[n-1]=x[n]$

$$x[n]=e^{jwn} \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n]=H(e^{jw})e^{jwn}$$

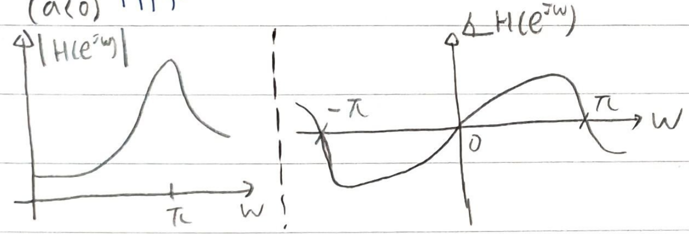
$$H(e^{jw})\cancel{e^{jwn}}-aH(e^{jw})e^{jw\cancel{(n-1)}}=\cancel{e^{jwn}}$$

$$\rightarrow \boxed{\begin{aligned} H(e^{jw}) &= \frac{1}{1-ae^{-jw}} \\ &= \frac{1}{1-a(\cos w-j\sin w)} \\ &= \frac{1}{1-a\cos w+ja\sin w} \end{aligned}}$$

$(a>0)$ LPF



$(a<0)$ HPF



* FIR nonrecursive DE

— Weighted Moving Average (MA)

$$\underbrace{y[n]=\sum_{k=-N}^{M} b_k\, x[n-k]}_{\text{Moving average filter}}$$

Ex) Three-point MA filter

$\rightarrow b_k=\frac{1}{3}$

$$y[n]=\frac{1}{3}(x[n-1]+x[n]+x[n+1])$$

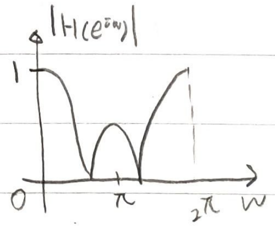
⇝ Blurring effect, LPF

- Impulse 응답

$$h[n]=\frac{1}{3}(\delta[n-1]+\delta[n]+\delta[n+1])$$

- Frequency 응답

$$\begin{aligned} H(e^{jw}) &= \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} \\ &= \frac{1}{3}\sum_{n=-\infty}^{\infty}(\delta[n-1]+\delta[n]+\delta[n+1])e^{-jwn} \\ &= \frac{1}{3}(e^{-jw}+1+e^{jw}) \\ &= \frac{1}{3}(1+2\cos w) \end{aligned}$$



$\angle H(e^{jw})=0$

⇝ LPF (그래프를 전체적으로 보면 LPF이다.)

=> 일반화

$(N+M-1)$-point moving average filter

$$y[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M} x[n-k]$$

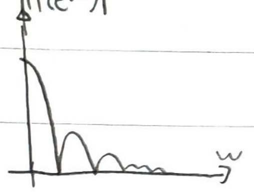
$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{jwn}$$

$$\vdots$$

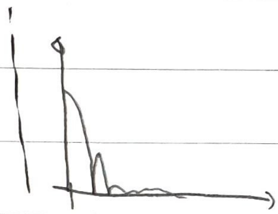
$$= \frac{1}{N+M+1}\sum_{n=-N}^{M} e^{-jwn}$$

$$= \frac{1}{N+M+1} e^{jw[(N-M)/2]} \frac{\sin[w(M+N+1)/2]}{\sin(w/2)}$$

$|H(e^{jw})|$



$M=N=16$



$M=N=32$

→ LPF, M과 N 큰값 사용하면

고주파는 줄어들고 저주파 성분들이 강조가 됨

* HPF

$$y[n] = \frac{x[n]-x[n-1]}{2}$$ → Edge detector

$$\rightarrow h[n] = \frac{1}{2}(\delta[n]-\delta[n-1])$$

$$H(e^{jw}) = \frac{1}{2}(1-e^{-jw})$$

$$= \frac{\boxed{j}\, e^{-jw/2}\,(e^{jw/2}-e^{-jw/2})}{2\boxed{j}} \rightsquigarrow \sin(\tfrac{w}{2})$$

$$= je^{-jw/2}\sin(\tfrac{w}{2})$$

$|H(e^{jw})|$