Report Lab 2

Exercise 2:

Question 1:
Minimizer Consistency: For all values of α0\alpha_0α0, the minimizer is consistently [−49,36], indicating that the gradient descent algorithm finds the same minimizer regardless of the initial step size.

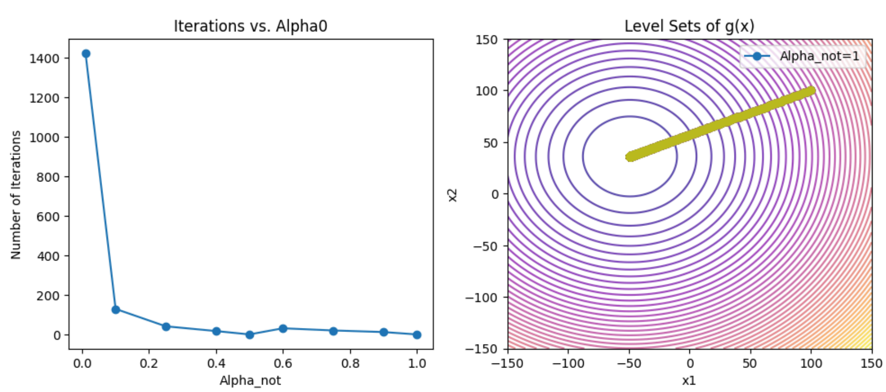
Objective Function Values: The objective function values are very small, approaching zero, with some variation depending on α0\alpha_0α0. The values are generally close to 0.00.00.0 for larger α0\alpha_0α0 and slightly higher for smaller α0\alpha_0α0.

Number of Iterations: Smaller values of α0\alpha_0α0 lead to a higher number of iterations required for convergence. Specifically:

- Large α0\alpha_0α0 values (like 1 and 0.5) result in very few iterations (1 or 13).
- Smaller α0\alpha_0α0 values (like 0.01) require many more iterations (up to 1426).

Comparison with Exact Line Search: For α0=1\alpha_0 = 1α0=1 and α0=0.5\alpha_0 = 0.5α0=0.5, the number of iterations is minimal and matches the exact line search results, suggesting these initial step sizes are quite effective. For other values of α0\alpha_0α0, especially smaller ones, the number of iterations increases significantly.

Plot and Analysis: A plot of the number of iterations versus α0\alpha_0α0 values would show a decreasing trend in the number of iterations as α0\alpha_0α0 increases. This highlights that larger step sizes are more efficient for convergence compared to smaller ones.



Question 2:

Minimizer Consistency: The minimizer is consistently [2,4][2, 4][2,4] for all α0\alpha_0α0 values, indicating that the gradient descent algorithm finds the same minimizer for different step sizes.

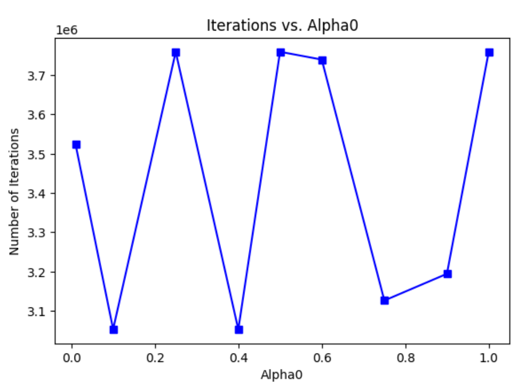
Objective Function Values: The objective function values are very close to zero, with slight variations based on α0\alpha_0α0. The values are in the range of 3.84×10−203.84 \times 10^{-20}3.84×10−20 to 4.23×10−204.23 \times 10^{-20}4.23×10−20.

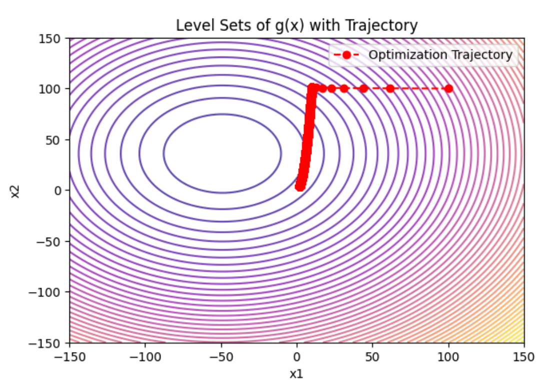
Number of Iterations: The number of iterations varies significantly based on α0\alpha_0α0:

- Larger values of α0\alpha_0α0 generally require more iterations (e.g., α0=1\alpha_0 = 1α0=1 and α0=0.5\alpha_0 = 0.5α0=0.5 have the highest iterations).
- Smaller values of α0\alpha_0α0 often require fewer iterations (e.g., α0=0.4\alpha_0 = 0.4α0=0.4, α0=0.1\alpha_0 = 0.1α0=0.1, and α0=0.01\alpha_0 = 0.01α0=0.01 have fewer iterations).

Comparison to Previous Results: Compared to the previous results, this function requires a significantly larger number of iterations for convergence. This suggests that the function f(x)f(x)f(x) from Exercise-1 might be more complex or less well-conditioned compared to the function used in the previous part.

Plot and Analysis: A plot of the number of iterations versus α0\alpha_0α0 would likely show a similar trend to previous results, with larger α0\alpha_0α0 values leading to more iterations. The objective function values are very close, indicating that the quality of the minimizer is similar across different step sizes.





Question 3:

After performing the above experiment it can be found out with two line search approaches (i.e., exact line search and the backtracking line search) is as follows:

The backtracking line search needs the number of iterations to be 3523709.

Backtracking line search needs much more iterations compared to the exact line search approach for the given function f(x). This may be the case because the step size is dynamic in nature allowing it to adjust to the local characteristics of the objective function.

Backtracking, the inexact line search method, is good with all kinds of problems because it doesn't need an exact step size; it figures out the right size as it goes. This is especially helpful when dealing with complicated problems that change a lot.

The inexact line search approach, specifically backtracking, demonstrates advantages in terms of adaptability and convergence speed, making it a preferred choice in many optimization scenarios.