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Title : Single Vehicle Pickup and Delivery TSP Problem

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Abstract

The Pickup and Delivery Problem TSP is a significant and commonly encountered problem in real-life logistics and transportation. It encompasses various scenarios where items need to be picked up from specific locations and delivered to designated destinations, often subject to constraints such as vehicle capacity, time windows, and route optimization. Given its complexity and practical relevance, the PDTSP has numerous variants tailored to different applications and challenges in the logistics domain.

In this project, we address a specific instance of the Pickup and Delivery Problem by considering 15 stations located in and around Andheri, Mumbai. Our objective is to determine the most efficient routes that minimize overall travel distance while ensuring all pickups and deliveries are completed in an optimal manner. We approach this problem by employing advanced optimization techniques to compute an exact optimal solution, demonstrating the applicability of these methods to real-world scenarios.

1 Introduction

The Traveling Salesman Problem (TSP) has many real-world applications, especially in its modified versions. One such variation is the Pickup and Delivery Problem (PDP). In PDP, similar to TSP, we deal with a network represented as either a directed or undirected graph. The network includes different types of nodes: a depot (often a single point), pickup nodes, and delivery nodes. Each of these nodes is associated with a specific quantity of a product, where pickup nodes supply a certain amount, and delivery nodes demand a certain amount.

The challenge is to determine the shortest possible route for a vehicle that starts and ends at the depot, ensuring all delivery requirements are met without exceeding the vehicle's capacity. In more complex scenarios, multiple vehicles may be involved, making PDP a broader version of the Vehicle Routing Problem (VRP) where vehicles must fulfill customer requests for both picking up and delivering goods.

2 Mathematical formulation

Problem Formulation

Given:

- $N = \{1, 2, \ldots, n\}$: Set of all locations (including pickup and delivery points).
- $P = \{1, 2, \ldots, m\}$: Set of pickup locations.
- $D = \{m + 1, m + 2, \ldots, 2m\}$: Set of delivery locations corresponding to each pickup.

- c_{ij}: Cost or distance of traveling from location i to location j, for all $i, j \in N$.
- $x_{ij} \in \{0, 1\}$: Binary decision variable, where $x_{ij} = 1$ if the route goes directly from location i to location j; otherwise, $x_{ij} = 0$.

Objective: Minimize the total travel cost.

$$\text{Minimize} \quad Z = \sum_{i \in N} \sum_{j \in N} c_{ij} \cdot x_{ij}$$

Subject to:

$$\sum_{j \in N} x_{ij} = 1 \quad \forall i \in N, \quad \text{(Each location is left exactly once)} \tag{1}$$

$$\sum_{i \in N} x_{ij} = 1 \quad \forall j \in N, \quad \text{(Each location is entered exactly once)} \tag{2}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in N \tag{3}$$

$$u_i - u_j + (n-1) \cdot x_{ij} \leq n - 2 \quad \forall i \neq j, \quad \text{(Subtour elimination)} \tag{4}$$

$$u_i \geq 0 \quad \forall i \in N, \quad \text{(Position variables for subtour elimination)} \tag{5}$$

$$x_{ij} = 0 \quad \text{if } i \in P \text{ and } j \in D, \quad \text{and } j < m + i, \quad \text{(Delivery must follow Pickup)} \tag{6}$$

Explanation

- The objective function minimizes the total travel cost.
- The first two constraints ensure that each location is visited exactly once.
- The third constraint ensures binary decision variables.
- The fourth and fifth constraints are used to eliminate subtours, enforcing that the vehicle does not form any disconnected loops.
- The final constraint ensures that each delivery location is visited only after its corresponding pickup location has been visited.

3 Places Taken into Account and Method Used to Solve the Problem

3.1 Programming Setup

- **Pyomo:**
  - Open-source Python-based optimization modeling language.
  - Allows users to model and solve complex optimization problems.
  - The abstract representation of the optimization model is passed to a solver interface that communicates with the solver.

  - The solver interface is responsible for translating the abstract model representation into a format the solver can understand and provide a solution.

- **CBC (Coin-or Branch and Cut):**
  - An open-source mixed integer linear programming solver written in C++.
  - Relies on other components from the COIN-OR repository, including the LP solver (CLP) and the Cut Generation Library (CGL) for generating cuts.
  - Utilizes the Branch and Cut Algorithm, along with various heuristics, to quickly obtain valid solutions.

ID	**Place**	**Demand**
0	IIT Bombay Mumbai	0
1	Versova Beach Mumbai	-200
2	Juhu Beach Mumbai	400
3	Andheri Sports Complex Mumbai	-200
4	Lokhandwala Market Mumbai	-100
5	Inorbit Mall Mumbai	-50
6	Mindspace Garden Mumbai	-50
7	Andheri Railway Station Mumbai	-300
8	Shivaji Park Mumbai	250
9	Powai Lake Mumbai	100
10	Mahalaxmi Racecourse Mumbai	50
11	Saki Naka Mumbai	-50
12	Oshiwara Mumbai	200
13	Dahisar Check Naka Mumbai	100
14	Oshiwara Police Station Mumbai	100

Table 1: Location Data with Respective Demands

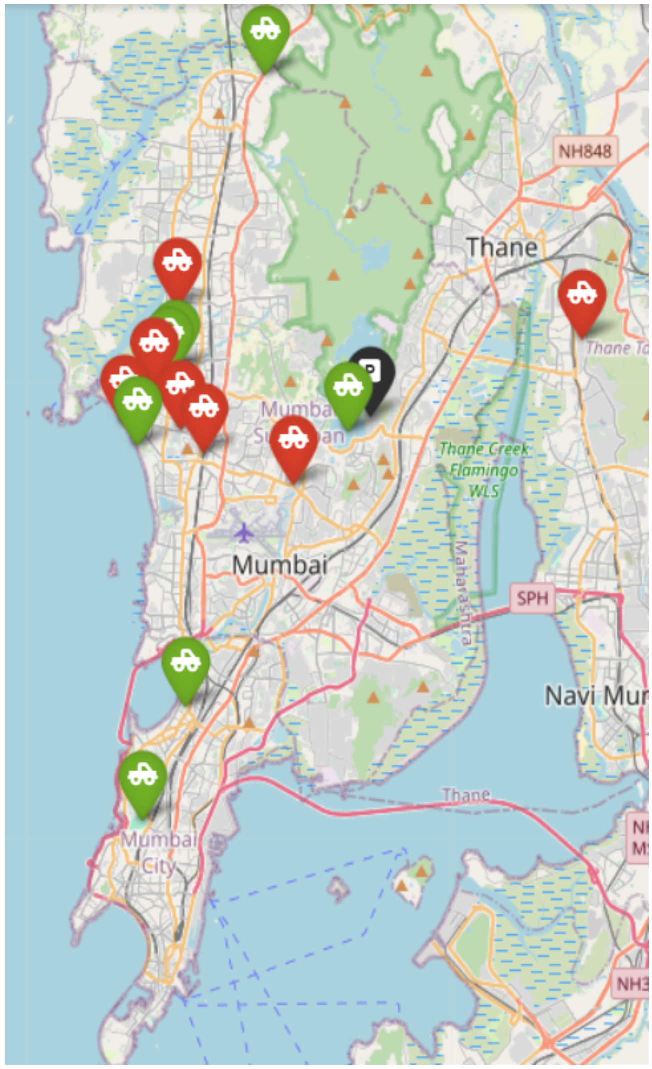

Figure 1: The +ve demand in Green and -ve demand in red

4 Data set Details

The selected locations are all situated around Andheri, Mumbai, with the depot established at IIT Bombay. In this context, a positive demand indicates that the vehicle is required to pick up items from the depot and deliver them to the specified location.

Conversely, a negative demand signifies that items need to be collected from the location and transported back to the depot.

5 Result

IIT Bombay Mumbai → Mindspace Garden Mumbai → Dahisar Check Naka Mumbai → Inorbit Mall Mumbai → Oshiwara Mumbai → Oshiwara Police Station Mumbai → Lokhandwala Market Mumbai → Versova Beach Mumbai → Juhu Beach Mumbai → Andheri Sports Complex Mumbai → Andheri Railway Station Mumbai → Mahalaxmi Racecourse Mumbai → Shivaji Park Mumbai → Saki Naka Mumbai → Powai Lake Mumbai → IIT Bombay Mumbai

6 Conclusion

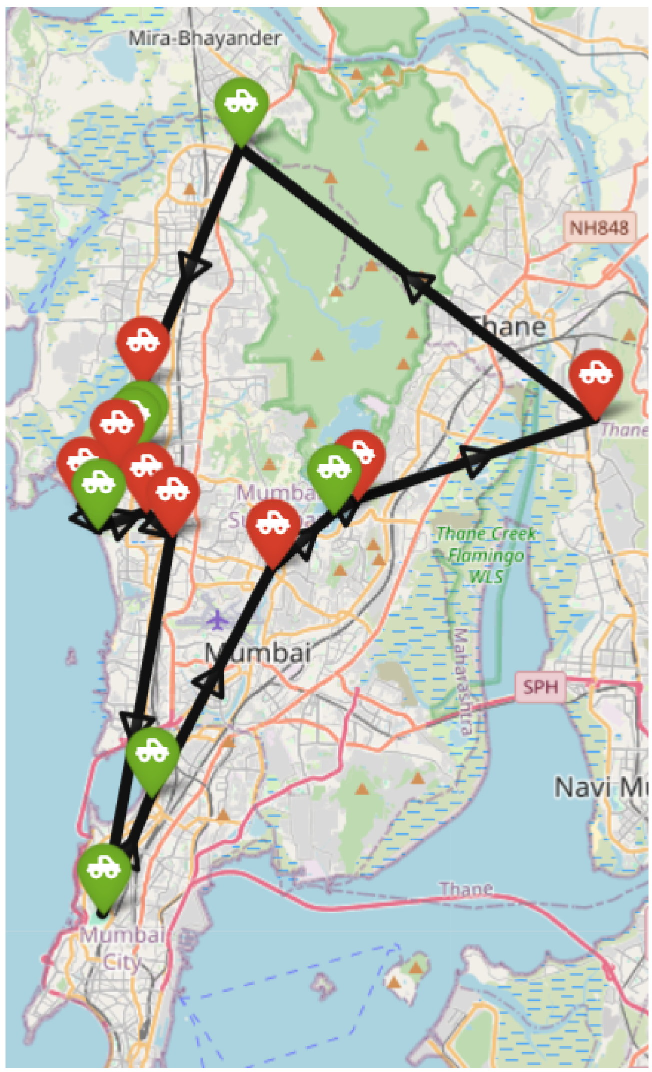

Figure 2: Optimized path of the vehicle and the arrow at the edge depicting the direction of the vehicle

The selected locations are all situated around Andheri, with the depot established at IIT Bombay. In this context, a positive demand indicates that the vehicle is required to pick up items from the depot and deliver them to the specified location. Conversely, a negative demand signifies that items need to be collected from the location and transported back to the depot.

The total distance that the vehicle needs to cover, when optimized, results in an objective value of **82**. This represents the minimized distance traveled to fulfill all pickup and delivery requirements efficiently.