Math AA HL

Test on Functions

Date: 1 November 2022

Paper 1

(without GDC)

P1: /30

Name of student: _____

1. [Maximum mark: 10]

Let $f(x)=\frac{2x+1}{x-1}+1$ and $g(x)=x-1$

(a) Write down the equations of the asymptotes of $y=f(x)$. [2]

(b) Find $(g\circ f)(x)$. [1]

(c) Find $f^{-1}(x)$. [3]

(d) Find $h(x)$, given that $(f\circ h)(x)=x$. [2]

(e) Find $k(x)$, given that $(k\circ f^{-1})(x)=g(x)$. [2]

2. [Maximum mark: 8]

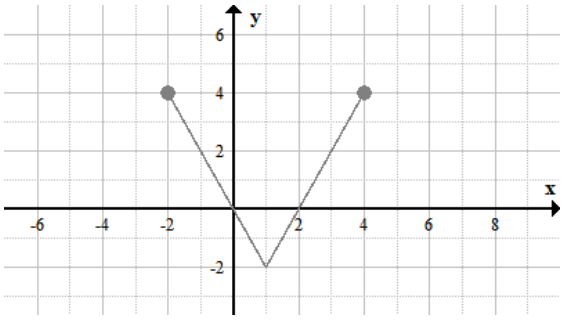
Let $f(x)=x^2+x$, $x\le -\frac{1}{2}$ and $g(x)=x^2$, $x\le 0$.

(a) Describe the sequence of transformations that map $g(x)$ to $f(x)$. [4]

(b) Find $f^{-1}(x)$ and state its domain. [4]

3. [Maximum mark: 5]

The following diagram shows the graph of the function $y=f(x)$.



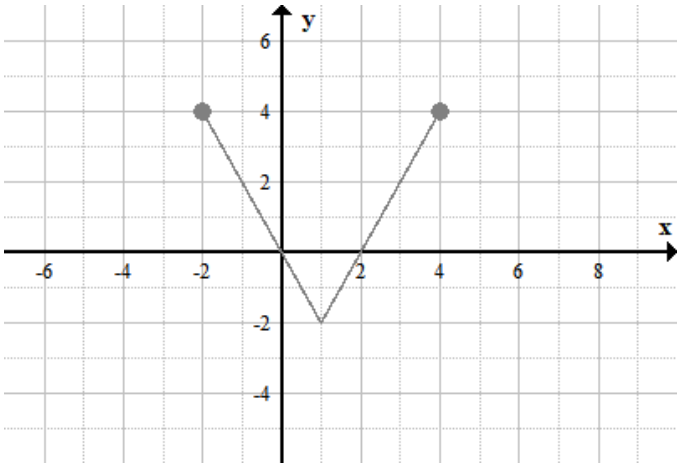
(a) Show that $(f\circ f)(1)=f(1)^2$. [3]

(b) The equation $|f(x)|=k$ has exactly 3 solutions. Find the value of k. [2]

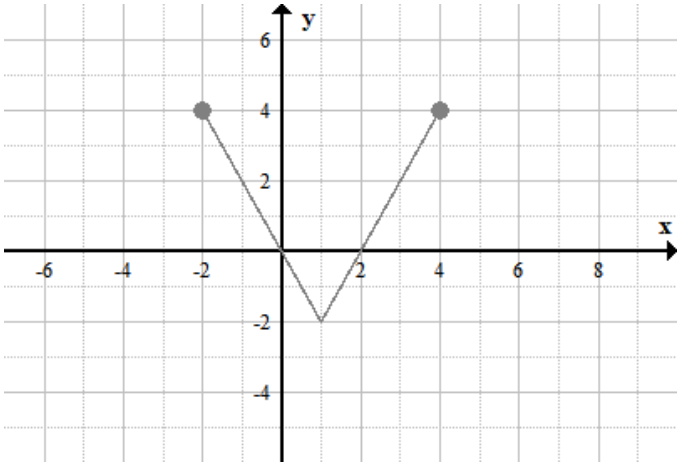
4. [Maximum mark: 7]

The following diagrams show the graph of the function $y = f(x)$.

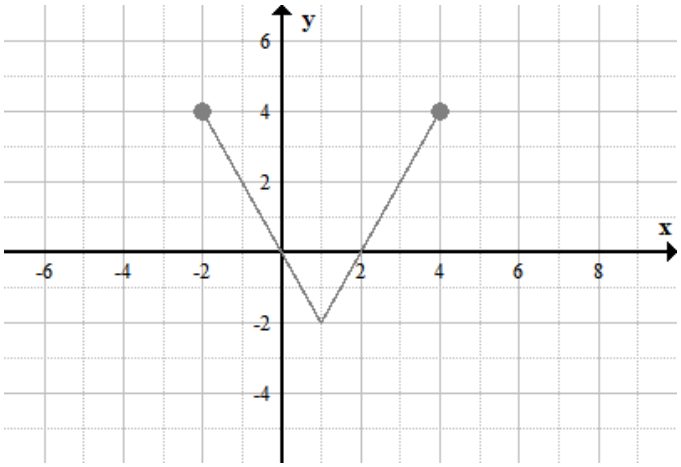
(a) Sketch the graph of $y = f(x+3)+2$. [2]



(b) Sketch the graph of $y = \left| f\left(\frac{x}{2}\right) \right|$. [3]



(c) Sketch the graph of $y = f(|x|+1)$. [2]



Paper 2

(with GDC)

P2: /30

Name of student: _____

1. [Maximum mark: 4]

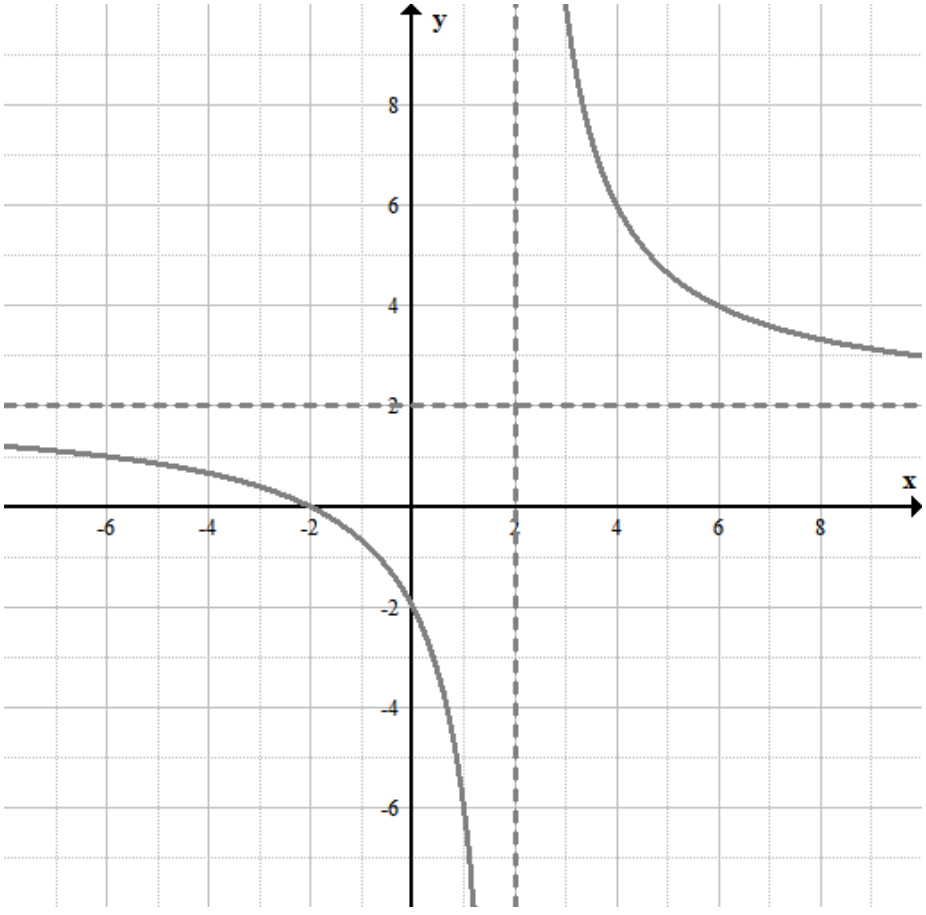
Determine whether each of the following functions is **even**, **odd** or **neither**. Justify your answer.

(a) $f(x) = x^5 - 2x^3 + 1$. [2]

(b) $f(x) = \dfrac{2x^2 + 1}{3x|x|}$. [2]

2. [Maximum mark: 5]

The following diagram shows the graph of a function $f(x)$



On the same diagram sketch the graph of $y = \dfrac{2}{f(x)}$. Indicate the asymptotes.

3. [Maximum mark: 10]

Let $f(x) = \dfrac{x-16}{2x^2-50x+312}$

Find all the asymptotes (vertical, horizontal or oblique) of

(a) $y = f(x)$. [3]

(b) $y = 6xf(x)$ [2]

(c) $y = \dfrac{1}{f(x)}$. [5]

4. [Maximum mark: 11]

Let $f(x) = \dfrac{\sqrt{x}}{x-1}$

(a) Write down

(i) the largest possible domain of f.

(ii) the range of f. [3]

(b) Find $f^{-1}(1)$. [2]

(c) Solve the equations

(i) $f(x) = x$.

(ii) $f^{-1}(x) = x$. [3]

(d) Sketch the graph of f^{-1}. [3]