II

I. 위계선형분석 모형들

Overview of HLM Two-Level School-Effects Models

	(1) One-Way ANOVA	(2) Means-as-Outcomes	(3) One-Way ANCOVA	(4) Random-Coefficient	(5) Intercept- and Slopes-as-Outcomes
<u>Level-1 model</u>:	$Y_{ij} = \beta_{0j} + r_{ij}$	$Y_{ij} = \beta_{0j} + r_{ij}$	$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + r_{ij}$	$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + r_{ij}$	$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + r_{ij}$
Independent variables	none	none	yes	yes	yes
<u>Level-2 models</u>:					
<u>Level-1 Intercept</u>:	$\beta_{0j} = \gamma_{00} + u_{0j}$	$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$	$\beta_{0j} = \gamma_{00} + u_{0j}$	$\beta_{0j} = \gamma_{00} + u_{0j}$	$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$
Independent variables	none	yes	none	none	yes
<u>Level-1 Slopes:</u>	none	none	$\beta_{1j} = \gamma_{10}$	$\beta_{1j} = \gamma_{10} + u_{1j}$	$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j}$
Independent variables			sometimes (fixed)	none (random)	yes
Pages references in Bryk/Raudenbush book:	23-24; 69-72	24-25; 72-75	25-26	26-27; 75-80	27-28; 80-85

위의 각 모형은 다음과 같은 요소들에 의해 구분된다.

(1) Whether predictors at level 1 or level 2;
(2) Whether level-2 predictors are fixed or random

II. One-Way ANOVA (fully unconditional) model

보통 우리가 검증해야 하는 첫 번째 모델이다.

이 모델을 통해 우리는 다음사항을 수행한다:

(1) Estimate grand mean for outcome (dependent) variable
(2) Estimate variance components at both levels
(3) Estimate intra-class correlations and reliability
(4) Test hypothesis whether all j units have same mean

Model:

- **Level-1 model:** $Y_{ij} = \beta_{0j} + r_{ij}$, $r_{ij} \sim N(0,\sigma^2)$ $\sigma^2 =$ **level-1 variance (within any school j)**
- **Level-2 model:** $\beta_{0j} = \gamma_{00} + u_{0j}$, $u_{0j} \sim N(0,\tau_{00})$ $\tau_{00} =$ **level-2 variance**

β_{0j} = **mean outcome for unit j (Level-1 coefficient)**

γ_{00} = **grand-mean outcome in the population (Level-2 coefficient)**

u_{0j} = **random effect associated with unit j**

r_{ij} = **level-1 error term**

■ **Combined model:** $Y_{ij} = \underbrace{\gamma_{00}}_{\text{Fixed effect}} + \underbrace{u_{0j} + r_{ij}}_{\text{Random effects}}$ **(ANOVA 와 비슷)**

- **$Var(Y_{ij}) = Var(\gamma_{00} + u_{0j} + r_{ij}) = \tau_{00} + \sigma^2$**
- **Intra-class correlation(ICC): $\rho = \tau_{00} / (\tau_{00} + \sigma^2)$ measures proportion of "variance between level-2 units"**

■ **Estimation:** $Y_{\cdot j}$ 나 $r_{\cdot j}$ 처럼 "$\cdot j$" 아래 첨자를 가진 기호는 심볼은 그 위에 ¯(bar)가 있는 집단 평균을 표시함.

■ **HLM 은**

- **Empirical Bayes estimates of randomly varying level-l coefficients, β_{qj}**
- **Weighted(generalized) least squares estimates of the level-2 coefficients, γ_{qs}**
- **MLE of variance and covariance components, σ^2 at Level-1, and T at Level 2.**

■ **Two possible estimators for β_{0j}**, true school mean:

- **Level-1 model: $Y_{ij} = \beta_{0j} + r_{ij}$, $r_{ij} \sim N(0,\sigma^2)$ σ^2 = level-1 variance**

(a) $\bar{Y}_{\cdot j}$ (sample mean) $= \beta_{0j} + \bar{r}_{\cdot j}$

$\bar{r}_{\cdot j} = \sum r_{ij}/n_j$ (이 값은 그룹마다 같을 수도 다를 수도 있다.)

$\bar{r}_{\cdot j}$ 의 분산[$Var(\bar{r}_{\cdot j}) = \sigma^2/n_j = V_j$] (error variance)

($V_j \rightarrow \beta_{0j}$ 에 대한 추정치로서 각 그룹 내 Y 값들에 대한 평균[$\bar{Y}_{\cdot j}$]의 분산)

■ **Level-2 model: $\beta_{0j} = \gamma_{00} + u_{0j}$, $u_{0j} \sim N(0,\tau_{00})$ τ_{00} = level-2 variance**

(b) $\bar{Y}_{\cdot j} = \gamma_{00} + u_{0j} + r_{\cdot j}$ $(=\bar{r}_{\cdot j})$

$Var(Y_{\cdot j}) = Var(u_{0j}) + Var(r_{\cdot j})$

$= \tau_{00} + V_j$

= parameter variance + error variance

$= \Delta_j$ (delta $_j$)

- 모든 Level-2 단위(e.g., 학교)에서 표본의 크기가 같을 때(다시말해 σ^2 / n_j **에서** n_j**=common "k"** $\therefore$ V_j **=common "V")**:
  $\Delta = \tau_{00} + V$ (constant)
  - Then, the unique, minimum-variance unbiased estimator of γ_{00}:
    - ✧ $\hat{\gamma}_{00} = \sum \bar{Y}_{\cdot j} / J$ **(각 그룹 내 Y 값의 평균들을 그룹 수로 나눈 값)**
- Level-2 단위(e.g., 학교)에서 표본의 크기가 다를 때: **$\Delta_j = \tau_{00} + V_j$ (variable),** 즉, 그룹에 따른 분산의 크기가 다름,
  - then the unique, minimum-variance unbiased estimator of γ_{00} :
    - ✧ $\hat{\gamma}_{00} = \sum \Delta_j^{-1} Y_{\cdot j} / \sum \Delta_j^{-1}$,
    - ✧ where Δ_j^{-1} = Precision($\bar{Y}_{\cdot j}$) 각 그룹의 Y 평균의 정밀도 Precision (:"$\bar{Y}_{\cdot j}$")

    Weighted least squares (maximum likelihood) estimator

    $\bar{Y}_{\cdot j}$에 대한 정밀도는 그 분산($\Delta_j = \tau_{00} + V_j$)에 대해 역수($1/\Delta_j$)를 취한 값이다. 그래서 이 값은 분산의 크기에 따라 변하는데 그 크기는 위에서 본 바와 같이 $V_j = \sigma^2/n_j$ 에 달려있고 V_j 는 Level-2 의 표본의 크기에 달려있다. (합의 기호 "$\sum$" 가 있어 단순한 분수 연산을 하면 안됨!)

    또한 분산Δ_j 는 컴퓨터에 주어지거나 표본으로부터 추정되어야 하는데 표본의 크기가 모든 Level-2 에서 같은 경우는 그럴 필요가 없다. 하지만 일반적인 식은 [$\sum \Delta_j^{-1} Y_{\cdot j} / \sum \Delta_j^{-1}$]가 되는데, 왜냐하면 Level-2 의 표본의 크기가 같은 경우, [$\sum \Delta_j^{-1} Y_{\cdot j} / \sum \Delta_j^{-1}$]는 [$\sum Y_{\cdot j} / J$]가 되기 때문이다.

- HLM 은 Random Level-1coefficients 에 대해 이 Level-1 모형의 $\boldsymbol{Y}_{.j}$ 와 Level-2 모형의 ^γ_{00} 를 동시에 고려하는 추정치 (a weighted combination, known as a **Bayes estimator**)를 제공한다:

$$\beta^*_{0j} = \lambda_j Y_{.j} + (1 - \lambda_j)\, \hat{\gamma}_{00} \ ,$$

where λ_j = reliability of least square estimator, $Y_{.j}$, for parameter, β_{0j}.

$= \mathrm{Var}(\beta_{0j}) / \mathrm{Var}(Y_{.j})$

$= \tau_{00} / (\tau_{00} + V_j)$, where $V_j = \sigma^2 / n_j$

= (parameter variance)/(parameter variance + error variance)

이 신뢰도(reliability)는 1) 집단(level-2) 평균값들이 집단간에 변이를 더 많이 보일 때(**τ_{00} 가 클 때)** 또는 집단 내(Level-1)의 표본 수가 많을 때 (**V_j 가 작을 때)**커진다(close to 1). 요약하면 HLM 은 $Y_{.j}$ 의 신뢰도가 높으면 $Y_{.j}$ 가 더 많이 가중되고 그 신뢰도가 낮으면 Level-2 모형에서 얻어지는 ^γ_{00} **값에 더 많은 가중치를 주는 방식으로** β^*_{0j} 를 추정한다. 이러한 이유로 β^*_{0j} 은 전체 평균 (grand mean) γ_{00} 로 집약되는 모습을 보여서 shrinkage estimator 라고 불린다. 그리고

When λ_j computed from known variances, β^*_{0j} known as a Bayes estimator

When λ_j computed from unknown variances, β^*_{0j} known as an empirical Bayes estimator

✧ 구간추정 Interval estimation:

- **95%CI (β_{0j}) = $\beta^*_{0j} \pm 1.96\, V^{*}_{j}{}^{1/2}$ (when variances, σ^2 and τ_{00}, are known)**

With unknown variances, we estimate reliabilities:

λ_j = reliability ($Y_{.j}$)

$= \tau_{00} / [\tau_{00} + (\sigma^2 / n_j)]$ for each school j

Overall reliability is $\lambda = \sum \lambda_j / J$

■ 가설검증(Hypothesis Testing)

Two types of hypothesis testing:

1. Single parameter tests
2. Multiple parameter tests (Differences across models)
   e.g., significance of single predictor variable in multiple equations
   e.g., significance of multiple predictors

Choice depends on particular hypothesis you want to test
Usually limit yourself to begin with single-parameter tests

One of the most common tests, for testing level-2 variance:

H_0: $\tau_{qq} = 0$

Illustration using HSB data (Book, pp. 69-72)

Review data descriptives

Model:

Level-1 model: $Y_{ij} = \beta_{0j} + r_{ij}$, $r_i \sim N(0,\sigma^2)$ $\sigma^2 =$ **level-1 variance**

Level-2 model: $\beta_{0j} = \gamma_{00} + u_{0j}$, $u_{0j} \sim N(0,\tau_{00})$ $\tau_{00} =$ **level-2 variance**

Running the model:

```
Program:                      HLM 6 Hierarchical Linear and Nonlinear Modeling
 Authors:                      Stephen Raudenbush, Tony Bryk, & Richard Congdon
 Publisher:                    Scientific Software International, Inc. (c) 2000
                                                     techsupport@ssicentral.com
                                                             www.ssicentral.com
 -------------------------------------------------------------------------------
 Module:      HLM2.EXE (6.02.25138.2)
-------------------------------------------------------------------------------
 SPECIFICATIONS FOR THIS HLM2 RUN
  Problem Title: no title
  The data source for this run  = C:\Program Files\HLM6\Examples\Chapter2\HSB.MDM
  The command file for this run = whlmtemp.hlm
  Output file name              = C:\Program Files\HLM6\Examples\Chapter2\hlm2.txt
  The maximum number of level-1 units = 7185
  The maximum number of level-2 units = 160
  The maximum number of iterations = 100
  Method of estimation: restricted maximum likelihood

 Weighting Specification
 -----------------------
                        Weight
                        Variable
           Weighting?   Name        Normalized?
 Level 1       no
 Level 2       no
 Precision     no

  The outcome variable is  MATHACH

  The model specified for the fixed effects was:
```

```
 ----------------------------------------------------
   Level-1                  Level-2
   Coefficients             Predictors
 ----------------------   ----------------
         INTRCPT1, B0       INTRCPT2, G00

 The model specified for the covariance components was:
 ---------------------------------------------------------
         Sigma squared (constant across level-2 units)

         Tau dimensions
               INTRCPT1
 Summary of the model specified (in equation format)
 ---------------------------------------------------
Level-1 Model
       Y = B0 + R

Level-2 Model
       B0 = G00 + U0

Iterations stopped due to small change in likelihood function
******* ITERATION 4 *******

 Sigma_squared =     39.14831

 Tau
 INTRCPT1,B0      8.61431

Tau (as correlations)
 INTRCPT1,B0  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                        0.901
 ----------------------------------------------------

The value of the likelihood function at iteration 4 = -2.355840E+004
```

```
The outcome variable is  MATHACH
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                        Standard             Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00          12.636972   0.244412    51.704        159    0.000
 ----------------------------------------------------------------------------


 The outcome variable is  MATHACH
 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                        Standard             Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00          12.636972   0.243628    51.870        159    0.000
 ----------------------------------------------------------------------------


 Final estimation of variance components:
 -----------------------------------------------------------------------------
 Random Effect            Standard      Variance     df    Chi-square  P-value
                          Deviation     Component
 -----------------------------------------------------------------------------
 INTRCPT1,       U0         2.93501       8.61431   159   1660.23259    0.000
  level-1,       R          6.25686      39.14831
 -----------------------------------------------------------------------------

 Statistics for current covariance components model
 --------------------------------------------------
 Deviance                       = 47116.793477
 Number of estimated parameters = 2
```

결과 요약:

1. Fixed effects: the same for all schools

   $\hat{\gamma}_{00} = 12.64$

   95%CI = 12.64 ± 1.96 (.024) = (12.17, 13.11)

2. Variance components (Random effects)

   $\hat{Var}(r_{ij}) = 39.15$

   $\hat{Var}(\beta_{0j}) = \hat{Var}(u_{0j}) = \hat{\tau}_{00} = 8.61$

   Range of plausible values among schools:

   $\hat{\gamma}_{00} \pm 1.96 (\hat{\tau}_{00})^{1/2}$

   12.64 ± 1.96 $(8.61)^{1/2}$ = (6.89, 18.39)

   Hypothesis testing: $H_0: \tau_{qq} = 0$

   만약에 u_{0j}가 모든 학교에서 "0" 이었다면 우리는 HLM 분석을 할 필요가 없고 OLS 를 수행해도 된다.

3. Intra-class correlation (집단내 상관계수)

   Intra-class correlation: $\hat{\rho} = \hat{\tau}_{00} / (\hat{\tau}_{00} + \hat{\sigma}^2)$

   = 8.61 / (8.61 + 39.15)

   = .18

   18%의 분산이 집단간 분산, 다시 집단내 상관에 의한 것임.

4. Reliability

◆ $\hat{\lambda}_j$ **= reliability ($\bar{Y}_{.j}$)**

= $\hat{\tau}_{00}$ /[$\hat{\tau}_{00}$ + ($\hat{\sigma}^2$ / n_j)] for each school j ($\hat{\tau}_{00}$ **가 크거나** 각 학교의 표본이 크면 이 값은 커진다!!)

■ This number indicates whether estimated differences across schools are reliable indicators of real differences among schools' population means.

■ Overall reliability is $\lambda = \sum \lambda_j / J$

첫 모형의 경우, reliability = .90, meaning within-school sample means are good estimate (각 학교의 표본크기는 평균이 45 명 정도였음! p.68)

■ We also use variance components to estimate the reliability of the sample mean for any school as an estimate of its population mean. Such reliability for a particular group is calculated as:

❑ $\lambda_j = \tau_{00} / (\tau_{00} + \sigma^2/n_j)$ where n_j is the sample size for group j.

❑ For example, school #1224 has 47 students, therefore:

❑ λ_{1224} = 8.61431 /(8.61431+39.14831/47)= .91183228

- Reliability ranges from 0 to 1 so .91 is pretty high.

❑ School #1308 has only 20 students, therefore:

❑ λ_{1308} = 8.61431 /(8.61431+39.14831/20)= .81484427

❑ HLM output includes only an average of such reliabilities: $\lambda = \Sigma\lambda_j /j$= 0.901.

III. Regression with Means-as-Outcomes (다시 말해 독립변수가 있다!)

Model:

Level-1 model: $Y_{ij} = \beta_{0j} + r_{ij}$

Level-2 model: $\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$

Combined model: $Y_{ij} = \underbrace{\gamma_{00} + \gamma_{01} W_j}_{\text{Fixed effects}} + \underbrace{u_{0j} + r_{ij}}_{\text{Random effects}}$

γ_{00} = intercept

γ_{01} = effect of W_j on β_{0j}

$u_{0j} = \beta_{0j} - \gamma_{00} - \gamma_{01} W_j$ = residual

τ_{00} = residual or conditional variance in β_{0j}, after controlling for W_j

■ **Estimation:**

As with the unconditional model, HLM produces a composite estimator for β_{0j}

$$\beta^{*}_{0j} = \lambda_j Y_{\cdot j} + (1 - \lambda_j)(\gamma_{00} + \gamma_{01} W_j)$$

- **Again this is an empirical Bayes or shrinkage estimator, but in this case, is shrunk toward a predicted value than the grand mean, so it is called a *conditional shrinkage estimator***

- **Interval estimation is the same as in the unconditional model:**
  - **95%CI (β_{0j}) = $\beta^*_{0j} \pm 1.96\ V^{*\ 1/2}_{j}$ (when variances, σ^2 and τ_{00}, are known)**

Hypothesis testing:

1. Single parameter test for fixed effects:

   H0: γ_{qs} = 0 (effect of level-2 predictor, W_{sj} , on particular level-2 parameter, β_{qj})

   Can be tested with two statistics, but generally with t-statistic with J – S_q –1 DOF(q=subscript for β S=subscript for W…L-2 독립변수의 숫자)

2. Single parameter test for variance components:

   H0: τ_{qq} = 0, where : τ_{qq} = Var (β_{qj})

   Can be tested with two statistics, but generally with Chi-square (X^2) with J – S_q –1 DOF

Illustration using HSB data (Book, pp. 72-75)

Module:	HLM2S.EXE (7.00.21103.1002)
Date:	12 February 2012, Sunday
Time:	16:24:51

Specifications for this HLM2 run

The maximum number of level-1 units = 7185
The maximum number of level-2 units = 160
The maximum number of iterations = 100

The outcome variable is MATHACH

Summary of the model specified

Level-1 Model

$MATHACH_{ij} = \beta_{0j} + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + \gamma_{01}*(MEANSES_j) + u_{0j}$

Mixed Model

$MATHACH_{ij} = \gamma_{00} + \gamma_{01}*MEANSES_j + u_{0j} + r_{ij}$

Final Results - Iteration 6

Iterations stopped due to small change in likelihood function

$\sigma^2 = 39.15708$

τ

INTRCPT1,β_0 2.63870

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.740

The value of the log-likelihood function at iteration 6 = -2.347972E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value

For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.649436	0.149280	84.736	158	<0.001
MEANSES, γ_{01}	5.863538	0.361457	16.222	158	<0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.649436	0.148377	85.252	158	<0.001
MEANSES, γ_{01}	5.863538	0.320211	18.311	158	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	1.62441	2.63870	158	633.51744	<0.001
level-1, *r*	6.25756	39.15708			

Statistics for current covariance components model
Deviance = 46959.446959
Number of estimated parameters = 2

Model:

Level-1 model: $Y_{ij} = \beta_{0j} + r_{ij}$

Level-2 model: $\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{MEAN SES})_j + u_{0j}$

Combined model: $Y_{ij} = \gamma_{00} + \gamma_{01}\ (\text{MEAN SES})_j + u_{0j} + r_{ij}$

Fixed effects

1. **Fixed effects: the same for all schools**

   $\hat{\gamma}_{00} = 12.65$

   $\hat{\gamma}_{01} = 5.86$

   $t = \hat{\gamma}_{01} / SE(\hat{\gamma}_{01}) = 5.86/.32 = 16.22$ (highly significant)

2. Variance components (Random effects)

   Range of plausible values among schools:

   $$\hat{\gamma}_{00} \pm 1.96 (\hat{\tau}_{00})^{1/2} = 12.65 \pm 1.96 (2.64)^{1/2} = (9.47, 15.83)$$

   이 범위가 Level-2 에서 독립변수를 도입하지 않은 첫째 모형 (Fully unconditional Model)에 비해 얼마나 좁아졌는지에 주목해야. 다시 말해 conditional on school MEANSES, Mean achievement among schools controlling for average SES 는 overall or observed mean achievement 에 비해 그 변이가 훨씬 덜 하다는 것에 주목 (이론적으로 타당한 독립변수를 가지고 종속변수에 대한 예측 오차를 줄이는 것이 통계적 분석의 목적)

   Hypothesis testing: Is residual variance, τ_{00}(왜 residual 이란 말이 추가 되었나?), significantly different than zero?

   Statistic: X^2 with 158 DOF = 633.52, $p < .001$ (무슨 뜻인가?)

3. **Variance explained**

Proportion of variance explained in β_{0j}

$$= \frac{\tau\hat{}_{00} \text{ (random ANOVA)} - \tau\hat{}_{00} \text{ (MEAN SES)}}{\tau\hat{}_{00} \text{ (random ANOVA)}}$$

= (8.61 – 2.64)/8.61

= 0.69

Meaning: Mean SES explains 69 percent of variance in mean achievement among schools.

4. Conditional intra-class correlation

$\rho\hat{} = \tau\hat{}_{00} / (\tau\hat{}_{00} + \sigma\hat{}^2)$

= 2.64 / (2.64 + 39.16)

= .06

Meaning:

1. **Variability between schools after controlling for mean SES is now 6 percent, compared to 18 percent in the unconditional model; much smaller variability remains.**
2. **After removing the effect of school MEAN SES, the correlation between pairs of scores in the same school (intra class correlation!) has changed from .18 to .06.**
3. **The degree of dependence among observations within schools that are of the same MEAN SES.**

5. Conditional reliability of the (weighted) least squares residuals, $u\hat{}_{0j}$(학교간 차이에 대한 추정치를 이것을 통해 볼 수 있음 왜냐 하면 지금 이 값이 MEANSES 를 도입해서 이전과 달라져 있음)

- **First recall,**
- $\lambda\hat{}_j$ **= reliability ($\bar{Y}_{\cdot j}$)**

  $= \tau\hat{}_{00} / [\tau\hat{}_{00} + (\sigma\hat{}^2 / n_j)]$ for each school j (각 학교의 표본이 크면 이 값은 커진다!!)

- This number indicates whether estimated differences across schools are reliable indicators of real differences among schools' population means.
- **Overall reliability is $\lambda = \sum \lambda_j / J$**

➢ $\boldsymbol{\hat{u}_{0j} = \bar{Y}_{\cdot j} - \hat{\gamma}_{00} - \hat{\gamma}_{01}(\text{MEAN SES})_j}$

= .74

이 reliability 는 MEAN SES 가 같은 학교들을 고려한 conditional reliability 이다.

우리는 이 residuals($\hat{u}_{0j}$) 에 대한 reliability 는 표본 평균(meanses 를 고려하지 않은 $\bar{Y}_{\cdot j}$)의 reliability 보다 작아 졌음을 알 수 있다. 왜 그럴까? (∵ 통제를 하면 표본 수가 작아지는 효과!) 하지만 Still highly reliable!

Means-as-outcomes model II (다른 독립변수를 고려함!) (Intercepts as outcomes)

LEVEL 1 MODEL

$\text{MATHACH}_{ij} = \beta_{0j} + r_{ij}$

LEVEL 2 MODEL

$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j) + u_{0j}$

- This model allows us to predict variation in the levels of math achievement using level-2 variables.
- If we would attempt to do this using regular OLS, we would be artificially inflating the sample size and pretend we have 7185 data points to evaluate the effect of type of school (Catholic vs public), when in fact it's only 160 schools.
- Aggregating the data to school level would be more acceptable, but we would not have any assessment of within-school variation.
  - ***Note, however, that the sample size for level 2 becomes important as soon as you try to include predictors at this level!***

```
Sigma_squared =      39.15135

 Tau
 INTRCPT1,B0      6.67771

Tau (as correlations)
 INTRCPT1,B0  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                        0.877
 ----------------------------------------------------
The value of the likelihood function at iteration 4 = -2.353915E+004

 The outcome variable is  MATHACH
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                        Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
   INTRCPT2, G00          11.393043   0.292887    38.899       158     0.000
     SECTOR, G01           2.804889   0.439142     6.387       158     0.000
 ----------------------------------------------------------------------------
```

- Here, we see a positive effect of Catholic schools on math achievement – the average achievement of Catholic schools is 2.8 units higher than for public schools.

- The intercept now is an average value for a public school student.

```
 The outcome variable is  MATHACH
 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                        Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
   INTRCPT2, G00          11.393043   0.292258    38.983       158     0.000
     SECTOR, G01           2.804889   0.435823     6.436       158     0.000
 ----------------------------------------------------------------------------

 Final estimation of variance components:
```

Random Effect		Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1,	U0	2.58413	6.67771	158	1296.76559	0.000
level-1,	R	6.25710	39.15135			

- As we did with earlier models, we can calculate the percentage of variance in math achievement explained by school type.
  Note that here we only explain level-2 variance, – level-1 variance remained the same (왜 그렇죠?).
- For level 2 variance:
  (8.61431 - 6.67771)/8.61431 = .22481197
- So 22% of school-level variance in math achievement was explained by type of school.

```
Statistics for current covariance components model
--------------------------------------------------
Deviance                       = 47078.295826
Number of estimated parameters = 2
```