Assignment 1

Machine Learning & Deep Learning for Data Science I

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PUBLISHED
October 12, 2022

0. Preparation

```
# Loading Packages
import os
from IPython.display import display, Math, Latex
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import matplotlib as mpl
import sklearn.linear_model as skl_lm
from scipy.stats import pearsonr
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.datasets import make_classification
from sklearn.model_selection import train_test_split
from sklearn.model_selection import KFold
from sklearn.model_selection import cross_val_score
from sklearn.metrics import mean_squared_error
from sklearn.metrics import accuracy_score
from sklearn.metrics import r2_score
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import LogisticRegression
from sklearn.linear_model import Ridge
from sklearn.linear_model import RidgeCV
from sklearn.linear_model import Lasso
from sklearn.linear_model import LassoCV
from sklearn.preprocessing import StandardScaler
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

Reference Repositiory: https://github.com/JWarmenhoven/ISLR-python

1. Question 1

[30 pts] In this problem, you will use the Carseats data set attached in the assignment (Carseats.csv) for linear regression.

Loading Dataset

```
# Set Work Directory
print("Before Directory: %s"%os.getcwd())
os.chdir("E:/OneDrive - SNU/r")
print("After Directory: %s"%os.getcwd())
```

```
Before Directory: e:\OneDrive - SNU\(B) 대학원\수업\2022 2학기\데이터사이언스를위한머신러닝과딥러닝
\과제1
After Directory: E:\OneDrive - SNU\r
```

```
# Read Dataset
carseats=pd.read_csv("Carseats.csv")
display(carseats.head(5))
# Check if there is missing value
print("Total missing variable: ",carseats.isnull().sum().sum()) # no missing value
```

	Sales	CompPrice	Income	Advertising	Population	Price	ShelveLoc	Age	Education	Urban	US
0	9.50	138	73	11	276	120	Bad	42	17	Yes	Yes
1	11.22	111	48	16	260	83	Good	65	10	Yes	Yes
2	10.06	113	35	10	269	80	Medium	59	12	Yes	Yes
3	7.40	117	100	4	466	97	Medium	55	14	Yes	Yes
4	4.15	141	64	3	340	128	Bad	38	13	Yes	No

```
Total missing variable:  0
```

```
# Data types
carseats.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 400 entries, 0 to 399
Data columns (total 11 columns):
 #   Column       Non-Null Count  Dtype  
---  ------       --------------  -----  
 0   Sales        400 non-null    float64
 1   CompPrice    400 non-null    int64  
 2   Income       400 non-null    int64  
 3   Advertising  400 non-null    int64  
 4   Population   400 non-null    int64  
 5   Price        400 non-null    int64  
 6   ShelveLoc    400 non-null    object 
 7   Age          400 non-null    int64  
 8   Education    400 non-null    int64  
 9   Urban        400 non-null    object 
 10  US           400 non-null    object 
dtypes: float64(1), int64(7), object(3)
memory usage: 34.5+ KB
```

(a)

[10 pts] Fit a multiple linear regression model to predict Sales using Price, Urban, and US. Report the R2 of the model.

```
# Sales: float, Price: income, Urban and US: Categorical variable
carseats.US=pd.Categorical(carseats.US, categories=["No", "Yes"])
carseats.Urban=pd.Categorical(carseats.Urban, categories=["No", "Yes"])
```

```
# Fit a multiple linear regression model
estimate = smf.ols('Sales ~  Urban + US + Price', carseats).fit()
estimate.summary()
```

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.239
Model:	OLS	**Adj. R-squared:**	0.234
Method:	Least Squares	**F-statistic:**	41.52
Date:	Sun, 09 Oct 2022	**Prob (F-statistic):**	2.39e-23
Time:	09:07:43	**Log-Likelihood:**	-927.66
No. Observations:	400	**AIC:**	1863.
Df Residuals:	396	**BIC:**	1879.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	13.0435	0.651	20.036	0.000	11.764	14.323
Urban[T.Yes]	-0.0219	0.272	-0.081	0.936	-0.556	0.512
US[T.Yes]	1.2006	0.259	4.635	0.000	0.691	1.710
Price	-0.0545	0.005	-10.389	0.000	-0.065	-0.044

Omnibus:	0.676	**Durbin-Watson:**	1.912
Prob(Omnibus):	0.713	**Jarque-Bera (JB):**	0.758
Skew:	0.093	**Prob(JB):**	0.684
Kurtosis:	2.897	**Cond. No.**	628.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
# Report R^2 of the model
print('R^2: ', round(estimate.rsquared,3))
```

```
R^2:  0.239
```

(b)

[5 pts] Write out the model in equation form, being careful to handle the qualitative variables properly. Provide an interpretation of each coefficient in the model.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon \, , \epsilon \sim \; iid \, N(0, \sigma^2)$$

$$E(Y) = 13.0435 - 0.0219X_1 + 1.2006X_2 - 0.0545X_3$$

where

$$Y = Sales$$

$$X_1 = \begin{cases} 1 & \text{if } Urban = Yes \\ 0 & \text{if } Urban = No \end{cases}$$

$$X_2 = \begin{cases} 1 & \text{if } US = Yes \\ 0 & \text{if } US = No \end{cases}$$

$$X_3 = Price$$

- Interpreation of the coefficient

1. β_0 13.0435: The overall average value of the Sales is 13.0435, if all other variables are equal to zero.

$$(i.e. \quad X_1, X_2, X_3 = 0)$$

2. β_1 -0.0219: The average Sales difference between Urban and Non Urban customers is estimated as -0.0219. (Even though the point estimates of the regression coefficient not equal to zero, the parameter estimate is not statistically significant under significance level of 0.05. Can't reject null hypothesis of $\beta_1 = 0$)
3. β_2 1.2006 : The average Sales difference between US and Non US customers is estimated as 1.2006.
4. β_3 -0.0545 : Under X_1, X_2 (Urban and US dummy) are controlled, the one unit increase in Price results in -0.545 unit of Sales decrease.

(c)

[5 pts] For which predictor variable j can you reject the null hypothesis H0 : β_j = 0? for which there is evidence of association with the outcome.

- Under the significance level of 0.05, the parameter β_1 estimate for the variable X_1 (Urban or Non-Urban) is not statistically significant as p-value is over 0.05 (0.936) and t-statistic is under critical region.
    - Null: H_0: β_1 = 0
    - Alternative: H_0: $\beta_1 \neq$ 0

(d)

[10 pts] Obtain 95% confidence intervals for the coefficient(s).

```
estimate.conf_int(alpha=0.05, cols=None).rename({0: 'Lower CI', 1: 'Upper CI'}, axis='columns')
```

	Lower CI	Upper CI
Intercept	11.763597	14.323341
Urban[T.Yes]	-0.555973	0.512141
US[T.Yes]	0.691304	1.709841
Price	-0.064764	-0.044154

2. Question 2

In class, we used the example of the logistic regression model to predict the probability of default using income and balance on the Default data set attached in the assignment (Default.csv). In this problem, we will estimate the test error of this logistic regression model using the validation set approach. Do not forget to set a random seed before beginning your analysis.

Loading Dataset

```
import random
random.seed(2022)

# Read Dataset
default=pd.read_csv("Default.csv")
display(default.head(5))
# Check if there is missing value
print("Total missing variable: ", default.isnull().sum().sum()) # no missing value
```

	default	student	balance	income
0	No	No	729.526495	44361.625074
1	No	Yes	817.180407	12106.134700
2	No	No	1073.549164	31767.138947
3	No	No	529.250605	35704.493935
4	No	No	785.655883	38463.495879

```
Total missing variable:  0
```

(a)

[10 pts] Fit a logistic regression model that uses income and balance to predict default. Report the log-likelihood of the model.

```
default.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 10000 entries, 0 to 9999
Data columns (total 4 columns):
 #   Column   Non-Null Count  Dtype  
---  ------   --------------  -----  
 0   default  10000 non-null  object 
 1   student  10000 non-null  object 
 2   balance  10000 non-null  float64
 3   income   10000 non-null  float64
dtypes: float64(2), object(2)
memory usage: 312.6+ KB
```

```
display(default['default'].value_counts())
display(default['student'].value_counts())
```

```
No     9667
Yes     333
Name: default, dtype: int64
```

```
No     7056
Yes    2944
Name: student, dtype: int64
```

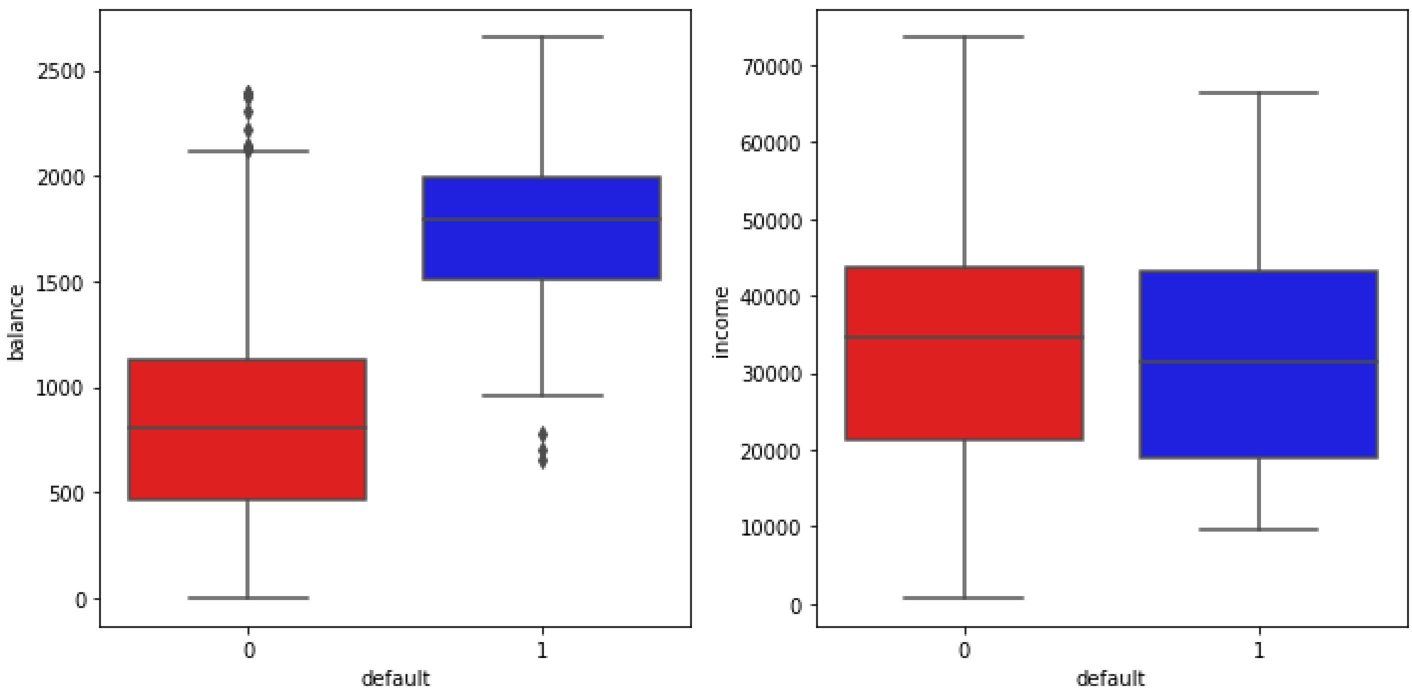
```
# Make dummy variable - No=0, Yes=1
default.default=pd.get_dummies(default.default, drop_first=True)
default.student=pd.get_dummies(default.student, drop_first=True)
display(default['default'].value_counts())
display(default['student'].value_counts())
```

```
0    9667
1     333
Name: default, dtype: int64

0    7056
1    2944
Name: student, dtype: int64
```

```
fig = plt.figure(figsize=(10,5))
gs = mpl.gridspec.GridSpec(1, 2)
ax1 = plt.subplot(gs[0])
ax2 = plt.subplot(gs[1])

pal = {0 :'red', 1:'blue'}
sns.boxplot(x='default', y='balance', data=default, orient='vertical', ax=ax1, palette=pal)
sns.boxplot(x='default', y='income', data=default, orient='vertical', ax=ax2, palette=pal)
gs.tight_layout(plt.gcf())
```




```
estimate2 = smf.logit('default ~  income + balance', data=default).fit()
estimate2.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.078948
         Iterations 10
```

```
Iterations 10
```

Logit Regression Results

Dep. Variable:	default	**No. Observations:**	10000
Model:	Logit	**Df Residuals:**	9997
Method:	MLE	**Df Model:**	2
Date:	Sun, 09 Oct 2022	**Pseudo R-squ.:**	0.4594
Time:	09:07:44	**Log-Likelihood:**	-789.48
converged:	True	**LL-Null:**	-1460.3
Covariance Type:	nonrobust	**LLR p-value:**	4.541e-292

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	-11.5405	0.435	-26.544	0.000	-12.393	-10.688
income	2.081e-05	4.99e-06	4.174	0.000	1.1e-05	3.06e-05
balance	0.0056	0.000	24.835	0.000	0.005	0.006

Possibly complete quasi-separation: A fraction 0.14 of observations can be perfectly predicted. This might indicate that there is complete quasi-separation. In this case some parameters will not be identified.

```python
print("Log-likelihood of logit model: ", round(estimate2.llf,3))
```

```
Log-likelihood of logit model:  -789.483
```

```python
estimate2.params
```

```
Intercept   -11.540468
income        0.000021
balance       0.005647
dtype: float64
```

```python
# Calculate Odds
odds_ratio = pd.DataFrame( {"Odds Ratio": estimate2.params,"Lower CI": estimate2.conf_int()
         [0],"Upper CI": estimate2.conf_int()[1]})
odds_ratio = np.exp(odds_ratio)
display (odds_ratio)
intercept=odds_ratio.iloc[0,0]
P = (intercept) / (1 + intercept)
print("p_o: ", round(P,15))
```

	Odds Ratio	**Lower CI**	**Upper CI**
Intercept	0.000010	0.000004	0.000023
income	1.000021	1.000011	1.000031
balance	1.005663	1.005215	1.006111

```
p_o:  9.728234005e-06
```

```python
odds_ratio.iloc[0,0]
```

```
9.728328644686842e-06
```

(b)

[5 pts] Write out the model in equation form and provide an interpretation of each coefficient in the trained model.

$$P(Y=1|X_1,X_2)=p=\frac{\exp(\beta_0+\beta_1X_1+\beta_2X_2)}{1+\exp(\beta_0+\beta_1X_1+\beta_2X_2)}$$

$$\ln(\frac{p}{1-p})=\beta_0+\beta_1X_1+\beta_2X_2=-11.5405+0.00002X_1+0.0056X_2$$

where

$$Y=\begin{cases}1 & \text{if } Default=Yes\\ 0 & \text{if } Default=No\end{cases}$$

$$X_1=Income$$

$$X_2=Balance$$

- Interpreation of the coefficient

1. β_0= -11.5405 (odds ratio: 0.000010): The probability of having the outcome Default is 0.00000972823, if all other variables are equal to zero

$$(i.e.\quad X_1,\,X_2=0)$$

2. β_1= 0.00002 (odds ratio: 1.000021): The one unit increase in Income, results in 1.000021 times increase in odds ratio of Default (increases odds of default by 0.0021%)
3. β_2= 0.0056 (odds ratio: 1.005663): The one unit increase in Balance, results in 1.005663 times increase in odds ratio of Default (increases odds of default by 0.5663%)

(c)

[5 pts] Perform 5-fold cross-validation using the model in Part (a), and estimate the test error of this model.

```
x = np.array(default.loc[ : , ['income','balance']]) # make array
y = default.loc[:,'default']

# Five-Fold cross-validation
k_fold=KFold(n_splits = 5, shuffle = True, random_state=2022)
```

Test error was calculated using Mean Squared Error

$$\text{Test Error of 5 folds: } CV_5=\sum_{k=1}^{5}\frac{n_k}{n}MSE_k$$

```
test_error = []

for train_index, test_index in k_fold.split(x):
    x_train, x_test = x[train_index], x[test_index]
    y_train, y_test = y[train_index], y[test_index]
```

```
    model = LogisticRegression() # Logistic Regression
    model.fit(x_train, y_train) # Model Training
    y_test_pred = model.predict(x_test)
    test_error.append(1-accuracy_score(y_test_pred, y_test)) # Test Error of each five folds

# Estimating Test Error
print("The test error of each five folds: ", np.round(test_error,3))
```

```
The test error of each five folds:  [0.031 0.025 0.026 0.029 0.031]
```

```
model = LogisticRegression() # Logistic Regression
test_error=cross_val_score(model, x, y, cv=k_fold, scoring='neg_mean_squared_error', n_jobs=-1)
```

```
size=[]
for train_index, test_index in k_fold.split(x):
    size.append(np.size(test_index))
print("The observations per each split: ", size)

# Using formula
testerror=[]
for folds in np.arange(0,5):
    testerror.append(np.negative(test_error)[folds]*size[folds])
testerrorvalue=round(sum(testerror)/len(default),4)
print("Test error of cross validation", testerrorvalue)
```

```
The observations per each split:  [2000, 2000, 2000, 2000, 2000]
Test error of cross validation 0.0286
```

(d)

[10 pts] Now consider a logistic regression model that predicts the probability of default using income, balance, and a dummy variable for student. Estimate the test error for this model using the 5-fold cross-validation set approach. Comment on whether or not including a dummy variable for student would lead to a reduction in the test error rate.

```
x1 = default.loc[ : , ['income','balance','student']] # make array
y1 = default.loc[:,'default']

# Five-Fold cross-validation
k_fold=KFold(n_splits = 5, shuffle = True,random_state=2022)
```

```
model = LogisticRegression() # Logistic Regression
test_error_withstu=cross_val_score(model, x1, y1, cv=k_fold, scoring='neg_mean_squared_error',
         n_jobs=-1)
```

```
size=[]
for train_index, test_index in k_fold.split(x):
    size.append(np.size(test_index))
print("The observations per each split: ", size)

# Using formula
testerror=[]
```

```
for folds in np.arange(0,5):
    testerror.append(np.negative(test_error_withstu)[folds]*size[folds])
testerrorvalue_withstudents=round(sum(testerror)/len(default),4)
print("Test error of cross validation with students", testerrorvalue_withstudents)
```

```
The observations per each split:  [2000, 2000, 2000, 2000, 2000]
Test error of cross validation with students 0.0317
```

Including a dummy variable for student would lead to almost no change in the test error rate. - Firstly, the strong negative linear correlation between student and income is statistically significant. Since the two variables are linearly correlatedm, even if the varaible student is inserted to the model it would lead to minor improvement in model prediction. (most of the areas that variable student can explain is also can be explained by variable income) - Secondly, the model's predictive ability would be negatively affected by statistically learning from train-dataset. Being a student and whether client would be default is correlated and it is statistically significant. As we use generalized linear model there is high possibility that "multicollinearity" would make parameter estimate unstable.

- As expected, the test error value with student variable is little(0.0001) higher than without student varaible.

```
print("Test error of cross validation", testerrorvalue)
print("Test error of cross validation with students", testerrorvalue_withstudents)

# Correlation
rho = default.corr()
pval = default.corr(method=lambda x, y: pearsonr(x, y)[1]) - np.eye(*rho.shape)
p = pval.applymap(lambda x: ''.join(['*' for t in [0.01,0.05,0.1] if x<=t]))
display(rho.round(2).astype(str) + p)
# significant negative linear relationship between student and income (rho=-0.75)

# Model including student variable
estimate3 = smf.logit('default ~  income + balance + student', data=default).fit()
display(estimate3.summary2().tables[1])
```

```
Test error of cross validation 0.0286
Test error of cross validation with students 0.0317
```

	default	student	balance	income
default	1.0***	0.04***	0.35***	-0.02**
student	0.04***	1.0***	0.2***	-0.75***
balance	0.35***	0.2***	1.0***	-0.15***
income	-0.02**	-0.75***	-0.15***	1.0***

```
Optimization terminated successfully.
         Current function value: 0.078577
         Iterations 10
```

	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
Intercept	-10.869045	0.492273	-22.079320	4.995499e-108	-11.833882	-9.904209
income	0.000003	0.000008	0.369808	7.115254e-01	-0.000013	0.000019
balance	0.005737	0.000232	24.736506	4.331521e-135	0.005282	0.006191

	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
student	-0.646776	0.236257	-2.737595	6.189022e-03	-1.109831	-0.183721

correlation 출처: https://stackoverflow.com/questions/25571882/pandas-columns-correlation-with-statistical-significance

3. Question 3

[40 pts] In this problem, you will predict the number of applications received using the other variables in the College data set attached in the assignment (College.csv).

Loading Dataset

```
import random
random.seed(2022)

# Read Dataset
college=pd.read_csv("College.csv")
display(college.head(5))
# Check if there is missing value
print("Total missing variable: ", college.isnull().sum().sum()) # no missing value
```

	Unnamed: 0	Private	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	R
0	Abilene Christian University	Yes	1660	1232	721	23	52	2885	537	7440	3
1	Adelphi University	Yes	2186	1924	512	16	29	2683	1227	12280	6
2	Adrian College	Yes	1428	1097	336	22	50	1036	99	11250	3
3	Agnes Scott College	Yes	417	349	137	60	89	510	63	12960	5
4	Alaska Pacific University	Yes	193	146	55	16	44	249	869	7560	4

```
Total missing variable:  0
```

Randomly split the data set into a training set and a test set by 90:10 ratio.
Data description: https://www.kaggle.com/faressayah/college-data

```
college.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 777 entries, 0 to 776
```

```
Data columns (total 19 columns):
 #   Column       Non-Null Count  Dtype  
---  ------       --------------  -----  
 0   Unnamed: 0   777 non-null    object 
 1   Private      777 non-null    object 
 2   Apps         777 non-null    int64  
 3   Accept       777 non-null    int64  
 4   Enroll       777 non-null    int64  
 5   Top10perc    777 non-null    int64  
 6   Top25perc    777 non-null    int64  
 7   F.Undergrad  777 non-null    int64  
 8   P.Undergrad  777 non-null    int64  
 9   Outstate     777 non-null    int64  
 10  Room.Board   777 non-null    int64  
 11  Books        777 non-null    int64  
 12  Personal     777 non-null    int64  
 13  PhD          777 non-null    int64  
 14  Terminal     777 non-null    int64  
 15  S.F.Ratio    777 non-null    float64
 16  perc.alumni  777 non-null    int64  
 17  Expend       777 non-null    int64  
 18  Grad.Rate    777 non-null    int64  
dtypes: float64(1), int64(16), object(2)
memory usage: 115.5+ KB
```

```
# Make dummy variable - No=0, Yes=1
# Priavate: A factor with levels Yes and No indicating private or public university
college.Private=pd.get_dummies(college.Private, drop_first=True)
display(college['Private'].value_counts())
```

```
1    565
0    212
Name: Private, dtype: int64
```

```
# Accept: Number of applications accepted
y=college.iloc[:,3]

# Sclaing - Standardization of Predictors
scaler = StandardScaler()
train_col=np.array(college.columns)
x1 = college[np.delete(train_col,[0,3]).tolist()]

x=pd.DataFrame(scaler.fit_transform(x1[x1.columns[range(1,x1.shape[1])]].to_numpy()))
x.columns=x1.columns[range(1,x1.shape[1])]
x.insert(loc=0,column="Private", value=x1["Private"])

# train test split
x_train, x_test, y_train, y_test = train_test_split(x, y, test_size=0.1, shuffle=True)
```

Ridge:

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}\beta_j^2 = RSS + \lambda\sum_{j=1}^{p}\beta_j^2$$

Lasso:

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}|\beta_j| = RSS + \lambda\sum_{j=1}^{p}|\beta_j|$$

In this sense, $\hat{\beta^R_{i,j}}$ will depend not only on the value of λ, but also on the scaling of the jth predictor (and other predictors). (RSS + β)
Therefore, it is best to apply ridge regression after $standardizing\, the\, predictors$ using the formula

$$\tilde{x_{ij}} = \frac{x_ij}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\bar{x_{ij}} - \bar{x}_j)^2}}$$

(ISLR, p. 239)

```
# Dataframe Dimensions
print("# of rows: ", college.shape[0])
print("# of train rows: ", x_train.shape[0])
print("# of test rows: ", x_test.shape[0])
```

```
# of rows:  777
# of train rows:  699
# of test rows:  78
```

(a)

[10 pts] Fit a linear model using least squares on the training set, and report the test error obtained.

The test error was calculated using Mean Squared Error

$$MSE_{train} = \frac{1}{699}\sum_{i=1}^{699}(\hat{Y_i} - Y_i)^2$$

$$MSE_{test} = \frac{1}{78}\sum_{i=1}^{78}(\hat{Y_i} - Y_i)^2$$

Linear Regression: scale equivariant - multiplying X_j by a constant c simply leads to a scale equivariant scaling of the least squares coefficient estimates by a factor of $\frac{1}{c}$. In other words, regardless of how the j_th predictor is scaled, $X_j\beta_j$ will remain the same (ISLR, p. 239). Therefore, this assignment going to use standardized predictors for the model (for convenience).

```
# Model
regr=LinearRegression()

model_ols= regr.fit(x_train,y_train)

y_train_pred=model_ols.predict(x_train)
y_test_pred=model_ols.predict(x_test)

print("Training R squared: ", round(regr.score(x_train,y_train),3))
print("Test R squared: ", round(regr.score(x_test,y_test),3))

print("Training Error: ", round(np.mean((y_train_pred-y_train)**2),3))
print("Test Error: ", round(np.mean((y_test_pred-y_test)**2),3))
print("Test Error: ", round(mean_squared_error(y_test_pred,y_test),3)) # alternate way
```

```
Training R squared:  0.952
Test R squared:  0.963
Training Error:  291415.502
Test Error:  192960.565
Test Error:  192960.565
```

(b)

[10 pts] Fit a ridge regression model on the training set, with λ chosen by 10-fold cross-validation. Report the test error obtained.

```
# ridge regression with default λ=1.0
ridreg = Ridge()
model_ridge=ridreg.fit(x_train, y_train)

y_train_pred=model_ridge.predict(x_train)
y_test_pred=model_ridge.predict(x_test)
print("Training Error: ", round(mean_squared_error(y_train_pred,y_train),3))
print("Test Error: ", round(mean_squared_error(y_test_pred,y_test),3))
```

```
Training Error:  291472.521
Test Error:  191707.194
```

```
# ridge regression with 10-fold cross validation
k_fold=KFold(n_splits = 10, shuffle = True, random_state=2022) # set random seed

# lambdas = np.arange(1,100, 1)
# lambdas = np.arange(1,20, 0.5)
lambdas = np.arange(2,10, 0.1)

# ridge
ridreg_cv = RidgeCV(alphas=lambdas, cv=k_fold)
model_ridge_cv=ridreg_cv.fit(x_train, y_train)

y_train_pred=model_ridge_cv.predict(x_train)
y_test_pred=model_ridge_cv.predict(x_test)

print("Training Error (with 10 folds): ", round(mean_squared_error(y_train_pred,y_train),3))
print("Test Error (with 10 folds): ", round(mean_squared_error(y_test_pred,y_test),3))

print("Training R Squared: ", round(r2_score(y_train, y_train_pred),2))
print("Testing R Squared: ", round(r2_score(y_test, y_test_pred),2))
print("Optimal Lambda: ", model_ridge_cv.alpha_)
print("Coefficients: ", model_ridge_cv.coef_)
```

```
Training Error (with 10 folds):  293156.156
Test Error (with 10 folds):  186375.172
Training R Squared:  0.95
Testing R Squared:  0.96
Optimal Lambda:  6.400000000000004
Coefficients:  [ 154.40316381 1606.59850184  942.98496287 -488.95762002  187.39331727
   63.85717783  -67.20841469  260.99875502  -16.86522954    4.24805295
  -32.17722811   75.26537243   12.02353776  -22.46635632  -83.30446315
 -161.72446977  -18.19291394]
```

```
# Coefficients with λ=6.41
display(pd.DataFrame({"Variable":x_train.columns,"Coefficients":model_ridge_cv.coef_}))
```

	Variable	Coefficients
0	Private	154.403164
1	Apps	1606.598502
2	Enroll	942.984963
3	Top10perc	-488.957620
4	Top25perc	187.393317
5	F.Undergrad	63.857178
6	P.Undergrad	-67.208415
7	Outstate	260.998755
8	Room.Board	-16.865230
9	Books	4.248053
10	Personal	-32.177228
11	PhD	75.265372
12	Terminal	12.023538
13	S.F.Ratio	-22.466356
14	perc.alumni	-83.304463
15	Expend	-161.724470
16	Grad.Rate	-18.192914

```
# ridge regression with optimal λ=6.41
ridreg = Ridge(alpha=6.41)
model_ridge=ridreg.fit(x_train, y_train)

y_train_pred=model_ridge.predict(x_train)
y_test_pred=model_ridge.predict(x_test)
print("Training Error: ", round(mean_squared_error(y_train_pred,y_train),3))
print("Test Error: ", round(mean_squared_error(y_test_pred,y_test),3))
```

```
Training Error:  293160.8
Test Error:  186366.688
```

(c)

[10 pts] Fit a lasso model on the training set, with λ chosen by 10-fold crossvalidation. Report the test error obtained, along with the number of non-zero coefficient estimates.

```
# lasso regression with default λ=1.0
lassoreg = Lasso()
model_lasso=lassoreg.fit(x_train, y_train)

y_train_pred=model_lasso.predict(x_train)
y_test_pred=model_lasso.predict(x_test)
```

```
y_test_pred=model_lasso.predict(x_test)
print("Training Error: ", round(mean_squared_error(y_train_pred,y_train),3))
print("Test Error: ", round(mean_squared_error(y_test_pred,y_test),3))
```

```
Training Error:  291537.881
Test Error:  190072.628
```

```
# ridge regression with 10-fold cross validation
k_fold=KFold(n_splits = 10, shuffle = True,random_state=2022) # set random seed
max_iter = 15000

# lambdas = np.arange(0.1,10,0.01) # Smallest Test Error calculated at 2.97
lambdas = np.arange(4,50,0.1)

lassoreg_cv = LassoCV(alphas=lambdas, cv=k_fold, max_iter=max_iter)
model_lasso_cv=lassoreg_cv.fit(x_train, y_train)

y_train_pred=model_lasso_cv.predict(x_train)
y_test_pred=model_lasso_cv.predict(x_test)

print("Training Error (with 10 folds): ", round(mean_squared_error(y_train_pred,y_train),3))
print("Test Error (with 10 folds): ", round(mean_squared_error(y_test_pred,y_test),3))

print("Training R Squared: ", round(r2_score(y_train, y_train_pred),2))
print("Testing R Squared: ", round(r2_score(y_test, y_test_pred),2))
print("Optimal Lambda: ", model_lasso_cv.alpha_)
print("Coefficients: ", model_lasso_cv.coef_)
```

```
Training Error (with 10 folds):  292747.201
Test Error (with 10 folds):  181011.81
Training R Squared:  0.95
Testing R Squared:  0.97
Optimal Lambda:  4.0
Coefficients:  [ 126.65222298 1632.17789857  979.5277818  -487.43215826  179.97073683
   -0.          -61.36589383  252.50632841   -8.20028126    0.
  -25.42427626   69.09512846    6.55921777  -15.98829039  -71.29649584
 -156.34592682  -13.15656712]
```

FYI: https://scikit-learn.org/stable/modules/grid_search.html#specifying-an-objective-metric
By default, parameter search uses the score function of the estimator to evaluate a parameter setting. (sklearn.metrics.r2_score for regression)

Even though the 10-fold cross-validation r2 score is lowest when $\lambda = 3$, I chose the lambda with less r2 score. Since the purpose of using lasso regression is selecting features (variable selection), lower lambdas are not estimating coefficients with 0. (so I used lambda with 4 which is closest to optimal lambda with some feature's coefficients are exactly equal to zero)

```
# Coefficients with λ=4
coefs_lam4=np.round(model_lasso_cv.coef_,3)
coefs=pd.DataFrame({"Variable":x_train.columns,"Coefficients":coefs_lam4})
display(coefs)

print("Training Error (with 10 folds): ", round(mean_squared_error(y_train_pred,y_train),3))
print("Test Error (with 10 folds): ", round(mean_squared_error(y_test_pred,y_test),3),"\n")


# Variables selected
```

```python
print("Variables selected")
display(coefs[coefs['Coefficients']!=0])

print("Variables not selected")
display(coefs[coefs['Coefficients']==0].Variable.to_numpy())
```

	Variable	Coefficients
0	Private	126.652
1	Apps	1632.178
2	Enroll	979.528
3	Top10perc	-487.432
4	Top25perc	179.971
5	F.Undergrad	-0.000
6	P.Undergrad	-61.366
7	Outstate	252.506
8	Room.Board	-8.200
9	Books	0.000
10	Personal	-25.424
11	PhD	69.095
12	Terminal	6.559
13	S.F.Ratio	-15.988
14	perc.alumni	-71.296
15	Expend	-156.346
16	Grad.Rate	-13.157

```
Training Error (with 10 folds):  292747.201
Test Error (with 10 folds):  181011.81

Variables selected
```

	Variable	Coefficients
0	Private	126.652
1	Apps	1632.178
2	Enroll	979.528
3	Top10perc	-487.432
4	Top25perc	179.971
6	P.Undergrad	-61.366
7	Outstate	252.506
8	Room.Board	-8.200
10	Personal	-25.424
11	PhD	69.095
12	Terminal	6.559

	Variable	Coefficients
13	S.F.Ratio	-15.988
14	perc.alumni	-71.296
15	Expend	-156.346
16	Grad.Rate	-13.157

```
Variables not selected
```

```
array(['F.Undergrad', 'Books'], dtype=object)
```

Coefficients of variable “F.Undergrad”, “Books” are equal to zero with lambda 4

(d)

[10 pts] Comment on the results obtained.

- How accurately can you predict the number of college applications received?
- Is there much difference among the test errors resulting from these three approaches?

Methods	Score	Value
OLS	Test Mean Squared Error	192960.565
Ridge	Test Mean Squared Error (with 10 folds)	186366.688
Lasso	Test Mean Squared Error (with 10 folds + variable selection)	181011.81

The lowest mean squared error was calculated using Lasso regression with two variables' coeficients are equal to zero. The test error of Lasso is lower compared to OLS (11948.76), Ridge (6593.88).

- Which model would you use?

The ridge and lasso regression is a shirinkage methods that constrains and regularizes the coefficient estimates. It is true that these two models are improving the fit compared to OLS, but they are not without potential disadvantages. They compromised a unbiasedness of the OLS and reduce variance, which may result in overfitting. Therefore, with caution, the lasso regression would be selected. Lasso regression can be used as a techniques to selecting important predictors out of an comparatively less important predictors.