Solid Rocket Motor

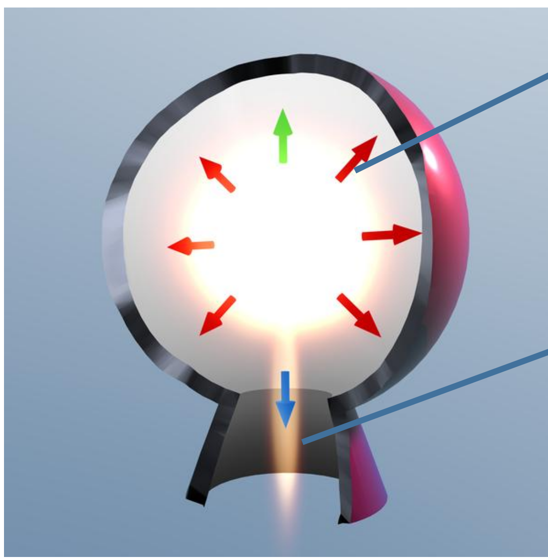
Part 2 Definitions and Fundamentals

Thrust

Propulsion: the act of changing the motion of a body

Jet Propulsion : reaction force by the momentum of ejected matter

Propellant Chemical Energy → combustion → **Internal Energy of Combustion Product** → acceleration / expansion → **Kinetic Energy of Exhaust Gases**



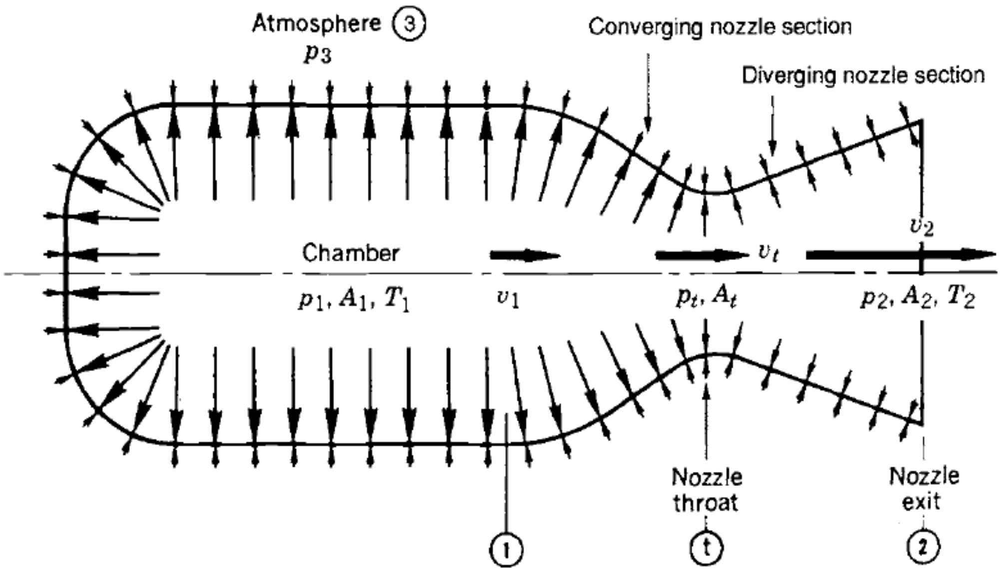
Combustion Chamber

Nozzle

Newton's second law

$$F = \frac{dM}{dt}$$

Thrust



$$F = \dot{m}v_2 + (p_2 - p_3)A_2$$

Momentum Thrust　Pressure Thrust

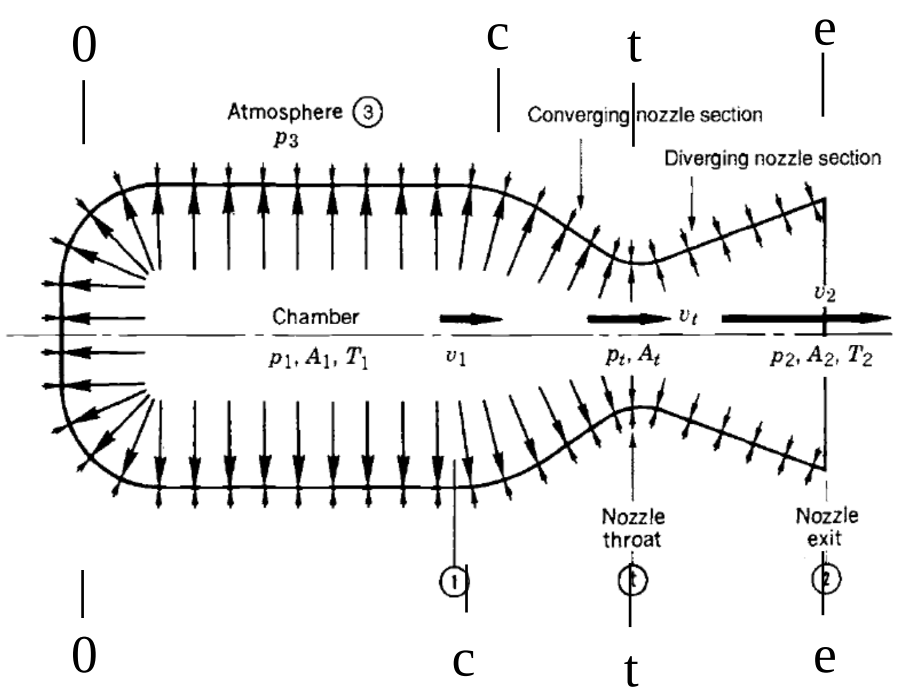
In vacuum space: $p_3 = 0$

$$F = \dot{m}v_2 + p_2 A_2$$

With optimum expansion ratio: $p_3 = p_2$

$$F = \dot{m}v_2$$

Thrust



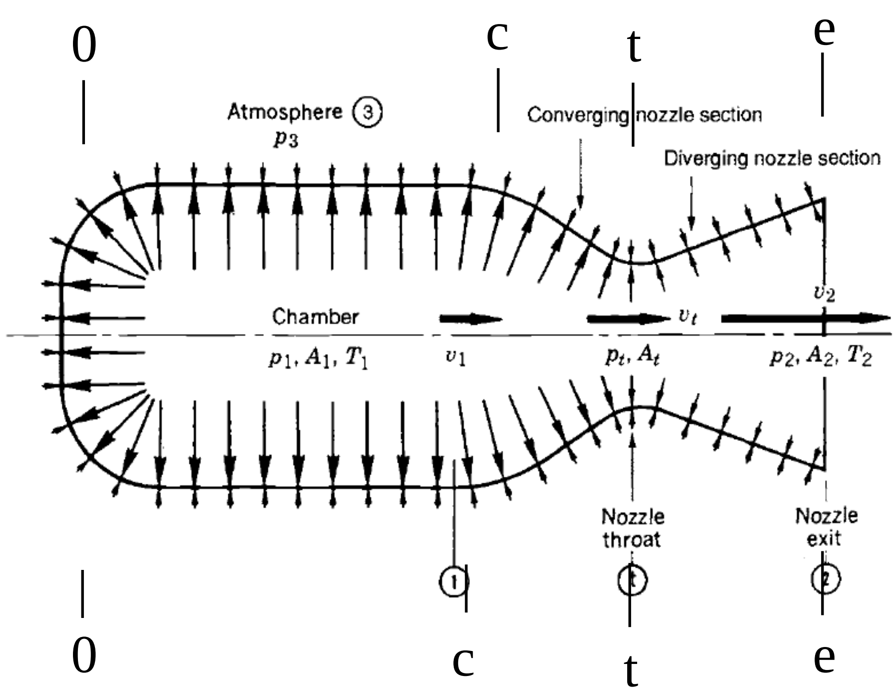
$$F = F_{in} + F_{ex}$$

$$\vec{F}_{in} = \int_{A_{in}} p_i \vec{n} dA$$

$$\vec{F}_{ex} = \int_{A_{ex}} p_a \vec{n} dA$$

$$F = \int_1^e p_i dA - \int_1^e p_a dA$$

Thrust

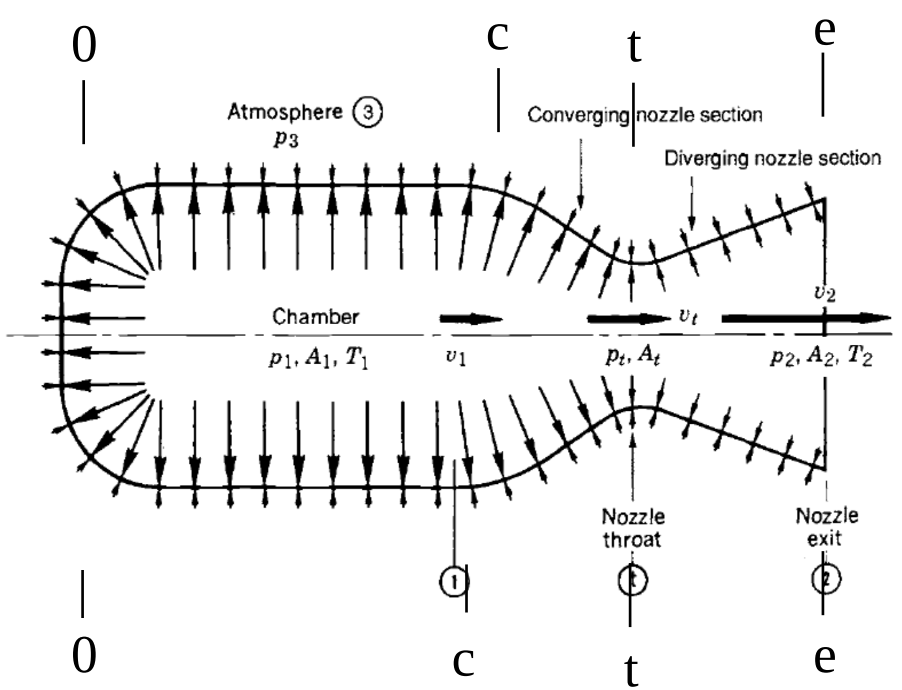


$$\vec{F}_{in} = \int_{A_{in}} p_i \vec{n} dA$$

$$= \int_1^c p_i dA + \int_c^e p_i dA \quad \text{as} \quad \left. \begin{aligned} \mathrm{d}p + \rho u \mathrm{d}u = 0 \\ \dot{m} = \rho u A \end{aligned} \right\} \Rightarrow \int_c^e A \mathrm{d}p_i = \int_c^e -\dot{m} \mathrm{d}u$$

$$F_{in} = \int_1^e p_i dA = p_e A_e - \int_c^e -\dot{m} du = p_e A_e + \dot{m}\left(u_e - u_c\right)$$

Thrust



$$F_{in} = \int_1^e p_i dA = p_e A_e - \int_c^e -\dot{m} du = p_e A_e + \dot{m}\left(u_e - u_c\right)$$

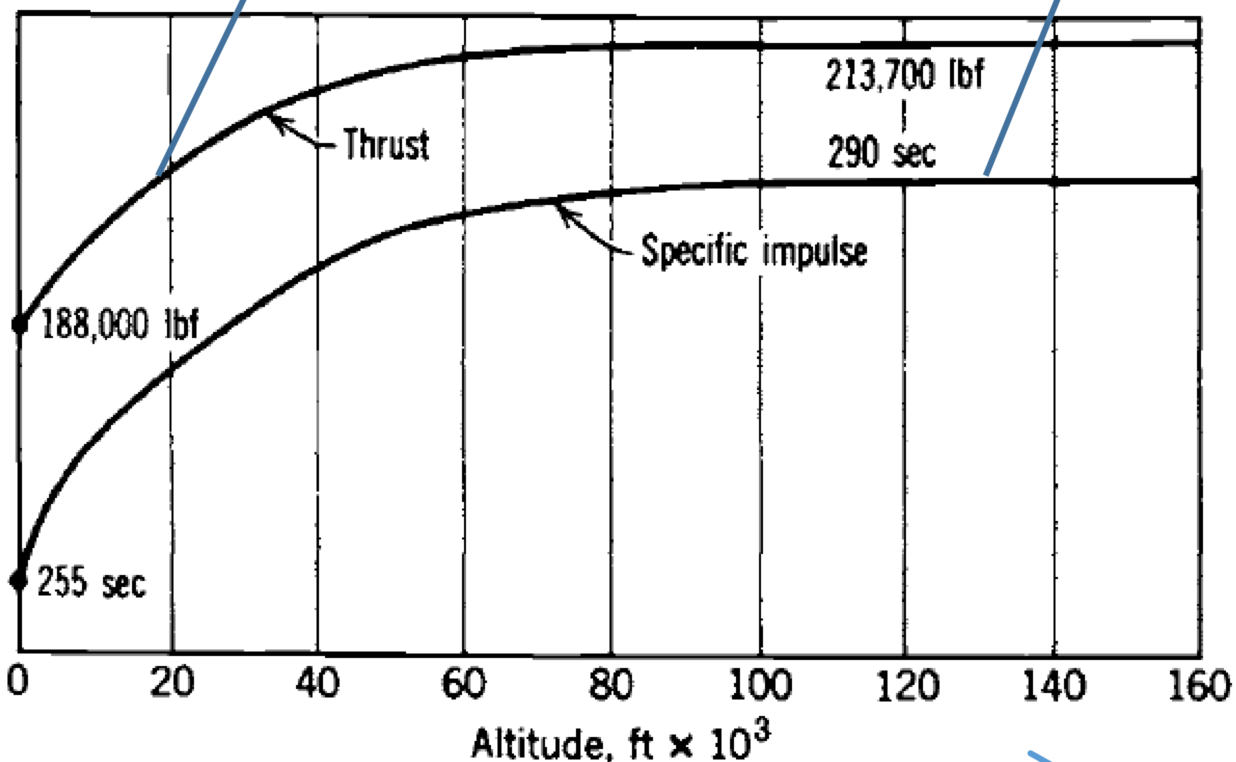
$$\vec{F}_{ex} = \int_{A_{ex}} p_a \vec{n} dA = \int_1^e p_a dA = p_a A_e$$

$$F = \dot{m}\left(u_e - u_c\right) + A_e\left(p_e - p_a\right) = \dot{m} u_e + A_e\left(p_e - p_a\right)$$

Thrust

$$F = \dot{m}v_2 + \left(p_2 - p_3\right) A_2$$

$$I_t = \int_0^t F\,dt$$



$p_3 \downarrow$

Thrust and Specific impulse VS altitude

Ideal Rocket

1. The working substance (or chemical reaction products) is *homogeneous*.
2. All the species of the working fluid are *gaseous*. Any condensed phases (liquid or solid) add a negligible amount to the total mass.
3. The working substance obeys the *perfect gas law*. $pV = RT$
4. There is no *heat transfer* across the rocket walls; therefore, the flow is adiabatic.
5. There is no appreciable *friction* and all *boundary layer* effects are neglected.
6. There are no *shock waves* or *discontinuities* in the nozzle flow.

Isentropic expansion

Ideal Rocket

7. The *propellant flow* is *steady* and *constant*. The expansion of the working fluid is uniform and steady, without vibration. Transient effects (i.e., start up and shut down) are of very short duration and may be neglected.

8. All exhaust gases leaving the rocket have an *axially directed velocity*.

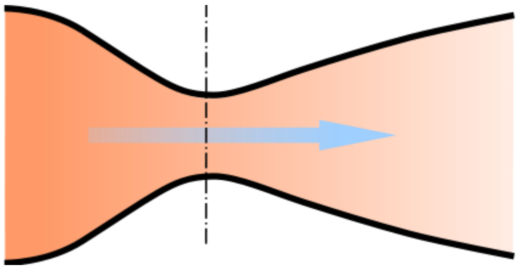
1D

9. The gas velocity, pressure, temperature, and density are all uniform across any section normal to the nozzle axis.

10. *Chemical equilibrium* is established within the rocket chamber and the gas composition does not change in the nozzle (frozen flow).

11. Stored propellants are at room temperature. Cryogenic propellants are at their boiling points.

Thermodynamic Relations



Adiabatic

ideal rocket 4.

No shaft-work

Conservation of energy

Without shocks or friction

ideal rocket 5. 6.

Isentropic flow

——Entropy change is zero

$$dS = 0$$

Internal thermal energy $h = c_p T$

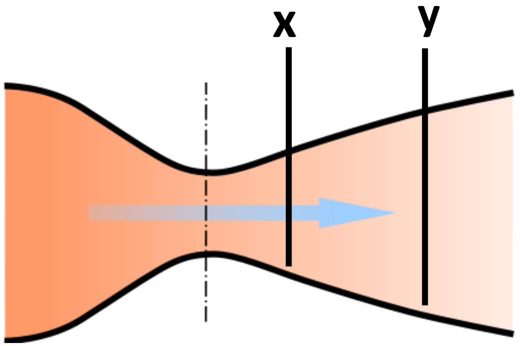
enthalpy

Flow work (kinetic energy) $\frac{u^2}{2}$

Total/Stagnation enthalpy

$\boxed{h_0 = c_p T + \frac{u^2}{2}}$ =constant

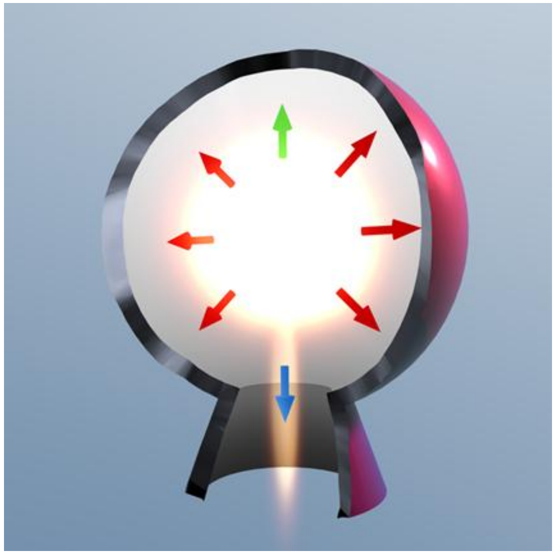
Thermodynamic Relations



Conservation of energy

$$h_0 = h + \frac{u^2}{2} = c_p T + \frac{u^2}{2} \quad \text{=constant} \quad \Rightarrow \quad h_{0x} = h_{0y}$$

$$\boxed{h_x - h_y = \frac{1}{2}\left(u_y^2 - u_x^2\right) = c_p\left(T_x - T_y\right)}$$

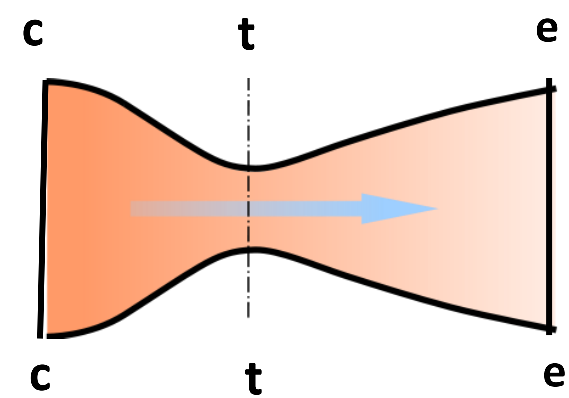


Conservation of mass

$$\boxed{\dot{m}_x = \dot{m}_y \equiv \dot{m} = Au\rho}$$

Perfect gas law $\quad p_x = \rho_x R T_x$

Nozzle exit velocity



For c-c and e-e cross section

$$H_0 = H_c + \frac{u_c^{\,2}}{2} = H_e + \frac{u_e^{\,2}}{2}$$

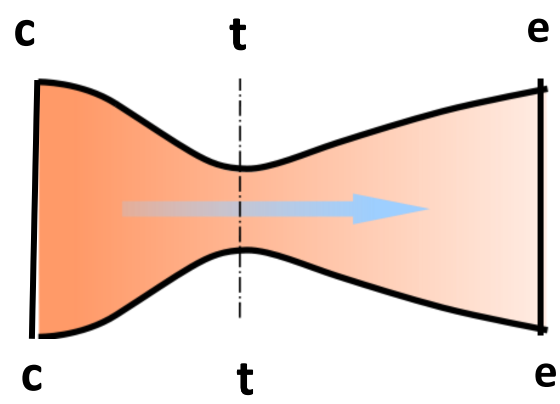
As $u_c \approx 0$, $H_c \approx H_0$

$$H_0 = H_e + \frac{u_e^{\,2}}{2} \quad \Rightarrow u_e = \sqrt{2\left(H_0 - H_e\right)}$$

$$H_0 = c_p T_f \qquad H_e = c_p T_e$$

$$u_e = \sqrt{2c_p(T_f - T_e)} = \sqrt{2c_p T_f\left(1 - \frac{T_e}{T_f}\right)}$$

Nozzle exit velocity



For c-c and e-e cross section

$$u_e = \sqrt{2c_p T_f \left(1 - \frac{T_e}{T_f}\right)}$$

For Isentropic flow

$$\frac{T_e}{T_f} = \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}$$

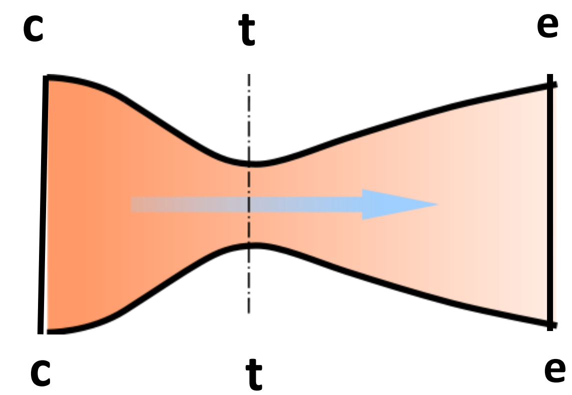
As

$$c_p = \frac{k}{k-1} R = \frac{k}{k-1} \frac{R_0}{\bar{M}}$$

$$u_e = \sqrt{\frac{2k}{k-1} \frac{R_0}{\bar{M}} T_f \left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}$$

Function of $k, \bar{M}, u_e, T_f, \frac{p_e}{p_c}$

Nozzle exit velocity



$$u_e = \sqrt{\frac{2k}{k-1}\frac{R_0}{\bar{M}}T_f\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}$$

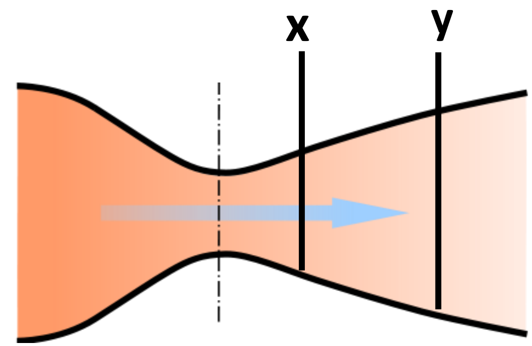
1. T_f : $\quad T_f \uparrow \Rightarrow u_e \uparrow$

2. $\bar{M}$: $\quad \bar{M} \downarrow \Rightarrow u_e \uparrow$

3. k : $\quad k \uparrow \Rightarrow \sqrt{\frac{2k}{k-1}} \downarrow$ *and* $\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right] \uparrow \Rightarrow u_e \downarrow$

4. $\frac{p_e}{p_c}$: $\quad \frac{p_e}{p_c} \downarrow \Rightarrow u_e \uparrow$

Thermodynamic Relations



Perfect gas law $p_x V_x = RT_x$

Conservation of energy $h_x - h_y = \frac{1}{2}(v_y^2 - v_x^2)/J = c_p(T_x - T_y)$

Conservation of mass $\dot{m}_x = \dot{m}_y \equiv \dot{m} = Av/V = Av\rho$

Velocity of sound $a = \sqrt{kRT}$

Mach number $M = v/a = v/\sqrt{kRT}$

Isentropic flow $T_x/T_y = (p_x/p_y)^{(k-1)/k} = (V_y/V_x)^{k-1}$

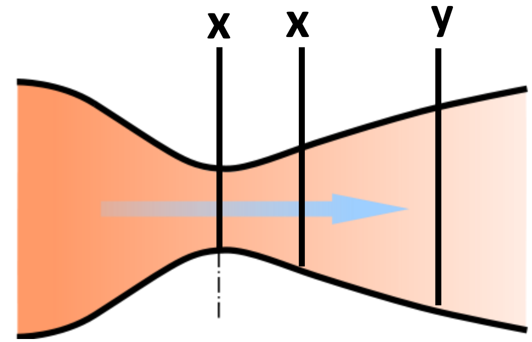
$$\frac{A_y}{A_x} = \frac{M_x}{M_y}\sqrt{\left\{\frac{1 + [(k-1)/2]M_y^2}{1 + [(k-1)/2]M_x^2}\right\}^{(k+1)/(k-1)}}$$

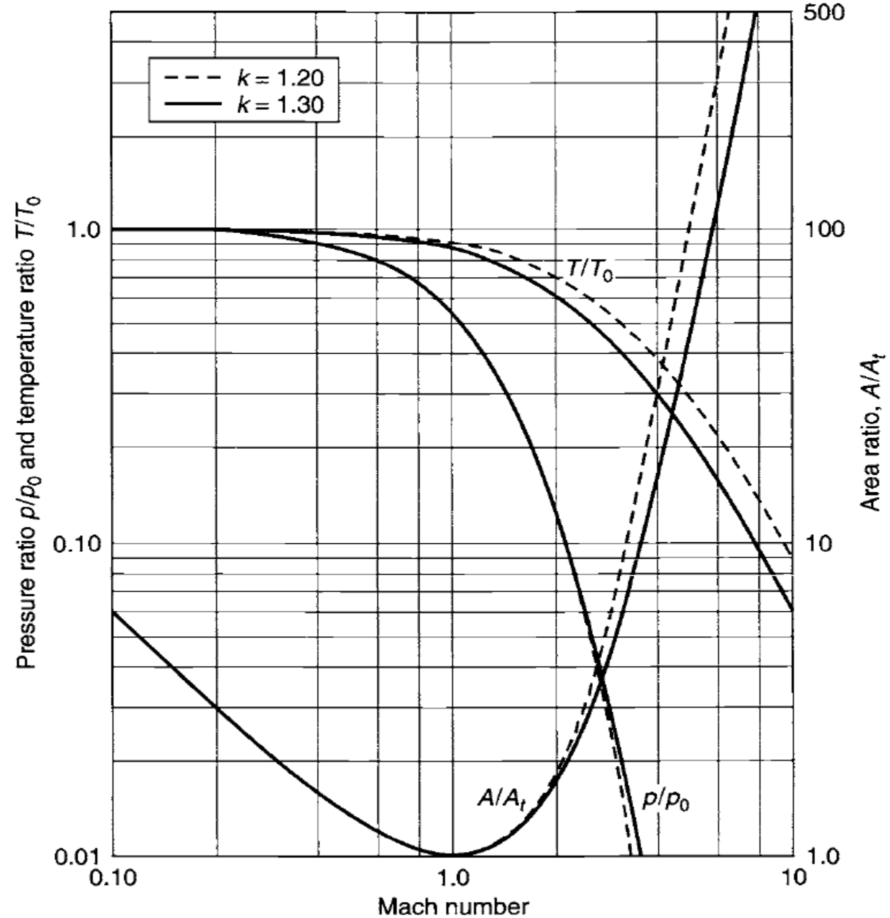
Thermodynamic Relations

$$\frac{A_y}{A_x} = \frac{M_x}{M_y}\sqrt{\left\{\frac{1+[(k-1)/2]M_y^2}{1+[(k-1)/2]M_x^2}\right\}^{(k+1)/(k-1)}}$$

For $A_x = A_t$, $M_x = 1.0$

$$\frac{A_y}{A_t} = \frac{1}{M_y}\sqrt{\left(\frac{1+\left[(k-1)/2\right]M_y^2}{1+(k-1)/2}\right)^{(k+1)/(k-1)}}$$





Total Impulse

$$I_t = \int_0^t F \, dt$$ (N·s)

proportional to the total energy released by all the propellant

Specific Impulse

I_{sp}: The total impulse per unit weight of propellant

$$I_s = \frac{\int_0^t F \, dt}{g_0 \int \dot{m} \, dt}$$ (s) time-averaged

For constant thrust and propellant flow

$$I_s = I_t/(m_p g_0) = I_t/w$$

$$= F/(\dot{m} g_0) = F/\dot{w}$$

Mass flow

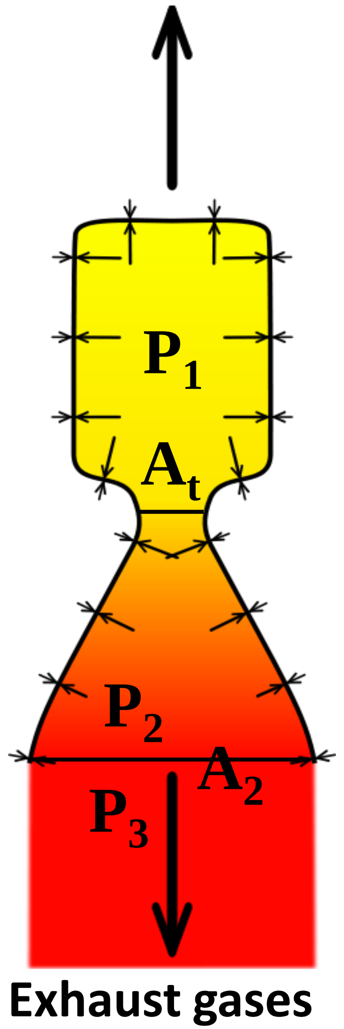
$$\dot{m} = \rho u A = \rho_t u_t A_t = const.$$

Effective exhaust velocity

$$c = F/\dot{m} \qquad \text{(m/s)}$$

$$F = \dot{m} v_2 + (p_2 - p_3) A_2$$

$$c = v_2 + (p_2 - p_3) A_2/\dot{m}$$

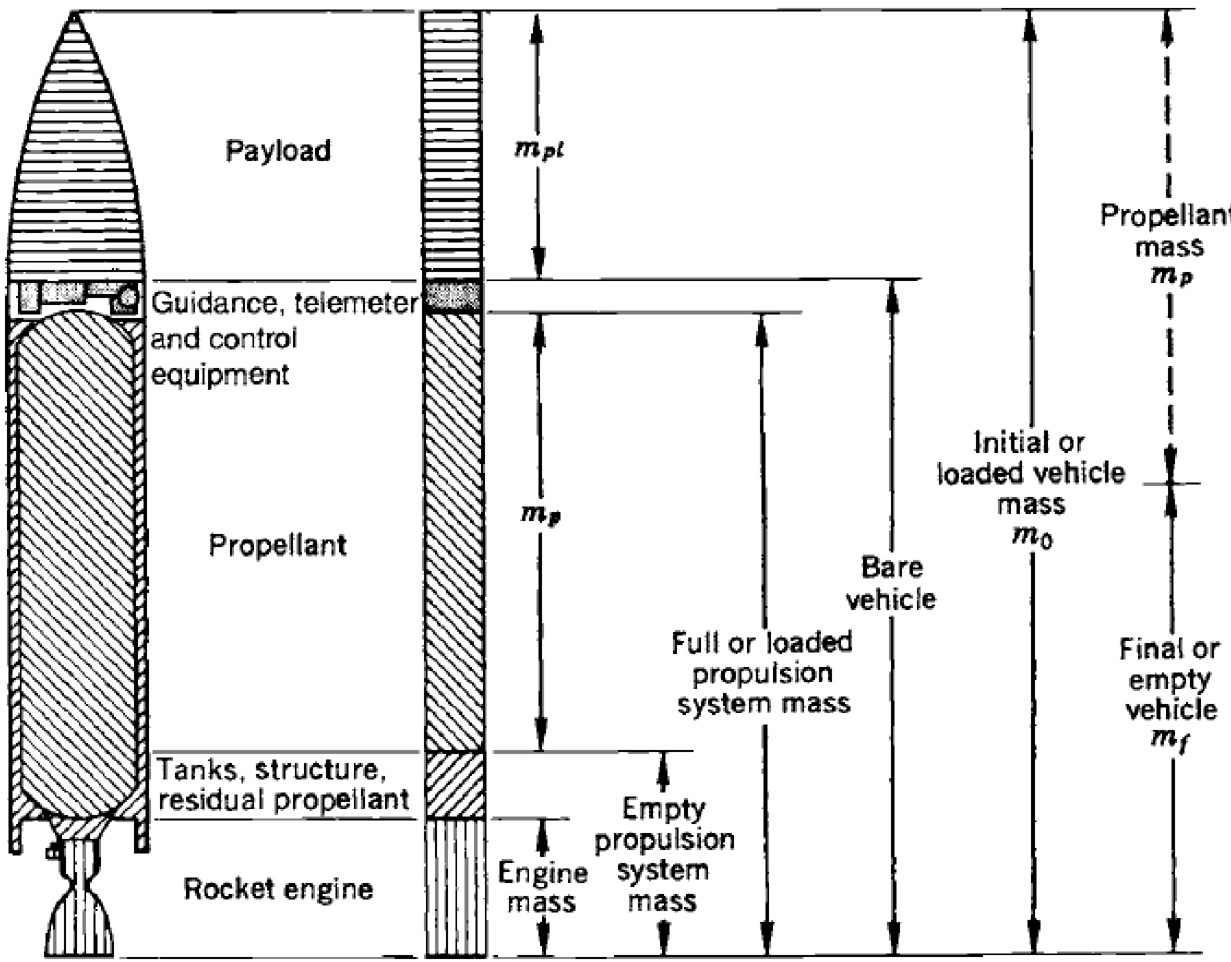


Characteristic velocity

$$c^* = p_1 A_t/\dot{m}$$

- related to the efficiency of the combustion
- independent of nozzle characteristics.

Definitions and Fundamentals



Mass ratio

$$\mathbf{MR} = m_f/m_0$$

- range from 60% to less than 10%.

Propellant mass fraction

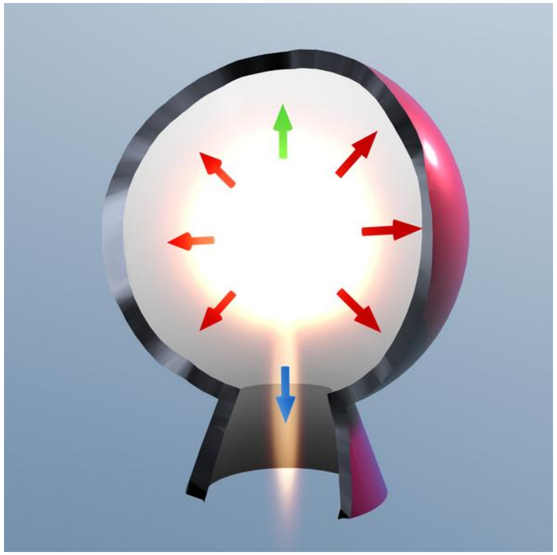
$$\zeta = m_p/m_0$$

$$m_0 = m_f + m_p$$

$$\zeta = (m_0 - m_f)/m_0$$

$$= 1 - \mathrm{MR}$$

- Indicates the quality of the design.



✓ exhaust velocity is not uniform over the entire exit cross-section

Impulse-to-weight ratio

$$\frac{I_t}{w_0} = \frac{I_t}{(m_f + m_p)g_0} = \frac{I_s}{m_f/m_p + 1}$$

- Thrust-to-weight ratio

Energy and Efficiencies

Power input to chemical engine

$$P_{\text{chem}} = \dot{m} Q_R$$

Combustion efficiency η_{comb}

Power of the jet

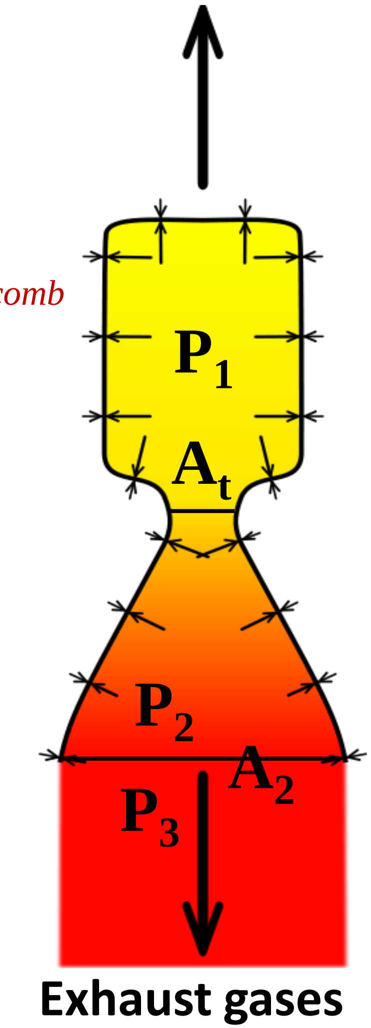
$$P_{\text{jet}} = \tfrac{1}{2}\dot{m}v^2 = \tfrac{1}{2}\dot{w}g_0 I_s^2 = \tfrac{1}{2}Fg_0 I_s = \tfrac{1}{2}Fv_2$$

Power transmitted to the vehicle

$$P_{\text{vehicle}} = Fu$$

Internal efficiency

$$\eta_{\text{int}} = \frac{\text{kinetic power in jet}}{\text{available chemical power}} = \frac{\tfrac{1}{2}\dot{m}v^2}{\eta_{\text{comb}} P_{\text{chem}}}$$



Energy and Efficiencies

Power input to chemical engine

$$P_{\text{chem}} = \dot{m} Q_R$$

Power of the jet

$$P_{\text{jet}} = \tfrac{1}{2}\dot{m}v^2 = \tfrac{1}{2}\dot{w}g_0 I_s^2 = \tfrac{1}{2}Fg_0 I_s = \tfrac{1}{2}Fv_2$$

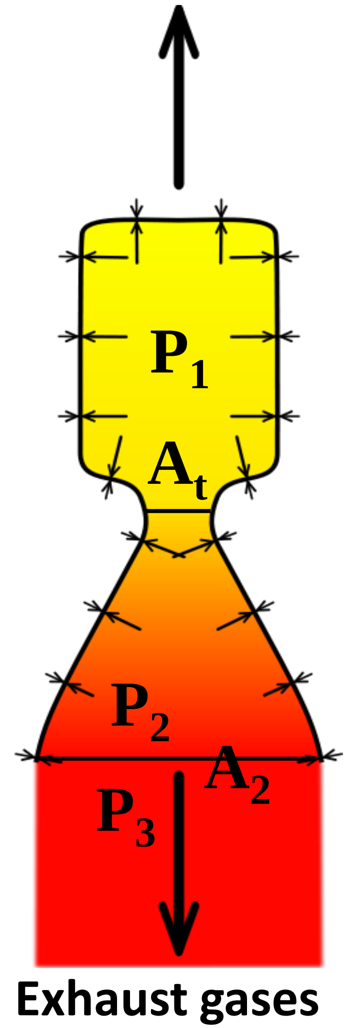
Power transmitted to the vehicle

$$P_{\text{vehicle}} = Fu$$

Propulsive efficiency

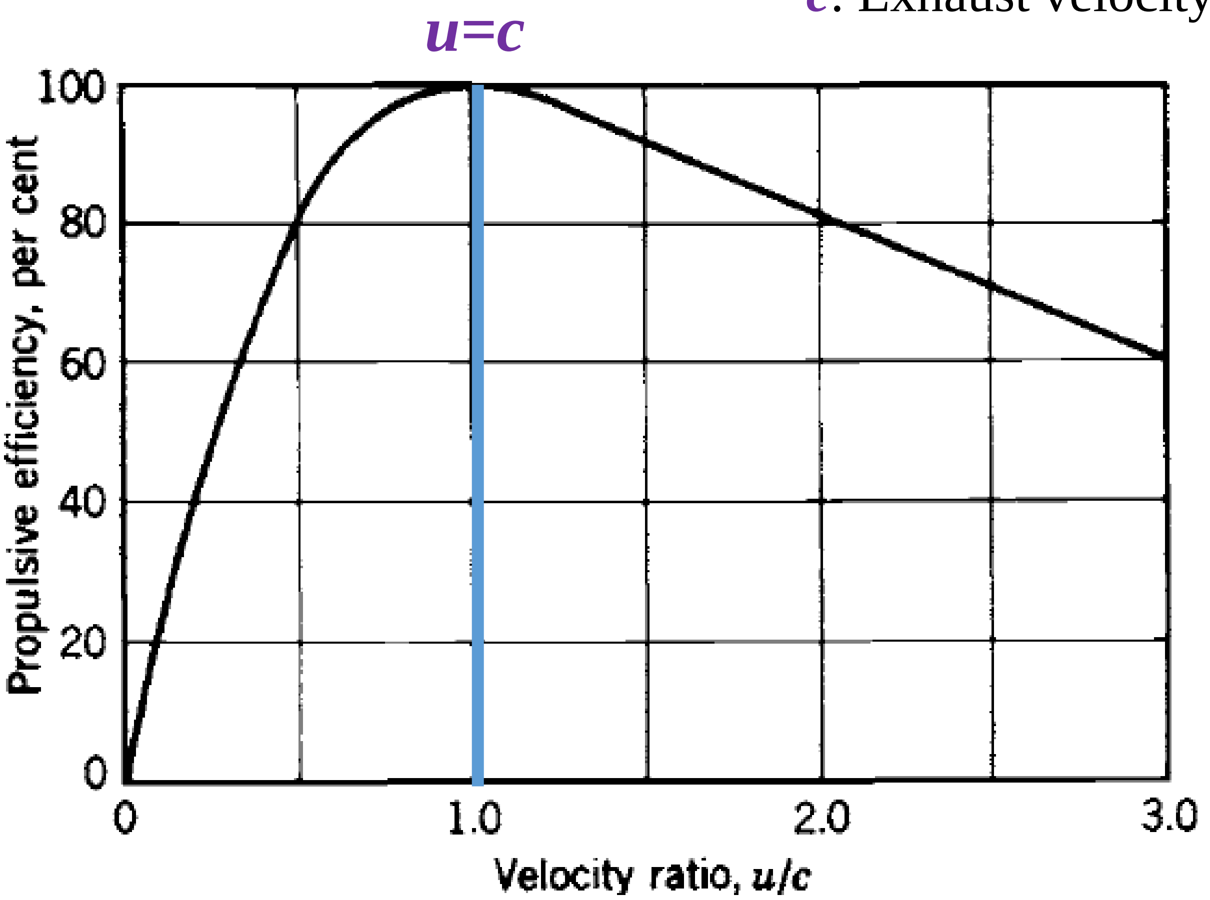
$$\eta_P = \frac{\text{vehicle power}}{\text{vehicle power} + \text{residual kinetic jet power}}$$

$$= \frac{Fu}{Fu + \frac{1}{2}(\dot{w}/g_0)(c-u)^2} = \frac{2u/c}{1+(u/c)^2}$$



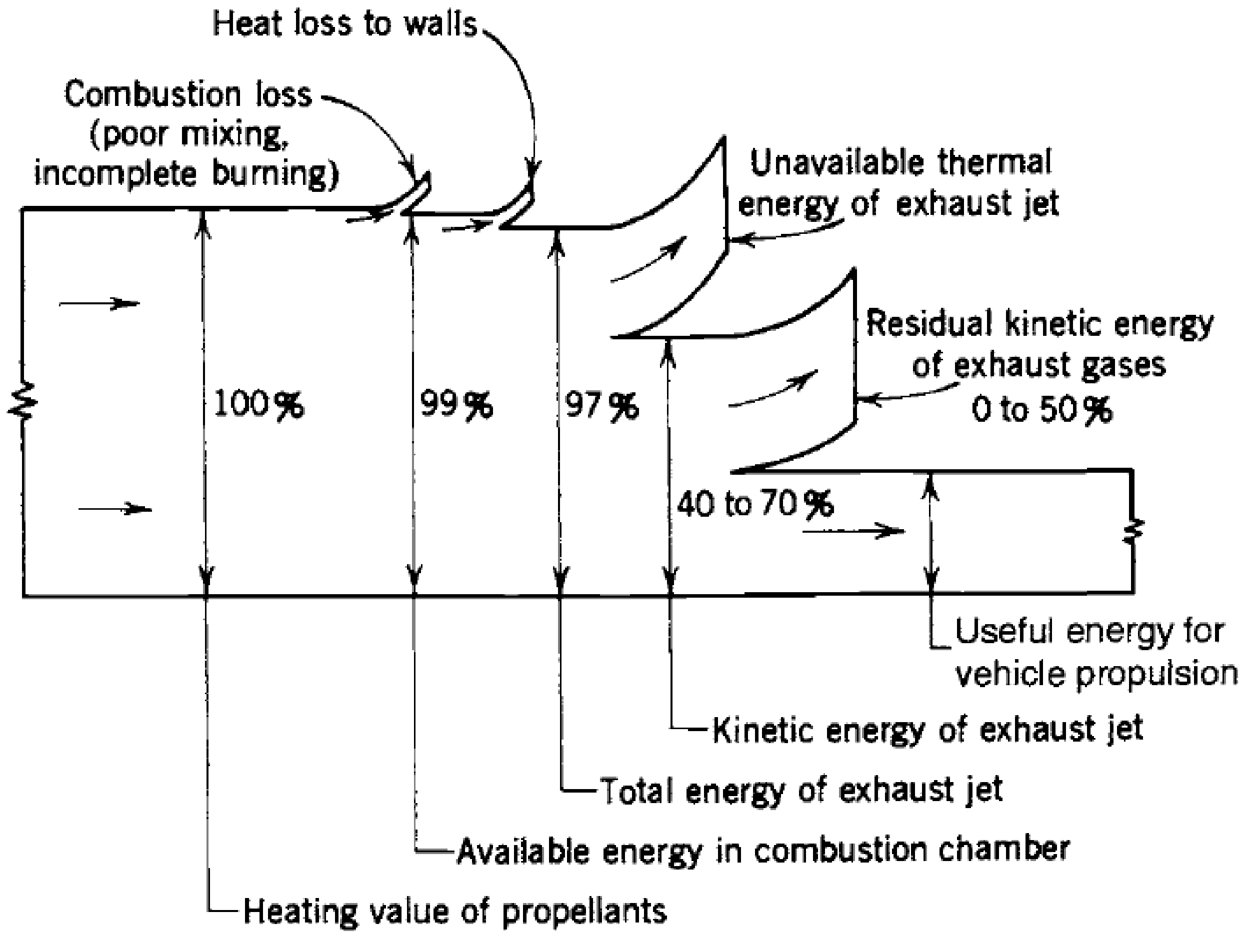
Energy and Efficiencies

u: vehicle velocity
c: Exhaust velocity



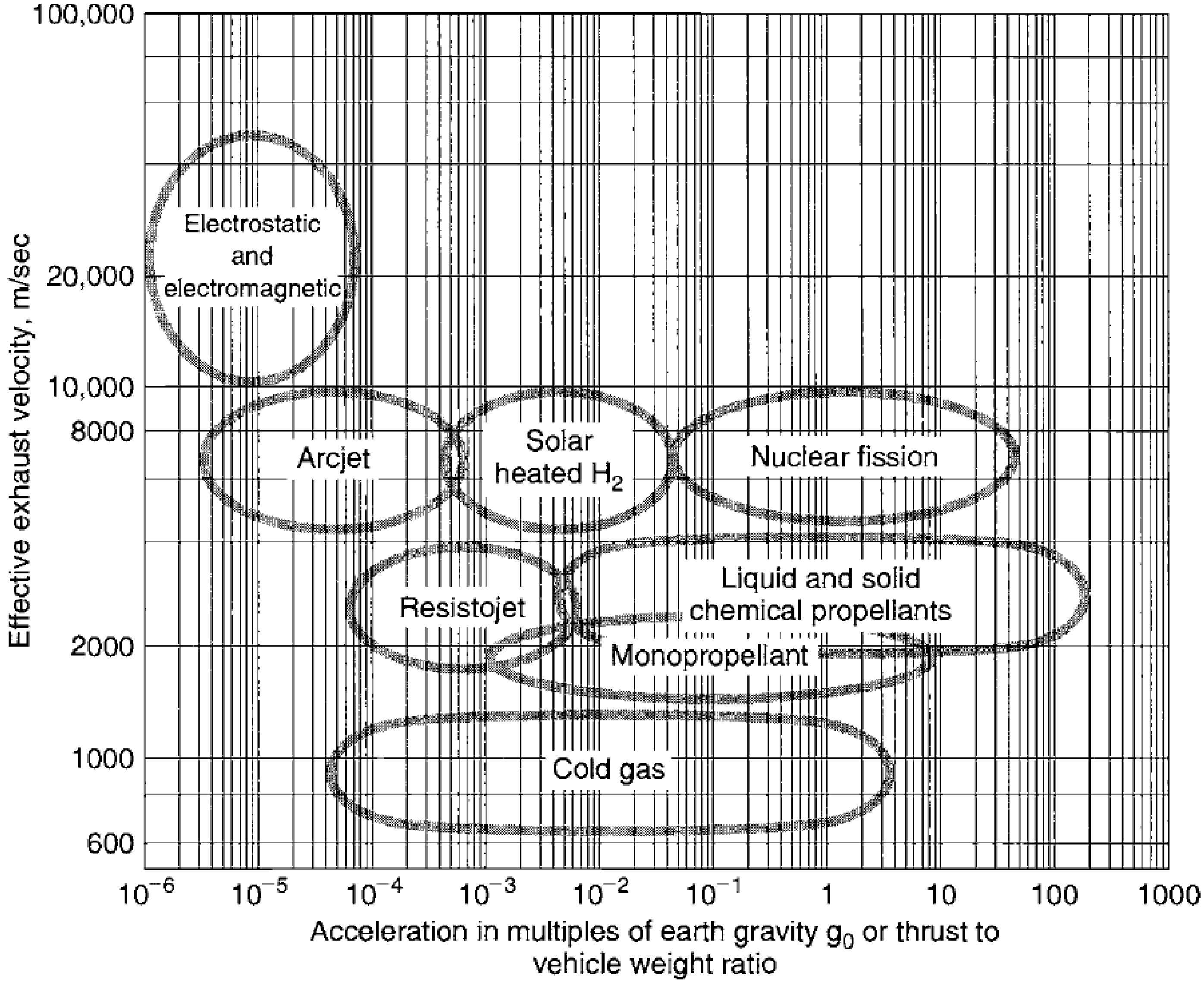
Propulsive efficiency at varying velocities

Energy and Efficiencies

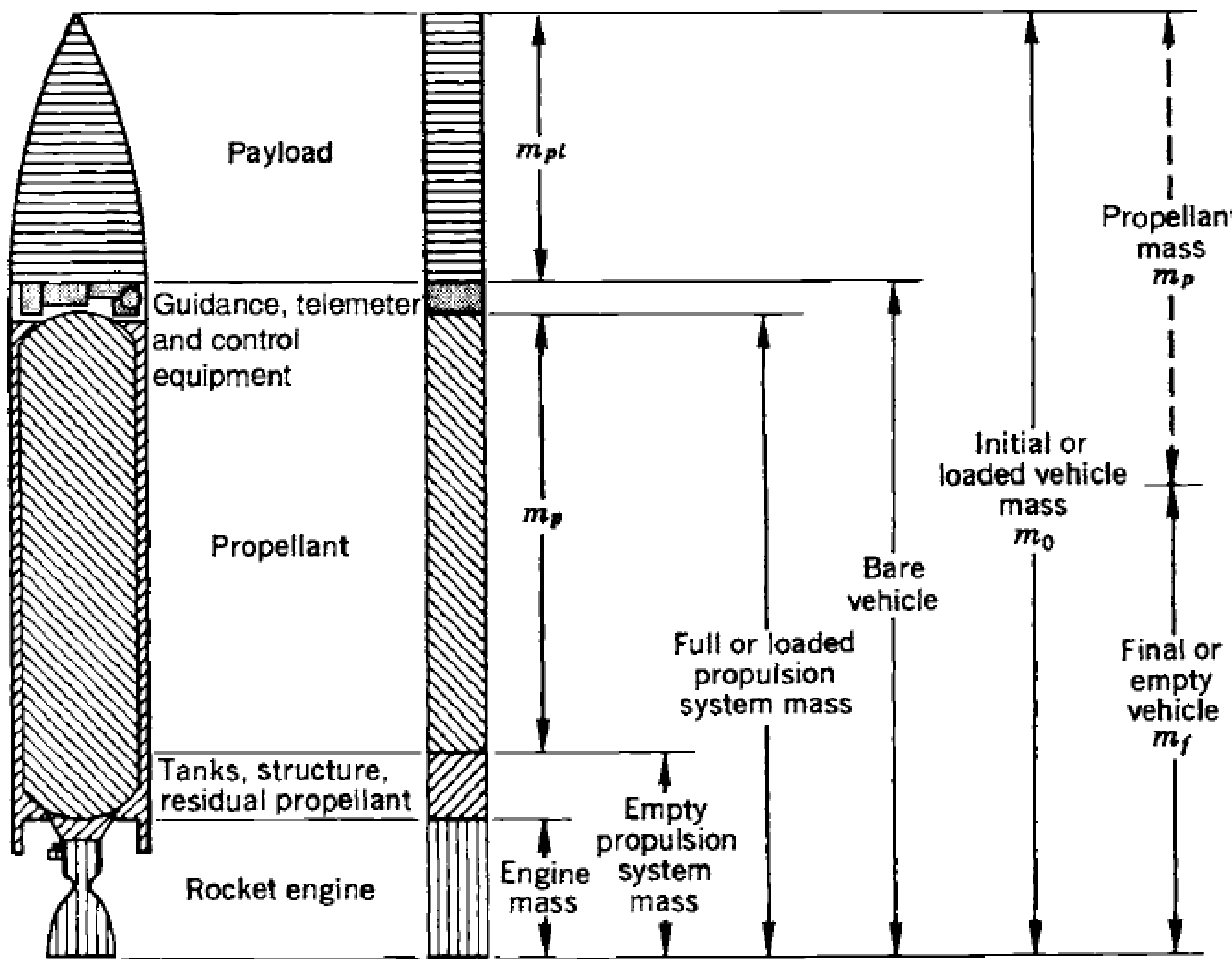


Typical energy balance diagram for a chemical rocket

TYPICAL PERFORMANCE VALUES



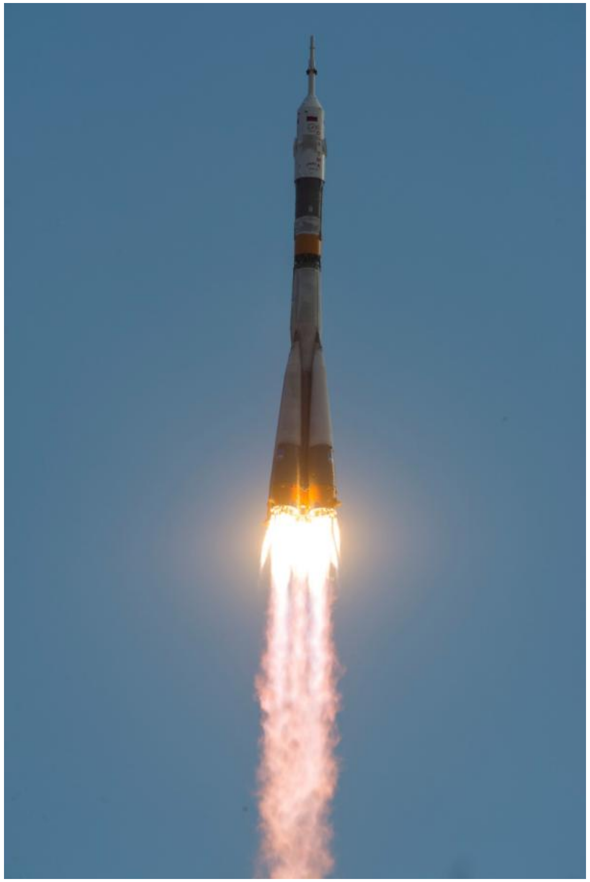
Flight Performance



Flight Performance

GRAVITY-FREE, DRAG-FREE

Newton's second low $F = m\frac{du}{dt}$



instantaneous mass

$$m = m_0 - \dot{m}t = m_0 - \frac{m_p}{t_p}t$$

$$= m_0\left(1 - \frac{m_p}{m_0}\frac{t}{t_p}\right)$$

Propellant mass fraction: $\zeta = \frac{m_p}{m_0}$

$$\boxed{m = m_0\left(1 - \zeta\frac{t}{t_p}\right)}$$

Flight Performance

GRAVITY-FREE, DRAG-FREE

Newton's second low

$$F = m\frac{du}{dt}$$

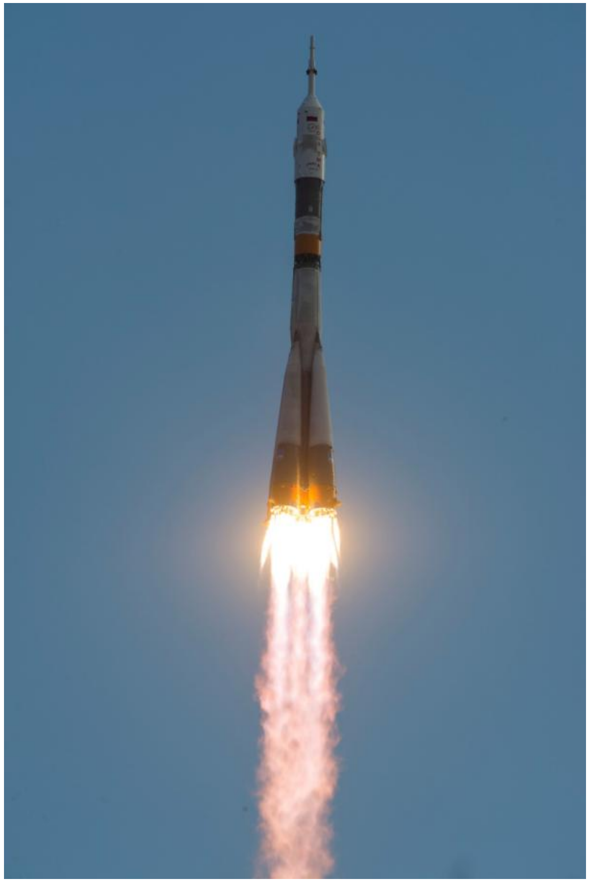
$$du = \frac{F}{m}dt$$

$$F = c\dot{m}$$

$$du = \frac{c\dot{m}}{m}dt$$

$$m = m_0\left(1 - \zeta\frac{t}{t_p}\right)$$

$$\boxed{du = \frac{c\zeta/t_p}{1 - \zeta t/t_p}dt}$$



Flight Performance

$$du = \frac{c\dot{m}}{m}dt = \frac{c\,m_p/t_p}{m_0\left(1-\zeta\,t/t_p\right)}dt = \frac{c\,\zeta/t_p}{1-\zeta\,t/t_p}dt$$

integration

$$\Delta u = -c\ \ln(1-\zeta) + u_0 = c\ln(m_0/m_f) + u_0$$

$$u_p = c\ln\left(\frac{1}{\mathrm{MR}}\right)$$

u_0=0

Vehicle mass ratio: $\mathbf{MR} = 1-\zeta = \dfrac{m_f}{m_0}$

$$u_p = \Delta u = -c\ln(1-\zeta) = -c\ln[m_0/(m_0-m_p)]$$

$$= -c\ln\mathbf{MR} = c\ln(1/\mathbf{MR})$$

$$= c\ln(m_0/m_f)$$

Tsiolkovski

$$\boxed{e^{\Delta u/c} = 1/\mathbf{MR} = m_0/m_f}$$

Flight Performance

Tsiolkovski

$$u_p = c \ln(1/\mathbf{MR}) = c \ln(m_0/m_f)$$

or

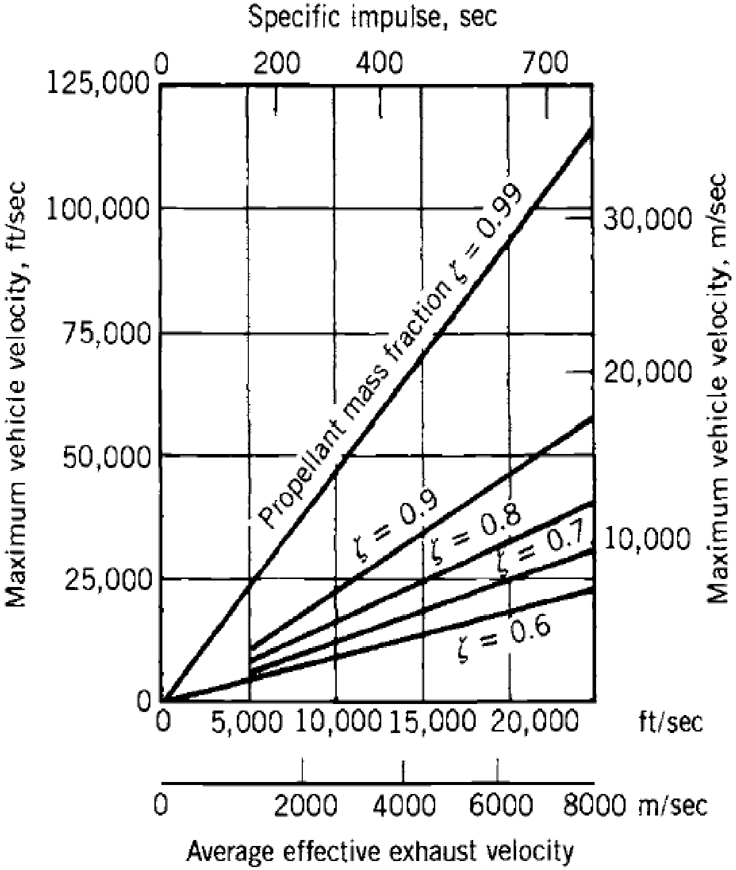
$$e^{\Delta u/c} = 1/\mathbf{MR} = m_0/m_f$$

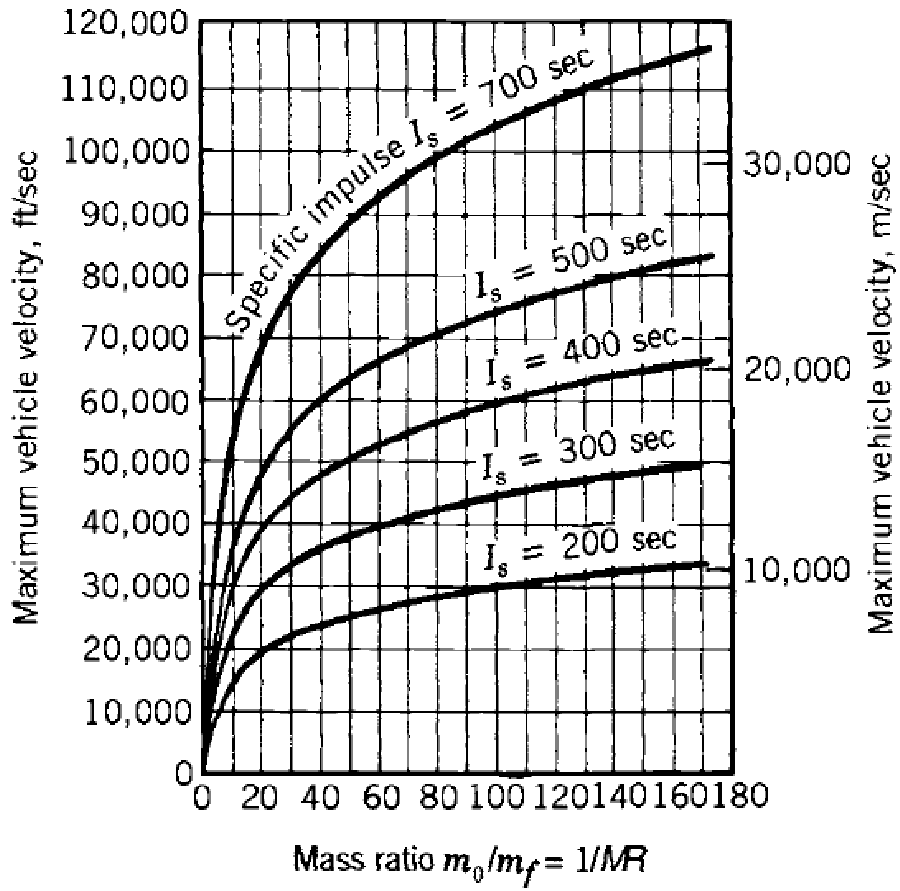
- propellant mass fraction has a logarithmic effect on the vehicle velocity
- The flight velocity increment u_p is proportional to the effective exhaust velocity c and, therefore, to the specific impulse

$$\left\{ \begin{array}{l} m_p = m_0 - m_f \\ e^{\Delta u/c} = 1/\mathbf{MR} = m_0/m_f \end{array} \right.$$

$$m_p = m_f(e^{\Delta u/c} - 1) = m_0(1 - e^{(-\Delta u/c)})$$

Flight Performance





- Single-state vehicles can have values of 1/MR up to about 20
- multistage vehicles can exceed 200

THE END