航天器控制原理

第九讲 相对轨道动力学

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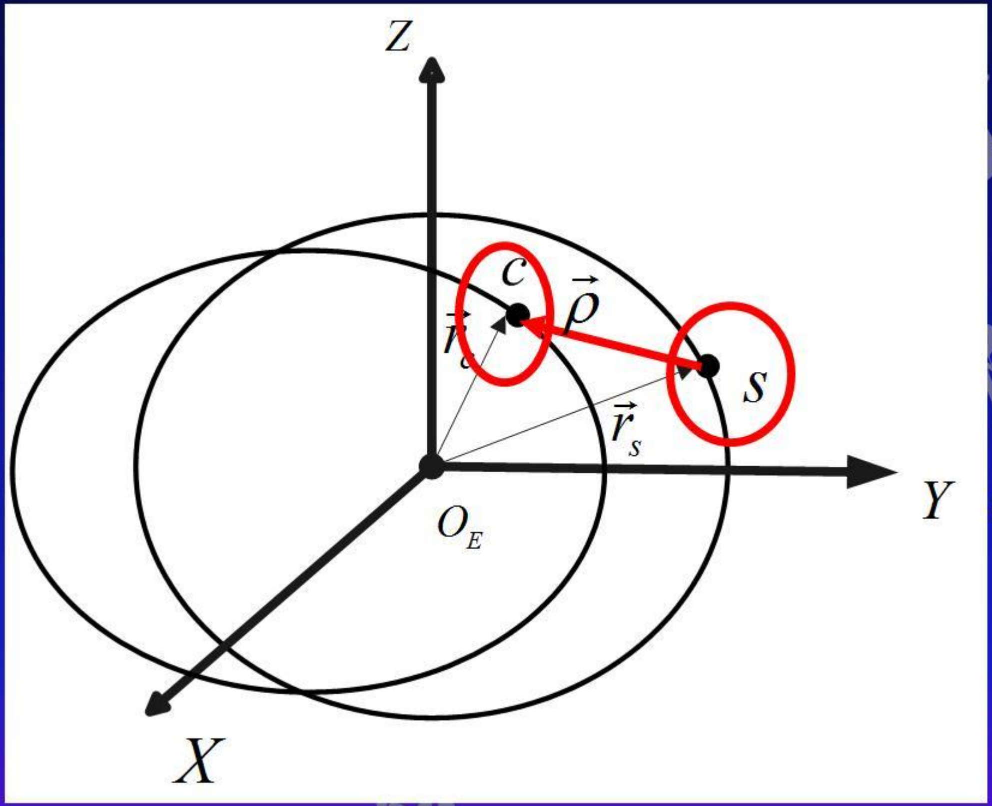
第九讲 相对轨道动力学

1、相对轨道动力学方程

2、相对轨道方程的求解

3、几种典型编队

1、相对轨道动力学方程

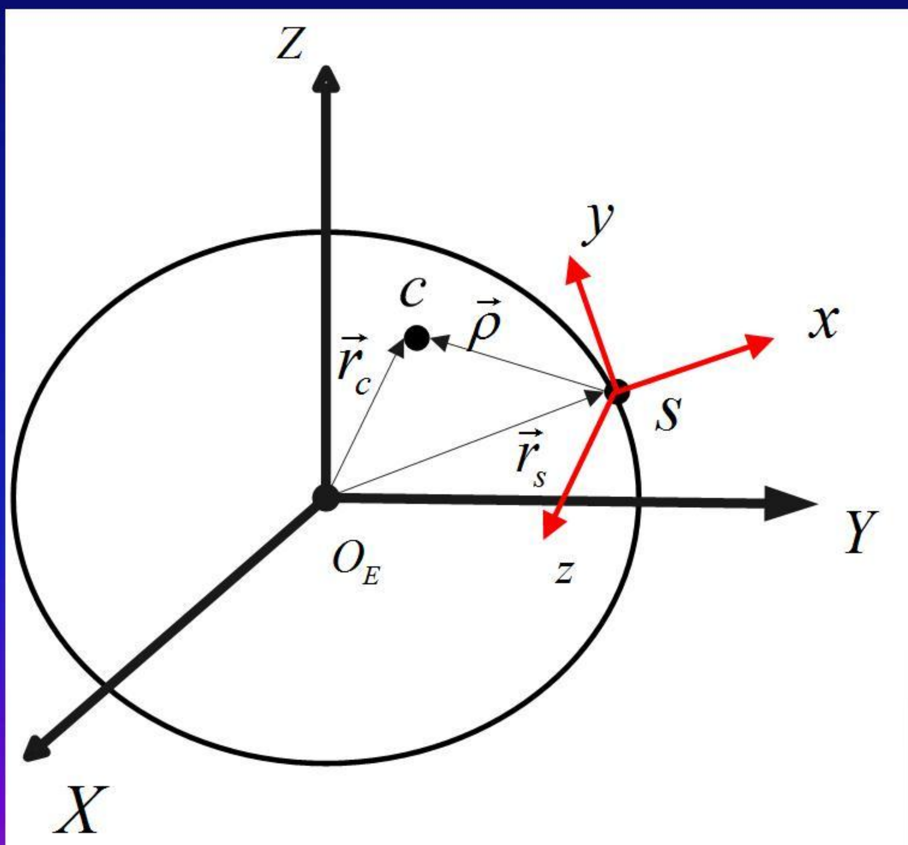


s 参考航天器

c 伴随航天器

相对运动方程

参考航天器轨道坐标系 *s-xyz* 圆轨道

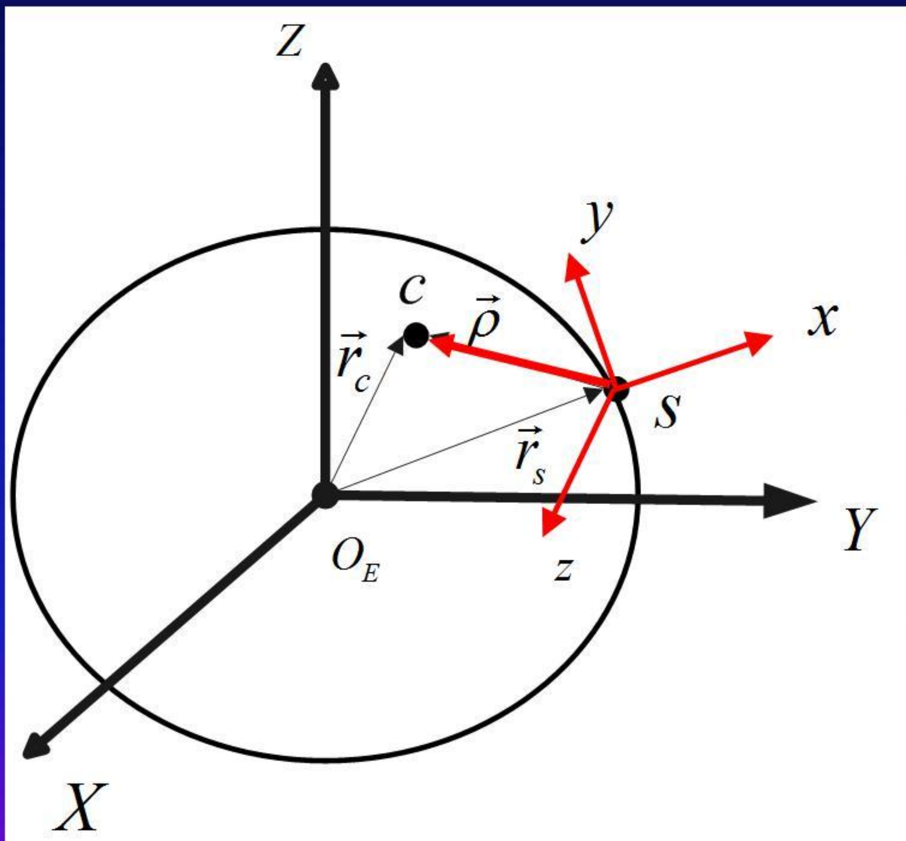


$$\vec{n} = \begin{bmatrix} 0 \\ 0 \\ n \end{bmatrix}$$

$$n = \sqrt{\frac{\mu}{a^3}}$$

$$\vec{r}_s = \begin{bmatrix} r_s \\ 0 \\ 0 \end{bmatrix}$$

相对位置矢量



$$\vec{\rho} = \vec{r}_c - \vec{r}_s$$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vec{r}_c = \begin{bmatrix} r_s + x \\ y \\ z \end{bmatrix}$$

相对运动方程

$$\vec{\rho} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\frac{d\vec{\rho}}{dt} = \underline{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}} + \underline{x}\dot{\vec{i}} + \underline{y}\dot{\vec{j}} + \underline{z}\dot{\vec{k}}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \vec{v} = \dot{\vec{\rho}}$$

$$\dot{\vec{i}} = \vec{n} \times \vec{i}$$

$$\dot{\vec{j}} = \vec{n} \times \vec{j}$$

$$\dot{\vec{k}} = \vec{n} \times \vec{k}$$

$$\frac{d\vec{\rho}}{dt} = \vec{v} + \vec{n} \times \vec{\rho}$$

相对运动方程

$$\frac{d\vec{\rho}}{dt}=\underline{\vec{v}}+\underline{\vec{n}\times\vec{\rho}}$$

$$\frac{d^2\vec{\rho}}{dt^2}=\underline{\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix}+\vec{n}\times\vec{v}}+\underline{\cancel{\dot{\vec{n}}\times\vec{\rho}}+\vec{n}\times\dot{\vec{\rho}}+\vec{n}\times(\vec{n}\times\vec{\rho})}$$

$\downarrow$

$$\frac{d^2\vec{\rho}}{dt^2}=\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix}+2\vec{n}\times\vec{v}+\vec{n}\times(\vec{n}\times\vec{\rho})$$

相对运动方程

$$\frac{d^2\vec{\rho}}{dt^2}=\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix}+2\vec{n}\times\vec{v}+\vec{n}\times(\vec{n}\times\vec{\rho})$$

$$\Downarrow$$

$$\frac{d^2\vec{\rho}}{dt^2}=\begin{bmatrix}\ddot{x}-2n\dot{y}-n^2x\\ \ddot{y}+2n\dot{x}-n^2y\\ \ddot{z}\end{bmatrix}$$

相对运动方程

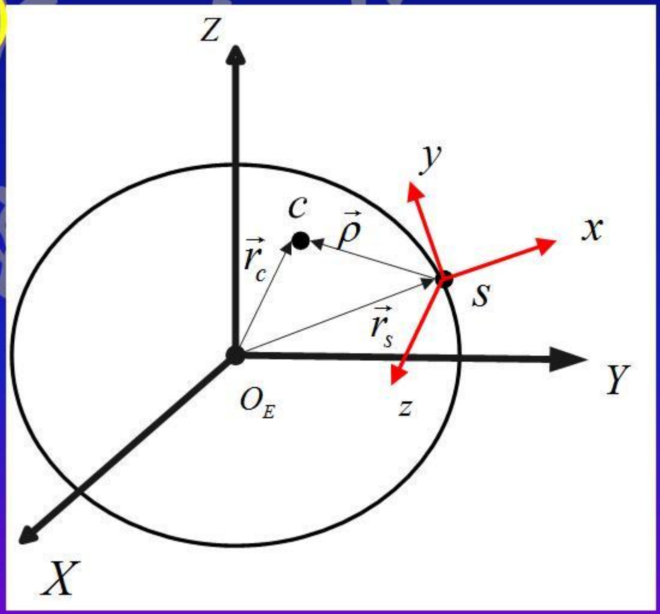
另一方面

$$\vec{\rho} = \vec{r}_c - \vec{r}_s$$

$$\frac{d^2\vec{\rho}}{dt^2} = \underline{\boldsymbol{E}_{oi}}(\ddot{\vec{r}}_{c(i)} - \ddot{\vec{r}}_{s(i)})$$

$$\ddot{\vec{r}}_{s(i)} + \frac{\mu}{r_s^3}\vec{r}_{s(i)} = \vec{f}_{s(i)}$$

$$\ddot{\vec{r}}_{c(i)} + \frac{\mu}{r_c^3}\vec{r}_{c(i)} = \vec{f}_{c(i)}$$



忽略航天器之间的万有引力

相对运动方程

$$\frac{d^2\vec{\rho}}{dt^2} = \boldsymbol{E}_{oi}(\ddot{\vec{r}}_{c(i)} - \ddot{\vec{r}}_{s(i)})$$

$$\ddot{\vec{r}}_{s(i)} + \frac{\mu}{r_s^3}\vec{r}_{s(i)} = \vec{f}_{s(i)} \Rightarrow \boldsymbol{E}_{oi}\ddot{\vec{r}}_{s(i)} + \frac{\mu}{r_s^3}\vec{r}_{s(o)} = \vec{f}_{s(o)}$$

$$\ddot{\vec{r}}_{c(i)} + \frac{\mu}{r_c^3}\vec{r}_{c(i)} = \vec{f}_{c(i)} \Rightarrow \boldsymbol{E}_{oi}\ddot{\vec{r}}_{c(i)} + \frac{\mu}{r_c^3}\vec{r}_{c(o)} = \vec{f}_{c(o)}$$

$$\frac{d^2\vec{\rho}}{dt^2} = -\frac{\mu}{r_c^3}\vec{r}_c + \frac{\mu}{r_s^3}\vec{r}_s + \vec{f}_c - \vec{f}_s$$

相对运动方程

$$\frac{d^2\vec{\rho}}{dt^2} = \underline{-\frac{\mu}{r_c^{\,3}}\vec{r}_c + \frac{\mu}{r_s^{\,3}}\vec{r}_s} + \underline{\vec{f}_c - \vec{f}_s}$$

$$= \frac{\mu}{r_s^{\,3}}\left[\vec{r}_s - \underline{\left(\frac{r_s}{r_c}\right)^3}\vec{r}_c\right] + \Delta\vec{f}$$

$$\vec{r}_c = \begin{bmatrix} r_s + x \\ y \\ z \end{bmatrix} \qquad r_c = \left[(r_s + x)^2 + y^2 + z^2\right]^{\frac{1}{2}}$$

$$= (r_s^{\,2} + 2r_s x + \rho^2)^{\frac{1}{2}}$$

$$\left(\frac{r_s}{r_c}\right)^3 = \left[1 + \frac{2x}{r_s} + \cancel{\left(\frac{\rho}{r_s}\right)^2}\right]^{-\frac{3}{2}} \approx 1 - \frac{3x}{r_s}$$

相对运动方程

$$\frac{d^2\vec{\rho}}{dt^2}=\frac{\mu}{r_s^{\ 3}}[\vec{r}_s-(\frac{r_s}{r_c})^3\vec{r}_c]+\Delta\vec{f}$$

$$\approx 1-\frac{3x}{r_s}$$

$$=\frac{\mu}{r_s^{\ 3}}[\vec{r}_s-(1-\frac{3x}{r_s})\vec{r}_c]+\Delta\vec{f}$$

$$=\frac{\mu}{r_s^{\ 3}}[\vec{r}_s-(1-\frac{3x}{r_s})(\vec{r}_s+\vec{\rho})]+\Delta\vec{f}$$

$$=\frac{\mu}{r_s^{\ 3}}(-\vec{\rho}+\frac{3x}{r_s}\vec{r}_s+\cancel{\frac{3x}{r_s}\vec{\rho}})+\Delta\vec{f}$$

相对运动方程

$$\frac{d^2\vec{\rho}}{dt^2}=\frac{\mu}{r_s^{\ 3}}(-\vec{\rho}+\frac{3x}{r_s}\vec{r}_s)+\Delta\vec{f}$$

$$=n^2\begin{bmatrix}2x\\-y\\-z\end{bmatrix}+\Delta\vec{f}$$

$$\|$$

$$\frac{d^2\vec{\rho}}{dt^2}=\begin{bmatrix}\ddot{x}-2n\dot{y}-n^2x\\\ddot{y}+2n\dot{x}-n^2y\\\ddot{z}\end{bmatrix}$$

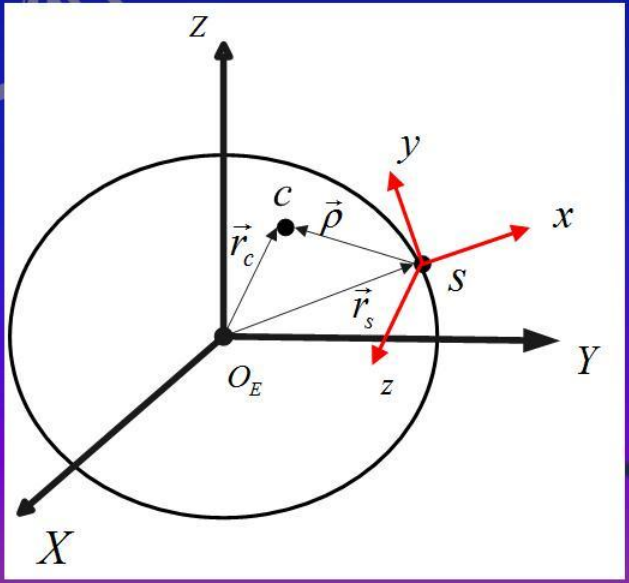
相对运动方程

$$\begin{bmatrix} \ddot{x}-2n\dot{y}-n^2x \\ \ddot{y}+2n\dot{x}-n^2y \\ \ddot{z} \end{bmatrix} = n^2\begin{bmatrix} 2x \\ -y \\ -z \end{bmatrix} + \Delta\vec{f}$$

$$\begin{cases} \ddot{x}-2n\dot{y}-3n^2x=\Delta f_x \\ \ddot{y}+2n\dot{x}=\Delta f_y \\ \ddot{z}+n^2z=\Delta f_z \end{cases}$$

Hill方程，C-W方程。

圆轨道，相对距离小。



2、相对轨道动力学方程的求解

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = \cancel{\Delta f_x} \\ \ddot{y} + 2n\dot{x} = \cancel{\Delta f_y} \\ \ddot{z} + n^2 z = \cancel{\Delta f_z} \end{cases}$$

$$\boldsymbol{X} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \qquad \dot{\boldsymbol{X}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

自由运动的解：

$$
\begin{cases}
\boxed{\begin{aligned}
x &= \frac{\dot{x}_0}{n}\sin nt - \left(2\frac{\dot{y}_0}{n} + 3x_0\right)\cos nt + 2\left(2x_0 + \frac{\dot{y}_0}{n}\right) \\
y &= 2\left(2\frac{\dot{y}_0}{n} + 3x_0\right)\sin nt + 2\frac{\dot{x}_0}{n}\cos nt - 3\left(\dot{y}_0 + 2nx_0\right)t + \left(y_0 - \frac{2}{n}\dot{x}_0\right)
\end{aligned}} \\
z = \frac{\dot{z}_0}{n}\sin nt + z_0\cos nt \\
\dot{x} = (3nx_0 + 2\dot{y}_0)\sin nt + \dot{x}_0\cos nt \\
\dot{y} = -2\dot{x}_0\sin nt + 2(2\dot{y}_0 + 3nx_0)\cos nt - 3(2nx_0 + \dot{y}_0) \\
\dot{z} = -z_0 n\sin nt + \dot{z}_0\cos nt
\end{cases}
$$

轨道平面内运动：

$$x=\frac{\dot{x}_0}{n}\sin nt-\left(2\frac{\dot{y}_0}{n}+3x_0\right)\cos nt+\underline{2\left(2x_0+\frac{\dot{y}_0}{n}\right)}$$

$$y=2\left(2\frac{\dot{y}_0}{n}+3x_0\right)\sin nt+2\frac{\dot{x}_0}{n}\cos nt\underline{-3\left(\dot{y}_0+2nx_0\right)t+\left(y_0-\frac{2}{n}\dot{x}_0\right)}$$

$$\Downarrow$$

$$\underline{x-2\left(2x_0+\frac{\dot{y}_0}{n}\right)}=\frac{\dot{x}_0}{n}\sin nt-\left(2\frac{\dot{y}_0}{n}+3x_0\right)\cos nt$$

$$\underline{y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+3\left(\dot{y}_0+2nx_0\right)t}=2\left(2\frac{\dot{y}_0}{n}+3x_0\right)\sin nt+2\frac{\dot{x}_0}{n}\cos nt$$

轨道平面内运动：

$$x-2\left(2x_0+\frac{\dot{y}_0}{n}\right)=\frac{\dot{x}_0}{n}\sin nt-\left(2\frac{\dot{y}_0}{n}+3x_0\right)\cos nt$$

$$y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+3\left(\dot{y}_0+2nx_0\right)t=2\left(2\frac{\dot{y}_0}{n}+3x_0\right)\sin nt+2\frac{\dot{x}_0}{n}\cos nt$$

$$[x-2\left(2x_0+\frac{\dot{y}_0}{n}\right)]^2+\frac{[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+3\left(\dot{y}_0+2nx_0\right)t]^2}{4}$$

$$=\underline{(\frac{\dot{x}_0}{n})^2+\left(2\frac{\dot{y}_0}{n}+3x_0\right)^2}$$

$$\frac{[x-2\left(2x_0+\frac{\dot{y}_0}{n}\right)]^2}{\underline{b}^2}+\frac{[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+3\left(\dot{y}_0+2nx_0\right)t]^2}{(\underline{2b})^2}=1$$

轨道平面内运动：

$$\frac{\left[x-2\left(2x_0+\frac{\dot{y}_0}{n}\right)\right]^2}{b^2}+\frac{\left[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+3\left(\dot{y}_0+2nx_0\right)t\right]^2}{(2b)^2}=1$$

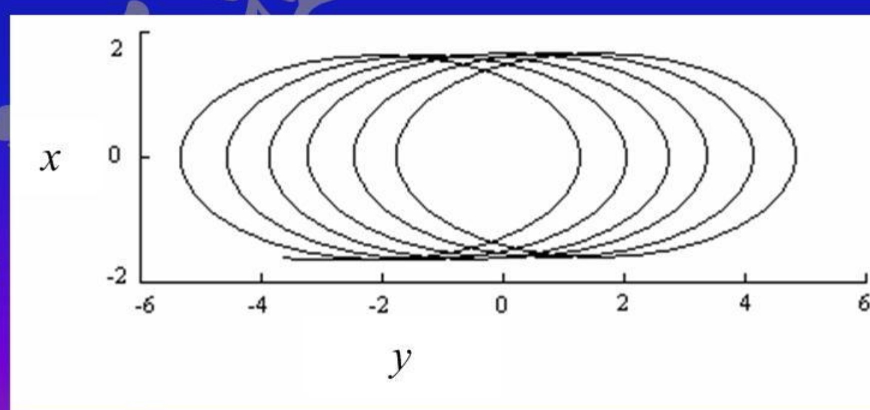
$$\Downarrow$$

$$\frac{\left[x-2\left(2x_0+\frac{\dot{y}_0}{n}\right)\right]^2}{b^2}+\frac{\left[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+\underline{3n\left(2x_0+\frac{\dot{y}_0}{n}\right)}t\right]^2}{(2b)^2}=1$$

封闭轨迹

必要条件

$$2x_0+\frac{\dot{y}_0}{n}=0$$

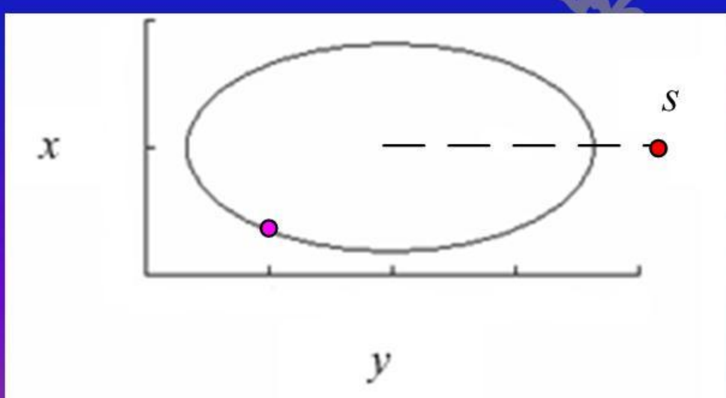


轨道平面内运动：

$$\frac{\left[x-2\underline{\left(2x_0+\frac{\dot{y}_0}{n}\right)}\right]^2}{b^2}+\frac{\left[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)+3n\underline{\left(2x_0+\frac{\dot{y}_0}{n}\right)}t\right]^2}{(2b)^2}=1$$

⇓

$$\frac{x^2}{b^2}+\frac{\left[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)\right]^2}{(2b)^2}=1 \qquad b^2=\left(\frac{\dot{x}_0}{n}\right)^2+\left(2\frac{\dot{y}_0}{n}+3x_0\right)^2$$



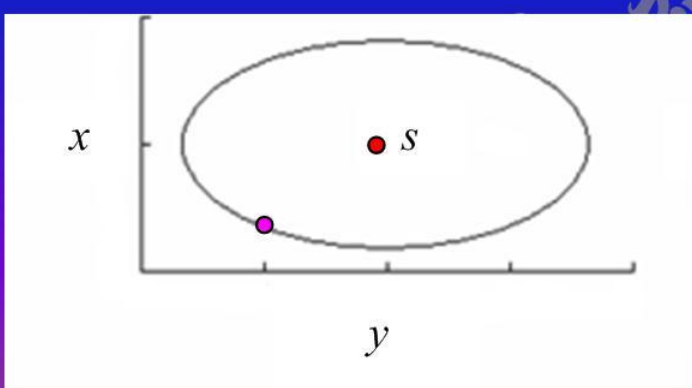
轨道平面内运动：

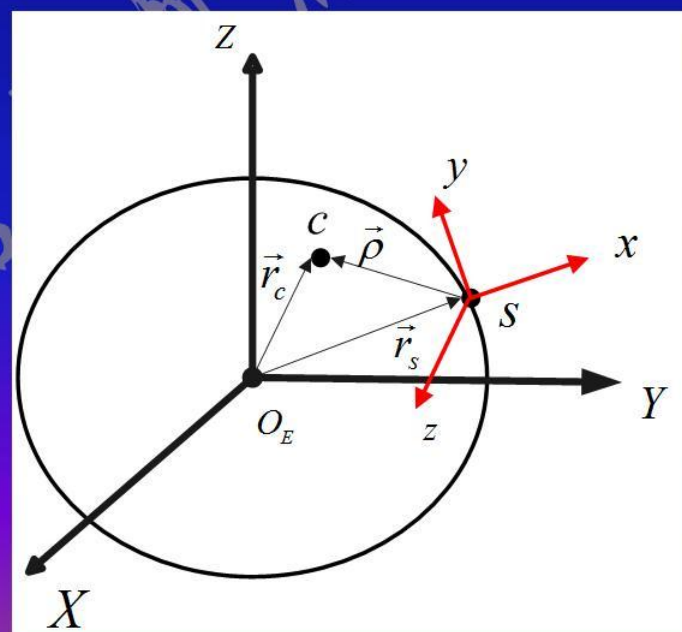
$$\frac{x^2}{b^2}+\frac{[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)]^2}{(2b)^2}=1$$

(1) y方向偏移

$$y_0-\frac{2}{n}\dot{x}_0=0$$

$$2x_0+\frac{\dot{y}_0}{n}=0$$





轨道平面内运动：

$$\frac{x^2}{b^2}+\frac{[y-\left(y_0-\frac{2}{n}\dot{x}_0\right)]^2}{(2b)^2}=1$$

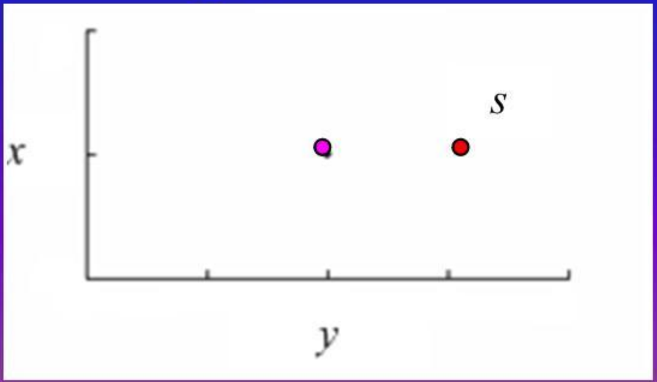
(2) 椭圆短半轴

$$(\frac{\dot{x}_0}{n})^2+\left(2\frac{\dot{y}_0}{n}+3x_0\right)^2=0$$

$$2x_0+\frac{\dot{y}_0}{n}=0$$

$$\Rightarrow \begin{cases}\dot{x}_0=0\\ \dot{y}_0=0\\ x_0=0\end{cases}$$

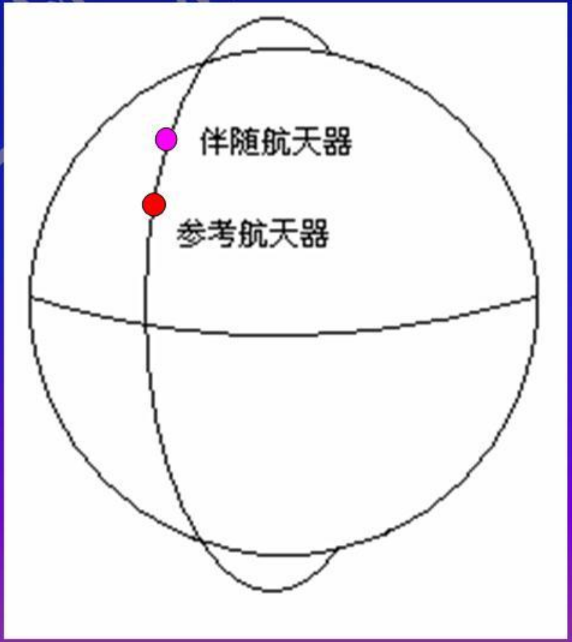
$$x^2+(y-y_0)^2=0$$



3、常用编队构型

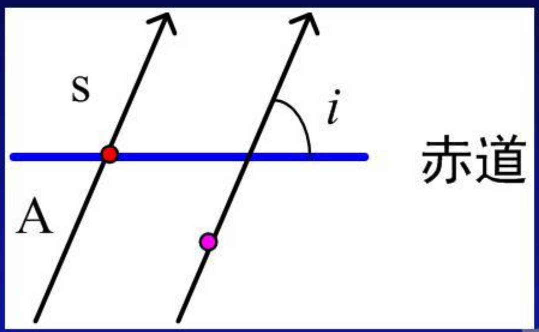
(1) 串行编队

$$\begin{cases} x_0 = 0 \\ y_0 = y_0 \\ \underline{z_0 = 0} \\ \dot{x}_0 = 0 \\ \dot{y}_0 = 0 \\ \underline{\dot{z}_0 = 0} \end{cases} \qquad \begin{cases} x(t) = 0 \\ y(t) = y_0 \\ z(t) = 0 \\ \dot{x}(t) = 0 \\ \dot{y}(t) = 0 \\ \dot{z}(t) = 0 \end{cases}$$

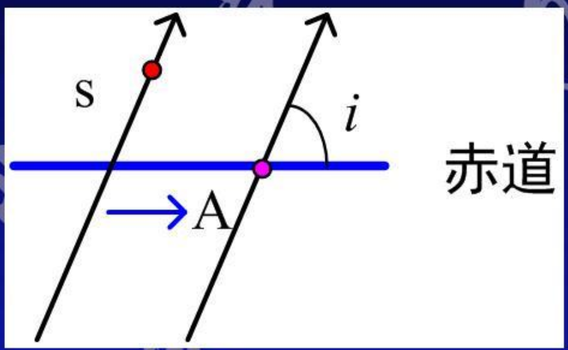


(2) 沿航向编队

两航天器星下点轨迹重合



Δt



$$\begin{cases} x_0 = 0 \\ y_0 = y_0 \\ z_0 = \dfrac{n_e}{n} y_0 \sin i \\ \dot{x}_0 = 0 \\ \dot{y}_0 = 0 \\ \dot{z}_0 = 0 \end{cases}$$

$$\begin{cases} x(t) = 0 \\ y(t) = y_0 \\ z(t) = \dfrac{n_e}{n} y_0 \sin i \cos nt \\ \dot{x}(t) = 0 \\ \dot{y}(t) = 0 \\ \dot{z}(t) = -n_e y_0 \sin i \sin nt \end{cases}$$

(3) 空间圆编队

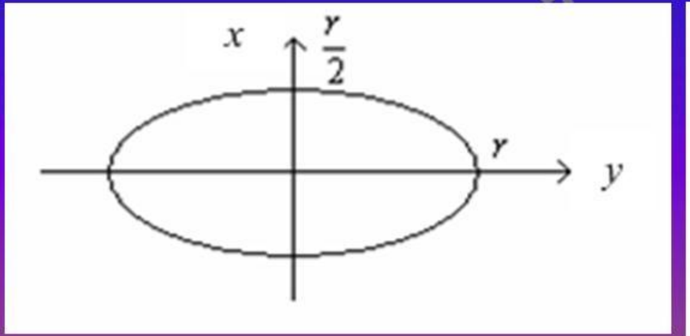
两航天器间距离保持不变

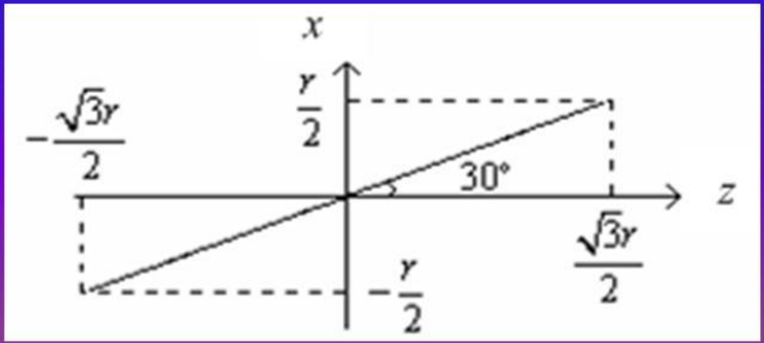
$$x^2+y^2+z^2=r^2$$

$$\dot{z}=\sqrt{3}\dot{x} \quad \text{或} \quad \dot{z}=-\sqrt{3}\dot{x}$$

$$y_0-\frac{2}{n}\dot{x}_0=0$$

$$2x_0+\frac{\dot{y}_0}{n}=0$$



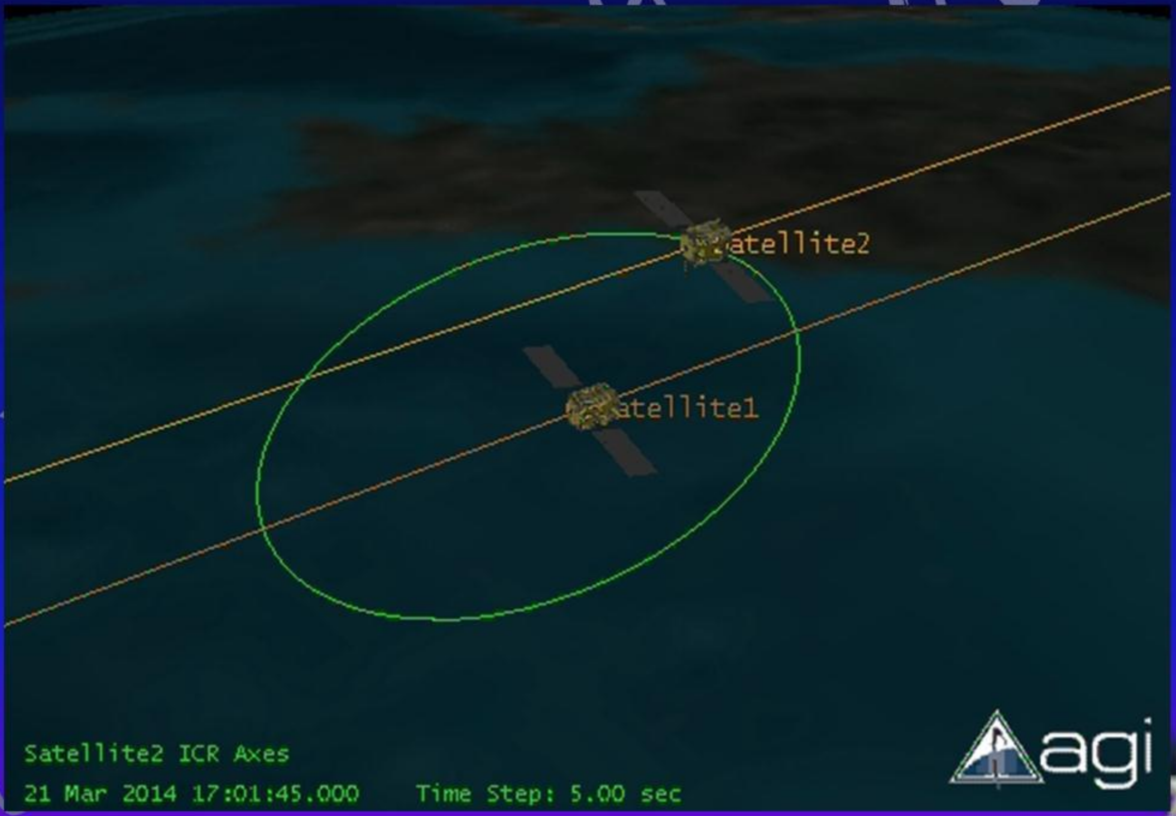


初始条件：

$$\begin{cases} x_0 = \dfrac{r}{2}\cos\theta \\ \dot{x}_0 = -\dfrac{nr}{2}\sin\theta \end{cases}$$

$$\begin{cases} y_0 = \dfrac{2}{n}\dot{x}_0 \\ \dot{y}_0 = -2nx_0 \end{cases}$$

$$\begin{cases} z_0 = \sqrt{3}x_0 \\ \dot{z}_0 = \sqrt{3}\dot{x}_0 \end{cases}$$

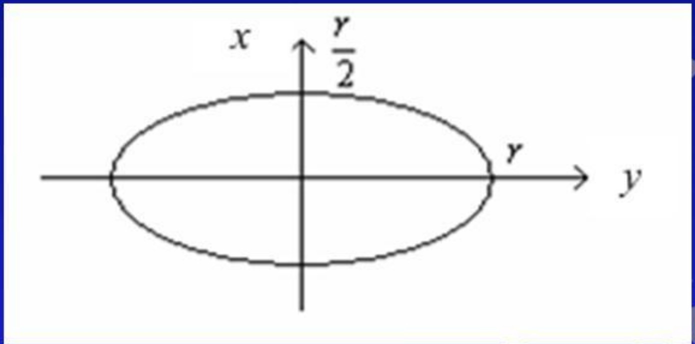


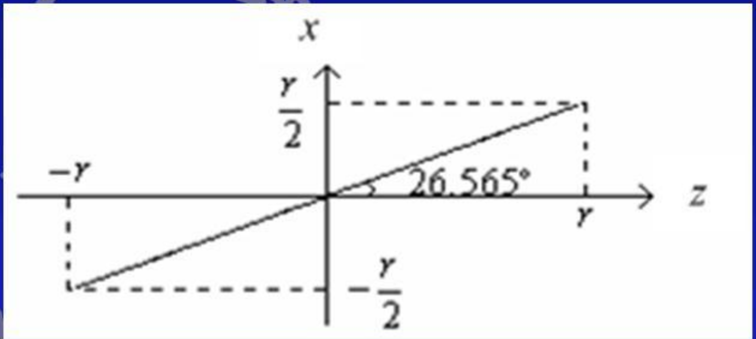
(4) 水平圆编队

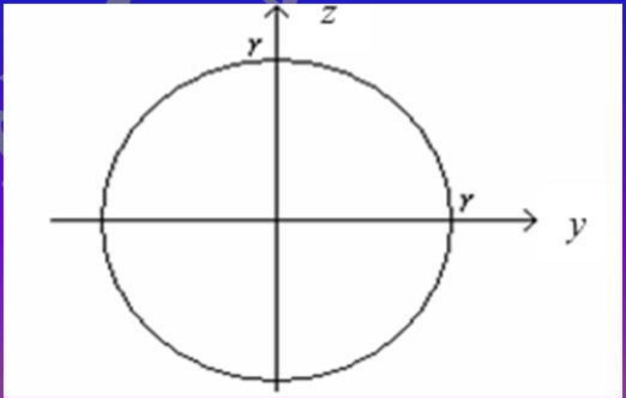
两航天器水平面内距离保持不变

$$y^2 + z^2 = r^2$$

$\dot{z} = 2\dot{x}$ 或 $\dot{z} = -2\dot{x}$







圆轨道，近距离。

$$\begin{cases} \ddot{x}-2n\dot{y}-3n^2x=0 \\ \ddot{y}+2n\dot{x}=0 \\ \ddot{z}+n^2z=0 \end{cases}$$

$$\begin{cases} \ddot{x}-2n\dot{y}-3n^2x=\Delta f_x \\ \ddot{y}+2n\dot{x}=\Delta f_y \\ \ddot{z}+n^2z=\Delta f_z \end{cases}$$

$$\ddot{\vec{r}}_{s(i)}+\frac{\mu}{r_s^{3}}\vec{r}_{s(i)}=\vec{f}_{s(i)}$$

$$\ddot{\vec{r}}_{c(i)}+\frac{\mu}{r_c^{3}}\vec{r}_{c(i)}=\vec{f}_{c(i)}$$