惯性导航基础

惯性导航基础

第五部分 —惯导系统的误差分析

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5 惯导系统的误差分析

5.0 惯性导航误差、误差分析的目的和方法

5.1 误差量的定义和误差的传播

5.2 平台姿态角误差方程

5.3 速度误差方程

5.4 位置误差方程

5.5 惯导系统的基本误差特性

组合导航原理与应用

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2.4 惯性导航系统的误差分析

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5.0 惯导系统的误差分析 — 惯导误差、误差分析的目的和方法

➤ 惯导系统的误差源主要包括：惯性仪表误差（包括陀螺的漂移和刻度因子误差、加速度计的零偏和刻度因子误差、模数转换误差等），惯性仪表安装误差，系统的初始条件误差（如平台对准误差，位置、速度初始值的装订误差），系统计算误差以及各种干扰引起的误差。

➤ 误差分析的目的是定性分析误差的变化规律和特性；定量估算惯导系统计算结果的准确程度。

➤ 误差分析首先要建立反映各个误差量之间有机联系的误差方程。

➤ 误差方程可依据系统中各力学量之间联系的方程通过微分处理来求取。一般地，所有误差源均可看成是对理想特性的小扰动，因而各个误差量都是对系统的一阶小偏差输入量。通常，在推导各误差量之间的关系时，可通过系统方程的微分处理并取一阶近似而忽略二阶以上的小量，得到系统的线性化误差方程。

惯导误差模型的用途

The INS error model has a number of uses. During INS testing, optimal estimation theory can be applied to process all of the available test data to obtain the best possible estimates of the sources of error of the INS under test. In an integrated GPS/INS, an error state space (indirect) Kalman filter uses the INS error model in the propagation step to predict the errors at the next measurement time. The INS error model is also used in the design of the GPS/INS Kalman filter to determine which error sources are important to include in the filter and which ones can be ignored.

The navigation errors committed by an inertial navigation system can be modelled by a set of first order, linear ordinary differential equations. These equations are derived using linear perturbation theory and allow the use of optimal estimation theory to estimate and remove the errors.

4.2 INS Error Sources

The major sources of error in an INS include initialization errors, system alignment errors, sensor errors, gravity model errors, quantization, and computational errors.

4.2.1 Initialization Errors

An INS integrates sensed accelerations to propagate initial estimates of position and velocity. Imperfections in the initial estimates will cause position and velocity errors.

4.2.2 System Alignment Errors

In order to properly resolve the components of sensed acceleration in the navigation frame, an INS must have knowledge of the orientation of the accelerometers with respect to the navigation frame. Typically an INS will perform an alignment process to determine the initial orientation. Errors in the initial alignment will cause position and velocity errors.

4.2.3 Sensor Errors

Despite sensor calibration, it is not uncommon for sensor characteristics to change over time and between turn-ons. These changes cannot all be

4/4/2018

6

compensated for and result in position and velocity errors. Sources of gyro and accelerometer errors include input axis misalignments, scale factor errors, non-linearities, bias instability from turn-on to turn-on and bias drift after turn-on.

4.2.3.1   Gyro Measurement Noise

Gyro measurement noise is caused by random errors in the angular velocity measurement. This is usually the predominant error source in ring laser and fiber optic gyros but is usually not included in error models for most mechanical gyros. This error is modelled as zero-mean, white Gaussian noise and causes angular random walk.

4.2.3.2   Gyro Drift (Bias)

There are two components in gyro drift. There is the gyro drift that varies from turn-on to turn-on (also referred to as bias repeatability or bias stability). This is modelled as a Gaussian random constant. The second part is bias drift or variability after turn-on (sometimes referred to as in-run stability) and is usually modelled as a random walk or first-order Markov process.

4.2.3.3   Gyro Scale Factor

Gyro scale factor error is an error in the calibrated scale factor of the gyro. This is modelled as a Gaussian random constant for each axis.

4.2.3.4 Gyro Misalignments

Gyro misalignments are errors in the alignment of the gyro input (or sensitive) axes from the orthogonal platform axes. This is modelled as a pair of Gaussian random constants for each gyro.

4.2.3.5 Gyro G-Sensitivity

Mechanical gyros may exhibit drifts that are linear and quadratic functions of acceleration. These drifts are usually not associated with ring laser or fiber optic gyros and are modelled as Gaussian random constants.

4.2.3.6 Accelerometer Measurement Noise

Accelerometer measurement noise is caused by random errors in the specific force measurement. This is modelled as white Gaussian noise and causes velocity random walk.

4.2.3.7 Accelerometer Bias

There are two components in accelerometer bias error. There is the bias repeatability that varies from turn-on to turn-on (also sometimes referred to as bias stability). This is modelled as a random constant. The second part is bias drift or variability after turn-on (sometimes referred to as in-run stability) and is usually modelled as a random walk or first-order Markov process.

4.2.3.8 Accelerometer Scale Factor

Accelerometer scale factor error is an error in the calibrated scale factor of the accelerometer. This is modelled as a Gaussian random constant for each axis.

4.2.3.9 Accelerometer Misalignments

Accelerometer misalignments are errors in the alignment of the accelerometer input (or sensitive) axes from the orthogonal platform axes. This is modelled as a pair of Gaussian random constants for each accelerometer.

### 4.2.3.10 Accelerometer Non-linearity

Accelerometers may exhibit errors that are quadratic functions of acceleration. This error source is modelled by six Gaussian random constants for each axis.

4.2.4 Gravity Model Errors

The Earth's gravity field cannot be modelled perfectly and so an INS designer must choose a gravity field model that meets the accuracy requirements of the particular application.

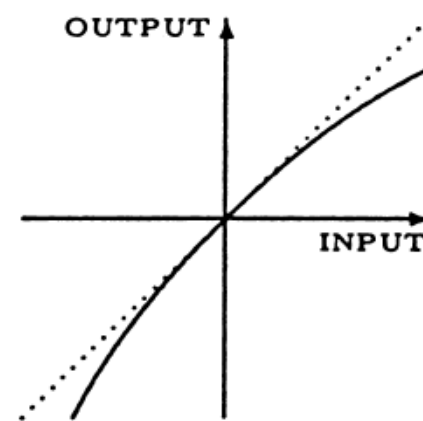### 4.2.5 Quantization and Computational Errors

The navigation equations are typically performed by a digital computer where quantization, overflow and numeric errors can occur.
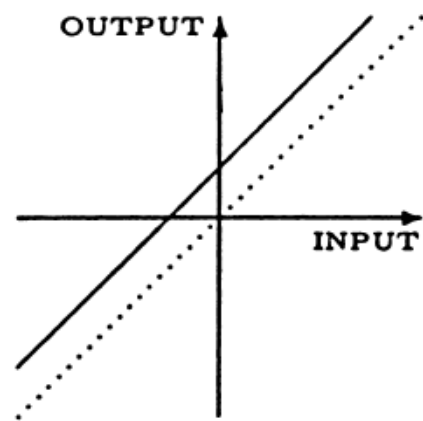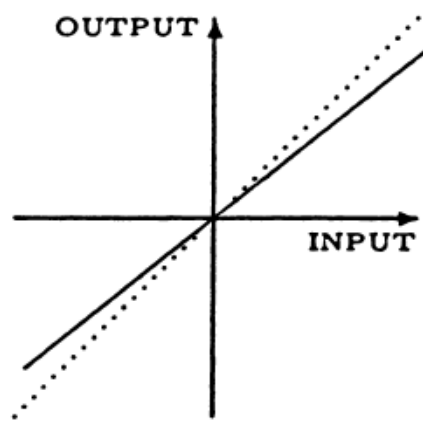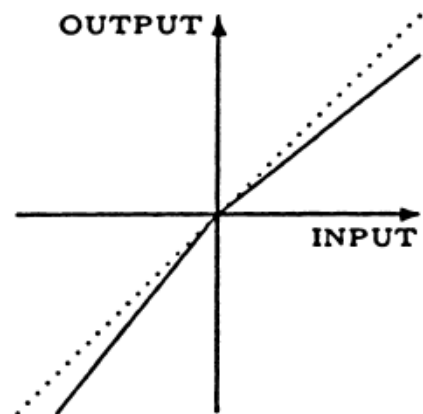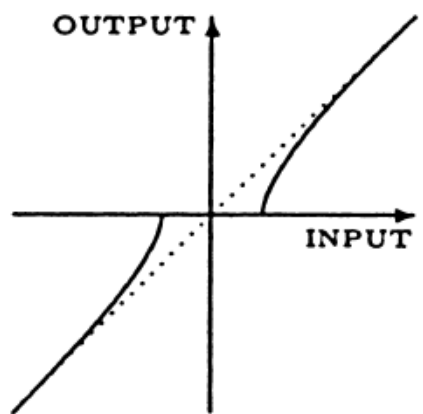
Fig. 6.1 Common input/output error types.
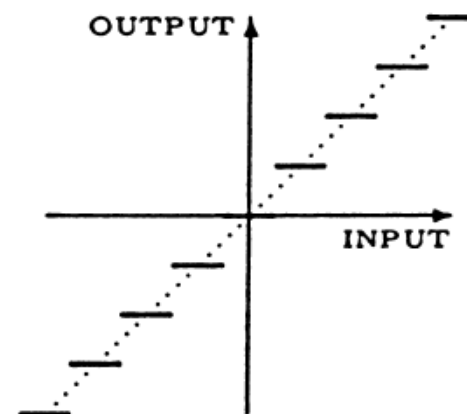
$$\mathbf{z}_{\text{output}} = S_{\text{nominal}}\{\mathbf{I} + \mathbf{M}\}\mathbf{z}_{\text{input}} + \mathbf{b}_z$$

$$\mathbf{z}_{\text{input}} = \frac{1}{S_{\text{nominal}}}\{\mathbf{I} + \mathbf{M}\}^{-1}\{\mathbf{z}_{\text{output}} - \mathbf{b}_z\}$$

$$= \frac{1}{S_{\text{nominal}}}\{\mathbf{I} - \mathbf{M} + \mathbf{M}^2 - \mathbf{M}^3 + \cdots\}\{\mathbf{z}_{\text{output}} - \mathbf{b}_z\}$$

$$\approx \frac{1}{S_{\text{nominal}}}\{\mathbf{I} - \mathbf{M}\}\{\mathbf{z}_{\text{output}} - \mathbf{b}_z\}$$

4/4/2018

The error model for a set of three orthogonal gyros can be defined

$$\delta\boldsymbol{\omega} = (\mathbf{S}_G + \mathbf{M}_G)\,\boldsymbol{\omega} + \delta\mathbf{b}_G + \delta\mathbf{nl}_G + \mathbf{n}_G \tag{4.14}$$

where the gyro scale factor error matrix is defined as:

$$\mathbf{S}_G = \begin{bmatrix} \delta SF_{G_x} & 0 & 0 \\ 0 & \delta SF_{G_y} & 0 \\ 0 & 0 & \delta SF_{G_z} \end{bmatrix} \tag{4.15}$$

and the gyro misalignment matrix is defined as:

$$\mathbf{M}_G = \begin{bmatrix} 0 & -XG_z & XG_y \\ YG_z & 0 & -YG_x \\ -ZG_y & ZG_x & 0 \end{bmatrix} \tag{4.16}$$

$\delta\mathbf{b}_G$ represents the gyro bias, $\delta\mathbf{nl}_G$ represents the gyro g-sensitive errors and $\mathbf{n}_G$ represents the gyro measurement noise.

角速度通道的测量模型为：

$$\begin{cases} N_x = E_{1x}(B_{0x} + B_{1x}f_x + B_{2x}f_y + B_{3x}f_z + \omega_x + E_{yx}\omega_y + E_{zx}\omega_z) \\ N_y = E_{1y}(B_{0y} + B_{1y}f_x + B_{2y}f_y + B_{3y}f_z + E_{xy}\omega_x + \omega_y + E_{zy}\omega_z) \\ N_z = E_{1z}(B_{0z} + B_{1z}f_x + B_{2z}f_y + B_{3z}f_z + E_{xz}\omega_x + E_{yz}\omega_y + \omega_z) \end{cases} \qquad (2\text{-}28)$$

式中，N_x、N_y、N_z 分别为陀螺 3 个通道单位时间内输出的脉冲数；

ω_x、ω_y、ω_z 分别为 3 个轴向的转动角速度；B_{0x}、B_{0y}、B_{0z} 分别为陀螺 3 通道

常值漂移项；B_{1x}、B_{1y}、B_{1z}，B_{2x}、B_{2y}、B_{2z}，B_{3x}、B_{3y}、B_{3z} 为陀螺 3 个轴

向视加速度的影响系数；E_{1x}、E_{1y}、E_{1z} 分别为陀螺 3 个通道输出的标度因数；

E_{yx}、E_{zx}，E_{xy}、E_{zy}，E_{xz}、E_{yz} 为陀螺的安装误差系数。

The error model for a set of three orthogonal accelerometers

$$\delta \mathbf{f} = (\mathbf{S}_A + \mathbf{M}_A)\,\mathbf{f} + \delta \mathbf{b}_A + \delta \mathbf{nl}_A + \mathbf{n}_A \tag{4.17}$$

where the accelerometer scale factor error matrix is defined as:

$$\mathbf{S}_A = \begin{bmatrix} \delta SF_{A_x} & 0 & 0 \\ 0 & \delta SF_{A_y} & 0 \\ 0 & 0 & \delta SF_{A_z} \end{bmatrix} \tag{4.18}$$

and the accelerometer misalignments matrix is defined as:

$$\mathbf{M}_A = \begin{bmatrix} 0 & -XA_z & XA_y \\ YA_z & 0 & -YA_x \\ -ZA_y & ZA_x & 0 \end{bmatrix} \tag{4.19}$$

$\delta \mathbf{b}_A$ represents the accelerometer bias, $\delta \mathbf{nl}_A$ represents the accelerometer non-linearity and $\mathbf{n}_A$ represents the accelerometer measurement noise.

加速度测量通道的测量模型为：

$$\begin{cases} N_{Ax} = K_{1x}(K_{0x} + f_x + K_{yx}f_y + K_{zx}f_z) \\ N_{Ay} = K_{1y}(K_{0y} + K_{xy}f_x + f_y + K_{zy}f_z) \\ N_{Az} = K_{1z}(K_{0z} + K_{xz}f_x + K_{yz}f_y + f_z) \end{cases} \qquad (2\text{-}27)$$

式中，N_{Ax}、N_{Ay}、N_{Az} 分别为 3 个加速度计单位时间内输出的脉冲数；f_x、f_y、f_z 分别为 3 个轴向的视加速度；K_{0x}、K_{0y}、K_{0z} 分别为 3 个加速度计偏值；K_{1x}、K_{1y}、K_{1z} 分别为 3 个加速度计输出的标度因数；K_{yx}、K_{zx}，K_{xy}、K_{zy} 和 K_{xz}、K_{yz} 为加速度计的安装误差系数。
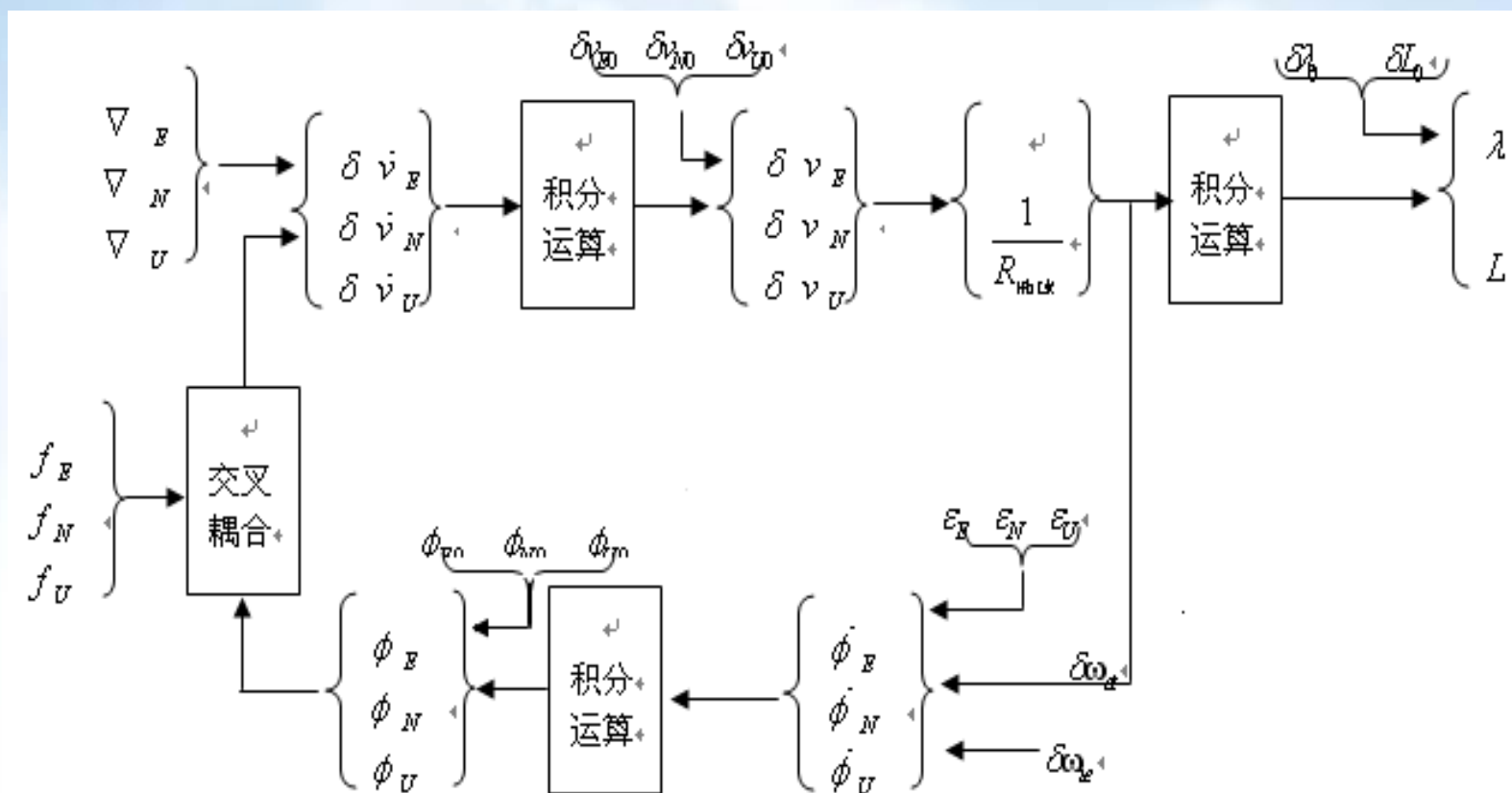
定义惯性导航的地理位置误差、速度误差以及平台系相对地理系（ENU 坐标系）误差角的分量分别为 $\delta L, \delta\lambda, \delta v_x(\delta v_E), \delta v_y(\delta v_N), \delta v_z(\delta v_U), \phi_x, \phi_y, \phi_z$，与之对应的初始给定误差为 $\delta L_0, \delta\lambda_0, \delta v_{x0}(\delta v_{E0}), \delta v_{y0}(\delta v_{N0}), \delta v_{z0}(\delta v_{U0}), \phi_{x0}, \phi_{y0}, \phi_{z0}$，各误差量的一阶导数为 $\dot{\delta L}, \dot{\delta\lambda}, \dot{\delta v_x}(\dot{\delta v_E}), \dot{\delta v_y}(\dot{\delta v_N}), \dot{\delta v_z}(\dot{\delta v_U}), \dot{\phi_x}, \dot{\phi_y}, \dot{\phi_z}$。用 $\nabla_x(\nabla_E), \nabla_y(\nabla_N), \nabla_z(\nabla_U)$ 表示东向、北向和天向加速度计的零位偏差，用 $\varepsilon_x, \varepsilon_y, \varepsilon_z$ 分别表示三个陀螺的漂移。

➢ 根据惯性导航系统的原理和导航参数与姿态角的计算过程，可得惯性导航误差传递的关系与方向示意图。

5.1 惯导系统的误差分析-惯导误差的传递

- ## 惯性导航的误差传递

系统误差的传递可以分成三段：第一段由平台误差角速率 $\dot{\phi}_x, \dot{\phi}_y, \dot{\phi}_z$ 通过一次积分并加上初始偏差，形成平台误差角 ϕ_x, ϕ_y, ϕ_z，从而引起加速度测量误差的交叉耦合误差，再加上加速度计的零偏误差，最后形成加速度误差 $\delta v_x(\delta v_E), \delta v_y(\delta v_N), \delta v_z(\delta v_U)$；第二段由加速度误差 $\delta v_x(\delta v_E), \delta v_y(\delta v_N), \delta v_z(\delta v_U)$ 通过一次积分并加上初始给定误差，形成速度误差 $\delta v_x(\delta v_E), \delta v_y(\delta v_N), \delta v_z(\delta v_U)$，而后除以地球曲率半径，再通过一次积分并加上初始给定误差，最后形成导航位置误差 $\delta L, \delta \lambda$；第三段由 $\dfrac{\delta v_x}{R}$、$\dfrac{\delta v_y}{R}$ 引起地理系相对地球坐标系的转动角速率误差（该角速度构成对陀螺仪的指令角速率误差），再加上陀螺本身的角速度漂移，以及地球自转角速度误差，最终形成平台系相对地理系的误差角速率 $\dot{\phi}_x, \dot{\phi}_y, \dot{\phi}_z$，构成了一次误差闭环传递。

- **建立误差方程的步骤：** 找到或建立有关参数的原始方程；将原始方程进行一阶微分处理，得到以误差量为基本参数的误差方程；惯导系统误差传播中三段误差传播方程的总和就是惯导系统的误差方程。

- **惯导系统误差方程：** 根据惯导系统速度方程（4-47）、位置变化率方程（4-52）和平台误差角的定义，可以建立速度误差方程、位置误差方程和平台姿态角误差方程，三个误差方程的总和就是系统的误差方程。

5.2 平台姿态角误差方程

平台姿态角误差的定义：平台姿态角误差是指陀螺稳定平台（或数学平台）模拟的导航坐标系（如地理坐标系）P与真实的导航（或地理）坐标系n之间的误差角。通常它是一个小角度。

$$C_n^P = \begin{bmatrix} 1 & \phi_z & -\phi_y \\ -\phi_z & 1 & \phi_x \\ \phi_y & -\phi_x & 1 \end{bmatrix} = I - [\overline{\phi}^n \times]$$

$$\overline{\phi}^n = \begin{bmatrix} \phi_x & \phi_y & \phi_z \end{bmatrix}^T$$

理想情况下，P 系与 n 系重合，平台坐标系 P 相对惯性空间的转动角速度 ω_{iP}^P 与地理坐标系 n 相对惯性空间的转动角速度 ω_{in}^n 之差为

$$\dot{\overline{\phi}}^n = \overline{\omega}_{iP}^P - \overline{\omega}_{in}^n = \overline{\omega}_{iP}^P - C_n^P \overline{\omega}_{in}^n = \overline{\omega}_{iP}^P - [I - (\overline{\phi}^n \times)]\overline{\omega}_{in}^n = \overline{\omega}_{iP}^P - \overline{\omega}_{in}^n + \overline{\phi}^n \times \overline{\omega}_{in}^n$$

考虑到造成 ω_{iP} 与 ω_{in} 不相等的原因是：$\overline{\varepsilon}^P \approx \overline{\varepsilon}^n$ 陀螺漂移率，计算 ω_{ie}^n 和 ω_{en}^n 有误差。有

$$\overline{\omega}_{iP}^P = \overline{\omega}_{in}^n + \delta\overline{\omega}_{ie}^n + \delta\overline{\omega}_{en}^n + \overline{\varepsilon}^P$$

可得

$$\dot{\overline{\phi}}^n = \delta\overline{\omega}_{ie}^n + \delta\overline{\omega}_{en}^n - (\overline{\omega}_{ie}^n + \overline{\omega}_{en}^n) \times \overline{\phi}^n + \overline{\varepsilon}^n$$

5.2 平台姿态角误差方程

$$\bar{\varepsilon}^n = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}, \quad \text{根据} \, \bar{\omega}_{ie}^n = \begin{bmatrix} 0 \\ \omega_{ie}\cos L \\ \omega_{ie}\sin L \end{bmatrix}, \quad \bar{\omega}_{en}^n = \begin{bmatrix} -\dfrac{v_y}{R_M + h} \\[2mm] \dfrac{v_x}{R_N + h} \\[2mm] \dfrac{v_x}{R_N + h}tgL \end{bmatrix} \, \text{有}$$

$$\delta\bar{\omega}_{ie}^n = \begin{bmatrix} 0 \\ -\omega_{ie}\sin L\,\delta L \\ \omega_{ie}\cos L\,\delta L \end{bmatrix}, \quad \delta\bar{\omega}_{en}^n = \begin{bmatrix} -\dfrac{\delta v_y}{R_M + h} \\[2mm] \dfrac{\delta v_x}{R_N + h} \\[2mm] \dfrac{\delta v_x}{R_N + h}tgL + \dfrac{v_x}{R_N + h}\sec^2 L\,\delta L \end{bmatrix}$$

5.2 平台姿态角误差方程

平台姿态角误差方程

$$\dot{\phi}_x = -\frac{\delta v_y}{R_M + h} + \left(\omega_{ie}\sin L + \frac{v_x tgL}{R_N + h}\right)\phi_y - \left(\omega_{ie}\cos L + \frac{v_x}{R_N + h}\right)\phi_z + \varepsilon_x$$

$$\dot{\phi}_y = \frac{\delta v_x}{R_N + h} - \left(\omega_{ie}\sin L + \frac{v_x tgL}{R_N + h}\right)\phi_x - \frac{v_y}{R_M + h}\phi_z - \omega_{ie}\sin L \delta L + \varepsilon_y$$

$$\dot{\phi}_z = \frac{\delta v_x}{R_N + h} tgL + \left(\omega_{ie}\cos L + \frac{v_x}{R_N + h}\right)\phi_x + \frac{v_y}{R_M + h}\phi_y + \left(\omega_{ie}\cos L + \frac{v_x \sec^2 L}{R_N + h}\right)\delta L + \varepsilon_z$$

5.3 速度误差方程

INS的速度方程及速度误差方程推导

由

$$\dot{\bar{v}}^n = \bar{f}^n - (2\bar{\omega}_{ie}^n + \bar{\omega}_{en}^n) \times \bar{v}^n + \bar{g}^n$$

得

$$\delta\dot{\bar{v}}^n = \delta\bar{f}^n - (2\delta\bar{\omega}_{ie}^n + \delta\bar{\omega}_{en}^n) \times \bar{v}^n - (2\bar{\omega}_{ie}^n + \bar{\omega}_{en}^n) \times \delta\bar{v}^n + \delta\bar{g}^n$$

式中

$$\delta\bar{f}^n = \bar{f}^p - \bar{f}^n = \bar{f}^n \times \bar{\phi}^n + \overline{\nabla}^p$$

$$\bar{f}^p = C_n^p \bar{f}^n + \overline{\nabla}^p = \left\lfloor I - \bar{\phi}^n \times \right\rfloor \bar{f}^n + \overline{\nabla}^p$$

$$\delta\bar{g}^n = \begin{bmatrix} 0 & 0 & -\delta g_z \end{bmatrix}^T, \quad \delta g_z = -2\omega_s^2 \delta h$$

5.3 速度误差方程

INS的速度误差方程

$$\delta \dot{v}_x = \left(\frac{v_y tgL}{R_N + h} - \frac{v_z}{R_N + h} \right) \delta v_x + \left(2\omega_{ie} \sin L + \frac{v_x tgL}{R_N + h} \right) \delta v_y - \left(2\omega_{ie} \cos L + \frac{v_x}{R_N + h} \right) \delta v_z$$

$$-f_z \phi_y + f_y \phi_z + \left(2\omega_{ie} v_z \sin L + 2\omega_{ie} v_y \cos L + \frac{v_x v_y \sec^2 L}{R_N + h} \right) \delta L + \nabla_x$$

$$\delta \dot{v}_y = -\left(\frac{2v_x tgL}{R_N + h} + 2\omega_{ie} \sin L \right) \delta v_x - \frac{v_z}{R_M + h} \delta v_y - \frac{v_y}{R_M + h} \delta v_z + f_z \phi_x - f_x \phi_z$$

$$-\left(2\omega_{ie} \cos L + \frac{v_x \sec^2 L}{R_N + h} \right) v_x \delta L + \nabla_y$$

$$\delta \dot{v}_z = \left(2\omega_{ie} \cos L + \frac{2v_x}{R_N + h} \right) \delta v_x + \frac{2v_y}{R_M + h} \delta v_y - f_y \phi_x + f_x \phi_y - 2\omega_{ie} v_x \sin L \delta L$$

$$+ \frac{2g_0}{R_M} \delta h + \nabla_z$$

5.4 位置误差方程

由 $\dot{L} = \dfrac{v_y}{R_M + h}$ 和 $\dot{\lambda} = \dfrac{v_x}{R_N + h} \sec L$，$\dot{\delta h} = \delta v_z$

INS的位置误差方程

$$
\left.
\begin{aligned}
\delta \dot{L} &= \frac{\delta v_y}{R_M + h} \\
\delta \dot{\lambda} &= \frac{\sec L}{R_N + h} \delta v_x + \frac{v_x}{R_N + h} \sec L \cdot tgL \delta L \\
\delta \dot{h} &= \delta v_z
\end{aligned}
\right\}
$$

5.5 惯导系统的基本误差特性－静基座误差方程

$$\delta \dot{v}_x = 2\delta v_y \omega_{ie} \sin L - \phi_y g + \nabla_x$$

$$\delta \dot{v}_y = -2\delta v_x \omega_{ie} \sin L + \phi_x g + \nabla_y$$

$$\delta \dot{L} = \frac{\delta v_y}{R_M}$$

$$\delta \dot{\lambda} = \frac{\delta v_x}{R_M} \sec L$$

$$\dot{\phi}_x = -\frac{\delta v_y}{R_M} + \phi_y \omega_{ie} \sin L - \phi_z \omega_{ie} \cos L + \varepsilon_x$$

$$\dot{\phi}_y = \frac{\delta v_x}{R_N} - \delta L \omega_{ie} \sin L - \phi_x \omega_{ie} \sin L + \varepsilon_y$$

$$\dot{\phi}_z = \frac{\delta v_x}{R_N} tgL + \delta L \omega_{ie} \cos L + \phi_x \omega_{ie} \cos L + \varepsilon_z$$

➢ **平台姿态角误差**：陀螺漂移；由平台误差角引起的交叉耦合误差；由导航参数引起的误差。

➢ **速度误差**：加速度零偏；由平台误差角引起的加速度计交叉耦合误差；在补偿有害加速度时引入的误差。

$$\dot{X}(t) = F(t)X(t) + W(t) \tag{4-87}$$

对系统误差方程（4-87）取拉氏变换得

$$X(s) = (sI - F)^{-1}[X(0) + W(s)] \tag{4-88}$$

系统的特征方程式为

$$\Delta(s) = |sI - F| = (s^2 + \omega_{ie}^2)\left[(s^2 + \omega_s^2)^2 + 4s^2\omega_{ie}^2\sin^2 L\right] = 0 \tag{4-89}$$

由 $s^2 + \omega_{ie}^2 = 0$ 得系统的一组特征根为

$$s_{1,2} = \pm j\omega_{ie} \tag{4-90}$$

由 $(s^2 + \omega_s^2)^2 + 4s^2\omega_{ie}^2\sin^2 L = 0$，计入 $\omega_s^2 \gg \omega_{ie}^2$，解得系统的另外二组特

征根

$$s_{3,4} = \pm j(\omega_s + \omega_{ie}\sin L)$$
$$s_{5,6} = \pm j(\omega_s - \omega_{ie}\sin L) \tag{4-91}$$

系统的特征根都是虚根，说明系统误差是周期振荡的，其固有振荡频率有三个，即 ω_{ie}，$\omega_s + \omega_{ie} \sin L$ 和 $\omega_s - \omega_{ie} \sin L$，$\omega_{ie} \sin L = \omega_f$ 称为付科角频率。对应于地球角速率 ω_{ie} 的固有振荡周期是 **24** 小时，对应于舒勒角频率 ω_s 和付科角频率 ω_f 的振荡周期分别是 $T_s = \dfrac{2\pi}{\omega_s} = 84.4$ 分钟，

$$T_f = \frac{2\pi}{\omega_{ie} \sin L} = 34 \text{ 小时}（\text{取} L = 45°）。$$

$$\omega_{ie} \approx 0.729 \times 10^{-4} \ rad/\sec$$

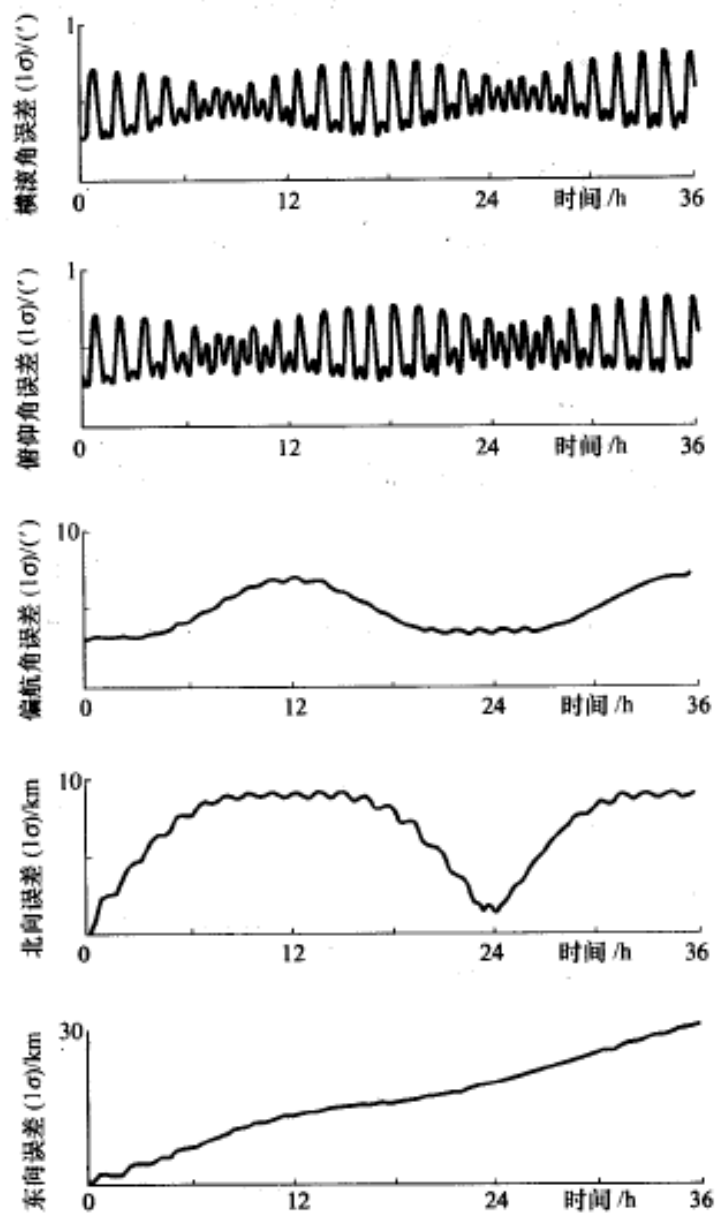$$\omega_s = \sqrt{g_0 / R} \approx 1.24 \times 10^{-3} \ rad/\sec$$

图 12.4　导航精度仿真曲线(36h)

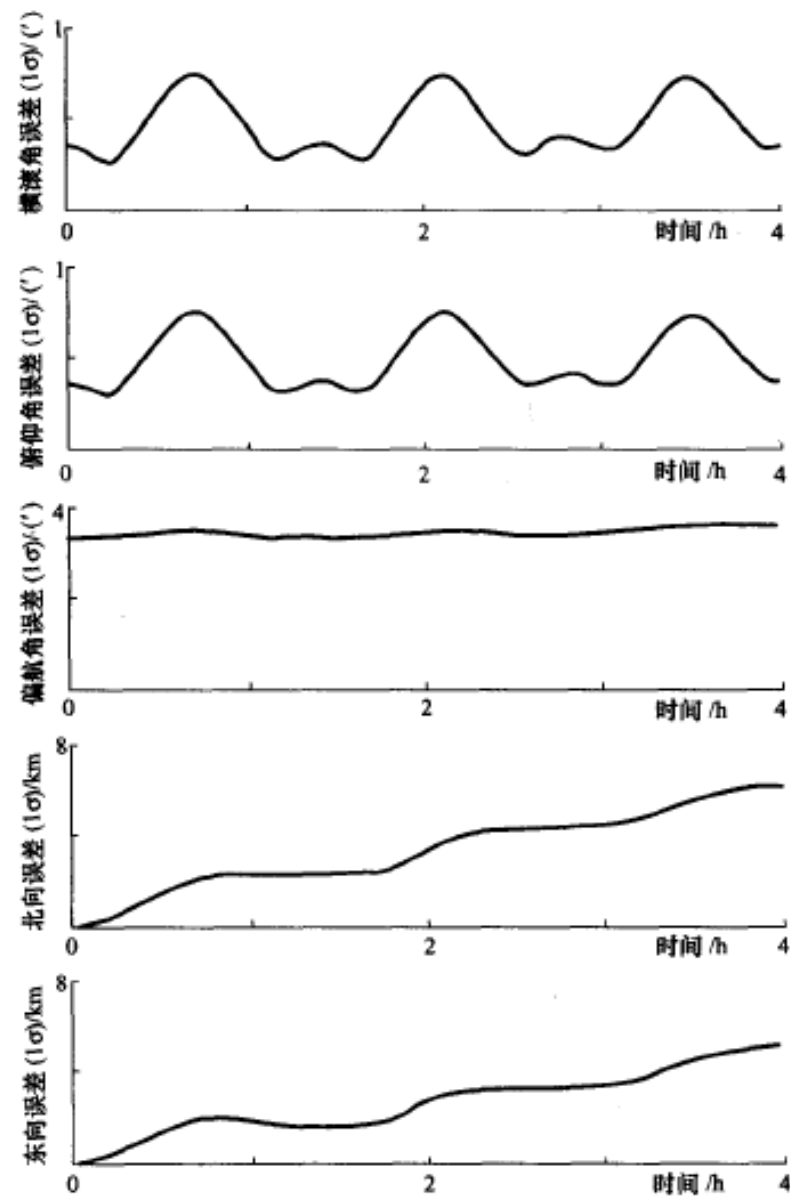图 12.5　导航精度仿真曲线(4h)

	∇_x	∇_y
$\delta V_x(t)$	$\dfrac{\sin \omega_s t}{\omega_s}\nabla_x$	
$\delta V_y(t)$	$\dfrac{g\sin L}{\omega_s^2 - \omega_{ie}^2}(\cos \omega_{ie}t - \cos \omega_s t)\varepsilon_x$	$\dfrac{\sin \omega_s t}{\omega_s}\nabla_y$
$\delta\lambda(t)$	$\dfrac{\sec L}{g}(1-\cos \omega_s t)\nabla_x$	
$\delta L(t)$		$\dfrac{1}{g}(1-\cos \omega_s t)\nabla_y$
$\phi_x(t)$		$-\dfrac{1}{g}(1-\cos \omega_s t)\nabla_y$
$\phi_y(t)$	$\dfrac{1}{g}(1-\cos \omega_s t)\nabla_x$	
$\phi_z(t)$	$\dfrac{tgL}{g}(1-\cos \omega_s t)\nabla_x$	

|	ε_x	ε_y	ε_z
$\delta V_x(t)$	$\dfrac{g\sin L}{\omega_s^2-\omega_{ie}^2}(\sin\omega_{ie}t-\dfrac{\omega_{ie}}{\omega_s}\sin\omega_s t)\varepsilon_x$	$(\dfrac{\omega_s^2-\omega_{ie}^2\cos^2 L}{\omega_s^2-\omega_{ie}^2}\cos\omega_s t-\dfrac{\omega_s^2-\sin^2 L}{\omega_s^2-\omega_{ie}^2}$ $\cos\omega_{ie}t-\cos^2 L)\varepsilon_y$	$R\sin L\cos L(\dfrac{\omega_s^2\cos\omega_{ie}t-\omega_{ie}^2\cos\omega_s t}{\omega_s^2-\omega_{ie}^2}$ $-1)\varepsilon_z$
$\delta V_y(t)$	$\dfrac{g\sin L}{\omega_s^2-\omega_{ie}^2}(\cos\omega_{ie}t-\cos\omega_s t)\varepsilon_x$	$\dfrac{g\sin L}{\omega_s^2-\omega_{ie}^2}(\sin\omega_{ie}t-\dfrac{\omega_{ie}}{\omega_s}\sin\omega_s t)\varepsilon_y$	$\dfrac{g\cos L}{\omega_s^2-\omega_{ie}^2}(\sin\omega_{ie}t-\dfrac{\omega_{ie}}{\omega_s}\sin\omega_s t)\varepsilon_z$
$\delta\lambda(t)$	$[\dfrac{tgL}{\omega_{ie}}(1-\cos\omega_{ie}t)-\dfrac{\omega_{ie}tgL}{\omega_s^2-\omega_{ie}^2}$ $(\cos\omega_{ie}t-\cos\omega_s t)]\varepsilon_x$	$[\dfrac{\sec L(\omega_s^2-\omega_{ie}^2\cos^2 L)}{\omega_s(\omega_s^2-\omega_{ie}^2)}\sin\omega_s t-$ $\dfrac{\omega_s^2 tg\sin L}{\omega_{ie}(\omega_s^2-\omega_{ie}^2)}\sin\omega_{ie}t-t\cos L])\varepsilon_y$	$[\dfrac{\omega_s^2\sin L}{\omega_{ie}(\omega_s^2-\omega_{ie}^2)}\sin\omega_{ie}t-$ $\dfrac{\omega_{ie}^2\sin L}{\omega_s(\omega_s^2-\omega_{ie}^2)}\sin\omega_s t-t\sin L]\varepsilon_z$
$\delta L(t)$	$\dfrac{\omega_s^2}{\omega_s^2-\omega_{ie}^2}(\dfrac{1}{\omega_{ie}}\sin\omega_{ie}t-\dfrac{1}{\omega_s}\sin\omega_s t)\varepsilon_x$	$[\dfrac{\omega_s^2\omega_{ie}\sin L}{\omega_s^2-\omega_{ie}^2}(\dfrac{1}{\omega_s^2}\cos\omega_s t-\dfrac{1}{\omega_{ie}}\cos\omega_{ie}t)$ $+\dfrac{\sin L}{\omega_{ie}}\varepsilon_y$	$[\dfrac{\omega_s^2\cos L}{\omega_{ie}(\omega_s^2-\omega_{ie}^2)}\cos\omega_{ie}t-\dfrac{\omega_{ie}^2\cos L}{\omega_s^2-\omega_{ie}^2}$ $\cos\omega_s t-\dfrac{\cos L}{\omega_{ie}}]\varepsilon_z$
$\phi_x(t)$	$\dfrac{1}{\omega_s^2-\omega_{ie}^2}(\omega_s\sin\omega_s t-\omega_{ie}\sin\omega_{ie}t)\varepsilon_x$	$\dfrac{\omega_{ie}\sin L}{\omega_s^2-\omega_{ie}^2}(\cos\omega_{ie}t-\cos\omega_s t)\varepsilon_y$	$(\cos\omega_s t-\cos\omega_{ie}t)\varepsilon_z$
$\phi_y(t)$	$\dfrac{\omega_{ie}\sin L}{\omega_s^2-\omega_{ie}^2}(\cos\omega_s t-\cos\omega_{ie}t)\varepsilon_x$	$[\dfrac{\omega_s^2-\omega_{ie}^2\cos^2 L}{\omega_s(\omega_s^2-\omega_{ie}^2)}\sin\omega_s t-\dfrac{\omega_{ie}\sin^2 L}{\omega_s^2-\omega_{ie}^2}$ $\sin\omega_{ie}t]\varepsilon_y$	$\dfrac{\omega_{ie}\sin L\cos L}{\omega_s^2-\omega_{ie}^2}(\sin\omega_{ie}t-\dfrac{\omega_{ie}}{\omega_s}\sin\omega_s t)$ ε_z
$\phi_z(t)$	$[\dfrac{\sec L}{\omega_{ie}}(1-\cos\omega_{ie}t)+\dfrac{\omega_{ie}\sin LtgL}{\omega_s^2-\omega_{ie}^2}$ $(\cos\omega_s t-\cos\omega_{ie}t)]\varepsilon_x$	$\dfrac{\omega_{ie}\sin L\cos L-\omega_s^2 tgL}{\omega_s^2-\omega_{ie}^2}(\dfrac{\sin\omega_{ie}t}{\omega_{ie}}-$ $\dfrac{\sin\omega_s t}{\omega_s t})\varepsilon_y$	$[\dfrac{\omega_s^2-\omega_{ie}^2\cos^2 L}{\omega_{ie}(\omega_s^2-\omega_{ie}^2)}\sin\omega_{ie}t-$ $\dfrac{\omega_{ie}\sin^2 L}{\omega_s(\omega_s^2-\omega_{ie}^2)}\sin\omega_s t]\varepsilon_z$
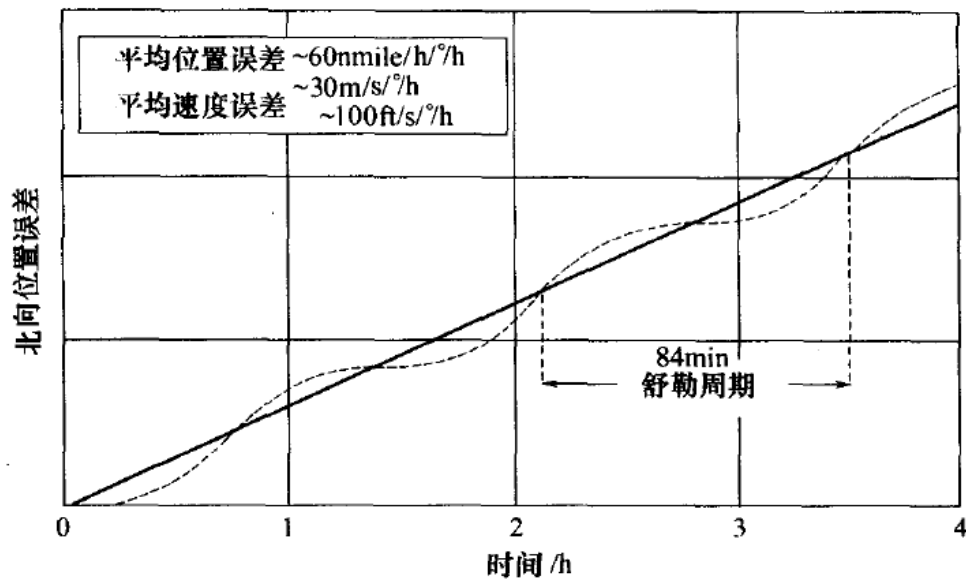
图 12.8　陀螺仪零偏引起的北向位置误差—时间曲线
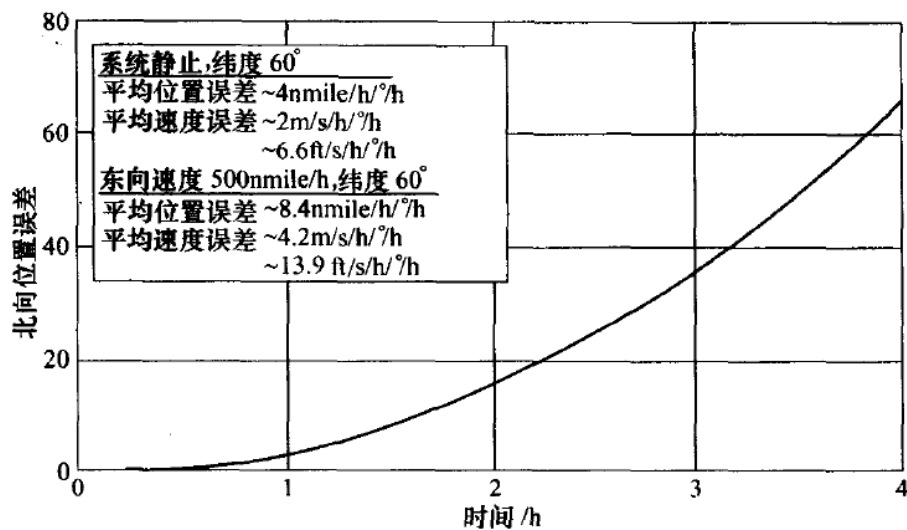
图 12.12　陀螺仪零偏垂直分量引起的北向位置误差—时间曲线

系统的误差传播特性可按公式（4-88）由各误差源与导航误差之间的传递函数关系求解。结果表明：东向加速度计零位误差 ∇_x 会引起经度误差 $\delta\lambda$ 和平台姿态误差 ϕ_y 的常值偏差，它产生的其他误差都是振幅由付科周期调制的舒勒周期振荡。北向加速度计零位误差 ∇_y 产生纬度误差 δL 和平台姿态误差 ϕ_x 的常值误差，它产生的其他误差都是振荡的。可以说，平台的姿态精度主要取决于加速度计的零位误差。北向和方位陀螺的常值漂移 ε_y 和 ε_z 都产生随时间增长的经度误差 $\delta\lambda$，它是系统中性质最严重的误差分量，ε_y 和 ε_z 产生的其他误差都是振荡形的。陀螺漂移的大小是决定系统精度的关键因素。

有些误差虽然从性质讲是振荡形的，但因振荡周期很长（如周期为 24 小时、周期为 84.4 分，或 34 小时），可能远远大于系统的一次工作时间。此时，在系统工作期间，误差仍应视为是随时间增长的。

如 $\phi_z = \dfrac{\varepsilon_z}{\omega_{ie}} \sin \omega_{ie} t$，其振荡周期是 24 小时，显然，在系统工作的几小时内，平台姿态误差 ϕ_z 是随时间增长的。

确定性误差的影响可以通过补偿措施加以减小，这时随机干扰就成为影响系统精度的主要误差源。由随机误差源造成的导航误差都是随时间振荡增长和发散的，发散速度与 $\sqrt{t}$ 成正比。

惯性导航系统的性能

➤ 安装在运载体上打的惯性导航系统在飞行过程中的导航精度，取决于导航开始时给系统提供的数据精度（即对准精度）、导航系统所用惯性敏感器的性能以及导航计算所能达到的精度。

➤ 由于惯导系统的误差随时间积累，通常用导航精度随时间的漂移来表示导航系统的性能。虽然惯导系统的误差增长并不是时间的线性函数，但常用n mile/h来表示系统漂移。

➤ 战略级惯导能通常指用于导航的敏感器所产生的导航精度小于1 n mile/h，这样的系统对陀螺和加速度计的典型要求为：陀螺漂移率为0.01°/h或精度更高，加速度计的测量精度为100ug。在战术应用场合，惯导系统用于短时间导航，或仅用于提供姿态参考。战术级惯导对对陀螺和加速度计漂移的典型要求分别为：1°/h和1mg。误差增长大约为1 n mile/min。

惯性导航的特点

1. 惯性导航具有高自主性、抗干扰性、高的短期精度、高数据输出率、完备的导航信息、适应范围广等特点。
2. 惯性导航的系统误差具有周期振荡的特性，某些导航参数误差具有随时间积累的特性。惯导系统精度主要由惯性器件精度决定，其中以陀螺仪的精度尤为突出。
3. 由于重力加速度随高度增加而减小，纯惯导系统高度通道工作不稳定，必须引入外部高度信息进行阻尼。
4. 惯性导航是一种航位推算导航，正常工作前需要初始化信息。初始化主要任务包括：给定初始速度和位置；惯导平台对准；惯性器件测漂和标定。准备时间长。

4/4/2018

惯性导航系统的主要研究方向

- 研制和采用新型、高精度惯性器件

- 惯性器件误差建模与器件误差补偿技术

- 高精度惯性导航和姿态计算算法研究

- 快速、高精度对准技术

- 与其他传感器或导航系统进行组合

- 估计理论在惯性导航中的应用

- 惯性导航技术应用

4/4/2018

思考题

- 画出SINS误差传播的框图，并说明其误差传播特点
- 推导INS的误差方程，并说明其误差特性

谢 谢 各 位 ！

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