航天器控制原理

第十一讲 航天器的姿态动力学

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第十一讲 航天器的姿态动力学

1、姿态动力学方程

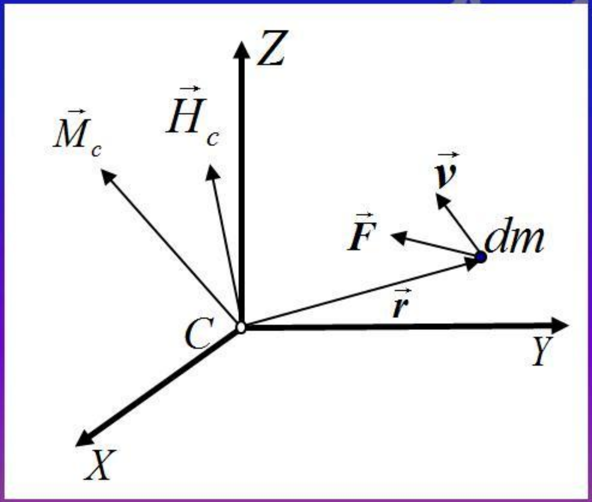
2、姿态运动方程的线性化

1、动力学方程

相对质心动量矩定理

$$\frac{d\vec{H}_c}{dt} = \vec{M}_c$$

质点系对质心的动量矩对时间的导数，等于该质点系所受全体外力对质心之矩的矢量和。



$$\vec{H}_c = \int_m \vec{r} \times \vec{v} dm$$

姿态动力学方程

$$\frac{d\vec{H}}{dt} = \vec{M}$$

$$\vec{H} = h_x\vec{i} + h_y\vec{j} + h_z\vec{k}$$

$$\frac{d\vec{H}}{dt} = \underline{\dot{h}_x\vec{i} + \dot{h}_y\vec{j} + \dot{h}_z\vec{k}} + h_x\frac{d\vec{i}}{\underline{dt}} + h_y\frac{d\vec{j}}{\underline{dt}} + h_z\frac{d\vec{k}}{\underline{dt}}$$

$$\dot{\vec{i}} = \vec{\omega}\times\vec{i}$$

$$\dot{\vec{j}} = \vec{\omega}\times\vec{j}$$

$$\dot{\vec{k}} = \vec{\omega}\times\vec{k}$$

$$\frac{d\vec{H}}{dt}=\dot{h}_x\vec{i}+\dot{h}_y\vec{j}+\dot{h}_z\vec{k}+h_x\frac{d\vec{i}}{dt}+h_y\frac{d\vec{j}}{dt}+h_z\frac{d\vec{k}}{dt}$$

$$=\dot{\vec{H}}+\vec{\omega}\times\vec{H}=\vec{M}$$

$$\begin{bmatrix}\dot{h}_x\\ \dot{h}_y\\ \dot{h}_z\end{bmatrix}+\begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}\begin{bmatrix}h_x\\ h_y\\ h_z\end{bmatrix}=\begin{bmatrix}M_x\\ M_y\\ M_z\end{bmatrix}$$

$$\begin{cases}\dot{h}_x+\omega_y h_z-\omega_z h_y=M_x\\ \dot{h}_y+\omega_z h_x-\omega_x h_z=M_y\\ \dot{h}_z+\omega_x h_y-\omega_y h_x=M_z\end{cases}$$

欧拉力矩方程

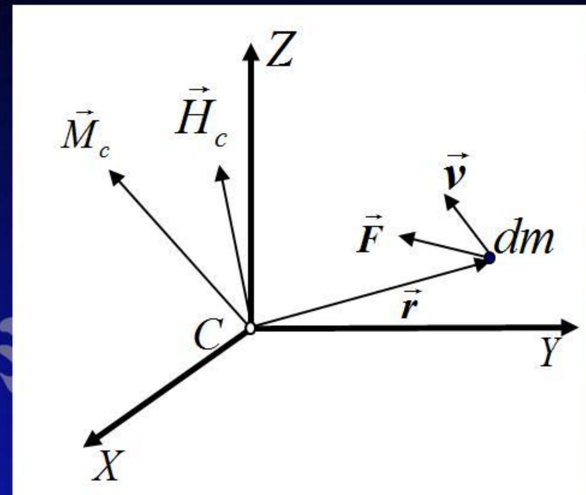
$$\vec{H} = \int_m \vec{r} \times \vec{v}\, dm$$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \cancel{\dot{\vec{r}}} + \vec{\omega} \times \vec{r}$$

$$= \vec{\omega} \times \vec{r}$$

$$\vec{H} = \int_m \vec{r} \times (\vec{\omega} \times \vec{r})\, dm$$



$$\vec{H} = \int_m \vec{r} \times (\vec{\omega} \times \vec{r}) dm$$

$$= \begin{bmatrix} \int_m [\omega_x \underline{(y^2 + z^2)} - \omega_y \underline{(xy)} - \omega_z \underline{(xz)}] dm \\ \int_m [-\omega_x \underline{(xy)} + \omega_y \underline{(x^2 + z^2)} - \omega_z \underline{(yz)}] dm \\ \int_m [-\omega_x \underline{(xz)} + \omega_y \underline{(yz)} + \omega_z \underline{(x^2 + y^2)}] dm \end{bmatrix}$$

$$I_x = \int_m (y^2 + z^2) dm \qquad I_{xy} = \int_m (xy) dm$$

$$I_y = \int_m (x^2 + z^2) dm \qquad I_{xz} = \int_m (xz) dm$$

$$I_z = \int_m (y^2 + x^2) dm \qquad I_{yz} = \int_m (yz) dm$$

$$\begin{cases} h_x = I_x \omega_x - I_{xy} \omega_y - I_{xz} \omega_z \\ h_y = -I_{xy} \omega_z + I_y \omega_y - I_{yz} \omega_z \\ h_z = -I_{xz} \omega_x - I_{yz} \omega_y + I_z \omega_z \end{cases}$$

$$\begin{cases} h_x = I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z \\ h_y = -I_{xy}\omega_x + I_y\omega_y - I_{yz}\omega_z \\ h_z = -I_{xz}\omega_x - I_{yz}\omega_y + I_z\omega_z \end{cases}$$

$$\begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \boldsymbol{I} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$I_{xy} = I_{yz} = I_{xz} = 0$ 主轴坐标系

$$\begin{cases} h_x = I_x\omega_x \\ h_y = I_y\omega_y \\ h_z = I_z\omega_z \end{cases}$$

航天器的本体坐标系为主轴坐标系：

$$\begin{cases} h_x = I_x\omega_x \\ h_y = I_y\omega_y \\ h_z = I_z\omega_z \end{cases} \Rightarrow \begin{cases} \dot{h}_x + \omega_y h_z - \omega_z h_y = M_x \\ \dot{h}_y + \omega_z h_x - \omega_x h_z = M_y \\ \dot{h}_z + \omega_x h_y - \omega_y h_x = M_z \end{cases}$$

$$\begin{cases} I_x \dfrac{d\omega_x}{dt} + \omega_y\omega_z(I_z - I_y) = M_x \\ I_y \dfrac{d\omega_y}{dt} + \omega_x\omega_z(I_x - I_z) = M_y \\ I_z \dfrac{d\omega_z}{dt} + \omega_x\omega_y(I_y - I_x) = M_z \end{cases}$$

姿态动力学方程

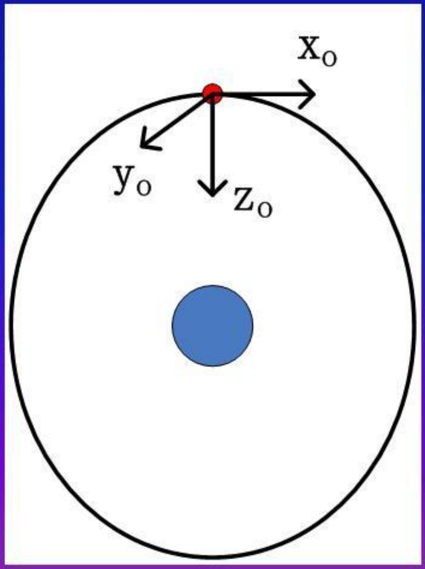
欧拉动力学方程

2、姿态运动方程的线性化

$$\begin{cases} I_x \dfrac{d\omega_x}{dt} + \omega_y \omega_z (I_z - I_y) = M_x \\ I_y \dfrac{d\omega_y}{dt} + \omega_x \omega_z (I_x - I_z) = M_y \\ I_z \dfrac{d\omega_z}{dt} + \omega_x \omega_y (I_y - I_x) = M_z \end{cases}$$

$$\vec{\omega} = \vec{\omega}_r + \vec{\omega}_e$$

$$= \begin{bmatrix} -\dot{\psi}\sin\theta\cos\varphi + \dot{\varphi}\cos\theta \\ \dot{\psi}\sin\varphi + \dot{\theta} \\ \dot{\psi}\cos\theta\cos\varphi + \dot{\varphi}\sin\theta \end{bmatrix} + A \begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix}$$



线性化运动方程

$$\vec{\omega} = \vec{\omega}_r + \vec{\omega}_e$$

$$= \begin{bmatrix} -\dot{\psi}\sin\theta\cos\varphi + \dot{\varphi}\cos\theta \\ \dot{\psi}\sin\varphi + \dot{\theta} \\ \dot{\psi}\cos\theta\cos\varphi + \dot{\varphi}\sin\theta \end{bmatrix} + \boldsymbol{A}\begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix}$$

$$\approx \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \varphi \\ \theta & -\varphi & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\varphi} - \omega_0\psi \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \omega_0\varphi \end{bmatrix}$$

$$\vec{\omega} = \begin{bmatrix} \dot{\varphi} - \omega_0 \psi \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \omega_0 \varphi \end{bmatrix} \Longrightarrow \begin{cases} I_x \dfrac{d\omega_x}{dt} + \omega_y \omega_z (I_z - I_y) = M_x \\ I_y \dfrac{d\omega_y}{dt} + \omega_x \omega_z (I_x - I_z) = M_y \\ I_z \dfrac{d\omega_z}{dt} + \omega_x \omega_y (I_y - I_x) = M_z \end{cases}$$

$$\begin{cases} I_x \ddot{\varphi} + (I_y - I_z - I_x)\underline{\omega_0 \dot{\psi}} + (I_y - I_z)\underline{\omega_0^2 \varphi} = M_x \\ \underline{I_y \ddot{\theta} = M_y} \\ I_z \ddot{\psi} - (I_y - I_z - I_x)\underline{\omega_0 \dot{\varphi}} + (I_y - I_x)\underline{\omega_0^2 \psi} = M_z \end{cases}$$

$$\begin{cases} I_x \ddot{\varphi} = M_x \\ I_y \ddot{\theta} = M_y \\ I_z \ddot{\psi} = M_z \end{cases}$$

$$M_x \longrightarrow \boxed{\frac{1}{I_x s^2}} \longrightarrow \varphi$$

$$M_y \longrightarrow \boxed{\frac{1}{I_y s^2}} \longrightarrow \theta$$

$$M_z \longrightarrow \boxed{\frac{1}{I_z s^2}} \longrightarrow \psi$$

$$\begin{cases} I_x \dfrac{d\omega_x}{dt} + \omega_y \omega_z (I_z - I_y) = M_x \\ I_y \dfrac{d\omega_y}{dt} + \omega_x \omega_z (I_x - I_z) = M_y \\ I_z \dfrac{d\omega_z}{dt} + \omega_x \omega_y (I_y - I_x) = M_z \end{cases}$$

$$\vec{\omega} = \begin{bmatrix} -\dot{\psi}\sin\theta\cos\varphi + \dot{\varphi}\cos\theta \\ \dot{\psi}\sin\varphi + \dot{\theta} \\ \dot{\psi}\cos\theta\cos\varphi + \dot{\varphi}\sin\theta \end{bmatrix} + A\begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix}$$