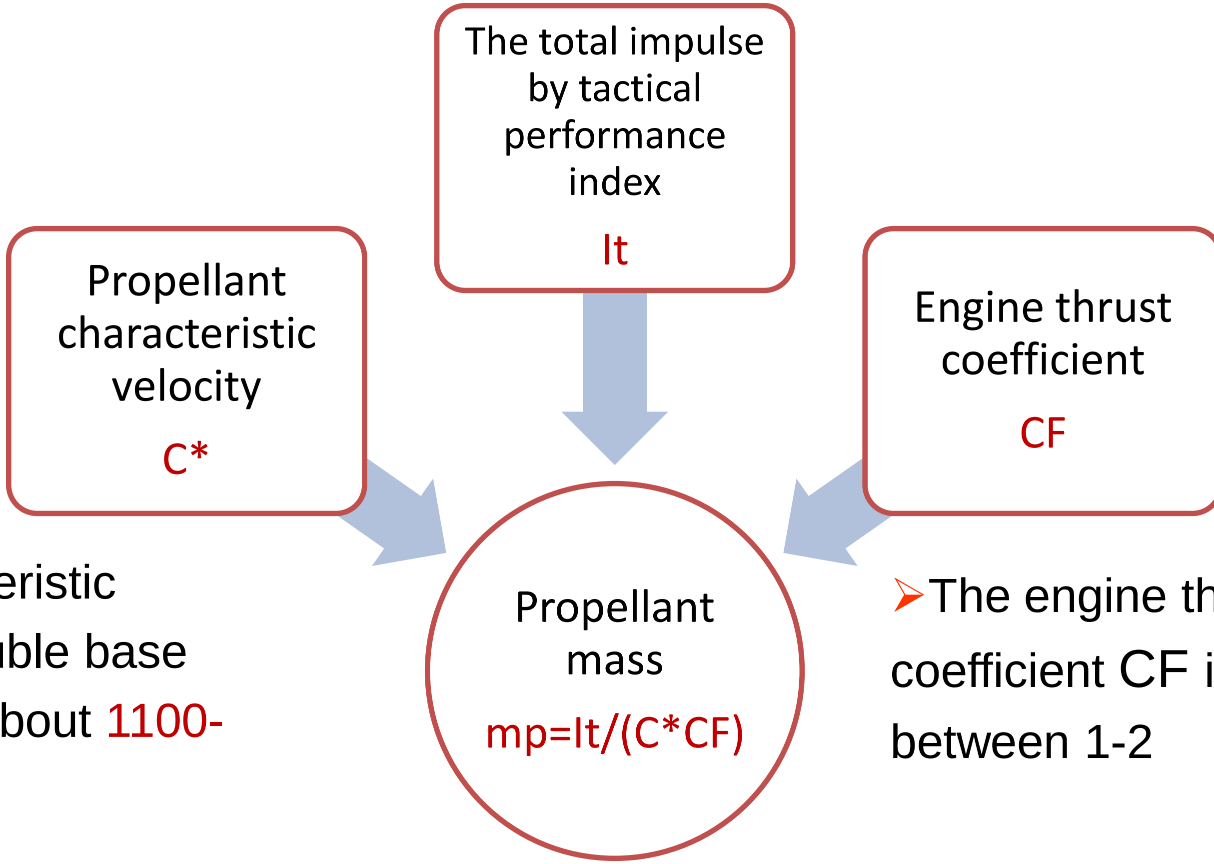
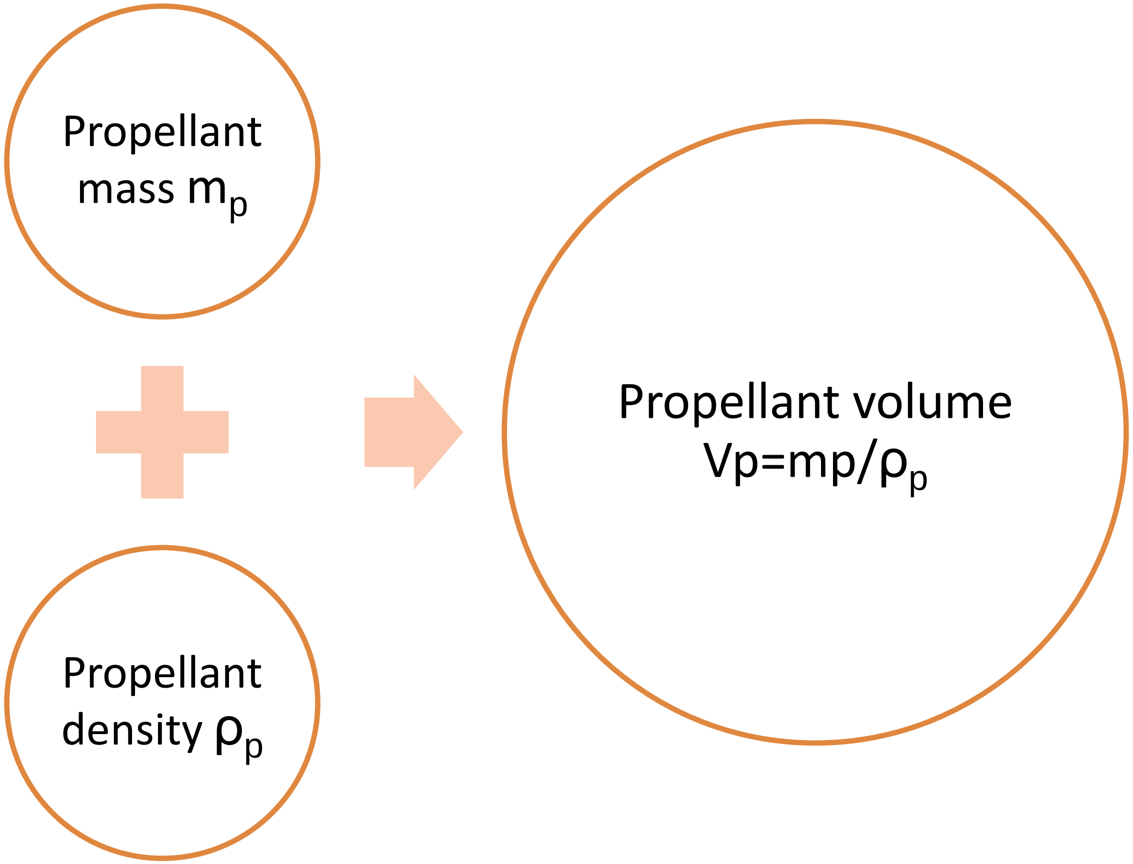
Propellant quality prediction



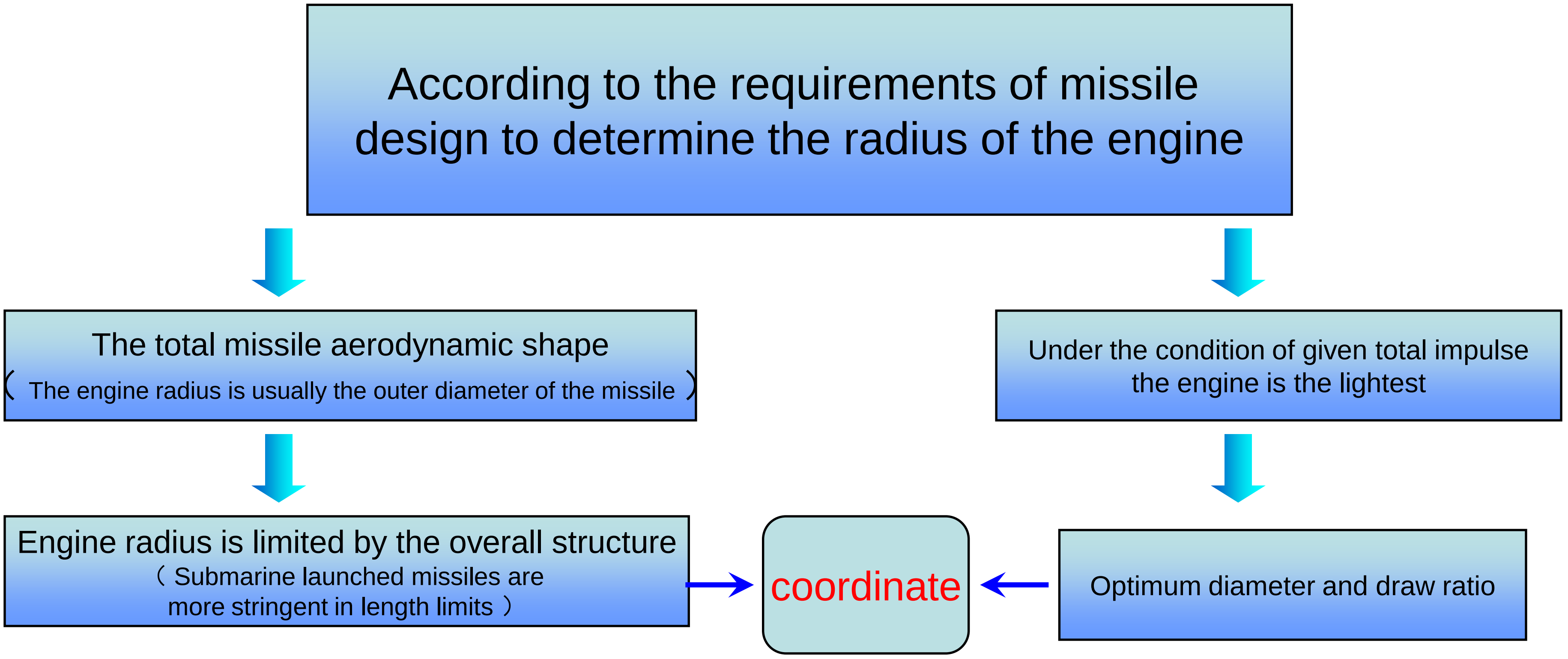
- The characteristic velocity of double base propellant is about 1100-1400m/s
- The characteristic velocity of composite propellant is about 1500-1800m/s
- The engine thrust coefficient CF is generally between 1-2
- In the overall design of the thrust coefficient can be estimated by 1.5

Propellant volume estimation



- The density of double base propellant is about： $1400\text{-}1600 kg/m^3$
- The density of composite propellant is about： $1650\text{-}1800 kg/m^3$

Engine radius determination



By the range of engine diameter and draw ratio from the overall structure , and the perspective of the optimal design of the engine, the aircraft (missile) need for coordination.

Engine radius determination
—The principle of determining the optimum radius

According to the principle of the lightest structure of the engine, the engine radius is determined according to the principle of the total engine.

Total impulse guaranteed by propellant mass m_p and specific impulse.

$$\frac{I_t}{m_m} \uparrow$$

$$m_m = m_p + m_n + m_c$$

The combustion chamber mass (MC) is related to the engine radius

Propellant mass （m_p） is determined

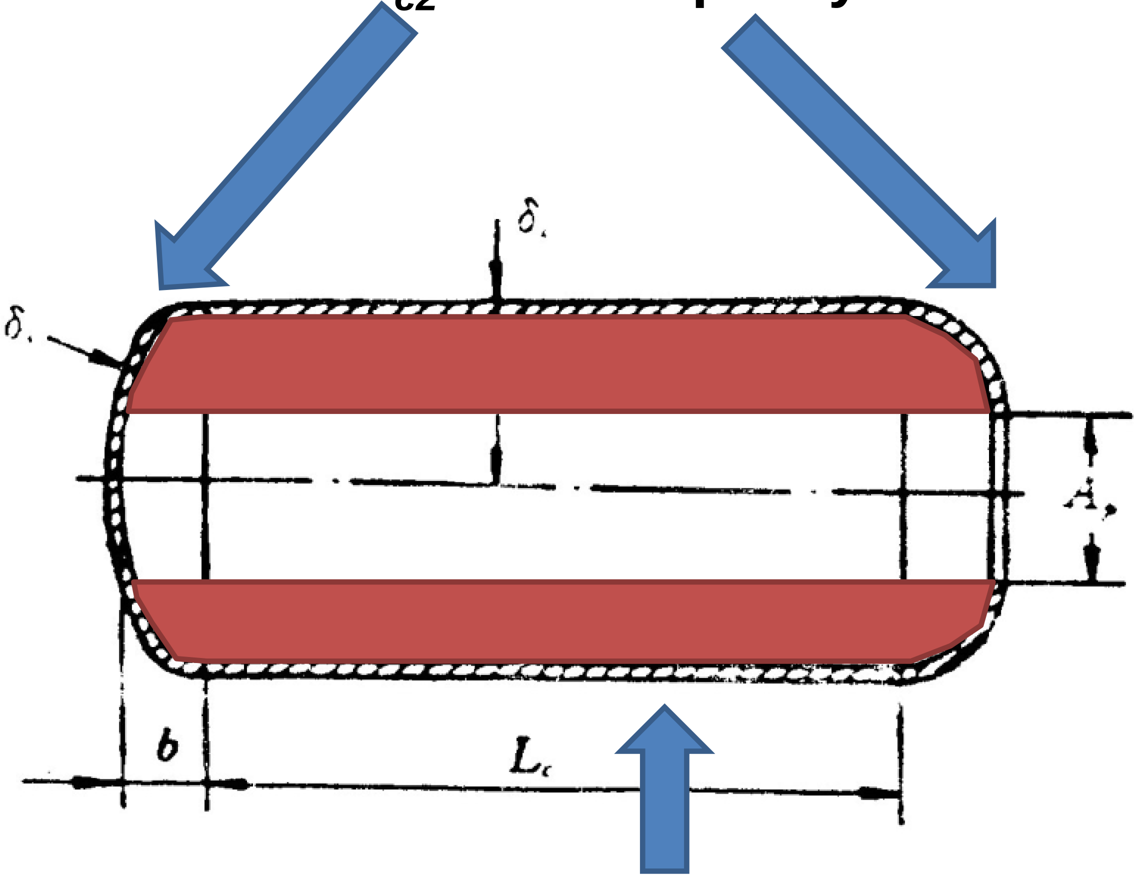
Nozzle mass（m_n）accounts for 10-30% of Structural quality

It is related to the mass flow rate and has little relation with the engine radius

Engine radius determination
—Method for determining optimum radius

■**According to the engine shell quality model to determine the optimal engine radius**

m_{c2}: **Head quality**



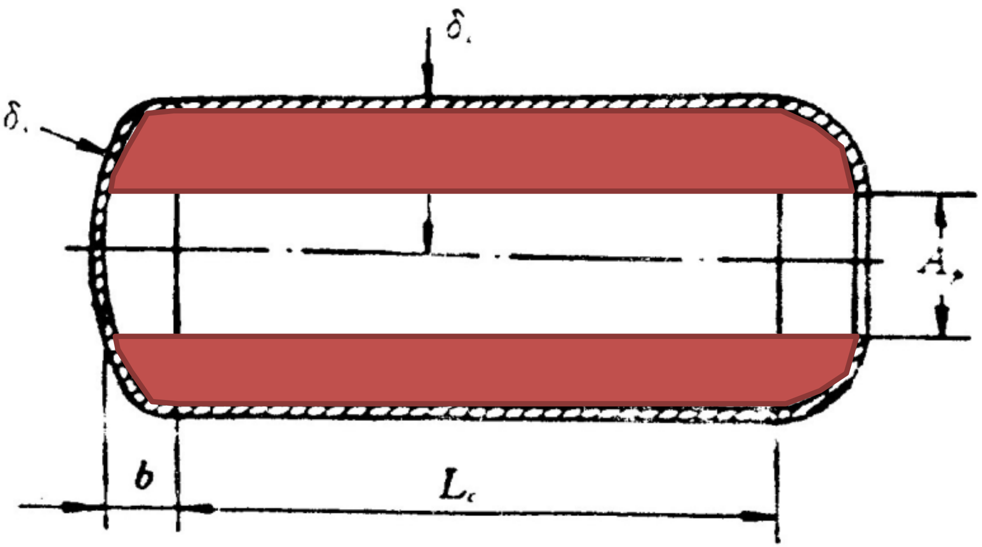
m_{c1}: **Cylinder section quality**

Engine radius determination
—Method for determining optimum radius

■Cylinder section quality （m_{c1}）

$$m_{c1}=2\pi r_c L_c \delta_c \rho_c$$

- Shell radius： r_c
- Shell wall thickness： δ_c
- Shell material density： ρ_c
- Cylinder length： L_c



In the preliminary calculation, the cylinder length is equal to the Length of column , $L_c = L_p$

Engine radius determination
—Method for determining optimum radius

■Cylinder section quality （m_{c1}） arrangement

Unknown quantity

$$m_{c1}=2\pi r_c L_c \delta_c \rho_c \Rightarrow \delta_c、L_c$$

- Cylinder length must be able to lay down the column：$L_c = L_p = \dfrac{V_p}{\pi r_c^2 - A_p}$
- The thickness of the shell must ensure the strength of the shell：$\delta_c = k_b P_c r_c / \sigma_b$ （Maximum stress strength theory）

In the formula： k_b is safety factor、

P_c is combustion chamber pressure、

σ_b is material tensile strength

$$m_{c1} = \frac{2\pi k_b P_c r_c^2 V_p}{\sigma_b (\pi r_c^2 - A_p)}$$

Engine radius determination
—Method for determining optimum radius

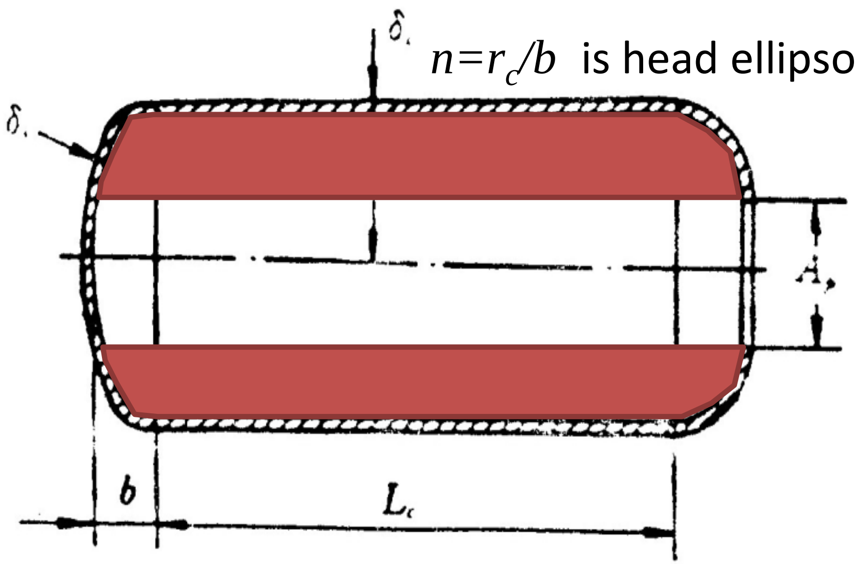
■Head quality （m_{c2}）

When the head is expanded into a plane, the head mass is:

$$m_{c2} = A_{c2}\delta_c\rho_c = 2\pi r_c^2[1+\frac{\ln(\sqrt{n^2-1}+n)}{n\sqrt{n^2-1}}]\delta_c\rho_c$$

In the formula： A_{c2} is ellipsoidal surface area

$n=r_c/b$ is head ellipsoid ratio



According to the design principle of equal thickness，ellipsoid ratio general fetch $n=2$;

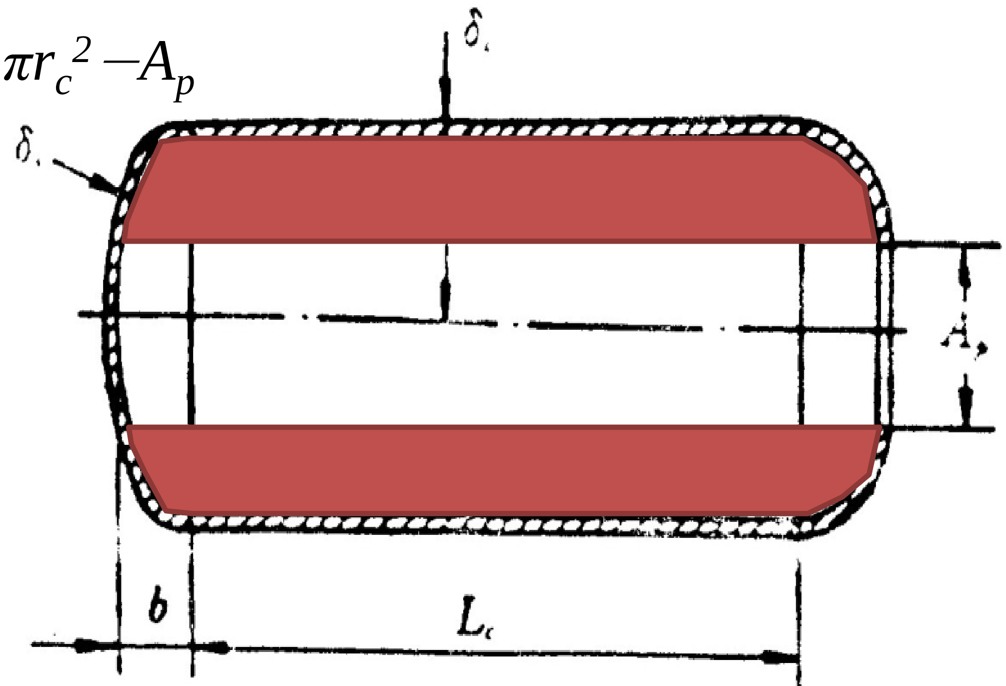
$$m_{c2}=2.76\,\pi r_c^2\delta_c\rho_c$$

Engine radius determination
—Method for determining optimum radius

■**Considering the head quality of rear open hole （m_{c2}）**

Considering the rear head open hole , the total area of front and rear head is:

$$A_{c2}=2.76\,\pi r_c^{2}-A_p$$



The total mass of front and rear head is：

$$m_{c2}=(2.76\pi\, r_c^{2}-A_p)\,\delta_c\,\rho_c$$

Engine radius determination
—Method for determining optimum radius

■Engine housing quality

$$m_c = m_{c1} + m_{c2} = \frac{k_b P_c}{\sigma_b / \rho_c}\left(\frac{2\pi r_c^2 V_p}{\pi r_c^2 - A_p} + 2.76\pi r_c^2 - A_p r_c\right)$$

- m_c is a function of r_c,
- For a certain volume of shell:：

$r_c\uparrow$—— Shell thickness $\delta_c\uparrow$—— $m_c\uparrow$

$r_c\downarrow$——Shell length $L\uparrow$—— $m_c\uparrow$

- *Therefore, there must exist an optimum radius to make the shell structure the lightest.*

Engine radius determination
—Method for determining optimum radius

■Optimum radius

$$m_c = \frac{k_b P_c}{\sigma_b / \rho_c}\left(\frac{2\pi r_c^2 V_p}{\pi r_c^2 - A_p} + 2.76\pi r_c^2 - A_p r_c\right)$$

$$\frac{dm_c}{dr_c} = 0$$

$$\Downarrow$$

$$(8.28\pi r_c^{*2} - A_p)(\pi r_c^{*2} - A_p)^2 = 4\pi r_c^{*2} A_p V_p$$

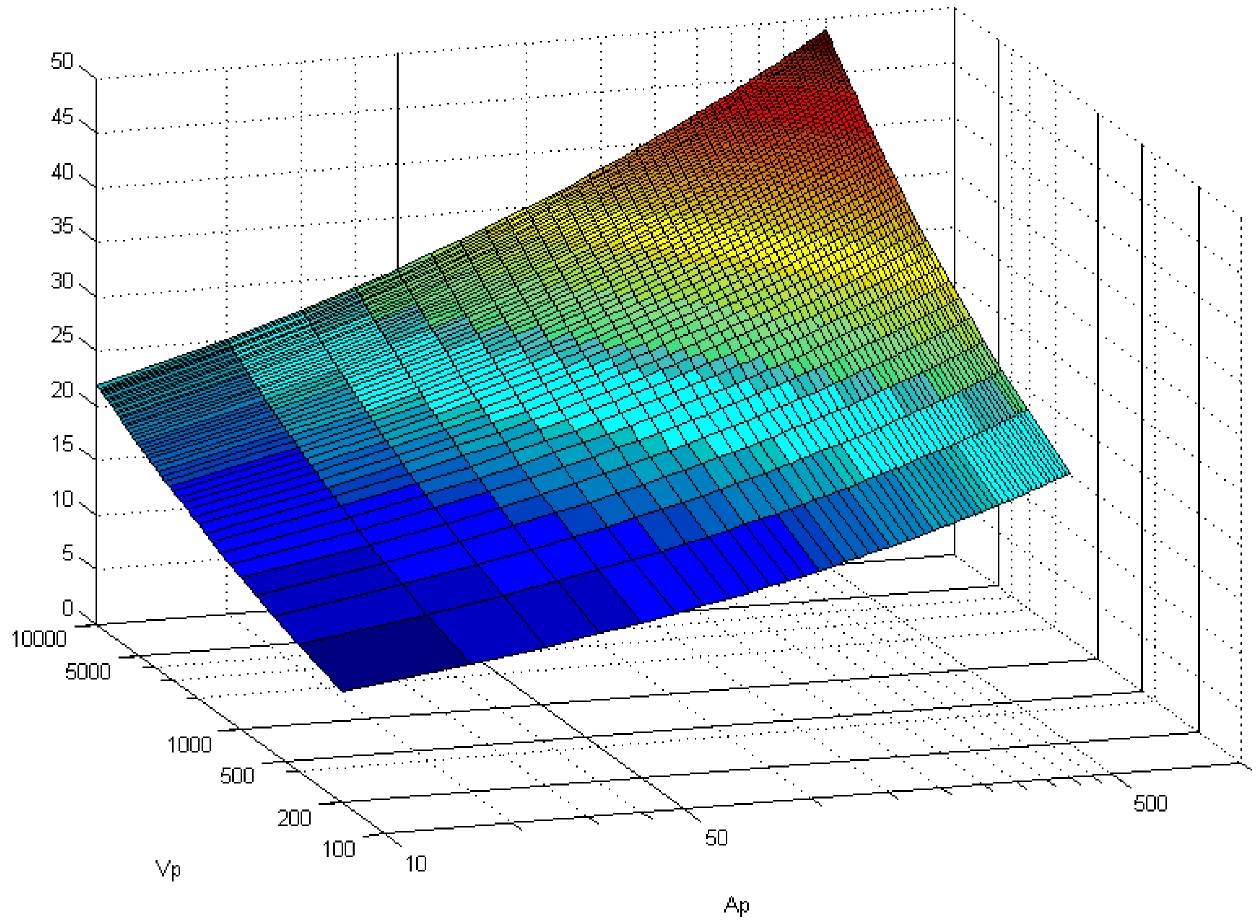
r_c^* **is the engine optimum diameter 。**

Engine radius determination
—Analysis of the optimum radius

■Analysis of the optimum radius

$$(8.28\pi r_c^{*2} - A_p)(\pi r_c^{*2} - A_p)^2 = 4\pi r_c^{*2} A_p V_p$$

The optimum radius is an implicit function of the six equation with respect to the volume of grain and the ventilation area of grain, it is very difficult to solve directly, but it is easy to solve by computer programming.



Engine optimum draw ratio

■Optimal draw ratio

According to the optimum engine radius r_c^*,
the best ratio of length to diameter λ^* can be calculated by the following formula:

$$\lambda^* = \frac{L_c^*}{r_c^*} = \frac{V_p}{(\pi r_c^{*2} - A_p) r_c^*}$$

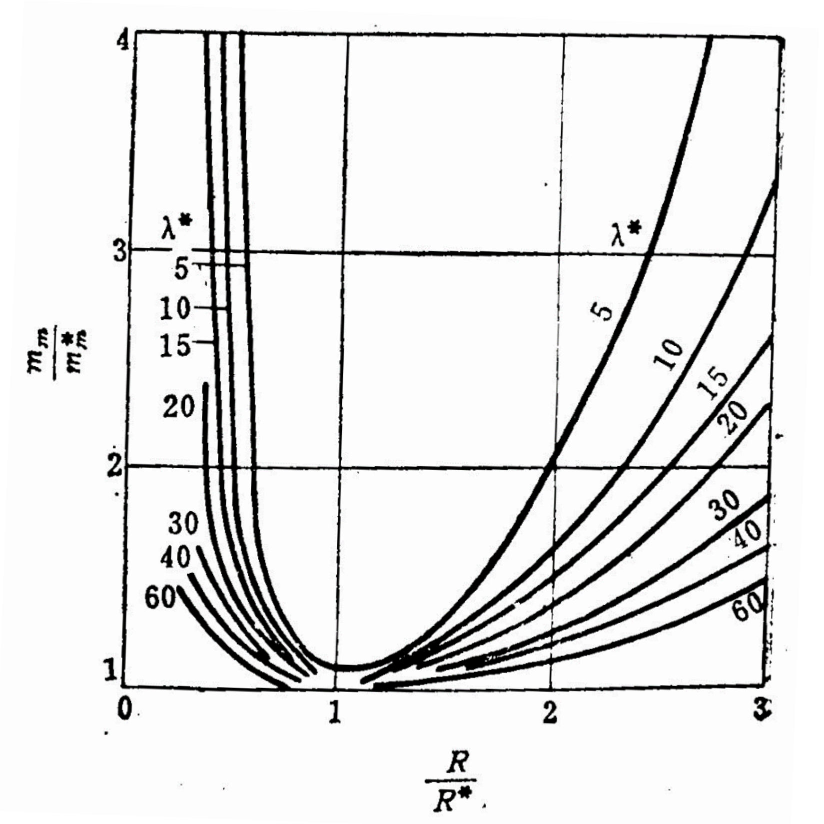
Sensitivity analysis of engine radius selection

■The influence of engine radius deviation from the optimum radius on engine structural quality

- When r_c deviates from the optimum value （$r_c \neq r_c^*$）， engine quality $m_m > m_m^*$， m_m/m_m^* is used to characterize the structural mass deviation from the optimum structural mass.

- m_m/m_m^* is a function of the best length diameter ratio λ^* and r_c/r_c^*.

$$m_m/m_m^* = \varphi(\lambda^*, r_c/r_c^*)$$

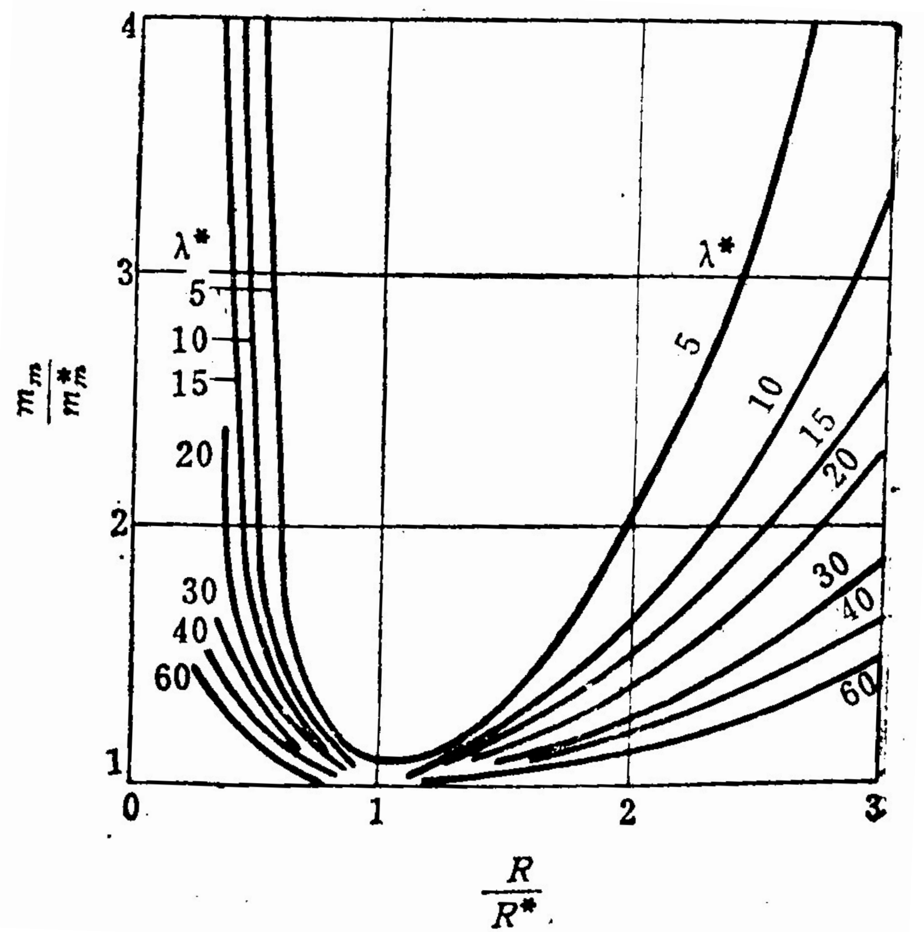


Sensitivity analysis of engine radius selection

■Selection principle of engine radius

- For smaller optimal length diameter ratio λ^* of engine ,it is best to use $r_c = r_c^*$;

- For bigger optimal length diameter ratio λ^* of engine , Select r_c can deviate from a certain distance r_c^* .



Selection of combustion chamber pressure

■（1） **To ensure that the drug can burn at low temperatures**

$$P_c = \left(C^* \rho_p a \frac{A_b}{A_t}\right)^{\frac{1}{1-n}}$$

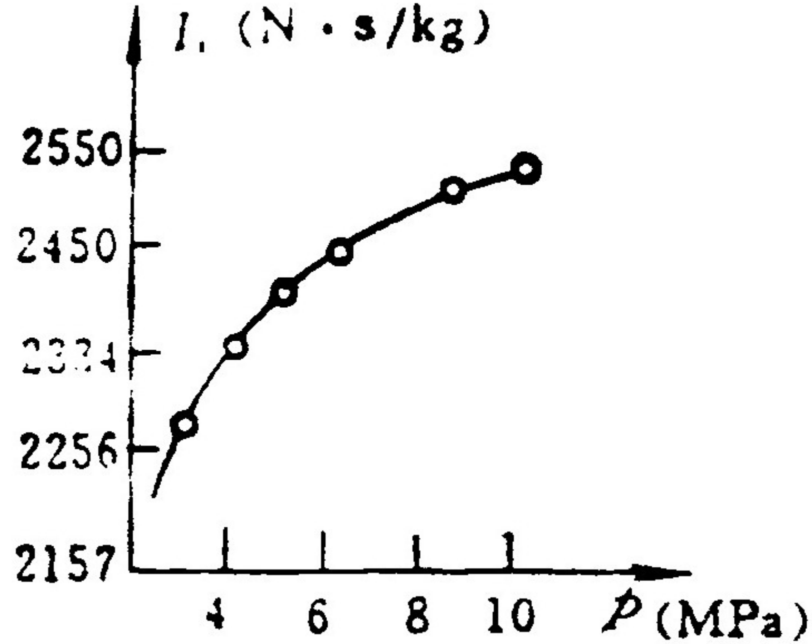
To ensure that the minimum equilibrium pressure is greater than or equal to the pressure of the selected propellant at the lowest temperature: $P_{egmin}(-TK) \geq P_{cr}(-TK)$

High Initial temperature	$\longrightarrow$	Low critical pressure	Critical pressure of double base propellant 4~6MPa
Low initial temperature	$\longrightarrow$	High critical pressure	Compound critical pressure 2~3MPa， or lower

Selection of combustion chamber pressure

■（2） **The impulse mass ratio is as large as possible**

$$\lambda = \frac{I_t}{m_m} = \frac{I_s m_p}{m_m} \uparrow$$



$$I_s = \sqrt{\frac{2k}{k-1} R T_0 [1 - (\frac{p_a}{p_c})^{\frac{k-1}{k}}]}$$

$$m_c = \frac{k_b P_c}{\sigma_b / \rho_c} (\frac{2\pi r_c^2 V_p}{\pi r_c^2 - A_p} + 2.76\pi r_c^2 - A_p r_c)$$

Selection of combustion chamber pressure

■ （2） **The impulse mass ratio is as large as possible**

$P_c\uparrow \longrightarrow$ *specific impulse* $I_s\uparrow \longrightarrow m_p\downarrow$

$\Downarrow$

There is an optimum pressure to make the impulse mass ratio λ maximum .

$\Uparrow$

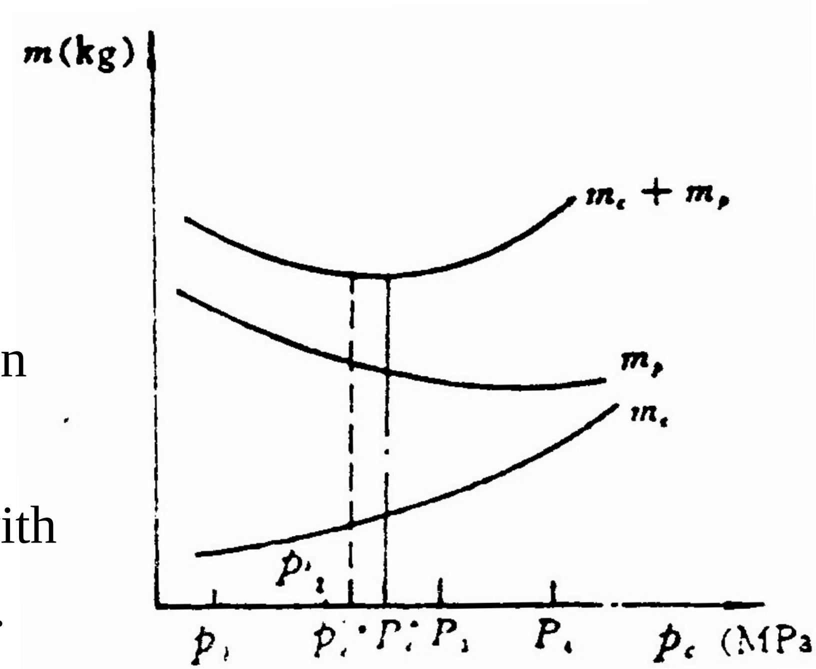
$P_c\uparrow \longrightarrow$ *Shell thickness* $\delta_c\uparrow \longrightarrow m_c\uparrow$

Selection of combustion chamber pressure

■（2） The impulse mass ratio is as large as possible

- The impulse mass ratio has maximum value when the derivation of m_p+m_c to p_c is zero.

$$\left(\frac{dm_c}{dP_c}+\frac{dm_p}{dP_c}\right)_I=0$$



  - It is difficult to solve the above equation analytically.
  - The chamber pressure can be chosen with the profile from the numerical solution.

Selection of combustion chamber pressure

■**Selection of combustion chamber pressure in composite propellant**

- Because the combustion performance of the composite propellant is better under the high pressure, the combustion chamber pressure of the composite propellant engine is generally larger than the critical pressure （4~10MPa）；
- Composite propellant booster or first stage engine combustion chamber pressure can be slightly higher （6~10MPa）；
- Due to the low external pressure, the combustion chamber pressure of composite propellant high altitude engine should be selected to be lower （4~5MPa）， Otherwise, the nozzle divergence ratio is too large, resulting in the increase of nozzle quality;

Selection of combustion chamber pressure

■Selection of combustion chamber pressure using double base propellant

- The specific impulse of double base propellant is smaller with the increase of working pressure, and the lower the working pressure is, the smaller the engine structural quality is, The working pressure at low temperature is usually higher than the critical pressure;
- In order to improve the shooting intensity, Some rockets, anti tank missiles and other tactical missile work time is very short, use the high burning rate and thin flesh grain . At the same time, in order to further increase the burning rate, the working pressure should be increased to increase the burning rate and shorten the working time.
- For small diameter tactical missiles, the increase of the chamber pressure has little effect on the shell wall thickness, therefore, the working pressure can be used relatively large （greater than 10MPa）

Prediction of nozzle throat radius

■According to the equilibrium pressure formula, the nozzle throat radius can be estimated

$$\dot{m} = \frac{P_c \cdot A_t}{C^*}$$

$$A_t = \frac{\dot{m} \cdot C^*}{P_c} = \frac{m_p \cdot C^*}{t \cdot P_c}$$

Selection of nozzle divergence ratio

■ Definition of nozzle divergence ratio

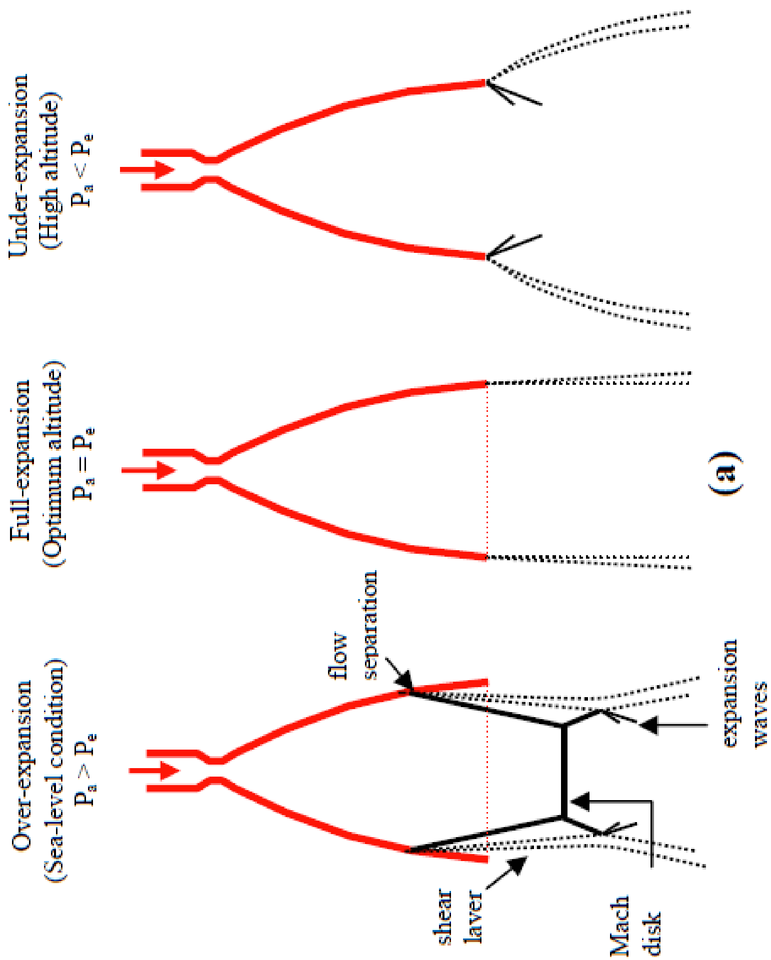
$$\varepsilon_A = Ae/At$$

There is a definite relationship between the nozzle expansion ratio ε_A and the pressure ratio P_e/P_c:

$$\varepsilon_A = \frac{A_e}{A_t} = \frac{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{\frac{k-1}{k+1}}}{\sqrt{\left(\frac{P_e}{P_c}\right)^{\frac{1}{k}} - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}}}$$

Selection of nozzle divergence ratio

■ （1） **According to the principle of maximum thrust select the nozzle divergence ratio**



Less expansion $P_e > P_a$,
thrust and specific impulse lower

Fully expanded $P_e = P_a$,
thrust and specific impulse maximum

Over expansion ， $P_e < P_a$
thrust and specific impulse lower

Selection of nozzle divergence ratio

■ **（1）According to the principle of maximum thrust select the nozzle divergence ratio**

1. Expansion ratio selection with little change in working height

Low altitude missile （Low altitude air defense missiles, anti tank missiles, ship to ship missiles, etc.）

↓

Little change in working height, External atmospheric pressure P_a unchanged

↓

The nozzle is designed to be fully expanded

$$P_e = P_a$$

Selection of nozzle divergence ratio

■ （1） **According to the principle of maximum thrust select the nozzle divergence ratio**

2. The choice of the expansion ratio when the height of the work varies greatly

Larger airspace missile （ ballistic missile , high altitude antimissile guided missile , large airspace air to air missile ） missile flight altitude varies greatly.

↓

fixed Pc and At， thrust F changed greatly

↓

to ensure Average thrust $\bar{F}$ Max

the outlet pressure of the nozzle is equal to the mean atmospheric pressure of flight airspace

$$P_e = \frac{1}{t_a}\int_0^{ta} P_a dt = \overline{P}_a$$

Selection of nozzle divergence ratio

(2) According to the principle of maximum ratio of engine to mass ratio select divergence ratio

In order to overcome the disadvantage of selecting the expansion ratio of the nozzle according to the principle of maximum thrust, Consideration should be given to the principle of maximum Engine impulse mass ratio λ_{mi} :

$\varepsilon_A \uparrow \Rightarrow I_s \uparrow \Rightarrow \lambda_{mi} \uparrow$

$\varepsilon_A \uparrow \Rightarrow m_n \uparrow \Rightarrow \lambda_{mi} \downarrow$

$\Rightarrow$ There is an optimal ε_A to make the impulse mass ratio λ_{mi} to the maximum.

Selection of nozzle divergence ratio

■ （3）Other factors affecting the expansion ratio of nozzle

- Under low temperature and low temperature conditions, due to the sensitivity of propellant to temperature, the working pressure of engine is lower, and the ambient pressure is higher. At this point, if the expansion ratio of nozzle is larger, it will lead to excessive expansion, and even cause flow separation and shock wave, resulting in loss of capacity.

- Nozzle expansion ratio needs to consider the engine structure constraints, for single nozzle outlet diameter is generally not greater than the engine diameter; For multi nozzle, the cross section of nozzle exit should be avoided.

- Preliminary design can be selected according to the same engine reference. A first stage engine and a low altitude engine, Usually take $\varepsilon_A=4\sim12$; For the overhead work last stage and apogee engine, Usually take $\varepsilon_A=15\sim100$.

The structure and selection of the column

■Principles of column form selection

- The selected shape should have enough combustion area, so as to meet the requirement of the mass flow rate of the thrust formula;

- The selected grain combustion surface changes or gas burning rate should be consistent with the quality of the generated thrust scheme;
- In the geometric structure should be burned completely, no residual drugs

- The selected shape of the drug requires sufficient strength on the structure, and the structure will not be damaged under the impact of various loads.

The structure and selection of the column

■Single thrust

- cigarette burning grain （ one-dimensional grain）；
- Inner hole burning grain （ two dimension grain）；
- Internal and external combustion grain （ two dimension grain）；
- Star shaped grain（ two dimension grain ）；
- Wing shaped grain （ Three dimension grain ）；

The structure and selection of the column

■Double thrust

- Wheel shaped grain（two dimension grain）；
- dendriform grain （two dimension grain）；
- star Tubular grain（Three dimension grain）；
- Blind hole shaped grain（Three dimension grain）；
- local Coated grain（Three dimension grain）；

THE END