Isomorphism In Game Development

517 Sania Ghulam Mohiuddin 519 Eshah Hamid

Punjab University College of Information Technology

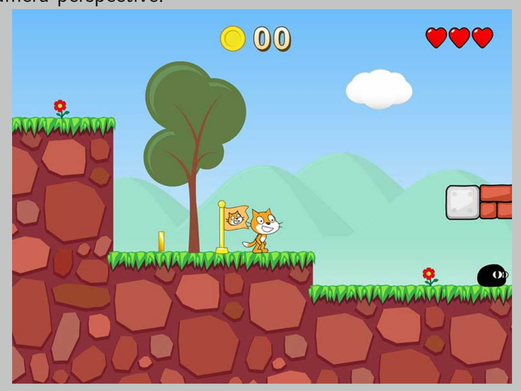
PUCIT
Punjab Univeristy
College of Information Technology

Isomorphism

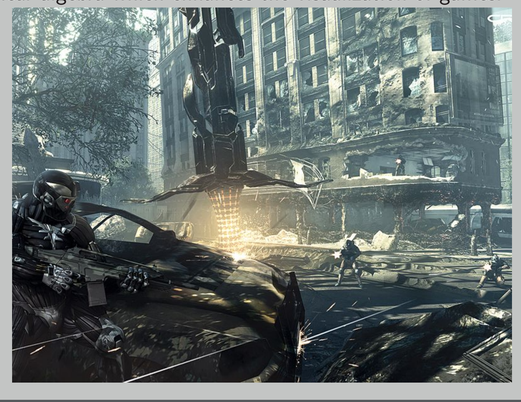
- Isomorphism is a word that comes from the greek composed of “isos” meaning same and “morphe” meaning shape.
- A one-to-one relation onto the map between two sets, which preserves the relations existing between elements in its domain.
- Isomorphic Vector Space: Two vector spaces V and W over the same field F are isomorphic if there is a bijection T : V → W which preserves addition and scalar multiplication, that is, for all vectors u and v in V , and all scalars c F, T(u + v) = T(u) + T(v) and T(cv) = cT(v).

2D and 3D Representation

- Traditionally games were on a 2D plane, following only one camera perspective.



- 3D Rendering: Process of converting 3D models into 2D images on a computer .
- Modern games are using rendering which is an application of linear algebra which enhances the visualization of games.

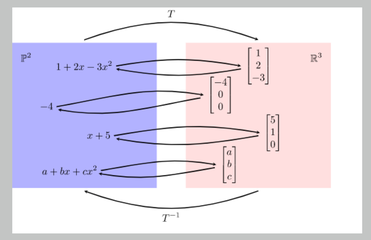


Vector Addition

One of the most common applications in games is vector addition in physics integration. Any object in a video game will likely have vector for position, velocity and acceleration.

Mapping in Isomorphism

In linear algebra, we call two vector spaces V and W isomorphic if there exist linear maps :VW and :WV .



Mathematical Implementation

The fact that T (and T1) are linear will allow us to translate questions related to linear combinations in one of the vector spaces to equivalent questions in the other vector space, then translate answers back to the original vector space. To make this statement concrete, consider the following problem:

$$Let p_1(x) = 3 - x + 2x^2 \quad \text{and} \quad p_2(x) = -1 + 3x + x^2 \tag{1}$$

$$Find p1(x) + p2(x). \tag{2}$$

The answer is, of course

$$p1(x) + p2(x) = 2 + 2x + 3x2 \tag{3}$$

$$T(p_1(x)) = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \tag{4}$$

$$T(p_2(x)) = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \tag{5}$$

Mathermatical implementation

$$T^{-1}\left(\begin{bmatrix}2\\2\\3\end{bmatrix}\right) = 2 + 2x + 3x^2 \tag{6}$$

$$p_1(x) + p_2(x) = (3 - x + 2x^2) + (-1 + 3x + x^2) \tag{7}$$

(8)

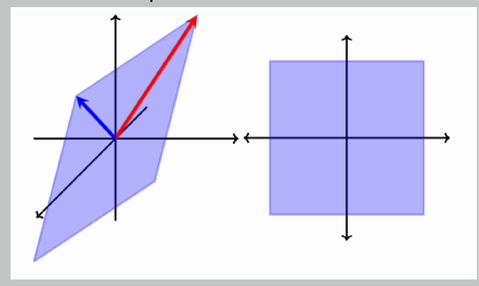
$$= T^{-1}\left(\begin{bmatrix}3\\-1\\2\end{bmatrix}\right) + T^{-1}\left(\begin{bmatrix}-1\\3\\1\end{bmatrix}\right) \tag{9}$$

$$= T^{-1}\left(\begin{bmatrix}3\\-1\\2\end{bmatrix} + \begin{bmatrix}-1\\3\\1\end{bmatrix}\right) \tag{10}$$

$$= T^{-1}\left(\begin{bmatrix}2\\2\\3\end{bmatrix}\right) \tag{11}$$

$$= 2 + 2x + 3x^2 \tag{12}$$

The span of any two linearly independent vectors in R3 is isomorphic to R2.



Linear Algebra In Interfaces

- A primary example for this linear algebra concept in gaming is HUD or Head Up Display that displays information in 2D representation in a 3D space.
- In video gaming, the HUD (heads-up display) or status bar is the method by which information is visually relayed to the player as part of a game’s user interface.

2D vs 3D

Conclusion

Through this application of linear algebra, 2D games can be displayed in a 3D space. The interaction techniques are improved to compete with the market regarding digital games keeping in view players’ demand. In a broader view, adopting this application brought visual benefits in game development.

Works cited

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