Comprehensive Modeling of Three-Phase Distribution Systems via the Bus Admittance Matrix

Sanij Gyawali

December 7, 2018

Abstract

Bus admittance matrix is far easier to form compared to bus impedance matrix and has countable benefits. So, this paper models all distribution side components and builds the admittance matrix that are in-fact suitable for Z-Bus load flow. This paper has detailed models of wye and delta connected constant-power, constant-current, and constant-impedance loads. All the series elements in distribution sides namely transmission lines, step-voltage regulators, and transformers are also modeled. The other half of the paper goes into the investigation of the invertibility of Y-Bus and feasibility of using Z-Bus load flow as a power flow solver. Right at end, paper provides online links where there are MATLAB scripts designed to model the components of some IEEE standard buses. IEEE 37 standard system is modeled using the online script and results are analyzed.

1 Introduction

Distribution systems may be modelled by an interconnected network represented by a Y- or Z- parameter network in which a common node is used for reference. Bus admittance parameter is preferred over bus impedance parameter. In a power system, Bus Admittance Matrix represents the nodal admittances of the various buses. With the help of the transmission line, each bus is connected to the various other buses. Admittance matrix is used to analyze the data that is needed in the load flow study of the buses. It explains the admittance and the topology of the network. The following are the advantages of the bus admittance matrix.

1. The data preparation of the bus admittance matrix is very simple.
2. The formation of the bus admittance matrix and their modification is easy.
3. The bus admittance matrix is a sparse matrix thus the computer memory requirement is less.

In the simplest form, ohms law in terms of admittance matrix can be written as shown below.

$$\mathbf{I} = [\mathbf{Y}]\mathbf{V}, \tag{1}$$

Where,
I is the current of the bus in the vector form.
Y is the admittance matrix
V is the vector of the bus voltage.

An accurately constructed bus admittance matrix is the basis of several applications such as,

1. Three phase load-flow study
2. Three phase optimal power flow analysis
3. Optimal system operation by selecting optimal regulator tap settings and optimal capacitor switching

The main contribution of this paper[1] is that it brings together the models for elements of distribution networks and constructs Y-Bus matrix so that studies like above can be easily made. In addition, the paper analyzes how each model affects the invertibility of the Y-Bus matrix. This is important because in order to utilize Z-Bus load flow, Y-Bus matrix should first be inverted. For certain transformer connections, Y-Bus happens to be singular. In [2], there is proposed a way to remove this singularity. This paper utilizes those methods to verify their effectiveness in removing the singularity. For a transformer connection, other than grounded Y to grounded Y, Y-Bus will be singular since there will be two isolated circuit sections. For distribution transformer, these sections the high voltage system and the transformer secondary have no connection to ground. The remedy suggested in [2] is adding a small shunt admittance from the isolated transformer sides to ground. This method is mathematically tested here and verified.

2 State of the Art

Three phase power system modeling is not a new topic. Paper [3] published in 1970s does a good work on this. But it misses to include models of voltage-dependent loads and SVRs. Our text book [4] which was first published in early 2000 does a good job to formulate ABCD matrices of transmission lines, distribution transformers, and SVRs. It misses to derive the corresponding admittance matrices from these ABCD parameters. As mentioned earlier, three-phase nodal admittance matrices of certain transformer connections, like delta-delta, is singular. Paper [3] happen to mention this but lacks in providing persuasive solution. An alternative approach to avoid Y-Bus singularity is to use equivalent current injections as in [5].

Despite the extensive literature on distribution network modeling [2]-[5], a precise unified Y-Bus modeling accounting for transmission lines with missing phases and SVRs, which are common components in IEEE feeders, is wanting. This paper attempts and succeeds in fulfilled this gap and is one of the foremost paper detailing on the theoretical side of Y-Bus invertibility. Below is the contributions and the progression pattern for this paper,

1. Detailed model on wye and delta constant-power, constant-current, and constant-impedance (ZIP) loads, three-phase transformers and transmission lines.
2. Novel nodal admittance model for SVR, ideal and non-ideal models.
3. List of the components causing singularities in the Y-Bus and solutions for the rectification. Also, provides proofs of Y-Bus invertibility showing the proposed solution works.
4. MATLAB scripts is provided online that takes as input the data files for the IEEE standard systems and models the loads, transmission lines, SVRs, and transformers. Further, the scripts develop Y-Bus for each feeder and implement Z-Bus method to compute the voltages.

3 Notations

A multiphase distribution model with Y-connected ZIP loads is being considered in this report. Representing the distribution network as a graph $(\mathcal{N}, \mathcal{E})$, with $\mathcal{N} := \{1, 2, \ldots, N\} \cup \{S\}$ being the set of nodes and $\mathcal{E}$ being the set of edges, let $\mathbf{v}_n$ be the vector of complex phase voltages at node n. The length of vector $\mathbf{v}_n$ is 1, 2 or 3 depending on the number of phases available at node n. S denotes the slack bus. Let $\mathbf{s}_{L_n}$, $\mathbf{i}_{L_n}$, $\mathbf{y}_{L_n}$ represent constant power, constant current and constant admittance component of the ZIP loads at node n respectively. Let the phase current injected at node n, $\mathbf{i}_n$ be the sum of currents from constant power load, $\mathbf{i}_{\mathrm{PQ}_n}$, constant impedance load $\mathbf{i}_{\mathrm{Z}_n}$ and constant current load $\mathbf{i}_{\mathrm{I}_n}$. Let us define vectors by stacking all these nodal quantities for nodes 1 to N by correspondingly dropping the subscript n. For instance $\mathbf{v}$ is the vector of phase voltages for all nodes obtained by stacking all vectors $\mathbf{v}_n$ for $n \in \mathcal{N}$. Let the length of $\mathbf{v}$ be J denoting the total number of phases across all nodes in the network. Let the set of indices for identifying a particular phase at any node be $\mathcal{J} := \{1, 2, \ldots, J\}$.

4 3-ϕ ZIP load models

The ZIP load modeling is exactly the same as in [4]. Since, the nodal net-current injection is taken as i, the direction is opposite to [4]. For any node with ZIP load in it, net current injection is,

$$i_n^{\phi}(v_n) = i_{P_{Q_n}}^{\phi}(v_n) + i_{I_n}^{\phi}(v_n) + i_{Z_n}^{\phi}(v_n)$$

TAble I below shows overall load models.

TABLE I

Current portion	Wye loads
$i^{\phi}_{\mathrm{PQ}_n}(\mathbf{v}_n)$	$-(s^{\phi}_{L_n}/v^{\phi}_n)^*$
$i^{\phi}_{\mathrm{I}_n}(\mathbf{v}_n)$	$-\frac{v^{\phi}_n}{\lvert v^{\phi}_n \rvert} i^{\phi}_{L_n}$
$i^{\phi}_{\mathrm{Z}_n}(\mathbf{v}_n)$	$-y^{\phi}_{L_n} v^{\phi}_n$

This ZIP model is only partly able to represent the dependencies of nodal injection currents on voltages. One example of this shortcoming is in capturing the high sensitivity of reactive powers on voltages. But in such cases, general models for static loads given in [6] can be used.

Before going into series elements modeling, for a series element between the edge n and m, a general equation can be written as,

$$\mathbf{i_{nm}} = \mathbf{Y^{n}_{nm}v_{n}} - \mathbf{Y^{m}_{nm}v_{m}} \tag{2}$$

5 Transmission line modeling

The transmission line between mode n and m can be depicted by following equation,

$$\mathbf{i_{nm}} = [\mathbf{\frac{1}{2}Y^{s}_{nm}} + \mathbf{Z^{-1}_{nm}}]\mathbf{v_n} - \mathbf{Z^{-1}_{nm}v_m} \tag{3}$$

Here, the order of matrix i and v depends upon the number of phases in the transmission line. For example, for 3-ϕ system, i and v will be the order 3 and Y_{nm} and Z_{nm} will be the order 3x3. Comparing equations 2 and 3, we can see,

$$Y^{n}_{nm} = [\frac{1}{2}Y^{s}_{nm} + Z^{-1}_{nm}] \quad and \quad Y^{m}_{nm} = Z^{-1}_{nm}$$

6 Step-Voltage Regulators

A SVR is basically an auto-transformer with adjustable turn ratios. These turn-ratios are dependent on the position of the taps. The parameter matrices like current gain and voltage gain are exactly as in [4]. The only difference is that here, a non-ideal SVR is also taken into consideration. A non-ideal SVR will have impedance matrix Z_R. So, the primary to secondary relation in SVR is,

$$\mathbf{v_n} = \mathbf{A_v v^{'}_n} + \mathbf{Z_R i_{nn'}} \tag{4}$$

$$\mathbf{i_{nn'}} = \mathbf{A_i i_{n'm}} \tag{5}$$

Where matrices A_v, A_i, and Z_R are voltage gain, current gain, and impedance matrix respectively. $n^{'}$ represent the secondary of the SVR. Since SVR is modeled between n and $n^{'}$ and transmission line is modeled between $n^{'}$ and m, an equivalent model between node n and $n^{'}$ can be generated in the form of equation (2), where,

$$\mathbf{Y^n_{nm}} = \mathbf{A_i F_R^{-1} Y^{n'}_{n'm} A_i^T} \tag{6}$$

$$\mathbf{Y^m_{nm}} = \mathbf{A_i F_R^{-1} Y^m_{n'm}} \tag{7}$$

Where, $F_R^{-T} = I - A_i^T Z_R F_R^{-1} Y^{n'}_{n'm}$ and for invertibility there is $A_v^{-1} = A_i^T$

7 Three-phase transformer

ABCD paramters are exactly the one derived in [4]. Additional work done is formation of Y-bus matrix. The corresponding nodal admittances of different transformer connection is listed in [5].

8 Y-bus construction and Z-bus Method

The Ohm's law for multiphase networks may be written as

$$\begin{bmatrix} \mathbf{i} \\ \mathbf{i}_s \end{bmatrix} = \begin{bmatrix} \mathbf{Y_{net}} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{v}_s \end{bmatrix} OR, \begin{bmatrix} \mathbf{i} \\ \mathbf{i}_s \end{bmatrix} = \begin{bmatrix} \mathbf{Y} & \mathbf{Y}_{NS} \\ \mathbf{Y}_{SN} & \mathbf{Y}_{SS} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{v}_s \end{bmatrix} \tag{8}$$

where $\mathbf{i}_s$ and $\mathbf{v}_s$ are the substation current injections and phase voltages; and submatrices $\mathbf{Y}, \mathbf{Y}_{NS}, \mathbf{Y}_{SN}$ and $\mathbf{Y}_{SS}$ have been partitioned suitably from the network overall admittance matrix. The matrix $\mathbf{Y}$ is called the bus admittance matrix of the network. Using the ZIP load decomposition and the first row of (8),

$$\mathbf{i}_{\mathrm{PQ}}(\mathbf{v}) + \mathbf{i}_{\mathrm{Z}}(\mathbf{v}) + \mathbf{i}_{\mathrm{I}}(\mathbf{v}) = \mathbf{Yv} + \mathbf{Y}_{NS}\mathbf{v}_S. \tag{9}$$

For constant impedance load, $\mathbf{i}_{\mathrm{Z}}(\mathbf{v}) = -\mathbf{Y}_L\mathbf{v}$, where $\mathbf{Y}_L = \mathrm{diag}(\mathbf{y}_L)$. The voltage dependence relation for $\mathbf{i}_{\mathrm{PQ}}(\mathbf{v})$ and $\mathbf{i}_{\mathrm{I}}(\mathbf{v})$ are from the ZIP model definition (not being provided here). Substituting these in (9), it may be shown that

$$\mathbf{v} = \mathbf{Z}[\mathbf{i}_{PQ}(\mathbf{v}) + \mathbf{i}_I(\mathbf{v})] + \mathbf{w}, \tag{10}$$

where $\mathbf{Z} := (\mathbf{Y} + \mathbf{Y}_L)^{-1}$ and $\mathbf{w} := -\mathbf{Z}\mathbf{Y}_{NS}\mathbf{v}$. Eq. (10) form the basis of Z-bus iterates. The voltage at $(t+1)$-th iteration may be found from the voltages at t-th iteration using

$$\mathbf{v}[t+1] = \mathbf{Z}[\mathbf{i}_{PQ}(\mathbf{v}[t]) + \mathbf{i}_I(\mathbf{v}[t])] + \mathbf{w}. \tag{11}$$

Note that for no constant current and constant power loads, the system becomes linear and the voltages can be found without iterations as $\mathbf{v} = \mathbf{w}$. No-load voltages can also be found in a similar fashion. Vector $\mathbf{w}$ thus contains the information of the effect of source voltage on all the node voltages of the network.

9 Y-Bus invertibility

The is unique thing that this paper does. Not much study has been done into the invertibility of Y-Bus matrix. Invertibility is must in order to utilize Z-Bus power flow tactics. For transmission line, Z_{nm}^{-1} exists. For SVR, it has been discussed that current and voltage gains are invertible. These facts support the invertibility of Y-Bus matrix. For transformer though, we know for sure that Y_G-Y_G supports Y-Bus invertibility but for other configurations, strategy from [2] is used.
In order to mathematically remedy the non-invertibility of Y for transformers other than wye-g-wye-g,strategy is to add a small shunt admittance from the isolated transformer sides to the ground. Below is an example remedy,

$$Y_2^{'} = Y_2 + \epsilon^{'} I$$
$$Y_4^{'} = Y_4 + \epsilon^{'} I$$

where, $\epsilon^{'}$ greater than 0 is a small shunt admittance (compared to $mag(y_t)$)

10 Numerical results

In this section, the paper gives links to free MATLAB scripts for modeling some standard IEEE systems. For this report, only IEEE-37 bus feeder will be taken into account and studied extensively. This feeder is an actual feeder in California, with a 4.8 kV operating voltage. It is characterized by delta configured, all line segments are underground, substation voltage regulation is two single-phase open-delta regulators, spot loads, and very unbalanced. This circuit configuration is fairly uncommon. Script for this system can be found at, https://github.com/hafezbazrafshan/three-phase-modeling

main.m

This script is the starting point. It contains all the necessary steps from modeling, to computation of voltages as well as comparisons between solutions. It calls other functions from here.

setupYbusIEEE37.m

This function uses the data files of the folder IEEE-37 feeder data provided from PES to create the bus admittance matrix. Other inputs for this script is regulator types which can be ideal or non-ideal and epsilon which is the value that is needed to adjust the delta connected transformer for invertibility. The main output from this is Ynet which is the bus admittance matrix including the slack bus that is further partitioned into Y (Bus admittance matrix excluding the slack bus), Y_NS (The portion of bus admittance matrix relating the network to the slack bus) and Y_SS (The portion of bus admittance matrix relating only to the slack bus).

computeNoLoadVoltage.m

This function computes some required no-load quantities for the network. The input for this function is the output from setupYbusIEEE37.m. Main outputs are slack bus complex voltage phasor and no load complex voltage profile of the network.

setupLoadsIEEE37.m

This function updates the network structure created previously with information on loads. The modeling of loads is based on the load type (constant-power, constant-current, constant-impedance, or any combination of these) as well as the connection (delta, wye, or a combination of these). The general load model is as follows:

```
iL_PQ= c(i,:)*[ fPQ( eVec1.' * v, sL(i,1)); fPQ(eVec2.'*v, sL(i,2))];
iL_I=c(i,:) *[ fI(eVec1.'*v, iL(i,1)); fI(eVec2.'*v, iL(i,2))];
iL_Y=c(i,:)*[fY(eVec1.'*v, yL(i,1)); fY(eVec2.'*v, yL(i,2))];
fv(i,1)=g(i,:)*[iL_PQ;iL_I;iL_Y];
```

The vector $c(i,:)$ has two binary entries. The first binary entry corresponds to wye connection. The second binary entry corresponds to delta. The vector $g(i,:)$ determines whether the load is constant-power, constant-current, or constant-impedance.
Input to this function would be output from previous setupLoadsIEEE37.m function. Outputs would be vectors of nominal power, nominal currents and nominal admittance of constant-power, constant-current and constant-impedance loads. CMat and gMat are also the outputs which are basically matrices to determine delta or wye connected loads and ZIP load combination respectively.

performZBus.m

This function computes the Z-Bus load flow. It taken input from above functions and outputs vector of voltages.

obtainVoltage.m

This function takes output from performZBus.m and maps the computed voltage vector to appropriate data structure in terms of buses and their phases.

These are the main functions but there are other functions that does some small tasks like loading other necessary data and some computational functions like converting degrees to radian or vice-versa.

Results

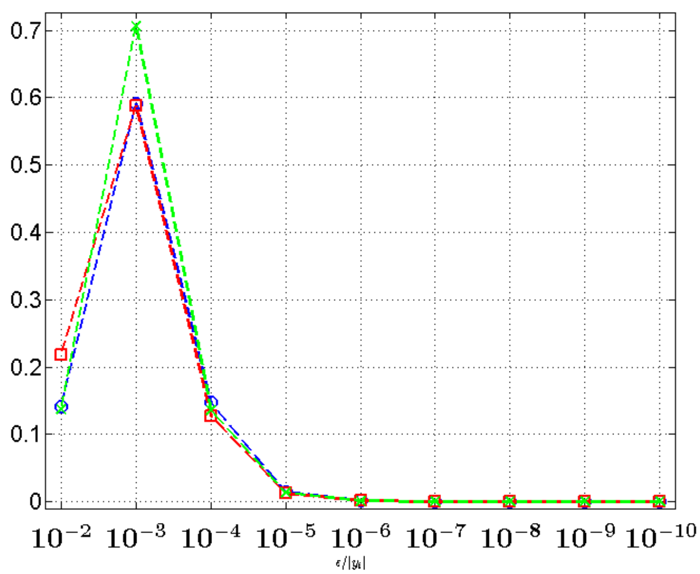

Figure 1: Maximum difference in compound voltage magnitudes obtained from load-flow runs on the IEEE 37-bus feeder

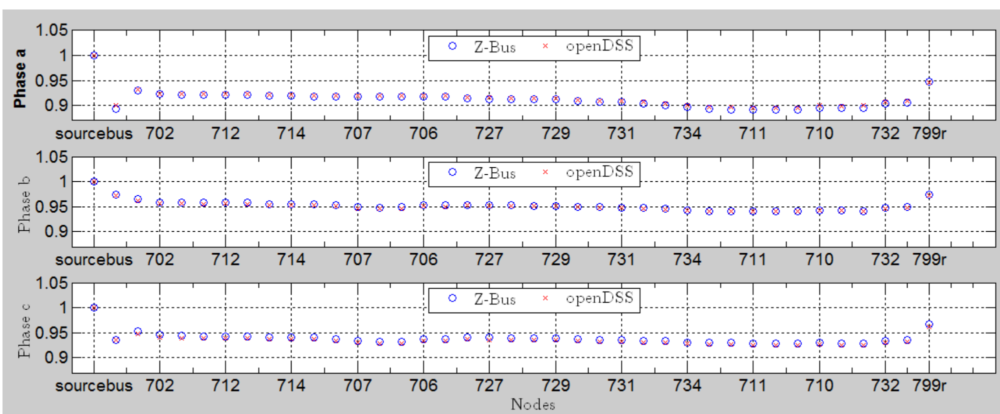

Figure 2: The IEEE 37-bus feeder voltage profile obtained from the Z-Bus method using non ideal SVRs(blue circles) and the voltage profile obtained from OpenDSS(red cross).

In figure 1, the y-axis is % per unit maximum difference in voltage magnitude and x-axis has epsilon by magnitude of leakage admittance. We can see couple of things from there. One is that the maximum difference occurs when the ratio of epsilon and leakage admittance is 10^{-3} and the difference gradually lowers as the ratio decreases. It validates the point made earlier that ϵ should be really small compared to y_t. Also phase c has the maximum voltage magnitude difference while phase a and b have quite similar curve.

In figure 2, we can see that all the voltage points from the load flow solver and the ones derived from the OpenDSS are on point. This help to validate the methodology of first forming Y-Bus and inverting it to run a load flow. The voltage profile given here is with using a non-ideal SVR i.e. $\mathbf{Z}_R$ is non-zero. The result with both ideal and non-ideal SVR is in the paper itself. There, it can be observed that the ideal SVR's voltage profile for phase **a** is little higher than other two.

11 Conclusion and discussion

A three phase distribution network was modeled and bus admittance matrix was extracted. In three phase modeling, transmission lines and transformer connections were just reviewed whereas new models for step-voltage regulators were derived. The extracted Y-bus matrix was tested for its invertibility and conditions were carefully laid out on each series element that guarantee the Y-bus invertibility. The non-singular characteristics of the Y-bus is important in deriving power flow solutions via the Z-bus method. Finally, MATLAB scripts were developed and the methodology was validated by comparing the Z-Bus power flow result with the OpenDSS result. Some of the future works can include,
(a) modeling for dynamic loads such as induction motors- Z,I,P load or any combination of ZIP loads cannot fully define induction motor load,
(b) modeling time-variant loads like water heater and building heating systems- all the models in this paper were time-invariant, so a time-variant load model should be next,
(c) distribution protection system model such as relay modelng, and
(d) Distributed Energy Resources modeling- as in furture, DERs are going to be an essential part of distribution system.

References

[1] M. Bazrafshan and N. Gatsis, "Comprehensive Modeling of Three-Phase Distribution Systems via the Bus Admittance Matrix," IEEE Trans. Power Syst., vol. 33, no. 2, pp. 20152029, Mar. 2018.

[2] M. L. Gorman and J. J. Grainger,"Transformer modelling for distribution system studies. II. Addition of models to Y-BUS and Z-BUS," in Proceedings of the 1991 IEEE Power Engineering Society Transmission and Distribution Conference, pp. 547552.

[3] M.-S. Chen and W. E. Dillion,“Power system modeling,” Proc. IEEE vol. 62, no. 7, pp. 901-915, Jul. 1974.

[4] W. H. Kersting,Distribution System Modeling and Analysis, IEEE Power Energy Mag., vol. 11, no. 3, pp. 106108, May 2013.

[5] T. H. Chen, M.S. Chen, T. Inoue, P. Kotas, and E. A. Chebli, “Three-phase cogenerator and transformer models for distribution system analysis,” IEEE Trans. Power Del., vol. 6, no. 4, pp. 1671-1681, Oct. 1991.

[6] “Standard load models for power flow and dynamic performance simulations,”IEEE Trans. Power Syst., vol. 10, no. 1, pp. 523-538, Feb. 1995