# Network Analysis Assignment 1

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## 1 Answers

Problem 1.
1.1)
For Air-traffic:
The number of nodes: 1226
The number of edges: 2410

For academic-collaboration:
The number of nodes: 5242
The number of edges: 14496

1.2)
If the question is asking about refactoring in the boiler template, then refactoring is done in the python file.

1.2.1)
For the air-traffic,
i)

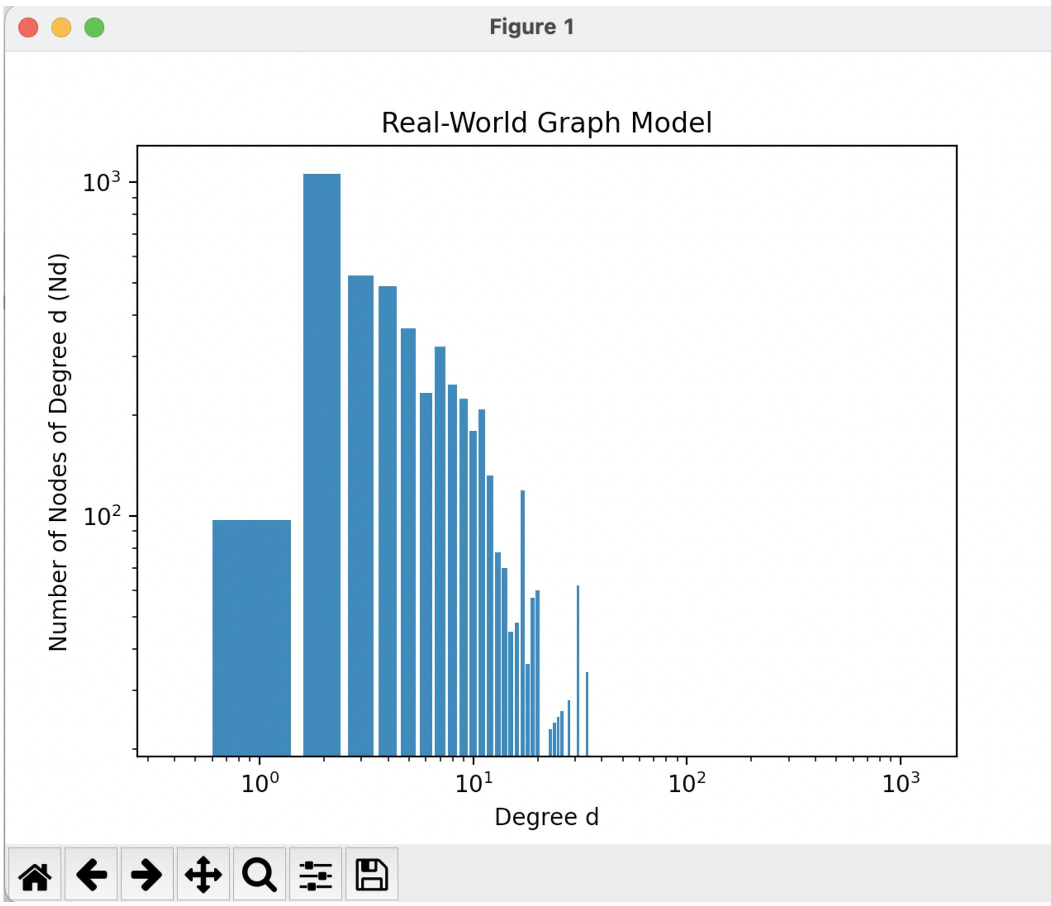


Figure 1: Air Traffic Real World Graph

ii)

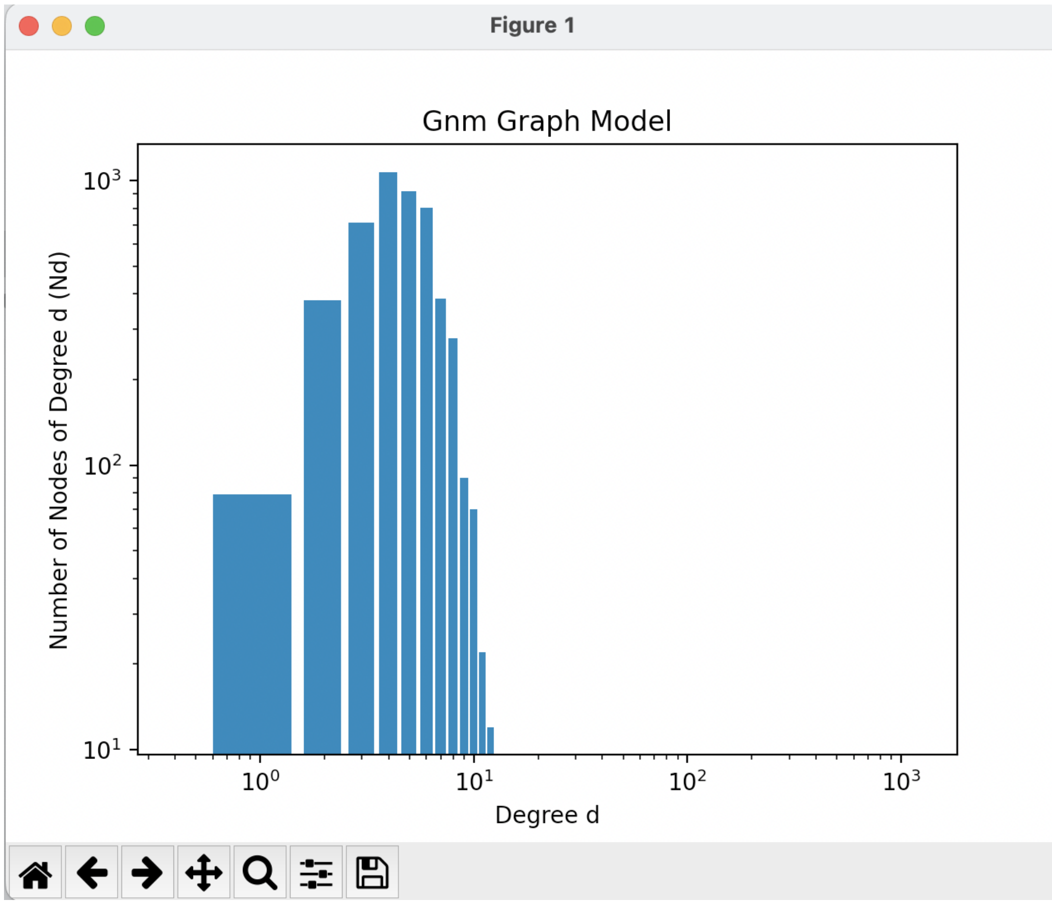


Figure 2: Gnm Graph For Air Traffic

iii)

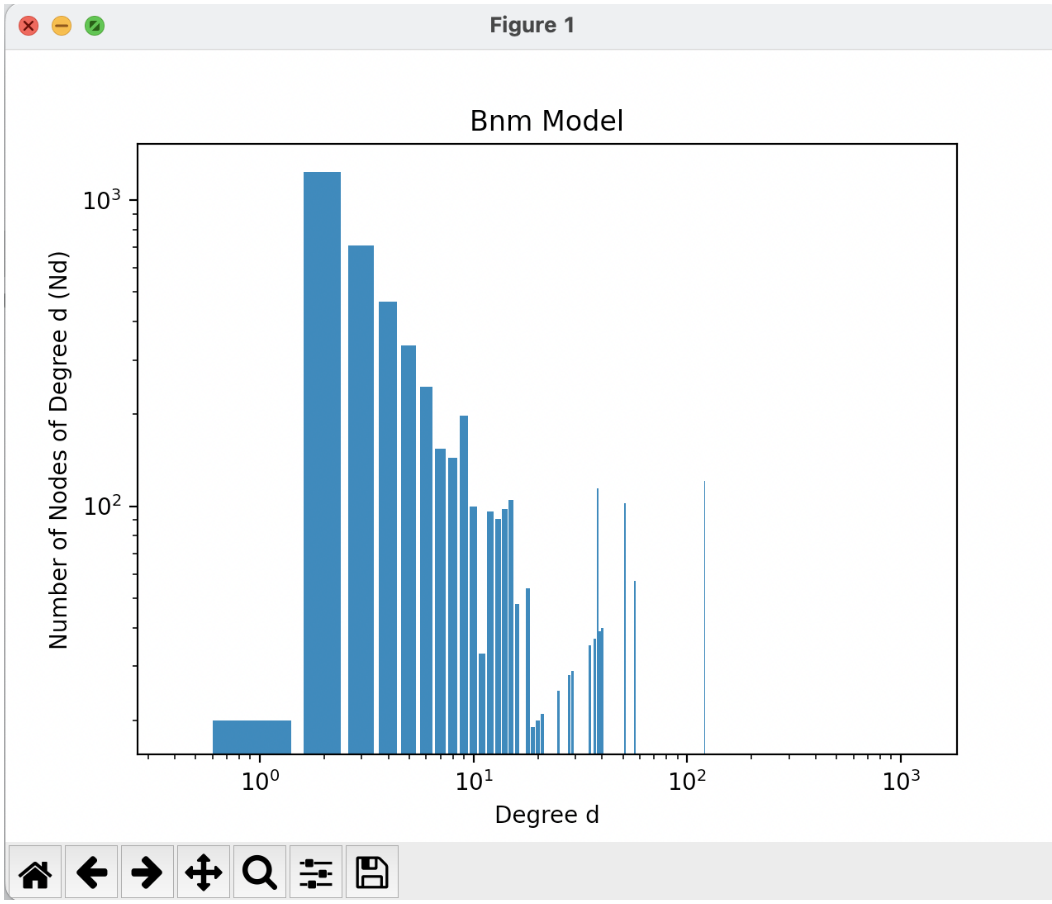


Figure 3: Bnm Graph For Air Traffic

For the academic-collaboration,
i)

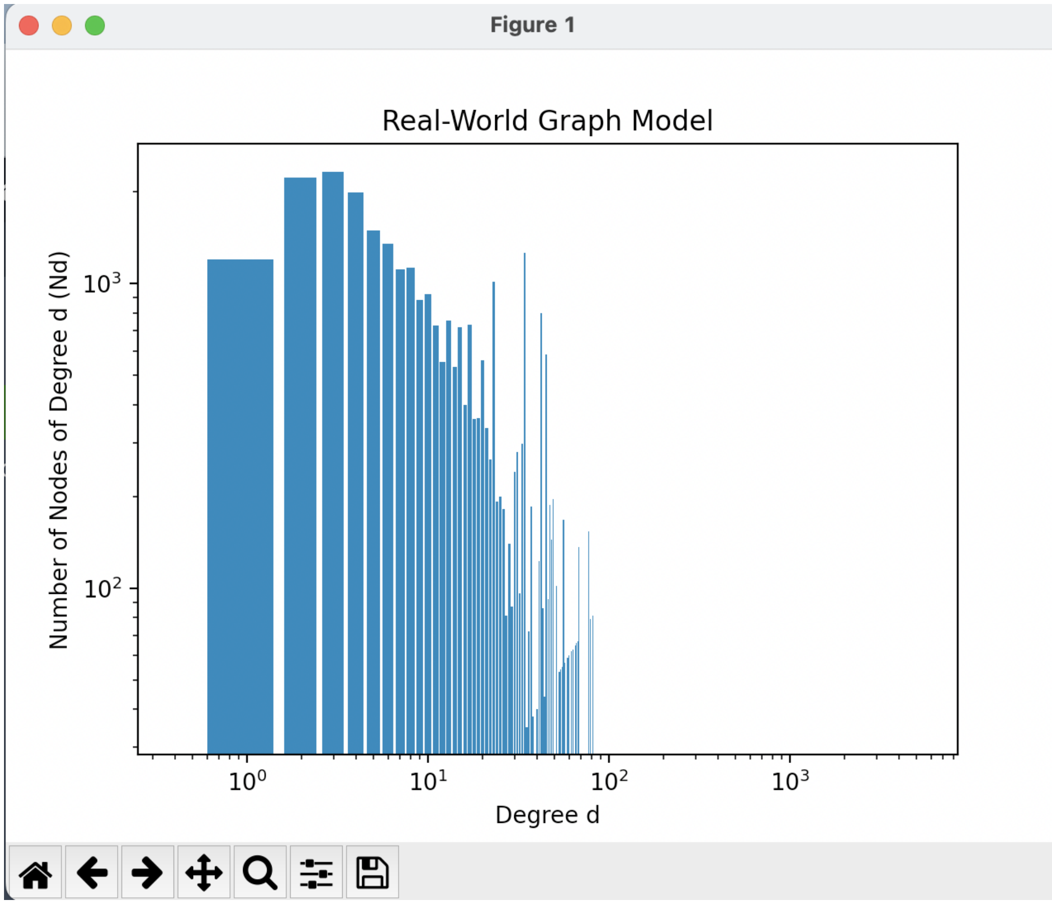


Figure 4: Academic Collaboration Real World Graph

ii)

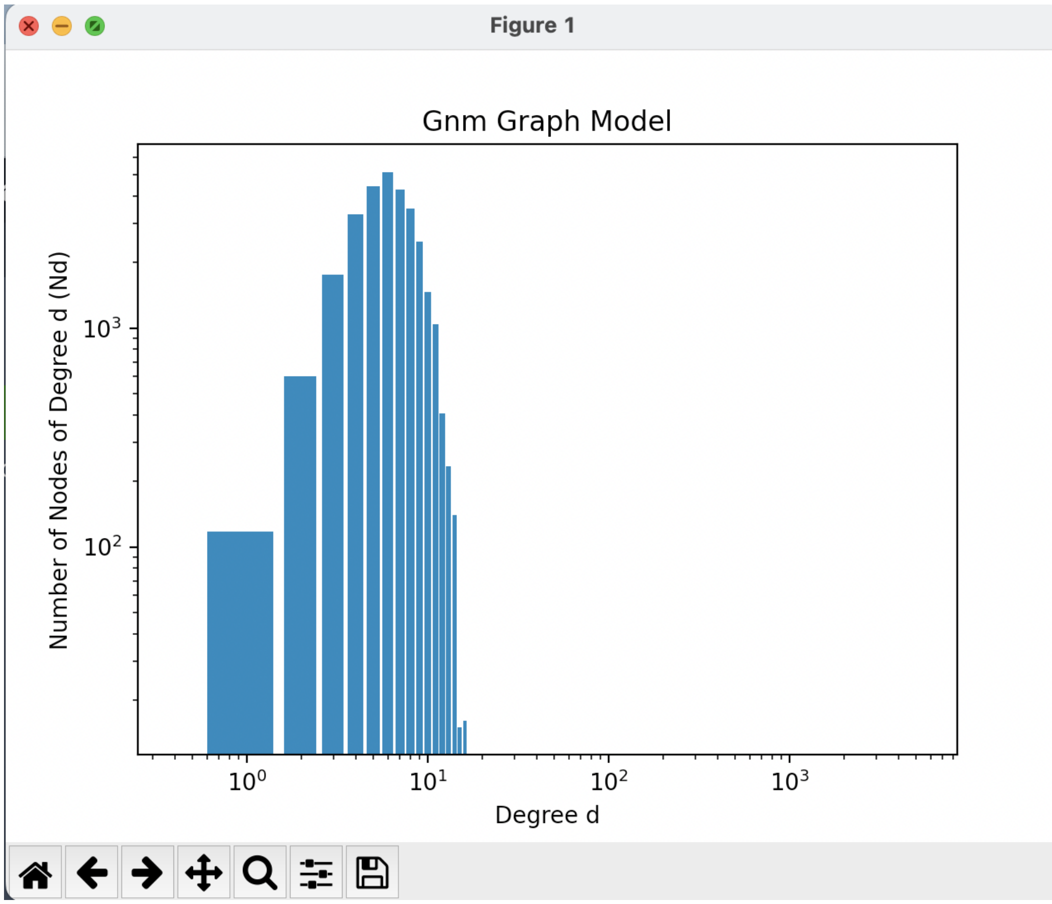


Figure 5: Gnm Graph For Academic Collaboration

iii)

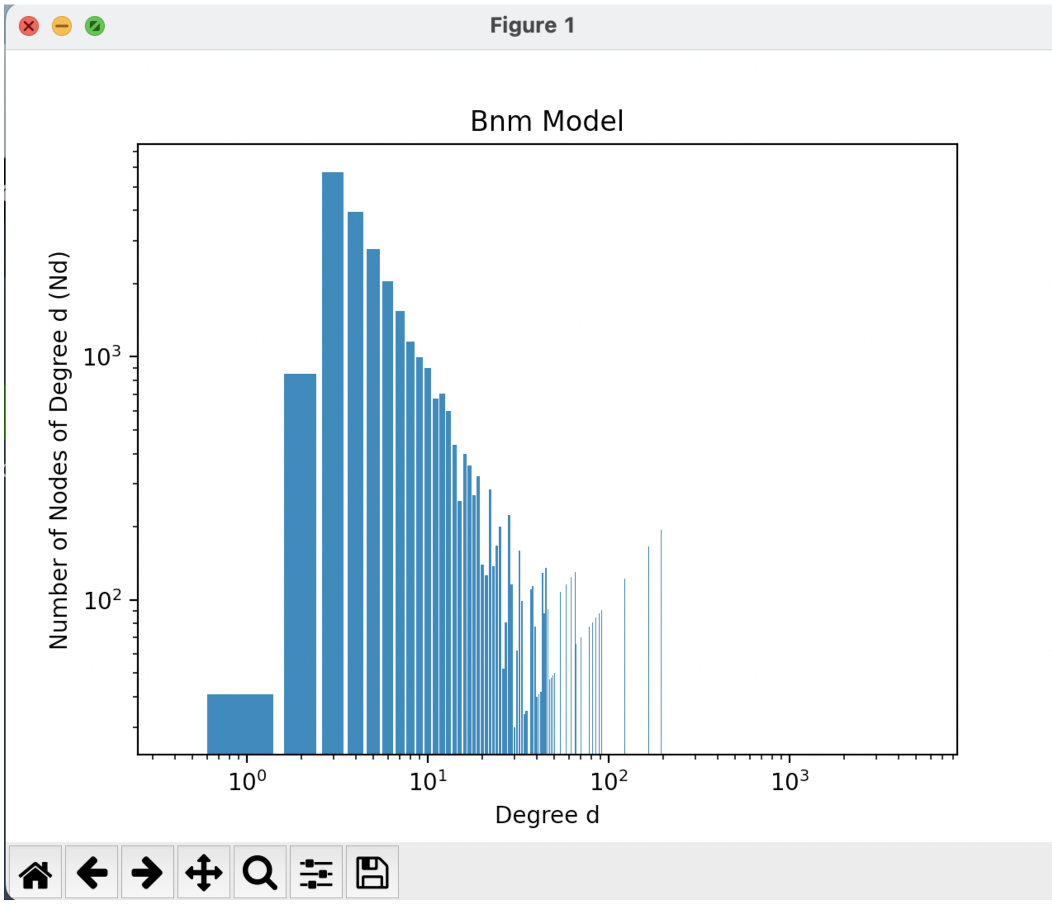


Figure 6: Bnm Graph For Academic Collaboration

1.2.2)
Are the real world degree distributions more similar to those of either random graph models?
The differeneces that I observe between the degree distrubutions of the random graphs and the real-world networks are:
i) The real-world network graphs are more real as the graph is slowing increasing and starts falling at a constant somewhat constant rate.
ii) graphs for gnm and the real world networks seems very similar.
iii) The Bnm graph seems to very fake because at the very first, degree d has considerable amount of number of nodes and it jumps suddenly. Then graph decreases at some point with some minor increase at the very end.

Yes, the real world degree distributions are more similar to the gnm graphs but not bnm graphs.

1.2.3)
The average clustering coefficients for :
For air-traffic:

i)the real world graph is 0.0675.

ii) the corresponding G(n,m) instance is 0.0034.

iii) the corresponding B(n,m) instance is 0.0215.

For academic-collaboration:

i)the real world graph is 0.5296.

ii) the corresponding G(n,m) instance is 0.0006.

iii) the corresponding B(n,m) instance is 0.0088.

For both cases, the real world average clustering coefficient is not similar to those of either random graph models.

Problem 2.
Given:
n denotes the number of nodes in the graph
$c \in [0, n-1]$
$p = \frac{c}{n-1}$ where $c \in [0, n-1]$

Now,
1.
let v $\in$ V where V denote the set of nodes in the graph.

The degree of a node v is given by:
$d_{\text{v}} = X_{\text{v},1} + X_{\text{v},2} + .. + X_{\text{v,n-1}}$ where $X_{\text{v,u}}$ denotes binary variable ( either 0 or

1 ) which tells us whether the edge between u and v exists or not.

Now, The expected degree of any/given vertex v is given by :
$E[d_{\text{v}}] = E[X_{\text{v},1} + X_{\text{v},2} + .. + X_{\text{v,n-1}}] = E[X_{\text{v},1}] + E[X_{\text{v},2}] + .. + E[X_{\text{v,n-1}}] = p + p + ... + p = p(n-1)$

As we know, $p = \frac{c}{n-1}$,
$E[d_{\text{v}}] = \frac{c}{(n-1)} * (n-1) = c$ where $c \in [0, n-1]$

Hence, the expected degree of any given vertex v is c where $c \in [0, n-1]$.

2.
Again, let v $\in$ V be one of the nodes.

If the node is isolated from the others nodes, it means that that node has no edge or neighboring node. This means that the isolated node has degree 0.

Now, the probability that a given node v has degree 0 is given by:
$P(k) = \binom{n-1}{k} p^k (1-p)^{\text{n-1-k}}$
$P(0) = \binom{n-1}{0} p^0 (1-p)^{\text{n-1-0}} = 1 * 1 * (1-p)^{\text{n-1}} = (1-p)^{\text{n-1}} = (1 - \frac{c}{n-1})^{\text{n-1}}$
where c $\in [0, n-1]$

Hence, the probability of that v is isolated is $(1 - \frac{c}{n-1})^{\text{n-1}}$ where c $\in [0, n-1]$

Connectivity:
My observations:
The graph is shown below and the graph looks similar to the f(x)=$\frac{1}{x}$ where $\forall x$ >0.

As the value of c is increasing, the value of probability is decreasing. So we can say that they are inversely proportional to each other in relations.

As figure is shown below,

Finally, the smallest value of c for which the graph becomes connected is 0.

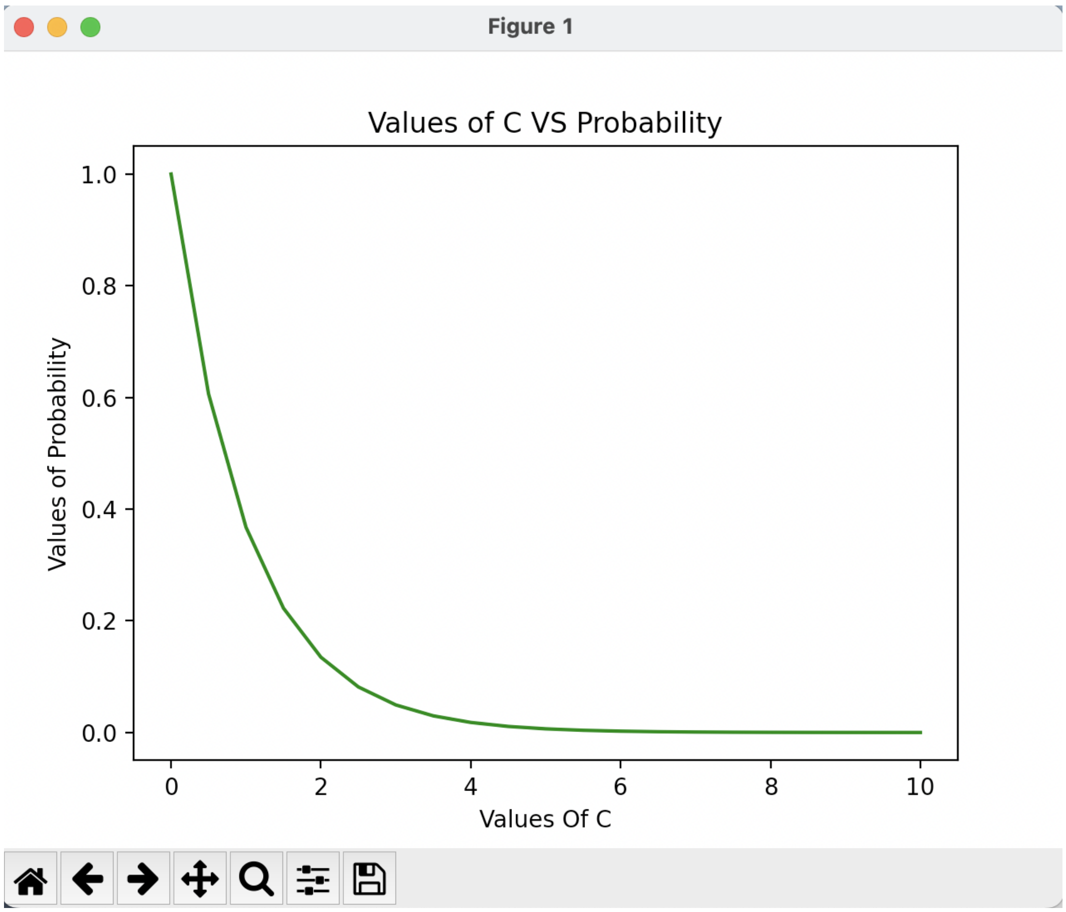


Figure 7: 2.Connectivity Picture