Network Analysis Assignment 3

Sanish Gurung

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1 Answers

1.
Consider the percolation (static view of epidemic) of the graph where X and Y be the two seed sets, such that X⊆Y in the stage 0 (initial stage).

let node x ∈X and let node y ∈Y

let (xi,xj) and (yi,yj) be the edges formed by nodes in X and Y in the percolation where (xi,xj) be the incoming link from xi to xj and (yi,yj) be the incoming link from yi to yj and also ∀i,j ∈ $\{1, 2, ..n\}$

Each edge is either open or blocked meaning that one node can infect the susceptible node obviously with some probability if the directed edge between the nodes exists.

Now,let n be the number of nodes in seed set X while n∈{0,1,..}
let n+m be the number of nodes in seed set Y while n∈{0,1,..} and m ∈{0,1,..}

let Z is the seed set of size m.

Case 1:
If n+m=n which means m=0, which further means that X=Y i.e. ∀x=∀y, then it is obvious that f(X)=f(Y).
Because the proof is given below:
X=X+Z
X=Y because set Z has no node
f(X)=f(Y) because if there are two same set in which one node map other node in the other set, then in the next stage, the resulting spread nodes are basically the same.

Case 2:
If n+m≠n that is m≠0, which means that X⊂Y, then it is obvious that f(X)⊂f(Y).
Because the proof is given below:
$X \subset X + Z$
$X \subset Y$ because X+Z=Y

$f(X) \subseteq f(Y)$ because the spread by nodes of X can be equal or less than the spread of Y depending on the structure of the network(epidemic).

Now, combining cases 1 and 2, we can prove that f(X)⊆f(Y).

In other words, if the additional nodes are infectious initially, we expect the virus to spread to more nodes eventually.

2.
Results:
Graphs with 0.1 and 0.5 probabilities are shown below.

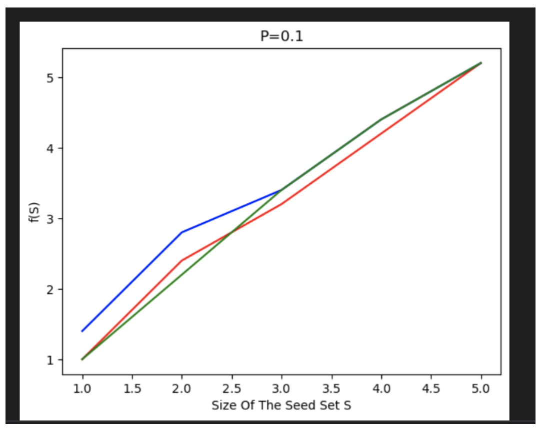

Figure 1: Size of the seed set VS Expected Spread with infection probability 0.1

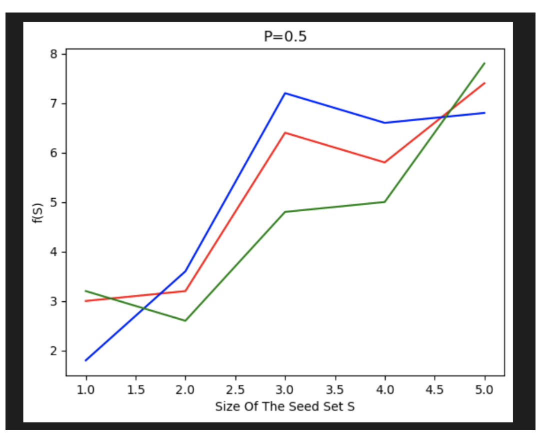

Figure 2: Size of the seed set VS Expected Spread with infection probability 0.5

Discussion:
With probability 0.1, the 3 different lines looks like the function f(x)=y while the 3 different lines for probability 0.5 gives different structures from time to time.

With probability 0.1, when the size of the seed set is increasing, the eventual spread is also increasing. While on the other hand,with probability 0.5, when the size of the seed is increasing, the eventual spread is increasing steady but not always meaning that there is a case where the spread decreased even though there is an increase in size of the the seed set. This happened because of setting recovery/removed time as 5 as it was asked in the question.

Obviously, the graph differs time to time depending on the randomly chosen candidate sets from the real-network in both cases with infection probabilities 0.1 and 0.5. Also, depending on the infection probability, the graphs will look very dissimilar.

Running time:
The number of minutes it takes to run my code in its entirety is less than 5 seconds.

I obtained it in Dual-Core Intel Core i5, 2 cores, 8 GBs RAM.