Eigen Values And Eigen Vectors.

Let A be a given nonzero square matrix of order n.If there exists a scalar λ and a non-zero vector X such that

$AX = \lambda X$

$AX = \lambda IX$

$(A - \lambda I)X = 0$ --- (1)

Then λ is called Eigen value of matrix A and X is called Eigen vector of A

Equation (1) is homogeneous system and it has non-trivial solution only if

$$|\mathbf{A} - \lambda\mathbf{I}| = \mathbf{0}$$

Def. Let A be nonzero square matrix. Then

1) $|A - \lambda I|$ is called Characteristic Polynomial of matrix A .i.e. Polynomial in λ.
Or $f(\lambda)$

2) $|A - \lambda I| = 0$ is called Characteristic Equation Of A.

3) Characteristic Roots or Eigen Values Of A :
The roots of the equation $|A - \lambda I| = 0$ are known as
Characteristic Roots or Eigen Values .

4) Characteristic Vectors or Eigen Vectors Of A :

We solve the system [A - λI] X = 0.
This system will have the non – trivial solution X.

X is called as Eigen vector or Latent vector or Character vector

associated with the root λ .

5) Spectrum Of A :

The set of all characteristic roots of the matrix A is called as Spectrum of A i.e. $\{\lambda_1, \lambda_2, \lambda_3, \dots\}$.

6) Trace of A :

Sum of the principle diagonal elements is called as the trace of A.

Properties Of Eigen Values :

1) A nxn matrix has at least one eigen value and at most n different eigen values.

2) Trace of A is equal to sum of the Eigen Values of A.

   i.e. $a_{11} + a_{22} + a_{33} + \ \dots = \lambda_1 + \lambda_2 + \lambda_3, \dots$

3) The product of eigen values of a matrix is equal to the determinant of the matrix A. i.e. $|A| = \lambda_1 . \lambda_2 . \lambda_3 . \dots \lambda_n$

4) The Eigen Values of an upper or lower triangular or diagonal matrix are the elements of on its main diagonal.

5) If λ is an eigen value of non – singular matrix A then $\frac{|A|}{\lambda}$ is an eigen value of Adj A

6) If $\lambda_i, i = 1, 2, \ldots, n$ are the eigen values of a non – singular matrix A then $\frac{1}{\lambda_i}$ are the eigen values of A^{-1}.

7) If $\lambda_i, i = 1, 2, \ldots, m$ are the eigen values of a matrix A then $\lambda_i{}^n$ are the eigen values of A^n where n is any positive integer.

8) If $\lambda_i, i = 1, 2, \ldots, n$ are the eigen values of a matrix A then $K\lambda_i$ are the eigen values of KA.

9) The eigen values of A and A' are same.

10) Zero is eigen value of a matrix iff it is singular matrix.

Properties Of Eigen Vectors :

1) If X is an eigen vector of a matrix A corresponding to an eigen Value λ then so is KX with $K \neq 0$.

2) If $\lambda_1, \lambda_2, \lambda_3 \ldots, \lambda_n$ are distinct eigen values of nxn matrix then corresponding eigen vectors $X_1, X_2, X_3, \ldots X_n$ form a linearly independent set.

3) Corresponding to n distinct eigen values , we get n independent eigen vectors. But when two or more eigen values are equal , it may not be possible to get linearly independent eigen vectors Corresponding to repeated roots.

4) An nxn matrix may have n or less than n linearly independent eigen vectors

5) Eigen vector of a square matrix can not correspond to two distinct eigen Values.

6) **orthogonal eigen vectors :** two eigen vectors X_1 and X_2 are said to be orthogonal if $X_1'X_2 = 0$.

7) Eigen vectors of a symmetric matrix corresponding to different eigen values are orthogonal.

Note : It may not be possible to find n linearly independent eigen vectors for an nxn matrix A when some of the roots are repeated.

But a symmetric matrix is an exception

A set of n linearly independent eigen vectors can be found for nxn symmetric matrix A even if some of the roots are repeated.

Solving Qubic equation

by Pawan Mall

Solving Cubic Equations
Super Trick

1) $x^3+13x^2+32x+20=0$

1, -1, 2, -2

$-1+\underline{13}-32+2\underline{0}$

$\times$

$$\begin{array}{r|rrrr} -1 & 1 & 13 & 32 & 20 \\ & \downarrow & -1 & -12 & -20 \\ \hline & 1 & 12 & +20 & \times \end{array}$$

$x^2+12x+20$

$$\begin{array}{cc} 1 & 2 \\ 1 & 10 \end{array}$$

$(x+2)(x+10)$

$(x+1)(x+2)(x+10)$
$=0$

Solving CUBIC EQUATIONS

SUPER TRICK

2) $x^3 - 7x^2 + 4x + 12 = 0$

$-1-7-4+12$

×

$(x+1)$

	1	−7	4	12
−1	1	−1	8	−12
	1	−8	12	×

$\begin{array}{cc} 1 & 6 \\ 1 & 2 \end{array}$

$(x-6)(x-2)(x+1)$

Solving CUBIC EQUATIONS

SUPER TRICK

3) $x^3 - 5x^2 - 2x + 24 = 0$

$(x+2)(x-3)(x-4) = 0$

	1	-5	-2	24
-2		-2	14	-24
	1	-7	12	X

$$\begin{array}{cc} 1 & 3 \\ 1 & 4 \end{array}$$

$(x-3)(x-4)$

$1 - 5 - 2 + 24$ X

$-1 - 5 + 2 + 24$ X

$8 - 20 - 4 + 24$ X

$-8 - 20 + 4 + 24$ ✓

Solving CUBIC EQUATIONS

SUPER TRICK

4) $2x^3 + 3x^2 - 11x - 6 = 0$

$2 + 3 - 11 - 6$ X

$-2 + 3 + 11 - 6$ X

$\underbrace{16 + 12}_{28} \underbrace{- 22 - 6}_{28}$ X

	2	3	−11	−6
2		4	14	6
	2	7	3	X

$$\begin{array}{ccc} 1 & & 3 \\ & \times & \\ 2 & & 1 \end{array}$$

$(x+3)(2x+1)(x-2)$

Q. Find the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

Q. Find the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

$\Rightarrow$ Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$

The characteristic eq^n is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{vmatrix} = 0$$

$$\boxed{\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0} \quad (1)$$

P = Sum of diagonals of A

$= 1 - 2 - 3$

$\boxed{P = -4}$

Q = Sum of minors of diagonals of A

$$= \begin{vmatrix} -2 & 6 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix}$$

$= 6 - 3 - 2$

$\boxed{Q = 1}$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{vmatrix}$$

$= 1(6) - 2(0) + 3(0)$

$\boxed{|A| = 6}$

eq^n (1) $\Rightarrow$

$$\boxed{\lambda^3 + 4\lambda^2 + \lambda - 6 = 0}$$

Roots are $(1, -2, -3)$

$\therefore \boxed{\lambda = 1, -2, -3}$

$\therefore$ **Eigenvalues are $1, -2, -3$**

i) when $\lambda = 1$

eqn (A) $\Rightarrow \begin{bmatrix} 0 & 2 & 3 \\ 0 & -3 & 6 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$0x_1 + 2x_2 + 3x_3 = 0$ —— (a)

$0x_1 - 3x_2 + 6x_3 = 0$ —— (b)

$0x_1 + 0x_2 - 4x_3 = 0$ —— (c)

from eqn (a) and (b)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ -3 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 2 \\ 0 & -3 \end{vmatrix}}$$

$$\frac{x_1}{21} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0}$$

$\therefore$ for $\lambda = 1$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

eqn (1) $\Rightarrow$

$$\boxed{\lambda^3 + 4\lambda^2 + \lambda - 6 = 0}$$

Roots are $(1, -2, -3)$

$\therefore \boxed{\lambda = 1, -2, -3}$

$\therefore$ Eigenvalues are $1, -2, -3$

Let $X = [x_1, x_2, x_3]$ be the eigen vectors corresponding to eigenvalue λ

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{—— (A)}$$

i) when $\lambda = 1$

eqⁿ (A) $\Rightarrow \begin{bmatrix} 0 & 2 & 3 \\ 0 & -3 & 6 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$0x_1 + 2x_2 + 3x_3 = 0$ — (a)

$0x_1 - 3x_2 + 6x_3 = 0$ — (b)

$0x_1 + 0x_2 - 4x_3 = 0$ — (c)

from eqⁿ (a) and (b)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ -3 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 2 \\ 0 & -3 \end{vmatrix}}$$

$$\frac{x_1}{21} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0}$$

$\therefore$ for $\lambda = 1$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

ii) when $\lambda = -2$

eqⁿ (A) $\Rightarrow \begin{bmatrix} 3 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$3x_1 + 2x_2 + 3x_3 = 0$ — (d)

$0x_1 + 0x_2 + 6x_3 = 0$ — (e)

$0x_1 + 0x_2 - x_3 = 0$ — (f)

from eqⁿ (d) and (e)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 2 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{12} = \frac{-x_2}{18} = \frac{x_3}{0}$$

$$\frac{x_1}{2} = \frac{x_2}{-3} = \frac{x_3}{0}$$

$\therefore$ for $\lambda = -2$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$

iii) when $\lambda = -3$

eqⁿ (A) $\Rightarrow \begin{bmatrix} 4 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$4x_1 + 2x_2 + 3x_3 = 0$ — (g)

$0x_1 + x_2 + 6x_3 = 0$ — (h)

$0x_1 + 0x_2 + 0x_3 = 0$

from eqⁿ (g) and (h)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & 2 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{9} = \frac{x_2}{-24} = \frac{x_3}{4}$$

$\therefore$ for $\lambda = -3$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -24 \\ 4 \end{bmatrix}$

Q. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

Q. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow$ Given, $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

$\therefore$ characteristic eq^n is:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\boxed{\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0} \quad \text{①}$$

P = sum of diagonals of A

$= 3+5+3$

$\boxed{P = 11}$ ✓

Q = sum of minors of diagonals of A

$= \begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix}$

$= 14 + 8 + 14 = 36$

$\boxed{Q = 36}$ ✓

$|A| = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{vmatrix} = 3(14) + 1(-2) + (-4)$

$= 36$

$\boxed{|A| = 36}$ ✓

eq^n ① $\Rightarrow$

$$\boxed{\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0}$$

$\therefore \boxed{\lambda = 2, 3, 6}$

$\boxed{\therefore \text{Eigen Values are } 2, 3, 6}$

Let $X = [x_1, x_2, x_3]$ be the eigen vectors corresponding to eigen value λ.

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 2$,

eqⁿ (2) ⇒

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 + x_3 = 0 \quad \text{— (3)}$$
$$-x_1 + 3x_2 - x_3 = 0 \quad \text{— (4)}$$
$$x_1 - x_2 + x_3 = 0 \quad \text{— (5)}$$

from eqⁿ (3) and (4)

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{-x_2}{0} = \frac{x_3}{2}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{-1}}$$

for $\lambda = 3$,

eqⁿ (2) ⇒

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 - x_2 + x_3 = 0 \quad \text{— (6)}$$
$$-x_1 + 2x_2 - x_3 = 0 \quad \text{— (7)}$$
$$x_1 - x_2 + 0x_3 = 0 \quad \text{— (8)}$$

from eqⁿ (6) and (7)

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{1} = \frac{x_3}{-1}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}}$$

eqⁿ (1) ⇒

$$\boxed{\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0}$$

$$\therefore \boxed{\lambda = 2, 3, 6}$$

$\boxed{\therefore \text{Eigen Values are } 2, 3, 6}$

Let $X = [x_1, x_2, x_3]$ be the eige vectors corresponding to eigen value λ.

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{— (2)}$$

Q. Find the eigen values and eigen vectors of the matrix:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q. Find the eigen values and eigen vectors of the matrix:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$\Rightarrow$ Given:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

characteristic eq^n is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\boxed{\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0} \quad \text{--- (1)}$$

P = Sum of diagonal of A

$= 8 + 7 + 3$

$\boxed{P = 18}$

Q = Sum of minors of diagonals of A

$$= \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$

$$= (21-16) + (24-4) + (56-36)$$

$\boxed{Q = 45}$

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix} = 8(21-16) + 6(-18+8) + 2(24-14)$$

$$= 0$$

eq^n (1) $\Rightarrow$

$$\boxed{\lambda^3 - 18\lambda^2 + 45\lambda = 0}$$

$\therefore \boxed{\lambda = 0, 3, 15}$

$\therefore$ Eigen values are 0, 3, 15.

Let $X = [x_1, x_2, x_3]$ be the eigen vectors corresponding to eigen value λ.

$$[A - \lambda I] X = 0$$

i) when $\lambda = 0$,

$$\Rightarrow \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1 - 6x_2 + 2x_3 = 0 \quad \text{--- (3)}$$
$$-6x_1 + 7x_2 - 4x_3 = 0 \quad \text{--- (4)}$$
$$2x_1 - 4x_2 + 3x_3 = 0 \quad \text{--- (5)}$$

from eq^n (4) and (5)

$$\frac{x_1}{\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{21-16} = \frac{-x_2}{-18+8} = \frac{x_3}{24-14}$$

$$\frac{x_1}{5} = \frac{x_2}{10} = \frac{x_3}{10}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$\boxed{\text{for } \lambda = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 2}$

ii) when $\lambda = 3$

$$(2) \Rightarrow \begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 - 6x_2 + 2x_3 = 0 \quad \text{--- (6)}$$
$$-6x_1 + 4x_2 - 4x_3 = 0 \quad \text{--- (7)}$$
$$2x_1 - 4x_2 + 0 = 0 \quad \text{--- (8)}$$

from eq^n (6) and (7)

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$\frac{x_1}{24-8} = \frac{-x_2}{-20+12} = \frac{x_3}{20-36}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$\boxed{\text{For } \lambda = 3,\ x_1 = 2,\ x_2 = 1,\ x_3 = -2}$

eq^n (1) $\Rightarrow$

$$\boxed{\lambda^3 - 18\lambda^2 + 45\lambda = 0}$$

$$\therefore \boxed{\lambda = 0, 3, 15}$$

$\therefore$ Eigen values are 0, 3, 15.

Let $x = [x_1, x_2, x_3]$ be the eigen vectors corresponding to eigen value λ.

$$[A - \lambda I]\, x = 0$$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{--- (2)}$$

i) when $\lambda = 0$,

$$(2) \Rightarrow \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1 - 6x_2 + 2x_3 = 0 \quad \text{—— (3)}$$
$$-6x_1 + 7x_2 - 4x_3 = 0 \quad \text{—— (4)}$$
$$2x_1 - 4x_2 + 3x_3 = 0 \quad \text{—— (5)}$$

from eq^n (4) and (5)

$$\frac{x_1}{\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{21-16} = \frac{-x_2}{-18+8} = \frac{x_3}{24-14}$$

$$\frac{x_1}{5} = \frac{x_2}{10} = \frac{x_3}{10}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$$\boxed{\text{for } \lambda = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 2}$$

ii) when $\lambda = 3$

$$(2) \Rightarrow \begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 - 6x_2 + 2x_3 = 0 \quad \text{—— (6)}$$
$$-6x_1 + 4x_2 - 4x_3 = 0 \quad \text{—— (7)}$$
$$2x_1 - 4x_2 + 0 = 0 \quad \text{—— (8)}$$

from eq^n (6) and (7)

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$\frac{x_1}{24-8} = \frac{-x_2}{-20+12} = \frac{x_3}{20-36}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$$\boxed{\text{for } \lambda = 3,\ x_1 = 2,\ x_2 = 1,\ x_3 = -2}$$

iii) when $\lambda = 15$

eq^n (2) $\Rightarrow$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-7x_1 - 6x_2 + 2x_3 = 0 \quad \text{—— (9)}$$
$$-6x_1 - 8x_2 - 4x_3 = 0 \quad \text{—— (10)}$$
$$2x_1 - 4x_2 - 12x_3 = 0 \quad \text{—— (11)}$$

from eq^n (9) and (11)

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ -4 & -12 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & 2 \\ 2 & -12 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & -6 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{72+8} = \frac{-x_2}{84-4} = \frac{x_3}{28+12}$$

$$\frac{x_1}{80} = \frac{x_2}{-80} = \frac{x_3}{40}$$

$$\frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$$\boxed{\text{for } \lambda = 15,\ x_1 = 2,\ x_2 = -2,\ x_3 = 1}$$

Q. Find the eigen values and eigen vectors of the matrix : $\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$

Q. Find the eigen values and eigen vectors of the matrix : $\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$

$\Rightarrow$ Given : $A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$

Characteristic eq^n is :

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -6 & -5-\lambda \end{vmatrix} = 0$$

$$-\lambda(-5-\lambda) + 6 = 0$$

$$5\lambda + \lambda^2 + 6 = 0$$

$$\boxed{\lambda^2 + 5\lambda + 6 = 0}$$

$$(\lambda+2)(\lambda+3) = 0$$

$$\therefore \boxed{\lambda = -2, -3}$$

$\therefore$ Eigen values are $-2, -3$.

Let $x = [x_1 \ x_2]$ be the eigen vector for the correspondig eigen values.

$$[A - \lambda I]\,x = 0$$

$$\begin{bmatrix} -\lambda & 1 \\ -6 & -5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{--- (1)}$$

Case i) when $\lambda = -2$

eq^n (1) $\Rightarrow \begin{bmatrix} 2 & 1 \\ -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$2x_1 + x_2 = 0 \quad \text{--- (2)}$$

$$-6x_1 - 3x_2 = 0 \quad \text{--- (3)}$$

from eq^n (2) $\Rightarrow 2x_1 + x_2 = 0$

$$2x_1 = -x_2$$

$$\frac{x_1}{1} = \frac{x_2}{-2}$$

$$\therefore \boxed{x_1 = 1}, \boxed{x_2 = -2}$$

For $\lambda = -2$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Case ii) when $\lambda = -3$

$$\text{eq}^n \ (1) \Rightarrow \begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x_1 + x_2 = 0 \quad \text{—— (4)}$$

$$-6x_1 - 2x_2 = 0 \quad \text{—— (5)}$$

from eq^n (4) $\Rightarrow 3x_1 + x_2 = 0$

$$3x_1 = -x_2$$

$$\frac{x_1}{1} = \frac{x_2}{-3}$$

$\therefore \boxed{x_1 = 1}, \boxed{x_2 = -3}$

$$\therefore \text{for } \lambda = -3, \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\boxed{\lambda^2 + 5\lambda + 6 = 0}$$

$$(\lambda + 2)(\lambda + 3) = 0$$

$$\therefore \boxed{\lambda = -2, -3}$$

$\therefore$ **Eigen values are $-2, -3$.**

Let $x = [x_1 \ x_2]$ be the eigen vector for the correspondig eigen values.

$$[A - \lambda I] x = 0$$

$$\begin{bmatrix} -\lambda & 1 \\ -6 & -5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{—— (1)}$$

Case i) when $\lambda = -2$

$$\text{eq}^n \ (1) \Rightarrow \begin{bmatrix} 2 & 1 \\ -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + x_2 = 0 \quad \text{—— (2)}$$

$$-6x_1 - 3x_2 = 0 \quad \text{—— (3)}$$

from eq^n (2) $\Rightarrow 2x_1 + x_2 = 0$

$$2x_1 = -x_2$$

$$\frac{x_1}{1} = \frac{x_2}{-2}$$

$\therefore \boxed{x_1 = 1}, \boxed{x_2 = -2}$

$$\text{For } \lambda = -2, \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Q. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

Q. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$\Rightarrow$ Let, $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$\therefore$ Characteristic eq^n is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$\boxed{\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0} \quad \text{—— ①}$$

P = Sum of diagonal elements

$= 3+2+5$

$\boxed{P = 10}$

Q = Sum of minors of diagonals of A

$$= \begin{vmatrix} 2 & 6 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 4 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 10 + 15 + 6$, $\boxed{Q = 31}$

$$|A| = \begin{vmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{vmatrix} = 3(10) - 1(0) + 4(0)$$

$= 30$

$\boxed{|A| = 30}$

eq^n ① $\Rightarrow$

$$\boxed{\lambda^3 - 10\lambda^2 + 31\lambda - 30 = 0}$$

$\therefore \boxed{\lambda = 2, 3, 5}$

$\therefore$ $\boxed{\text{Eigen Values are } 2, 3, 5.}$

Let $X = [x_1 \ x_2 \ x_3]$ be the eigen vectors corresponding to eigen values.

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{—— ②}$$

For $\lambda = 2$,

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + 4x_3 = 0$$

$$0x_1 + 0x_2 + 6x_3 = 0$$

$$0x_1 + 0x_2 + 3x_3 = 0$$

from 1st two eq^{n}s

$$\frac{x_1}{\begin{vmatrix} 1 & 4 \\ 0 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 4 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{6} = \frac{-x_2}{6} = \frac{x_3}{0}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{0}}$$

For $\lambda = 3$,

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 + x_2 + 4x_3 = 0$$

$$0x_1 - x_2 + 6x_3 = 0$$

$$0x_1 + 0x_2 + 2x_3 = 0$$

from 1st two eq^{n}s

$$\frac{x_1}{\begin{vmatrix} 1 & 4 \\ -1 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 4 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}}$$

$$\frac{x_1}{10} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0}}$$

For $\lambda = 5$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + 4x_3 = 0$$

$$0x_1 - 3x_2 + 6x_3 = 0$$

$$0x_1 + 0x_2 + 0x_3 = 0$$

from 1st two eq^{n}s

$$\frac{x_1}{\begin{vmatrix} 1 & 4 \\ -3 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & 4 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix}}$$

$$\frac{x_1}{18} = \frac{-x_2}{-12} = \frac{x_3}{6}$$

$$\boxed{\frac{x_1}{3} = \frac{x_2}{2} = \frac{x_3}{1}}$$

Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$\Rightarrow$ Take,

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Characteristic eq^n is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$\boxed{\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0} \quad \text{--- (1)}$$

P = Sum of diagonals of A

$= -2 + 1 + 0$

$\boxed{P = -1}$

Q = Sum of minors of diagonals of A

$$= \begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}$$

$= (-12) + (-3) + (-6)$, $\boxed{Q = -21}$

$$|A| = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix} = -2(-12) - 2(-6) - 3(-3)$$

$= 24 + 12 + 9$

$\boxed{|A| = 45}$

(1) $\Rightarrow$ $\boxed{\lambda^3 + \lambda^2 - 21\lambda - 45 = 0}$

$\therefore$ $\boxed{\lambda = 5, -3, -3}$

$\therefore$ $\boxed{\text{Eigen Values are } 5, -3, -3}$

Let $X = [x_1, x_2, x_3]$ be the eigen vectors corresponding to eigen values λ.

Consider $[A - \lambda I]X = 0$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{--- (2)}$$

for $\lambda = 5$, eqⁿ ② $\Rightarrow$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-7x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 - 4x_2 - 6x_3 = 0$$
$$-x_1 - 2x_2 - 5x_3 = 0$$

Solving last 2 eqⁿs.

$$\frac{x_1}{\begin{vmatrix} -4 & -6 \\ -2 & -5 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & -6 \\ -1 & -5 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & -4 \\ -1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{8} = \frac{-x_2}{-16} = \frac{x_3}{-8}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{-1}$$

$\therefore$ $\boxed{\text{Eigen Vectors are } (1, 2, -1)}$

for $\lambda = -3$, eqⁿ ② $\Rightarrow$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 + 4x_2 - 6x_3 = 0$$
$$-x_1 - 2x_2 + 3x_3 = 0$$

Take, $\boxed{x_1 + 2x_2 - 3x_3 = 0}$ ✓

Let $\boxed{x_2 = 0}$ and $\boxed{x_3 = 1}$

$\therefore x_1 + 0 - 3 = 0$, $\boxed{x_1 = 3}$

$\therefore$ $\boxed{\text{Eigen Vectors are } (3, 0, 1)}$

Now, Let $\boxed{x_3 = 0}$ and $\boxed{x_2 = -1}$

$\therefore x_1 + 2(-1) + 0 = 0$

$\boxed{x_1 = 2}$

$\boxed{\therefore \text{Eigen vectors are } (2, -1, 0)}$

<u>Result</u>

Eigen Values $= (5, -3, -3)$

Eigen Vectors $= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

Q. find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$\Rightarrow$ Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

characteristic eqn is -

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\boxed{\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0} \quad \text{①}$$

P = sum of diagonals elements of A

$= 6 + 3 + 3$

$\boxed{P = 12}$

Q = Sum of minors of diagonals of A

$$= \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$$

$= (9-1) + (18-4) + (18-4)$

$\boxed{Q = 36}$

$$|A| = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 6(8) + 2(-4) + 2(-4)$$

$= 48 - 8 - 8$

$\therefore \boxed{|A| = 32}$

① $\Rightarrow \boxed{\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0}$

$\therefore \boxed{\lambda = 2, 2, 8}$

$\boxed{\therefore \text{Eigen Values are } 2, 2, 8}$

Let $x = [x_1, x_2, x_3]$ be the eigen vectors corresponding to the given eigen values λ.

Consider,

$$[A - \lambda I]\, x = 0$$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

i) for $\lambda=2$

$$\begin{bmatrix}4 & -2 & 2\\ -2 & 1 & -1\\ 2 & -1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$4x_1-2x_2+2x_3=0$$
$$-2x_1+x_2-x_3=0$$
$$2x_1-x_2+x_3=0$$

Take, $2x_1-x_2+x_3=0$

Let $\boxed{x_2=0}$, $\boxed{x_1=-1}$

$\therefore 2(-1)-0+x_3=0$

$\therefore \boxed{x_3=2}$

$\therefore$ Eigen Vectors are

for $\lambda=2$, $X=\begin{bmatrix}-1\\0\\2\end{bmatrix}$ ✓

ii) For $\lambda=2$

Take,

$$2x_1-x_2+x_3=0$$

Let $\boxed{x_3=0}$, $\boxed{x_1=1}$

$\therefore 2(1)-x_2+0=0$

$\therefore \boxed{x_2=2}$

$\therefore$ Eigen Vectors are

for $\lambda=2$, $X=\begin{bmatrix}1\\2\\0\end{bmatrix}$ ✓

iii) for $\lambda=8$

$$\begin{bmatrix}-2 & -2 & 2\\ -2 & -5 & -1\\ 2 & -1 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$-2x_1-2x_2+2x_3=0 \quad \text{— (2)}$$
$$-2x_1-5x_2-x_3=0 \quad \text{— (3)}$$
$$2x_1-x_2-5x_3=0 \quad \text{— (4)}$$

from eq^n (2) and (3)

$$\frac{x_1}{\begin{vmatrix}-2 & 2\\ -5 & -1\end{vmatrix}}=\frac{-x_2}{\begin{vmatrix}-2 & 2\\ -2 & -1\end{vmatrix}}=\frac{x_3}{\begin{vmatrix}-2 & -2\\ -2 & -5\end{vmatrix}}$$

$$\frac{x_1}{2+10}=\frac{-x_2}{2+4}=\frac{x_3}{10-4}$$

$$\frac{x_1}{12}=\frac{x_2}{-6}=\frac{x_3}{6}$$

$$\boxed{\frac{x_1}{2}=\frac{x_2}{-1}=\frac{x_3}{1}}$$

$\therefore$ for $\lambda=8$, eigenvectors are $X=\begin{bmatrix}2\\-1\\1\end{bmatrix}$ ✓

For $\lambda = 2$,

eq^n (2) $\Rightarrow$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - x_2 + x_3 = 0$ —— (3)

$-x_1 + 3x_2 - x_3 = 0$ —— (4)

$x_1 - x_2 + x_3 = 0$ —— (5)

from eq^n (3) and (4)

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{-x_2}{0} = \frac{x_3}{2}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{-1}}$$

Eigen vectors are $(1, 0, -1)$

for $\lambda = 3$,

eq^n (2) $\Rightarrow$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 - x_2 + x_3 = 0$ —— (6)

$-x_1 + 2x_2 - x_3 = 0$ —— (7)

$x_1 - x_2 + 0x_3 = 0$ —— (8)

from eq^n (6) and (7)

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{1} = \frac{x_3}{-1}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}}$$

Eigen vectors are $(1, 1, 1)$

For $\lambda = 6$,

eq^n (2) $\Rightarrow$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 - x_2 + x_3 = 0$ —— (9)

$-x_1 - x_2 - x_3 = 0$ —— (10)

$x_1 - x_2 - 3x_3 = 0$ —— (11)

from eq^n (9) and (10)

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -3 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -3 & -1 \\ -1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{-x_2}{4} = \frac{x_3}{2}$$

$$\boxed{\frac{x_1}{1} = \frac{x_2}{-2} = \frac{x_3}{1}}$$

Eigen vectors are $(1, -2, 1)$

$\boxed{\text{Transpose}} \rightarrow$ Interchange Rows & Columns

Transpose $\rightarrow$ Interchange Rows & Columns

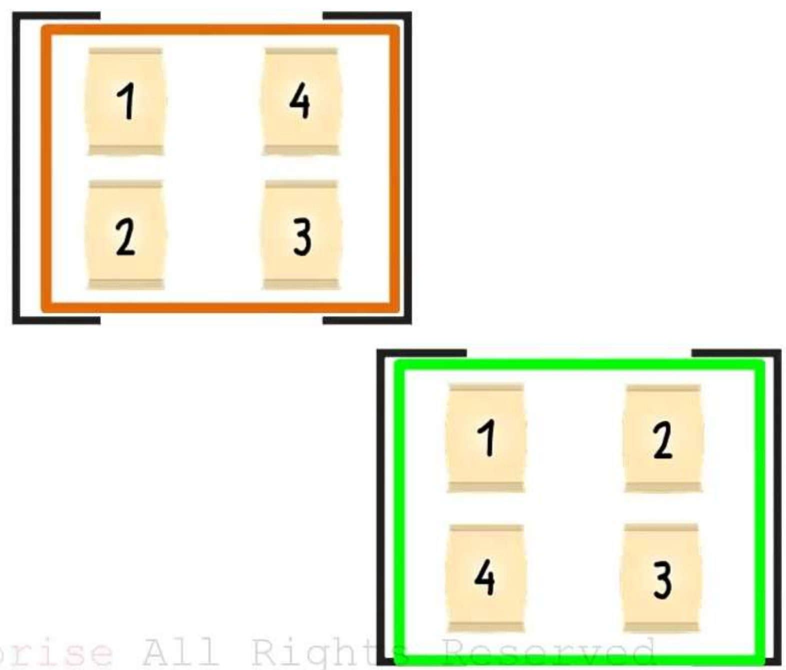
$$A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

Transpose $\longrightarrow$ Interchange Rows & Columns

$$A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$



Transpose $\rightarrow$ Interchange Rows & Columns

$$A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 4 \end{bmatrix}$$

$$B^T = ?$$

Transpose $\rightarrow$ Interchange Rows & Columns

$$A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 4 \end{bmatrix}$$

Orthogonal Matrices .

A square matrix A is said to be **Orthogonal Matrix** if

$AA^T = A^T A = I$ OR $AA^{'} = A^{'}A = I$

Where A^T = Transpose of A

Orthogonal Matrices .

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Orthogonal Matrices .

A square matrix A is said to be **Orthogonal Matrix** if

$AA^T = A^T A = I$ OR $AA' = A'A = I$

Where A^T = Transpose of A

Properties of Orthogonal Matrix :

Let A be an orthogonal matrix ,

- $AA^T = I$.
- $AA^T = I \Longrightarrow A^{-1} = A^T$
- Product of two orthogonal matrices is a orthogonal matrix.
- Transpose of orthogonal matrix is a orthogonal matrix.
- Inverse of orthogonal matrix is a orthogonal matrix.
- If A is a orthogonal matrix then $|A| = \pm 1$

EXAMPLES

Ex 1 : show that $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal.

A is orthogonal if $AA' = A'A = I$.

$$A\,A' = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

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EXAMPLES

Ex 1 : show that A = $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal.

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$$A\,A' = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & 2+2-4 & 2-4+2 \\ 2+2-4 & 4+1+4 & 4-2-2 \\ 2-4+2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

EXAMPLES

Ex 1 : show that $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal.

A is orthogonal if AA′ = A′A = I.

$$A\,A' = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & 2+2-4 & 2-4+2 \\ 2+2-4 & 4+1+4 & 4-2-2 \\ 2-4+2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}$$

$\therefore$ A is orthogonal matrix.

--𝒬-- --

Ex 2 : show that A = $\begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ is orthogonal.

$$A' = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$$

A is orthogonal if AA' = A'A = I.

$$cos^2\theta + sin^2\theta = 1$$

$$A\,A' = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} cos^2\theta + sin^2\theta & 0 & -sin\theta cos\theta + cos\theta sin\theta \\ 0 & 1 & 0 \\ -sin\theta cos\theta + cos\theta sin\theta & 0 & cos^2\theta + sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}$$

$\therefore$ A is orthogonal matrix.

Definition: Proper orthogonal Matrix:

An orthogonal matrix A is said to be proper orthogonal if $|A| = 1$

Ex. $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{cos\,\theta} & \boldsymbol{-sin\,\theta} \\ \boldsymbol{sin\,\theta} & \boldsymbol{cos\,\theta} \end{bmatrix}$

Definition: Improper orthogonal Matrix:

An orthogonal matrix A is said to be proper orthogonal if $|A| = -1$

Ex. $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{cos\,\theta} & \boldsymbol{-sin\,\theta} \\ \boldsymbol{-sin\,\theta} & \boldsymbol{-cos\,\theta} \end{bmatrix}$

Definition: Proper orthogonal Matrix:

An orthogonal matrix A is said to be proper orthogonal if $|A| = 1$

Ex. $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{\cos\theta} & \boldsymbol{-\sin\theta} \\ \boldsymbol{\sin\theta} & \boldsymbol{\cos\theta} \end{bmatrix}$

$|A| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1 \Rightarrow |A| = 1$

Definition: Improper orthogonal Matrix:

An orthogonal matrix A is said to be Improper orthogonal if $|A| = -1$

Ex. $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{\cos\theta} & \boldsymbol{-\sin\theta} \\ \boldsymbol{-\sin\theta} & \boldsymbol{-\cos\theta} \end{bmatrix}$

$|A| = \begin{vmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{vmatrix} = -\cos^2\theta - \sin^2\theta = -\{\cos^2\theta + \sin^2\theta\} = -1$

$|A| = -1$

Definition: Proper orthogonal Matrix:

An orthogonal matrix A is said to be proper orthogonal if $|A| = 1$

Ex. $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{\cos\theta} & \boldsymbol{-\sin\theta} \\ \boldsymbol{\sin\theta} & \boldsymbol{\cos\theta} \end{bmatrix}$

$$|A| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1 \Rightarrow |A| = 1$$

Definition: Improper orthogonal Matrix:

An orthogonal matrix A is said to be Improper orthogonal if $|A| = -1$

Ex. $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{\cos\theta} & \boldsymbol{-\sin\theta} \\ \boldsymbol{-\sin\theta} & \boldsymbol{-\cos\theta} \end{bmatrix}$

$$|A| = \begin{vmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{vmatrix} = -\cos^2\theta - \sin^2\theta = -\{\cos^2\theta + \sin^2\theta\} = -1$$

$$|A| = -1$$

Understanding Change of Basis

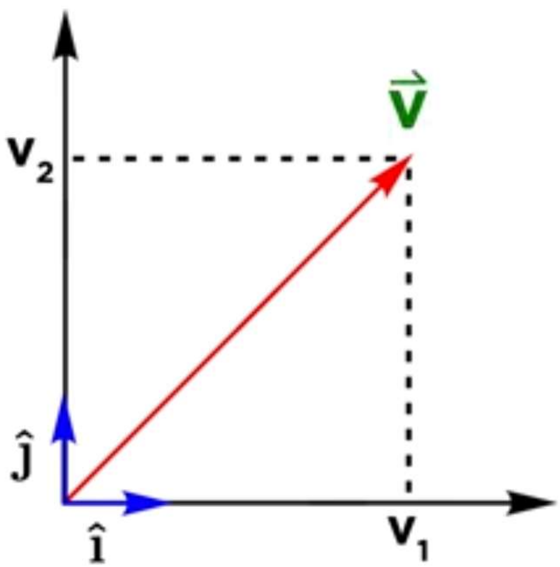
$$\mathbb{R}^3 \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

these three vectors form a **basis** for this vector space

Understanding Change of Basis

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= v_1 \hat{\imath} + v_2 \hat{\jmath}$$



we can express any vector in this space as a linear combination of these **basis vectors**

Understanding Change of Basis

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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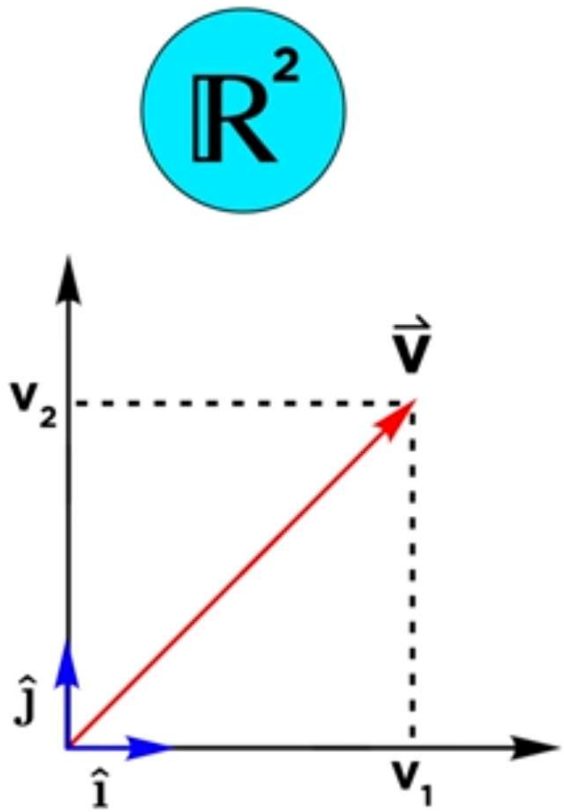


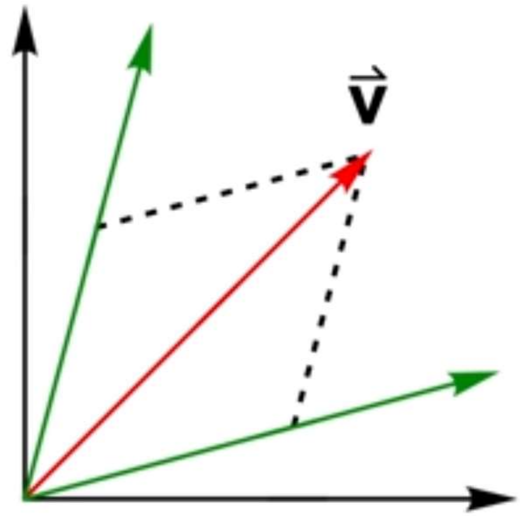
we can express any vector in this space as a linear combination of these basis vectors

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= v_1 \hat{\imath} + v_2 \hat{\jmath}$$



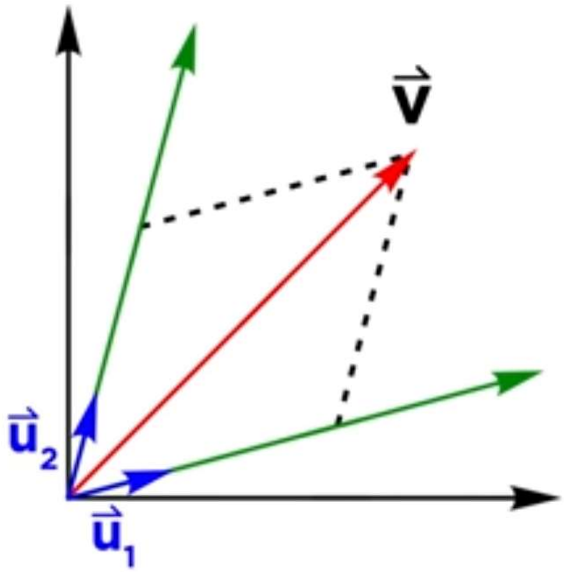
we could also express this vector in terms of some **other basis**

Understanding Change of Basis

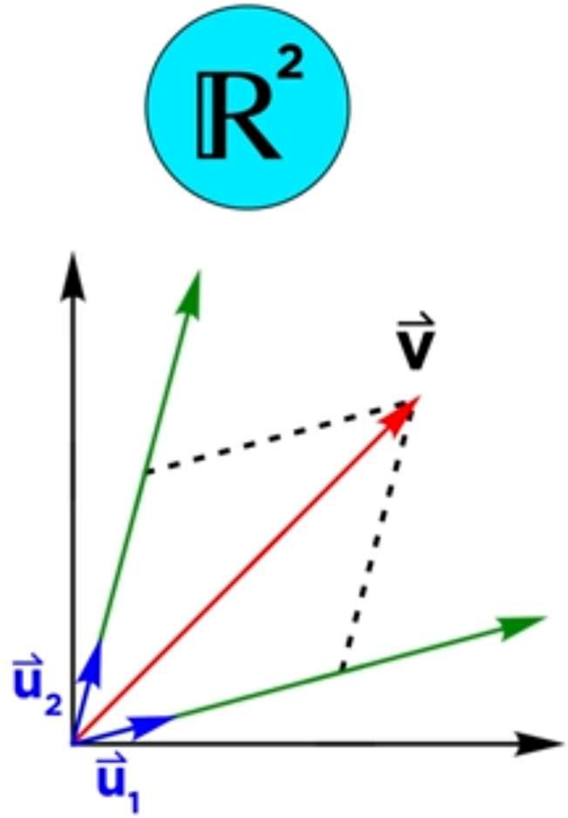
$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\vec{v} = v_1'\vec{u}_1 + v_2'\vec{u}_2$$



Understanding Change of Basis



$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

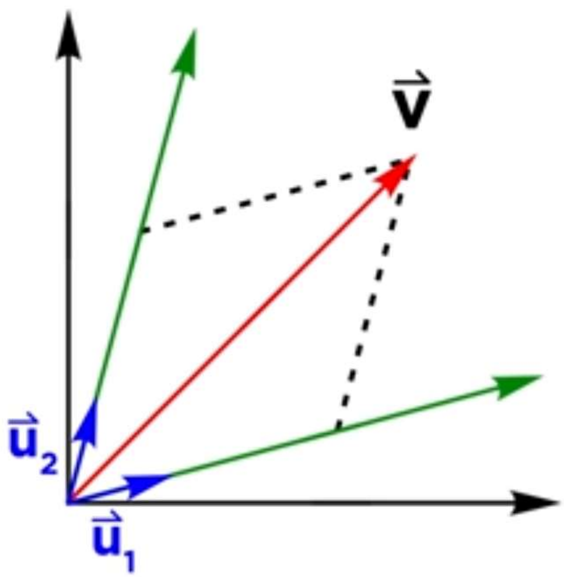
$$\vec{v} = v_1'\vec{u}_1 + v_2'\vec{u}_2$$

these are the
new coordinates
for this basis

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



$$\vec{v} = v_1'\vec{u}_1 + v_2'\vec{u}_2$$

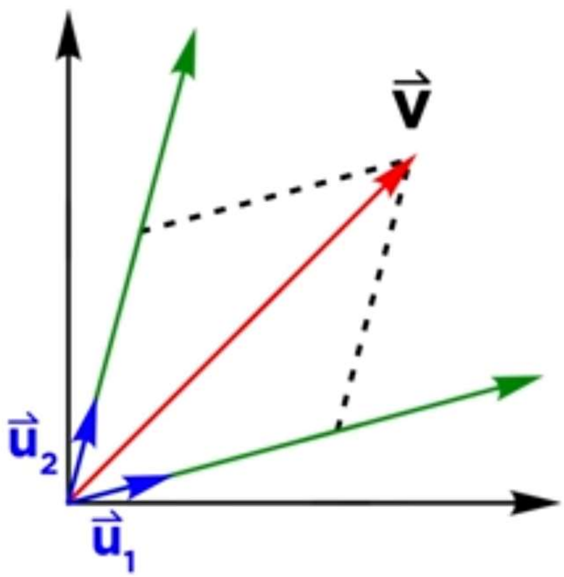
$$\vec{v} = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

Understanding Change of Basis

$\mathbb{R}^2$

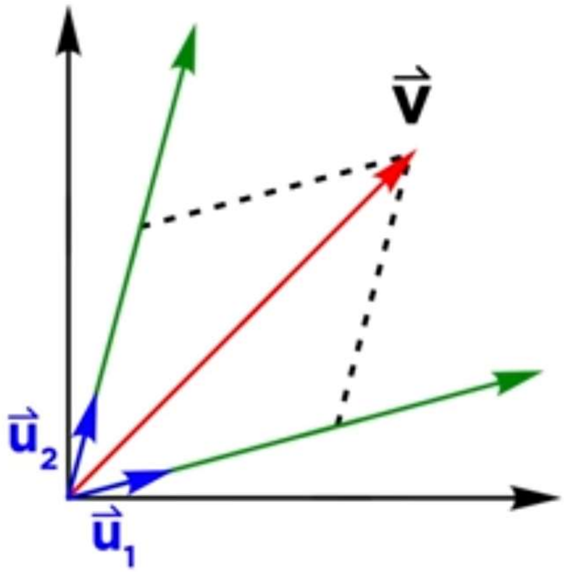
$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$



Understanding Change of Basis

$\mathbb{R}^2$



$$\vec{u}_1 = \begin{bmatrix}1\\2\end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix}3\\3\end{bmatrix}$$

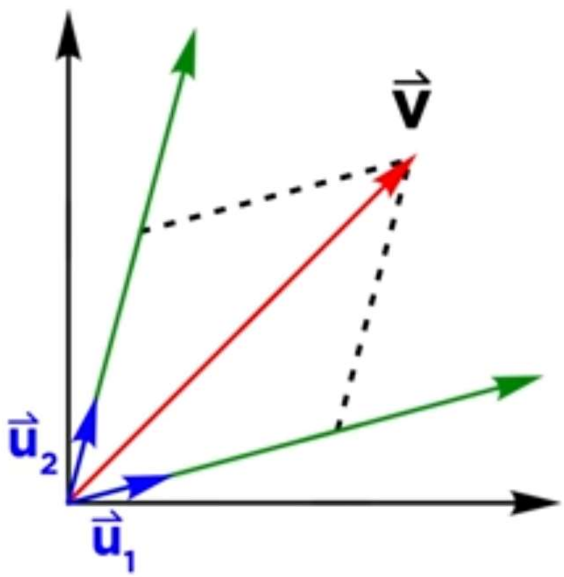
$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

$$\vec{u}_1 = \begin{bmatrix}1\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

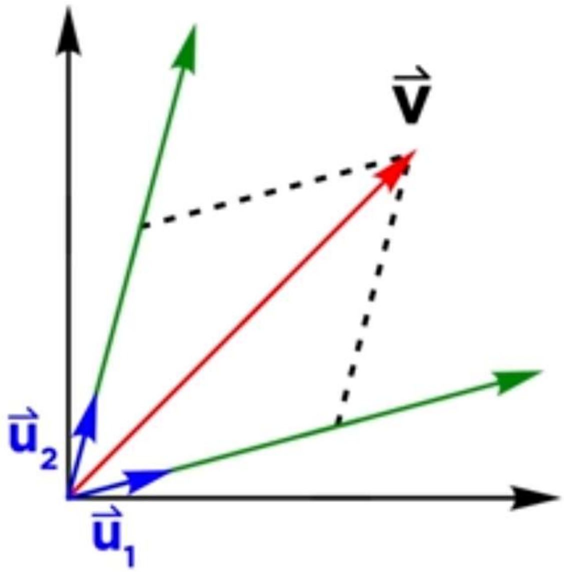
$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \hat{\imath} + 2\,\hat{\jmath}$$

$$\vec{u}_2 = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 3\,\hat{\imath} + 3\,\hat{\jmath}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



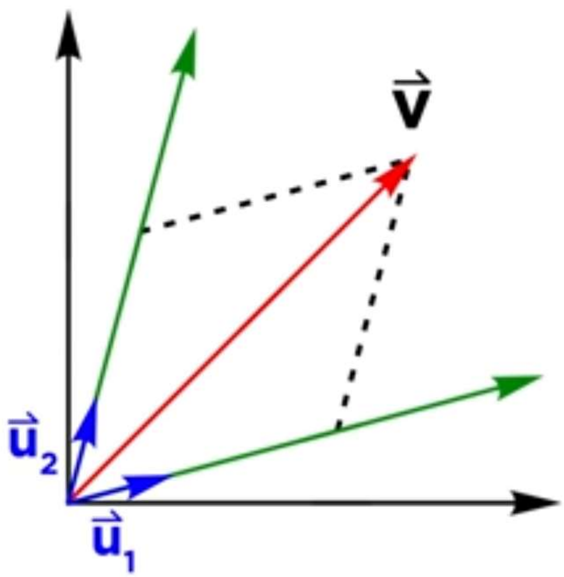
$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

$$v_1'(\hat{\imath} + 2\hat{\jmath}) + v_2'(3\hat{\imath} + 3\hat{\jmath}) = v_1\hat{\imath} + v_2\hat{\jmath}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

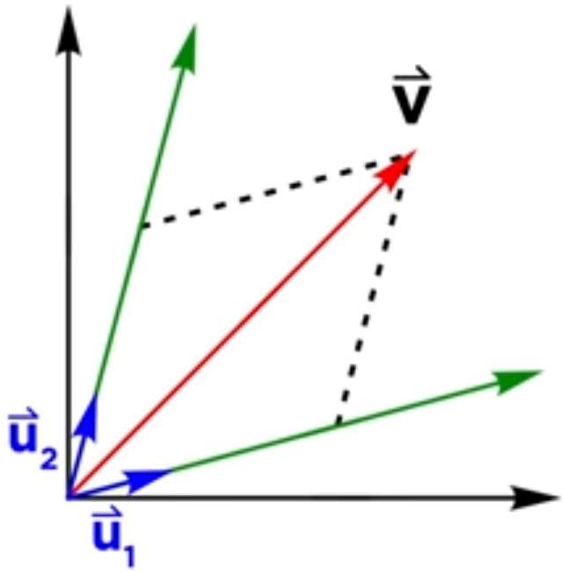
$$v_1'(\hat{\imath} + 2\hat{\jmath}) + v_2'(3\hat{\imath} + 3\hat{\jmath}) = v_1\hat{\imath} + v_2\hat{\jmath}$$

$$(v_1' + 3v_2')\hat{\imath} + (2v_1' + 3v_2')\hat{\jmath} = v_1\hat{\imath} + v_2\hat{\jmath}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

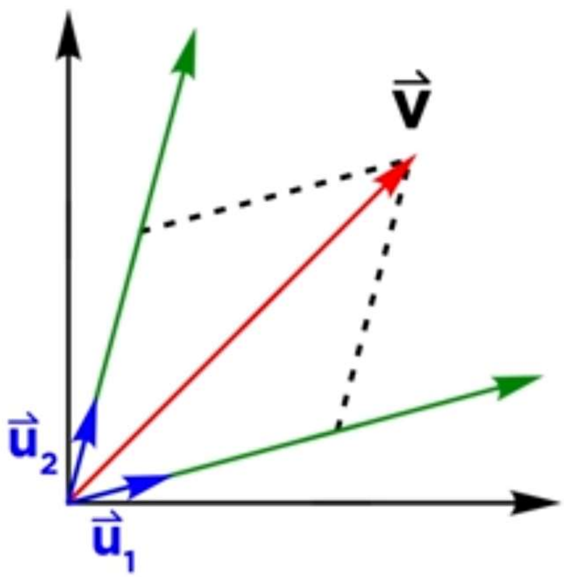
$$v_1'(\hat{\imath} + 2\hat{\jmath}) + v_2'(3\hat{\imath} + 3\hat{\jmath}) = v_1\hat{\imath} + v_2\hat{\jmath}$$

$$(v_1' + 3v_2')\hat{\imath} + (2v_1' + 3v_2')\hat{\jmath} = v_1\hat{\imath} + v_2\hat{\jmath}$$

$$\begin{bmatrix} v_1' + 3v_2' \\ 2v_1' + 3v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$



$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

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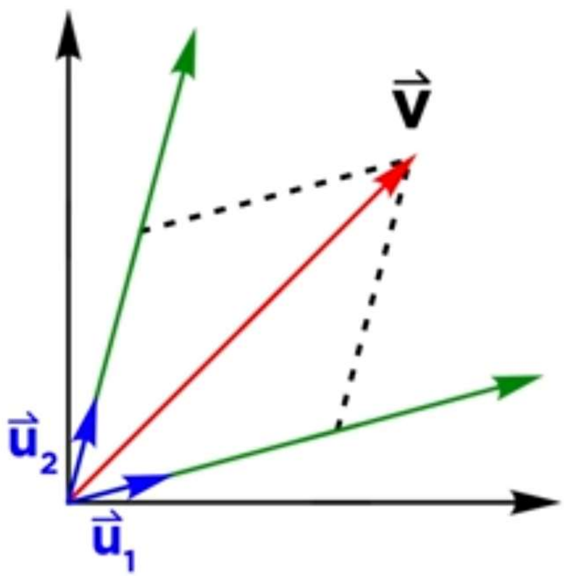
$$(v_1' + 3v_2')\hat{\imath} + (2v_1' + 3v_2')\hat{\jmath} = v_1\hat{\imath} + v_2\hat{\jmath}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$



$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\hat{\imath} + v_2\hat{\jmath}$$

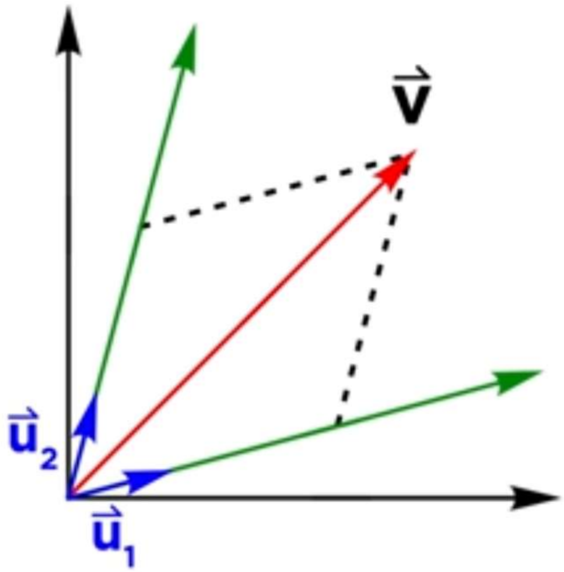
this equation relates the
standard basis coefficients
to the new basis coefficients

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

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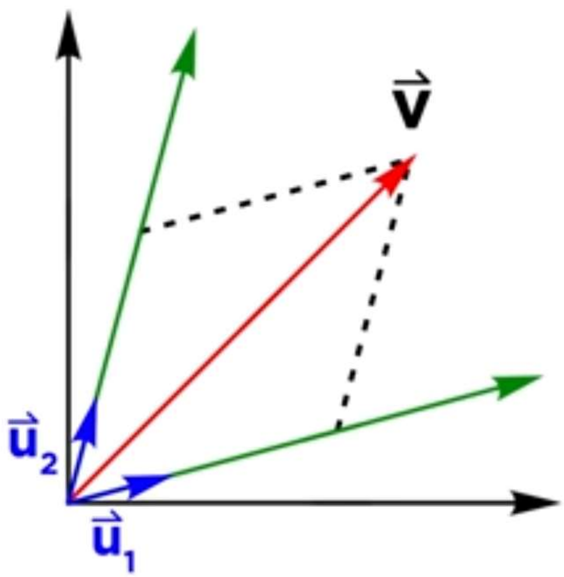
to the new basis coefficients

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

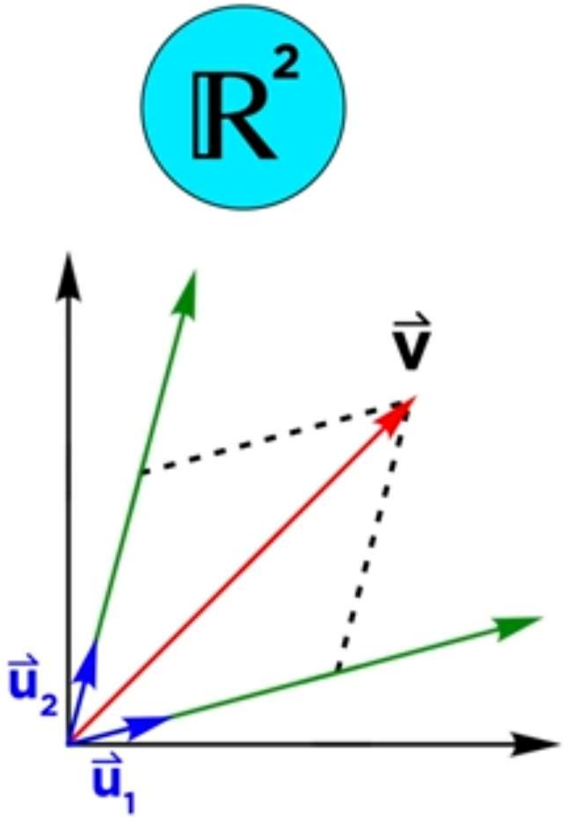


$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

this is called the
transition matrix (U)

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis



$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

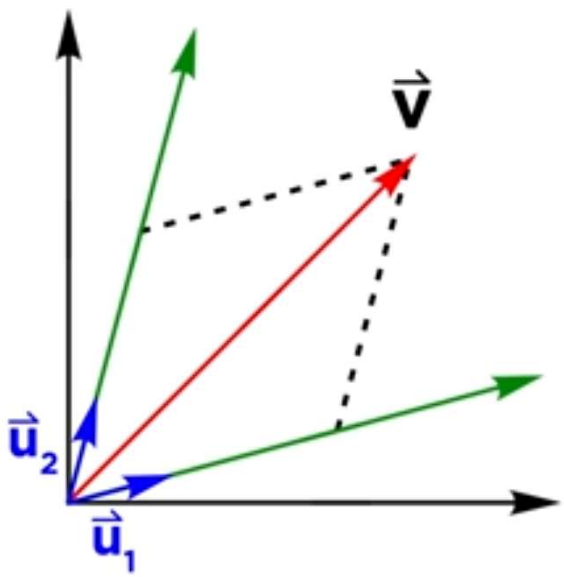
$$v_1' \vec{u}_1 + v_2' \vec{u}_2 = v_1 \hat{\imath} + v_2 \hat{\jmath}$$

$$\vec{v} = U \vec{v}'$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$



$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

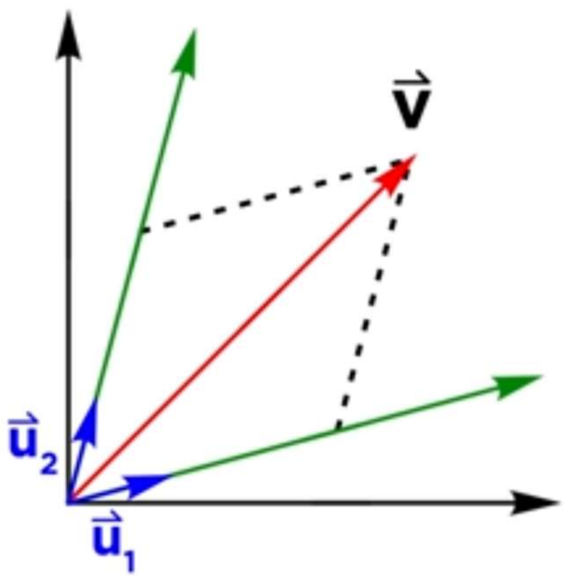
$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

U has columns made of the new basis vectors

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$



$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$v_1'\vec{u}_1 + v_2'\vec{u}_2 = v_1\,\hat{\imath} + v_2\,\hat{\jmath}$$

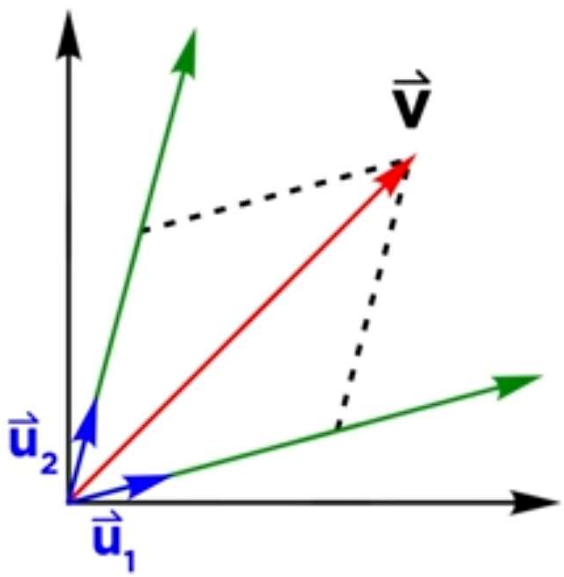
U has columns made of the new basis vectors

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



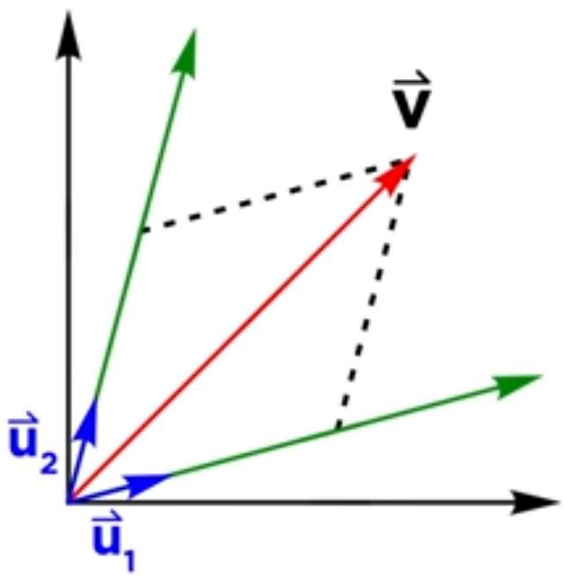
let's find the **coefficients** in this new basis

$$\vec{v} = U\,\vec{v}'$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



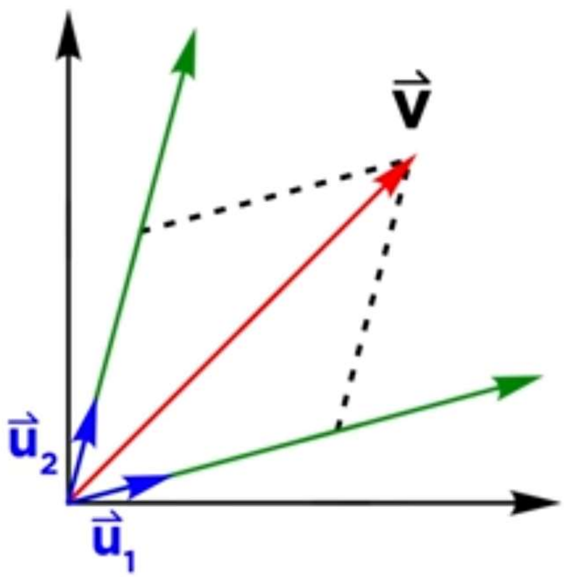
let's find the **coefficients** in this new basis

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



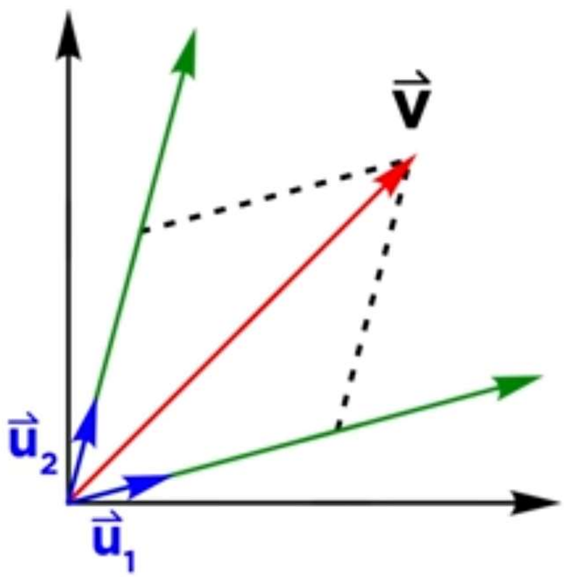
we could use the **transition matrix**

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



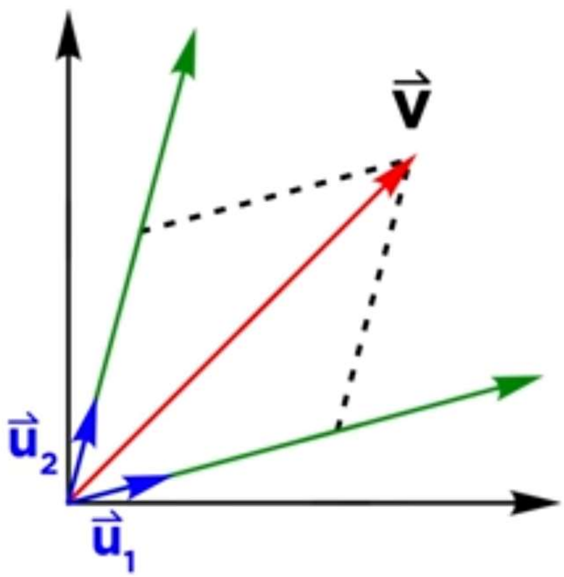
instead let's
isolate this term

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

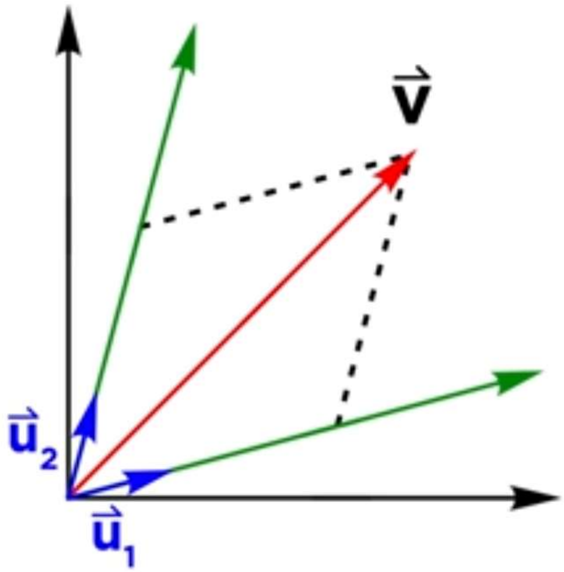


$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



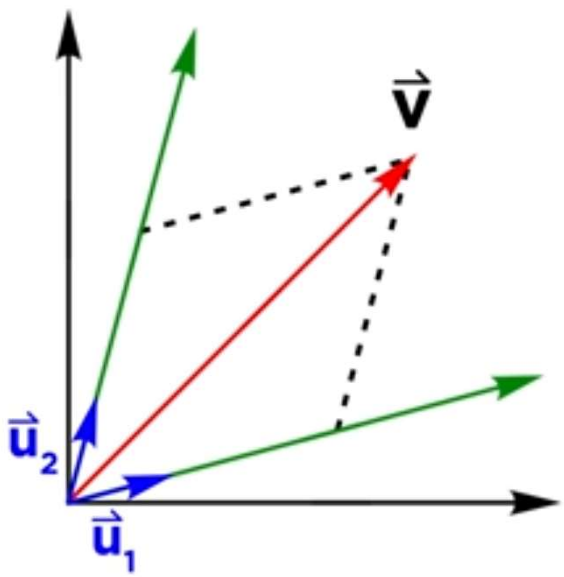
$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$A\,A^{-1} = I$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

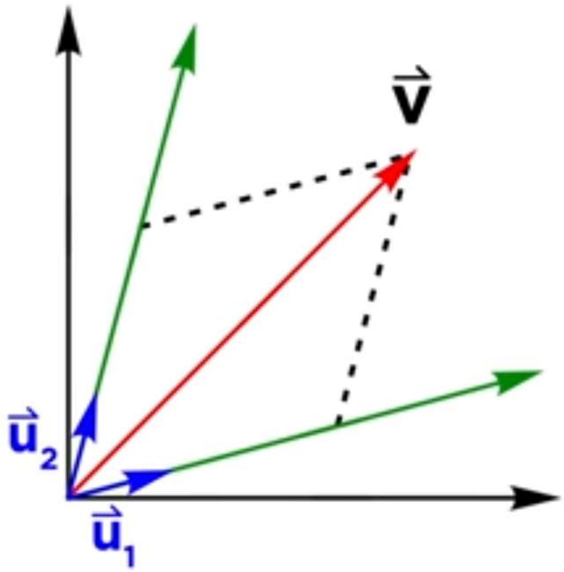


$$U^{-1}\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix} \right)$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



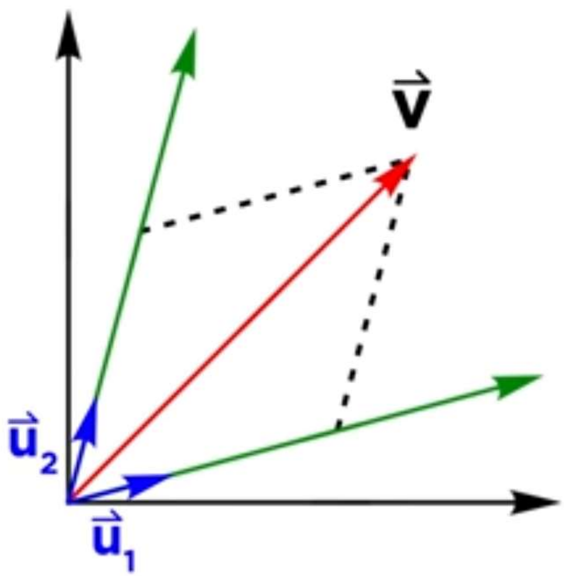
$$U^{-1}\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U \begin{bmatrix} v_1' \\ v_2' \end{bmatrix} \right)$$

matrix multiplication
is not commutative

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

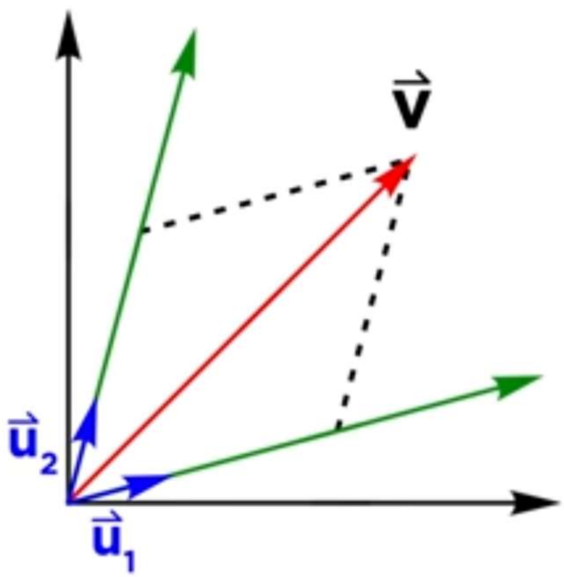


$$U^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = U^{-1}\,U\begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

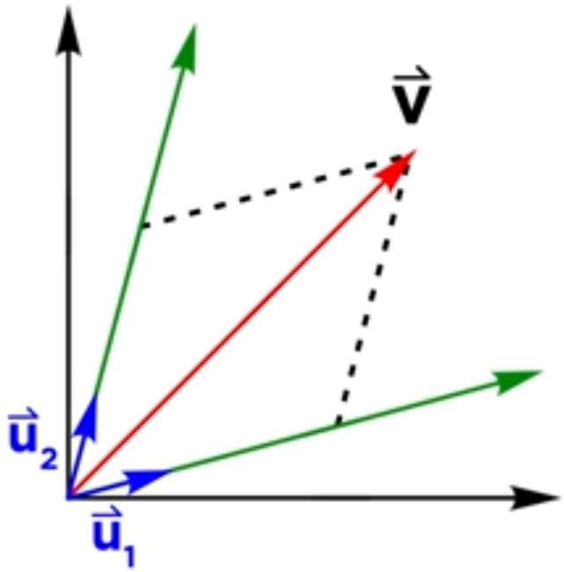


$$U^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



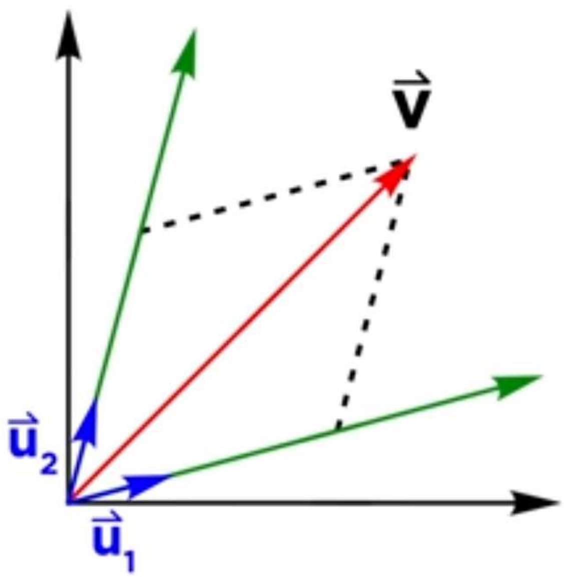
$$U^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



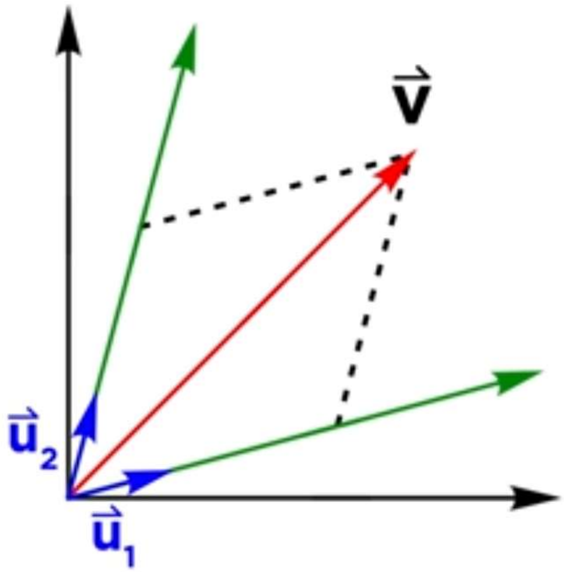
$$U^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \qquad U^{-1} = \begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix} \frac{1}{|U|}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



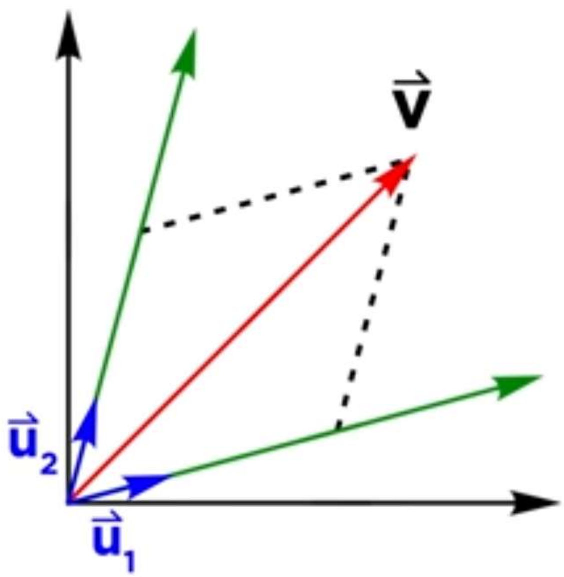
$$-\frac{1}{3}\begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \quad U^{-1} = \begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}\frac{1}{-3}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

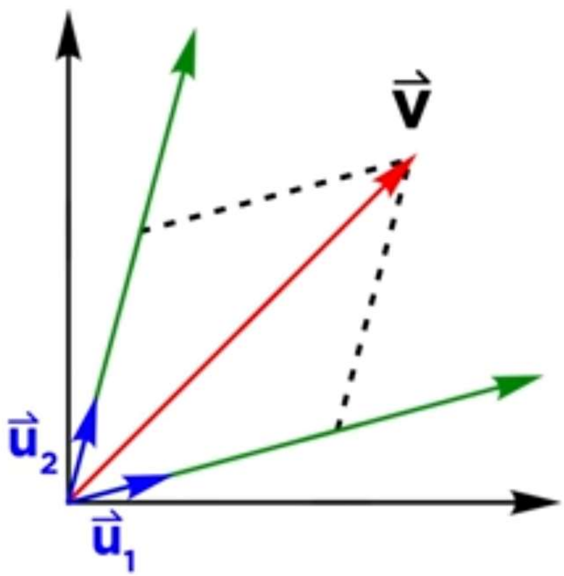


$$-\frac{1}{3}\begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$-\frac{1}{3}\begin{bmatrix} (3)(2) + (-3)(3) \\ (-2)(2) + (1)(3) \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$



$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

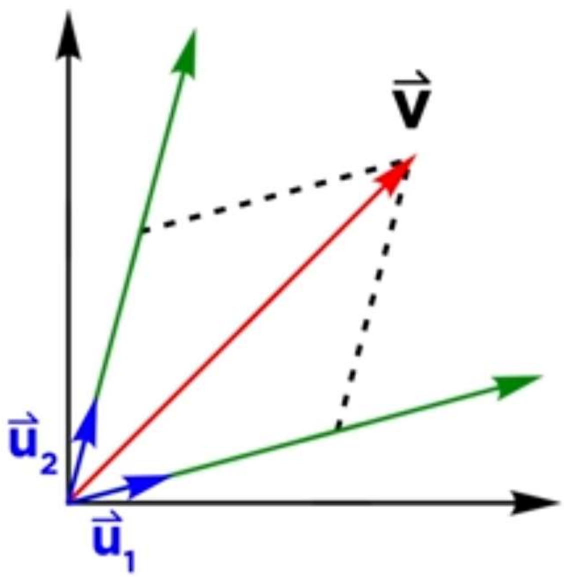
$$-\frac{1}{3}\begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$-\frac{1}{3}\begin{bmatrix} -3 \\ -1 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



$$-\frac{1}{3}\begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

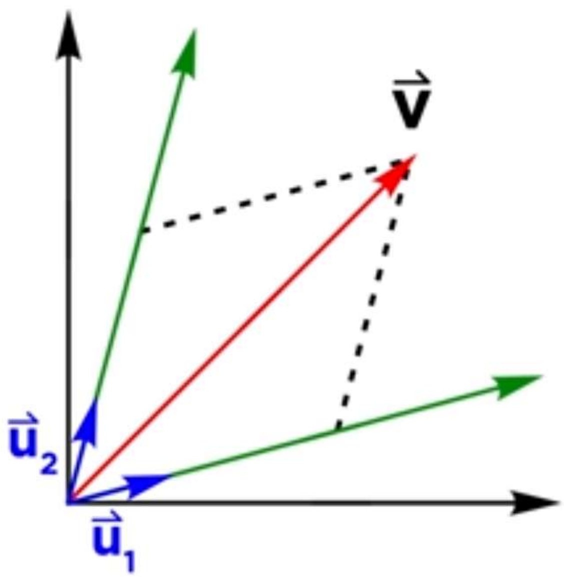
$$\begin{bmatrix} 1 \\ 1/3 \end{bmatrix} = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

coordinates in the new basis

Understanding Change of Basis

$\mathbb{R}^2$

$$\vec{v} = \begin{bmatrix}2\\3\end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix}1\\2\end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix}3\\3\end{bmatrix}$$



$$-\frac{1}{3}\begin{bmatrix}3 & -3\\-2 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}v_1'\\v_2'\end{bmatrix}$$

$$\vec{v} = \vec{u}_1 - \frac{1}{3}\vec{u}_2$$

coordinates in the new basis

Relating Two Non-Standard Bases

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}$$

this transition matrix relates coordinates in this basis to the standard basis only

Relating Two Non-Standard Bases

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}$$

relating two non-standard bases requires an extra step using the standard basis as a stepping stone

Relating Two Non-Standard Bases

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \qquad \vec{w}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \vec{w}_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \qquad W = \begin{bmatrix} -1 & 3 \\ -1 & 0 \end{bmatrix}$$

relates to the **standard basis** (for U)

relates to the **standard basis** (for W)

Relating Two Non-Standard Bases

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \qquad \vec{w}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \vec{w}_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \qquad W = \begin{bmatrix} -1 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\vec{v} = U\,\vec{v}_u \qquad \vec{v} = W\,\vec{v}_w$$

Relating Two Non-Standard Bases

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \qquad \vec{w}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \vec{w}_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} \qquad W = \begin{bmatrix} -1 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\vec{v} = U\,\vec{v}_u \qquad \vec{v} = W\,\vec{v}_w$$

$$U\,\vec{v}_u = W\,\vec{v}_w$$

Relating Two Non-Standard Bases

$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{u}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $\vec{w}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $\vec{w}_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$U = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}$ $W = \begin{bmatrix} -1 & 3 \\ -1 & 0 \end{bmatrix}$

$\vec{v} = U\,\vec{v}_u$ $\vec{v} = W\,\vec{v}_w$

$U\,\vec{v}_u = W\,\vec{v}_w$

$\vec{v}_u = U^{-1}W\,\vec{v}_w$ $\vec{v}_w = W^{-1}U\,\vec{v}_u$

CHECKING COMPREHENSION

(press pause for more time)

1) Give the transition matrix for the following basis:

$\langle 0, 1, 1\rangle \quad \langle -2, 1, 0\rangle \quad \langle -1, 0, -1\rangle$

2) Given the vector $\langle 5, 4\rangle$ write this in terms of the basis:

$u_1 = \langle 2, -1\rangle \quad u_2 = \langle 1, 4\rangle$

CHECKING COMPREHENSION

(press pause for more time)

1) Give the transition matrix for the following basis:

<0, 1, 1>   <-2, 1, 0>   <-1, 0, -1>

$$\begin{bmatrix} 0 & -2 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

2) Given the vector <5, 4> write this in terms of the basis:

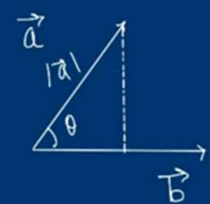
$u_1 = \langle 2, -1 \rangle \quad u_2 = \langle 1, 4 \rangle$

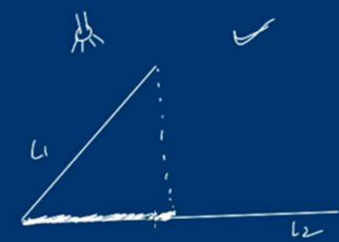
$(16/9)u_1 + (13/9)u_2$

Projection

Projection of one Vector on other

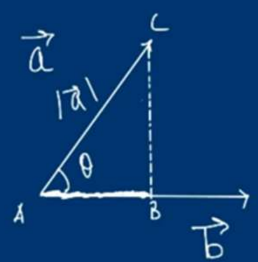
projection of $\vec{a}$ on $\vec{b}$





Projection of one Vector on other

Projection of $\vec{a}$ on $\vec{b}$



$$\cos\theta = \frac{B}{H} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|}$$

$$AB = AC \cdot \cos\theta$$

$$AB = |\vec{a}| \cos\theta$$

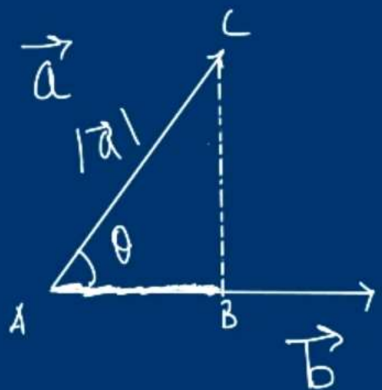
$AB \rightarrow$ Projection of $\vec{a}$ on $\vec{b}$

$$AB = \cancel{|\vec{a}|}\, \frac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}\,|\vec{b}|}$$

$$AB = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \quad \left\{ \begin{array}{l} \text{Projection of } \vec{a} \\ \text{on } \vec{b} \end{array} \right\}$$

Projection of one Vector on other

projection of $\vec{a}$ on $\vec{b}$



$$\cos\theta = \frac{B}{H} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|}$$

$$AB = AC \cdot \cos\theta$$

$$AB = |\vec{a}| \cos\theta$$

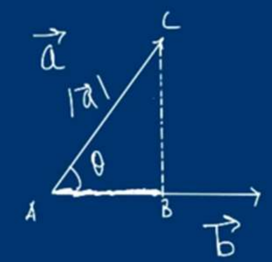
$$AB = \cancel{|\vec{a}|}\,\frac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}\,|\vec{b}|}$$

AB → Projection of $\vec{a}$ on $\vec{b}$

$$AB = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \quad \left\{ \begin{array}{l} \text{Projection of } \vec{a} \\ \text{on } \vec{b} \end{array} \right\}$$

Projection of one Vector on other

projection of $\vec{a}$ on $\vec{b}$



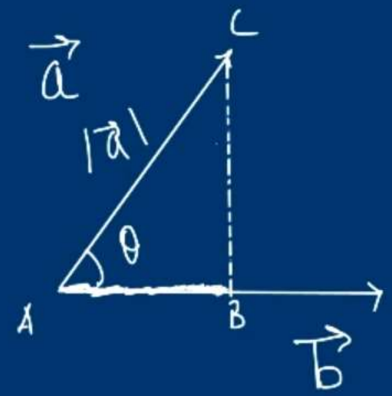
$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

a

Projection of one Vector on other

projection of $\vec{a}$ on $\vec{b}$



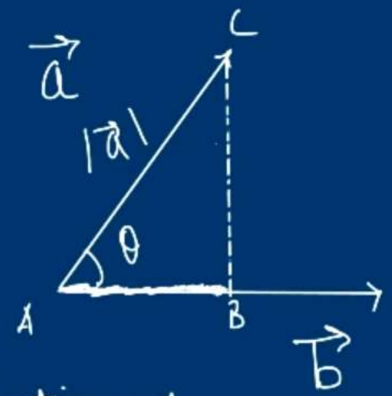
$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$ ✓

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$ ✓

a

Projection of one Vector on other

projection of $\vec{a}$ on $\vec{b}$



$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

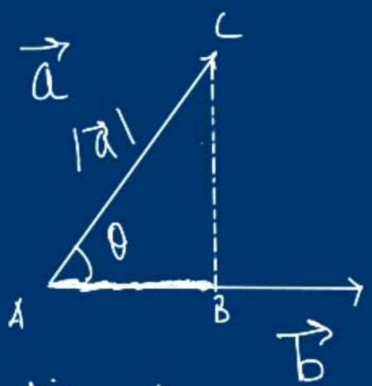
$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

Projection of

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ on $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

Projection of one Vector on other

projection of $\vec{a}$ on $\vec{b}$



$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

Projection of

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ on $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|b|} = \frac{2\times 1 + 1\times 1 + 3\times 1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{6}{\sqrt{3}}$$

Dot Product of 2 Vectors

[2]

```
a = np.array([2, 3])

b = np.array([4, 4])

a_dot_b = np.dot(a, b)
```

[3]

```
print(a_dot_b)
```

```
20
```

[4]

```
c = np.array([40, 20, 35])

d = np.array([53, 24, 68])

c_dot_d = np.dot(c, d)
```

```
print(c_dot_d)
```

```
4980
```

5.1.5. Vector Operations - in Python - Part 2.ipynb

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Cross Product of 2 Vectors

```
a = np.array([2, 3])

b = np.array([4, 4])

a_cross_b = np.cross(a, b)
```

[7]
```
print(a_cross_b)
```

```
-4
```

```
c = np.array([5, 10, 20])


```

[6]

```

b = np.array([4, 4])

a_cross_b = np.cross(a, b)
```

[7]

```
print(a_cross_b)
```

```
-4
```

[8]

```
c = np.array([5, 10, 20])

d = np.array([18, 32, 50])

c_cross_d = np.cross(c, d)
```

```
print(c_cross_d)
```

```
[-140  110  -20]
```

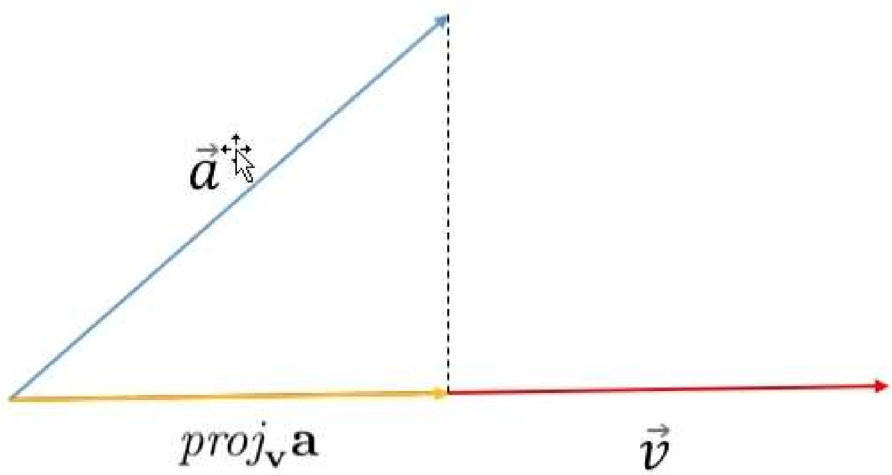
[8]

```
c = np.array([5, 10, 20])

d = np.array([18, 32, 50])

c_cross_d = np.cross(c, d)
```

```
print(c_cross_d)
```

```
[-140  110  -20]
```

Projection of Vector



$$proj_{\mathbf{v}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$$

Projection of "a" vector on "v" vector

$$proj_{\mathbf{v}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$$

```
a = np.array([2, 5])

v = np.array([8, -6])

# magnitude of "v" vector
magnitude_of_v = np.sqrt(sum(v**2))

proj_of_a_on_v = (np.dot(a,v)/magnitude_of_v**2)*v
```

$$proj_{\mathbf{v}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$$

```python
a = np.array([2, 5])

v = np.array([8, -6])

# magnitude of "v" vector
magnitude_of_v = np.sqrt(sum(v**2))

proj_of_a_on_v = (np.dot(a,v)/magnitude_of_v**2)*v

print('Projection of a vector on v vector = ', proj_of_a_on_v)
```

```
Projection of a vector on v vector =  [-1.12  0.84]
```