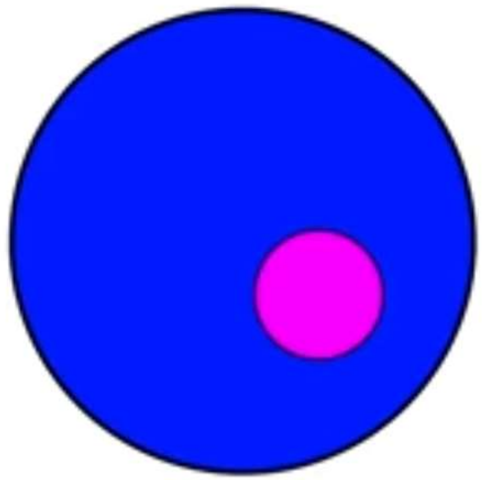
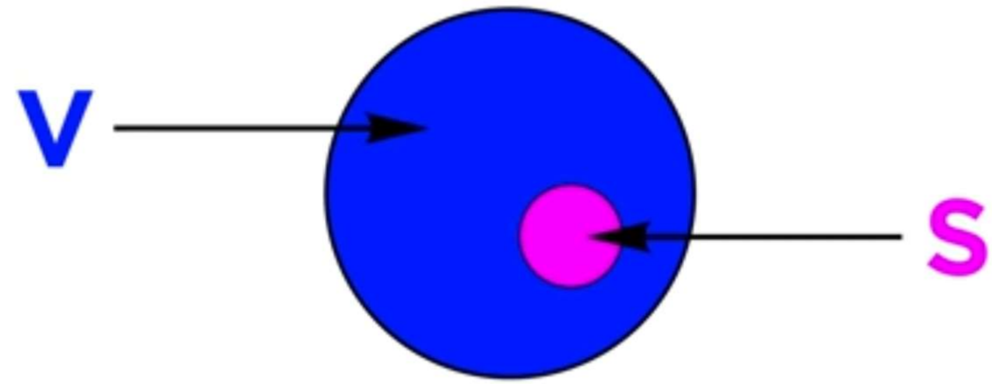
Vector Subspace

vector
space



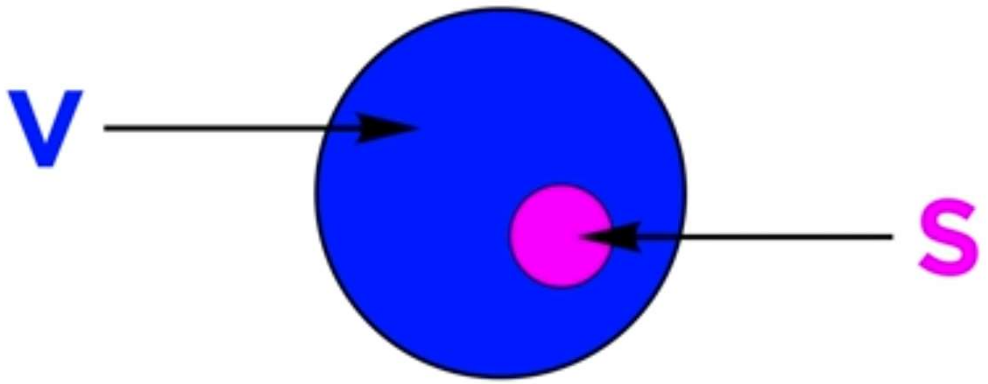
subspace

Properties of Subspaces



every element of S is also an element of V

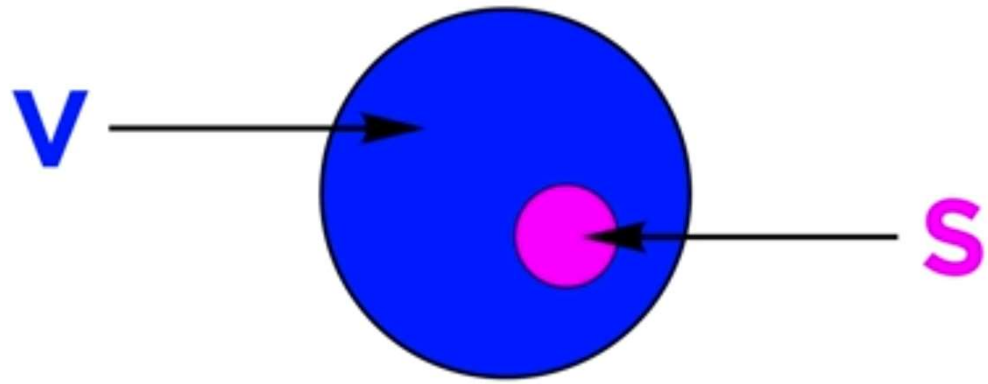
Properties of Subspaces



1) given $\vec{a} \in S$ and scalar c, then $c\vec{a} \in S$

2) given $\vec{a} \in S$ and $\vec{b} \in S$, then $\vec{a} + \vec{b} \in S$

Properties of Subspaces

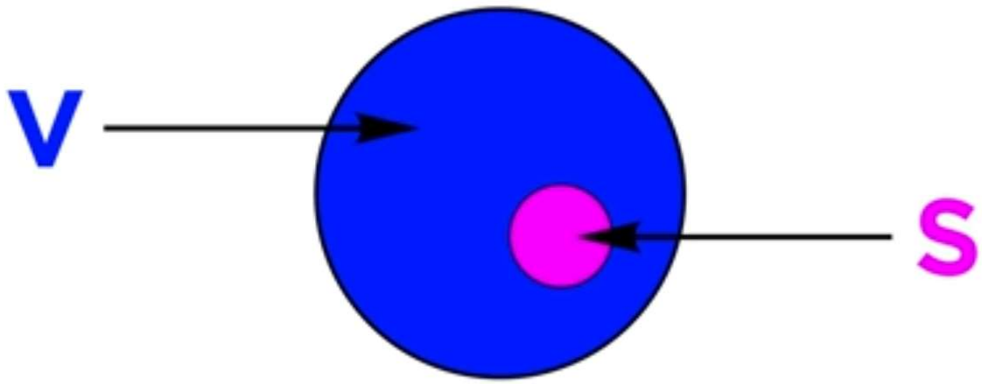


if S is closed then it is a vector space

1) given $\vec{a} \in S$ and scalar c, then $c\vec{a} \in S$

2) given $\vec{a} \in S$ and $\vec{b} \in S$, then $\vec{a} + \vec{b} \in S$

Properties of Subspaces

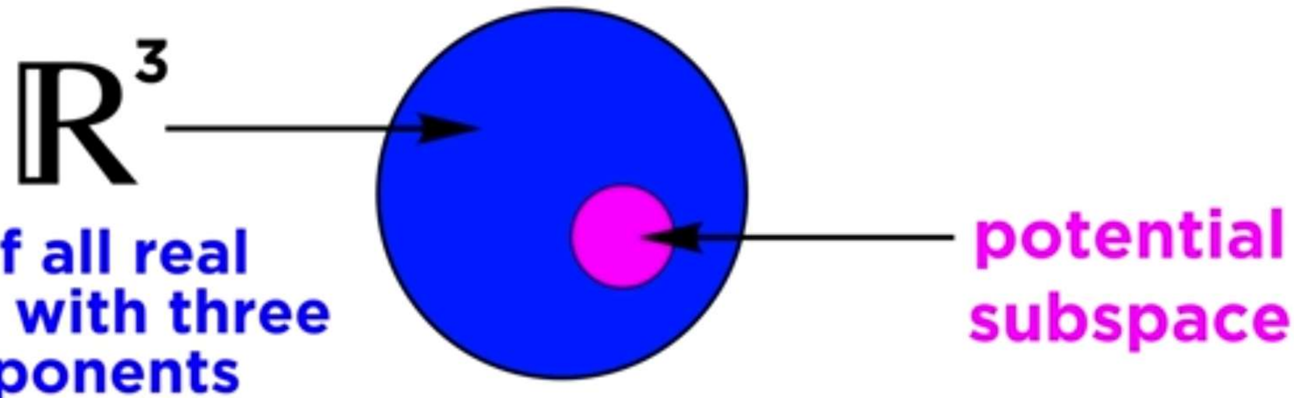


if S is closed then it is a vector space
and it is therefore a subspace of V

1) given $\vec{a} \in S$ and scalar c, then $c\vec{a} \in S$

2) given $\vec{a} \in S$ and $\vec{b} \in S$, then $\vec{a} + \vec{b} \in S$

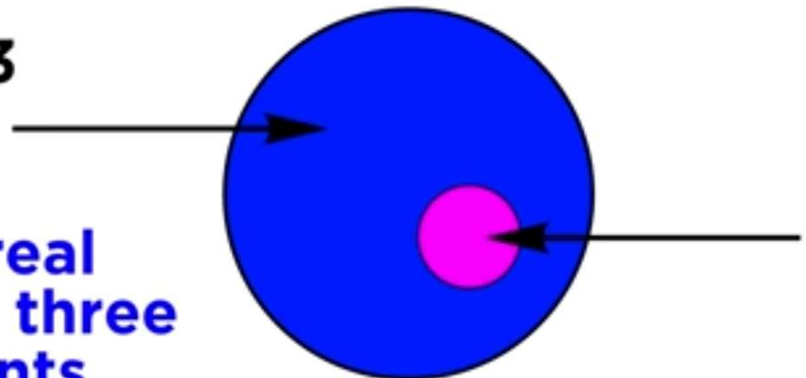
Examples of Subspaces



$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Examples of Subspaces

$\mathbb{R}^3$



S

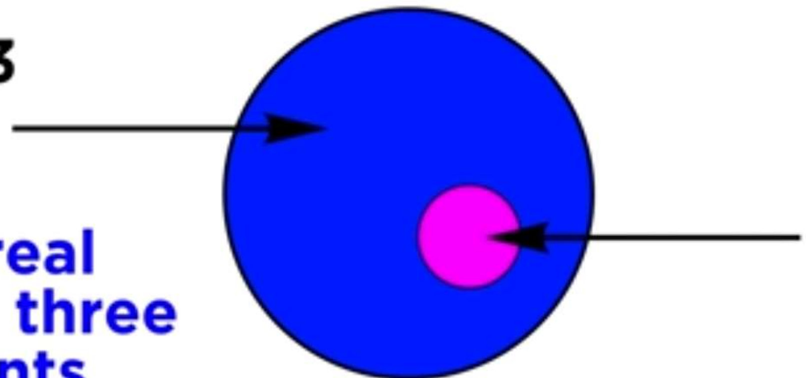
set of all real vectors with three components

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Examples of Subspaces

$\mathbb{R}^3$



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$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

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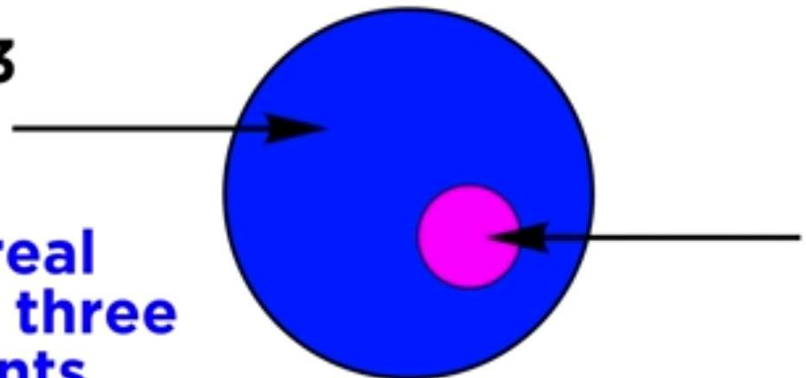
every element in S is necessarily also in $\mathbb{R}^3$

Examples of Subspaces

$\mathbb{R}^3$



S

set of all real vectors with three components

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}$$

let's check if S is closed

Examples of Subspaces

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}$$

$$c\vec{x} = c\begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}$$

1) given $\vec{x} \in S$ and scalar c, then $c\vec{x} \in S$

2) given $\vec{x} \in S$ and $\vec{y} \in S$, then $\vec{x} + \vec{y} \in S$

Examples of Subspaces

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}$$

$$c\vec{x} = c\begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} = \begin{bmatrix} cx \\ 0 \\ -(cx) \end{bmatrix}$$

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1) given $\vec{x} \in S$ and scalar c, then $c\vec{x} \in S$ ✓

2) given $\vec{x} \in S$ and $\vec{y} \in S$, then $\vec{x} + \vec{y} \in S$

Examples of Subspaces

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y \\ 0 \\ -y \end{bmatrix}$$

1) given $\vec{x} \in S$ and scalar c, then $c\vec{x} \in S$

2) given $\vec{x} \in S$ and $\vec{y} \in S$, then $\vec{x} + \vec{y} \in S$

Examples of Subspaces

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y \\ 0 \\ -y \end{bmatrix}$$

$$\vec{x} + \vec{y} = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} + \begin{bmatrix} y \\ 0 \\ -y \end{bmatrix} = \begin{bmatrix} x + y \\ 0 \\ -(x + y) \end{bmatrix}$$

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Understanding Span

$$\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n \in V$$

Understanding Span

$$\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n \in V$$

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots a_n\vec{v}_n$$

Understanding Span

$$\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n \in V$$

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots a_n\vec{v}_n$$

this is a linear combination

Understanding Span

$$\vec{v}_1, \vec{v}_2, \dots \vec{v}_n \in V$$

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \dots a_n\vec{v}_n$$

set of all linear combinations is the span

Understanding Span

$\mathbb{R}^3$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3$$

Understanding Span

$\mathbb{R}^3$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

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Understanding Span

$\mathbb{R}^3$

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$$\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{bmatrix} 2a \\ a \\ -a \end{bmatrix} + \begin{bmatrix} 0 \\ 2b \\ 2b \end{bmatrix} + \begin{bmatrix} -c \\ -c \\ -c \end{bmatrix} = \begin{bmatrix} 2a - c \\ a + 2b - c \\ -a + 2b - c \end{bmatrix}$$

Understanding Span

$\mathbb{R}^3$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

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the span of any number of elements of vector space V is also a subspace of V

Understanding Span

$\mathbb{R}^3$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathrm{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{bmatrix} 2a \\ a \\ -a \end{bmatrix} + \begin{bmatrix} 0 \\ 2b \\ 2b \end{bmatrix} + \begin{bmatrix} -c \\ -c \\ -c \end{bmatrix} = \begin{bmatrix} 2a - c \\ a + 2b - c \\ -a + 2b - c \end{bmatrix}$$

this is the smallest subspace of V
that contains this set of elements

Understanding Span

$\mathbb{R}^3$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{bmatrix} 2a \\ a \\ -a \end{bmatrix} + \begin{bmatrix} 0 \\ 2b \\ 2b \end{bmatrix} + \begin{bmatrix} -c \\ -c \\ -c \end{bmatrix} = \begin{bmatrix} 2a - c \\ a + 2b - c \\ -a + 2b - c \end{bmatrix}$$

span is important for describing vector spaces

CHECKING COMPREHENSION

(press pause for more time)

1) True or False:

The set of matrices of the form $\begin{bmatrix} a & b \\ -b & c \end{bmatrix}$ is a subspace of $\mathbb{R}^{2 \times 2}$

2) If $\vec{v}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, find their span.

CHECKING COMPREHENSION

(press pause for more time)

1) True or False:

The set of matrices of the form $\begin{bmatrix} a & b \\ -b & c \end{bmatrix}$ is a subspace of $\mathbb{R}^{2 \times 2}$

True

2) If $\vec{v}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, find their span.

$$\begin{bmatrix} 5a \\ a - b \end{bmatrix}$$

MULTIVARIATE CALCULUS
– INTRODUCTION

Business – sales – Pricing
Marketing
Brand value

$F = ma$

MULTIVARIATE CALCULUS

MULTI → More than 1

VARIATE → Variable

$f(x)$ $f(x, y)$

$f(x, y, z)$

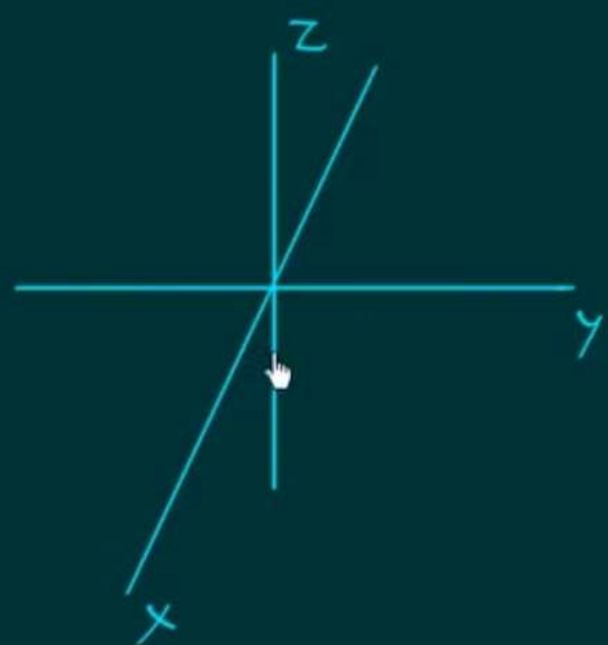
$f(u, v, w, x)$

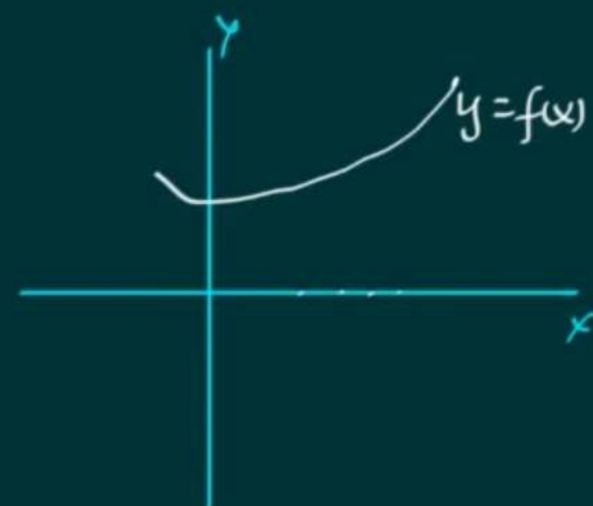
CALCULUS
1. Limit & continuity
2. Differentiation – Max & Minima, Approximation, Tangents & Normal
3. Integration

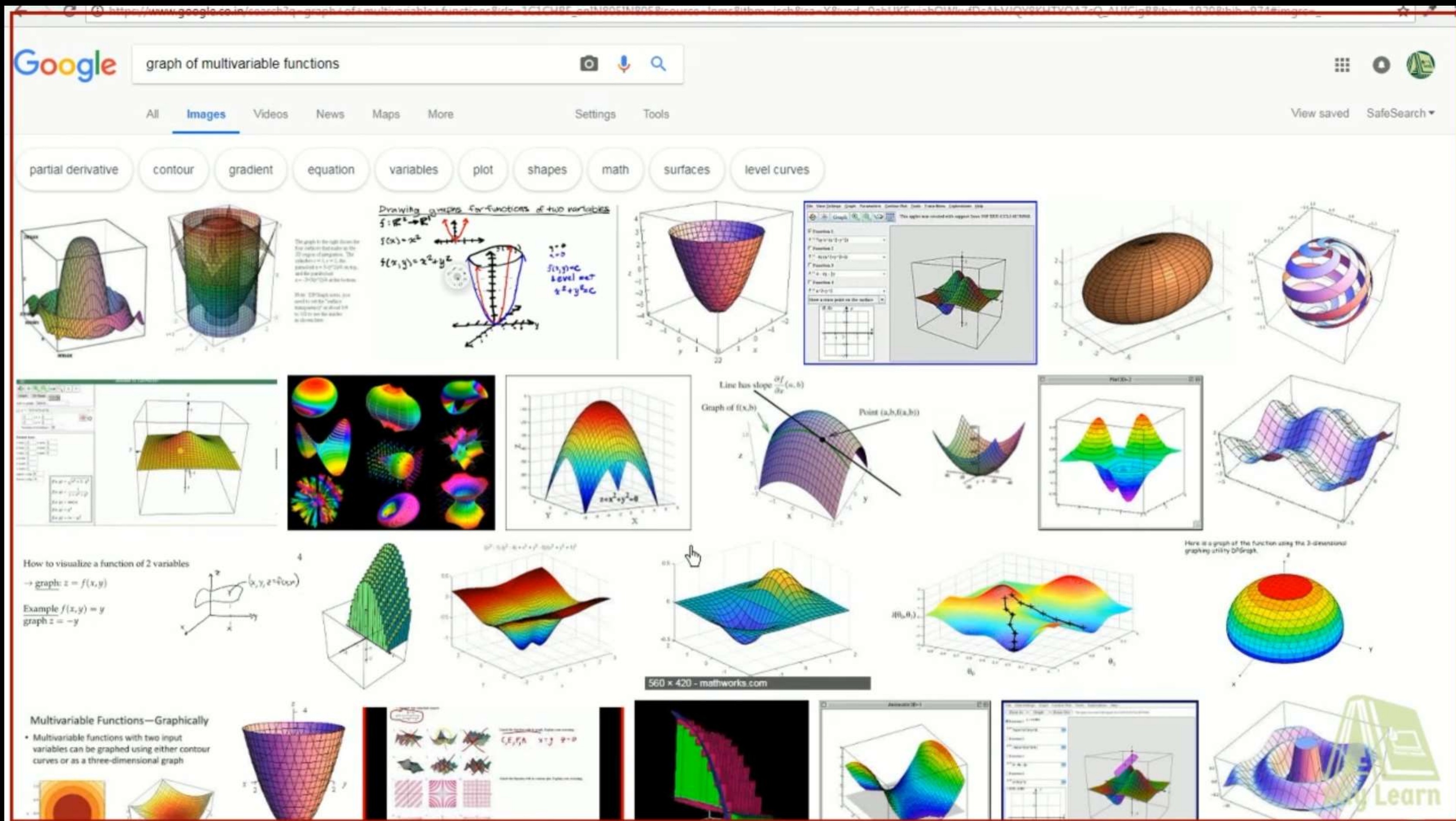
Functions

① $f = x + y$ $\quad f(x,y) = x + y$

② $f(x,y) = x^2 + y^2$ $\quad$ 3-Dimensional $\quad$ 1 variable → 2D



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surfaces
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How to visualize a function of 2 variables
Multivariable Functions—Graphically
Multivariable functions with two input variables can be graphed using either contour curves or as a three-dimensional graph
560 × 420 - mathworks.com

FUNCTIONS OF SEVERAL VARIABLES

Function of n variable –

A function of n variable is a rule that assigns to each n-tuples $(x_1, x_2 \dots x_n)$ in a set D a unique $f(x_1, x_2 \dots x_n)$.

The set D is called Domain of function and the corresponding values of $f(x_1, x_2 \dots x_n)$ constitute the Range of f.

In particular function of two variables is a rule that assigns to each ordered pair (x, y) in a set D a unique number $f(x, y)$.

$$f : D \to \mathbb{R}$$
$$(x, y) \mapsto f(x, y)$$

$$\underbrace{D \subseteq R \times R}_{\text{plane}}$$

$$z = f(x, y)$$

Dependent Variable — Independent variables

$$z = x^2 + y^2$$

Note – Often the rule will be given as formula, and unless otherwise stated, we will assume that the domain is the largest set of points in the plane (or in R^3) for which the formula is defined or real valued.

Derivative

Tells us how fast something is changing

⇓

Rate of Change

Average Rate of Change | Instantaneous Rate of change

Rate of Change

#1 Whenever something is changing ①

#2 Changing w.r.t. something else ②

#3 Defined a $\boxed{\text{RELATIONSHIP}} = \frac{\Delta 1}{\Delta 2}$

⇓

Rate of Change

Why Bother?

Travelled 100km, 200km more to go

& imagine NO speedometer

Try to find time to cover 200km

⇓

Not an easy problem to solve

⇓

So much easy to define a Relationship between Distance & time

⇓

Every field has its own rate of change
We just call them by different names e.g.
Run Rate, Stepness, Interest Rate

Derivative

① —— 200km —— ②

$$\text{SPEED} = \frac{\text{Change in Distance } ① \; ②}{\text{Change in time } ① \; ②} = \frac{200-0}{5-0} = \frac{40\,km}{hr}$$

You Need **TWO** points to find Speed

Our formula gives us AVERAGE speed

What determines if driver will survive or not

⇓

Speed at the time of collosion matters

⇓

central question behind derivate

⇓

Whats speed at time of collosion **OR** speed at an Instant **OR** Instantaneous Speed

PAINFUL to FIND

$$\underset{(t=5s)}{\text{SPEED}} = \frac{\text{Change in Distance}}{\text{Change in time}} = \frac{0}{0} = \text{Meaningless}$$

You just have ONE point & hence CAN'T use the formula

Derivative

Solution is slightly Subtle & has 3 Steps

Step #1 Choose some point as 2nd point. Only criteria, it is extremely close to point where you need to find speed

Exp For speed $(t=5)$, Choose $4.999 - 5$

Step #2 Find Avg. speed over this small interval

$$\frac{\text{Change in Distance } ① \,\&\, ②}{0.001} = \frac{\Delta D}{\Delta t}$$

Step #3. Choose interval to be extremely close to 0, but NOT 0

$$\boxed{\text{Speed}_{(t=5)} = \lim_{\Delta t \to 0} \frac{\Delta D}{\Delta t}}$$

Created imaginary numbers called as infinitesimals

< Reals numbers, but > 0

Power of generalization

$\boxed{Y = x^2}$ $\boxed{D = \frac{1}{2} g t^2}$

Step #1 2nd point $x + \Delta x$

Step #2. $\dfrac{\Delta y}{\Delta x} = \dfrac{(x+\Delta x)^2 - x^2}{\Delta x} = \dfrac{(\Delta x)^2 + 2x\Delta x}{\Delta x}$

Step #3 (i) $\Delta x \neq 0$, $\dfrac{\Delta y}{\Delta x} = \Delta x + 2x$

(ii) $\Delta x \to 0$, $\dfrac{\Delta y}{\Delta x} = \boxed{2x}$

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x}$$

$\dfrac{dy}{dx}$ is called derivative of y w.r.t. x

$$\frac{d}{dx}1 = 0$$

$$\frac{d}{dx}x = 1$$

$$2x^{2-1}$$

$$2x^1$$

$$2x$$

$$\frac{d}{dx}x^2 = 2x$$

$$\frac{d}{dx}2x = 2 \times 1 = 2$$

$$\frac{d}{dx}(1+x^2) = 0+2x$$
$$= 2x$$

$$\frac{d}{dx}(1-x^2) = 0-2x$$
$$= -2x$$

$$\boxed{\frac{d}{dx}x^n = nx^{n-1}}$$

$$\frac{d}{dx} e^{x} = e^{x} \times 1 = e^{x}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^{2} x$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^{2}}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^{2}}}$$

$$\frac{d}{dx} e^{f(x)} = e^{f(x)} \times \frac{d}{dx} f(x)$$

$$\frac{d}{dx} \sin f(x) = \cos f(x) \frac{d}{dx} f(x)$$

$$\frac{d}{dx} \cos f(x) = -\sin f(x) \frac{d}{dx} f(x)$$

$$\frac{d}{dx} [f(x)]^{n} = n[f(x)]^{n-1} \frac{d}{dx} f(x)$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^{2}}}$$

$$\frac{d}{dx} \log x = \frac{1}{x}$$

x^{-1}

$$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^{2}}$$

$-1x^{-1-1}$

$-1x^{-2}$

$\frac{-1}{x^{2}}$

$$\frac{d}{dx} a^{x} = a^{x} \log a$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

Concept of Chain Rule:

$y=(x^3+1)^6$, $y'=?$

$y' = \frac{dy}{dx}$

let $(x^3+1) = u$

$y = u^6$

$\frac{dy}{du} = 6\times u^5 = 6u^5$ — (i)

$u = x^3+1$

$\frac{du}{dx} = 3x^2+0$

$= 3x^2$

$\frac{du}{dx} = 3x^2$ — (ii)

$\frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx} = 6u^5 \times 3x^2$

$\frac{dy}{dx} = 18u^5 \times x^2$

$= \underline{18(x^3+1)^5 \times x^2}$

Concept of Chain Rule:

$y = \sin(\cos(\log_e x))$

$\frac{dy}{dx} = ?$

let $u = \cos(\log_e x)$

$y = \sin u$

$\frac{dy}{du} = \cos u$ — (i)

$u = \cos(\log_e x)$

let $\log_e x = v$

$u = \cos v$

$\frac{du}{dv} = -\sin v$ — (ii)

$v = \log_e x$

$\frac{dv}{dx} = \frac{1}{x}$ — (iii)

$\frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{\cancel{dv}} \times \frac{\cancel{dv}}{dx} = \cos u \times (-\sin v) \times \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} =$

ncept of Chain Rule:

$$y=(\sin x)^5$$

let $\sin x = u$

$y = u^5$

$\frac{dy}{du} = 5u^4.$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times \cos x$$

$$= 5(\sin x)^4 \times \cos x.$$

i) If u and v are functions of two independent variable x and y then the determinant:

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$ is called the jacobian of u, v w.r.t. x, y.

And is denoted by:

$$J\left(\frac{u, v}{x, y}\right) \text{ or } \frac{\partial(u, v)}{\partial(x, y)}$$

ii) Similarly jacobian of u, v, w w.r.t. x, y, z.

$$J\left(\frac{u, v, w}{x, y, z}\right) = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{vmatrix}$$

1) Reciprocal Property:

If $J = \frac{\partial(u, v)}{\partial(x, y)}$ and $J' = \frac{\partial(x, y)}{\partial(u, v)}$ then $\boxed{J \cdot J' = 1}$

i.e., $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} = 1$

2) When the variable groups are in chain form:

If u, v are function of r, s and r, s are function of x, y then:

$u, v \longrightarrow r, s \longrightarrow x, y$

$$\frac{\partial(u, v)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(r, s)} \times \frac{\partial(r, s)}{\partial(x, y)}$$

3) Functionally Related:

If u, v are functionally related to x, y then

$$\boxed{\frac{\partial(u,v)}{\partial(x,y)} = 0}$$

OR

If u, v, w are functionally related to x, y, z then

$$\boxed{\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0}$$

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If $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$ then $\boxed{J \cdot J' = 1}$

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2) When the variable groups are in chain form:

If u, v are function of r, s and r, s are function of x, y then:

$u, v \longrightarrow r, s \longrightarrow x, y$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$$

3) Functionally Related: