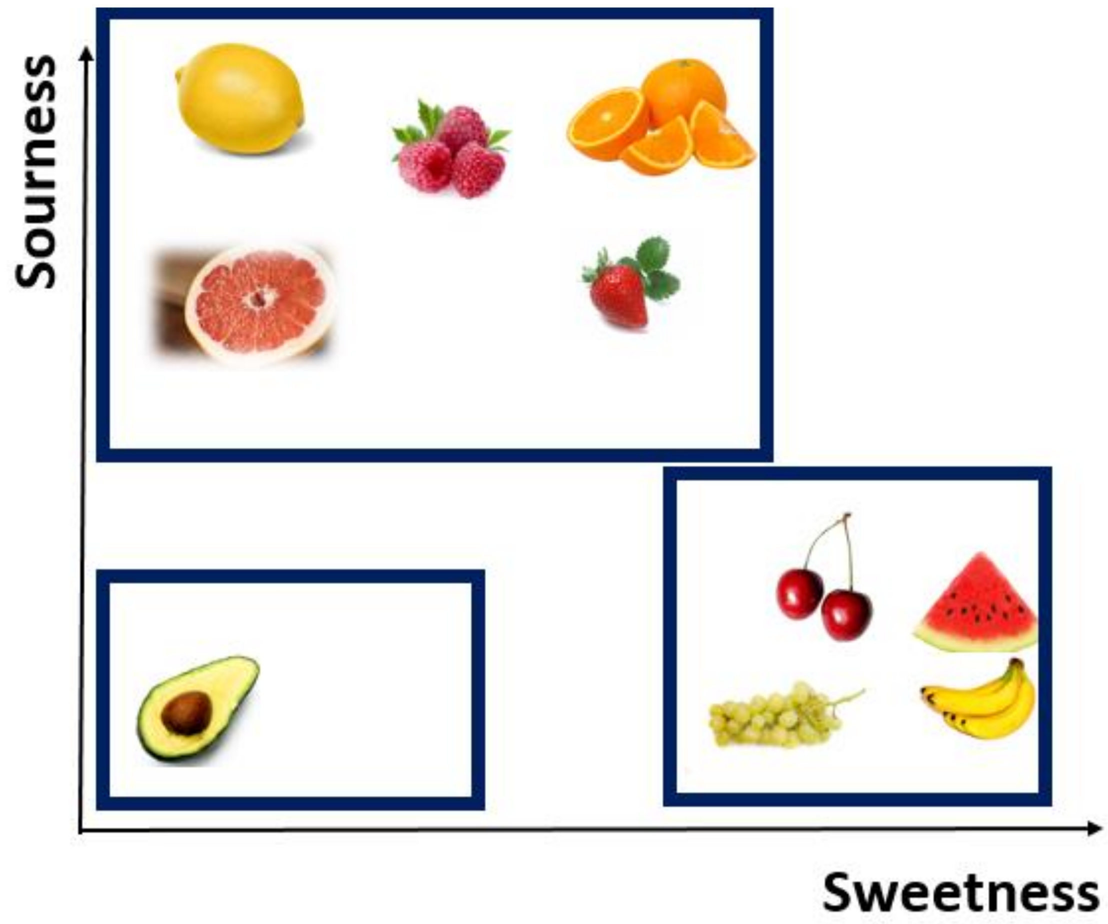
INT247
Machine Learning Foundations

Lecture #2.1

K-Nearest Neighbour and Bayesian Learning

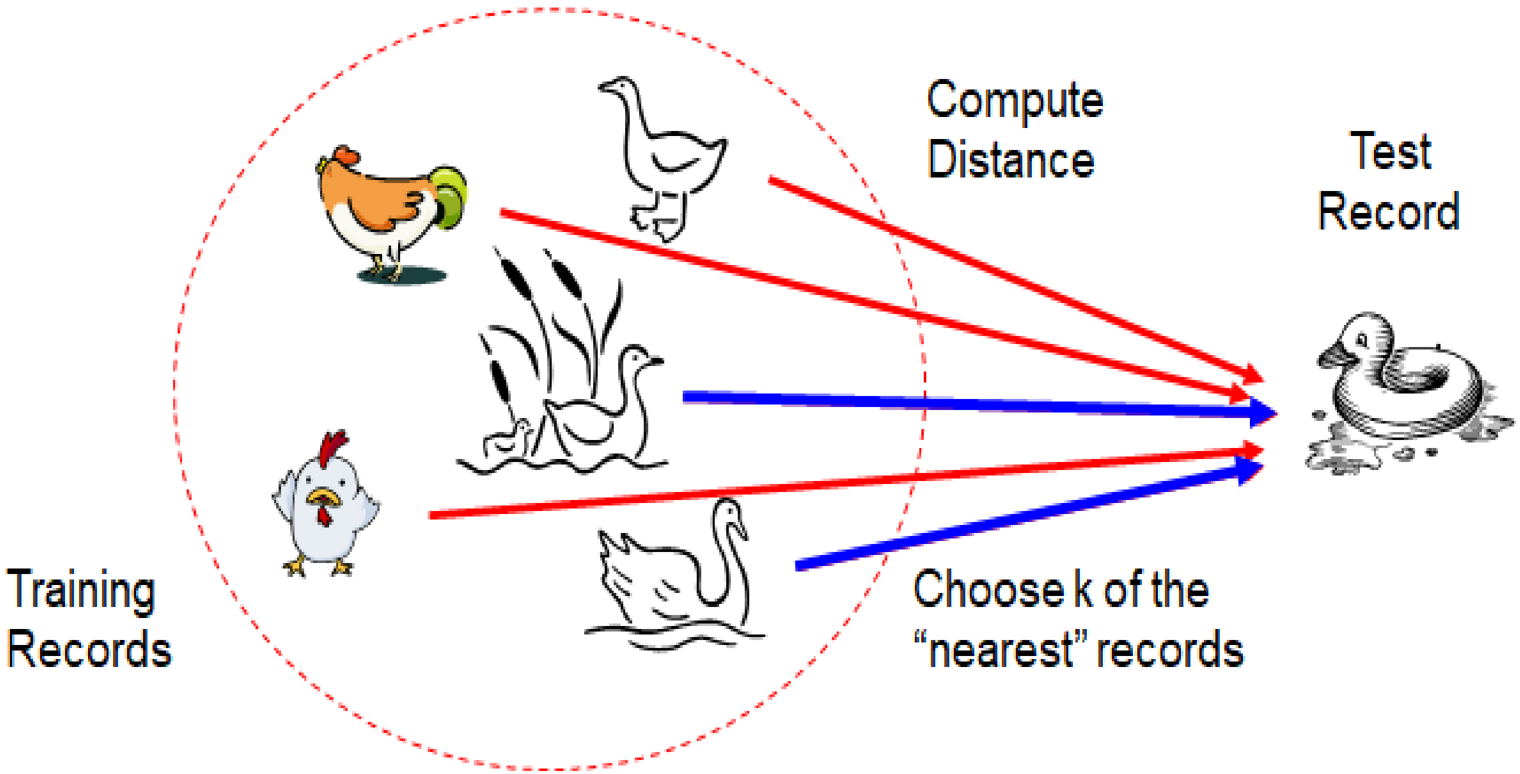
K- Nearest Neighbour Algorithm

- **Lazy Learner**
- **Uses K “closest” points for performing classification.**



K- Nearest Neighbour Algorithm

- **If it walks like a duck, quacks like a duck, then it's probably a duck**



K- Nearest Neighbour Algorithm

- **Requires three things:**
  - **The set of stored records.**
  - **Distance Metric to compute distance between records.**
  - **The value of K, the number of nearest neighbours to retrieve .**
- **To classify an unknown record:**
  - **Compute distance to other training records.**
  - **Identify k nearest neighbours.**
  - **Use class labels of nearest neighbours to determine the class label of unknown record.**

Compute Distance

- **Euclidean distance**

$$d(p,q) = \sqrt{\sum_{i} (p_i - qi)^2}$$

Numerical Example

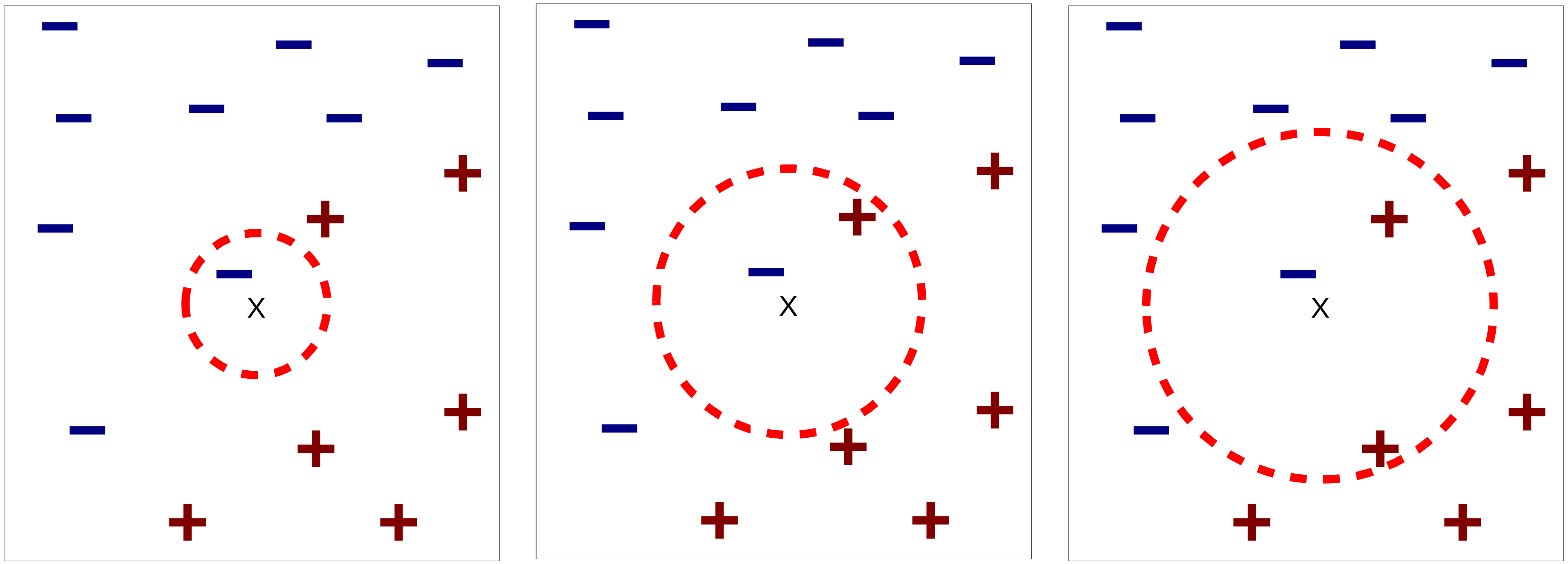
- Consider a dataset with 2 attributes:
  - Acid durability (x1)
  - Strength (x2)

  to classify whether a special paper tissue is good or bad (y).

x1	x2	y
7	7	Bad
7	4	Bad
3	4	Good
1	4	Good

- Classify paper tissue with x1=3 and x2=7 considering K=3.

K- Nearest Neighbour Algorithm



(a) 1-nearest neighbor (b) 2-nearest neighbor (c) 3-nearest neighbor

Note:

- **If K is too small, sensitive to noise points.**
- **If K is too large, neighbourhood may include points from other classes.**

Scaling Issues

- **Attributes may have to be scaled to prevent distance measures from being dominated by one of the attributes.**
- **Examples:**
  - **Height of a person may vary from 1.5m to 1.8m**
  - **weight of a person may vary from 90lb to 300lb**
  - **income of a person may vary from $10K to $1M**

Advantages of KNN

- **No assumption about data.**
- **Simple algorithm.**
- **High accuracy.**
- **Versatile – useful for classification or regression.**

Disadvantages of KNN

- **Computationally expensive.**
- **High memory requirement.**
- **Store all the training data.**
- **Sensitive to irrelevant features and the scale of the data.**

Bayesian Learning

- Probabilistic approach to inference.
- Quantities of interest are governed by probability distributions.
- Optimal decisions can be made by reasoning about these probabilities together with observed training data.

Bayesian Learning

- **Initial knowledge of many probabilities is required.**
- **Significant computational costs required.**

Bayes Theorem

- **Direct method of calculating the probability of best hypothesis.**

$$P(h|D) = \frac{P(D|h).P(h)}{P(D)}$$

- **P(h): prior probability of (h) hypothesis .**
- **P(D): prior probability of (D) observation.**
- **P(D|h): Probability of observing D in which h holds.**
- **P(h|D): posterior probability of h, reflects confidence that h holds after D has been observed.**

MAP (Maximum a Posteriori) Hypothesis

- $h_{MAP} =$

$$h_{MAP} = \arg\max_{h \in H} P(h|D)$$
$$= \arg\max_{h \in H} \frac{P(D|h)P(h)}{P(D)}$$
$$= \arg\max_{h \in H} P(D|h)P(h)$$

- **P(D) can be dropped, because it is a constant independent of h.**

Example

- Consider a medical diagnosis problem where:
  - P(cancer)=0.008
  - P(pos|cancer)=0.98
  - P(pos|~cancer)=0.03
- If a new patient comes in with a positive test result, what is the probability that he has cancer?

$$P(pos|cancer).P(cancer)=0.98*0.008=0.0078$$

$$P(pos|\sim cancer).P(\sim cancer)=0.03*0.992=0.0298$$

Thus, $h_{MAP} = \sim cancer$

Brute Force MAP Learning

- **In order to specify a learning problem of the algorithm, values for P(h) and P(D|h) must be specified.**
- **Assumptions:**
  - **Training data D is noise free (i.e. $d_i = c(x_i)$)**
  - **Target concept c is contained in H i.e.**

$$(\exists \boldsymbol{h} \in \boldsymbol{H})[(\forall \boldsymbol{x} \in \boldsymbol{X})[\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{c}(\boldsymbol{x})]]$$

  - **No reason to believe that any hypothesis is more probable than any other.**

Learning a Real-Valued function

- **Consider any real-value target function f.**
- **Training examples <x_i,d_i>, where d_i is noisy training value.**
- **$d_i=f(x_i)+e_i$**
- **Where e_i is random variable drawn independently for each x_i according to some Gaussian distribution with mean=0.**
- **The maximum likelihood hypothesis h_{ML} is defined as:**

$$\begin{aligned} h_{\mathrm{ML}} &= \operatorname{argmax}_{h \in H} p(D \mid h) \\ &= \operatorname{argmax}_{h \in H} \prod_{i=1}^{m} p(d_i \mid h) \end{aligned}$$

$$h_{\mathrm{ML}} = \operatorname{argmin}_{h \in H} \sum_{i=1}^{m} (d_i - h(x_i))^2$$

Example of Naïve Bayes Classifier

- **Dataset with 3 classes:**
  - **Parrot**
  - **Dog**
  - **Fish**
- **4 features:**
  - **Swim**
  - **Wings**
  - **Green Color**
  - **Dangerous Teeth**

Example of Naïve Bayes Classifier

Swim	Wings	Green Color	Dangerous Teeth	Animal Type
50	500/500	400/500	0	Parrot
450/500	0	0	500/500	Dog
500/500	0	100/500	50/500	Fish

- **Table shows a frequency table of dataset.**

Example of Naïve Bayes Classifier

- **Considering the dataset predict the type of animal if:**
  - **Swim=True**
  - **Wings=False**
  - **Green=True**
  - **Teeth=False**

Example of Naïve Bayes Classifier

- **Use naïve bayes approach for multiple evidances:**

$$P(H|E_1, E_2 \ldots EN) = \frac{P(E_1|H) * P(E_2|H) \ldots * P(E_N|H) * P(H)}{P(E_1, E_2, \ldots EN)}$$

Example of Naïve Bayes Classifier

- **For hypothesis testing for animal to be a dog:**

P(Dog|Swim , Green)=P(Swim|Dog)*P(Green|Dog)*P(Dog)/P(Swim, Green)

=0.9*0*0.333/P(Swim, Green)

=0

Example of Naïve Bayes Classifier

- **For hypothesis testing for animal to be a Parrot:**

P(Parrot|Swim , Green)=P(Swim|Parrot)*P(Green|Parrot)*P(Parrot)/ P(Swim, Green)

=0.1*0.8*0.333/P(Swim, Green)

=0.0264/P(Swim, Green)

Example of Naïve Bayes Classifier

- **For hypothesis testing for animal to be a Fish:**

P(Fish|Swim , Green)=P(Swim|Fish)*P(Green|Fish)*P(Fish)/P(Swim, Green)

=1*0.2*0.333/P(Swim, Green)

=0.0666/P(Swim, Green)

Example of Naïve Bayes Classifier

- **For all the three cases, denominator is same.**
- **The value of P(Fish|Swim, Green) is greater than P(Parrot|Swim, Green).**
- **Thus, Class of animal is Fish.**

Exercise

- **Predict the class of animal if:**
  - **Swim=True**
  - **Wings=False**
  - **Green=True**
  - **Teeth=True**


COMING UP