Miami University

Department of Electrical and Computer Engineering

An perspective analytic of a Planar Motion of Quadrotor

Authors
Mirza Sanita Haque
Md Rakibul Ahasan

Advisor
Dr. Bryan Van Scoy

November 4, 2022

Contents

1 Introduction

As in the name itself, a quadrotor consists of multiple rotors which stabilize themselves using a controller and fly in upward, downward, back, and forth directions. In another way, an effective controlling system enhance quadrotor uses in different applications. One of the major application is the Unmanned Aerial Vehicle(UAV) which has two types fixed wings(FW) and rotating wings(RW). The fixed wing concept of a UAV requires constant wind flow over its span to make it move forward but it does not support stationary motion. Because of this rotating wing UAV contribute vastly to military support, wireless communication, distance learning, and many more. Using the basic principle of quadrotor, it can instantly hover, fly in any direction, and with convenient speed and angular momentum.

The most challenging part of designing a quadrotor is its controller. This controller should have a stable position in all cases of the thrust and torque produced by the rotors. The planar motion is the simple position of a quadrotor means going up and down. Due to its simplicity, the mathematical model of the PID controller is easy.

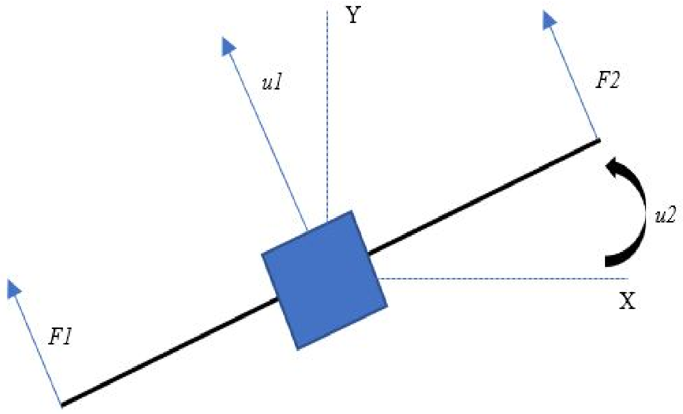

Figure 1.1: A Simple Planator Quadrotor.

Fig1.1 is defining the basic properties of a quadrotor. The coordinate (X, Y) defines the base position and we are expecting the quadrotor will hover in the Y-Axis without any issue. The net thrust force u_1 is equal to the sum of the force F_1 and F_2 exerted from the rotors. This is initiating the hovering of the quadrotor. This force will be implied on two positions at the same time the net torque will be denoted by u_2. We need to stable the position of quadrotor using input of u_1 and u_2. The angular position of the quadrotor will be zero as we will be working in the hovering aspect of a quadrotor meaning to say in the initial stage the quadrotor will be going up and down. Other important parameters will be the mass of the quadrotor, its acceleration, and its span.

In the subsequent section of this report, we will discuss the modeling and simulation of the planar motion of a quadrotor. Thereafter we will show the linearization and estimation aspect and finally the control aspect of the quadrotor with machine learning.

2 Quadrotor Basics

The name quadrotor came from having four rotors and it usually consists of frame, motors, ESC (electronic speed controller), propeller, battery, Flight Controller, rc receiver and sensor. The sensor plays important steps to identify the system states.

2.1 Sensors

There are many sensors and the important one are listed below:

1. Ultrasound: It is in the bottom part of the quadrotor which use high frequency sound to measure the distance of quadrotor from the ground. It is the main driver to measure the altitude of the quadrotor.
2. Camera: This is also in the bottom part of the quadrotor. This sensor take photos and using image processing techniques it measures horizontal motion and speed of the quadrotor.
3. Pressure Sensor: This is a supplement sensor which compliments the ultrasound sensor. If quadrotor is hovering low altitude then the pressure will be high and subsequently if the quadrotor hovering high then the pressure is low.
4. IMU sensor: This inertial measurement unit sensor is crucial as it measures the linear acceleration and angular rate. This feed the system state roll, pitch and yaw in the feedback loop to more stable hovering of quadrotor.

2.2 Actuators/Motor Principle

Another important parameter are rotor position of planator and quadrotor. In planator the rotor is place in the opposite side showed in Fig 2.1. Where force is in the upward direction and the generating torque will cancel out each other and if other state does not change then the planator will hover in upward direction.

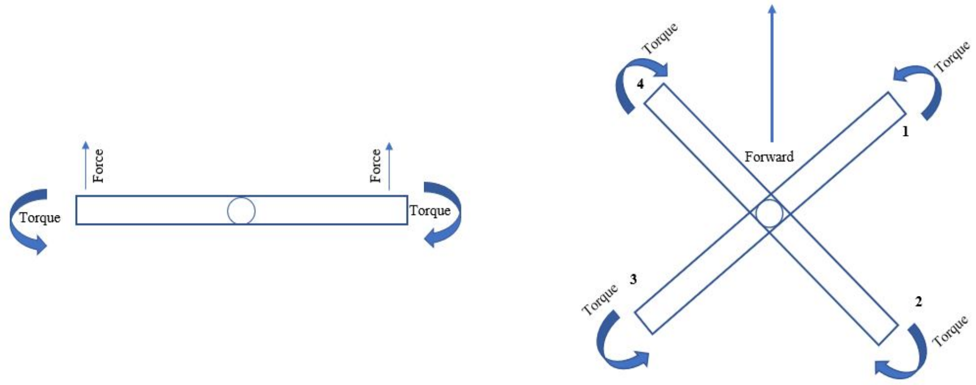

Figure 2.1: Force and Torque state of a Planar Quadrotor and a Quadrotor

For the quadrotor perspective the rotor are in 'X' position. To perfectly hovering the quadrotor the motor in 1,3 will spin in one direction and the motors in 2,4 will hover in opposite direction. An efficient maneuvers of the rotor's will make the quadrotors move in any direction.

2.3 Control Mechanism of a quadrotor

Let us look on below Fig 2.2 where quadrotor input is taken from 4 motors and the expected output we are looking for that the hovering will be perfect. During this the output state is measured through the different sensors and fed back to the system. This system state input are crosschecked with maneuver points or reference point and joint input is plugged to the control system. In the control system the motor mix algorithm is established with combination of thrust, yaw, roll and pitch. This algorithm is established such a way that each thrust, yaw, roll and pitch can control independently. The quadrotor is an under-actuated system which has 4 motors with 6 degree of freedom. The output is classfied to transitional direction and rotational direction. The transitional direction are up/down, left/rigt and forward/backward. The rotational direction can be roll, pitch or yaw.

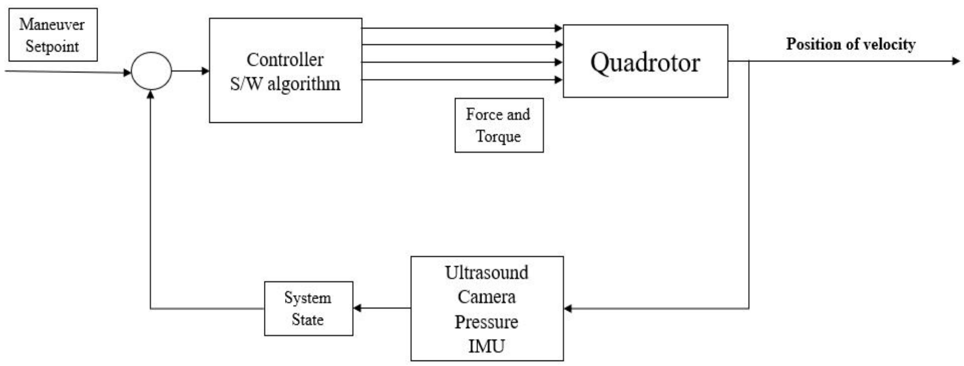

Figure 2.2: Control Principle

3 Mathematical Modeling

3.1 Qualitative Explanation

A quadrotor is a highly unstable system with complex dynamics. From Fig 1.1 lets consider $F1$ and $F2$ are two perpendicular force and the net thrust force $u1$ can calculated from below equation.

$$u_1 = F1 + F2 \tag{3.1}$$

The force is imposed from different location hence any imbalance force will generate a torque. The $u2$ is the next thrust torque which calculated from below equation where L is the distance from center of force towards the middle point.

$$u_2 = \frac{L}{2}(F1 - F2) \tag{3.2}$$

hence the overall momentum of the system is dependent of the input vector $u = [u1\ u2]$ and the output is defined as $u = [x\ y]$. We will be working in simple quadrotor hence the angle will be considered as zero in the initial state.

3.2 Equation of Motion

We have defined the input and output vector and now let's observe the mathematical model of the system. From Newtons second law of motion in the horizontal and vertical direction can be written as below

$$\ddot{x} = \frac{-u_1 \sin(\phi)}{m} \tag{3.3}$$

$$\ddot{y} = -g + \frac{u_1 \cos(\phi)}{m} \tag{3.4}$$

The torque equation of center of mass can be defined from below

$$\ddot{\phi} = \frac{u_2}{J} \tag{3.5}$$

In matrix from it will be

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{-\sin(\phi)}{m} & 0 & 0 \\ \frac{\cos(\phi)}{m} & 0 & -1 \\ 0 & \frac{1}{m} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ g \end{bmatrix} \tag{3.6}$$

3.3 Equilibrium Point Determination

We can define the state vector as $\begin{bmatrix} x & y & \phi & \ddot{x} & \ddot{y} & \ddot{\phi} \end{bmatrix}^T$ where x y us the co-ordinate of the quadrotor initial position and ϕ is the angle. and are the linear and horizontal velocities respectively and $\dot{\phi}$ is the angular velocity. In the initial state of stationary position of system is $\begin{bmatrix} x & y & \phi & 0 & 0 & 0 \end{bmatrix}^T$ as there will be not angular or horizontal or vertical velocity. Putting this in the eq 3.6 will give as below.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-\sin(\phi)}{m} & 0 & 0 \\ \frac{\cos(\phi)}{m} & 0 & -1 \\ 0 & \frac{1}{m} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ g \end{bmatrix}$$

Solving this equation w.r.t the above condition of equilibrium points we get the below:

$$\mathrm{u}_1 \sin(\phi) = 0$$
$$\mathrm{u}_1 \cos(\phi) = mg$$
$$\mathrm{u}_2 = 0$$

Now $u_1 \sin(\phi) = 0$ when $\phi = 0$, in such case the state vector will transform as below

$$X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{3.7}$$

Since the equilibrium angle of the quadrotor is $\phi = 0$, the equilibrium state $u_1 = mg$. Here, it is necessary to mention the physical parameter of the quadrotors.

Quadrotor mass m=0.18 kg
Acceleration of gravity g=9.8 m/s
Span of quadrotor L=0.086 m
The moment of inertia of quadrotor J=2.5*$10^{-4} kgm^2$

3.4 Linearization

In Fig1.1 considering net thrust as u_1, net momentum u_2 and gravitational acceleration as input. The co-ordinates of quadrotors x and y also consider to be the output state. Now below states needs to linearize:

$$\ddot{x} = \frac{-u_1 \sin(\phi)}{m} = f_1(x, y, \phi, \dot{x}, \dot{y}, \dot{\phi}, u_1, u_2, g) \tag{3.8}$$

$$\ddot{y} = -g + \frac{u_1 \cos(\phi)}{m} = f_2(x, y, \phi, \dot{x}, \dot{y}, \dot{\phi}, u_1, u_2, g) \tag{3.9}$$

$$\ddot{\phi} = \frac{u_2}{J} = f_3(x, y, \phi, \dot{x}, \dot{y}, \dot{\phi}, u_1, u_2, g) \tag{3.10}$$

Linearize the first state will give us below outcome which will be simplified in eq 3.11.

$\delta\ddot{x} = \frac{\partial f_1}{\partial x}|\delta x + \frac{\partial f_1}{\partial y}|\delta y + \frac{\partial f_1}{\partial \phi}|\delta\phi + \frac{\partial f_1}{\partial \dot{x}}|\delta\dot{x} + \frac{\partial f_1}{\partial \dot{y}}|\delta\dot{y} + \frac{\partial f_1}{\partial \dot{\phi}}|\delta\dot{\phi}$

$\delta\ddot{x} = -\frac{u_1}{m} cos(\phi)$

$$\delta\ddot{x} = -\frac{mg}{m}\delta\phi = -9.8\delta\phi \tag{3.11}$$

In similar manner we will linearize other two state

$\delta\ddot{y} = \frac{\partial f_2}{\partial x}|\delta x + \frac{\partial f_2}{\partial y}|\delta y + \frac{\partial f_2}{\partial \phi}|\delta\phi + \frac{\partial f_2}{\partial \dot{x}}|\delta\dot{x} + \frac{\partial f_2}{\partial \dot{y}}|\delta\dot{y} + \frac{\partial f_2}{\partial \dot{\phi}}|\delta\dot{\phi} + \frac{\partial f_2}{\partial u_1}|\delta u_1 + \frac{\partial f_2}{\partial g}|\delta g$

$\delta\ddot{y} = -\frac{u_1}{m} sin(\phi) + \frac{cos(\phi)}{m}\delta u_1 - \delta g$

when ϕ=0 then it becomes

$$\delta\ddot{y} = -\delta g + \frac{\delta u_1}{m} \tag{3.12}$$

$\delta\ddot{\phi} = \frac{\partial f_3}{\partial u_2}|\delta u_2$

$$\delta\ddot{\phi} = \frac{\delta u_2}{g} \tag{3.13}$$

3.5 State Space Realization

From equation 3.11, 3.12 and 3.13 we have the linearized system. Let us draw those equation in state space realization form

The state-equation:

$$\begin{bmatrix} \partial\dot{x} \\ \partial\dot{y} \\ \partial\dot{\phi} \\ \partial\ddot{x} \\ \partial\ddot{y} \\ \partial\ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -9.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial x \\ \partial y \\ \partial\phi \\ \partial\dot{x} \\ \partial\dot{y} \\ \partial\dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & -1 \\ 0 & \frac{1}{J} & 0 \end{bmatrix} \begin{bmatrix} \partial u_1 \\ \partial u_2 \\ \partial g \end{bmatrix} \tag{3.14}$$

and the output equation

$$\begin{bmatrix} \partial x \\ \partial y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \partial x \\ \partial y \\ \partial\phi \\ \partial\dot{x} \\ \partial\dot{y} \\ \partial\dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial u_1 \\ \partial u_2 \\ \partial g \end{bmatrix} \tag{3.15}$$

From eq 3.14 and 3.15 we can deduced system matrix A, control matrix B, output matrix C and feed-forward matrix D.

A= $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -9.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, B= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & -1 \\ 0 & \frac{1}{J} & 0 \end{bmatrix}$,

C= $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ D= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

As we have the matrix we can deduce the transfer function from below formula in matrix form

$$H(s) = C(SI - A)^{-1}B + D \tag{3.16}$$

Hence the open loop transfer function matrix is

$$\big(H(s)\big) = \begin{pmatrix} 0 & -\frac{g}{Js^4} & 0 \\ \frac{1}{ms^2} & 0 & -\frac{1}{s^2} \end{pmatrix} \tag{3.17}$$

The transfer function indicates that the poles are at the origin and there are no zeros. We can can inject the physical parameter in eq 3.17

$$\big(H(s)\big) = \begin{pmatrix} 0 & -\frac{39200}{s^4} & 0 \\ \frac{50}{9s^2} & 0 & -\frac{1}{s^2} \end{pmatrix} \tag{3.18}$$

Output vector $Y_{out}(s) = H(s)U(s)$ where

H(s)= $\begin{bmatrix} X(s) & Y(s) \end{bmatrix}^T$

U(s)= $\begin{bmatrix} U_1(s) & U_1(s) & \frac{g}{s} \end{bmatrix}^T$