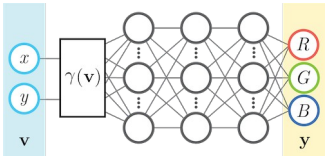
About the Project

This project explores coordinate-based neural networks for image reconstruction. A fully connected multilayer perceptron is implemented from scratch using NumPy, where the input is a 2D pixel coordinate (x, y) and the output is the corresponding RGB color value (r, g, b).



To investigate how the network captures spatial and frequency details, I compared several input feature mapping strategies $\gamma(v)$:

- No mapping: $\gamma(v) = v$.
- Basic mapping: $\gamma(v) = \left[\cos(2\pi v), \sin(2\pi v)\right]^T$.
- Gaussian Fourier feature mapping: $\gamma(v) = \left[\cos(2\pi Bv), \sin(2\pi Bv)\right]^T$, where each entry in $B \in R^{m \times d}$ is sampled from $N(0, \sigma^2)$.

Implementation Highlights:

- Mappings are implemented in the helper functions `get_B_dict` and `input_mapping`.
- The basic mapping can be considered a case where $B \in R^{2 \times 2}$ is the indentity matrix.
- For this project, d is 2 because the input coordinates in two dimensions.

Setup

```
from google.colab import drive
drive.mount("/content/drive")

import os
datadir = "path/"
if not os.path.exists(datadir):
  !ln -s "path/" $datadir
os.chdir(datadir)
!pwd
```

Imports

```
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import os, imageio
import cv2
import numpy as np
```

```
from base64 import b64encode
from IPython.display import HTML

from models.neural_net import NeuralNetwork

# makes sure your NeuralNetwork updates as you make changes to the .py
file
%load_ext autoreload
%autoreload 2

# sets default size of plots
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0)
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

Helper Functions

Image Data and Feature Mappings

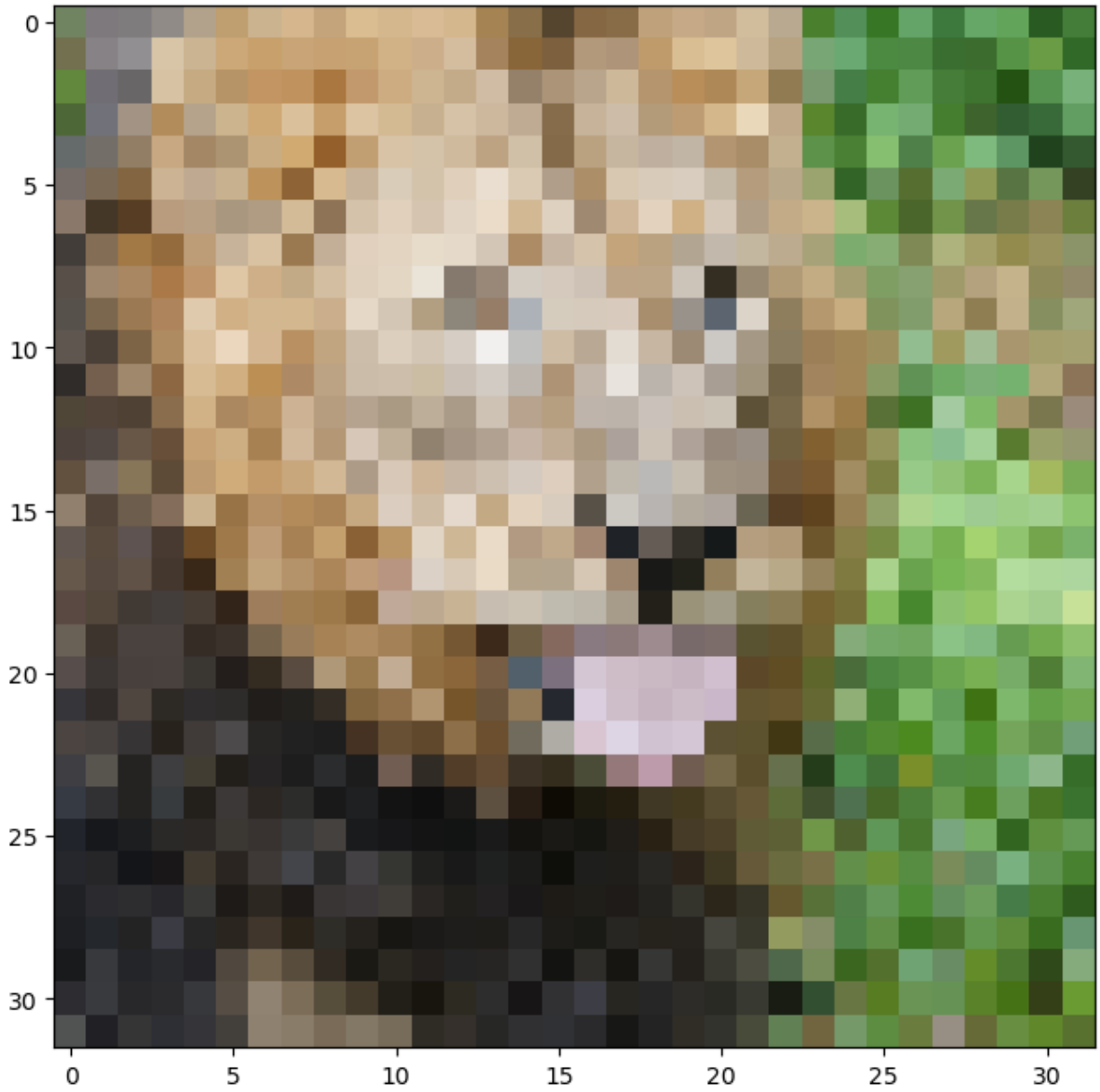
```
# Data loader - already done for you
def get_image(size=512, \

image_url='https://bmild.github.io/fourfeat/img/lion_orig.png'):

  # Download image, take a square crop from the center
  img = imageio.imread(image_url)[..., :3] / 255.
  c = [img.shape[0]//2, img.shape[1]//2]
  r = 256
  img = img[c[0]-r:c[0]+r, c[1]-r:c[1]+r]

  if size != 512:
    img = cv2.resize(img, (size, size))

  plt.imshow(img)
  plt.show()

  # Create input pixel coordinates in the unit square
  coords = np.linspace(0, 1, img.shape[0], endpoint=False)
  x_test = np.stack(np.meshgrid(coords, coords), -1)
  test_data = [x_test, img]
  train_data = [x_test[::2, ::2], img[::2, ::2]]

  return train_data, test_data

# Create the mappings dictionary of matrix B -  you will implement
this
def get_B_dict(size, scale = 1.0):
    mapping_size = size // 2
```

```
    # mapping_size = size (for extra credit)
    B_dict = {}
    B_dict['none'] = None

    # Basic mapping
    basic_mapping = np.eye(2)
    B_dict['basic'] = basic_mapping

    # Gaussian mapping
    gauss_mapping = np.random.normal(scale=scale, size =
(mapping_size, 2))
    B_dict['gauss_1.0'] = gauss_mapping

    return B_dict

# Given tensor x of input coordinates, map it using B
def input_mapping(x, B):
    if B is None:
        return x
    else:
        proj = 2 * np.pi * (x @ B.T)
        cos_term = np.cos(proj)
        sin_term = np.sin(proj)
        mapped_features = np.concatenate([cos_term, sin_term], axis=-
1)
        return mapped_features
```

MSE Loss and PSNR Error

```
# Loss function and evaluation metric
def mse(y, p):
  return np.mean((y - p) ** 2)

def psnr(y, p):
  return -10 * np.log10(2.*mse(y, p))

# Display image
size = 32
train_data, test_data = get_image(size)
```

```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```




```
# Set the hyperparameters
num_layers = 4

hidden_size = 256
epochs = 1000
learning_rate = 0.1
output_size = 3
B_dict = get_B_dict(size)

print('B_dict items:')
for k,v in B_dict.items():
    print('\t',k,np.array(v).shape)
```

```
B_dict items:
      none ()
      basic (2, 2)
      gauss_1.0 (16, 2)
```

```
# Apply the input feature mapping to the train and test data
def get_input_features(B_dict, mapping):
  # mapping is the key to the B_dict, which has the value of B
  # B is then used with the function `input_mapping` to map x
  y_train = train_data[1].reshape(-1, output_size)
  y_test = test_data[1].reshape(-1, output_size)
  X_train = input_mapping(train_data[0].reshape(-1, 2),
B_dict[mapping])
  X_test = input_mapping(test_data[0].reshape(-1, 2), B_dict[mapping])
  return X_train, y_train, X_test, y_test
```

Plotting and video helper functions

```
def plot_training_curves(train_loss, train_psnr, test_psnr):
  # plot the training loss
  plt.subplot(2, 1, 1)
  plt.plot(train_loss)
  plt.title('MSE history')
  plt.xlabel('Iteration')
  plt.ylabel('MSE Loss')

  # plot the training and testing psnr
  plt.subplot(2, 1, 2)
  plt.plot(train_psnr, label='train')
  plt.plot(test_psnr, label='test')
  plt.title('PSNR history')
  plt.xlabel('Iteration')
  plt.ylabel('PSNR')
  plt.legend()

  plt.tight_layout()
  plt.show()

def plot_reconstruction(p, y):
  p_im = p.reshape(size,size,3)
  y_im = y.reshape(size,size,3)

  plt.figure(figsize=(12,6))

  # plot the reconstruction of the image
  plt.subplot(1,2,1), plt.imshow(p_im), plt.title("reconstruction")

  # plot the ground truth image
  plt.subplot(1,2,2), plt.imshow(y_im), plt.title("ground truth")
```

```
  print("Final Test MSE", mse(y, p))
  print("Final Test psnr",psnr(y, p))

def plot_reconstruction_progress(predicted_images, y, N=8):
  total = len(predicted_images)
  step = total // N
  plt.figure(figsize=(24, 4))

  # plot the progress of reconstructions
  for i, j in enumerate(range(0,total, step)):
      plt.subplot(1, N+1, i+1)
      plt.imshow(predicted_images[j].reshape(size,size,3))
      plt.axis("off")
      plt.title(f"iter {j}")

  # plot ground truth image
  plt.subplot(1, N+1, N+1)
  plt.imshow(y.reshape(size,size,3))
  plt.title('GT')
  plt.axis("off")
  plt.show()

def plot_feature_mapping_comparison(outputs, gt):
  # plot reconstruction images for each mapping
  plt.figure(figsize=(24, 4))
  N = len(outputs)
  for i, k in enumerate(outputs):
      plt.subplot(1, N+1, i+1)
      plt.imshow(outputs[k]['pred_imgs'][-1].reshape(size, size, -1))
      plt.title(k)
  plt.subplot(1, N+1, N+1)
  plt.imshow(gt)
  plt.title('GT')
  plt.show()

  # plot train/test error curves for each mapping
  iters = len(outputs[k]['train_psnrs'])
  plt.figure(figsize=(16, 6))
  plt.subplot(121)
  for i, k in enumerate(outputs):
      plt.plot(range(iters), outputs[k]['train_psnrs'], label=k)
  plt.title('Train error')
  plt.ylabel('PSNR')
  plt.xlabel('Training iter')
  plt.legend()
  plt.subplot(122)
  for i, k in enumerate(outputs):
      plt.plot(range(iters), outputs[k]['test_psnrs'], label=k)
  plt.title('Test error')
  plt.ylabel('PSNR')
```

```
  plt.xlabel('Training iter')
  plt.legend()
  plt.show()
```

Experiment Runner

```
def NN_experiment(X_train, y_train, X_test, y_test, input_size,
num_layers,\
                  hidden_size, output_size, epochs,\
                  learning_rate, loss_function, opt='SGD', b1=0.9,
b2=0.999, eps=1e-8,
                  batch_size = 32):

    # Initialize a new neural network model
    hidden_sizes = [hidden_size] * (num_layers - 1)
    current_lr = learning_rate
    net = NeuralNetwork(input_size, hidden_sizes, output_size,
num_layers, opt, loss_function)

    # Variables to store performance for each epoch
    train_loss = np.zeros(epochs)
    train_psnr = np.zeros(epochs)
    test_psnr = np.zeros(epochs)
    predicted_images = np.zeros((epochs, y_test.shape[0],
y_test.shape[1]))

    # For each epoch...
    for epoch in tqdm(range(epochs)):
        # Shuffle the dataset
        idx = np.arange(X_train.shape[0])
        np.random.shuffle(idx)
        X_train_shuf = X_train[idx]
        y_train_shuf = y_train[idx]

        if opt == 'Adam':
          # 1. Forward pass
          y_pred_train = net.forward(X_train_shuf)
          # 2. Backward pass
          epoch_loss = net.backward(y_train_shuf)
          # 3. Update the weights
          net.update(lr=current_lr, b1=b1, b2=b2, eps=eps)

        elif opt == 'SGD': # mini-batch SGD
          num_train = X_train_shuf.shape[0]
          num_batches = (num_train + batch_size - 1) // batch_size
          epoch_loss = 0.0

          for b in range(num_batches):
              start = b * batch_size
              end = min(start + batch_size, num_train)
```

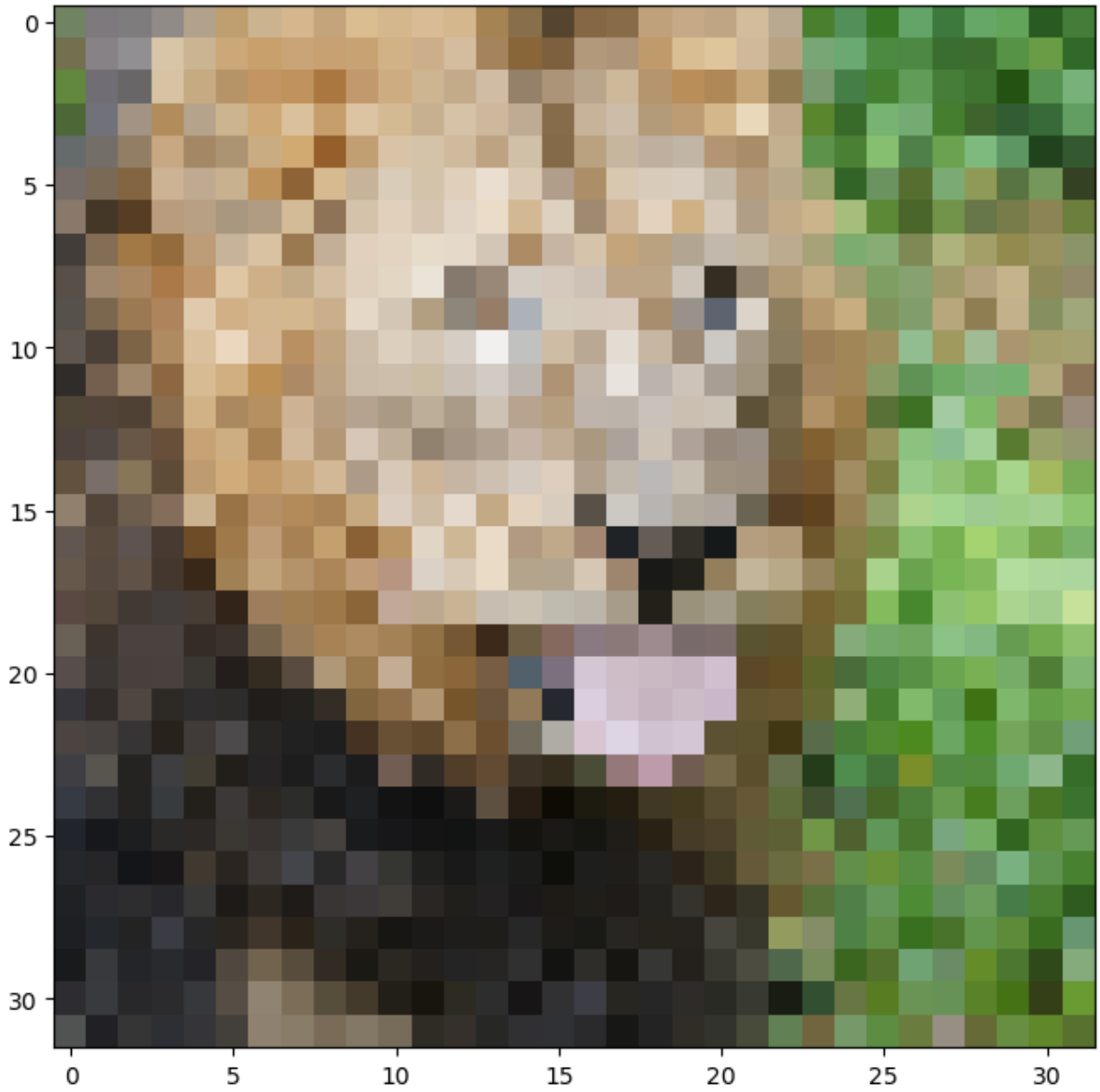
```
            X_batch = X_train_shuf[start:end]
            y_batch = y_train_shuf[start:end]

            # 1. Forward pass
            y_pred_train_batch = net.forward(X_batch)

            # 2. Backward pass
            train_loss_val = net.backward(y_batch)

            # 3. Update the weights
            net.update(lr=current_lr)

            epoch_loss += train_loss_val

        epoch_loss /= num_batches

      train_loss[epoch] = epoch_loss

      # Testing
      y_pred_train = net.forward(X_train)
      train_psnr[epoch] = psnr(y_train, y_pred_train)

      y_pred_test = net.forward(X_test)
      test_psnr[epoch] = psnr(y_test, y_pred_test)
      predicted_images[epoch] = y_pred_test.copy()

  return net, train_psnr, test_psnr, train_loss, predicted_images
```

Low Resolution Reconstruction

Low Resolution Reconstruction - SGD - None Mapping

```
# Display image
size = 32
train_data, test_data = get_image(size)
```

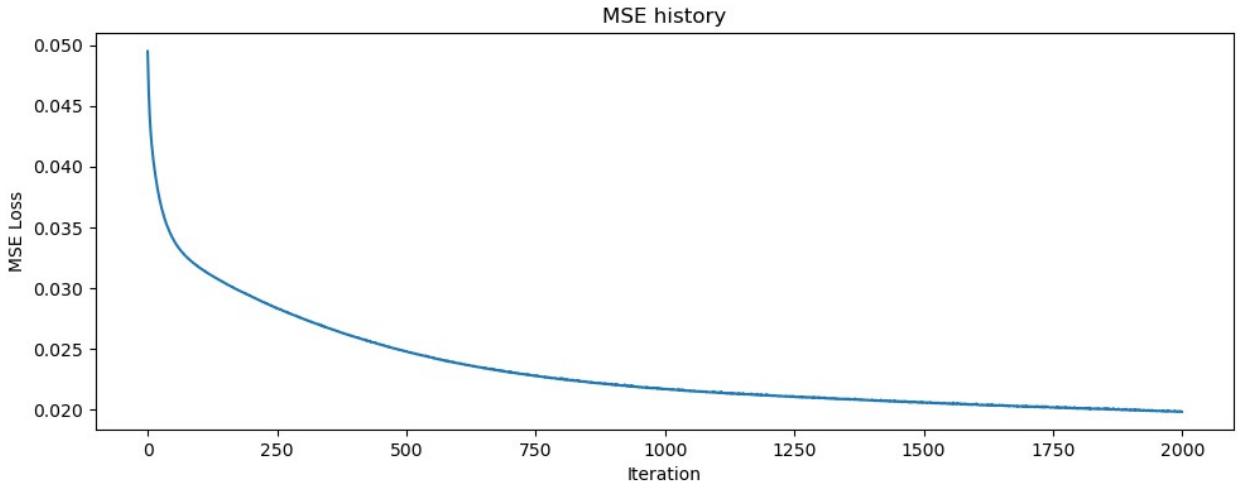
```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```

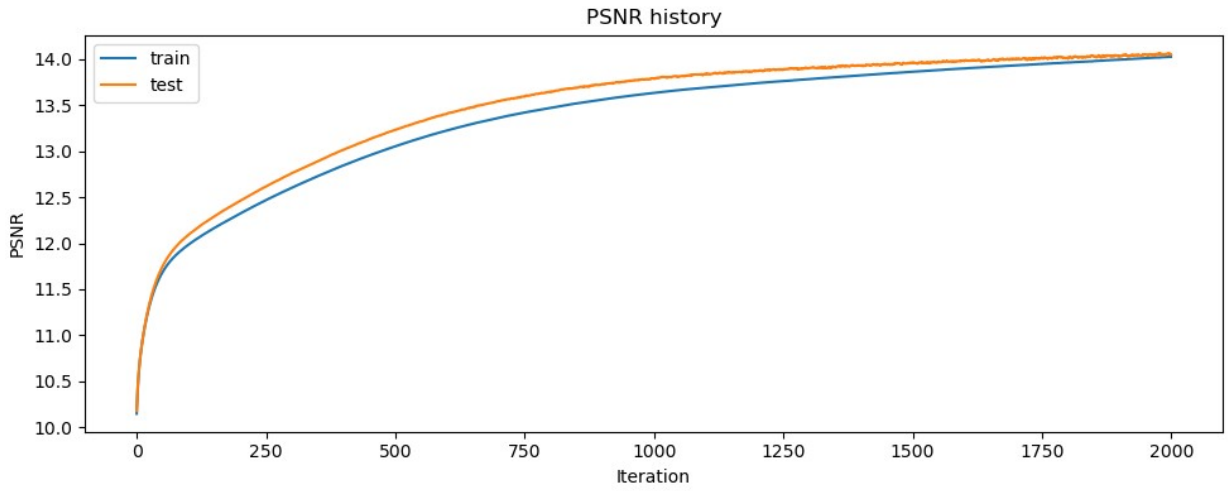



```
B_dict = get_B_dict(size)

print('B_dict items:')
for k,v in B_dict.items():
    print('\t',k,np.array(v).shape)

B_dict items:
	none ()
	basic (2, 2)
	gauss_1.0 (16, 2)

# Set the hyperparameters
num_layers = 4
```

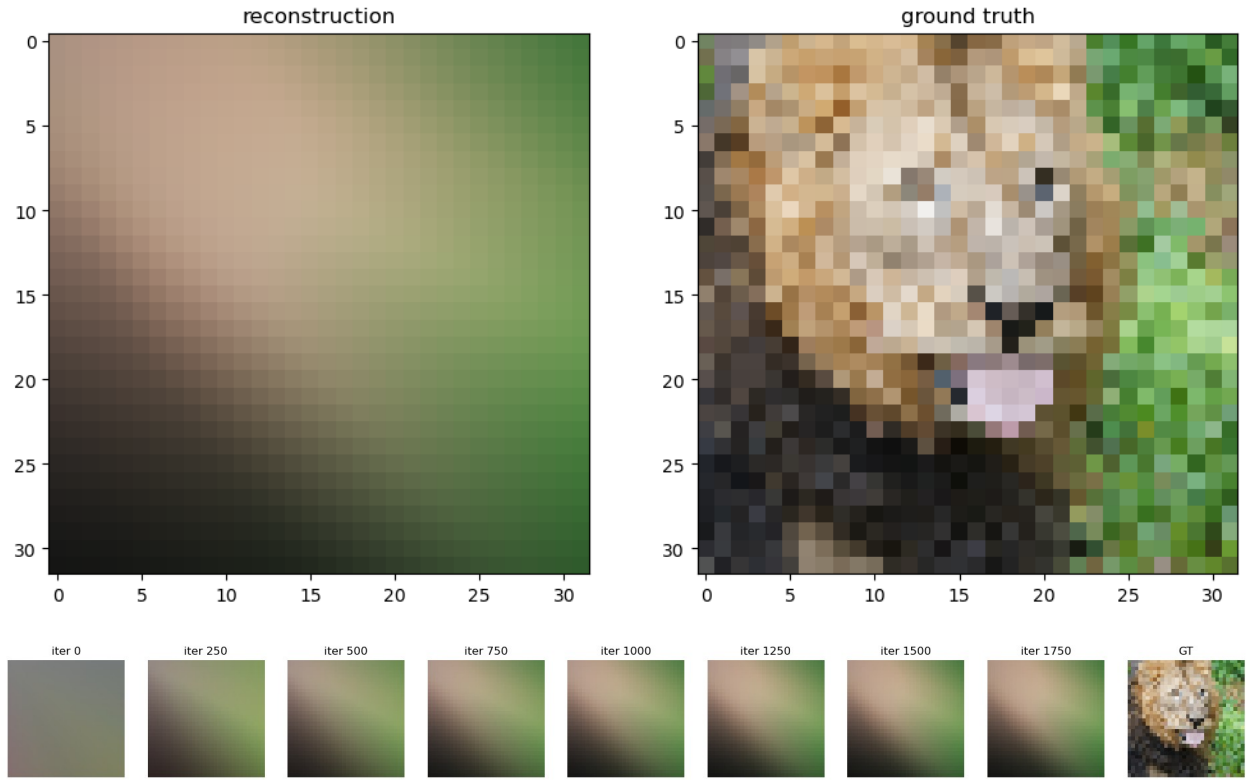
```
output_size = 3

# Adjust these
hidden_size = 256
epochs = 2000
learning_rate = 0.02

# Get input features
B_dict = get_B_dict(size=512)
X_train, y_train, X_test, y_test = get_input_features(B_dict,
mapping='none')
input_size = X_train.shape[1]

# Run NN experiment on input features
net, train_psnr, test_psnr, train_loss, predicted_images =
NN_experiment(
                    X_train, y_train, X_test, y_test, input_size,
num_layers,
                    hidden_size, output_size, epochs, learning_rate,
'mse', opt="SGD",
                    batch_size=32)

# Plot results of experiment
plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

{"model_id":"9d0b159ccd9f496cbfebfff29eb34175","version_major":2,"version_minor":0}






```
Final Test MSE 0.01968779947165195
Final Test psnr 14.047728270869522
```



Low Resolution Reconstruction - SGD - Various Input Mapping Stategies

```
def train_wrapper(mapping, size, num_layers, hidden_size, output_size,
epochs, learning_rate, loss_function, opt='SGD'):

    B_dict = get_B_dict(size)
    X_train, y_train, X_test, y_test =
get_input_features(B_dict,mapping)
    input_size = X_train.shape[1]

    net, train_psnrs, test_psnrs, train_loss, predicted_images =
NN_experiment(
                    X_train, y_train, X_test, y_test, input_size,
num_layers,
                    hidden_size, output_size, epochs, learning_rate,
loss_function, opt, batch_size=32)

    return {
        'net': net,
        'train_psnrs': train_psnrs,
        'test_psnrs': test_psnrs,
        'train_loss': train_loss,
        'pred_imgs': predicted_images
    }

outputs = {}
for k in tqdm(B_dict):
```

```
  print("training", k)
  outputs[k] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mse', opt='SGD')
```

{"model_id":"099e8b8c4a3c425099b388ee9fad170b","version_major":2,"version_minor":0}

```
training none
```

{"model_id":"3dc7c3e141e94231ad8931e52ed54478","version_major":2,"version_minor":0}

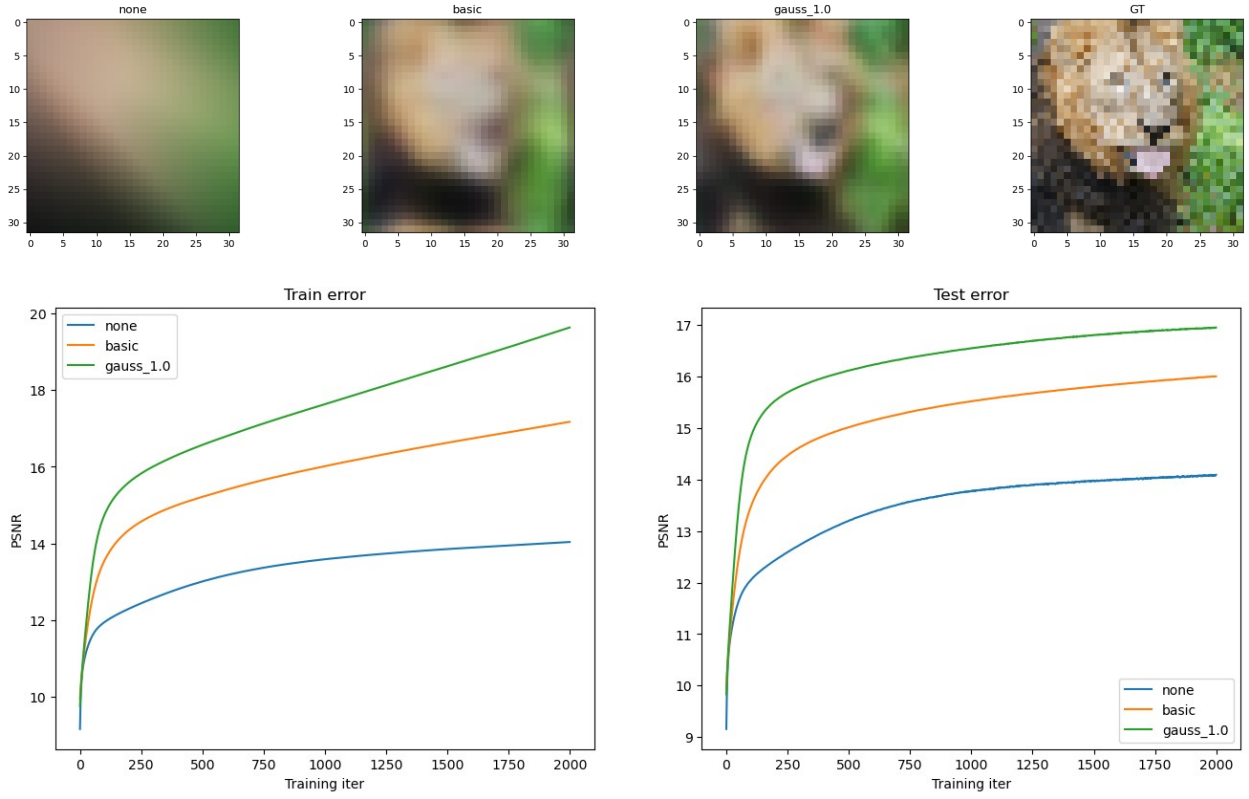
```
training basic
```

{"model_id":"2d4c1b64800d437f980e10aab16b5ca6","version_major":2,"version_minor":0}

```
training gauss_1.0
```

{"model_id":"74467db8ee4e4d598632ca3b05b6a1dd","version_major":2,"version_minor":0}

```
plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```



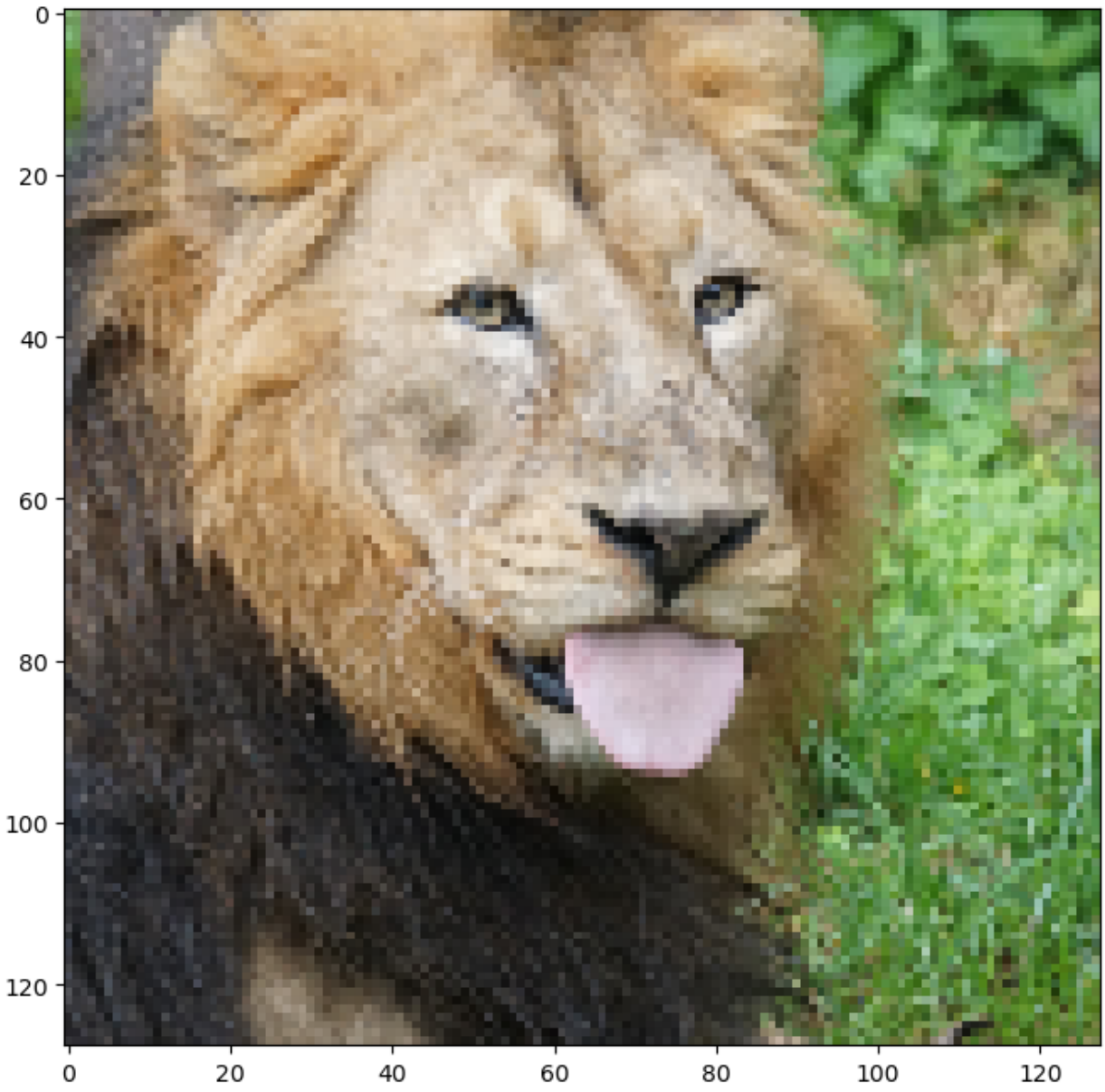
High Resolution Reconstruction

High Resolution Reconstruction - SGD - Various Input Mapping Stategies

Repeat the previous experiment, but at the higher resolution.

```
# load hi-res image
size = 128
train_data, test_data = get_image(size)
```

```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```




```
# Set the hyperparameters
num_layers = 4
output_size = 3

# Adjust these
hidden_size = 256
epochs = 300
learning_rate = 0.2

outputs = {}
for k in tqdm(B_dict):
  print("training", k)
```

```
  outputs[k] = train_wrapper(k,size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mse', opt='SGD')
```


```
{"model_id":"7d15f574c1da4048bebd1144ad84bf59","version_major":2,"vers
ion_minor":0}
```


```
training none
```


```
{"model_id":"d276d18812c245648f309c327e9203ce","version_major":2,"vers
ion_minor":0}
```

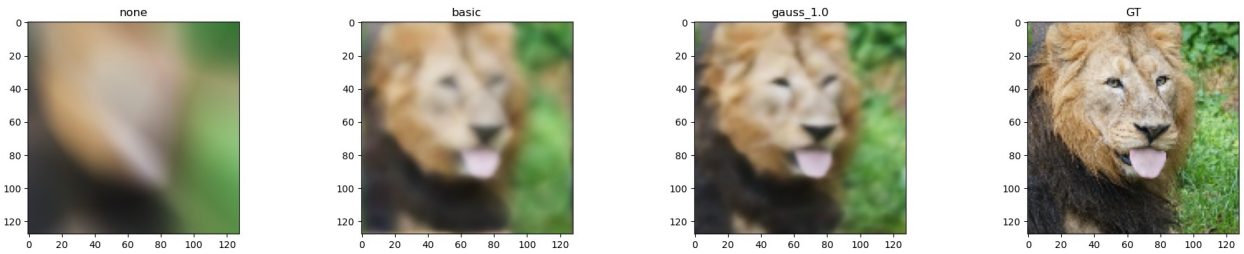

```
training basic
```

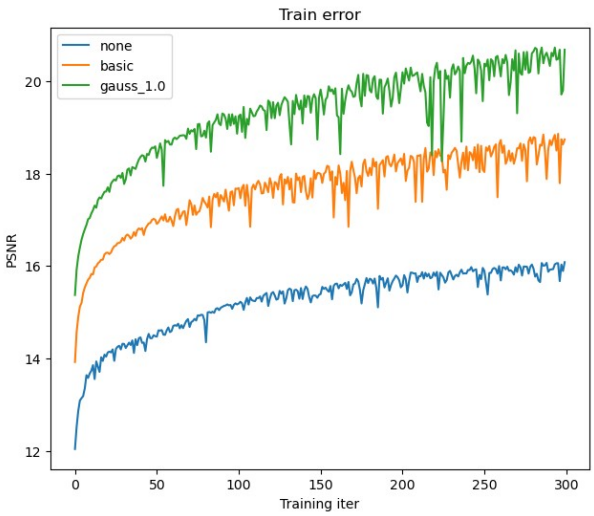

```
{"model_id":"4771dd159f294b968f7e1e53209583fd","version_major":2,"vers
ion_minor":0}
```

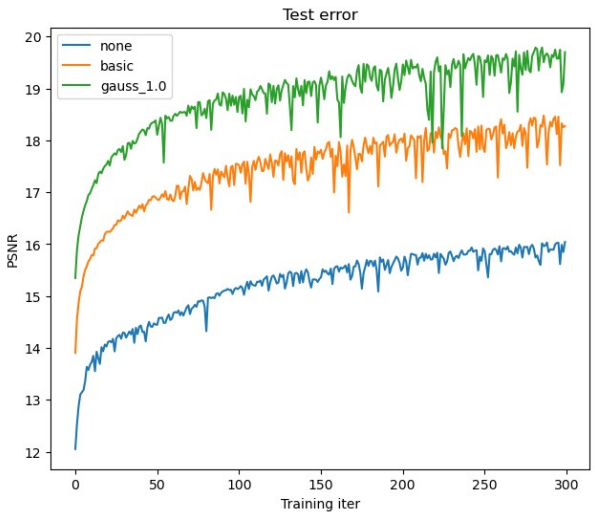

```
training gauss_1.0
```


```
{"model_id":"36bdd029b5c84c6cb3b3225928aff0e2","version_major":2,"vers
ion_minor":0}
```


```
X_train, y_train, X_test, y_test =
get_input_features(get_B_dict(size), "none")  # for getting y_test

plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```



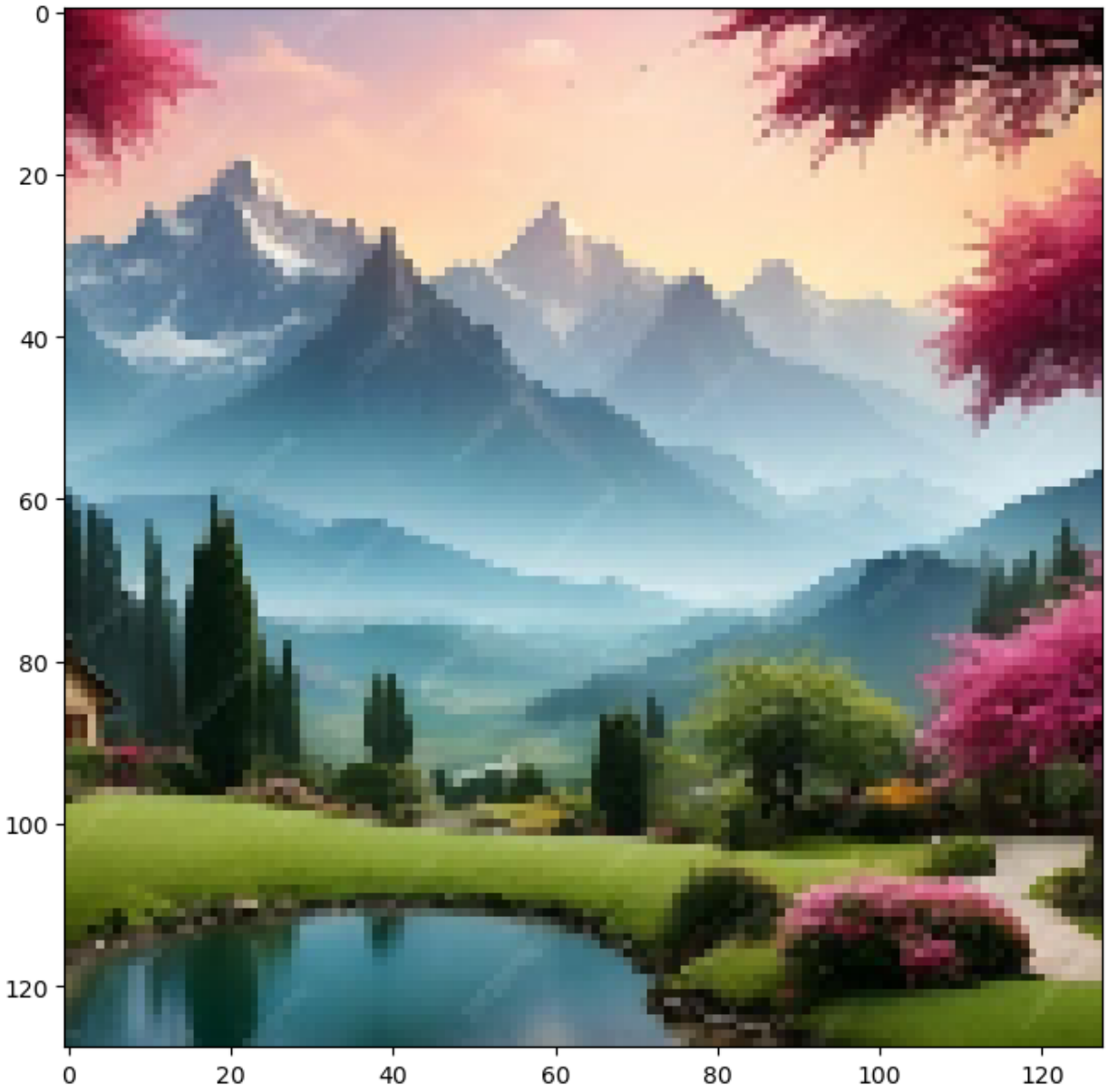




High Resolution Reconstruction - Image of your Choice

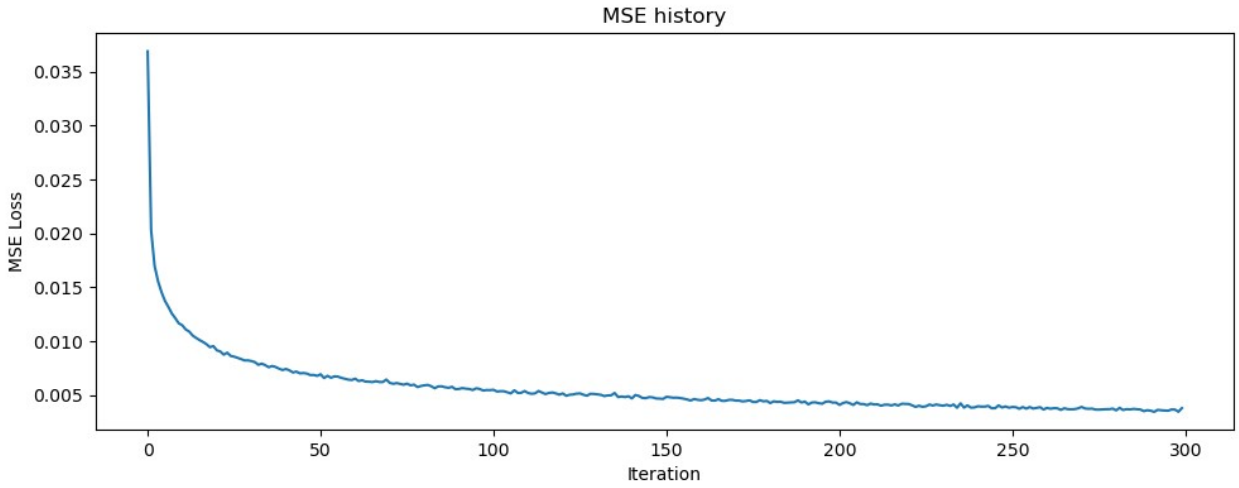
```
size = 128

train_data, test_data = get_image(size,
image_url="https://i.pinimg.com/736x/0c/b4/6f/0cb46fc064b18cede98279089d39e19b.jpg")
```

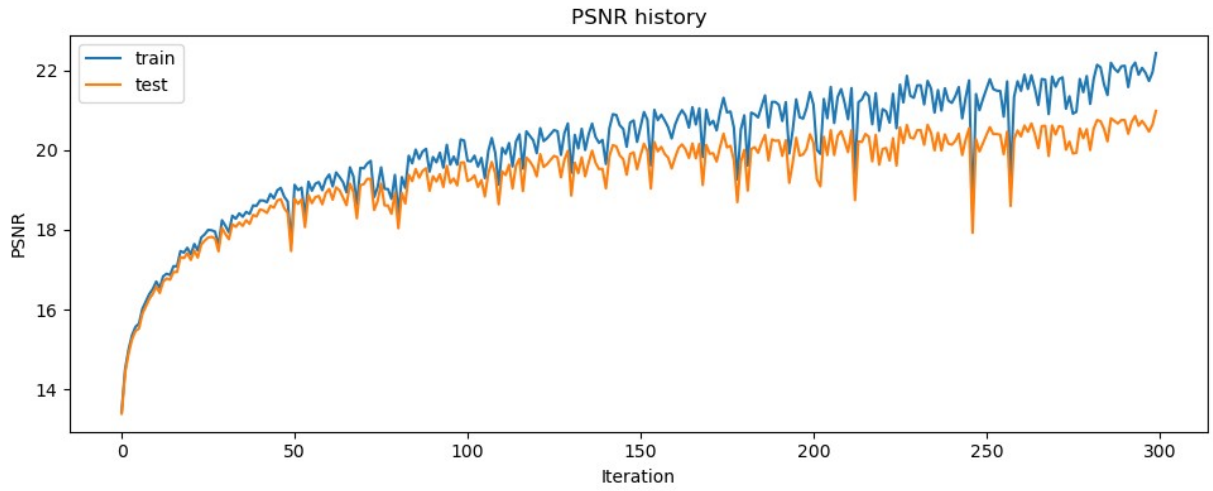
```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting with ImageIO v3 the behavior of this function will switch to that of iio.v3.imread. To keep the current behavior (and make this warning disappear) use `import imageio.v2 as imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```



```
# Set the hyperparameters
num_layers = 4
output_size = 3

# Adjust these
hidden_size = 256
epochs = 300
learning_rate = 0.2

# Get input features
B_dict = get_B_dict(size)
X_train, y_train, X_test, y_test =
get_input_features(B_dict,mapping="gauss_1.0")
input_size = X_train.shape[1]

# Run NN experiment on input features
net, train_psnr, test_psnr, train_loss, predicted_images =
NN_experiment(
                    X_train, y_train, X_test, y_test, input_size,
num_layers,
                    hidden_size, output_size, epochs, learning_rate,
'mse', opt="SGD",
                    batch_size=32)

# Plot results of experiment
plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

{"model_id":"f283deee84e54aed84524e013f6091eb","version_major":2,"version_minor":0}





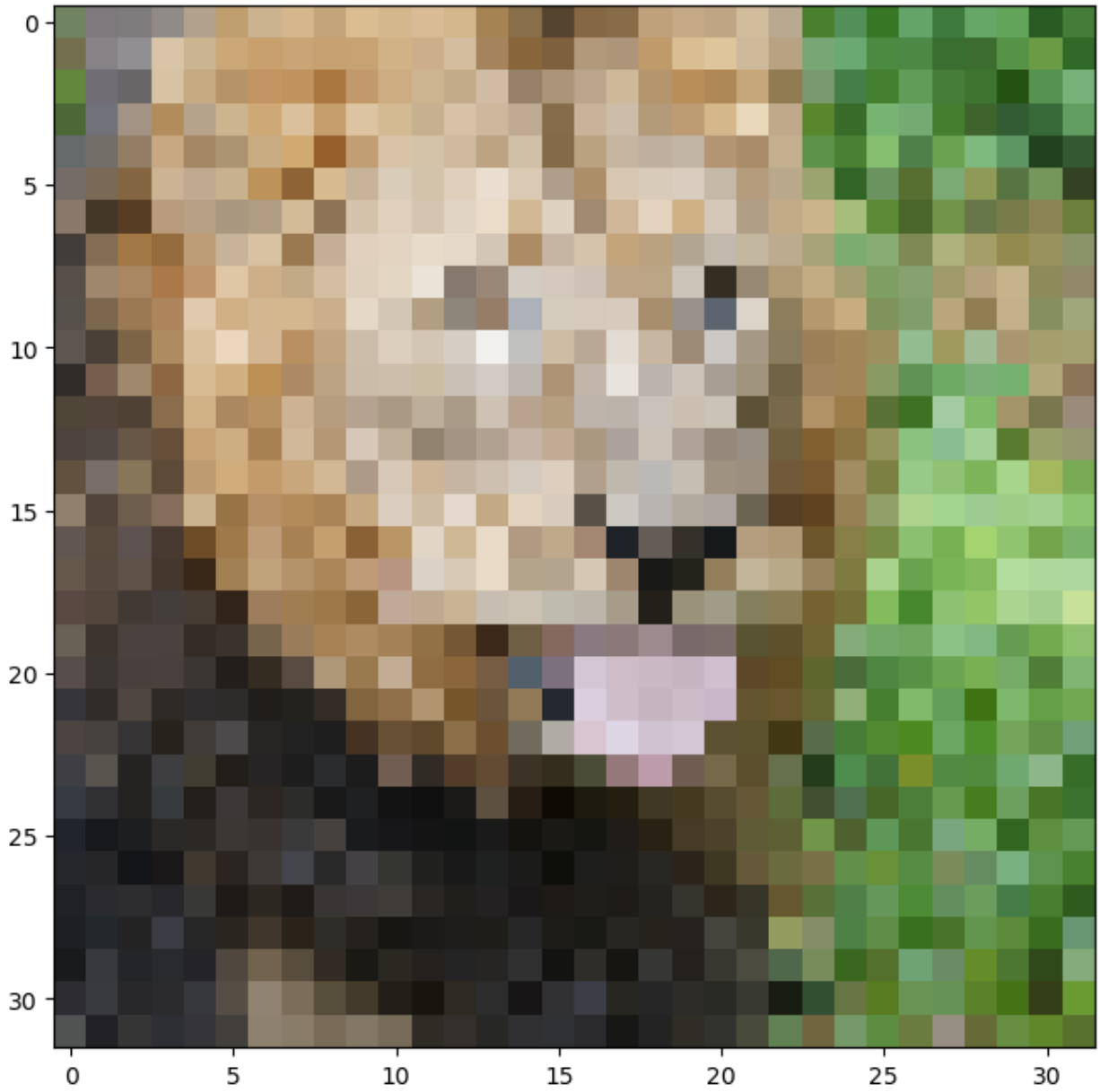

```
Final Test MSE 0.003990793849716292
Final Test psnr 20.97910710088206
```

1. Additional Experiments - Adam Optimizer

Low Resolution Reconstruction - Adam - None Mapping

```
# load low-res image
size = 32
train_data, test_data = get_image(size)
```

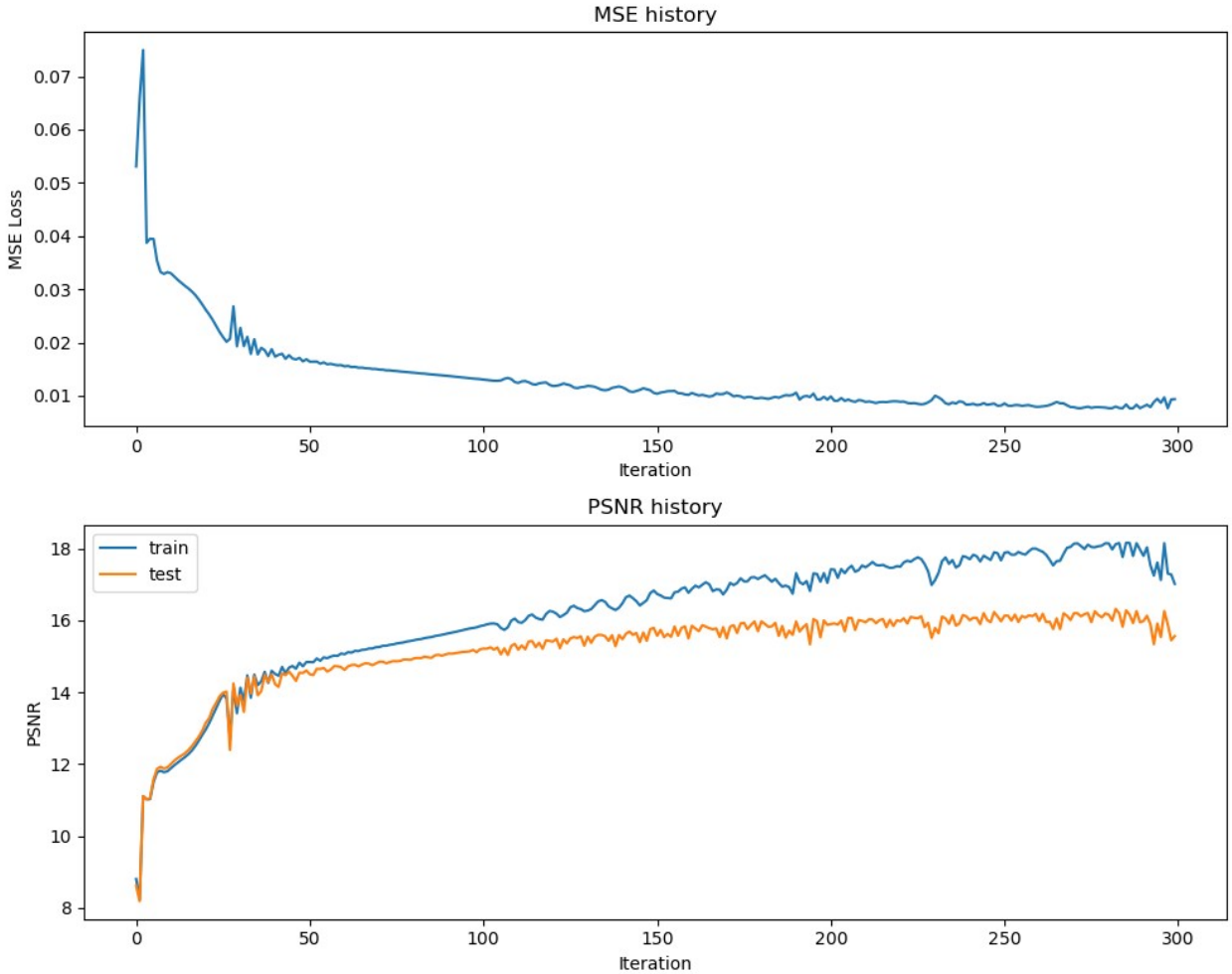
```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```



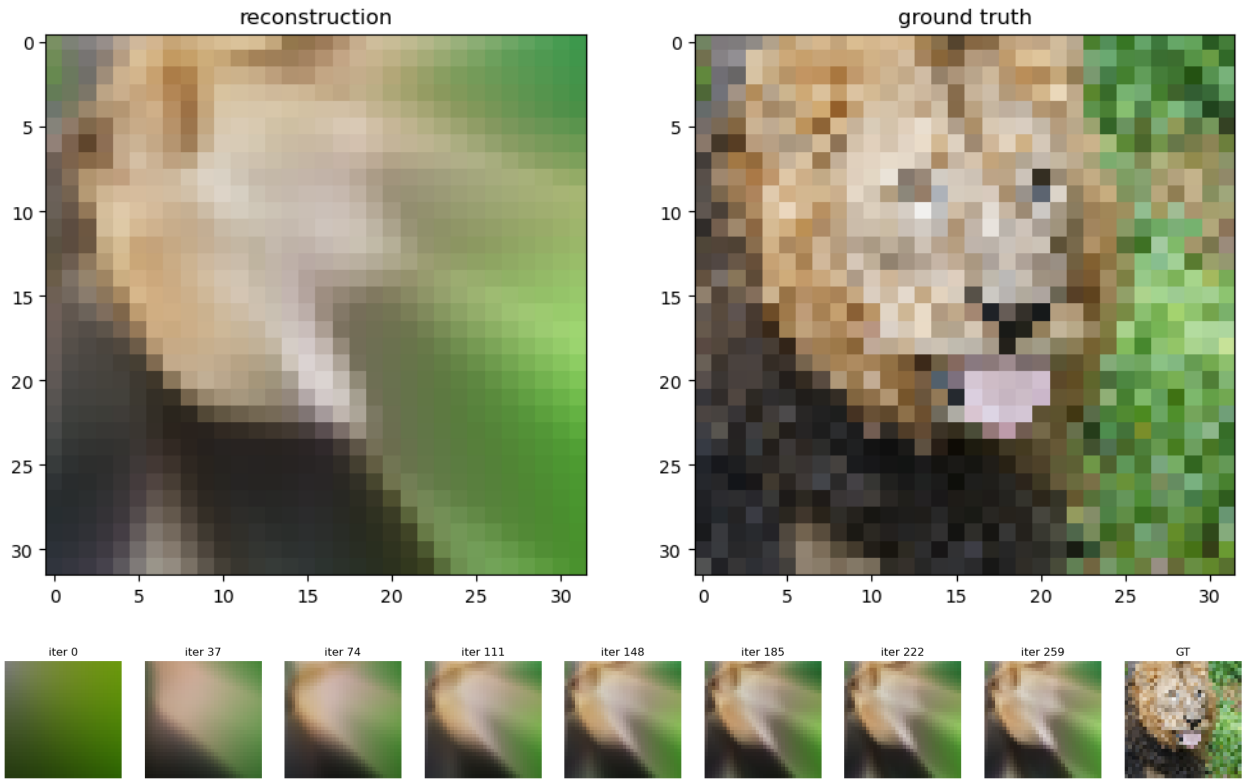

```
# Set the hyperparameters
num_layers = 4
output_size = 3

# Adjust these
hidden_size = 256
epochs = 300
learning_rate = 0.0075

# get input features
B_dict = get_B_dict(size)
X_train, y_train, X_test, y_test =
get_input_features(B_dict,mapping='none')
```

```
input_size = X_train.shape[1]

# run NN experiment on input features
net, train_psnr, test_psnr, train_loss, predicted_images =
NN_experiment(
                    X_train, y_train, X_test, y_test, input_size,
num_layers,
                    hidden_size, output_size, epochs, learning_rate,
'mse', opt="Adam",
                    b1=0.9, b2=0.999, eps=1e-8)

# plot results of experiment
plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

{"model_id":"94d167fabab440f1b98a85981fda2856","version_major":2,"version_minor":0}



```
Final Test MSE 0.013894503187029448
Final Test psnr 15.56126981613346
```

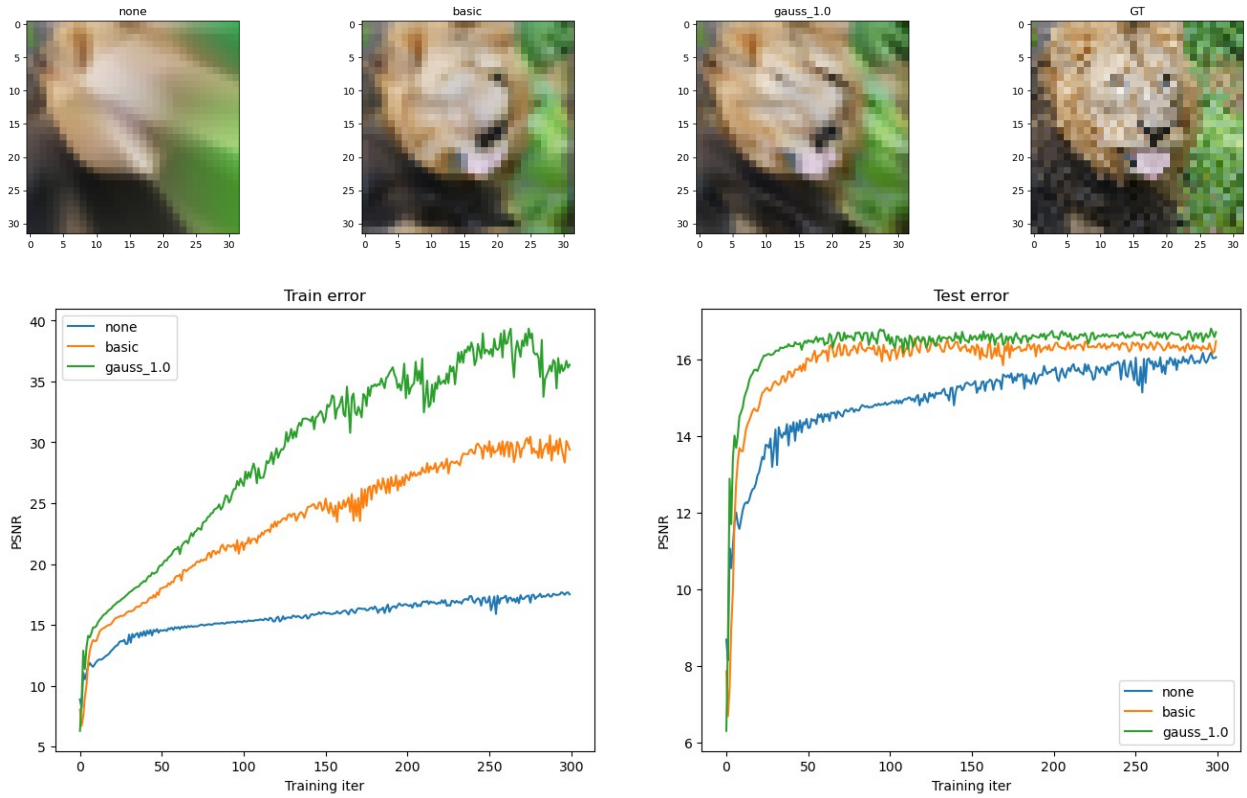


Low Resolution Reconstruction - Adam - Various Input Mapping Stategies

```
# start training
outputs = {}
for k in tqdm(B_dict):
  print("training", k)
  outputs[k] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mae', opt='Adam')

{"model_id":"b7b21b9d07cf4eb7a604f95b95b1195c","version_major":2,"vers
ion_minor":0}

training none

{"model_id":"bc31e1ece2264a8388acacfaf69264fe","version_major":2,"vers
ion_minor":0}

training basic

{"model_id":"6666d090ac084c4282a2be0243142746","version_major":2,"vers
ion_minor":0}

training gauss_1.0
```

{"model_id":"c46641c9a4e546c18fd139f9c17dd9d2","version_major":2,"version_minor":0}

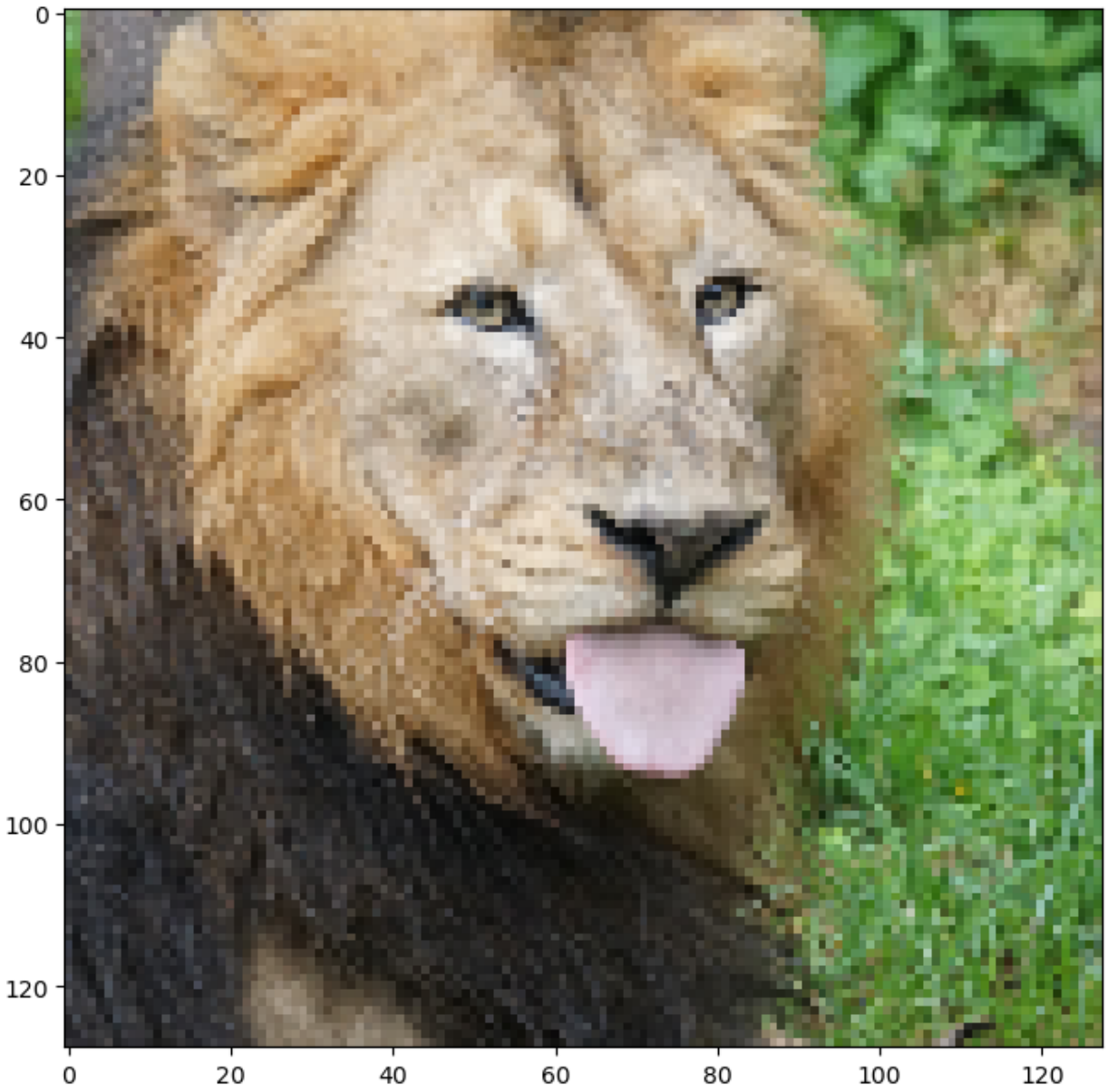
```
plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```



High Resolution Reconstruction - Adam - Various Input Mapping Stategies

```
# load hi-res image
size = 128
train_data, test_data = get_image(size)
```

```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting with ImageIO v3 the behavior of this function will switch to that of iio.v3.imread. To keep the current behavior (and make this warning disappear) use `import imageio.v2 as imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```




```
# Set the hyperparameters
num_layers = 4
output_size = 3

# Adjust these
hidden_size = 256
epochs = 300
learning_rate = 0.0075

# start training
outputs = {}
for k in tqdm(B_dict):
  print("training", k)
```

```
  outputs[k] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mse', opt='Adam')
```


```
{"model_id":"5e3e1ce35cdf4893ac36b3a9242e116f","version_major":2,"vers
ion_minor":0}
```


```
training none
```


```
{"model_id":"c58274bc208a47eda3f440c1da833c82","version_major":2,"vers
ion_minor":0}
```

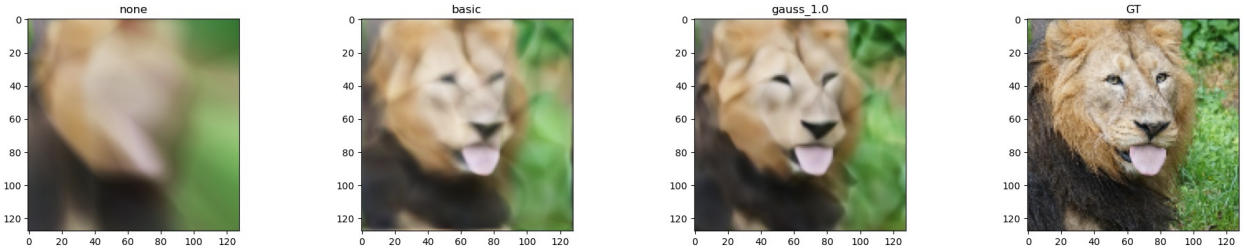

```
training basic
```

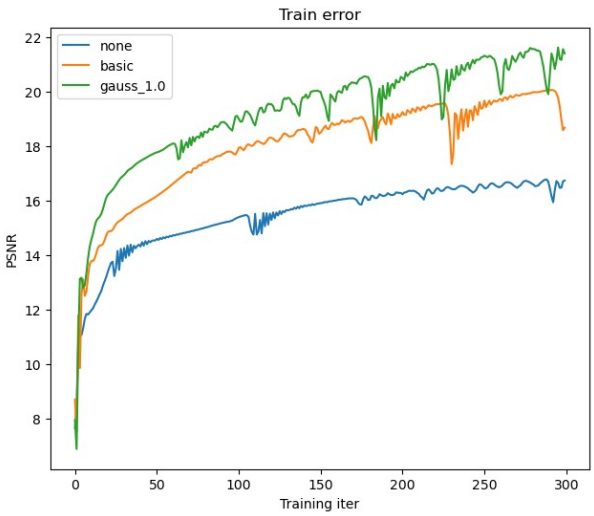

```
{"model_id":"5f35dcd342f947a58c090d75a7f33bc6","version_major":2,"vers
ion_minor":0}
```

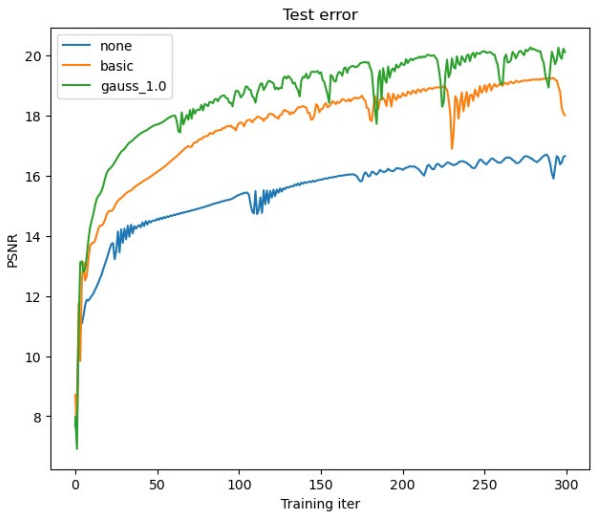

```
training gauss_1.0
```


```
{"model_id":"0ef2cfad70a745a1ac8cbfe0cf8776f2","version_major":2,"vers
ion_minor":0}
```


```
X_train, y_train, X_test, y_test =
get_input_features(get_B_dict(size), "none")  # for getting y_test

plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```





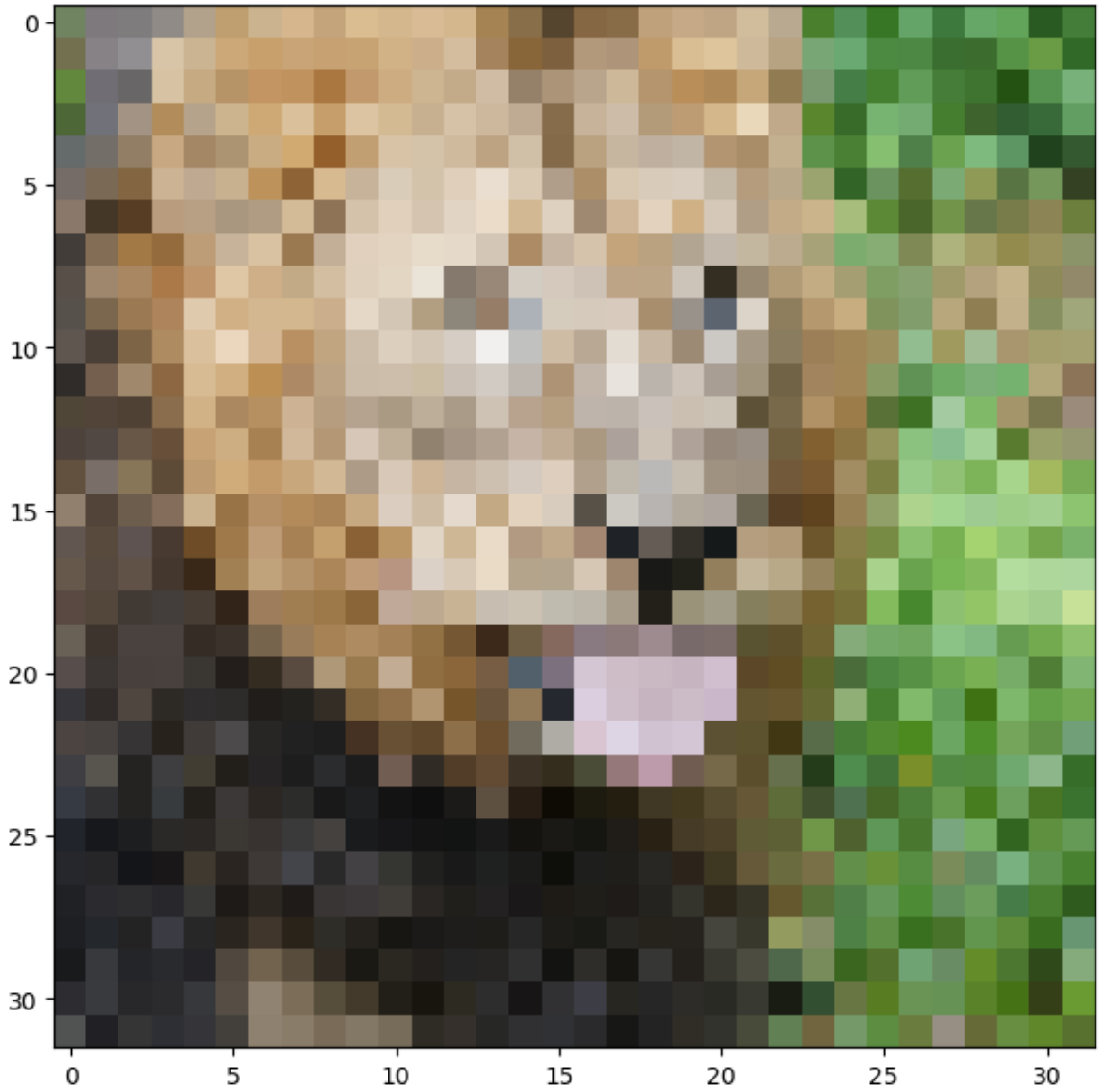


2. Additional Experiments - Deeper Network

Low Resolution Reconstruction

```
size = 32
train_data, test_data = get_image(size)
```

```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```




```
num_layers = 6
output_size = 3

hidden_size = 512
epochs = 300
learning_rate = 0.2

outputs = {}
for k in tqdm(B_dict):
  print("training", k)
  outputs[k] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mse', opt='SGD')
```

```
{"model_id":"669a9bb41a31461a89e44b5177761970","version_major":2,"version_minor":0}
```

```
training none
```

```
{"model_id":"2fc977efee0e4c899add0fbe5c8725ef","version_major":2,"version_minor":0}
```

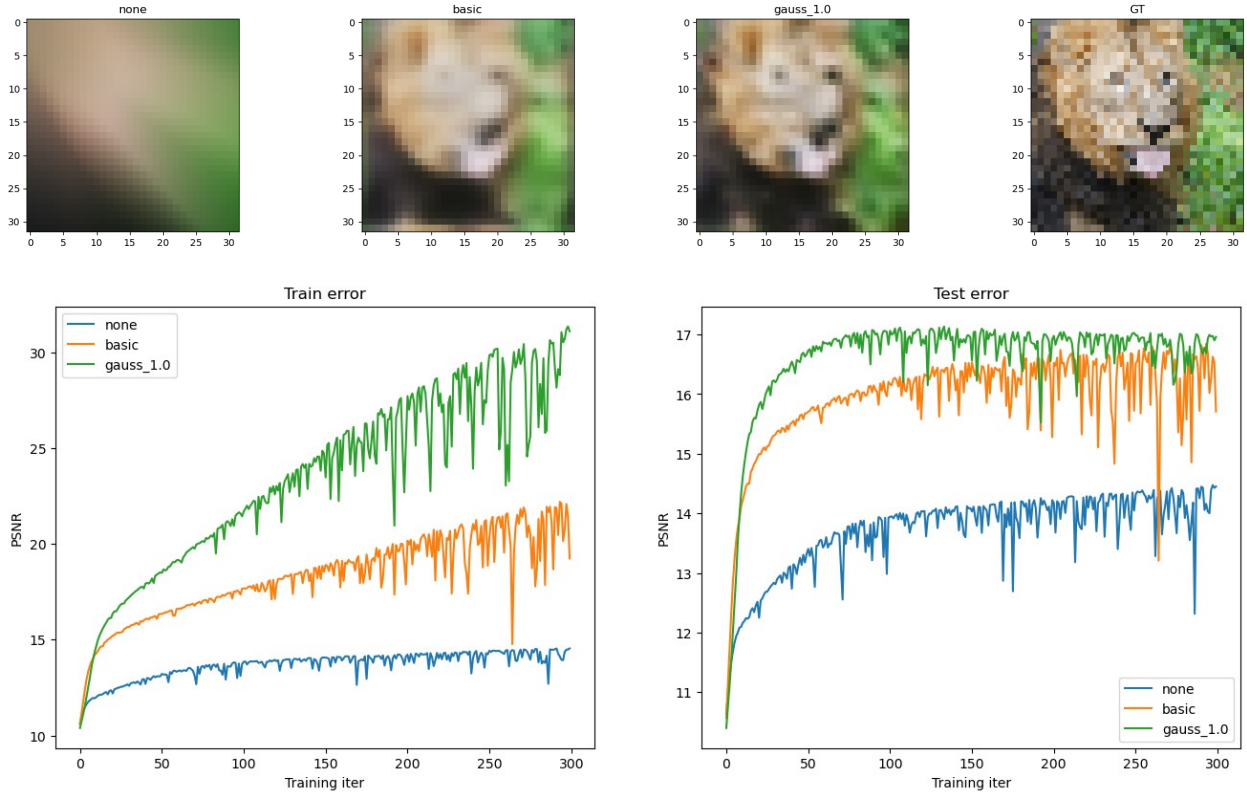
```
training basic
```

```
{"model_id":"3881f0c385c94f32ac9e026773d1df77","version_major":2,"version_minor":0}
```

```
training gauss_1.0
```

```
{"model_id":"4a4b66bc5d094a16a54ffaaf51c83763","version_major":2,"version_minor":0}
```

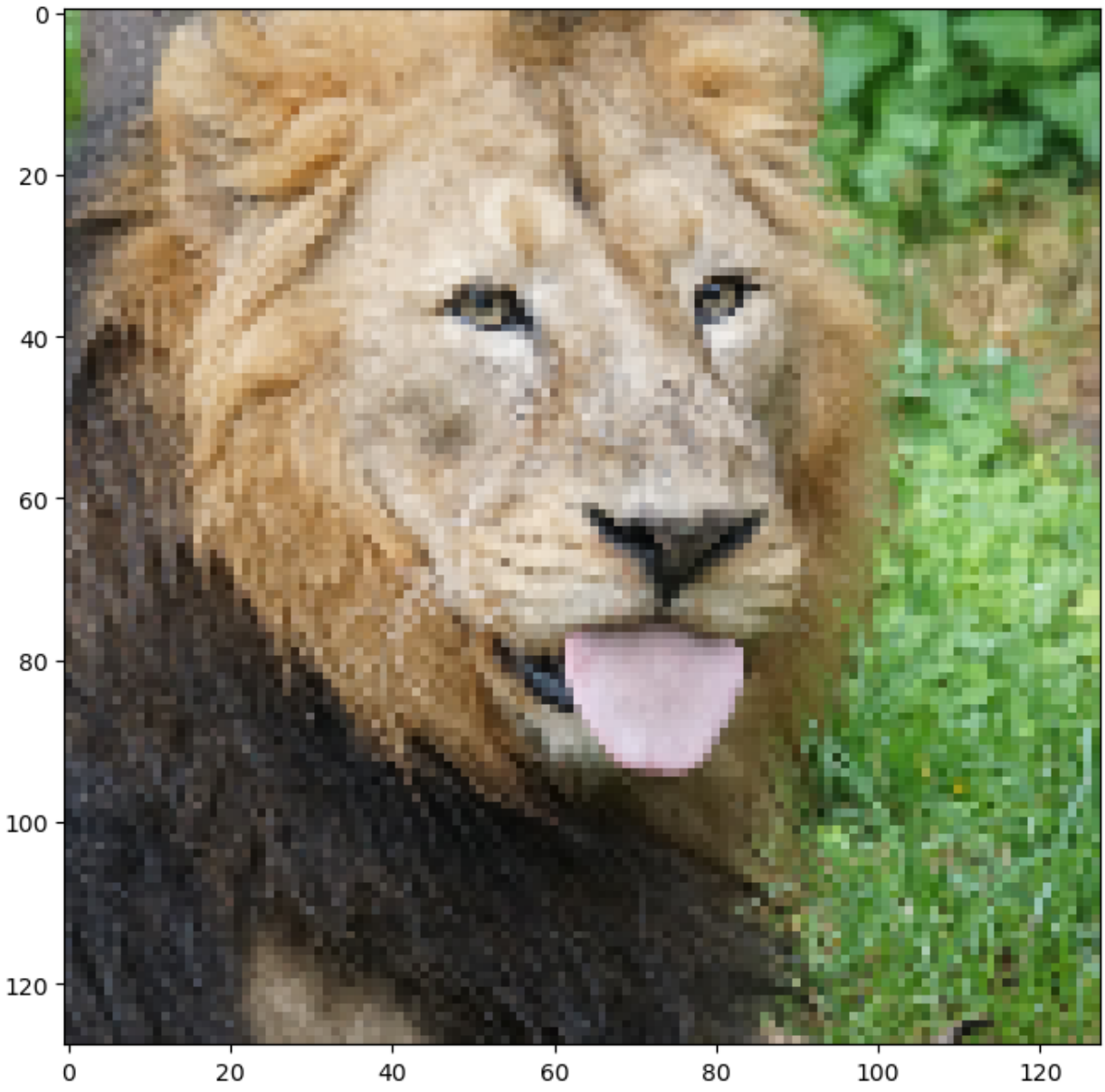
```
X_train, y_train, X_test, y_test =
get_input_features(get_B_dict(size), "none")  # for getting y_test

plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```



High Resolution Reconstruction

```
size = 128
train_data, test_data = get_image(size)
```

```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting with ImageIO v3 the behavior of this function will switch to that of iio.v3.imread. To keep the current behavior (and make this warning disappear) use `import imageio.v2 as imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```




```
num_layers = 6
output_size = 3
```

```
# Adjust these
hidden_size = 768
epochs = 300
learning_rate = 0.2

outputs = {}
B_dict = get_B_dict(size)
for k in tqdm(B_dict):
  print("training", k)
  outputs[k] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mse', opt='SGD')
```

{"model_id":"fa685c60588e4cf69e46f991d7053eea","version_major":2,"vers
ion_minor":0}

```
training none
```

{"model_id":"6a1d5c46cc35482491acb38ef3c5ffcb","version_major":2,"vers
ion_minor":0}

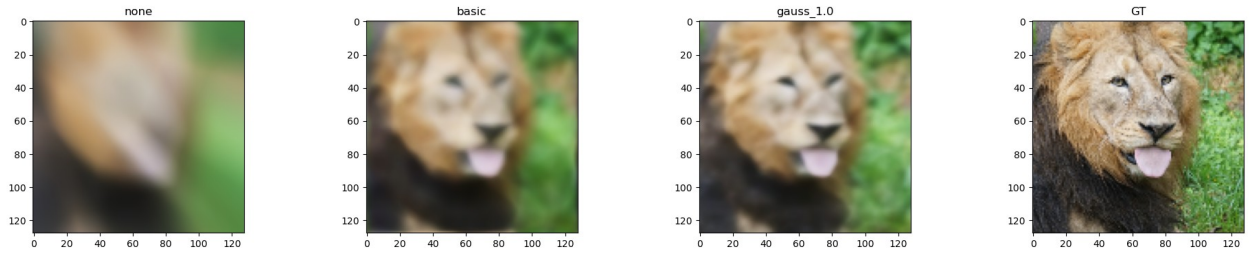
```
training basic
```

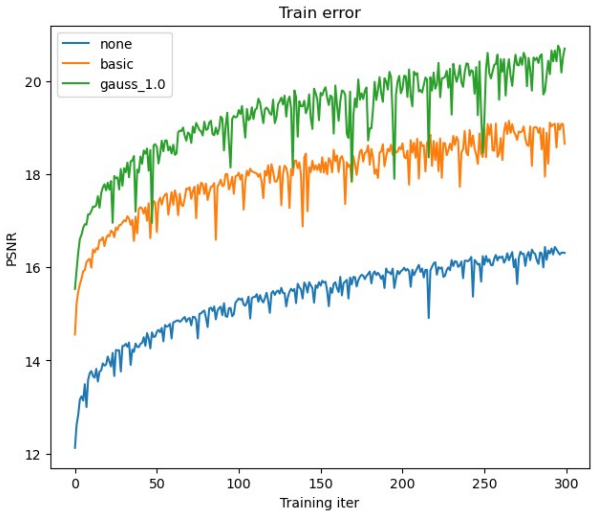
{"model_id":"f083cbb982454641bbdbbfa8c4b9c20c","version_major":2,"vers
ion_minor":0}

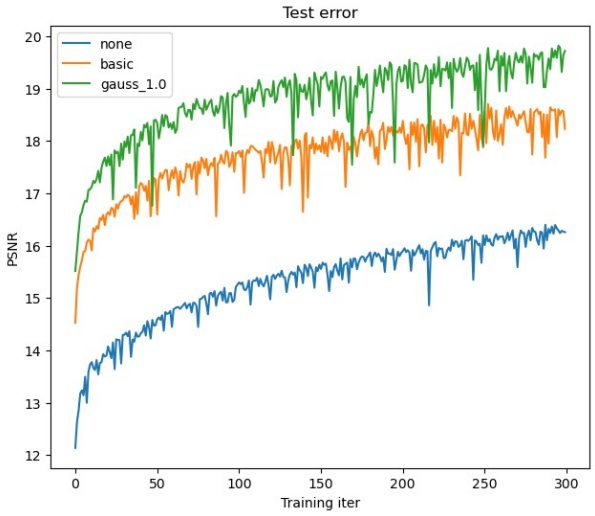
```
training gauss_1.0
```

{"model_id":"b4f5d10a44ff4acdbff3b55b15284e71","version_major":2,"vers
ion_minor":0}

```
X_train, y_train, X_test, y_test =
get_input_features(get_B_dict(size), "none")
plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```



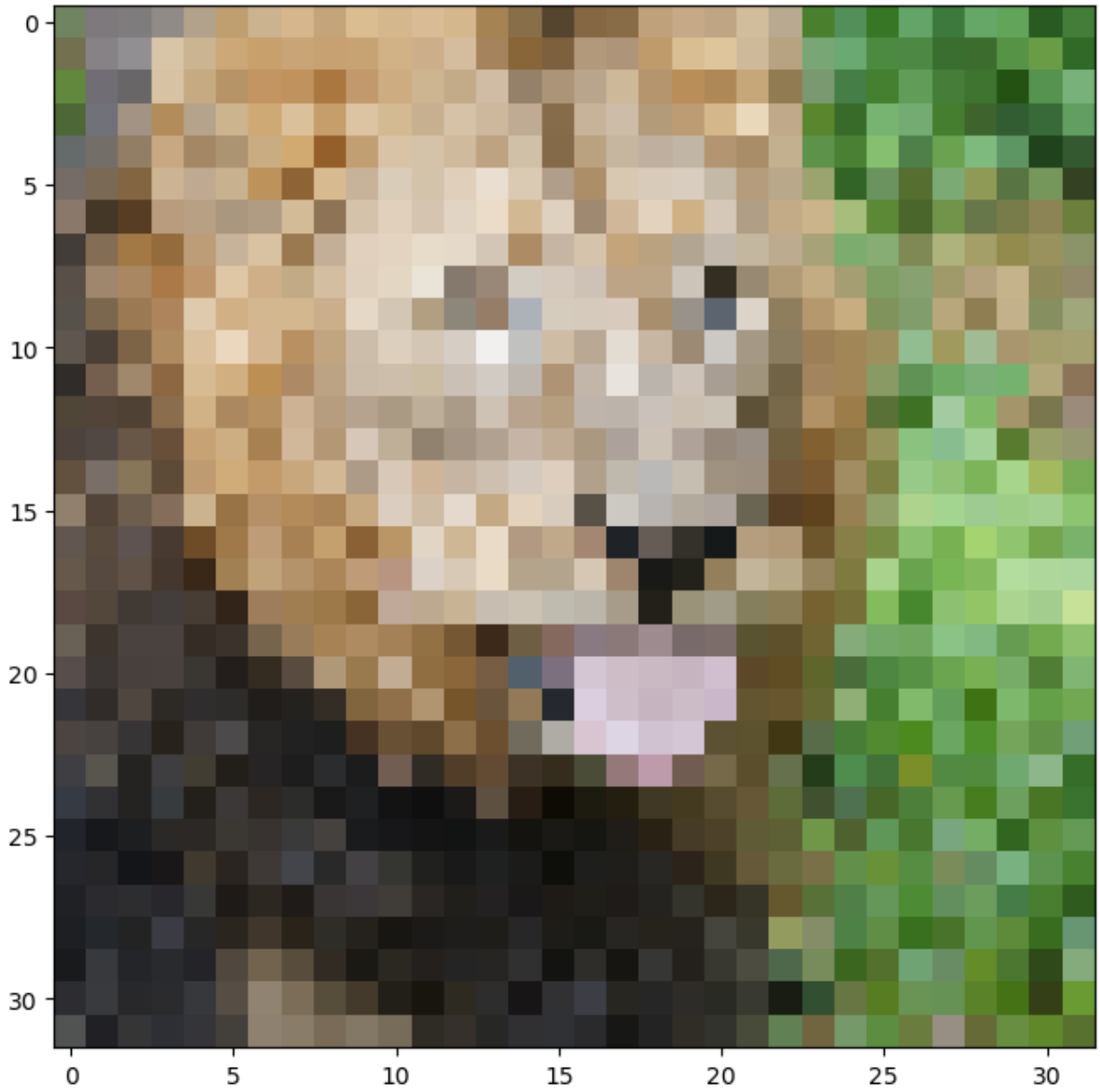




3. Additional Experiments - Alternative Losses

```
size = 32
train_data, test_data = get_image(size)
num_layers = 3
output_size = 3
hidden_size = 256
epochs = 300
learning_rate = 0.2


outputs = {}
B_dict = get_B_dict(size=256)
k = 'gauss_1.0'

## mae
outputs['mae'] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mae', opt='SGD')
outputs['mse'] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'mse', opt='SGD')
outputs['huber'] = train_wrapper(k, size, num_layers, hidden_size,
output_size, epochs, learning_rate, 'huber', opt='SGD')
```

```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```

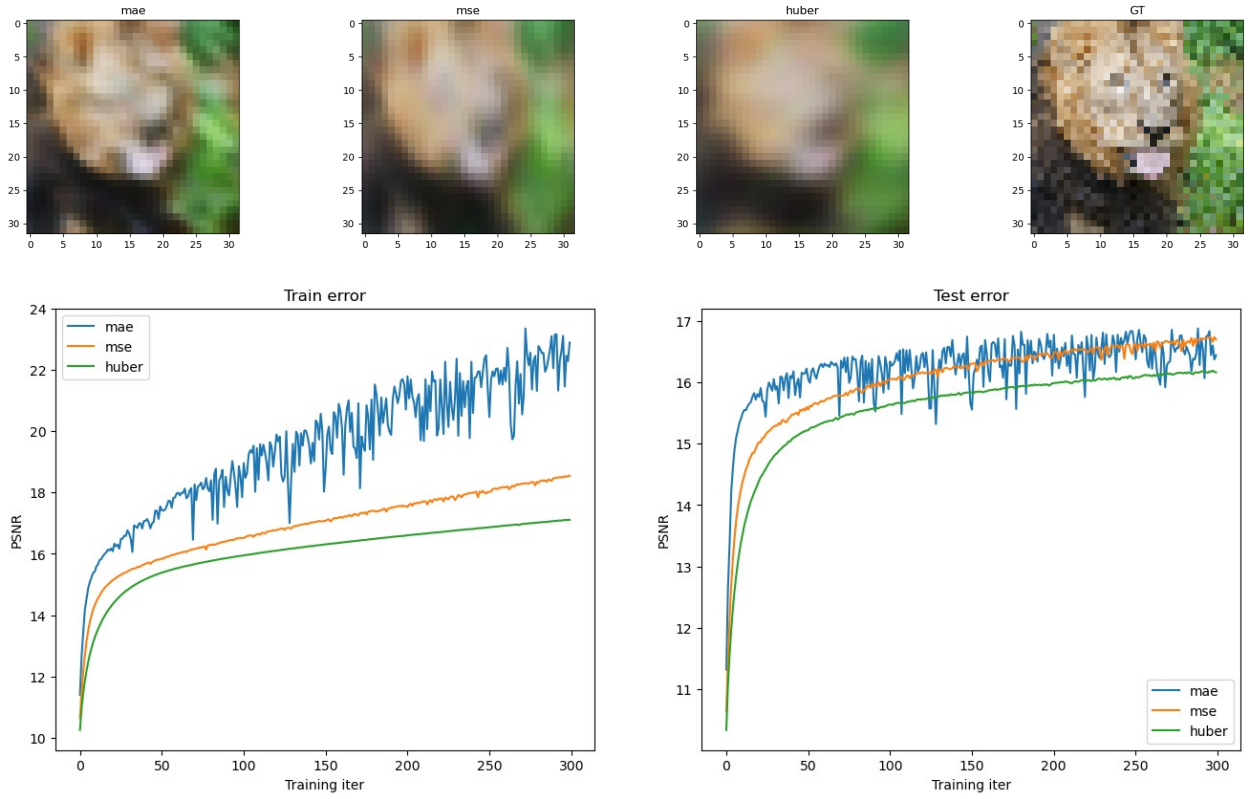


{"model_id":"cf1f5b8457e34f47a622977b4d194988","version_major":2,"version_minor":0}

{"model_id":"6ba55f8bf3bc49c6bbad9c2269885e6e","version_major":2,"version_minor":0}

{"model_id":"c0ef768e6c9f4d949d6038d1cfedde65","version_major":2,"version_minor":0}

```
X_train, y_train, X_test, y_test =
get_input_features(get_B_dict(size), "none")
plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3))
```



4. Additional Experiments - Gaussian Fourier Feature mapping hyperparameters

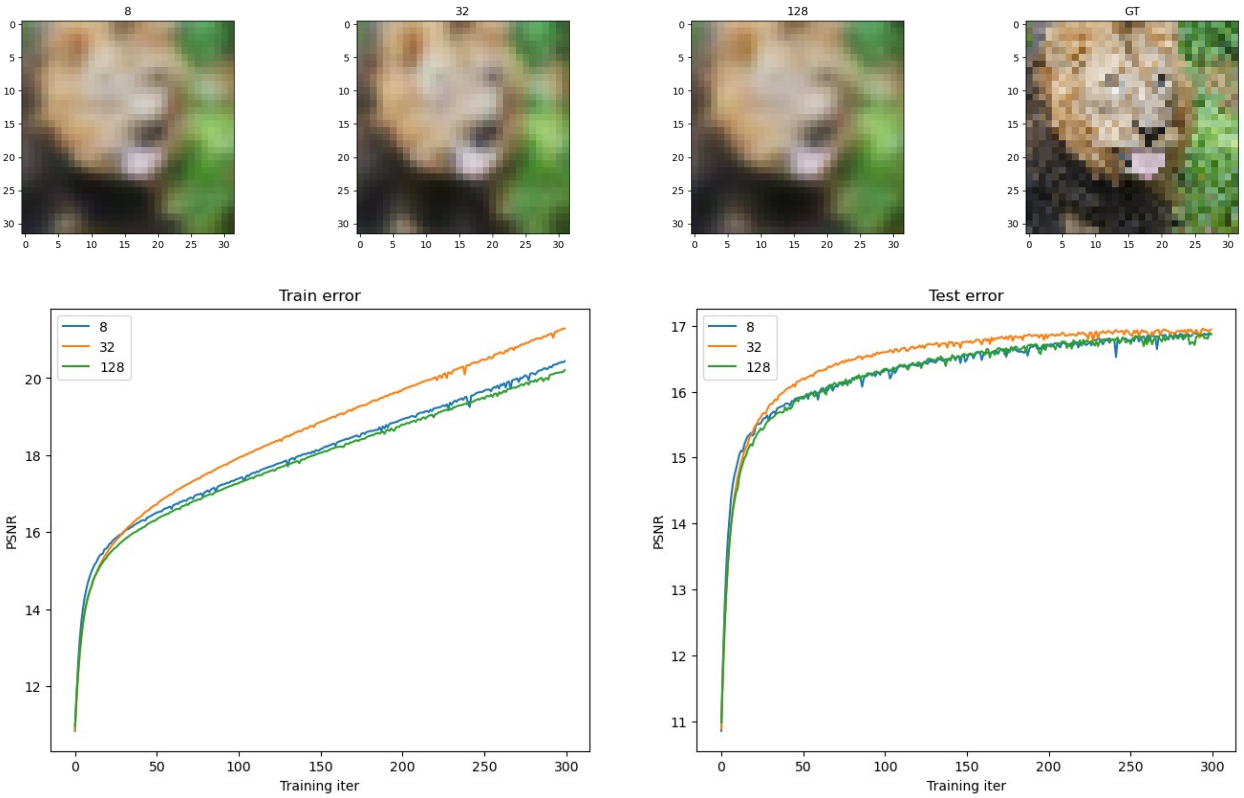
```
mapping_sizes = [8, 32, 128]
gaussian_scales = [1, 2]

size = 32

hyperparameter_configs = []
for scale in gaussian_scales:
    outputs = {}
    print('For mapping size:', scale)
    for m_size in mapping_sizes:
        B_dict = get_B_dict(scale, m_size)
        outputs[m_size] = train_wrapper('gauss_1.0', size, num_layers,
hidden_size, output_size, epochs, learning_rate, 'mse', opt='SGD')

    X_train, y_train, X_test, y_test =
get_input_features(get_B_dict(size), "none")
    plot_feature_mapping_comparison(outputs,
y_test.reshape(size,size,3))
```

```
For mapping size: 1
```

{"model_id":"7d226ab076fd4dd4bcd94442c1cc8971","version_major":2,"version_minor":0}

{"model_id":"d6377b795dd54238860089d8f7be0c1d","version_major":2,"version_minor":0}

{"model_id":"49584400b2514af88cb4fcca1c09374e","version_major":2,"version_minor":0}

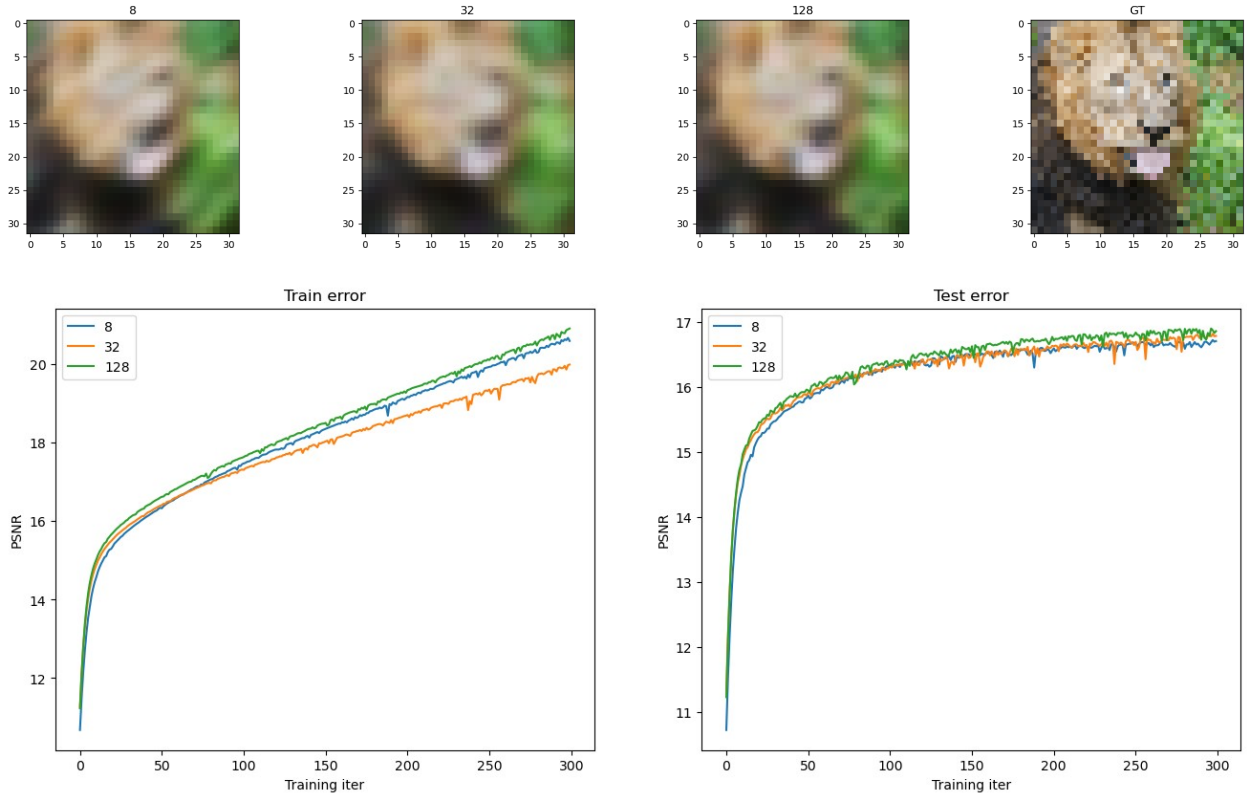


For mapping size: 2

{"model_id":"00265c56f4474408ad6d0774678ecd49","version_major":2,"version_minor":0}

{"model_id":"474f43abe59a43f9aa0fab088c784d50","version_major":2,"version_minor":0}

{"model_id":"8454747e5c9e403489ab0cf6ab2819b4","version_major":2,"version_minor":0}

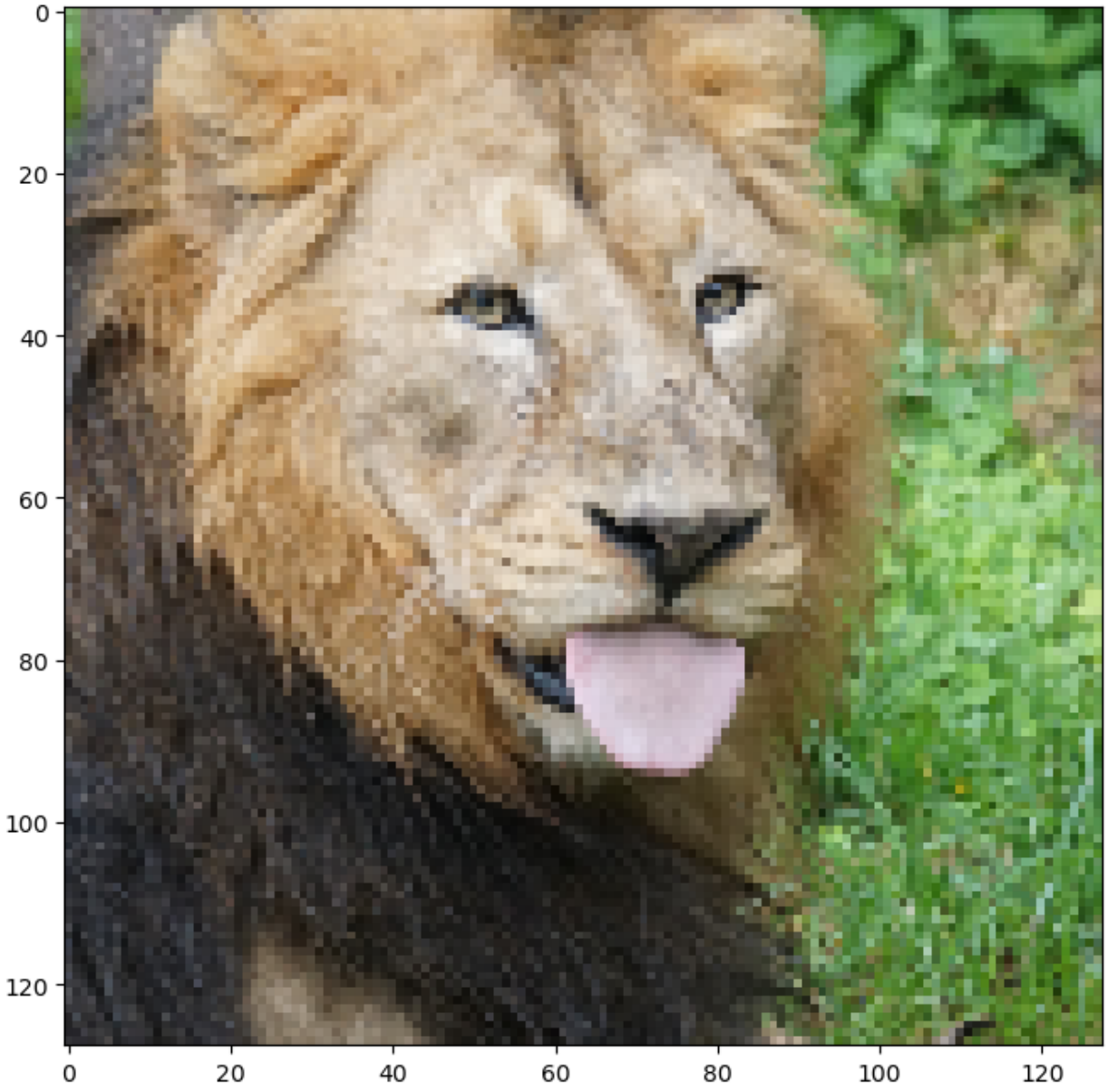


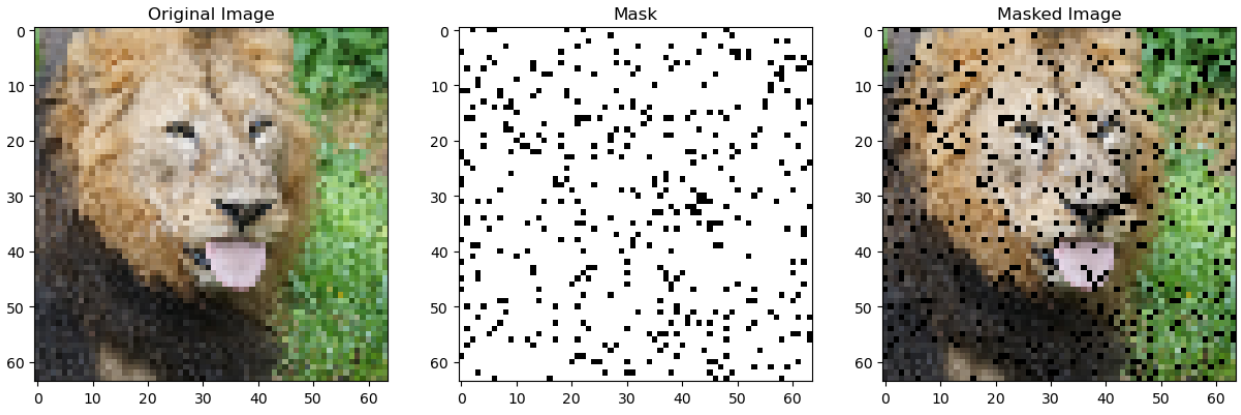
5. Additional Experiments - Image inpainting or restoration scenarios

```
size = 128
def get_inpainting_data(size=512, mask_type='random', mask_ratio=0.1):

    train_data_orig, test_data = get_image(size)
    train_coords, train_img = train_data_orig
    test_coords, test_img = test_data

    train_size = train_img.shape[0]  # Size of downsampled training
image
    mask = np.ones((train_size, train_size), dtype=np.float32)

    if mask_type == 'random':
        random_mask = np.random.random((train_size, train_size)) >
mask_ratio
        mask = random_mask.astype(np.float32)

    elif mask_type == 'center':
        center = train_size // 2
        radius = int(train_size * np.sqrt(mask_ratio) / 2)
        y, x = np.ogrid[:train_size, :train_size]
```

```
        center_mask = (x - center)**2 + (y - center)**2 <= radius**2
        mask[center_mask] = 0.0

    elif mask_type == 'checkerboard':
        block_size = max(1, int(train_size * np.sqrt(mask_ratio) / 4))
        for i in range(0, train_size, block_size*2):
            for j in range(0, train_size, block_size*2):
                if i + block_size <= train_size and j + block_size <=
train_size:
                    mask[i:i+block_size, j:j+block_size] = 0.0

                if i + block_size*2 <= train_size and j + block_size*2
<= train_size:
                    mask[i+block_size:i+block_size*2,
j+block_size:j+block_size*2] = 0.0


    mask_3c = np.stack([mask] * 3, axis=-1)
    masked_img = train_img * mask_3c

    train_gt = train_img.copy()

    inpainting_train_data = [train_coords, masked_img, mask_3c]


    plt.figure(figsize=(15, 5))
    plt.subplot(1, 3, 1)
    plt.imshow(train_gt)
    plt.title('Original Image')

    plt.subplot(1, 3, 2)
    plt.imshow(mask, cmap='gray')
    plt.title('Mask')

    plt.subplot(1, 3, 3)
    plt.imshow(masked_img)
    plt.title('Masked Image')
    plt.show()

    return inpainting_train_data, test_data, train_gt


train_data, test_data, train_gt = get_inpainting_data(size)
```

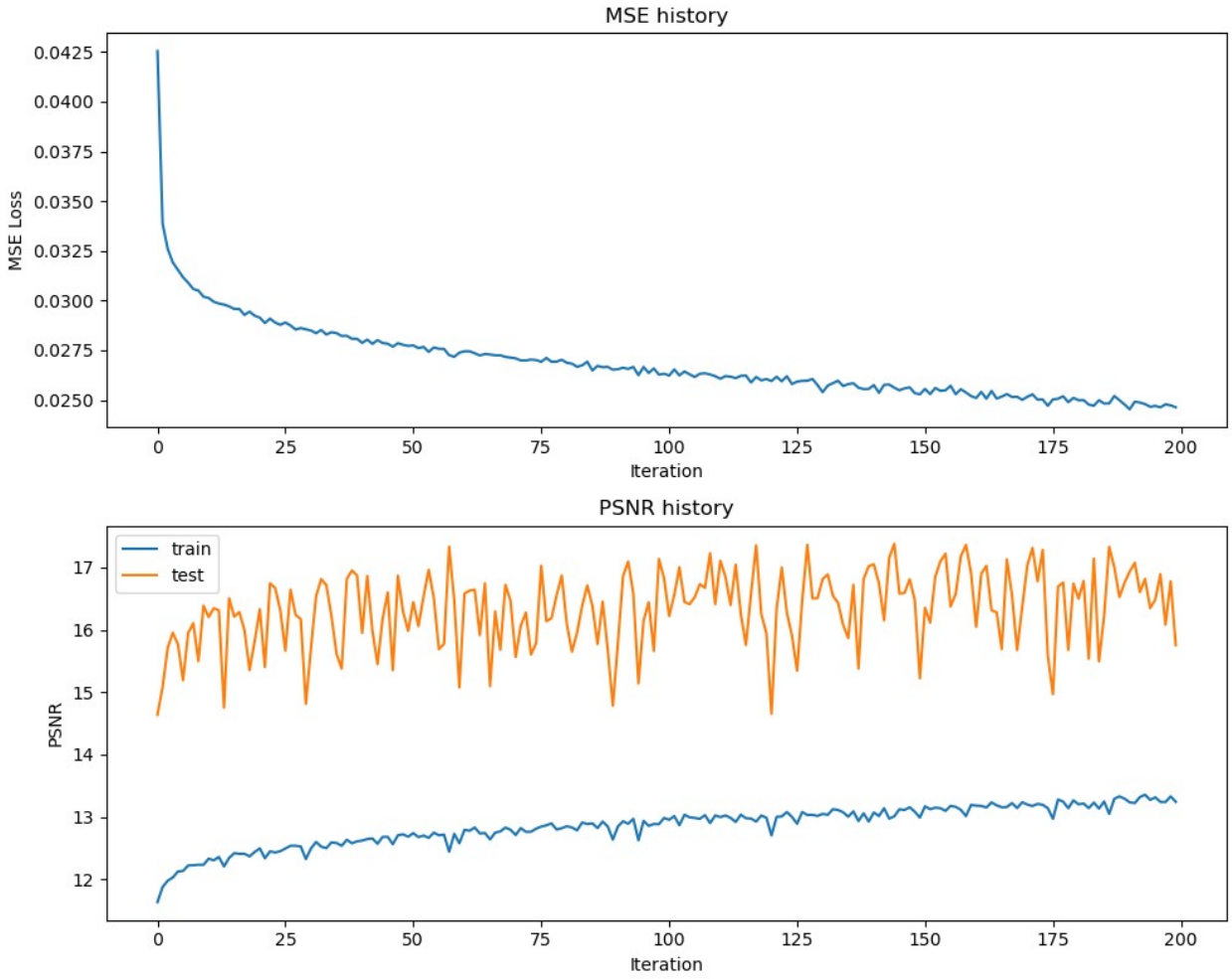
```
/tmp/ipykernel_1671925/2979632171.py:6: DeprecationWarning: Starting
with ImageIO v3 the behavior of this function will switch to that of
iio.v3.imread. To keep the current behavior (and make this warning
disappear) use `import imageio.v2 as imageio` or call
`imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```


0
20
40
60
80
100
120

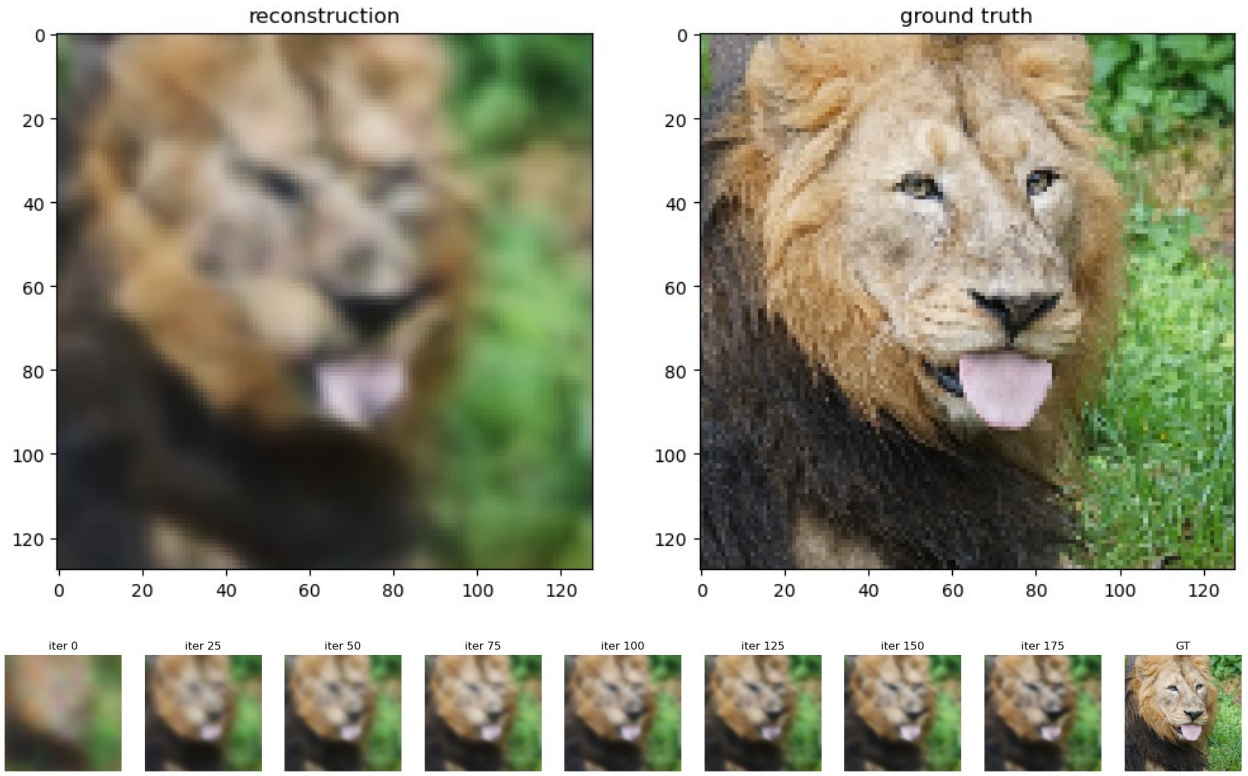

Original Image
Mask
Masked Image
0
10
20
30
40
50
60

```
num_layers = 4
output_size = 3

hidden_size = 256
epochs = 200
learning_rate = 0.2

B_dict = get_B_dict(size)
X_train, y_train, X_test, y_test =
get_input_features(B_dict,mapping="gauss_1.0")
input_size = X_train.shape[1]

net, train_psnr, test_psnr, train_loss, predicted_images =
NN_experiment(
                    X_train, y_train, X_test, y_test, input_size,
num_layers,
                    hidden_size, output_size, epochs, learning_rate,
'mse', opt="SGD",
                    batch_size=32)


plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

{"model_id":"0d854f96c4494bdaac17da1bf3160544","version_major":2,"version_minor":0}




```
Final Test MSE 0.013294226003214523
Final Test psnr 15.753069467841119
```


reconstruction
ground truth
0
20
40
60
80
100
120
iter 0
iter 25
iter 50
iter 75
iter 100
iter 125
iter 150
iter 175
GT