Project Report

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```r
library(tidymodels)
library(tidyverse)
library(bonsai)
library(themis)
library(readxl)
library(stringr)
library(lubridate)
library(ggplot2)
library(dplyr)
library(ggrepel)
library(scales)
library(colorblindr)
library(colorspace)
```

1. Introduction and Data

Data was collected from following resources:

UNESCO Institute for Statistics (UIS): Educational indicators data collected from https://sdg4-data.uis.unesco.org/. This data provides comprehensive educational metrics for various countries across multiple years and was last updated in September 2024. Data from 2013 to 2024

TidyTuesday GitHub Repository: Data related to the International Mathematical Olympiad (IMO) collected from https://github.com/rfordatascience/tidytuesday/blob/master/data/2024/2024-09-24/readme.md. The IMO data tracks country-level performance, including scores, medals, and rankings, and was also updated in September 2024. Data from 1959 to 2024

DataBank: https://databank.worldbank.org/source/education-statistics-%5e-all-indicators# other educational series was collected from this site.

After data cleaning and feature engineering, the dataset contains `1,142 observations` and `38 columns`. However, not all columns were used for model fitting.

Columns Used:

Year: Year of the IMO

team_size_all: Total team size

team_size_male: Male team size

team_size_female: Female team size

Value_gross_enr_ratio_for_tertirary_edu: Gross enrollment ratio for tertiary education

Value_gov_expen_as_perc_of_GDP: Government expenditure as a percentage of GDP

Value_literacy_rate: Literacy rate

UIS.X.USCONST.FSGOV: Government expenditure on education (constant US$ in millions)

UIS.X.US.FSGOV: Government expenditure on education (US$ in millions)

NY.GDP.PCAP.CD: GDP per capita (current US$)

SE.XPD.TOTL.GB.ZS: Expenditure on education as a percentage of total government expenditure (%)

SE.XPD.CUR.TOTL.ZS: Current expenditure as a percentage of total expenditure in public institutions (%)

OECD.TSAL.1.E10: Annual statutory teacher salaries in public institutions (USD, primary education, 10 years of experience)

SL.TLF.ADVN.ZS: Labor force with advanced education (% of total labor force)

IT.NET.USER.P2: Internet users (per 100 people)

SE.TER.GRAD.SC.ZS: Percentage of graduates from tertiary education graduating in Natural Sciences, Mathematics, and Statistics (both sexes, %).

Units of Analysis

- **Countries**: Each country represents a unit of analysis in this dataset, with attributes related to educational performance (such as completion rates, expenditure) and their success in the IMO (average score, team size, medals won).
- **Time**: The dataset spans multiple years, allowing for the analysis of trends over time in both education indicators and IMO performance.

Topic Description

This project aims to explore how a country's education system impacts its performance in the **International Mathematical Olympiad (IMO)**. I am particularly interested in studying the relationship between **government spending on education, literacy rates** and a country's success in the IMO.

Why This Topic?

This topic interests me because I want to understand how a country's investment in education and the quality of its education system influence its ability to succeed in international academic competitions like the IMO. By exploring these relationships, I hope to identify the factors that most strongly contribute to winning medals or achieving high scores in the IMO.

Expectations

I expect to find a **positive relationship** between a country's investment in education and its success in the IMO.

```
education_data <- read_csv('data/education_data.csv')
imo_data <- read_csv('data/imo_data.csv')
```

Main Outcome/Target (Y Variable):

The main outcome or target variable in this analysis is the **average score per contestant for each country in a given year**. This is calculated by summing the scores from problems 1 to 7 for each country's team and dividing the total by the number of participants (team_size_all). This variable represents how well the entire team from each country performed in the International Mathematical Olympiad (IMO).

This outcome is a good fit for the study because it provides a clear measure of how well a country's education system prepares students for international competitions. By using the average score, the analysis captures the performance of the whole team, not just the top individual performers. This is important for understanding the impact of educational investments, such as government spending on education, literacy rates, and school completion rates, on a country's success in the IMO.

The average score per contestant gives a more detailed and fair comparison between countries. It helps to evaluate the overall strength of the team, making it a useful measure for examining how education systems contribute to performance in international competitions.

By focusing on the average score, this analysis can effectively explore the connection between educational investments and a country's overall performance in the IMO, making it a suitable target for this project.

2.Exploratory Data Analysis

2.1 Data Cleaning

```
literacy_rate_by_country_and_region <- read_csv("data/literacy_data.csv")

country_and_region_data <- read_xlsx(
  "data/P_Data_Extract_From_Education_Statistics_-_All_Indicators_Metadata.xlsx",
  sheet = "Country - Metadata")

youth_literacy_rate <- literacy_rate_by_country_and_region |>
  filter(Series == "Youth literacy rate, population 15-24 years, both sexes (%)")

youth_literacy_rate <- pivot_longer(youth_literacy_rate,
                                   cols = c('2009 [YR2009]',
                                            '2010 [YR2010]',
                                            '2011 [YR2011]',
                                            '2012 [YR2012]',
                                            '2013 [YR2013]',
                                            '2014 [YR2014]',
                                            '2015 [YR2015]',
                                            '2016 [YR2016]',
                                            '2017 [YR2017]',
                                            '2018 [YR2018]',
                                            '2019 [YR2019]'),
                                   names_to = "Year",
                                   values_to = "Literacy_Rate") |>
  mutate(Year = str_replace(Year, " \\[YR[0-9]+\\]", "")) |>
  select("Country Code", "Country Name", Year, Literacy_Rate)

youth_literacy_rate <- youth_literacy_rate |>
  mutate(Year = as.double(Year),
         Literacy_Rate = as.double(Literacy_Rate))


education_data_joined <- education_data |>
  left_join(youth_literacy_rate,
```

```
            by = c("Country" = "Country Name", "Year" = "Year"),
            suffix = c("", "_new"))

education_data_updated <- education_data_joined |>
  mutate(Value_literacy_rate = coalesce(Value_literacy_rate,
                                        Literacy_Rate)) |>
  select(-Literacy_Rate)
```

```
education_with_region <- education_data_updated |>
  left_join(country_and_region_data, by = c("Country" = "Long Name"))

education_full <- education_with_region |>
  left_join(youth_literacy_rate,
            by = c("Region" = "Country Name", "Year" = "Year"),
            suffix = c("", "_region")) |>
  left_join(youth_literacy_rate,
            by = c("Country" = "Country Name", "Year" = "Year"),
            suffix = c("", "_country")) |>
  left_join(youth_literacy_rate,
            by = c("Income Group" = "Country Name", "Year" = "Year"),
            suffix = c("", "_income"))

education_data_updated <- education_full |>
  mutate(Value_literacy_rate = coalesce(Value_literacy_rate,
                                        Literacy_Rate_country,
                                        Literacy_Rate)) |>
  select(-Literacy_Rate_country, -Literacy_Rate)

education_data_updated <- education_data_updated |>
  mutate(Value_literacy_rate = coalesce(Value_literacy_rate,
                                        Literacy_Rate_income)) |>
  select(Country,
         Year,
         `Country Code`,
         Value_gross_enr_ratio_for_tertiary_edu,
         Value_gov_expen_as_perc_of_GPP,
         Value_literacy_rate,
         Region,
         `Income Group`)
```

2.2 Merging Educational Data:

The educational indicators from the UNESCO Institute for Statistics were split across multiple variables (e.g., primary and secondary education completion rates, government expenditure on education). These were merged into a single dataset, ensuring all relevant indicators were available for each country and year. The merging process involved handling mismatched country names between the datasets. For example, differences such as "Kyrgyz Republic" vs. "Kyrgyzstan" were corrected manually to ensure proper alignment of the data.

2.3 Combining IMO Data with Educational Data:

The educational data (which now included literacy rates, completion rates, and government expenditure) was merged with the IMO performance data (e.g., team scores, medals won) to create a comprehensive dataset. This allowed for the analysis of the relationship between a country's educational indicators and its performance in the IMO.

2.4 Creating New Variables :

`Medal_Efficiency` This variable was created by dividing the total number of medals (gold, silver, and bronze) won by a country by its team size (team_size_all). It measures how efficiently a country converts its team into medals, providing insights into performance relative to team size.

`Gov_Investment_Per_Medal` This variable measures the amount of government expenditure on education required to produce one IMO medal. It was created by dividing the government expenditure as a percentage of GDP by the total number of medals won.

`Lit_Performance_Ratio` This variable measures the ratio between a country's youth literacy rate and its average IMO score or total number of medals won, helping to explore the link between literacy and performance.

These variables were created before the training-test split to avoid any issues related to leakage between the datasets.

```
imo_data <- imo_data |>
  rowwise() |>
  mutate(total_score = sum(c_across(p1:p7), na.rm = TRUE)) |>
  ungroup()

imo_data <- imo_data |>
  mutate(average_score_per_contestant = total_score / team_size_all)

imo_data <- imo_data |>
```

```r
  mutate(medal_Efficiency = ifelse(
    team_size_all > 0,
    (awards_gold + awards_silver + awards_bronze) / team_size_all,
                                  NA))

combined_data <- imo_data |>
  left_join(education_data_updated, by = c("country" = "Country",
                                          "year" = "Year"))

combined_data <- combined_data |>
  mutate(Gov_Investment_Per_Medal = ifelse(
    (awards_gold + awards_silver + awards_bronze) > 0,
    Value_gov_expen_as_perc_of_GPP /
      (awards_gold + awards_silver + awards_bronze),
                                          NA),
         Lit_Performance_Ratio = ifelse(
           average_score_per_contestant > 0,
           Value_literacy_rate / average_score_per_contestant,
                                       NA))

combined_data <- combined_data |>
  filter(year > 2008 & year < 2020)


write_csv(combined_data, "data/combined_data.csv")
```

2.5 Excluded Observations

Observations from years prior to 2009 and after 2019 were excluded due to insufficient data availability.

Additionally, certain features were excluded due to a significant number of missing values (approximately 1,500 NAs out of 1,999 total observations). These features included:

1. Completion rate, primary education, both sexes (%)
2. Completion rate, lower secondary education, both sexes (%)
3. Completion rate, upper secondary education, both sexes (%)

Since no relevant data was available to fill the missing values, these features were omitted from the analysis.

2.6 Handling missing data

The summary showed that the literacy_rate feature had about 1,500 missing values, indicating that we lacked sufficient data. To address this, I sourced an additional dataset for literacy rates from the World Bank (https://databank.worldbank.org/source/education-statistics-%5e-all-indicators). When missing data was'nt found for a specific country, the missing NA values were replaced with regional data. After all of these if we have NA's it will be replaced with mean value Other NA's from numeric features will be imputed with mean value

Also added the following features to improve performance:

Government expenditure on education, constant US$ (millions)

Government expenditure on education, US$ (millions)

GDP per capita (current US$)

Expenditure on education as % of total government expenditure (%),

Current expenditure as % of total expenditure in public institutions (%)

Annual statutory teacher salaries in public institutions in USD. Primary. 10 years of experience

Labor force with advanced education (% of total labor force)

Internet users (per 100 people)

Percentage of graduates from tertiary education graduating from Natural Sciences, Mathematics and Statistics programmes, both sexes (%)'

```
education_data_new <- read_csv('data/New_Education_Data.csv')

education_data_new_1 <- education_data_new |>
  filter(
    Series %in%
      c('Government expenditure on education, constant US$ (millions)',
        'Government expenditure on education, US$ (millions)',
        'GDP per capita (current US$)',
        'Expenditure on education as % of total government expenditure (%)',
        'Current expenditure as % of total expenditure in public institutions (%)',
        'Annual statutory teacher salaries in public institutions in USD. Primary. 10 years o
        'Labor force with advanced education (% of total labor force)',
        'Internet users (per 100 people)',
        'Percentage of graduates from tertiary education graduating from Natural Sciences, Ma
```

```r
educational_data_2 <-  pivot_longer(education_data_new_1,
                                    cols = c('2009 [YR2009]',
                                             '2010 [YR2010]',
                                             '2011 [YR2011]',
                                             '2012 [YR2012]',
                                             '2013 [YR2013]',
                                             '2014 [YR2014]',
                                             '2015 [YR2015]',
                                             '2016 [YR2016]',
                                             '2017 [YR2017]',
                                             '2018 [YR2018]',
                                             '2019 [YR2019]'),
                                    names_to = "Year",
                                    values_to = "Value") |>
  mutate(Year = str_replace(Year, " \\[YR[0-9]+\\]", ""))
```

```r
SE.TER.GRAD.SC.ZS <- educational_data_2 |>
  filter(`Series Code` == "SE.TER.GRAD.SC.ZS") |>
  mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)


IT.NET.USER.P2 <- educational_data_2 |>
  filter(`Series Code` == "IT.NET.USER.P2") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

SL.TLF.ADVN.ZS <- educational_data_2 |>
  filter(`Series Code` == "SL.TLF.ADVN.ZS") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

UIS.X.US.FSGOV <- educational_data_2 |>
  filter(`Series Code` == "UIS.X.US.FSGOV") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

UIS.X.USCONST.FSGOV <- educational_data_2 |>
```

```
  filter(`Series Code` == "UIS.X.USCONST.FSGOV") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

NY.GDP.PCAP.CD <- educational_data_2 |>
  filter(`Series Code` == "NY.GDP.PCAP.CD") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

SE.XPD.TOTL.GB.ZS <- educational_data_2 |>
  filter(`Series Code` == "SE.XPD.TOTL.GB.ZS") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

SE.XPD.CUR.TOTL.ZS <- educational_data_2 |>
  filter(`Series Code` == "SE.XPD.CUR.TOTL.ZS") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)

OECD.TSAL.1.E10 <- educational_data_2 |>
  filter(`Series Code` == "OECD.TSAL.1.E10") |>
    mutate(Year =  as.numeric(Year),
         Value = as.numeric(Value)) |>
  select(Year, `Country Name`, `Country Code`, Value)
```

```
combined_educ_data <- read_csv("data/combined_data.csv")

combined_educ_data <- combined_educ_data |>
  left_join(SE.TER.GRAD.SC.ZS,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(SE.TER.GRAD.SC.ZS = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(IT.NET.USER.P2,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
```

```
  mutate(IT.NET.USER.P2 = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(SL.TLF.ADVN.ZS,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(SL.TLF.ADVN.ZS = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(UIS.X.US.FSGOV,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(UIS.X.US.FSGOV = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(UIS.X.USCONST.FSGOV,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(UIS.X.USCONST.FSGOV = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(NY.GDP.PCAP.CD,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(NY.GDP.PCAP.CD = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(SE.XPD.TOTL.GB.ZS,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(SE.XPD.TOTL.GB.ZS = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(SE.XPD.CUR.TOTL.ZS,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
```

```
  mutate(SE.XPD.CUR.TOTL.ZS = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)

combined_educ_data <- combined_educ_data |>
  left_join(OECD.TSAL.1.E10,
            by = c("country" = "Country Name", "year" = "Year"),
            suffix = c("", "_new")) |>
  mutate(OECD.TSAL.1.E10 = as.numeric(Value)) |>
  select(-`Country Code_new`, -Value)


write_csv(combined_educ_data, "data/combined_educ_data.csv")
```

2.7 Split data

The following variables were removed because they directly contribute to the outcome variable: -medal_Efficiency, -awards_gold, -awards_bronze, -awards_silver, -awards_honorable_mentions, -total_score, -p1, -p2, -p3, -p4, -p5, -p6.

The following variables were removed because they are categorical: -country, -p7, -leader, -deputy_leader, -Country Code, -Region, -Income Group.

```
combined_educ_data <-read_csv("data/combined_educ_data.csv")

education_numeric_data <- combined_educ_data |>
  select(-country, -p7, -leader, -deputy_leader,
         -`Country Code`,-Region, -`Income Group`, -medal_Efficiency,
         -awards_gold, -awards_bronze, -awards_silver,
         -awards_honorable_mentions, -medal_Efficiency,
         -total_score, -p1, -p2, -p3, -p4, -p5, -p6)
```

```
set.seed(1234)
educ_data_split <- initial_split(education_numeric_data, prop = 3/4,
                                 strata = Value_gov_expen_as_perc_of_GPP)
train_data <- training(educ_data_split)
test_data <- testing(educ_data_split)

edu_recipe <- recipe(average_score_per_contestant ~ ., data = train_data) |>
  step_nzv(all_predictors()) |>
  step_impute_mean(all_numeric(), -all_outcomes()) |>
 step_impute_mode(all_nominal()) |>
```

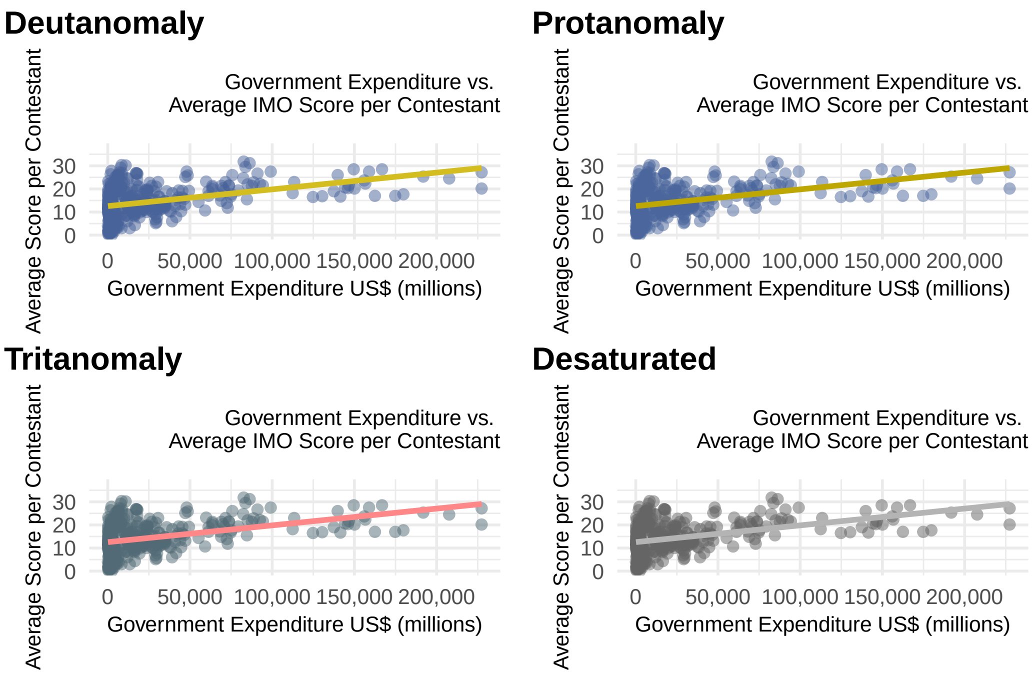
```
  step_unknown(all_nominal(), -all_outcomes()) |>
 step_normalize(all_numeric_predictors())
```

2.8 Data Visualization

2.8.1.Scatter Plot: Government Expenditure vs. Average Score Per Contestant

This plot shows the relationship between government spending on education (US$ (millions)) and the average score achieved by a country's team in the IMO.

```
scat_plot <- ggplot(train_data, aes(x = UIS.X.US.FSGOV,
                                    y = average_score_per_contestant)) +
  geom_point(alpha = 0.5, color = "#615e9b") +
  geom_smooth(method = "lm", color = "#ff9e1b", se = FALSE) +
  ggtitle("\nGovernment Expenditure vs. \n Average IMO Score per Contestant") +
  xlab("Government Expenditure US$ (millions)") +
  ylab("Average Score per Contestant") +
  scale_x_continuous(labels = label_comma()) +
  theme_minimal() +
  theme(
    plot.title = element_text(hjust = 1, size = 8, margin = margin(b = 10)),
    axis.title = element_text(size = 8),
    axis.text = element_text(size = 8) ,
    plot.margin = margin(t = 10,  b = 10)
  )


cvd_grid(scat_plot)
```



Interpretation:There is a positive correlation between government expenditure and the average IMO score per contestant. As government expenditure increases, the average IMO score tends to increase, as indicated by the orange regression line

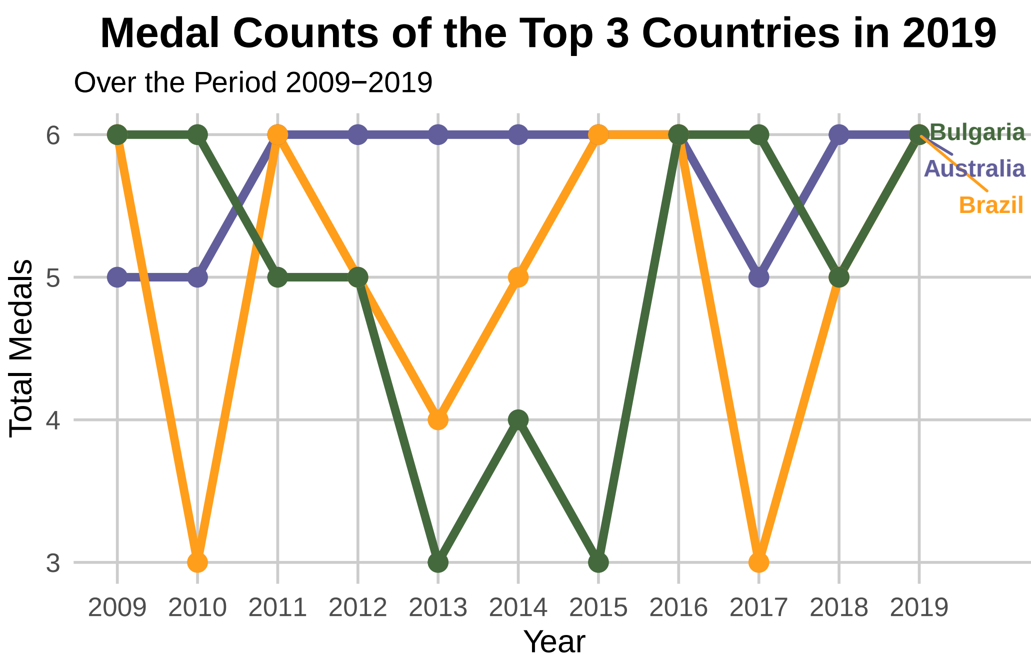
2.8.2.Line Plot: Medal Counts of the Top 3 Countries in 2019 Over the Period 2009–2019

This line plot displays the total number of medals won by the top 3 countries from 2009 to 2019, selected based on their medal counts in 2019. Each line represents a country and tracks its medal achievements over time. The colors of the lines correspond to different countries, and the labels for each country are positioned next to the last point (2019) for easy identification. This visualization allows us to observe the trend and consistency of each country's performance in terms of medal counts over the 10-year period.

```
library(ggrepel)

top_countries_2019 <- combined_data |>
  filter(year == 2019) |>
  group_by(country) |>
  summarize(total_medals_2019 = sum(awards_gold + awards_silver + awards_bronze,
                                    na.rm = TRUE)) |>
  arrange(desc(total_medals_2019)) |>
  slice_head(n = 3) |>
```

```
  pull(country)


medal_data <- combined_data |>
  filter(country %in% top_countries_2019, year >= 2009, year <= 2019) |>
  group_by(year, country) |>
  summarize(total_medals = sum(awards_gold + awards_silver + awards_bronze,
                               na.rm = TRUE)) |>
  ungroup()


country_colors <- setNames(c("#615e9b", "#ff9e1b", "#44693d"),
                           top_countries_2019)


ggplot(medal_data, aes(x = year, y = total_medals, color = country,
                      group = country)) +
  geom_line(size = 1.5) +
  geom_point(size = 3) +
  scale_color_manual(values = country_colors) +
  scale_x_continuous(breaks = seq(2009, 2019, by = 1),
                     labels = as.character(seq(2009, 2019, by = 1))) +
  labs(
    title = "Medal Counts of the Top 3 Countries in 2019 ",
    subtitle = "Over the Period 2009-2019",
    x = "Year",
    y = "Total Medals",
    color = "Country"
  ) +
  theme_minimal() +
  theme(
    plot.title = element_text(size = 16, face = "bold", hjust = 0.5),
    axis.title = element_text(size = 12),
    axis.text = element_text(size = 10),
    panel.grid.major = element_line(color = "gray80", size = 0.5),
    panel.grid.minor = element_blank(),
    legend.position = "none"
  ) +

  geom_text_repel(data = medal_data %>% filter(year == 2019),
                  aes(label = country),
                  nudge_x = 0.9,
```

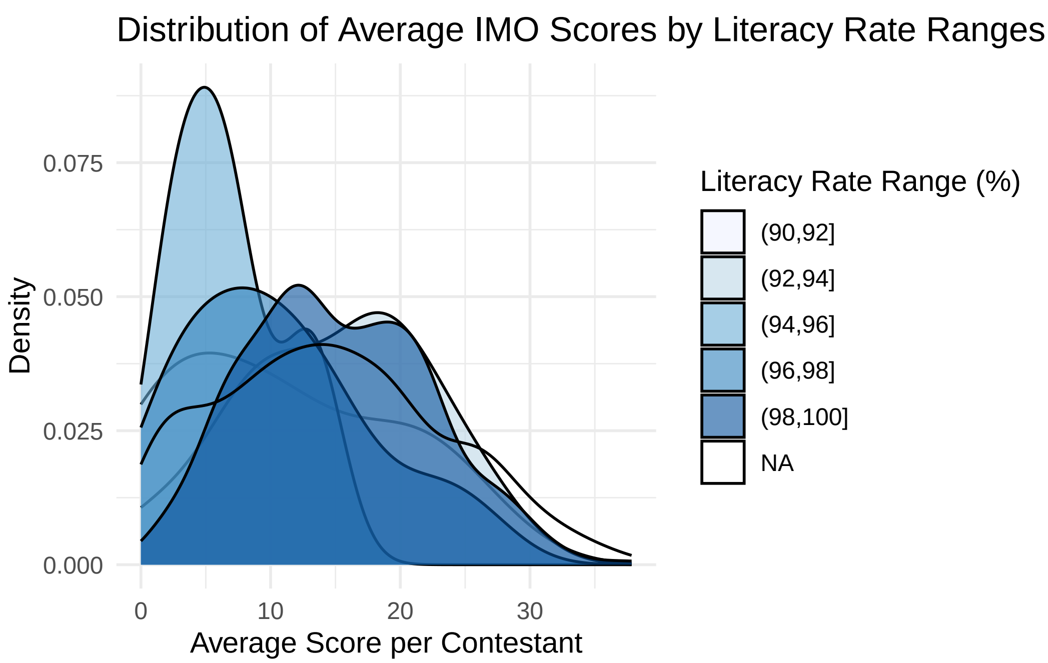
```
                     direction = "y",
                     size = 3, fontface = "bold", color = country_colors)
```



2.8.3.Density plot: Distribution of Average IMO Scores by

Literacy Rate Ranges

```
ggplot(train_data, aes(x = average_score_per_contestant,
                       fill = cut(Value_literacy_rate,
                                  breaks = seq(90, 100, by = 2)))) +
  geom_density(alpha = 0.6) +
  scale_fill_brewer(palette = "Blues", name = "Literacy Rate Range (%)") +
  labs(
    title = "Distribution of Average IMO Scores by Literacy Rate Ranges",
    x = "Average Score per Contestant",
    y = "Density"
  ) +
  theme_minimal()
```



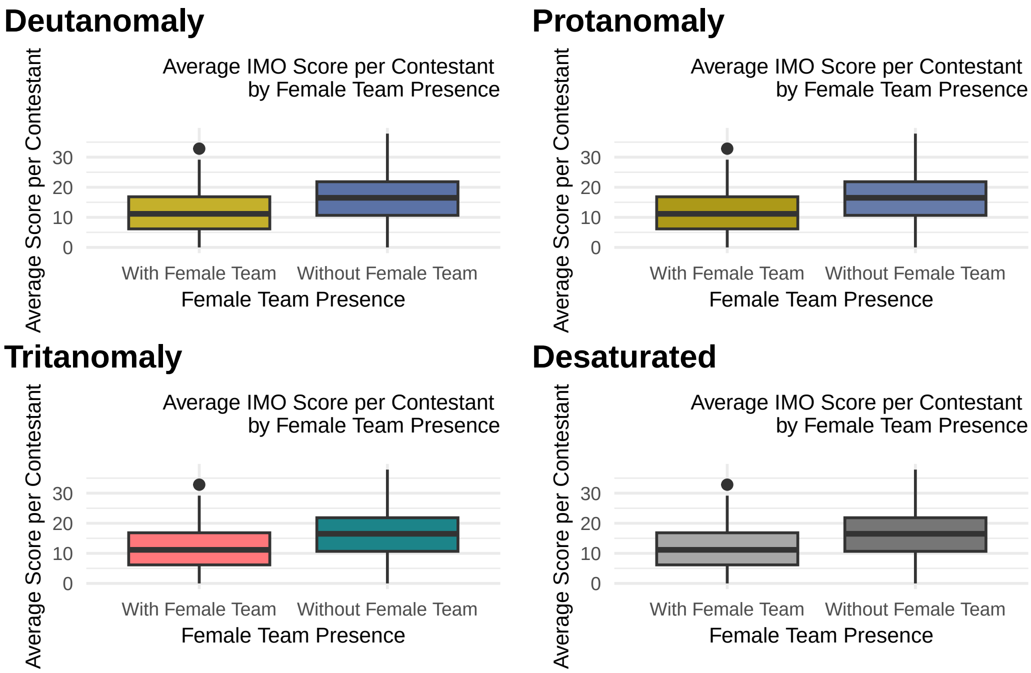
Interpretation:

The plot suggests that literacy rate does not strongly impact the distribution of average IMO scores per contestant. Countries with both lower and higher literacy rates show similar distributions of average scores, implying that literacy rate alone does not significantly influence IMO performance.

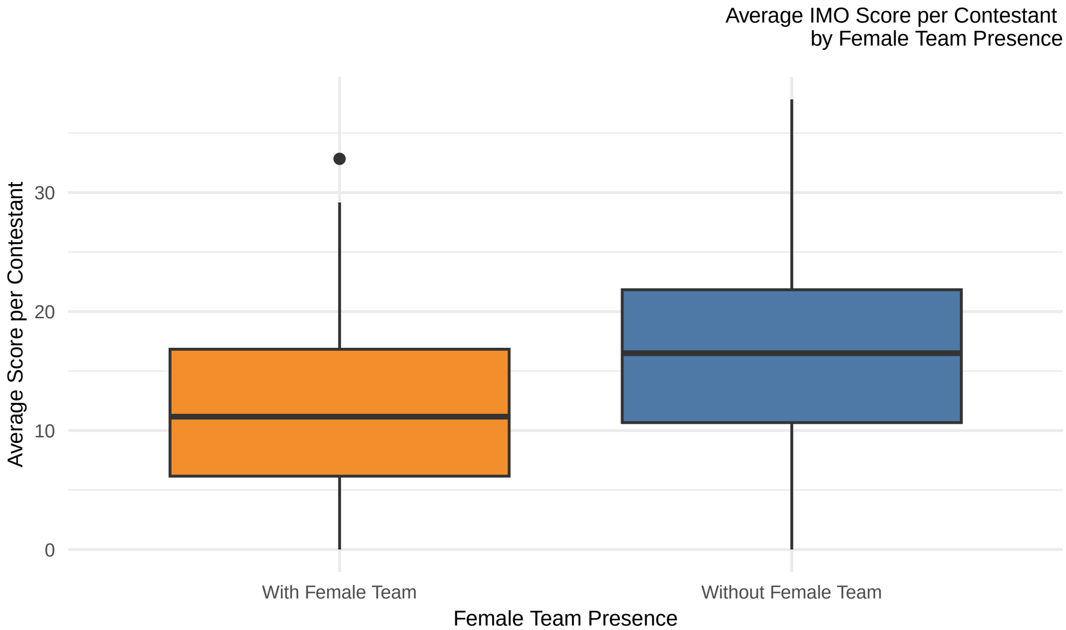
2.8.4.Boxplot: Compares the average IMO scores between countries with and without female team members:

```
train_data_fem <- train_data |>
  mutate(
    team_size_female = ifelse(is.na(team_size_female), 0, team_size_female),
    has_female_team = ifelse(team_size_female > 0,
                             "With Female Team", "Without Female Team")
  )

boxplot_1 <- ggplot(train_data_fem, aes(x = has_female_team,
                                         y = average_score_per_contestant,
                                         fill = has_female_team)) +
  geom_boxplot() +
  labs(
    title = "\nAverage IMO Score per Contestant \n by Female Team Presence",
    x = "Female Team Presence",
```

```
    y = "Average Score per Contestant"
  ) +
  scale_fill_manual(values = c("With Female Team" = "#F28E2B",
                               "Without Female Team" = "#4E79A7")) +
  theme_minimal() +
  theme(legend.position = "none",
    plot.title = element_text(hjust = 1, size = 8, margin = margin(b = 10)),
    axis.title = element_text(size = 8),
    axis.text = element_text(size = 7) ,
    plot.margin = margin(t = 5,  b = 5)
  )


cvd_grid(boxplot_1)
```




```
boxplot_1
```



The plot suggests a slight association between the absence of female team members and higher average IMO scores, although the difference is not very large.

3. Evaluation Metric

RMSE and R-squared were used as evaluation metrics due to the regression nature of the task.

4.Fit Models

4.1 Data Preprocessing

`step_nzv(all_predictors())` to remove near-zero variance predictors

`step_impute_mean(all_numeric(), -all_outcomes())` to impute missing values for numeric predictors

`step_impute_mode(all_nominal())` to impute missing categorical values

`step_normalize(all_numeric_predictors())` to ormalize numeric predictors

4.2 Choose Hyperparameters; Fit and Test Models

4.2.1 Linear Regression

```r
library(tidymodels)
library(Metrics)
library(dplyr)

lm_spec <- linear_reg() |>
  set_engine("lm")

lm_workflow <- workflow() |>
  add_recipe(edu_recipe) |>
  add_model(lm_spec)


lm_fit <- fit(lm_workflow, data = train_data)

summary_model <- extract_fit_parsnip(lm_fit) |>
  tidy()
print(summary_model)
```

```
# A tibble: 19 x 5
   term                                  estimate std.error statistic  p.value
   <chr>                                    <dbl>     <dbl>     <dbl>    <dbl>
 1 (Intercept)                             14.5       0.221    65.8   0
 2 year                                    -0.292     0.244    -1.19  2.33e- 1
 3 team_size_all                            0.250     0.385     0.650 5.16e- 1
 4 team_size_male                           2.62      0.389     6.73  3.06e-11
 5 team_size_female                        -0.389     0.233    -1.67  9.56e- 2
 6 Value_gross_enr_ratio_for_tertiary_edu   0.992     0.268     3.70  2.27e- 4
 7 Value_gov_expen_as_perc_of_GPP          -0.266     0.274    -0.973 3.31e- 1
 8 Value_literacy_rate                      0.485     0.279     1.74  8.29e- 2
 9 Gov_Investment_Per_Medal                -2.15      0.257    -8.38  2.29e-16
10 Lit_Performance_Ratio                   -1.44      0.244    -5.91  5.07e- 9
11 SE.TER.GRAD.SC.ZS                       -0.176     0.233    -0.757 4.50e- 1
12 IT.NET.USER.P2                          -0.103     0.361    -0.286 7.75e- 1
13 SL.TLF.ADVN.ZS                          -0.290     0.234    -1.24  2.14e- 1
14 UIS.X.US.FSGOV                           3.28      1.56      2.10  3.58e- 2
15 UIS.X.USCONST.FSGOV                     -2.14      1.57     -1.37  1.72e- 1
16 NY.GDP.PCAP.CD                           0.511     0.355     1.44  1.51e- 1
```

```
17 SE.XPD.TOTL.GB.ZS                          0.368       0.243      1.52  1.30e- 1
18 SE.XPD.CUR.TOTL.ZS                         0.310       0.228      1.36  1.75e- 1
19 OECD.TSAL.1.E10                           -0.0988      0.250     -0.395 6.93e- 1
```

```r
y_pred <- predict(lm_fit, new_data = test_data) |>
  pull(.pred)

y_pred <- ifelse(y_pred < 0, 1e-6, y_pred)

mse_train <- mean(
  (train_data$average_score_per_contestant - predict(lm_fit,
                                                     new_data = train_data) |>
                   pull(.pred))^2)
r2_train <- caret::R2(predict(lm_fit, new_data = train_data) |> pull(.pred),
                      train_data$average_score_per_contestant)

rmse_train <- sqrt(mse_train)

mse_test <- mean((test_data$average_score_per_contestant - y_pred)^2)
rmse_test <- sqrt(mse_test)
r2_test <- caret::R2(y_pred, test_data$average_score_per_contestant)

msle_test <- msle(test_data$average_score_per_contestant, y_pred)
rmsle_test <- sqrt(msle_test)

cat("Training MSE:", mse_train, "\n")
```

```
Training MSE: 40.69752
```

```r
cat("Training RMSE:", rmse_train, "\n")
```

```
Training RMSE: 6.379461
```

```r
cat("Training R-squared:", r2_train, "\n")
```

```
Training R-squared: 0.3768742
```

```r
cat("Test MSE:", mse_test, "\n")
```

```
Test MSE: 41.71663
```

```r
cat("Test RMSE:", rmse_test, "\n")
```

```
Test RMSE: 6.458842
```

```r
cat("Test R-squared:", r2_test, "\n")
```

```
Test R-squared: 0.3984548
```

```r
cat("Mean Squared Log Error (MSLE):", msle_test, "\n")
```

```
Mean Squared Log Error (MSLE): 0.399063
```

```r
cat("Root Mean Squared Log Error (RMSLE):", rmsle_test, "\n")
```

```
Root Mean Squared Log Error (RMSLE): 0.6317143
```

4.2.2 Gradient Boosting

`trees (500 to 3000, step 500)`: This range was chosen to balance computational efficiency with predictive accuracy. Smaller numbers of trees (e.g., 500) allow for faster training and provide a baseline for performance, while larger numbers (up to 3000) enable the model to capture more complex patterns in the data, covering a broad range to explore optimal tree count.

`tree_depth (1 to 5)`: Tree depth controls the complexity of each decision tree. A shallow depth (e.g., 1) promotes simpler and faster models, reducing the risk of overfitting, while deeper trees (up to 5) allow for capturing more intricate patterns in the data, providing a balanced exploration of model complexity.

`learn_rate (0.01, 0.05, 0.1)`: The learning rate determines how quickly the model adjusts during training. A smaller rate (e.g., 0.01) ensures careful and incremental adjustments, minimizing the risk of overshooting optimal solutions, while a larger rate (e.g., 0.1) speeds up training, with values chosen to balance accuracy and convergence speed.

`iter = 100`: The number of iterations (100) ensures a comprehensive exploration of the parameter space, allowing the model to evaluate a wide range of potential combinations and converge on the most effective hyperparameters.

```r
boost_edu <- boost_tree(mode = "regression",
                         engine = "lightgbm",

                         trees = tune(),

                         tree_depth = tune(),

                         learn_rate = tune())


boost_wf <- workflow() |>
  add_recipe(edu_recipe) |>
  add_model(boost_edu)
```

```
```{r cv-bayes-r}
#| eval: false

folds <- vfold_cv(train_data,
               v = 6)

boost_grid <- crossing(
 trees =  seq(500, 3000, by = 500),
 tree_depth = 1:5,
 learn_rate = c(0.01, 0.05, 0.1)
)

boost_cv_edu <- tune_grid(boost_wf,
                      resamples = folds,
                      grid = boost_grid,
                      metrics = metric_set(yardstick::rmse)
                      )

boost_params <- extract_parameter_set_dials(boost_wf)

boost_params <- boost_params |>
  update(trees = trees(range = c(1000, 3000)))

set.seed(756)
boost_cv_bayes_edu <- boost_wf |>
  tune_bayes(
    resamples = folds,
    param_info = boost_params,
```
```

````
    initial = boost_cv_edu,
    iter = 50,
    metrics = metric_set(yardstick::rmse),
    control = control_bayes(no_improve = 15)
  )

save(boost_cv_bayes_edu, file = "data/boost_cv_bayes_edu.RData")
```
````

```
load(file = "data/boost_cv_bayes_edu.RData")
```

```
collect_metrics(boost_cv_bayes_edu) |>
  arrange(desc(mean))
```

```
# A tibble: 131 x 10
   trees tree_depth learn_rate .metric .estimator  mean     n std_err .config
   <dbl>      <int>      <dbl> <chr>   <chr>      <dbl> <int>   <dbl> <chr>
 1   500          1       0.01 rmse    standard    5.82     6   0.127 Preproces~
 2  1000          1       0.01 rmse    standard    5.58     6   0.130 Preproces~
 3  1500          1       0.01 rmse    standard    5.49     6   0.129 Preproces~
 4  2000          1       0.01 rmse    standard    5.45     6   0.127 Preproces~
 5  2500          1       0.01 rmse    standard    5.43     6   0.126 Preproces~
 6   500          1       0.05 rmse    standard    5.43     6   0.125 Preproces~
 7  3000          1       0.01 rmse    standard    5.40     6   0.123 Preproces~
 8  1000          1       0.05 rmse    standard    5.35     6   0.116 Preproces~
 9   500          1       0.1  rmse    standard    5.35     6   0.117 Preproces~
10  1500          1       0.05 rmse    standard    5.34     6   0.110 Preproces~
# i 121 more rows
# i 1 more variable: .iter <int>
```

````
```{r eval-bayes-r}
#| eval: true

boost_wf_best_bayes <- boost_wf |>
  finalize_workflow(select_best(boost_cv_bayes_edu,
                                metric = "rmse")) |>
  fit(train_data)

edu_aug_bayes_train <- boost_wf_best_bayes |>
  augment(new_data = train_data)
````

```
boost_metrics_train <- metrics(
  edu_aug_bayes_train,
  truth = average_score_per_contestant,
  estimate = .pred
)

edu_aug_bayes <- boost_wf_best_bayes |>
  augment(new_data = test_data)


boost_metrics <- metrics(
  edu_aug_bayes,
  truth = average_score_per_contestant,
  estimate = .pred
)

print(boost_metrics)
print(boost_metrics_train)
```

```
# A tibble: 3 x 3
  .metric .estimator .estimate
  <chr>   <chr>          <dbl>
1 rmse    standard       4.71
2 rsq     standard       0.684
3 mae     standard       3.11
# A tibble: 3 x 3
  .metric .estimator .estimate
  <chr>   <chr>          <dbl>
1 rmse    standard       3.13
2 rsq     standard       0.852
3 mae     standard       1.76
```

4.2.3 GAM

Smoothing Basis (`bs = "cr"`): Cubic regression splines were chosen because they efficiently model nonlinear patterns while reducing the risk of overfitting.

Maximum Degrees of Freedom (`k = 10`): This setting controls the complexity of the spline, ensuring the model stays simple and easy to interpret while capturing enough flexibility to fit the data well.

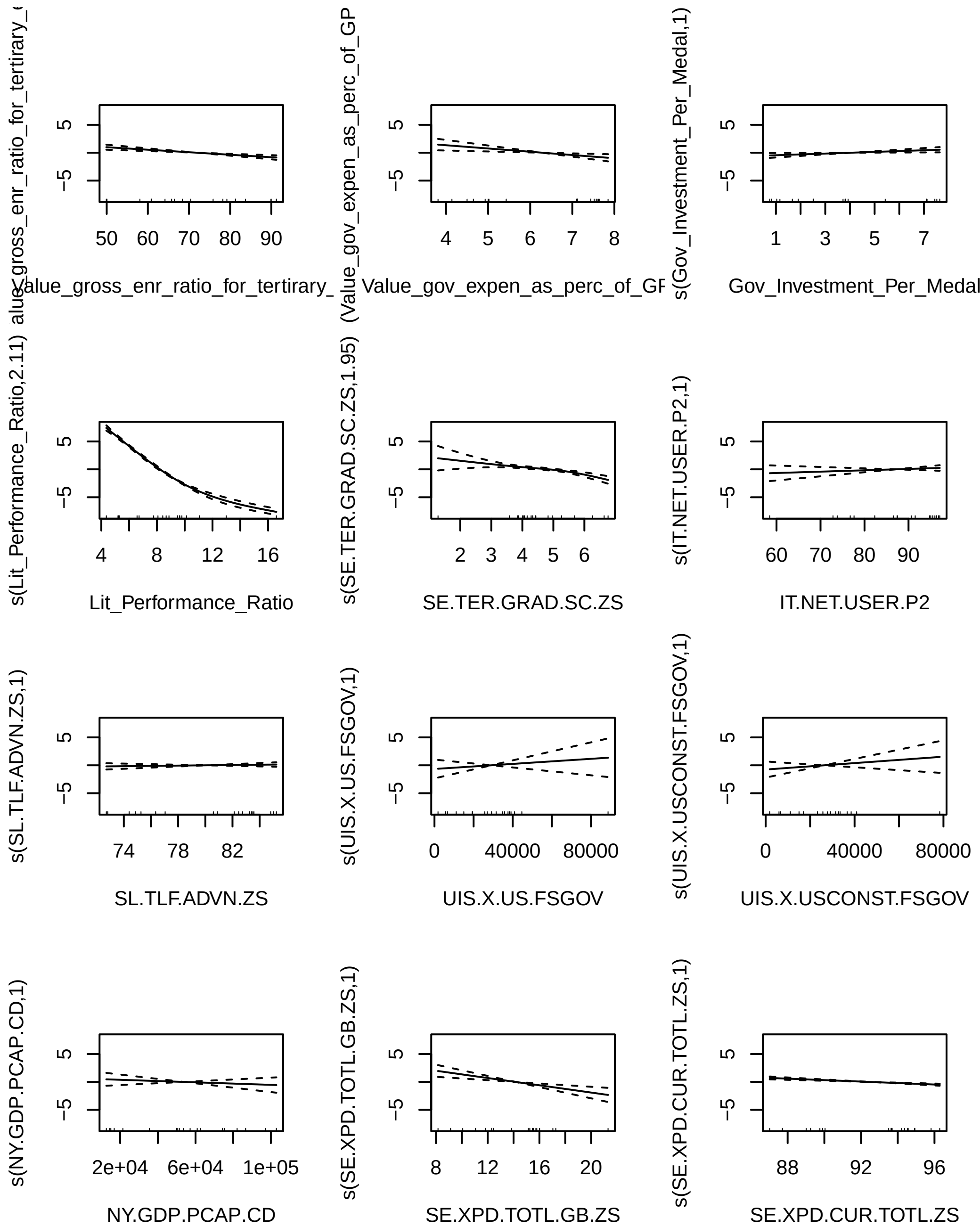
```r
library(mgcv)


gam_formula <- average_score_per_contestant ~
  team_size_all+  team_size_male+ team_size_female+
  s(Value_gross_enr_ratio_for_tertiary_edu, k = 10, bs = "cr") +
  s(Value_gov_expen_as_perc_of_GPP, k = 10, bs = "cr") +
  Value_literacy_rate +
  s(Gov_Investment_Per_Medal, k = 10, bs = "cr") +
  s(Lit_Performance_Ratio, k = 10, bs = "cr") +
  s(SE.TER.GRAD.SC.ZS, k = 10, bs = "cr") +
  s(IT.NET.USER.P2, k = 10, bs = "cr") +
  s(SL.TLF.ADVN.ZS, k = 10, bs = "cr") +
  s(UIS.X.US.FSGOV, k = 10, bs = "cr") +
  s(UIS.X.USCONST.FSGOV , k = 10, bs = "cr") +
  s(NY.GDP.PCAP.CD, k = 10, bs = "cr") +
  s(SE.XPD.TOTL.GB.ZS, k = 10, bs = "cr") +
  s(SE.XPD.CUR.TOTL.ZS, k = 10, bs = "cr") +
  s(OECD.TSAL.1.E10, k = 10, bs = "cr")

preproc_form <- average_score_per_contestant ~ .
```

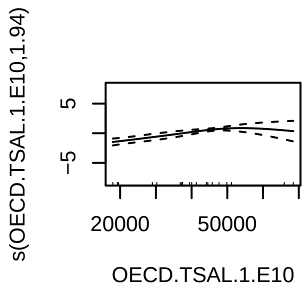
```r
gam_mod <- gen_additive_mod()  |>
  set_engine("mgcv") |>
  set_mode("regression")

gam_pre <- recipe(preproc_form, data = train_data)

gam_wf <- workflow() |>
  add_recipe(gam_pre) |>
  add_model(gam_mod, formula = gam_formula)

gam_fit <- gam_wf |>
  fit(train_data)
```

Partial Dependency Plots

```r
par(mfrow = c(2, 3))
gam_fit_pd <- gam_fit |>
```

```r
extract_fit_engine() |>
  plot()
```






```r
library(yardstick)
mls_test_gam <- gam_fit |>
  augment(new_data = test_data)
actual <- mls_test_gam$average_score_per_contestant
predicted <- mls_test_gam$.pred
valid_data <- mls_test_gam[!is.na(actual) & !is.na(predicted), ]

rmsle_gam <- rmsle(
  actual = valid_data$average_score_per_contestant,
  predicted = valid_data$.pred)


results <- tibble(
  truth = valid_data$average_score_per_contestant,
  estimate = valid_data$.pred
)

r2_gam_yardstick <- rsq(results, truth = truth, estimate = estimate)
rmse_test_gam <- yardstick::rmse(results, truth = truth, estimate = estimate)

mls_train_gam <- gam_fit |>
  augment(new_data = train_data)
actual <- mls_train_gam$average_score_per_contestant
predicted <- mls_train_gam$.pred
valid_data1 <- mls_train_gam[!is.na(actual) & !is.na(predicted), ]

rmsle_gam_train <- rmsle(
  actual = valid_data1$average_score_per_contestant,
  predicted = valid_data1$.pred)


results_train <- tibble(
  truth = valid_data1$average_score_per_contestant,
  estimate = valid_data1$.pred
```

```
)
r2_gam_yardstick_traom <- rsq(results_train, truth = truth, estimate = estimate)
rmse_train_gam <- yardstick::rmse(results_train, truth = truth, estimate = estimate)
cat("rmse train:", rmse_train_gam$.estimate, "\n")
```

```
rmse train: 1.336476e-05
```

```
cat("R-squared (R2)  train:", r2_gam_yardstick_traom$.estimate, "\n")
```

```
R-squared (R2)  train: 1
```

```
cat("R-squared (R2) test", r2_gam_yardstick$.estimate, "\n")
```

```
R-squared (R2) test 0.7718495
```

```
cat("rmse test:", rmse_test_gam$.estimate, "\n")
```

```
rmse test: 1.403301
```

5.Comparing Models

Overfitting vs. Underfitting

Linear Regression: Tends to underfit since it assumes simple linear relationships. The R-squared values (~0.37 and ~0.39) show it doesn't capture the complexity of the data well. Training RMSE: 6.379461 Test RMSE: 6.458842

Gradient Boosting: Strikes a good balance by adjusting hyperparameters to avoid both overfitting and underfitting. It performs better, with a test R-squared of 0.684. and test rmse is 4.7092575 train R-squared is 0.851623 and train rmse is 3.133

GAM: Handles nonlinear relationships best, giving the highest test R-squared (0.772) and the lowest rmse test 1.403301. However, model was overfitted train R-squared was 1 and train rmse was 0.00001336

Bias vs. Variance

Linear Regression: Has high bias (makes simple assumptions) but low variance (predictions don't change much between datasets). This makes it consistent but not very accurate.

Gradient Boosting: Reduces bias by iteratively improving predictions and balances variance well with proper tuning.

GAM: Reduces bias by modeling complex patterns but can have higher variance depending on the smoothing settings.

Flexibility vs. Interpretability

Linear Regression: Very simple and easy to interpret, but lacks flexibility for capturing complex relationships.

Gradient Boosting: Flexible enough to model complex data but harder to interpret without extra tools like feature importance analysis.

GAM: Combines flexibility with decent interpretability, especially through partial dependency plots.

Key Takeaways

Gradient Boosting is a strong choice for balancing flexibility and accuracy, making it practical for predictions.

GAM is the most accurate and excels at modeling complex relationships, but it needs careful tuning to avoid overfitting.

Linear Regression is a good starting point for understanding basic relationships but doesn't handle complex data well.

Each model has its strengths depending on the need: simplicity (Linear Regression), flexibility and reliability (Gradient Boosting), or detailed nonlinear modeling (GAM).

There is a positive linear relationship between government expenditure on education (US$ in millions) and the average score per contestant. However, none of the models used performed ideally in capturing the impact of government expenditure and literacy rate on winning medals in the Math Olympiad.

6. Ethical Implications

The model could show bias if the data favors certain countries, like those with more government spending or higher literacy rates. This might lead to unfair decisions, like giving more resources to already advantaged countries.