CS 6850 Final Paper: Cascade Models with Transaction Costs and Temporal Components

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1 Abstract

Threshold models are powerful tools for analyzing information cascades within networks. This study extends the classical Watts threshold model [3] by incorporating two critical dimensions: transaction costs and temporal components. Transaction costs capture the penalties or effort associated with adopting a neighbor's behavior, while temporal components introduce time windows, representing when neighbors are available to influence decisions. By integrating these extensions, we develop a mathematical framework to examine the interplay between these factors and their impact on cascade behavior. We analyze the frequency, size, and nature of cascades, differentiating between gross information cascades and trend shift cascades. Our results offer a nuanced understanding of cascading phenomena, which is critical for applications ranging from financial markets to social networks.

2 Background

Information cascades occur when individuals imitate others' behaviors, often disregarding their private information. These cascades play a pivotal role in various domains, including technology adoption, financial trading, and the propagation of misinformation. The Watts threshold model provides a foundational approach to understanding such dynamics, positing that individuals adopt behaviors if a threshold proportion of their neighbors have already done so. While insightful, the model does not account for key real-world complexities such as economic costs or fluctuating availability of connections.

In this paper, we extend the Watts threshold model by introducing transaction costs, which influence economic decision-making, and temporal components, which account for the dynamic nature of connections. By incorporating these features, our model captures realistic constraints faced by individuals in networks, enhancing its applicability to scenarios like financial markets, where transaction fees and time-limited opportunities significantly shape behaviors. Below, we examine the two papers that extend the Watts threshold model through distinct approaches, each offering unique insights into cascading behaviors within networks.

2.1 Observing Cascade Behavior Depending on the Network Topology and Transaction Costs

Kim et al. [2] explore the impact of network topology and transaction costs on information cascades in a financial market setting. Information cascades are defined as instances where traders imitate the decisions of others rather than acting based on their private signals, leading to herding behavior. The paper employs a computational model to simulate cascading behaviors among agents under varying network structures and transaction costs, providing a quantitative framework for analyzing market dynamics.

The model assumes a network of artificial traders, or nodes, who sequentially decide to buy, sell, or hold a single asset. Each trader has access to private information about the asset (referred to as private signals) and can also observe the decisions of directly connected peers in the network. The network edges represent communication pathways, allowing for information sharing among connected traders. Two key types of transaction costs are incorporated into the model:

- Trading Fees (T_f): These represent the cost that a trader incurs when buying or selling an asset.
- Search Costs (S_c): These represent the effort or difficulty required to access and interpret peers' trading decisions.

The study examines three distinct network topologies:

1. **Spatially-Clustered Networks**: The nodes are connected in localized, stable clusters with connectivity following a normal distribution.
2. **Random Networks**: Connections between nodes are randomly assigned, resulting in a Poisson distribution of connectivity.
3. **Scale-Free Networks**: Connectivity is dominated by highly connected hubs with a power-law distribution.

The authors evaluate cascade behaviors using two key metrics:

- **Gross Information Cascades (I)**: Defined as the total number of instances where agents imitate trends, irrespective of their private signals:

$$I = \sum_{i=1}^{N} \delta(\chi_{i,1} \neq \chi_{i,0}),$$

  where $\chi_{i,1}$ is the final decision of trader i, $\chi_{i,0}$ is the initial decision, and $\delta(\cdot)$ is the indicator function.
- **Trend Shift Cascades (C)**: Represent the magnitude of shifts in market trends due to cascading behavior. This is quantified as:

$$C = \left| \sum_{i=1}^{N} \delta(\chi_{i,0} = \text{buy}) - \sum_{i=1}^{N} \delta(\chi_{i,1} = \text{buy}) \right| + \left| \sum_{i=1}^{N} \delta(\chi_{i,0} = \text{sell}) - \sum_{i=1}^{N} \delta(\chi_{i,1} = \text{sell}) \right| - \sum_{i=1}^{N} \delta(\chi_{i,1} = \text{hold}).$$

As for key findings, lower trading fees (T_f) and search costs (S_c) increase the frequency of gross information cascades (I), as agents are more likely to adopt peer trends when the costs of trading and information retrieval are low. Additionally, higher trading fees (T_f) lead to "no-trade cascades", where agents refrain from trading to avoid incurring costs. This behavior reflects a market equilibrium characterized by inactivity and diminished cascading behavior. Furthermore, the structure of the network significantly affects the dynamics of the cascade. Scale-free networks foster broader, market-wide cascades due to the influence of central, highly connected hubs, where the decisions of a single hub can propagate widely across the network.

2.2 Effects of Temporal Correlations on Cascades: Threshold Models on Temporal Networks

Backlund et al. [1] explore how temporal correlations in network connections influence cascading behaviors. Unlike traditional static networks, temporal networks exhibit edges that activate and deactivate over time, capturing the intermittent nature of real-world interactions. The authors adapt the Watts threshold model to include these temporal dynamics, allowing for a more realistic analysis of cascades where influence is time dependent.

The authors begin with a static network topology, represented by either:

1. **Random Networks**: Connections between nodes are distributed uniformly, leading to a Poisson degree distribution.

2. **Scale-Free Networks**: Highly connected nodes (hubs) dominate the connectivity structure, resulting in a distribution of power-law degrees.

To model temporal dynamics, each edge e_{ij} between nodes i and j is assigned an *activation sequence*, which determines whether the edge is active ($e_{ij}(t) = 1$) or inactive ($e_{ij}(t) = 0$) at time t. These activations are governed by a probabilistic process that reflects real-world "burstiness", where activity occurs in short, intense periods rather than being uniformly spread over time.

The probability of an edge e_{ij} being active at time t is given as:

$$P(e_{ij}(t) = 1) = p_{\text{on}},$$

where p_{on} is the constant activation probability for simplicity. In future work, the activation probability can be generalized to follow distributions that better mimic bursty temporal correlations observed in real-world networks.

At each discrete time step t, nodes make adoption decisions based on their active neighbors. A node i adopts a behavior if the fraction of its active neighbors who have already adopted the behavior exceeds its threshold ϕ_i. This rule can be expressed mathematically as:

$$\frac{\sum_{j \in N_i(t)} a_j(t)}{|N_i(t)|} \geq \phi_i,$$

where:

- $N_i(t)$: The set of active neighbors of node i at time t,
- $a_j(t)$: An indicator variable where $a_j(t) = 1$ if node j has adopted the behavior and $a_j(t) = 0$ otherwise,
- ϕ_i: The adoption threshold for node i, representing the proportion of active neighbors required for adoption.

The authors identify several important effects of temporal correlations on cascade dynamics. The first effect is **impact of burstiness**, where cascades are less likely to spread broadly in networks where edges are only active for short, intense bursts. The intermittent nature of connections limits the continuous influence required for a cascade to propagate. The second effect is **threshold effect** where in networks where the adoption threshold ϕ_i is high, cascades are even less likely to occur. Nodes require a greater proportion of their active neighbors to adopt the behavior, and short activation bursts make it harder to satisfy this condition. The third effect is **role of timing** where the timing of edge activation plays a decisive role in cascade propagation. If the activation of edges aligns well with node adoption decisions, cascades can spread efficiently. Conversely, misalignment of edge activation and node adoption hinders cascade propagation, leading to abrupt halts. The fourth effect is **network topology comparisons** where in random networks, cascades require longer and sustained periods of activity to propagate, as the uniform distribution of connections spreads influence evenly but slowly. In scale free networks, cascades are more efficient due to the presence of hubs. Hubs act as critical nodes that can quickly propagate behaviors to a large fraction of the network once they adopt the behavior.

The study highlights the critical role of temporal correlations in determining cascade size and spread. Short, bursty activations reduce the likelihood of large cascades, particularly in networks with higher adoption thresholds. Additionally, the effectiveness of scale-free networks in triggering cascades is diminished when temporal constraints limit hub activity. However, a key limitation of this study is the absence of economic considerations, such as transaction costs, in the model. In real-world scenarios, nodes, or agents may refrain from adopting behaviors due to prohibitive costs, regardless of temporal alignment or peer influence. Integrating transaction costs into the temporal threshold model would provide a more comprehensive framework for understanding cascading behaviors in systems where both time and economic factors influence decision-making. By combining the temporal correlations explored in this study with economic constraints such as transaction fees and search costs, as we propose in our model, we can better capture the interplay between timing, costs, and influence propagation in networks.

3 Our Model

To address the limitations and areas for further research identified in the previous studies [2], we propose an extended model that integrates key concepts from both works. Specifically, we adapt the model from Kim et al. [2], which focuses on transaction costs and network topologies, and incorporate a probabilistic activation mechanism inspired by Backlund et al. [1], where edges activate dynamically over time. This results in a more realistic framework for modeling information cascades, accounting for both economic costs and temporal network dynamics.

In our proposed model, agents are represented as nodes in a static network topology. These agents make sequential decisions to **buy**, **sell**, or **hold** an asset based on:

1. Their private signals about the asset's value,
2. The observed behavior of their active neighbors,
3. The economic costs associated with trading or accessing information.

We analyze three widely studied network structures–which we also examined in our preliminary research–to understand how different connectivity patterns influence cascading behaviors: (1) Random Networks, (2) Scale-Free Networks, and (3) Spatially-Clustered Networks.

3.1 Simulation Structure

To simulate temporal dynamics, each connection e_{ij} between agents i and j has a probability of being active at any given time step. Specifically $P(e_{ij}(t) = 1) = p_{edge}$, where p_{edge} is a constant that represents the probability of an edge being active during a time step t. Inactive edges ($e_{ij}(t) = 0$) prevent agents from observing or influencing their neighbors during that time step.

We introduce two types of costs that affect agent behavior: trading fee (T_f), which is the cost incurred when an agent buys or sells the asset, and search cost (S_c), which is the effort or cost required to access and interpret the trading behavior of neighbors.

Each simulation consists of 10 discrete time steps. During each time step:

- Active edges are determined probabilistically using p_{edge}. Only active edges allow agents to observe their neighbors' behaviors.
- Agents observe the behavior of their active neighbors (if any) and combine this information with their private signals to make decisions about whether to **buy**, **sell**, or **hold** the asset.
- Decisions are influenced by the trading fee T_f, which imposes a penalty for executing a trade, and the search cost S_c, which limits the accessibility of neighbor behaviors.

At the end of each run, two key metrics are recorded:

1. These are the variables for the below metrics:

$$\begin{aligned} N &: \text{Total number of agents (1000)} \\ p_{edge} &: \text{Probability an edge is active.} \\ \tau &: \text{search cost (fraction of neighbors not observed) from 0 to 1} \\ \theta &: \text{Trading fee from 0 to 0.4} \end{aligned}$$

2. **Gross Information Cascades (I)**: This measures the total number of instances where agents imitate trends, irrespective of their private signals. Formally:

$$I = \sum_{i=1}^{N} \delta(\chi_{i,1} \neq \chi_{i,0}) = \begin{cases} N \cdot p_{\text{edge}} \cdot (1-\tau) \cdot (1-\theta), & \text{if } \theta \leq 0.3 \\ 0, & \text{if } \theta > 0.3 \end{cases},$$

   where $\chi_{i,1}$ represents the final decision of agent i, $\chi_{i,0}$ is the initial decision, and $\delta(\cdot)$ is the indicator function.

3. **Cascade Size (C_{size})**: This quantifies how polarized the decisions are across the network, similar to the gross information cascades metric:

$$C_{\text{size}} = \sum_{i=1}^{N} \Big(\sum_{i_{neighbors}} buying - \sum_{i_{neighbors}} selling \Big) = \delta(\chi_{i,1} \neq \chi_{i,0}) = \begin{cases} N \cdot p_{\text{edge}} \cdot (1-\tau) \cdot (1-\theta), & \text{if } \theta \leq 0.3 \\ 0, & \text{if } \theta > 0.3 \end{cases}.$$

We derived the above metrics as approximations based off of the related literature and the changes we made. Once the trading fee is greater than 0.3, all agents hold (and no cascades occur) because they cannot yield a profit [2]. As such our equations are piecewise to approximate this phenomenon. These parameters allow us to explore how temporal edge activation and cost constraints affect the spread and scale of cascades across different network topologies.

The following is a snippet of the code we use for our simulation.

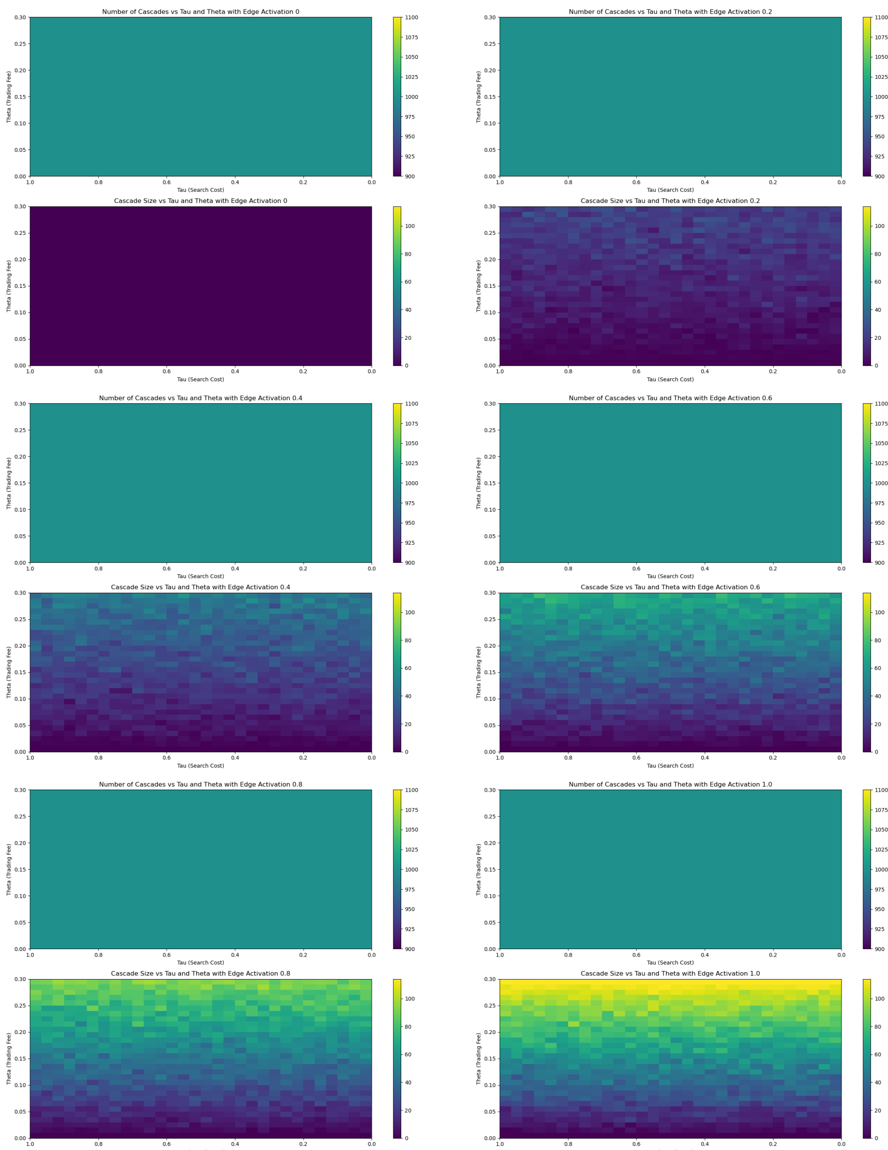
```
N = 1000 # Number of agents
L = 4000 # Number of total links
P = 50 # Fixed asset price
p = 0.5 # Probability of fundamental value
q = 0.65 # Deciding probability of private signal
eta_min = 0.7 # Minimum relative influence
eta_max = 3.0 # Maximum relative influence
network_type = "cluster" # Network type: 'cluster', 'random', 'scale_free'
...
def simulate_cascades_with_edge_activation(theta, tau, edge_activation_prob, time_steps=10):
"""
Simulate decision cascades with edge activation over multiple time steps.

Parameters:
- theta: Trading fee (affects utility calculation)
- tau: Search cost (affects whether neighbors are considered)
- edge_activation_prob: Probability that an edge is active at a given time step
- time_steps: Number of time steps for the simulation

Returns:
- cascade_count: Number of agents who changed their decision from their initial state
- cascade_size: Magnitude of decision imbalances across the network
"""
# Initialize agent properties
agents = {
    i: {
        "private_signal": 100 if random.random() < q else 0,
        "decision": "hold", # Current decision
        "initial_decision": "hold", # Updated at each time step
        "connected_decisions": [],
    }
    for i in range(N)
}

cascade_count = 0
cascade_size = 0
trend_shift_cascade = 0
# Simulate multiple time steps
for t in range(time_steps):
    for node in sorted(relative_influence, key=relative_influence.get, reverse=True):
        agent = agents[node]

        # Set the initial decision before considering external sources
        agent["initial_decision"] = agent["decision"]

        # Access connected agents' decisions, considering edge activation
        connected_nodes = list(G.neighbors(node))
```

```
        observed_decisions = [
            agents[n]["decision"]
            for n in connected_nodes
            if random.random() < edge_activation_prob and random.random() > tau # Edge
                activation + search cost
        ]
        agent["connected_decisions"] = observed_decisions

        # Calculate probabilities for decisions
        buy_count = observed_decisions.count("buying")
        sell_count = observed_decisions.count("selling")
        total_influence = sum(
            relative_influence[n] for n in connected_nodes if agents[n]["decision"]
        )
        if total_influence == 0: # No connected decisions observed
            total_influence = 1 # Avoid division by zero

        # Update decision based on utility
        fv = 100 if agent["private_signal"] == 100 else 0
        def utility(fv, decision):
            if decision == "buying":
                return fv - P - theta * P
            elif decision == "selling":
                return P - fv - theta * P
            else: # holding
                return 0

        # Determine agent's decision at this time step
        new_decision = "hold"
        if utility(fv, "buying") > max(utility(fv, "selling"), utility(fv, "hold")):
            new_decision = "buying"
        elif utility(fv, "selling") > max(utility(fv, "buying"), utility(fv, "hold")):
            new_decision = "selling"

        # Update the agent's decision
        agent["decision"] = new_decision

    # Analyze results
    # 1. Cascade Count: Count agents who changed their decision from the initial state
    cascade_count += sum(
        1 for agent in agents.values() if agent["initial_decision"] != agent["decision"] and
            agent["decision"] != "hold"
    )

    # 2. Cascade Size: Magnitude of decision imbalances across the network
    cascade_size += sum(
        abs(agent["connected_decisions"].count("buying") -
            agent["connected_decisions"].count("selling"))
        for agent in agents.values()
    )

return cascade_count, cascade_size
```

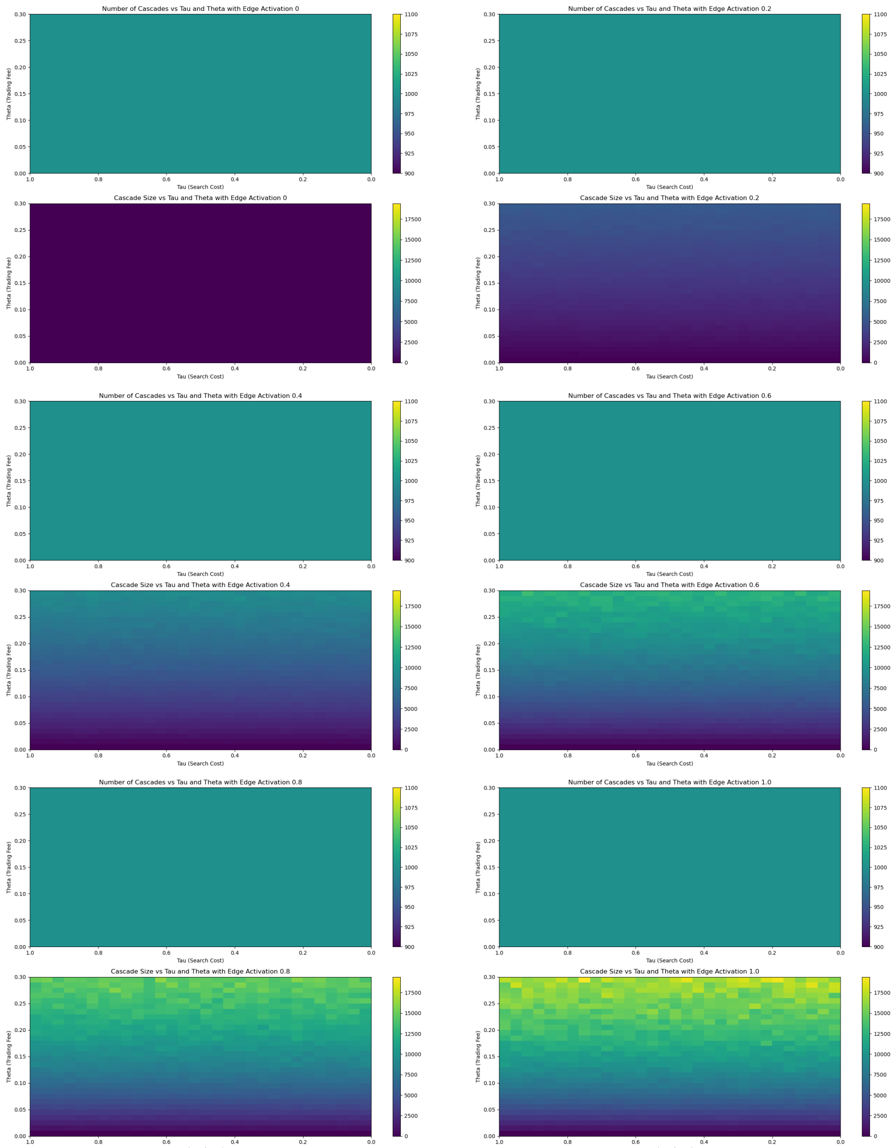
4 Results

In this section, we analyze the outcomes of our simulations across three distinct network topologies—*random networks*, *scale-free networks*, and *spatially-clustered networks*. The following plots illustrate the results for the three topologies across different edge activation probabilities:

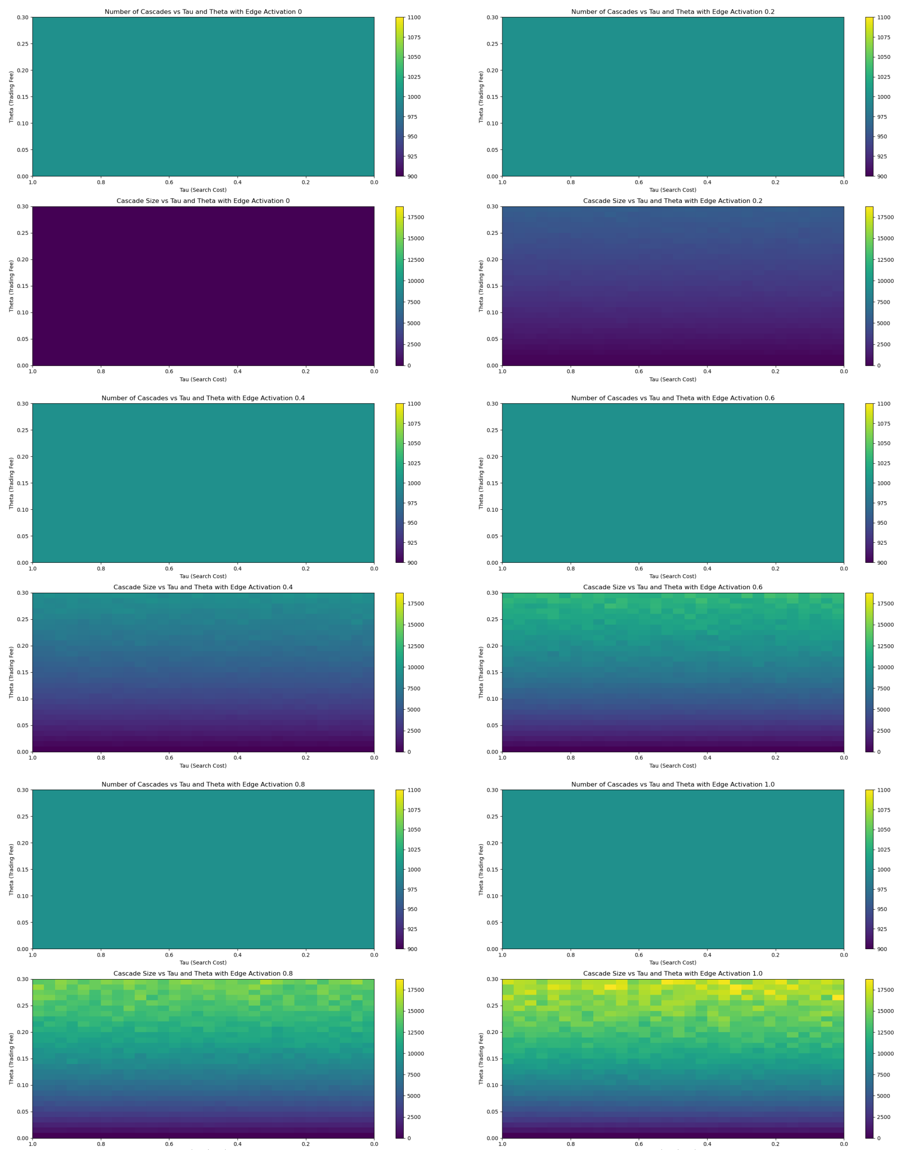
4.1 Random Networks



4.2 Scale-Free Networks



4.3 Spatially-Clustered Networks



4.4 Analysis of Results

Random networks, characterized by uniform connections, serve as the baseline for evaluating cascade behavior.

- The number of cascades (I) did not change with respect to T_f or θ, S_c or τ, p_{edge}, or network topology.
- Edge activation probability (p_{edge}) determines the temporal connectivity of networks. High p_{edge} led to bigger cascade sizes, though the distribution of cascade size to trading fee and search cost remained the same across edge activation probabilities.
- The cascade size C_{size} changed with respect to the trading fee S_c or θ but not with respect to S_c or τ.

Scale-free networks, dominated by hubs, exhibit distinct cascade behavior driven by their structural centralization.

- The number of cascades (I) remained unaffected by variations in trading fees (θ), search costs (τ), edge activation probability (p_{edge}), or network topology.
- Temporal connectivity, determined by the edge activation probability (p_{edge}), influences cascade sizes. Higher p_{edge} values resulted in larger cascade sizes; however, the relationship between cascade size, trading fee, and search cost remained consistent across different activation probabilities.
- Cascade size (C_{size}) exhibited sensitivity to changes in trading fees (θ), but showed no significant dependence on search costs (τ) across the simulations.

Spatially-clustered networks, with localized connections, impose structural limitations on cascade spread.

- The number of cascades (I) showed no noticeable variation with changes in trading fees (θ), search costs (τ), edge activation probability (p_{edge}), or the network's structural type.
- The edge activation probability (p_{edge}) controls the temporal connectivity of the network. Larger values of p_{edge} resulted in increased cascade sizes, yet the pattern of cascade size in relation to trading fees and search costs remained unchanged across activation probabilities.
- Cascade size (C_{size}) demonstrated a clear dependency on trading fees (θ), while it remained largely unaffected by variations in search costs (τ).

This is reflected in the equations for the metrics.

$$I = \begin{cases} N \cdot p_{\text{edge}} \cdot (1-\tau) \cdot (1-\theta), & \text{if } \theta \leq 0.3 \\ 0, & \text{if } \theta > 0.3 \end{cases},$$

$$C_{\text{size}} = \begin{cases} N \cdot p_{\text{edge}} \cdot (1-\tau) \cdot (1-\theta), & \text{if } \theta \leq 0.3 \\ 0, & \text{if } \theta > 0.3 \end{cases}.$$

We can see that when the active edge probability (p_{edge}) increases, both I and C_{size} should increase linearly according to the equations which is reflected in the results. For trading fees, as T_f or θ increases, both I and C_{size} should decrease, which is also reflected in the results. According to the equations, when the search cost S_c or τ increases, I and C_{size} should decrease. However, we didn't see this reflected in our results; thus, this may be a flaw of our model that we didn't account for.

A comparison across all three network topologies reveals the following key insights:

1. **Impact of Costs**: T_f dampens cascading behaviors across all networks.
2. **Temporal Connectivity (p_{edge})**: Higher edge activation probabilities enhance cascade spread in all networks. At low p_{edge}, cascades are almost entirely suppressed in spatially-clustered networks, whereas scale-free networks retain limited activity due to hub influence.
3. **Structural Differences**:
   - Random networks exhibit uniform cascade spread, heavily influenced by temporal connectivity.
   - Scale-free networks facilitate larger and more resilient cascades due to hub dominance.
   - Spatially-clustered networks restrict cascades to localized regions, with temporal connectivity playing a critical role in enabling broader propagation.

5 Overall Insights

In this paper, we extended the classical Watts threshold model to account for transaction costs and temporal components, offering a more comprehensive framework for analyzing information cascades in networks. By incorporating trading fees and search costs alongside temporal edge activation, our model captures the interplay between economic constraints and network connectivity, providing insights into the propagation of gross cascades and trend shift cascades across random, scale-free, and spatially-clustered networks.

Our findings highlight the significant role of trading fees in dampening cascading behavior, with higher costs leading to reduced information spread and diminished cascade sizes. Temporal connectivity, governed by edge activation probabilities, was found to compensate for cost constraints by facilitating cascade propagation, particularly in random and scale-free networks. Spatially-clustered networks, however, exhibited the most pronounced limitations, with cascades restricted to localized regions despite increased temporal connectivity. These results emphasize the critical influence of network structure, temporal dynamics, and economic costs on cascade behavior.

6 Future Work

While our study successfully integrates transaction costs and temporal edge activations, there remain opportunities for further research to enhance and expand this work. First, the current mathematical approximations for cascade metrics were developed without explicitly accounting for the influence of network topology. Future work could refine these equations to better represent the unique structural properties of random, scale-free, and spatially-clustered networks, ensuring that the underlying dynamics are accurately modeled. Additionally, extending this analysis to other network types, such as small-world or hierarchical networks, may yield further insights into how diverse topologies impact cascading behavior under economic and temporal constraints.

Second, the model assumes a constant edge activation probability (p_{edge}), limiting its applicability to networks with uniform temporal dynamics. Incorporating bursty or non-uniform temporal activations, as observed in real-world systems, could provide a more realistic representation of information propagation. Exploring how temporal heterogeneity interacts with economic costs across different network structures would be a valuable avenue for future research.

Third, while our study focuses on agent-based decision-making driven by private signals and peer influence, introducing adaptive behavior or learning mechanisms could further enrich the model. Agents might adjust their thresholds or trading strategies over time based on observed outcomes, introducing dynamic decision-making processes that reflect real-world systems more accurately.

Lastly, the integration of external shocks or global signals, such as news events or sudden market changes, could provide a deeper understanding of how cascades are initiated and disrupted in dynamic environments. By simulating such scenarios, future research could explore resilience strategies for networks and identify critical thresholds where cascading behaviors emerge or collapse.

In conclusion, our study represents a significant step toward modeling realistic cascading behavior under economic and temporal constraints. By addressing the outlined limitations and exploring these future directions, this work can be expanded into a robust framework with applications across domains such as financial markets, social influence propagation, and epidemic spreading. Ultimately, advancing our understanding of information cascades in complex networks will enable more effective strategies for managing and mitigating their impacts in real-world systems.

References

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