Module 2: AC Circuits (8 hours)
Representation of sinusoidal waveforms, peak and rms values, phasor representation, real power, reactive power, apparent power, power factor. Analysis of single-phase ac circuits consisting of R, L, C, RL, RC, RLC combinations (series and parallel), resonance. Three phase balanced circuits, voltage and current relations in star and delta connections.

AC Fundamentals

Alternating current is the current which constantly changes in amplitude, and which reverses direction at regular intervals

In 1831 Michael Faraday discovered the fundamental relationship between the voltage and magnetic flux in a circuit. This relationship is now known as Faraday's law of electromagnetic induction, which states: If the flux linking a conductor forming a loop changes with time, a voltage is induced at its terminal. The magnitude of the induced voltage is proportional to the rate of change in the magnetic flux.

Mathematically the law is stated as follows: $E = N\,\frac{d\varphi}{dt}$

Including Lenz's law $E = -N\,\frac{d\varphi}{dt}$

where, E = induced voltage (in volts) N = number of turns in the conductor φ = Flux linking the conductor

Note that the induced voltage E depends on the **rate of change of flux** linking the conductor. The flux linkage can vary either by a magnetic field that is changing with time (due to AC current) or by moving the conductor in a stationary magnetic field. Fig. 1 shows a conductor being moved in a magnetic field. When the conductor is moved horizontally (right to left) a voltage is induced in the conductor due to the changing flux linkage. If the conductor forms a closed loop, the induced voltage would cause a current to flow though the conductor.

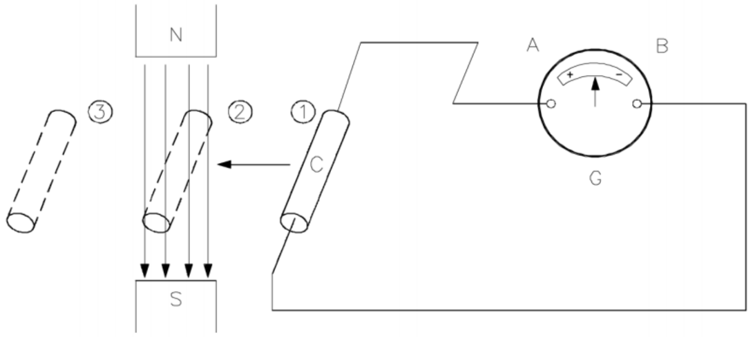


Fig 1 : A conductor moving in a magnetic field

Note that the polarity of the induced voltage and the direction of the current through the conductor would reverse when the conductor is moved back from left to right. Furthermore, no voltage would be induced if the conductor in Fig. 1 is moved vertically because the flux through the conductor would not change in the vertical movement.

Now we know that, $\varphi = BA\cos\theta$

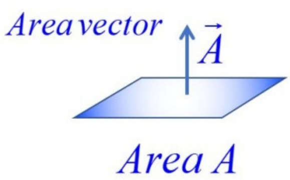
Where B = magnetic flux density

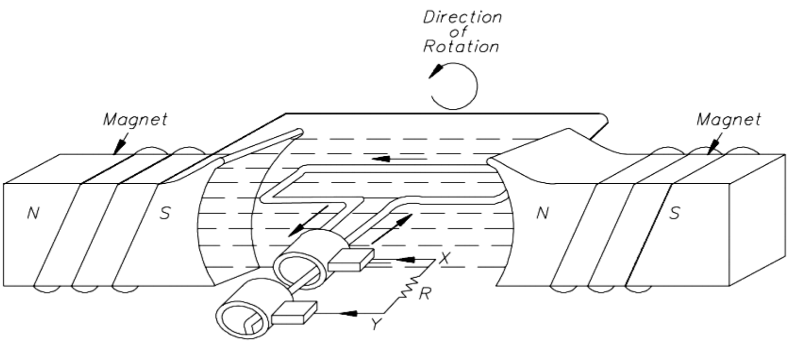
A= area of the coil (length x width)

θ = angle between area vector and flux density

To change φ we can change B or A or angle θ between them .

Here we have to note that direction of area vector is perpendicular to the coil.





If the coil rotating with an angular velocity of ω ,then at time t

$$\theta = \omega t$$

And $\qquad \varphi = B\,A\cos\omega t$

Now from Faraday's law of induction

$$E = -N\frac{d\varphi}{dt}$$

$$E = -N\frac{d(B\,A\cos\omega t)}{dt}$$

$$E = -N\,B\,A\,\omega\,(-\sin\omega t)$$

$$E = N\,B\,A\,\omega\sin\omega t$$

The value of E will be maximum when $\sin\omega t = 1$ [as the range of sin is from -1 to +1]

$\therefore\ E_{max} = N\,B\,A\,\omega$

And $\qquad E = E_{max}\sin\omega t = E_{max}\sin\theta$

From this equation ,

For $\theta = 0°$ i,e, area vector parallel to flux density and coil is perpendicular to the coil

$E = 0$ $\qquad \longrightarrow$ B(magnetic flux)

$\qquad\qquad \longrightarrow$ A(area vector)

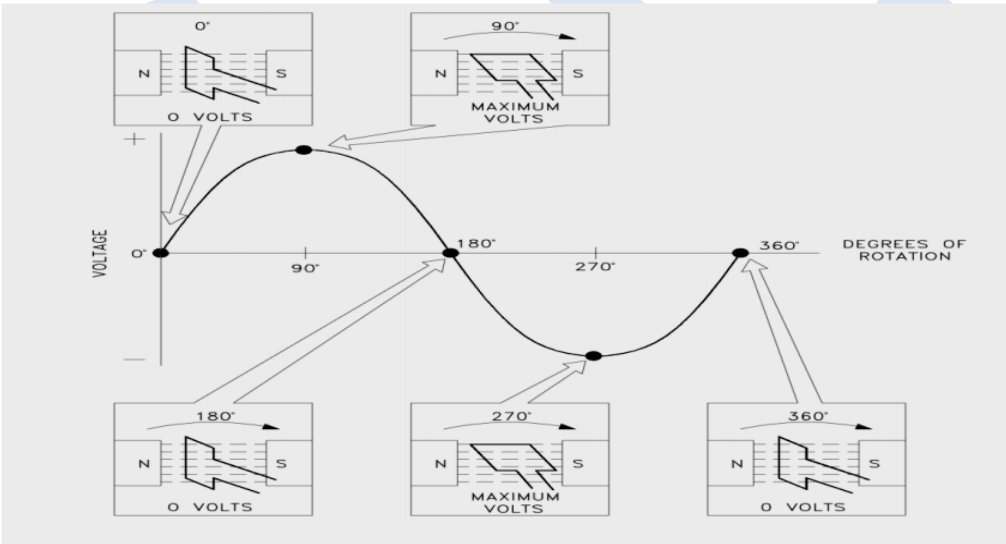
For $\theta = 90°$, $E = E_{max}$ $\qquad$ A $\uparrow$ $\longrightarrow$ B

For $\theta = 180°$, $E = 0$ $\qquad$ A $\longleftarrow\longrightarrow$ B

For $\theta = 270°$, $E = -\ E_{max}$ $\qquad$ A $\downarrow$ $\longrightarrow$ B

For $\theta = 360°$, $E = 0$ $\qquad \longrightarrow$ B

$\qquad\qquad \longrightarrow$ A



<u>Terminology</u>

Frequency(f): It is the complete cycles that occurred in one second.Unit is Hertz(Hz)

Time period(T): it is the duration of time required for the quantity to complete one cycle.T= 1/f

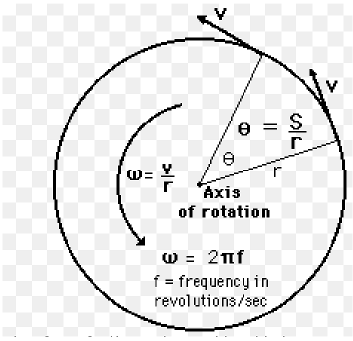
Amplitude: The amplitude of a sine wave is the maximum value, positive or negative,that it can attain.

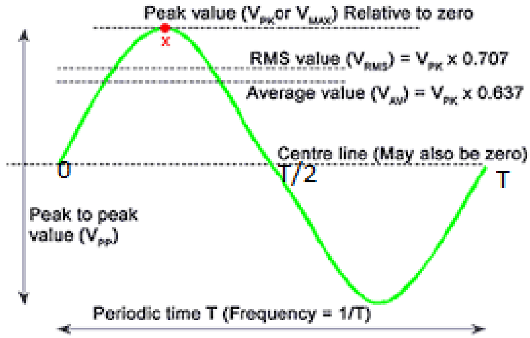
Instantaneous value: the instantaneous value of an alternating voltage or current at one particular instant.

Average value: the average value of an alternating current or voltage is the average of all the instantaneous values during one alternation.

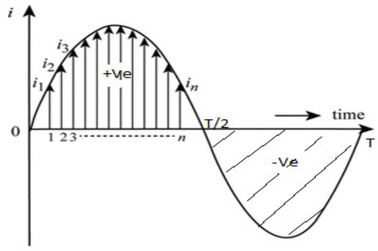
Peak value: It is the maximum value of voltage or current.

Angular velocity: the rate of change of angular position of a rotating body





Average value of sine wave

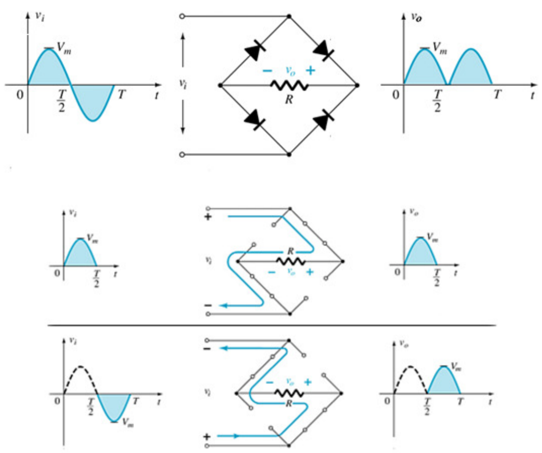


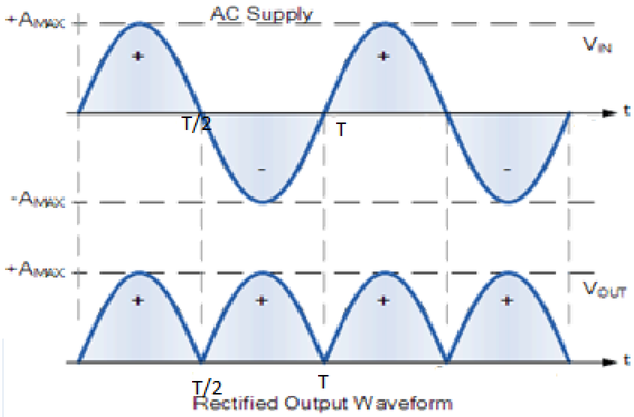
I_{avg} = Area under the curve/T

= [area(0-T/2) + area (T/2 -T)] /T

= [+ Ve + (-Ve)] = 0

Full wave rectified ac sine wave :





Average or mean value = $\frac{area\ under\ curve}{length\ of\ base}$

In this condition the curve repeating afterT/2 time period. So area under half cycle will provide the average value.

$$V_{avg} = \frac{one\ half\ of\ the\ curve}{T/2}$$

Now ,v =V_m sin wt =V_m sin 2πft

$$Vavg = \frac{\int_0^{T/2} vdt}{T/2} = \frac{2}{T}\int_0^{T/2} Vm \sin{(wt)}\, dt \quad = \frac{2Vm}{T}\left[\frac{-coswt}{w}\right]^{T/2}$$

$$= \frac{2Vm}{Tw}\ [\ -\cos\frac{2\pi}{\cancel{T}}\ \frac{\cancel{T}}{2}\ \ -(-\cos\frac{2\pi}{T}\ 0)]$$

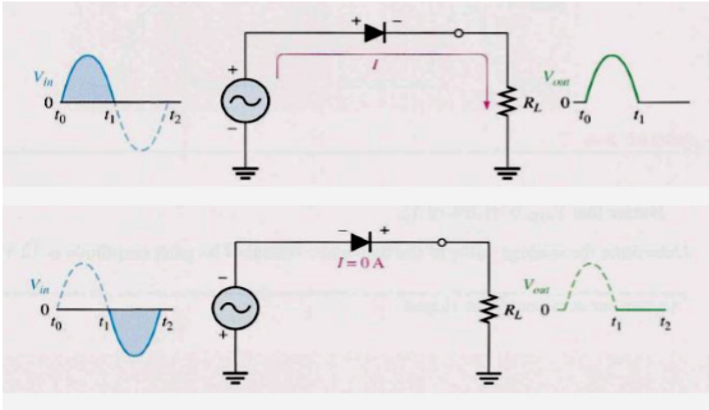
$$= \frac{2Vm}{Tw}\ [1+1]\ = \frac{4Vm}{\left(\frac{2\pi}{T}\right)T} \ = \frac{2Vm}{\pi}$$

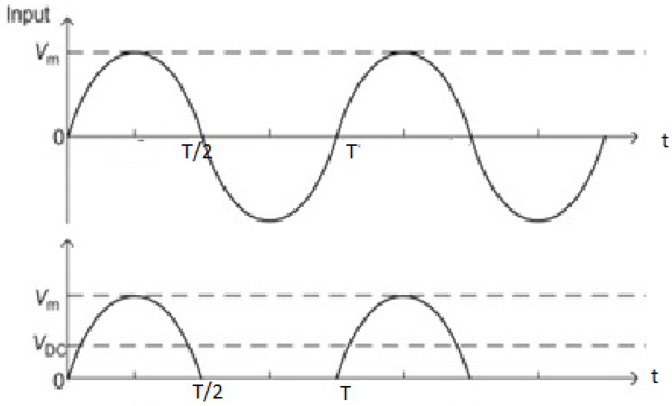
For full wave rectified sine wave

$$V_{avg} = \frac{2Vm}{\pi} = 0.637V_m$$

Similarly I_{avg} = 0.637 I_m

Half wave rectified ac sine wave:





The rectified signal repeat itself after time period T

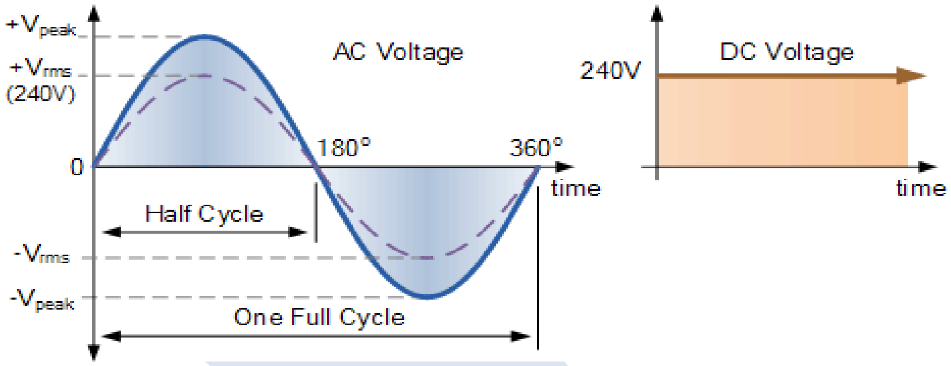
$$\therefore \quad V_{avg} = \frac{1}{T}\int_0^{\frac{T}{2}} Vm\,\sin(wt)dt \;+\; \int_{T/2}^{T} 0\,dt$$

$$= \frac{1}{T}\,Vm\,\left[\frac{-coswt}{w}\right]^{T/2} = \frac{Vm}{\pi}$$

Root mean square (RMS) value of a sinusoid:

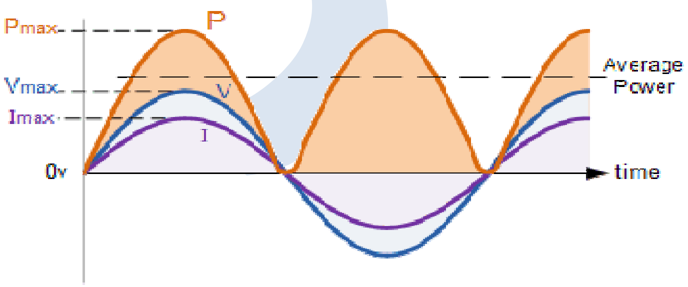
The rms value or effective value of a alternating current or voltage is given by the steady current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

It has a relation with the power .It is that value of voltage or current which is the power producing part.



We know that,

$v = V_m$ sinwt and $i = I_m$ sinwt and power P = vi



Power will be always positive ,as

P at negative cycle = (-v) x(-i) = vi

P= vi= V_m sinwt * I_m sinwt = $V_m I_m \sin^2 wt$

$$= V_m I_m \left(\frac{1-cos2wt}{2}\right) = \frac{Vm\,Im}{2} - \frac{Vm\,Im}{2}\cos 2wt$$

So there are two parts , dc part and ac part. The frequency of ac part is twice than the voltage or current frequency.

Average value of P:

$$P_{avg} = \frac{1}{T}\int_0^T vi\;dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{Vm\,Im}{2} - \frac{Vm\,Im}{2}\cos 2wt\right)dt \;=\; \frac{1}{T}\,\frac{Vm\,Im}{2}\left[\int_0^T (1-cos2wt)dt\right]$$

$$= \frac{Vm\,Im}{2T}\left[\,t\,\Big|^{T} - \frac{sin2wt}{2w}\Big|^{T}\right]$$

$$= \frac{Vm\,Im}{2T}\,[T-0] = \frac{\boldsymbol{Vm\,Im}}{\mathbf{2}}$$

Now $\quad V_m = I_m R$

$P_{avg} = I_m^2 R/2 = \frac{Im}{\sqrt{2}} \frac{Im}{\sqrt{2}} R$

$= \underline{Amp} . (\underline{Amp \times ohm}) = watt$

The ampere part ($\frac{Im}{\sqrt{2}}$) is producing the energy in the circuit.This part is called RMS current.

$P_{avg} = I_{rms} . I_{rms} R = (I_{rms})^2 . R$..........(1)

$I_{rms} = \frac{Im}{\sqrt{2}}$, $V_{rms} = \frac{Vm}{\sqrt{2}}$

Again , $\quad P_{avg} = \frac{1}{T}\int_0^T (Vm \sin wt . Im \sin wt)dt \quad = \frac{1}{T}\int_0^T (Im\, R \sin wt . Im \sin wt)dt$

$= \frac{1}{T}\int_0^T (Im^2 \sin^2 wt)\, dt\, R$

Equating with equation 1,

$\frac{Im^2}{2} = \frac{1}{T}\int_0^T (Im^2 \sin^2 wt)\, dt$

$\frac{Im}{\sqrt{2}} = \sqrt{\frac{1}{T}\int_0^T (Im^2 \sin^2 wt)\, dt} = I_{rms}$

$\boldsymbol{I_{rms} = \sqrt{\frac{1}{T}\int_0^T (Im^2 \sin^2 wt)\, dt}} = \frac{Im}{\sqrt{2}} = 0.707\, I_m$

<u>Form factor(K_f)</u> : It is the ratio of the rms value to the average value of an alternating quantity

Form factor$=\frac{\text{rms value}}{\text{average value}}$

Form factor for sinusoidal wave , $K_f = \frac{I_{rms}}{I_{avg}} = \frac{0.707\, Im}{0.637\, Im} = 1.11$

<u>Peak factor</u> :It is the ratio of peak or maximum value to the rms value of an alternating quantity.

Peak factor $= \frac{\text{maximum value}}{\text{rms value}}$

Peak factor for sinusoidal wave $= \frac{Im}{0.707\, Im} = 1.414$

Ex1. An alternating current has rms value of 50 A and frequency 60 Hz.find the time taken to reach 50 A for the first time.

sol^n $\quad I_{rms} = 50A$, $\quad I_m = 50 \times \sqrt{2} = 70.71A$

$i = I_m \sin 2\pi ft = 70.71 \sin 2\pi 60t$

Now i = 50

$\therefore 50 = 70.71 \sin 2\pi 60t$

$\sin 120\pi t = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$t = \frac{1}{(120 \times 4)}$ $= \mathbf{2.08\ ms}$

Ex2. An alternating voltage is given by the equation $v = 282.84 \sin (377t + \frac{\pi}{6})$ .find the i) rms value ii) frequency iii)time period.

Soln:

i) $V_m = 282.84$ V

$V_{rms} = \frac{282.84}{\sqrt{2}} = 200$ V

ii) $w = 2\pi f = 377$

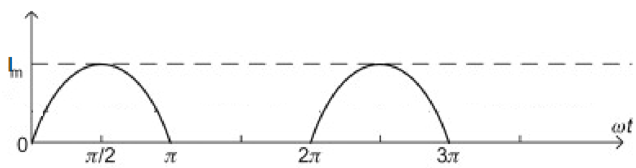
$f = \frac{377}{2\pi} = 60$ Hz

iii) $T = \frac{1}{f} = \frac{1}{60} = 0.0167$ s

Ex3. Find the rms,average,peak factor ,form factor of half wave rectified alternating current.

Sol: $I_{avg} = \frac{1}{2\pi}\int_0^{\pi} Im \sin wt\, d(wt)$

$= \frac{Im}{2\pi}[-\cos wt]^{\pi} = \frac{Im}{\pi} = 0.318 I_m$



$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(Im^2 \sin^2 (wt))\, d(wt)}$

$= \frac{Im}{\sqrt{2\pi}}\sqrt{\int_0^{\pi}(\sin^2 (wt))\, d(wt)}$

$= \frac{Im}{\sqrt{4\pi}}\sqrt{\int_0^{\pi}(1 - \cos 2(wt))\, d(wt)}$

$=\frac{Im}{\sqrt{4\pi}} \sqrt{[wt - \frac{1}{2} sinwt]_0^{\pi}} = \frac{Im}{\sqrt{4\pi}} \times \sqrt{\pi} = \frac{Im}{2} = 0.5\, Im$

Peak factor = $\frac{Im}{0.5\ \text{Im}}$ = 2

Form factor = $\frac{I_{rms}}{I_{avg}} = \frac{0.5\ \text{Im}}{0.318\text{Im}}$ = 1.57

Ex4.Write the mathematical expression for a 50Hz sinusoidal voltage supplied for domestic purpose at 230V.

Sol: Vrms = 230v , Vm = √2 x Vrms = √2 x 230 = 325.27 V

Expression for instantaneous value

V= Vm sinwt = 325.27 sin2π50t = 327.27 sin314t

Hw1.The equation of an alternating current is i= 141.4sin314t . what is the rms value of current and frequency?[100A,50Hz]

Hw2.An ac has frequency 50Hz and rms current 25A.Write equation of instantaneous current and find i)current at time 0.0025s ii)time at which current is 14.14amp. [25A,0.00131s]

Hw3. An alternating voltage is given by v = 141.4sin314t. Find i) frequency ii)rms value iii)average value iv)the instantaneous value of voltage when t is 3ms v) the time taken for the voltage to reach 100V for the first time after passing through zero value. [50Hz, 100V, 90V, 114.4V, 2.5ms]

Hw4. An alternating voltage is v = 100sin 100t . Find i)amplitude ii) time period and frequency iii) angular velocity iv)form factor v)peak factor. [100V,63ms,63.7V,1.11,1.4142]

PHASOR REPRESENTATION OF ALTERNATING QUANTITIES :

Assumed that alternating voltages and currents follow sine law and generators are designed to give emfs having sine waveform. The above said assumption makes the calculations simple. The method of representing alternating quantities by waveform or by the equations giving instantaneous values is quite cumbersome.

For solution of ac problems it is advantageous to represent a sinusoidal quantity (voltage or current) by a line of definite length rotating in counter clockwise direction with the same angular velocity as that of the sinusoidal quantity. Such a rotating line is called the phasor.

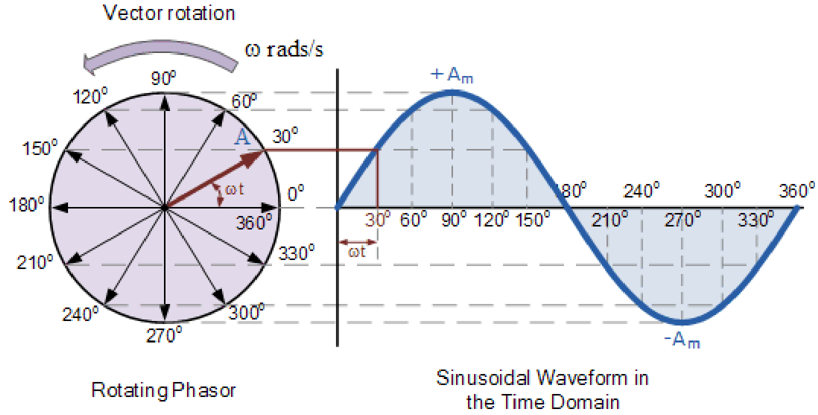
Basically a rotating vector, simply called a "**Phasor**" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase"). A phasor is a vector that has an arrow head at one end which signifies partly the maximum value of the vector quantity (V or I) and partly the end of the vector that rotates.

Generally, vectors are assumed to pivot at one end around a fixed zero point known as the "point of origin" while the arrowed end representing the quantity, freely rotates in an anti-clockwise direction at an angular velocity, (ω) of one full revolution for every cycle. This anti-clockwise rotation of the vector is considered to be a positive rotation. Likewise, a clockwise rotation is considered to be a negative rotation.

Although the both the terms vectors and phasors are used to describe a rotating line that itself has both magnitude and direction, the main difference between the two is that a vectors magnitude is the "peak value" of the sinusoid while a phasors magnitude is the "rms value" of the sinusoid. In both cases the phase angle and direction remains the same.

The phase of an alternating quantity at any instant in time can be represented by a phasor diagram, so phasor diagrams can be thought of as "functions of time". A complete sine wave can be constructed by a single vector rotating at an angular velocity of ω = 2π*f*, where *f* is the frequency of the waveform. Then a **Phasor** is a quantity that has both "Magnitude" and "Direction".

Generally, when constructing a phasor diagram, angular velocity of a sine wave is always assumed to be: ω in rad/sec. Consider the phasor diagram below.



As the single vector rotates in an anti-clockwise direction, its tip at point A will rotate one complete revolution of 360° or 2π representing one complete cycle. If the length of its moving tip is transferred at different angular intervals in time to a graph as shown above, a sinusoidal waveform would be drawn starting at the left with zero time. Each position along the horizontal axis indicates the time that has elapsed since zero time, t = 0. When the vector is horizontal the tip of the vector represents the angles at 0°, 180° and at 360°.

Phase and Phase Angle:

By phase of an alternating current is meant the fraction of the time period of that alternating current that has elapsed since the current last passed through the zero position of reference.

The phase angle of any quantity means the angle the phasor representing the quantity makes with the reference line (which is

taken to be at zero degrees or radians).

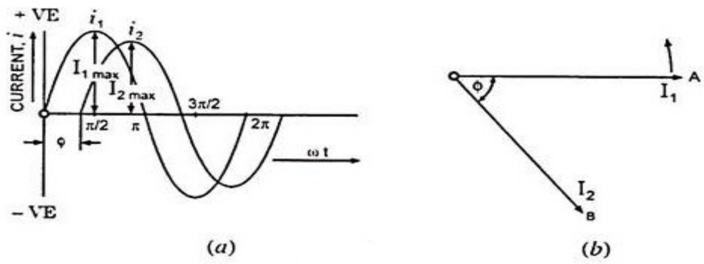
For

example the

phase angle of

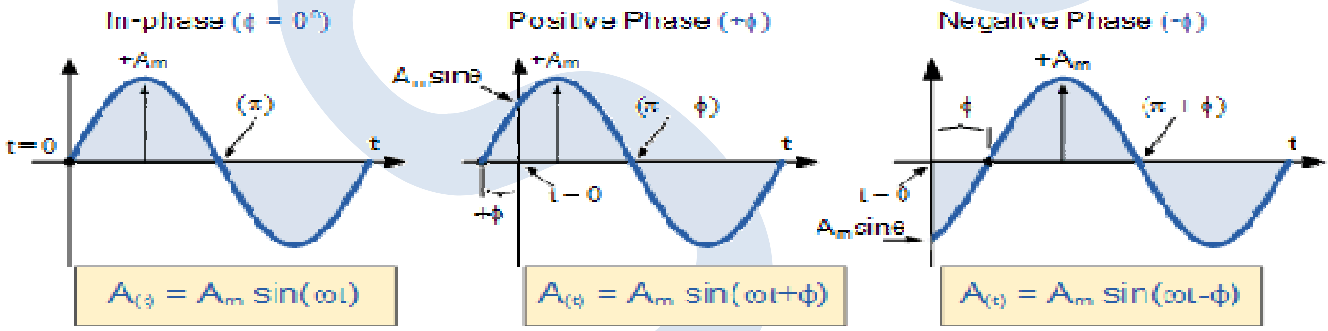
current I_2, in

Fig. is $(-\phi)$.

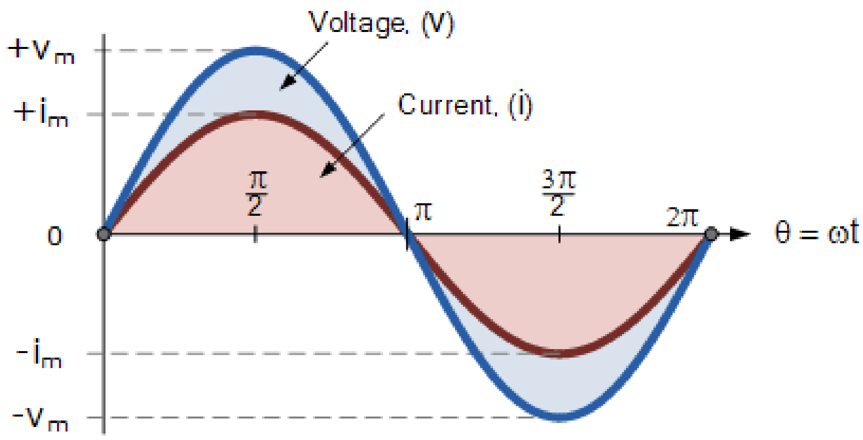


Phase difference: The phase difference between the two electrical quantities is defined as the angular phase difference between the maximum possible value of the two alternating quantities having the same frequency. In other words, the two alternating quantities have phase difference when they have the same frequency, but they attain their zero value at the different instant. The angle between zero points of two alternating quantities is called angle of phase differences. For the above fig the phase difference between I_1 and I_2 is φ.

Phase Relationship of a Sinusoidal Waveform:

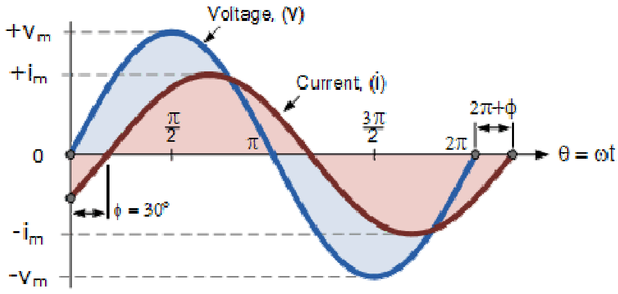


In phase quantity:



When both the quantities reach the max and min value at the same time.

out-of-phase quantity:



When both the quantities reach the max and min value at different time. The voltage waveform above starts at zero along the horizontal reference axis, but at that same instant of time the current waveform is still negative in value and does not cross this reference axis until 30° later. So we can say that the two waveforms are now 30° out-of phase. The current waveform can also be said to be "lagging" behind the voltage waveform by the phase angle, φ. Then in our example above the two waveforms have a **Lagging Phase Difference** so the expression for both the voltage and current above will be given as.

Voltage $(v(t)) = V_m \sin wt$

Current $(i(t)) = I_m \sin(wt - \varphi)$

where, i lags v by angle φ

leading phasor: when the max and min of a quantity reaches earlier thanthe other quantity. Measured in anticlockwise direction

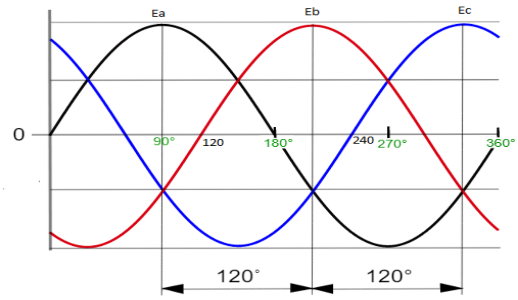
lagging phasor: When the max and min value of a quantity reaches later than the other quantity.Measured in clockwise direction.

In the fig. Eb is leading with respect to Ec by 120° and lagging with respect to Ea by 120°

Now if Eb = Emax sinwt ,

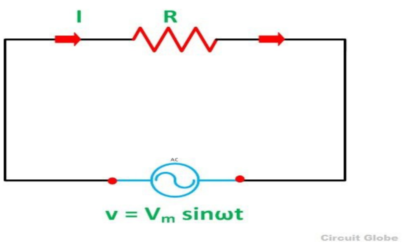
Then Ea = Emax sin (wt +120) and

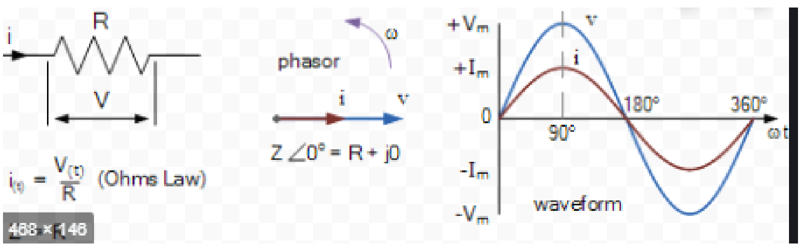
Ec = Emax sin (wt – 120)



Analysis of single-phase ac circuit

Pure resistive circuit

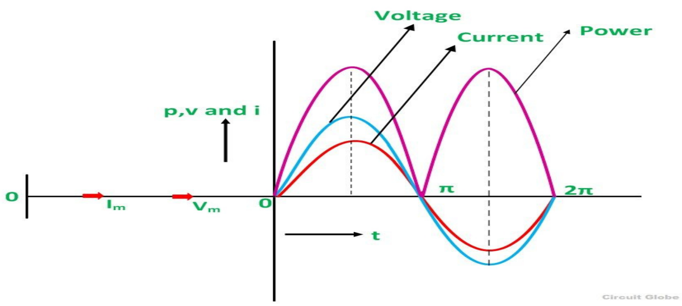




$v = V_m \sin wt$, $i = \frac{Vm}{R} \sin wt$ => $i = I_m \sin wt$

so there is no phase difference between voltage and current signal.

Power in Pure Resistive Circuit



Instantaneous power, p= vi

$$p = (V_m \sin\omega t)(I_m \sin\omega t)$$

$$p = \frac{V_m I_m}{2}\, 2\sin^2\omega t = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}(1 - \cos 2\omega t)$$

$$p = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos 2\omega t$$

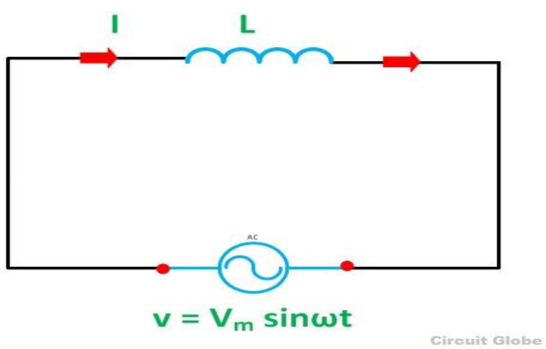
The average power consumed in the circuit over a complete cycle is given by

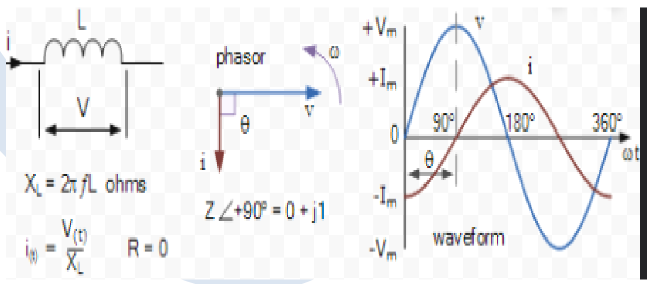
$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} - \text{average of } \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\omega t$$

Since average of $\frac{Vm\, Im}{2}$ coswt over a complete cycle is zero,

$$P = \frac{Vm}{\sqrt{2}}\frac{Im}{\sqrt{2}} = V_{rms}\, I_{rms}$$

AC through pure inductive circuit





For inductance we know that ,

$v = L\frac{di}{dt}$ => Vm sinwt = $L\frac{di}{dt}$ => $di = \frac{Vm}{L}$ sinwt dt

Integrating both sides of the equation

$$\int di = \int \frac{V_m}{L}\sin\omega t\, dt \quad \text{or}$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t) \quad \text{or}$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - {}^{\pi}/_{2}\right) = \frac{V_m}{X_L}\sin\left(\omega t - {}^{\pi}/_{2}\right)$$

where, $X_L = \omega L$ is the opposition offered to the flow of alternating current by a pure inductance and is called inductive reactance.

The value of current will be maximum when sin (ωt – π/2) = 1

Therefore,

$$I_m = \frac{V_m}{X_L}$$

$$i = I_m \sin\left(\omega t - {}^{\pi}/_{2}\right)$$

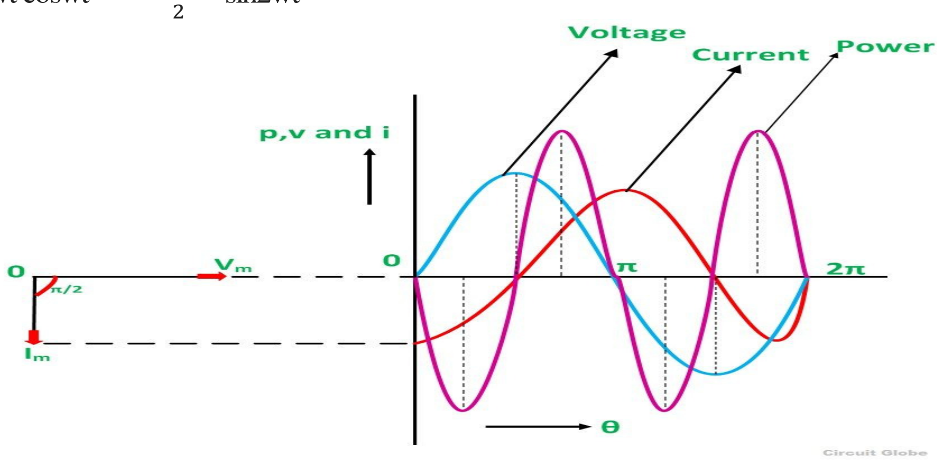
so the current lags behind the voltage by $\frac{\pi}{2}$.

Power in Pure Inductive Circuit:

Instantaneous power in the inductive circuit is given by

$P = vi = V_m \sin wt \; I_m \sin(wt - \frac{\pi}{2})$

Or $p = -V_m I_m \sin wt \cos wt = -\frac{VmIm}{2} \sin 2wt$



The power measured by wattmeter is the average value of p which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. In the purely inductive circuit, during the first quarter cycle, the power supplied by the source, is stored in the magnetic field set up around the coil. In the next quarter cycle, the magnetic field diminishes and the power that was stored in the first quarter cycle is returned to the source. This process continues in every cycle, and thus, no power is consumed in the circuit. Hence in a pure inductive circuit power absorbed is zero.

Ex1. A coil of inductance 0.05 H is connected to a supply of 220V,50 Hz .calculate current in the coil.

Sol: inductive reactance of the coil $X_L = wL = 2\pi fL = 15.7\Omega$

Current in coil , $I = \frac{V}{X_L} = \frac{220}{15.7} = 14$ A

Ex2. The voltage and current through a circuit element are

v= 100 sin(314t +45°)volt , i= 10 sin (314t + 315°) ampere

i)identify the circuit element ii)Find the value iii)obtain expression for power

sol: v= 100 sin(314t +45°)volt

i= 10 sin (314t + 315° - 360) = i= 10 sin (314t - 45°)

i) from the equation for voltage and current it is observed that the circuit current lags behind the applied voltage by 90°.It means the circuit element is an inductor.

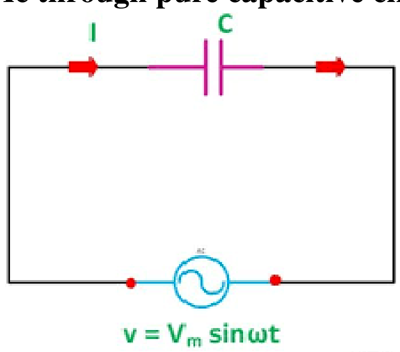
ii) Inductive reactance of the inductor, $X_L = \frac{Vrms}{Irms} = \frac{100/\sqrt{2}}{10/\sqrt{2}} = 10\ \Omega$

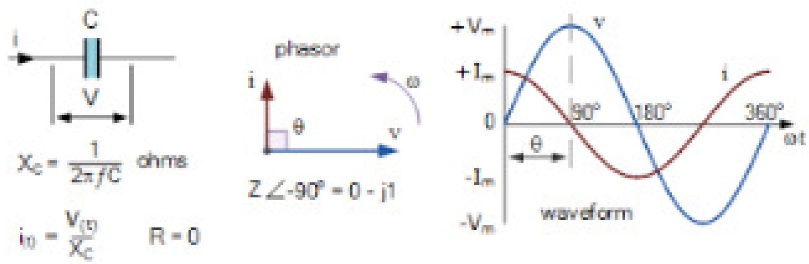
and $L = \frac{X_L}{w} = 10/314 = 0.0318$ H

iii)expression forpower is given as , $p = -\frac{VmIm}{2} \sin 2wt = \frac{-100 \times 10}{2} \sin(2 \times 314t)$

$= -500 \sin 628t$

Ac through pure capacitive circuit:





We know that for capacitor,

$i = \frac{dq}{dt}$ and q= c v , $v = V_m \sin wt$

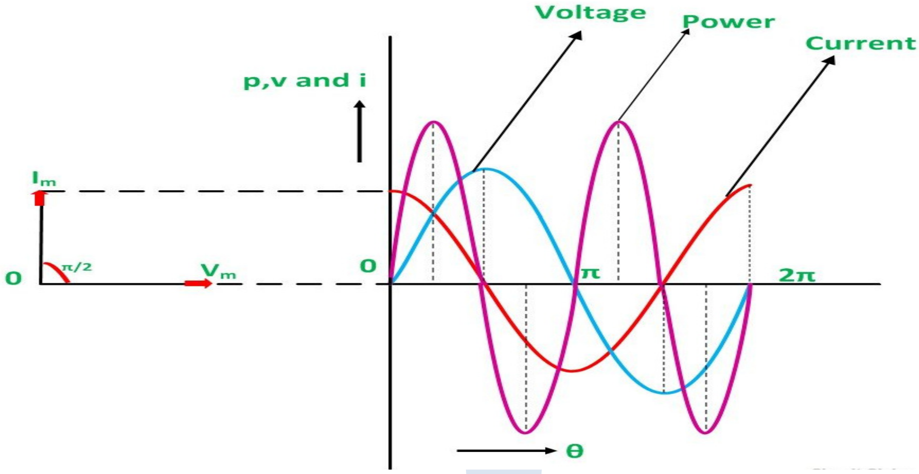
so $i = c\frac{dv}{dt} = c\frac{d}{dt}V_m \sin wt = V_m wc \cos wt = \frac{V_m}{\frac{1}{wc}} \sin(wt + \frac{\pi}{2})$

$= \frac{V_m}{X_c} \sin(wt + \frac{\pi}{2})$ where $X_c = \frac{1}{wc}$ =capacitive reactance

The value of i will be maximum when value of sin is 1. So $I_m = \frac{V_m}{X_c}$

$i = I_m \sin(wt + \frac{\pi}{2})$

Power in pure capacitor circuit



Instantaneous power is given by p = vi

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + {}^{\pi}/_{2}))$$

$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} \sin 2\omega t \quad \text{or}$$

Now average power $P = \frac{V_m I_m}{2}$ x average of sin2wt over a complete cycle =0

Hence power absorbed in pure capacitive circuit is zero.

When the voltage is increased, the capacitor gets charged and reaches or attains its maximum value and, therefore, a positive half cycle is obtained and when the voltage level decreases the capacitor gets discharged, and the negative half cycle is formed. This cycle of charging and discharging of capacitor continues.

Ex1.A capacitor of 100µF is connected across a 200V ,50Hz single phase supply .Calculate i)the reactance of the capacitor ii)rms value of the current and iii)maximum current.

Sol: C=100µF , V=200v, f= 50Hz

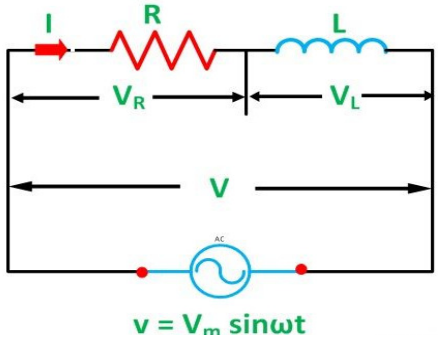
i)reactance of capacitor $Xc = \frac{1}{wc} = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 1 \times 10^{-4}}$ =31.83 Ω

ii) rms value of current $I = V/X_c$ = 200/31.83 = 6.283 A

iii) Maximum value of current , Im = √2 x 6.283 = 8.886

<u>AC through series R-L circuit</u>:

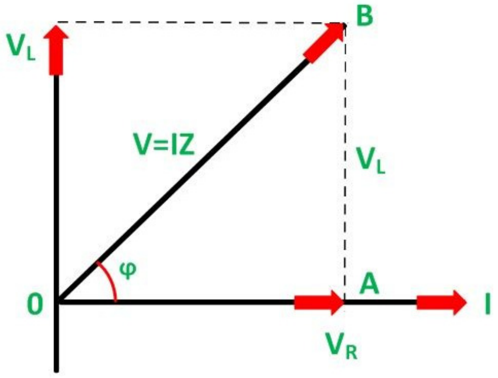
A circuit that contains pure resistance R ohms connected in series with a coil having pure inductance of L (Henry) is known as **R L Series Circuit**. When an AC supply voltage V is applied the current, I flows in the circuit. I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other



Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L
- V – Total voltage of the circuit

Phasor Diagram of the RL Series Circuit



Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step.

- Current I is taken as a reference.
- The Voltage drop across the resistance V_R = IR is drawn in phase with the current I.
- The voltage drop across the inductive reactance V_L =IX_L is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

Now,

In right angle triangle OAB

$$V_R = IR \text{ and } V_L = IX_L \text{ where } X_L = 2\pi fL$$

Applied voltage $V = \sqrt{V_R^2 + V_L^2} = \sqrt{((IR)^2 + (IX_L)} = I\sqrt{R^2 + X_L^2}$

$$V = IZ \text{ where } Z = \text{impedance } = \sqrt{R^2 + X_L^2}$$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

From phasor diagram it is also evident that the current lags behind the applied voltage V by angle φ.Which is given by,

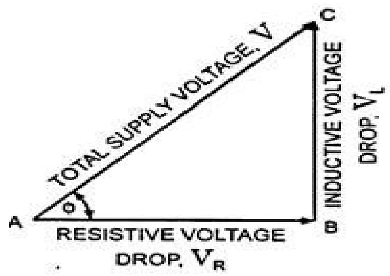
$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

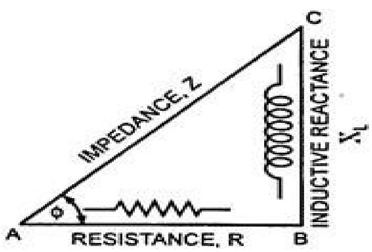
Now $v = V_m \sin wt$ and $i = I_m \sin(wt-\varphi)$

Impedance Triangle:

If a triangle ABC is drawn so that AB = V_R/I = R, BC = V_L/I = X_L and AC = V/I = Z, it is a triangle similar to that produced by the voltage triangle. Such a triangle is called an impedance triangle, which is most useful in letting one see at a glance how R, X, and Z are related to each other.



(a) Voltage Triangle



(b) Impedance Triangle

The angle between Z and R sides of the impedance triangle is known as phase angle of the circuit and cos of this angle is known as power factor of the circuit.

Power factor = Cos ϕ = R/Z

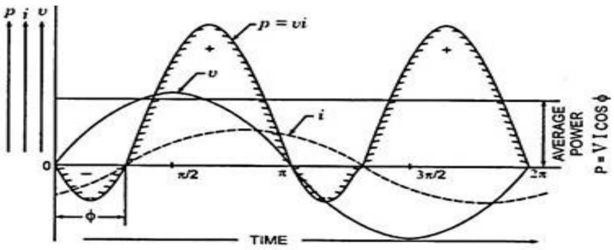
Power in Resistance—Inductance (R-L) Circuit:

p = Power consumed by resistance

$= I^2 R = I\,(I\,R) = V/Z.\,I\,R$

$= V\,I\,R/Z = V\,I\cos\phi$,

Since from impedance triangle $\cos\phi = R/Z$



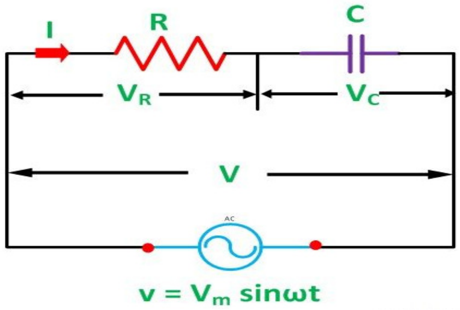
Wave Diagram

Power consumed by inductance is zero

So the power in an ac circuit is given by the product of rms values of current and voltage and cosine of the phase angle between voltage and current. Cosine of the phase angle between the voltage and current, $\cos\phi$ is known as the power factor of the circuit, and is equal to R/Z which is obvious from impedance triangle

AC through series R-C circuit:

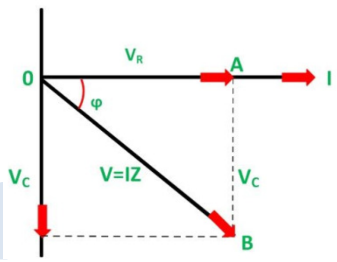
A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.**



Where,

- V_R – voltage across the resistance R
- V_C – voltage across the capacitor C
- V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit



Steps to draw a Phasor Diagram

The following steps are used to draw the phasor diagram of RC Series circuit

- Take the current I (r.m.s value) as a reference vector
- Voltage drop in resistance $V_R = IR$ is taken in phase with the current vector
- Voltage drop in capacitive reactance $V_C = IXC$ is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees in pure capacitive circuit)
- The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now, $V_R = I_R$ and $V_C = IX_C$ Where, $X_C = I/2\pi fC$

In right triangle OAB

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \qquad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Phase angle

From the phasor diagram shown above it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the phase angle.

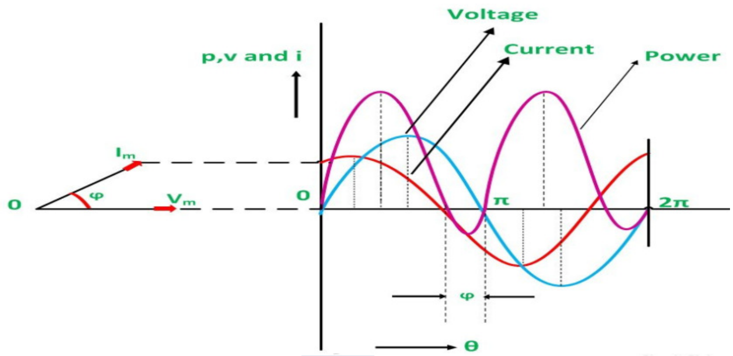
$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Now $v = V_m \sin wt$ and current leading the voltage by angle φ

$I = I_m \sin(wt + \varphi)$

Waveform and Power Curve of the RC Series Circuit



Power consumed by the circuit is given by

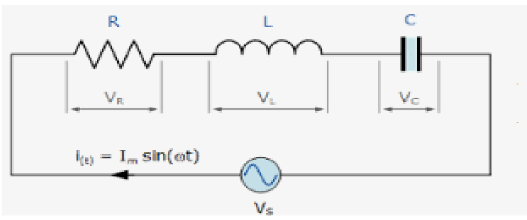
$P = VI\cos\varphi$ (power consumed by resistance)

Where, cosϕ is called the power factor of the circuit.

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Ac through series RLC circuit

Series RLC circuits consist of a resistance, a capacitance and an inductance connected in series across an alternating supply.



In a pure ohmic resistor the voltage waveforms are "in-phase" with the current. In a pure inductance the voltage waveform "leads" the current by 90^o. In a pure capacitance the voltage waveform "lags" the current by 90^o.

In the RLC Series Circuit

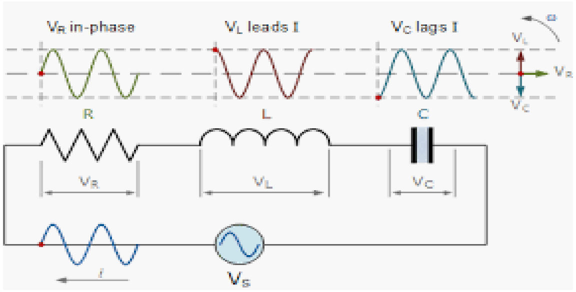
$$X_L = 2\pi fL \text{ and } X_C = 1/2\pi fC$$

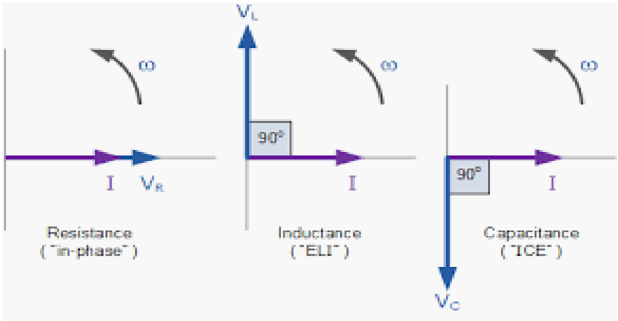
When the AC voltage is applied through the RLC Series Circuit the resulting current I flows through the circuit, and thus the voltage across each element will be

- $V_R = IR$ that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across the capacitor C and it lags the current I by an angle of 90 degrees.

This Phase Difference, Φ depends upon the reactive value of the components being used. If the inductive reactance is greater than the capacitive reactance, i.e $X_L > X_C$, then the RLC circuit has lagging phase angle and if the capacitive reactance is greater than the inductive reactance, i.e $X_C > X_L$ then the RLC circuit have leading phase angle and if both inductive and capacitive are the same, i.e $X_L = X_C$ then circuit will behave as purely resistive circuit. Series RLC circuits are classed as second-order circuits because they contain two energy storage elements, an inductance L and a capacitance C.

For the series RLC circuit

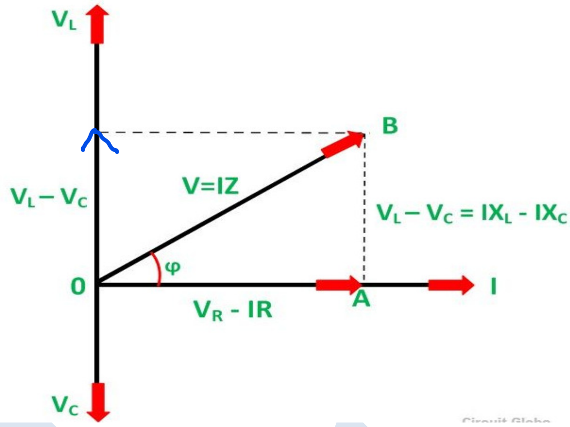




Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC Series Circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.

Steps to draw the Phasor Diagram of the RLC Series Circuit



- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90 degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vector V_L and V_C are opposite to each other.

From the phasor

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Impedance Triangle of RLC Series Circuit

In the voltage triangle the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle. The impedance triangle of the RL series circuit, when ($X_L > X_C$) is shown below fig 1,and Impedance triangle is shown below when the circuit acts as a RC series circuit ($X_L < X_C$) fig 2

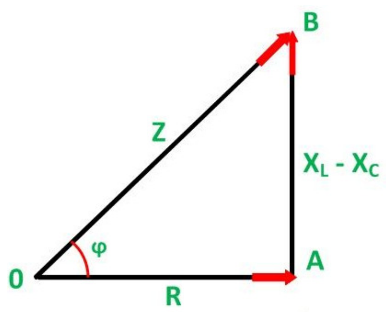


fig1

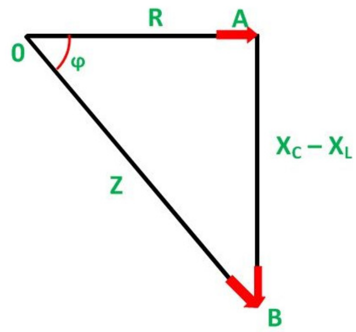
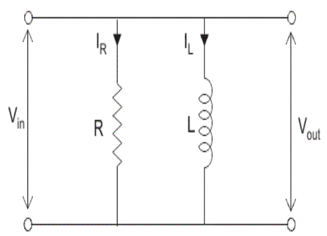


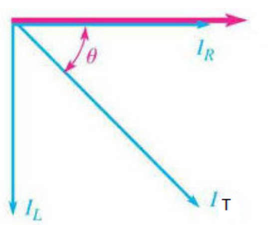
fig 2

If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a lagging phase angle.
When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as a capacitive and the phase angle will be leading.

RL Parallel Circuit

In **RL parallel circuit** resistor and inductor are connected in parallel with each other and this combination is supplied by a voltage source, V_{in}. The output voltage of circuit is V_{out}. Since the resistor and inductor are connected in parallel, the input voltage is equal to output voltage but the currents flowing in resistor and inductor are different.

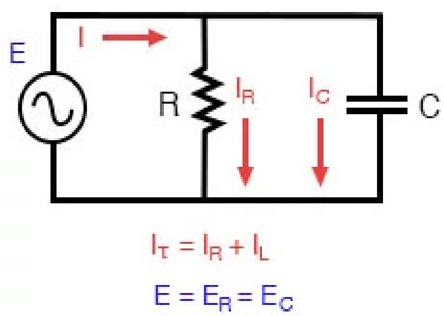


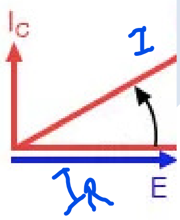


Let us say
I_T = the total current flowing from U87 voltage source in amperes.
I_R = the current flowing in the resistor branch in amperes.
I_L = the current flowing in the inductor branch in amperes.
θ = angle between I_R and I_T.
So the total current I_T,

$$I_T^2 = I_R^2 + I_L^2$$

RC Parallel Circuit

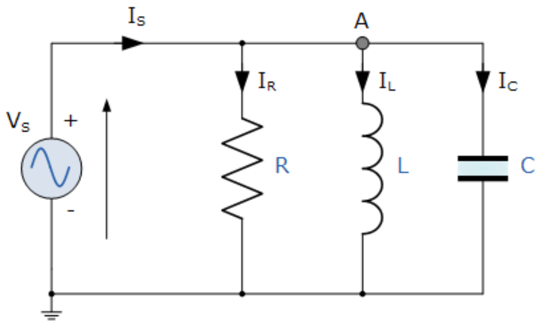


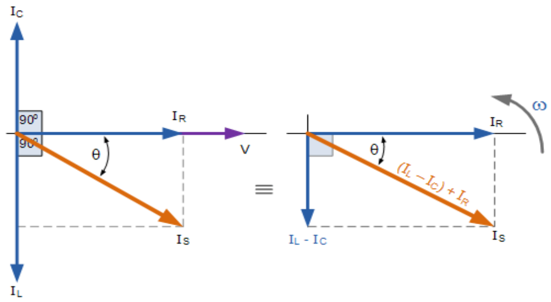


$$I^2 = I_C^2 + I_R^2$$

Parallel RLC Circuit

The Parallel RLC Circuit is the exact opposite to the series circuit. In a RLC circuit resistor, inductor and capacitor are connected in parallel to each other.





In the above parallel RLC circuit, we can see that the supply voltage, V_S is common to all three components whilst the supply current I_S consists of three parts. The current flowing through the resistor, I_R, the current flowing through the inductor, I_L and the current through the capacitor, I_C. The total current drawn from the supply will not be the mathematical sum of the three individual branch currents but their vector sum. Like the series RLC circuit, we can solve this circuit using the phasor or vector method but this time the vector diagram will have the voltage as its reference with the three current vectors plotted with respect to the voltage.

From the Current Triangle

$$I_S^2 = I_R^2 + (I_L - I_C)^2$$

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\therefore \; I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$\text{where:} \quad I_R = \frac{V}{R}, \quad I_L = \frac{V}{X_L} \quad I_C = \frac{V}{X_C}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

Conductance, Admittance and Susceptance

Admittance (Y) : Admittance is the reciprocal of impedance, Z and is given the symbol Y. In AC circuits admittance is defined as the ease at which a circuit composed of resistances and reactances allows current to flow when a voltage is applied taking into account the phase difference between the voltage and the current.

$$Y = \frac{1}{Z} \; [S]$$

Conductance (G) : Conductance is the reciprocal of resistance, R and is given the symbol G. Conductance is defined as the ease at which a resistor (or a set of resistors) allows current to flow when a voltage, either AC or DC is applied.

$$G = \frac{1}{R} \; [S]$$

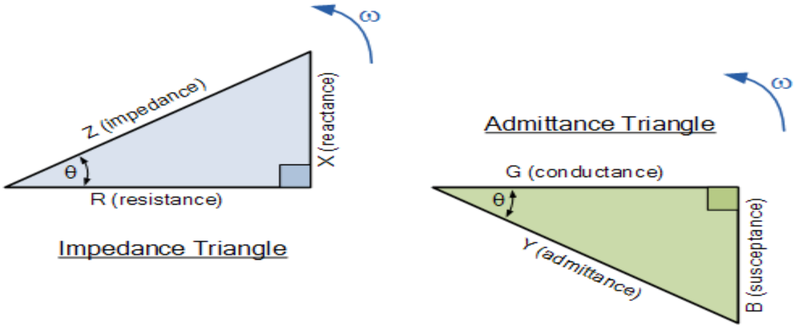
Susceptance (B) : Susceptance is the reciprocal of of a pure reactance, X and is given the symbol B. In AC circuits susceptance is defined as the ease at which a reactance (or a set of reactances) allows an alternating current to flow when a voltage of a given frequency is applied.

$$B_C = \frac{1}{X_C} \; [S] \qquad B_L = \frac{1}{X_L} \; [S]$$

The units used for **conductance**, **admittance** and **susceptance** are all the same namely Siemens (S), which can also be thought of as the reciprocal of Ohms or ohm^{-1}

In AC series circuits the opposition to current flow is impedance, Z which has two components, resistance R and reactance, X and from these two components we can construct an impedance triangle. Similarly, in a parallel RLC circuit, admittance, Y also has two components, conductance, G and susceptance, B. This makes it possible to construct an **admittance triangle**

Admittance Triangle for a Parallel RLC Circuit:



Real Power, Reactive Power, Apparent Power, Power Factor:

Real Power in AC Circuits : Real power (P), also known as true or active power, performs the "real work" within an electrical circuit. Real power, measured in watts, defines the power consumed by the resistive part of a circuit. Then real power, (P) in an AC circuit is the same as power, P in a DC circuit. So just like DC circuits, it is always calculated as I^2*R, where R is the total resistive component of the circuit. As resistances do not produce any phasor difference (phase shift) between voltage and current waveforms, all the useful power is delivered directly to the resistance and converted to heat, light and work. Then the power consumed by a resistance is real power which is fundamentally the circuit's average power**.**

To find the corresponding value of the real power the rms voltage and current values are multiplied by the cosine of the phase angle, Φ as shown.

$$\text{Real Power} \quad P = I^2R = V*I*\cos(\Phi) \quad \text{Watts, (W)}$$

But as their is no phase difference between the voltage and the current in a resistive circuit, the phase shift between the two waveforms will be zero (0). Then

$$P = V_{rms}\, I_{rms}$$

Where real power (P) is in watts, voltage (V) is in rms volts and current (I) is in rms amperes.

Reactive Power in AC Circuits

Reactive power (Q), (sometimes called watt-less power) is the power consumed in an AC circuit that does not perform any useful work but has a big effect on the phase shift between the voltage and current waveforms. Reactive power is linked to the reactance produced by inductors and capacitors and counteracts the effects of real power. Reactive power does not exist in DC circuits. The power stored by an inductor in its magnetic field tries to control the current, while the power stored by a capacitors electrostatic field tries to control the voltage. The result is that capacitors "generate" reactive power and inductors "consume" reactive power. This means that they both consume and return power to the source so none of the real power is consumed.

To find reactive power, the rms voltage and current values are multiplied by the sine of the phase angle, Φ as shown.

$$\text{Reactive Power} \quad Q = I^2X = V*I*\sin(\Phi) \quad \text{volt-amperes reactive, (VAr's)}$$

As there is a 90^o phase difference between the voltage and the current waveforms in a pure reactance (either inductive or capacitive), multiplying V*I by sin(Φ) gives a vertical component that is 90^o out-of-phase with each other, so:

$$Q = V_{rms} \times I_{rms} \times \sin\phi$$

$$\sin\left(90^o\right) = 1$$

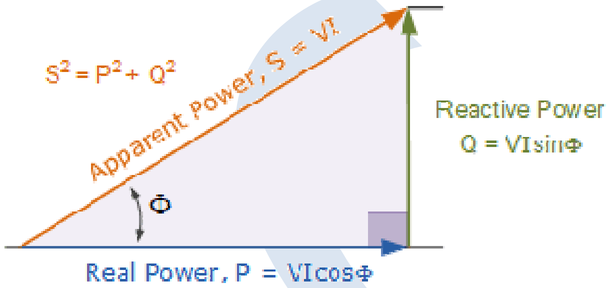
$$Q = V_{rms} \times I_{rms} \times 1$$

$$\therefore Q = V_{rms} \times I_{rms} \quad (\text{VAr})$$

Apparent Power(complex power) in AC Circuits

We have seen above that real power is dissipated by resistance and that reactive power is supplied to a reactance. Current and voltage waveforms are not in-phase due to the difference between a circuits resistive and reactive components. Total power in an AC circuit, both dissipated and absorbed/returned is referred to as apparent power.The combination of reactive power and true power is called apparent power. The product of the rms voltage, V applied to an AC circuit and the rms current, I flowing into that circuit is called the "volt-ampere product" (VA) given the symbol S and whose magnitude is known generally as apparent power. This complex Power is not equal to the algebraic sum of the real and reactive powers added together, but is instead the vector sum of P and Q given in volt-amps (VA). It is complex power that is represented by the power triangle. The rms value of the volt-ampere product is known more commonly as the apparent power as, "apparently" this is the total power consumed by a circuit even though the real power that does the work is a lot less.

Power Triangle of an AC Circuit



Where,

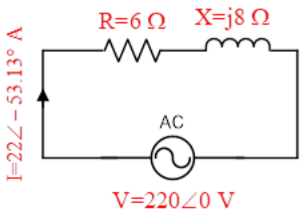
- **P** is the Real power that performs work measured in watts, W.
- **Q** is the Reactive power measured in volt-amperes reactive, VAr.
- **S** is the Apparent power measured in volt-amperes, VA.

Role of Active Power and Reactive Power

There is an important relationship between active and reactive power and the active power (P) is called true power and reactive power (Q) is called imaginary power.

If current passes through inductor the energy stored by the inductor $= \frac{1}{2} LI^2$

Now consider the following R-L circuit as shown on Figure-1. All the value of current and voltage are also shown in the Figure

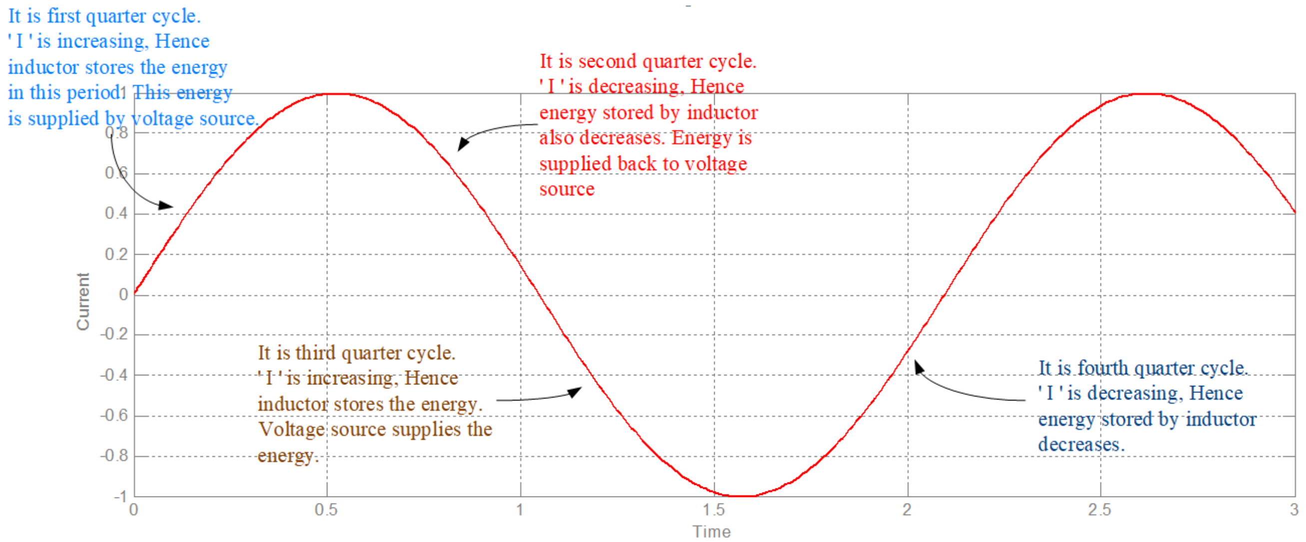


Active power consumed by the circuit is $I^2R = 22^2 x6 = 2904$ Watt. What is the meaning of this power? Please note that "Watt= Joule/second". Therefore it means that resistor in the circuit is consuming 2904 Joule energy per second and dissipating it in the air. It is not storing any energy. So you can say it is the true power, or actual power which is used. (If you will keep your finger near the resistor, you will find it hot, because it is dissipating energy in the air, i.e. electrical energy is converted into thermal energy).

Now what about inductor?

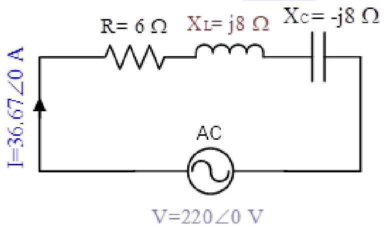
It is an AC Circuit, current is changing continuously. Therefore in the first quarter cycle inductor consumes energy, because current is increasing (energy stored by the inductor is $\frac{1}{2} LI^2$.

In next quarter cycle, energy is released by the inductor because there is a decrease in current. In next quarter cycle (third quarter) current is increasing (in reverse direction), so again energy is stored by the inductor. In next quarter cycle (fourth quarter) current is decreased, so, again energy is released by the inductor. This procedure is explained in Figure



So, inductor is consuming energy in a quarter cycle (taking it from voltage source) and giving back energy in next quarter cycle to the voltage source or you can say it is exchanging energy with the voltage source. In this case, it can be said that Inductor is consuming reactive power and voltage source is generating the reactive power. But note that it is an energy exchange between voltage source and inductor. There is no power consumed by inductor. This is the reason reactive power is called imaginary power.

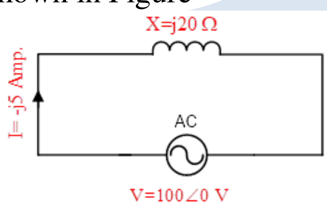
Now take another example. Consider following RLC circuit



In this circuit, value of inductive reactance and capacitive reactance is same. If you will calculate Power Factor (PF) of this circuit, then you will find it unity. Energy stored by the inductor is $\frac{1}{2}LI^2$, Energy stored by a capacitor is given by $\frac{1}{2}CV_c^2$.

In AC circuit both 'I' and 'V_c' are changing continuously. If you will draw the wave form of current 'I' and 'V_c' then you will find that when 'I' is increasing at the same time 'V_c' is decreasing and vice-versa. It implies that in a particular quarter cycle if inductor is storing the energy, at the same time capacitor is releasing the energy. In next quarter cycle reverse will happen, i.e. capacitor will store the energy and inductor will release the energy. So, there is an energy exchange continuously between inductor & capacitor. **Inductor consumes reactive power and capacitor generates reactive power.** But it is an energy exchange between two elements. No true power is consumed or generated; this is the reason reactive power (Q) is called imaginary power.

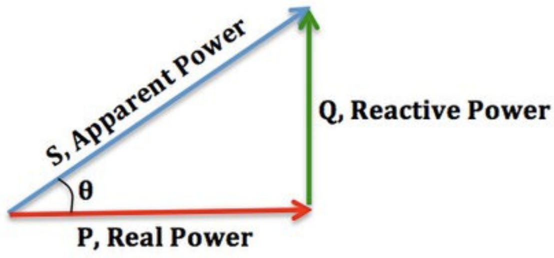
Consider the circuit diagram shown in Figure

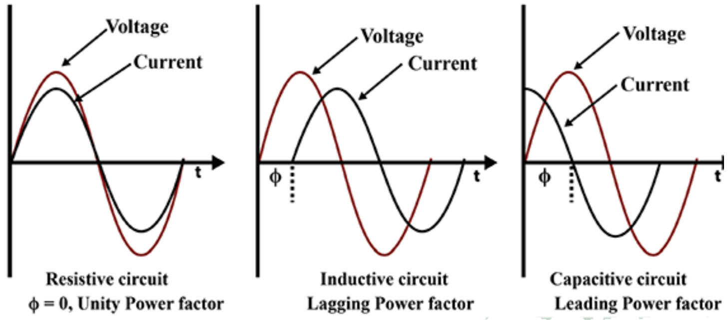


Value of current shown in the figure can be verified by the readers, it is –j5 A. If one ammeter is inserted in the circuit, what will be its reading? Its reading will be 5 Amp., while no true power is consumed by circuit. So, for the purpose of energy exchange, between voltage source and inductor, current flows in the circuit. You can understand that inductor is consuming reactive power (imaginary power), no true power is consumed despite that flow of current is necessary. If wattmeter is inserted in this circuit (current coil in series with voltage source and pressure coil across voltage source), its reading can be found to be zero.

Power factor:

In electrical engineering, the power factor of an AC electrical power system is defined as the ratio of the real power absorbed by the load to the apparent power flowing in the circuit, and is a dimensionless number in the closed interval of −1 to 1





Power factor = Active power/Apparent power
= VI.COS φ / VI
= COS φ

Unity power factor: Unity power factor is considered as a perfect scenario, during which apparent power and the real power shall be in phase. When the load is purely resistive, the current flow to the load will be linear and hence the phase shift between the voltage and current will be zero and cos Φ will be unity.

Leading power factor: Power factor is considered to be leading if the current leads voltage. Capacitive loads cause the current to lead the voltage so as the power factor.

Lagging power factor: Power factor is considered to be leading if current lags voltage. Inductive loads cause the current to lag the voltage so as the power factor.

Importance of power factor:

We know that $P = VI\cos\varphi \Rightarrow I = \frac{P}{V\cos\varphi}$

For constant power at constant voltage ,the current drawn by the circuit increases with decrease in power factor>so at low power factor ,the current drawn is more,causing the following disadvantages

i)greater conductor size ii) poor efficiency due to copper loss increases iii)larger voltage drop iv)higher amount of reactive power

To improve the Power factor of an ac circuit, a capacitor is connected across the circuit.

Phasor Algebra of AC Circuits

We have seen that ac circuits cannot be solved by use of simple algebra since geometrical relations are to be taken into consideration. Though phasor diagram method is quite satisfactory for fairly simple circuits but becomes complicated when a circuit or network is made up of several branches. AC circuits can be conveniently solved by use of complex algebra. The results obtained by this method are of the same order of accuracy as those obtained by trigonometrical methods although calculations are usually much simpler. The system enables equations representing alternating voltages and currents and their phase relationships to be expressed in simple algebraic form. It is based upon the idea that a phasor can be resolved into two components along two axis at right angles to each other. Complex Numbers were introduced to allow complex equations to be solved with numbers that are the square roots of negative numbers, √-1.In electrical engineering this type of number is called an "imaginary number" and to distinguish an imaginary number from a real number the letter " j " known commonly as the j-operator, is used. Thus the letter "j" is placed in front of a real number to signify its imaginary number operation. the application of j-operator to phasor causes its rotation through 90° in the positive (counter-clockwise) direction. Each successive multiplication or power of " j ", j^2, j^3 etc, will force the vector to rotate through a fixed angle of 90° in an anticlockwise direction It is assigned the numerical value √-1. Complex Numbers represent points in a two dimensional complex or s-plane that are referenced to two distinct axes. The horizontal axis is called the "real axis" while the vertical axis is called the "imaginary axis".

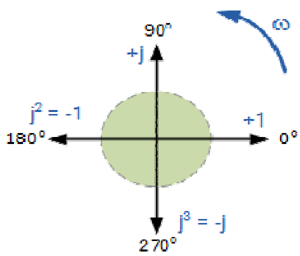
Vector Rotation of the j-operator

90° rotation: $j^1 = \sqrt{-1} = +j$

180° rotation: $j^2 = \left(\sqrt{-1}\right)^2 = -1$

270° rotation: $j^3 = \left(\sqrt{-1}\right)^3 = -j$

360° rotation: $j^4 = \left(\sqrt{-1}\right)^4 = +1$



Complex Numbers using the Cartesian or Rectangular Form.

complex number is represented by a real part and an imaginary part that takes the generalised form of

$$Z = x + jy$$

- Where:

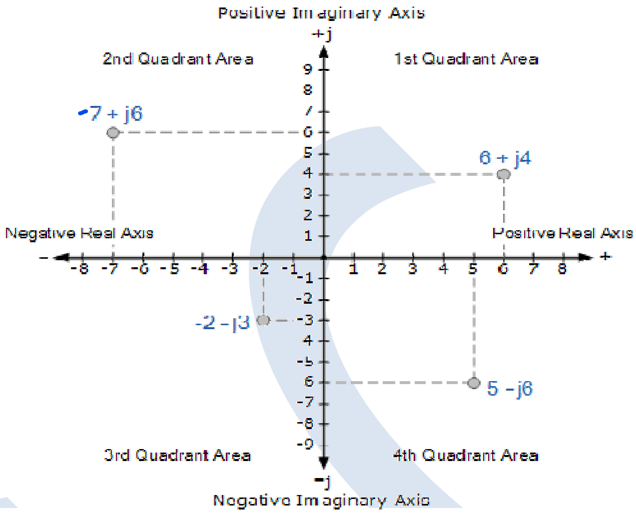
  Z - is the Complex Number representing the Vector

  x - is the Real part or the Active component

  y - is the Imaginary part or the Reactive component

  j - is defined by √-1

In the rectangular form, a complex number can be represented as a point on a two dimensional plane called the **complex** or **s-plane**. So for example, Z = 6 + j4 represents a single point whose coordinates represent 6 on the horizontal real axis and 4 on the vertical imaginary axis of a complex plane with four quadrants called an **Argand Diagram**



Complex Numbers can also have "zero" real or imaginary parts such as: Z = 6 + j0 or Z = 0 + j4. In this case the points are plotted directly onto the real or imaginary axis. Also, the angle of a complex number can be calculated using simple trigonometry . the relevant angles can be found from:

$$\tan^{-1}(\text{imaginary component} \div \text{real component})$$

Addition and Subtraction of Complex Numbers

$$A = x + jy \qquad B = w + jz$$

$$A + B = (x + w) + j(y + z)$$

$$A - B = (x - w) + j(y - z)$$

Multiplication and Division of Complex Numbers

$$A \times B = (4 + j1)(2 + j3)$$

$$= 8 + j12 + j2 + j^2 3$$

$$\text{but } j^2 = -1,$$

$$= 8 + j14 - 3$$

$$A \times B = 5 + j14$$

$$\frac{A}{B} = \frac{4 + j1}{2 + j3}$$

Multiply top & bottom by Conjugate of 2 + j3

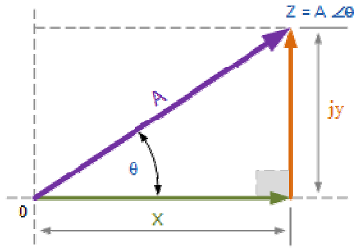
$$\frac{4 + j1}{2 + j3} \times \frac{2 - j3}{2 - j3} = \frac{8 - j12 + j2 - j^2 3}{4 - j6 + j6 - j^2 9}$$

$$= \frac{8 - j10 + 3}{4 + 9} = \frac{11 - j10}{13}$$

$$\frac{11}{13} + \frac{-j10}{13} = 0.85 - j0.77$$

Complex Numbers using Polar Form

Unlike rectangular form which plots points in the complex plane, the **Polar Form** of a complex number is written in terms of its magnitude and angle. Thus, a polar form vector is presented as: $Z = A \angle \pm\theta$, where Z is the complex number in polar form, A is the magnitude of the vector and θ is its angle of A which can be either positive or negative.



$$A^2 = x^2 + y^2$$

$$A = \sqrt{x^2 + y^2}$$

Also, $x = A.\cos\theta$, $y = A.\sin\theta$ $\qquad \theta = \tan^{-1}\frac{y}{x}$

Polar Form Multiplication and Division

$$Z_1 \times Z_2 = A_1 \times A_2 \angle \theta_1 + \theta_2$$

$$\frac{Z_1}{Z_2} = \left(\frac{A_1}{A_2}\right) \angle \theta_1 - \theta_2$$

Converting Polar Form into Rectangular Form, (P→R)

$6\angle 30^0 = x + jy$

However,

$x = A.\cos\theta \quad y = A.\sin\theta$

Therefore,

$$\begin{aligned} 6\angle 30^0 &= (6\cos\theta) + j(6\sin\theta) \\ &= (6\cos 30^o) + j(6\sin 30^o) \\ &= (6 \times 0.866) + j(6 \times 0.5) \\ &= 5.2 + j3 \end{aligned}$$

Converting Rectangular Form into Polar Form, (R→P)

$(5.2 + j3) = A\angle\theta$

where: $A = \sqrt{5.2^2 + 3^2} = 6$

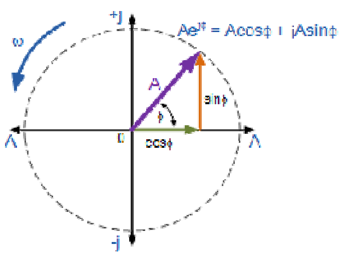
and $\theta = \tan^{-1}\frac{3}{5.2} = 30^o$

Hence, $(5.2 + j3) = 6\angle 30^0$

Complex Numbers using Exponential Form

$Z = Ae^{j\phi}$

$Z = A(\cos\phi + j\sin\phi)$



<u>Complex Number Forms</u>

$$Z = x + jy = A\angle\theta = A(\cos\phi + j\sin\phi)$$

Now let two sin signal V_a and V_b, and V_b leads V_a by 90°.And the maximum value for both is V_m.Then

$V_a = V_m$ sinwt and $V_b = V_m \sin(wt + \frac{\pi}{2}) = V_m$ coswt

In exponential form,

$V_b = V_a\, e^{\frac{j\pi}{2}} = V_a(\cos(\pi/2) + j\sin(\pi/2))$

And , $V_b = jV_a = j V_m$ sinwt

Now, $V_a = V_m$ sinwt

$\frac{d}{dt}V_a = \frac{\mathbf{d}}{\mathbf{dt}}\mathbf{V_m\ sinwt} = V_m$ w coswt $=$ **V_m w j sinwt** $= V_a w J$

Comparing both side,

In ac system $\frac{d}{dt} = wj$

For Capacitor, C $\frac{dv_c}{dt}$ = i_c => (C j w) (v_c) = (i_c) = mho x volt = amp

Ohm = $\frac{1}{mho}$ = $\frac{1}{jwC}$ = $\frac{-j}{-j^2wc}$ = $\frac{-j}{wc}$ = $-jX_c$

∴ Capacitive reactance = $-jX_c$

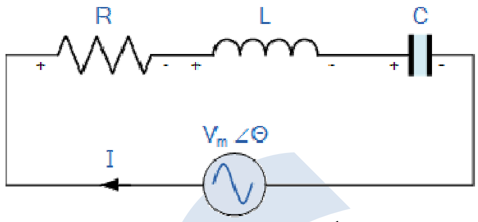
For Inductor, L $\frac{di_L}{dt}$ = V_L = (L j w) i_L = V_L = ohm x amp = volt

∴ Inductive reactance = jwl = jX_L

Resonance

Series Resonance Circuit

Resonance is defined as the condition in a circuit containing at least one inductor and one capacitor,when the supply voltage and the supply current are in phase.thus at resonance the equivalent impedance of the circuit is purely resistive and power factor is unity.



Z = $\sqrt{R^2 + (X_L - X_C)^2}$ where X_L = wL = 2πfL and X_C = $\frac{1}{2\pi fc}$

From defination of resonance ,for impedance (z) of the circuit become purely resistive ,the total reactance is zero.

$(X_L - X_C) = 0$ => $X_L = X_C$ => $2\pi f_0 L = \frac{1}{2\pi f_0 C}$ => $\mathbf{f_0 = \frac{1}{2\pi\sqrt{LC}}}$ =resonant frequency

At resonance, $I_0 X_L = I_0 X_C$ => $V_{L0} = V_{C0}$

V = $\sqrt{V_R^2 + (V_{L0} - V_{C0})^2}$ = $\sqrt{V_R^2}$ = V_R

& $\cos\varphi_0 = 1$, φ_0 = angle between V and I at resonance.

Since the impedance has a minimum value ,the current has a maximum value at resonance.the series resonant circuit is also called 'acceptor circuit'.

Properties of series resonant circuit

i)supply voltage and supply current are in phase so that $\varphi_0 = 0$, $\cos\varphi_0 = 1$

ii) the circuit impedance Z_0 is minimum and Z_0 = R

iii) The supply current I_0 is a maximum and $I_0 = \frac{V}{R}$

iv)X_{L0} = X_{C0} , & V_{L0} = V_{C0}

v)The power absorbed by the circuit is maximum

<u>Variation of different quantities with frequency</u>

X_L vs f curve cuts X_C vs f curve at apoint where f =f_0 ,the graph of net reactance X (= X_L –X_C) is obtained from X_L vs f and -X_C vs f curves.It is hyperbola.this curve crosses the frequency axis at a point f =f_0

R is independant of frequency.

The impedance is minimum at

resonance given by Z = R.

The value of Z increases on either

Side of resonance

At frequency below resonance XC > XL ,the impedance is capacitive and power factor is leading .at frequency above resonace XL > XC and the impedance is inductive and power factor is lagging. The curve Z vs F is drawn from R vs f and X vs f curve. The current versus frequency curve is known as resonance curve or response curve. The current has a maximum value at resonance and decreases on either side of resonace .

$$I = \frac{V}{Z} \propto \frac{1}{Z} \quad , I = \frac{V}{R} \text{ at resonance.}$$

Q factor in series resonance:

Q factor of a series rlc circuit is defined as the voltage magnification produced in the circuit at resonance.

$$\text{Voltage magnification} = \frac{\text{Voltage across L at resonance}}{\textit{supply voltage at resonance}} = \frac{V_{L0}}{V} = \frac{V_{C0}}{V}$$

Now , $V = V_{R0}$ at resonance

$$\therefore \frac{V_{L0}}{V} = \frac{V_{L0}}{V_{R0}} = \frac{X_{L0} I_0}{R I_0} = \frac{X_{L0}}{R} = \frac{w_0 L}{R} = Q$$

$$\therefore Q = \frac{w_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \frac{1}{2\pi\sqrt{LC}} L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q factor is also referred as the magnification factor of the circuit.

- Frequency for maximum voltage across L & C:

  Frequency at which V_C is maximum is $f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$

  Frequency at which V_L is maximum is $f_L = \frac{1}{2\pi} \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}$

Bandwidth and selectivity in series resonance circuit.

The bandwidth of series resonant circuit is given by the band of frequencies which lie between two point on either side of resonant frequency where the current falls to $\frac{1}{\sqrt{2}}$ (or 0.707) of its maximum value at resonance.

A & B are the points at which current falls to $\frac{1}{\sqrt{2}}$ of I_0 (maximum value)

Bandwidth = BW = $\Delta f = (f_2 - f_1)$ Hz

f_1 and f_2 are the frequencies at points A and B .Point A is called lower cut off frequency point and Poin B is called upper cutoff frequency point.

Power at point A and B

$$P_A = P_B = I^2R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_0^2 R = \frac{1}{2} \text{(power at resonance)}$$

For this reason the two points A & B on the response curve are known as half power points.

Determination of bandwidth and Selectivity:

Let w1 and w2 be the lower half power and upper half power frequency.then

$$w_1 = -\propto + \sqrt{\propto^2 + w_0^2} \quad \text{and} \quad w_2 = \propto + \sqrt{\propto^2 + w_0^2} \quad \text{where } \propto = \frac{R}{2L} \quad \text{and } w_0 = \sqrt{\frac{1}{LC}}$$

$$w_{bw} = w_2 - w_1 = 2\propto = \frac{R}{L}$$

and bandwidth , $\Delta f = (f_2 - f_1) = \frac{w_2 - w_1}{2\pi} = \frac{R}{2\pi L}$ Hz

Lower half frequency $f_1 = f_0 - \frac{\Delta f}{2} = f_0 - \frac{R}{4\pi L}$

Upper half frequency $f_2 = f_0 + \frac{\Delta f}{2} = f_0 + \frac{R}{4\pi L}$

From the above equations it reveals that the bandwidth of series RLC circuit depends solely upon the R/L ratio.The bandwidth is independant of the circuit capacitance.

The ratio of the bandwidth to the resonant frequency is defined as the selectivity of the ciecuit.

$$\therefore \text{selectivity} = \frac{f_2 - f_1}{f_0}$$

Again , $w_1 w_2 = \propto^2 + w_0^2 - \propto^2 = w_0^2$

$$\frac{w_1}{2\pi}\frac{w_2}{2\pi} = \frac{w_0^2}{(2\pi)^2}$$

$f_1 f_2 = f_0^2$ i.e the resonant frequency f0 is the root of the product of the half power frequencies of a series resonant circuit.

Resonance in Parallel circuit:

Under resonance as the pf is unity ,

the reactive power component of

the total current is zero

From the phasor diag,

$I_C = I_L \sin\varphi$

If resistance is negligible, $w_0 = \sqrt{\frac{1}{LC}}$ and $\mathbf{f_0 = \frac{1}{2\pi\sqrt{LC}}}$

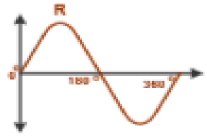
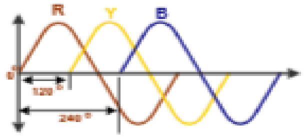
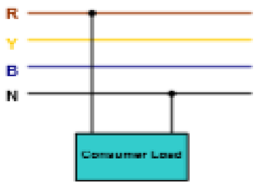
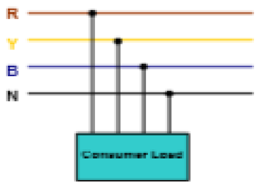
As the net reactive component of current is zero at resonance,so the supply current is equal to the active component of the current.

$$I = I_L \cos\varphi = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$

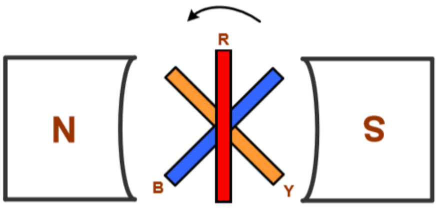
Thus at resonance the net impedance is given by $\frac{L}{CR}$ and is known as dynamic impedance.Current is minimum at resonance as its reactive part is zero.The circuit is called rejector circuit.

At resonance , $Z_0 = \frac{L}{CR}$. This impedance is a pure resistance because it is independent of frequency.lower the resistance of coil higer the value of Z_0. This resonance is often referred to as current resonance because the current circulating between two branches is many time greater than the line current taken from supply.The current taken from supply can be greatly magnified in this circuit.

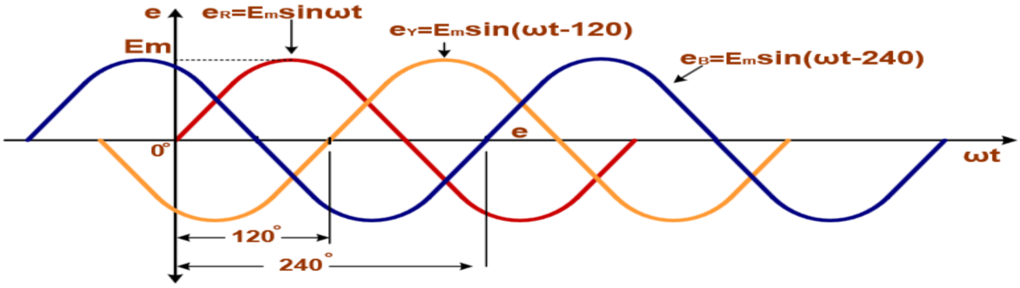
Comparison between single phase and three phase

Basis for Comparison	Single Phase	Three Phase
Definition	The power supply through one conductor.	The power supply through three conductors.
Wave Shape		
Number of wire	Require two wires for completing the circuit	Requires four wires for completing the circuit
Voltage	Carry 230V	Carry 415V
Phase Name	Split phase	No other name
Network	Simple	Complicated
Loss	Maximum	Minimum
Power Supply Connection		
Efficiency	Less	High
Economical	Less	More
Uses	For home appliances.	In large industries and for running heavy loads.

Generation of three phase EMF



- According to Faraday's law of electromagnetic induction, we know that whenever a coil is rotated in a magnetic field, there is a sinusoidal emf induced in that coil.
- Now, we consider 3 coil C1(R-phase), C2(Y-phase) and C3(B-phase), which are displaced 1200 from each other on the same axis. This is shown in above fig
- The coils are rotating in a uniform magnetic field produced by the N and S pols in the counter clockwise direction with constant angular velocity.
- According to Faraday's law, emf induced in three coils. The emf induced in these three coils will have phase difference of 1200. i.e. if the induced emf of the coil C1 has phase of 00, then induced emf in the coil C2 lags that of C1 by 1200 and C3 lags that of C2 1200



Waveform of Three Phase EMF

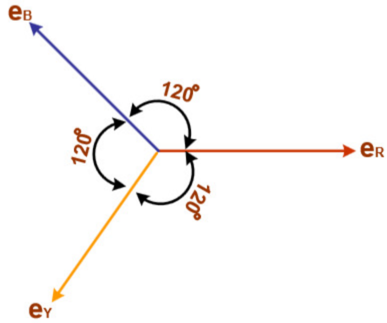
Thus, we can write,

$$e_R = E_m \sin\omega t$$

$$e_Y = E_m \sin\left(\omega t - 120^0\right)$$

$$e_B = E_m \sin\left(\omega t - 240^0\right)$$

The above equation can be represented by their phasor diagram as in the Fig below .,



Phasor Diagram of Three Phase EMF

Important definitions

Phase Voltage: It is defined as the voltage across either phase winding or load terminal. It is denoted by V_{ph}. Phase voltage V_{RN}, V_{YN} and V_{BN} are measured between R-N, Y-N, B-N for star connection and between R-Y, Y-B, B-R in delta connection.

Line voltage: It is defined as the voltage across any two-line terminal. It is denoted by VL. Line voltage VRY, VYB, VBR measure between R-Y, Y-B, B-R terminal for star and delta connection both.

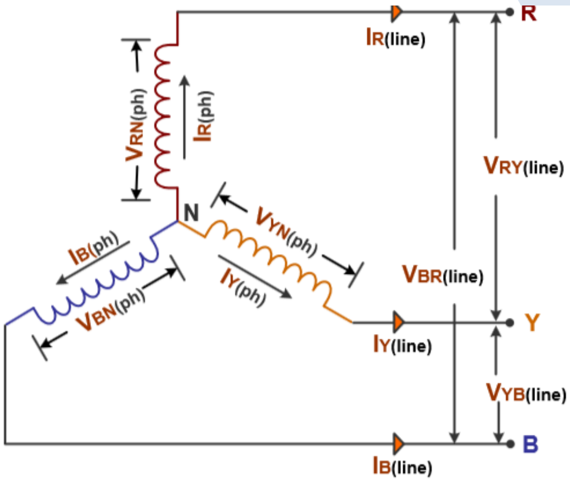


Figure Three Phase Star Connection System

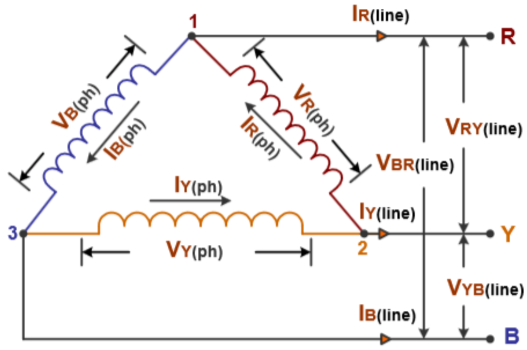


Figure Three Phase Delta Connection System

Phase current: It is defined as the current flowing through each phase winding or load. It is denoted by I_{ph}. Phase current $I_{R(ph)}$, $I_{Y(ph)}$ and $I_{B(Ph)}$ measured in each phase of star and delta connection. respectively.

Line current: It is defined as the current flowing through each line conductor. It denoted by I_L. Line current I_R(line), I_Y(line), and I_B((line) are measured in each line of star and delta connection.

Phase sequence: The order in which three coil emf or currents attain their peak values is called the phase sequence. It is customary to denoted the 3 phases by the three colours. i.e. red (R), yellow (Y), blue (B).

Balance System: A system is said to be balance if the voltages and currents in all phase are equal in magnitude and displaced from each other by equal angles.

Unbalance System: A system is said to be unbalance if the voltages and currents in all phase are unequal in magnitude and displaced from each other by unequal angles.

<u>Relation between line and phase values for voltage and current in case of balanced delta connection.</u>

Delta (Δ) or Mesh connection, starting end of one coil is connected to the finishing end of other phase coil and so on which giving a closed circuit.

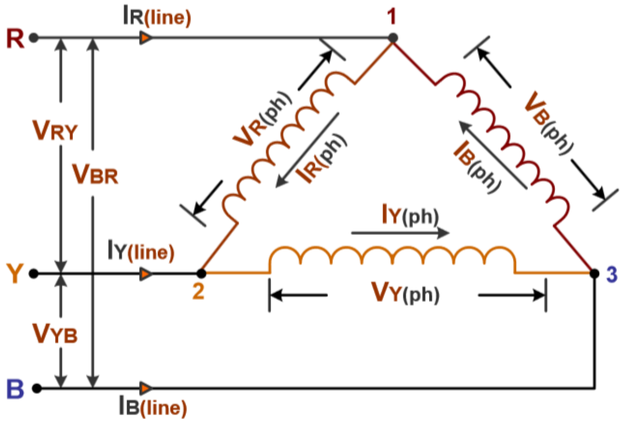


Figure Three Phase Delta Connection

$$\text{Line voltage, } V_{RY} = V_{YB} = V_{BR} = V_L$$

$$\text{Phase voltage, } V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$$

$$\text{Line current, } I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$$

$$\text{Phase current, } I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$$

Relation between line and phase voltage

For delta connection line voltage V_L and phase voltage V_{ph} both are same.

$$V_{RY} = V_{R(ph)}$$
$$V_{YB} = V_{Y(ph)}$$
$$V_{BR} = V_{B(ph)}$$
$$\therefore V_L = V_{ph}$$

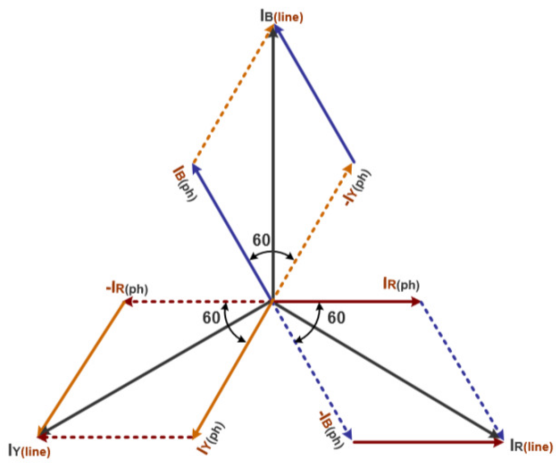
Line voltage = Phase Voltage

Relation between line and phase current

For delta connection,

$$I_{R(line)} = I_{R(ph)} - I_{B(ph)}$$
$$I_{Y(line)} = I_{Y(ph)} - I_{R(ph)}$$
$$I_{B(line)} = I_{B(ph)} - I_{Y(ph)}$$

i.e. current in each line is vector difference of two of the phase currents



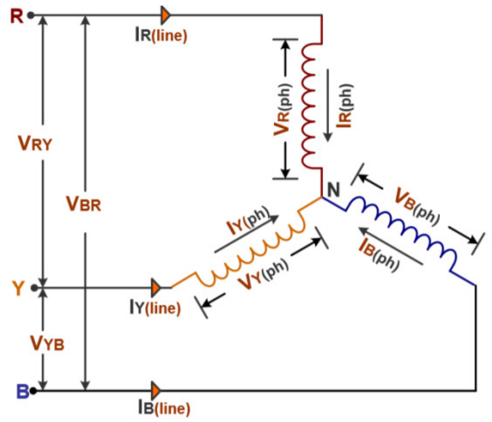
Thus, in delta connection Line current = √3 Phase current

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$
$$P = 3V_{ph}I_{ph}\cos\phi$$
$$P = 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$$
$$\therefore P = \sqrt{3}V_L I_L\cos\phi$$

Relation between line and phase values for voltage and current in case of balanced star connection.

In the Star Connection, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point.



Circuit Diagram of Three Phase Star Connection

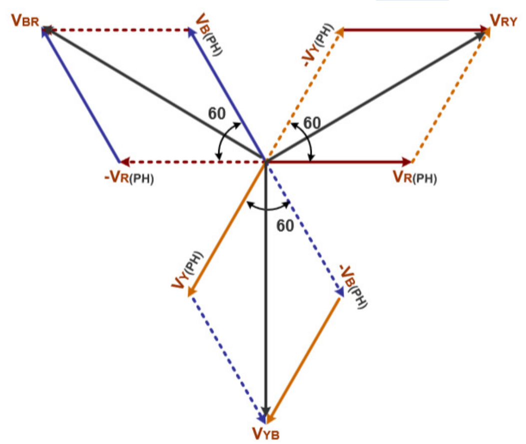
Relation between line and phase voltage

$$I_{R(line)} = I_{R(ph)}$$
$$I_{Y(line)} = I_{Y(ph)}$$
$$I_{B(line)} = I_{B(ph)}$$
$$\therefore \quad I_L = I_{ph}$$
$$\text{Line Current} = \text{Phase Current}$$

Relation between line and phase voltage

$$V_{RY} = V_{R(ph)} - V_{Y(ph)}$$
$$V_{YB} = V_{Y(ph)} - V_{B(ph)}$$
$$V_{BR} = V_{B(ph)} - V_{R(ph)}$$

i.e. line voltage is vector difference of two of the phase voltages. Hence,



Phasor Diagram of Three Phase Star Connection

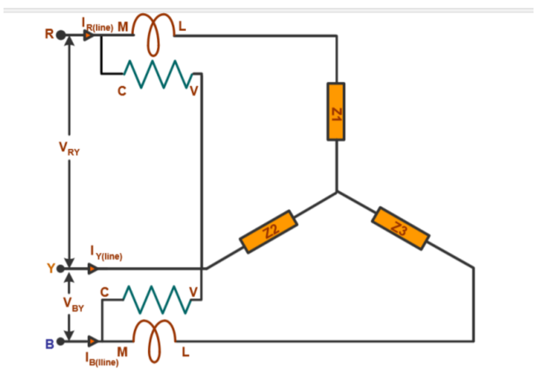
Thus, in star connection Line voltage = 3Phase voltage

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$
$$P = 3V_{ph}I_{ph}\cos\phi$$
$$P = 3\left(\frac{V_L}{\sqrt{3}}\right)I_L\cos\phi$$
$$\therefore P = \sqrt{3}V_L I_L\cos\phi$$

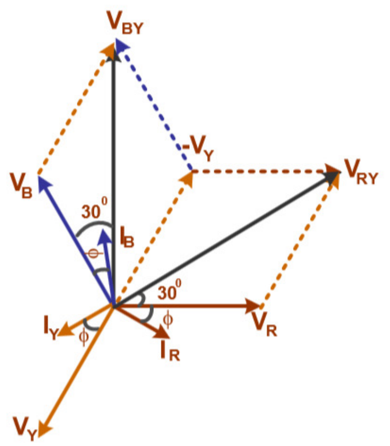
<u>Measurement of power in balanced 3-phase circuit by two-watt meter method</u>

This is the method for 3-phase power measurement in which sum of reading of two wattmeter gives total power of system.



Circuit Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

The load is considered as an inductive load and thus, the phasor diagram of the inductive load is drawn below



The three voltages V_{RN}, V_{YN} and V_{BN}, are displaced by an angle of 120^0 degree electrical as shown in the phasor diagram. The phase current lag behind their respective phase voltages by an angle φ.

The power measured by the Wattmeter, W_1 and W_2.

Reading of wattmeter, $W_1 = V_{RY} I_R \cos\phi_1 = V_L I_L \cos(30+\phi)$

Reading of wattmeter, $W_2 = V_{BY} I_B \cos\phi_2 = V_L I_L \cos(30-\phi)$

Total power, P = W_1+W_2

$$\therefore P = V_L I_L \cos(30+\phi) + V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos(30+\phi) + \cos(30-\phi)\right]$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\cos 30 \cos\phi\right]$$
$$= V_L I_L \left[2\left(\frac{\sqrt{3}}{2}\right)\cos\phi\right]$$
$$= \sqrt{3} V_L I_L \cos\phi$$

Thus, the sum of the readings of the two wattmeter is equal to the power absorbed in a 3phase balanced system.

<u>Determination of Power Factor from Wattmeter Readings</u>

- As we know that

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

Now,

$$W_1 - W_2 = V_L I_L \cos(30+\phi) - V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\left(\frac{1}{2}\right)\sin\phi\right] = V_L I_L \sin\phi$$

$$\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\therefore \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

- Power factor of load given as,

$$\therefore \cos\phi = \cos\left(\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right)$$

Effect of power factor on wattmeter reading:

- From the Fig. 5.6, it is clear that for lagging power factor $\cos\phi$, the wattmeter readings are

$$W_1 = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30-\phi)$$

- Thus, readings W_1 and W_2 will very depending upon the power factor angle ϕ.

p.f	ϕ	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	Remark
$\cos\phi = 1$	0^0	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	Both equal and +ve
$\cos\phi = 0.5$	60^0	0	$\frac{\sqrt{3}}{2} V_L I_L$	One zero and second total power
$\cos\phi = 0$	90^0	$-\frac{1}{2} V_L I_L$	$\frac{1}{2} V_L I_L$	Both equal but opposite

Problems

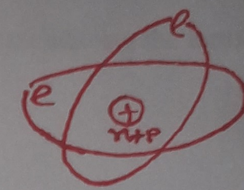
1)

MOD-1

Electric charge :

We know that an atom may be regarded as the smallest particle of an element. Atoms are composed of two parts ,namely central nucleus and the surrounding or orbital electrons.

Again the nucleus of an atom is largely a cluster of two types of particles calied the protons and neutrons.



Electron is of negative charge having magnitude 1.602 x 10^{-19} c.

Proton is of positive charge having magnitude 1.602 x 10^{-19} c .

The presence of equal number of proton and electron makes an atom neutrally charged.

Charge is an electrical property of atom measured in coulombs.

Coulomb is large unit for charges. In one coulomb of charge there are 6.24x10^{18} electrons.

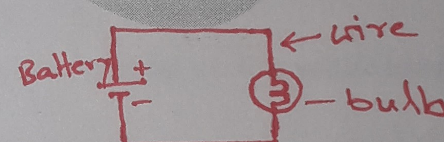
If an atom losses an electron it is called positive charged ion (+ve)

If an electron is added to an atom it is called negative charged ion (-ve)

A body having a number of ionized atoms is said to be electrically charged.

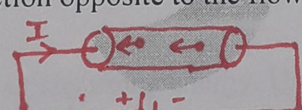
Electricity : The Invisible energy which constitutes the flow of electrons through a circuit to do work is called electricity. It is used for lighting, heating, cooling ,radio and TV broadcasting ,computers, transportation etc.

Electric circuit : For communicating or transferring energy from one point to another we require interconnection of electrical devices. An electric circuit is an interconnection of electrical elements.A simple electric circuit is shown in the figure. This consists of three basic elements a battery ,a lamp and connecting wires.



Electric current : electric current is the rate of change of charge with time, measured in amperes(A).

When a conducting wire is connected to a battery the charges are compelled to move i.e. positive charge in one direction and negative charge in opposite direction. This motion of charges creates electric current.so in conductor current flows in the direction opposite to the flow of electron.



Current due to flow of charges in a conductor

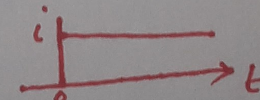
Mathematically,

$$i = \frac{dq}{dt} \Rightarrow q = \int_{t_0}^{t} i\,dt$$

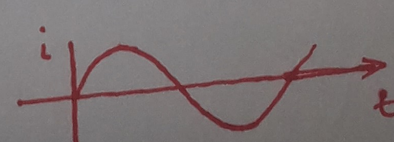
Unit of current is ampere.1A = 1 coulomb/sec

- As charges can vary with time in several ways, there are two types of current exists.

direct current(dc): a direct current is a current that remains constant with time.



alternating current(ac) :AC current is a current that



varies sinusoidally with time

Problems

1) the total charge entering a terminal is given by $q=5t\sin4\pi t$ mc.calculate the current at t= 0.5s

Sol: $i = dq/dt = d(5t\sin4\pi t)/dt = 5\sin4\pi t + 20\pi t\cos4\pi t$

At t=0.5s, $i = 5\sin2\pi + 10\pi\cos2\pi = 0+10\pi = 31.42$ mA

2)Determine the total charge entering a terminal between t=1s and t=2s, if the current passing the terminal is $i=(3t^2-t)$A

Sol: $q=\int_{t=1}^{2} i\,dt = \int_1^2 (3t^2-t)\,dt = \left(t^3-\frac{t^2}{2}\right)\Big|_1^2$

$= (8-2)-\left(1-\frac{1}{2}\right) = 5.5C$ Ans

Hw1) The total charge entering a terminal is $q=(10-10e^{-2t})$mc .find the current at t=0.5s.[ans 7.36mA]

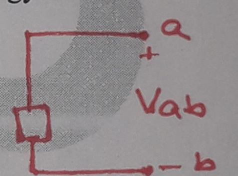
Hw2)The current flowing through an element is $i = 2A, \quad 0<t<1$

$= 2t^2, \quad t>1$

calculate the charge entering the element from t=0 to t=2s [ans 6.667c]

- Electric potential (or voltage): voltage (or potential difference) is the energy required to move a unit charge through an element.

The voltage V_{ab} between two points a and b in an electric circuit is the energy needed to move a unit charge from a to b.



Mathematically, $V_{ab} = \frac{dW}{dq}$

Where W= energy in joules (J) q= charge in coulomb(c)

Unit of voltage is volt named after physicist Alessandro Volta, who invented the first voltaic battery.

$$1V = \frac{1J}{1C} = \frac{1\,newton\ meter}{coulomb}$$

Like electric current a constant voltage is called a dc voltage and a sinusoidally time varying voltage is called ac voltage.

voltage from a battery is DC voltage and AC voltage is produced by an ac electric generator.

- Power and energy :

power is the time rate of expanding or absorbing energy. Unit of power is watt.

Mathematically $$p=\frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} = v.i$$

P = v i =instantaneous power

If Power has a positive sign then power is being delivered to or absorbed by the element, whereas if Power has negative sign then power is being supplied by element.

According to law of conservation of energy the algebraic sum of power in a circuit at any instant of time, must be zero. This confirms that total power supplied to the circuit must equal to the total power absorbed.

- Energy :it is the capacity to do work. unit is Joule

Mathematically $W = \int_{t0}^{t} p\,dt = \int_{t0}^{t} vi\,dt$

The electric power utility companies measures energy in what-hour(Wh)

1Wh = 3600 J

The relationship between power and energy

$$\text{power} = \frac{energy}{time} \Rightarrow \text{Energy} = \text{power x time} = \text{p x t}$$

- Ohm's law:

It states that the current through any conductor is directly proportional to the potential difference between its ends, when all physical conditions (temperature, length, cross sectional area) are constant.

$$I \propto v \Rightarrow I = \frac{1}{R}V$$

R=Resistance of the conductor

Hw). Limitation of Ohm's Law

Resistance of metallic conductor :

$$R = \rho \frac{l}{a}$$

ρ = Resistivity of the conductor

l=Length a=cross sectional area unit of resistance

unit of resistance : ohm

Problems

1) A heater element is made of nichrome wire having resistivity equal to 100×10^{-8} ohm-meter.The diameter of the wire is 0.4mm.calculate the length of the wire required to get a resistance of 40 ohm.

sol^n : R= 40 ohm . ρ= 100×10^{-8} d= 0.4mm = 4×10^{-4} m

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (4 \times 10^{-4})^2 = 4\pi \times 10^{-8} \text{ m}^2$$

$$l = \frac{R\,a}{\rho} = \frac{40 \times 4\pi}{100} = 5.03 \text{ m}$$

Hw1)A coil consists of 2000 turns of copper wire having cross sectional area of 0.8 mm^2 .the mean length per turn is 80cm,and the resistivity of copper is 0.2μΩ-m. find resistance of the coil. [ans ~~40~~Ω] 400Ω

- Circuit element

linear and nonlinear element : a linear element shows linear characteristics of voltage versus current. thus the parameters do not change with voltage or current. example :resistor, inductor, capacitor etc

For non linear element, the current passing through it does not change linearly with the linear change in applied voltage across it at a particular temperature and frequency. Example: diodes, transistors, thyristor etc.

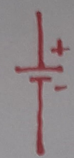
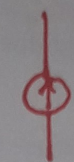
Non linear element does not obey the ohm law.
Unilateral and bilateral element : if the magnitude of the current passing through an element is effected due to change in polarity of the applied voltage, the element is called a unilateral element.
example :diode ,transistor etc
on the other hand if the current magnitude remains the same even if the polarity of applied voltage is reversed, it is called bilateral element.
Example: resistor, inductor etc
Active and passive element : an active element is capable of generating energy while a passive element is not .
example of active element: generator, batteries, amplifiers ,voltage and current source,
example of passive element: resistor, inductor etc

- Sources: these generally deliver power to the circuit .

Types: i) Independent and dependent sources

ii) Ideal and practical sources

Independent and dependent sources: an independent source is an active element that provides a specific voltage or current that is completely independent of other circuit element

independent AC voltage independent DC voltage independent current source

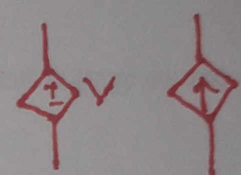
independent sources are represented by circle symbols.

A dependent or controlled source is an active element in which the source quantity is controlled by another voltage or current.

types

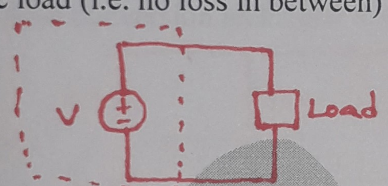
i)voltage controlled voltage source ii)current controlled voltage source

iii)voltage controlled current source iv)current controlled current source

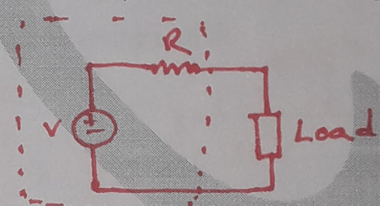


Dependent sources are usually represented by Diamond shaped symbol.

Ideal and practical voltage source: ideal voltage source is that, where all the voltage value will appear across the load (i.e. no loss in between)



ideal voltage source

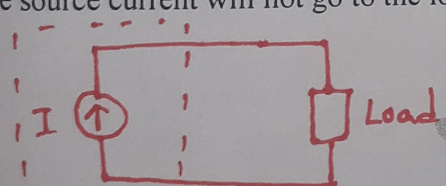


practical voltage source

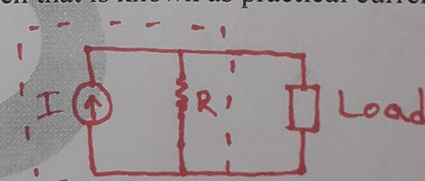
In practical voltage source the total voltage will not appear across the load. Some voltage will be drop in the series resistor. **Internal resistance of ideal voltage source is '0' .**

Ideal and practical current source: ideal current source is that which transfer all the current to the load. Internal resistance of Ideal current source is infinite. Example :solar cell.

If all the source current will not go to the load then that is known as practical current source



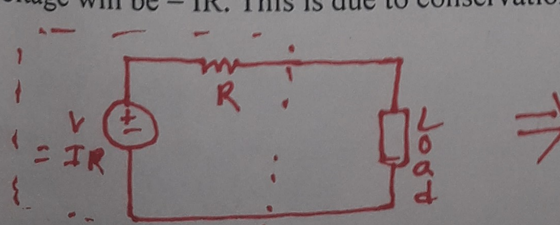
ideal current source



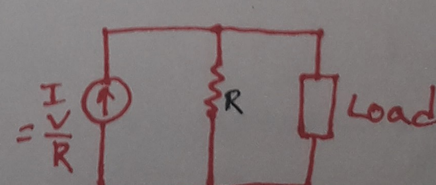
practical current source

- Source conversion: source conversion says that, if there is a voltage source with its internal resistance in series, it can be converted into a current source with its internal resistance in parallel and vice versa for current to voltage source conversion.

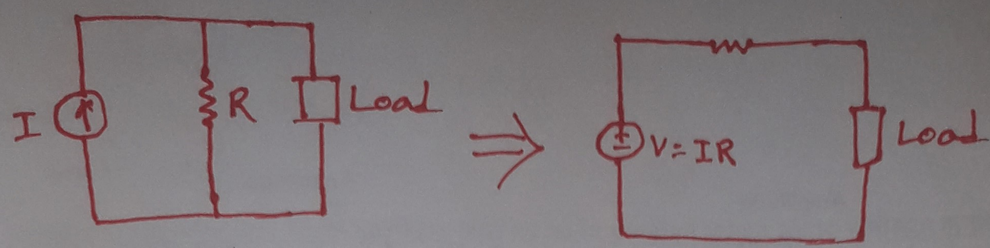
In voltage to current source conversion the current will be $= \frac{V}{R}$ and for current to voltage source conversion the voltage will be = IR. This is due to conservation of power in the circuit.



Voltage source



current source

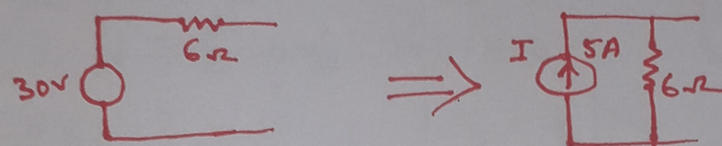


current source voltage source

Problems:

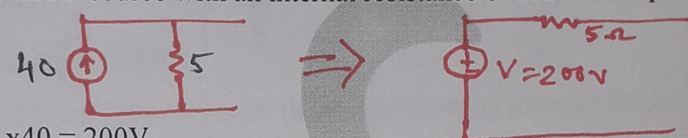
1) Convert a voltage source of 30 volt along with an internal resistance of 6 ohm to a current source

Sol:



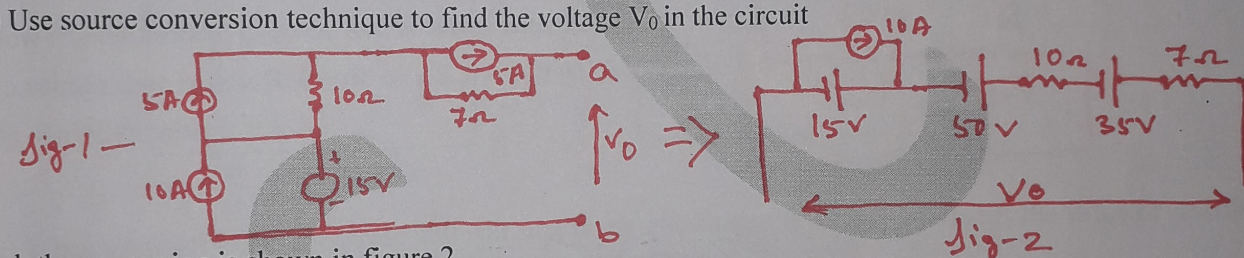
The value of $I = \frac{V}{R} = \frac{30}{6} = 5A$

2) Convert the 40 amps current source with an internal resistance of 5Ω to an equivalent voltage source



V=IR =5 x40 = 200V

3) Use source conversion technique to find the voltage V_0 in the circuit

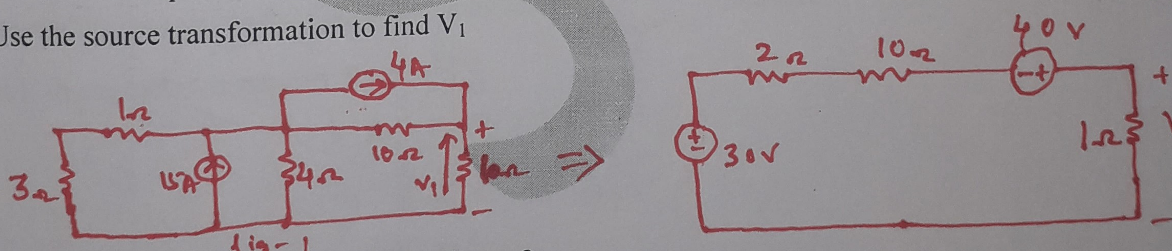


Sol: the conversion is shown in figure 2

When current and voltage source are connected in parallel, current source does not mean anything .so

output voltage $V_0 = 15 + 50 + 35 = 100V$

4) Use the source transformation to find V_1



Sol: The transformed circuit is given by figure 2.

Resistance 3 Ω and 1Ω are in series, with 4Ω resistance in parallel. So equivalent resistance across 15amps current source is $(3 + 1)||4 = 2\Omega$

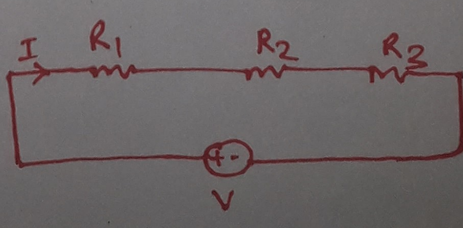
current flowing through circuit of figure 2

$$I = \frac{30+40}{2+10+1} = \frac{70}{13}$$

Voltage drop in 1Ω resistance $V_1 = \frac{70}{13} \times 1 = 5.38V$

- **Resistance in series** :Resistance are said to be in series when they are connected in such a way that some current flows through them.

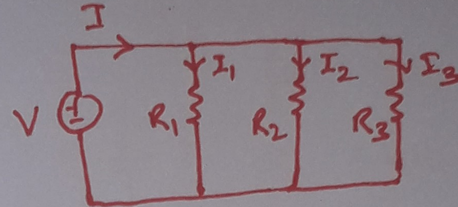
for series connection



equivalent $R_{eq} = R_1 + R_2 + R_3$

and $I = \frac{V}{Req}$

➢ <u>Resistance in parallel</u> : resistance are said to be in parallel when same voltage appears across the resistances.



In parallel connection equivalent resistance

will be $\frac{1}{Req} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}$

$I = \frac{V}{Req}$, $I_1 = \frac{V}{R1}$, $I_2 = \frac{V}{R2}$, $I_3 = \frac{V}{R3}$, $I = I_1 + I_2 + I_3$

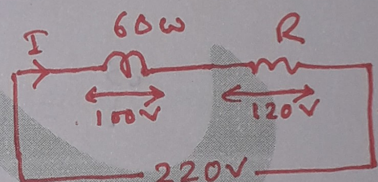
Problems

1) A 100 volt 60 watt bulb is to be operated from a 220 volt supply .what is the resistance to be connected in series with the bulb to glow normally?

Sol: p= 60W , V= 100V

Now p=VI , => $I = \frac{60}{100} = 0.6A$

Lamp will operate normally on 220 volt also if the current flowing through the lamp remains the rated current that is 0.6 amps.



Now let resistance R is connected in series with the lamp.

So voltage drop across R should be e 220 – 100 =120 V

As current flowing is 0.6 A

$R = \frac{120}{0.6} = 200\ \Omega$

2) When a resistor is placed across 230 volt supply (dc) the current is 12 amps. What is the value of resistor that must be connected in parallel to increase the load current to 16 amps?

Sol: resistance of the given resistor $= R_1 = \frac{230}{12} = 19.167$

Now equivalent resistance of parallel combination $Req = \frac{230}{16} = 14.373\Omega$

Let R_2 is connected in parallel $Req = \frac{R1R2}{R1+R2}$

$$R_2 R_1 = R_{eq}(R_1 + R_2)$$

$$R_2 = \frac{Req\ R1}{R1-Req} = \frac{14.375 \times 19.167}{19.167-14.375}$$

$$= 57.5\ \Omega$$

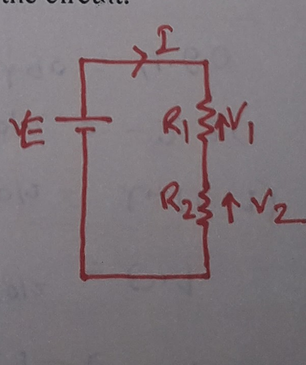
➢ <u>Voltage divider theorem:</u> in a series circuit the portion of applied EMF developed across each resistor is the ratio of that resistors value to the total series resistance in the circuit.

$V_1 = IR_1$

now $I = \frac{E}{R1+R2}$ $\therefore V_1 = \frac{E}{R1+R2} \times R_1$

similarly $V_2 = \frac{E}{R1+R2} \times R_2$

and for n number of resistors $V_i = \frac{Ri}{R1+R2.....+Rn} \times E$

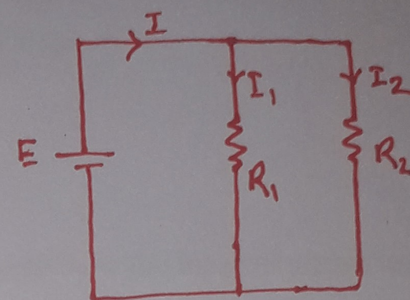


➢ Current limiting resistors: sometimes a resistor is included in series with an electrical circuit or electronic device to drop the supply voltage Down to desired value. This resistor can be treated as current limiting resistor.

Current divider rule: it is used to find out the current in the resistance in parallel circuit

$$\frac{1}{Req} = \frac{1}{R1} + \frac{1}{R2} \quad => \quad Req = \frac{R1R2}{R1+R2}$$

$$I = \frac{E}{Req} = E\,\frac{R1+R2}{R1R2} \quad => \quad E = I\,\frac{R1R2}{R1+R2}$$



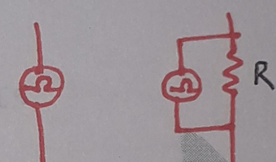
$$\text{Now, } I_1 = \frac{E}{R1}\ ,\ I_2 = \frac{E}{R2}$$

$$I_1 = I\,\frac{R1R2}{R1+R2}\,\frac{1}{R1} = I\,\frac{R2}{R1+R2}$$

$$\text{Similarly } I_2 = I\,\frac{R1}{R1+R2}$$

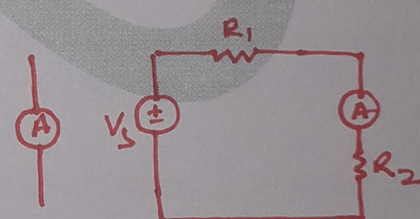
➢ Measuring devices

Ohm meter: it is a device which can measure the resistance of an element. while measuring the resistance of an element it should be disconnected from any other circuit.



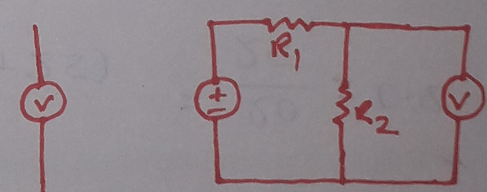
symbol and connection

Ammeter: it is a device which measures the current flowing through an element. It is always connected in series with the circuit element. *The internal resistance of ammeter is 0 ohm.*



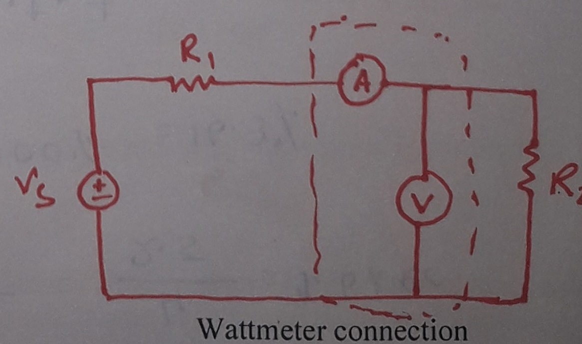
symbol and connection diagram

Voltmeter: it is a device which measures the voltage across an element. It is always connected in parallel to the element. *The internal resistance of Voltmeter is infinite ohm.*



symbol and connection diagram

Wattmeter :it measures the power dissipated by a circuit element. it measures the current flowing through the load and simultaneously the voltage across it and multiplies the two to provide a reading of power dissipated by the load.



Wattmeter connection

Voltage measuring coil is known as potential coil and current measuring coal is known as current coil

Resistance: the resistance R of an element denotes its ability to resist the flow of electric current. Unit of resistance is ohm. It is represented by the symbol R and the symbol is 

Resistance R offered by a conductor depends on

i) Directly with the length of the conductor (l)

ii) Varies inversely with the cross sectional area(a) of the conductor

iii) Nature of the material (resistivity ρ)of the conductor

iv) Depends on temperature

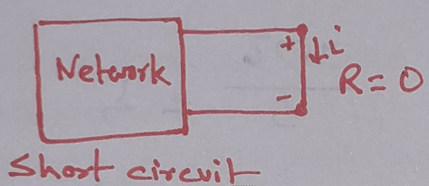
Mathematically, $R = \rho \frac{l}{a}$

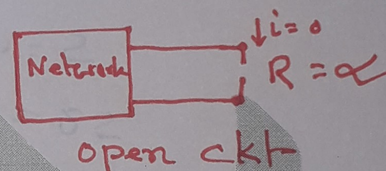
Good conductors such as copper and Aluminium have low resistivity while insulators, such as mica and paper have high resistivity.

Power dissipated by a resistor: $p = vi = i^2R = v^2/R$

➢ Open circuit and short circuit :

A short circuit is a circuit element with zero resistance, and open circuit is a circuit element with infinite resistance.





conductance (G) : it is the ability of an element to conduct electric current. it can be written as $G = \frac{1}{R}$. unit is mho or siemens .

➢ Inductors:

The electrical element that stores energy in association with the flow of current is called inductor(energy stored in magnetic field). Unit is Henry. symbol 

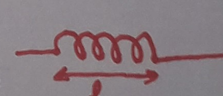
In ideal condition the voltage across an inductor is proportional to the rate of change of current in it.

$$V \propto \frac{di}{dt}$$

$$V = L\frac{di}{dt} \quad \text{....L= inductance}$$

So inductance is the property of inductor which opposes the change in current flowing through it.

Inductance of a coil is given by $L = \frac{N^2 \mu A}{l}$



where ,N= number of turns, A= cross sectional area , μ=permeability of the core

the energy stored in an inductor is $= \frac{1}{2} Li^2$

Important properties of an inductor: as voltage across an inductor is zero when the current is constant an inductor acts like a short circuit to DC.

Capacitors

A capacitor is a passive element which stores energy in the electric field produced by it. a capacitor consists of two conducting plates separated by an insulator (or dielectric). In practice the plates maybe Aluminium foil while the dielectric may be air, ceramic, paper or mica. The capacitor is set to store the electrical charge. the amount of charge stored is directly proportional to the applied voltage.

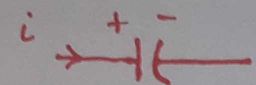
$q = Cv$ ..... where C=proportionality constant known as capacitance ,unit is Farad(F).

for parallel plate capacitor the capacitance is given by $C = \frac{\varepsilon A}{d}$

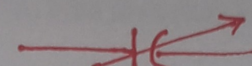
where, A= surface area of each plate, d= distance between plates , ε= permittivity of the dielectric

Different type of capacitors: ceramic capacitor, polyester capacitor, electrolytic capacitor

symbol:



fixed capacitor



variable capacitor

The current voltage relationship : $i = c\frac{dv}{dt}$ => $v = \frac{1}{C}\int i\,dt$

Energy stored in capacitor is $= \frac{1}{2}Cv^2$

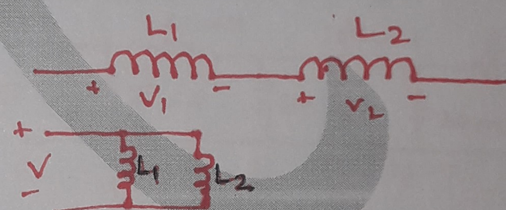
the current through the capacitor is zero when constant voltage is applied across it. So a capacitor acts like open circuit in DC.

Series and parallel connection of inductors and capacitors

Inductors

If two inductors are connected in series then the equivalent inductance is $L_{eq} = L_1 + L_2$

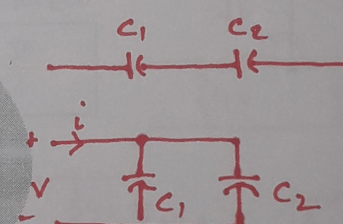
If two inductors are connected in parallel then the equivalent inductance is $L_{eq} = \frac{L1L2}{L1+L2}$



Capacitor

If two capacitors having capacitanceC1 and C2 are connected in series, then equivalent capacitance is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

If two capacitors are connected in parallel then equivalent capacitance is $C_{eq} = C_1 + C_2$



Problems

1) what is the voltage across a 3μF capacitor if the charge on the plate is 0.12mC?how much energy is stored ?

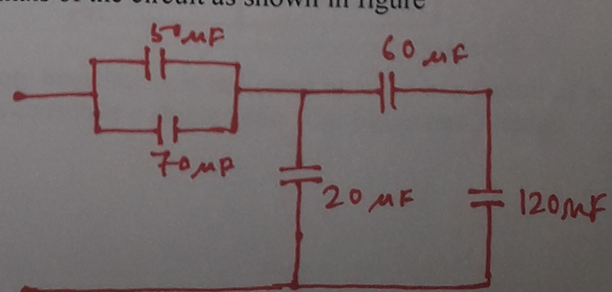
by definition of capacitor we know that $C = \frac{q}{v}$ => $v = \frac{q}{C} = \frac{0.12\ X\ 10^{\wedge}-3}{3X10^{\wedge}-6} = 40v$

Energy stored $= \frac{1}{2}Cv^2 = \frac{1}{2}x3x10^{-6}\ x\ 40^2 = 2.4mj$

2) The voltage across a 5μF capacitor is v(t)= 10cos6000t volt. Calculate the current through it .

Sol: we know that $i = c\frac{dv}{dt}$ = 10 x 5 x 10-6 x 6000 (-sin6000t) = -0.3sin6000t A

3) Find the equivalent capacitance seen at the terminals of the circuit as shown in figure



Soln: 50μF & 70μF are in parallel.

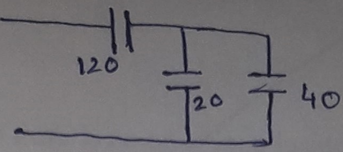
$C_{eq1} = 50 + 70 = 120\mu F$

60μF & 120μF are in series.

$C_{eq2} = \frac{60 \times 120}{60 + 120} = 40\mu F$

Now, 20 || 40

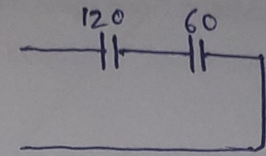
$C_{eq}3 = 20 + 40 = 60\mu F$



Now 120 || 60

$$C_{eq} = \frac{120 \times 60}{120 + 60} = 40\mu F$$

Ans.



4)The current through a 0.1H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Sol: We know that, $V = L\frac{di}{dt} = 0.1 \times \frac{d}{dt}(10te^{-5t})$

$$= 0.1 \times 10\,[e^{-5t} + (-)5te^{-5t}]$$

$$V = e^{-5t}(1 - 5t)$$

$$\text{Energy stored} = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.1 \times (10te^{-5t})^2$$

$$= 5t^2e^{-10t} \text{ Jule.}$$

5)Calculate the equivalent inductance for the inductive ladder network as shown in figure

Sol: 40 and 20 in series , so L_{eq1} =40+20=60

30 and 60 in parallel in next figure 2

$$L_{eq}2 = \frac{30 \times 60}{30 + 60} = 20$$

100mH & 20mH in series in fig 3

$L_{eq}3 = 100 + 20 = 120mH$

40 || 120 in fig 4

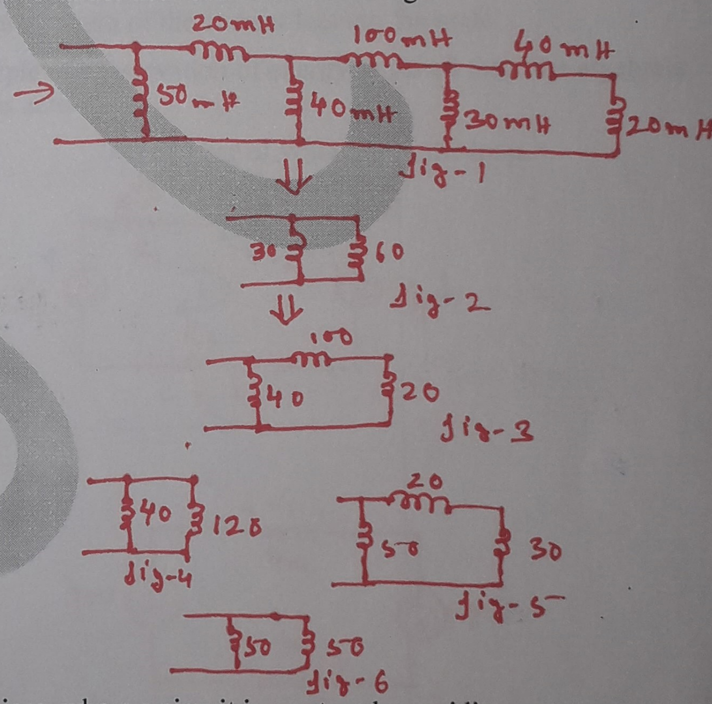
$$L_{eq}4 = \frac{40 \times 120}{40 + 120} = 30mH$$

20 & 30 are in series in fig 5

$L_{eq}5 = 20 + 30 = 50mH$

50 & 50 are parallel in fig - 6

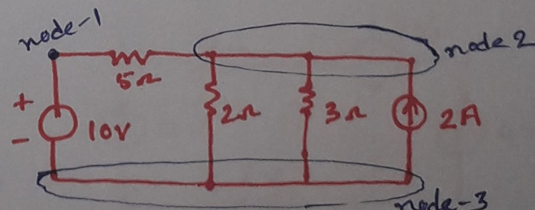
$$L_{eq} = \frac{50 \times 50}{50 + 50} = 25mH$$



➢ <u>Nodes,branches loop and mesh</u> :

A network is an interconnection of elements or devices, where a circuit is a network providing one or more closed path.

Branch: a branch represents a single element such as a source or passive element. For the circuit shown there are five branches namely 10V voltage source,2A current source & 3 resistor



Node: it is a point of connection between two or more branches. It is indicated by a dot in a circuit .if a connecting wire connects two nodes, that two nodes constitute a single node. The figure has 3 nodes.

Loop: it is any closed path in a circuit. If there is no other loop inside the loop, then that loop is called independent loop or mesh. For the figure 6 loops and 3 number of meshes.

For a network with the 'b' branches, 'n' nodes& 'l' independent loops,

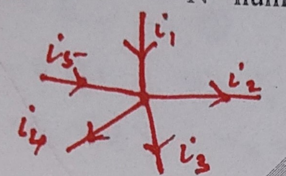
$$b = l + n - 1$$

- Kirchoff's Laws

Kirchoff's first law is based on the law of conservation of charge .Charge cannot be created but must be conserved.

Kirchoff's current law(KCL): at any instant of time, the algebraic sum of current at node is zero.

i.e $\sum_{n=1}^{N} I_N = 0$ N= number of branch connected to the nodes



by taking entering currents as positive and living currents as negative for node 1

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0$$

$$\Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$$

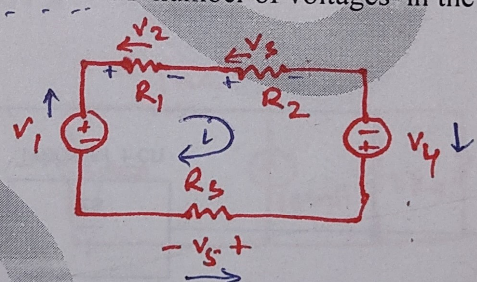
i.e. the sum of the current entering a node is equal to the sum of the current leaving the node.

Kirchoff's Voltage law(KVL): it is based on principle of conservation of energy. it stated that "the algebraic sum of all voltages around a closed path (or loop) is zero

i.e $\sum_{m=1}^{M} V_m = 0$ ...... M=number of voltages in the loop.

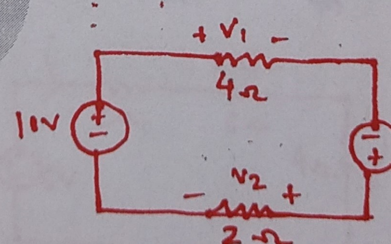
Applying KVL,

$$V_1 - V_2 - V_3 + V_4 - V_5 = 0$$



problems

1) find v_1 & v_2 in the circuit shown in the figure



Sol: applying KVL

10 -4i +8 -2i =0 => I=3 A

v_1 = 3x 4 =12v ; v_2 = 3 x 2 =6v

2)Determine v_0 and i in the circuit shown in figure

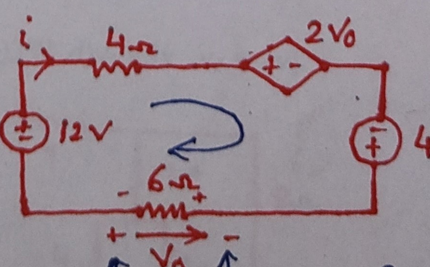
Sol: applying KVL

12 – 4i -2v_0 + 4 -6i =0

Now from the fig v_0 = -6i = -6i

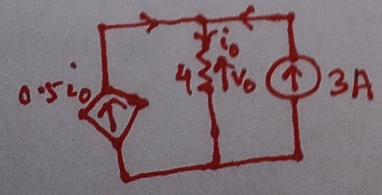
So, 12 – 4i +12i + 4 -6i =0 =>i =-8A

v_0=(-6) (-8)=48 v



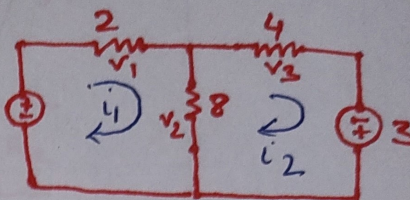
sign are given & current in following (-) to(+) so
$V_0 = -6i$

3)Find i_0 & v_0



Sol: applying KCL , $0.5i_0 + 3 = i_0$ => $i_0 = 6A$ and $v_0 = 4i_0 = 24V$

4) Find current and voltage:



Loop-1, $5 = 2i_1 + 8(i_1 - i_2)$

$5 = 10i_1 - 8i_2$

$15 = 30i_1 - 24i_2$ ---(1)

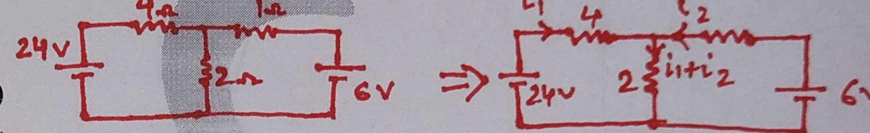
Loop-2, $-3 + (i_2 - i_1)8 + 4i_2 = 0$

$24i_2 - 16i_1 = 6$ ...(2)

Solving 1 & 2, $14i_1 = 21$, => $i_1 = 1.5A$, $i_2 = \cancel{1.5}\ 1.25$ A

$V_1 = 3V$, $V_2 = 2V$, $V_3 = 5V$

5) Find current in 2Ω resistor



Sol: applying KVLin loop 1

$4I_1 + 2(I_1 + I_2) = 24$.............(1)

for loop 2, $I_2 + 2(I_1+I_2) = 6$.........(2)

solving equation 1 & 2

$I_1 = 30/7$ A , $I_2 = -6/7$A

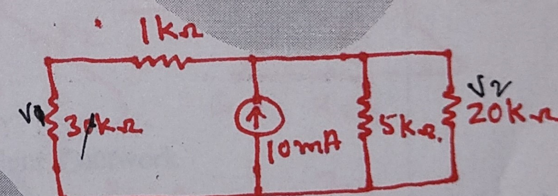
Current in 2 ohm resistor, $I_1 + I_2 = 30/7 + (-6/7) = 24/7$ A

HW1) For the circuit shown in the figure find

a)v_1 &v_2

b)power dissipated in 3~~0~~kΩ and 20kΩ resistor

c) power supplied by the current source



[a)15v,20v b)75~~M~~mW,20~~M~~mW c)200 ~~M~~mW)]

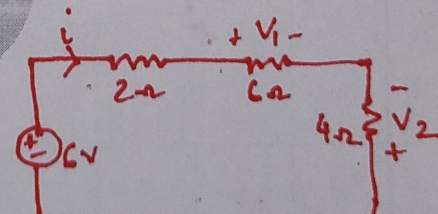
HW2)for the circuit shown in the figure find

a)the equivalent resistance seen by the source

b)i=?

c) power delivered by the source

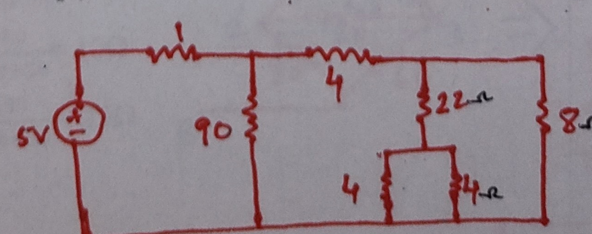
d) the voltage v_1 & v_2



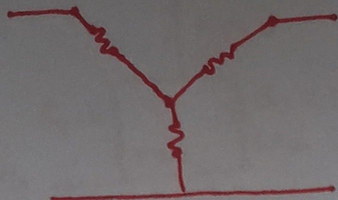
[a)12Ω b) 0.5A c)3W d)v_1 =3v ,v_2 =-2v]

HW3) Find the equivalent resistance seen by the source and the current i in the circuit shown in figure.

[ans R_{eq} = 10Ω, i=0.5A]

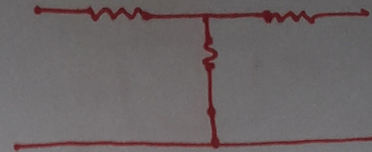


➢ Star delta transformation.

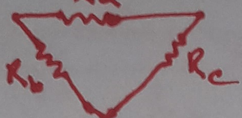


STAR(Y)

OR

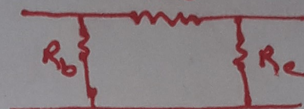


Tea(T)



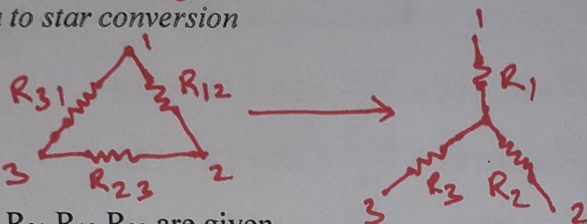
Delta

or



Pi(π)

Delta to star conversion



now R_{31},R_{12},R_{23} are given

$$R1 = \frac{R12\ R13}{R12+R23+R31}$$

$$R2 = \frac{R21\ R23}{R12+R23+R31}$$

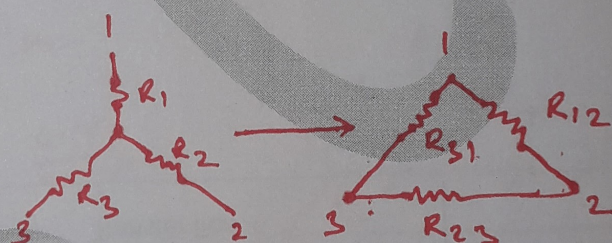
$$R3 = \frac{R31\ R32}{R12+R23+R31}$$

Star to delta conversion

$$R12 = R1 + R2 + \frac{R1R2}{R3}$$

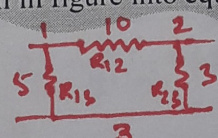
$$R23 = R2 + R3 + \frac{R2R3}{R1}$$

$$R31 = R3 + R1 + \frac{R3R1}{R2}$$

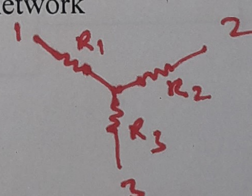


Problems

1)Convert the π network shown in figure into equivalent T network



⟹



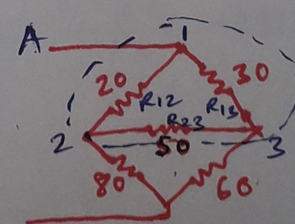
from delta to star conversion

$$R1 = \frac{R12\ R13}{R12+R23+R31} = \frac{10x5}{10+5+3} = \frac{50}{18} = 2.7\Omega$$

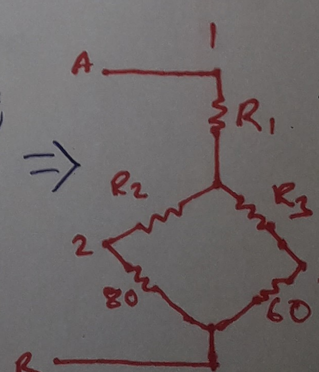
$$R2 = \frac{R21\ R23}{R12+R23+R31} = \frac{3x10}{10+5+3} = 1.67\Omega$$

$$R3 = \frac{R31\ R32}{R12+R23+R31} = \frac{3x5}{10+5+3} = 0.8\Omega$$

2) Find R seen from AB of the network



⟹



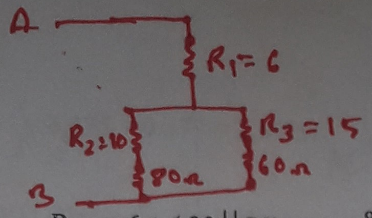
$$R1 = \frac{R12\ R13}{R12+R23+R31} = \frac{20x30}{20+30+50} = 6\Omega$$

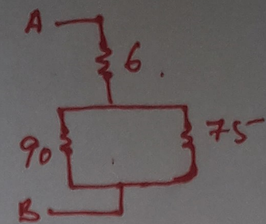
$$R2 = \frac{R21\ R23}{R12+R23+R31} = \frac{20x50}{20+30+50} = 10\Omega$$

$$R3 = \frac{R31\ R32}{R12+R23+R31} = \frac{30x50}{20+30+50} = 15\Omega$$

Now the network can be drawn as

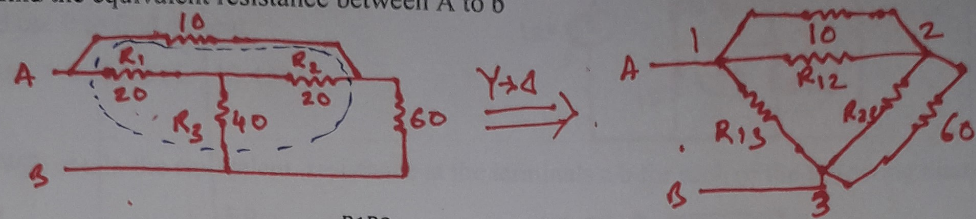
$\Delta \rightarrow Y$





$R_{AB} = 6 + (90 \,||\, 75) = 6 + \frac{90 \times 75}{90+75} = 46.9\Omega$

3) find the equivalent resistance between A to b

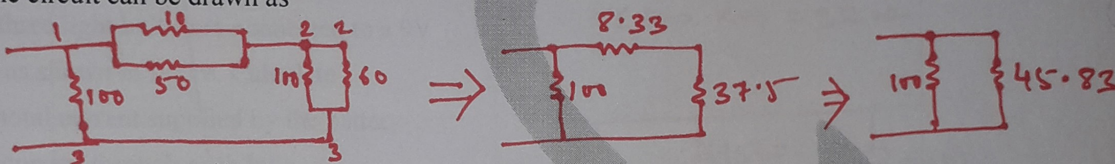


$$R12 = R1 + R2 + \frac{R1R2}{R3} = 20 + 20 + \frac{20 \times 20}{40} = 50\Omega$$

$$R23 = R2 + R3 + \frac{R2R3}{R1} = 20 + 40 + \frac{20 \times 40}{20} = 100\Omega$$

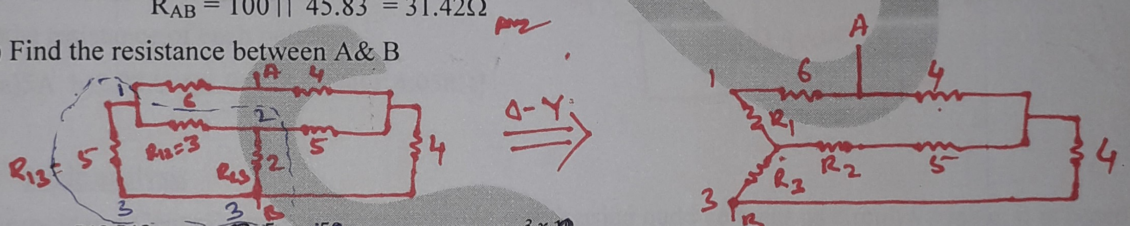
$$R31 = R3 + R1 + \frac{R3R1}{R2} = 100\Omega$$

Now the circuit can be drawn as



$R_{AB} = 100 \,||\, 45.83 = 31.42\Omega$

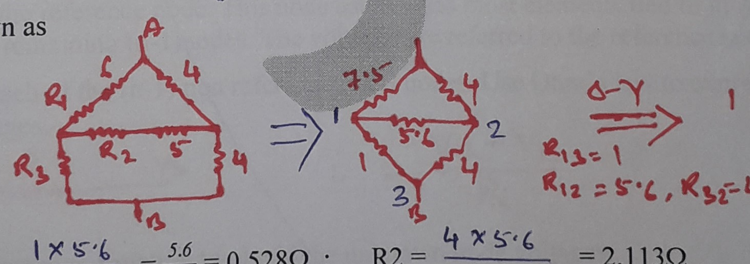
4) Find the resistance between A& B

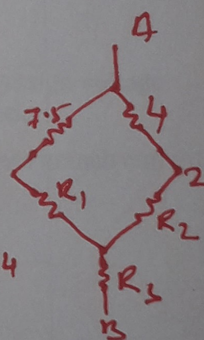


$R1 = \frac{R12\,R13}{R12+R23+R31} = \frac{3 x 5}{2+5+3} = \frac{15}{18} = 1.5\Omega$; $R2 = \frac{3 \times 2}{2+5+3} = 0.6\Omega$

$R3 = \frac{2 x 5}{2+5+3} = 1\Omega$

Now it can be drawn as





$R1 = \frac{R_{13}\,R_{12}}{R_{12}+R_{23}+R_{31}} = \frac{1 \times 5.6}{1+5.6+4} = \frac{5.6}{10.6} = 0.528\Omega$; $R2 = \frac{4 \times 5.6}{10.6} = 2.113\Omega$

$R3 = \frac{1 \times 4}{10.6} = 0.377\Omega$

now the network can be drawn as

$R_{AB} = \{(7.5 + 0.528) \,||\, (4 + 2.13)\} + 0.377$

$= (8.028 \,||\, 6.13) + 0.377$

~~= 3.475 + 0.377~~

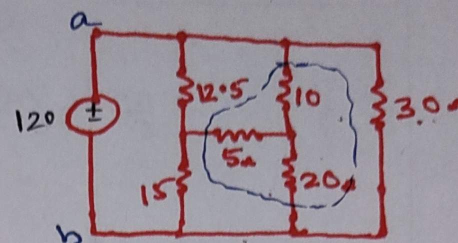
$= \frac{49.21}{14.158} + 0.377$

$= 3.475 + 0.377 = 3.852\Omega$

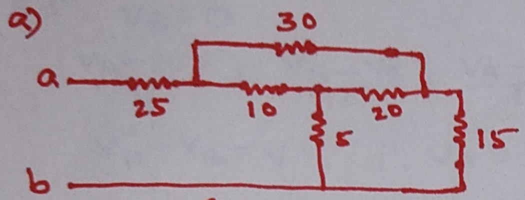
Ans

HW1) Obtain the equivalent resistance R_{ab} for the circuit shown in figure and use it to find current.

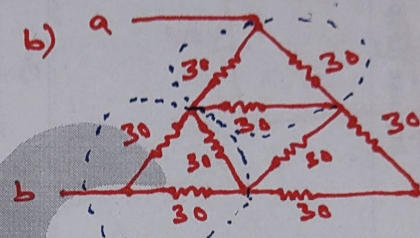
[Rab $= 9.632\Omega$, i = 12.458A]



HW2) obtain the equivalent resistance at the terminals a b for each of the following circuit



(Ans Rab = 36.25Ω)



(Ans Req = ~~32Ω~~ 33.33

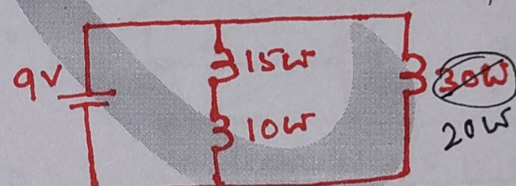
HW3) three light bulbs are connected to a 9V battery as shown in figure. Calculate

a) The total current supplied by the battery

b) The current through each bulb

c)the resistance of each bulb

[a)5A b)2.78A c)1.941Ω,1.294Ω ,4.058Ω]



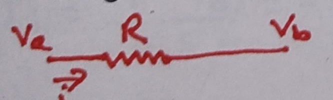
➤ Node analysis

it provides a General procedure for analysing circuit using node voltages as circuit variable. it is based on KCL .

Steps to determine node voltages:

i) Select a node as the reference node. This node usually has most elements tied to it. Assign voltage $v_1, v_2 \ldots\ldots v_{n-1}$ to the remaining (n-1)nodes. The voltages are referred to the reference node.

ii) Apply KCL to each of the (n-1) non reference notes nodes. Use Ohm's law to express branch current in terms of node voltages.



$$i = \frac{V_a - V_b}{R}$$

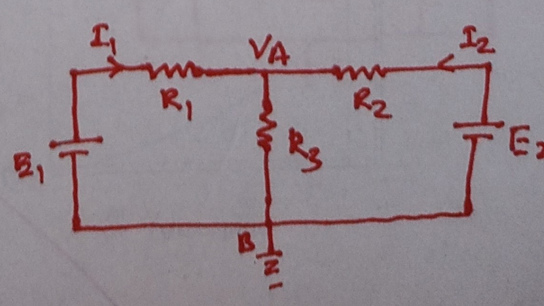
iii) Solve the simultaneous equation to obtain the unknown node voltages

Note: reference node is having zero potential and given by symbol of earth ground as shown ($\frac{\perp}{=}$)

Explanation

Consider B as reference node,

$V_B = 0$



$$I_1 = \frac{E_1 - V_A}{R_1} \quad , \quad I_2 = \frac{E_2 - V_A}{R_2}$$

$$I_3 = \frac{V_A - V_B}{R_3}$$

now applying KCL at A

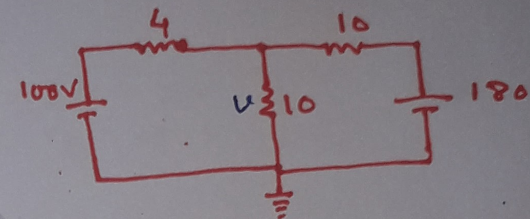
$$-I_1 - I_2 + I_3 = 0$$

$$-\left(\frac{E_1 - V_A}{R_1}\right) - \left(\frac{E_2 - V_A}{R_2}\right) + \frac{V_A}{R_3} = 0$$

This is the nodal form of the network

Problems

1) Find out V



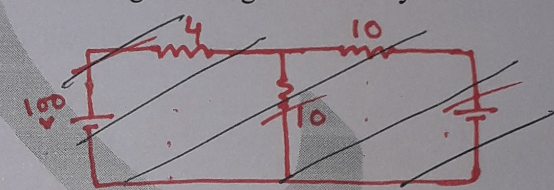
Soln. $V_B = 0$

$$\frac{V_A - 100}{4} + \frac{V_A - V_B}{10} + \frac{V_A - 180}{10} = 0$$

$$V_A - V_B = V, \quad \therefore V_A = V$$

$$\therefore V = 95.55\,V$$

2) find the current in different branches of the network shown in figure using Nodal analysis

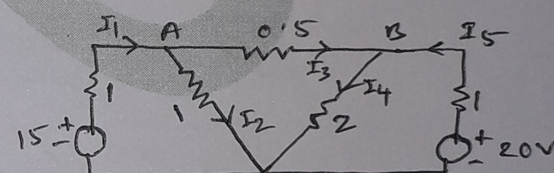


Sol: At node A

$$\frac{V_A - 15}{1} + \frac{V_A - V_B}{0.5} + \frac{V_A - 0}{1} = 0$$

$$4V_A - 2V_B = 15 \quad \text{--- (1)}$$

At node B



$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$3.5V_B - 2V_A = 20 \quad \text{--- (2)}$$

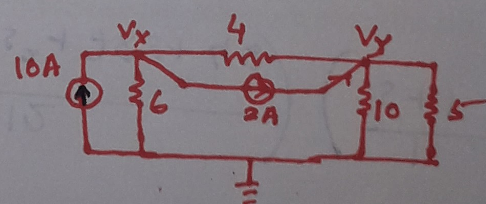
Solving equation 1 and 2, $V_B = 11V$, $V_A = 9.25V$

$$I_1 = \frac{V_A - 15}{1} = -5.75A \qquad I_4 = \frac{V_B}{2} = 5.5A$$

$$I_2 = \frac{V_A}{1} = 9.25A \qquad I_5 = \frac{V_B - 20}{1} = -9A$$

$$I_3 = \frac{V_A - V_B}{0.5} = -3.5A$$

3) find v_x & v_y using Nodal analysis



At node x

$$-10 + \frac{V_x}{6} + 2 + \frac{V_x - V_y}{4} = 0$$

$$5V_x - 3V_y = 96 \quad \text{--- (1)}$$

At node y

$$\frac{V_y - V_x}{4} + \frac{V_y}{10} + \frac{V_y}{5} = 0$$

$$5V_x - 11V_y = -40 \quad \text{--- (2)}$$

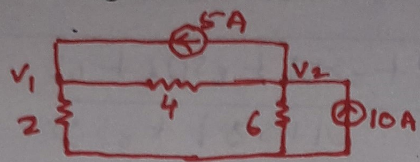
solving equation 1 and 2

$$-3V_y + 11V_y = 96 + 40$$

$$8V_y = 136$$

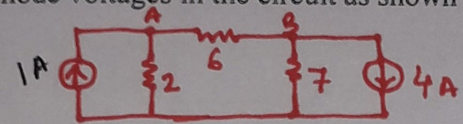
$$V_y = 17V \quad \& \quad V_x = 29.4V$$

HW1) calculate the node voltage in the circuit shown.



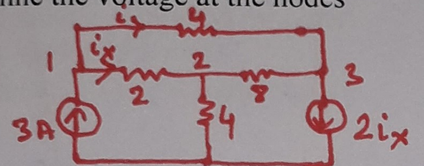
[Ans. $V_1 = 13.33\,V$, $V_2 = 20\,V$]

HW2) obtain the node voltages in the circuit as shown in figure



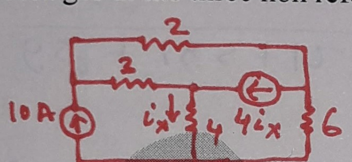
[$V_A = -2\,V$, $V_B = -14\,V$]

HW3) determine the voltage at the nodes



[$V_1 = 4.8\,V$, $V_2 = 2.4\,V$, $V_3 = -2.4\,V$]

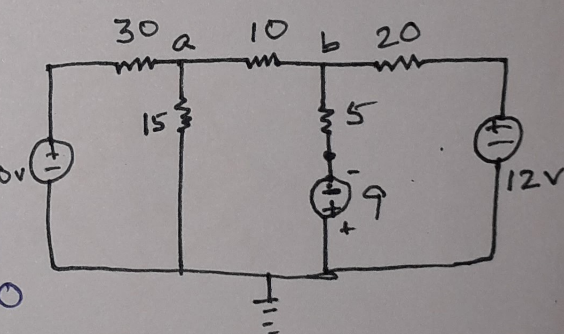
HW4) find the voltages at the three non reference nodes in the circuit as shown in figure



[$V_1 = 80\,V$, $V_2 = -64\,V$, $V_3 = 156\,V$]

Note 1: if a voltage source is connected between the reference node and non reference node, then the voltage at the non reference node is equal to the voltage of the voltage source.

Ex. Find the voltages at node a and b using Nodal analysis



Sol: at node a, $\frac{V_a - 10}{30} + \frac{V_a}{15} + \frac{V_a - V_b}{10} = 0$

$6V_a - 3V_b = 10$ --- (1)

At node b $\frac{V_b - 12}{20} + \frac{V_b - V_a}{10} + \frac{V_b - (-9)}{5} = 0$

$2V_a - 7V_b = 24$ --- (2)

by calculation $v_a = -0.0556V$; $v_b = -3.445V$

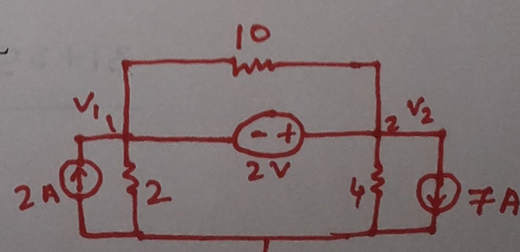
Note 2: *Supernode* : if the voltage source is connected between two non reference nodes, the two non reference nodes form a supernode.

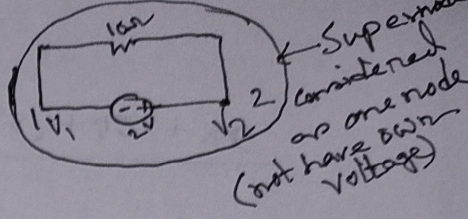
properties of a supernode:

i) it has no voltage of its own

ii) it requires the application of both KCL and KVL

Ex. For the circuit shown in figure find the node voltages





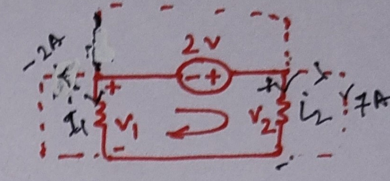
Sol: here supernode contains 2V volt source,

node 1 and 2, and the 10Ω resistance.

applying KCL at supernode , $2 = i_1 + i_2 + 7$

$$2 = (v_1/2) + (v_2/4) + 7$$

$$8 = 2v_1 + v_2 + 28 \quad \Rightarrow 2v_1 + v_2 = -20 \text{..........}(1)$$



applying KVL in supernode $v_1 + 2 - v_2 = 0 \quad \Rightarrow v_1 - v_2 = -2 \text{...........}(2)$

solving 1 and 2 , $v1 = -7.33V$, $v2 = -5.33V$ Ans

➢ *Mesh analysis*: it is based on KVL. it provides a General procedure for analysing circuits using mesh current as a circuit variable.

Steps to determine mesh analysis:

i)select meshes and assign mesh currents

ii)apply kvl to each of the mesh

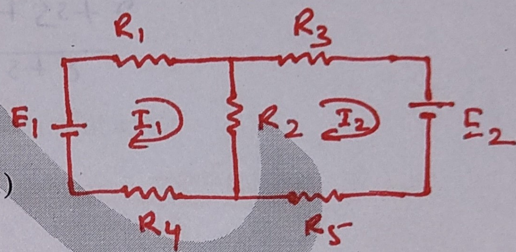
iii)solve the equations to get mesh current

explanation :

number of mesh =2

current in meshes : I_1 & I_2

applying KVL at mesh 1



$$-E_1 + I_1R_1 + (I_1 - I_2)R_2 + I_1R_4 = 0$$

$$E_1 = (R_1 + R_2 + R_4) - I_2 R_2 \text{ ....}(1)$$

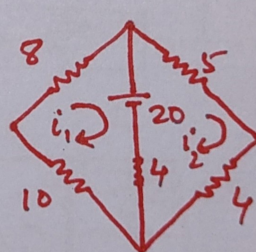
for mesh 2

$$E_2 + I_2R_5 + I_2R_3 + (I_2 - I_1)R_2 = 0$$

$$E_2 = I_2R_2 - (R_2 + R_3 + R_5)I_2 \text{..................}(2)$$

Problems:

1) find the loop currents



Solⁿ, loop - I

$$20 + (I_1 - I_2)4 + 10I_1 + 8I_1 = 0$$

$$22I_1 - 4I_2 = -20 \quad \text{—— } (1)$$

Loop - 2, $5I_2 + 4I_2 + (I_2 - I_1)4 - 20 = 0$

$$-4I_1 + 13I_2 = 20$$

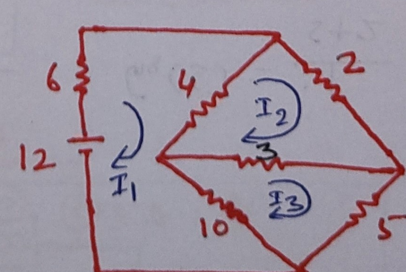
from eq 1 & 2, $I_1 = -0.667A$, $I_2 = 1.33A$

2)find loop currents

Sol:number of mesh 3

Mesh currents I_1, I_2, I_3

for loop 1 ,



$$6I_1 + 4(I_1 - I_2) + 10(I_1 - I_3) - 12 = 0$$

$$\Rightarrow 10I_1 - 2I_2 - 5I_3 = 6 \text{ --- } (1)$$

for loop 2,

$$(I_2 - I_3)3 + (I_2 - I_1)4 + 2I_2 = 0$$

$$4I_2 - 9I_2 + 3I_3 = 0 \text{ --- } (2)$$

for loop 3

$$(I_3 - I_2)3 + 5I_3 + 10(I_2 - I_1) = 0$$

$$18I_3 - 3I_2 - 10I_1 = 0$$

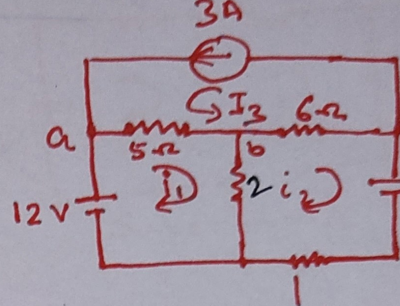
solving 1 ,2&3 , I_1 =11.42A ,I_2 =10.84 , I_3 = 17.30A

3)find current through 5Ω resistance

number of mesh 3

currents in the mesh = I_1,I_2,I_3

loop 3 contains a current source ,so I_3 =3A



for loop 1

$$(I_1+3)5+(I_1-I_2)2-12=0$$

$$7I_1-2I_2+3=0 \quad --(1)$$

for loop 2

$$(I_2+3)6+6+I_2\cdot 1+(I_2-I_1)2=0$$

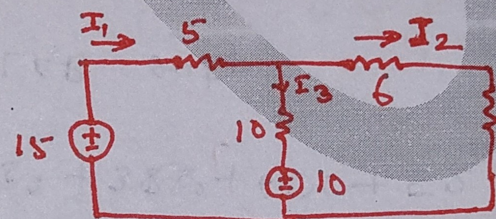
$$9I_2+24-2I_1=0 \quad ---(2)$$

Solving 1 and 2 .... I1 = -1.275 , I2 =-2.95A

Current through 5Ω resistor = -1.275 + 3 = 1.725 in the direction of a to b

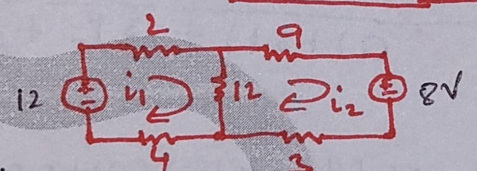
HW1) Find I_1,I_2,I_3 using mesh analysis

[I_1=1A ,I_2 = 1A ,I_3 =- 0A]



HW2)calculate I_1,I_2 in figure

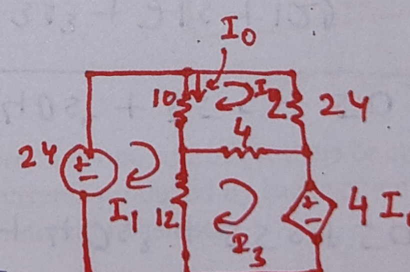
$$\left[\begin{array}{l} i_1 = \frac{2}{3}A \\ i_2 = 0A \end{array}\right]$$



HW3) find I_0 using mesh analysis

[Hints: $I_0 = (I_1 - I_2)$

$4I_0$ = dependent voltage source]



for loop 3

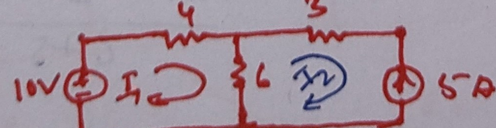
$$4(I_3-I_2)+12(I_3-I_1)+4I_0=0$$

$$4(I_3-I_2)+12(I_3-I_1)+4(I_1-I_2)=0$$

[ans I0 = 1.5A]

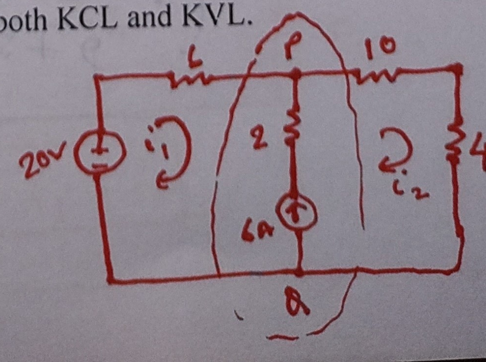
HW4)find loop currents

$[I_1 = -2A \ , I_2 = -5A]$



<u>Note</u> : *Supermesh* : A supermesh results when two meshes have a current source in common. A supermesh has no currents of its own. It requires the application of both KCL and KVL.

Ex 1. Solve the loop currents i_1,i_2 using mesh analysis

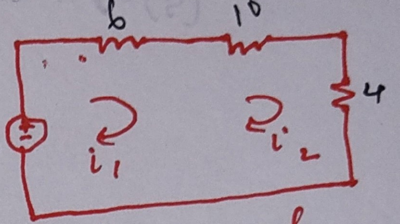


Sol:there is asupermesh formed as 6 amps source common to both meshes.

applying KVL in supermesh

$$-20+6i_1+10i_2+4i_2=0$$

$$+6i_1+14i_2=20 \quad \text{---} (1)$$



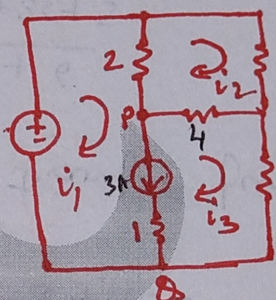
Supermesh.

applying KCL at node p

$$i_1+6=i_2 \quad \text{---} (2)$$

by solving equation 1 and 2, $i_1 = -3.2A$, $i_2 = 2.8A$

Ex2. Use mesh analysis to determine the loop currents as shown in figure



Sol:by applying KVL in supermesh

$$-6+2(i_1-i_2)+4(i_3-i_2)+8i_3=0$$

$$\Rightarrow -2i_1+6i_2-12i_3=-6 \quad \text{---} (1)$$

KVL in mesh 2

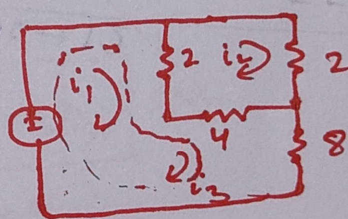
$$2i_2+4(i_2-i_3)+2(i_2-i_1)=0$$

$$2i_1-8i_2+4i_3=0 \quad \text{---} (2)$$

KCL at node Q

$$3+i_3=i_1$$

$$i_1-i_3=3 \quad \text{---} (3)$$



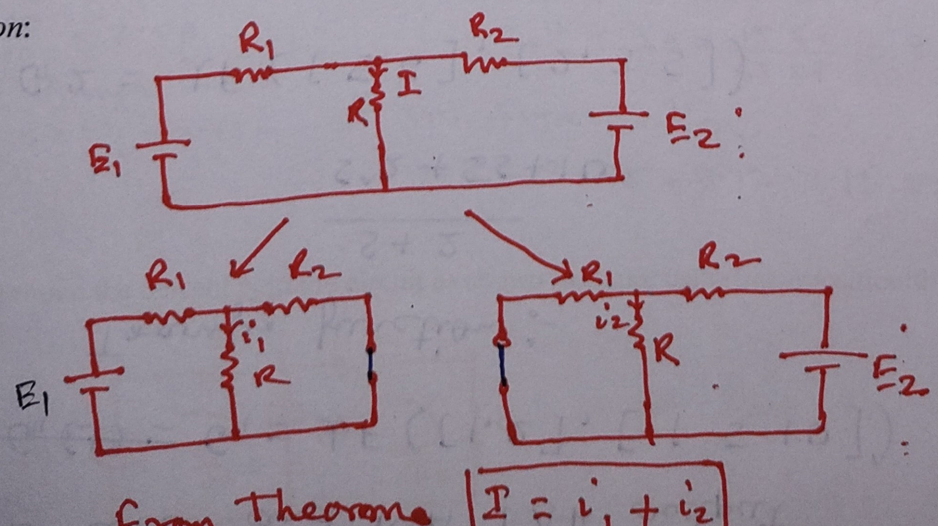
Supermesh.

By solving 1 2 and 3 , $i_1 = 3.47A$, $i_2 = 1.105A$, $i_3 = 0.474A$

Network theorem

Superposition theorem: in any linear network containing more than one source (voltage or current) the total current in any branch is the algebraic sum of the individual currents produced by each source acting alone. Meanwhile all other sources are replaced by their respective internal resistance.

Explanation:



from Theorem $\boxed{I = i_1 + i_2}$

Note: i) internal resistance of voltage sources is zero.

ii) internal resistance of current sources is infinite.

Steps

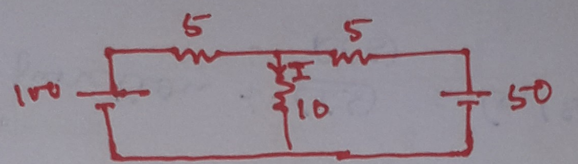
i)turn off (replace by internal resistance) all independent source except one source. find the output due to that active source.

ii) repeat step one for each of the independent sources

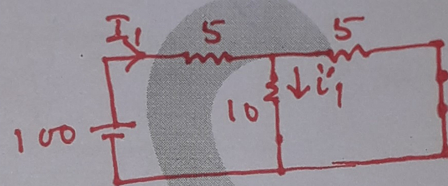
iii)find the total contribution by adding algebraically

Problems

1) find I



step 1, from the figure

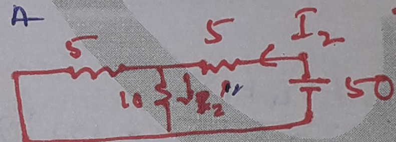


$R_{eq} = 5 + (5 || 10) = 8.33\ \Omega$

$I_1 = \frac{100}{8.33} = 12A$

by current division rule

Step 2, $i_1' = 12 \times \frac{5}{10+5} = 12 \times \frac{1}{3} = 4A$



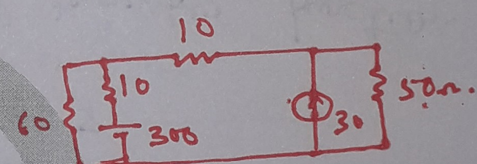
$R_{eq} = 8.33\ \Omega$, $I_2 = \frac{50}{8.33} = 6A$

$i'' = 6 \times \frac{5}{15} = 2A$

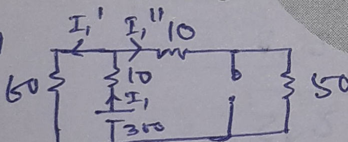
from superposition theorem

$I = i_1' + i_2'' = 2 + 4 = 6A$

2) Find current through 50 Ω resistance



Step-1



10Ω & 50Ω are in series.

$R_1 = 50 + 10 = 60\Omega$

$R_{eq} = 10 + (60 || 60)$

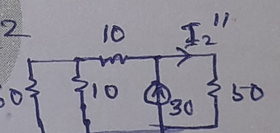
$= 10 + 30 = 40\Omega$.

$I_1 = \frac{300}{40} = 7.5A$

Current through 50Ω

$I'' = 7.5 \times \frac{60}{120} = 3.75A$

Step-2



$R_1 = 10 + (60 || 10) = 18.57\Omega$

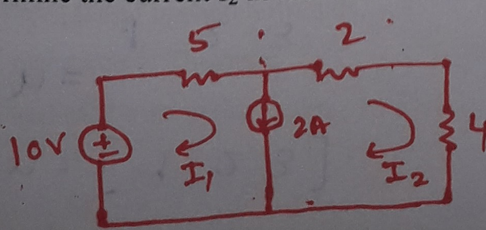
Current through 50Ω

$I_2'' = 30 \times \frac{18.57}{68.57} = 8.12A$

From Superposition,

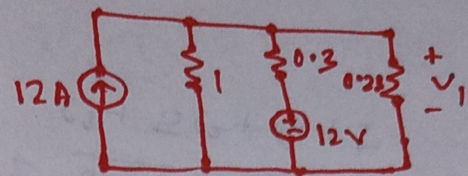
$I = 3.75 + 8.12 = 11.87A$ Ans.

Hw1) Determine the current I_2 in the circuit as shown in figure using superposition theorem [I_2 = 0A]



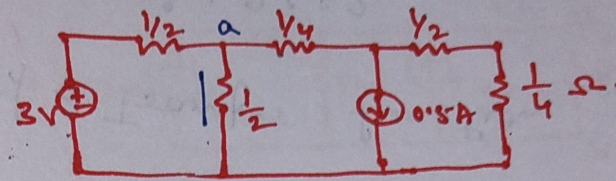
Hw2) determine the voltage across 0.23Ω resistor in the circuit as shown

[5.99V]



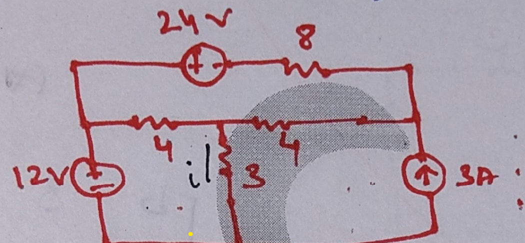
Hw3) find the current i through (ab) $\frac{1}{2}\Omega$ resistor by superposition theorem

Ans - 1·137



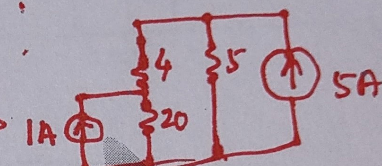
Hw4)find i using superposition

[2A]



Hw5)find the current through 20Ω using superposition

Ans - 1·17



Thevenin theorem :it states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th},where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals with the independent sources are turned off.

explanation

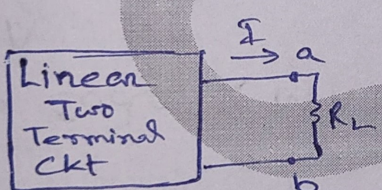


fig-1 Original ckt

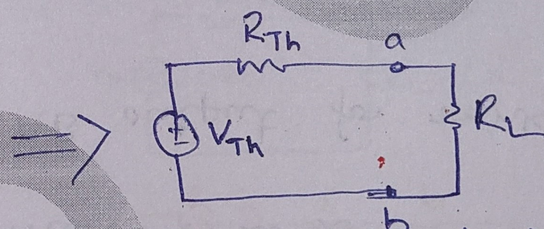


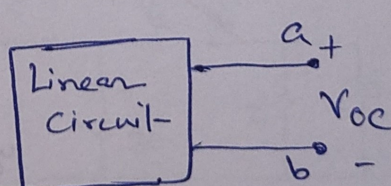
fig2 - Thevenin equivalent ckt.

figure 1 shows a linear two terminal circuit and a load resistor connected across the terminals a and b. This circuit can be replaced by an equivalent circuit consisting of V_{Th} in series with R_{Th} as shown in figure 2

To find V_{Th} :-

Open the Load terminals and measure or calculate the open circuit voltage(V_{oc}) across the terminal as shown in figure.

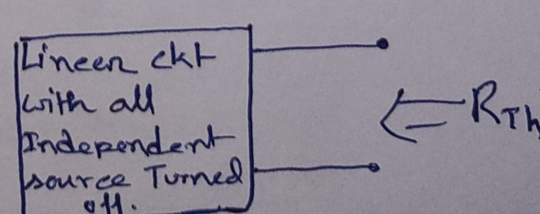
this $V_{oc} = V_{Th}$



finding V_{Th}

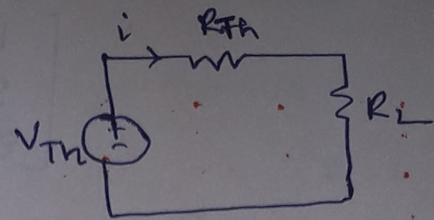
to find R_{Th} :-

Open the load resistor when the linear circuit has no independent sources. by looking through the opened terminals, the internal resistance is Thevenin resistance (R_{Th}) as shown in the figure.



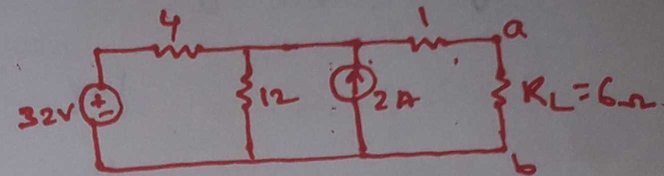
Finding R_{Th}

Draw the Thevenin's equivalent circuit with V_{Th} , R_{Th} & R_L to find out the current.



Problems

1) find the thevenin equivalent circuit of the circuit shown in figure at terminals a to b. then find the current through $R_L = 6\Omega$



sol: To find V_{Th}

using Nodal analysis

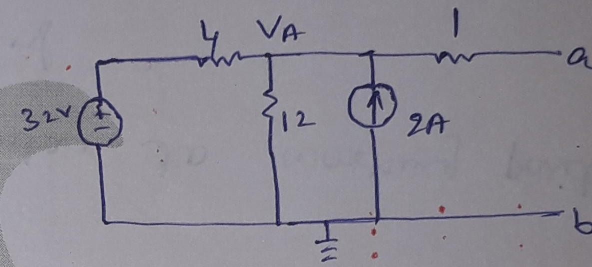
$$\frac{V_A - 32}{4} + \frac{V_A - 0}{12} - 2 = 0$$

$$\Rightarrow 3V_A - 96 + V_A - 24 = 0$$

$$\Rightarrow V_A = 30V$$

Now, $V_{oc} = V_A = 30V$

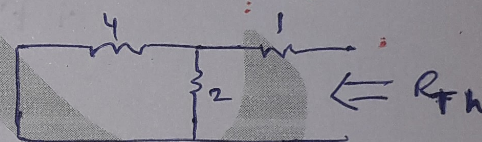
& $V_{Th} = V_{oc} = 30V$



To find R_{Th}

$$R_{Th} = 1 + (4 \| 12)$$

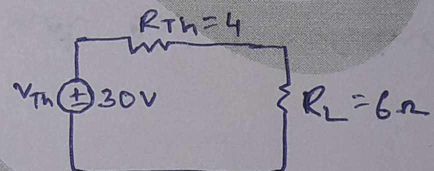
$$= 4\Omega.$$



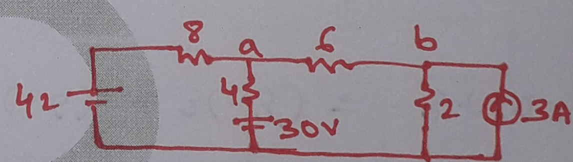
thevenin equivalent circuit

$$i = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{30}{4+6} = 3\ A\ \text{Ans.}$$



2) Find the current through 6 ohm resistor using Thevenin's Theorem



sol: to find V_{Th}

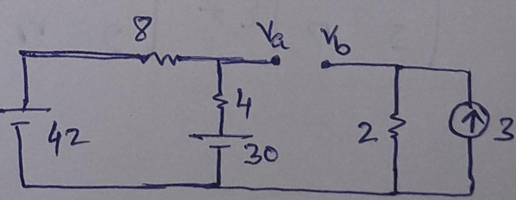
using Nodal analysis at node a

$$\frac{V_a - 42}{8} + \frac{V_a - 30}{4} = 0$$

$$\Rightarrow V_a - 42 + 2V_a - 60 = 0$$

$$\Rightarrow 3V_a = 102$$

$$V_a = 34V$$



at node b,

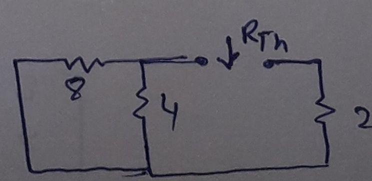
$$\frac{V_b - 0}{2} - 3 = 0 \Rightarrow V_b = 6V$$

Now $V_{ab} = V_a - V_b = 34 - 6 = 28V = V_{oc} = V_{Th}$

finding R_{Th}

$$R_{Th} = 2 + (8 \| 4)$$

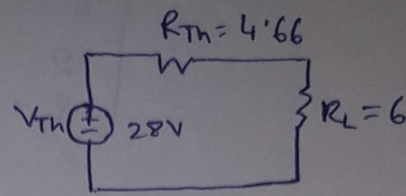
$$= 4.66\Omega.$$



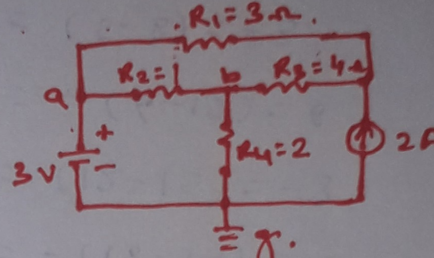
Thevenin's equivalent circuit

$$i = \frac{28}{6+4.66}$$

$$= 2.625\,A \text{ Ans.}$$



3) Find out the current through $R_L = R_2 = 1\Omega$ resistor using Thevenin's theorem and hence calculate the voltage across V_{cg}



Sol: finding V_{Th} across ab terminals

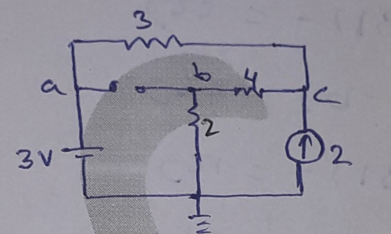
at node a, $V_a = 3V$

at node b

$$\frac{V_b - V_c}{4} + \frac{V_b - 0}{2} = 0$$

$$V_b - V_c + 2V_b = 0$$

$$3V_b - V_c = 0 \Rightarrow V_c = 3V_b \quad \text{--- (1)}$$



at node c

$$\frac{V_c - V_a}{3} + \frac{V_c - V_b}{4} - 2 = 0$$

$$\Rightarrow \frac{3V_b - 3}{3} + \frac{3V_b - V_b}{4} - 2 = 0$$

$$\Rightarrow \frac{3V_b - 3}{3} + \frac{V_b}{2} - 2 = 0$$

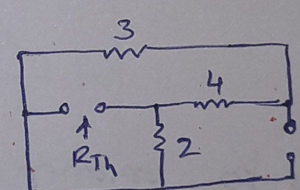
$$\Rightarrow 6V_b - 6 + 3V_b - 12 = 0$$

$$\Rightarrow 9V_b = 18, \Rightarrow V_b = 2V$$

now $V_{oc} = V_a - V_b = 3 - 2 = 1V$

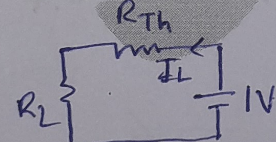
Finding R_{Th}

$$R_{Th} = (3+4) \| 2 = 1.55\Omega$$



Thevenin's equivalent circuit

$$I_L = \frac{1}{1.55+1} = 0.39A \text{ Ans}$$



part 2

$V_c = 3V_b = 6V$, $\Rightarrow V_b = 2V$ & $V_a = 3V$

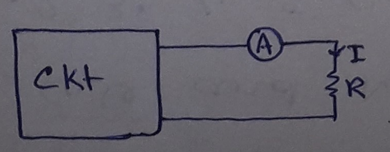
$V_{bg} = V_{ag} - V_{ab} = 3 - (1 \times 0.39) = 2.61V$

$I_{bg} = 2.61/2 = 1.305A$

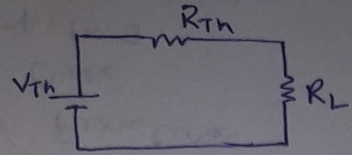
$Ic_b = 1.305 = 0.39 = 0.915A$

$V_{cg} = (4 \times 0.915) + (2 \times 1.305) = 6.27V$

4)The circuit consists of independent DC source and resistances. Measurement are taken by connecting an ammeter in series with the resistor R and the results are shown in the table



R	I
10Ω	2A
20Ω	1.5A
?	0.6A



Sol: the circuit can be replaced by an equivalentThevenin's circuit as shown in the figure

$$I = \frac{V_{Th}}{R_{Th}+R}$$

from the table

$$\frac{V_{Th}}{R_{Th}+10} = 2 \quad \Rightarrow V_{Th} - 2R_{Th} = 20 \text{ -- } (1)$$

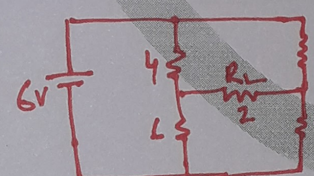
$$\& \quad \frac{V_{Th}}{R_{Th}+20} = 1.5 \quad \Rightarrow V_{Th} - 1.5R_{Th} = 30 \text{ -- } (2)$$

solving equation 1 and 2, V_{Th} =60V , R_{Th} =20Ω

now when I = 0.6A

$$0.6 = \frac{V_{Th}}{R_{Th}+R} = \frac{60}{20+R} \quad \Rightarrow R = 80\Omega \text{ Ans}$$

5)find the current through 2 ohm resistor using Thevenin's Theorem



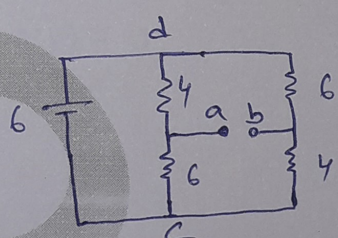
sol: finding V_{Th}

at node a

$$\frac{V_a - V_d}{4} + \frac{V_a}{6} = 0$$

Now $V_d = 6$

$\therefore V_a = 3.6V$



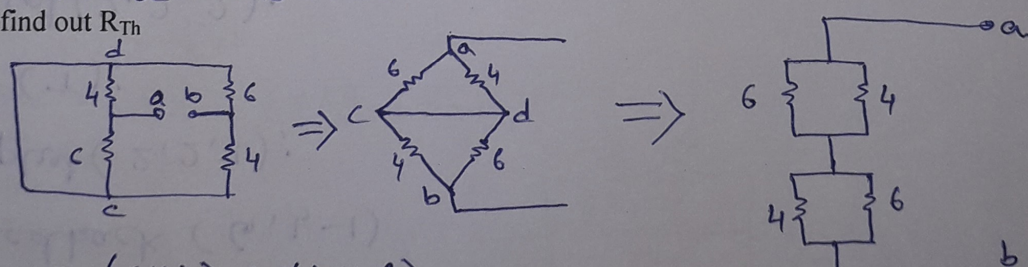
at node b

$$\frac{V_b - 6}{6} + \frac{V_b}{4} = 0 \Rightarrow V_b = 2.4V$$

$$\therefore V_{ab} = V_a - V_b = 3.6 - 2.4 = 1.2V$$

$$V_{Th} = V_{oc} = V_{ab} = 1.2V$$

To findfind out R_{Th}



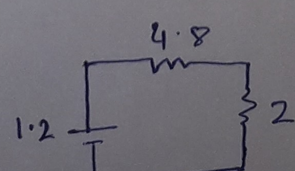
$$R_{ab} = (6 \| 4) + (4 \| 6)$$

$$= 4.8\Omega$$

Thevenin's circuit

$$I = \frac{1.2}{4.8+2}$$

$$= 0.176A \text{ Ans}$$



<u>**special cases**</u> :

case 1: if the circuit contains Independent and dependent sources, then V_{Th} can be calculated opening the terminals. but to calculate R_{Th} the following steps to follow

i)connect a test voltage source (1 volt) and find the resultant current(i).

then $R_{Th} = 1/i$

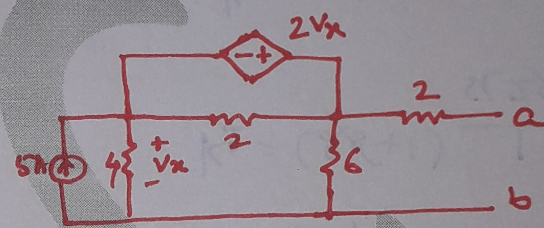
ii)connect a test current source (1A)and find the resulting voltage(V) .

then $R_{Th} = V/1$

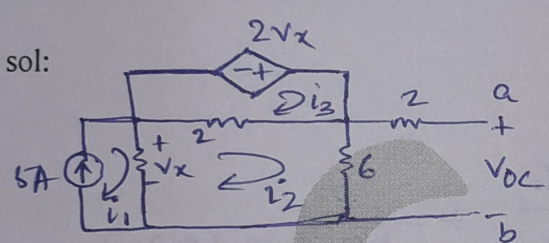
case 2 : if the circuit only contains dependent sources then the V_{Th} =0 and the R_{Th} can be calculated as discussed in the previous case.

<u>Problems</u>

1)Find the Thevenin equivalent of the circuit as shown in the figure



sol:



$V_x = 4(i_1 - i_2)$ & $i_1 = 5A$

for mesh 2

$2(i_2 - i_3) + 6i_2 + 4(i_2 - i_1) = 0$

$12i_2 - 2i_3 = 20$ — (1) [as $i_1 = 5A$]

for mesh 3

$-2V_x + 2(i_3 - i_2) = 0$

$-2[4(i_1 - i_2)] + 2i_3 - 2i_2 = 0$

$6i_2 + 2i_3 = 40$ — (2)

by solving equation 1 and 2

$i_2 = \frac{10}{3} A$

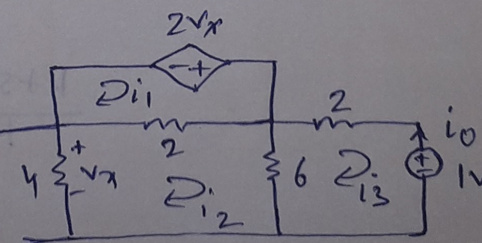
$V_{oc} = 6i_2 = 6 \times \frac{10}{3} = 20V$

To find R_{Th} $V_x = -4i_2$

For mesh 1

$-2V_x + 2(i_1 - i_2) = 0$

$2i_1 + 6i_2 = 0$ — (1) [Putting $V_x = -4i_2$]



For mesh 2

$2(i_2 - i_1) + 6(i_2 - i_3) + 4i_2 = 0$

$-2i_1 + 2i_2 - 6i_3 = 0$ — (II)

For mesh 3, $2i_3 + 1 + 6(i_3 - i_2) = 0$

$$-6i_2 + 8i_3 = -1$$

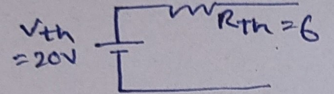
$$6i_2 - 8i_3 = 1 \quad \text{—— (iii)}$$

By solving 1, 2 and 3, i1=0.167A, i2 = -0.056A, i3 = -0.167A

And, $i_0 = -i_3 = 0.167A$

$$\therefore R_{Th} = \frac{1}{i_0} = \frac{1}{0.167} = 6\Omega$$

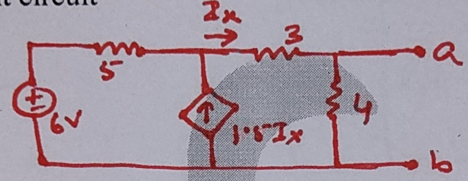
Thevenin equivalent



Hw1) Find the Thevenin equivalent circuit

[~~5.55~~V, 0.44Ω)

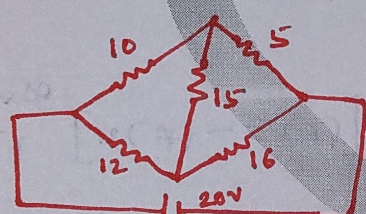
5.33



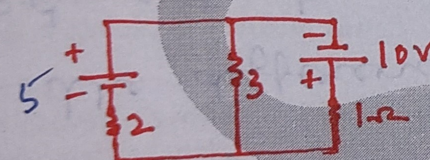
Hw2)find current through 15 ohm resistance

using Thevenin's Theorem

Ans 0.1890



Hw3)find current through 3 ohm resistor in figure



$V_{Th} = 5V$

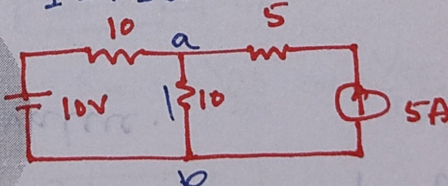
$R_{Th} = 0.66$

$I = 1.366$ A

Hints: change 5V source to current source.

Hw4)find the thevenin equivalent of the network

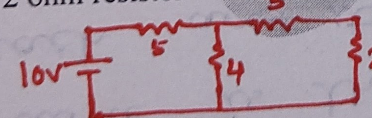
[60V,10Ω]



Hw5)find current through 2 ohm resistor

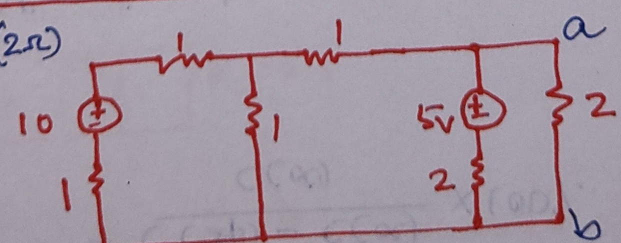
using Thevenin

[0.6154A)



Hw6) draw Thevenin equivalent (2Ω)

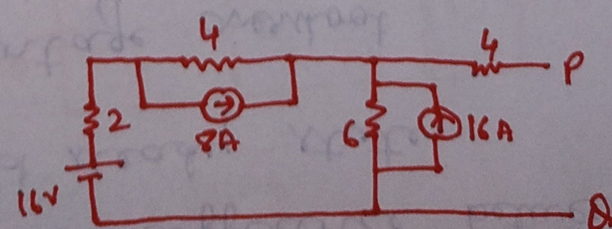
[45/11 V,10/11 Ω]



Hw7)find V_{Th} and R_{Th}

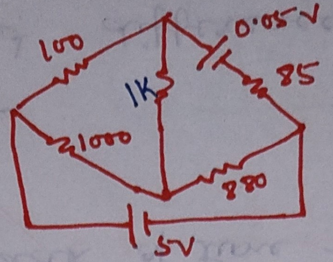
[hints :for V_{Th} convert current source to voltage source and apply mesh analysis]

[72V,7Ω]



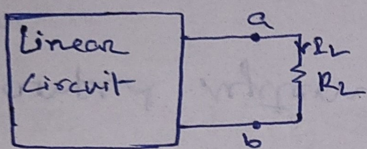
Hw8)find I_L using Thevenin's

$[I_L = -10.625]\,\mu A$

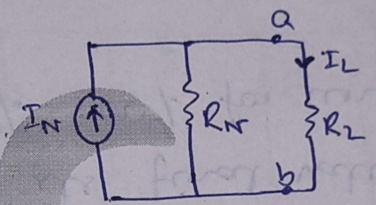


➢ Norton's theorem : it states that "a linear two terminal circuit can be replaced by an equivalent circuit consisting of a current source 'I_N' in parallel with resistance 'R_N' where I_N is the short circuit current through the terminals and R_N is the equivalent resistance at the terminals when the independent sources are turned off.

Explanation



original circuit



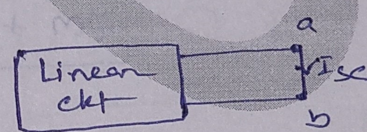
Norton equivalent circuit

the load current is given by

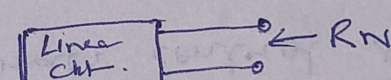
$$I_L = \frac{R_N}{R_L + R_N} \times I_N$$

steps

For finding I_N :
short the load terminal A and B
and find the current through it. that is Norton
current or short circuit current(I_N or I_{sc})



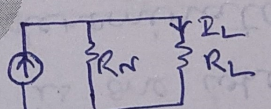
For finding R_N:
Same is the Thevenin's resistance(R_{Th}).
open load terminal and look through this,
the equivalent resistance resulting will be
Norton resistance(R_N) ,while the independent
source must be removed.



$$R_{Th} = R_N = \frac{V_{Th}}{I_N} \text{ or. } \frac{V_{oc}}{I_{sc}}$$

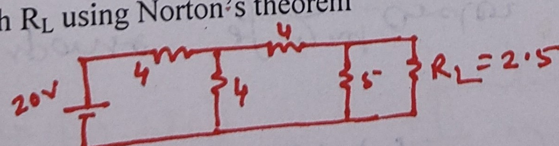
Draw the Norton equivalent circuit and find out I_L

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$



problems

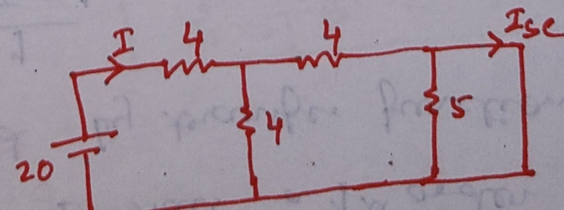
1)find I_L through R_L using Norton's theorem



sol: finding I_N

$$I = \frac{20}{4 + (4 \| 4)}$$

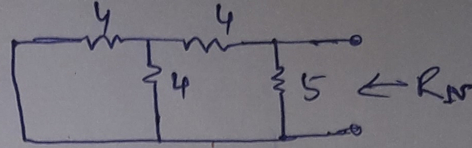
$$= 3.34A$$



$$I_N = 3.34 \times \frac{4}{4+4} = 1.67A$$

$$I_L = 1.67 \times \frac{2.73}{2.73 + 2.5} = 0.87A$$

finding R_N



$R_N = \{(4 \| 4) + 4\} \| 5$

$= 2.73\,\Omega$

2) Find Norton equivalent circuit

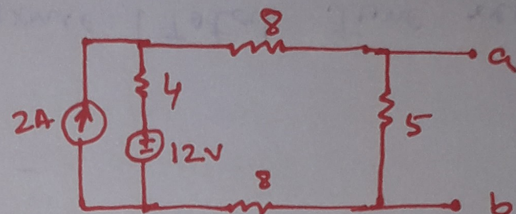


fig-1

To find I_N

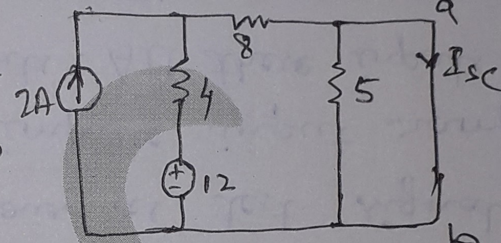


fig-2

$I_1 = 2A$

Loop-2

$8I_2 + 8I_2 - 12 + 4(I_2 - I_1) = 0$

$\Rightarrow 16I_2 - 12 + 4(I_2 - I_1) = 0$

$\Rightarrow 20I_2 = 20 \Rightarrow I_2 = 1A$

Now $I_{sc} = I_N = I_2 = 1A$

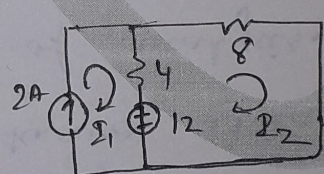
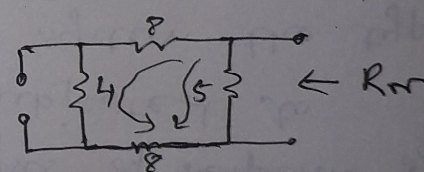


fig-3

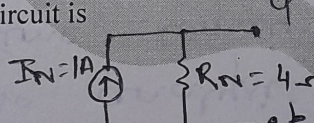
to find R_N

$R_N = (8 + 4 + 8) \| 5$

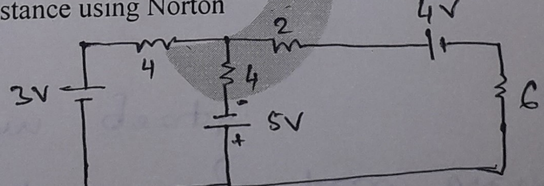
$= 4\,\Omega$



So Norton equivalent circuit is



3) find current in 6 ohm resistance using Norton

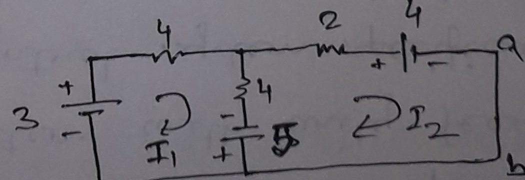


Sol: finding I_N

loop 1

$-3 + 4I_1 + 4(I_1 - I_2) - 5 = 0$

$\Rightarrow 2I_1 - I_2 = 2$ --- (1)

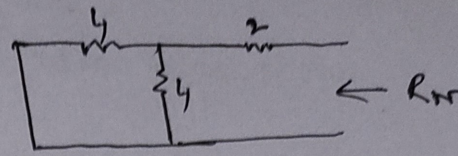


loop 2

$5 + (I_2 - I_1)4 + 2I_2 + 4 = 0$

$6I_2 - 4I_1 = -9$ --- (2)

solving 1 and 2 , I2 = 1.25A from b to a finding R_N



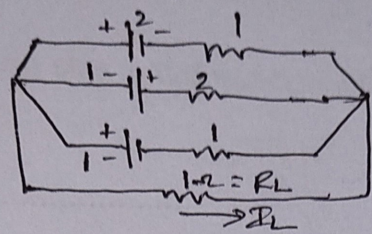
$R_N = 2 + (4 \| 4)$

$= 4\,\Omega$

$I_L = 1.25 \times \frac{4}{4+6} = 0.5\,A$

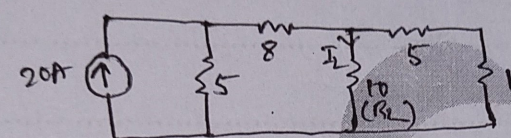
Hw1)find I_L using Norton

[0.714A]



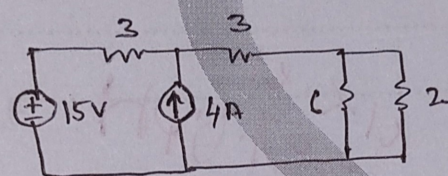
Hw2)find I_L through 10 ohm resistor using Norton

Ans 2.238A



Hw3)Find load current through 2 ohm resistance using Norton

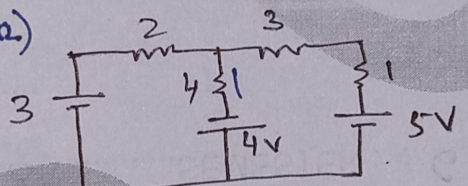
[2.7A]



Hw4)find Norton equivalent circuit (4Ω)

[~~69/16~~ A , 4/3Ω]

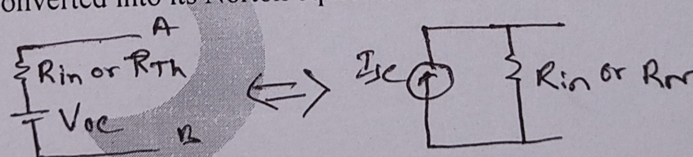
23/16



Hints: $I_{sc} = \frac{23}{4}$

➢ equivalence of Thevenin's and Norton theorem

A Thevenin equivalent can be converted into its Norton equivalent and vice versa. it can be understood by source conversion theory.



$I_{sc} = \frac{V_{oc}}{R_{in}}$ ..... which is Norton's current

& $V_{oc} = V_{Th} = I_{sc} R_{in}$ ....which is Thevenin voltage

➢ Time domain analysis of first order RL RC circuit

The circuit consisting of a single energy-storing element (either single inductor or single capacitor) in addition to a resistor is known as first order circuit.

Points to Remember

i) Current through a resistor changes instantly if the voltage changes instantly and vice versa

ii) In case of inductor current through it cannot changes instantaneously

iii) In case of a capacitor the voltage across it cannot changes instantaneously

TRANSFORMER

Q1. Show that for a Single phase Transformer $E_p = 4.44 \phi_m f N_p$ where the Symbol have their usual meaning.

EMF EQUATION

Q2: A 200 KVA transformer has 400 turns on the primary and 40 turn on the Secondary winding. The primary is Connected to 2KV, 50 Hz Supply. Find the full load primary and Secondary currents, Secondary emf and the maximum flux in the Core. Neglect leakage drop and no-load primary current.

Q3: A Single phase 50 Hz Core type transformer core of Cross-Sectional area of 400 Sq. cms. The permissible flux density is 1 wb/m^2. Calculate the number of turn on the high & low voltage side for a 3000/220V ratio.

Q4: Explain what will happen to transformer if we give DC Supply to it.

Q5: Explain the working of a transformer on no-load.

Q6: Draw and explain the phasor diagram of a Single-phase transformer under lagging p.f.

(or)

EFFICIENCY:

Q7: prove that the efficiency of a transformer is maximum when iron loss = Copper loss.

Q8. Following test data were obtained on a 20kVA, 50Hz, 1 ph 2000/200 V for

No-load test = 200 V, 1A, 120 W.

Short-Circuit test = 60V, 10A, 300 W.

Find (i) efficiency of the transformer at $\frac{1}{2}$ of the full load and 0.8 pf lagging.

(ii) maximum efficiency and the load at which it occures, pf = 0.8 lag.

Q9: Determine the full-load efficiency at unity p.f for the 4 kVA, 200/400 V, 50 Hz Single phase Transformer of which the following are test figures.

O.C test :- 200V, 0.8A, 70 W.

S.C test :- 17.5 V, 9A, 50 W.

Q10. A 20 kVA, 2000/200V, Single phase transformer has a primary resistance of 2.1 Ω and secondary resistance of 0.02 Ω. If the total Iron loss equals 200W Find the efficiency on.

(i) Full load & p.f of 0.5 lag

(ii) Half load & p.f of 0.8 lag.

Q11. The efficiency at unity of a 6600/384 V, 200 kVA, Single phase transformer is 98% at full load and at $\frac{1}{2}$ load. Calculate the full load cu loss and core loss.

REFERRED VALUES

Q12. A 2200/250 volt transformer has primary resistance and reactance of 5 Ω and 6.2 Ω respectively. The secondary resistance and reactance values are 0.03 Ω and 0.06 Ω.

Calculate:

i) equivalent resistance referred to primary side.
ii) equivalent resistance referred to secondary side.
iii) equivalent reactance referred to primary side.
iv) equivalent reactance referred to secondary side.

Q13. A 220/110 V Transformer is having no load current of 0.9A at 0.12 pf (lag) and a secondary current of 95A at 0.27 pf (lag). Find the primary current.

Q14. Draw & explain the phasor diagram of a single phase transformer under loaded condition

Test :-

Q15. Why is the open circuit test on a transformer conducted at rated load.

Q16. The open-circuit & short circuit tests on a 4KVA, 200/400 V, 50 Hz single phase transformer gave the following results:

OC test on the LV side : 200V, 1A, 100W.
SC test with the LV side shorted :- 15V, 10A, 85W

i) Determine the parameter of the equivalent circuit
ii) Draw the equivalent circuit referred to the LV side.