

MOD - I

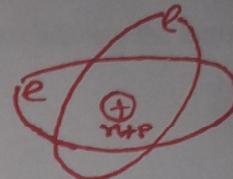
Electric charge :

We know that an atom may be regarded as the smallest particle of an element. Atoms are composed of two parts ,namely central nucleus and the surrounding or orbital electrons.

Again the nucleus of an atom is largely a cluster of two types of particles called the protons and neutrons.

Electron is of negative charge having magnitude 1.602×10^{-19} C.

Proton is of positive charge having magnitude 1.602×10^{-19} C .



The presence of equal number of proton and electron makes an atom neutrally charged.

Charge is an electrical property of atom measured in coulombs.

Coulomb is large unit for charges. In one coulomb of charge there are 6.24×10^{18} electrons.

If an atom losses an electron it is called positive charged ion (+ve)

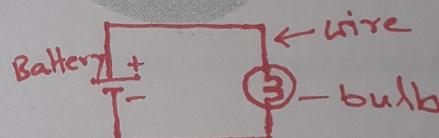
If an electron is added to an atom it is called negative charged ion (-ve)

A body having a number of ionized atoms is said to be electrically charged.

Electricity : The Invisible energy which constitutes the flow of electrons through a circuit to do work is called electricity. It is used for lighting, heating, cooling ,radio and TV broadcasting ,computers, transportation etc.

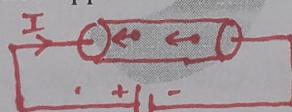
Electric circuit : For communicating or transferring energy from one point to another we require interconnection of electrical devices. An electric circuit is an interconnection of electrical elements. A simple electric circuit is shown in the figure. This consists of

three basic elements a battery ,a lamp and connecting wires.



Electric current : electric current is the rate of change of charge with time, measured in amperes(A).

When a conducting wire is connected to a battery the charges are compelled to move i.e. positive charge in one direction and negative charge in opposite direction. This motion of charges creates electric current.so in conductor current flows in the direction opposite to the flow of electron.



Current due to flow of charges in a conductor

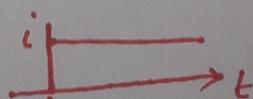
Mathematically,

$$i = \frac{dq}{dt} \Rightarrow q = \int_{t_0}^t i dt$$

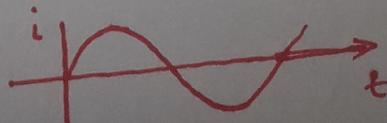
Unit of current is ampere. $1A = 1$ coulomb/sec

➤ As charges can vary with time in several ways, there are two types of current exists.

direct current(dc): a direct current is a current that remains constant with time.



alternating current(ac) :AC current is a current that



varies sinusoidally with time

Problems

1) the total charge entering a terminal is given by $q=5tsin4\pi t$ mc. calculate the current at $t=0.5s$

$$\text{Sol: } i = \frac{dq}{dt} = d(5tsin4\pi t)/dt = 5sin4\pi t + 20\pi tcos4\pi t$$

$$\text{At } t=0.5s, i = 5\sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

2) Determine the total charge entering a terminal between $t=1s$ and $t=2s$, if the current passing the terminal is $i=(3t^2-t)$ A

$$\begin{aligned}\text{Sol: } q &= \int_{t=1}^2 i dt = \int_1^2 (3t^2 - t) dt = \left(t^3 - \frac{t^2}{2}\right) \Big|_1^2 \\ &= (8 - 2) - (1 - \frac{1}{2}) = 5.5 \text{ C} \quad \text{Ans}\end{aligned}$$

Hw1) The total charge entering a terminal is $q=(10 - 10e^{-2t})$ mc .find the current at $t=0.5s$. [ans 7.36mA]

Hw2) The current flowing through an element is $i = 2A, 0 < t < 1$
 $= 2t^2, t > 1$

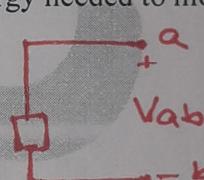
calculate the charge entering the element from $t=0$ to $t=2s$ [ans 6.667c]

➤ Electric potential (or voltage): voltage (or potential difference) is the energy required to move a unit charge through an element.

The voltage V_{ab} between two points a and b in an electric circuit is the energy needed to move a unit charge from a to b.

$$\text{Mathematically, } V_{ab} = \frac{dW}{dq}$$

Where W = energy in joules (J) q = charge in coulomb(c)



Unit of voltage is volt named after physicist Alessandro Volta, who invented the first voltaic battery.

$$1V = \frac{1J}{1C} = \frac{1 \text{ newton meter}}{\text{coulomb}}$$

Like electric current a constant voltage is called a dc voltage and a sinusoidally time varying voltage is called ac voltage.

voltage from a battery is DC voltage and AC voltage is produced by an ac electric generator.

➤ Power and energy:

power is the time rate of expanding or absorbing energy. Unit of power is watt.

$$\text{Mathematically } p = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} = v.i$$

$$P = v i = \text{instantaneous power}$$

If Power has a positive sign then power is being delivered to or absorbed by the element, whereas if Power has negative sign then power is being supplied by element.

According to law of conservation of energy the algebraic sum of power in a circuit at any instant of time, must be zero. This confirms that total power supplied to the circuit must equal to the total power absorbed.

➤ Energy :it is the capacity to do work. unit is Joule

$$\text{Mathematically } W = \int_{t_0}^t pdt = \int_{t_0}^t vidt$$

The electric power utility companies measures energy in what-hour(Wh)

$$1 \text{ Wh} = 3600 \text{ J}$$

The relationship between power and energy

$$\text{power} = \frac{\text{energy}}{\text{time}} \Rightarrow \text{Energy} = \text{power} \times \text{time} = P \times t$$

➤ Ohm's law:

It states that the current through any conductor is directly proportional to the potential difference between its ends, when all physical conditions (temperature, length, cross sectional area) are constant.

$$I \propto V \Rightarrow I = \frac{1}{R}V$$

R=Resistance of the conductor

Hw). Limitation of Ohm's Law

Resistance of metallic conductor :

$$R = \rho \frac{l}{a}$$

ρ = Resistivity of the conductor

l =Length

a =cross sectional area unit of resistance

unit of resistance : ohm

Problems

1) A heater element is made of nichrome wire having resistivity equal to 100×10^{-8} ohm-meter. The diameter of the wire is 0.4mm.calculate the length of the wire required to get a resistance of 40 ohm.

$$\text{sol}^n : R = 40 \text{ ohm} . \rho = 100 \times 10^{-8} \quad d = 0.4 \text{ mm} = 4 \times 10^{-4} \text{ m}$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (4 \times 10^{-4})^2 = 4\pi \times 10^{-8} \text{ m}^2$$

$$l = \frac{R a}{\rho} = \frac{40 \times 4 \pi}{100} = 5.03 \text{ m}$$

Hw1)A coil consists of 2000 turns of copper wire having cross sectional area of 0.8 mm^2 .the mean length per turn is 80cm, and the resistivity of copper is $0.2 \mu\Omega\text{-m}$. find resistance of the coil. [ans 40Ω] **400Ω**

➤ Circuit element

linear and nonlinear element : a linear element shows linear characteristics of voltage versus current. thus the parameters do not change with voltage or current. example :resistor, inductor, capacitor etc

For non linear element, the current passing through it does not change linearly with the linear change in applied voltage across it at a particular temperature and frequency. Example: diodes, transistors, thyristor etc.

Non linear element does not obey the ohm law.

Unilateral and bilateral element : if the magnitude of the current passing through an element is effected due to change in polarity of the applied voltage, the element is called a unilateral element.

example :diode ,transistor etc

on the other hand if the current magnitude remains the same even if the polarity of applied voltage is reversed, it is called bilateral element.

Example: resistor, inductor etc

Active and passive element : an active element is capable of generating energy while a passive element is not .

example of active element: generator, batteries, amplifiers ,voltage and current source,

example of passive element: resistor, inductor etc

➤ Sources: these generally deliver power to the circuit .

Types: i) Independent and dependent sources

ii) Ideal and practical sources

Independent and dependent sources: an independent source is an active element that provides a specific voltage or current that is completely independent of other circuit element



independent AC voltage



independent DC voltage



independent current source

independent sources are represented by circle symbols.

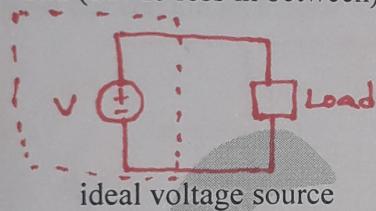
A dependent or controlled source is an active element in which the source quantity is controlled by another voltage or current.

types

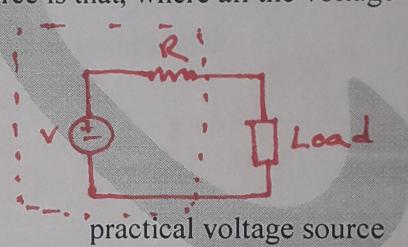
- i)voltage controlled voltage source
- ii)current controlled voltage source
- iii)voltage controlled current source
- iv)current controlled current source

Dependent sources are usually represented by Diamond shaped symbol.

Ideal and practical voltage source: ideal voltage source is that, where all the voltage value will appear across the load (i.e. no loss in between)



ideal voltage source

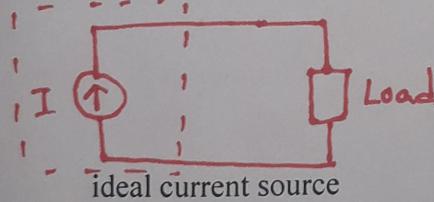


practical voltage source

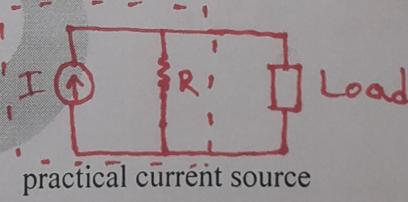
In practical voltage source the total voltage will not appear across the load. Some voltage will be drop in the series resistor. **Internal resistance of ideal voltage source is '0'.**

Ideal and practical current source: ideal current source is that which transfer all the current to the load. Internal resistance of Ideal current source is infinite. Example :solar cell.

If all the source current will not go to the load then that is known as practical current source



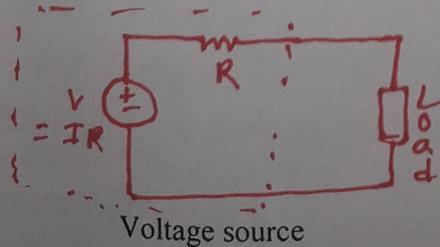
ideal current source



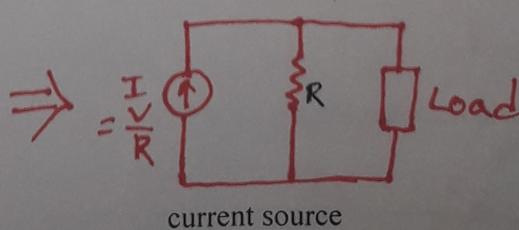
practical current source

➤ Source conversion: source conversion says that, if there is a voltage source with its internal resistance in series, it can be converted into a current source with its internal resistance in parallel and vice versa for current to voltage source conversion.

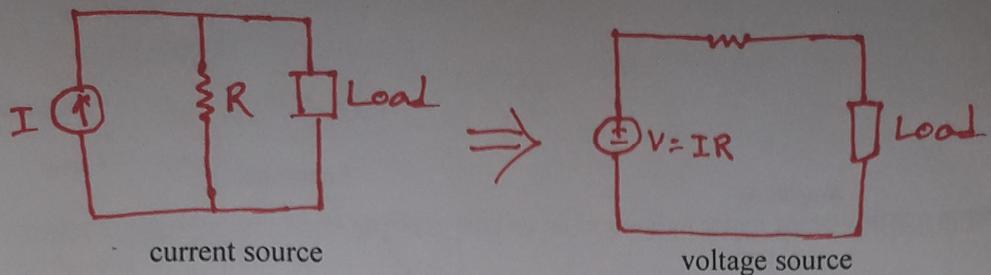
In voltage to current source conversion the current will be $= \frac{V}{R}$ and for current to voltage source conversion the voltage will be $= IR$. This is due to conservation of power in the circuit.



Voltage source



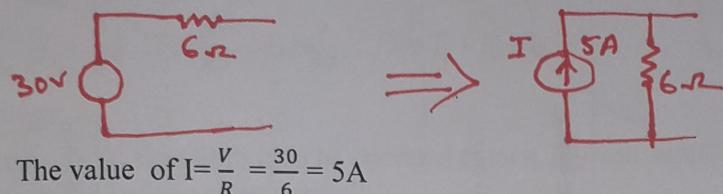
current source



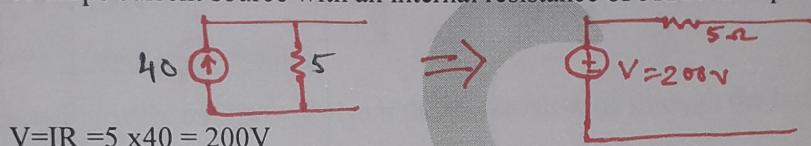
Problems:

1) Convert a voltage source of 30 volt along with an internal resistance of 6 ohm to a current source

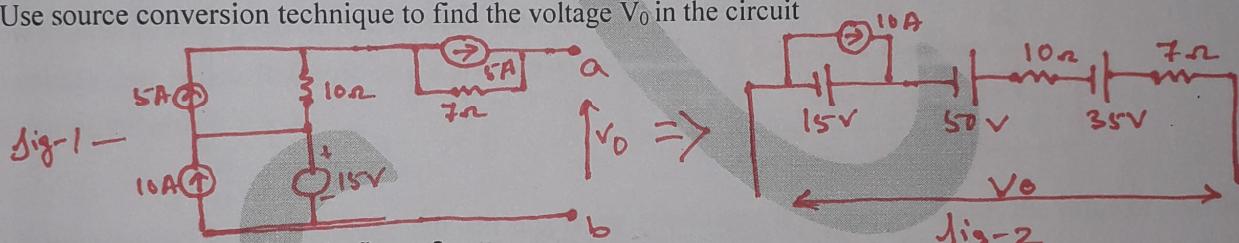
Sol:



2) Convert the 40 amps current source with an internal resistance of 5Ω to an equivalent voltage source



3) Use source conversion technique to find the voltage V_0 in the circuit

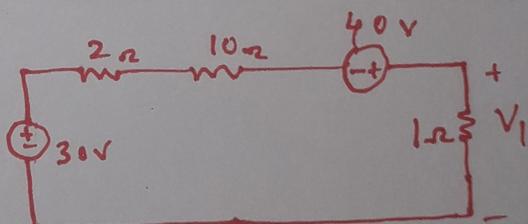
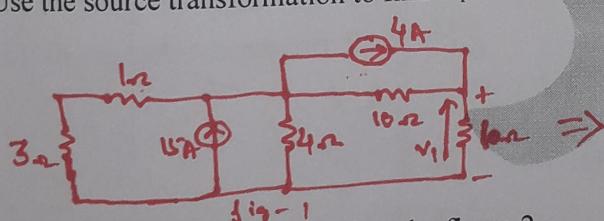


Sol: the conversion is shown in figure 2

When current and voltage source are connected in parallel, current source does not mean anything .so

$$\text{output voltage } V_0 = 15 + 50 + 35 = 100V$$

4) Use the source transformation to find V_1



Sol: The transformed circuit is given by figure 2.

Resistance 3Ω and 1Ω are in series, with 4Ω resistance in parallel. So equivalent resistance across 15amps current source is $(3+1) \parallel 4 = 2\Omega$

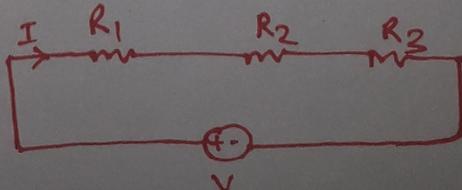
current flowing through circuit of figure 2

$$I = \frac{30+40}{2+10+1} = \frac{70}{13}$$

$$\text{Voltage drop in } 1\Omega \text{ resistance } V_1 = \frac{70}{13} \times 1 = 5.38V$$

➤ Resistance in series :Resistance are said to be in series when they are connected in such a way that some current flows through them.

for series connection



equivalent $R_{eq} = R_1 + R_2 + R_3$

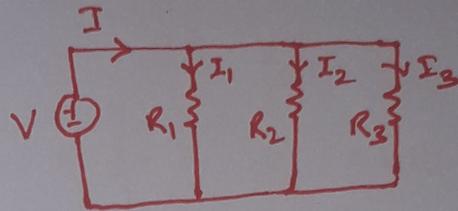
and $I = \frac{V}{R_{eq}}$

➤ Resistance in parallel: resistances are said to be in parallel when same voltage appears across the resistances.

In parallel connection equivalent resistance

will be $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$I = \frac{V}{R_{eq}}, I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}, I = I_1 + I_2 + I_3$$



Problems

1) A 100 volt 60 watt bulb is to be operated from a 220 volt supply. What is the resistance to be connected in series with the bulb to glow normally?

Sol: $P = 60W, V = 100V$

$$\text{Now } P = VI, \Rightarrow I = \frac{60}{100} = 0.6A$$

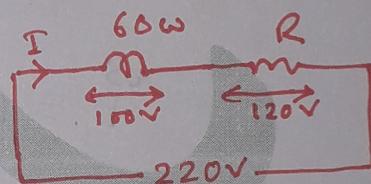
Lamp will operate normally on 220 volt also if the current flowing through the lamp remains the rated current that is 0.6 amps.

Now let resistance R is connected in series with the lamp.

So voltage drop across R should be $220 - 100 = 120V$

As current flowing is 0.6 A

$$R = \frac{120}{0.6} = 200\Omega$$



2) When a resistor is placed across 230 volt supply (dc) the current is 12 amps. What is the value of resistor that must be connected in parallel to increase the load current to 16 amps?

Sol: resistance of the given resistor $= R_1 = \frac{230}{12} = 19.167$

Now equivalent resistance of parallel combination $R_{eq} = \frac{230}{16} = 14.373\Omega$

Let R_2 is connected in parallel $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$$R_2 R_1 = R_{eq}(R_1 + R_2)$$

$$R_2 = \frac{R_{eq} R_1}{R_1 - R_{eq}} = \frac{14.375 \times 19.167}{19.167 - 14.375} \\ = 57.5\Omega$$

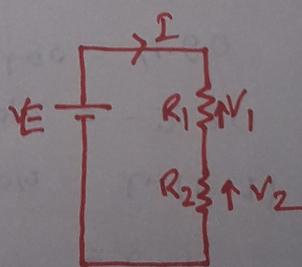
➤ Voltage divider theorem: in a series circuit the portion of applied EMF developed across each resistor is the ratio of that resistor's value to the total series resistance in the circuit.

$$V_1 = IR_1$$

now $I = \frac{E}{R_1 + R_2}$ $\therefore V_1 = \frac{E}{R_1 + R_2} \times R_1$

similarly $V_2 = \frac{E}{R_1 + R_2} \times R_2$

and for n number of resistors $V_i = \frac{R_i}{R_1 + R_2 + \dots + R_n} \times E$



- Current limiting resistors: sometimes a resistor is included in series with an electrical circuit or electronic device to drop the supply voltage down to desired value. This resistor can be treated as current limiting resistor.

Current divider rule: it is used to find out the current in the resistance in parallel circuit

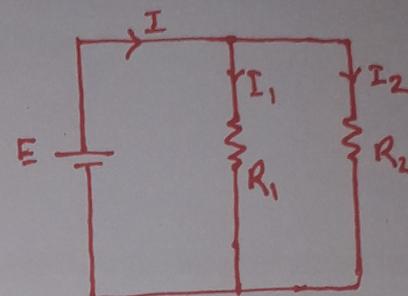
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I = \frac{E}{R_{eq}} = E \frac{R_1 + R_2}{R_1 R_2} \Rightarrow E = I \frac{R_1 R_2}{R_1 + R_2}$$

Now, $I_1 = \frac{E}{R_1}$, $I_2 = \frac{E}{R_2}$

$$I_1 = I \frac{R_1 R_2}{R_1 + R_2} \frac{1}{R_1} = I \frac{R_2}{R_1 + R_2}$$

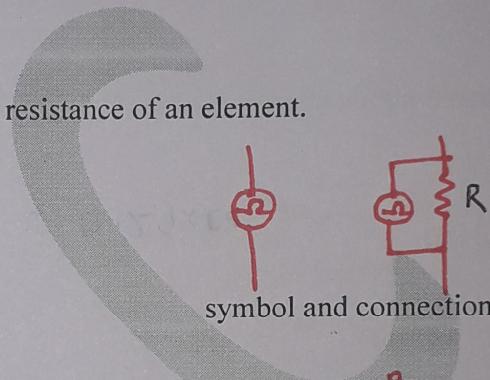
Similarly $I_2 = I \frac{R_1}{R_1 + R_2}$



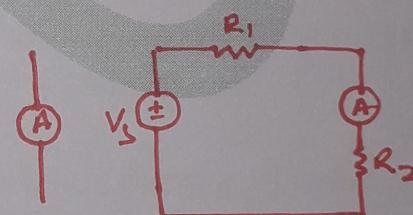
➤ Measuring devices

Ohm meter: it is a device which can measure the resistance of an element.

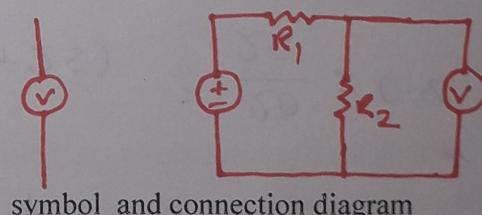
while measuring the resistance of an element it should be disconnected from any other circuit.



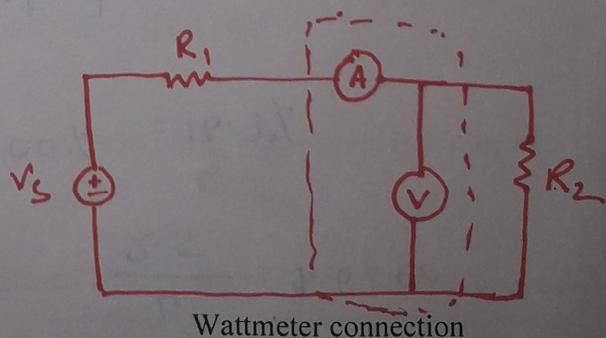
Ammeter: it is a device which measures the current flowing through an element. It is always connected in series with the circuit element. *The internal resistance of ammeter is 0 ohm.*



Voltmeter: it is a device which measures the voltage across an element. It is always connected in parallel to the element. *The internal resistance of Voltmeter is infinite ohm.*



Wattmeter: it measures the power dissipated by a circuit element. it measures the current flowing through the load and simultaneously the voltage across it and multiplies the two to provide a reading of power dissipated by the load.



Voltage measuring coil is known as potential coil and current measuring coil is known as current coil

Resistance: the resistance R of an element denotes its ability to resist the flow of electric current. Unit of resistance is ohm. It is represented by the symbol R and the symbol is Resistance R offered by a conductor depends on



- Directly with the length of the conductor (l)
- Varies inversely with the cross sectional area (a) of the conductor
- Nature of the material (resistivity ρ) of the conductor
- Depends on temperature

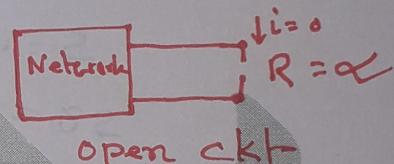
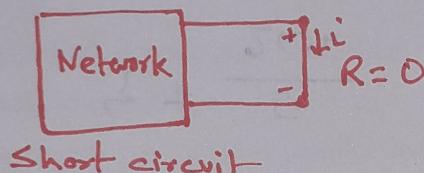
Mathematically, $R = \rho \frac{l}{a}$

Good conductors such as copper and Aluminium have low resistivity while insulators, such as mica and paper have high resistivity.

Power dissipated by a resistor: $p = vi = i^2R = v^2/R$

➤ Open circuit and short circuit:

A short circuit is a circuit element with zero resistance, and open circuit is a circuit element with infinite resistance.



conductance (G): it is the ability of an element to conduct electric current. it can be written as $G = \frac{1}{R}$. unit is mho or siemens .

➤ Inductors:

The electrical element that stores energy in association with the flow of current is called inductor(energy stored in magnetic field). Unit is Henry. symbol



In ideal condition the voltage across an inductor is proportional to the rate of change of current in it.

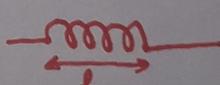
$$V \propto \frac{di}{dt}$$

$$V = L \frac{di}{dt} \quad \dots L = \text{inductance}$$

So inductance is the property of inductor which opposes the change in current flowing through it.

Inductance of a coil is given by $L = \frac{N^2 \mu A}{l}$

where , N = number of turns, A = cross sectional area , μ =permeability of the core



the energy stored in an inductor is $= \frac{1}{2} Li^2$

Important properties of an inductor: as voltage across an inductor is zero when the current is constant an inductor acts like a short circuit to DC.

Capacitors

A capacitor is a passive element which stores energy in the electric field produced by it. a capacitor consists of two conducting plates separated by an insulator (or dielectric). In practice the plates maybe Aluminium foil while the dielectric may be air, ceramic, paper or mica. The capacitor is set to store the electrical charge. the amount of charge stored is directly proportional to the applied voltage.

capacitance, unit is Farad(F).

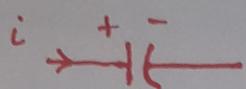
$Q = CV$... where C=proportionality constant known as

for parallel plate capacitor the capacitance is given by $C = \frac{\epsilon A}{d}$

where, A= surface area of each plate, d= distance between plates , ϵ = permittivity of the dielectric

Different type of capacitors: ceramic capacitor, polyester capacitor, electrolytic capacitor

symbol:



fixed capacitor



variable capacitor

The current voltage relationship : $i = C \frac{dv}{dt} \Rightarrow v = \frac{1}{C} \int idt$

Energy stored in capacitor is $= \frac{1}{2} Cv^2$

the current through the capacitor is zero when constant voltage is applied across it. So a capacitor acts like open circuit in DC.

Series and parallel connection of inductors and capacitors

Inductors

If two inductors are connected in series then

the equivalent inductance is $L_{eq} = L_1 + L_2$

If two inductors are connected in parallel then

the equivalent inductance is $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

Capacitor

If two capacitors having capacitance C_1 and C_2

are connected in series, then equivalent

capacitance is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

If two capacitors are connected in parallel

then equivalent capacitance is $C_{eq} = C_1 + C_2$

Problems

1) what is the voltage across a $3\mu F$ capacitor if the charge on the plate is $0.12mC$? how much energy is stored ?

by definition of capacitor we know that $C = \frac{q}{v} \Rightarrow v = \frac{q}{C} = \frac{0.12 \times 10^{-3}}{3 \times 10^{-6}} = 40V$

Energy stored $= \frac{1}{2} Cv^2 = \frac{1}{2} \times 3 \times 10^{-6} \times 40^2 = 2.4mJ$

2) The voltage across a $5\mu F$ capacitor is $v(t) = 10\cos 6000t$ volt. Calculate the current through it .

Sol: we know that $i = C \frac{dv}{dt} = 10 \times 5 \times 10^{-6} \times 6000 (-\sin 6000t) = -0.3\sin 6000t A$

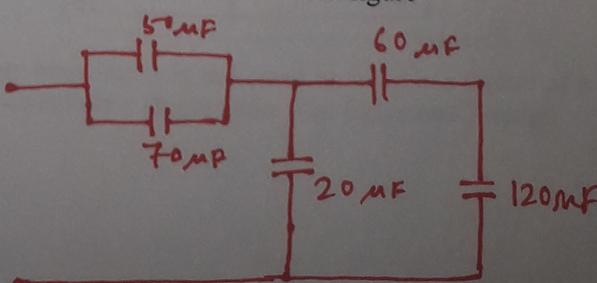
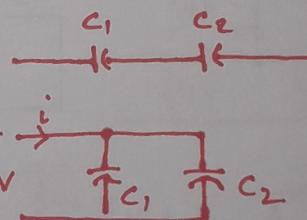
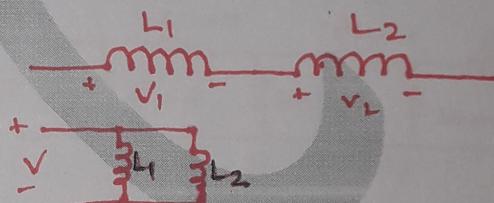
3) Find the equivalent capacitance seen at the terminals of the circuit as shown in figure

Soln: $50\mu F$ & $70\mu F$ are in parallel .

$$C_{eq1} = 50 + 70 = 120 \mu F$$

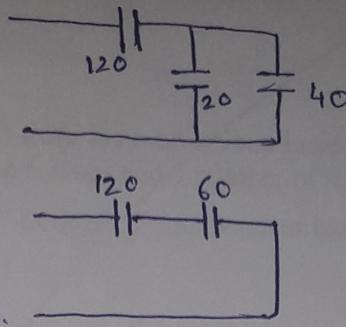
$60\mu F$ & $120\mu F$ are in series .

$$C_{eq2} = \frac{60 \times 120}{60 + 120} = 40 \mu F$$



Now, 201140

$$C_{eq} = 20 + 40 = 60 \mu F$$



Now 1201160

$$C_{eq} = \frac{120 \times 60}{120 + 60} = 40 \mu F$$

Ans.

4) The current through a 0.1H inductor is $i(t) = 10t e^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Sol: We know that, $V = L \frac{di}{dt} = 0.1 \times \frac{d}{dt} (10t e^{-5t})$

$$= 0.1 \times 10 [e^{-5t} + (-5t) e^{-5t}]$$

$$V = e^{-5t} (1 - 5t)$$

$$\text{Energy stored} = \frac{1}{2} L i^2 = \frac{1}{2} \times 0.1 \times (10t e^{-5t})^2$$
$$= 5t^2 e^{-10t} \text{ Joule.}$$

5) Calculate the equivalent inductance for the inductive ladder network as shown in figure

Sol: 40 and 20 in series, so $L_{eq1} = 40 + 20 = 60$

30 and 60 in parallel in next figure 2

$$\text{Leg 2} = \frac{30 \times 60}{30 + 60} = 20$$

100 mH & 20 mH in series in fig 3

$$\text{Leg 3} = 100 + 20 = 120 \text{ mH}$$

40 || 120 in fig 4

$$\text{Leg 4} = \frac{40 \times 120}{40 + 120} = 30 \text{ mH}$$

20 & 30 are in series in fig 5

$$\text{Leg 5} = 20 + 30 = 50 \text{ mH}$$

50 & 50 are parallel in fig 6

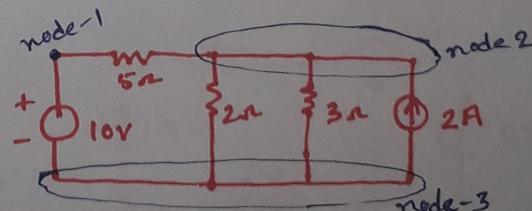
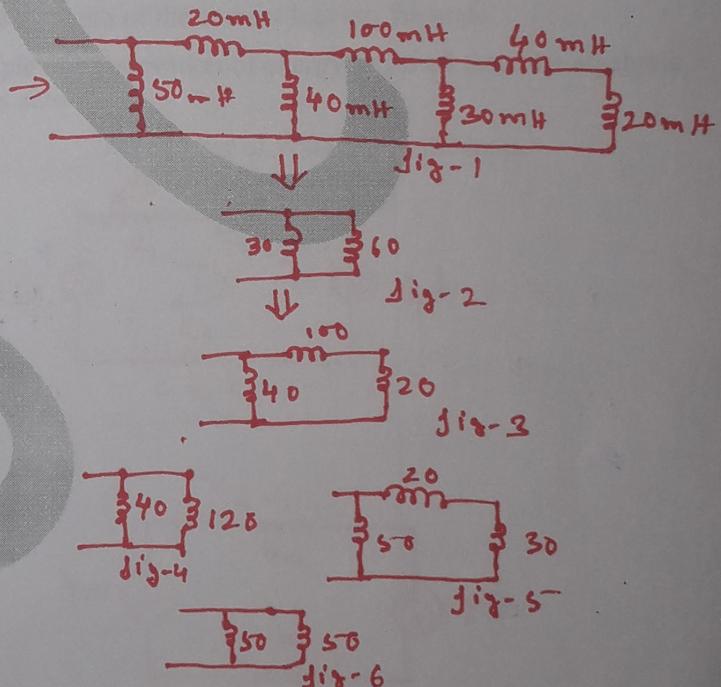
$$\text{Leg 6} = \frac{50 \times 50}{50 + 50} = 25 \text{ mH}$$

> Nodes, branches, loop and mesh:

A network is an interconnection of elements or devices, where a circuit is a network providing one or more closed path.

Branch: a branch represents a single element such as a source or passive element. For the circuit shown there are five branches namely 10V voltage source, 2A current source & 3 resistor

Node: it is a point of connection between two or more branches. It is indicated by a dot in a circuit. If a connecting wire connects two nodes, that two nodes constitute a single node. The figure has 3 nodes.



Loop: it is any closed path in a circuit. If there is no other loop inside the loop, then that loop is called independent loop or mesh. For the figure 6 loops and 3 number of meshes.

For a network with the 'b' branches, 'n' nodes & 'l' independent loops,

$$b = l + n - 1$$

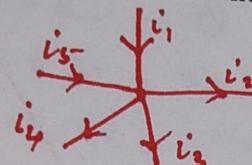
> Kirchoff's Laws

Kirchoff's first law is based on the law of conservation of charge. Charge cannot be created but must be conserved.

Kirchoff's current law (KCL): at any instant of time, the algebraic sum of current at node is zero.

$$\text{i.e. } \sum_{n=1}^N i_n = 0$$

N = number of branch connected to the nodes



by taking entering currents as positive and living currents as negative for node 1

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0$$

$$\Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$$

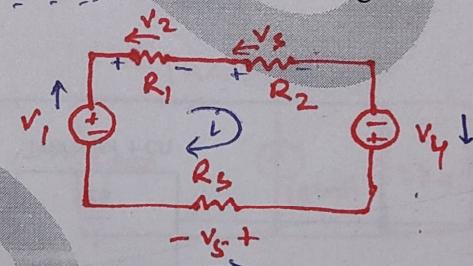
i.e. the sum of the current entering a node is equal to the sum of the current leaving the node.

Kirchoff's Voltage law (KVL): it is based on principle of conservation of energy. It stated that "the algebraic sum of all voltages around a closed path (or loop) is zero"

Applying KVL,

$$V_1 - V_2 - V_3 + V_4 - V_5 = 0$$

M = number of voltages in the loop.



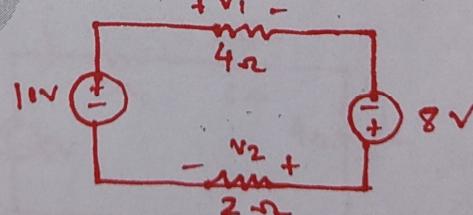
problems

1) find v_1 & v_2 in the circuit shown in the figure

Sol: applying KVL

$$10 - 4i + 8 - 2i = 0 \Rightarrow I = 3 \text{ A}$$

$$v_1 = 3 \times 4 = 12 \text{ V} ; v_2 = 3 \times 2 = 6 \text{ V}$$



2) Determine v_0 and i in the circuit shown in figure

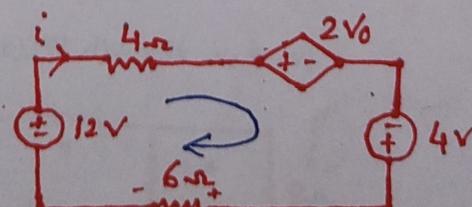
Sol: applying KVL

$$12 - 4i - 2v_0 + 4 - 6i = 0$$

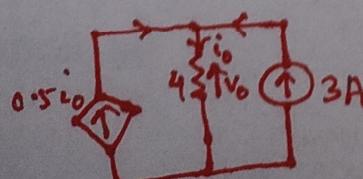
$$\text{Now from the fig } v_0 = 6i \Rightarrow -6i$$

$$\text{So, } 12 - 4i + 12i + 4 - 6i = 0 \Rightarrow i = -8 \text{ A}$$

$$v_0 = (-6)(-8) = 48 \text{ V}$$



3) Find i_0 & v_0



Sign are given & current in following (-) to (+) so
 $v_0 = -6i$

Sol: applying KCL, $0.5i_0 + 3 = i_0 \Rightarrow i_0 = 6A$ and $v_0 = 4i_0 = 24V$

4) Find current and voltage:

$$\text{LOOP-1}, 5 = 2i_1 + 8(i_1 - i_2)$$

$$5 = 10i_1 - 8i_2$$

$$15 = 30i_1 - 24i_2 \quad \dots \dots \textcircled{1}$$

$$\text{LOOP-2}, -3 + (i_2 - i_1) 8 + 4i_2 = 0$$

$$24i_2 - 16i_1 = 6 \quad \dots \dots \textcircled{2}$$

Solving $\textcircled{1} + \textcircled{2}$, $14i_1 = 21 \Rightarrow i_1 = 1.5A$, $i_2 = 1.5A$
 $v_1 = 3V$, $v_2 = 2V$, $v_3 = 5V$

5) Find current in 2Ω resistor

Sol: applying KVLL in loop 1

$$4I_1 + 2(I_1 + I_2) = 24 \dots \dots \textcircled{1}$$

$$\text{for loop 2, } I_2 + 2(I_1 + I_2) = 6 \dots \dots \textcircled{2}$$

solving equation 1 & 2

$$I_1 = 30/7 A, I_2 = -6/7 A$$

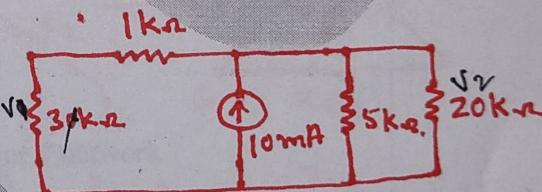
Current in 2Ω resistor, $I_1 + I_2 = 30/7 + (-6/7) = 24/7 A$

HW1) For the circuit shown in the figure find

a) v_1 & v_2

b) power dissipated in $3k\Omega$ and $20k\Omega$ resistor

c) power supplied by the current source



[a) 15v, 20v b) 75MW, 20MW c) 200MW]

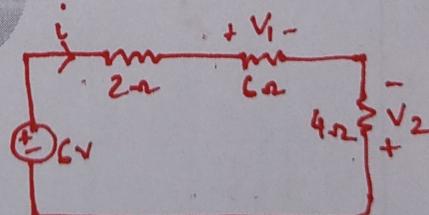
HW2) for the circuit shown in the figure find

a) the equivalent resistance seen by the source

b) $i = ?$

c) power delivered by the source

d) the voltage v_1 & v_2



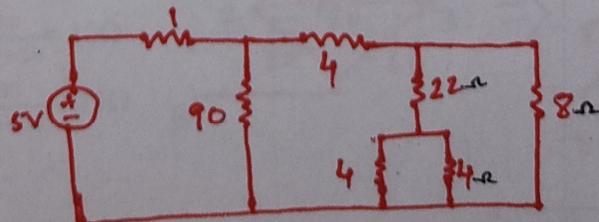
[a) 12Ω b) 0.5A c) 3W d) $v_1 = 3V, v_2 = -2V$]

HW3) Find the equivalent resistance

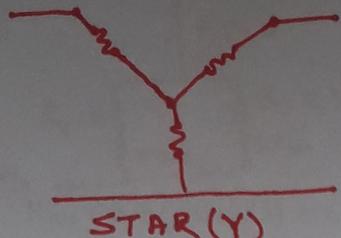
seen by the source and the current

i in the circuit shown in figure.

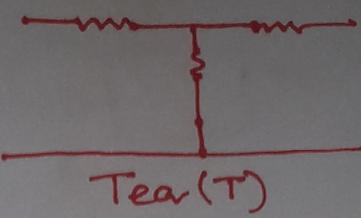
[ans $R_{eq} = 10\Omega, i = 0.5A$]



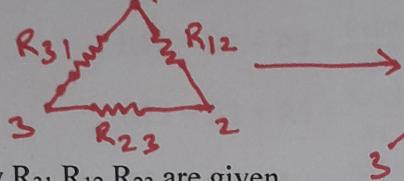
➤ Star delta transformation.



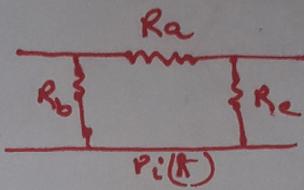
OR



Delta to star conversion



OR



now R_{31}, R_{12}, R_{23} are given

$$R_1 = \frac{R_{12} R_{13}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{21} R_{23}}{R_{12} + R_{23} + R_{31}}$$

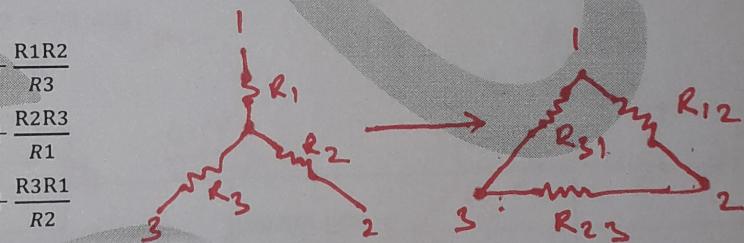
$$R_3 = \frac{R_{31} R_{32}}{R_{12} + R_{23} + R_{31}}$$

Star to delta conversion

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

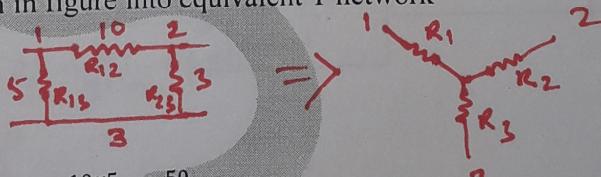
$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$



Problems

1) Convert the π network shown in figure into equivalent T network



from delta to star conversion

$$R_1 = \frac{R_{12} R_{13}}{R_{12} + R_{23} + R_{31}} = \frac{10 \times 5}{10 + 5 + 3} = \frac{50}{18} = 2.7\Omega$$

$$R_2 = \frac{R_{21} R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{3 \times 10}{10 + 5 + 3} = 1.67\Omega$$

$$R_3 = \frac{R_{31} R_{32}}{R_{12} + R_{23} + R_{31}} = \frac{3 \times 5}{10 + 5 + 3} = 0.8\Omega$$

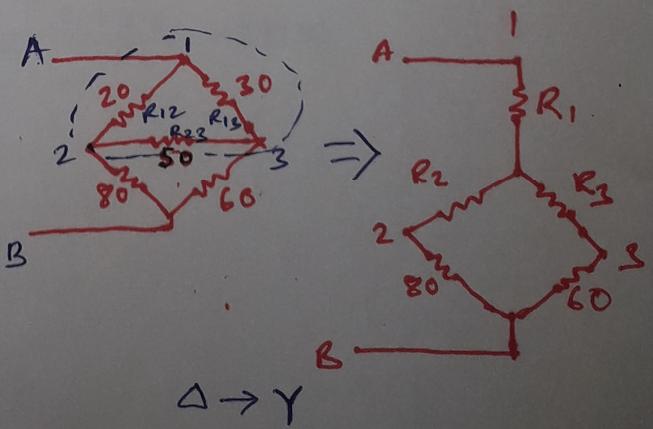
2) Find R seen from AB of the network

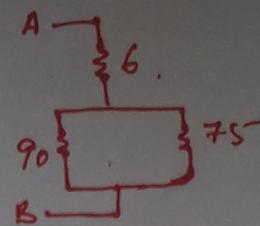
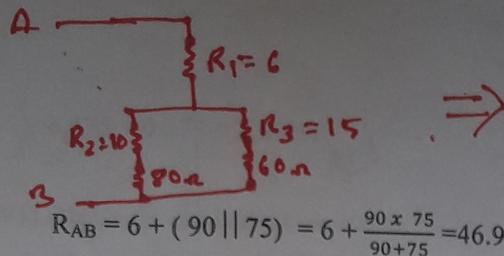
$$R_1 = \frac{R_{12} R_{13}}{R_{12} + R_{23} + R_{31}} = \frac{20 \times 30}{20 + 30 + 50} = 6\Omega$$

$$R_2 = \frac{R_{21} R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{20 \times 50}{20 + 30 + 50} = 10\Omega$$

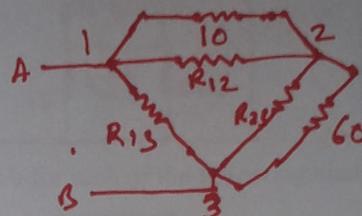
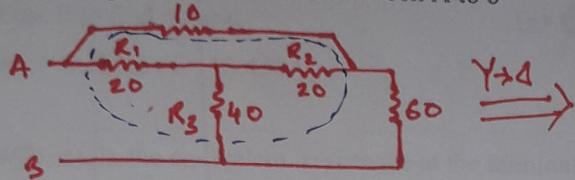
$$R_3 = \frac{R_{31} R_{32}}{R_{12} + R_{23} + R_{31}} = \frac{30 \times 50}{20 + 30 + 50} = 15\Omega$$

Now the network can be drawn as





3) find the equivalent resistance between A to b

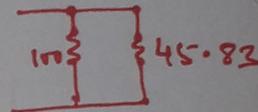
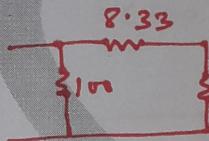
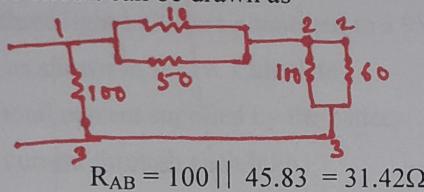


$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} = 20 + 20 + \frac{20 \times 20}{40} = 50\Omega$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} = 20 + 40 + \frac{20 \times 40}{20} = 100\Omega$$

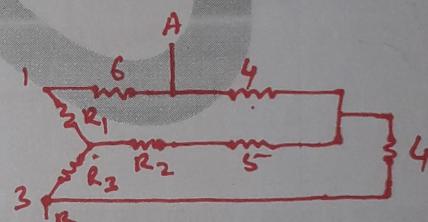
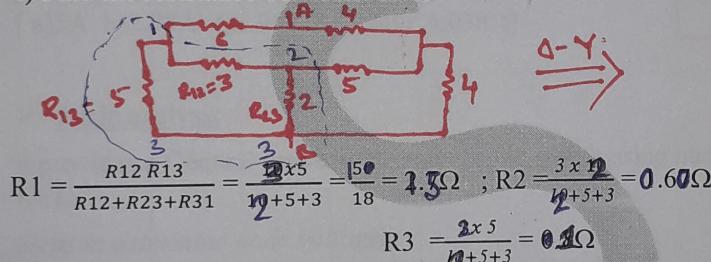
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} = 100\Omega$$

Now the circuit can be drawn as



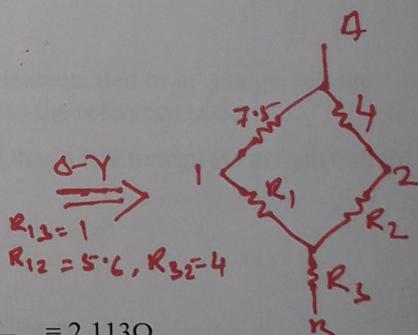
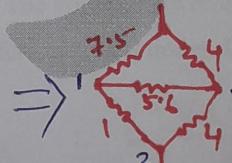
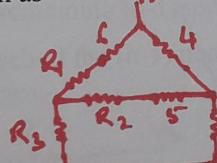
$$R_{AB} = 100 \parallel 45.83 = 31.42\Omega$$

4) Find the resistance between A & B



$$R_3 = \frac{2 \times 5}{2+5+3} = 0.4\Omega$$

Now it can be drawn as



$$R_1 = \frac{R_{13} R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{1 \times 5.6}{1+5.6+4} = \frac{5.6}{10.6} = 0.528\Omega ; \quad R_2 = \frac{4 \times 5.6}{10.6} = 2.113\Omega$$

$$R_3 = \frac{1 \times 4}{10.6} = 0.377\Omega$$

now the network can be drawn as

$$R_{AB} = \{(7.5 + 0.528) \parallel (4 + 2.113)\} + 0.377$$

$$= (8.028 \parallel 6.13) + 0.377$$

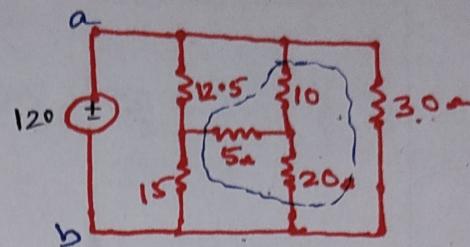
$$= 3.475 + 0.377$$

$$= \frac{49.21}{14.158} + 0.377$$

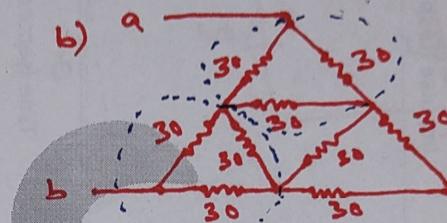
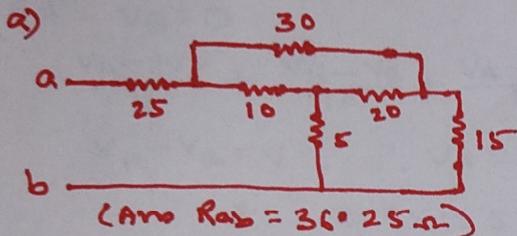
$$= 3.475 + 0.377 = 3.852\Omega$$

Ans

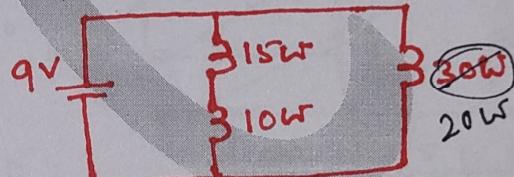
- HW1) Obtain the equivalent resistance R_{ab} for the circuit shown in figure and use it to find current.
 $[R_{ab} = 9.632\Omega, i = 12.458A]$



- HW2) obtain the equivalent resistance at the terminals a b for each of the following circuit



- HW3) three light bulbs are connected to a 9V battery as shown in figure. Calculate
a) The total current supplied by the battery
b) The current through each bulb
c) the resistance of each bulb
 $[a) 5A \ b) 2.78A \ c) 1.941\Omega, 1.294\Omega, 4.058\Omega]$

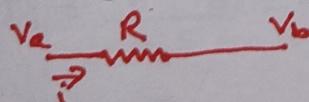


➤ Node analysis

it provides a General procedure for analysing circuit using node voltages as circuit variable. it is based on KCL .

Steps to determine node voltages:

- Select a node as the reference node. This node usually has most elements tied to it. Assign voltage v_1, v_2, \dots, v_{n-1} to the remaining $(n-1)$ nodes. The voltages are referred to the reference node.
- Apply KCL to each of the $(n-1)$ non reference notes nodes. Use Ohm's law to express branch current in terms of node voltages.



$$i = \frac{v_a - v_b}{R}$$

- iii) Solve the simultaneous equation to obtain the unknown node voltages

Note: reference node is having zero potential and given by symbol of earth ground as shown (\ominus)

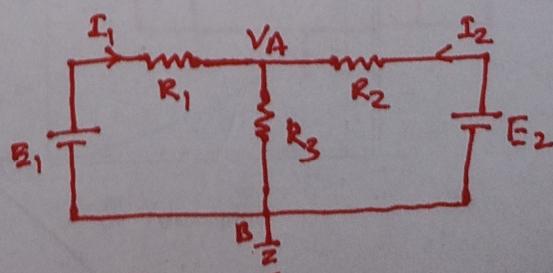
Explanation

Consider B as reference node,

$$v_B = 0$$

$$I_1 = \frac{E_1 - V_A}{R_1}, \quad I_2 = \frac{E_2 - V_A}{R_2}$$

$$I_3 = \frac{V_A - V_B}{R_3}$$



now applying KCL at A

$$-I_1 - I_2 + I_3 = 0$$

$$-\left(\frac{E_1 - V_A}{R_1}\right) - \left(\frac{E_2 - V_A}{R_2}\right) + \frac{V_A}{R_3} = 0$$

This is the nodal form of the network

Problems

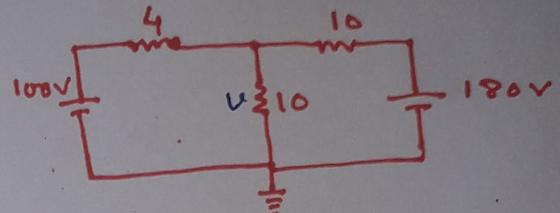
1) Find out V

$$\text{Soln. } V_B = 0$$

$$\frac{V_A - 150}{4} + \frac{V_A - V_B}{10} + \frac{V_A - 180}{10} = 0$$

$$V_A - V_B = V, \therefore V_A = V$$

$$\therefore V = 95.55 \text{ V}$$



2) find the current in different branches of the network shown in figure using Nodal analysis

Sol: At node A

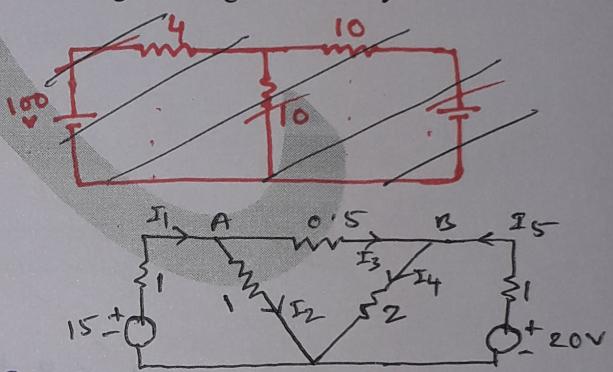
$$\frac{V_A - 15}{1} + \frac{V_A - V_B}{0.5} + \frac{V_A - 0}{1} = 0$$

$$4V_A - 2V_B = 15 \quad \dots \textcircled{1}$$

At node B

$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$3.5V_B - 2V_A = 20 \quad \dots \textcircled{2}$$



Solving equation 1 and 2, $V_B = 11 \text{ V}$, $V_A = 9.25 \text{ V}$

$$I_1 = \frac{V_A - 15}{1} = -5.75 \text{ A}$$

$$I_4 = \frac{V_B}{2} = 5.5 \text{ A}$$

$$I_2 = \frac{V_A}{1} = 9.25 \text{ A}$$

$$I_5 = \frac{V_B - 20}{1} = -9 \text{ A}$$

$$I_3 = \frac{V_A - V_B}{0.5} = -3.5 \text{ A}$$

3) find v_x & v_y using Nodal analysis

At node x

$$-10 + \frac{v_x}{6} + 2 + \frac{v_x - v_y}{4} = 0$$

$$5v_x - 3v_y = 96 \quad \dots \textcircled{1}$$

At node y

$$\frac{v_y - v_x}{4} + \frac{v_y}{10} + \frac{v_y}{5} = 0$$

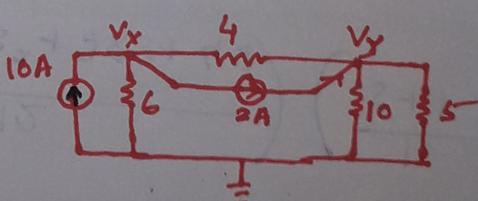
$$5v_x - 11v_y = -40 \quad \dots \textcircled{2}$$

solving equation 1 and 2

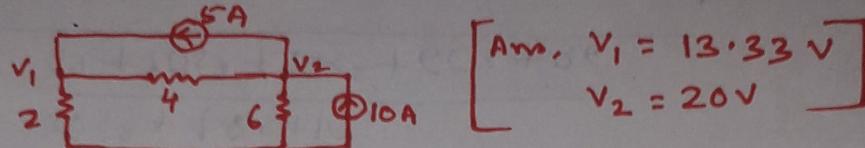
$$-3v_y + 11v_y = 96 + 40$$

$$8v_y = 136$$

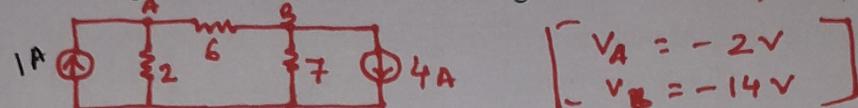
$$v_y = 17 \text{ V} \quad \& \quad v_x = 29.4 \text{ V}$$



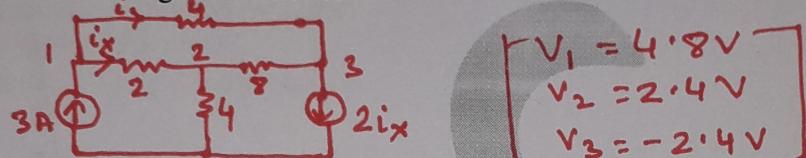
HW1) calculate the node voltage in the circuit shown.



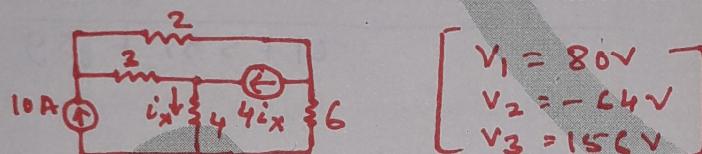
HW2) obtain the node voltages in the circuit as shown in figure



HW3) determine the voltage at the nodes



HW4) find the voltages at the three non reference nodes in the circuit as shown in figure



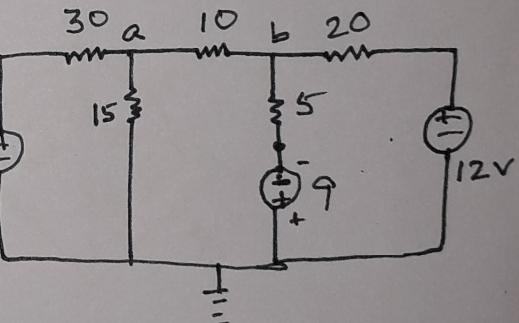
Note 1: if a voltage source is connected between the reference node and non reference node, then the voltage at the non reference node is equal to the voltage of the voltage source.

Ex. Find the voltages at node a and b using Nodal analysis

Sol: at node a,

$$\frac{V_a - 10}{30} + \frac{V_a}{15} + \frac{V_a - V_b}{10} = 0$$

$$6V_a - 3V_b = 10 \quad \dots \textcircled{1}$$



At node b

$$\frac{V_b - 12}{20} + \frac{V_b - V_a}{10} + \frac{V_b - (-9)}{5} = 0$$

$$2V_a - 7V_b = 24 \quad \dots \textcircled{2}$$

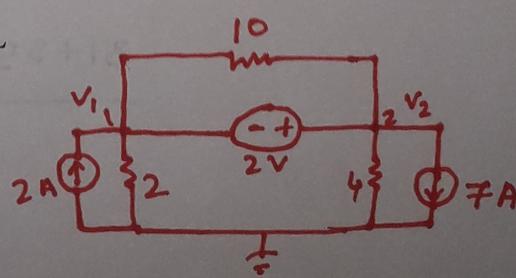
by calculation $V_a = -0.0556V$; $V_b = -3.445V$

Note 2: Supernode: if the voltage source is connected between two non reference nodes, the two non reference nodes form a supernode.

properties of a supernode:

- i) it has no voltage of its own
- ii) it requires the application of both KCL and KVL

Ex. For the circuit shown in figure find the node voltages



Sol: here supernode contains 2V volt source, node 1 and 2, and the 10Ω resistance.

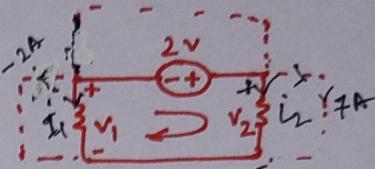
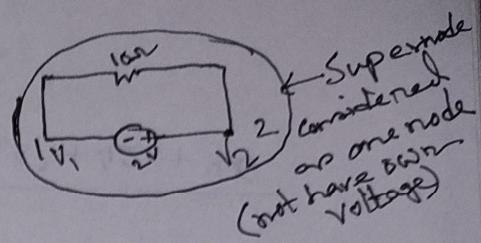
applying KCL at supernode, $2 = i_1 + i_2 + 7$

$$2 = (v_1 / 2) + (v_2 / 4) + 7$$

$$8 = 2v_1 + v_2 + 28 \Rightarrow 2v_1 + v_2 = -20 \dots\dots\dots(1)$$

applying KVL in supernode $v_1 + 2 - v_2 = 0 \Rightarrow v_1 - v_2 = -2 \dots\dots\dots(2)$

solving 1 and 2, $v_1 = -7.33V$, $v_2 = -5.33V$



➤ Mesh analysis: it is based on KVL. it provides a General procedure for analysing circuits using mesh current as a circuit variable.

Steps to determine mesh analysis:

- select meshes and assign mesh currents
- apply kvl to each of the mesh
- solve the equations to get mesh current

explanation :

number of mesh = 2

current in meshes : I_1 & I_2

applying KVL at mesh 1

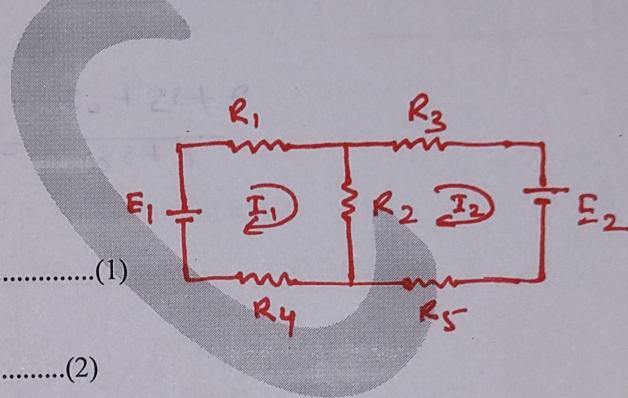
$$-E_1 + I_1 R_1 + (I_1 - I_2) R_2 + I_1 R_4 = 0$$

$$E_1 = (R_1 + R_2 + R_4) - I_2 R_2 \dots\dots\dots(1)$$

for mesh 2

$$E_2 + I_2 R_5 + I_2 R_3 + (I_2 - I_1) R_2 = 0$$

$$E_2 = I_2 R_2 - (R_2 + R_3 + R_5) I_2 \dots\dots\dots(2)$$



Problems:

1) find the loop currents

soM, Loop - 1

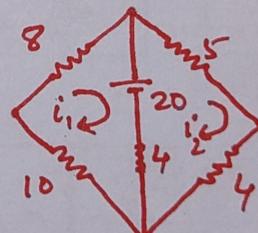
$$20 + (I_1 - I_2) 4 + 10I_1 + 8I_1 = 0$$

$$22I_1 - 4I_2 = -20 \quad \text{--- (1)}$$

Loop - 2, $5I_2 + 4I_2 + (I_2 - I_1) 4 - 20 = 0$

$$-4I_1 + 13I_2 = 20$$

from eqn 1 & 2, $I_1 = -0.667A$, $I_2 = 1.33A$



2) find loop currents

Sol: number of mesh 3

Mesh currents I_1, I_2, I_3

for loop 1,

$$6I_1 + 4(I_1 - I_2) + 10(I_1 - I_3) - 12 = 0$$

$$\Rightarrow 10I_1 - 2I_2 - 5I_3 = 6 \quad \text{--- (1)}$$

for loop 2,

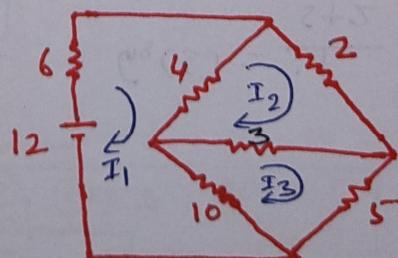
$$(I_2 - I_3) 3 + (I_2 - I_1) 4 + 2I_2 = 0$$

$$4I_2 - 9I_2 + 3I_3 = 0 \quad \text{--- (2)}$$

for loop 3

$$(I_3 - I_2) 3 + 5I_3 + 10(I_2 - I_1) = 0$$

$$18I_3 - 3I_2 - 10I_1 = 0$$



solving 1, 2 & 3, $I_1 = 11.42A$, $I_2 = 10.84$, $I_3 = 17.30A$

3) find current through 5Ω resistance

number of mesh 3

currents in the mesh = I_1, I_2, I_3

loop 3 contains a current source, so $I_3 = 3A$

for loop 1

$$(I_1 + 3)5 + (I_1 - I_2)2 - 12 = 0$$

$$7I_1 - 2I_2 + 3 = 0 \quad \dots \textcircled{1}$$

for loop 2

$$(I_2 + 3)6 + 6 + I_2 \cdot 1 + (I_2 - I_1)2 = 0$$

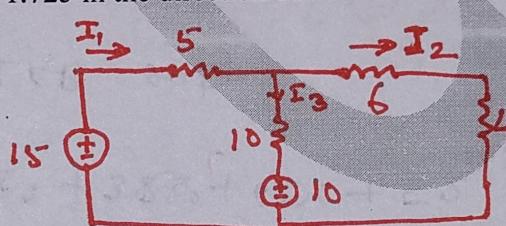
$$9I_2 + 24 - 2I_1 = 0 \quad \dots \textcircled{2}$$

Solving 1 and 2 $I_1 = -1.275$, $I_2 = -2.95A$

Current through 5Ω resistor = $-1.275 + 3 = 1.725$ in the direction of a to b

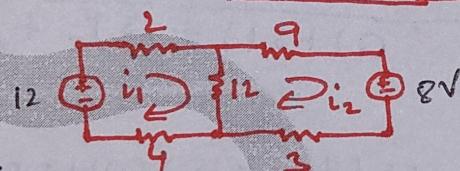
HW1) Find I_1, I_2, I_3 using mesh analysis

$$[I_1 = 1A, I_2 = 1A, I_3 = 0A]$$



HW2) calculate I_1, I_2 in figure

$$\begin{bmatrix} i_1 = \frac{2}{3} A \\ i_2 = 0A \end{bmatrix}$$



HW3) find I_0 using mesh analysis

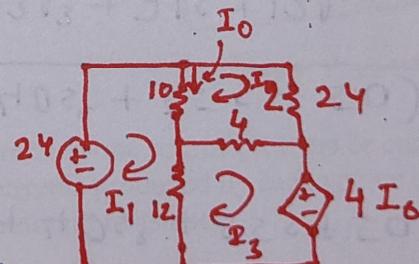
$$[\text{Hints: } I_0 = (I_1 - I_2)]$$

$$4I_0 = \text{dependent voltage source}$$

for loop 3

$$4(I_3 - I_2) + 12(I_3 - I_1) + 4I_0 = 0$$

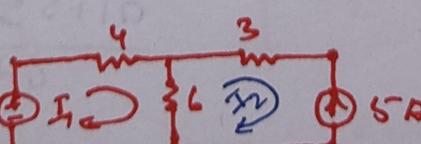
$$4(I_3 - I_2) + 12(I_3 - I_1) + 4(I_1 - I_2) = 0$$



$$[\text{ans } I_0 = 1.5A]$$

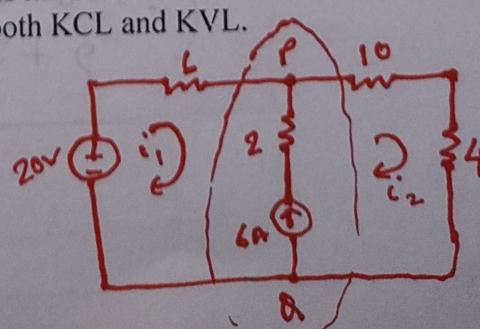
HW4) find loop currents

$$[I_{12} = 2A, I_{22} = -5A]$$



Note : Supermesh : A supermesh results when two meshes have a current source in common. A supermesh has no currents of its own. It requires the application of both KCL and KVL.

Ex 1. Solve the loop currents i_1, i_2 using mesh analysis



Sol: there is a supermesh formed as 6 amps source common to both meshes.

applying KVL in supermesh

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$+6i_1 + 14i_2 = 20 \quad \dots \textcircled{1}$$

applying KCL at node p

$$i_1 + 6 = i_2 \quad \dots \textcircled{2}$$

by solving equation 1 and 2, $i_1 = -3.2A$, $i_2 = 2.8A$

Ex2. Use mesh analysis to determine the loop currents as shown in figure

Sol: by applying KVL in supermesh

$$-6 + 2(i_1 - i_2) + 4(i_3 - i_2) + 8i_3 = 0$$

$$\Rightarrow -2i_1 + 6i_2 - 12i_3 = -6 \quad \dots \textcircled{1}$$

KVL in mesh 2

$$2i_2 + 4(i_2 - i_3) + 2(i_2 - i_1) = 0$$

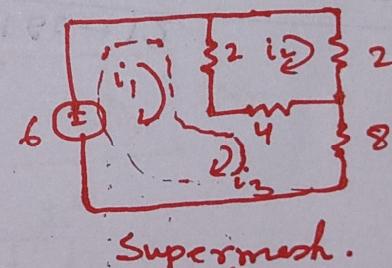
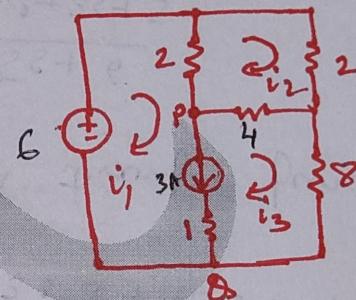
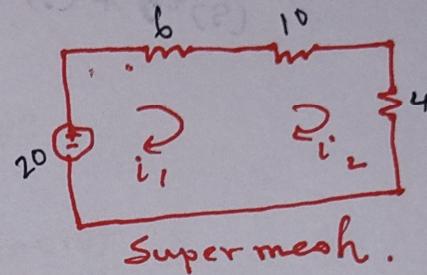
$$2i_2 - 8i_2 + 4i_3 = 0 \quad \dots \textcircled{2}$$

KCL at node Q

$$3 + i_3 = i_1$$

$$i_1 - i_3 = 3 \quad \dots \textcircled{3}$$

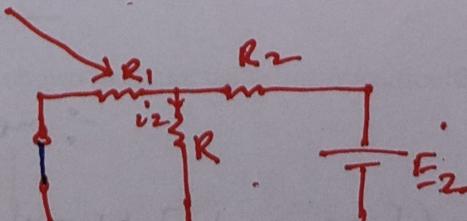
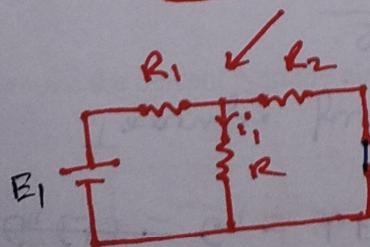
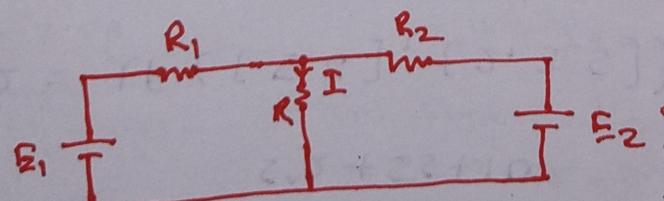
By solving 1 2 and 3, $i_1 = 3.47A$, $i_2 = 1.105A$, $i_3 = 0.474A$



Network theorem

Superposition theorem: in any linear network containing more than one source (voltage or current) the total current in any branch is the algebraic sum of the individual currents produced by each source acting alone. Meanwhile all other sources are replaced by their respective internal resistance.

Explanation:



From Theorems

$$I = i_1 + i_2$$

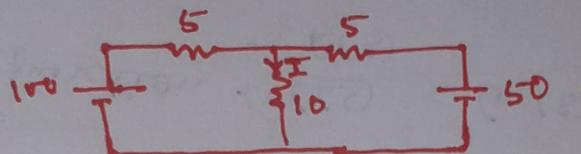
- Note: i) internal resistance of voltage sources is zero.
ii) internal resistance of current sources is infinite.

Steps

- turn off (replace by internal resistance) all independent source except one source. find the output due to that active source.
- repeat step one for each of the independent sources
- find the total contribution by adding algebraically

Problems

1) find I



step 1, from the figure

$$R_{eq} = 5 + (5 \parallel 10) = 8.33 \Omega$$

$$I_1 = \frac{100}{8.33} = 12A$$

by current division rule

$$\text{Step 2, } i'_1 = 12 \times \frac{5}{10+5} = 12 \times \frac{1}{3} = 4A$$

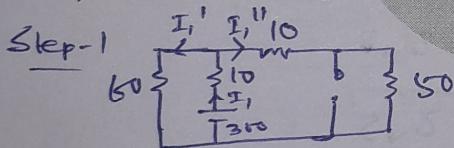
$$R_{eq} = 8.33 \Omega, I_2 = \frac{50}{8.33} = 6A$$

$$i''_1 = 6 \times \frac{5}{15} = 2A$$

from superposition theorem

$$I = i'_1 + i''_1 = 2 + 4 = 6A$$

2) Find current through 50Ω resistance



10Ω & 50Ω are in series.

$$R_1 = 50 + 10 = 60\Omega$$

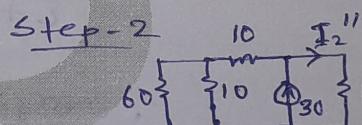
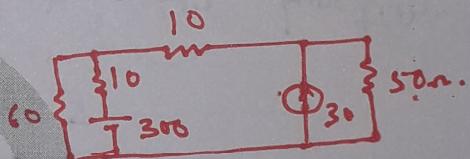
$$R_{eq} = 10 + (60 \parallel 60)$$

$$= 10 + 30 = 40\Omega$$

$$I_1 = \frac{30}{40} = 7.5A$$

Current through 50Ω

$$I''_1 = 7.5 \times \frac{60}{120} = 3.75A$$



$$R_1 = 10 + (60 \parallel 60) = 18.57\Omega$$

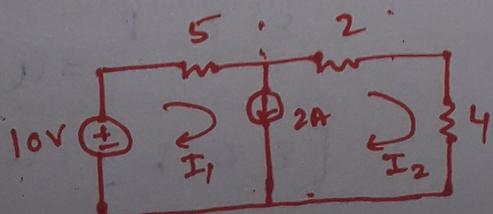
Current through 50Ω

$$I''_2 = 30 \times \frac{18.57}{68.57} = 8.12A$$

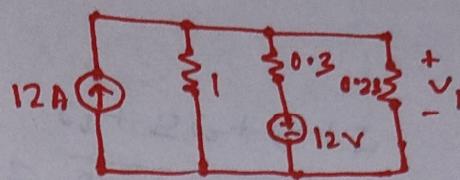
From Superposition,

$$I = 3.75 + 8.12 = 11.87A$$

Hw1) Determine the current I_2 in the circuit as shown in figure using superposition theorem [$I_2 = 0A$]



Hw2) determine the voltage across 0.23Ω resistor in the circuit as shown
[5.99V]

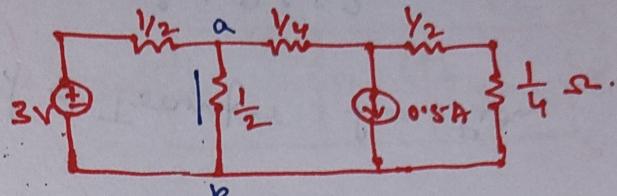


Hw3) find the current i through (ab)
 $\frac{1}{2}\Omega$ resistor by superposition theorem

$$\text{Ans} - 1.137$$

Hw4) find i using superposition

[2A]



Hw5) find the current through 20Ω using superposition

$$\text{Ans} - 1.17$$

Thevenin theorem : it states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} , where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals with the independent sources are turned off.

explanation

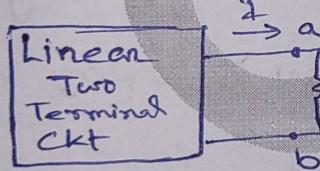


fig-1 Original ckt

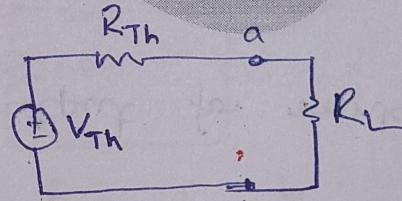


fig2 - Thevenin equivalent ckt.

figure 1 shows a linear two terminal circuit and a load resistor connected across the terminals a and b. This circuit can be replaced by an equivalent circuit consisting of V_{Th} in series with R_{Th} as shown in figure 2

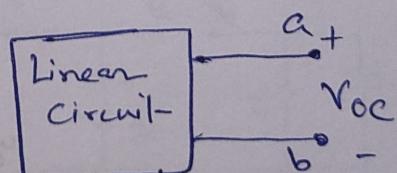
To find V_{Th} :-

Open the Load terminals and measure
or calculate the open circuit voltage(V_{oc})
across the terminal as shown in figure.

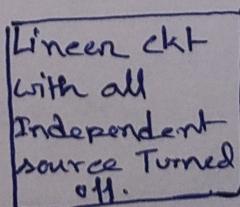
this $V_{oc} = V_{Th}$

to find R_{Th} :-

Open the load resistor when the linear circuit has no independent sources.
by looking through the opened terminals,
the internal resistance is Thevenin
resistance (R_{Th}) as shown in the figure.

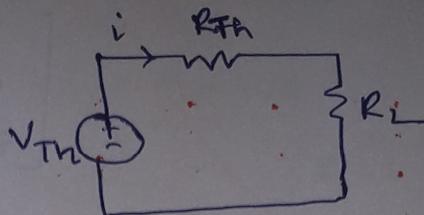


Finding V_{Th}



Finding R_{Th}

Draw the Thevenin's equivalent circuit with V_{Th} , R_{Th} & R_L to find out the current.



Problems

- 1) find the thevenin equivalent circuit of the circuit shown in figure at terminals a to b.
then find the current through $R_L = 6\Omega$

sol: To find V_{Th}

using Nodal analysis

$$\frac{V_A - 32}{4} + \frac{V_A - 0}{12} - 2 = 0 \\ \Rightarrow 3V_A - 96 + V_A - 24 = 0 \\ \Rightarrow V_A = 30V$$

Now, $V_{OC} = V_A = 30V$
& $V_{Th} = V_{OC} = 30V$

To find R_{Th}

$$R_{Th} = 1 + (4//12) \\ = 4\Omega$$

thevenin equivalent circuit

$$i = \frac{V_{Th}}{R_{Th} + R_L} \\ = \frac{30}{4 + 6} = 3A$$

- 2) Find the current through 6 ohm resistor using Thevenin's Theorem

sol: to find V_{Th}

using Nodal analysis at node a

$$\frac{V_a - 42}{8} + \frac{V_a - 30}{4} = 0 \\ \Rightarrow V_a - 42 + 2V_a - 60 = 0$$

$$\Rightarrow 3V_a = 102$$

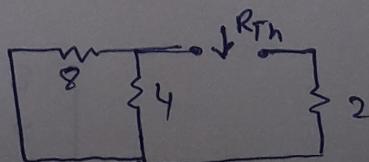
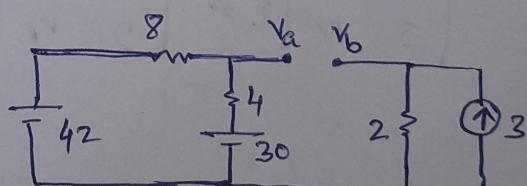
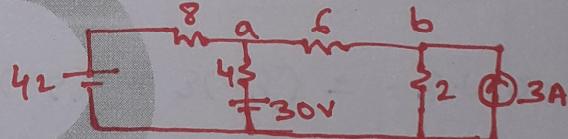
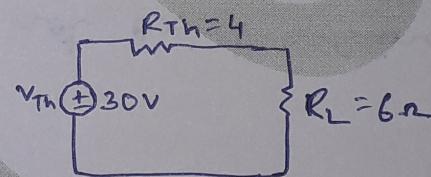
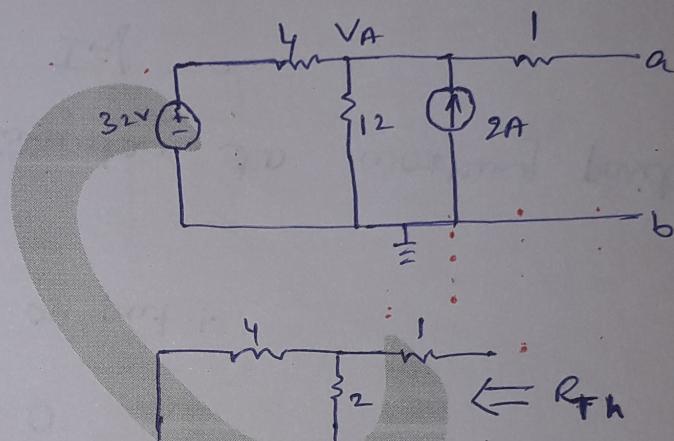
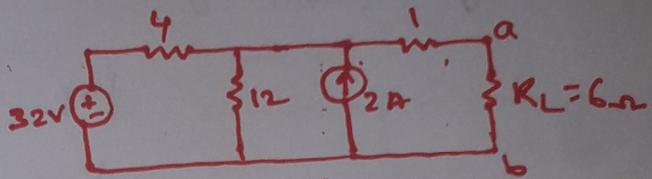
at node b,
 $V_a = 34V$

$$\frac{V_b - 0}{2} - 3 = 0 \Rightarrow V_b = 6V$$

$$\text{Now } V_{ab} = V_a - V_b = 34 - 6 = 28V = V_{OC} = V_{Th}$$

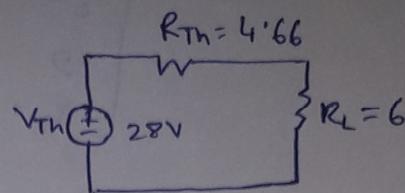
finding R_{Th}

$$R_{Th} = 2 + (8//4) \\ = 4.66\Omega$$



Thevenin's equivalent circuit

$$I = \frac{28}{6+4.66} \\ = 2.625 \text{ A Am.}$$



3) Find out the current through $R_L = R_2 = 1\Omega$ resistor using

Thevenin's theorem and hence calculate the voltage across V_{cg}

Sol: finding V_{Th} across ab terminals

$$\text{at node a, } V_a = 3V$$

at node b

$$\frac{V_b - V_c}{4} + \frac{V_b - 0}{2} = 0$$

$$V_b - V_c + 2V_b = 0$$

$$3V_b - V_c = 0 \Rightarrow V_c = 3V_b \quad \text{---(1)}$$

at node c

$$\frac{V_c - V_a}{3} + \frac{V_c - V_b}{4} - 2 = 0$$

$$\Rightarrow \frac{3V_b - 3}{3} + \frac{3V_b - V_b}{4} - 2 = 0$$

$$\Rightarrow \frac{3V_b - 3}{3} + \frac{V_b}{2} - 2 = 0$$

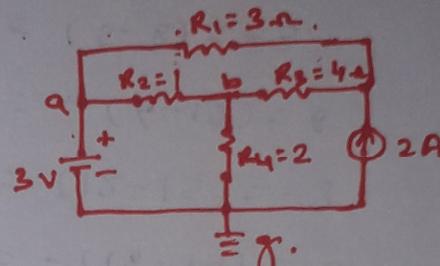
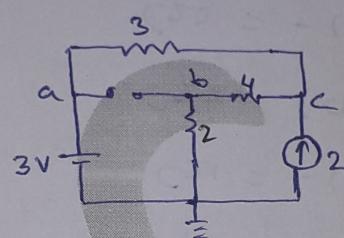
$$\Rightarrow 6V_b - 6 + 3V_b - 12 = 0$$

$$\Rightarrow 9V_b = 18, \Rightarrow V_b = 2V$$

$$\text{now } V_{oc} = V_a - V_b = 3 - 2 = 1V$$

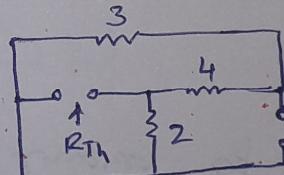
Finding R_{Th}

$$R_{Th} = (3+4) // 2 = 1.55\Omega$$



Thevenin's equivalent circuit

$$I_L = \frac{1}{1.55+1} = 0.39A$$



part 2

$$V_c = 3V_b = 6V, \Rightarrow V_b = 2V \quad \& \quad V_a = 3V$$

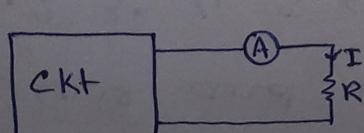
$$V_{bg} = V_{ag} - V_{ab} = 3 - (1 \times 0.39) = 2.61V$$

$$I_{bg} = 2.61/2 = 1.305A$$

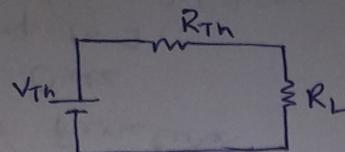
$$I_{cb} = 1.305 = 0.39 = 0.915A$$

$$V_{cg} = (4 \times 0.915) + (2 \times 1.305) = 6.27V$$

4) The circuit consists of independent DC source and resistances. Measurement are taken by connecting an ammeter in series with the resistor R and the results are shown in the table



R	I
10Ω	2A
20Ω	1.5A
?	0.6A



Sol: the circuit can be replaced by an equivalent Thevenin's circuit as shown in the figure

$$I = \frac{V_{Th}}{R_{Th} + R}$$

from the table

$$\frac{V_{Th}}{R_{Th} + 20} = 2 \Rightarrow V_{Th} - 2R_{Th} = 20 \quad \text{--- (1)}$$

$$\text{&} \frac{V_{Th}}{R_{Th} + 30} = 1.5 \Rightarrow V_{Th} - 1.5R_{Th} = 30 \quad \text{--- (2)}$$

solving equation 1 and 2, $V_{Th} = 60V$, $R_{Th} = 20\Omega$

now when $I = 0.6A$

$$0.6 = \frac{V_{Th}}{R_{Th} + R} = \frac{60}{20 + R} \Rightarrow R = 80\Omega \text{ Am}$$

5) find the current through 2 ohm resistor using Thevenin's Theorem

sol: finding V_{Th}

at node a

$$\frac{V_a - V_d}{4} + \frac{V_a}{6} = 0$$

$$\text{Now } V_d = 6$$

$$\therefore V_a = 3.6V$$

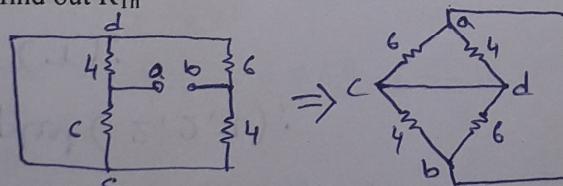
at node b

$$\frac{V_b - 6}{6} + \frac{V_b}{4} = 0 \Rightarrow V_b = 2.4V$$

$$\therefore V_{ab} = V_a - V_b = 3.6 - 2.4 = 1.2V$$

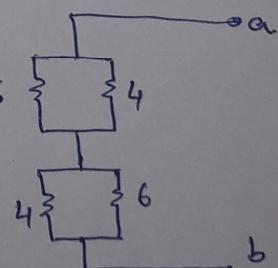
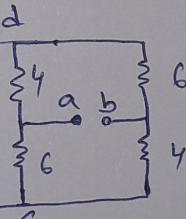
$$V_{Th} = V_{OC} = V_{ab} = 1.2V$$

To find R_{Th}



$$R_{ab} = (6||4) + (4||6)$$

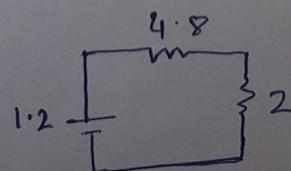
$$= 4.8\Omega$$



Thevenin's circuit

$$I = \frac{1.2}{4.8 + 2}$$

$$= 0.176A$$



special cases :

case 1: if the circuit contains Independent and dependent sources, then V_{Th} can be calculated opening the terminals. but to calculate R_{Th} the following steps to follow

i) connect a test voltage source (1 volt) and find the resultant current(i).

$$\text{then } R_{Th} = 1/i$$

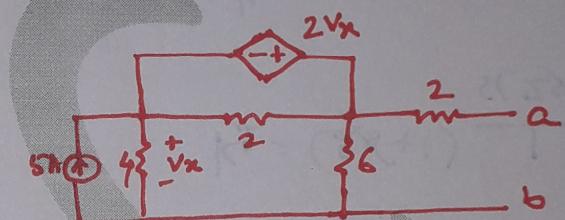
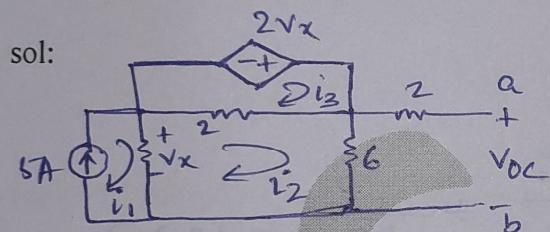
ii) connect a test current source (1A) and find the resulting voltage(V).

$$\text{then } R_{Th} = V/1$$

case 2 : if the circuit only contains dependent sources then the $V_{Th}=0$ and the R_{Th} can be calculated as discussed in the previous case.

Problems

1) Find the Thevenin equivalent of the circuit as shown in the figure



$$V_x = 4(i_2 - i_3) \quad \text{if } i_1 = 5A$$

for mesh 2

$$2(i_2 - i_3) + 6i_2 + 4(i_2 - i_1) = 0$$

$$12i_2 - 2i_3 = 20 \quad \text{--- (1)} \quad [\text{as } i_1 = 5A]$$

for mesh 3

$$-2Vx + 2(i_3 - i_2) = 0$$

$$-2[4(i_1 - i_2)] + 2i_3 - 2i_2 = 0$$

$$6i_2 + 2i_3 = 40 \quad \text{--- (2)}$$

by solving equation 1 and 2

$$i_2 = \frac{10}{3} A$$

$$V_{oc} = 6i_2 = 6 \times \frac{10}{3} = 20V$$

To find R_{Th} $V_x = -4i_2$

For mesh 1

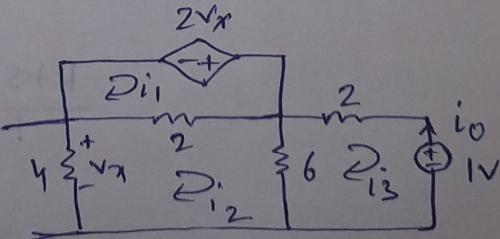
$$-2Vx + 2(i_1 - i_2) = 0$$

$$2i_1 + 6i_2 = 0 \quad \text{--- (1)} \quad [\text{Putting } V_x = -4i_2]$$

For mesh 2

$$2(i_2 - i_1) + 6(i_2 - i_3) + 4i_2 = 0$$

$$-2i_1 + 2i_2 - 6i_3 = 0 \quad \text{--- (11)}$$



For mesh 3, $2i_3 + 1 + 6(i_3 - i_2) = 0$

$$-6i_2 + 8i_3 = -1$$

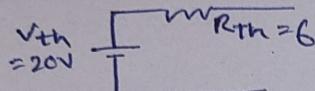
$$6i_2 - 8i_3 = 1 \quad \text{--- (ii)}$$

By solving 1, 2 and 3, $i_1 = 0.167A$, $i_2 = -0.056A$, $i_3 = -0.167A$

And, $i_o = -i_3 = 0.167A$

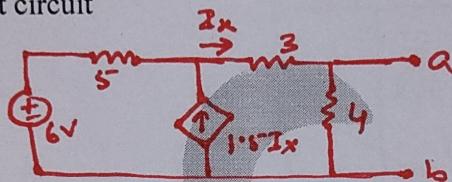
$$\therefore R_{Th} = \frac{1}{i_o} = \frac{1}{0.167} = 6\Omega$$

Thevenin equivalent



Hw1) Find the Thevenin equivalent circuit

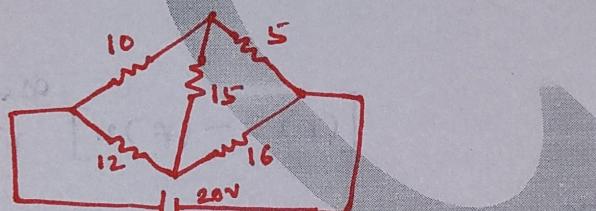
[5.55V, 0.44Ω)
5.33



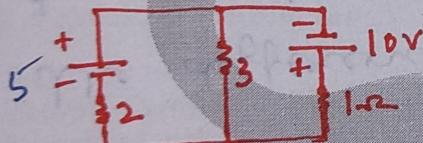
Hw2) find current through 15 ohm resistance

using Thevenin's Theorem

$$Ans 0.1890$$



Hw3) find current through 3 ohm resistor in figure



$$V_{Th} = 5V$$

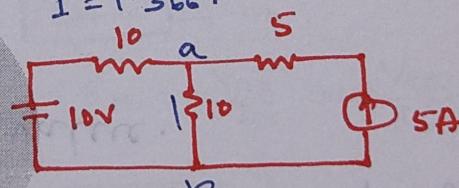
$$R_{Th} = 0.66$$

$$I = 1.366 A$$

Hints: change 5V source to current source.

Hw4) find the thevenin equivalent of the network

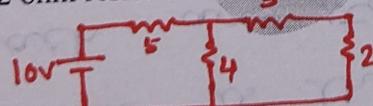
[60V, 10Ω]



Hw5) find current through 2 ohm resistor

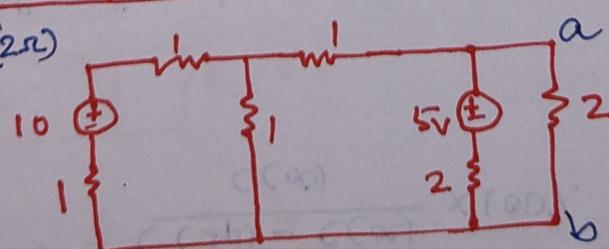
using Thevenin

[0.6154A)



Hw6) draw Thevenin equivalent (2Ω)

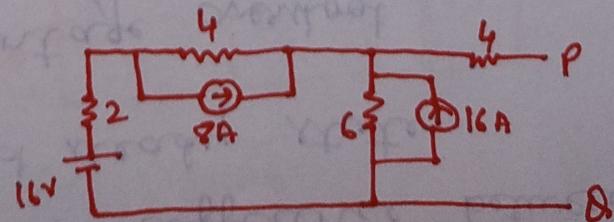
[45/11 V, 10/11 Ω]



Hw7) find V_{Th} and R_{Th}

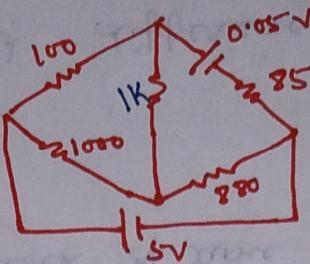
[hints : for V_{Th} convert current source to voltage source and apply mesh analysis]

[72V, 7Ω]



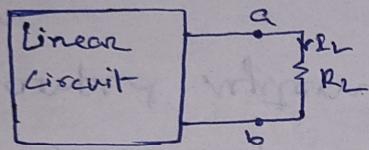
Hw8) find I_L using Thevenin's

$$[I_L = -10.625] \text{ mA}$$

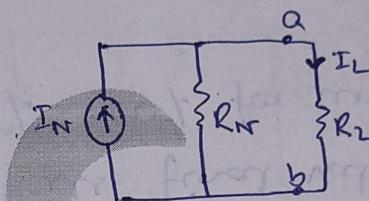


- Norton's theorem : it states that "a linear two terminal circuit can be replaced by an equivalent circuit consisting of a current source 'IN' in parallel with resistance 'RN' where IN is the short circuit current through the terminals and RN is the equivalent resistance at the terminals when the independent sources are turned off.

Explanation



original circuit



Norton equivalent circuit

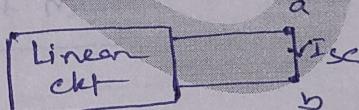
the load current is given by

$$I_L = \frac{R_N}{R_L + R_N} \times I_N$$

steps

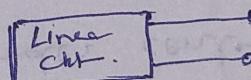
For finding I_N :

short the load terminal A and B
and find the current through it. that is Norton current or short circuit current (I_N or I_{sc})



For finding R_N :

Same is the Thevenin's resistance (R_{Th}).
open load terminal and look through this,
the equivalent resistance resulting will be
Norton resistance (R_N), while the independent
source must be removed.



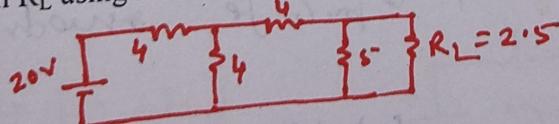
$$R_{Th} = R_N = \frac{V_{Th}}{I_N} \text{ or } \frac{V_{oc}}{I_{sc}}$$

Draw the Norton equivalent circuit and find out I_L

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

problems

1) find I_L through R_L using Norton's theorem

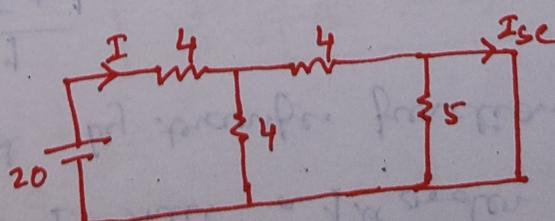


sol: finding I_N

$$\begin{aligned} I &= \frac{20}{4 + (4||4)} \\ &= 3.34 \text{ A} \end{aligned}$$

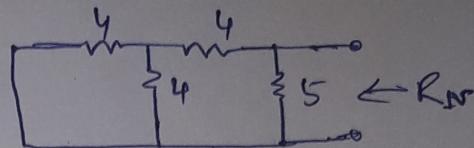
$$I_N = 3.34 \times \frac{4}{4+4} = 1.67 \text{ A}$$

$$I_L = 1.67 \times \frac{2.73}{2.73 + 2.5} = 0.87 \text{ A}$$



finding R_N

$$R_N = \{ (4 || 4) + 4 \} || 5 \\ = 2.73 \Omega$$



2) Find Norton equivalent circuit

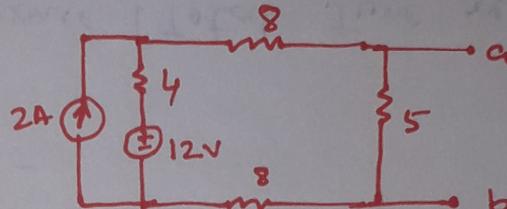


fig-1

To find I_N

$$I_1 = 2A$$

Loop-2

$$8I_2 + 8I_2 - 12 + 4(I_2 - I_1) = 0 \\ \Rightarrow 16I_2 - 12 + 4(I_2 - I_1) = 0$$

$$\Rightarrow 20I_2 = 20 \Rightarrow I_2 = 1A$$

$$\text{Now } I_{SC} = R_N = I_2 = 1A$$

to find R_N

$$R_N = (8 + 4 + 8) || 5 \\ = 4 \Omega$$

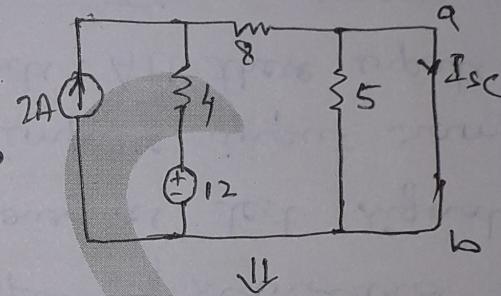


fig-2

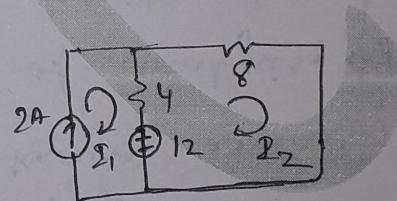
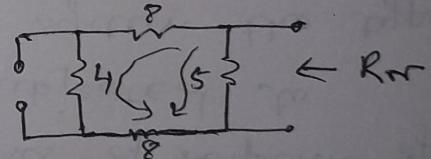
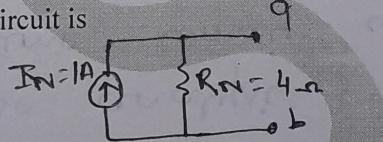
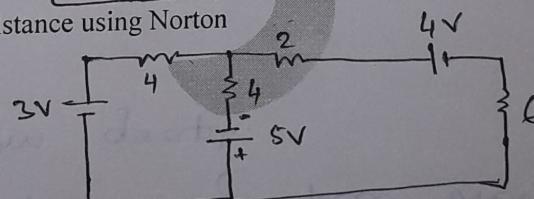


fig-3

So Norton equivalent circuit is



3) find current in 6 ohm resistance using Norton



Sol: finding I_N

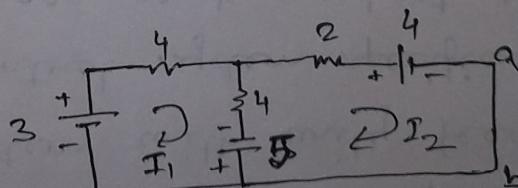
loop 1

$$-3 + 4I_1 + 4(I_1 - I_2) - 5 = 0 \\ \Rightarrow 2I_1 - I_2 = 2 \quad \text{--- ①}$$

loop 2

$$5 + (I_2 - I_1)4 + 2I_2 + 4 = 0 \\ 6I_2 - 4I_1 = -9 \quad \text{--- ②}$$

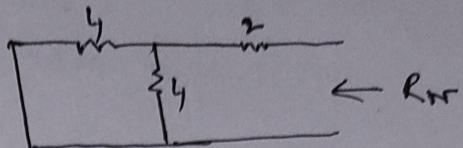
solving 1 and 2, $I_2 = 1.25A$ from b to a



finding R_N

$$R_N = 2 + (4 \parallel 4)$$

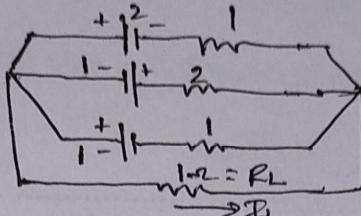
$$= 4 \Omega$$



Hw1) find I_L using Norton

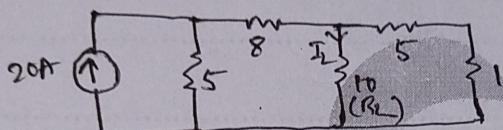
[0.714A]

$$I_L = 1.25 \times \frac{4}{4+6} = 0.5 A$$



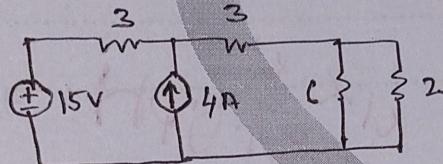
Hw2) find I_L through 10 ohm resistor using Norton

$\text{Ans } 2.238 A$



Hw3) Find load current through 2 ohm resistance using Norton

[2.7A]

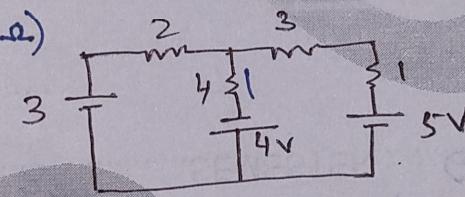


Hw4) find Norton equivalent circuit (4Ω)

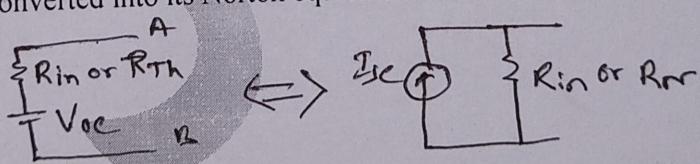
~~[69/16 A, 4/3Ω]~~

~~23/16~~

Hint: $I_{SC} = \frac{23}{4}$



➤ equivalence of Thevenin's and Norton theorem
A Thevenin equivalent can be converted into its Norton equivalent and vice versa. It can be understood by source conversion theory.



$$I_{SC} = \frac{V_{OC}}{R_{IN}}$$
 which is Norton's current

$$\& V_{OC} = V_{TH} = I_{SC} R_{IN}$$
 which is Thevenin voltage

➤ Time domain analysis of first order RL RC circuit

The circuit consisting of a single energy-storing element (either single inductor or single capacitor) in addition to a resistor is known as first order circuit.

Points to Remember

- Current through a resistor changes instantly if the voltage changes instantly and vice versa
- In case of inductor current through it cannot change instantaneously
- In case of a capacitor the voltage across it cannot change instantaneously