Physics.

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Mechanics

① What are scalar and vector field? Physical significance of grad and curl?

→ Scalar field → it is something that has a particular value at every point in space.

Vector field → it is same as scalar field but except for only having a value at every point in space, it has a value and directions at every point in space.

$$\text{Grad } \phi = \nabla\phi = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)\phi$$

$$= \hat{i}\frac{\delta\phi}{\delta x} + \hat{j}\frac{\delta\phi}{\delta y} + \hat{k}\frac{\delta\phi}{\delta z}$$

$$\text{Curl } \gamma = \vec{\nabla}\times\vec{\gamma} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ x & y & z \end{vmatrix}$$

② What is solenoidal and irrotational vector?

→ $\vec{\nabla}\cdot\bar{A} = 0 \Rightarrow \bar{A}$ is solenoidal vector

$\vec{\nabla}\times\bar{A} = 0 \Rightarrow \bar{A}$ is irrotational vector.

③ Differential form of 1D simple harmonic (free vibration) and damped harmonic oscillation and derive their solution. What is overdamped and critical damped motion? Explain in brief.

→ Simple harmonic motion:-

$F = -kx$

Also, $F = m\frac{d^2x}{dt^2}$

$$\Rightarrow m\frac{d^2x}{dt^2} + kx = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{kx}{m} = 0$$

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Simple harmonic motion:-

$F = -kx$ Also, $F = m\frac{d^2x}{dt^2}$

$\Rightarrow m\frac{d^2x}{dt^2} + kx = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{kx}{m} = 0.$

$\Rightarrow x + \omega^2 x = 0$ where, $\omega = \sqrt{\frac{k}{m}}$

Multiply by $2x$.

$\Rightarrow 2x.x + 2\omega^2 x.x = 0$

$\Rightarrow \frac{d}{dx}(x^2 + \omega^2 x^2) = 0$

Now integrating,

$\Rightarrow x^2 + \omega^2 x^2 = C^2$ where, $C = A\omega$

$\Rightarrow \cancel{\frac{x}{\sqrt{A^2 - x^2}} \; 7\omega}$

$\Rightarrow x^2 + \omega^2 x^2 = A^2\omega^2$

$\Rightarrow x^2 = A^2\omega^2 - \omega^2 x^2$

$\Rightarrow x^2 = (A^2 - x^2)\omega^2$

$\Rightarrow \omega = \frac{x}{\sqrt{A^2 - x^2}} \Rightarrow \boxed{\frac{d}{dt}\left(\frac{x}{\sqrt{A^2 - x^2}}\right) = \omega} \rightarrow$ differential form.

$\Rightarrow \sin^{-1}\left(\frac{x}{A}\right) = \omega t + \phi$

$\Rightarrow \boxed{x = A\sin(\omega t + \phi)}$

$\because T = \frac{2\pi}{\omega}$ is the time period for one second $f = \frac{1}{T}$ is the frequency of oscillation.

Damped harmonic oscillation $\rightarrow$ moving particle gradually loses its KE on interaction with resistive forces like air / friction.

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Damped harmonic oscillation:

$f = \mu \vec{v}$ $\qquad$ μ = damping coefficient.

$\therefore f = -kx - \mu v$

$\Rightarrow m\frac{d^2x}{dt^2} = -kx - \mu\frac{dx}{dt} \Rightarrow m\frac{d^2x}{dt^2} + kx + \mu\frac{dx}{dt} = 0$

$\Rightarrow \boxed{\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = 0}$ (dividing by m)

here, $\omega_0 = \sqrt{\frac{k}{m}}$, $\boxed{\delta = \frac{\mu}{2m}}$ is the damping ratio.

There are 3 conditions of damping oscillation:-

i) if $\delta > 1$, damping is exponential and it stops very quickly this is called overdamped motion.

ii) if $\delta = 1$, motion is critically damped. the system returns to equilibrium position very quickly.

iii) if $\delta < 1$, motion is underdamped. The motion slowly decreases to zero and there is notable oscillation. The angular frequency is $\boxed{\omega_1 = \omega_0\sqrt{1-\delta^2}}$ and the decaying given by $\boxed{\lambda = \omega_0 \delta.}$

Overdamped motion → An overdamped system moves slowly toward equilibrium. If the system contained high loses, for ex- if the spring-mass expt. were conducted in a viscous fluid, the mass could slowly return to its rest posn without ever overshooting.

Critical damped motion → damping of an oscillation causes it to return as quickly as possible to its equil. posn without oscillating back & forth about this posn. It is defined as the threshold b/w overdamping and underdamping.

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(4) What is logarithmic decrement? Q-factor for damped vibration?

→ It measures the rate of at which the amplitude of damped oscillatory motion dies away (or falls off).

Let $a_1, a_2, a_3, a_4, \ldots$ be the successive amplitudes at time $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \frac{7T}{4}, \ldots$ respectively.

here, T = time prd. of oscillation.

$\therefore x = Ce^{-bt}\sin(\omega' t + \phi)$ $\qquad \omega' = \sqrt{\omega_0^2 - b^2}$

$a_1 = Ce^{-bT/4}$ $\qquad a_3 = Ce^{-5bT/4}$

$a_2 = Ce^{-3bT/4}$ $\qquad a_4 = Ce^{-7bT/4}$ etc.

So, we can write:- $\frac{a_1}{a_2} = \frac{a_2}{a_3} = \ldots = \frac{a_{n-1}}{a_n} = e^{bT/2} = d$ (const.)

d = decrement. The 'log' of this decrement is called logarithmic decrement.

$\therefore \lambda = \ln d = \ln e^{bT/2} = \frac{bT}{2}$

$\therefore \boxed{\lambda = \frac{bT}{2}}$

$\therefore \lambda = \ln \frac{a_1}{a_2} = \ln \frac{a_2}{a_3} = \ldots$

This logarithmic decrement is defined as the natural logarithmic of the ratio of 2 successive amplitudes of damped oscillations.

Quality factor:- The ratio of the max. value of restoring force to the max. value of damping force is called quality factor.

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Max. value of restoring force, $= KA$ A = amplitude of vibration
$= m\omega_0^2 A$

Now, $x = Ce^{-bt}\sin(\omega' t + \phi)$ $\omega' = \sqrt{\omega_0^2 - b^2}$

$$\frac{dx}{dt} = Ce^{-bt}\omega'\cos(\omega' t + \phi) - Cbe^{-bt}\sin(\omega' t + \phi)$$

If the body is excited by giving a velocity V_0 suddenly in mean posn., i.e.,
at $t = 0$, $\frac{dx}{dt} = v_0$ and $\phi = 0$.

We have, $V_0 = C\omega'$ or $C = \frac{V_0}{\omega}$

$$\therefore x = \frac{V_0}{\omega'} e^{-bt}\sin\omega' t$$

and $V = \frac{V_0}{\omega'} e^{-bt}\omega'\cos(\omega' t) - be^{-bt}\frac{V_0}{\omega'}\sin\omega' t$

Now, for small damping, b is very small
$V = V_0 e^{-bt}\cos\omega' t$
$\therefore (V)_{max} = V_0 e^{-bt} = C\omega' e^{-bt} = A\omega'$ A = amplitude $= Ce^{-bt}$

$\therefore$ Max. damping force $= \beta V_{max}$
$= \beta A\omega' = (2bm)A\omega_0$ [for small b, $\omega' = \omega_0$

$\therefore$ Quality factor $(Q) = \frac{m\omega_0^2 A}{2bm\omega_0 A}$

$$\boxed{Q = \omega_0 \tau}$$

$\tau = \frac{1}{2b}$ = Relaxation time.

Relation b/w Quality factor and logarithmic decrement :-

$Q = \frac{\omega}{2k}$ hence, required relation is :- $\boxed{Q = \frac{\omega T}{4\lambda}}$

$\lambda = k\frac{T}{2}$

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(5) Difference b/w amplitude resonance & velocity resonance. Derive the resonant frequency in both case.

Velocity Resonance	Amplitude resonance
i) Velocity amplitude of the forced oscillator is the max. at a particular ν of the applied force.	i) Amplitude of forced oscillator is the max. for a particular ν of the applied force.
ii) It occurs at ν, $\omega = q$ ω = natural ν of oscillator	ii) It occurs at resonant freq. $q = \sqrt{\omega^2 - 2k^2}$ k = damping constant
iii) At applied ν, $q = 0$, the velocity amplitude = 0.	iii) At applied ν, $q = 0$, amplitude of forced oscillator is $\frac{f_0}{k}$ f_0 = force amplitude k = force constant.
iv) phase of forced oscillator w.r.t. that of applied force is zero	iv) phase of forced oscillator w.r.t. that of applied force is $\pi/2$.

Derivation

Amplitude Resonance.

Amplitude of forced vibration:- $A_0 = \dfrac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2}}$ —(i)

A_0 will be max. if denominator will be min.

$\therefore \dfrac{d}{d\omega}\left[(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2\right] = 0$

$\Rightarrow 2(\omega_0^2 - \omega^2)(-2\omega) + 8b^2\omega = 0$

$\Rightarrow -4\omega\left[(\omega_0^2 - \omega^2) - 2b^2\right] = 0$

$\because \omega \neq 0$

$(\omega_0^2 - \omega^2) - 2b^2 = 0$

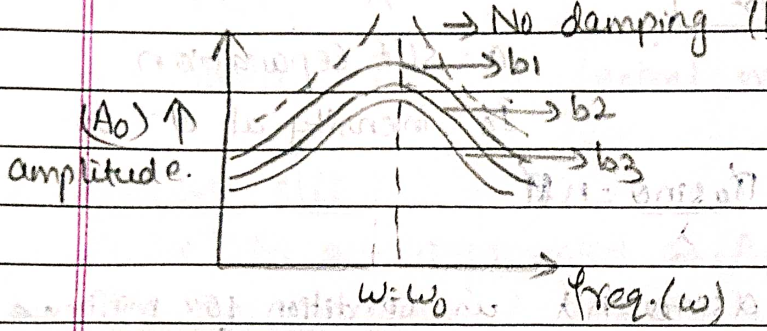
Or $\boxed{\omega = \sqrt{\omega_0^2 - 2b^2}}$ → Resonant freq.

—(ii)

$\therefore (A_0)_{max.}$ = Putting value of (ii) in (i), we get,

$$\boxed{(A_0)_{max.} = \frac{f_0}{2b\sqrt{\omega_0^2 - b^2}}}$$

* In amplitude resonance, PE ($\frac{1}{2}Kx^2$) of driven system is max.



$b_1 < b_2 < b_3$.

Velocity Resonance

From particular integral solⁿ, we have,

$x = Ae^{i\omega t} = A_0 e^{i(\omega t - \phi)}$ —(i)

Now, if impressed periodic force is $F_0 \sin\omega t$, the eqⁿ. of forced vibration becomes eqⁿ.(i)

$$x = A_0 \sin(\omega t - \phi)$$

hence, velocity of particle, $v = \frac{dx}{dt} = A_0 \omega \cos(\omega t - \phi)$

Velocity amplitude becomes $\boxed{V_0 = A_0 \omega = \frac{f_0 \omega}{\sqrt{(\omega_0^2 - \omega^2) + 4b^2\omega^2}}}$

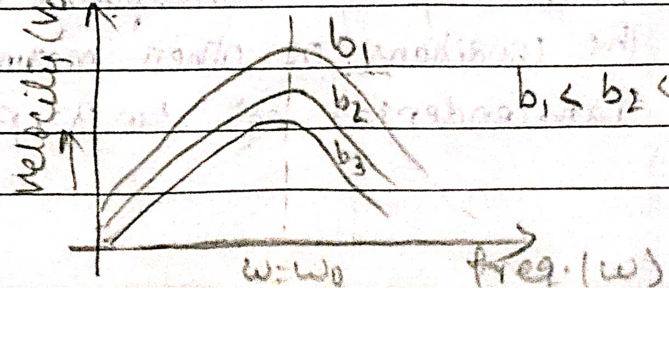
(where, $f_0 = \frac{F_0}{m}$)

V_0 will be max. if denominator will be min. when $\omega = \omega_0$

$\therefore \boxed{(V_0)_{max.} = \frac{f_0}{2b}}$

So, velocity resonance occurs when the natural ν of the ~~applied~~ body is equal to the ν of the applied periodic force.

* In velocity resonance, KE ($\frac{1}{2}mv^2$) of driven system is max.



$b_1 < b_2 < b_3$

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Optics

1) Write down the single slit and double slit Fraunhofer diffraction intensity expression. Derive the condition for diffraction minima and maxima for single slit diffraction from intensity expression.

→ Single Slit.

Intensity distribution due to single slit diffraction pattern is given by $\boxed{I = I_0 \frac{\sin \beta^2}{\beta^2}}$, $\beta = \frac{\pi a \sin\theta}{\lambda}$

a : slit separation

I_0 : intensity at $\theta = 0$.

For $\beta = n\pi$, we have, $\frac{\pi a \sin\theta}{\lambda} = n\cancel{\pi}$

$\therefore$ $a\sin\theta = n\lambda$ is condition for minima
($n = \pm 1, \pm 2, \pm 3, \ldots$ etc.)

Posⁿ of 2ⁿᵈary minima can be obtained from :- $\theta = \sin^{-1}\left(\frac{n\lambda}{a}\right)$

Central bright maximum:-

for $\beta = 0$, $I = \frac{\sin\beta}{\beta} = 1$ so, $I = I_0$

So, for $\beta = 0$, we get central bright maximum.

~~Posⁿ of 2ⁿᵈary m~~ In order to determine the positions of maxima, we differentiate intensity expression w.r.t. β and set it equal to zero. Thus,

$$\frac{dI}{d\beta} = I_0 \left\{ \frac{2\sin\beta \cdot \cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3} \right\}$$

$$= I_0 \frac{\sin 2\beta}{\beta^3} \{\beta - \tan\beta\} = 0$$

Condition $\beta = 0$ corresponds to maxima.

The conditions of other maxima are roots of following transcendental eqⁿ. $\tan\beta = \beta$ (maxima).

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The root $\beta = 0$, corresponds to central maximum. The other roots can be found by determining the points of intersection of the curve $y = \beta$ and $y = \tan\beta$. Intersection occurs at $\beta = 1.43\pi$, $\beta = 2.461\pi$, etc. are known as 1st maximum, 2nd maximum etc.

$\because \left\{\frac{\sin(1.43\pi)}{1.43\pi}\right\}^2$ is about 0.0496, Intensity of 1st maximum is about 4.96% of central maxima.

Double slit

Double slits are represented as A_1B_1 and A_2B_2 in fig. below. Slits are narrow & rectangular in shape.

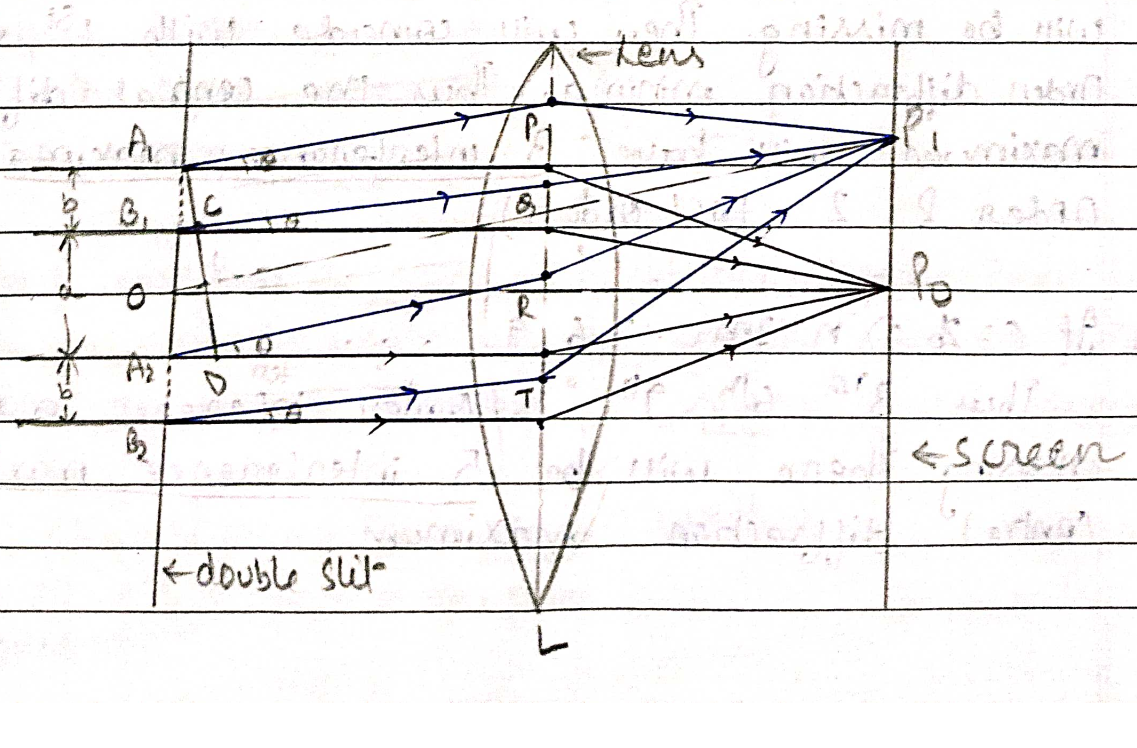
Now, Intensity of diffracted wave due to double slit at P_1 is :-

$$I = A^2 = 4A^2 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma$$

$$\boxed{I = \left(4I_0 \frac{\sin^2\beta}{\beta^2}\right) \times \cos^2\gamma} \qquad \text{where, } (A^2 = I_0)$$

where, $\beta = \frac{\pi a \sin\theta}{\lambda}$ (phase diff. due to single slit diffracted wavelets)

$\gamma = \frac{\pi(a+b)\sin\theta}{\lambda}$ (phase diff. due to interference of the diffracted waves from 2 slits).



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Condition for maxima and minima:-
Diffraction maxima:- for $\beta = \frac{\pi b \sin\theta}{\lambda} = \pm(2m-1)\frac{\pi}{2}$

or $\boxed{b\sin\theta = \pm(2m-1)\frac{\lambda}{2}}$

Diffraction minima:- $\boxed{b\sin\theta = \pm m\lambda}$

(2) What is missing order? Calculate the missing order for a given slit width & opaque space width.

→ If the conditions for maxima of interference pattern & minima of diffraction pattern are simultaneously satisfied for a given value of θ, then the corresponding interference maxima will be missing.

We have,
$(b+c)\sin\theta = n\lambda$ $(n = 0, 1, 2, \ldots)$
Direction of intensity maxima.

& $b\sin\theta = m\lambda$ $(m = 1, 2, 3, \ldots)$ Direc? of diff. minima.

$\therefore \boxed{\frac{b+c}{b} = \frac{n}{m}}$

(i) If $c = b \Rightarrow n = 2m = 2, 4, 6, \ldots$ (for $m = 1, 2, 3, \ldots$)
Thus, 2nd, 4th, 6th ... order interference maxima will be missing. They will coincide with 1st, 2nd, 3rd ... order diffraction minima. Thus the central diffraction maximum will have 3 interference maxima (the zero-order & 2 first-order)

(ii) If $c = 2b \Rightarrow n = 3m = 3, 6, 9, \ldots$
Thus, 3rd, 6th, 9th, ... order interference maxima will be missing. There will be 5 interference maxima in the central diffraction maximum.

iii) $c=3b \Rightarrow n=4m = 4, 8, 12, \ldots$

Thus, 4^{th}, 8^{th}, 12^{th}, interference maxima will be missing. There will be 7 int. maxima in the central diffraction maximum.

iv) $c=0 \Rightarrow n=m$

Thus, all the orders of int. maxima will be missing. The diffraction pattern will be same as that of a single slit of width $= b$.

(2) Differentiate b/w Fresnel and Fraunhofer class of diffraction.

→

Fraunhofer diffraction	Fresnel diffraction
i) Source & screen on which the pattern is observed are at $\boxed{\text{infinite dist.}}$ from aperture or the obstacle causing diffra^n.	i) Source or screen or both are $\boxed{\text{finite distances}}$ from the aperture or obstacle causing diffraction.
ii) 2 lens are required to get diffrac^n pattern. One is used to make light from light source parallel before it falls on aperture & other is used to focus the light after diffrac^n on the screen.	ii) No lens is required to make the rays parallel or convergent.
iii) Incident wavefront is plane. 2^{nd}ary wavelets originate from blocked portions of wave front.	iii) Incident wavefront is not plane but is either spherical or cylindrical.
iv) 2^{nd}ary wavelets are in same phase at every point in the plane of aperture.	iv) phase is not same.

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v) Resultant diffraction pattern on the screen are due to the interference b/w parallel rays which are brought into focus with the help of convex lens.	v) Wavelets are divided into small elements / zones called Fresnel's zone. Resultant effect at any point on screen is combined effect of all 2° waves originating from various zones.

(4) What is grating? What is resolving power & write down the expression of resolving power for a grating.

→ A diffraction grating is an optical element that divides (disperses) light composed of lots of different wavelengths (e.g. white light) into light components by wavelengths. The simplest type of grating is one with a large no. of evenly spaced parallel slits.

Resolving power of grating → defined as ratio of wavelength of a line in the spectrum to the least difference in the wavelength of the next line that can just be seen as separate.

Mathematically, it's equal to $\frac{\lambda}{d\lambda}$; where, $d\lambda$ = diff. of 2 wavelength which can be resolved by optical system.

For grating, resolving power is capacity to form separate diffracn. maxima of 2 wavelengths which are very close to each other.

Direcn. of nth primary maximum for wavelength λ is given by :- $(a+b)\sin\theta_n = n\lambda$ —(i)

Direcn. of n^{th} primary maximum for wavelength $(\lambda+d\lambda)$ is given by :- $(a+b)\sin(\theta_n+d\theta_n) = n(\lambda+d\lambda)$ — (ii)

The two lines will appear just resolved if angle of diffraction $(\theta_n + d\theta_n)$ also corresponds to direcⁿ of 1^{st} 2° minimum after nth primary maximum.
For angle of diffraction θ changes by an amount $d\theta$ the path diff. b/w the corres. points of 2 consecutive slit changes by λ/N where N = total no. of lines in grating.
Now,
The condition for 1^{st} 2° minimum in n^{th} order spectrum can be obtained by introducing additional path diff. λ/N to eqⁿ (i), we have,

$$(a+b)\sin(\theta_n + d\theta_n) = n\lambda + \frac{\lambda}{N} \quad \text{--- (iii)}$$

Now, comparing (ii) & (iii), we have,

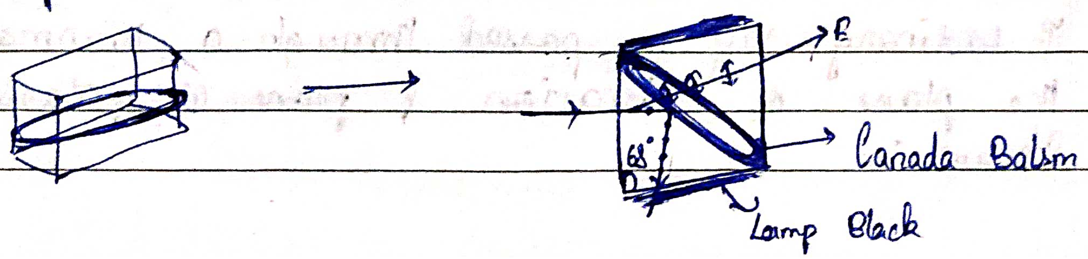
$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

Or, $n\,d\lambda = \frac{\lambda}{N}$ $\Rightarrow$ $\boxed{\frac{\lambda}{d\lambda} = nN}$ is required expression for resolving power of grating.

5) Principle of Nicol prism with diagram.

Double refraction

When an ordinary ray of light travels in a calcite crystal and enters the canada Balsam layer it eliminates one of the rays by total internal reflection, i.e; the ordinary rays (O-ray) are eliminated and only the extra-ordinary ray (E-ray) is transmitted through the prism.



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(6) State Brewster's law and Malus law.

→ Brewster's law → Acc. to Brewster's law the tangent of the angle of polarisation is numerically equal to the R.I. of the medium.

Mathematically, $\boxed{\mu = \tan(i_p)}$

where, i_p = angle of polarisation

μ = R.I. of the medium concerned.

Malus law → Intensity of polarized light transmitted through the analyzer varies as the sq. of the cosine of the angle b/w the plane of transmission of the analyzer and the plane of the polarizer.

Mathematically, $\boxed{I = I_0 \cos^2 \theta}$

I_0 = Original intensity.

I = intensity of transmitted light through analyser

θ = angle b/w the axes of polarised sheet.

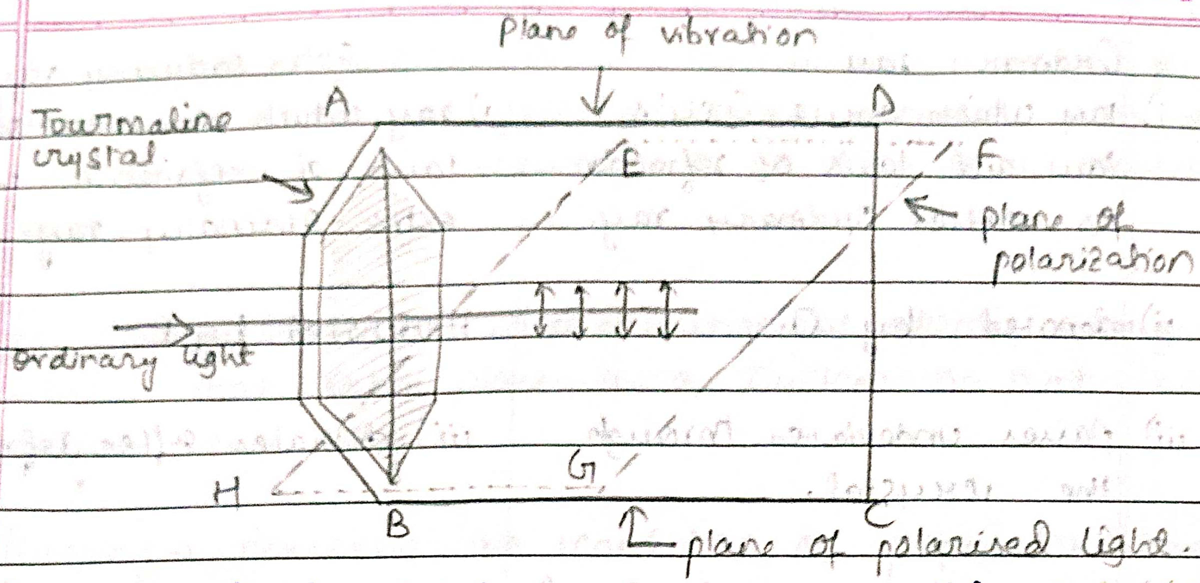
(7) What are plane of polarization and plane of vibration?

→ Plane of polarization → The magnetic vector and the direction of propagation vector form a plane which for historical reasons, called plane of polarization.

Otherwise, it's defined as the plane ⊥r to the plane of vibration & containing no electric vector vibration.

Plane of vibration → The vibrating electric vector and the direction of wave propagation constitute a plane, called plane of vibration.

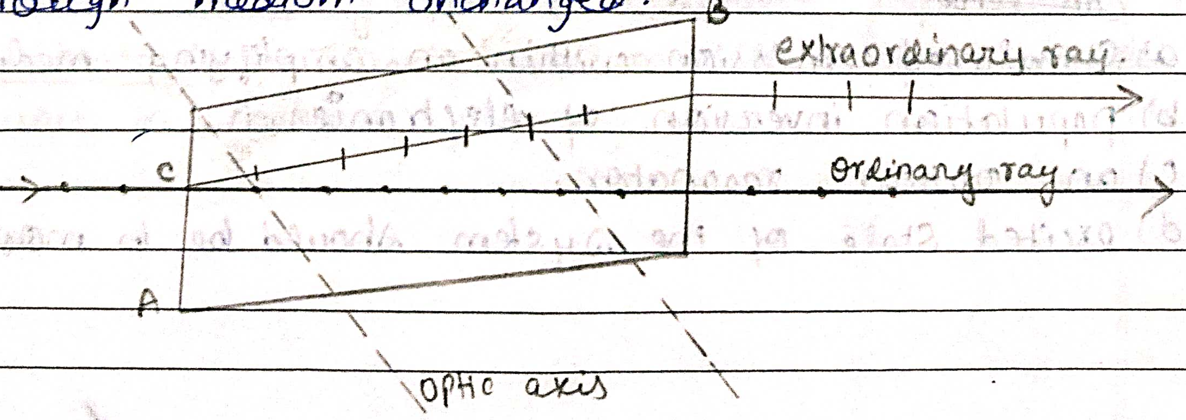
* If ordinary ray is passed through a tourmaline crystal, the plane of vibration & plane of polarisation is shown :-



The plane ABCD containing crystallographic axis AB in which vibrations of electric vector are restricted is Plane of vibrations.
EFGH represents plane of polarization where magnetic vectors are restricted. A plane ⊥ to plane of vibration

(8) Define double refraction. Difference b/w O-Ray & E-ray.

→ Double refraction (birefringence) →
an optical property in which a single ray of unpolarized light entering an anisotropic medium is split into 2 rays, each travelling in a different direction.
One ray (called extraordinary ~~light~~ ray) is bent, or refracted, at an angle as it travels through the medium; the other ray (called Ordinary ray) passes through medium unchanged.



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Ordinary ray	Extra-Ordinary ray
i) ray which obeys Snell's law and laws of refraction is called Ordinary ray.	i) ray which does not obey the laws of refraction is called extra-ordinary ray.
ii) denoted by O.	ii) denoted by E.
iii) passes undeviated through the crystal.	iii) deviates after refraction.
iv) Inside the crystal, Speed of O-ray is less than E-ray.	iv) Inside the crystal, Speed of E-ray is more than O-ray.
v) Speed is constant in the medium.	v) Speed is constant in the medium.
vi) gives spherical wave front.	vi) gives ellipsoidal wavefront.

Q) Write down the principles of LASER. What is population inversion and pumping. Elaborate any 2 process of pumping.

→ Principle of laser is based on 4 Separate features:-

~~a) Stimulated emission within an amplifying medium.~~
~~Acc. to quantum mechanics, an e within an atom/lattice can have~~

a) Stimulated emission within an amplifying medium.
b) population inversion of electronics.
c) an optical resonator.
d) excited state of the system should be in most-stable state.

Stimulated emission → process by which an incoming photon of a specific frequency can interact with an excited atomic e^-, causing it to drop to a lower energy level.

Population inversion → redistribution of atomic energy levels that takes place in a system so that laser action can occur.

Pumping → Achieving the condition of population inversion is pumping.
Pumping is the method of raising the particles from a lower energy state to a higher energy state.

Any 2 process of pumping:-
i) Optical pumping → Light is used to achieve population inversion. Solid state lasers use this method. Generally, for optical pumping Xe flash lamps are used ∵ these materials have very broadband absorption, a sufficient amount of energy is absorbed from the emission band of the flash lamp & population inversion is achieved.
Ex- ruby, Nd: YAG Laser

ii) Thermal Pumping → We can achieve population inversion by heating the laser medium. Heat acts as the pump source in thermal pumping.

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⑩ Derive the Einstein's Equation.

→ $N_2 = N_1 \exp\left(\frac{E_1 - E_2}{KT}\right)$ —(i)

where, $\frac{N_2}{N_1} = \exp\left\{\frac{-(E_2 - E_1)}{KT}\right\}$ —(ii)

If $E_2 > E_1$ then $N_2 > N_1$

when thermal equilibrium is reached,

$$N_1 B_{21} U_v = N_2 A_{21} + N_2 B_{21} U_v.$$

$\Rightarrow U_v = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$ —(iii)

$\Rightarrow U_v = \frac{A_{21}}{\left(\frac{N_1}{N_2}\right) B_{12} - B_{21}}$ —(iv)

Acc. to eq. (ii), we have,

$N_2 = N_1 \exp\left\{\frac{-(E_2 - E_1)}{KT}\right\}$

$\frac{N_1}{N_2} = \exp\left\{\frac{(E_2 - E_1)}{KT}\right\}$ —(v)

Substituting (v) in (iv), we have,

$U_v = \frac{A_{21}}{\exp\left\{\frac{(E_2 - E_1)}{KT}\right\} B_{12} - B_{21}}$ —(vi)

Acc. to Planck's law of radiation,

$U_v = \frac{8\pi h v^3}{c^3}\left[\frac{1}{\exp(hv/KT) - 1}\right]$ —(vii)

Taking $E_2 - E_1 = h\nu$ rearranging eq^n^ (vii), we have,

$$U_\nu = \frac{A_{21}}{B_{12}} \left[\frac{1}{\exp\left(\frac{h\nu}{kT}\right) - \frac{B_{21}}{B_{12}}} \right] \quad \text{— (viii)}$$

Comparing eq^n^ (vii) & (viii), we have,

$\frac{B_{21}}{B_{12}} = 1 \Rightarrow B_{12} = B_{12}$ & $\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3}$

$$A_{21} = B_{12} \frac{8\pi h\nu^3}{c^3} \quad \text{— (ix)}$$

$$A_{21} = B_{21} \frac{8\pi h\nu^3}{c^3} \quad \text{— (x)}$$

eq^n^ (ix) & (x) are Einstein's eq^n^

A_{21}, B_{12}, B_{21} are called Einsteins coefficients.
These coefficients are related to one another.

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Electromagnetism

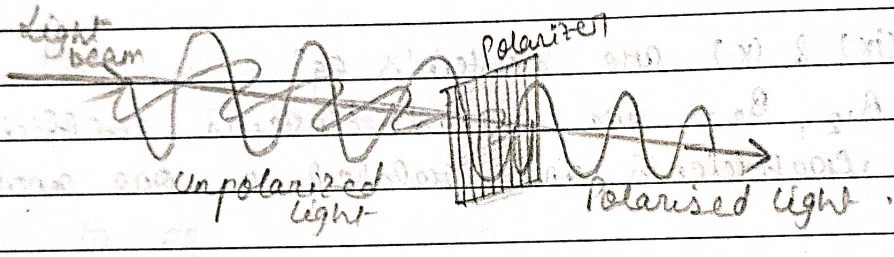
(1) Write down Clausius-Mossotti Equation mentioning all parameters.

→ $\boxed{\dfrac{\varepsilon_r - 1}{\varepsilon_r + 2} = \dfrac{N\alpha}{3\varepsilon_0}}$

ε_r = dielectric constant of material
ε_0 → permitivity of free space
α → molecular polarizability.
N → no. density of molecule.

(2) What is Polarization? Discuss different types of polarization.

→ It is a property applying to transverse waves that specifies the geometrical orientation of the oscillations.
In transverse wave, the direcⁿ of oscillation is ⊥r to the direcⁿ of motion of wave.



Types of polarization:-

- Linear Polarization → $\bar{E}$ of light is limited to a single plane along the direcⁿ of propagation.
- Circular Polarization → There are 2 linear components in the $\bar{E}$ of light that are ⊥r to each other such that their amplitudes are equal, but phase difference is π/2. The propagation of occurring $\bar{E}$ will be in a circular motion.
- Elliptical Polarization → $\bar{E}$ of light follows an elliptical propagation. The amplitude & phase diffⁿ b/w the two linear components are not equal.

(3) Write down all the Maxwell's Eqⁿ. Derive the 1st 3 laws.

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→ Differential form of Maxwell's equation:-

1st eq^n :- $\vec{\nabla} \cdot \bar{D} = \rho$

2nd eq^n :- $\vec{\nabla} \cdot \bar{B} = 0$

3rd eq^n :- $\vec{\nabla} \times \bar{E} = -\frac{\delta \bar{B}}{\delta t}$

4th eq^n :- $\nabla \times \vec{H} = J + \frac{\delta \bar{D}}{\delta t}$

Physical significance:-

Maxwell's 1st eq^n:-

$\because \int_v \rho dv = Q$ = total charge enclosed by the volume v so we can write the net outward flux of electric disp. vector through the surface enclosing a volume is equal to the net charge contained within that vol.

Maxwell's 2nd eq^n:-

The net outward flux of magnetic Induction B through any closed surface is equal to zero.

Maxwell's 3rd eq^n:-

The electromotive force (emf: $e = \int_c \bar{E} \cdot d\bar{l}$) around a closed path is equal to -ve rate of change of magnetic flux linked with the path ($\because$ magnetic flux, $\phi = \int_s \bar{B} \cdot d\bar{s}$)

Maxwell's 4th eq^n:-

The magneto motive force (m.m.f. $e = \int_c H \cdot dl$) around a closed path is equal to the conduction current plus displacement current through any surface bounded by the path.

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Conversion of Maxwell's eq^n from integral to differential form:-

1) $\boxed{\oint_s \bar{E}.d\bar{s} = \frac{q}{\epsilon_0}}$ —(i)

Let ρ = charge density

$q = \oint_v \rho dV$ —(ii)

dV = small vol. considered

Acc. to Gauss divergence theorem,

$\oint_s \bar{E}.d\bar{s} = \oint_v (\bar{\nabla}.\bar{E}) dV$ —(iii)

Putting (ii) & (iii) in (i), we get,

$$\boxed{\oint_v (\bar{\nabla}.\bar{E}) dV = \frac{1}{\epsilon_0}\oint_v \rho dV}$$

Integral form	Differential form
i) $\oint_s \bar{E}.d\bar{s} = \frac{q}{\epsilon_0}$	div $\bar{E} = \frac{\rho}{\epsilon_0}$
ii) $\oint_s \bar{B}.d\bar{s} = 0$	div $\bar{B} = 0$
iii) $\oint_l \bar{E}.d\bar{l} = -\frac{\delta\phi_B}{\delta t}$	curl $\bar{E} = -\frac{\delta\bar{B}}{\delta t}$
iv) $\oint_l \bar{B}.d\bar{l} = \mu_0 i$	curl $\bar{B} = \mu_0(\bar{J} + \bar{J}_d) = \left(\bar{J} + \epsilon_0 \frac{\delta\bar{E}}{\delta t}\right)$

4) What is displacement vector?

→ The change in the position vector of an Object is known as the displ. vector.

5) What is displacement current? Write modified Ampere's law.

→ Displacement current → Acc. to Maxwell, its not only current in a conductor that produces a magnetic field. A changing $\bar{E}$ in vacuum or in a dielectric also produces a $\bar{B}$. This implies that a changing $\bar{E}$ is equivalent to a current, which flows till the $\bar{E}$ is changing. This equivalent current produces the same B as a conventional current in a conductor. This equivalent current is known as displ. current.

Modified Ampere's Law

$$\boxed{\oint \bar{B}\cdot d\bar{l} = \mu_0 (i + i_d) = \mu_0 \left(i + \varepsilon_0 \frac{\delta \phi_E}{\delta t}\right)}$$

⑥ Define magnetization. What is relative permeability? The relation b/w relative permeability & susceptivility.

→ Magnetization, also termed as magnetic polarization, is a vector quantity that measures the density of permanent or induced dipole moment in a given magnetic material.
Magnetization results from motion of e^- in atoms or the spin of e^- or the nuclei.

Relative permeability → is the ratio of permeability of the substance to absolute permeability. The ratio of permeability of a substance to the permeability of vacuum is relative permeability. $\boxed{\mu_r = \mu\mu_0}$

Relation :- $\boxed{\mu_r = (1 + \chi_m)}$

$\hookrightarrow$ Susceptibility.

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(7) Weiss theory of ferromagnetism.
→ Weiss proposed a concept of ferromagnetic domains to explain the phenomenon of ferromagnetism. Ferromagnetic domains are the small regions in a ferromagnetic material within which all the magnetic dipoles are aligned in a particular direct. parallel to each other.

As transformer core, what is preferred between soft iron and steel and why?
→ In transformers, soft iron cores are utilised bcz. they have excellent magnetic permeability, which focuses magnetic lines of force and reduce energy loss.
Bcz. soft iron has high permeability, it allows full coupling of magnetic flux from the main coil to the 2ndary coil in core of transformer.

(8) What is Curie and curie-weiss law?
→ Curie law → magnetization in a paramagnetic material is directly proportional to the applied magnetic field. If the object is heated, the magnetization is viewed to be inversely proportional to the temp.

Curie-weiss law → one of the imp. laws in electromagnetism that says that the magnetic susceptibility is above the curie temp. point of a ferromagnet in the paramagnetic region. The magnetic moment is the quantity of a magnet that determines its torque in an external magnetic field.

(9) Ferromagnetism and antiferromagnetism.

Ferromagnetism	Antiferromagnetism.
i) Presence of magnetic domain that are aligned in the same direction in magnetic materials.	Presence of magnetic domains aligned in opposite direc^n^ in magnetic materials
ii) have a value for net magnetic moment	have a zero net magnetic moment.
iii) Ex - iron, nickel, cobalt and their metal alloys.	transition metal oxides.
↑↑↑↑↑	↑↓↑↓↑
iv) it has its applications in transformers, electromagnets, and magnetic tape recording.	antiferromagnetic materials improve the way informatio is written & read electrica in devices.

Quantum Mechanics

① 1-D and 3-D time dependent and time independent Schrodinger Equation.

→ In 1-D

Time dependent Schrödinger's Equation:-

i) $-\left(\frac{\hbar^2}{2m}\right)\frac{\delta^2\psi}{\delta x^2} = i\hbar\frac{\delta\psi}{\delta t}$ [for a free particle i.e., $V(x)=0$.]

ii) $i\hbar\frac{\delta\psi}{\delta t} = \left[\frac{\hbar^2 k^2}{2m} + V(x)\right]\psi$

Time independent Schrödinger's Equation:-

i) $-\left(\frac{\hbar^2}{2m}\right)\frac{\delta^2\psi}{\delta x^2} = i\hbar\frac{\delta\psi}{\delta t}$ for a free particle i.e., $V(x)$

ii) $i\hbar\frac{\delta\psi}{\delta t} = \left[-\frac{\hbar^2 k^2}{2m} + V(x)\right]\psi$ [for $V(x) \neq 0$]

iii) $-\left(\frac{\hbar^2}{2m}\right)\frac{d^2\psi}{dx^2} = E\psi$ (for a free particle)

iv) $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi(x) = 0$ [for $V(x) \neq 0$]

In 3-D

Time Dependent Schrödinger's Equation:-

i) $i\hbar\frac{\delta\psi}{\delta t} = -\frac{\hbar^2}{2m}\nabla^2\psi$ (for a free particle)

ii) $i\hbar\frac{\delta\psi}{\delta t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\psi$

Time independent Schrödinger's Equation:-

i) $\left[-\frac{\hbar^2}{2m}\nabla^2\right]\psi = E\psi$ (for a free particle)

ii) $\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\psi = E\psi$ [for $V(x) \neq 0$]

② Write down the 3D momentum operator, Energy operator and Hamiltonian operator.

→ Momentum operator

$$\hat{p}_x = \frac{\hbar}{i} \frac{\delta}{\delta x}$$

$$\hat{p}_y = \frac{\hbar}{i} \frac{\delta}{\delta y}$$

$$\hat{p}_z = \frac{\hbar}{i} \frac{\delta}{\delta z}$$

∴ in 3D, the operator for the momentum $\bar{p}$ can be represented by :-

$$\boxed{\hat{p} = \frac{\hbar}{i} \vec{\nabla}}$$

Energy operator

$$\boxed{\hat{E} = i\hbar \frac{\delta}{\delta t}}$$

Hamiltonian Operator

$$\boxed{H = \frac{-\hbar^2}{2m} \nabla^2 + V(\bar{r})}$$

③ 1-D potential box, Energy and wave function (full derivation).

→ Particle in a one dimensional box.

[i.e., in an infinitely deep one dimensional potential well]

Lets consider a free particle of rest mass 'm' is trapped in an infinitely deep potential energy well of width L. Since the box is 1-D, the particle moves along a straight line, say along the x-axis with its P.E. $V(x) = 0$, except at the boundaries (i.e., $x=0$ & $x=L$) where it is infinitely large.

Mathematically, the potential well can be expressed as,

$$V(x) = 0 \quad \text{at} \quad 0 < x < L$$
$$= \infty \quad \text{at} \quad x \leq 0 \text{ and } x \geq L.$$

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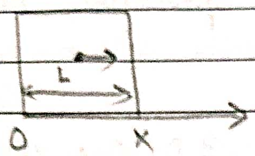
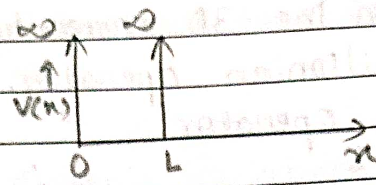


Fig(1) 1-D potential Box



Fig(2) The potential energy curve for the particle confined within an infinitely deep potential well with boundaries at $x=0$ & $x=L$.

As the potential well is infinitely deep, a particle can not exist outside of the wall (i.e., box) and is reflected back whenever it reaches the end of the range.

$\because$ the P.E. is independent of time, the 1-D time independent Schrödinger wave eq^n. in

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0.$$

Now, the time independent Schrödinger wave eq^n. for a free particle in the region where $V(x)=0$ is given by,

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}E\,\psi(x) = 0$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + K^2\psi(x) = 0 \quad \text{—— (i)}$$

[where, $K^2 = \frac{2mE}{\hbar^2}$ or $E = \frac{\hbar^2 K^2}{2m}$]

The ~~generalisation~~ sol^n. of this eq^n is given by,

$$\psi(x) = A\cos kx + B\sin kx \quad \text{—— (ii)}$$

Now, we know, from the boundary conditions:-

i) at $x=0$, $\psi(x)=0$

at $x=L$, $\psi(x)=0$

Thus, where, $x=0$, we get from eq^n (ii)

$$0 = A\cdot 1 + 0$$

$$\Rightarrow A = 0$$

So, eq^n (ii) now becomes

$$\psi(x) = B\sin kx \quad \text{— (iii)}$$

Again, at $x = L$, $\psi(x) = 0$.

Applying this boundary condition into eq^n (iii), we have,

$$B\sin kL = 0$$

$\because B \neq 0$ $\quad \sin kL = 0 = \sin n\pi$

$$\Rightarrow k = \frac{n\pi}{L}, \text{ where } n = 1, 2, 3, \text{etc.}$$

Hence, the permissible wave function for this motion are given from eq^n (iii)

$$\psi_n(x) = B\sin\frac{n\pi x}{L}$$

Energy Eigen values

Energy of the particle, $E = \frac{\hbar^2 k^2}{2m}$

$$\because k = \frac{n\pi}{L} \Rightarrow E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \frac{n^2 h^2}{8mL^2}$$

Normalized wave function (i.e., eigen function)

$$\psi_n(x) = B\sin\frac{n\pi x}{L}$$

The normalization condition is:-

$$\int_{-\infty}^{\infty} \psi_m^*(x)\,\psi_n(x)\,dx = 1$$

$\because \psi_n(x) = 0$ outside the box

$$\int_0^L \psi_n^*(x)\,\psi_n(x)\,dx = 1 \Rightarrow \int_0^L B^2 \sin^2\frac{n\pi x}{L} = 1$$

$$\Rightarrow \frac{B^2}{L}\int_0^L \left(1 - \cos\frac{2n\pi x}{L}\right)dx = 1$$

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$\Rightarrow \frac{B^2}{2}L - \frac{B^2}{2}\int_0^L \cos\frac{2n\pi x}{L}\,dx = 1$

$\Rightarrow \frac{B^2}{2}L = 1 \qquad \left[\because \int_0^L \cos\frac{2n\pi x}{L}\,dx = 0\right]$

$\Rightarrow B = \sqrt{\frac{2}{L}}$

So, the normalized wave function or eigen function

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$$

(4) Energy of Quantum harmonic Oscillator.

→ $E_n = \left(n+\frac{1}{2}\right)\hbar\omega \qquad n = 0, 1, 2, \ldots$

$E_0 = \frac{1}{2}\hbar\omega$

$E_1 = \frac{3}{2}\hbar\omega$

$E_2 = \frac{5}{2}\hbar\omega \ldots$

(6) What is Bohr magneton?

→ In atomic physics, Bohr magneton (Symbol μ_B) is a physical constant and the natural unit for expressing the magnetic moment of an e^- caused by its orbital or spin angular momentum.

$$\boxed{\mu_B = \frac{e\hbar}{2m_e}}$$

μ_B = Bohr magneton

e = elementary charge

$\hbar$ = reduced Planck constant

m_e = e^- rest mass.

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(7) What is Black Body Radiation? Conditions for Black Body Radiations.

→ refers to the spectrum of light emitted by any heated object. ex - including the heating elements of a toaster and the filament of a light bulb.

Conditions → A blackbody is a body that absorbs all wavelengths of light. No light is reflected & so, at low temp. it appears black. Emission from a blackbody its temp. dependent & at high temp., a blackbody will emit a spectrum of light.
Ex - Sun (a high temp. black body.)

(8) Wien's Displacement Law.

→ States that the black-body radiation curve for diff. ~~wave~~ temp. will peak at diff. wavelengths that are inversely proportional to the temp. Hotter objects emit radiations of shorter wavelengths & hence they appear blue

$$\boxed{\lambda_{max.} = \frac{b}{T}}$$

λ_{max} = wavelength peak
b = constant of proportionality
T = absolute temperature.

(9) Write down ~~Wien's law~~ & Rayleigh Jeans Law. What Ultraviolet catastrophe?

→ Rayleigh - Jeans law -

1. A black body in thermal equilibrium will emit radiation in all ν ranges, and as the ν ↑, so does the energy of the radiation emission.
2. The black body is a matter that can absorb & emit all energy that comes into contact with it.
3. Greater the intensity, the higher the temp. of matter.

4. The intensity & energy of the emitted radiation ↑ as the temp. rises.
5. As temp. ↑, the higher intensity shifts to the lower wavelength side.

Mathematically, $\boxed{\rho(\lambda, T) = \frac{2ckbT}{\lambda^4}}$

T = temp.
~~λ = wavelength~~
c = Speed of light in vacum
K = Boltzmann's const.

Ultraviolet catastrophe
1. Rayleigh's law predicts infinite intensity as wavelength tends to 0.
2. Bcz. intensity is a finite quantity, the impossible event of infinite intensity predicted by the wave model of electromagnetic radiation was dubbed the 'Ultraviolet catastrophe'.

(10) State Planck's Radiation Law.
→ It describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temp. T, when there is no net flow of matter or energy b/w the body and its environment.

$$\boxed{B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\left(e^{\frac{h\nu}{k_B T}} - 1\right)}}$$

B = Spectral radiance of body
ν = frequency
T = absolute temp.
K_B = Boltzmann const.
h = planck const.
C = Speed of light in medium

Statistical Mechanics

(1) What are micro state and macro state?

→ Microstate → In statistical mechanics, a microstate describes a specific detailed microscopic configuration of a system that the system visits in the course of its thermal fluctuations.

Macrostate → A macrostate of the system is defined by specifying the external parameters, and any other constraints to which the system is subject.
For ex- if we are dealing with an isolated system then the macrostate might be specified by giving the values of the vol. & the constant total energy.
For a many-particle system, there are generally a very great no. of microstates which are consistent with a given macrostate.

(2) Compare b/w FD, BE & MB statics.

Category	Maxwell-Boltzman Statics	Bose-Einstein Statics	Fermi-Dirac Statics.
No. of particles in a cell	any no. of particles can occupy a single phase space cell.	any no. of particles can occupy a single phase space cell	a phase space cell can't accomodate more than one particle.
Nature of particles.	Particles are identical but distinguishable ex- gas molecules.	Particles are identical but distinguishable ex- photon particles.	Particles are identical but indistinguishable ex- electron.
Spin	particles can have any spin.	zero / integral spin are called Boson.	Odd half integral spin called Fermions.

Category	M.B. Statistics	B.E. Statistics	F.D. Statistics
Energy Distribution.	$n_i = \cancel{g_i e^{-\alpha} e^{-\beta E_i}}$ $\frac{g_i}{e^{\alpha} e^{-\beta E_i}}$	$n_i = \frac{g_i}{e^{\alpha} e^{-\beta E_i} - 1}$	$n_i = \frac{g_i}{e^{\alpha} e^{-\beta E_i} + 1}$
Microstate & Macrostate	A macrostate can have more than one microstate.	each macrostate has one microstate	each macrostate has one microstate
Wave function	No specification in wave function of the particles.	wave function of a particle can be represented by symmetric wave function.	wave function of a particle can be represented by anti-symm. wave function

(3) Write down distribution function for fermion & boson particle
→ Fermi Dirac Statistics → applicable to the identical & indistinguishable particles of half-integral spin. These particles obey $\boxed{\text{Pauli exclusion principle}}$ & are called fermions. Ex- e^-, protons, neutrons, etc.

$$\boxed{n_i = \frac{g_i}{e^{\alpha} e^{-\beta E_i} + 1}}$$

Bose Einstein's Statistics → applicable to the identical & indistinguish particles of zero or integral spin. These particles are called bosons. ex- photons, phonons.

$$\boxed{n_i = \frac{g_i}{e^{\alpha} e^{-\beta E_i} - 1}}$$