MOD-I

① Background of Image Processing :-

- Digital image processing refers to processing digital images using computers.
- Involves applying mathematical & algorithmic techniques to improve, analyze, and extract info. from images.

② Digital Image Representation :-

- It is made up of a grid of small dots called pixels (Pixels - Short for picture elements).
- Each pixel represents a color / intensity value.
- It is typically represented as ~~as~~ 2D matrix of pixel value.
- Intensity ~~value~~ Levels :- Brightness value of a pixel. For grayscale, typically 0 (black) to 255 (white) for 8-bit images.
- Color images : Represented using 3 color channels (Red, Green, Blue - RGB), each with its own Itensity.
- File formats - JPEG, PNG, BMP, TIFF in compressed / uncompressed forms.

③ Fundamental Steps in Image Processing :-

1. Image Acquisition - capturing an image using camera / sensor
2. Preprocessing - Removing noise & enhancing image quality
3. Segmentation - Dividing the image into regions / objects.
4. Representation and Description - converting image data into a usable form.
5. Recognition - Identifying objects or patterns.
6. Interpretation - Understanding the image content.

④ Elements of Digital Image Processing :-

1. Image Acquisition - 1st step
   - Image is captured from real world using input device.
   - Device used :- Cameras, Scanners, etc.
   - converting into digital form using ADC.

2. Image Storage - after capturing, the image needs to be stored digitally for future use.
   - Temporary :- RAM (during processing)
   Permanent :- Hard drives, SSDs, etc.
   - Format :- JPEG, PNG, etc. (compression/resol.)

3. Image Processing - core element where the image is improved / analyzed.
   - improving image quality (brightness)
   - Removing blurs, noise
   - reducing file size (compression)
   - analyzing shapes & str.
   - filtering (sharpen)

4. Image Communication - Sharing / transmitting images from one device / syst. to another.
   - methods :- wired / wireless transmissi- internet (email, websites, social media)
   - maintaining quality during transmissi-
   - error detection & correction during ..

5. Image Display - Presenting the processed image to the user for viewing.
   - Device :- TV, projector, phones, etc.
   - refresh rate & brightness.

MOD-II

① **Digital Image Formation**

- process of creating a digital image from real-world scene using mathematical model & digital techniques

② i) **A Simple Image Model:-**

A digital image can be represented as a 2D function $f(x, y)$

x, y = Spatial coordinates of images

$f(x,y)$ = intensity / gray level at point (x,y).

for grayscale images:-

intensity values range from 0 (black) to 255 (white) for 8 bit images.

for color images:-

there are 3 values for each pixel - R, G, B (Red Green Blue)

ii) **Geometric Model** - used to modify the pos^n, size or orientation of an image.

a) **Translation** - Shifts the image from 1 pos^n to another

Formula:- $x' = x + T_x$, $y' = y + T_y$

where T_x and T_y are translation distances.

b) **Scaling** - changes the size of the image (zoom in / out)

Formula:- $x' = S_x \cdot x$, $y' = S_y \cdot y$ S_x, S_y = Scaling factor

c) **Rotation** - Rotates the image around the origin or a point.

Formula:- $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$.

θ = angle of rotation.

③ **Perspective Projection** -

- used to create a 3D view on a 2D image.
- Distant objects appear smaller, giving a depth effect.
- Formula (Simplified):- $x' = \frac{x}{z}$, $y' = \frac{y}{z}$

z = depth or distance from the viewer.

(4) Sampling and Quantization:-
Process of converting an analog image to a digital image.

a) Sampling → selecting values at specific intervals (Spatial resolution).
→ Divides image into a grid of pixels.
→ Higher sampling = higher resolution.

- Think of an image as a big paintaing.
- Sampling means we pick certain points (pixels) from it in a grid pattern.
- The more points (higher sampling rate) we pick, the better the image quality.

Ex:- If you sample a photo at 10x10 pixels, it looks blocky.
If you sample a 1000x1000 pixels, it looks clear.

Quantization → Assigning fixed no. of intensity (brightness or color) levels to each sampled pixel. It affects intensity resolution. (no. of brightness levels).
Ex:- If we allow only 2 values → Black or White (1-Bit Image)

If we allow 256 values → Smooth grayscale (8-bit images)

Uniform Quantization	Non-uniform Quantizn
- Divides full range into equal-sized intervals.	- Divides the range into unequal intervals.
- Interval size - small.	- varies depending on region of interest
- complexity - simple.	- more complex.
- Less accurate	- more accurate.
- uniform brightness distribution.	- non-uniform.
- Ex:- each level differs by 10. (eg. 0,10,20,...)	- Levels: 0-5, 6-15, 16-30, 31-60 ... (non-equal gaps).

MOD-III

Mathematical Preliminaries

① **Neighbour of Pixels** -
A pixel has neighbors that influence its value during filtering & processing.

- For a pixel at (x, y):

- 4-Neighbours (N4):
  Left $(x-1, y)$
  Right $(x+1, y)$
  Top $(x, y-1)$
  Bottom $(x, y+1)$

- 8-Neighbours (N8):
  Includes the 4-neighbours + diagonals:-
  $(x-1, y-1), (x+1, y-1), (x-1, y+1), (x+1, y+1)$

- Diagonal neighbours:-
  Only the diagonal neighbours.

② Connectivity in Digital Images:-
- defines how pixels are considered connected based on their posⁿ & values.
- It is imp. for identifying objects, regions, or boundaries in an image.

To say that 2 pixels are "connected," two things must be true:-
1. They must have similar intensity or gray level (eg. both white or both black).
2. They must be neighbours - either directly or diagonally.

Types of connectivity:-
• 4-connectivity (4-connected neighbourhood)
• A pixel is connected to its 4 immediate neighbours: Top, Bottom, left, Right.

Ex:- If pixel is at posⁿ. (x, y), neighbors are:-
(x-1, y), (x+1, y), (x, y-1), (x, y+1)

```
  0
0 x 0
  0
```

• 8-connectivity (8-connected neighbourhood)
Includes 4 direct neighbors + 4 diagonal neighbors.
Neighbors of (x, y):
• (x-1, y-1), (x-1, y), (x-1, y+1)
• (x, y-1), x, (x, y+1)
• (x+1, y-1), (x+1, y), (x+1, y+1)

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0 0 0
0 x 0
0 0 0

- m-connectivity (Mixed connectivity)
- A hybrid model that avoids ambiguity in diagonal connections.
- combines 4-connectivity & some diagonal connections only if they don't form a "bridge" b/w separate objects.

(3) Relations - a way to define a connection b/w 2 pixels.
A relation R on a set of pixels S is a rule that says whether 2 pixels (p, q) are related or not.
Ex:- "Pixel p is a neighbor of q"
"Pixel p has the same intensity as q"
"Pixel p is connected to q"
Mathematically,
If $(p, q) \in R$, it means pixel p is related to pixel q.

Equivalence Relation
Special type of relation that satisfies three properties:-
Reflexive - every pixel is related to itself: $(p, p) \in R$
Symmetric:- If p is related to q, then q is related to p: $(p, q) \in R \Rightarrow (q, p) \in R$

Transitive:- If p is related to q, and q is related to r, then p is related to r:-
$(p,q) \in R$ and $(q,r) \in R \Rightarrow (p,r) \in R$.

Transitive closure - of a relation R is the smallest relation that includes R & also satisfies transitivity.
If you know pixel A is related to B, and B is related to C, then Transitive closure says: A is related to C too.

use

- Helps identify all pixels that are part of a connected region.
- Builds the complete connected component from a seed pixel.

(4) Distance Measures → used to measure how far two pixels are from each other.

Help in - object detection, Segmentation, Region growing, Morphological operati

(i) Euclidean Distance (D _e)

- It's the st. line dist. b/w 2 pixels.
- Most natural & used when diagonal movt is allowed.

Formula :- $D_e(p,q) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Ex:- Between (1,1) and (4,5):

$D_e = \sqrt{(4-1)^2+(5-1)^2} = \sqrt{9+16} = 5$

ii) City Block Distance (D₄ or Manhattan Distance.

- Measures dist. only along horizontal & vertical lines
- No diagonal movt. allowed.
- Based on 4-connectivity.

Formula:-

$D_4(p,q) = |x_1 - x_2| + |y_1 - y_2|$

Ex:- Between (1,1) and (4,5):

$D_4 \; |4-1| + |5-1| = 3+4 = 7.$

iii) Chessboard distance (D₈)

- Measure dist. by allowing diagonal movt.
- Based on 8-connectivity.

Formula:-

$D_8(p,q) = \max(|x_1 - x_2|, |y_1 - y_2|)$

Ex:- B/w (1,1) and (4,5):

$D_8 = \max(3,4) = 4.$

⑤ Arithmetic and Logical Operations

- To manipulate & process images pixel by pixel.
- useful for enhancement, masking, highlighting features, etc.

Arithmetic Operations:-

Addition. $g(x,y) = f(x,y) + h(x,y)$ - Brightening image, merging 2 images
Subtraction. $g(x,y) = f(x,y) - h(x,y)$ - Edge detection, bckg. subtracⁿ.
Multiplication $g(x,y) = f(x,y) \times h(x,y)$: Masking, adjusting contrast.
Division. $g(x,y) = f(x,y) \div h(x,y)$: Normalizing pixel values.

where:-
$f(x,y)$: 1st image.
$h(x,y)$: 2nd image / scalar
$g(x,y)$: output image.

Logical operations - mainly used in binary (black & white) images or masks.

- AND → Pixel = 1 if both pixels are 1
Ex:- Extract overlapping regions of 2 images.
- OR → Pixel = 1 if any 1 pixel is 1
combine regions from 2 images.
- NOT → Pixel = inverse (1 becomes 0, 0 becomes 1)
Ex:- Invert an image (black ↔ white)
- XOR → Pixel = 1 if pixels are different.
Highlight diff. b/w images.

Ex:- Let's say 2 binary images A & B:-
A = [1 0 1]
B = [1 1 0]
- A AND B = [1 0 0]
A OR B = [1 1 1]
A XOR B = [0 1 1]
NOT A = [0 1 0]

(6) Fourier Transformation (FT)
- mathematical technique ~~is~~ used to convert an image from the spatial domain to the ν domain.
→ In spatial domain, we view images as pixels.
→ In ν domain, we represent images in terms of sine & cosine waves (ν).

Why??
→ to analyze texture & patterns
→ to perform image filtering
→ to compress images
→ to remove periodic noise.

Properties
Linearity
Scaling
rotation
Periodicity
Energy is preserved

Mathematical formula (2D continuous FT):

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \cdot e^{-j2\pi(ux+vy)}\,dx\,dy.$$

where : $f(x,y)$ = input image in spatial domain.
$F(u,v)$ = transformed image in ν domain.
(u,v) = ν coordinates
j = imaginary unit.

Inverse Fourier Transform:-
To convert ν domain image back to spatial domain.

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) \cdot e^{j2\pi(ux+vy)}\,du\,dv$$

Discrete Fourier Transform (DFT):-
∵ digital images are discrete (not continuous)
we use the DFT version:-

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \cdot e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$M \times N$ = size of the image.

Application Noise Removal, compression.

Blurry image may have low-ν dominance.
Sharp image will contain many high-ν comp.

Discrete cosine & sine Transform
Image compression (eg. JPEG).
DCT:-

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$$F(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{(2x+1)u\pi}{2\pi}\right]$$

$$\cos\left[\frac{(2y+1)u\pi}{2N}\right]$$

- Only real no.
- energy compaction
- Discrete Sine Transform (DST)
  - Similar to DCT but uses sine functions.
  - Less commonly used than DCT.

MOD-4 :-

Image Enhancement

- Process of improving the visual appearance of an image or making it more suitable for analysis by highlighting imp. features.
- It can be performed in 2 main Domain:- ① domain. Spatial domain

1) Spatial Domain Method - Direct manipulation of pixels in image.
- Each pixel has a value (intensity / color).
- It involve modifying these pixel values directly to achieve image enhancement.
- $g(x,y) = T[f(x,y)]$

where:- $f(x,y)$ - Original image

$g(x,y)$ - enhanced image.

T - transform. fun. applied to each pixel.

- Common Spatial Domain Techniques:-
- Brightness adjustment - Add a const. to all pixel value
- contrast adjustment - Multiply all pixel values by a factor.
- Histogram Equaliz". - Improve contrast using the pixel distribution.

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→ Smoothing - reduce [illegible]
→ Sharpening - highlight edges & details - Laplacian

- Spatial filtering - use a filter mask to process each pixel based on its neighbors.
- Advantages - simple, easy to implement, fast for small filters.
- Limitations - less effective for complex problems.

2) Frequency Domain Method.

- analyze & modify image based on rate at which pixel values change, rather than modifying pixel values directly.
- Transform the image from spatial domain to ν domain using Fourier Transform.
- Apply a filter to modify specific ν
- Inverse transform it back to spatial domain to get the processed image.
- Adv. - remove periodic noise
  - sharpen / smooth images.
- common filters :- Low-pass filter - allows low ν, blocks high ones (Image Smoothing)
  - High-pass - allows high ν, blocks low ones (Edge detection)
  - Band-pass - allows a specific range of ν - detail enhancement
  - Notch filter - removes specific ν - noise reduction.

Spatial	Frequency
works on Pixels	frequencies (patterns)
Technique - filtering, masking	Fourier Transform & filters
Best for - Simple enhancements	Noise removal.
Visualiz^n - easy	Harder.
Processing Speed - fast for small filters.	Slower unless using FFT

3) Contrast Enhancement

- Technique to improve visibility of features in an image by expanding the range of intensity values (brightness)
- If an image looks dull, grayish – contrast enhancement helps make it clearer.
- Two common methods → Linear contrast Stretching
  → Non-Linear "

Linear – Linearly maps original pixel values to a new wider range (like 0 to 255).

Formula :- $S = \frac{(r - r_{min}) \times (S_{max} - S_{min})}{r_{max} - r_{min}} + S_{min}$

r = input pixel value.
r_min, r_max = min. & max. of input ~~rang~~ image.
S_min, S_max = desired output range (usually 0 to 255)
S = new pixel value.

- used – when you want to stretch all pixel values proportionally. Image is uniformly dark/light.

Ex:- Original Pixel Range	Output Pixel Range
50 to 150	0 to 255

A pixel with value 100 will become:-

$$S = \frac{(100 - 50) \times (255 - 0)}{150 - 50} = 127.5$$

Non-Linear – uses non-linear functions (logarithmic/power-law functions)
– to stretch contrast non-uniformly.

Techniques:-

i) Log Stretching :- $S = c \cdot \log(1 + r)$ – enhances dark regions
ii) Power-law (γ) :- $S = c \cdot r^{\gamma}$ – control brightness/contrast
iii) Exponential Stretching :- $S = c \cdot (e^{ar} - 1)$ – enhance bright/dark zones.

r – input pixel (normalized 0-1)
S = output pixel
c, γ, a :- constants.

- used when want to highlight specific ranges
- image has uneven lighting / hidden features.

47. Histogram Processing -
- Graphical repⁿ that shows how frequently each intensity (gray level) appears in an image.
- method to enhance image contrast using histogram of pixel intensity values.
- to improve image brightness & contrast
  - enhance dark, light images.
- Histogram → bar graph of pixel intensities (x-axis : intensity ; y-axis : ν)

Equalization → spreads out pixel values evenly across the range.

Specification → Matches histogram of one image to another.

- Techniques →

Histogram Equalization - Spread intensities uniformly.
Histogram Stretching - Expand intensity range.
Histogram Matching - Match another histogram.

- Ex:- If an image is too dark, most of its pixel values will lie in the lower range (0-50).
  Histogram equalizⁿ. will spread them across 0-255, making it brighter & clearer.

- Adv. - Automatic contrast improvement
  - Makes hidden details visible.

Disadv. → may not work well for images with complex lighting.

5> **Smoothing** – (Noise Reduction)
- Technique used to reduce noise & blur unnecessary details in an image.
- make the image clearer & less noisy
- prepare image for further processing (edge detection)
- Techniques → Image Averaging
  → Median filter
  → Gaussian filter (Low-Pass filter)

Image Averaging (ex. Satellite, medical imaging) → combine multiple noisy images of the same scene.
→ noise gets averaged out.

Smoothed Image = $\frac{1}{n} \sum_{i=1}^{n} \text{Image}_i$

Mean filter (Blurs edges too) – Each filter value - replaced with avg. of its neighbours.
- uses a kernel (3×3) to slide over the image.

New Pixel = $\frac{1}{N} \sum$ Neighbour Pixels.

Low-Pass filter (more natural blur) (better edge preservation) → weighted avg. where central pixels get more importance.
→ smooths the image without destroying edges

$$G(x, y) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$

6> **Image Sharpening** – highlights edges & fine details
- High-Pass Filtering → keeps high-ν components (edges, fine details)
  → removes low-ν background
- High-Boost Filtering → enhances edges while preserving overall str.

$g(x, y) = A \cdot f(x, y) - \text{Low-pass}(f(x, y))$

- Derivative filtering :- uses gradient operators (Sobel, Laplacian) to detect & enhance edges.

7> Homomorphic filtering →
- Enhances image by separating illumination & reflectance components
- Performed in logarithmic & ν domain
- enhances contrast & compresses dynamic range.

8> Enhancement in frequency domain :-
→ Image is transformed using Fourier Transform
→ filters are applied to modify ν components.

- Low-Pass filtering
  Removes high ν (noise, edges)
  Smooths the image.
- High-Pass filtering
  Removes low ν (bckg)
  Enhances fine details & edges.

MOD-5

(1) Image Restoration :-

- Process of recovering an original image that has been degraded by known distortions like blurring, noise or motion.
- Unlike enhancement (which is subjective), Restoration is objective, based on mathematical models.

2) Degradation Model -

General model of Image degradation :-

$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$

where :- $f(x, y) \rightarrow$ original image.

$h(x, y) \rightarrow$ Degradation fn. (eg. blur)

$* \rightarrow$ convolution operator.

$\eta(x, y) \rightarrow$ Additive noise

$g(x, y) \rightarrow$ observed degraded image.

3) Discrete Model -

In digital Images (discrete), the above model becomes

$g = Hf + \eta$

where :- $H \rightarrow$ matrix form of degradation

$f \rightarrow$ vector form of original image

$g \rightarrow$ " " degraded image.

useful for algebraic restoration methods.

4) Algebraic Approach to Restoration -

Techniques solve for the original image using linear algebra.

Unconstrained Restoration - Directly inverse the degradation model.

$\hat{f} = H^{-1} g$

Problem:- If H is not invertible or is ill-conditioned, results are unstable.

Constrained Restoration - Adds a constraint to improve stability.

5) Constrained Least Square Restoration

Minimizes a cost fun^n that balances fidelity & smoothness:-

$$J = \|g - Hf\|^2 + \gamma \|Cf\|^2$$

where:-

$|g - Hf|^2$ → restoration error.

$|Cf|^2$ → constraint (eg - smoothness)

γ → regulariz^n parameter.

C → high-pass operator (like Laplacian)

This gives a stable sol^n even with noisy data.

6) Restoration by Homomorphic Filtering:-

Seperates illumination & reflectance in images.

low ν high ν

Steps → take logarithm of image.

→ Apply high-pass filter in ν domain.

→ Apply exponential to get restored image.

useful in non-uniform illumination correction.

7) Geometric Transformation -

- used to correct geometric distortion like rotation, translation or warping.

Spatial Transformation → uses a mapping function:

$$(x', y') = T(x, y)$$

(x, y) → coordinates in i/p image

(x', y') → " " o/p image.

ex:- Rotation, Translation, Scaling.

Gray Level Interpolation:- ∵ transformed coordinates (x', y') may not fall on actual pixels, Interpolation is used to estimate pixel values.
Common method:-
Nearest neighbor (fast but blocky)
Bilinear interpolation (smooth)
Bicubic interpolation (more accurate).

MOD-6.

Image Segmentation

Process of dividing an image into meaningful regions like objects, boundaries, or features.

- Simplifies image analysis by isolating areas of interest.

•) Point Detection - Identifies isolated pixels with diff. intensity than their neighbors. Typically uses Laplacian filter / difference masks to detect these points.

•) Line Detection - Detect lines in specific directions (horizontal, vertical, diagonal) uses line masks like.

direction	Mask (3x3)
horizontal	-1 -1 -1 ; 2 2 2 ; -1 -1 -1
vertical	-1 2 -1 ; -1 2 -1 ; -1 2 -1

•) Edge Detection - Detects boundaries where intensity changes sharply.
common operators.

- Sobel (gradient magnitude)
- Prewitt
- Roberts
- Laplacian
- Canny (most accurate, multi-steps)

•) Combined Detection - combines point, line, edge detecⁿ results.

Gives better structural info. about objects.

•) Edge Linking & Boundary Detection.
once edges are detected, they need to be connected to form boundaries.

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Local Processing - uses neighborhood info. to link edges.

Eg. If two edge pixels are close & have similar gradient direcⁿ., they are linked.

Global Processing (Hough Transform) -
Detects lines, circles, etc by transforming pixels into parameter space.
Useful for detecting shapes even in noisy images.

•> Thresholding - Segments image based on intensity values.

Foundation
Assumes that obj. & bcg. have diff. gray levels.

Simple Global Thresholding
- One fixed threshold T is chosen.
- If $f(x, y) > T$, then object; else bcg.

Optimal Thresholding
Automatically finds the best threshold.
eg. Otsu's method - minimizes intra-class variance.

•> Region Oriented Segmentation
Segments based on similarity of pixels in a region.

- Basic Formulation -
  - Starts from a seed point & grows/splits regions based on criteria.
- Region Growing (by Pixel Aggregation)
  Begins with seed pixel and adds neighbors if similar in intensity.

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- **Region Splitting & Merging**

Splits the image into smaller regions. Then merges adjacent regions if they meet similarity criteria.