Maths Assignment.

Date 09/02/23
Page

Assignment - II

① ~~find the evolute of the parabola~~ $\cancel{y^2 = 4ax}$

$\Rightarrow$

① If λ is the eigen value of A matrix, the P.T. λ^2 is the eigen value of A^2.

$\rightarrow$ λ is given eigen value of A $\Rightarrow A\alpha = \lambda\alpha$

$A(A\alpha) = A(\lambda\alpha) \Rightarrow A^2\alpha = \lambda(A\alpha)$

$\Rightarrow A^2\alpha = \lambda\lambda\alpha \Rightarrow A^2\alpha = \lambda^2\alpha$

$\therefore \lambda^2$ is given eigen value of A^2.

② Find the eigen values & corr. eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

The characteristic eq^n. is $|(A - \lambda I_3)| = 0$ $\quad \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)[(2-\lambda)(1-\lambda)] - 1[(1-\lambda)] + 1[0] = 0$

$\Rightarrow (2-\lambda)[2 - 2\lambda - \lambda + \lambda^2] - 1(1-\lambda) = 0$

$\Rightarrow (2-\lambda)(\lambda^2 - 3\lambda + 2) - 1 + \lambda = 0$

$\Rightarrow 2\lambda^2 - 6\lambda + 4 - \lambda^3 + 3\lambda^2 - 2\lambda - 1 + \lambda = 0$

$\Rightarrow -\lambda^3 + 5\lambda^2 - 7\lambda + 3 = 0$

$\Rightarrow \cancel{\lambda(-\lambda^2 + 5\lambda - 7) + 3 = 0}$

-6λ

$$\begin{array}{r|rrrr} 3 & -1 & 5 & -7 & 3 \\ & 0 & -3 & 6 & -3 \\ \hline & -1 & 2 & -1 & 0 \end{array}$$

$\Rightarrow (\lambda - 3) = 0$

$(-\lambda^2 + 2\lambda - 1) = 0$

$\cancel{(-\lambda^2 - \lambda - 1) = 0}$

$\cancel{\Rightarrow -\lambda(\lambda - 2) - 1(\lambda}$

$-\lambda^2 - \lambda - \lambda - 1 = 0$

$-\lambda(\lambda + 1) - 1(\lambda + 1) = 0$

$\Rightarrow \lambda = -1$ or $\lambda = -1$

Date 10/02/23

Subspace.

(1) Let $V = R^2$, show that W is a subspace of V
where, i) $W = \{(a,b,0) : a,b \in R\}$
ii) $W = \{(a,b,c) : a+b+c = 0\}$
iii) $W = \{(a,a,0) : a \in R\}$

Sol^n: i) Let $v, w \in W$

$v = (a_1, b_1, 0)$ $\quad w = (a_2, b_2, 0)$

$\boxed{av+bw} = a(a_1, b_1, 0) + b(a_2, b_2, 0)$
$= (aa_1, ab_1, 0) + (a_2b, bb_2, 0)$
$= (aa_1 + a_2b, ab_1 + bb_2, 0)$

So, $av+bw \in W$ for all a, b are real
$\therefore W$ is subspace of V.

ii) Let $v, w \in W$

$v = (a_1, b_1, c_1)$ $\quad w = (a_2, b_2, c_2)$

$\boxed{av+bw} = a(a_1, b_1, c_1) + b(a_2, b_2, c_2)$
$= (aa_1, ab_1, ac_1) + (ba_2, bb_2, bc_2)$
$= (aa_1 + a_2b, ab_1 + bb_2, ac_1 + bc_2) \in W$

So, $av+bw \in W$ for a, b, c are real
$\therefore W$ is subspace of V.

Now,
$(aa_1 + ba_2) + (ab_1 + bb_2) + (ac_1 + bc_2)$
$= a(a_1 + b_1 + c_1) + b(a_2 + b_2 + c_2)$
$= a \cdot 0 + b \cdot 0 = 0.$

iii) $v = (a_1, a_1, 0)$; $w = (a_2, a_2, 0)$

$\boxed{av+bw} = a(a_1, a_1, 0) + b(a_2, a_2, 0)$
$= (aa_1, aa_1, 0) + (a_2b, a_2b, 0)$
$= (aa_1 + ba_2, aa_1 + ba_2, 0)$

So, $av+bw \in W$, for all a is real.
$\therefore W$ is subspace of V.

Mapping.

(2) Show that the mapping T be linear $T : R^2 \to R^2$ defined by $T(x, y) = (x-y,\ x+y,\ 2x)$

Sol. Let $X, Y \in R^2$

$X = (x_1, y_1) \qquad Y = (x_2, y_2)$

i) $T(X) = (x_1 - y_1,\ x_1 + y_1,\ 2x_1)$

$T(Y) = (x_2 - y_2,\ x_2 + y_2,\ 2x_2)$

$$aX + bY = a(x_1, y_1) + b(x_2, y_2)$$
$$= (ax_1, ay_1) + (bx_2, by_2)$$
$$= (\underbrace{ax_1 + bx_2}_{x},\ \underbrace{ay_1 + by_2}_{y})$$

$$T(aX + bY) = (ax_1 + bx_2 - ay_1 - by_2,\ ax_1 + bx_2 + ay_1 + by_2,\ 2ax_1 + 2bx_2)$$
$$= \cancel{ax_1 - ay_1}$$
$$= a(x_1 - y_1,\ x_1 + y_1,\ 2x) + b(x_2 - y_2,\ x_2 + y_2,\ 2y)$$
$$= aT(X) + bT(Y)$$

Basic Dimensions

(Q) Show that the vectors $(1,-2,3),(2,3,1),(-1,3,2)$ from a basis of R^3.

Sol:
$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & 1 \\ -1 & 3 & 2 \end{vmatrix}$$

$$\Delta = 1(6-3)+2(4+1)+3(6+3)$$
$$= 3+10+27$$
$$= 40$$

$\because \Delta \neq 0 \Rightarrow$ linearly independent.

$\because$ dimension of R^3 = no. of linearly independent vectors containing R^3

So, S is a basis of R^3.

Q) P.T. $S = \{(1,1,2),(1,2,5),(5,3,4)\}$ is not a basis of R^3.

Sol:
$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 5 & 3 & 4 \end{vmatrix}$$

$$\Delta = 1(8-15)-1(4-25)+2(3-10)$$
$$= -7+21-14$$
$$= 0$$

So, S is linearly dependent.
So, S is not a basis of IR^3.

Q) Show that $S = \{(1,1,1),(1,-1,5)\}$ is not a basis of IR^3.

Sol:
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 5 \end{vmatrix}$$

For a basis of R^3, must contain exactly 3 elements.
$\because R^3$ is of dimension 3

So, S is not a basis of IR^3.

Rolle's theorem

Q) $f(x) = 2x^3 + x^2 - 4x - 2$ points $\underset{a}{-\sqrt{2}} \leq x \leq \underset{b}{\sqrt{2}}$

Soln:- $f(x) = 2x^3 + x^2 - 4x - 2$

Let $f(a) = 2a^3 + a^2 - 4a - 2$

$= 2(-\sqrt{2})^3 + (-\sqrt{2})^2 - 4(-\sqrt{2}) - 2$

$= -4\sqrt{2} + 2 + 4\sqrt{2} - 2$

$= 0$

$f(a) = 0$

Let $f(b) = 2b^3 + b^2 - 4b - 2$

$= 2(\sqrt{2})^3 + (\sqrt{2})^2 - 4(\sqrt{2}) - 2$

$= 4\sqrt{2} + 2 - 4\sqrt{2} - 2$

$= 0$

$f(b) = 0$

$f(a) = f(b)$

$\because f(a) = f(b) \Rightarrow$ Rolle's theorem is valid.

$f(x) = 2x^3 + x^2 - 4x - 2$

$f'(x) = 6x^2 + 2x - 4$

Let $f'(c) = 6c^2 + 2c - 4$

$= 6c^2 + 6c - 4c - 4$

$= 6c(c+1) - 4(c+1)$

$= (6c-4)(c+1)$

Now, $f'(c) = 0$.

$\Rightarrow (6c-4)(c+1) = 0$

$\Rightarrow c = \frac{2}{3}$ or $c = -1$

Q) $f(x) = \cos x$ in $[-\pi/2, \pi/2]$

$f(x) = \cos x$

let $f(a) = \cos a = \cos(-\frac{\pi}{2}) = 0$

$f(b) = \cos b = \cos(\frac{\pi}{2}) = 0$

$\because f(a) = f(b) \Rightarrow$ Rollo's theorem is valid.

Now, $f(x) = \cos x$

$f'(x) = -\sin x$

$f'(c) = -\sin c$

$\because f'(c) = 0 \Rightarrow -\sin c = 0 \Rightarrow c = \sin^{-1} 0 \Rightarrow \boxed{c = 0}$

Mean Value Theorem $\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$

Q) Verify lagrange's mean value theorem for the function $f(x) = x^2 - 2x + 4$ in the interval $[1, 5]$

$f(x) = x^2 - 2x + 4$

$f(a) = a^2 - 2a + 4 = 1 - 2 + 4 = 3$

$f(b) = b^2 - 2b + 4 = 25 - 10 + 4 = 19$

$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{19 - 3}{5 - 1} = \frac{16}{4} = 4.$

$f'(x) = 2x - 2$

$f'(c) = 2c - 2$

$4 = 2c - 2$

$\Rightarrow c = \frac{6}{2} = 3.$

Caley Hemilton

Q) $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ Solve by caley Hemilton theorem.

Acc. to caley Hemilton theorem, $A - \lambda I = 0$.

$\therefore \begin{bmatrix} 0-\lambda & 0 & 1 \\ 3 & 1-\lambda & 0 \\ -2 & 1 & 4-\lambda \end{bmatrix}$

$\Delta = -\lambda[(1-\lambda)(4-\lambda)] + 1[3 + 2(1-\lambda)] = 0$

$\Rightarrow -\lambda(4 - \lambda - 4\lambda + \lambda^2) + (3 + 2 - 2\lambda) = 0$

$\Rightarrow -4\lambda + \lambda^2 + 4\lambda^2 - \lambda^3 + 5 - 2\lambda = 0$

$\Rightarrow -\lambda^3 + 5\lambda^2 - 6\lambda + 5 = 0$

$\Rightarrow -\lambda^3 + 5\lambda^2 - 6\lambda + 5 = 0$

$\Rightarrow -(\lambda^3 - 5\lambda^2 + 6\lambda - 5) = 0$

$\Rightarrow \lambda^3 - 5\lambda^2 + 6\lambda - 5 = 0$

Now, this can be written as:- $A^3 - 5A^2 + 6A = 0$

$A^3 = A^2 \cdot A$

$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0+0-2 & 0+0+1 & 0+0+4 \\ 0+3-0 & 0+1+0 & 3+0+0 \\ -0+3-8 & 0+1+4 & -2+0+16 \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 & 4 \\ 3 & 1 & 3 \\ -5 & 5 & 14 \end{bmatrix}$

$A^3 = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 1 & 3 \\ -5 & 5 & 14 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0+3-8 & 0+1+4 & -2+0+16 \\ 0+3-6 & 0+1+3 & 3+0+12 \\ 0+15-28 & 0+5+14 & -5+0+56 \end{bmatrix}$

$= \begin{bmatrix} -5 \\ -3 \\ -13 \end{bmatrix}$

Now,

$$= \begin{bmatrix} -5 & 5 & 14 \\ -3 & 4 & 15 \\ -13 & 19 & 51 \end{bmatrix}$$

Now, we have, $A^3 - 5A^2 + 6A - 5$

$$\begin{bmatrix} -5 & 5 & 14 \\ -3 & 4 & 15 \\ -13 & 19 & 51 \end{bmatrix} - \begin{bmatrix} -10 & 5 & 20 \\ 15 & 5 & 15 \\ -25 & 25 & 70 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 6 \\ 18 & 6 & 0 \\ -12 & 6 & 24 \end{bmatrix}$$

$$-5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5+10+0-5 & 5-5+0-0 & 14-20+6-0 \\ -3-15+18-0 & 4-5+6-5 & 15-15+0-0 \\ -13+25-12+0 & 19-25+6+0 & 51-70+24-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, Caley Hemilton theorem (proved)

$(1,2),(1,3),(1,4)$
$(2,3),(2,4),(3,4)$

Laplace's expansion.

Q) P.T. $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$

By laplace's expansion :-

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ 1 & 0 & a & b & c \\ 2 & -a & 0 & d & e \\ 3 & -b & -d & 0 & f \\ 4 & -c & -e & -f & 0 \end{array} = (-1)^{1+2+1+2}\begin{vmatrix} 0 & a \\ -a & 0 \end{vmatrix}\begin{vmatrix} 0 & f \\ -f & 0 \end{vmatrix} + (-1)^{1+2+1+3}$$

$$\begin{vmatrix} 0 & b \\ -a & d \end{vmatrix}\begin{vmatrix} -d & f \\ -e & 0 \end{vmatrix} + (-1)^{1+2+1+4}\begin{vmatrix} 0 & c \\ -a & e \end{vmatrix}\begin{vmatrix} -d & 0 \\ -e & f \end{vmatrix}$$

$$+ (-1)^{1+2+2+3}\begin{vmatrix} a & b \\ 0 & d \end{vmatrix}\begin{vmatrix} -b & f \\ -c & 0 \end{vmatrix} + (-1)^{1+2+2+4}\begin{vmatrix} a & c \\ 0 & e \end{vmatrix}\begin{vmatrix} -b & 0 \\ -c & -f \end{vmatrix}$$

$$+ (-1)^{1+2+3+4}\begin{vmatrix} b & c \\ d & e \end{vmatrix}\begin{vmatrix} -b & -d \\ -c & -e \end{vmatrix}$$

$= (-1)^6 (a^2f^2) + (-1)^7 (abef) + (-1)^8 (acdf) + (-1)^8 (adcf)$

$+ (-1)^9 (aebf) + (-1)^{10} [(be - dc)(be - cd)]$

$= a^2f^2 - abef + acdf + adcf - aebf + (be - dc)^2$
$= a^2f^2 - abef + acdf + adcf - aebf + b^2e^2 + d^2c^2 - 2bedc$
$= a^2f^2 + b^2e^2 + d^2c^2 - 2aebf - 2bedc + 2acdf$
$= (af)^2 + (be)^2 + (dc)^2 - 2(aebf) - 2(bedc) + 2(acdf)$
$= (af - be + cd)^2$

(proved)

Gauss-Jo

Q) Find A^{-1}

Now,

Gauss-Jorden Elimination method

Q) Find A^{-1} by this method

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{bmatrix}$$

This can be written as:- $AX = I$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A | I$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 4 & 0 & 1 & 0 \\ 3 & 3 & 7 & 0 & 0 & 1 \end{array}\right]$$

∴ To make value of lower triangle zero we perform row elementary operation.

$R_2' \to R_2 - 2R_1$

$R_3' \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right]$$

New A   New I

Now,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} x_{11} + x_{21} + 2x_{31} \\ +2x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} x_{11} + x_{21} + 2x_{31} &= 1 \\ 2x_{21} &= -2 \\ x_{31} &= -3 \end{aligned}$$

$\Rightarrow x_{31} = -3$

$x_{21} = -2/2 = -1$

$\therefore x_{11} = 1 - x_{21} - 2x_{31} = 1 - (-1) - (-6) = 8.$

again:- $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_{12} + x_{22} + x_{32} \\ 2x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\Rightarrow x_{32} = 0$

$x_{22} = 1/2$

$x_{12} = -1$

also:- $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_{13} + x_{23} + 2x_{33} \\ 2x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\Rightarrow x_{13} = 4 - 0 = 4$

$x_{23} = 0$

$x_{33} = -2$

$\because AX = I$

$\Rightarrow A^{-1} = XI = X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$

$\Rightarrow A^{-1} = \cancel{\begin{bmatrix} 8 & -1 & A \\ -1 & 1/2 & -2 \\ -3 & 0 & 1 \end{bmatrix}}$

$A^{-1} = \begin{bmatrix} 8 & -1 & 4 \\ -1 & 1/2 & 0 \\ -3 & 0 & -2 \end{bmatrix}$ Ans.

Cramer's

Q) Solve by

$3x + y +$

$x - y$

$x + 2y$

Here, we

$D_1 =$

$D_2 =$

$D_3 =$

by

$x =$

$y =$

$z =$

He

Cramer's Rule

Q) Solve by Cramer's rule :-

$3x + y + z = 4$

$x - y + 2z = 6$

$x + 2y - z = -3$

Here, coefficient determinant $D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix}$

$D = 3(1-4) - 1(-1-2) + 1(2+1)$

$= -9 + 3 + 3 = -3 \neq 0$

$D_1 = \begin{vmatrix} 4 & 1 & 1 \\ 6 & -1 & 2 \\ -3 & 2 & -1 \end{vmatrix} = 4(1-4) - 1(-6+6) + 1(12-3)$

$= 4(-3) - 1(0) + 9$

$= -12 + 9 = -3.$

$D_2 = \begin{vmatrix} 3 & 4 & 1 \\ 1 & 6 & 2 \\ 1 & -3 & -1 \end{vmatrix} = 3(-6+6) - 4(-1-2) + 1(-3-6)$

$= 3(0) - 4(-3) + 1(-9)$

$= +12 - 9 = 3.$

$D_3 = \begin{vmatrix} 3 & 1 & 4 \\ 1 & -1 & 6 \\ 1 & 2 & -3 \end{vmatrix} = 3(3-12) - 1(-3-6) + 4(2+1)$

$3(-9) - 1(-9) + 4(3)$

$-27 + 9 + 12 = -6$

by Cramer's rule :-

$x = \frac{D_1}{D} = \frac{-3}{-3} = 1$

$y = \frac{D_2}{D} = \frac{3}{-3} = -1$

$z = \frac{D_3}{D} = \frac{-6}{-3} = 2.$

Hence, $x = 1$, $y = -1$, $z = 2$. Ans.

Eigen vector.

Q) Find the eigen values & the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

We know, $\det(A - \lambda I_3) = 0$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow 2-\lambda\left[(3-\lambda)(2-\lambda)-2\right] - 2\left[2-\lambda-1\right] + 1\left[2-(3-\lambda)\right] = 0$

$\Rightarrow (2-\lambda)(6-3\lambda-2\lambda+\lambda^2-2) - 2(1-\lambda) + 1(\lambda-1) = 0$

$\Rightarrow (2-\lambda)(\lambda^2-5\lambda+4) - 2 + 2\lambda + \lambda - 1 = 0$

$\Rightarrow 2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda - 3 + 3\lambda = 0$

$\Rightarrow -\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$

~~$\Rightarrow -\lambda(\lambda^2 - 7\lambda + 11) + 5 = 0$~~

5	-1	7	-11	5
	0	-5	10	-5
	-1	2	-1	0

$(\lambda-5)(-\lambda^2+2\lambda-1) = 0$

$\Rightarrow (\lambda-5)(-\lambda^2+\lambda+\lambda-1) = 0$

$\Rightarrow (\lambda-5)\{-\lambda(\lambda-1)+1(\lambda-1)\} = 0$

$\Rightarrow (\lambda-5)(-\lambda+1)(\lambda-1) = 0$

$\Rightarrow \lambda = 5,\ \lambda = 1,\ \lambda = 1$

$\therefore$ The req. eigen value are $\lambda = 1, 1, 5$

Now, for $\lambda = 1$; $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

L-Hospital Rule

If $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ reduce to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Also, if $f(x)$ & $g(x)$ differentiable

then, $\left[\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}\right]$

Q) $\lim\limits_{x\to 2} \frac{x^6 - 24x - 16}{x^3 + 2x - 12}$

$= \frac{(2)^6 - 24(2) - 16}{(2)^3 + 2(2) - 12} = \frac{0}{0}$

So, $\lim\limits_{x\to 2} \frac{6x^5 - 24}{3x^2 + 2}$

$= \frac{6(2)^5 - 24}{3(2)^2 + 2} = 12.$ Ans.

Q) $\lim\limits_{x\to\infty} \frac{\ln x}{x}$

$= \lim\limits_{x\to\infty} \frac{1/x}{1} = \lim\limits_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

Q) $\lim\limits_{x\to\infty} \frac{x + \cos x}{x}$

$\because [-1 < \cos x \leq 1]$ $\lim\limits_{x\to\infty} \frac{1 + (-\sin x)}{1}$

$\because [-1 < \sin x \leq 1]$

Bcz. Sinx value can be b/w -1 & 1 only,

so, value of sinx will fluctuate.

So, L-Hospital Rule fails here. (So, doesn't exist)

Hermitian Matrix

Sq. matrix is called Hermitian if $A^* = A$

where, $A^* = (\bar{A})^T$

Ex. $A = \begin{bmatrix} 1 & 1+2i & 3i \\ 1-2i & 2 & 2i \\ -3i & -2i & 4 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 1 & 1-2i & -3i \\ 1+2i & 2 & -2i \\ 3i & 2i & 4 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 1 & 1+2i & 3i \\ 1-2i & 2 & 2i \\ -3i & -2i & 4 \end{bmatrix} = A$

Skew- Hermitian Matrix

if $A = -(\bar{A})^T$

Ex. $A = \begin{matrix} \cancel{0} & \cancel{1-3i} & \cancel{-2i} \\ \cancel{-1+3i} & \cancel{0} & \cancel{-2-4i} \\ \cancel{-2i} & \cancel{2-4i} & \cancel{0} \end{matrix}$ $\quad A = \begin{bmatrix} 0 & 1+3i & 2i \\ -1+3i & 0 & 2+4i \\ 2i & -2+4i & 0 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 0 & 1-3i & -2i \\ 1-3i & 0 & 2-4i \\ -2i & 2-4i & 0 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 0 & 1-3i & -2i \\ 1-3i & 0 & 2-4i \\ -2i & 2-4i & 0 \end{bmatrix} = -A$

$\Rightarrow A = -(\bar{A})^T$

Solving equation Solution problem

Q) $x + y + z = 3$

$2x - y + 3z = 4$

$5x - y + \lambda z = \mu$

$Ax + B$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 5 & -1 & \lambda \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 4 \\ \mu \end{bmatrix}$

$|A| \neq 0$ if unique sol^n?

$1(-\lambda+3) - 1(2\lambda - 15) + 1(-2+5) \neq 0$

$\Rightarrow -\lambda + 3 - 2\lambda + 15 + 3 \neq 0$

$\Rightarrow -3\lambda \neq -21$

$\Rightarrow \lambda \neq \frac{21}{3} = 7$

$\Rightarrow \lambda \neq 7$

$$A = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & \lambda & \mu \end{array}\right] \xrightarrow[R_2' = R_2 - 2R_1]{R_3' = R_3 - 5R_1}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & 2 \\ 0 & -6 & 2 & \mu - 15 \end{array}\right] \xrightarrow{R_2' \to R_2 \times -\frac{1}{3}}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1/3 & -2/3 \\ 0 & -6 & 2 & \mu - 5 \end{array}\right] \xrightarrow{R_3' \to R_3 + 6R_2}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & 0 & \mu - 11 \end{array}\right]$$

So, $\mu - 11 = 0$

$\mu = 11$

eq. has ∞ solⁿ, if $\mu \neq 11$
Rank of AB $\neq$ Rank of A

i) unique solⁿ, $\lambda \neq 7$
ii) No sol. if $\lambda = 7$, $\mu \neq 11$
iii) ∞ sol. if $\lambda = 7$, $\mu = 11$

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* Orthogonal = $A \times A^T = I$
* Idempotent = $A^2 = A$
* Involuntary = $A^2 = I$
* Skew-Symmetric = $A^T = -A$
* Symmetric = $A^T = A$

Date ______
Page ______

Gamma and Beta function

(1) $\Gamma 1 = 1$

(2) $\Gamma n+1 = n\Gamma n$

(3) $\Gamma n+1 = n!$

(4) $\Gamma n \, \Gamma 1-n = \frac{\pi}{\sin n\pi}$

(5) $e^{-m}x^{n-1} = \frac{(n-1)!}{k^n}$

(6) $\Gamma \frac{1}{2} = \sqrt{\pi}$

(1) $\beta(m,n) = \beta(n,m)$

(2) $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

(3) $\beta(m,n) = \frac{1}{2}\int_0^{\pi/2} \sin\theta^{2m-1} \cos\theta^{2n-1}$

(4) $\boxed{\beta\left(\frac{1}{2},\frac{1}{2}\right) = \pi}$

* Relation:- $\boxed{\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}}$

Q) $\int_0^{\pi/2} \sin^7\theta \cdot \cos^5\theta \, d\theta$

$\frac{1}{2}\int_0^{\pi/2} \sin^{2\times 4-1}\theta \cos^{2\times 3-1}\theta$

$\frac{1}{2}\beta(m,n)$

$\frac{1}{2}\beta(4,3) = \frac{1}{2}\frac{\Gamma 4 \, \Gamma 3}{\Gamma 4+3}$

$= \frac{1}{2}\frac{\Gamma 3+1 \, \Gamma 2+1}{\Gamma 7} = \frac{1}{2}\frac{\Gamma 3+1 \, \Gamma 2+1}{\Gamma 6+1}$

$= \frac{1}{2}\left(\frac{3! \; 2!}{6!}\right)$

$= \frac{1}{2}\frac{\cancel{3}\times\cancel{2}\times\cancel{2}}{6\times5\times\cancel{4}\times\cancel{3}\times\cancel{2}} = \frac{1}{60}\times\frac{1}{2} = \frac{1}{120}$

Q) $\Gamma 7$

$\Gamma 6+1 = 6!$

Q) $\Gamma\frac{1}{3}\,\Gamma\frac{2}{3}$

$= \Gamma\frac{1}{3}\,\Gamma 1-\frac{1}{3}$

$= \frac{\pi}{\sin\frac{1}{3}\pi}$

$= \frac{2\pi}{\sqrt{3}}$

Maxima
Q) find fo
$f(x)=$
find
$\to f(x)$
$f'(x)$
$f''(x)$
$f'''(x)$
for
for
for
wh
Gi

Maxima and Minima

Q) find for which values of x, the function $f(x) = x^3 - 6x^2 + 12x - 3$ has max^m or min. to find the global max. & min. close interval $[-1, 3]$

$\rightarrow$ $f(x) = x^3 - 6x^2 + 12x - 3$

$f'(x) = 3x^2 - 12x + 12 \quad - \cancel{(x-2)}^2$

$f''(x) = 6x - 12$

$f'''(x) = 6$

for max^m or min^m point, $f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 12 = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2, 2$

for $x = 2$

$f''(2) = 6 \times 2 - 12 = 0$

$f'''(2) = 6 > 0$

for global max^m & min^m

$f'(x) = (x-2)^2$

when, $x < 2$, $f'(x) > 0$ i.e., f increases

$x > 2$, $f'(x) > 0$ i.e., f increases

$\therefore f(x)$ is increasing on both sides

Global max^m $= \max \{f(-1), f(3)\}$

$= \max \{-22, 6\} = 6$

min^m $= -22$

$(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)$

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ 1 & 0 & a & b & c \\ 2 & -a & 0 & d & e \\ 3 & -b & -d & 0 & f \\ 4 & -c & -e & -f & 0 \end{array} = (af - be + cd)^2$$

$$(-1)^{1+2+1+2}\begin{vmatrix}0 & a\\ -a & 0\end{vmatrix}\begin{vmatrix}0 & f\\ -f & 0\end{vmatrix} + (-1)^{1+2+1+3}\begin{vmatrix}0 & b\\ -a & d\end{vmatrix}\begin{vmatrix}-d & f\\ -e & 0\end{vmatrix}$$

$$+ (-1)^{1+2+1+4}\begin{vmatrix}0 & c\\ -a & e\end{vmatrix}\begin{vmatrix}-d & 0\\ -e & -f\end{vmatrix} + (-1)^{1+2+2+3}\begin{vmatrix}a & b\\ 0 & d\end{vmatrix}\begin{vmatrix}-b & f\\ -c & 0\end{vmatrix}$$

$$\cancel{+ (-1)^{1+2+2+4}\begin{vmatrix}a & c\\ 0 & e\end{vmatrix}\begin{vmatrix}0 & f\\ -f & 0\end{vmatrix} + (-1)^{1+2+3+4}\begin{vmatrix}b & c\\ d & e\end{vmatrix}}$$

$$\cancel{+ (-1)^{1+2+2+4}\begin{vmatrix}-b & 0\\ -c & -f\end{vmatrix}}$$

$$+ (-1)^{1+2+2+4}\begin{vmatrix}a & c\\ 0 & e\end{vmatrix}\begin{vmatrix}-b & 0\\ -c & -f\end{vmatrix} + (-1)^{1+2+3+4}\begin{vmatrix}b & c\\ d & e\end{vmatrix}\begin{vmatrix}-b & -d\\ -c & -e\end{vmatrix}$$

$$= +(a^2f^2) - (abef) + (acdf) + (adcf) - (aebf)$$
$$+ (be-cd)(be-cd)$$

$$= a^2f^2 - 2abef + 2acdf + b^2e^2 + c^2d^2 - 2becd$$
$$= a^2f^2 + b^2e^2 + c^2d^2 - 2abef - 2becd + 2acdf$$
$$= (af - be + cd)^2$$ (hence proved)

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$
$$[a + (-b) + c]^2$$