(3) Lagrange's Interpolation

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 +$$

$$\frac{(x-x_2)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)\ \_\_\_\_} y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)\ \_\_\ \_\_} y_4$$

(4) Newton's Divided Diff.

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
		$\frac{y_1 - y_0}{x_1 - x_0} = \Delta y_0$	$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$	

$$\boxed{f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0}$$

MOD-2

1 Newton's forward.

$x_0 =$
$x_1 = x_0 + h$
$x_2 = x_1 + h$
$\vdots$
$x_n = x_{n-1} + h$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	....
			$\vdots$		

$$\boxed{p = \frac{x - x_0}{h}}$$

2 ~~Newton's B~~

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots$$

2 Newton's Backward

Formula :-

$$y = f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n.$$

$$\boxed{p = \frac{x - x_n}{h}}$$

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MOD-③

① Trapezoidal rule.

a =
b =
n =
h =

formula:-

$$\int_b^a f(x)\,dx \approx \frac{h}{2}\; f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_n)$$

Error Term.

$$E_T = \boxed{\frac{-(b-a)^3}{12n^2} f''(\xi)}$$

order:- $O(h^2)$

$x_0 =$
$x_1 = x_0 + h$
$x_2 = x_1 + h$
$x_3 = x_2 + h$

x_i	$f(x_i)$

② Simpson's 1/3rd Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots f(x_n)\right]$$

$$\boxed{E_T = \frac{-(b-a)^5}{180n^4} f''(\xi)}$$

order:- $O(h^4)$

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(4) Gauss- Seidel Iterative method.

Rewrite diagonally dominant.

$25x + y - 5z = 19$
$x + 20y + z = -18$
$3x + 4y + 8z = 7$

$x = \frac{19 - y - 5z}{25}$

$y =$

$z =$

Initialy :- $x = 0,\ y = 0,\ z = 0$

1st Iterat":

2nd Iter

3rd "

4th Iter

where value of x, y, z are equal stop.

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MOD- IV

① Gauss Elimination Method.

$AX = B$

$[A:B]$

Make Lower triangle 0. $\begin{bmatrix} \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 0 & 0 & \cdot \end{bmatrix}$

Compute :- x, y, z.

② Matrix Inversion

Identity Matrix, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$[A : I]$

$A^{-1} = [\quad]$

$AX = B$

$X = A^{-1} B$

$= [\quad][\quad]$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\quad]$

$x = -$
$y = -$
$z = -$

③ LU factorization

$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$LU = [\quad][\quad]$

$AX = B$

$LUX = B$

Let $UX = Y$

$LY = B$

$[\quad]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = [\quad]$

$\to y_1 =$
$y_2 =$
$y_3 =$

$A = LU$

$\hookrightarrow$ values of l_{21}, u_{11} —

$UX = Y$

$x =$
$y =$
$z =$

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MOD- V

1) Bisection method

$f(x) = -ve$
$x = a, \quad f(x) = f(a)$

$f(x) = +ve$
$x = b, \quad f(x) = f(b)$

2) Regula-Falsi Method.

$$\boxed{x_i = \frac{a f(b) - b f(a)}{f(b) - f(a)}}$$

3) Newton's - Raphson method

$$\boxed{x_i = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$x_1 = \frac{39}{15} = 2.6$

$f(x_1) = (2.6)^3 - 4(2.6) = -1.824$ (-ve)

So, $a = 2.6$ $\quad f(a) = -1.824$

$b = 3$

$x_2 =$ put all values.

$f(x_2) =$

Similarly :- $x_3 =$

$f(x_3) =$

$x_4 =$

$f(x_4) =$

(3) Newton - Raphson method

$f(x_0) = x^3 - 4x - 9$ $\quad [2, 3]$

$f'(x_0) = 3x^2 - 4$ $\quad f(0)$

$f(1)$

$f(2)$

$f(3)$ - - -

$a = 2$

$b = 3$ $\quad$ Initial approx. $x_0 =$

$f(a) = -9$

$f(b) = 6$

$$\boxed{x_1 = x - \frac{f(x_0)}{f'(x_0)}}$$

MOD - V

1) Bisection method

$x^3 - x = 1$

$x^3 - x - 1 = 0$

$f(0) = -1$ (-ve)

$f(1) = -1$ (-ve)

$f(2) = 5$ (+ve) stop.

$f(x) = -ve$
$x = a$ & $f(x) = f(a)$

$f(x) = +ve$
$x = b$ & $f(x) = f(b)$

$(1.5)^3 - (1.5) - 1$

$a = 1, \; b = 2$

a	b	f(x)	f(b)	$x = \frac{a+b}{2}$	f(x)
1	2	-1	5	1.5	0.875

2) Regula-Falsi Method

$f(x) = x^3 - 4x - 9 = 0$

$f(0) = -9$

$f(1) = -12$

$f(2) = -9$ -ve

$f(3) = 6$ +ve stop

$$\boxed{x_i = \frac{a f(b) - b f(a)}{f(b) - f(a)}}$$

$a = 2$

$b = 3$

$f(a) = -9$

$f(b) = 6.$

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MOD - VI

① Euler's method :-

$$\frac{dy}{dx} = (x_n, y_n) \qquad y(x_0) = y_0$$

$$x_n = x_{n-1} + h$$
$$y_n = y_{n-1} + hf(x_n + y_{n-1})$$

② Runge-Kutta method :-

$$\frac{dy}{dx} = f(x, y) \qquad y(x_0) = y_0$$

4^{th} order formula :

$$K_1 = h \cdot f(x_n, y_n)$$
$$K_2 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right)$$
$$K_3 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{K_2}{2}\right)$$
$$K_4 = h \cdot f(x_n + h, y_n + K_3)$$

$$\boxed{y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)}$$

③ Predictor - corrector.

i) $y_{n+1} = y_n + h \cdot f(x_n, y_n)$ Euler's fnl-

ii) $y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$ → Trapezoidal rul.

④ Finite Difference Method.

$$\boxed{\frac{d^2y}{dx^2} \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}}$$