Assignment - 2

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1.> Knapsack Capacity = 11

i ↓	W	V	0	1	2	3	4	5	6	7	8	9	10	11
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
2	2	6	0	1	6	7	7	7	7	7	7	7	7	7
3	5	18	0	1	6	7	7	18	19	24	25	25	25	25
4	6	22	0	1	6	7	7	18	22	27	28	29	29	40
5	7	28	0	1	6	7	7	18	22	28	29	34	35	40

Conditions:-

Create a Table $V[n][N] \rightarrow V(1\ldots n, 0\ldots N)$

$n \longrightarrow$ no. of objects ; $n = 5$
$N \rightarrow$ capacity of Knapsack ; $N = 11$

1> Make $V[i][0] = 0$; $0 < i \leq n$

2> if $j < W_i$ then, $V[i][j] = V[i-1][j]$

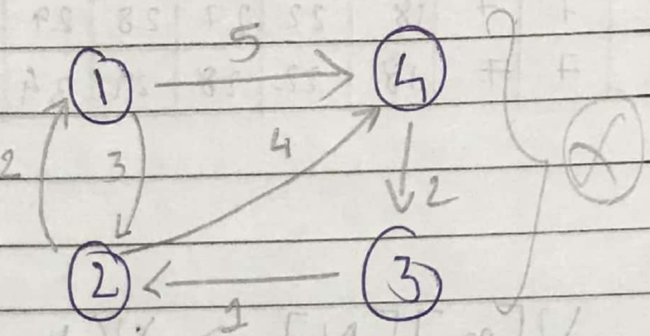
3> else $j \geq W_i$ then, $V[i][j] = \max(V[i-1][j],\ V[i-1][j-W_i] + V_i)$

eg. $V[3][6]$; here $6 > 5$;

$\therefore V[i][j] = \max(V[i-1][j] ; V[i-1][j-W_i] + V_i)$
$\therefore V[3][6] = \max(V[2][6] ; V[2][1] + V_3)$
$\therefore V[3][6] = \max(7 ; 1 + 18)$
$\therefore V[3][6] = \max(7 ; 19)$
$\therefore V[3][6] = 19$ .//

$\therefore$ Answer: Choosing object 3 & 4. $\left(\frac{MC = 40}{TW = 11}\right)$

2.> Equation for Chained Matrix Multiplication using Dynamic Programming, also find out optimal sequence for multiplication: $A1[18 \times 4]$, $A2[4 \times 13]$, $A3[13 \times 7]$ and $A4[7 \times 15]$ (also given optimal parenthesization of matrices



A => $T(n) = \frac{{}^{2n}C_n}{n+1}$ $\quad$ ($\frac{n(n-1)}{2}$ → elements (excluding zero)

$T(4) = \frac{{}^{8}C_4}{5} = \frac{70}{5} = 14$ cases are possible

Using Dynamic Programming;

1> $M[i][j] = 0$, if $i = j$

2> if, $j = i+1$, $M[i][j] = P_{i-1} \cdot P_i \cdot P_{i+1}$

3> if, $i < j$, $M[i][j] = \min\left(M[i][K] + M[K+1][j] + P_{i-1} \cdot P_K \cdot P_j\right)$

with $i \leq K < j$

m	1	2	3	4
1	0	936	868	1864
2	-	0	364	784
3	-	-	0	1365
4	-	-	-	0

S	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

- $P_0 = 18$
  $P_1 = 4$
  $P_2 = 13$
  $P_3 = 7$
  $P_4 = 15$

⟨2⟩ $M[1][2] = P_0 \cdot P_1 \cdot P_2 = 18 \times 4 \times 13 = 936$
$M[2][3] = P_1 \cdot P_2 \cdot P_3 = 4 \times 13 \times 7 = 364$
$M[3][4] = P_2 \cdot P_3 \cdot P_4 = 13 \times 7 \times 5 = 1365$

⟨3⟩ $M[1][3] \Rightarrow K \Rightarrow i \leq K < j$
$1 \leq K < 3$
$\therefore K = 1$ or 2

For $K = 1$

$M[1][3] = M[1][1] + M[2][3] + P_0 \cdot P_1 \cdot P_3$
$= 0 + 364 + 18 \times 4 \times 7 = \boxed{868}$

or $(K = 2)$

$M[1][3] = M[1][\cancel{2}] + M[3][3] + P_0 \cdot P_2 \cdot P_3$
$= 936 + 1365 + 18 \times 13 \times 7 = 3{,}939$

$\therefore \min(868, 3939) = 868$

$\therefore M[1][3] = 868$ (same $M[2][4]$)

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$M[1][4]$; $k = 1, 2, 3$

$k = 1$

$M[1][1] + M[2][4] + P_0 \cdot P_1 \cdot P_4 = 0 + 784 + 1080 = 1864$

$k = 2$

$M[1][2] + M[3][4] + P_0 \cdot P_2 \cdot P_4 = 936 + 1365 + 3510 = 5811$

$k = 3$

$M[1][3] + M[4][4] + P_0 \cdot P_3 \cdot P_4 = 868 + 0 + 1890 = 2,758$

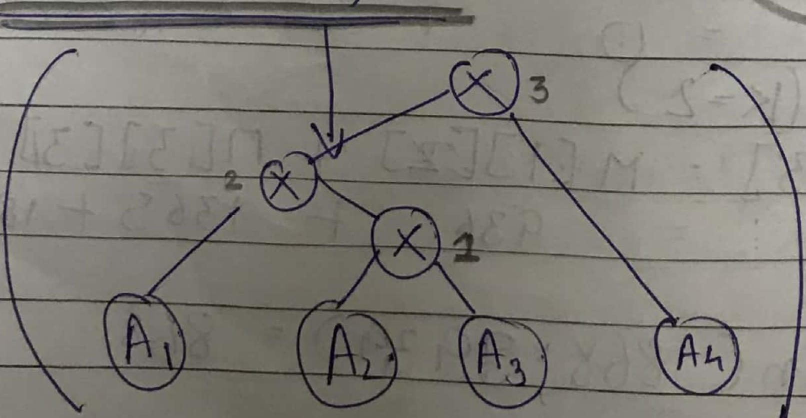
$\therefore M[1][4] = \min(1864; 5811; 2,758)$

$\therefore M[1][4] = 1864$

$\therefore$ The parenthesis is:-

$\Rightarrow (A_1 \; A_2 \; A_3) \cdot A_4$

$\Rightarrow \underline{\underline{((A_1) \cdot (A_2 \cdot A_3)) \cdot A_4}}$ —— Answer



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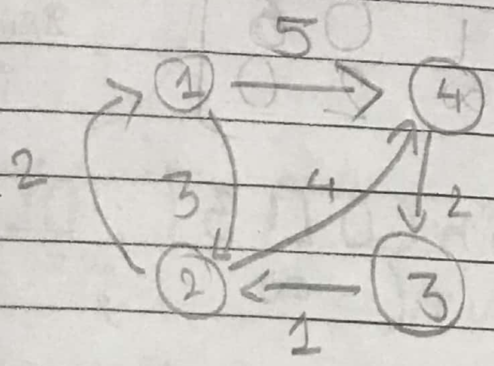
3.) Solve using Floyd's Algorithm



[All pairs shortest path problem]

(Using Dijkshtra's Algo
$O(n^3)$)

=>

$$D_0 = \begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 5 \\ 2 & 2 & 0 & \infty & 4 \\ 3 & \infty & 1 & 0 & \infty \\ 4 & \infty & \infty & 2 & 0 \end{array}$$

Intermidiate point

any shortest path via vertex 1

$$D_1 = \begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 5 \\ 2 & 2 & 0 & \infty & 4 \\ 3 & \infty & 1 & 0 & \infty \\ 4 & \infty & \infty & 2 & 0 \end{array}$$

Check path / distance:
(1) directly
(2) via i ($i = 1, 2, 3, 4$)

(here $i = 1$;)

$D^0[2,3] \rightarrow \infty$ & $D^0[2 \rightarrow 1 \rightarrow 3] \rightarrow \infty$

$\therefore D_0[2,3] \rightarrow \infty$

$D^0[2,4] \rightarrow 4 < D^0(2,1,4) \rightarrow 2+5 = 7$

$\Rightarrow D_0[2,4] = 4$

$D^0[3,2] \rightarrow 1 < D^0[3,1,2] \rightarrow \infty$

$D^0[3,4] \rightarrow \infty$ & $D^0[3,1,4] \rightarrow \infty$

(here $i=2$) ↓

$$D^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 5 \\ 2 & 2 & 0 & \infty & 4 \\ 3 & 3 & 1 & 0 & 6 \\ 4 & \infty & \infty & 2 & 0 \end{array}$$

→ prepared D^1 & copied 2 row - column as it is

$D^2[1,3] \rightarrow \min(D^1[1,3] \; ; \; D[1,2] + D^1[2,3])$
$\min(\infty \; ; \; 2+\infty = \infty) = \infty$

$D^2[1,4] \rightarrow \min(D^1(1,4) \; ; \; D^1(1,2) + D^1(2,4))$
$\min(5 \; ; \; 3+4 = 7)$
5

$D^2[3,1] \rightarrow \min(D^1(3,1) \; ; \; D^1(3,2) + D^1(2,3))$
$\min(\infty \; ; \; 1+2=3)$
3

$D^2[3,4] \rightarrow \min(D^1(3,4) \; ; \; D^1(3,2) + D^1(2,4))$
$\min(\infty \; ; \; 1+5=6)$
6

$D^2[4,1] \rightarrow \min(D^1(4,1) \; ; \; D^1(4,2) + D^1(2,4))$
$\min(\infty \; ; \; \infty+2) = \infty$

$D^2[4,3] \rightarrow \min(D^1(4,3) \; ; \; D^1(4,2) + D^1(2,3))$
$\min(2 \; ; \; \infty+\infty) = 2$

(here $i = 3$ ↓)

$$D^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 5 \\ 2 & 2 & 0 & \infty & 4 \\ 3 & 3 & 1 & 0 & 6 \\ 4 & 5 & 3 & 2 & 0 \end{array}$$

$D^3[1,2] \rightarrow \min(D^2[1,2] \; ; \; D^2[1,3] + D^3[3,2])$

$\min(3 \; ; \; \infty + 1) = 3$

$D^3[1,4] \rightarrow \min(D^2[1,4] \; ; \; D^2[1,3] + D^2[3,4])$

$\min(5 \; ; \; \infty + 6) = 5$

$D^3[2,1] \rightarrow \min(D^2[2,1] \; ; \; D^2[2,3] + D^2[3,1])$

$\min(2 \; ; \; \infty + 3) = 2$

$D^3[2,4] \rightarrow \min(D^2[2,4] \; ; \; D^2[2,3] + D^2[3,4])$

$\min(4 \; ; \; \infty + 6) = 4$

$D^3[4,1] \rightarrow \min(D^2[4,1] \; ; \; D^2[4,3] + D^2[3,1])$

$\min(\infty \; ; \; 2 + 3 = 5) = 5$

$D^3[4,2] \rightarrow \min(D^2[4,2] \; ; \; D^2[4,3] + D^2[3,2])$

$\min(\infty \; ; \; 2 + 1 = 3) = 3$

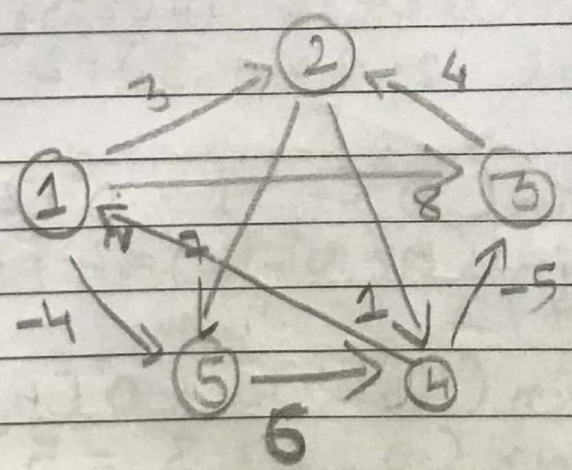
(Here $i = 4$)

$$D^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 7 & 5 \\ 2 & 2 & 0 & 6 & 4 \\ 3 & 3 & 1 & 0 & 6 \\ 4 & 5 & 3 & 2 & 0 \end{array}$$

— Answer

$$\therefore D^{K}[i][j] = \min\{D^{K-1}[i][j]; D^{K-1}[i][K] + D^{K-1}[K][j]\}$$

4) Solve using Floyd's Algorithm.



$$D^0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & \infty & \infty & -4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \infty & 4 & 0 & \infty & \infty \\ 4 & 2 & \infty & -5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \end{array}$$

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$$D^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & \infty & -4 \\ \infty & 0 & \infty & 1 & \cancel{4}\,7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D^{K}[i,j] = \min\left(D^{K-1}[i,j]\ ;\ D^{K-1}[i,k] + D^{K-1}[K,j]\right)$$

$$D^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D^5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 3 & -1 & 4 & \cancel{+2} \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix} \end{array}$$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & -1 & -5 & 0 & +2 \\
2 & 3 & 0 & -4 & 1 & -1 \\
3 & 7 & 4 & 0 & 5 & 3 \\
4 & 2 & -1 & -5 & 0 & -2 \\
5 & 8 & 5 & 1 & 6 & 0
\end{array}
$$

$D^5 = \begin{bmatrix} 0 & -1 & -5 & 0 & +2 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$ // — Answer

5.> Find any one Longest Common Subsequence of given two strings using Dynamic Programming.

S_1 = abbacdcba S_2 = bcdbbcaa

⇒ Table C(C[1...m, 1...n)

m → length of S_1 ⇒ 9

n → length of S_2 ⇒ 8

bcdba

S1:> $c[i][0] = 0$ & $c[0][j] = 0$

S2:> if $x_i = y_j$ ⇒ $c[i,j] = c[i-1, j-1] + 1$

S3:> else; $c[i,j] = \max(c[i-1,j] ; c[i,j-1])$

∴ Here, $m = 9$

$n = 8$

LCS	$y_j \rightarrow$	b	c	d	b	b	c	a	a
X_i	0	0	0	0	0	0	0	0	0
a	0	0	0	0	0	0	0	1	1
b	0	1	1	1	1	1	1	1	1
b	0	1	1	1	2	2	2	2	2
a	0	1	1	1	2	2	2	3	3
c	0	1	2	2	2	2	3	3	3
d	0	1	2	3	3	3	3	3	3
c	0	1	2	3	3	4	4	4	4
b	0	1	2	3	4	4	4	4	4
a	0	1	2	3	4	4	4	5	5

↳ Length of LCS

Tracing back the table we get,

$\boxed{LCS = b\ c\ d\ b\ a.}$ //

6.> Determine ~~an~~ LCS of :

A = [0 ; 0 ; 1 ; 0 ; 1 ; 0 ; 1] &
B = [1 ; 0 ; 1 ; 1 ; 0 ; 1 ; 1 ; 0]

⇒ m → length of A = 7

n → length of B = 8

	X	0	0	1	0	1	0	1
Y	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1	1
0	0	1	1	1	2	2	2	2
1	0	1	1	2	2	3	3	3
1	0	1	1	2	2	3	3	4
0	0	1	2	2	3	3	4	4
1	0	1	2	3	3	4	4	5
1	0	1	2	3	3	4	4	5
0	0	1	2	3	4	4	5	5

↳ length of LCS

1. 01010

2. 10101

∴ We have 2 LCS which are:
① 01010 & ② 10101.