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Class → 3C02

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Q1 ⇒ normal distribution
mean $= \theta_1$
variance $= \theta_2$
Sample $(x_1, x_2, x_3 \dots, x_n)$
Likelihood $= L(\theta_1, \theta_2 | x_1, x_2, x_3 \dots x_n)$ is:
$\prod_{i=1}^{n} 1/\sqrt{2\pi\theta_2}\, e^{-(x_i-\theta_1)^2/2\theta_2}$

Take log:

$$l(\theta_1, \theta_2 | x_1, x_2 \dots x_n) = \sum_{i=1}^{n}\left[-\tfrac{1}{2}\ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2}\right]$$

$$\Rightarrow \frac{\partial l}{\partial \theta_1} = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$, taking derivative

$$\sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Rightarrow \frac{\partial l}{\partial \theta_2} = \sum_{i=1}^{n}\left[-1/2\theta_2 + \frac{(x_i-\theta_1)^2}{2\theta_2^2}\right] = 0$$

$$1/\theta_2^2 \sum_{i=1}^{n} (x_i - \theta_1)^2 = n/\theta_2$$

So, likelihood estimates are :- $\hat{\theta}_1 = 1/n \sum_{i=1}^{n} x_i$

Q2 → $\theta \in [0,1)$ Binomial distribution
$m > 0$

$$pmf = {}^{m}C_x \, \theta^x (1-\theta)^{m-x}$$

$x \to$ no of success.
$m \to$ no of trials.
$\theta \to$ possibility of success.

$$l = (\theta | x_1, x_2, \ldots) = \prod_{i=1}^{n} \theta^{x_i} (1-\theta)^{m-x_i}$$

taking log,

$$l(\theta | x_1, x_2, \ldots, x_n) = \cancel{\prod_{i=1}^{n} {}^{m}C_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}}$$

$$= \sum_{i=1}^{m} \left[\ln \binom{m}{x_i} + x_i \ln\theta + (m - x_i) \right]$$

$$\frac{dl}{d\theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right]$$

$$\sum_{i=1}^{n} \left[x_i/\theta - m - x_i/1-\theta \right] = 0$$

$$\sum_{i=1}^{n} \left[x_i/\theta - \frac{m - x_i}{1-\theta} \right] = 1/\theta \sum_{i=1}^{n} x_i - 1/1-\theta \sum_{i=1}^{n} (m - x_i) = 0$$

$$1/\theta \sum_{i=1}^{n} x_i = 1/1-\theta \sum_{i=1}^{n} (m - x_i)$$

$$(1-\theta) \sum_{i=1}^{n} x_i = \theta \sum_{i=1}^{n} (m - x_i)$$

$$(1-\theta) \sum_{i=1}^{n} x_i = \theta \left(mn - \sum_{i=1}^{n} x_i \right)$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{mn}$$

Max likelihood $(\theta) = \frac{\sum_{i}^{n} x_i}{mn}$

~~so, estimate θ = sample mean~~