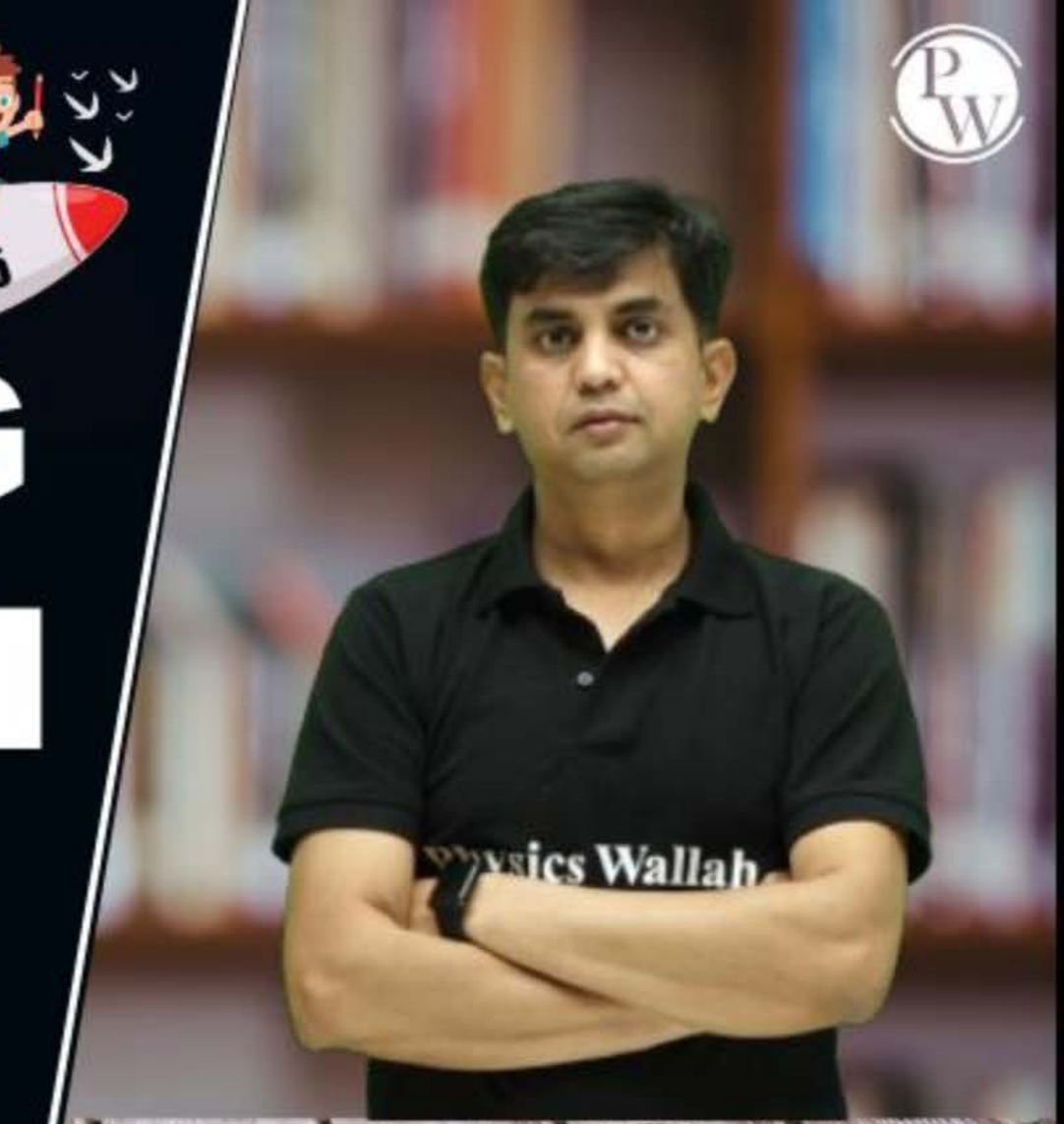
CS & IT ENGINEERING

Computer Network

Error Control

Lecture No. - 06



By - Abhishek Sir

Recap of Previous Lecture

Topic

CRC

Topics to be Covered

Topic 2D Parity

Topic Hamming Code

ABOUT ME

Hello, I'm Abhishek

- GATE CS AIR - 96
- M.Tech (CS) - IIT Kharagpur
- 12 years of GATE CS teaching experience

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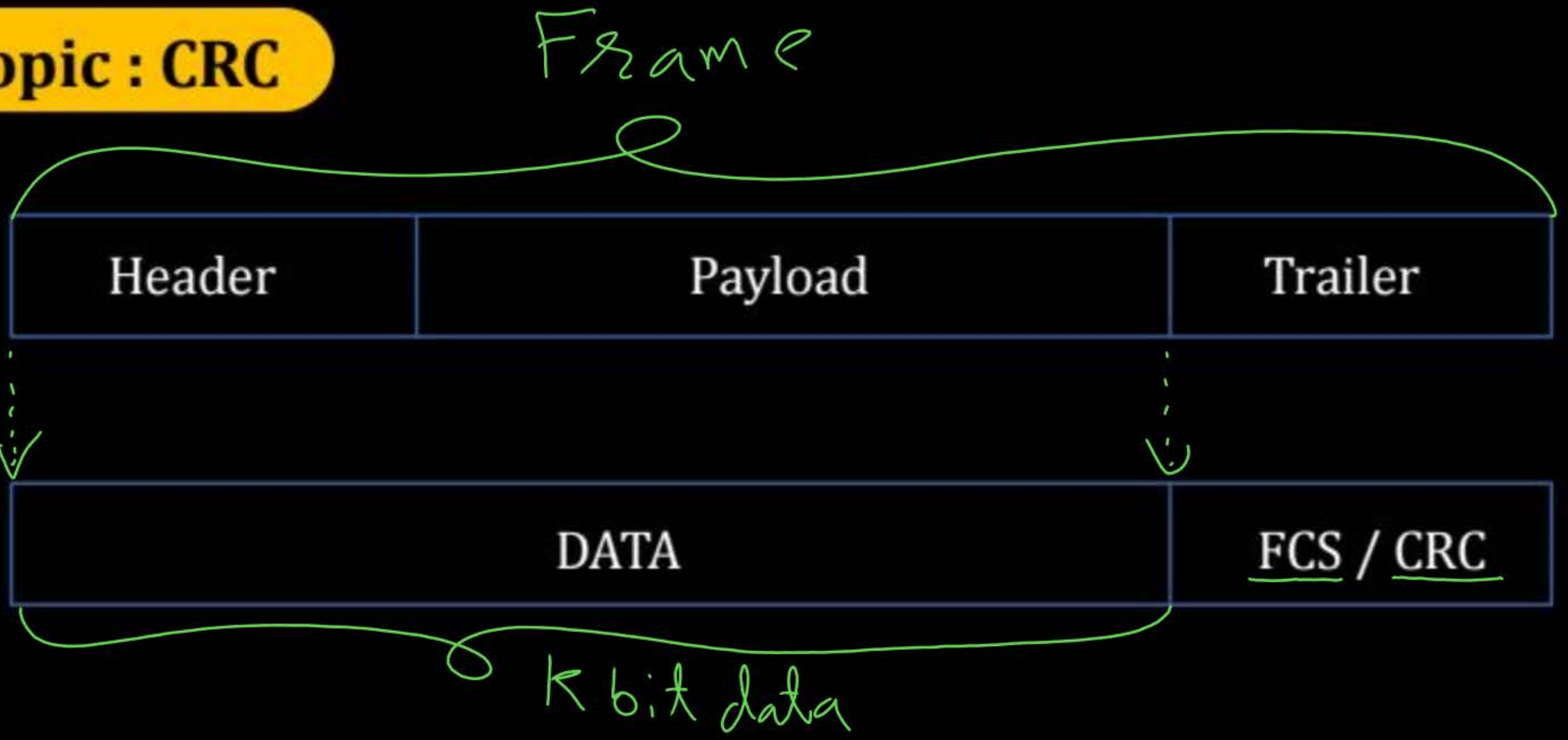
Topic : CRC



CRC-32 : [32-bit / 4 byte]

$G(X) = X^{32} + \ldots\ldots + 1$

Topic : Cyclic Code

CRC [$G(X) = X^3 + 1$] and 3 data bits

```
1001)001000
     1001
     ----
      001
```

```
1001)100000
     1001
     ----
      100
```

Data	-->	Codeword
000	-->	000000
001	-->	001001
010	-->	010010
011	-->	011011
100	-->	100100
101	-->	101101
110	-->	110110
111	-->	111111

2^3

Block Code
+
Linear Code
+
Cyclic Code

Topic : Valid Codewords vs Invalid Codewords

k-bits input (data) → n-bits output (codeword)

→ Number of parity bits = $(n - k)$

→ Number of valid codewords = 2^k

→ Number of invalid codewords = $(2^n - 2^k)$

Topic : Error Detection and Correction

Types of error correction techniques :

1. 2D Parity
2. Hamming Code

Detection only

① CRC
→ one bit parity

② Checksum

Topic : 2D Parity

→ Both sender and receiver must agree on same block size and same parity (either even or odd parity)

→ Block Size = m * n
[m rows & n columns]

Number of Data bits = (m-1) * (n-1)

Number of parity bits = [m + n – 1]

Topic : 2D Parity

Suppose Block Size = 4 x 6 and "Even Parity"

Data = "1 0 0 1 1 0 1 1 0 1 1 0 0 1 0"

Sender

1	0	0	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
0	1	1	0	0	0

Parity of parity

Topic : 2D Parity

Suppose Block Size = 4 x 6 and "Even Parity"

Data = "1 0 0 1 1 0 1 1 0 1 1 0 0 1 0"

Sender

1	0	0	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
0	1	1	0	0	0

Msb

Transmitted Data = "1 0 0 1 1 1 0 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0"

Lsb

Topic : 2D Parity

Suppose Block Size = 4 x 6 and "Even Parity"

Data = "1 0 0 1 1 0 1 1 0 1 1 0 0 1 0"

CASE I : "No any error"

Receiver

1	0	0	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
0	1	1	0	0	0

Receiver Concluded "No error detected"

Sender

1	0	0	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
0	1	1	0	0	0

Transmitted Data = "1 0 0 1 1 1 0 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0"

Received Data = "1 0 0 1 1 1 0 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0"

Topic : 2D Parity

if receiver finds all row wise and column wise parities are balanced

then receiver concluded “No any error detected”

else

receiver concluded “Error detected”

Topic : 2D Parity

Suppose Block Size = 4 x 6 and "Even Parity"

Data = "1 0 0 1 1 0 1 1 0 1 1 0 0 1 0"

CASE II : Single bit error

Receiver Concluded "Error Detected"

Successfully Corrected Single bit Error

Receiver

1	0	0	1	1	1
0	1	0	0	1	1
1	0	0	1	0	0
0	1	1	0	0	0

Sender

1	0	0	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
0	1	1	0	0	0

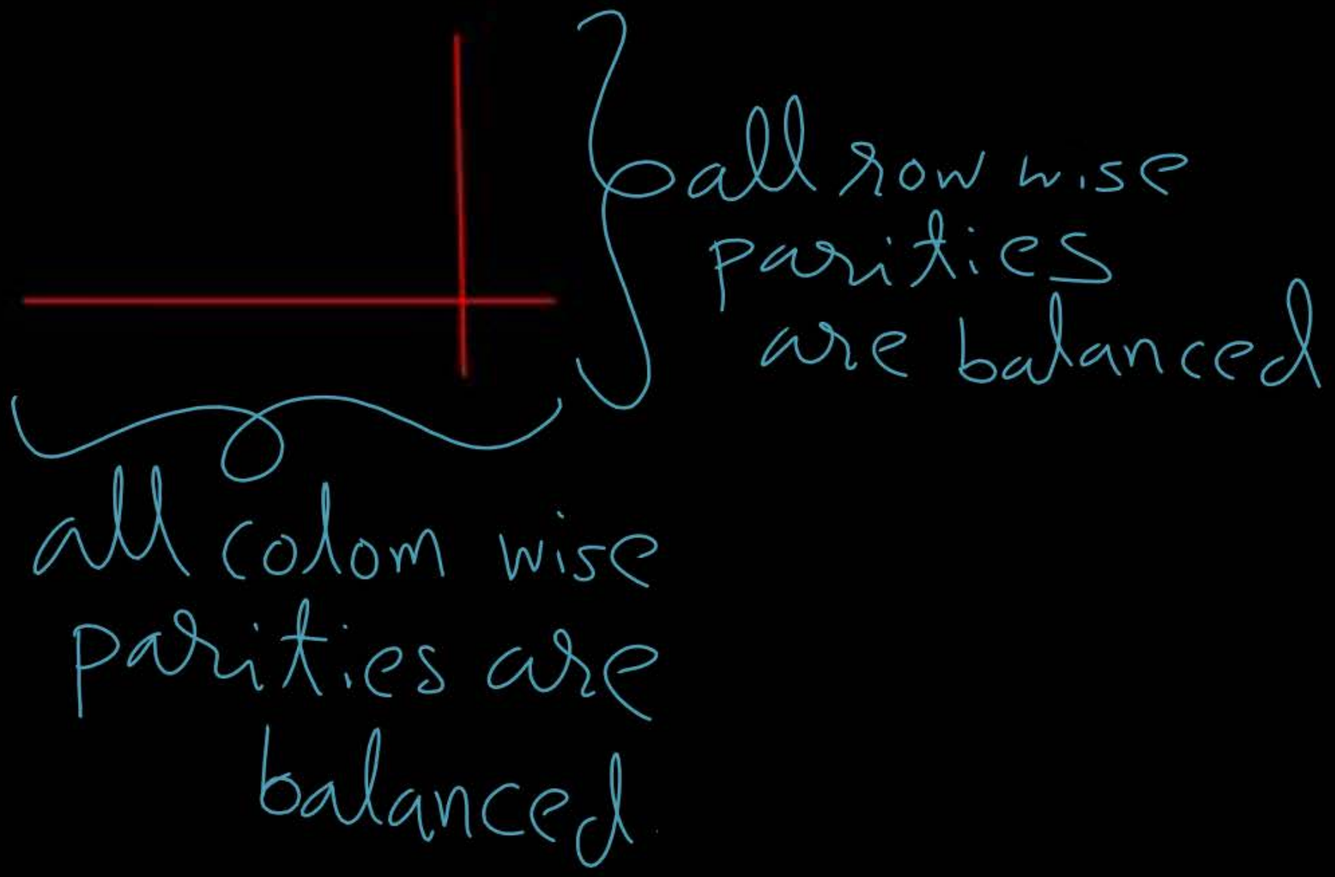
Transmitted Data = "1 0 0 1 1 1 0 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0"

Received Data = "1 0 0 1 1 1 0 1 0 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0"

Topic : 2D Parity

max(0,0)

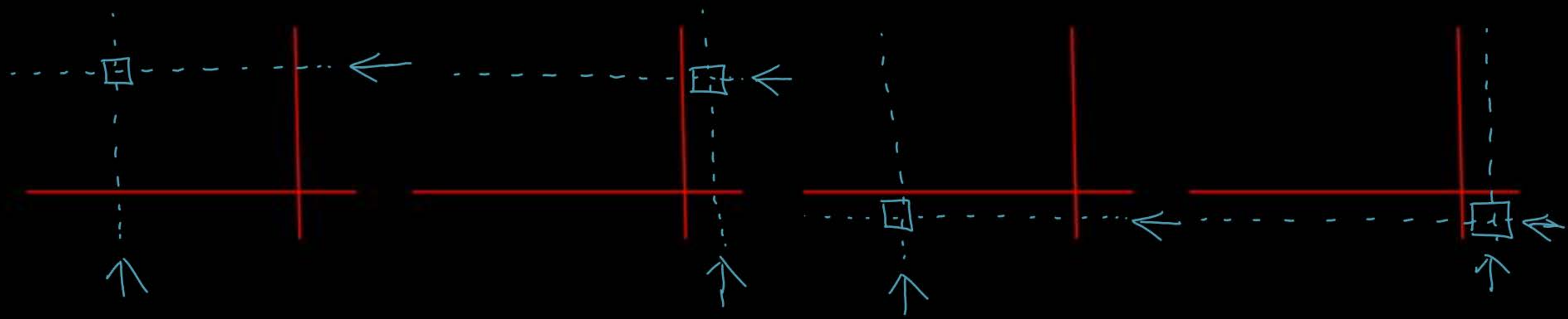
CASE I : No any error



Topic : 2D Parity

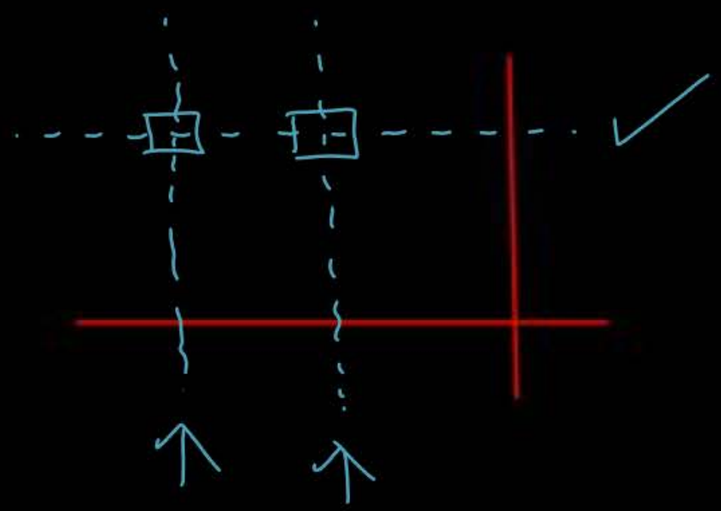
max(1, 1)

CASE II : One-bit error

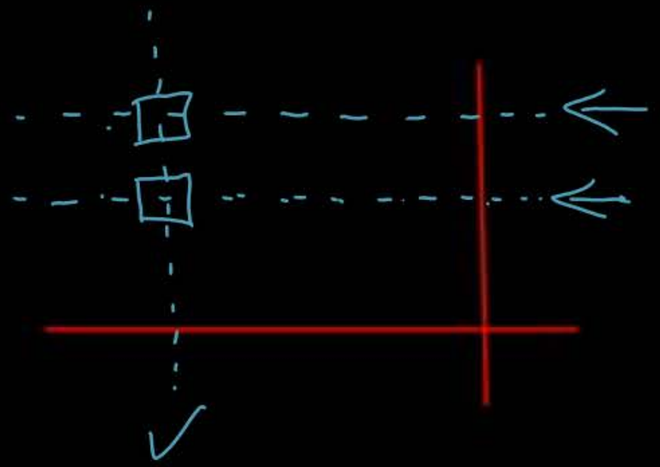


Topic : 2D Parity

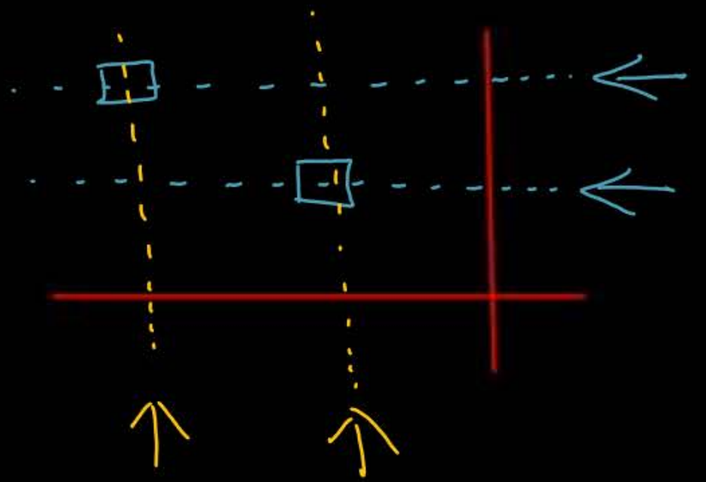
CASE III : Two-bit error



max(0,2)



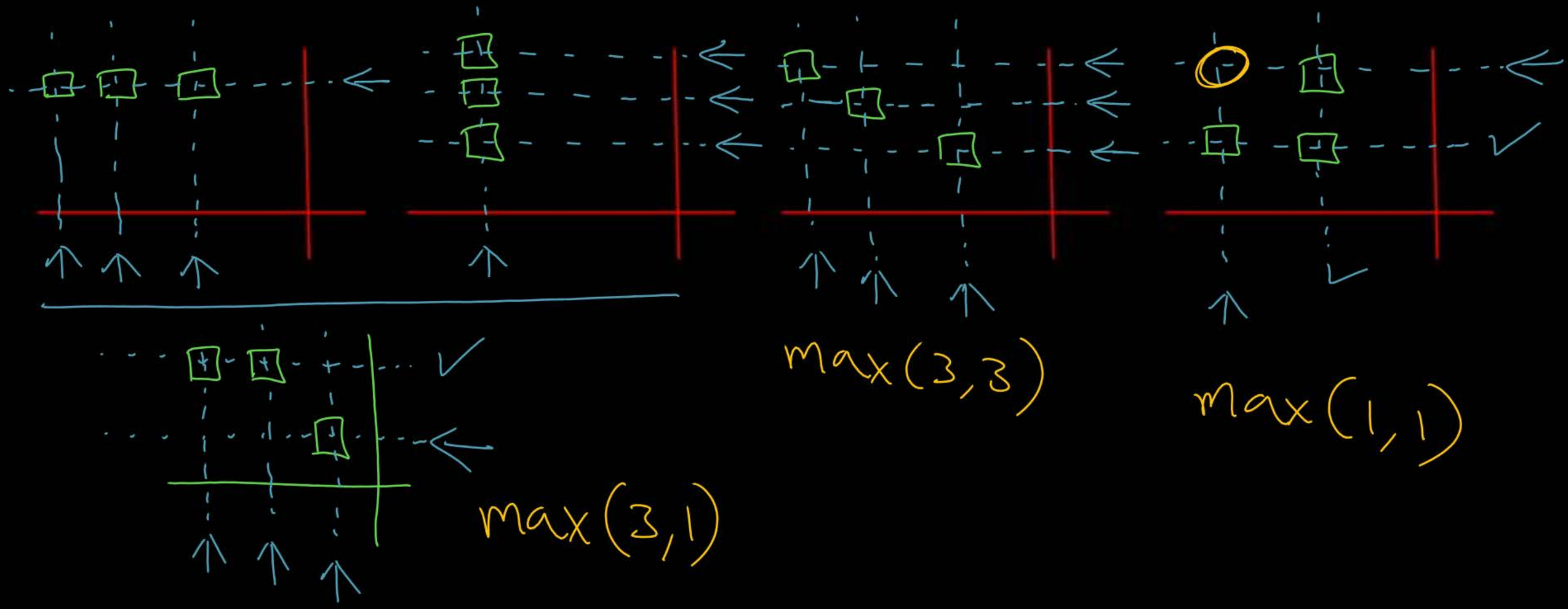
max(2,0)



max(2,2)

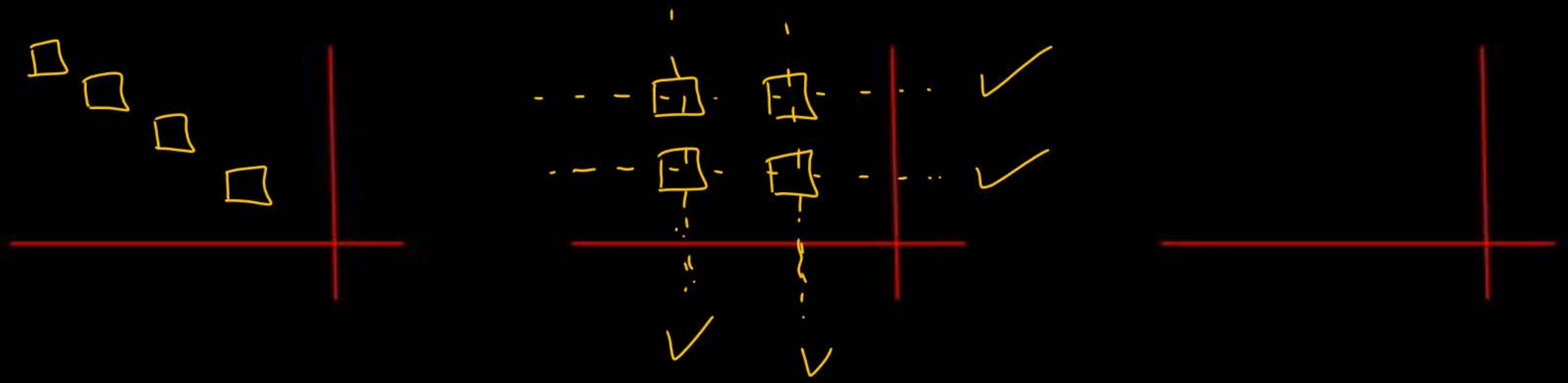
Topic : 2D Parity

CASE IV : Three-bit error



Topic : 2D Parity

CASE V : Four-bit error



Topic : 2D Parity

Minimum number of bits corrupted in the Block

= Maximum [total number of row wise parity unbalanced,
total number of column wise parity unbalanced]

#Q. Data transmitted on a link uses the following 2D parity scheme for error detection:

Each sequence of 28 bits is arranged in a 4 x 7 matrix (rows r_0 through r_3, and columns d_7 through d_1) and is padded with a column d_0 and row r_4 of parity bits computed using the Even parity scheme. Each bit of column d_0 (respectively, row r_4) gives the parity of the corresponding row (respectively, column). These 40 bits are transmitted over the data link.

The table shows data received by a receiver and has n corrupted bits. What is the minimum possible value of n?

[GATE 2008]

IISC
H.W.

(A) 1 (B) 2 (C) 3 (D) 4

	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0
r_0	0	1	0	1	0	0	1	1
r_1	1	1	0	0	1	1	1	0
r_2	0	0	0	1	0	1	0	0
r_3	0	1	1	0	1	0	1	0
r_4	1	1	0	0	0	1	1	0

Topic : 2D Parity

→ Receiver **detect** and **correct** "**all single bit error**"

→ In case of burst error,
receiver may able to detect "**burst error**"

odd parity

odd parity

Topic : Hamming Code

→ Single bit error-correcting code

→ Both transmitter and receiver must agree on same parity
[either "Even Parity" or "Odd Parity"]

→ Number of data bits = m

→ Number of parity bits = r

→ Code length (n) = [m + r] bits

→ Hamming (n, m)

Topic : Hamming Code

→ Parity bit placed at position = 2^i
[where i = 0, 1, 2, 3 . . .]

d_7	d_6	d_5	d_4	R_3	d_3	d_2	d_1	R_2	d_0	R_1	R_0
12	11	10	9	8	7	6	5	4	3	2	1

→ Minimum number of parity bits required

$$2^r > (m + r)$$

→ Minimum code length = 3
[contains only one data bit and two parity bit]

Topic : Hamming Code

→ Even or Odd Parity

d_7	d_6	d_5	d_4	R_3	d_3	d_2	d_1	R_2	d_0	R_1	R_0
12	11	10	9	8	7	6	5	4	3	2	1
				2^3				2^2		2^1	2^0

R_0 = d_0 d_1 d_3 d_4 d_6 [1 = 3, 5, 7, 9, 11]

R_1 = d_0 d_2 d_3 d_5 d_6 [2 = 3, 6, 7, 10, 11]

R_2 = d_1 d_2 d_3 d_7 [4 = 5, 6, 7, 12]

R_3 = d_4 d_5 d_6 d_7 [8 = 9, 10, 11, 12]

2 mins Summary

Topic 2D Parity

Topic Hamming Code

THANK - YOU