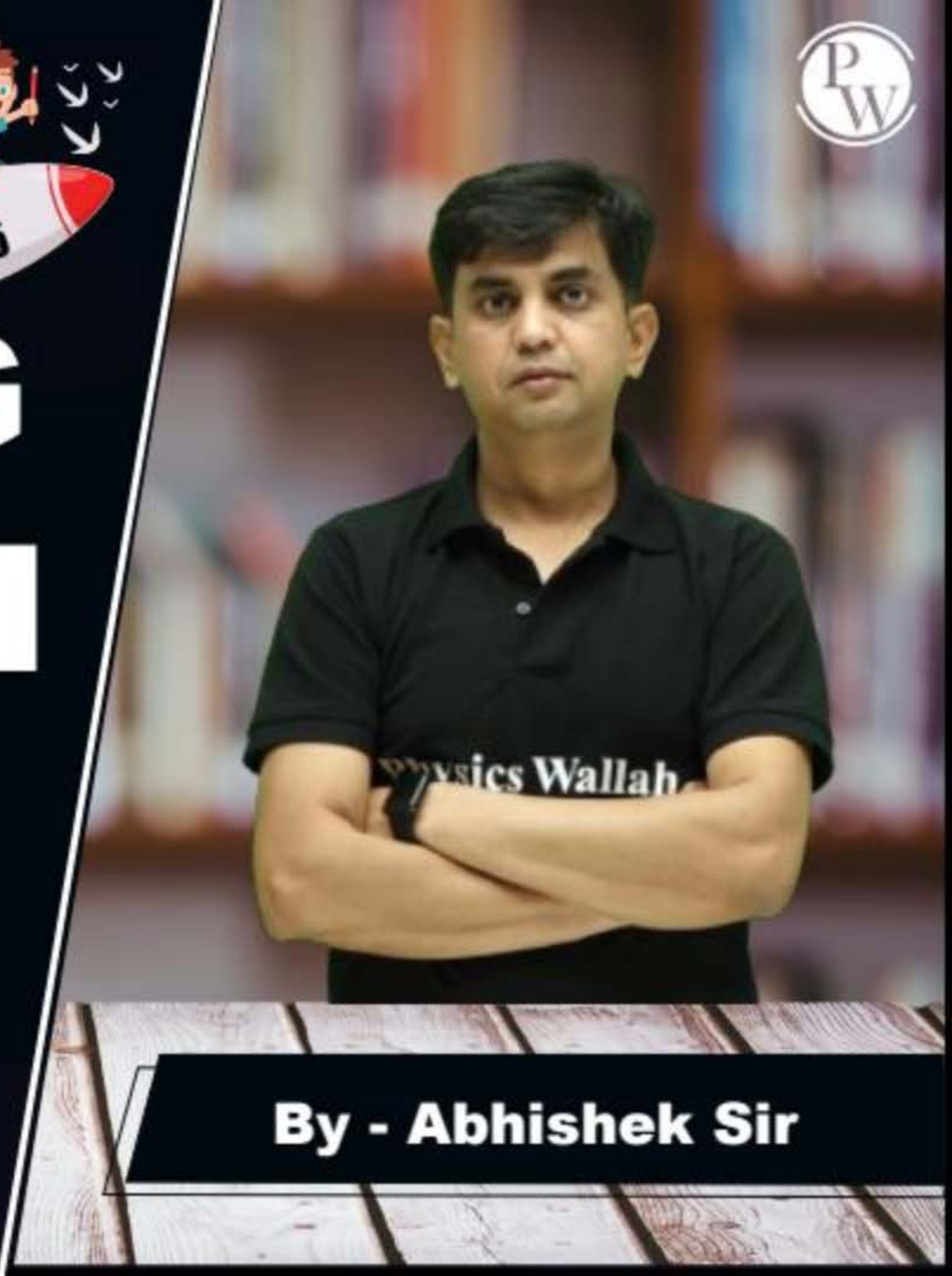
CS & IT ENGINEERING

Computer Network

Error Control

Lecture No. - 07



By - Abhishek Sir

Recap of Previous Lecture

Topic 2D Parity

Topic Hamming Code

Topics to be Covered

Topic Hamming Code

Topic Hamming Distance

ABOUT ME

Hello, I'm Abhishek

- GATE CS AIR - 96
- M.Tech (CS) - IIT Kharagpur
- 12 years of GATE CS teaching experience

Telegram Link : https://t.me/abhisheksirCS_PW

@ABHISHEKSIRCS_PW

Topic : 2D Parity

Minimum number of bits corrupted in the Block

= Maximum [total number of row wise parity unbalanced,
total number of column wise parity unbalanced]

#Q. Data transmitted on a link uses the following 2D parity scheme for error detection:

Each sequence of 28 bits is arranged in a 4 x 7 matrix (rows r_0 through r_3, and columns d_7 through d_1) and is padded with a column d_0 and row r_4 of parity bits computed using the Even parity scheme. Each bit of column d_0 (respectively, row r_4) gives the parity of the corresponding row (respectively, column). These 40 bits are transmitted over the data link.

The table shows data received by a receiver and has n corrupted bits. What is the minimum possible value of n?

[GATE 2008]

(A) 1 (B) 2 (C) 3 (D) 4

Ans: C

Even Parity

	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0
r_0	0	1	0	1	0	0	1	1
r_1	1	1	0	0	1	1	1	0
r_2	0	0	0	1	0	1	0	0
r_3	0	1	1	0	1	0	1	0
r_4	1	1	0	0	0	1	1	0

1

1 2 3

$$\text{Ans} = \max(1, 3)$$
$$= 3$$

Topic : Hamming Code

→ Single bit error-correcting code

→ Both transmitter and receiver must agree on same parity [either "Even Parity" or "Odd Parity"]

→ Number of data bits = m

→ Number of parity bits = r

→ Code length (n) = [m + r] bits

→ Hamming (n, m)

Topic : Hamming Code

→ Parity bit placed at position = 2^i
[where i = 0, 1, 2, 3 . . .]

				2^3				2^2		2^1	2^0
d_7	d_6	d_5	d_4	R_3	d_3	d_2	d_1	R_2	d_0	R_1	R_0
12	11	10	9	8	7	6	5	4	3	2	1

→ Minimum number of parity bits required

$$2^r > (m + r)$$

→ Minimum code length = 3
[contains only one data bit and two parity bit]

m	r
1	2
2	3
3	3
4	3
5	4
6	4
7	4
11	4
12	5
26	5
27	6

Topic : Hamming Code

→ Even or Odd Parity

d_7	d_6	d_5	d_4	R_3	d_3	d_2	d_1	R_2	d_0	R_1	R_0
12	11	10	9	8	7	6	5	4	3	2	1
8 + 4	8 + 2 + 1	8 + 2	8 + 1		4 + 2 + 1	4 + 2	4 + 1		2 + 1		

R_0 = d_0 d_1 d_3 d_4 d_6 [1 = 3, 5, 7, 9, 11]

R_1 = d_0 d_2 d_3 d_5 d_6 [2 = 3, 6, 7, 10, 11]

R_2 = d_1 d_2 d_3 d_7 [4 = 5, 6, 7, 12]

R_3 = d_4 d_5 d_6 d_7 [8 = 9, 10, 11, 12]

Topic : Hamming Code

Example 1 :

$d_3\ d_2\ d_1\ d_0$

Consider data bits are "1 1 0 1" and "Even Parity", generate hamming code?

1	1	0	0	1	1	0
7	6	5	4	3	2	1

Topic : Hamming Code

Example 1 :

Consider data bits are "1 1 0 1" and "Even Parity", generate hamming code?

1	1	0	**0**	1	**1**	**0**
7	6	5	4	3	2	1

Ans : 1100110

Topic : Hamming Code

Example 2 :

d_3 d_0

Consider data bits are "0 0 1 1" and "Even Parity", generate hamming code?

0	0	1	1	1	1	0
7	6	5	4	3	2	1

Topic : Hamming Code

Example 2 :

Consider data bits are "0 0 1 1" and "Even Parity", generate hamming code?

<u>0</u>	<u>0</u>	<u>1</u>	**<u>1</u>**	<u>1</u>	**<u>1</u>**	**<u>0</u>**
7	6	5	4	3	2	1

Ans : 0011110

Topic : Hamming Code

Example 3 :

7 bit

Consider data bits are "1110110" and "Even Parity", generate hamming code?

1	1	1	1	0	1	1	0	0	1	1
11	10	9	8	7	6	5	4	3	2	1

$R_0 = 1$

$R_1 = 1$

$R_2 = 0$

$R_3 = 1$

Topic : Hamming Code

Example 3 :

Consider data bits are "1110110" and "Even Parity", generate hamming code?

1	1	1	**1**	0	1	1	**0**	0	**1**	**1**
11	10	9	8	7	6	5	4	3	2	1

Ans : 11110110011

Topic : Hamming Code

Example 4 :

8 bit

Consider data bits are "10101101" and "Even Parity", generate hamming code?

1	0	1	0	0	1	1	0	1	1	0	0
12	11	10	9	8	7	6	5	4	3	2	1

$R_0 = 0$

$R_1 = 0$

$R_2 = 1$

$R_3 = 0$

Topic : Hamming Code

Example 4 :

Consider data bits are “10101101” and “Even Parity”, hamming code :

1	0	1	0	**0**	1	1	0	**1**	1	**0**	**0**
12	11	10	9	8	7	6	5	4	3	2	1

Ans : 101001101100

Topic : Hamming Code

Even Parity

At receiver :

CASE I : No any error

Transmitted Codeword = 1 0 1 0 0 1 1 0 1 1 0 0

Received Codeword = 1 0 1 0 0 1 1 0 1 1 0 0

1	0	1	0	**0**	1	1	0	**1**	1	**0**	**0**
12	11	10	9	8	7	6	5	4	3	2	1

R_0 ✓
R_1 ✓
R_2 ✓
R_3 ✓

Topic : Hamming Code

if receiver finds all the parity bits are balanced

then receiver concluded "No any error detected"

else

receiver concluded "Error detected"

Topic : Hamming Code

At receiver :

CASE I : No any error

1	0	1	0	**0**	1	1	0	**1**	1	**0**	**0**
12	11	10	9	8	7	6	5	4	3	2	1

Receiver Concluded : "No any error detected"

[Receiver finds all parity bits are balanced]

Topic : Hamming Code

Even Parity

At receiver :

CASE II : One-bit error

Transmitted Codeword = 1 0 1 0 0 1 1 0 1 1 0 0

Received Codeword = 1 0 1 0 0 0 1 0 1 1 0 0

1	0	1	0	0	0	1	0	1	1	0	0
12	11	10	9	8	7	6	5	4	3	2	1

R_0 ×
R_1 ×
R_2 ×
R_3 ✓

R_3	R_2	R_1	R_0
0	1	1	1

Topic : Hamming Code

At receiver :

CASE II : One-bit error

Receiver Concluded : "Error detected"

[Receiver finds parity bits R_0, R_1, and R_2 are unbalanced]

Error Position = 7 [$R_3\ R_2\ R_1\ R_0$ = 0 1 1 1] = 7

Corrected Codeword = 1 0 1 0 0 1 1 0 1 1 0 0

[Receiver successfully corrected single-bit error]

Topic : Hamming Code

Even Parity

At receiver :

CASE III : Two-bit error

Transmitted Codeword = 1 0 1 0 0 1 1 0 1 1 0 0

Received Codeword = 1 0 1 0 0 0 1 1 1 1 0 0

1	0	1	0	0	0	1	1	1	1	0	0
12	11	10	9	8	7	6	5	4	3	2	1

R_0 ✓
R_1 ✗
R_2 ✓
R_3 ✓

R_3	R_2	R_1	R_0
0	0	1	0

Topic : Hamming Code

At receiver :

CASE III : Two-bit error

Receiver Concluded : "Error detected"

[Receiver finds only parity bits R_1 is unbalanced]

Error Position = 2 [R_3 R_2 R_1 R_0 = 0 0 1 0]

Corrected Codeword = 1 0 1 0 0 0 1 1 1 1 1 0

[Receiver can not correct burst error]

#Q. Assume that a 12-bit Hamming codeword consisting of 8-bit data and 4 check bits is $d_8\ d_7\ d_6\ d_5\ C_8\ d_4\ d_3\ d_2\ C_4\ d_1\ C_2\ C_1$, where the data bits and the check bits are given in the following tables:

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check its			
c_8	c_4	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of x and y?

[GATE 2021]

H.W.

IIT-B

(A) x is 0 and y is 0

(B) x is 0 and y is 1

(C) x is 1 and y is 0

(D) x is 1 and y is 1

Topic : Hamming Code

- Using **r** parity bit,

  Maximum code length = $(2^r - 1)$

  Maximum data bits = $(2^r - r - 1)$

- Code rate = $(2^r - r - 1) / (2^r - 1)$

Topic : Hamming Code

Hamming Code (with Even Parity) and 3 data bits :

Data	-->	Codeword
000	-->	000000
001	-->	000111
010	-->	011001
011	-->	011110
100	-->	101010
101	-->	101101
110	-->	110011
111	-->	110100

2 mins Summary

Topic Hamming Code

Topic ~~Hamming Distance~~

THANK - YOU