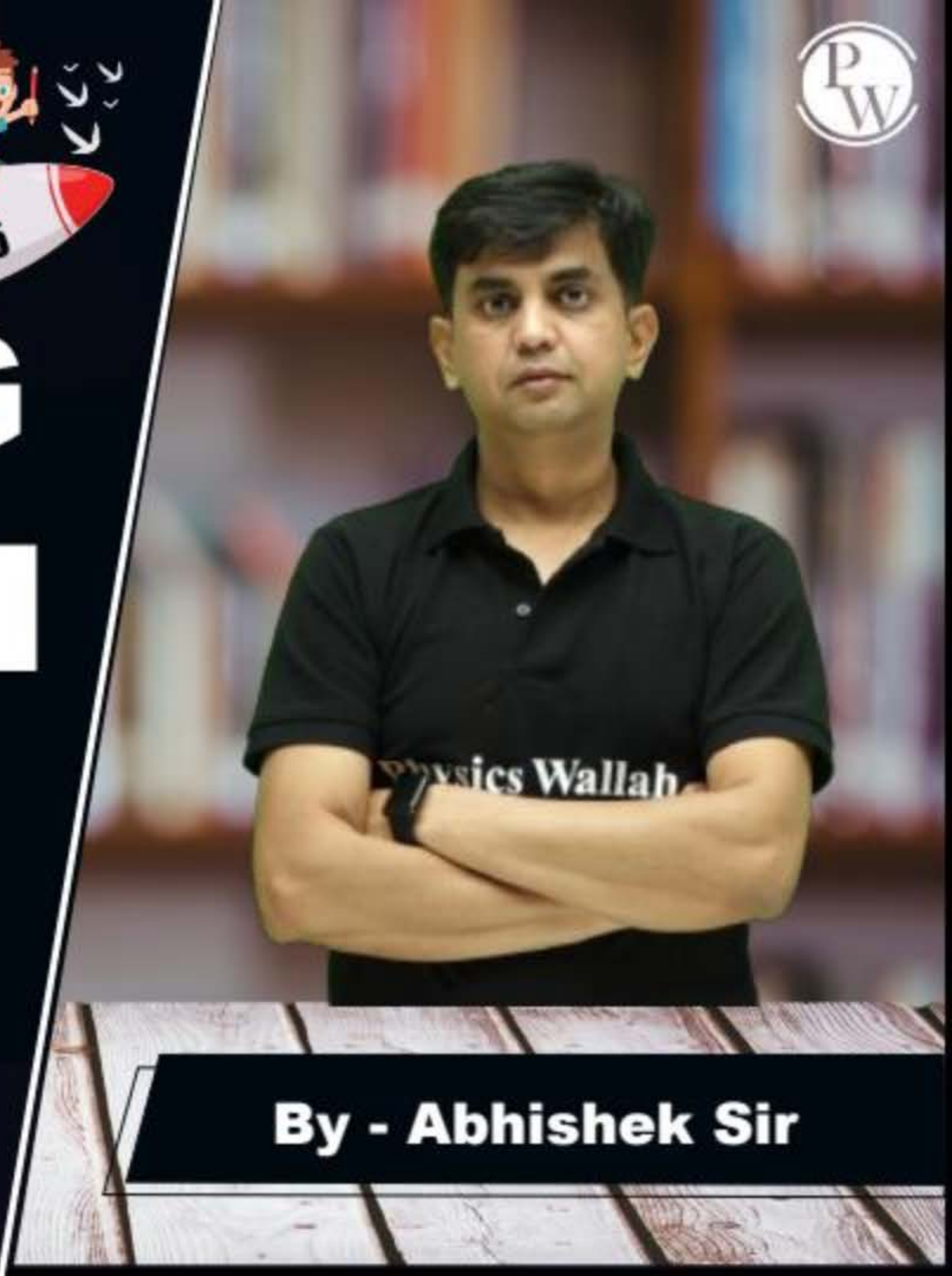
CS & IT ENGINEERING

Computer Network

Error Control

Lecture No. - 05



By - Abhishek Sir

Recap of Previous Lecture

Topic CRC

Topics to be Covered

Topic CRC

ABOUT ME

Hello, I'm Abhishek

- GATE CS AIR - 96
- M.Tech (CS) - IIT Kharagpur
- 12 years of GATE CS teaching experience

Telegram Link : https://t.me/abhisheksirCS_PW

@ABHISHEKSIRCS_PW

Topic : CRC

CASE II : Error Included

Transmitter transmit : $\boxed{[\,M(X) * X^n\,] + [\,R(X)\,]}$

Receiver received : $\boxed{[\,M(X) * X^n\,] + [\,R(X)\,] + \underbrace{[\,E(X)\,]}}$

Topic : Error Polynomial

$E(X)$: Error Polynomial Function

→ Coefficient are either Zero or One

Data : m bits   CRC : n bits

Codeword : $(m + n)$ bits

$$\boxed{\text{Degree}(E(X)) < (m + n)}$$

Topic : Error Polynomial

For single bit error :

$$E(X) = X^{i}$$

Where $i = 0$ to $(m + n - 1)$

Topic : Error Polynomial

Transmitter transmited :

$$X^{10} + X^7 + X^6 + X^5 + X^3 + X + 1$$

1 0 0 1 1 1 0 1 0 1 1

Receiver received :

$$X^{10} + X^7 + X^6 + X^5 + X + 1$$

1 0 0 1 1 1 0 0 0 1 1

$$E(X) = X^3$$

Topic : Error Polynomial

Transmitter transmited :

$X^{10} + X^7 + X^6 + X^5 + X^3 + X + 1$

1 0 0 1 1 1 0 1 0 1 1

Receiver received :

$X^{10} + X^7 + X^6 + X^5 + X^4 + X^3 + X + 1$

1 0 0 1 1 1 1 1 0 1 1

$E(X) = X^4$

Topic : Error Polynomial

For two bit error :

$$E(X) = (X^i + X^j)$$

Where i & j are 0 to $(m + n - 1)$ and $i > j$

Topic : Error Polynomial

Transmitter transmited :

$$X^{10} + X^7 + X^6 + X^5 + X^3 + X + 1$$

1 0 0 1 1 1 0 1 0 1 1

Receiver received :

$$X^{10} + X^9 + X^7 + X^6 + X^5 + X^3 + 1$$

1 1 0 1 1 1 0 1 0 0 1

$$E(X) = X^9 + X$$

Topic : CRC

CASE II : Error Included

Receiver protocol :

$$\boxed{[\, M(X) * X^n + R(X) + E(X) \,]} \text{ [Modulo-2 Division] } \boxed{[\, G(X) \,]}$$

$$\boxed{\frac{(A+B+C)}{G} = \frac{(A+B)}{G} + \frac{C}{G}}$$

CASE II : Error Included

Receiver protocol :

$\boxed{[\,M(X) * X^n + R(X)\,] \quad [\text{Modulo-2 Division}] \quad [\,G(X)\,]}$ ⟹ Always lead zero remainder

$+ \boxed{[\,E(X)\,] \quad [\text{Modulo-2 Division}] \quad [\,G(X)\,]}$ ⟹ When this lead Non-zero remainder

Topic : CRC

$$\boxed{\frac{E(x)}{G(x)} = \frac{E(x)}{x^n + \ldots + 1}}$$

CASE II : Error Included

Receiver protocol :

[E(X)] [Modulo-2 Division] [G(X)]

→ G(X) have (n+1) terms (length)

→ If E(X) causes at-most n length burst error then above equation always lead non-zero remainder

All Single bit Error : $E(x) = x^i$

$$\frac{E(x)}{G(x)} = \frac{x^i}{x^n + \dots + 1} \qquad 0 \le i \le (m+n-1)$$

Suppose $G(x) = x^3 + 1$

$$E(x) = x^6 + x^3$$

$$\frac{E(x)}{G(x)} = \frac{x^6 + x^3}{x^3 + 1} = \frac{x^3(x^3+1)}{(x^3+1)}$$

$$G(x) = x^3 + 1$$

$$E(x) = x^5 + x^3$$

$$\frac{E(x)}{G(x)} = \frac{x^5 + x^3}{x^3 + 1} = \frac{x^3(x^2+1)}{x^3+1}$$

Topic : CRC Property

→ CRC can detect any length burst error,
up-to the degree of generator polynomial function

$$\text{degree}[G(x)] = n$$

Topic : CRC Property

$$\frac{E(x)}{G(x)} = \frac{\text{odd term}}{\text{Even term}}$$

→ For odd number of errors,
E(X) must have odd number of terms

$E(x) = x^7 + x^5 + x^4$

$G(x) = x^3 + 1$

→ if (X+1) is a factor of G(X)
then CRC can detect "**all odd number of errors**"

→ if $(1+x)$ is not a factor of $G(x)$
then CRC always detect
"all even number of errors"

Topic : CRC Property

→ "(X+1) is a factor of G(X)"
means "G(X) is completely divisible by (X+1)"
and "G(X) has even number of terms"

$$G(X) = F_1(X) * F_2(X)$$

$$\boxed{G(X) = (X+1) * F_2(X)}$$

$(x+1) * (x^4 + x^3 + x)$

$x^5 + x^4 + x^2 + x^4 + x^3 + x$

$x^5 + x^3 + x^2 + x$

#Q. Let G(X) be the generator polynomial used for CRC checking. What is the condition that should be satisfied by G(X) to detect odd number of bits in error?

[GATE 2009]

IIT-R

(A) G(X) contains more than two terms

(B) G(X) does not divide $1 + X^k$, for any k not exceeding the frame length

(C) $(1 + X)$ is a factor of G(X) [G(X) has an even number of terms]

(D) G(x) has an odd number of terms

Topic : CRC Property

→ For two-bit error,

E(X) must have two terms only

$$E(X) = (X^i + X^j)$$

Where i & j are 0 to (m + n – 1) $i > j$

$$E(X) = X^j\,(X^{(i-j)} + 1)$$

→ To detect any two-bit error,

G(X) does not divide by $(X^k + 1)$

[for any k from 0 to (m + n – 1)]

Topic : CRC

Example 3:

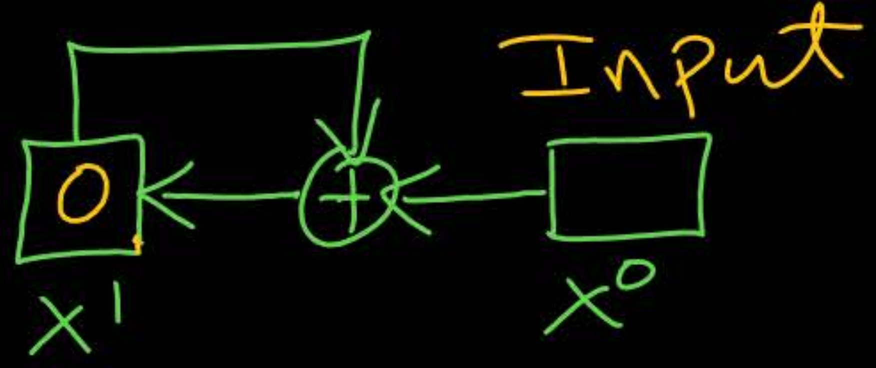
G(X) = X + 1 Divisor = 11

Message (DATA) = 1 0 1 1 1 0 1

CRC = 1

```
11)10111010
   11
   --
    11
    11
    ---
      11
      11
      ----
         10
         11
         --
          1
```

Topic : CRC



Example 3 :

G(X) = X + 1

Message (DATA) = $\underbrace{1\ 0\ 1\ 1\ 1\ 0\ 1}_{0}$ 1 CRC = 1

One-bit CRC [$\underbrace{\text{CRC - 1}}_{0}$] is same as **one-bit party** with "**even parity**".

Topic : CRC

$$
\begin{array}{r}
1001\,)\,10011001000 \\
1001 \\
\hline
1001 \\
1001 \\
\hline
\underbrace{000}_{CRC}
\end{array}
$$

Example 4:

$G(X) = X^3 + 1$ | Divisor = 1001

Message (DATA) = 1 0 0 1 1 0 0 1

CRC = 000

Topic : CRC

$$
\begin{array}{r}
1111\,)\,11110001000 \\
1111 \\
\hline
1000 \\
1111 \\
\hline
\underbrace{111}_{\text{CRC}}
\end{array}
$$

Example 5:

$G(X) = X^3 + X^2 + X + 1$

Message (DATA) = 1 1 1 1 0 0 0 1

CRC = 111

Topic : CRC Property

→ CRC can be "all zero bits" and can be "all one bits"

Topic : CRC

Frame

Header	Payload	Trailer

DATA	FCS / CRC

(n=32) bit

4 byte

CRC-32 : [32-bit / 4 byte]

$G(X) = X^{32} + \ldots\ldots + 1$

2 mins Summary

Topic CRC

THANK - YOU