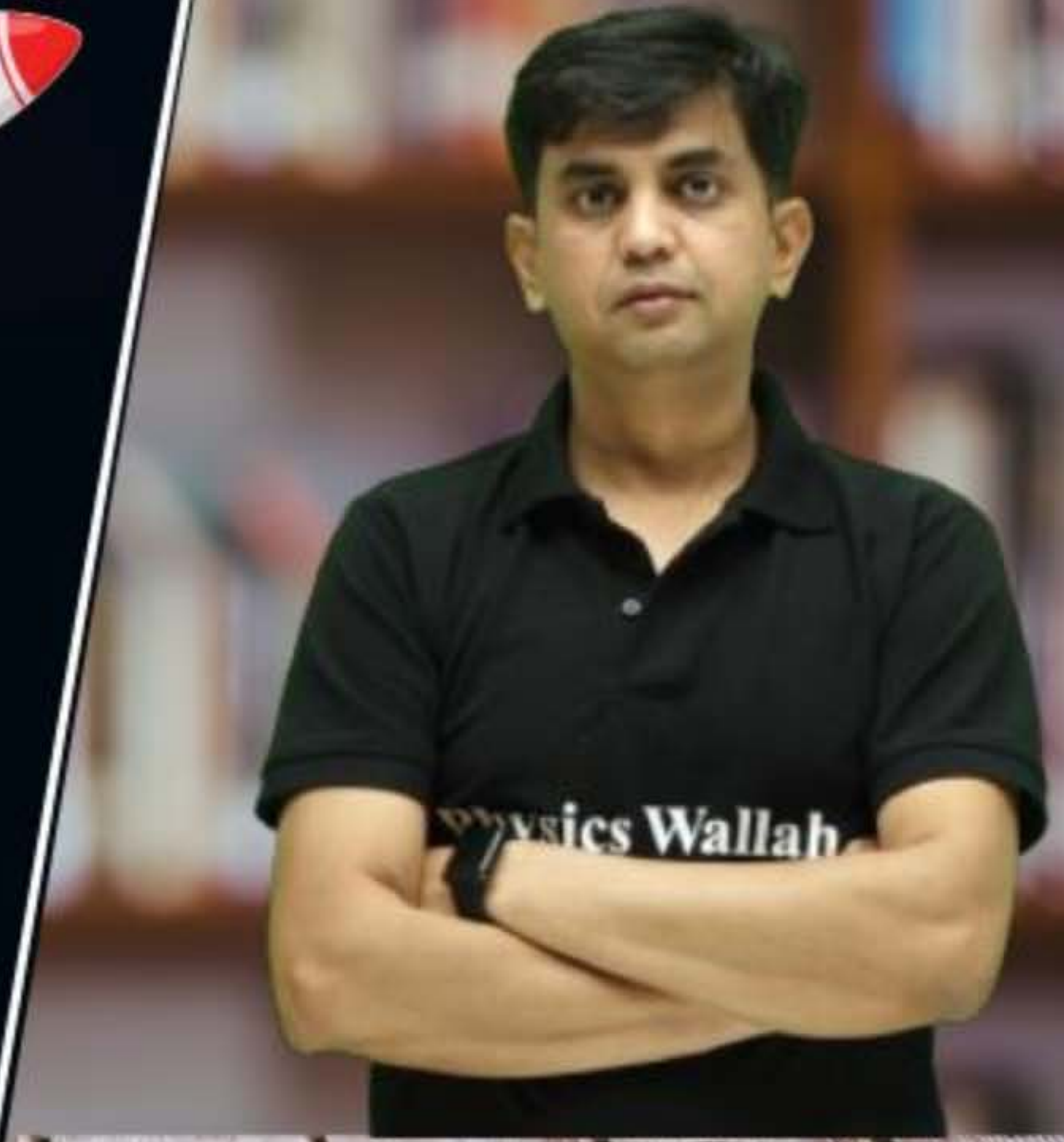
CS & IT ENGINEERING

Computer Network

Error Control

Lecture No. - 08



By - Abhishek Sir

Recap of Previous Lecture

Topic Hamming Code

Topics to be Covered

Topic Hamming Distance

ABOUT ME

Hello, I’m Abhishek

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#Q. Assume that a 12-bit Hamming codeword consisting of 8-bit data and 4 check bits is $d_8\ d_7\ d_6\ d_5\ C_8\ d_4\ d_3\ d_2\ C_4\ d_1\ C_2\ C_1$, where the data bits and the check bits are given in the following tables:

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_8	c_4	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of x and y?

[GATE 2021]

IIT-B

(A) x is 0 and y is 0

(B) x is 0 and y is 1

(C) x is 1 and y is 0

(D) x is 1 and y is 1

$\boxed{\text{Ans: A}}$

Even Parity

1	1	0	X	Y	0	1	0	0	1	1	0
12	11	10	9	8	7	6	5	4	3	2	1

$x = 0$

$Y = 0$

Solution :-

			0	R_3				R_2		R_1	
1	1	0	X	**Y**	0	1	0	**0**	1	**1**	**0**
12	11	10	9	8	7	6	5	4	3	2	1

=> Identify even or odd parity

→ Parity $\mathbf{R_1}$ is balanced using "even parity"

=> Identify value of variable X

→ As per parity $\mathbf{R_0}$, X should be "zero bit"

0

=> Identify value of variable Y

→ If X is "zero", then as per parity $\mathbf{R_3}$, Y should be "zero bit"

Hamming Code (with Even Parity) and 3 data bits :

valid

Data $d_2 d_1 d_0$	-->	Codeword $d_2 d_1 R_2 d_0 R_1 R_0$
000	-->	000000
001	-->	000111
010	-->	011001
011	-->	011110
100	-->	101010
101	-->	101101
110	-->	110011
111	-->	110100

1	1	0	0	1	1
6	5	4	3	2	1

Block code + Linear code

Not cyclic code

Topic : Hamming Weight

→ The weight of a codeword is the number of nonzero elements

Hamming weight of a binary string = Number of one's in that string

Example :-

Hamming Weight["1011101"] = 5

Topic : Hamming Distance

→ Metric for comparing two binary strings of equal length

Hamming distance between two binary strings of equal length

= Number of positions at which the corresponding bits are different

Topic : Hamming Distance

→ Let suppose A and B are two binary strings of equal length

$d(A,B)$ = Hamming Distance between A and B

= Hamming Weight of [A bit-wise XOR B]

Example :- A = 1011101 and B = 0110110

$$\begin{array}{rl} A = & 1\,0\,1\,1\,1\,0\,1 \\ B = & 0\,1\,1\,0\,1\,1\,0 \\ \hline \text{Result} = & 1\,1\,0\,1\,0\,1\,1 \end{array}$$

bit-wise XOR

$d(A,B)$ = Hamming Weight[Result] = 5

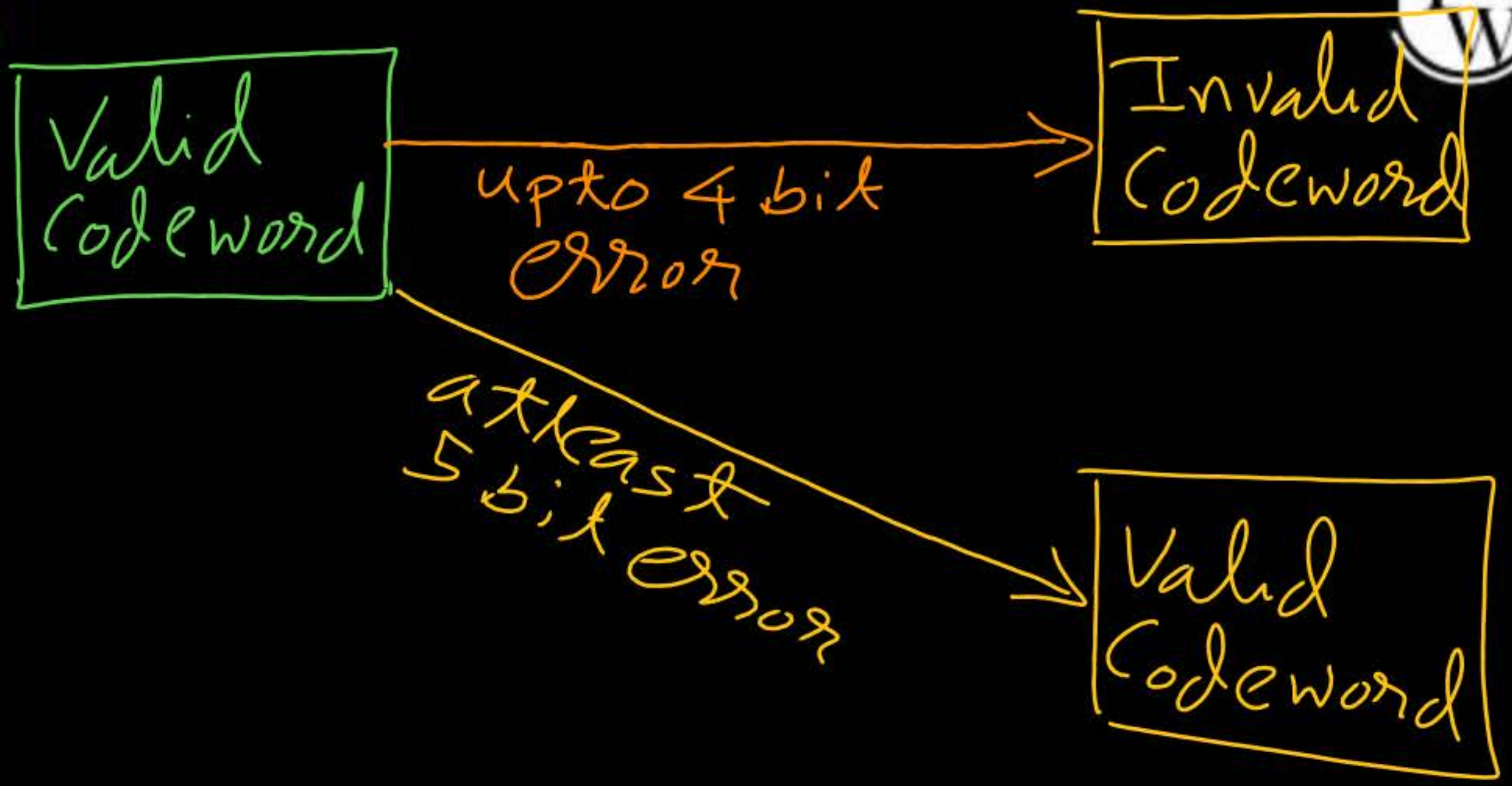
$$\begin{array}{rl} str_1 = & 101011101 \\ str_2 = & 011011101 \\ \hline \text{Result} = & 110000000 \end{array}$$

$d(str_1, str_2) = 2$

Topic : Hamming Distance

Let suppose set of valid codewords :

Codeword C_1 : 00000 00000
Codeword C_2 : 00000 11111
Codeword C_3 : 11111 00000
Codeword C_4 : 11111 11111



minimum Hamming Distance =

$= \text{Minimum}[\, d(C_1, C_2), d(C_1, C_3), d(C_1, C_4), d(C_2, C_3), d(C_2, C_4), d(C_3, C_4) \,]$

$= \min [5, 5, 10, 10, 5, 5]$

$= 5$

Topic : Hamming Distance

CASE I : No any error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00000 00000 (Valid Codeword)

Error Detected : "No"

Corrected Codeword :

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

Topic : Hamming Distance

CASE II : One-bit error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00000 00001 (Invalid Codeword)

Error Detected : "Yes"

Corrected Codeword : 00000 00000 (Nearest Valid Codeword)

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

Topic : Hamming Distance

CASE III : Two-bit error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00000 00011 (Invalid Codeword)

Error Detected : "Yes"

Corrected Codeword : 00000 00000 (Nearest Valid Codeword)

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

Topic : Hamming Distance

CASE IV : Three-bit error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00000 00111 (Invalid Codeword)

Error Detected : "Yes"

Corrected Codeword : 00000 11111 (Nearest Valid Codeword)

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

Topic : Hamming Distance

CASE V : Four-bit error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00000 01111 (Invalid Codeword)

Error Detected : "Yes"

Corrected Codeword : 00000 11111 (Nearest Valid Codeword)

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

Topic : Hamming Distance

CASE VI : Five-bit error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00000 11111 (Valid Codeword)

Error Detected : "No"

Corrected Codeword :

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

CASE VII : Five-bit error

Transmitted Code = 00000 00000 (Valid Codeword)

Received Code = 00001 01111 (Invalid Codeword)

Error Detected : "Yes"

Corrected Codeword : 00000 11111 (Nearest Valid Codeword)

Codeword C_1 : 00000 00000

Codeword C_2 : 00000 11111

Codeword C_3 : 11111 00000

Codeword C_4 : 11111 11111

Topic : Hamming Distance

if (minimum) Hamming Distance is D

then receiver ~~can~~ always detect upto (D-1) bits error

and receiver ~~can~~ always correct upto Floor[(D-1)/2] bits error

$$\left\lfloor \frac{(D-1)}{2} \right\rfloor$$

Topic : Hamming Distance

To detect up to x - bits error

-> minimum Hamming Distance should be $(x+1)$

To correct up to y - bits error

-> minimum Hamming Distance should be $(2y+1)$

#Q. An error correcting code has the following code words:

00000000, 00001111, 01010101, 10101010, 11110000

What is the maximum number of bit errors that can be corrected ?

[GATE 2007]

IIT-K

(A) 0

(B) 1

(C) 2

(D) 3

$\boxed{\text{Ans: B}}$

Solution :-

$$C_1 = 00000000$$
$$C_2 = 00001111$$
$$C_3 = 01010101$$
$$C_4 = 10101010$$
$$C_5 = 11110000$$

minimum Hamming Distance (D) =

Minimum[$d(C_1, C_2)$, $d(C_1, C_3)$, $d(C_1, C_4)$, $d(C_1, C_5)$,
$d(C_2, C_3)$, $d(C_2, C_4)$, $d(C_2, C_5)$,
$d(C_3, C_4)$, $d(C_3, C_5)$, $d(C_4, C_5)$]

$= \min[4, 4, 4, 4, 4, 4, 8, 8, 4, 4]$

$= 4$

→ $\boxed{D = 4}$

→ Receiver can correct upto Floor[(D-1)/2] bits error

$$\left\lfloor \frac{(D-1)}{2} \right\rfloor = \left\lfloor \frac{(4-1)}{2} \right\rfloor = \left\lfloor \frac{3}{2} \right\rfloor = 1$$

#Q. Consider a binary code that consists only four valid codewords as given below.

00000, 01011, 10101, 11110

Let minimum Hamming distance of code be p and maximum number of erroneous bits that can be corrected by the code be q. The value of p and q are:

$q = \left\lfloor \frac{(p-1)}{2} \right\rfloor$

[GATE 2017]

$\underbrace{\hphantom{GATE 2017}}_{\text{IIT-R}}$

(A) p = 3 and q = 1

(B) p = 3 and q = 2

(C) p = 4 and q = 1

(D) p = 4 and q = 2

$\boxed{\text{Ans: A}}$

Solution :-

$$C_1 = 00000$$
$$C_2 = 01011$$
$$C_3 = 10101$$
$$C_4 = 11110$$

minimum Hamming Distance (p) =

Minimum[$d(C_1, C_2)$, $d(C_1, C_3)$, $d(C_1, C_4)$, $d(C_2, C_3)$, $d(C_2, C_4)$, $d(C_3, C_4)$]

$\min[3, 3, 4, 4, 3, 3]$

$\boxed{P = 3}$

→ $\boxed{p = 3}$

→ Receiver can correct upto Floor[(p-1)/2] bits error

→ q = Floor[(p-1)/2] $= \left\lfloor \frac{(P-1)}{2} \right\rfloor = \left\lfloor \frac{(3-1)}{2} \right\rfloor = 1$

→ $\boxed{q = 1}$

Topic : Cyclic Code

One-bit parity (with even parity) and 3 data bits

Always Detect all single bit error

No any provision for error correction

Data	-->	Valid Codeword
000	-->	0000
001	-->	0011
010	-->	0101
011	-->	0110
100	-->	1001
101	-->	1010
110	-->	1100
111	-->	1111

[Block code + Linear code + Cyclic code]

$\min^m$ H.D. = 2

Topic : Linear Code

Hamming Code (with Even Parity) and 3 data bits :

Always detect upto two bit error

Always correct single bit error

$$\frac{(3-1)}{2} = \left\lfloor \frac{2}{2} \right\rfloor = 1$$

Data	-->	valid Codeword
000	-->	000000
001	-->	000111
010	-->	011001
011	-->	011110
100	-->	101010
101	-->	101101
110	-->	110011
111	-->	110100

[Block code + Linear code]

[Not cyclic code]

min^m H.D. = 3

Topic : Hamming Distance

(minimum) Hamming Distance of the linear code is

= minimum weight of its nonzero codewords

Topic : Hamming Distance

→ (minimum) Hamming Distance in:

1. One-bit parity is always "2".
   → Always detect "single bit error"

2. Hamming Code is always "3".
   → Always correct "single bit error"
   → Always detect "upto two bit error"

Topic : Block Code

Set of Codewords in :

→ CRC is "Cyclic Code"

→ Hamming Code is "Linear Code, but not Cyclic"

2 mins Summary

Topic Hamming Distance

THANK - YOU