



INDIAN INSTITUTE OF TECHNOLOGY ROPAR

Department of Mathematics

First Semester of the Academic Year 2023-2024

MA-718: EVOLUTIONARY GAME THEORY

- (1) In a penalty kick in soccer, the kicker has two alternative strategies: either he can kick left or kick right. Similarly, the goalie's strategy can be to dive left or right to block the kick. Assign a payoff of 1 for the player that is successful and -1 for the player that is unsuccessful, assuming a correct dive blocks the kick. Calculate the payoff matrix for the interaction.
- (2) Find the dominant strategy and pure strategy Nash equilibrium for each player in the game defined in (1).
- (3) Given the following normal form game:

		Player 2	
		<i>Movie</i>	<i>Theatre</i>
Player 1	<i>Movie</i>	(a, b)	$(0, 0)$
	<i>Theatre</i>	$(0, 0)$	(c, d)

Each player chooses to go either to movie or to theatre. Player 1 prefers to go to movie with player 2 over theatre. Player 2 prefers to go to theatre over movie with player 1. Ending up at different choices they get 0 payoff. Find the restriction on a , b , c and d .

- (4) Consider the game between two firms to open a plant or not.

		Firm 2	
		<i>Build</i>	<i>Not</i>
Firm 1	<i>Build</i>	$(1, 1)$	$(3, 0)$
	<i>Not</i>	$(0, 3)$	$(2, 2)$

Find all pure strategy Nash equilibria.

- (5) In the game below:

		Player 2	
		<i>Left</i>	<i>Right</i>
Player 1	<i>Up</i>	$(2, 1)$	$(1, 1)$
	<i>Down</i>	$(0, 1)$	$(0, 2)$

Which player has a dominant strategy?

- (6) In the game below:

		Player 2	
		<i>Left</i>	<i>Right</i>
Player 1	<i>Left</i>	(3, 3)	(1, 1)
	<i>Right</i>	(1, 4)	(1, 1)

Which of the following outcomes are Pareto-optimal?

Given below are a few hypothetical payoff matrices: For each of the 4 games :

		Player Y	
		<i>A</i>	<i>B</i>
Player X	<i>A</i>	(2, 4)	(0, 1)
	<i>B</i>	(2, 5)	(3, 3)

		Player Y	
		<i>A</i>	<i>B</i>
Player X	<i>A</i>	(5, 0)	(3, 2)
	<i>B</i>	(3, 2)	(1, 4)

		Player Y	
		<i>A</i>	<i>B</i>
Player X	<i>A</i>	(6, 6)	(4, 0)
	<i>B</i>	(0, 4)	(5, 5)

		Player Y	
		<i>A</i>	<i>B</i>
Player X	<i>A</i>	(6, 6)	(0, 5)
	<i>B</i>	(5, 0)	(4, 4)

- (7) Find all dominant strategies for either player.
- (8) Find all Pareto optimal outcomes.
- (9) Find all pure-strategy Nash equilibrium outcomes.
- (10) In a prey-predator interaction, the predator hunts for prey with choices of staying active (searching for prey) or passive (waiting for prey). Likewise, the prey strategy is to avoid predators with similar choices (active or passive). Find the dominant strategy, the best response for each player, and Nash equilibrium, if any. The payoff matrix for the same is given below.
- (11) Find the beneficial action for both players in a game with payoff for their actions defined below

		Prey	
		<i>Active</i>	<i>Passive</i>
Predator	<i>Active</i>	$(2.5, -0.9)$	$(4, -1)$
	<i>Passive</i>	$(2, -0.8)$	$(0, 0)$

		Player 2	
		<i>A</i>	<i>B</i>
Player 1	<i>A</i>	$(1, 1)$	$(0, 0)$
	<i>B</i>	$(0, 0)$	$(1, 1)$

- (12) In a vote for a hike in salary three legislators cast vote either in favor or against of yearly hike in salary of 2000€. Legislators voting for a hike raise may experience some loss of face. Suppose the legislators estimate that the loss of face is worth 1000 €. Find the consequence of all three legislators voting at the same time.
- (13) Two clubs charge their own price for a glass of cola, either 2€, 4€, or 5€. Approximately 6000 colas per month are drunk in a club by tourists, who can drink at one of the two clubs randomly, and 4000 colas per month are drunk by the locals who go to the club with the lowest price and split evenly when both clubs offer the same price. What prices should the club select?
- (14) Let Pulkit's actions be P1, P2, P3, P4, and actions for Manish are M1, M2, and M3. The payoff bimatrix of the two-person game is as follows:

		Manish		
		<i>M1</i>	<i>M2</i>	<i>M3</i>
Pulkit	<i>P1</i>	$(1, 3)$	$(2, 2)$	$(1, 2)$
	<i>P2</i>	$(2, 3)$	$(2, 3)$	$(2, 1)$
	<i>P3</i>	$(1, 1)$	$(1, 2)$	$(3, 2)$
	<i>P4</i>	$(1, 2)$	$(3, 1)$	$(2, 3)$

Find the best response for both Pulkit and Manish.

- (15) In the following game in matrix form between Shelia and Thomas, find if any of them has a dominant strategy. Justify.

		Thomas	
		<i>C</i>	<i>D</i>
Shelia	<i>A</i>	$(10, 16)$	$(14, 24)$
	<i>B</i>	$(15, 20)$	$(6, 12)$

- (16) Find strictly dominant and very weakly dominant strategy for the given payoff matrix between two players.

		Player 2		
		x	y	z
Player 1	a	(1, 2)	(2, 2)	(5, 1)
	b	(4, 1)	(3, 5)	(3, 3)
	c	(5, 2)	(4, 4)	(7, 0)
	d	(2, 3)	(0, 4)	(3, 0)

- (17) Find strategies that form pure-strategy Nash equilibria for the payoff matrix in (16).
- (18) Two friends split one dollar between themselves. They agree to receive shares s_1 and s_2 such that $0 \leq s_1, s_2 \leq 1$ conditioned that $s_1 + s_2 \leq 1$. If $s_1 + s_2 > 1$, then both of them get 0. Find the strictly dominant strategy.
- (19) Two stores produce items with a production cost $c > 0$ per item. The stores set selling prices p_1 and p_2 ($p_1, p_2 > 0$). If $p_1 \neq p_2$, customers buy from the one store that sells at the lowest price, else 50% of the customers buy from each store (when $p_1 = p_2$), D_0 is the total demand. The profit of a store i is
- (a) 0 if $p_i > p_j$
 - (b) $D_0 \frac{p_i - c}{2}$, if $p_i = p_j$
 - (c) $D_0(p_i - c)$, if $p_i < p_j$

Find the pure-strategy Nash equilibrium.

- (20) Two candidates (A and B) stand for the general secretary position and the winner is the one who receives the majority vote out of the vote from 3 voters. The pure strategy for voters is such that if A wins, voter 1 gets a payoff of 1, and voters 2 and 3 get payoffs of 0; while if B wins, voter 1 gets 0 and voters 2 and 3 get payoff 1. Clearly voter 1 favours A , and 2 and 3 favour B . Find all pure-strategy Nash equilibria.