Week 2 HW problems.

Problem 1. (HW_2)

$S = 5 + 9 + 13 + ... + 89$ in sigma notation.

We have 5 - it is the first number of this count of arithmetic sequences. So

$a_1 = 5.$

The last a_n - could be defined as $a_n = 89.$

we do not know the value of n yet.

but also we can use the formule to find number of terms (n)

$a_n = ...$ $9-5=4$; $13-9=4$. it means that each step each itterraton adds on 4. So $d=4$.

Thus $d=4$. this can be defined as a common difference $\Rightarrow d=4$.

So the formula looks like this:

$$a_n = a_1 + (n-1)\cdot d$$

$n-1 \rightarrow$ meens that value n is increesed by each further step in the arithmetic sequences.

So let's count: $a_1 = 5$; $a_n = 89$; $d = 4$

$89 = 5 + (n-1)\cdot 4$

~~$89 - 5 = 4n - 4$~~

~~$84 = 4n - 4$~~

~~$89 + 4 = 4n$~~

$84 = (n-1)\cdot 4$

$21 = n-1$

$22 = n.$

So we have

$$S = \sum_{k=1}^{22} 5 + (k-1)\times 4$$

Problem 12: Shifting index in Sigma Notation. (HW_2)

Rewrite $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k$ as a sum starting from $k=0$

So we need defind k as a start poind equals 0.

$$\sum_{k=3}^{12} 4\cdot\left(\frac{1}{2}\right)^k - 4\cdot\left(\frac{1}{2}\right)^0 = -7\sum_{k=0} 4\cdot\left(\frac{1}{2}\right)^k =$$

$\left| \begin{matrix} 4\cdot\left(\frac{1}{2}\right)^1 \\ 4\cdot\left(\frac{1}{2}\right)^2 \end{matrix} \right|$ 12 - means we a doing 12 itterations, (terms)

$$= 13\cdot 4 - 7 + \cancel{\xi}\sum_{k=0}^{12}\left(\frac{1}{2}\right)^k = 45 + \sum_{k=0}^{12}\left(\frac{1}{2}\right)^k$$

$\sum_{k=3}^{12}\left(\frac{1}{2}\right)^{k-3}\frac{1}{2}$ (and here the question is (How does the shifting conducted?),

we can consider j as $j=k-3$. and $\Rightarrow$

$k=3$ and $j=0$

$k=12$ $j=9$ $(12-3)$

$$=\sum_{j=0}^{9} 4\cdot\left(\frac{1}{2}\right)^{j+3} = 4\cdot\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4$$ because So before 12,

So now we can observe same sum of each sigma expressions:

$$\sum_{k=0}^{12} 4\cdot\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j =$$

$$=\sum_{j=0}^{9}\left(\frac{1}{2}\right)^j\frac{1}{2}; \quad = \sum_{j=0}^{9}\left(\frac{1}{2}\right)^{j+1}$$ so the sum starting

from $k=0$ is
$$S=\sum_{k=0}^{9}\left(\frac{1}{2}\right)^k\times\frac{1}{2}$$

$$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k = \sum_{k=0}^{9}\left(\frac{1}{2}\right)^k\times\frac{1}{2}$$

Problem 2 (HW_2)

$$\sum_{k=3}^{15} (2k+1)$$ where $k=1$ is start.

we need to defind value, assuming that $k=0$ or 1.

We can observe that $2k+1$ second value $k=1$

$(2k+1)=2 \quad j=1 \quad k=3 \quad j=1.$

$$2\cdot 3+1 = (2(1+\ldots)+1)$$

must be same in value

$j+2$

$$\sum_{j=1}^{x} (2(j+2)+1) = \sum_{j=1} 2j+4+1 = \sum_{j=1} 2j+5$$

x we need to find. $2(15)+1 = 2j+5.$

$31-5=2j$

$26=2j$ so $$\sum_{j=1}^{13} (2j+5)$$

$13=j$

another way is ↳

$$\sum_{k=3}^{15} (2k+1) = \sum_{j=3}^{13} 2(j+2)+1 = \sum_{j=1}^{13} (2j+5)$$

Problem 4 (OR 3) (HW_2)

$a_1 = 12$, $a_n = a_{n-1}+d$; If $a_{10}=57$. $d=?$

we assum that every value of a_n is related with antecedent value like a_1+x, a_2+x2 etc.

So this is arithmetic sequence;

$a_2 = a_{2-1} + d \;=\; a_2 = 12 + d$

$\cancel{a_3 = a_2 +}$ $\quad a_2 = 12 + d$

$a_3 = a_2 + d = 12 + d + d = 12 + 2d.$

So logically $\rightarrow$ $a_{10} = 12 + 9d$

$a_{10} = 57 = 12 + 9d$

$$d = \frac{57 - 12}{9} = \frac{45}{9} = 5.$$

$d = 5.$

now let's see another way of thinking.

$a_n = a_1 + (n-1)d$

$a_{10} = 12 + (10-1)d = 12 + 9d = 57$

$9d = 45$

$d = 5$ same.

now let's find a_{25}: $a_{25} = 12 + (25-1) \cdot 5 = 132$

Problem 3 (OR 4). (HW-2)

Find the sum of all multiples of 7 between 100 and 1000.

It seems like 7 is a ~~sum~~ factor of the sequence and we need to find sum of such factors. between 100 and 1000.

100 $\longrightarrow$ 1000

first we need to do is to define first enter:

$$\begin{array}{r|l} 100 & 7 \\ \hline -7 & 14 \\ 30 & \\ -28 & \\ 2 & \end{array}$$

$100 - 2 = 98$. it means that the remainder of the division is 2. We see 14 times of facters, and 98 is a start point

$98 + 7 = 105$ now let's see the end of

$105+7 = 112$ this point: $1000 \div 7 = 142$, remainder 6

$112+7 = 119$ etc.

accordiy to same way

$1000 - 6 = 994$. this is the last value.

So now we see:

$a_1 = 105$ $a_{127} = 994$ $d=7$. $(a_2 = a_1 + 7)$ so: this formula: $a_n = a_{n-1} + d$

$\sum_{n=1}^{128} a_1 + (n-1) \cdot 7$ let's assume that:

$S_{127} = a_1 + \underbrace{a_1 + d}_{a_2} + \underbrace{a_1 + 2d}_{a_3} + \dots + \underbrace{a_1 + 127d}_{a_{128}} =$

$= \underline{128}\ a_1 + d_1 + 2d + \dots \overset{63}{|} \dots + 126d + 127d =$ 126

$= 63 \cdot 127d$

$= 128\, a_1 + 128 \cdot 63 \cdot 7 + 64 \cdot 7 = 13440 + 56448 + 448 =$

$= 70\,336.$

another way of solution.

$$n = \frac{a_n - a_1}{d} = \frac{994 - 105}{7} = 128.$$

Sum: $S = \frac{n}{2}(a_1 + a_n) = \frac{128}{2}(105 + 994) = 64 \times 1{,}099 = 70336.$

also we can depict:

$S_{128} = \underbrace{128}_{n} a_1 + \underbrace{128}_{n} \cdot \underbrace{63 \cdot d}_{(\frac{n}{2}-1)} + \underbrace{64}_{\frac{n}{2}} \cdot d = a_1 n + n\left(\frac{n}{2} - 1\right)d + \frac{n}{2} \cdot d =$

also: $= n\left(a_1 + \frac{d}{2}(n-2+1)\right) - n\left(a_1 + \frac{d}{2}(n+1)\right) =$

= now we reduce to a common divizor

$= n(2a_1 + d(n-1)) = \frac{n}{2}(2a_1 + (n-1)d) =$

every following value in this term is:

$a_n = a_1 + (n-1)d. = \frac{n}{2}(a_1 + a_n) = \frac{a_1 + a_n}{2} \cdot n = S_n.$

Problem 16. (HW_2)

Find the GCD and LCM of $24x^3y^2z^5$ and $36x^5y^3z^2$

GCD → 12 $\quad \frac{24}{12} = 2 \quad \frac{36}{12} = 3 \quad 12x^3y^2z^2$

LCM → 72 $\quad 72x^5y^3z^5$

another way of solution:

gcd(24, 36) = 12 $\quad$ lcm(24, 36) = 72

vary $x^3y^2z^2$ $\quad$ vary $x^5y^3z^5$

$12x^3y^2z^2$ $\quad$ $72x^5y^3z^6$

Problem: 17. (HW_2)

Factor $x^4 - 13x^2 + 36$. Let's assum: $x^2 = a^2 - 13a + 36$

$(a-9)(a-4)$ $\quad$ | $a^2 - b^2 = (a+b)(a-b)$

$a^2 - 4a - 9a + 36$

$a^2 - 13a + 36$

$(x^2-9)(x^2-4) = (x-3)(x+3)(x-4)(x+4)$

Problem 18. Binomial Products. (HW_2)

$(2x+3y)^5$

$(2x)^5 + 5(2x)^4 \cdot 3y + 10(2x)^3 \cdot (3y)^2 + 10(2x)^2(3y)^3 +$

$+5(2x) \cdot (3x)^4 + (3y)^5 =$

$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

Problem 19 (HW_2)

$$
\begin{array}{l|l}
-\;6x^3 + 11x^2 - 31x + 15 & 3x-2 \\
\quad 6x^2 - 4x & 2x^2 - 31x \; - 7 \\
\hline
\qquad\quad 15x^2 - 31x & \\
\qquad -\;15x^2 - 10x & \\
\hline
\qquad\qquad\quad 21x + 15 & \\
\qquad\qquad\quad 21x + 14 & \\
\hline
\qquad\qquad\qquad\quad 1 &
\end{array}
$$

it seem like:

$(3x-2) \cdot (2x^2 + 31x - 7) + 1$