LAB 2: ELECTRON DIFFRACTION

LAB GOALS

In this lab, we will demonstrate that electrons behave like waves, and we will measure the properties of the waves. The fact that electrons behave like waves, rather than localized particles, is at the heart of quantum mechanics and the foundation of all of chemistry.

This is a 1 week lab. You will work with partners for this lab. There is a prelab lecture on BrightSpace. There will be a 10 minute quiz at the beginning of lab.

Upload your Jupyter notebook to BrightSpace as by PDF by the end of your lab day.

PRELAB

Watch the prelab video and read the prelab text below. There will be a 10 minute quiz at the beginning of class.

WAVES VERSUS PARTICLES

In classical mechanics, particles are each assigned a definite position and velocity. Particles follow trajectories governed by Newton's laws, $F = m\, a$. Light on the other hand behaves like a wave, governed by Maxwell's wave equation.

A traveling wave has a velocity v and wavelength λ (or a wave-vector $k = 2\,\pi/\lambda$).

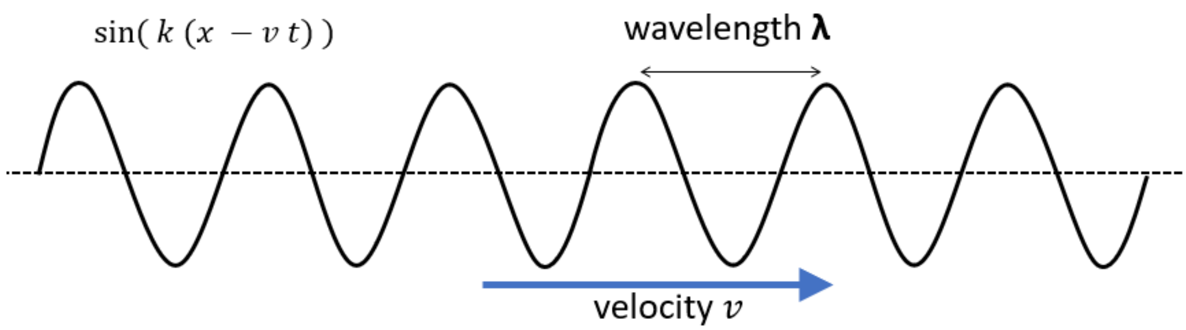


Figure 1 Traveling wave

Light for example always travels at the speed of light $c = 299{,}792{,}458\ m/s$.

An important property of waves is their ability to interfere with each other. If two waves are "in-phase", meaning that their maximum are aligned, then they can constructively interfere. If two waves are "out-of-phase", meaning that one wave's maximum is at another wave's minimum, then they cancel each other out.

The left side of Fig 2 shows a wave traveling through a single slit. The right side shows waves coming from two slits. The two waves interfere with each other to create an interference pattern. The detector on the right measures the intensity (amplitude squared) of the waveforms and detects the interference pattern. This interference pattern was confirmation that light indeed is a wave.

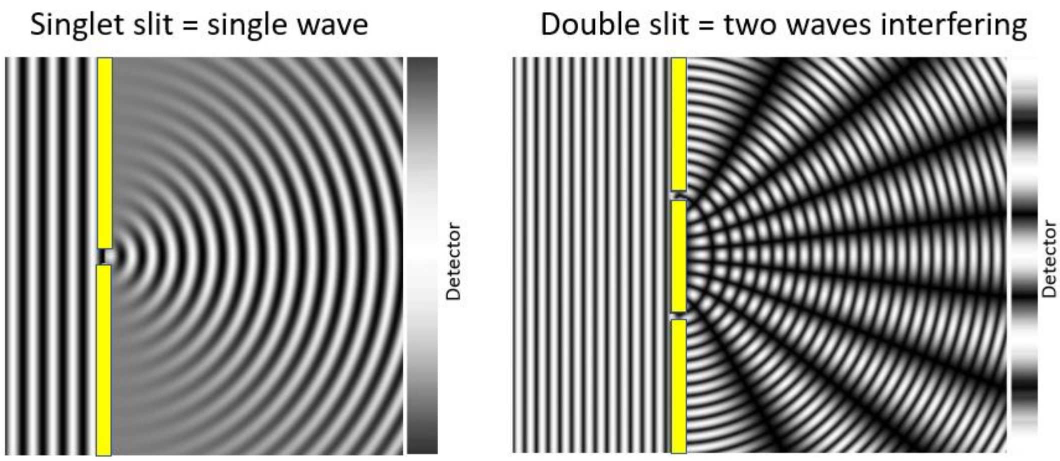


Figure 2: Waves coming from one slit and two slits. An interference pattern is detected from two interfering waves coming from the double slit.

In contrast, if we instead throw particles at the two slits, we would only detect particles behind each slit.

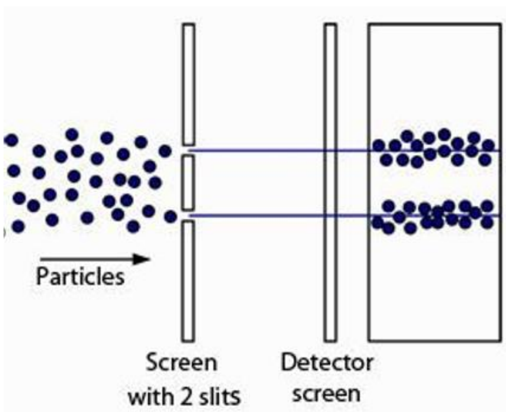


Figure 3: Particles

Interference is a signature of waves. In this lab, you will investigate whether electrons also behave like a wave.

HISTORY OF ELECTRON DIFFRACTION

In the beginning of the 20^{th} century, wave-like and particle-like behavior were thought to be incompatible. Objects either acted like a wave or a particle. This notion started to unravel when measurements (see for example the photoelectric effects and ultraviolet catastrophe in black-body radiation) started to suggest that light is made up of discrete energy packets called photons. The energy of a photon is related to its frequency by Planck's constant, $E = h\, v$, and its momentum is inversely proportional to its wavelength $p = h/\lambda$.

And matter (e.g. electrons, protons, neutrons) was treated as particles following Newton's laws, but this was incompatible with the observations that electrons occupied discrete energy levels. If classical mechanics holds for electrons, then electrons orbiting around a nucleus would be equivalent to planets orbiting a star. At small length and energy scales, clearly Newton's laws were not sufficient.

In his 1924 graduate thesis, Louis de Broglie (pronounced de Broy) proposed that perhaps all matter, not just light, behaves as waves. If light can sometime behave like a particle, why cannot matter also sometimes behave like a wave? That wavelength is given by the de Broglie relation

$$\lambda_{\text{de Broglie}} = \frac{h}{p} \tag{1}$$

The energy of a particle is related to its momentum and mass by

$$E = \frac{p^2}{2\, m}, \tag{2}$$

and therefore, an electron's wavelength is related to its energy by

$$\boldsymbol{\lambda}_{\textbf{de Broglie}} = \frac{\boldsymbol{h}}{\sqrt{\boldsymbol{2\, m\, E}}} \tag{3}$$

Here h is Planck's constant, and m is the mass of the particle. So why can we not just perform the two slit experiment with electrons and try to observe interference? The two slit experiment only works when the slits are separated by a distance d similar to the wavelength. Visible light has a wavelength of approximately $1\ \mu m\ (10^{-6} m)$. Electrons instead were predicted to have wavelengths on the order of $10\, pm\ (10^{-12} m)$. That is too small of a slit to make.

Fortunately, we can borrow ideas from X-ray scattering of crystals, since X-rays also have wavelengths that are this small. In 1913, W.H. and W.L. Bragg showed that X-rays incident on a crystal are reflected at specific angles. The diffraction of the X-rays by the crystal can be understood by picturing a crystal as containing a periodic set of planes, with each plane reflecting a wave like a simple plane mirror.

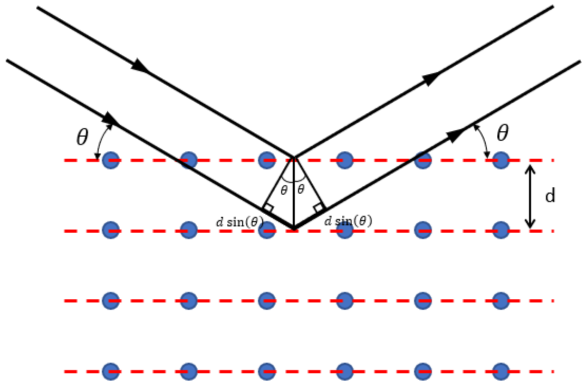


Figure 4: Bragg scattering from planes in a crystal. The extra path length traveled by wave hitting second plane is $2d\ sin(\theta)$.

Fig 4 shows a 2D example of a crystal with the planes separated by a distance d. A plane wave enters from the top and can reflect off of the different planes. The two output rays constructively interfere when the path length difference of the two rays is an integer multiple of the wavelength, which gives the Bragg condition for the deflected waves,

$$2\,\boldsymbol{d}\,\mathbf{sin}(\boldsymbol{\theta}_{\mathbf{Bragg}}) = \boldsymbol{n}\,\boldsymbol{\lambda}_{\mathbf{Bragg}}, \tag{4}$$

where n is an integer (we will always use n=1), and d is the distance between the lattice planes. If this condition is not met, then the reflections from all of the planes will deconstructively interfere, canceling each other out and resulting in no reflection.

In 1927, Davisson and Germer accelerated electrons in a vacuum towards crystalline nickel. If electrons are in fact waves, then we should also observe Bragg reflections from the crystal, just as we do for X-rays. The angles of these reflections will tell us about the wavelength of the electron by the Bragg condition.

In this lab, we are going to perform a similar experiment to test de Broglie's prediction.

ELECTRON DIFFRACTION TUBE

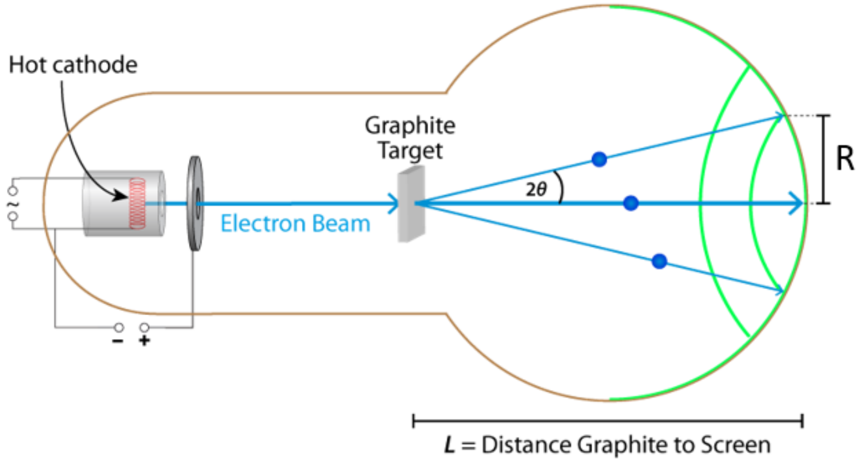


Figure 5: Electron diffraction tube

Electrons are emitted by heating a tungsten filament (in red) in a vacuum. The electrons are accelerated towards a graphite target by applying a large voltage on electrode with a hole in it. For example, if a voltage of 5 kV is applied, then the electrons are accelerated to an energy of 5000 $e \cdot$V, where e is the electron charge.

The polycrystaline graphite is a conglomerate of a large number of small crystal domains, where each domain is large enough to embody a “true” crystal structure, but all the domains are oriented randomly with respect to each other. A beam incident on a bulk sample will find many domains oriented at the correct Bragg angle for the given beam energy, as shown in Fig 4.

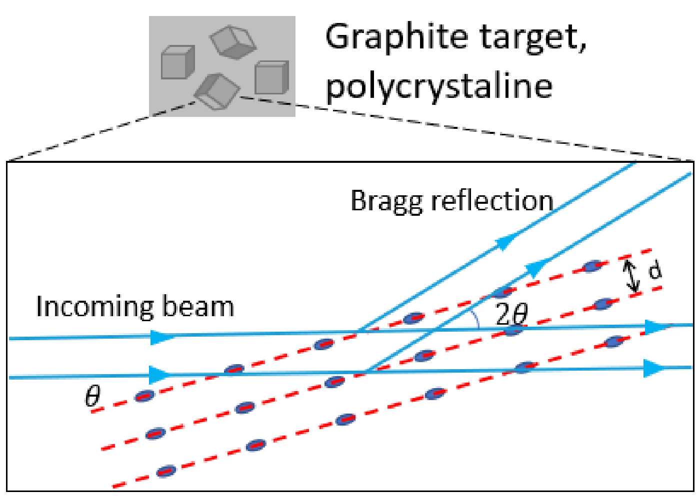


Figure 6: Bragg reflection from a crystal.

Confirm for yourself that the deflected angle of the Bragg reflection is $2\theta_{\text{Bragg}}$.

Since the crystal domains are randomly oriented, the Bragg reflection is actually a cone. If you rotate the crystal in Fig 6 about the axis of the incoming beam, the Bragg reflection will still occur. **Convince yourself that in a polycrystaline material, the diffracted electrons form a cone, and a ring with radius R on the fluorescent screen.**

For a tube of length L, the Bragg angle is given by

$$\mathbf{2\theta_{Bragg}} = \mathbf{\tan^{-1}}\left(\frac{\boldsymbol{R}}{\boldsymbol{L}}\right) \tag{5}$$

Graphite is a hexagonal lattice of carbon atoms. If the electrons diffract as a wave, we expect to see multiple rings on the fluorescent screen corresponding to different crystal planes. There are other crystal planes, but the dominant lattices are shown in Fig 7 and having spacings

$$d_{11} = 0.123 \text{ nm}, \quad d_{10} = 0.213 \text{ nm}.$$

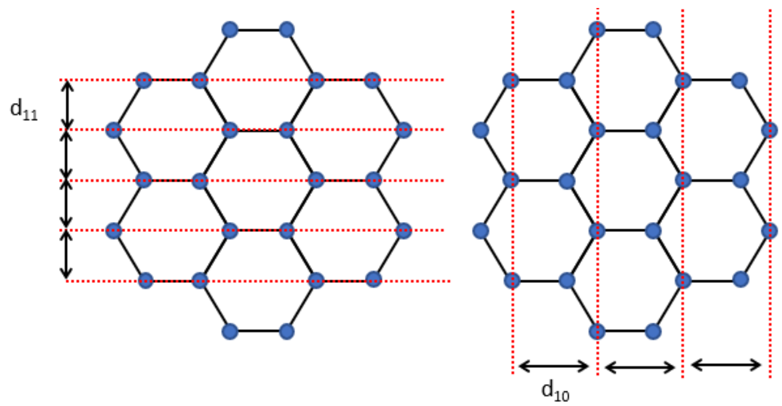


FIGURE 7: TWO CRYSTAL PLANES OF GRAPHITE

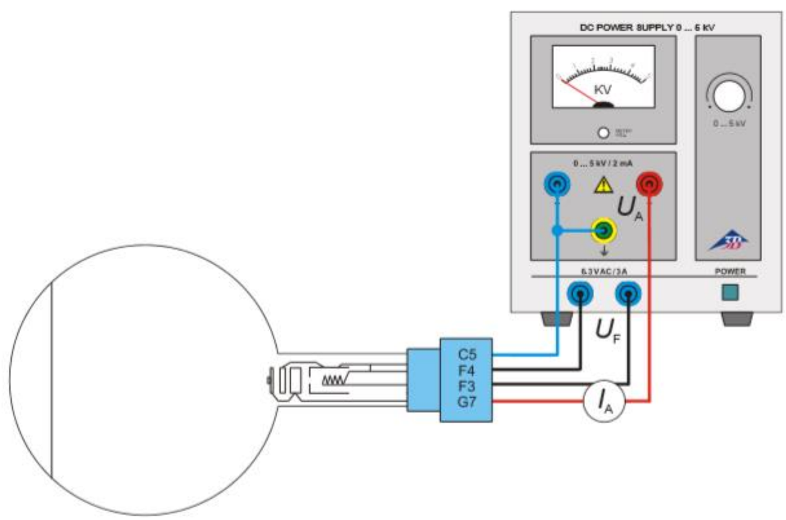
LAB PROCEDURE

Label each problem in your Jupyter notebook with "# Problem X" in a MarkDown cell. Perform all calculations in the Jupyter notebook. Submit a pdf by the end of your lab day.

Useful equations and constants are found on the last page of this handout.

Warning. Be careful with the high voltage supply. Do not turn on unless voltage is turned all the way down and all the connections are correct.

1) Make sure the power supply is correctly connected to the electron diffraction tube. Follow the diagram below.



- F4 and F3 are low voltage (6.3VAC) and connected to the pure tungsten heater filament. The filament ejects electrons when it heats up.
- G7 and C5 are connected to the cathode and anode. A large voltage difference accelerates the electrons to the graphite foil.

PROBLEM 1: QUALITATIVE

2) Make sure voltage is turned all the way down (CCW). Turn on the power. Tune the voltage over the whole range to observe the rings.

3) How do we know that these rings are coming from electrons, and not something else like x-rays? According to electromagnetic theory, a particle with a charge q moving with velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{B}$ experiences a force $\boldsymbol{F} = q\,(\boldsymbol{v} \times \boldsymbol{B})$ that deflects the path. But light is

unaffected by a magnetic field. There is a magnet nearby the apparatus. Place the magnetic near the apparatus. **Are we dealing with light or electrons?**

4) Before being quantitative, we are first going to see if what we observe agrees qualitatively with de Broglie's prediction. **According to de Broglie's prediction, should the ring diameter increase or decrease with the voltage?**

PROBLEM 2: DE BROGLIE'S PREDICTION AT A SINGLE VOLTAGE

5) Now we will measure the diameters of the two rings to test de Broglie's predictions at one specific voltage. Go to a voltage of 5.00kV. Measure the diameters of the two rings with the caliper and set them as new variables diameter1, diameter2.

HINT: You are going to be dealing with a lot of units, which can get confusing. All the constants will use are in SI, and so it will be useful to have all the variables in Python defined in SI units as well, i.e. meters. Define variables mm, um, nm, etc to help convert.

```
mm = 1e-3
diameter_1 = 100*mm
print(diameter_1) # diameter is in SI units of m
```

```
0.1
```

6) Use the lattice constants $d_{11} = 0.123\text{ nm}$, $d_{10} = 0.213\text{ nm}$ to convert these diameters to Bragg wavelengths **lambda_bragg** of the electron. You will have to figure out which lattice constant corresponds to which ring. The two wavelengths should be similar. The distance between the graphite target and screen is $L = 125$ mm.

HINT: The output of arctan is radians. If you want to convert to degrees, then you need to multiply by $180/\pi$. The argument of sin and cos also must be radians.

```
import numpy as np #only need to run once in Jupyter notebook
theta = np.arctan(1) #output of arctan is radians
degrees = theta * (180/np.pi) #convert radians to degrees
print(degrees)
```

```
45.0
```

7) Use de Broglie's hypothesis $\lambda = h/p$ to calculate the expected electron wavelength at 5kV, **lambda_electron** . Useful constants are found at the end of these notes and defined here:

```
h = 6.626e-34 #Planck's constant in SI
electron_mass = 9.109e-31 # kg, which is mass in SI
electron_volts = 1.602e-19 # Joules, which is energy in SI
```

HINT: an electron accelerated between two electrodes with a voltage difference of 1kV has an energy of 1keV = 1000eV.

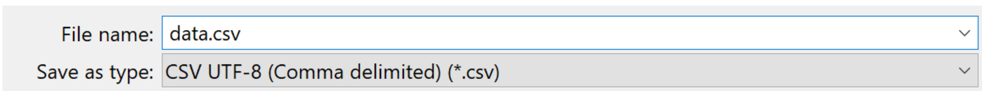
8) In order to be quantitative about the agreement of the model and your measurement, we will need to use uncertainity analysis. Estimate an uncertainty for your ring diameter measurments, σ_D. Now convert this into an uncertainty σ_λ for the Bragg wavelength λ_{Bragg}. The standard way to do this is error propagation, but all the derivatives can be cubersome. Just use $\frac{\sigma_\lambda}{\lambda} \approx \frac{\sigma_D}{D}$, which is often a good approximation and happens to be very accurate in our case.

9) Start a text cell for this discussion. How close is the Bragg wavelength and the theoretical electron wavelength? Do they agree within the uncertainity of the measured Bragg wavelength? If they are more than 50% different, then you likely have a bug in your code or problems with units. If the two values are not within the uncertainity, can you think of other uncertainities in the measurement besides the diameter measurment?

Problem 3: Measuring Planck’s Constant

We will now assume that de Broglie’s hypothesis is true, but that we do not know the value of Planck’s constant. In this part of the lab, you will measure the ring diameters for multiple voltages and then fit the curve with de Broglie’s model to extract a value for Planck’s constant.

10) Measure the diameter of the larger ring for at least 5 voltages and record values in a new Excel file. Label the columns “voltage” and “diameter”. Save Excel file as a CSV file named ‘data.csv’. You can do that by selecting “Save As” and selecting the CSV type.



11) Import the data into Jupyter. See Section 8 of the python tutorial “python_tutorial.ipynb” (same as last week) for help. Plot the diameter vs energy. As always, label your axes.

12) Convert the diameters into Bragg angles in a new variable called ‘theta_bragg’ in units radians. Use the Bragg angles to calculate the Bragg wavelengths λ_{Bragg}. Make a plot of of λ_{Bragg} in units picometers versus energy in units keV.

13) Now let us assume that we do not know Planck’s constant h. Use de Broglie’s formula $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\, m\, E}}$ with Planck’s constant h as a fitting parameter to fit your data. Read Section 9 on “Fitting Data” in the python tutorial for help in fitting. You will need to define a Python function to use as the model for the curve fitting. You will also need to import a library for the curve_fitting() function.

    Hint: If your fitted value of h is 2x larger or smaller than the actual value, then you probably have a bug in your code. Check your units!

14) We would like to be more quantitative about the agreement between your fitted value of Planck's constant h and the actual value.   Calculate your 95% confidence interval for the fitted value of h?

HINT:
The curve_fit() function also outputs the covariance.

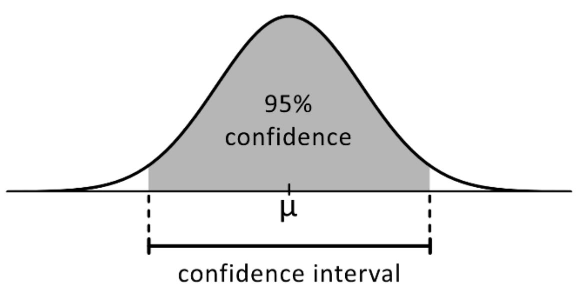
```
# Perform the fit with a guess
params, covariance = curve_fit(fit_func, x_data, y_data2, p0=[2])
```

A list of standard errors of the fitted parameters is given by

```
error = np.sqrt(np.diagonal(covariance))
```

Since you are only fitting 1 parameter h, the list will only have 1 element.

The standard error is the one standard deviation confidence internal for our fitted parameter, or 68% confidence.   The fitted value plus or minus 1.96 times its standard error gives the 95% confidence internal. Then we can say that there is only a 5% chance that the actual value of the fitted parameter is outside the 95% confidence interval range *if the model is true and the measurements only had random error.* If the actual value is outside this range, then likely either the model is wrong, or our measurements had systematic errors we did not account for.



15) Start a text cell. Write your value of Planck's constant and the confidence interval.   How many standard deviations away is your measured value from the actual value?   Do not be surprised if it is many standard deviations away.   That is because we may have a systematic errors in our experiment. What are some other sources of uncertainity in this measurement that could cause this discrepancy?

Congratulations! If you did this experiment before 1927, you would have won a Nobel prize. The 1937 Nobel prize in physics was awarded to Davisson and Thomson for "their experimental discovery of the diffraction of electrons by crystals."

POSTLAB

Make a text cell with # Postlab and answer the following questions. Label them with the question number.

16) What if you repeated this experiment, but instead of using electrons you used X-rays that have the same wavelength? What do you expect to see? One reason that it is easier to use X-rays is that you do not need to do it in vacuum.

17) Consider a 5 kg bowling ball traveling at 5 m/s. Calculate the wavelength of the bowling ball? Why do you not observe interference of the ball from the pins? There is a reason we don't see quantum effects in our everyday life.

18) We have measured the wave-like properties of an electron. However, what if we performed a double-slit experiment with electrons, but this time we only accelerating a single electron at a time. Instead of the fluorescent screen, we use a sensitive detector that can detect one electron at a time. Each time an electron arrives, you hear a click and see where it landed on the screen. If you took enough data, would you still observe an interference pattern shown below?

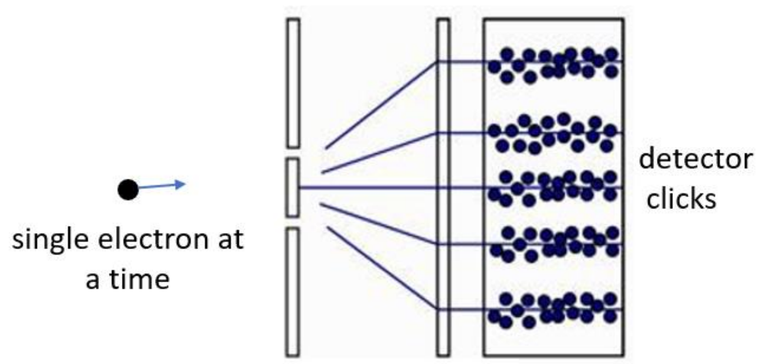


Figure 8: Double slit with one electron at at time.

HINT: This is tricky. I encourage you guys to talk to each other and the TA. This question really gets to heart of why quantum mechanics is so weird.

19) In a hydrogen atom, the average radius of an electron from the proton is 1 Bohr $a_0 = 52.9$ pm. Let's consider a very simple model of an electron traveling in a circle around a proton. The electron constructively interferes with itself, and so the circumference must be equal to an integer multiple of the wavelength. Use this model estimate the energy of the ground state electron. Compare it to the actual ground state binding energy 13.6 eV. (Hint: It is pretty close.)

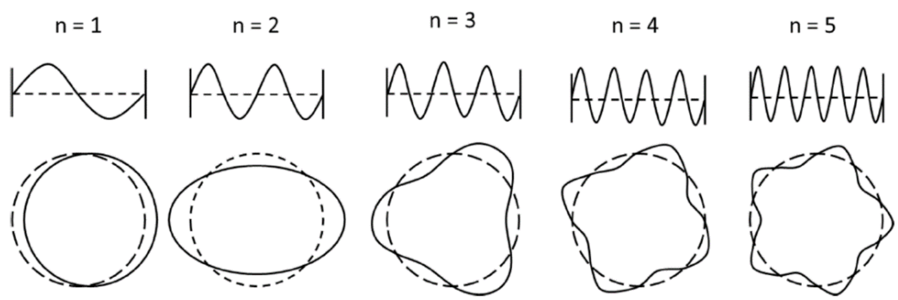


Figure 9: Electron traveling around proton forms standing waves. In the ground state (n=1), the electron's wavelength fits once around the circumference.

SUBMITTING YOUR JUPYTER LAB NOTEBOOK AS A PDF

Select File/ Print Preview. The notebook will open the preview in a separate tab as an html document. Print the preview and select to Save as PDF. Upload the PDF to BrightSpace.

The lab must be submitted to BrightSpace by the end of your lab day. You are welcome to send the Jupyter notebook to yourself and finish at home.

USEFUL CONSTANTS

electron mass = $9.109 \times 10^{-31}\ kg$

electron volts = $1.602 \times 10^{-19}\ Joules$

Planck's constant = h = $6.626 \times 10^{-34}\ Joules \cdot second$

Distance from graphite target to fluorescent screen L= 125 $\pm$2 mm

Glass bulb diameter = 130 mm diameter

Graphite lattice constants $d_{11} = 0.123\ \text{nm}$, $d_{10} = 0.213\ \text{nm}$

Useful equations:

$$\mathbf{2\theta_{Bragg} = \tan^{-1}\left(\frac{\mathit{R}}{\mathit{L}}\right)},$$

R is ring radius, and L is distance between target and where you measure ring

$$2\,\boldsymbol{d}\,\mathbf{sin}\left(\boldsymbol{\theta}_{\mathbf{Bragg}}\right) = \boldsymbol{\lambda}_{\mathbf{Bragg}},$$

d is a lattice constant of a crystal

$$\boldsymbol{\lambda}_{\mathbf{de\ Broglie}} = \frac{\boldsymbol{h}}{\sqrt{\mathbf{2}\,\boldsymbol{m}\,\boldsymbol{E}}}$$

E is energy of particle, m is the mass, and h is Planck's constant