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# EXACT DISTRIBUTIONS OF *R*2 AND ADJUSTED *R*2

IN A LINEAR REGRESSION MODEL

WITH MULTIVARIATE *t* ERROR TERMS

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In this paper we consider a linear regression model when error terms obey a

multivariate *t* distribution, and examine the eﬀects of departure from normality of error terms on the exact distributions of the coeﬃcient of determination (say, *R*2)

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and adjusted *R* (say, *R* ). We derive the exact formulas for the density function,

distribution function and *m*-th moment, and perform numerical analysis based on the exact formulas. It is shown that the upward bias of *R*2 gets serious and the standard error of *R*2 gets large as the degrees of freedom of the multivariate *t* error

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distribution (say, *ν*0) get small. The conﬁdence intervals of *R* and *R* are examined,

and it is shown that when the values of *ν*0 and the parent coeﬃcient of determination (say, Φ) are small, the upper conﬁdence limits are very large, relative to the value of Φ.

*Key words and phrases*: Adjusted *R*2, Exact distribution, Interval estimation, Mul- tivariate *t* error terms, *R*2.

## Introduction

To measure goodness of ﬁt of an estimated linear regression model, the coeﬃcient of determination (say, *R*2) and the adjusted coeﬃcient of determination (say, *R*2) have traditionally been used (see Section 2 for *R*2 and *R*2). Thus, there are many studies on the small sample properties of *R*2 and *R*2. For example, Barten (1962) suggests a modiﬁed version of *R*2 to reduce its bias, and Press and Zellner (1978) discuss the reason why the study of *R*2 is important in the case of ﬁxed regressors and they perform Bayesian analysis of *R*2. Also, Cramer (1987) derives the exact formulas for the ﬁrst two moments of *R*2 and *R*2, and shows that *R*2 is seriously biased upward in small samples while *R*2 is more unreliable than *R*2 in terms of standard deviation.

Although it is assumed that the model is correctly speciﬁed in the above studies, Carrodus and Giles (1992) examine the small sample properties of *R*2 when the indepen- dence of error terms is mistakenly assumed. Also, using asymmetric linear loss functions, Ohtani (1994) examines the risk performances of *R*2 and *R*2 when the relevant regres- sors are omitted in the speciﬁed model and when irrelevant regressors are included in the speciﬁed model. Ohtani and Hasegawa (1993) examine the bias and mean squared error (MSE) performances when the proxy variables are used instead of unobservable regressors and the error terms obey a multivariate *t* distribution.

Although there are many studies on the small sample properties of *R*2 and *R*2, the studies on the exact distribution of *R*2 and *R*2 per se are few. Although Ohtani (1994) derives the exact distribution and density functions of *R*2 and *R*2, he assumes that the error terms obey a normal distribution. As is discussed in Fama (1965) and Blattberg and

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Gonedes (1974), there exist many economic data that may be generated by distributions with fatter tails than a normal distribution. One example of such distributions is a multivariate *t* distribution. To examine the eﬀects of departure from normality of error terms on the sampling performances of estimators and test statistics, the multivariate *t* distribution has often been used. Some examples are Zellner (1976), Ullah and Zinde- Walsh (1984), Giles (1991), and Namba and Ohtani (2002). Although Srivastava and Ullah (1995) examined the sampling properties of *R*2 and *R*2 under a general non-normal error distribution, their analysis is based on the large sample asymptotic expansions. In this paper we consider a linear regression model when error terms obey a mul- tivariate *t* distribution, and examine the eﬀects of departure from normality of error terms on the exact distributions of *R*2 and *R*2. In Section 2 the model and estimators are presented, and in Section 3 the exact formulas for the density function, distribution function and *m*-th moment are derived. In Section 4 we evaluate means, standard errors, density functions, and conﬁdence intervals of *R*2 and *R*2 numerically. The numerical re- sults show that the upward bias of *R*2 gets serious and the standard error of *R*2 gets large as the degrees of freedom of the multivariate *t* error distribution (say, *ν*0) get small. It is also shown that when the values of *ν*0 and the parent coeﬃcient of determination (say, Φ, which is deﬁned in Section 4) are small, the upper conﬁdence limits of *R*2 and *R*2 are vary large. Finally, the 95% conﬁdence intervals of *R*2 for Φ = 0*.*5 are shown.

## Model and estimators

We consider the following linear regression model:

(2.1) *y* = *ℓβ*0 + *Xβ* + *u,*

where *y* is an *n* 1 vector of observations of the dependent variable, *ℓ* is an *n* 1 vector consisting of ones, *X* is an *n* (*k* 1) matrix of non-stochastic regressors, *β*0 is an intercept, *β* is a (*k* 1) 1 vector of regression coeﬃcients, and *u* is an *n* 1 vector of error terms.

*− × ×*

*× −*

*× ×*

As to the error terms, we assume that *u* obeys a multivariate *t* distribution with location parameter 0, scale parameter *σ*2, and degrees of freedom parameter *ν*0. Then, as is shown in Zellner (1976), the density function of *u* is written as:

(2.2)

where (2.3)

*∞*

*IG*

*N*

*p*(*u*) = ∫ *p* (*u|τ* ) *p* (*τ* ) d*τ,*

0

*p* (*u|τ* ) = 1 exp.*−u′u* Σ*,*

*N*

(2*π*)*n/*2 *τn*

2*τ* 2

(2.4)

*pIG τ*

Γ(

2)

2

*τ*

0

*−* 2

*.*

*ν*0*/*

( ) = 2 . *ν*0*σ*2 Σ*ν*0 */*2

*−*(*ν* +1) exp.

*ν*0*σ*2 Σ

*τ* 2

We assume that *ν*0 *>* 2 so that the ﬁrst two moments of *u* may exist. Then, we have E[*u*] = 0 and E[*uu′*] = *σ*2 *In* = [*ν*0*/*(*ν*0 2)] *σ*2 *In*.

*u*

*−*

We assume without loss of generality that all the regressors are measured as devi- ations from their sample means (i.e., *X′ℓ* = 0). Then, the ordinary least squares (OLS) estimators of *β*0 and *β* are:

^

(2.5)

(2.6)

*β*0 = *ℓ′y/n* = *y, β*^ = *S−*1*X′y,*

where *S* = *X′X*. The associated residual vector is:

(2.7)

*e* = *y −* (*ℓy* + *Xβ*^)*.*

Since *y′y − ny*2 = *β*^*′Sβ*^ + *e′e*, the sample coeﬃcient of determination is written as:

*−*

^ ^

(2.8)

*R*2 = 1 *e′e*

*y′y − ny*2

= *β′Sβ .*

*β*^*′Sβ*^ + *e′e*

*− −*

Also, the adjusted coeﬃcient of determination is:

(2.9)

*R*2 = 1 *n −* 1 (1 *R*2)*.*

*n − k*

If we deﬁne a formally general estimator as:

*h*

(2.10)

*R*2 = *hR*2 + 1 *− h,*

where *h ≥* 1, then *R*2 reduces to *R*2 when *h* = 1, and to *R*2 when *h* = (*n −* 1)*/*(*n − k*). Since 0 *≤ R*2 *≤* 1, we see that 1 *− h ≤ R*2 *≤* 1.

*h*

*h*

## Exact density and distribution functions

If we assume temporarily that *τ* is ﬁxed, then the error terms obey a normal dis- tribution with E[*u*] = 0 and E[*uu′*] = *τ* 2 *In*. As is shown in Ohtani (1994), the density function of *R*2 , given *τ* , is:

*h*

(3.1)

*∞*

( ) = *− − −*

Σ *w* (*λ*)*i* (*n* 1)*/*2 *i*+1*p c|τ h*

*i*=0 *B*((*k −* 1)*/*2 + *i,* (*n − k*)*/*2)

*×* (*c* + *h −* 1)(*k−*1)*/*2+*i−*1 (1 *− c*)(*n−k*)*/*2*−*1*,*

where *wi*(*λ*) = [(*λ/*2)*i/i*!] exp( *λ/*2) and *λ* = *β′Sβ/τ* 2.

*−*

Using (2.4) and (3.1), the density function of *R*2 can be obtained as follows:

*h*

(3.2)

*p*(*c*) = ∫ *∞ p*(*c|τ* ) *p* (*τ* ) d*τ*

0

*IG*

*∞*

= *− − −*

Σ 1 (*n* 1)*/*2 *i*+1*h*

*i*=0 *B*((*k −* 1)*/*2 + *i,* (*n − k*)*/*2) *i*!

*×* (*c* + *h −* 1)(*k−*1)*/*2+*i−*1 (1 *− c*)(*n−k*)*/*2*−*1

2 . *ν*0*σ*2 Σ*ν*0 */*2. *β′Sβ* Σ*i* ∫ *∞*

*×* Γ(

2)

2

2

*τ*

0

exp

*−*

2

d*τ.*

*ν*0*/*

0

*τ* 2

*−*(*ν* +1)*−*2*i*

. *β′Sβ* + *ν*0*σ*2 Σ

Making use of the change of variable, *t* = (*β′Sβ* + *ν*0*σ*2)*/*(2*τ* 2), and performing some manipulations, we obtain the following distribution:

(3.3)

0

*B*((*k −* 1)*/*2 + *i,* (*n − k*)*/*2) *i*! Γ(*ν*0*/*2)(*ν*0 + *θ*)*ν*0 */*2+*i*

*p*(*c*) = Σ

*θi νν*0 */*2 Γ(*ν*0*/*2 + *i*)

*× h−*(*n−*3)*/*2*−i* (*c* + *h −* 1)(*k−*1)*/*2+*i−*1 (1 *− c*)(*n−k*)*/*2*−*1*,*

*∞*

*i*=0

where *θ* = *β′Sβ/σ*2, and *B*(*·, ·*) is the beta function.

The distribution function of *R*2 is:

*h*

∫

(3.4)

0

*F* (*c*0

) = *c*0

1*−h*

= Σ

*∞*

*i*=0

*B*((*k −* 1)*/*2 + *i,* (*n − k*)*/*2) *i*! Γ(*ν*0*/*2)(*ν*0 + *θ*)*ν*0 */*2+*i*

*p*(*c*) d*c*

*θi νν*0 */*2 Γ(*ν*0*/*2 + *i*)

*× h−*(*n−*3)*/*2*−i*

*c*0

*c*

∫ (

1*−h*

+ *h −*

1)(*k−*1)*/*2+*i−*1 (1

*− c*)(*n−k*)*/*2*−*1

d*c.*

Making use of the change of variable, *t* = (*c* + *h* 1)*/h*, and performing some manipu- lations, (3.4) reduces to:

*−*

*∞*

(3.5)

0

*− · ·*

0

*i*! Γ(*ν /*2)(*ν*

+ *θ*)*ν*0*/*2+*i*

*c*0

*i*=0

0

0

*F* (*c* ) = Σ

*θi νν*0 */*2 Γ(*ν*0*/*2 + *i*)

*I ∗* ((*k −* 1)*/*2 + *i,* (*n − k*)*/*2)*.*

where *c∗*0 = (*c*0 + *h* 1)*/h*, and *Ia*( *,* ) is the incomplete beta function ratio. When *β* = 0 (i.e., *θ* = 0), we see that the distribution function reduces to:

(3.6)

*F* (*c*0) = *Ic∗*0 ((*k −* 1)*/*2*,* (*n − k*)*/*2)*.*

Putting *λ*1 = *λ*2 = 0 in eq. (18) in Ohtani (1994) which is the distribution function when the error terms obey a normal distribution, and comparing with (3.6), we see that when *β* = 0, the distribution function is robust to the change of the error distribution from a normal distribution to a multivariate *t* distribution. However, when *β* = 0, this robustness does not hold.

*̸*

Also, the formula for the *m*-th moment of *R*2 is:

*h*

(3.7)

E[(

) ] = ∫ 1

( ) d

2 *m*

*R*

*h*

1*−h*

= Σ

*∞*

0

*i*=0

*B*((*k −* 1)*/*2 + *i,* (*n − k*)*/*2) *i*! Γ(*ν*0*/*2)(*ν*0 + *θ*)*ν*0 */*2+*i*

*cm p c c*

*θi νν*0 */*2 Γ(*ν*0*/*2 + *i*)

*− − −* ∫ 1

(*n*

*× h*

(*n*

3)*/*2

*i*

1*−h*

*cm*

*c*

(*k*

1)*/*2+*i*

1

*k*)*/*2

1

*c.*

( + 1) *−*

*−* (1

) *− −* d

Again, making use of the change of variable, *t* = (*c* + *h* 1)*/h*, the integral in (3.7)

*−*

*h −*

*− c*

reduces to:

(3.8)

∫ 1[

+ (1

)] ( )

*−*

[(1 ) ] d

*th h*

*m*

0

Σ

*m th* (*k−*1)*/*2+*i−*1

*− t h* (*n−k*)*/*2*−*1 *h t*

∫ 1

*−*

=

*r*=0

*m*

Σ

*mCr h*(*n−*3)*/*2+*r*+*i* (1 *− h*)*m−r*

*t*(*k−*1)*/*2+*r*+*i−*1 (1 *t*)(*n−k*)*/*2*−*1 d*t*

0

= *mCr h*(*n−*3)*/*2+*r*+*i* (1 *− h*)*m−r B*((*k −* 1)*/*2 + *r* + *i,* (*n − k*)*/*2)*.*

*r*=0

Thus, using the formula, *B*(*a, b*) = Γ(*a*)Γ(*b*)*/*Γ(*a* + *b*), we ﬁnally obtain the expectation of (*R*2 )*m*:

*h*

*ν*

(3.9)

E[(*R*2 )*m*] = Σ

Γ((*n −* 1)*/*2 + *i*) Γ(*ν*0*/*2 + *i*) *θi*

*ν*0*/*2 0

*h*

*∞*

*i*=0

Γ((*k −* 1)*/*2 + *i*)Γ(*ν*0*/*2) *i*! (*ν*0 + *θ*)*ν*0 */*2+*i*

*m*

*m*

*r*

Γ((*n −* 1)*/*2 + *r* + *i*)

*×* Σ *C hr* (1 *− h*)*m−r* Γ((*k −* 1)*/*2 + *r* + *i*) *.*

*r*=0

## Numerical analysis

In this section we perform numerical analysis based on the exact formulas given in (3.3), (3.5) and (3.9). We deﬁne Φ as follows:

(4.1)

*u*

Φ = *β′Sβ β′Sβ* + *nσ*2

= *θ , θ* + *nν*0*/*(*ν*0 *−* 2)

which is called the parent coeﬃcient of determination (see Press and Zellner (1978), Cramer (1987) and Ohtani and Hasegawa (1993)). Note that the relationship between Φ and *R*2 is given by plim*n* Φ = plim*n R*2. In the numerical evaluations, we ﬁrst decided the value of Φ, and then calculated the value of *θ* through *θ* = *nν*0Φ*/*[(*ν*0 2)(1 Φ)]. The parameter values used in the numerical evaluations were *k* = 3, 4, 5, 6, 7, 8, *n* = 10, 20, 30, 40, *ν*0 = 3, 5, 10, 30, 100, (normal), and various values of Φ. The numerical evaluations were executed on a personal computer, using the FORTRAN code. The inﬁnite series in the exact formulas converged rapidly with the convergence tolerance of 10*−*12.

*∞*

*−*

*−*

*→∞ →∞*

Tables 1 and 2 show the mean, standard error (denoted as ’S.E.’) and 95% conﬁdence

interval of *R*2 and *R*2 when *k* = 5 and *n* = 20, where ’*c*

*L*

) = *P* (

’ and ’*cU*

’ denote the conﬁdence

limits such that *P* (*R*2 *< cL*

2

*R < cL*

) = 0*.*025 and *P* (*R*2 *> cU*

) = *P* (*R*2

*> cU* ) =

0*.*025, where *P* (*A*) is the probability of an event *A*. Figures 1 and 2 show the density functions of *R*2 and *R*2 when *k* = 5, *n* = 20 and Φ = 0*.*6.

We see from Tables 1 and 2 that *R*2 is seriously biased upward in small samples,

and *R*2 is more unreliable than *R*2 in terms of standard error. In particular, the upward bias of *R*2 gets serious and the standard error of *R*2 gets large as the degrees of freedom of the multivariate *t* error distribution get small. The phenomena are also seen from Figure 1. This indicates that as the tails of the error distribution get fatter, *R*2 becomes more unreliable. Also, we see from Figures 1 and 2 that the density function of *R*2 is ﬂatter than that of *R*2 though the modes of the density functions of *R*2 are smaller than those of *R*2.

We see from Tables 1 and 2 that the conﬁdence intervals of *R*2 and *R*2 are con- siderably wide, and the conﬁdence intervals get wide as the degrees of freedom of the multivariate *t* error distribution get small. This phenomenon is also expected from Fig-

ures 1 and 2. In particular, when the values of *ν*0 and Φ are small, the upper conﬁdence

limits of *R*2 and *R*2 are vary large. For example, when *ν* = 5 and Φ = 0.2, the upper

0

2

conﬁdence limit of *R*2 is *cU* = 0*.*7684, and that of *R* is *c* = 0*.*7067. This indicates that

*U*

even when the estimated values of *R*2 and *R*2 are more than 0.7, the parent coeﬃcient

of determination may be just 0.2. We see from Table 2 that when the value of Φ is small, the lower conﬁdence limits of *R*2 can be negative though the absolute value of *cL* becomes small. This phenomenon is caused by the shift to the right of the density function when *ν*0 decreases, as is shown in Figure 2.

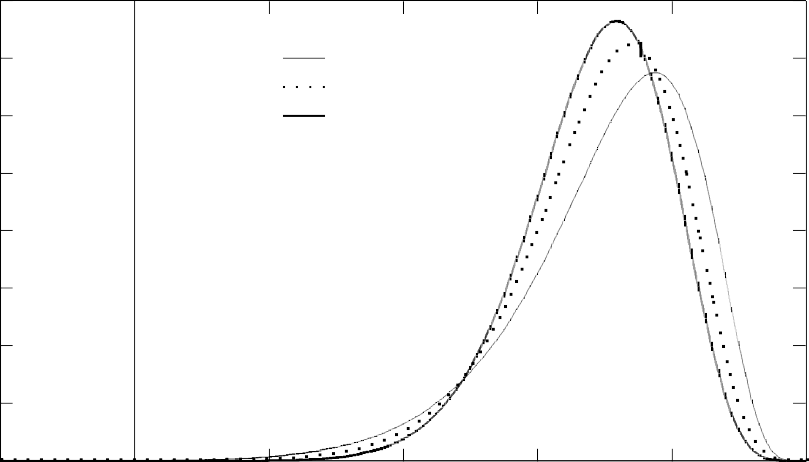
Finally, we show 95% conﬁdence intervals for Φ = 0*.*5 and for some values of *k* and *n* in Table 3. Although there is no deﬁnite reason why Φ = 0*.*5 is selected, we can conﬁrm at the conﬁdence coeﬃcient 0.95 that the parent coeﬃcient of determination is at least more than half if the value of *R*2 exceeds the upper limit given in Tbale 3. Since *ν*0 = and *ν*0 = 3 are two extreme values, we can conﬁrm at least Φ = 0*.*5 if the value of *R*2 is larger than the upper limit for *ν*0 = 3 even if the true value of *ν*0 is larger than 3, and we may doubt Φ = 0*.*5 if the value of *R*2 is less than the lower limit for *ν*0 = *∞*.

*∞*

Table 1. Mean, standard error and 95% conﬁdence interval of *R*2 for *k* = 5 and *n* = 20

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *ν*0 | Φ | Mean | S.E. | *cL* | *cU* |
|  | 0.8 | 0.8633 | 0.1028 | 0.5984 | 0.9731 |
| 5 | 0.6 | 0.7307 | 0.1525 | 0.3832 | 0.9298 |
|  | 0.4 | 0.5881 | 0.1774 | 0.2464 | 0.8688 |
|  | 0.2 | 0.4226 | 0.1762 | 0.1485 | 0.7684 |
|  | 0.8 | 0.8522 | 0.0823 | 0.6535 | 0.9563 |
| 10 | 0.6 | 0.7043 | 0.1334 | 0.4175 | 0.9014 |
|  | 0.4 | 0.5513 | 0.1613 | 0.2536 | 0.8297 |
|  | 0.2 | 0.3885 | 0.1640 | 0.1420 | 0.7307 |
|  | 0.8 | 0.8476 | 0.0650 | 0.7001 | 0.9439 |
| 30 | 0.6 | 0.6914 | 0.1163 | 0.4526 | 0.8802 |
|  | 0.4 | 0.5323 | 0.1493 | 0.2645 | 0.8051 |
|  | 0.2 | 0.3717 | 0.1574 | 0.1394 | 0.7114 |
|  | 0.8 | 0.8464 | 0.0583 | 0.7183 | 0.9387 |
| 100 | 0.6 | 0.6875 | 0.1094 | 0.4675 | 0.8722 |
|  | 0.4 | 0.5265 | 0.1448 | 0.2696 | 0.7969 |
|  | 0.2 | 0.3666 | 0.1554 | 0.1387 | 0.7056 |
|  | 0.8 | 0.8459 | 0.0553 | 0.7206 | 0.9351 |
| *∞*  (normal) | 0.6  0.4 | 0.6860  0.5241 | 0.1063  0.1429 | 0.4514  0.2283 | 0.8633  0.7796 |
|  | 0.2 | 0.3645 | 0.1545 | 0.0866 | 0.6732 |

*4.0*



*0 = 10*

*0 = 30*

*0 = 1*

*(normal)*

*3.5*

*3.0*

*2.5*

*2.0*

*1.5*

*1.0*

*0.5*

*0.0*

*0:2 0.0 0.2 0.4 0.6 0.8 1.0*

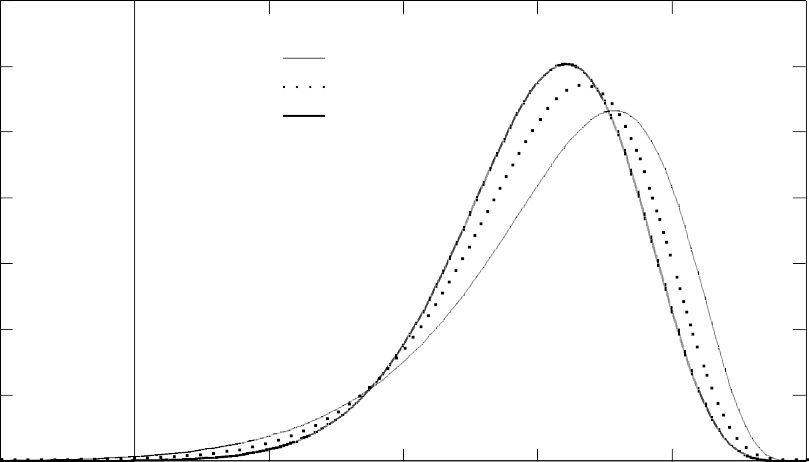
Figure 1. Density functions of *R*2 for *k* = 5, *n* = 20, and Φ = 0*.*6

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Table 2. Mean, standard error and 95% conﬁdence interval of *R* for *k* = 5 and *n* = 20

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *ν*0 | Φ | Mean | S.E. | *cL* | *cU* |
|  | 0.8 | 0.8269 | 0.1302 | 0.4913 | 0.9659 |
| 5 | 0.6 | 0.6589 | 0.1932 | 0.2187 | 0.9110 |
|  | 0.4 | 0.4783 | 0.2247 | 0.0454 | 0.8338 |
|  | 0.2 | 0.2687 | 0.2232 | *−*0.0785 | 0.7067 |
|  | 0.8 | 0.8128 | 0.1042 | 0.5611 | 0.9447 |
| 10 | 0.6 | 0.6254 | 0.1689 | 0.2621 | 0.8751 |
|  | 0.4 | 0.4316 | 0.2043 | 0.0546 | 0.7843 |
|  | 0.2 | 0.2254 | 0.2078 | *−*0.0868 | 0.6589 |
|  | 0.8 | 0.8070 | 0.0823 | 0.6202 | 0.9290 |
| 30 | 0.6 | 0.6091 | 0.1473 | 0.3066 | 0.8482 |
|  | 0.4 | 0.4076 | 0.1891 | 0.0684 | 0.7532 |
|  | 0.2 | 0.2041 | 0.1994 | *−*0.0901 | 0.6345 |
|  | 0.8 | 0.8055 | 0.0739 | 0.6432 | 0.9224 |
| 100 | 0.6 | 0.6042 | 0.1386 | 0.3255 | 0.8382 |
|  | 0.4 | 0.4002 | 0.1835 | 0.0748 | 0.7427 |
|  | 0.2 | 0.1977 | 0.1968 | *−*0.0910 | 0.6270 |
|  | 0.8 | 0.8049 | 0.0701 | 0.6461 | 0.9177 |
| *∞*  (normal) | 0.6  0.4 | 0.6022  0.3972 | 0.1346  0.1810 | 0.3052  0.0226 | 0.8268  0.7208 |
|  | 0.2 | 0.1951 | 0.1957 | *−*0.1570 | 0.5860 |

*3.5*



*0 = 10*

*0 = 30*

*0 = 1*

*(normal)*

*3.0*

*2.5*

*2.0*

*1.5*

*1.0*

*0.5*

*0.0*

*0:2 0.0 0.2 0.4 0.6 0.8 1.0*

Figure 2. Density functions of *R*2 for *k* = 5, *n* = 20, and Φ = 0*.*6

Table 3. 95% conﬁdence interval when Φ = 0*.*5

*ν*0 = *∞ ν*0 = 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *k* | *n* | *cL* | *cU* | *cL* | *cU* |
|  | 10 | 0.2770 | 0.8678 | 0.2933 | 0.9658 |
| 3 | 20 | 0.3187 | 0.7558 | 0.2361 | 0.9374 |
|  | 30 | 0.3445 | 0.7056 | 0.2170 | 0.9270 |
|  | 40 | 0.3617 | 0.6760 | 0.2073 | 0.9295 |
|  | 10 | 0.3470 | 0.9008 | 0.3676 | 0.9741 |
| 4 | 20 | 0.3489 | 0.7762 | 0.2743 | 0.9421 |
|  | 30 | 0.3638 | 0.7199 | 0.2432 | 0.9302 |
|  | 40 | 0.3758 | 0.6871 | 0.2273 | 0.9318 |
|  | 10 | 0.4205 | 0.9310 | 0.4443 | 0.9819 |
| 5 | 20 | 0.3794 | 0.7963 | 0.3125 | 0.9468 |
|  | 30 | 0.3831 | 0.7342 | 0.2694 | 0.9334 |
|  | 40 | 0.3900 | 0.6981 | 0.2472 | 0.9342 |
|  | 10 | 0.4983 | 0.9575 | 0.5241 | 0.9889 |
| 6 | 20 | 0.4104 | 0.8161 | 0.3508 | 0.9515 |
|  | 30 | 0.4026 | 0.7483 | 0.2954 | 0.9366 |
|  | 40 | 0.4041 | 0.7090 | 0.2671 | 0.9365 |
|  | 10 | 0.5822 | 0.9791 | 0.6083 | 0.9948 |
| 7 | 20 | 0.4418 | 0.8354 | 0.3893 | 0.9562 |
|  | 30 | 0.4222 | 0.7624 | 0.3215 | 0.9398 |
|  | 40 | 0.4184 | 0.7199 | 0.2869 | 0.9388 |
|  | 10 | 0.6755 | 0.9940 | 0.6996 | 0.9987 |
| 8 | 20 | 0.4737 | 0.8544 | 0.4279 | 0.9608 |
|  | 30 | 0.4419 | 0.7763 | 0.3476 | 0.9430 |
|  | 40 | 0.4327 | 0.7308 | 0.3067 | 0.9412 |

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