Vibration Response of Functionally Graded Nano plates using the Non-local Theory

Project Report Submitted in Partial Fulfillment of the Requirements for the Degree

of

Bachelor of Technology

in

Mechanical Engineering

Submitted by

SANJAY (2010110563)

Under the Supervision of

Dr. Ankit Gupta.

Assistant Professor

SHIV NADAR
INSTITUTION OF EMINENCE DEEMED TO BE
UNIVERSITY
DELHI NCR

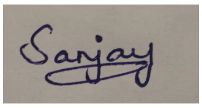
Department of Mechanical Engineering

May 2024

DECLARATION

I/We declare that this written submission represents our ideas in our own words and where others' ideas or words have been included, we have adequately cited and referenced the original sources. I/We also declare that we have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in our submission. I/We understand that any violation of the above will be cause for disciplinary action by the University and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

Sanjay



Place:Shiv Nadar

Institute of

EMinence

Date:13/05/2024

ACKNOWLEDGEMENTS

I would like to take this opportunity to acknowledge the contribution of and thank the various individuals and organisations without whose contribution this project could not be completed successfully.

First, I would like to thank our faculty advisor, guide and mentor, Dr. Ankit Gupta, for giving me this opportunity to work under his esteemed guidance and supervision as well as constantly providing us with his guidance, support and motivation that pushed me to achieve more with this project. I am extremely grateful for his strong faith in our abilities and for providing us with everything - knowledge, guidance and resources that allowed us to complete the project.

Lastly, I would like to express our sincere gratitude to Shiv Nadar Institute of Eminence For granting me access to the equipment in their labs on a priority basis so that I can conduct the coding and analysis section of the project.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

V	Volume Fraction of nano plates
E	Modulus of Elasticity
k	Thermal conductivity coefficient
α	Thermal expansion coefficient
T	Temperature,K
σ	Stress, N/m^2
σ^T	Thermal Stress
ε	Strain
Q	Resultant Shear Force
U_p	Strain Energy
U_T	Strain Energy due to thermal Load
K	Stiffness Matrix
M	Mass matrix

LIST OF ABBREVIATIONS

2D	Two dimensions
3D	Three dimensions
FGM	Functionally Graded Materials
FSDT	First order shear Deformation Theory
HSDT	Higher order shear Deformation Theory
CPT	Classical Plate theory

CHAPTER 1

1. INTRODUCTION

1.1 Functionally Graded Materials

The rapid advancement of nanotechnology has ignited widespread fascination with nanostructures, propelled by their extraordinary electronic, physical, chemical, and mechanical characteristics. Within this burgeoning landscape, Functionally Graded Materials (FGMs) have emerged as prominent players in nanostructure applications, distinguished by their distinctive composition and the seamless transition of material properties across their surfaces. In stark contrast to conventional laminated composites, FGMs offer a novel solution to mitigate stress concentration challenges through their gradual property gradient, thereby enhancing structural integrity and performance in diverse applications.

1.1.1 Motivation:

The motivation behind the selection of Functionally Graded Materials (FGMs) is deeply rooted in their exceptional and multifaceted characteristics, which offer a myriad of advantages across engineering and material science domains. One of the standout features driving their preference is their distinct stress-strain behavior, which diverges from the uniform response observed in homogeneous materials. This non-uniform behavior empowers engineers and researchers to finely optimize FGMs for a wide array of loading scenarios and structural designs, ensuring optimal performance and reliability in diverse applications. Furthermore, the ability to precisely control thermal conductivity sets FGMs apart from their homogeneous counterparts, making them indispensable in industries where resistance to thermal gradients is paramount. It is the combination of these intrinsic qualities that ignites the fervent interest and unwavering preference for FGMs, positioning them at the forefront of innovation and advancement in modern engineering and material science pursuits.

1.1.2 Applications of Functionally Graded Materials:

Functionally graded nanoplate materials offer a vast array of potential applications, owing to their adaptable properties that can be customized based on their unique composition and structural makeup. These applications span across diverse sectors, including:

i) Aerospace: FGMs find utility in aerospace applications where lightweight yet robust materials are essential for spacecraft and aircraft components. ii) Nuclear: In nuclear engineering, FGMs play a crucial role in the fabrication of reactor components, offering enhanced thermal and mechanical properties under extreme conditions. iii) Civil: In civil engineering, FGMs contribute to the development of advanced structural materials for buildings, bridges, and infrastructure, offering improved durability and performance. iv) Automotive: FGMs are increasingly integrated into automotive manufacturing processes, where they enhance the strength-to-weight ratio and fuel efficiency of vehicle components. v) Biosensors: FGMs serve as key components in biosensor technologies, facilitating precise detection and analysis of biological substances in medical and environmental applications.

As the demand for FGMs in various industries continues to grow, there arises a need for more accurate plate theories to predict and analyze the response of FG plates under different operational conditions. This underscores the ongoing research efforts aimed at advancing our understanding and utilization of FGMs in diverse engineering and scientific domains

CHAPTER 2

2. LITERATURE REVIEW

2.1 An overview of research previously done

- Previously there were researches where conventional continuum theories were used with First order Shear deformation theories to find the Vibration response of the Functionally Graded Nano plates namely the classical plate theory First Order Shear Deformation theory
- The w displacement in many cases were considered just as a function with w_o and without any shear deformations.

$$u(x, y, z) = u_0 - \frac{\partial wo}{\partial x} + f(z)\varphi 1,$$

$$v(x, y, z) = v_0 - \frac{\partial wo}{\partial y} + f(z)\varphi 2,$$

$$w(x,y,z) = w_0$$

- There are also cases were displacement was taken just as a linear function in z
  f(z) = z

In the project I'm dedicated to, the displacement in w doesn't follow a constant or linear pattern in relation to z; rather, it can take on various forms such as polynomial, exponential, trigonometric, and more.

A number of investigations dealing with dynamic response of functionally graded nanostructures had been studied in the scientific literature extensively. Using the finite element method (FEM),{natarajan2012size} employed Eringen's differential theory to analyze vibration behavior of functionally graded nanoplates based on first-order shear deformation theory (FSDT).{hosseini2013exact} proposed an analytical approach in combination with Mindlin and nonlocal elasticity plate theories to study free vibration response of thick circular/annular FGM nanoplates under various boundary conditions. Resonance of FGM micro/nanoplate by considering the nonlocal elasticity theory, strain gradient theory, and the small-scale effects is investigated by{nami2014resonance}. The same authors developed an analytical solution for vibration analysis of FGM rectangular nanoplates. Nguyena et al.\cite{nguyen2015efficient}proposed quasi-3D theory for free vibration, bending, and buckling analysis of FGM nanoplates taking into account the thickness stretching effect and by using an efficient computational approach. Hosseini and Jamalpoor{daikh2021vibration} analyzed dynamic behavior of double FGM viscoelastic nanoplates in thermal environment and resting on Pasternak elastic foundation by considering surface effects. Based on three-dimensional (3D) nonlocal elasticity theory, free vibration of exponential FGM simply supported micro/nanoplates is investigated by Salehipour et al{zare2015natural}.They have also used modified couple stress to investigate free vibration response of FGM micro/nanoplates. Salehipour proposed a modified nonlocal theory for vibration analysis of FGM simply supported rectangular micro/nanoplates using FSDT and 3D. Zara {zare2015natural} developed an analytical method to study vibration response of FGM rectangular nanoplates with various boundary conditions. Belkorissat proposed a new nonlocal refined four variable theory for temperature-dependent free vibration of FGM nanoplates.

Further research has been undertaken to explore the small and large amplitude vibration analyses of functionally graded (FG) plates within thermal environments. These investigations encompass porosities, elastic foundations, and a spectrum of thermal and mechanical loading scenarios. It becomes imperative to meticulously examine the behavioural responses of plates due to the extensive utility of FG nano-plates in thermal environments across diverse fields.

These can be solved using Higher order shear deformation Theory.

The function which I have taken here is

$g=\sin\left(\frac{c.z}{h}\right)cos\left(\frac{c.z}{h}\right)$

$Mu=\left(\frac{-c.z}{h}\right)cos(c)$

f=g+Mu.z

2.2 Aim and Objective:

Through an extensive review of existing literature, a critical gap in knowledge was identified, providing the foundation for the aims and objectives of the project. AIM: The primary aim of this research is to delve into the intricate influences of microstructural deflection and geometric imperfections on the vibrational response of Functionally Graded Nano plates, employing the Non-Local Theory as a guiding framework.

OBJECTIVES:

1. To conduct a comprehensive investigation into the effects of microstructural deflections on the vibrational behavior of Functionally Graded Nano plates, elucidating their role in shaping the dynamic response of these complex structures.
2. To analyze and evaluate the impact of geometric imperfections on the vibrational characteristics of Functionally Graded Nano plates, considering their influence on mode shapes, natural frequencies, and overall structural integrity.
3. To explore the role of elastic foundation conditions in governing the vibrational properties of Functionally Graded Nano plates, examining how varying

foundation parameters affect structural stability and dynamic response.

4. To investigate the thermal environment's influence on the vibrational behavior of Functionally Graded Nano plates, studying temperature-dependent material properties and their effects on structural dynamics.
5. To develop and propose a novel shear strain function, denoted as f(z), tailored to capture the intricate mechanical behavior of Functionally Graded Nano plates, thereby enhancing the accuracy and reliability of analytical predictions and simulations.

CHAPTER 3

3. CONCEPTUAL ANALYSIS

The Nonlocal elasticity theory:

The stress tensor at a point is a function of the strain tensor at all the points of the continuum. Hence the nonlocal constitutive equation can be defined by the following relation as

$$(1-\mu\nabla^2)\sigma_{ij}^{NL} = \sigma_{ij}^{L} = C : \epsilon$$

Where $\sigma_{ij}^{NL}, \sigma_{ij}^{L}$ are nonlocal and local stress tensors. C and ε represent the elastic constants and strain respectively.

Higher order Shear Deformation Theory:

The higher-order shear deformation theories (HSDTs) represent a sophisticated approach that meticulously considers the influence of shear deformation effects within structural elements. One of the notable advantages of HSDTs lies in their ability to ensure that the transverse shear stresses on the top and bottom surfaces of the plate are zero, thus eliminating the necessity for a shear correction factor. This intrinsic characteristic underscores the robustness of HSDTs in accurately capturing the complex behavior of thin-walled structures, particularly in scenarios where precise stress distributions are paramount. By providing a comprehensive framework that accounts for shear deformation effects without the need for additional corrective measures, HSDTs offer an efficient and accurate method for analyzing the mechanical response of various structural configurations. This distinctive feature distinguishes HSDTs as a valuable tool in structural analysis and design, empowering engineers to gain deeper insights into the performance of thin-walled elements under various loading conditions.

$$u(x, y, z) = u_0 - z\frac{\partial wo}{\partial x} + f(z)\frac{\partial wo}{\partial x}$$

$$v(x, y, z) = v_0 - \frac{\partial wo}{\partial y} + f(z)\frac{\partial wo}{\partial y},$$

$$w(x,y,z) = w_b + w_s + g(z)\varphi$$

In the classical thin plate theory (CPT), the displacement field is derived by setting the function f(z)f(z) equal to zero. This simplistic assumption allows for a straightforward representation of the plate's behavior under loading conditions. Conversely, the first-order shear deformation plate theory (FSDT) introduces a slightly more nuanced approach by setting f(z) equal to a constant value. This adjustment accounts for the effects of transverse shear deformation within the plate while maintaining a simplified analytical framework. However, in the higher-order shear deformation theory (HSDT), the function f(z)f(z) takes on a more complex form, allowing for a more detailed characterization of the plate's behavior. By incorporating this variable function of zz, HSDT offers a more comprehensive understanding of the structural response, particularly in situations where shear deformation effects are significant. This flexibility in choosing the form of f(z)f(z) enables engineers and researchers to tailor their analyses to specific structural configurations and loading conditions, thereby enhancing the accuracy and applicability of the theory.

3.2 Conventional Continuum theory Vs Non Local theory

The Conventional Continuum theory is based on the assumption that a material can be treated as a continuous medium without any internal structure or interactions at a distance, whereas Non-local continuum theory acknowledges that material behavior can be influenced by distant points within the material, beyond just the local state.

Mathematically,conventional continuum mechanics typically involves partial differential equations (PDEs) that describe how physical quantities (e.g., stress, strain, velocity) vary with respect to space and time,in the other case of non local continuum theory, introduce non-local operators or integral terms into the governing equations to account for the influence of material points at a distance.

Conventional Continuum Mechanics forms the basis of classical engineering mechanics, assuming that the behaviour of a material at a point depends solely on the surrounding material properties and gradients of the field variables at that point. It's founded on the principle of local equilibrium and relies on differential equations, like the Navier-Stokes equations or the equations of elasticity, to describe the material's behaviour. However, as structures and materials become smaller or exhibit extreme conditions, the assumptions of continuum mechanics start to break down.

Non-local theories, such as non-local elasticity or non-local plasticity, step away from this local equilibrium assumption by considering integral rather than differential equations, incorporating non-local effects and interactions between distant material points. These theories introduce a length scale parameter, allowing the influence of material points separated by a distance to affect each other's behavior, capturing the material's non-local responses accurately. They are especially relevant in nanoscale mechanics, where material behavior is significantly influenced by interactions at small length scales that are not captured by classical continuum theories. Non-local theories bridge the gap by offering a more comprehensive description of material behavior in situations where continuum assumptions no longer hold.

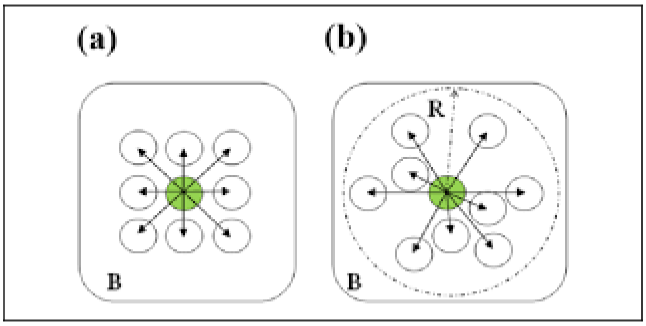


Figure 1.1 CCT vs NLT

3.3 HSDT VS FSDT:

HSDT (Higher-Order Shear Deformation Theory) and FSDT (First-Order Shear Deformation Theory) are two different approaches used in the field of structural engineering and mechanics to model the behaviour of thin-walled structures like beams and plates. They differ primarily in how they account for the effects of transverse shear deformation in these structures. In FSDT, the distribution of the transverse shear stress with respect to the thickness coordinate is assumed constant. Thus, a shear correction factor, which is hard to find because it depends on many parameters, is required to compensate for the error because of this assumption in FSDT. To avoid the use of shear correction factors, HSDT has been used.

First-order Shear Deformation Theory (FSDT) simplifies the analysis of thin plate and shell structures by neglecting transverse shear strains while allowing non-zero transverse shear stresses within the structure's thickness. FSDT assumes a linear variation of shear stress through the thickness, making it computationally efficient and easier to apply. It's suitable for thicker structures where shear effects might not significantly influence behavior. However, FSDT tends to be less accurate for thin structures or situations where precise transverse shear stress distribution is crucial for accurate predictions.

In contrast, Higher-order Shear Deformation Theory (HSDT) accounts for transverse shear strains within the structure's thickness, offering improved accuracy in predicting the behavior of thin to moderately thick structures. HSDT considers a parabolic variation of shear stress through the thickness, providing better results where shear effects play a critical role. Despite its higher accuracy, HSDT involves more complex mathematical formulations and demands higher computational resources compared to FSDT. Its strengths lie in accurately capturing shear effects, making it suitable for scenarios where detailed stress analysis in thin structures is necessary.

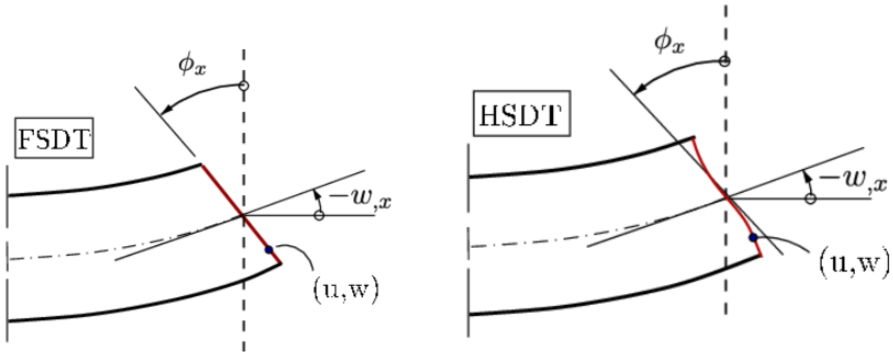


Figure 1.2 HSDT Vs FSDT

MATERIAL PROPERTIES:

Material	E (GPa)	μ	ρ (kg/m³)
Aluminium (Al)	70	0.3	2707
Alumina (Al_2O_3)	380	0.3	3800

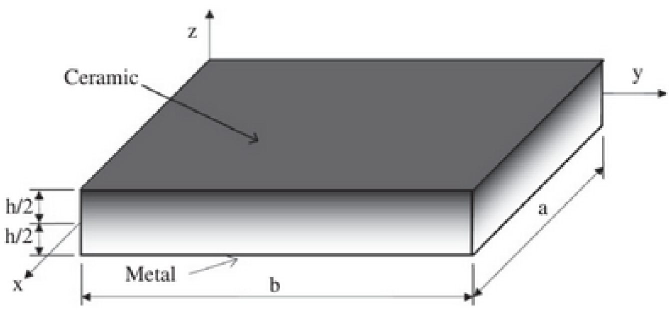


Fig 1.3 Geometry and coordinates of FG plates.

CHAPTER 4

4. MATHEMATICAL ANALYSIS:

Mathematical formulation:

The equations of motion of the functionally graded material sandwich nanoplates are derived by using the higher shear deformation theory and the Hamilton's variational principle, and solved using the Navier's solutions

M= Mass Matrix
[M11, M12, M13, M14, M15]
[M21, M22, M23, M24, M25]
[M31, M32, M33, M34, M35]
[M41, M42, M43, M44, M45]
[M51, M52, M53, M54, M55]

K=Stiffness Matrix
[K11, K12, K13, K14, K15]
[K21, K22, K23, K24, K25]
[K31, K32, K33, K34, K35]
[K41, K42, K43, K44, K45]
[K51, K52, K53, K54, K55]

We need to find the two matrices to solve the eigenvalue problem to get the frequencies.

Where all these values of Matrix are dependent on The coefficients A_{ij}, B_{ij}, D_{ij}, C_{ij}, F_{ij}, H_{ij}, J_{ij} etc; are defined as

$$A_{ij}, B_{ij}, D_{ij}, C_{ij}, F_{ij}, H_{ij} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} Q_{ij}^n [1, z, z^2, f(z), zf(z), f(z)^2]\, dz \qquad i,j=1,2,6$$

$$J_{ii} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} Q_{ii}^n [\frac{df(z)}{dz}^2]\, dz, (i, j = 4, 5)$$

Now the coefficients A_{ij}, B_{ij}, D_{ij}, C_{ij}, F_{ij}, H_{ij}, J_{ij} are basically a functions of Q_{ij} where

Q_{ij} can be represented by:

i) Stress- Strain Relations [3]

$$[1-\mu\nabla^2]\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{Bmatrix}^{(n)} = \begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix}^{(n)}\begin{Bmatrix}\varepsilon_{xx}-\alpha^{(n)}(z)T^{(n)}(z)\\ \varepsilon_{yy}-\alpha^{(n)}(z)T^{(n)}(z)\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}$$

$Q^n_{11}= Q^n_{22}= E^n/ 1-\nu^2$

$Q^n_{12}= \nu Q^n_{11}$

$Q^n_{55}= Q^n_{66}= E^n/2(1+\nu)$

Here Q_{ij} is a function on E(z):

$E(z)= E_m+(E_c-E_m)*V(z)$

Where V(z): $V^1(z)= \left(\frac{z-h0}{h1-h0}\right)^k \quad h_0\leq z \leq h_1$

$V^2(z)= 1 \quad h_1 \leq z \leq h_2$

$V^3(z)=\left(\frac{z-h2}{h2-h3}\right)^k \quad h_2\leq z \leq h_3$

Since we are also finding it under the thermal environment we also have things which alter the Matrix constants with respect to temperature.

The values are $A_{ij}^T, B_{ij}^{T,} D_{ij}^T, C_{ij}^T, F_{ij}^T, H_{ij}^T$

$$A^T_{ij}, B^T_{ij}, D^T_{ij}, C^T_{ij}, F^T_{ij}, H^T_{ij} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \frac{\alpha^{(n)}(z,T)E^{(n)}(z,T)\Delta T}{1-\nu}\,dz \quad \text{for i,j=1,2}$$

We use Hamilton's principle to find the equations of Motion of FGM sandwich nanoplates.

δ ∫(U-T)dt=0 with limits t1 to t2;

Where U =Strain Energy and T= Kinetic energy;

i)U=U□+U□

$$U_T = \frac{1}{2}\int_V [\sigma^n_{xxxx} + \sigma^n_{yy}\epsilon_{yy} + \sigma^n_{xy}\gamma_{xy} + \sigma^n_{yz}\gamma_{yz} + \sigma^n_{xz}\gamma_{xz}\, dV$$

$$U_T = \frac{1}{2}\int_V [\sigma^{(T)n}_{xx} d_{xx} + 2\sigma^{(T)n}_{xy} d_{xy} + \sigma^{(T)n}_{yy} d_{yy}\, dV$$

ii) T=

$$T = \frac{1}{2}\int_0^L \int_V \rho(z,T)((\frac{du}{dt})^2 + (\frac{dv}{dt})^2 + (\frac{dw}{dt})^2)\, dA dx$$

These equations of motion will be useful in finding the values which are required to find the values in the stiffness and mass matrices. With the values obtained we will be able to incorporate the values in the code to get the plots and values for different non local parameter values.

Now, so then we have all the required values we now go for the eigenvalue analysis to find the frequencies.

CHAPTER 5

5. MATLAB CODE:

```
clear all
clc
%input paramter
P=1; %volume fraction index
Nu = 0.3;
rho0 = 1;
E0=10^9;
Em= 70*10^9;
Ec= 151*10^9;
mrho=2702;
crho=3000;
m = 1;
n = 1;
h = 1;
% t = 1;
mu=0; %Non-local parameter
c=2;
a=10*h;
b=a;
syms z
E=Em+(Ec-Em)*(0.5+z/h)^P;
D0=Ec*h^3/(12*(1-Nu^2));
K0=0;
K1=0;
k0=(K0*D0)/a^4;
k1=(K1*D0)/a^2;
syms x y
lambda = (m*pi)/a;
Beta = (n*pi)/b;
L=1+mu*(lambda^2+Beta^2);
alphac=3.27*10^-6;
alpham=12.8*10^-6;
%%%%
%%%%%Displacement field
g=sin(c*z/h)*cos(c*z/h);
Mu=(-c/h)*cos(c);
```

```
w=g+Mu*z;
h0=-h/2;
h1=-h/6;
h2=h/6;
h3=h/2;
%temperature gradients
delT1 = 0;
delT2 = 0;
delT3 = 0;
R1(z)= mrho +(crho-mrho)*((z-h0)/(h1-h0))^P;
R2= mrho +(crho-mrho)*1;
R3(z)= mrho +(crho-mrho)*((z-h3)/(h2-h3))^P;
Q111= Em/(1-Nu^2);
Q112= E/(1-Nu^2);
Q113= Ec/(1-Nu^2);
Q221= Em/(1-Nu^2);
Q222= E/(1-Nu^2);
Q223= Ec/(1-Nu^2);
Q661= Em/(2*(1+Nu));
Q662= E/(2*(1+Nu));
Q663= Ec/(2*(1+Nu));
Q121= Nu*Q111;
Q122= Nu*Q112;
Q123= Nu*Q113;
Q441= Em/(2*(1+Nu));
Q442= E/(2*(1+Nu));
Q443= Ec/(2*(1+Nu));
%%Matrix coeffecients
A11=(int(Q111,z,-h/2,-h/4) + int(Q112,z,-h/4,h/4) + int(Q113,z,h/4,h/2));
A22=(int(Q221,z,-h/2,-h/4) + int(Q222,z,-h/4,h/4) + int(Q223,z,h/4,h/2));
A66=(int(Q661,z,-h/2,-h/4) + int(Q662,z,-h/4,h/4) + int(Q663,z,h/4,h/2));
A12=(int(Q121,z,-h/2,-h/4) + int(Q122,z,-h/4,h/4) + int(Q123,z,h/4,h/2));
B11=(int(Q111*z,z,-h/2,-h/4) + int(Q112*z,z,-h/4,h/4) + int(Q113*z,z,h/4,h/2));
B22=(int(Q221*z,z,-h/2,-h/4) + int(Q222*z,z,-h/4,h/4) + int(Q223*z,z,h/4,h/2));
B66=(int(Q661*z,z,-h/2,-h/4) + int(Q662*z,z,-h/4,h/4) + int(Q663*z,z,h/4,h/2));
B12=(int(Q121*z,z,-h/2,-h/4) + int(Q122*z,z,-h/4,h/4) + int(Q123*z,z,h/4,h/2));
C11=(int(Q111*w,z,-h/2,-h/4) + int(Q112*w,z,-h/4,h/4) + int(Q113*w,z,h/4,h/2));
C22=(int(Q221*w,z,-h/2,-h/4) + int(Q222*w,z,-h/4,h/4) + int(Q223*w,z,h/4,h/2));
C66=(int(Q661*w,z,-h/2,-h/4) + int(Q662*w,z,-h/4,h/4) + int(Q663*w,z,h/4,h/2));
C12=(int(Q121*w,z,-h/2,-h/4) + int(Q122*w,z,-h/4,h/4) + int(Q123*w,z,h/4,h/2));
D11=(int(Q111*z^2,z,-h/2,-h/4) + int(Q112*z^2,z,-h/4,h/4) +int(Q113*z^2,z,h/4,h/2));
D22=(int(Q221*z^2,z,-h/2,-h/4) + int(Q222*z^2,z,-h/4,h/4) +int(Q223*z^2,z,h/4,h/2));
```

```
D66=(int(Q661*z^2,z,-h/2,-h/4) + int(Q662*z^2,z,-h/4,h/4) +int(Q663*z^2,z,h/4,h/2));
D12=(int(Q121*z^2,z,-h/2,-h/4) + int(Q122*z^2,z,-h/4,h/4) +int(Q123*z^2,z,h/4,h/2));
F11=(int(Q111*z*w,z,-h/2,-h/4) + int(Q112*z*w,z,-h/4,h/4)
+int(Q113*z*w,z,h/4,h/2));
F22=(int(Q221*z*w,z,-h/2,-h/4) + int(Q222*z*w,z,-h/4,h/4)
+int(Q223*z*w,z,h/4,h/2));
F66=(int(Q661*z*w,z,-h/2,-h/4) + int(Q662*z*w,z,-h/4,h/4)
+int(Q663*z*w,z,h/4,h/2));
F12=(int(Q121*z*w,z,-h/2,-h/4) + int(Q122*z*w,z,-h/4,h/4)
+int(Q123*z*w,z,h/4,h/2));
H11=(int(Q111*w^2,z,-h/2,-h/4) + int(Q112*w^2,z,-h/4,h/4)
+int(Q113*w^2,z,h/4,h/2));
H22=(int(Q221*w^2,z,-h/2,-h/4) + int(Q222*w^2,z,-h/4,h/4)
+int(Q223*w^2,z,h/4,h/2));
H66=(int(Q661*w^2,z,-h/2,-h/4) + int(Q662*w^2,z,-h/4,h/4)
+int(Q663*w^2,z,h/4,h/2));
H12=(int(Q121*w^2,z,-h/2,-h/4) + int(Q122*w^2,z,-h/4,h/4)
+int(Q123*w^2,z,h/4,h/2));
J44=(int(Q441*diff(w)^2,z,-h/2,-h/4) + int(Q442*diff(w)^2,z,-h/4,h/4)
+int(Q443*diff(w)^2,z,h/4,h/2));
J66=(int(Q661*diff(w)^2,z,-h/2,-h/4) + int(Q662*diff(w)^2,z,-h/4,h/4)
+int(Q663*diff(w)^2,z,h/4,h/2));
At11=(int((alphac*Em*delT1)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2)/(1-Nu),z,-h/
4,h/4) +int((alphac*Em*delT3)/(1-Nu),z,h/4,h/2));
At22=(int((alphac*Em*delT1)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2)/(1-Nu),z,-h/
4,h/4) +int((alphac*Em*delT3)/(1-Nu),z,h/4,h/2));
Bt11=(int((alphac*Em*delT1*z)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*z)/(1-Nu),
z,-h/4,h/4)+int((alphac*Em*delT3*z)/(1-Nu),z,h/4,h/2));
Bt22=(int((alphac*Em*delT1*z)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*z)/(1-Nu),
z,-h/4,h/4)+int((alphac*Em*delT3*z)/(1-Nu),z,h/4,h/2));
Dt11=(int((alphac*Em*delT1*z^2)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*z^2)/(1-
Nu),z,-h/4,h/4)+int((alphac*Em*delT3*z^2)/(1-Nu),z,h/4,h/2));
Dt22=(int((alphac*Em*delT1*z^2)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*z^2)/(1-
Nu),z,-h/4,h/4)+int((alphac*Em*delT3*z^2)/(1-Nu),z,h/4,h/2));
Ct11=(int((alphac*Em*delT1*w)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*w)/(1-Nu)
,z,-h/4,h/4)+int((alphac*Em*delT3*w)/(1-Nu),z,h/4,h/2));
Ct22=(int((alphac*Em*delT1*w)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*w)/(1-Nu)
,z,-h/4,h/4)+int((alphac*Em*delT3*w)/(1-Nu),z,h/4,h/2));
Ft11=(int((alphac*Em*delT1*z*w)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*z*w)/(1
-Nu),z,-h/4,h/4)+int((alphac*Em*delT3*z*w)/(1-Nu),z,h/4,h/2));
Ft22=(int((alphac*Em*delT1*z*w)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*z*w)/(1
-Nu),z,-h/4,h/4)+int((alphac*Em*delT3*z*w)/(1-Nu),z,h/4,h/2));
```

```
Ht11=(int((alphac*Em*delT1*w^2)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*w^2)/(
1-Nu),z,-h/4,h/4)+int((alphac*Em*delT3*w^2)/(1-Nu),z,h/4,h/2));
Ht22=(int((alphac*Em*delT1*w^2)/(1-Nu),z,-h/2,-h/4)+int((alphac*Ec*delT2*w^2)/(
1-Nu),z,-h/4,h/4)+int((alphac*Em*delT3*w^2)/(1-Nu),z,h/4,h/2));
I0=(int(R1,z,-h/2,-h/4) + int(R2,z,-h/4,h/4) + int(R3,z,h/4,h/2));
I1=(int(z*R1,z,-h/2,-h/4) + int(z*R2,z,-h/4,h/4) + int(z*R3,z,h/4,h/2));
I2=(int(z^2*R1,z,-h/2,-h/4) + int(z^2*R2,z,-h/4,h/4) + int(z^2*R3,z,h/4,h/2));
I3=(int(w*R1,z,-h/2,-h/4) + int(w*R2,z,-h/4,h/4) + int(w*R3,z,h/4,h/2));
I4=(int(z*w*R1,z,-h/2,-h/4) + int(z*w*R2,z,-h/4,h/4) + int(z*w*R3,z,h/4,h/2));
I5=(int(w^2*R1,z,-h/2,-h/4) + int(w^2*R2,z,-h/4,h/4) + int(w^2*R3,z,h/4,h/2));
%%%%
[ty1,ty2,ty3,ty4]=deal(0);
for m=1:1
  for n=1:1
     if bitget(m,1)&&bitget(n,1)
        lambda=m*pi/a;
        Beta=n*pi/b;
        K=zeros(5,5);
        K(1,1) =K(1,1) +(A11+ At11)*lambda^2 + (A66+At22)*Beta^2;
        K(1,2) =lambda*Beta*(A12+A66);
        K(1,3) =-(B11+Bt11)*lambda^3 - lambda*(Beta)^2*(B12+2*B66+Bt22);
        K(1,4) =(C11+Ct11)*lambda^2 +(C66+Ct22)*Beta^2;
        K(1,5) =lambda*Beta*(C12+C66);
        K(2,2) =(A66+ At11)*lambda^2 + (A22+At22)*Beta^2;
        K(2,5) =(C66+Ct11)*lambda^2 +(C22+Ct22)*Beta^2;
        K(2,3) =-(B22+Bt22)*Beta^3 - Beta*(lambda)^2*(B12+2*B66+Bt11);
        K(2,4) =K(1,5);
        K(3,3)
=(D11+Dt11)*lambda^4+(2*D12+4*D66+Dt11+Dt22)*(lambda^2)*(Beta^2)+(D22+
Dt22)*Beta^4+L*(At11*(lambda)^2+At22*(Beta)^2);
        K(3,4) =-(F11+Ft11)*lambda^3-lambda*(Beta)^2*(F12+2*F66+Ft22);
        K(3,5) =-(F22+Ft22)*Beta^3-Beta*(lambda)^2*(F12+2*F66+Ft11);
        K(4,4) =J44+(H11+Ht11)*lambda^2+(H66+Ht22)*Beta^2;
        K(4,5) =lambda*Beta*(H12+H66);
        K(5,5) =J44+(H66+Ht11)*lambda^2+(H22+Ht22)*Beta^2;
        q0=1;
        Qmn=q0;
        load=[0 0 Qmn 0 0]';
        u=K\load;
        ry1=vpa(u(1)*cos(m*pi*x/a)*sin(n*pi*y/a));
        ry2=vpa(u(2)*sin(m*pi*x/a)*cos(n*pi*y/b));
        ry3=vpa(u(3)*sin(m*pi*x/a)*sin(n*pi*y/b));
```

```
            ry4=vpa(u(4)*sin(m*pi*x/a)*sin(n*pi*y/b));
            ty1=ty1+ry1;
            ty2=ty2+ry2;
            ty3=ty3+ry3;
            ty4=ty4+ry4;
        end
    end
end
U_DEF=ty1;
V_DEF=ty2;
Wb_DEF=ty3;
Ws_DEF=ty4;
w_def=ty3+ty4;
Non_WDef=double(10*Ec*h^3*subs(w_def,{x,y},{a/2,b/2})/(a^4*q0));
m=1; n=1;
K= zeros(5,5);
M= zeros(5,5);
K(1,1) = (A11+ At11)*lambda^2 + (A66+At22)*Beta^2;
K(1,2) = lambda*Beta*(A12+A66);
K(1,3) = -(B11+Bt11)*lambda^3 - lambda*(Beta)^2*(B12+2*B66+Bt22);
K(1,4) = (C11+ Ct11)*lambda^2 + (C66 + Ct22)*Beta^2;
K(1,5) = lambda*Beta*(C12+C66);
K(2,2) = (A66+ At11)*lambda^2 + (A22+At22)*Beta^2;
K(2,3) = -(B22+Bt22)*Beta^3 - Beta*(lambda)^2*(B12+2*B66+Bt11);
K(2,4) = K(1,5);
K(2,5) = (C66+Ct11)*lambda^2 +(C22+Ct22)*Beta^2;
K(3,3) =
(D11+Dt11)*lambda^4+(2*D12+4*D66+Dt11+Dt22)*(lambda^2)*(Beta^2)+(D22+Dt
22)*Beta^4+k0+k1*L*(At11*(lambda)^2+At22*(Beta)^2);
K(3,4) = -(F11+Ft11)*lambda^3-lambda*(Beta)^2*(F12+2*F66+Ft22);
K(3,5) = -(F22+Ft22)*Beta^3-Beta*(lambda)^2*(F12+2*F66+Ft11);
K(4,4) = J44+(H11+Ht11)*lambda^2+(H66+Ht22)*Beta^2;
K(4,5) = lambda*Beta*(H12+H66);
K(5,5) = J44+(H66+Ht11)*lambda^2+(H22+Ht22)*Beta^2;
M(1,1) = L*I0;
M(1,3) = -lambda*L*I1;
M(1,4) = L*I3;
M(1,5) = L*I3;
M(2,2) = L*I0;
M(2,3) = -Beta*L*I1;
M(3,3) = (L*I0)-(L*I2*(lambda^2+Beta^2));
M(3,4) = -lambda*L*I4;
```

```
M(3,5) = -Beta*L*I4;
M(4,4) = L*I5;
M(5,5) = L*I5;
M(1,2)=0; M(1,4)=0; M(1,5)=0; M(4,5)=0;
omn = eig(K,M);
frqm = abs(sqrt(omn));
frq1 = (frqm*(a^2/h)*sqrt(rho0/E0));
frq=(10*frqm*sqrt(mrho/Em));
disp(min(frq1));
c = 2.5;
% g = (c*z)/(h*(((c^2*z^2))/h^2)+1);
% Mu = -(8*c*h^2)/(c^3*h^3+8*h^3);
g=sin(c*z/h)*cos(c*z/h);
Mu=(-c/h)*cos(c);
f = g + Mu*z;
phix = diff(diff(Ws_DEF, x));
phiy = diff(diff(Wb_DEF, y));
u = U_DEF - z*diff(Wb_DEF, x) + f*phix;
v = V_DEF - z*diff(Wb_DEF, y) + f*phiy;
w = Wb_DEF;
epsx=diff(U_DEF,x)-z*diff(diff(Wb_DEF,x),x)+[g + Mu*z]*(phix);
epsy=diff(V_DEF,y)-z*diff(diff(Wb_DEF,y),y)+[g + Mu*z]*(phiy);
gamaxy=diff(U_DEF,y)+diff(V_DEF,x)-2*z*diff(diff(Wb_DEF,x),y)+[g +
Mu*z]*(diff(phix,y)+diff(phiy,x));
gamaxz= (diff(g, z) + Mu)*diff(Ws_DEF, x);
Gammayz = (diff(g, z) + Mu)*diff(Wb_DEF, y);
% Z = z_values/h;
%
% % Plot
% plot(Gammayz_values_double, Z);
% xlabel('Gamma yz');
% ylabel('Z/h');
%
epsx=diff(U_DEF,x)-Z*diff(diff(Wb_DEF,x))-4*Z^3/(3*h^2)*diff(diff(Ws_DEF,x));
%
epsy=diff(V_DEF,y)-Z*diff(diff(Wb_DEF,y))-4*Z^3/(3*h^2)*diff(diff(Ws_DEF,y));
%
gamaxy=diff(u(2),x)+diff(u(1),y)-2*Z*diff((diff(Wb_DEF,x)),y)-8*Z^3/(3*h^2)*diff((
diff(Ws_DEF,x)),y);
% gamaxz=(1-4*Z^2/h^2)*diff(Ws_DEF,x);
% gamayz=(1-4*Z^2/h^2)*diff(Wb_DEF,y);
strain=[epsx epsy gamaxy gamaxz Gammayz]';
```

```
E(z)=Em+(Ec-Em)*(0.5+z/h)^P;
coef=E(z)/(1-Nu)*[1 Nu 0 0 0; Nu 1 0 0 0; 0 0 (1-Nu)/2 0 0; 0 0 0 (1-Nu)/2 0; 0 0 0 0
(1-Nu)/2];
sigma=coef*strain;
sigmazz=subs(sigma,{x,y},{a/2,0});
sigmayz=h/(q0*a)*sigmazz(5);
tt=-0.5:0.01:0.5;
tt = 0; % Set the z value for calculation
kk1 = double(subs(sigmayz, z, tt));
z_values = linspace(-h/2, h/2);
kk = subs(sigmayz, z, z_values);
% Plot the results
plot(kk, z_values);
xlabel('Sigma yz');
ylabel('Z/h');
title('Stress Sigma yz vs. Z/h');
grid on;
```

CHAPTER 6

6. RESULTS AND DISCUSSION:

$$\overline{w}(z)=\frac{10\,hE_0}{a^2q_0}w\left(\frac{a}{2},\frac{b}{2},z\right),\quad \overline{\sigma}_x(z)=\frac{10\,h^2}{a^2\,q_0}\sigma_x\left(\frac{a}{2},\frac{b}{2},z\right),\quad \overline{\tau}_{yz}(z)=\frac{h}{a}\,\tau_{yz}\left(\frac{a}{2},0,z\right),\quad \overline{\tau}_{xz}(z)=\frac{h}{a}\,\tau_{xz}\left(0,\frac{b}{2},z\right),$$

$$\overline{\omega}=\omega a^2\Big/h\sqrt{\rho_0/E_0},\quad \overline{\lambda}=N_xa^2\Big/100h^3,\quad K_w=k_0a^4\Big/D_c,\quad K_s=k_1a^2\Big/D_c,\quad D_c=E_ch^3\Big/\left(12\left(1-\upsilon^2\right)\right),$$

$$E_0=1\;GPa,\quad \rho_0=1\;kg\Big/m^3$$

TABLE NO:1			
k=1			
a/h	**μ^2**	Paper	My Results
10	0	0.093	0.0829
	1	0.085	0.0754
	2	0.0787	0.0728
	3	0.0737	0.0709
	4	0.0695	0.0695
	5	0.0659	0.0682

TABLE NO:2					
		k=1		k=5	
		Sobhy	Present	Sobhy	Present
a/h	**μ**				
5	0	3.0186	3.3595	2.42443	2.665
	0.5	2.94677	2.9219	2.36674	2.3328
	1	2.75859	2.5731	2.2156	2.1523
	1.5	2.5119	2.3232	2.01747	1.9512
	2	2.25648	2.1326	1.81232	1.7976
10	0	0.8225	0.8527	0.66485	0.658

	0.5	0.80292	0.7888	0.64903	0.6255
	1	0.75165	0.7536	0.60758	0.5969
	1.5	0.68443	0.7225	0.55325	0.5716
	2	0.61384	0.6446	0.49669	0.529
20	0	0.21083	0.2219	0.17077	0.1645
	0.5	0.20851	0.2195	0.16671	0.1521
	1	0.19267	0.2072	0.15606	0.1407
	1.5	0.17544	0.1799	0.1421	0.1375
	2	0.1576	0.1683	0.12765	0.1353

TABLE NO:3						
	Paper	Present	Paper	Present	Paper	Present
μ	(1-1-1)		(1-2-1)		(1-2-1)	
0	6.1093	6.4848	6.3821	6.3802	6.2465	6.677
0.5	5.804	6.1867	6.0602	6.121	5.9334	6.27
1	5.5362	5.6293	5.7776	6.006	5.6588	5.9018
1.5	5.2985	5.4961	5.5268	5.7743	5.415	5.8649
2	5.0856	5.2909	5.302	5.6627	5.1966	5.6536

TABLE NO:4						
a/h	μ	k	Kw	Ks	FGM 1-1-1	FGM 1-2-1
5	0	0	0	0	1.213	1.5672
			10	10	1.331	1.5932
		2	0	0	1.1974	1.5468
			10	10	1.2186	1.5812
10		0	0	0	1.1821	1.5274
			10	10	1.3018	1.5576

		2	0	0	1.1681	1.5119
			10	10	1.188	1.5455
100		0	0	0	1.1725	1.5149
			10	10	1.392	1.5499
		2	0	0	1.1389	1.5009
			10	10	1.1794	1.5343
5	0.1	0	0	0	1.5128	1.5088
			10	10	1.5452	1.5386
		2	0	0	1.5064	1.4891
			10	10	1.5399	1.5222
10		0	0	0	1.5126	1.5126
			10	10	1.5424	1.5424
		2	0	0	1.5149	1.4972
			10	10	1.5486	1.5305
100		0	0	0	1.5148	1.5148
			10	10	1.5447	1.5447
		2	0	0	1.5185	1.5007
			10	10	1.5523	1.5341

TABLE NO:5					
a/h	P	σ_x		σ_xy	
		Paper	Present	Paper	Present
10	1	1.5062	1.4952	0.6081	0.64
	2	1.4147	1.4178	0.5421	0.5267
	4	1.1985	0.9366	0.5666	0.5061

Stresses vs z/h Graphs:

σ_x vs z/h

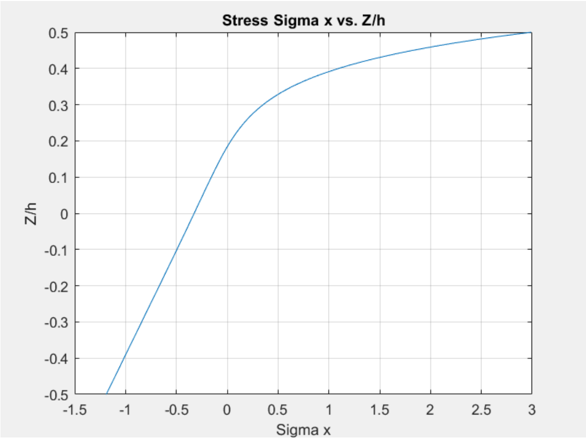


Figure 1.4 σ_x vs z/h

σ_{xy} vs z/h

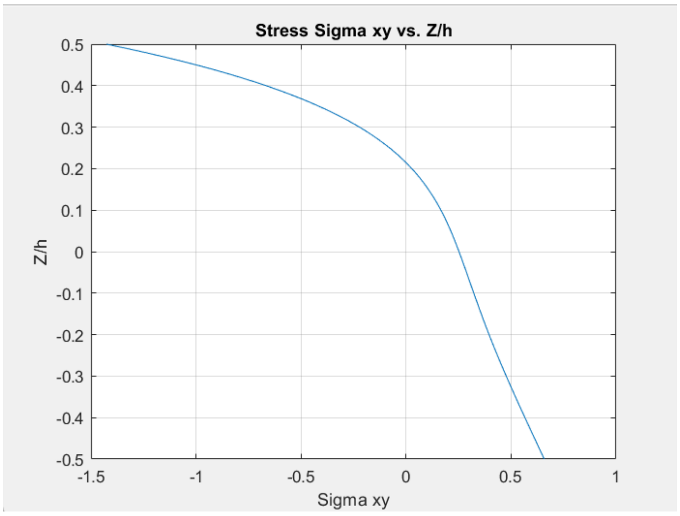


Figure 1.5 σ_{xy} vs z/h

σ_{yz} vs z/h

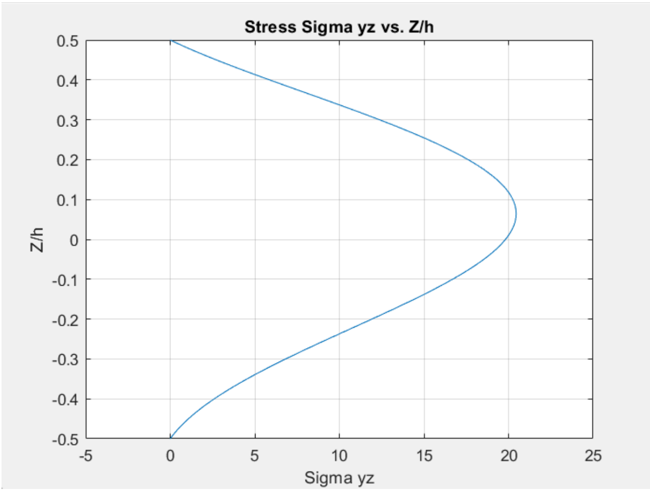


Figure 1.6 σ_{yz} vs z/h

CHAPTER 7

7. CONCLUSION AND SUMMARY

Ultimately, starting with an examination of research paper reviews and dissecting the concepts they contained laid a strong groundwork for crafting mathematical formulations. This served as an invaluable direction as I embarked on the Matlab coding phase, resulting in error-free code and outputs closely aligned with test values. However, the lengthy compilation process highlighted the necessity for time reduction, paving the path to streamline the code for enhanced ease of use and faster compilation.This study presents a novel analytical methodology aimed at examining the dynamic behavior of functionally graded sandwich nanoplates under free vibration conditions. The study assumes a continuous variation of material properties across the thickness of the functionally graded layers, following a straightforward power-law distribution. Various configurations of these sandwich nanoplates are explored within the analysis. To compute the natural frequencies of the sandwich nanoplates, the study integrates Hamilton's principle with Navier's solution. This approach adopts a generalized higher-order shear deformation plate theory, effectively incorporating the influence of thermal loading through strain energies. Furthermore, the paper proposes a precise solution for the nonlinear temperature distribution along the thickness of the sandwich nanoplate. This solution accounts for factors such as thermal conductivity, power-law index variations, and the specific arrangement of the sandwich layers. The investigation underscores the significant impact of temperature distribution, power-law index alterations, nonlocal parameters, and plate geometry on the dimensionless frequency of functionally graded sandwich nanoplates.

Additionally, the study introduces a straightforward higher-order shear deformation theory tailored to analyze the bending and dynamic characteristics of functionally graded plates. This innovative theory directly integrates shear deformation effects, obviating the need for cumbersome shear correction factors. Comparative assessments suggest that this novel approach yields results on par with established higher-order shear deformation theories.

CHAPTER 8

8. FUTURE WORK

As the present research culminates, it offers the foundational code essential for conducting vibration analyses, marking a significant milestone in the exploration of functionally graded materials (FGMs) and sandwich plate structures. Looking ahead, this study presents promising avenues for further investigation and expansion. One avenue involves extending the developed code to accommodate sandwich plates with varying degrees of porosity, incorporating diverse porosity models to capture the complex interplay between material composition and structural behavior. Moreover, the scope of the research can be broadened by exploring an array of FGM plate configurations, including but not limited to (1-1-1), (1-2-1), (2-1-1), and (1-0-1) designs, to elucidate their dimensionless frequency characteristics. Additionally, the comparative analysis can be enriched by incorporating different theoretical frameworks, enabling a comprehensive evaluation of results and facilitating a parametric study to delve deeper into the nuanced influences of various parameters on the vibrational response of FGM and sandwich plate structures. Through these extensions and refinements, the research endeavors to offer valuable insights into the dynamic behavior of these advanced materials and structures, paving the way for advancements in engineering design and innovation.

References

[1] https://www.researchgate.net/figure/Local-theory-of-classical-continuum-mechanics-and-nonlocal-continuum-mechanics-theory_fig1_349158325

[2] https://www.researchgate.net/figure/Color-online-Undeformed-and-deformed-geometries-of-an-edge-of-a-plate-in-various-plate_fig2_259176061.

[3] Ahmed Amine Daikh1 , Ahmed Drai 2, Ismail Bensaid3 ,Mohammed Sid Ahmed Houari 4,5 and Abdelouahed Tounsi 5,6, On vibration of functionally graded sandwich nanoplates in the thermal environment.

[4] Eringen, A. Cemal. "Nonlocal polar elastic continua." *International journal of engineering science* 10.1 (1972): 1-16.

[5] Zare, Mojtaba, Reza Nazemnezhad, and Shahrokh Hosseini-Hashemi. "Natural frequency analysis of functionally graded rectangular nanoplates with different boundary conditions via an analytical method." *Meccanica* 50 (2015): 2391-2408.

[6] Daikh, Ahmed Amine, et al. "On vibration of functionally graded sandwich nanoplates in the thermal environment." *Journal of Sandwich Structures & Materials* 23.6 (2021): 2217-2244.

[7] Nguyen, Ngoc-Tuan, et al. "An efficient computational approach for size-dependent analysis of functionally graded nanoplates." *Computer Methods in Applied Mechanics and Engineering* 297 (2015): 191-218.

[8] Nami, Mohammad Rahim, and Maziar Janghorban. "Resonance behavior of FG rectangular micro/nano plate based on nonlocal elasticity theory and strain gradient theory with one gradient constant." *Composite Structures* 111 (2014): 349-353.

[9] Akavci, S. S. "Mechanical behavior of functionally graded sandwich plates on elastic foundation." *Composites Part B: Engineering* 96 (2016): 136-152.

[10] Sarangan, Suganyadevi, and B. N. Singh. "Higher-order closed-form solution for the analysis of laminated composite and sandwich plates based on new shear deformation theories." *Composite Structures* 138 (2016): 391-403.

[11] Thai, Huu-Tai, and Seung-Eock Kim. "A simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded plates." *Composite Structures* 96 (2013): 165-173.

[12] Natarajan, Sundararajan, et al. "Size-dependent free flexural vibration behavior of functionally graded nanoplates." *Computational Materials Science* 65 (2012): 74-80.