Distribution assumptions

GARCH MODELS IN PYTHON

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Why make assumptions

- Volatility is not directly observable
- GARCH model use residuals as volatility shocks

$$r_t = \mu_t + \epsilon_t$$

- Volatility is related to the residuals:

$$\epsilon_t = \sigma_t * \zeta(WhiteNoise)$$

Standardized residuals

- Residual = predicted return - mean return

$$residuals = \epsilon_t = r_t - \mu_t$$

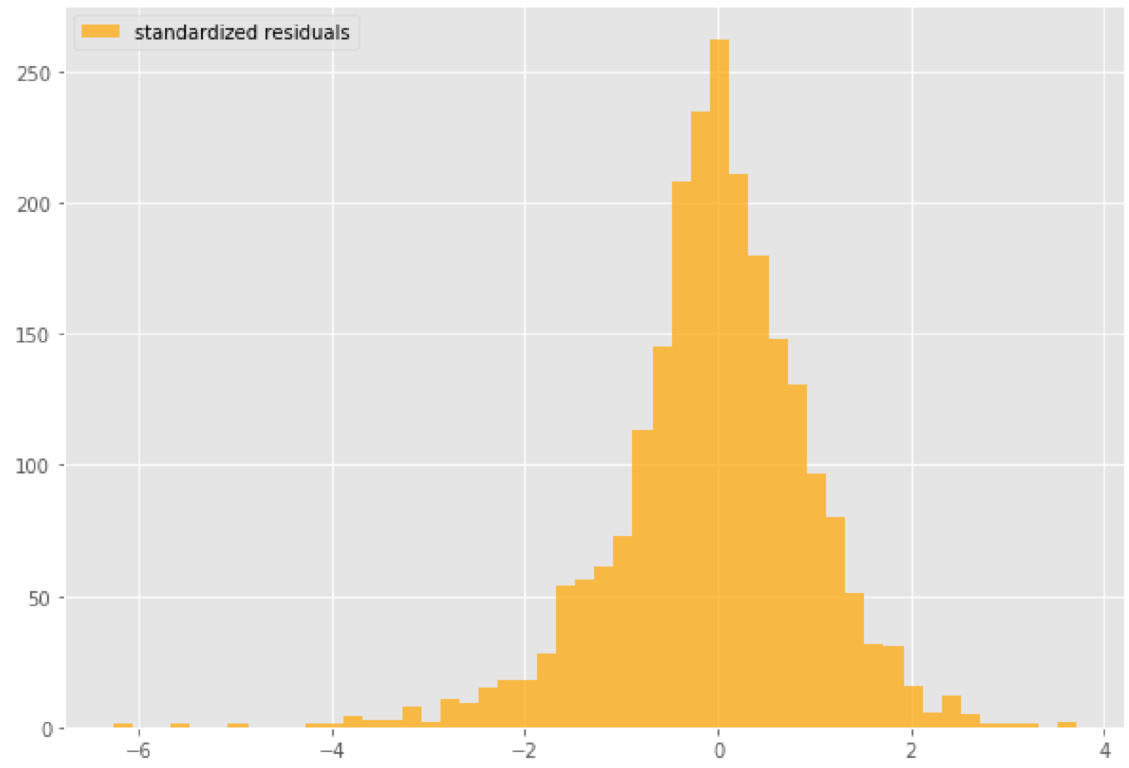
- Standardized residual = residual / return volatility

$$std\, Resid = \frac{\epsilon_t}{\sigma_t}$$

Residuals in GARCH

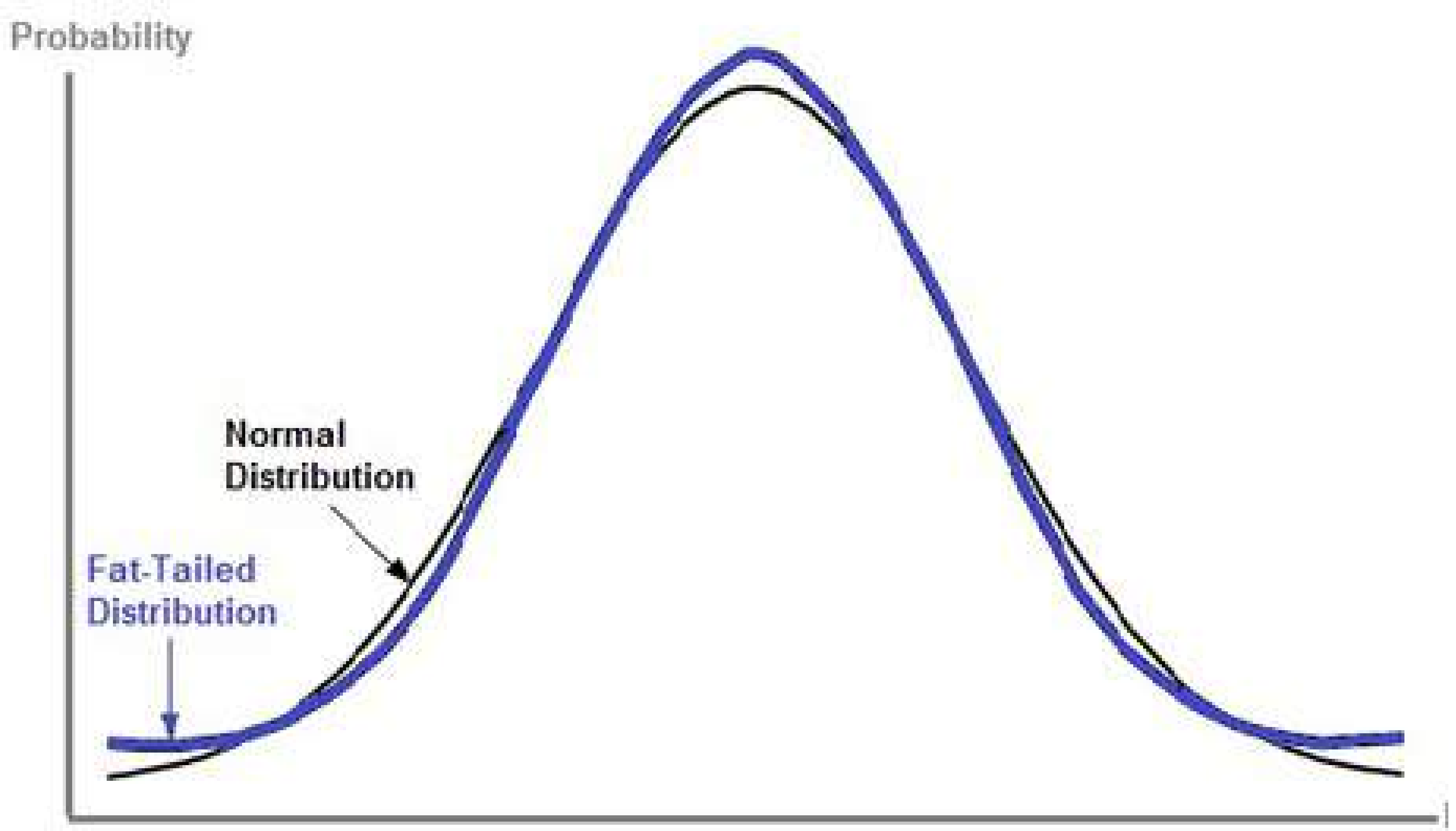
```
gm_std_resid = gm_result.resid / gm_result.conditional_volatility
```

```
plt.hist(gm_std_resid, facecolor = 'orange',label = 'standardized residuals')
```



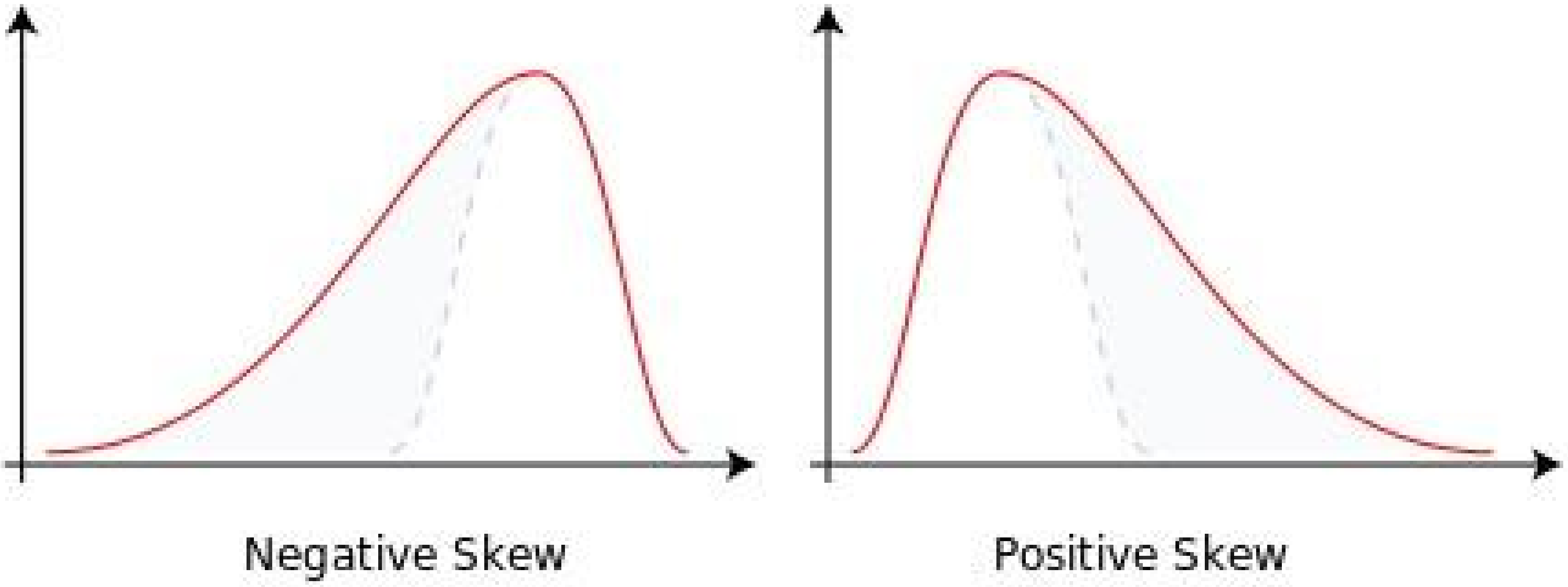
Fat tails

- Higher probability to observe large (positive or negative) returns than under a normal distribution

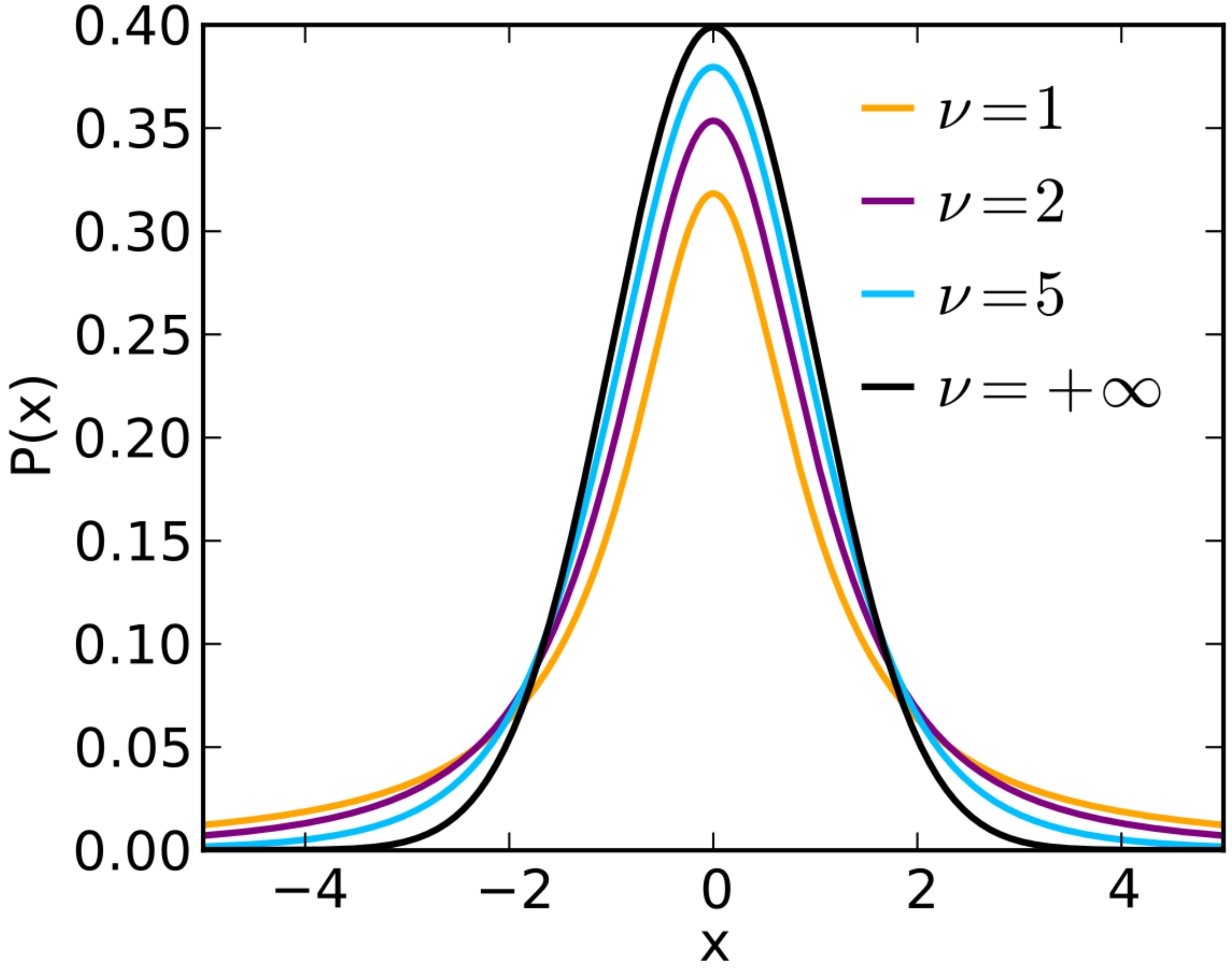


Skewness

- Measure of asymmetry of a probability distribution



Student's t-distribution



ν parameter of a Student's t-distribution indicates its shape

GARCH with t-distribution

```
arch_model(my_data, p = 1, q = 1,
           mean = 'constant', vol = 'GARCH',
           dist = 't')
```

```
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             4.9249      0.507      9.709  2.766e-22 [  3.931,  5.919]
========================================================================
```

GARCH with skewed t-distribution

```
arch_model(my_data, p = 1, q = 1,
           mean = 'constant', vol = 'GARCH',
           dist = 'skewt')
```

```
                              Distribution
========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
------------------------------------------------------------------------
nu             5.2437      0.575      9.118  7.681e-20   [  4.117,  6.371]
lambda        -0.0822  2.541e-02     -3.235  1.216e-03 [ -0.132,-3.241e-02]
========================================================================
```

Let's practice!

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Mean model specifications

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Constant mean by default

- `constant` mean: generally works well with most financial return data

```
arch_model(my_data, p = 1, q = 1,
           mean = 'constant', vol = 'GARCH')
```

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                 Return   R-squared:                      -0.001
Mean Model:             Constant Mean   Adj. R-squared:                 -0.001
Vol Model:                      GARCH   Log-Likelihood:               -2771.96
Distribution:                  Normal   AIC:                           5551.93
Method:            Maximum Likelihood   BIC:                           5574.95
                                        No. Observations:                 2336
Date:                Fri, Dec 20 2019   Df Residuals:                     2332
Time:                        05:26:46   Df Model:                            4
                               Mean Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
mu             0.0772  1.445e-02      5.345  9.031e-08 [4.892e-02,  0.106]
```

Zero mean assumption

- `zero` mean: use when the mean has been modeled separately

```
arch_model(my_data, p = 1, q = 1,
           mean = 'zero', vol = 'GARCH')
```

```
                       Zero Mean - GARCH Model Results
==============================================================================
Dep. Variable:                    Return   R-squared:                       0.000
Mean Model:                    Zero Mean   Adj. R-squared:                  0.000
Vol Model:                         GARCH   Log-Likelihood:               -2786.65
Distribution:                     Normal   AIC:                           5579.30
Method:               Maximum Likelihood   BIC:                           5596.57
                                           No. Observations:                 2336
Date:                   Fri, Dec 20 2019   Df Residuals:                     2333
Time:                           05:36:28   Df Model:                            3
```

Autoregressive mean

- `AR` mean: model the mean as an autoregressive (AR) process

```
arch_model(my_data, p = 1, q = 1,
          mean = 'AR', lags = 1, vol = 'GARCH')
```

```
                           AR - GARCH Model Results
==================================================================================
Dep. Variable:                        Return   R-squared:                    0.001
Mean Model:                               AR   Adj. R-squared:               0.000
Vol Model:                             GARCH   Log-Likelihood:            -2690.07
Distribution:        Standardized Student's t   AIC:                       5392.13
Method:                   Maximum Likelihood   BIC:                       5426.66
                                               No. Observations:             2335
Date:                       Fri, Dec 20 2019   Df Residuals:                 2329
Time:                               05:39:58   Df Model:                        6
                                Mean Model
============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
----------------------------------------------------------------------------
Const          0.0877  1.293e-02      6.783  1.181e-11    [6.234e-02,  0.113]
Return[1]     -0.0541  2.060e-02     -2.625  8.670e-03 [-9.444e-02,-1.369e-02]
```

Let's practice!

GARCH MODELS IN PYTHON

Volatility models for asymmetric shocks

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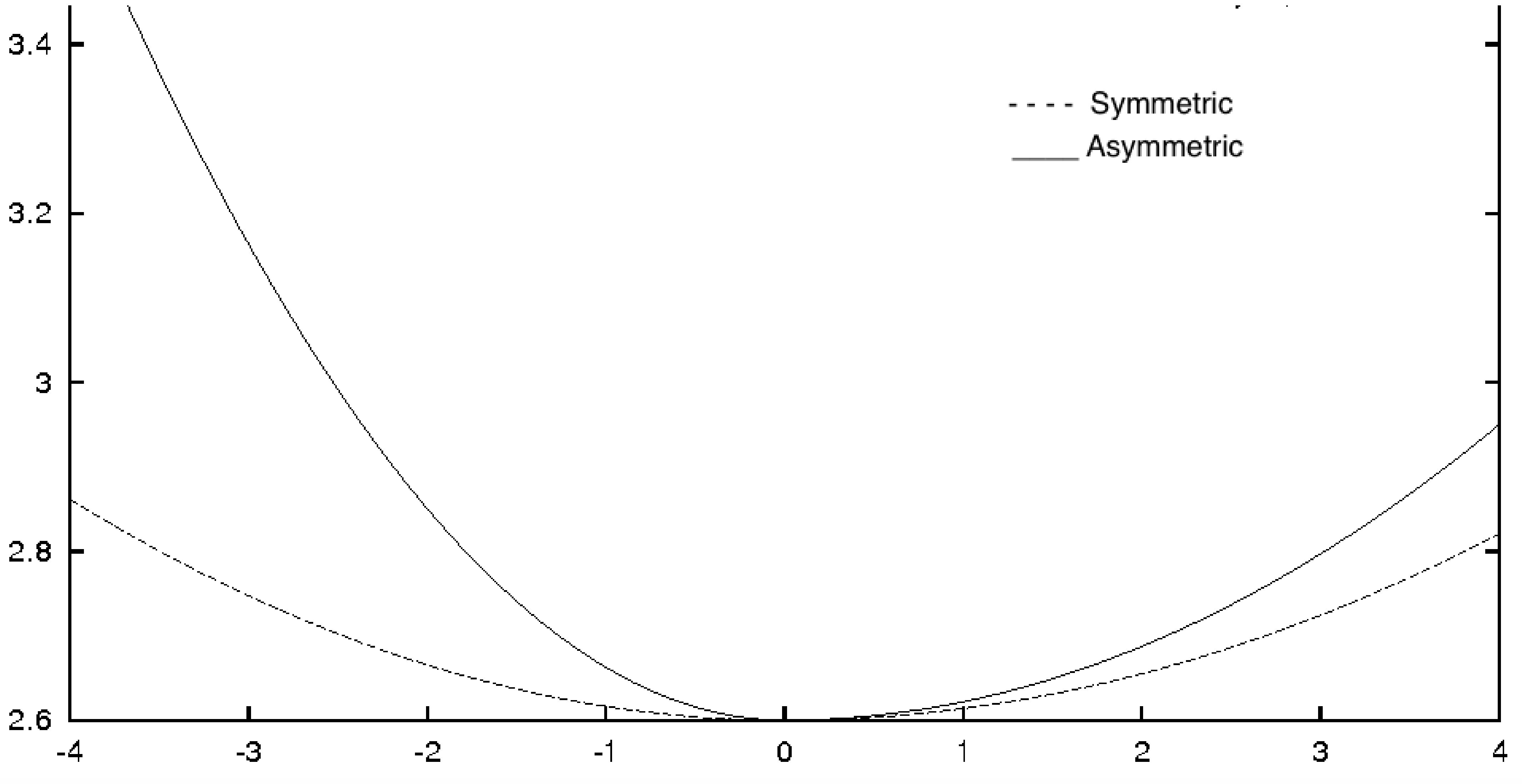
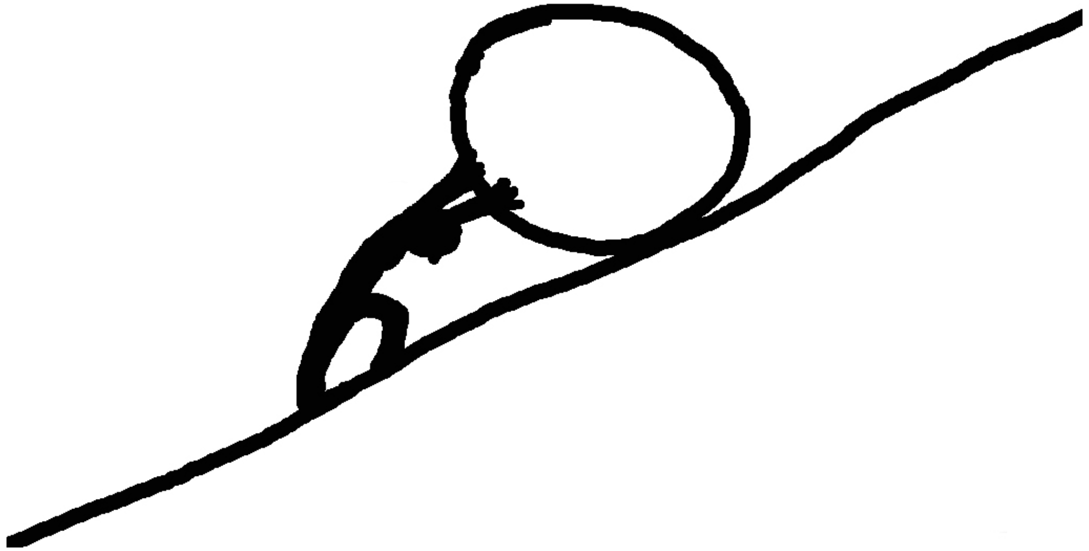
Asymmetric shocks in financial data

News impact curve:



Leverage effect

- Debt-equity Ratio = Debt / Equity
- Stock price goes down, debt-equity ratio goes up
- Riskier!



GJR-GARCH

$$\sigma_t^2 = \omega + \left(\alpha + \gamma I_{t-1}\right)\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

$$I_{t-1} := \begin{cases} 0 & \text{if } r_{t-1} \geq \mu \\ 1 & \text{if } r_{t-1} < \mu \end{cases}$$

GJR-GARCH in Python

```
arch_model(my_data, p = 1, q = 1, o = 1,
           mean = 'constant', vol = 'GARCH')
```

```
                   Constant Mean - GJR-GARCH Model Results
==================================================================================
Dep. Variable:                      Return   R-squared:                     -0.000
Mean Model:                  Constant Mean   Adj. R-squared:                -0.000
Vol Model:                       GJR-GARCH   Log-Likelihood:              -2641.12
Distribution:      Standardized Student's t   AIC:                          5294.23
Method:                 Maximum Likelihood   BIC:                          5328.77
                                             No. Observations:                2336
Date:                     Tue, Dec 10 2019   Df Residuals:                    2330
Time:                             11:19:41   Df Model:                           6
                                Mean Model
==========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
--------------------------------------------------------------------------
mu             0.0554  1.227e-02      4.521  6.163e-06 [3.141e-02,7.949e-02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
---------------------------------------------------------------------------
omega          0.0298  5.609e-03      5.317  1.054e-07  [1.883e-02,4.082e-02]
alpha[1]       0.0000  2.338e-02      0.000      1.000 [-4.583e-02,4.583e-02]
gamma[1]       0.3267  4.852e-02      6.733  1.663e-11      [  0.232,  0.422]
beta[1]        0.8121  2.257e-02     35.978 1.835e-283      [  0.768,  0.856]
```

EGARCH

- A popular option to model asymmetric shocks
- Exponential GARCH
- Add a conditional component to model the asymmetry in shocks similar to the GJR-GARCH
- No non-negative constraints on alpha, beta so it runs faster

EGARCH in Python

```
arch_model(my_data, p = 1, q = 1, o = 1,
           mean = 'constant', vol = 'EGARCH')
```

```
                     Constant Mean - EGARCH Model Results
==========================================================================================
Dep. Variable:                      Return   R-squared:                          -0.000
Mean Model:                  Constant Mean   Adj. R-squared:                     -0.000
Vol Model:                          EGARCH   Log-Likelihood:                    -2628.40
Distribution:      Standardized Student's t   AIC:                                5268.79
Method:                 Maximum Likelihood   BIC:                                5303.33
                                             No. Observations:                     2336
Date:                     Tue, Dec 10 2019   Df Residuals:                         2330
Time:                             11:19:42   Df Model:                                6
                               Mean Model
==========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
--------------------------------------------------------------------------
mu             0.0493  9.578e-03      5.146  2.663e-07 [3.051e-02,6.806e-02]
                            Volatility Model
==============================================================================
                 coef    std err          t      P>|t|        95.0% Conf. Int.
------------------------------------------------------------------------------
omega         -0.0202  7.350e-03     -2.743  6.094e-03 [-3.457e-02,-5.753e-03]
alpha[1]       0.1707  2.279e-02      7.490  6.874e-14     [  0.126,  0.215]
gamma[1]      -0.2360  2.598e-02     -9.087  1.019e-19     [ -0.287, -0.185]
beta[1]        0.9547  9.191e-03    103.869      0.000     [  0.937,  0.973]
```

Which model to use

GJR-GARCH or EGARCH?

Which model is better depends on the data

Let's practice!

GARCH MODELS IN PYTHON

GARCH rolling window forecast

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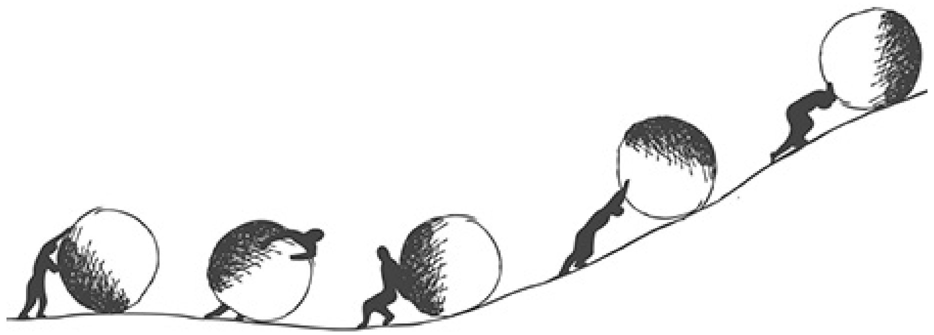
Rolling window for out-of-sample forecast

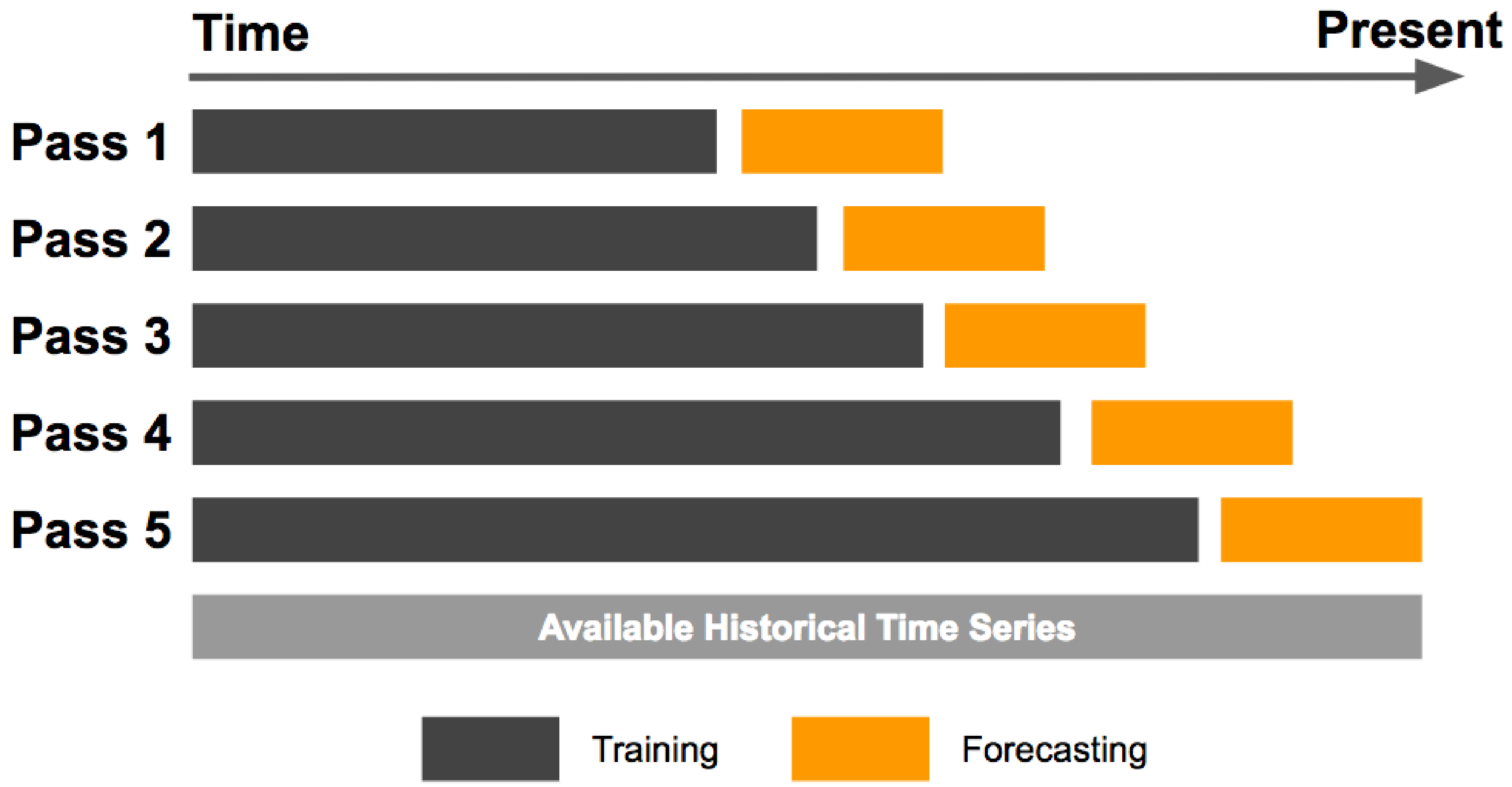
An exciting part of financial modeling: **predict the unknown**

Rolling window forecast: repeatedly perform model fitting and forecast as time rolls forward



Expanding window forecast

Continuously add new data points to the sample



Motivations of rolling window forecast

- Avoid lookback bias
- Less subject to overfitting
- Adapt forecast to new observations

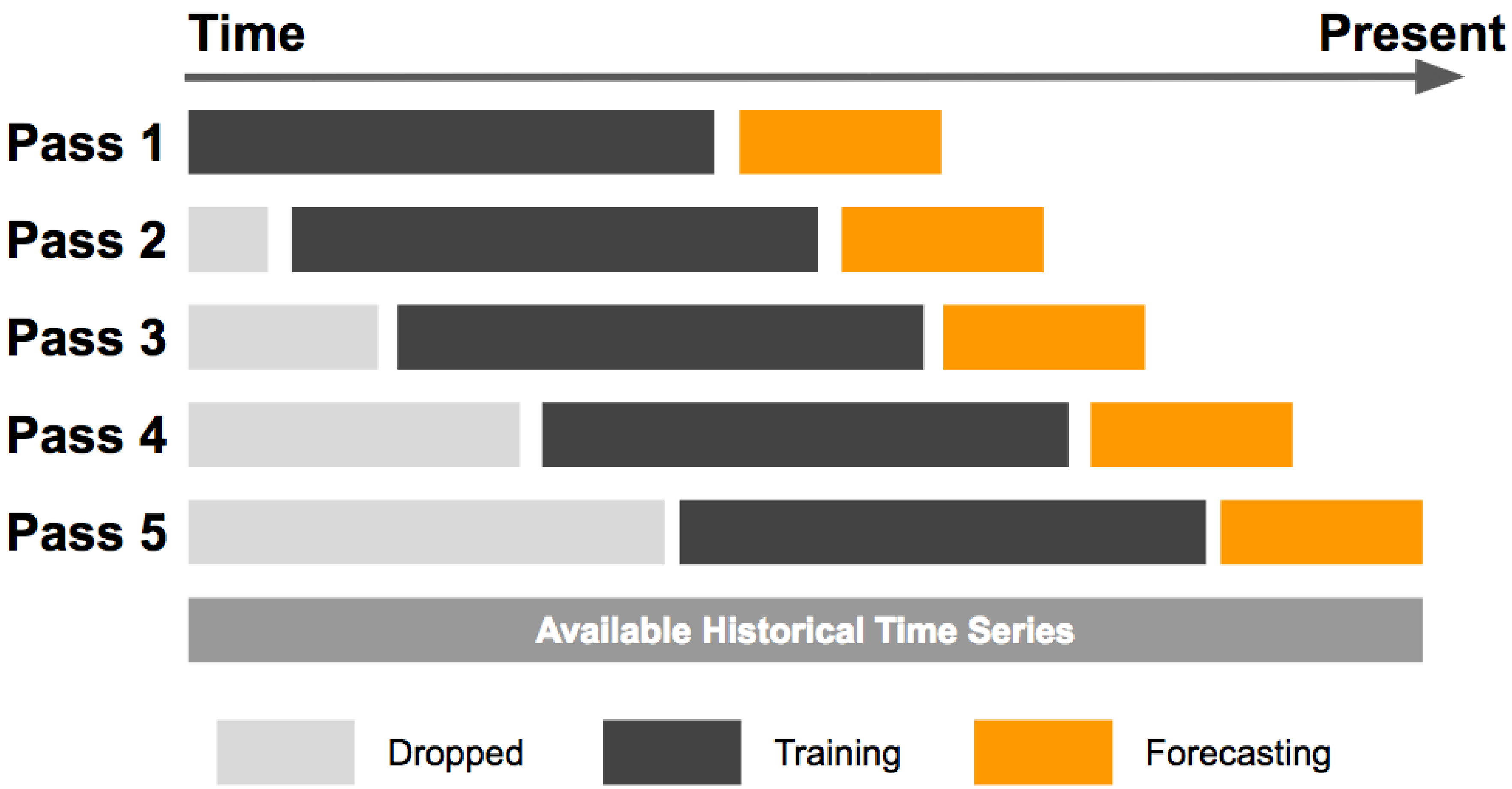
Implement expanding window forecast

Expanding window forecast:

```
for i in range(120):
    gm_result = basic_gm.fit(first_obs = start_loc,
                             last_obs = i + end_loc, disp = 'off')
    temp_result = gm_result.forecast(horizon = 1).variance
```

Fixed rolling window forecast

New data points are added while old ones are dropped from the sample



Implement fixed rolling window forecast

Fixed rolling window forecast:

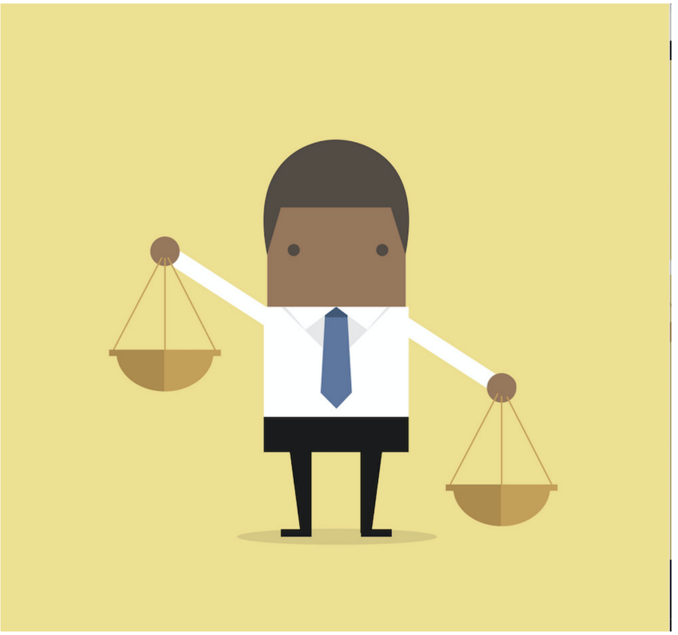
```
for i in range(120):
    # Specify rolling window range for model fitting
    gm_result = basic_gm.fit(first_obs = i + start_loc,
                             last_obs = i + end_loc, disp = 'off')
    temp_result = gm_result.forecast(horizon = 1).variance
```

How to determine window size

Usually determined on a case-by-case basis

- Too wide window size: include obsolete data that may lead to high bias
- Too narrow window size: exclude relevant data that may lead to higher variance

The optimal window size: trade-off to balance bias and variance



Let's practice!

GARCH MODELS IN PYTHON