

Why do we need GARCH models

GARCH MODELS IN PYTHON



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Course overview

GARCH: Generalized AutoRegressive Conditional Heteroskedasticity

- Chapter 1: GARCH Model Fundamentals
- Chapter 2: GARCH Model Configuration
- Chapter 3: Model Performance Evaluation
- Chapter 4: GARCH in Action

What is volatility

- Describe the dispersion of financial asset returns over time
- Often computed as the standard deviation or variance of price returns
- The higher the volatility, the riskier a financial asset



How to compute volatility

- Step 1: Calculate returns as percentage of price changes

$$return = \frac{P_1 - P_0}{P_0}$$

- Step 2: Calculate the sample mean return

$$mean = \frac{\sum_{i=1}^n return_i}{n}$$

- Step 3: Calculate the sample standard deviation

$$volatility = \sqrt{\frac{\sum_{i=1}^n (return_i - mean)^2}{n - 1}} = \sqrt{variance}$$

Compute volatility in Python

Use pandas `pct_change()` method:

```
return_data = price_data.pct_change()
```

Use pandas `std()` method:

```
volatility = return_data.std()
```

Volatility conversion

- Convert to monthly volatility from daily:
(assume 21 trading days in a month)

$$\sigma_{monthly} = \sqrt{21} * \sigma_d$$

- Convert to annual volatility from daily:
(assume 252 trading days in a year)

$$\sigma_{annual} = \sqrt{252} * \sigma_d$$

The challenge of volatility modeling

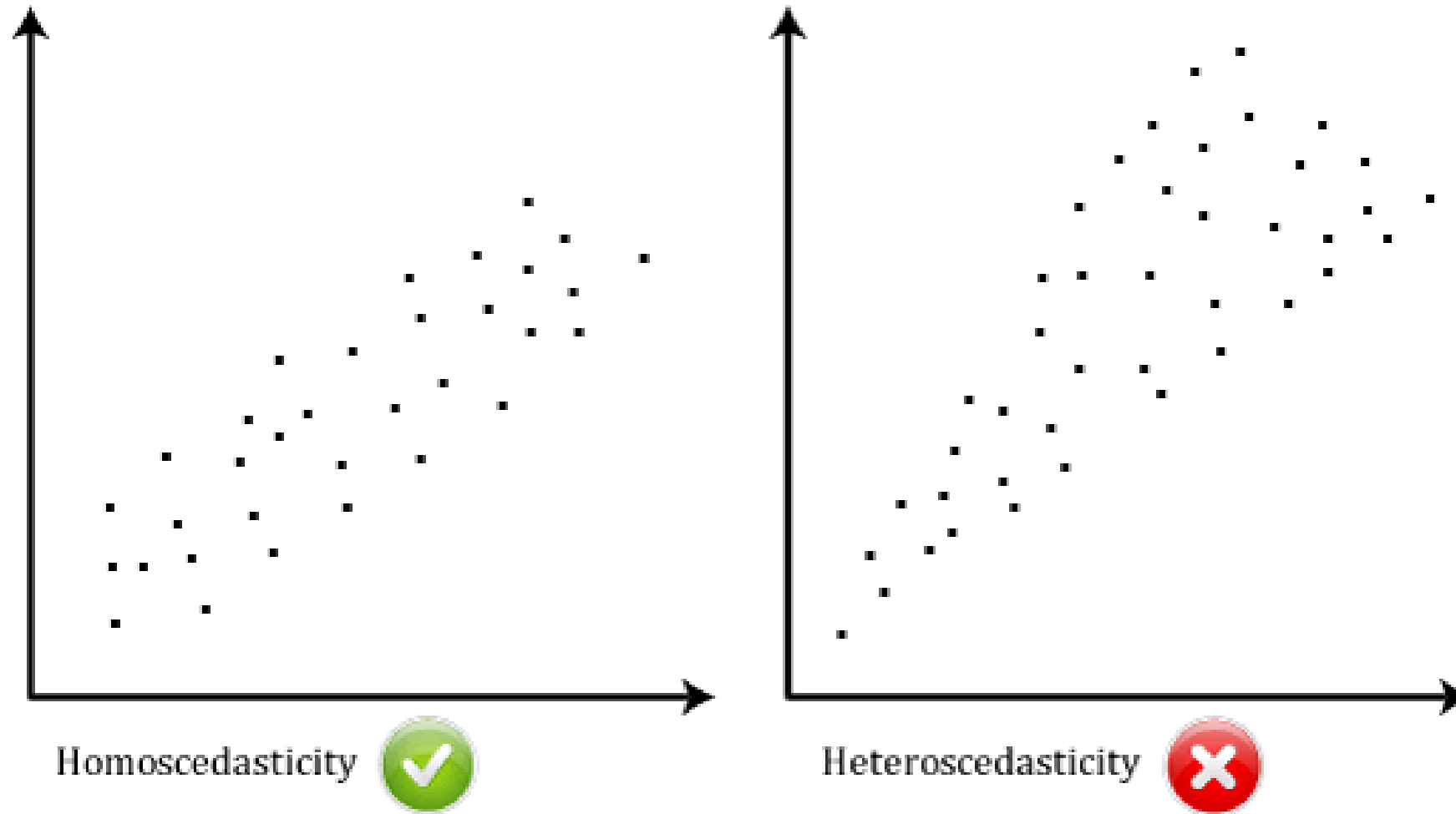
Heteroskedasticity:

- In ancient Greek: "different" (hetero) + "dispersion" (skedasis)
- A time series demonstrates varying volatility systematically over time



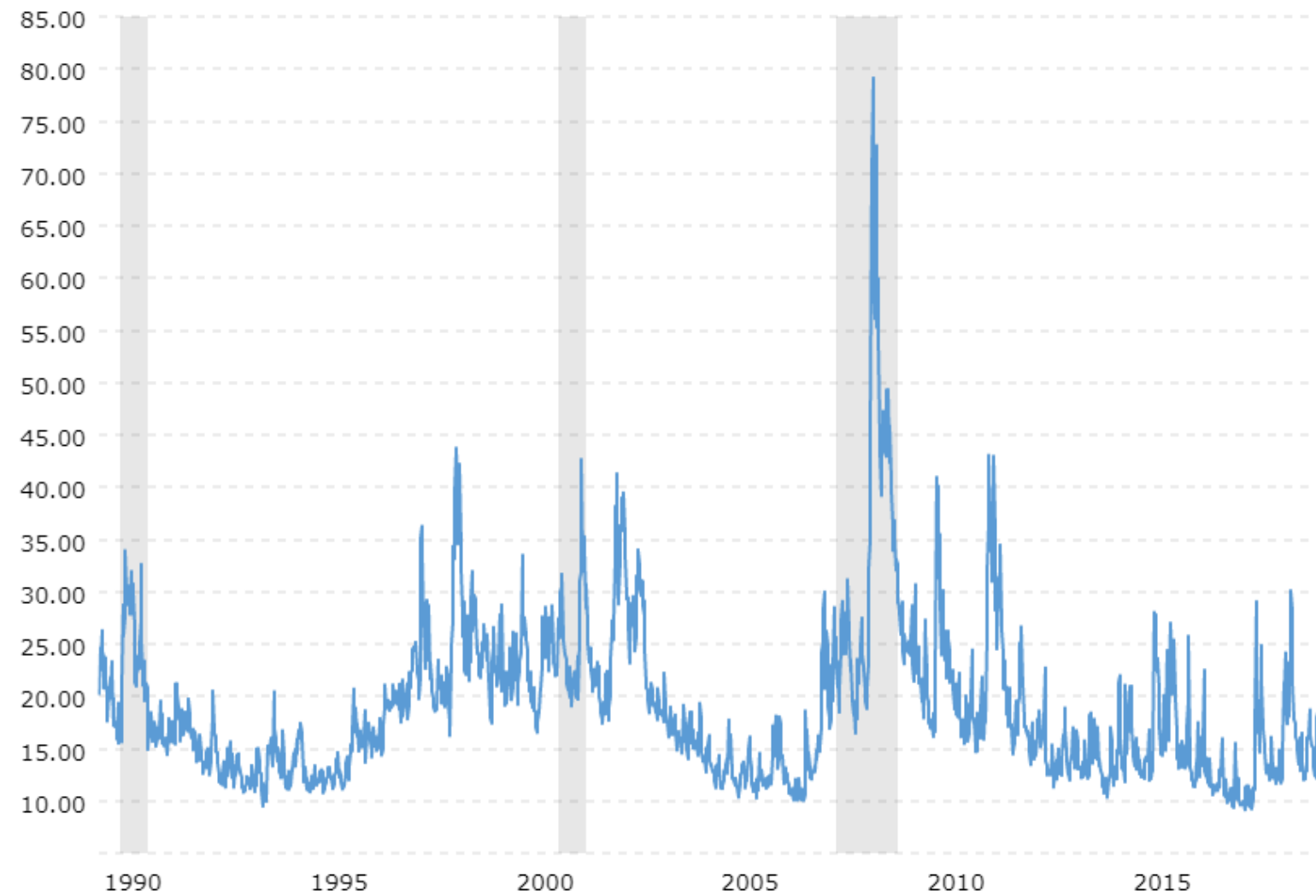
Detect heteroskedasticity

Homoskedasticity vs Heteroskedasticity



Volatility clustering

VIX historical prices:



Let's practice!

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What are ARCH and GARCH

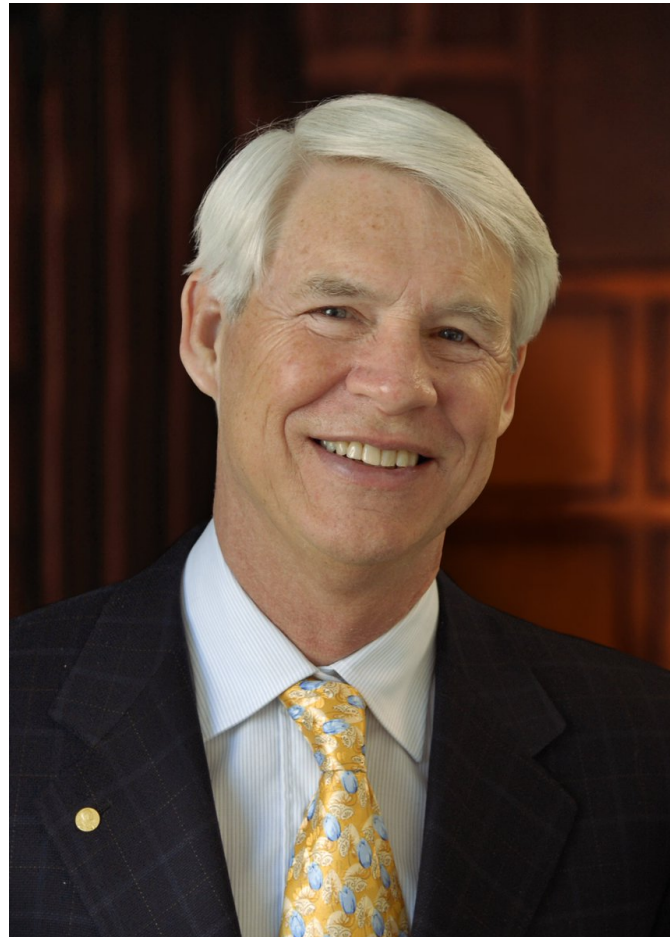
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First came the ARCH

- Auto Regressive Conditional Heteroskedasticity
- Developed by Robert F. Engle (Nobel prize laureate 2003)



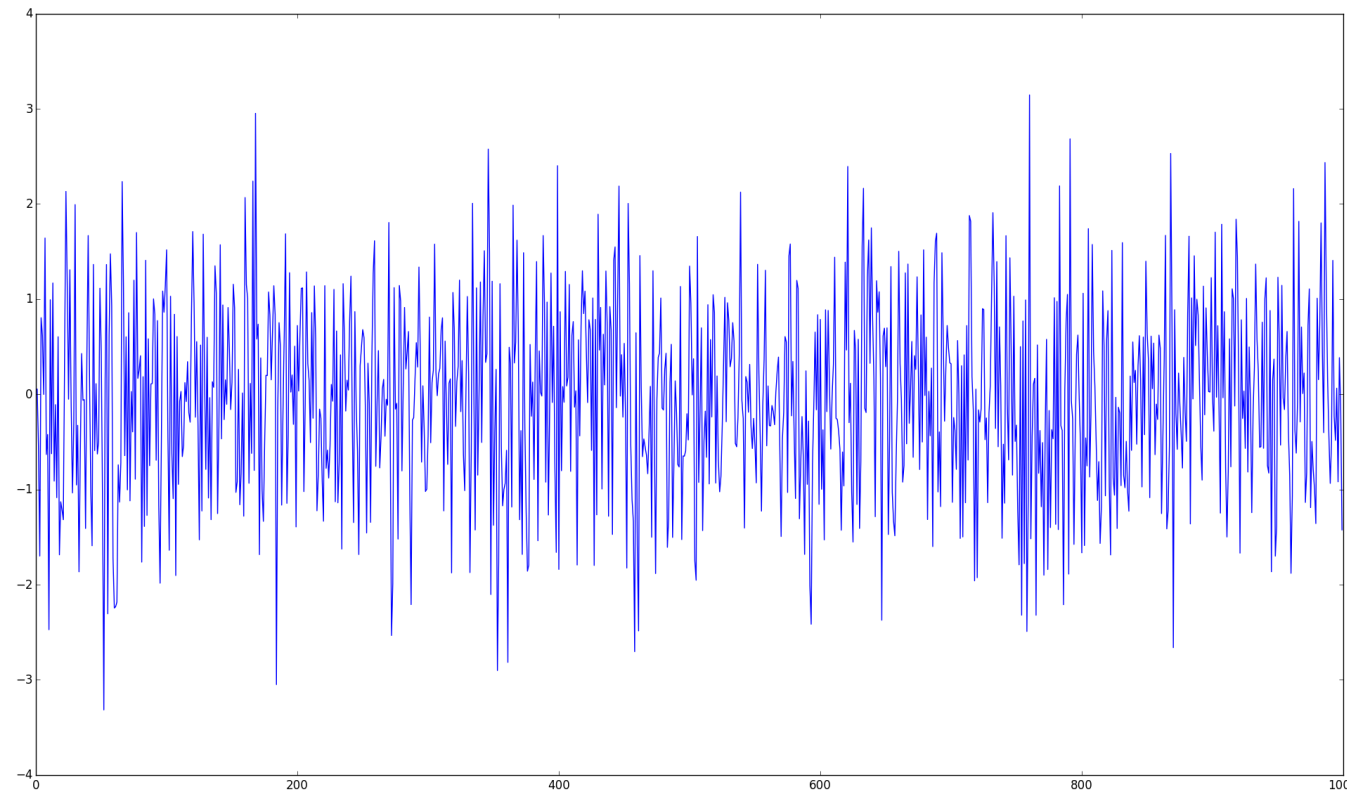
Then came the GARCH

- "Generalized" ARCH
- Developed by Tim Bollerslev (Robert F. Engle's student)



Related statistical terms

White noise (z): Uncorrelated random variables with a zero mean and a finite variance



Residual = predicted value - observed value

Model notations

Expected return:

$$\mu_t = \textit{Expected}[r_t | I(t - 1)]$$

Expected volatility:

$$\sigma^2 = \textit{Expected}[(r_t - \mu_t)^2 | I(t - 1)]$$

Residual (prediction error):

$$r_t = \mu_t + \epsilon_t$$

Volatility is related to the residuals:

$$\epsilon_t = \sigma_t * \zeta(\textit{WhiteNoise})$$

Model equations: ARCH

$$ARCH(p) : \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2$$

$$ARCH(1) : \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2$$

Model equations: GARCH

$$GARCH(p, q) : \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

$$GARCH(1, 1) : \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Model intuition

- Autoregressive: predict future behavior based on past behavior
- Volatility as a weighted average of past information



GARCH(1,1) parameter constraints

To make the GARCH(1,1) process realistic, it requires:

- All parameters are non-negative, so the variance cannot be negative.

$$\omega, \alpha, \beta \geq 0$$

- Model estimations are "mean-reverting" to the long-run variance.

$$\alpha + \beta < 1$$

long-run variance:

$$\omega / (1 - \alpha - \beta)$$

GARCH(1,1) parameter dynamics

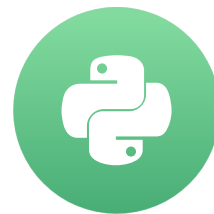
- The larger the α , the bigger the immediate impact of the shock
- The larger the β , the longer the duration of the impact

Let's practice!

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How to implement GARCH models in Python

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Python "arch" package

```
from arch import arch_model
```



¹ Kevin Sheppard. (2019, March 28). bashtage/arch: Release 4.8.1 (Version 4.8.1). Zenodo.
<http://doi.org/10.5281/zenodo.2613877>

Workflow

Develop a GARCH model in three steps:

1. Specify the model
2. Fit the model
3. Make a forecast

Model specification

Model assumptions:

- Distribution: "normal" (default), "t", "skewt"
- Mean model: "constant" (default), "zero", "AR"
- Volatility model: "GARCH" (default), "ARCH", "EGARCH"

```
basic_gm = arch_model(sp_data['Return'], p = 1, q = 1,  
                      mean = 'constant', vol = 'GARCH', dist = 'normal')
```

Model fitting

Display model fitting output after every n iterations:

```
gm_result = gm_model.fit(update_freq = 4)
```

```
Iteration:      4,   Func. Count:      34,   Neg. LLF: 2783.005885607893
Iteration:      8,   Func. Count:      61,   Neg. LLF: 2771.9886612376513
Iteration:     12,   Func. Count:      85,   Neg. LLF: 2771.963828246998
Optimization terminated successfully.   (Exit mode 0)
      Current function value: 2771.9638282462456
      Iterations: 12
      Function evaluations: 85
      Gradient evaluations: 12
```

Turn off the display:

```
gm_result = gm_model.fit(dis = 'off')
```

Fitted results: parameters

Estimated by "maximum likelihood method"

```
print(gm_result.params)
```

```
mu          0.077239
omega       0.039587
alpha[1]    0.167963
beta[1]     0.786467
Name: params, dtype: float64
```

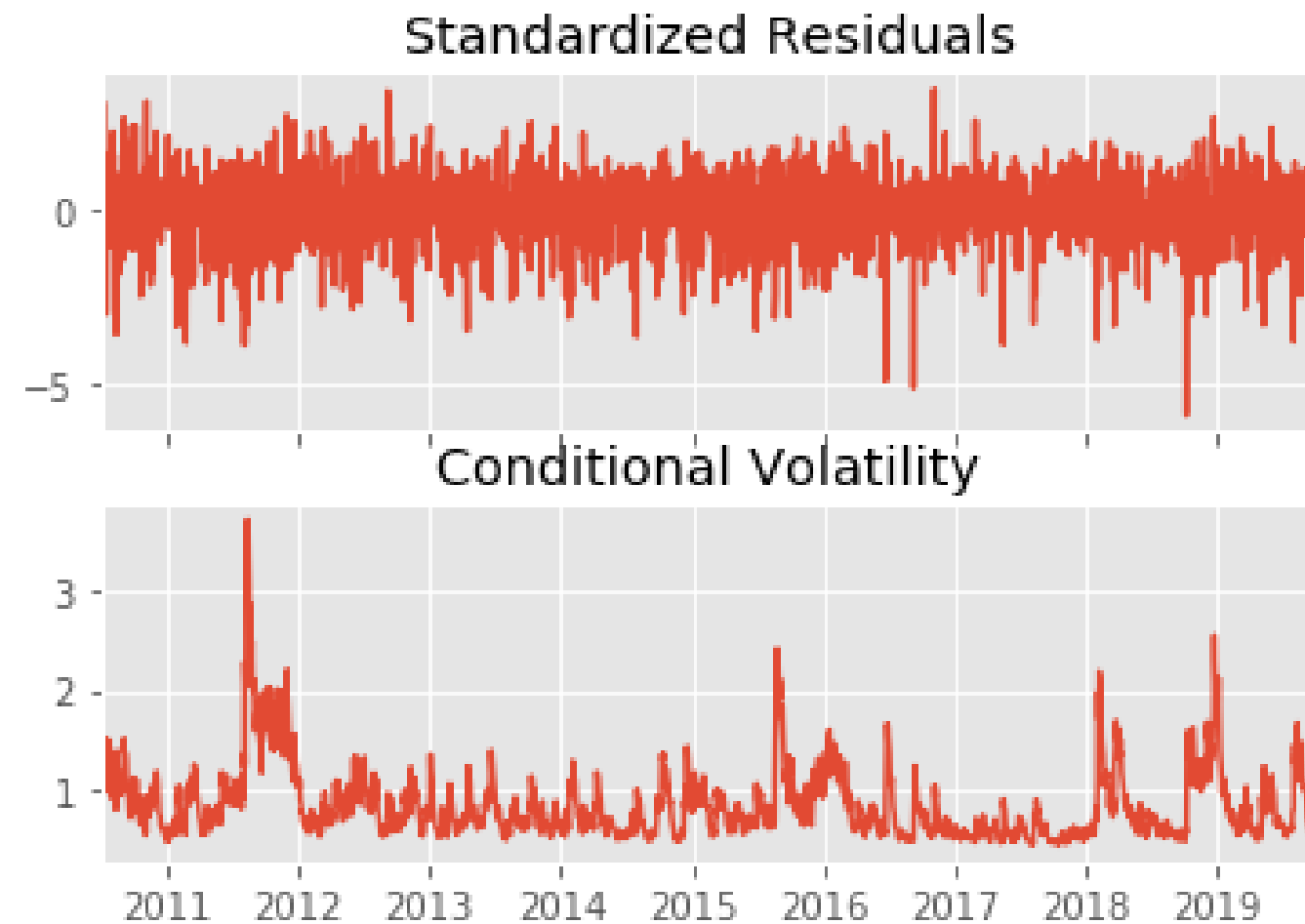
Fitted results: summary

```
print(gm_result.summary())
```

```
=====
                        Constant Mean - GARCH Model Results
=====
Dep. Variable:          Return    R-squared:          -0.001
Mean Model:            Constant Mean  Adj. R-squared:      -0.001
Vol Model:             GARCH        Log-Likelihood:    -2771.96
Distribution:          Normal       AIC:              5551.93
Method:               Maximum Likelihood  BIC:              5574.95
                                     No. Observations:      2336
Date:                 Mon, Dec 02 2019  Df Residuals:       2332
Time:                 12:54:53         Df Model:          4
                        Mean Model
=====
                        coef      std err          t      P>|t|      95.0% Conf. Int.
-----
mu                   0.0772   1.445e-02      5.345   9.031e-08 [4.892e-02, 0.106]
                        Volatility Model
=====
                        coef      std err          t      P>|t|      95.0% Conf. Int.
-----
omega                0.0396   9.181e-03      4.312   1.619e-05 [2.159e-02,5.758e-02]
alpha[1]             0.1680   2.690e-02      6.243   4.284e-10 [ 0.115, 0.221]
beta[1]              0.7865   2.722e-02     28.897   1.303e-183 [ 0.733, 0.840]
=====
```

Fitted results: plots

```
gm_result.plot()
```



Model forecasting

```
# Make 5-period ahead forecast  
gm_forecast = gm_result.forecast(horizon = 5)
```

```
# Print out the last row of variance forecast  
print(gm_forecast.variance[-1:])
```

	h.1	h.2	h.3	h.4	h.5
Date					
2019-10-10	0.994079	0.988366	0.982913	0.977708	0.972741

h.1 in row "2019-10-10": 1-step ahead forecast made using data up to and including that date

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