Assignment 4: Applied data science

ANSWERS:

1. Solution are described in the following sections:
   a. Using the rat example from class, together with R or Python, iterate the associated Markov matrix to obtain the transition probabilities two steps ahead, five steps ahead, 10 steps ahead, and 25 steps ahead:

```
Markov transition matrix when step =  2
       A   B      C
A 0.3125 0.5 0.1875
B 0.2500 0.5 0.2500
C 0.1875 0.5 0.3125
```

```
Markov transition matrix when step =  5
          A   B         C
A 0.2578125 0.5 0.2421875
B 0.2500000 0.5 0.2500000
C 0.2421875 0.5 0.2578125
```

```
Markov transition matrix when step =  10
          A   B         C
A 0.2502441 0.5 0.2497559
B 0.2500000 0.5 0.2500000
C 0.2497559 0.5 0.2502441
```

```
Markov transition matrix when step =  25
     A   B    C
A 0.25 0.5 0.25
B 0.25 0.5 0.25
C 0.25 0.5 0.25
```

   b. Consider a three-state system with the following transition probabilities.

1	0	0
0.25	0.50	0.25
0	0	1

This system is consistent with two absorbing states: Room A and Room C. Using R or Python, iterate this Markov matrix to obtain the transition probabilities two steps ahead, five steps ahead, 10 steps ahead, and 25 steps ahead..

```
Markov transition matrix when step =  2
       A     B      C
A 1.0000 0.000 0.0000
B 0.4375 0.125 0.4375
C 0.0000 0.000 1.0000
```

```
Markov transition matrix when step =  5
          A        B         C
A 1.0000000 0.000000 0.0000000
B 0.4921875 0.015625 0.4921875
C 0.0000000 0.000000 1.0000000
```

```
Markov transition matrix when step =  10
          A            B         C
A 1.0000000 0.0000000000 0.0000000
B 0.4997559 0.0004882812 0.4997559
C 0.0000000 0.0000000000 1.0000000
```

```
Markov transition matrix when step =  25
    A            B   C
A 1.0 0.000000e+00 0.0
B 0.5 1.490116e-08 0.5
C 0.0 0.000000e+00 1.0
```

```
Markov transition matrix when step =  1000
    A             B   C
A 1.0  0.000000e+00 0.0
B 0.5 4.666318e-302 0.5
C 0.0  0.000000e+00 1.0
```

```
Markov transition matrix when step =  1e+05
    A B   C
A 1.0 0 0.0
B 0.5 0 0.5
C 0.0 0 1.0
```

c. Challenging question: Consider the maze presented in class, and add two rooms to the right of Room C, labeling them Room D and Room E. In this situation, treat Room A as an absorbing state. If the rat is in any room other than Rooms A or E, it has probability 0.5 of remaining in that room, probability 0.25 of moving left and probability 0.25 of moving right. For Room E, assume probability 0.5 of remaining in that room, and probability 0.5 of moving left. Write out the matrix of Markov transition probabilities. Iterate this matrix forward as many times as is necessary for you to determine empirically its limit. Based on this limit, what can you say about the evolution of the system if the rat begins in Room C? Is there a general conclusion you can draw:

```
     A    B    C    D    E
A 1.00 0.00 0.00 0.00 0.00
B 0.25 0.50 0.25 0.00 0.00
C 0.00 0.25 0.50 0.25 0.00
D 1.00 0.00 0.25 0.50 0.25
E 0.00 0.00 0.00 0.50 0.50
```

```
Markov transition matrix when step =  2
         A       B       C       D        E
A 1.000000 0.00000 0.00000 0.00000 0.000000
B 0.453125 0.21875 0.21875 0.09375 0.015625
C 0.125000 0.21875 0.31250 0.25000 0.093750
```

```
D 0.015625 0.09375 0.25000 0.40625 0.234375
E 0.000000 0.03125 0.18750 0.46875 0.312500
```

```
Markov transition matrix when step =  5
           A         B         C         D          E
A 1.00000000 0.0000000 0.0000000 0.0000000 0.00000000
B 0.58105469 0.1049805 0.1425781 0.1206055 0.05078125
C 0.26708984 0.1425781 0.2255859 0.2441406 0.12060547
D 0.09570312 0.1206055 0.2441406 0.3461914 0.19335938
E 0.04492188 0.1015625 0.2412109 0.3867188 0.22558594
```

```
Markov transition matrix when step =  10
          A          B          C         D          E
A 1.0000000 0.00000000 0.00000000 0.0000000 0.00000000
B 0.6801729 0.05514336 0.09392357 0.1123104 0.05844975
C 0.4154892 0.09392357 0.16745377 0.2108231 0.11231041
D 0.2447290 0.11231041 0.21082306 0.2797642 0.15237331
E 0.1862793 0.11689949 0.22462082 0.3047466 0.16745377
```

```
Markov transition matrix when step =  25
          A          B          C          D          E
A 1.0000000 0.00000000 0.00000000 0.00000000 0.00000000
B 0.8246035 0.02672794 0.04935541 0.06444496 0.03486817
C 0.6759350 0.04935541 0.09117290 0.11909175 0.06444496
D 0.5766219 0.06444496 0.11909175 0.15561785 0.08422358
E 0.5417537 0.06973634 0.12888992 0.16844716 0.09117290
```

```
Markov transition matrix when step =  1000
  A            B            C            D            E
A 1 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
B 1 9.900928e-19 1.829453e-18 2.390295e-18 1.293619e-18
C 1 1.829453e-18 3.380388e-18 4.416690e-18 2.390295e-18
D 1 2.390295e-18 4.416690e-18 5.770684e-18 3.123071e-18
E 1 2.587237e-18 4.780591e-18 6.246143e-18 3.380388e-18
```

```
Markov transition matrix when step =  1e+07
  A             B             C            D             E
A 1  0.000000e+00  0.000000e+00  0.00000e+00  0.000000e+00
B 1 7.410985e-323 1.333977e-322 1.72923e-322 9.387247e-323
C 1 7.410985e-323 1.333977e-322 1.72923e-322 9.387247e-323
D 1 7.410985e-323 1.333977e-322 1.72923e-322 9.387247e-323
E 1 7.410985e-323 1.333977e-322 1.72923e-322 9.387247e-323
```

Due to memory limitation, manual prediction for step = n is as follows:

Markov Transition matrix when step = n

	A	**B**	**C**	**D**	**E**
A	1	0	0	0	0
B	1	0	0	0	0
C	1	0	0	0	0
D	1	0	0	0	0
E	1	0	0	0	0

ANSWERS:

2. Solution are described in the following sections:
   a. Read this dataset into R or Python. You will see that it is smaller than the earlier version of the dataset but has an additional variable called "prior_union". For each individual in the sample, this variable is prior union status, for which a 0 indicates "not in union" and a 1 indicates "in union". Using R or Python, create a two-by-two table that relates prior union status to current union status. Your results should match what was presented in class. Again using R or Python, create a two-by-two matrix of Markov transition probabilities based on these results (either in percentage or decimal format):

```
Summation table:
      0    1
0 14758 2086
1  2110 2812
```

```
As percentage:
          0         1
0 0.8761577 0.1238423
1 0.4286875 0.5713125
```

Calculate Markov matrix using markovchain library:

```
          0         1
0 0.8761577 0.1238423
1 0.4286875 0.5713125
```

   b. Using R or Python, estimate the logit model presented in class, using prior union status as a characteristic. Present the results in a "nice" table:

```
Call:
glm(formula = union ~ age + grade + smsa + south + black + year
+
    prior_union, family = "binomial", data = union_ori)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7413  -0.5600  -0.4959  -0.3628   2.4232

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.296040   0.385301  -5.959 2.54e-09 ***
age          0.023661   0.006152   3.846  0.00012 ***
grade        0.046393   0.007849   5.911 3.40e-09 ***
smsa         0.047586   0.043301   1.099  0.27179
south       -0.662996   0.042135 -15.735  < 2e-16 ***
black        0.579656   0.043245  13.404  < 2e-16 ***
year        -0.011370   0.006686  -1.701  0.08903 .
prior_union  2.120996   0.037795  56.119  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)
```

```
    Null deviance: 23211  on 21765  degrees of freedom
Residual deviance: 18925  on 21758  degrees of freedom
AIC: 18941

Number of Fisher Scoring iterations: 4
```

```
Regression Results for logit
=============================================
                   Dependent variable:
                 ----------------------------
                           union
---------------------------------------------
age                      0.024***
                          (0.006)

grade                    0.046***
                          (0.008)

smsa                       0.048
                          (0.043)

south                    -0.663***
                          (0.042)

black                    0.580***
                          (0.043)

year                      -0.011*
                          (0.007)

prior_union              2.121***
                          (0.038)

Constant                 -2.296***
                          (0.385)

---------------------------------------------
Observations              21,766
Log Likelihood          -9,462.543
Akaike Inf. Crit.       18,941.090
=============================================
Note:             *p<0.1; **p<0.05; ***p<0.01
```

c. Using R or Python, reset your “prior_union” variable so that it takes on value 0 for all observations:

```
> head(union_2c$pred_01)
   [1] 0.1984455 0.2043759 0.2104370 0.2145504 0.2187219 0.2249275
   > head(union_2c$pred_00)
   [1] 0.8015545 0.7956241 0.7895630 0.7854496 0.7812781 0.7750725
```

d. Using R or Python, reset your “prior_union” variable so that it takes on value 1 for all observations

```
> head(union_pred$pred_11)
[1] 0.6736977 0.6817509 0.6896969 0.6949331 0.7001193 0.7076127
> head(union_pred$pred_10)
[1] 0.3263023 0.3182491 0.3103031 0.3050669 0.2998807 0.2923873
```

e. In a two-by-two table, find the average values for the four predictions created above in a manner consistent with that presented in class:

```
> print(p_transition)
          0         1
0 0.8714181 0.1285819
1 0.4629799 0.5370201
```

f. use the Markov matrix calculated in 2e. to iterate the system forward until the Markov matrix has converged to its limit:

```
Markov transition matrix when step =  2
          0         1
0 0.8189003 0.1810997
1 0.6520786 0.3479214
```

```
Markov transition matrix when step =  5
          0         1
0 0.7851106 0.2148894
1 0.7737439 0.2262561
```

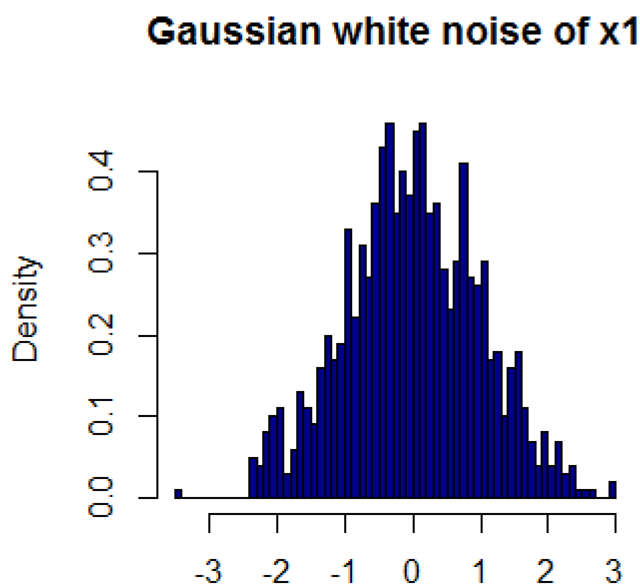
```
Markov transition matrix when step =  10
          0         1
0 0.7826680 0.2173320
1 0.7825388 0.2174612
```

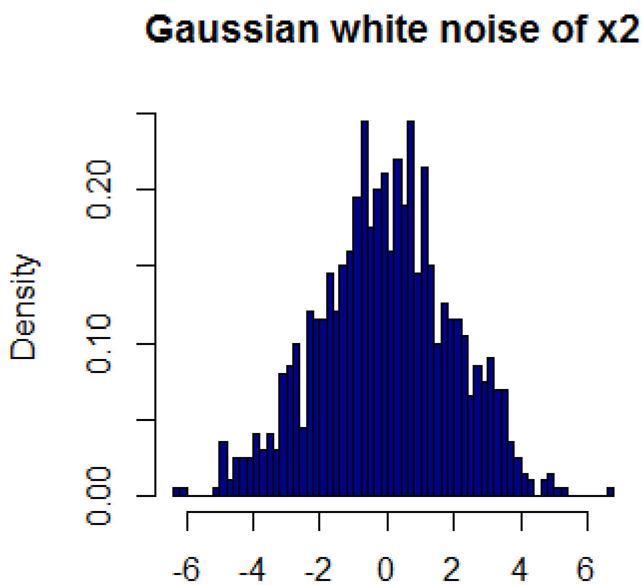
```
Markov transition matrix when step =  25
          0         1
0 0.7826399 0.2173601
1 0.7826399 0.2173601
```

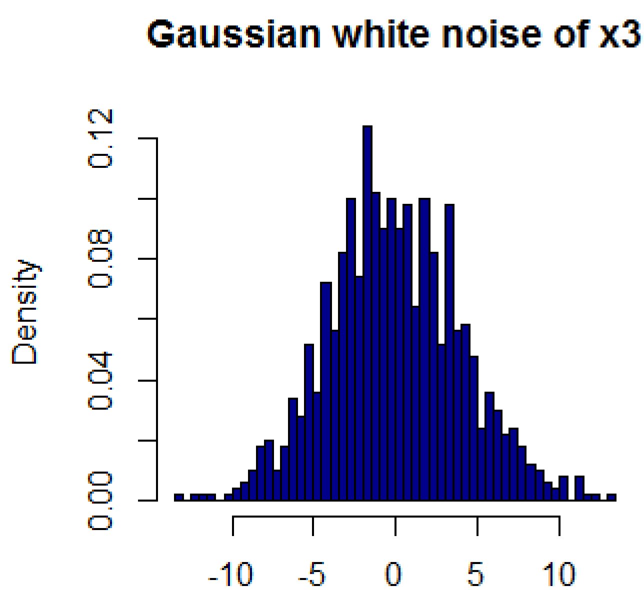
```
Markov transition matrix when step =  1e+05
          0         1
0 0.7826399 0.2173601
1 0.7826399 0.2173601
```

ANSWERS:

3. Solution are described in the following sections:
    a. Generate and plot three Gaussian white noise random variables with 1,000 draws, the first with variance 1, the second with variance 2, the third with variance 4:







b. Linear model Summary of X2 ~ X1:

```
> summary(linear.model)
```

```
Call:
lm(formula = X2 ~ X1)

Residuals:
    Min      1Q  Median      3Q     Max
-3.6607 -0.6669  0.0042  0.6520  3.1544

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.052297   0.031885   1.640    0.101
X1          0.006434   0.031455   0.205    0.838

Residual standard error: 1.008 on 998 degrees of freedom
Multiple R-squared:  4.193e-05,      Adjusted R-squared:
 -0.00096
F-statistic: 0.04184 on 1 and 998 DF,  p-value: 0.838
```

c. repeat b. above 1,000 times, each time recording the estimated value of the slope coefficient of the bivariate regression. Generate a histogram of your 1,000 replications: