CS222 - Algorithm Design

Dr. Arpita Korwar

Assignment – 2 **:** *Fibonacci series using recursion*

Authors :

- Prakhar Mathur 1906328
- Sanjay Marreddi 1904119
- Rishabh Tripathi 1904129

Problem Statement

1. : Recall the Fibonacci series:

$$\begin{aligned} F_0 &= 0, \\ F_1 &= 1, \\ F_n &= F_{n-1} + F_{n-2}, \qquad \forall n \geq 2. \end{aligned}$$

Implement a recursive function that computes the nth Fibonacci number F_n.

In a line graph, map

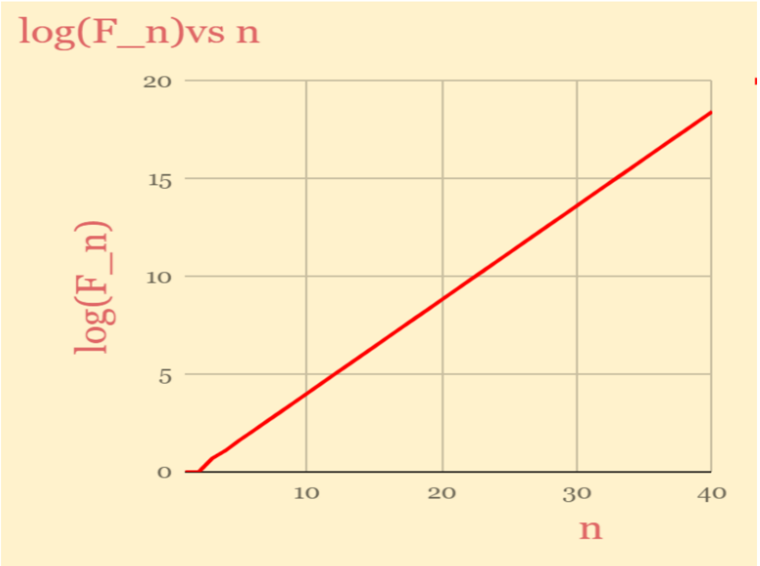
1. $n, \log(F_n)$ and
2. $n, \log(T(n))$, where $T(n)$ is the time taken to compute F_n.

Conclude that the Fibonacci series and the time taken grows exponentially. What are the slopes of the two lines? Make a guess about F_n as a function of n. Make a guess about $T(n)$ as a function of n.

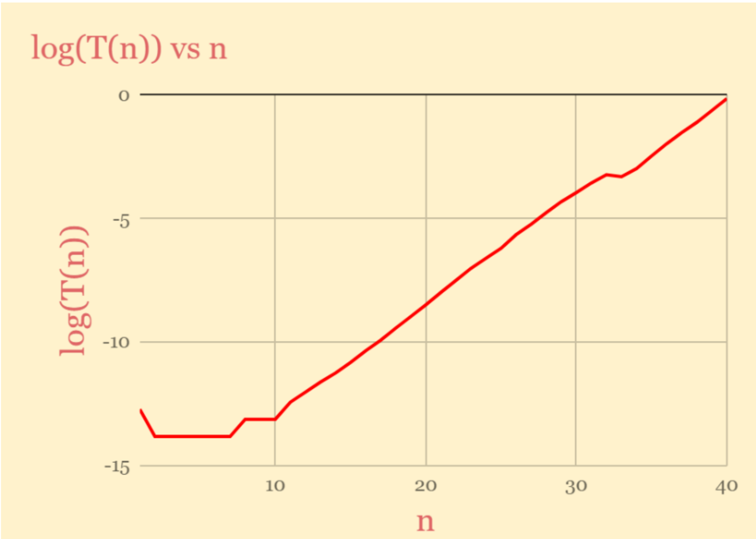
The values of $\mathbf{n}, \log(F_n)$, $\log\,(T(n))$ obtained using our code are as follows:

n	log(F_n)	log(T(n))
1	0	-12.7169
2	0	-13.8155
3	0.693147	-13.8155
4	1.098612	-13.8155
5	1.609438	-13.8155
6	2.079442	-13.8155
7	2.564949	-13.8155
8	3.044522	-13.1224
9	3.526361	-13.1224
10	4.007333	-13.1224
11	4.488636	-12.4292
12	4.969813	-12.0238
13	5.451038	-11.6183
14	5.932245	-11.2506
15	6.413459	-10.8198
16	6.89467	-10.3498
17	7.375882	-9.92369
18	7.857094	-9.43348
19	8.338306	-8.96348
20	8.819518	-8.48763
21	9.300729	-7.98657
22	9.781941	-7.50015
23	10.26315	-7.01201
24	10.74437	-6.60469
25	11.22558	-6.20317
26	11.70679	-5.64731
27	12.188	-5.22432
28	12.66921	-4.75367
29	13.15042	-4.31527
30	13.63164	-3.9446
31	14.11285	-3.55501
32	14.59406	-3.21953
33	15.07527	-3.30101
34	15.55648	-2.97434
35	16.0377	-2.46199
36	16.51891	-1.96811
37	17.00012	-1.51752
38	17.48133	-1.10221
39	17.96254	-0.62338
40	18.44375	-0.14186

Line graph of **n vs** $\mathbf{log}(\boldsymbol{F_n})$:



Line graph of **n vs** $\mathbf{log}(\boldsymbol{T}(\boldsymbol{n}))$:



The slopes of both the above graphs are almost **constant** i.e., **logarithms** of $\boldsymbol{T}(\boldsymbol{n})$ and $\boldsymbol{F_n}$ are constant which implies that they are **exponential** in nature.

So, we can conclude that the Fibonacci series and the time taken grows exponentially.

The slopes of the two lines are as follows:

> $\mathbf{n}$ **vs** $\log(\boldsymbol{F_n})$ $\boldsymbol{has\ a\ slope\ of}$ 0.4804468921

> $\mathbf{n}$ **vs** $\log(\boldsymbol{T}(\boldsymbol{n}))$ $\boldsymbol{has\ a\ slope\ of}$ 0.3846896294

Let us guess/compute the $\boldsymbol{F_n}$ *and* ***T(n)*** *as a function of n using above values:*

Since we observed that slopes are constant,

d(log($\boldsymbol{F_n}$))/dn = *constant* ($Here\ it\ is$ 0.4804468921)

∫d(log($\boldsymbol{F_n}$)) = ∫0. 4804468921dn

log($\boldsymbol{F_n}$) = 0. 4804468921**n** + **k** *(Integration constant)*

$\boldsymbol{F_n}$ = e^(0. 4804468921**n**+**k**)

$\boldsymbol{F_n}$ = A*(e^(0. 4804468921**n**)) (*A is constant)*

In the similar way, the function for T(n) will be of the form:

$\boldsymbol{T}(\boldsymbol{n})$ = B*(e^(0.3846896294 **n**)) (*B is constant)*

--- The End ---