Assignment 2
CS 230 /CS 561: Probability and Statistics for CS
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INSTRUCTIONS: Solve all problems. Please submit your solutions for assignment in google classroom.

I Conditional Probability

1. For the monty hall problem analyse if there are 4 doors instead of 3 doors.
2. Repeat the above exercise with python and report the distribution obtained over several runs.
3. Show with an appropriate counter example that pairwise independence for several events is not sufficient to claim full independence
4. Show with an appropriate counter example the condition

$$P(E_1, \ldots, E_n] = \Pi_{i=1}^{n} P(E_i)$$

   is not enough to claim pairwise independence.
5. Suppose A and B are independent, show that following events are independent
   (a) A and B^c
   (b) A^c and B
   (c) A^c and B^c
6. Prove that conditional probability obeys all the axioms of probability
7. Show with an appropriate counter example that unconditional independence doesn't necessarily imply conditional independence
8. Show with an appropriate counter example that conditional independence doesn't necessarily imply unconditional independence

II Discrete Random Variables

1. Prove that $\lim_{n\to\infty} \lim_{m\to\infty} (2^{m-n}) \neq \lim_{m\to\infty} \lim_{n\to\infty} (2^{m-n})$
2. Write a python code to implement monte carlo integration of $\int_0^1 \sqrt{sin(x)}dx$. Note: please report a distribution and not a single value.
3. Compute the expectation and variance for the following discrete random variables

(a) Bernoulli

(b) Binomial

(c) Geometric

(d) Poisson

4. In python, plot the probability mass function and cumulative distribution function of the following random variables

   (a) Binomial, $n = 30, p = 0.2$

   (b) Geometric, $k = 1 \ldots, 20, p = 0.2$

5. Sheldon Ross edition 10, Chapter 2, Exercise 6

6. Sheldon Ross edition 10, Chapter 2, Exercise 8

7. Sheldon Ross edition 10, Chapter 2, Exercise 9

8. Sheldon Ross edition 10, Chapter 2, Exercise 13

9. Sheldon Ross edition 10, Chapter 2, Exercise 17

10. In the previous question, let X_i, denote the number of times that ith type outcome occurs, $i = 1, 2, \ldots, r$. For $0 \leq j \leq n$, use the definition of conditional probability to find $P(X_i = x_i, i = 1, 2, \ldots, r-1 | X_r = j)$

11. Sheldon Ross edition 10, Chapter 2, Exercise 23

12. Sheldon Ross edition 10, Chapter 2, Exercise 24