

Assignment 1
CS 230 /CS 561: Probability and Statistics for CS
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INSTRUCTIONS: Solve all problems. Please submit your solutions for assignment in google classroom.

1. A probability space is the collection of outcome space, events and probability function. Given events A, B in a probability space such that $A \subseteq B$. Prove that $P(A) \leq P(B)$. [1 mark]
2. Given an event A and let its complement be A^c . Prove that $P(A) = 1 - P(A^c)$. [1 mark]
3. Let A_1, A_2, \dots, A_n be set of finite events (not necessarily mutually disjoint). Prove that

$$P\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i)$$

[2 marks]

4. Section 1.2 Problem 9(a) Pg 54, Chapter 1. Bertsekas-Tsitsiklis's book (2nd edition). [2 marks]
5. Suppose you have an event A , which you approximate from "below" as follows $A_1 \subset A_2 \subset A_3 \dots$ and let $A = \bigcup_{i \geq 1} A_i$. Using the probability axioms, prove that

$$P(A) = \lim_{n \rightarrow \infty} P(A_n)$$

[3 marks]

6. Suppose you have an event A , which you approximate from "above" as follows $A_1 \supseteq A_2 \supseteq A_3 \dots$ and let $A = \bigcap_{i \geq 1} A_i$. Prove from Q5 that

$$P(A) = \lim_{n \rightarrow \infty} P(A_n)$$

[3 marks]