

# Look into Monthly Production of Toyota Passenger cars

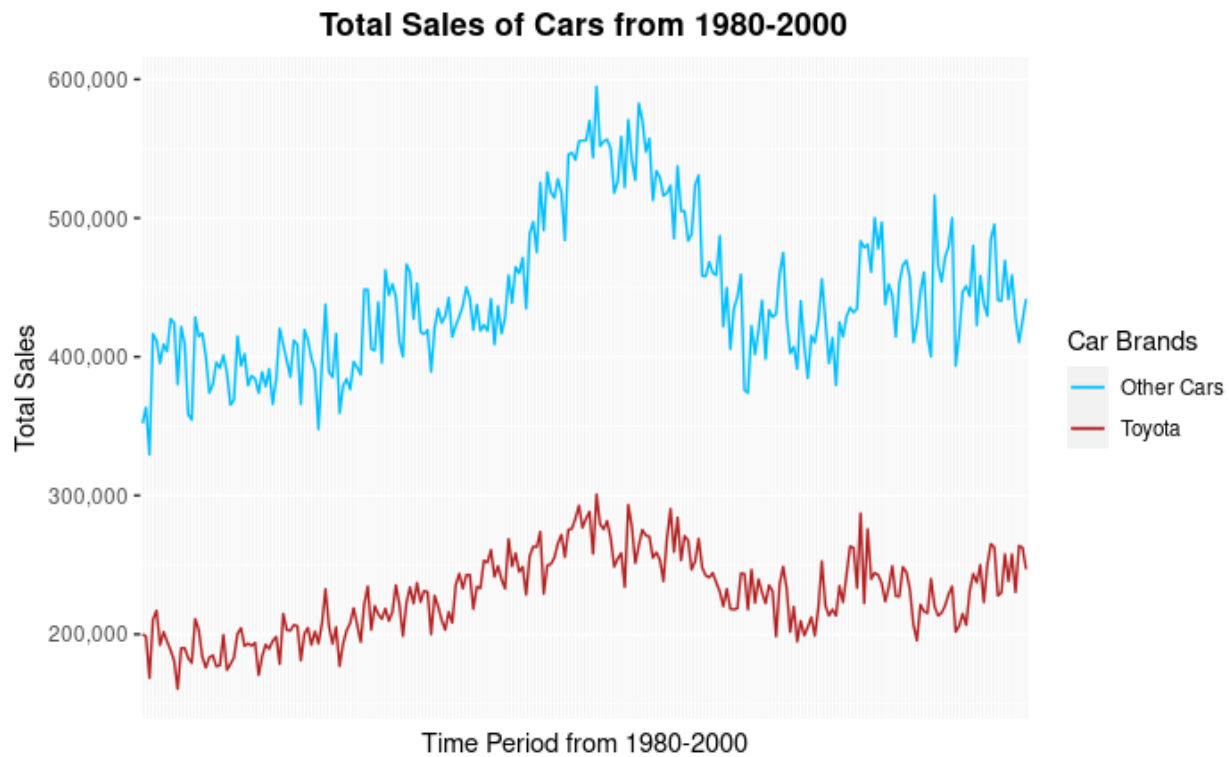
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01/06/2021

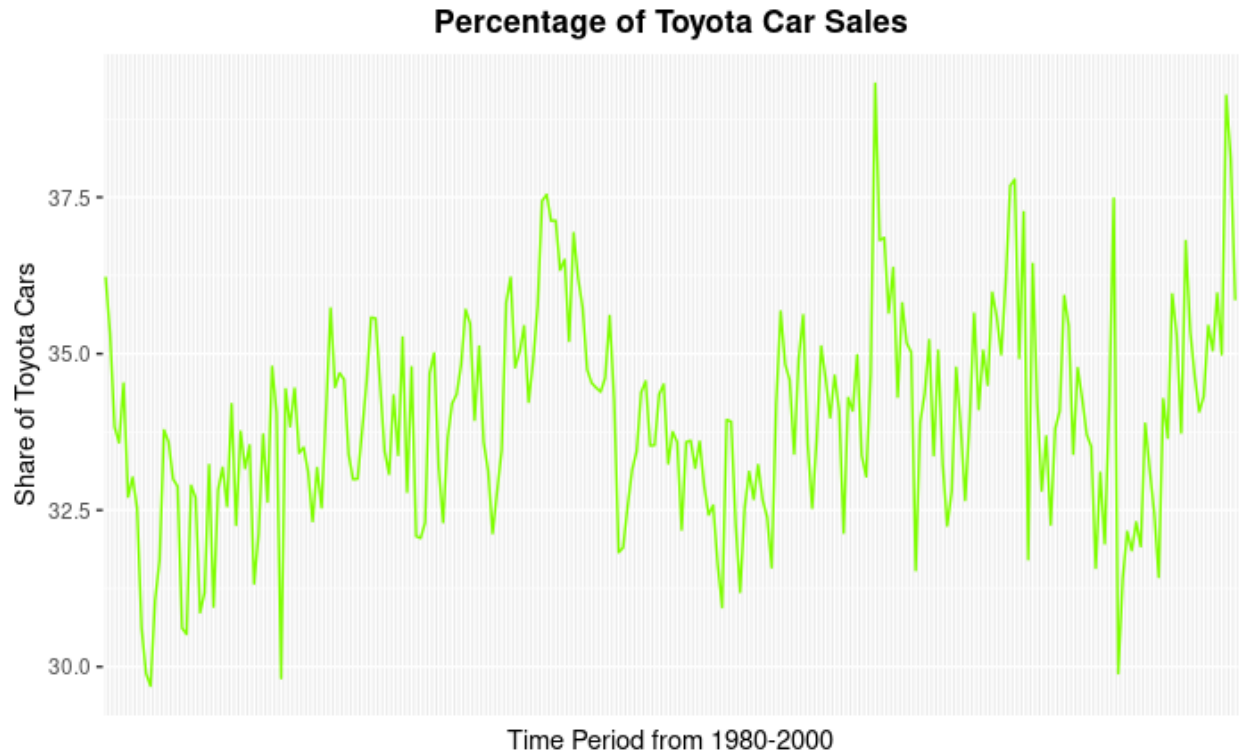
## Questions

- (a). Make time series plots of the variables  $y_t$  and  $x_t$ , and also of the share of Toyota in all produced passenger cars, that is  $y_t/(y_t + x_t)$ . What conclusions do you draw from these plots?

Answers:



There is no defining trend in the above plot



The is no great flucation in the numbers

- (b)
  - (i) Perform the Augmented Dickey-Fuller (ADF) test for  $y_t$ . In the ADF test equation, include a constant ( ) and three lags of  $y_t$ , as well as the variable of interest,  $y_{t-1}$ . Report the coefficient of  $y_{t-1}$  and its standard error and t-value, and draw your conclusion.

Answers:

Table 1	Estimate	Std. Error	t value
LagTOYOTA	0.07478	0.03673	2.036

- (ii) Perform a similar ADF test for  $x_t$ .

Answer:

Table 2	Estimate	Std. Error	t value
LagOthers	0.05290	0.03356	1.576

- (c). Perform the two-step Engle-Granger test for cointegration of the time series  $y_t$  and  $x_t$ . In step 1, regress  $y_t$  on a constant and  $x_t$ . In step 2, perform a regression of the residuals  $e_t$  in the model  $e_t = \alpha_0 + \alpha_1 e_{t-1} + \alpha_2 e_{t-2} + \alpha_3 e_{t-3} + \epsilon_t$ . What is your conclusion?

Answers:

step 1:

$$y_t = 28428.168 + 0.4501 x_t$$

step 2:

$$dE = 191.27 + 0.28956 \cdot dE_t + 0.586 \cdot dE_{t-1} + 0.14 \cdot dE_{t-2} + 0.096 \cdot dE_{t-3}$$

using Engle-Granger for cointegration test we get

Engle-Granger Cointegration Test alternative: cointegrated

lag	EG	p.value
3.00	-4.73	0.01

Hence we could conclude that both variable are cointegrated.

- **(d).** Construct the first twelve sample autocorrelations and sample partial autocorrelations of  $yt$  and use the outcomes to motivate an AR(12) model for  $yt$ . Check that only the lagged terms at lags 1 to 5, 10, and 12 are significant, and estimate the following model:  $yt = \alpha + \sum_{j=1}^5 \alpha_j yt-j + \alpha_6 yt-10 + \alpha_7 yt-12 + \epsilon_t$  (recall that the estimation sample is Jan 1980 - Dec 1999).

Table3	Estimate	Std. Error	t value	Pr(>
(Intercept)	518.49151	897.80065	0.578	0.564165
LagDelta_1	-0.61038	0.06391	-9.550	< 0.0000000000000002 ***
LagDelta_2	-0.30488	0.07550	-4.038	0.0000737 ***
LagDelta_3	-0.24722	0.07634	-3.238	0.001383 **
LagDelta_4	-0.23912	0.07793	-3.069	0.002412 **
LagDelta_5	-0.18718	0.07952	-2.354	0.019426 *
LagDelta_6	-0.10445	0.08001	-1.305	0.193076
LagDelta_7	-0.13765	0.08012	-1.718	0.087161 .
LagDelta_8	0.04058	0.07999	0.507	0.612411
LagDelta_9	0.02838	0.07869	0.361	0.718745
LagDelta_10	-0.26206	0.07675	-3.415	0.000756 ***
LagDelta_11	-0.03910	0.07596	-0.515	0.607243
LagDelta_12	0.23236	0.06400	3.631	0.000349 ***

- **(e).** Extend the model of part (d) by adding the Error Correction (EC) term ( $yt - 0.45xt$ ), that is, estimate the ECM  $yt = \alpha + (yt-1 - 0.45xt-1) + \sum_{j=1}^5 \alpha_j yt-j + \alpha_6 yt-10 + \alpha_7 yt-12 + \epsilon_t$  (estimation sample is Jan 1980- Dec 1999). Check that the EC term is significant at the 5% level, but not at the 1% level.

Table 4	Estimate	Std. Error	t value	Pr(>
(Intercept)	-3039.85443	1922.07272	-1.582	0.11512
LagDelta_1	-0.54566	0.06449	-8.462	0.000000000000000305
LagDelta_2	-0.24207	0.07489	-3.232	0.00141
LagDelta_3	-0.19619	0.07321	-2.680	0.00789
LagDelta_4	-0.18636	0.06931	-2.689	0.00769
LagDelta_5	-0.09950	0.05918	-1.681	0.09406
LagDelta_10	-0.26582	0.05002	-5.314	0.00000025230739777
LagDelta_12	0.25515	0.05278	4.834	0.00000244194347778
ErrorCorrection	0.12523	0.06136	2.041	0.04239

- **(f).** Use the models of parts (d) and (e) to make two series of 12 forecasts of monthly changes in production of Toyota passenger cars in 2000. At each month, you should use the data that are then available, for example, to forecast production for September 2000 you can use the data up to and including August 2000. However, do not re-estimate the model and use the coefficients as obtained in parts (d) and (e). For each of the two forecast series, compute the values of the root mean squared error

(RMSE) and of the mean absolute error (MAE). Finally, give your interpretation of the outcomes.

Table 5	AR(12)	ECM
RMSE	14443.15	27657.25
MAE	11351.77	21305.06

hence, Not Ideal to predict the sale of TOYOTA

## Methods

### Load the required Library

```
library(readxl)
library(ggplot2)
options(scipen=999)
options(warn = -1)
```

### Load the Data

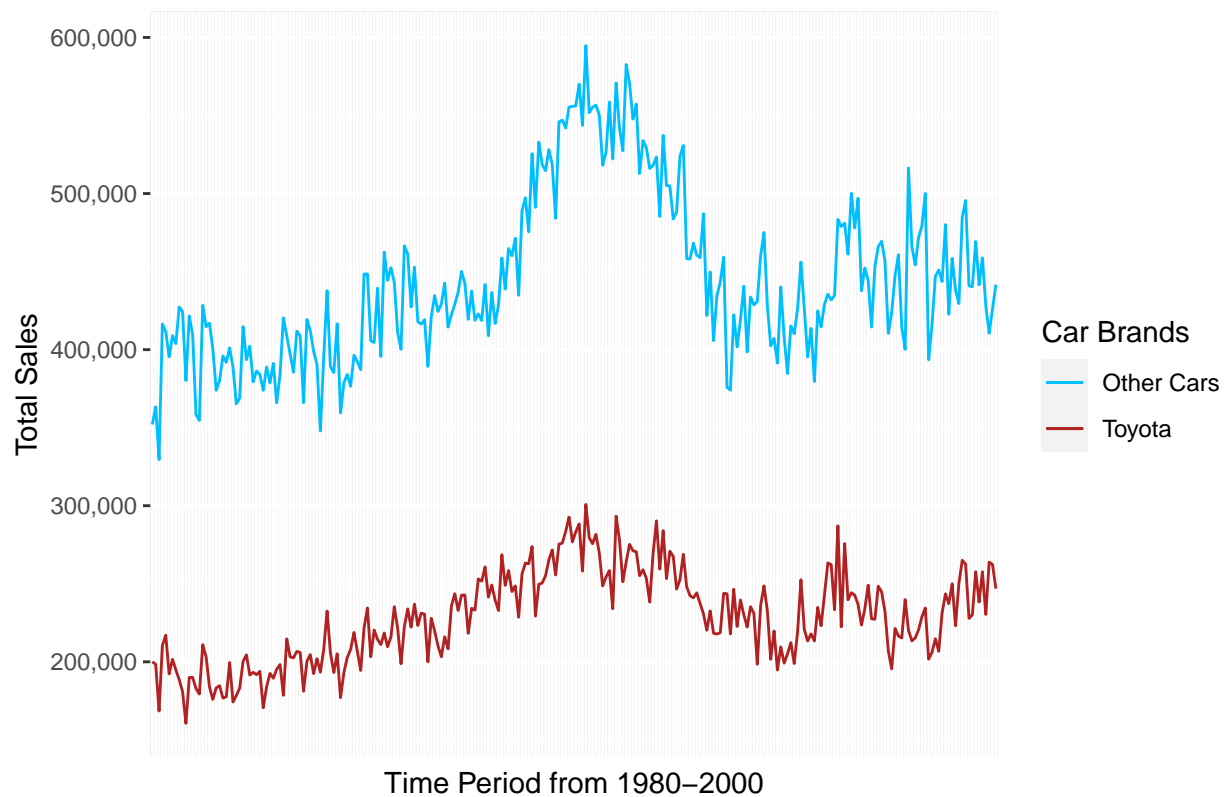
```
data <- read_xlsx('TestExer6-CARS-round2.xlsx')

head(data)
```

```
## # A tibble: 6 x 5
##   `YYYY-MM` TOYOTA  OTHER TOYOTA_SA OTHER_SA
##   <chr>      <dbl>  <dbl>      <dbl>    <dbl>
## 1 1980M01   175734 315111   200015.  352054.
## 2 1980M02   200479 377893   198443.  363393.
## 3 1980M03   200373 385236   168488.  329570.
## 4 1980M04   211636 404110   210573.  416497.
## 5 1980M05   208527 371930   217037.  411574.
## 6 1980M06   203901 420137   192241.  395403.
```

```
ggplot(data, aes(x = `YYYY-MM`))+
  geom_line(aes(y = OTHER_SA, group = 1, color = "Other Cars"))+
  geom_line(aes(y = TOYOTA_SA, group = 1, color = "Toyota"))+
  labs(y = "Total Sales", x = "Time Period from 1980-2000", title = "Total Sales of Cars from 1980-2000",
       color = "Car Brands")+
  scale_color_manual(values = c(
    "Other Cars" = "deepskyblue1",
    "Toyota" = "firebrick"))+
  theme(axis.text.x = element_blank(),
        axis.ticks.x = element_blank(),
        plot.title = element_text(hjust = 0.5, face = 'bold'))+
  scale_y_continuous(labels = scales::comma)
```

## Total Sales of Cars from 1980–2000



```
dataCarShare <- subset(data, select =c('YYYY-MM', 'TOYOTA_SA', "OTHER_SA"))
```

```
head(dataCarShare)
```

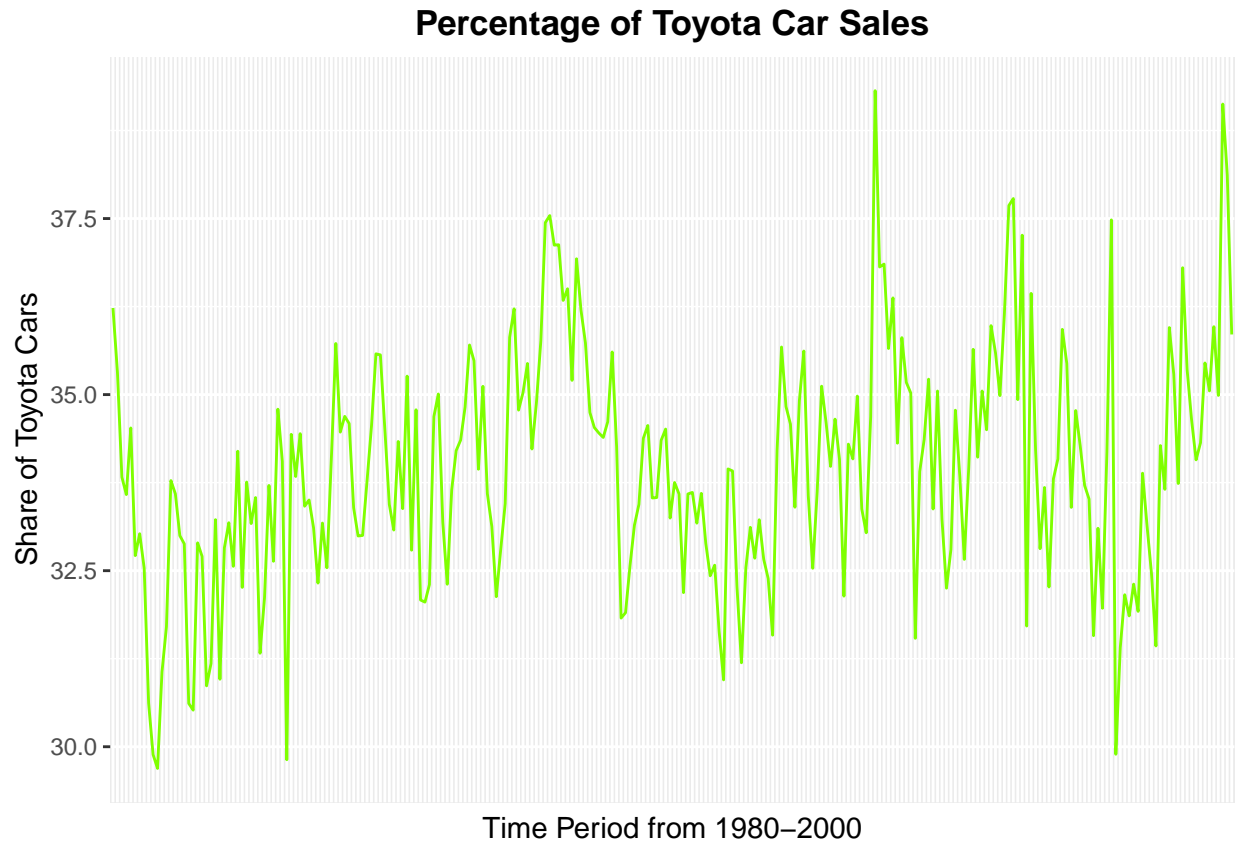
```
## # A tibble: 6 x 3
##   `YYYY-MM` TOYOTA_SA OTHER_SA
##   <chr>      <dbl>    <dbl>
## 1 1980M01    200015.   352054.
## 2 1980M02    198443.   363393.
## 3 1980M03    168488.   329570.
## 4 1980M04    210573.   416497.
## 5 1980M05    217037.   411574.
## 6 1980M06    192241.   395403.
```

```
dataCarShare$ShareTOYOTA <- dataCarShare$TOYOTA_SA/(data$TOYOTA_SA + data$OTHER_SA)*100
```

```
head(dataCarShare)
```

```
## # A tibble: 6 x 4
##   `YYYY-MM` TOYOTA_SA OTHER_SA ShareTOYOTA
##   <chr>      <dbl>    <dbl>      <dbl>
## 1 1980M01    200015.   352054.      36.2
## 2 1980M02    198443.   363393.      35.3
## 3 1980M03    168488.   329570.      33.8
## 4 1980M04    210573.   416497.      33.6
## 5 1980M05    217037.   411574.      34.5
## 6 1980M06    192241.   395403.      32.7
```

```
ggplot(dataCarShare, aes(`YYYY-MM`, ShareTOYOTA, group = 1)) +
  geom_line(col = 'chartreuse1') +
  labs(y = 'Share of Toyota Cars', x = 'Time Period from 1980-2000', title = 'Percentage of Toyota Car S
  theme(axis.text.x = element_blank(),
        axis.ticks.x = element_blank(),
        plot.title = element_text(hjust = 0.5, face = 'bold'))
```



```
library(tseries)

tseries::adf.test(data$TOYOTA_SA, k = 3)

##
## Augmented Dickey-Fuller Test
##
## data: data$TOYOTA_SA
## Dickey-Fuller = -2.5096, Lag order = 3, p-value = 0.3612
## alternative hypothesis: stationary

tseries::adf.test(data$OTHER_SA, k = 3)

##
## Augmented Dickey-Fuller Test
##
## data: data$OTHER_SA
## Dickey-Fuller = -2.0658, Lag order = 3, p-value = 0.5481
## alternative hypothesis: stationary
```

```

library(aTSA)

coint.test( data$TOYOTA_SA,data$OTHER_SA, nlag = 3)

## Response: data$TOYOTA_SA
## Input: data$OTHER_SA
## Number of inputs: 1
## Model: y ~ X + 1
## -----
## Engle-Granger Cointegration Test
## alternative: cointegrated
##
## Type 1: no trend
##      lag      EG p.value
##      3.00   -4.73    0.01
## -----
## Type 2: linear trend
##      lag      EG p.value
##      3.00    1.21    0.10
## -----
## Type 3: quadratic trend
##      lag      EG p.value
##      3.000  -0.305   0.100
## -----
## Note: p.value = 0.01 means p.value <= 0.01
##       : p.value = 0.10 means p.value >= 0.10
for (i in 2:nrow(data)){
  data$DeltaTOYOTA[1] <- 0
  data$DeltaTOYOTA[i] <- data$TOYOTA_SA[i] - data$TOYOTA_SA[i-1]
}

data$LagDelta_1 <- dplyr::lead(data$DeltaTOYOTA, n = 1)
data$LagDelta_2 <- dplyr::lead(data$DeltaTOYOTA, n = 2)
data$LagDelta_3 <- dplyr::lead(data$DeltaTOYOTA, n = 3)

data$LagTOYOTA <- dplyr::lead(data$TOYOTA_SA, n = 1)

model1 <- lm(DeltaTOYOTA~LagDelta_1+LagDelta_2+LagDelta_3+LagTOYOTA, data)

summary(model1)

##
## Call:
## lm(formula = DeltaTOYOTA ~ LagDelta_1 + LagDelta_2 + LagDelta_3 +
##     LagTOYOTA, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -45616 -10266    264    9029   39114
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept) -16669.26905   8450.46348  -1.973    0.0497 *
## LagDelta_1   -0.65786     0.06342 -10.373 < 0.0000000000000002 ***

```

```

## LagDelta_2      -0.34576      0.07303  -4.735      0.00000372 ***
## LagDelta_3      -0.09677      0.06388  -1.515      0.1311
## LagTOYOTA       0.07478      0.03673   2.036      0.0428 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15450 on 244 degrees of freedom
## (3 observations deleted due to missingness)
## Multiple R-squared:  0.3119, Adjusted R-squared:  0.3006
## F-statistic: 27.65 on 4 and 244 DF,  p-value: < 0.00000000000000022

for (i in 2:nrow(data)){
  data$DeltaOthers[1] <- 0
  data$DeltaOthers[i] <- data$OTHER_SA[i] - data$OTHER_SA[i-1]
}

data$LagOtherDelta_1 <- dplyr::lead(data$DeltaOthers, n = 1)
data$LagOtherDelta_2 <- dplyr::lead(data$DeltaOthers, n = 2)
data$LagOtherDelta_3 <- dplyr::lead(data$DeltaOthers, n = 3)

data$LagOthers <- dplyr::lead(data$OTHER_SA, n = 1)

model2 <- lm(DeltaOthers~LagOtherDelta_1+LagOtherDelta_2+LagOtherDelta_3+LagOthers, data)

summary(model2)

##
## Call:
## lm(formula = DeltaOthers ~ LagOtherDelta_1 + LagOtherDelta_2 +
##     LagOtherDelta_3 + LagOthers, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -80708 -17962  -2401   16367   82867
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -22953.12547  15030.68236  -1.527    0.128
## LagOtherDelta_1  -0.57608    0.06370  -9.044 < 0.0000000000000002 ***
## LagOtherDelta_2  -0.37494    0.07046  -5.322    0.000000233 ***
## LagOtherDelta_3  -0.10087    0.06423  -1.570    0.118
## LagOthers       0.05290    0.03356   1.576    0.116
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26220 on 244 degrees of freedom
## (3 observations deleted due to missingness)
## Multiple R-squared:  0.2696, Adjusted R-squared:  0.2576
## F-statistic: 22.52 on 4 and 244 DF,  p-value: 0.0000000000000007666

model3 <- lm(data$TOYOTA_SA~data$OTHER_SA)

summary(model3)

##
## Call:

```



```

## lm(formula = data$TOYOTA_SA ~ data$OTHER_SA)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -40684 -11089    558    9837   50631
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept)  28428.16819   8686.84193   3.273    0.00122 **
## data$OTHER_SA    0.45013     0.01941  23.196 < 0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 16460 on 250 degrees of freedom
## Multiple R-squared:  0.6828, Adjusted R-squared:  0.6815
## F-statistic: 538 on 1 and 250 DF, p-value: < 0.00000000000000022

data$Errors <- model3$residuals

for (i in 2:nrow(data)){
  data$DeltaErrors[i] <- 0
  data$DeltaErrors[i] <- data$Errors[i] - data$Errors[i-1]
}

data$LagDeltaErrors_1 <- dplyr::lead(data$DeltaErrors, n = 1)
data$LagDeltaErrors_2 <- dplyr::lead(data$DeltaErrors, n = 2)
data$LagDeltaErrors_3 <- dplyr::lead(data$DeltaErrors, n = 3)

data$LagErrors <- dplyr::lead(data$Errors, n = 1)

model4 <- lm(DeltaErrors~LagDeltaErrors_1+LagDeltaErrors_2+LagDeltaErrors_3+LagErrors,data)

summary(model4)

##
## Call:
## lm(formula = DeltaErrors ~ LagDeltaErrors_1 + LagDeltaErrors_2 +
##      LagDeltaErrors_3 + LagErrors, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -54952  -8553     85    7812   38534
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept)  191.26861   816.15563   0.234    0.8149
## LagDeltaErrors_1 -0.58609    0.06343  -9.240 < 0.0000000000000002 ***
## LagDeltaErrors_2 -0.14254    0.07105  -2.006    0.0459 *
## LagDeltaErrors_3 -0.09610    0.06219  -1.545    0.1236
## LagErrors      0.28956    0.06374   4.543  0.00000874 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12870 on 244 degrees of freedom

```

```

## (3 observations deleted due to missingness)
## Multiple R-squared: 0.2683, Adjusted R-squared: 0.2563
## F-statistic: 22.37 on 4 and 244 DF, p-value: 0.0000000000000009446

data$LagDelta_4 <- dplyr::lead(data$DeltaTOYOTA, n = 4)
data$LagDelta_5 <- dplyr::lead(data$DeltaTOYOTA, n = 5)
data$LagDelta_6 <- dplyr::lead(data$DeltaTOYOTA, n = 6)
data$LagDelta_7 <- dplyr::lead(data$DeltaTOYOTA, n = 7)
data$LagDelta_8 <- dplyr::lead(data$DeltaTOYOTA, n = 8)
data$LagDelta_9 <- dplyr::lead(data$DeltaTOYOTA, n = 9)
data$LagDelta_10 <- dplyr::lead(data$DeltaTOYOTA, n = 10)
data$LagDelta_11 <- dplyr::lead(data$DeltaTOYOTA, n = 11)
data$LagDelta_12 <- dplyr::lead(data$DeltaTOYOTA, n = 12)

data$ErrorCorrection <- (data$TOYOTA_SA - (0.45*(data$OTHER_SA)))

model6 <- lm(DeltaTOYOTA~LagDelta_1+LagDelta_2+LagDelta_3+LagDelta_4+LagDelta_5+LagDelta_6+LagDelta_7+LagDelta_8+LagDelta_9+LagDelta_10+LagDelta_11+LagDelta_12, data = data)

summary(model6)

##
## Call:
## lm(formula = DeltaTOYOTA ~ LagDelta_1 + LagDelta_2 + LagDelta_3 +
##     LagDelta_4 + LagDelta_5 + LagDelta_6 + LagDelta_7 + LagDelta_8 +
##     LagDelta_9 + LagDelta_10 + LagDelta_11 + LagDelta_12, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -43343  -8616   -228    8170   40343
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept)  518.49151   897.80065   0.578    0.564165
## LagDelta_1   -0.61038    0.06391  -9.550 < 0.0000000000000002 ***
## LagDelta_2   -0.30488    0.07550  -4.038    0.0000737 ***
## LagDelta_3   -0.24722    0.07634  -3.238    0.001383 **
## LagDelta_4   -0.23912    0.07793  -3.069    0.002412 **
## LagDelta_5   -0.18718    0.07952  -2.354    0.019426 *
## LagDelta_6   -0.10445    0.08001  -1.305    0.193076
## LagDelta_7   -0.13765    0.08012  -1.718    0.087161 .
## LagDelta_8    0.04058    0.07999   0.507    0.612411
## LagDelta_9    0.02838    0.07869   0.361    0.718745
## LagDelta_10  -0.26206    0.07675  -3.415    0.000756 ***
## LagDelta_11  -0.03910    0.07596  -0.515    0.607243
## LagDelta_12   0.23236    0.06400   3.631    0.000349 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13760 on 227 degrees of freedom
## (12 observations deleted due to missingness)
## Multiple R-squared: 0.4638, Adjusted R-squared: 0.4355
## F-statistic: 16.37 on 12 and 227 DF, p-value: < 0.00000000000000022

model5 <- lm(DeltaTOYOTA~LagDelta_1+LagDelta_2+LagDelta_3+LagDelta_4+LagDelta_5+LagDelta_10+LagDelta_12, data = data)

```

```

summary(model5)

##
## Call:
## lm(formula = DeltaTOYOTA ~ LagDelta_1 + LagDelta_2 + LagDelta_3 +
##     LagDelta_4 + LagDelta_5 + LagDelta_10 + LagDelta_12 + ErrorCorrection,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -41336  -9247    639    8662   37740
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept)  -3039.85443   1922.07272  -1.582    0.11512
## LagDelta_1    -0.54566    0.06449   -8.462 0.00000000000000305 ***
## LagDelta_2    -0.24207    0.07489   -3.232    0.00141 **
## LagDelta_3    -0.19619    0.07321   -2.680    0.00789 **
## LagDelta_4    -0.18636    0.06931   -2.689    0.00769 **
## LagDelta_5    -0.09950    0.05918   -1.681    0.09406 .
## LagDelta_10   -0.26582    0.05002   -5.314 0.00000025230739777 ***
## LagDelta_12    0.25515    0.05278    4.834 0.00000244194347778 ***
## ErrorCorrection  0.12523    0.06136    2.041    0.04239 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13780 on 231 degrees of freedom
## (12 observations deleted due to missingness)
## Multiple R-squared:  0.4535, Adjusted R-squared:  0.4345
## F-statistic: 23.96 on 8 and 231 DF,  p-value: < 0.00000000000000022

library(Metrics)

rmse(data$DeltaTOYOTA, model5$fitted.values)

## [1] 14443.15

rmse(data$DeltaOthers, model6$fitted.values)

## [1] 27657.25

mae(data$DeltaTOYOTA, model5$fitted.values)

## [1] 11351.77

mae(data$DeltaOthers, model6$fitted.values)

## [1] 21305.06

```