

# JEE-Main-22-07-2021-Shift-2 (Memory Based)

## PHYSICS

**Question:** Find the ratio of de Broglie wavelength of an electron and a proton having same Kinetic Energy. Given that the mass of electron is  $m_e$  and proton is  $m_p$ .

**Options:**

(a)  $\frac{m_p}{m_e}$

(b)  $\frac{m_e}{m_p}$

(c)  $\sqrt{\frac{m_p}{m_e}}$

(d)  $\sqrt{\frac{m_e}{m_p}}$

**Answer:** (c)

**Solution:**

$$\text{de Broglie wavelength } \lambda = \frac{h}{\sqrt{2mk}}$$

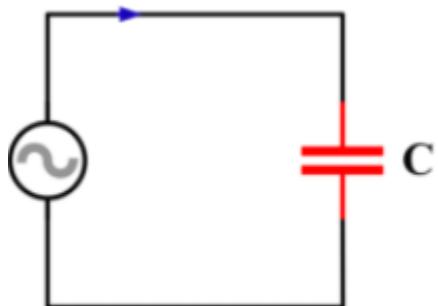
$$\text{for electron, } \lambda_e = \frac{h}{\sqrt{2m_e k}} \dots (\text{i})$$

$$\text{for proton, } \lambda_p = \frac{h}{\sqrt{2m_p k}} \dots (\text{ii})$$

from equation (i) and (ii)

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

**Question:** If an AC source is connected to a capacitor in series. Then which of the graphs relating current and emf is correct?

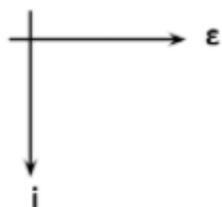


**Options:**

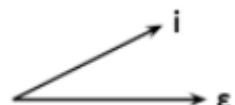
(a)



(b)



(c)

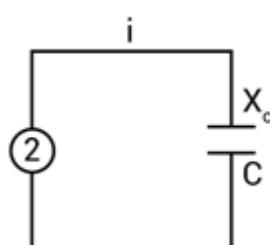


(d)



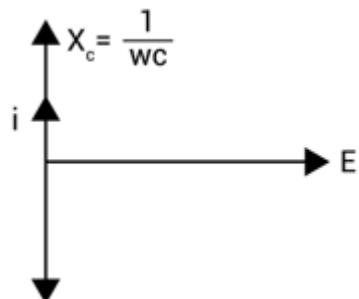
**Answer:** (a)

**Solution:**



$$i = \frac{\mathcal{E}}{x_C} \hat{j}$$

$$i = \frac{\varepsilon_0 \sin \omega t}{\frac{1}{\omega C}} \hat{j}$$

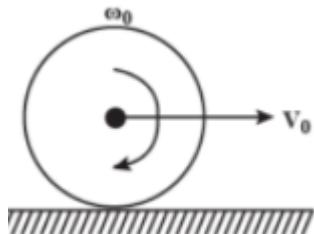


$$i = \varepsilon_0 \omega C \sin \omega t \hat{j}$$

$$i = \varepsilon \omega C \hat{j}$$

option A is correct.

**Question:** A wheel is undergoing pure rolling on a horizontal surface as shown. The point on rim of wheel at a same horizontal level as center has velocity  $\sqrt{x}V_0$ . The x is

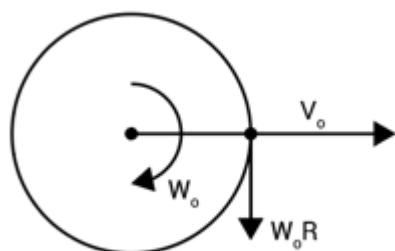


**Options:**

- (a) 1
- (b) 2
- (c) 3
- (d) 4

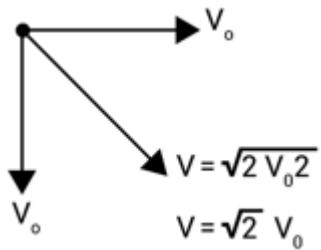
**Answer:** (b)

**Solution:**



In pure rolling

$$V_0 = \omega_0 R$$



Therefore  $x = 2$ .

**Question:** If a particle is performing SHM such that  $X = A \sin(\omega t) + B \cos(\omega t)$ . Then the equation of SHM can also be represented as  $x = C \sin[\omega t + \phi]$ . The  $C$  &  $\phi$  are

**Options:**

(a)  $\sqrt{A^2 + B^2}, \tan^{-1}\left[\frac{B}{A}\right]$

(b)  $\sqrt{A^2 + B^2}, \tan^{-1}\left[\frac{A}{B}\right]$

(c)  $A + B, \tan^{-1}\left[\frac{B}{A}\right]$

(d) None

**Answer:** (a)

**Solution:**

$$X = A \sin \omega t + B \cos (\omega t)$$

$$X = \sqrt{A^2 + B^2} \left\{ \frac{A}{\sqrt{A^2 + B^2}} \sin \omega t + \frac{B}{\sqrt{A^2 + B^2}} \cos (\omega t) \right\}$$

$$X = \sqrt{A^2 + B^2} \{ \cos \phi \sin \omega t + \sin \phi \cos \omega t \}$$

$$\begin{aligned} \sin \phi &= \frac{B}{\sqrt{A^2 + B^2}} \\ \cos \phi &= \frac{A}{\sqrt{A^2 + B^2}} \\ \tan \phi &= \frac{B}{A} \\ \phi &= \tan^{-1}\left(\frac{B}{A}\right) \end{aligned}$$

$$X = \sqrt{A^2 + B^2} \sin(\omega t + \phi)$$

$$\text{Therefore, } C = \sqrt{A^2 + B^2}, \phi = \tan^{-1}\left(\frac{B}{A}\right)$$

**Question:** The intensity of sunlight at a place on earth is  $0.92 \text{ W m}^{-2}$ . Find the amplitude of magnetic field at that place.

**Options:**

- (a)  $4.6 \times 10^{-8} T$
- (b)  $5.6 \times 10^{-8} T$
- (c)  $6.7 \times 10^{-8} T$
- (d)  $8.77 \times 10^{-8} T$

**Answer:** (d)**Solution:**

$$I = 0.92 \text{ W/m}^2$$

$$I = \frac{1}{2} C \frac{B_0^2}{u_0}$$

$$0.92 = \frac{1}{2} \times \frac{3 \times 10^8 \times B_0^2}{1.26 \times 10^{-6}}$$

$$B_0 = \sqrt{\frac{0.92 \times 2 \times 1.26 \times 10^{-6}}{3 \times 10^8}} T$$

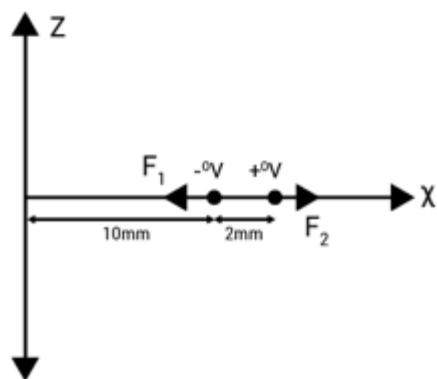
$$B_0 = 0.877 \times 10^{-7} T$$

$$B_0 = 8.77 \times 10^{-8} T$$

**Question:** A long line charge of linear charge density  $4 \times 10^{-5} \text{ C/m}$  is placed along z-axis. An electric dipole of length 2 mm is placed along x-axis. The negative end of dipole is at 10 mm and positive end of dipole is at 12 mm. The force experienced by dipole is 4 N. Find charge of dipole

**Options:**

- (a)  $0.5 \mu C$
- (b)  $0.33 \mu C$
- (c)  $5 \mu C$
- (d)  $8 \mu C$

**Answer:** (b)**Solution:**

$$\lambda = 4 \times 10^{-5} \text{ C/m}$$

$$\text{Electric field due to infinitely long charged wire } E = \frac{\lambda}{2\pi \epsilon_0 x}$$

$$F_{net} = F_1 - F_2$$

$$4 = \frac{q\lambda}{2\pi\epsilon_0 \times 10 \times 10^{-3}} - \frac{q\lambda}{2\pi\epsilon_0 \times 12 \times 10^{-3}}$$

$$4 = q \times \left( \frac{2q}{4\pi\epsilon_0 \times 10^{-2}} - \frac{2\lambda}{4\pi\epsilon_0 \times 12 \times 10^{-3}} \right)$$

$$4 = q \times \left( \frac{2 \times 4 \times 10^{-5} \times 2 \times 10^9}{10^{-2}} - \frac{2 \times 4 \times 10^{-5} \times 9 \times 10^3}{12 \times 10^{-3}} \right)$$

$$4 = q \times 12 \times 10^{-6}$$

$$q = \frac{1}{3} \times 10^{-6} C$$

$$q = 0.33 \mu C$$

**Question:** The angle of Dip in plane is  $\delta'$  and true Dip in magnetic meridian is  $\delta$ . Then

**Options:**

- (a)  $\delta' > \delta$
- (b)  $\delta' < \delta$
- (c)  $\delta' = \delta$
- (d) None

**Answer:** (a)

**Solution:**

$\delta'$  is apparent dip, let  $\alpha$  be the angle made by dip circle with magnetic meridian.

Now rotating dip circle by  $90^\circ$  from this position. It will now make angle  $90 - \alpha$  with the magnetic meridian. In this case apparent dip is  $\delta''$

**Now we have relation**

$$\cot^2 \delta' + \cot^2 \delta'' = \cot^2 \delta \dots (i)$$

Where,  $\delta$  is true dip in magnetic meridian

From eq (i)

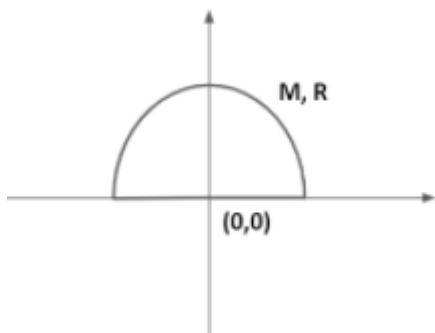
$$\cot^2 \delta' < \cot^2 \delta$$

or  $\cot \delta' < \cot \delta$

$$\Rightarrow \delta' > \delta$$

**Question:** A semi-ring has mass m and radius R as shown. The COM of semi ring lies on Y

axis at a distance of  $\frac{xR}{\pi}$ . Find (x).



**Options:**

- (a) 1
- (b) 2
- (c) 3
- (d) 4

**Answer:** (b)**Solution:**

The center of mass of the uniform semicircular ring is at  $\frac{2R}{\pi}$  hence the value of x is 2.

**Question:** If a ring and a solid cylinder of same mass and radius are released from the top of an inclined plane. If the time taken by ring to come down is  $t_1$  and time taken by cylinder is  $t_2$  while both perform pure rolling, then choose the correct option.

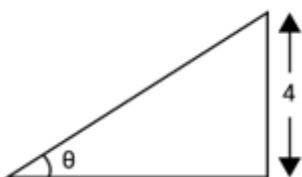
**Options:**

- (a)  $t_1 > t_2$
- (b)  $t_1 < t_2$
- (c)  $t_1 = t_2$
- (d) Can't be related

**Answer:** (a)**Solution:**

$$t = \sqrt{\frac{2h}{g \sin^2 \theta} \left(1 + \frac{k^2}{R^2}\right)}$$

$$t \propto \left(1 + \frac{k^2}{R^2}\right)^{1/2}$$



For ring

$$\frac{k^2}{R^2} = 1$$

For solid cylinder

$$\frac{k^2}{R^2} = \frac{1}{2}$$

$$t_1 \propto (1+1)^{\frac{1}{2}}$$

$$t_2 \propto \left(1 + \frac{1}{2}\right)^{\frac{1}{2}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{2}{\frac{3}{2}}} = \sqrt{\frac{4}{3}} \Rightarrow t_1 > t_2$$

**Question:** Identify the correct statement/statements

Statement -1: Ferromagnetic converts to Paramagnetic on Heating.

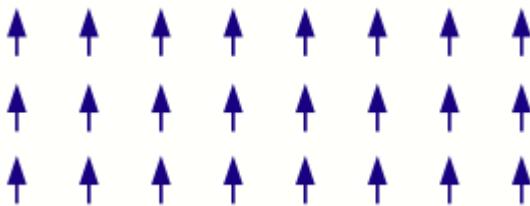
Statement-2: Domains of Ferromagnetic material increase on Heating.

**Options:**

- (a) 1
- (b) 2
- (c) Both 1 and 2
- (d) Can't say

**Answer:** (a)

**Solution:**



Ferromagnetic material: All the molecular magnetic dipoles are pointed in the same direction

When the ferromagnetic material heated , all magnetic dipoles are disturbed and get disoriented. Due to which the net magnetic dipole moment becomes very weak, so they behave as paramagnetic material.

**Question:** Find average kinetic energy of a monatomic gas molecule, assuming ideal behaviour.

(Given T is the temperature of the gas and k is Boltzmann constant)

**Options:**

- (a)  $\frac{1}{2}kT$
- (b)  $\frac{3}{2}kT$
- (c)  $\frac{5}{2}kT$
- (d)  $kT$

**Answer:** (b)

**Solution:**

The average kinetic energy of a molecule is given by

$$\frac{1}{2}f kT$$

Where f is degree of freedom

For monoatomic gas f = 3 so average kinetic energy is

$$\frac{3}{2}kT$$

**Question:** P is travelling along the vector  $\vec{A} = \hat{i} + \hat{j}$ , Q is moving along the vector  $\vec{B} = \hat{j} + \hat{k}$ , R is moving along the vector  $\vec{C} = -\hat{i} + \hat{j}$ . All three particles collide at a point and after collision, P moves along  $\vec{A} \times \vec{B}$  and Q moves along  $\vec{B} \times \vec{C}$ , then the angle between the direction of P and Q after collision-

**Options:**

(a)  $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(b)  $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c)  $\sin^{-1}\left(\frac{1}{3}\right)$

(d)  $\cos^{-1}\left(\frac{1}{3}\right)$

**Answer:** (d)

**Solution:**

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\vec{\alpha}_1 = \vec{A} \times \vec{B} = \hat{i}(1-0) - \hat{j}(1-0) + \hat{k}(1-0)$$

$$= -\hat{i} - \hat{j} + \hat{k}$$

$$\vec{\alpha}_1 = \vec{A} \times \vec{B} = \hat{i}(1-0) - \hat{j}(1-0) + \hat{k}(1-0)$$

$$= -\hat{i} - \hat{j} + \hat{k}$$

$$\vec{\alpha}_2 = \vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(0+1) + \hat{k}(+1)$$

$$= -\hat{i} - \hat{j} + \hat{k}$$

Angle between  $\vec{\alpha}_1$  &  $\vec{\alpha}_2$

$$\vec{\alpha}_1 \cdot \vec{\alpha}_2 = (\hat{i} - \hat{j} + \hat{k})(-\hat{i} - \hat{j} + \hat{k})$$

$$= -1 + 1 + 1$$

$$\vec{\alpha}_1 \cdot \vec{\alpha}_2 = 1$$

$$|\alpha_1| = \sqrt{3}$$

$$|\alpha_2| = \sqrt{3}$$

$$\vec{\alpha}_1 \cdot \vec{\alpha}_2 = \alpha_1 \alpha_2 \cos \theta$$

$$\alpha_1 \alpha_2 \cos \theta = \vec{\alpha}_1 \cdot \vec{\alpha}_2$$

$$(\sqrt{3})(\sqrt{3}) \cos \theta = 1$$

$$\cos \theta = \frac{1}{3}$$

$$\theta = \cos^{-1} \left( \frac{1}{3} \right)$$

**Question:** A TV transmitter has range of 1500 km. What is the approximate height of the TV transmission tower?

**Options:**

- (a) 175 km
- (b) 200 km
- (c) 150 km
- (d) 125 km

**Answer:** (a)

**Solution:**

Range  $d = 1500$  km, Radius of earth  $R_e = 6400$  km

$$d = \sqrt{2hR_e}$$

$$h = \frac{d^2}{2R_e} = \frac{(1500 \times 10^3)^2}{2 \times 6400 \times 10^3} \approx 175 \text{ km}$$

**Question:** A ray of light is incident on a prism at an angle of incidence  $45^\circ$  and ray is undergoing minimum deviation such that the angle of deviation is  $30^\circ$ . Find the angle of prism

**Answer:**  $-60^\circ$

**Solution:**

**When set at minimum deviation position,  $A - 2i - d$**

$$i - 45^\circ, d - 30^\circ$$

$$A - 2 \times 45 - 30 - 60^\circ$$

**Question:** In 5 min, a body changes temperature from  $75^\circ C$  to  $65^\circ C$  at room temperature  $25^\circ C$ . What temperature will it have after another 5 min?

**Options:**

- (a)  $60^\circ C$
- (b)  $57^\circ C$
- (c)  $55^\circ C$
- (d)  $53^\circ C$ .

**Answer:** (b)

**Solution:**

$$T_1 = 75^\circ C, T_2 = 65^\circ C, t = 5 \text{ min}, T_0 = 25^\circ C, T = ?$$

$$\frac{T_1 - T_2}{t} = k \left( \frac{(T_1 + T_2)}{2} - T_0 \right) \Rightarrow \frac{10}{t} = 45k \quad \dots (\text{i})$$

$$\frac{T_2 - T}{t} = k \left( \frac{(T + T_2)}{2} - T_0 \right) \Rightarrow \frac{65 - T}{t} = k \left( \frac{(T + 65)}{2} - 25 \right) \dots (\text{ii})$$

$$\text{From (i) and (ii)} \frac{10}{65 - T} = \frac{45}{\left( \frac{(T + 65)}{2} - 25 \right)}$$

on solving for  $T, T = 57^\circ C$

**Question:** A section of a railway track is clamped between two nails. In summers it gets heated up and as a result, some elastic energy is stored in it. If  $\Delta T = 10^\circ C, \alpha = 10^{-6}^\circ C^{-1}$  = coefficient of linear expansion,  $Y = 10^{11} Nm^{-2}$  = Young's modulus,  $A = 0.01 m^2$  = Area of cross section. Find the energy stored per unit length in the railway track.

**Options:**

(a)  $5 \times 10^{-2} Jm^{-1}$

(b)  $3 \times 10^{-2} Jm^{-1}$

(c)  $2 \times 10^{-2} Jm^{-1}$

(d)  $10^{-2} Jm^{-1}$

**Answer:** (a)

**Solution:**

$$\alpha = 10^{-6}^\circ C^{-1}, \Delta T = 10^\circ C, Y = 10^{11} N/m^2, A = 0.01 m^2 = 10^{-2} m^2$$

$$\ell = \alpha L \Delta T \Rightarrow \frac{\ell}{L} = \alpha \Delta T \text{ (strain)}$$

$$= (10 \times 10^{-6})$$

$$E_{\text{per unit length}} = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{area}$$

$$= \frac{1}{2} \times 10^{11} \times (10 \times 10^{-6})^2 \times 10^{-2}$$

$$= 5 \times 10^{-2} J/m$$

**Question:** Mass number of a nucleus is 184. It undergoes alpha decay. If Q value is 5.5 MeV, find kinetic energy of the emitted alpha particle.

**Options:**

(a) **0.4 MeV**

(b) **5 MeV**

(c) **5.4 MeV**

(d) **0.1 MeV**

**Answer:** (c)

**Solution:**

$$M \rightarrow X + \alpha$$

$$E = \frac{p^2}{2m}$$

**Let Total energy = T**

$$T = E_\alpha + E_X$$

$$M = M_\alpha + M_X$$

$$M_X = 180$$

**Conservation of momentum**

$$P_\alpha = P_X$$

$$ME_\alpha = M_X E_X$$

$$E_\alpha = \frac{M_X}{M} T$$

$$E_\alpha = \frac{184}{180} \times 5.5 = 5.4 \text{ MeV}$$

**Question:** A pendulum has time period of small oscillations as T. What will be the new time period by length of pendulum is reduced to  $\frac{1}{16}$ <sup>th</sup> of its initial value?

**Options:**

(a)  $\frac{T}{2}$

(b)  $\frac{T}{4}$

(c)  $\frac{T}{8}$

(d)  $\frac{T}{16}$

**Answer:** (b)

**Solution:**

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$T' = 2\pi \sqrt{\frac{L'}{g}}$$

$$L' = \frac{L}{16}$$

$$\therefore T' = 2\pi \sqrt{\frac{L}{16g}} = \frac{1}{4} \left( 2\pi \sqrt{\frac{L}{g}} \right)$$

$$T' = \frac{T}{4}$$

**Question:** Find projection of  $\vec{A}$  on  $\vec{B}$  given  $\vec{A} = \hat{i} + \hat{j}$  &  $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

**Options:**

(a)  $\frac{\sqrt{3}}{2}$

(b)  $\frac{2}{\sqrt{3}}$

(c)  $\frac{3}{2}$

(d) 2

**Answer:** (b)

**Solution:**

We know that

Projection of  $\vec{a}$  on  $\vec{b}$  is

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Given:

$$\vec{A} = \hat{i} + \hat{j}$$

$$\vec{B} = \hat{i} + \hat{j} + \hat{k}$$

$$\therefore \text{Projection} = \frac{(\hat{i} + \hat{j}) \cdot (\hat{i} + \hat{j} + \hat{k})}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$= \frac{1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

**Question:** A bullet of mass 4 gm is shot from a gun of mass 4 kg. Bullet has speed  $50 \text{ ms}^{-1}$ .

Find the impulse of bullet and velocity of recoil of gun?

**Options:**

(a)  $200 \text{ kg m/s}, 5 \text{ m/s}$

(b)  $0.2 \text{ kg m/s}, 5 \text{ m/s}$

(c)  $0.2 \text{ kg m/s}, 5 \text{ cm/s}$

(d)  $200 \text{ kg m/s}, 5 \text{ cm/s}$

**Answer:** (b)

**Solution:**

$|\text{Impulse}| = |\text{Change in momentum}|$

$$\text{Mass of bullet} = 4 \text{ gm} = 4 \times 10^{-3} \text{ Kg}$$

Here initial velocity of bullet = 0

Here final velocity of bullet = 50 m/s

$$\therefore |\Delta \vec{P}| = m |(\vec{V}_f - \vec{V}_i)|$$

$$= 4 \times 10^{-3} (50 - 0)$$

$$= 2 \times 10^{-1} \text{ kgm/s}$$

$$= 0.2 \text{ kgm/s}$$

By conservation of linear momentum

$$0 = (4 \times 10^{-3})(50) - 4(V)$$

Here V=recoil velocity of gun

$$\therefore V = 0.05 \text{ m/s} = 5 \text{ cm/s}$$

**Question:** An 80 kg object brought down by 80 cm at constant speed. Find work done by external agent.

**Options:**

(a) Zero

(b) +640J

(c) -640J

(d) 645

**Answer:** (c)

**Solution:**

$$m = 80 \text{ kg}$$

$$\Delta h = -80 \text{ cm} = -0.8 \text{ m} [\text{height decreased}]$$

$$g = 10 \text{ m/s}^2$$

Work done = Change in potential energy

[K.E. doesn't change]

$$W = \Delta U$$

$$= mg(\Delta h)$$

$$= 80(10)(-0.8)$$

$$= -640 \text{ J}$$

**Question:** Q. A body is projected from surface of earth with escape velocity. Find the time taken by body to attain height  $h$  is ( $M$  = mass of earth,  $R$  = Radius of earth,  $G$  = Gravitational constant)

**Options:**

$$(a) \frac{2 \left[ (R+h)^{\frac{3}{2}} - R^{\frac{3}{2}} \right]}{3\sqrt{2GM}}$$

$$(b) \frac{\left[ (R+h)^{\frac{3}{2}} - R^{\frac{3}{2}} \right]}{3\sqrt{GM}}$$

$$(c) \frac{\left[ (R+h)^{\frac{1}{2}} - R^{\frac{1}{2}} \right]}{3\sqrt{2GM}}$$

(d) None

**Answer:** (a)

**Solution:**

**Total energy of body will remain zero**

**T.E=0**

**At a distance 'h' from surface of earth**

**M=mass of earth**

**M=mass of body**

**R=Radius of earth**

$$\frac{1}{2}mv^2 - G \frac{Mm}{h} = 0$$

$$v = \sqrt{\frac{2GM}{h}}$$

$$\frac{dh}{dt} = \sqrt{\frac{2GM}{h}}$$

$$\Rightarrow \sqrt{h} dh = \sqrt{2GM} dt$$

$$\Rightarrow \int_R^{(R+h)} \sqrt{h} dh = \sqrt{2GM} \int_0^t dt$$

$$\frac{2}{3} \left[ h^{3/2} \right]_R^{R+h} = \sqrt{2GM} t$$

$$\Rightarrow \frac{2}{3} \frac{\left[ (R+h)^{3/2} - R^{3/2} \right]}{\sqrt{2GM}} = t$$

$$\Rightarrow t = \frac{2}{3} \frac{\left[ (R+h)^{3/2} - R^{3/2} \right]}{3\sqrt{2GM}}$$

**Question:** A cell of emf E is attached in a circuit having 5 ohms and 2 ohms resistor one by one. PD across 5 ohms comes out to be 1.25 V and across 2 ohms as 1 volt. If  $E = \frac{x}{10}$ , then find x.

**Answer:** (a)

**Solution:**

**Let the internal resistance of cell be 'r'. Then when we connect  $5\Omega$  in the circuit**

$$i_1 = \frac{E}{5+r}$$

And P.D across  $5\Omega$  ohm will be

$$V_5 = 5i_1 = 5 \left( \frac{E}{5+r} \right) = 1.25 \dots (1)$$

When we connect  $2\Omega$

$$i_2 = \frac{E}{2+r}$$

& P.D across  $2\Omega$  is

$$V_2 = 2i_2 = 2 \left( \frac{E}{2+r} \right) = 1 \dots (2) \text{ Given}$$

1) can be written as

$$\frac{5E}{5+r} = 1.25 = \frac{5}{4}$$

$$\Rightarrow \frac{E}{5+r} = \frac{1}{4}$$

$$\Rightarrow 4E = 5+r \dots (3)$$

2) can be written as

$$\frac{2E}{2+r} = 1$$

$$\Rightarrow 2E = 2+r \dots (4)$$

Solving 3 and 4 we get

$$E = \frac{3}{2} = \frac{15}{10}$$

$$\therefore x = 15$$

# **JEE-Main-22-07-2021-Shift-2 (Memory Based)**

## **CHEMISTRY**

**Question:** More dissolved oxygen is found in?

**Options:**

- (a) Boiling water
- (b) Water at 4°C
- (c) Water at 80°C
- (d) Polluted water

**Answer:** (b)

**Solution:** As temperature increases, solubility of gas decreases.

**Question:** Match the following.

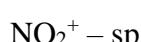
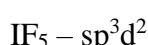
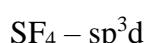
<b>Compound</b>	<b>Hybridisation</b>
I. SF <sub>4</sub>	(A) sp <sup>3</sup> d <sup>2</sup>
II. IF <sub>5</sub>	(B) sp <sup>3</sup> d
III. NO <sub>2</sub> <sup>+</sup>	(C) sp <sup>3</sup>
IV. NH <sub>4</sub> <sup>+</sup>	(D) sp
	(E) sp <sup>3</sup> d <sup>3</sup>

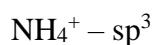
**Options:**

- (a) I → (B); II → (A); III → (D); IV → (C)
- (b) I → (C); II → (B); III → (A); IV → (D)
- (c) I → (D); II → (B); III → (C); IV → (A)
- (d) I → (B); II → (C); III → (A); IV → (D)

**Answer:** (a)

**Solution:**





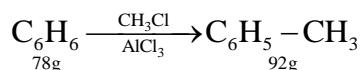
**Question:** Methylation of 10 g of benzene gives 9.2 g of toluene, the % yield is:

**Options:**

- (a) 92 %
- (b) 78 %
- (c) 90 %
- (d) 70 %

**Answer:** (b)

**Solution:**

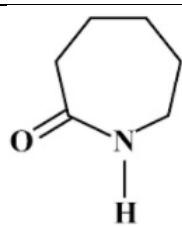


$$10 \text{ g benzene gives } \frac{92}{78} \times 10 \text{ g}$$

$$\% \text{ yield} = \frac{9.2}{920/78} \times 100 = 78 \%$$

**Question:** Identify the structure of isoprene, neoprene, acrylonitrile, caprolactam are

Column I	Column-II
I. 	(A) isoprene
II. 	(B) acrylonitrile
III. 	(C) neoprene
IV.	(D) Caprolactam

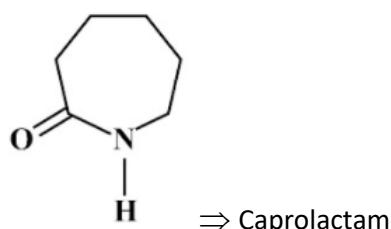
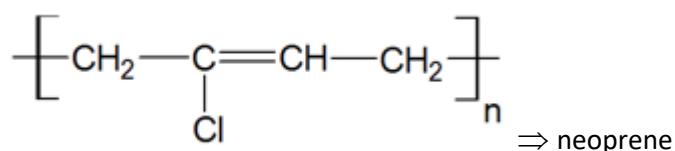
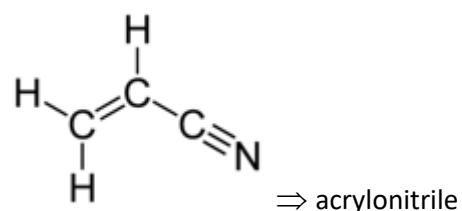
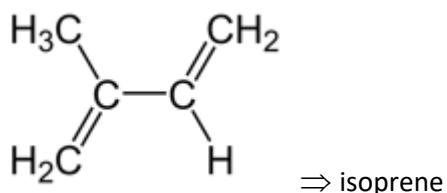


**Options:**

- (a) I → (B); II → (A); III → (D); IV → (C)
- (b) I → (C); II → (B); III → (A); IV → (D)
- (c) I → (D); II → (B); III → (C); IV → (A)
- (d) I → (A); II → (B); III → (C); IV → (D)

**Answer:** (d)

**Solution:**



**Question:** Isotope of hydrogen which emits low energy  $\beta$ -particle with half-life value greater than 12 years is:

**Options:**

- (a) Protium
- (b) Tritium
- (c) Deutrium
- (d) None of the above

**Answer:** (c)

**Solution:** Half-life of tritium is 12.33 yrs

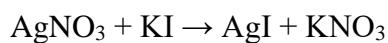
**Question:** When  $\text{AgNO}_3$  is added to KI solution, the solution produced is:

**Options:**

- (a)  $\text{KI}/\text{NO}_3^-$
- (b)  $\text{AgI}/\text{Ag}^+$
- (c)  $\text{AgI}/\text{NO}_3^-$
- (d)  $\text{AgNO}_3/\text{NO}_3^-$

**Answer:** (b)

**Solution:** When  $\text{AgNO}_3$  is added to KI solution,  $\text{Ag}^+$  is an excess so we get accumulation of  $\text{Ag}^+$  ions outside AgI, giving positively charged sol



**Question:** Which statement is not true for DI Mendeleev?

**Options:**

- (a) He wrote the book Principles of chemistry
- (b) He invented accurate barometer
- (c) He proposed periodic table when structure of atoms were unknown
- (d) Element with atomic number 101 is named after his name

**Answer:** (b)

**Solution:** Other options are true, only (b) is wrong.

**Question:** Which of the following 0.06 M solution has lowest freezing point?

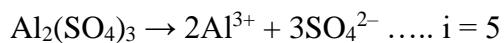
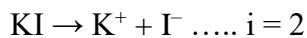
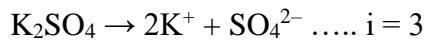
**Options:**

- (a)  $\text{K}_2\text{SO}_4$

- (b) KI
- (c) C<sub>6</sub>H<sub>12</sub>O<sub>6</sub>
- (d) Al<sub>2</sub>(SO<sub>4</sub>)<sub>3</sub>

**Answer:** (d)

**Solution:**



More the value of i, greater will be depression in freezing point

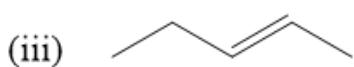
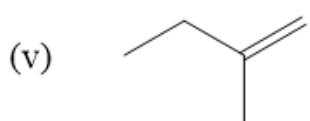
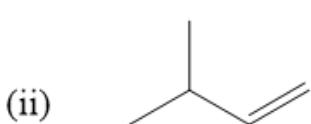
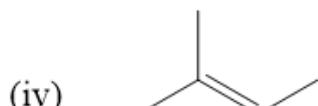
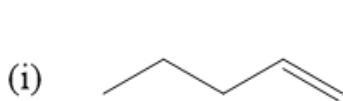
**Question:** Number of acyclic structural isomers for pentene are :

**Options:**

- (a) 2
- (b) 3
- (c) 4
- (d) 5

**Answer:** (d)

**Solution:**



**Question:** Thiamine and pyridoxine are respectively:

**Options:**

- (a) Vitamin B<sub>1</sub> and Vitamin B<sub>6</sub>
- (b) Vitamin B<sub>2</sub> and Vitamin B<sub>12</sub>
- (c) Vitamin C and Vitamin A
- (d) Vitamin C<sub>1</sub> and Vitamin D

**Answer:** (a)

**Solution:** Thiamine: Vitamin B<sub>1</sub>

Pyridoxine: Vitamin B<sub>6</sub>

**Question:** Which of the following is most reducing agent in group 15 elements?

**Options:**

- (a) AsH<sub>3</sub>
- (b) SbH<sub>3</sub>
- (c) BiH<sub>3</sub>
- (d) PH<sub>3</sub>

**Answer:** (c)

**Solution:** E-H bond energy decreases down the group.

So, best reducing agent in group 15 is BiH<sub>3</sub>

**Question:** Total number of unpaired electron [Co(NH<sub>3</sub>)<sub>6</sub>]Cl<sub>2</sub> and [Co(NH<sub>3</sub>)<sub>6</sub>]Cl<sub>3</sub>

**Options:**

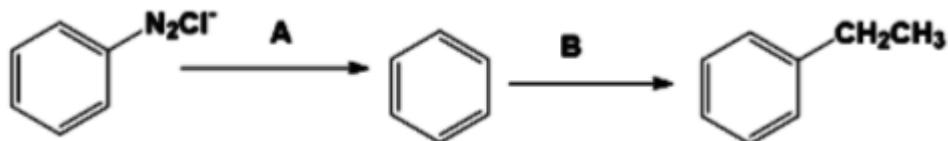
- (a) 3, 0
- (b) 1, 2
- (c) 1, 0
- (d) 2, 1

**Answer:** (c)

**Solution:**

[Co(NH<sub>3</sub>)<sub>6</sub>]Cl<sub>2</sub> contains 1 unpaired electron while [Co(NH<sub>3</sub>)<sub>6</sub>]Cl<sub>3</sub> contains 0 unpaired electron

**Question:**



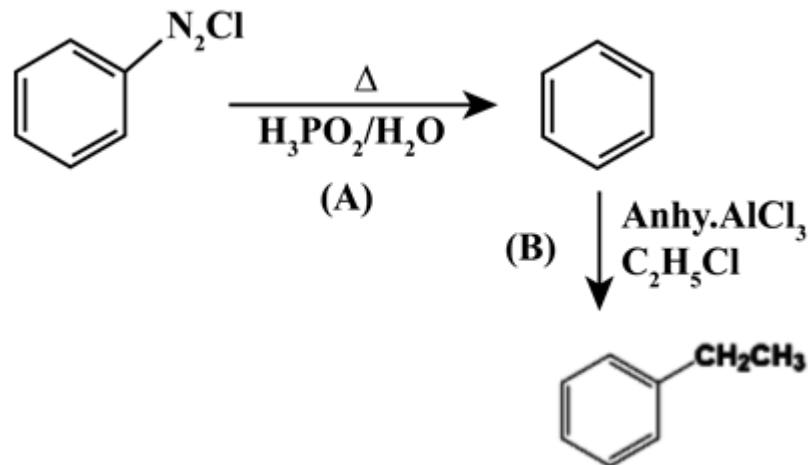
A and B respectively are

**Options:**

- (a) A  $\rightarrow \text{H}_3\text{PO}_2/\text{H}_2\text{O}$ ; B  $\rightarrow$  Anh.  $\text{AlCl}_3$ ,  $\text{C}_2\text{H}_5\text{Cl}$
- (b) A  $\rightarrow \text{KMnO}_4/\text{H}_2\text{O}$ ; B  $\rightarrow \text{H}_2\text{O}$
- (c) A  $\rightarrow \text{HCl}$ ; B  $\rightarrow \text{C}_2\text{H}_5\text{OH}$
- (d)  $\text{Zn}(\text{Hg})$ ,  $\text{HCl}$ ; B  $\rightarrow$  Anh.  $\text{AlCl}_3$

**Answer:** (a)

**Solution:**



**Question:** Which of the following pair is paramagnetic as well as coloured?

**Options:**

- (a)  $\text{Mn}^{7+}$ ,  $\text{Mn}^{2+}$
- (b)  $\text{Mn}^{2+}$ ,  $\text{Cu}^{2+}$
- (c)  $\text{Mn}^{7+}$ ,  $\text{Cu}^{2+}$
- (d)  $\text{Mn}^{7+}$ ,  $\text{Cu}^{2+}$

**Answer:** (b)

**Solution:**  $\text{Mn}^{2+}$  and  $\text{Cu}^{2+}$  both are paramagnetic as well as coloured

$\text{Mn}^{2+}$  – 2 unpaired electrons and light pink colour

$\text{Cu}^{2+}$  – 1 unpaired electrons and blue colour

**Question:** Which of the following about  $\text{B}_2\text{H}_6$  is correct?

**Options:**

- (a) Diborane is obtained by  $\text{NaBH}_4 + \text{I}_2$
- (b) It is a planar molecule
- (c) Boron are  $\text{sp}^2$  hybridised
- (d) Contains one 3c-3e bond

**Answer:** (a)

**Solution:** It is non-planar molecule in which B atoms are  $\text{sp}^3$  hybridised. Molecule contains two 3c-2e bonds

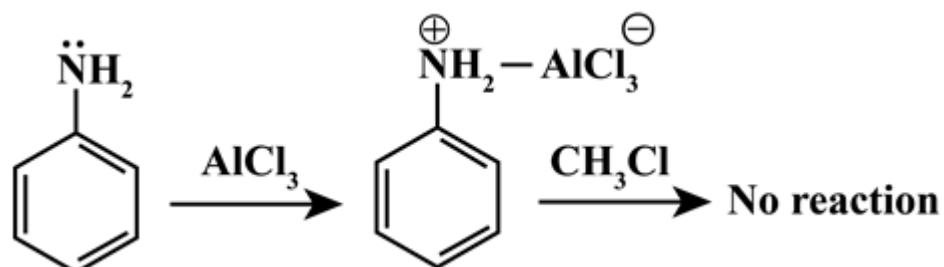
**Question:** Which compound does not give Friedel craft reaction?

**Options:**

- (a) Benzene
- (b) Aniline
- (c) Toluene
- (d) Ethyl benzene

**Answer:** (b)

**Solution:** Lone pair of  $\ddot{\text{N}}\text{H}_2$  combines with  $\text{AlCl}_3$  due to which further reaction is not possible



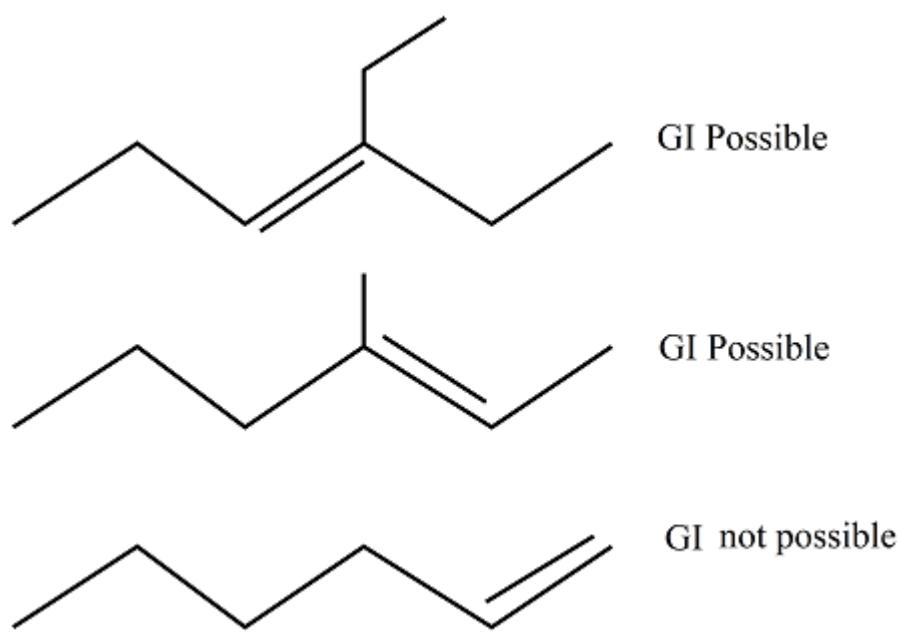
**Question:** Which of the following

**Options:**

- (a) 3-ethye hex-3-ene
- (b) 3-methyl hex-2-ene
- (c) hex-1-ene
- (d) None of these

**Answer:** (c)

**Solution:**



**Question:** If concentration of glucose ( $C_6H_{12}O_6$ ) in blood is 0.72 g/L. Its molarity is:

**Answer:** 0.004

**Solution:**

$$\text{Molarity} = \frac{w}{M \times V}$$

M of glucose = 180 g/mol

$$\text{Molarity} = \frac{0.72}{180} = 0.004 \text{ M}$$

**Question:** Number of electrons in p-orbitals of Vanadium

**Answer:** 12.00

**Solution:**  $^{23}\text{V} = 1\text{s}^2, 2\text{s}^2, 2\text{p}^6, 3\text{s}^2, 3\text{p}^6, 4\text{s}^2, 3\text{d}^3$

Total p electrons = 12

**Question:** For  $\text{N}_2\text{O}_4 \rightarrow 2\text{NO}_2$   $K_P = 476$  find  $K_C$  at 288 K

Answer: 2.00

$$K_P = K_C(RT)^{\Delta n_g}$$

$$47.6 = K_C[0.0821 \times 288]$$

$$K_C = \frac{47.6}{23.65} = 2$$

# **JEE-Main-22-07-2021-Shift-2 (Memory Based)**

## **MATHEMATICS**

**Question:** The number of solution of  $\sin^7 x + \cos^7 x = 1$ ,  $x \in (0, 4\pi]$

**Options:**

- (a) 7
- (b) 11
- (c) 9
- (d) 5

**Answer:** (d)

**Solution:**

$$\sin^7 x + \cos^7 x = 1$$

If  $\sin x \neq 0$  and  $\cos x \neq 0$

$$\sin^7 x < \sin^2 x \text{ and } \cos^7 x < \cos^2 x$$

Adding both we get

$$\sin^7 x + \cos^7 x < \sin^2 x + \cos^2 x$$

$$\Rightarrow \sin^7 x + \cos^7 x < 1$$

Hence  $\sin^7 x + \cos^7 x = 1$  is only possible

When  $\sin x = 1$  and  $\cos x = 0$

or  $\cos x = 1$  and  $\sin x = 0$

$$\Rightarrow x = 0, \frac{\pi}{2}, \frac{5\pi}{2}, 2\pi, 4\pi$$

**Question:** Let  $s_n$  denote sum of first  $n$ -terms of ap,  $s_{10} = s_{30}$ ,  $s_5 = 140$  then  $s_{20} - s_6$

**Options:**

- (a) 1872
- (b) 1842
- (c) 1852
- (d) 1862

**Answer:** (d)

**Solution:**

$$s_{10} = 5[2a + 9d] = 530 \Rightarrow 2a + 9d = 106$$

$$s_5 = \frac{5}{2}[2a + 4d] = 140 \Rightarrow 2a + 4d = 56$$

$\Rightarrow d = 10$  and  $a = 8$

$$\therefore s_{20} - s_6 = 10[2a + 19d] - 3[2a + 5d]$$

$$= 14a + 175d = 1862$$

**Question:** If the domain  $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$  is the integral  $(\alpha, \beta)$  then  $\alpha + \beta$  is

**Options:**

(a)  $\frac{1}{2}$

(b)  $\frac{3}{2}$

(c) 1

(d) 2

**Answer:** (b)

**Solution:**

$$f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$$

$$0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \leq \frac{\pi}{2}$$

$$0 < \frac{2x-1}{2} \leq 1$$

$$0 < 2x - 1 \leq 2$$

$$1 < 2x \leq 3$$

$$\frac{1}{2} < x \leq \frac{3}{2} \quad \dots(1)$$

$$x^2 - x + 1 = x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$$

$$\text{Also, } x^2 - x + 1 = x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$x^2 - x + 1 \leq 1$$

$$x^2 - x \leq 0$$

$$x(x-1) \leq 0$$

$$x \in [0,1] \quad \dots (2)$$

From (1) and (2)

$$x \in \left( \frac{1}{2}, 1 \right]$$

$$\alpha = \frac{1}{2}, \beta = 1$$

$$\alpha + \beta = \frac{1}{2} + 1 = \frac{3}{2}$$

**Question:**  $11^n > 10^n + 9^n$  number of integers satisfy the relation in  $\{1, 2, 3, \dots, 100\}$

**Answer:** 96

**Solution:**

$$f(x) = \left( \frac{10}{11} \right)^x + \left( \frac{9}{11} \right)^x$$

$$f'(x) = \left( \frac{10}{11} \right)^x \ln \frac{10}{11} + \left( \frac{9}{11} \right)^x \ln \frac{9}{11} < 0$$

$f(x)$  is decreasing

$$f(4) > 1 \text{ and } f(5) < 1$$

Hence  $x = 1, 2, 3, 4$  will not satisfy

$x = 5, 6, 7, \dots, 100$  will satisfy

Number of integers = 96

**Question:** 0, 2, 4, 6, 8 number of numbers  $> 10000$  which can be formed, if repetition not allowed.

**Answer:** 96

**Solution:**

Number of numbers formed =  $4 \times 4 \times 3 \times 2 \times 1 = 96$

**Question:**  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , number of matrices 'B' which can be formed such that  $AB = BA$ ;

B can have elements {1, 2, 3, 4, 5}

**Answer:**  $5^5$

**Solution:**

$$\text{Let } B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore AB = BA$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow d = b, e = a, f = c, g = h$$

So,  $i$  can be selected in 5 ways and each pair by 5, so total ways =  $5^5$

**Question:** 4 die rolled, numbers are first in  $2 \times 2$  matrices. Find the probability that the matrices is non-singular & all entries are different.

**Answer:**  $\frac{80}{81}$

**Solution:**

$$\text{Let } a = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

For A to be singular,  $ad = bc$

$$\therefore a, d = 1, 6 \text{ and } b, c = 3, 2 \Rightarrow 8$$

$$a, d = 2, 6 \text{ and } b, c = 4, 3 \Rightarrow 8$$

So, total case of singular matrix =  $8 + 8 = 16$

$$\therefore \text{Total non-singular matrix} = 6 \times 5 \times 4 \times 3 - 16$$

$$\therefore \text{Probability} = \frac{344}{360} = \frac{43}{45}$$

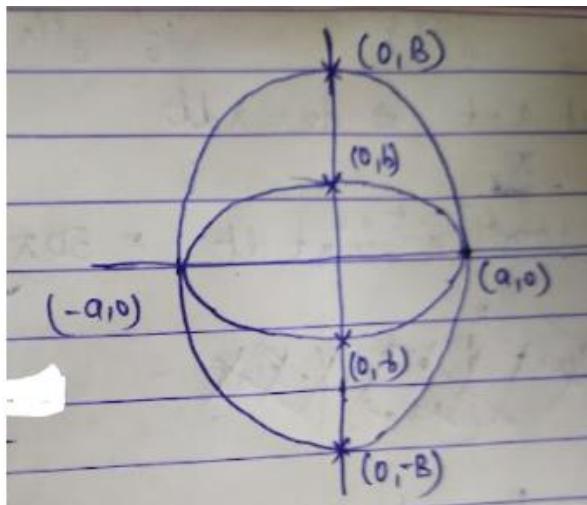
**Question:** If  $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; a > b$ ,  $E_2$  is an ellipse which touches  $E_1$  at the end of major axis and end of minor axis  $E_1$  are the foci of  $E_2$ . The eccentricity of both ellipses are equal. Find 'e'

**Options:**

- (a)
- (b)
- (c)
- (d)

**Answer:** ()

**Solution:**



$$\text{Let } E_2 : \frac{x^2}{a^2} + \frac{y^2}{B^2} = 1$$

Where  $B > a$

$$\because e_1 = e_2 \Rightarrow 1 - \frac{b^2}{a^2} = 1 - \frac{a^2}{B^2} \Rightarrow B^2 = \frac{a^4}{b^2}$$

$$\text{Also, foci of } E_2 = Be = b \Rightarrow e = \frac{b}{B} = \frac{b^2}{a^2} = 1 - e^2$$

$$\therefore e = \frac{\sqrt{5}-1}{2}$$

**Question:** If  $36x^2 + 36y^2 - 108x + 120x + C = 0$  the circle does not cut/touch coordinate axis.

Find the range of 'c'.

**Options:**

(a)  $(100, 165)$

(b)  $(100, 156)$

(c)

(d)

**Answer:** ()**Solution:**

$$36x^2 + 36y^2 - 108x + 120y + C = 0$$

$$x^2 + y^2 - \frac{108x}{36} + \frac{120y}{36} + \frac{C}{36} = 0$$

$$x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$$

Circle does not touch or cut axes

$$\Rightarrow g^2 - C < 0 \text{ and } f^2 - C < 0$$

$$\Rightarrow \left(\frac{3}{2}\right)^2 - \frac{C}{36} < 0 \text{ and } \left(\frac{10}{6}\right)^2 - \frac{C}{36} < 0$$

$$\Rightarrow \frac{9}{4} - \frac{C}{36} < 0 \text{ and } C > 36\left(\frac{100}{36}\right)$$

$$\Rightarrow C > \frac{36 \times 5}{4} \text{ and } \Rightarrow C > 100 \quad \dots(2)$$

$$\Rightarrow C > 81 \quad \dots(1)$$

Also  $g^2 + f^2 - C > 0$ 

$$\Rightarrow \left(\frac{3}{2}\right)^2 + \left(\frac{3}{3}\right)^2 - \frac{C}{36} > 0$$

$$\Rightarrow \frac{C}{36} < \frac{9}{4} + \frac{25}{9}$$

$$\Rightarrow C < 81 + 100$$

$$\Rightarrow C < 181 \quad \dots(3)$$

From (1), (2) and (3)

$$C \in (100, 181)$$

**Question:** If  $f(1) + f(2) + f(3) = 3$

$$A = \{0, 1, 2, 3, \dots, 9\}$$

No. of objective functions  $f : A \rightarrow A$  which satisfy?

**Answer:**  $3! \times 7!$

**Solution:**

$$f(1) + f(2) + f(3) = 3$$

Only possible combination is 0 + 1 + 2

So total bijective function =  $3! \times 7!$

**Question:** If  $P: y^2 = \alpha x$ ;  $L: 2x + y = k$

$L$  is a tangent to  $x^2 - y^2 = 3$  and  $P$ . Find ' $\alpha$ '

**Options:**

- (a) -24
- (b) 24
- (c) -19
- (d) 19

**Answer:** (b)

**Solution:**

$2x + y = k$  is tangent to  $x^2 - y^2 = 3$

$$y = -2x + k$$

$$c^2 = a^2 m^2 - a^2$$

$$\Rightarrow k^2 = 3(-2)^2 - 3$$

$$\Rightarrow k^2 = 9$$

$$\Rightarrow k = \pm 3$$

$$y = -2x + 3 \text{ and } y = -2x - 3$$

If  $y = mx + c$  is tangent to  $y^2 = 4ax$  then  $c = \frac{a}{m}$

$$\Rightarrow 3 = \frac{\alpha}{4(-2)} \text{ or } -3 = \frac{\alpha}{4(-2)}$$

$$\Rightarrow \alpha = -24 \text{ or } \alpha = 24$$

**Question:** If 'n' is the number of solutions of  $z^2 + 3\bar{z} = 0$ , where  $z \in C$ , then find  $\sum_{k=0}^{\infty} \frac{1}{n^k}$

**Answer:**  $\frac{1}{2}$

**Solution:**

$$z = x + iy$$

$$(x + iy)^2 + 3(x - iy) = 0$$

$$x^2 - y^2 + 2ixy + 3x - 3iy = 0$$

$$x^2 - y^2 + 3x + i(2xy - 3y) = 0$$

$$\Rightarrow x^2 - y^2 + 3x = 0 \text{ and } y(2x - 3) = 0$$

$$\text{If } y = 0 \quad x^2 + 3x = 0 \quad \Rightarrow y = 0 \text{ or } x = \frac{3}{2}$$

$$\Rightarrow x = 0, -3$$

$$\text{And if } x = \frac{3}{2}$$

$$y^2 = \left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right)$$

$$= \frac{9}{4} + 3\left(\frac{3}{2}\right) = \frac{9}{4} + \frac{9}{2}$$

$$= \frac{27}{4}$$

Hence 3 solutions

$$\sum_{k=0}^1 \frac{1}{3^k} = \frac{1}{3} + \frac{1}{3^2} + \dots \infty$$

$$= \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$$

**Question:**  $\omega$  be a cube root of unity  $r_1, r_2, r_3$  are the numbers obtained on the dice then the probability of  $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ ?

**Options:**

(a)  $\frac{1}{18}$

(b)  $\frac{1}{9}$

(c)  $\frac{2}{9}$

(d)  $\frac{1}{3}$

**Answer:** (c)

**Solution:**

$$r_1, r_2, r_3 \in \{1, 2, 3, 4, 5, 6\}$$

$r_1, r_2, r_3$  are of the form  $3k, 3k+1, 3k+2$

$$\text{Required Probability} = \frac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \frac{6 \times 8}{216} = \frac{2}{9}$$

**Question:** The number of elements in the set  $\{x \in R : (|x|-3)(|x+4|)=6\}$  is equal to

**Options:**

(a) 3

(b) 4

(c) 2

(d) 1

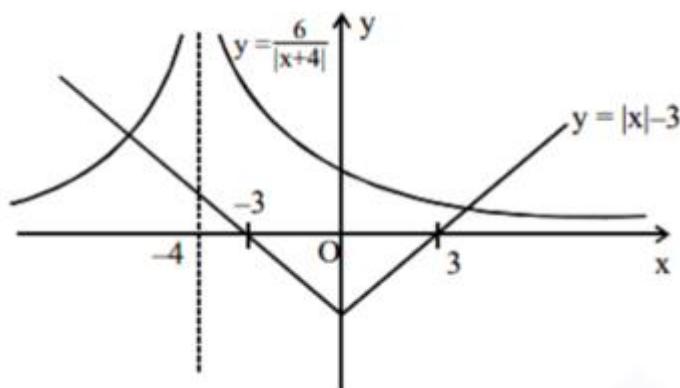
**Answer:** (c)

**Solution:**

$$x \neq -4$$

$$(|x|-3)(|x+4|)=6$$

$$\Rightarrow |x|-3 = \frac{6}{|x+4|}$$



Number of solutions = 2

**Question:**  $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}} dx$

**Answer:**  $\frac{2}{9}$

**Solution:**

$$I = \int \frac{e^x(2-x^2)}{1-\sqrt{1-x^2}} dx$$
$$= \int e^x \left( \frac{1}{(1-x)\sqrt{1-x^2}} + \frac{\sqrt{1+x}}{\sqrt{1-x}} \right) dx$$

Let  $\sqrt{\frac{1+x}{1-x}} = f(x)$

Then  $f'(x) = \frac{1}{(1-x)\sqrt{1-x^2}}$

$$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$$
$$= e^x f(x) + c$$

$$\Rightarrow I = e^x \sqrt{\frac{1+x}{1-x}} + c$$

$$r_1, r_2, r_3 \in \{1, 2, 3, 4, 5, 6\}$$

$r_1, r_2, r_3$  are of the form  $3k, 3k+1, 3k+2$

$$\text{Required Probability} = \frac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \frac{6 \times 8}{216} = \frac{2}{9}$$

**Question:**  $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{2}-\left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1+4\pi^2}$ ,  $\alpha \in R[x]$  is greatest integer

**Options:**

- (a)  $50(e-1)$
- (b)  $150(e^{-1}-1)$
- (c)  $200(1-e^{-1})$
- (d)  $100(1-e)$

**Answer:** (c)

**Solution:**

$$\int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{ \frac{x}{\pi} \right\}}} dx = 100 \int_0^\pi \frac{\sin^2 x}{e^{\frac{x}{\pi}}} dx$$

$$\text{Let } \frac{x}{\pi} = t \Rightarrow dx = \pi dt$$

$$I = 100\pi \int_0^1 e^{-t} \sin^2 \pi t dt = 50\pi \int_0^1 (1 - \cos 2\pi t) e^{-t} dt$$

$$I = 50\pi \left[ \left( -e^{-t} \right)_0^1 - \left( \frac{e^{-t}}{1+4\pi^2} \left\{ -\cos 2\pi t + 2\pi \sin 2\pi t \right\} \right)_0^1 \right]$$

$$= 50\pi \left[ \left( 1 - \frac{1}{e} \right) - \frac{1}{(1+4\pi^2)} \left\{ \frac{-1}{e} + 1 \right\} \right]$$

$$= 50\pi \left( 1 - \frac{1}{e} \right) \left( \frac{4\pi^2}{1+4\pi^2} \right) = \frac{200\pi^3}{1+4\pi^2} (1 - e^{-1})$$

$$\therefore \alpha = 200(1 - e^{-1})$$