ADS Homework 3

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1 Problem 4.1

2 Problem 4.2

2.1 a)

Here,
$T(n) = 36T(n/6) + 2n$
Using master method, we get,
a = 36, b = 6
$n^{\log_b a} = n^2$
From Case 1,
$f(n) = O(n^{2-\epsilon})$
When $\epsilon = 1$,
$f(n) = O(n)$ which is asymptotically/polynomially smaller than $n^{\log_b a}$, So,
$T(n) = \theta(n^2)$

2.2 b)

Here,
$T(n) = 5T(n/3) + 17n^{1.2}$
Using master method, we get,
a = 5, b = 3
$n^{\log_b a} = n^{1.46497}$
From Case 1,
$f(n) = O(n^{2-\epsilon})$
When $\epsilon = 0.26497$,
$f(n) = O(n^{1.2})$ which is asymptotically/polynomially smaller, So,
$T(n) = \theta(n^{1.46497})$

2.3 c)

Here,
$T(n) = 12T(n/2) + n^2 \log n$
Using master method, we get,

a = 12, b = 2
$n^{\log_b a} = n^{3.58496}$
From Case 1,
$f(n) = O(n^{2-\epsilon})$
As $n^2 \log n$ is less than $n^{3.58496}$,
So,
$T(n) = \theta(n^{3.58496})$

2.4 d)

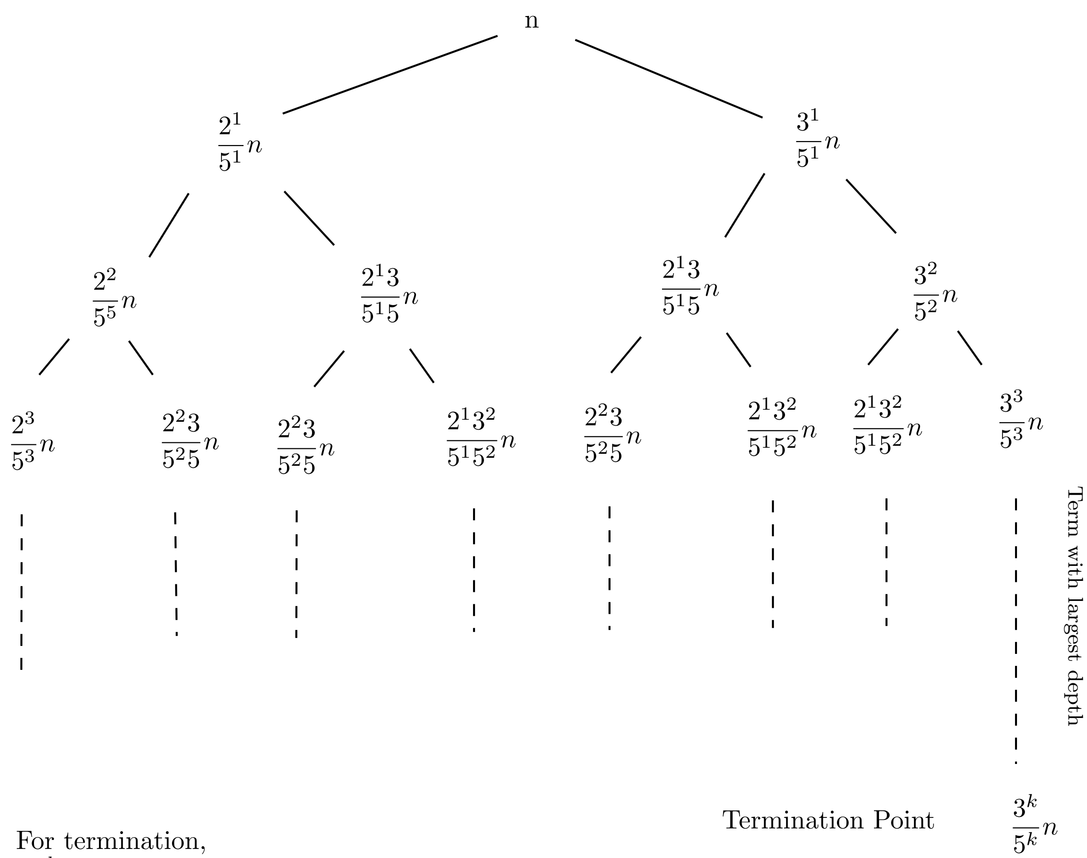
Here,
$T(n) = 3T(n/5) + T(n/2) + 2^n$

As we presume the time complexity for individual cases, the time complexity of 2^n is maximum here in comparison to other logarithmic and polynomial functions, i.e,

$O(2^n) > O(n^{\log_5 3}) > O(1)$
So,
$T(n) = O(2^n)$

2.5 e)

Here,
$T(n) = T(2n/5) + T(3n/5) + \theta(n)$



For termination,

$$\frac{3^k}{5^k} = \frac{1}{n}$$

so,

$$k = \log_{5/3} n$$

Using recursion tree method, we find,
Cost of each level = n
Maximum depth of the recursion = $\log_{5/3} n$
So, the order of time complexity is given by,
$T(n) = O(n \log_{5/3} n)$