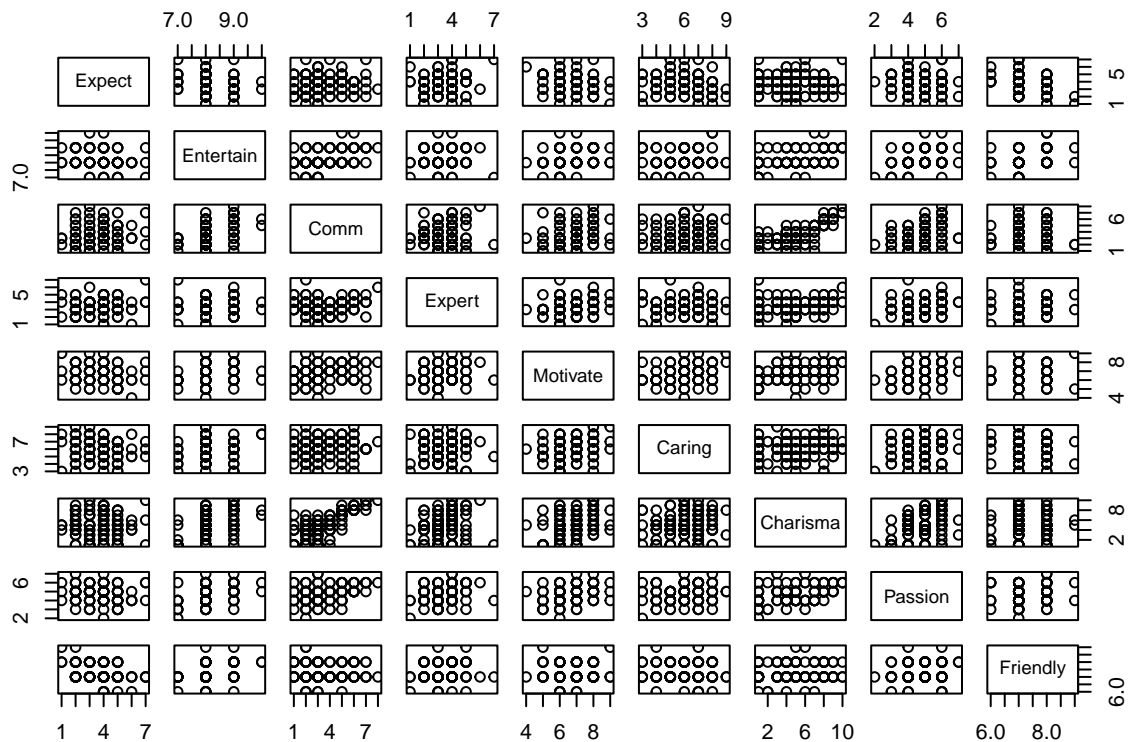# Teacher Evaluation data-Factor Analysis and PCA

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Factor Analysis

Scatterplot matrix

```r
teacher <- read.csv("Teacher_Evaluation.csv")
library(car)
pairs(teacher)
```


Scatter plot matrix for teacher evaluation data is shown in above Figure. Factor analysis tries to explain the covariances or correlation of the observed variables by means of a few common factors. According to Figure most of the variables show linear relationship between other variables implying factor analysis is likely to be helpful for understanding these data.

Factor Analysis - PC method

```r
teacher<-read.csv("Teacher_Evaluation.csv")
teacher.cor<-as.matrix(cor(teacher))
teacher.mat=as.matrix(teacher)
## PC Method
library(psych)
teacher.fa.pc <- lapply(1:4,function(nf)fa(teacher.mat,nfactors=nf,
```

```
                  rotate="none",fm = 'pa',max.iter = 1))

teacher.fa.pc[[4]]$STATISTIC
```

```
## [1] 21.61055
```

```
teacher.fa.pc[[4]]$PVAL
```

```
## [1] 0.001424149
```

Factor analysis with PC method - Test statistics:

No of Factors	Test statistics	p-value
1	93.62929	2.792113e-09
2	50.50468	0.0001104178
3	31.54493	0.001625939
4	21.61055	0.001424149

From PC method it it is unable to find significant adequate number of factors to explain the variables.

Factor Analysis - ML method (Without varimax rotation)

```
#ML method
teacher<-read.csv("Teacher_Evaluation.csv")
teacher.cor<-cor(teacher)
teacher.cor<-as.matrix(teacher.cor)

cat("Unrotated factor analysis with m=1,2,3 by covariance matrix")
```

```
## Unrotated factor analysis with m=1,2,3 by covariance matrix
```

```
teacher.fa1<-lapply(1:4,function(nf) factanal(covmat=teacher.cor,factors=nf,rotation="none",n.obs=120))

#teacher.fa1[1]
#teacher.fa1[2]
teacher.fa1[3]
```

```
## [[1]]
##
## Call:
## factanal(factors = nf, covmat = teacher.cor, n.obs = 120, rotation = "none")
##
## Uniquenesses:
##    Expect Entertain      Comm    Expert  Motivate    Caring  Charisma   Passion
##     0.786     0.570     0.199     0.909     0.631     0.807     0.273     0.623
##  Friendly
##     0.005
##
## Loadings:
##           Factor1 Factor2 Factor3
## Expect            -0.345  -0.307
## Entertain  0.505   0.404   0.105
## Comm       0.881          -0.161
## Expert     0.219   0.187
## Motivate   0.404           0.452
```

```
## Caring      0.172               0.401
## Charisma    0.846    0.107
## Passion     0.500               0.344
## Friendly             0.997
##
##              Factor1 Factor2 Factor3
## SS loadings    2.238   1.337   0.624
## Proportion Var 0.249   0.149   0.069
## Cumulative Var 0.249   0.397   0.466
##
## Test of the hypothesis that 3 factors are sufficient.
## The chi square statistic is 20.61 on 12 degrees of freedom.
## The p-value is 0.0564
```

```r
cat("Residual matrix for m=3")
```

```
## Residual matrix for m=3
```

```r
pred1<-teacher.fa1[[3]]$loadings%*%t(teacher.fa1[[3]]$loadings)+
diag(teacher.fa1[[3]]$uniquenesses)
round(teacher.cor-pred1,digits=3)
```

```
##           Expect Entertain   Comm Expert Motivate Caring Charisma Passion
## Expect     0.000    -0.021  0.006  0.206   -0.007 -0.006   -0.015  -0.008
## Entertain -0.021     0.000  0.015 -0.147   -0.024  0.010   -0.019   0.054
## Comm       0.006     0.015  0.000 -0.028    0.003  0.020    0.000  -0.017
## Expert     0.206    -0.147 -0.028  0.000    0.057 -0.018    0.043   0.100
## Motivate  -0.007    -0.024  0.003  0.057    0.000  0.036    0.005  -0.035
## Caring    -0.006     0.010  0.020 -0.018    0.036  0.000   -0.032  -0.024
## Charisma  -0.015    -0.019  0.000  0.043    0.005 -0.032    0.000   0.011
## Passion   -0.008     0.054 -0.017  0.100   -0.035 -0.024    0.011   0.000
## Friendly   0.000     0.000  0.000  0.001    0.000  0.000    0.000   0.000
##           Friendly
## Expect       0.000
## Entertain    0.000
## Comm         0.000
## Expert       0.001
## Motivate     0.000
## Caring       0.000
## Charisma     0.000
## Passion      0.000
## Friendly     0.000
```

Factor Analysis - ML method (With varimax rotation)

```
## Rotated factor loadings
```

```
## [[1]]
##
## Call:
## factanal(factors = nf, covmat = teacher.cor, n.obs = 120, rotation = "varimax")
##
## Uniquenesses:
##     Expect Entertain      Comm    Expert  Motivate    Caring  Charisma   Passion
##      0.786     0.570     0.199     0.909     0.631     0.807     0.273     0.623
##   Friendly
##      0.005
```

```
## 
## Loadings:
##           Factor1 Factor2 Factor3
## Expect            -0.399  -0.225
## Entertain  0.507   0.373   0.182
## Comm       0.884           0.105
## Expert     0.209   0.182   0.120
## Motivate   0.266           0.538
## Caring                     0.437
## Charisma   0.824           0.218
## Passion    0.394   0.115   0.456
## Friendly   0.131   0.975  -0.164
## 
##                 Factor1 Factor2 Factor3
## SS loadings       2.012   1.315   0.872
## Proportion Var    0.224   0.146   0.097
## Cumulative Var    0.224   0.370   0.466
## 
## Test of the hypothesis that 3 factors are sufficient.
## The chi square statistic is 20.61 on 12 degrees of freedom.
## The p-value is 0.0564


## Residual matrix for m=3

##           Expect Entertain   Comm Expert Motivate Caring Charisma Passion
## Expect     0.000   -0.021  0.006  0.206   -0.007 -0.006   -0.015  -0.008
## Entertain -0.021    0.000  0.015 -0.147   -0.024  0.010   -0.019   0.054
## Comm       0.006    0.015  0.000 -0.028    0.003  0.020    0.000  -0.017
## Expert     0.206   -0.147 -0.028  0.000    0.057 -0.018    0.043   0.100
## Motivate  -0.007   -0.024  0.003  0.057    0.000  0.036    0.005  -0.035
## Caring    -0.006    0.010  0.020 -0.018    0.036  0.000   -0.032  -0.024
## Charisma  -0.015   -0.019  0.000  0.043    0.005 -0.032    0.000   0.011
## Passion   -0.008    0.054 -0.017  0.100   -0.035 -0.024    0.011   0.000
## Friendly   0.000    0.000  0.000  0.001    0.000  0.000    0.000   0.000
##           Friendly
## Expect       0.000
## Entertain    0.000
## Comm         0.000
## Expert       0.001
## Motivate     0.000
## Caring       0.000
## Charisma     0.000
## Passion      0.000
## Friendly     0.000
```

Factor analysis without varimax rotation ML method Test statistics:

No of Factors	Test statistics	p-value
1	84.21	8.56e-08
2	42.5	0.00152
3	20.61	0.0564

Factor analysis with varimax rotation ML method Test statistics:

No of Factors	Test statistics	p-value
1	84.21	8.56e-08
2	42.5	0.00152
3	20.61	0.0564

From two method with and without varimax rotation give the different results.

Large Sample Test for the number of common factors:

$$H_0 : \boldsymbol{\Sigma} = \mathbf{LL'} + \boldsymbol{\Psi}$$
$$H_a : \boldsymbol{\Sigma} \quad \text{any other positive definite matrix}$$

Test statistic:

$$n - 1 - \frac{2p + 4m + 5}{6} ln \frac{|\hat{L}\hat{L}' + \hat{\Psi}|}{|S_n|}$$

For method with and without varimax rotation have similar results

For both methods p-value is 0.0564>0.05. We do not reject H_0. Hence we can conclude that three factor model is adequate by the test.

For both methods the off diagonal values of the residual matrix are close to zero. So again we can conclude that three factor model is adequate to explain the nine variables.

Factor 1: Variables Entertaining, Communicate effectively and Charismatic are included in the first factor. (engaging skills)

Factor 2: Only Friendly and easy-going variable is belong to second factor

Factor 3: Generally all nine variables are included in third factor. But motivation has higher contribution.

$\boldsymbol{\Sigma} = \mathbf{LL'} + \boldsymbol{\Psi}$

Factor loading matrix:

```
##
## Loadings:
##           Factor1 Factor2 Factor3
## Expect            -0.399  -0.225
## Entertain  0.507   0.373   0.182
## Comm       0.884           0.105
## Expert     0.209   0.182   0.120
## Motivate   0.266           0.538
## Caring                     0.437
## Charisma   0.824           0.218
## Passion    0.394   0.115   0.456
## Friendly   0.131   0.975  -0.164
##
##                Factor1 Factor2 Factor3
## SS loadings      2.012   1.315   0.872
## Proportion Var   0.224   0.146   0.097
## Cumulative Var   0.224   0.370   0.466
```

Specific variance matrix:

```
diag(teacher.fa2[[3]]$uniquenesses)
```

```
##             [,1]      [,2]     [,3]      [,4]      [,5]      [,6]      [,7]
##  [1,] 0.7856282 0.0000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000
##  [2,] 0.0000000 0.5703421 0.000000 0.0000000 0.0000000 0.0000000 0.0000000
##  [3,] 0.0000000 0.0000000 0.198612 0.0000000 0.0000000 0.0000000 0.0000000
##  [4,] 0.0000000 0.0000000 0.000000 0.9088882 0.0000000 0.0000000 0.0000000
##  [5,] 0.0000000 0.0000000 0.000000 0.0000000 0.6305646 0.0000000 0.0000000
##  [6,] 0.0000000 0.0000000 0.000000 0.0000000 0.0000000 0.8065978 0.0000000
##  [7,] 0.0000000 0.0000000 0.000000 0.0000000 0.0000000 0.0000000 0.2725809
##  [8,] 0.0000000 0.0000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000
```

```
##  [9,] 0.0000000 0.0000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000
##            [,8]   [,9]
##  [1,] 0.0000000 0.000
##  [2,] 0.0000000 0.000
##  [3,] 0.0000000 0.000
##  [4,] 0.0000000 0.000
##  [5,] 0.0000000 0.000
##  [6,] 0.0000000 0.000
##  [7,] 0.0000000 0.000
##  [8,] 0.6233554 0.000
##  [9,] 0.0000000 0.005
```

Correlation/covariance matrix:

```
##              Expect Entertain    Comm   Expert Motivate   Caring Charisma
## Expect      0.99999  -0.15597 0.07307 -0.08557 -0.14016 -0.09924 -0.00653
## Entertain  -0.15597   1.00000 0.43239  0.19568  0.26762  0.10687  0.46938
## Comm        0.07307   0.43239 1.00000  0.18025  0.28335  0.08596  0.74832
## Expert     -0.08557   0.19568 0.18025  1.00001  0.13711  0.06401  0.20411
## Motivate   -0.14016   0.26762 0.28335  0.13711  1.00001  0.24870  0.33984
## Caring     -0.09924   0.10687 0.08596  0.06401  0.24870  1.00000  0.13388
## Charisma   -0.00653   0.46938 0.74832  0.20411  0.33984  0.13388  1.00001
## Passion    -0.12178   0.32573 0.38585  0.15795  0.36121  0.21884  0.42807
## Friendly   -0.34391   0.40076 0.00692  0.18511  0.03730 -0.05533  0.10316
##             Passion Friendly
## Expect     -0.12178 -0.34391
## Entertain   0.32573  0.40076
## Comm        0.38585  0.00692
## Expert      0.15795  0.18511
## Motivate    0.36121  0.03730
## Caring      0.21884 -0.05533
## Charisma    0.42807  0.10316
## Passion     1.00000  0.08880
## Friendly    0.08880  1.00000
```

Principal Component Analysis

```
teacher.pc <- princomp(teacher,cor=TRUE) # used CORRELATION matrix
summary(teacher.pc,loadings=TRUE)
```

```
## Importance of components:
##                          Comp.1    Comp.2    Comp.3    Comp.4     Comp.5
## Standard deviation     1.6971850 1.1994392 1.0788557 1.0121525 0.83976703
## Proportion of Variance 0.3200485 0.1598505 0.1293255 0.1138281 0.07835652
## Cumulative Proportion  0.3200485 0.4798990 0.6092245 0.7230526 0.80140915
##                           Comp.6     Comp.7     Comp.8     Comp.9
## Standard deviation     0.80475053 0.74992883 0.58757842 0.48171840
## Proportion of Variance 0.07195816 0.06248814 0.03836093 0.02578362
## Cumulative Proportion  0.87336730 0.93585544 0.97421638 1.00000000
##
## Loadings:
##           Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
## Expect     0.109  0.639  0.257  0.114  0.417         0.481  0.284
## Entertain -0.412 -0.253  0.181 -0.262  0.340         0.436 -0.593
## Comm      -0.444  0.291  0.187 -0.302         0.133 -0.301        -0.692
## Expert    -0.216  0.136  0.183  0.840               -0.221 -0.379
```

```
## Motivate  -0.341          -0.420  0.156 -0.424  0.560  0.425
## Caring    -0.177          -0.715          0.633          -0.216
## Charisma  -0.481  0.200  0.180 -0.164                    -0.361  0.168  0.702
## Passion   -0.405          -0.108  0.116 -0.215 -0.783  0.282  0.259
## Friendly  -0.178 -0.618  0.319  0.231  0.274  0.190          0.559
```

Proportions of variances

	Factor 1	Factor 2	Factor 3	Factor 4
FA Proportion Var	0.26	0.09	0.04	0.03
FA Cumulative Var	0.26	0.35	0.39	0.42
PC Proportion Var	0.3200485	0.1598505	0.1293255	0.1138281
PC Cumulative Var	0.3200485	0.4798990	0.6092245	0.7230526