Homework 7

Problem 1: Population Growth

The file `population.mat` contains two 21×1 vectors: `t` and `N`. The entries of `t` represent time in years since 1900. For example, $t = 0$ represents the year 1900, $t = -90$ represents the year 1810 and $t = 110$ represents the year 2010. The entries of N represent the population of the United States in millions at the corresponding year. If you download `population.mat` and save it in the same folder as your code, then you can access both of these vectors with the code `load('population.mat')`. You should *not* upload this `.mat` file to scorelator. Scorelator has its own copy of `population.mat`.

You may find some or all of the following difference schemes useful in this problem:

$$
\begin{aligned}
f'(x) &\approx \frac{f(x+\Delta x) - f(x)}{\Delta x}, \\
f'(x) &\approx \frac{f(x) - f(x-\Delta x)}{\Delta x}, \\
f'(x) &\approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}, \\
f'(x) &\approx \frac{-3f(x) + 4f(x+\Delta x) - f(x+2\Delta x)}{2\Delta x}, \\
f'(x) &\approx \frac{3f(x) - 4f(x-\Delta x) + f(x-2\Delta x)}{2\Delta x}.
\end{aligned}
$$

The first two are first order approximations and the last three are second order approximations.

For each of the following, use a central difference scheme if possible.

(a) Estimate $\frac{\mathrm{d}N}{\mathrm{d}t}$ at the year 1810 using a second order accurate difference scheme. Save your answer in `A1.dat`.

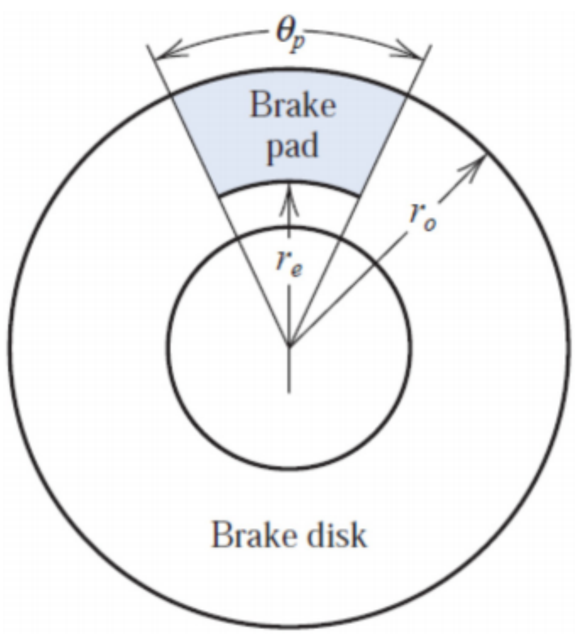
(b) Estimate $\frac{\mathrm{d}N}{\mathrm{d}t}$ at the year 1910 using a second order accurate difference scheme. Save your answer in `A2.dat`.

(c) Estimate $\frac{\mathrm{d}N}{\mathrm{d}t}$ at the year 2010 using a second order accurate difference scheme. Save your answer in `A3.dat`.

(d) The per capita growth rate of the US population at time t is given by $\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}t}$. Use a second order accurate difference scheme to approximate $\frac{\mathrm{d}N}{\mathrm{d}t}$ at all 21 times in `t`, then use these approximations to estimate the per capita growth rate at all 21 times in `t`. Save the *average* of these per capita growth rates in `A4.dat`. You may find the `mean` command helpful.

Problem 2: Brake Pads

Brake pads tend to get extremely hot, and it is important to be able to accurately predict this temperature. As a first approximation, we can assume that a brake pad is a section of an annulus, as shown in the following diagram:



Here, r_e represents the radius at which the pad-disc contact begins, r_o represents the radius at which contact ends and θ_p represents the angular width of the brake pad.

The disc (attached to your vehicle's wheel) is moving faster farther from the center, and the brake pad therefore gets hotter as you move toward the outside. If we let $T(r)$ denote the temperature of the brake pad at radius r, then the average temperature of the brake pad is given by

$$\bar{T} = \frac{T_{total}}{A} = \frac{\int_{r_e}^{r_o} rT(r)\theta_p \, \mathrm{d}r}{\int_{r_e}^{r_o} r\theta_p \, \mathrm{d}r}.$$

The exact formula for $T(r)$ is difficult to obtain, but it is easy to measure the temperature at various points on any given brake pad. In particular, the file `brake_pad.mat` contains two 11×1 vectors `r` and `T`. The vector `r` contains radius values (in feet) and the vector `T` contains corresponding temperatures (in degrees Fahrenheit). Do not upload this `.mat` file to scorelator. Scorelator has its own copy of `brake_pad.mat`. The brake pad that this data came from has $r_e = 0.308$, $r_o = 0.478$ and $\theta_p = 0.7051$.

(a) Calculate T_{total} using the left rectangle rule and save your answer in `A5.dat`. Calculate A using the left rectangle rule and save your answer in `A6.dat` (**Note:** This used to say the right rectangle rule, but they should both be left). Using these two approximations, calculate $\bar{T}$ and save it in `A7.dat`.

(b) Calculate T_{total} using the trapezoidal rule and save your answer in `A8.dat`. Calculate A using the trapezoidal rule and save your answer in `A9.dat`. Using these two approximations, calculate $\bar{T}$ and save it in `A10.dat`.