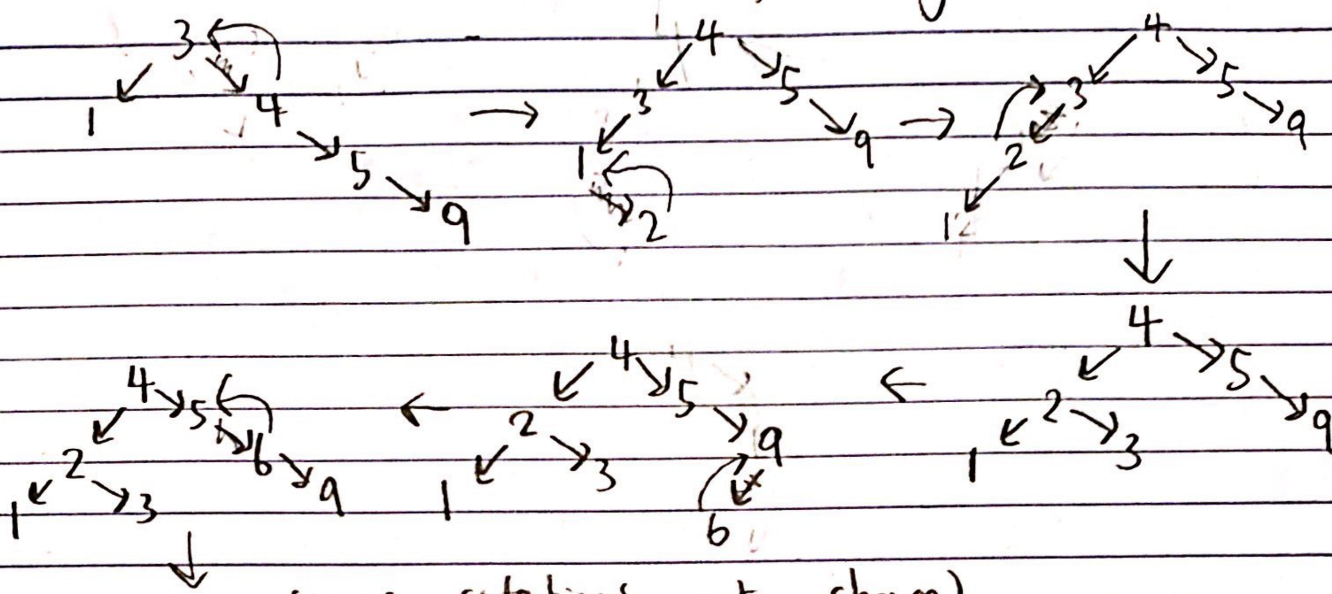
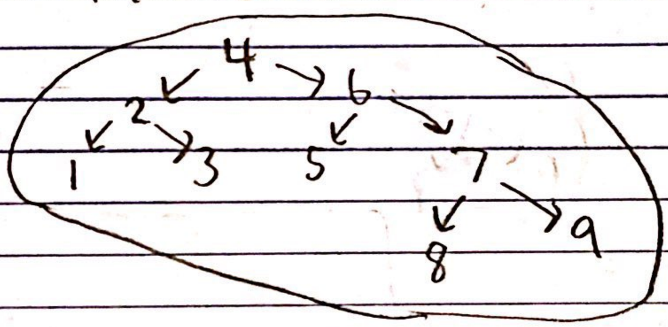
Homework 4: AVL trees, testing, ... Sanjeev Janarthanan

1.



... (more rotations not shown)



Homework 4

2a)

$$(0)\quad \sum_{i=0}^{n-1}\left(\sum_{j=0}^{i-1} j + \sum_{j=0}^{n^2-1} 5i\right)$$

Simplifying $\sum_{j=0}^{i-1} j$

$$\sum_{j=0}^{i-1} j = \frac{i(i-1)}{2}$$ 1. Gauss's identity

Simplifying $\sum_{j=0}^{n^2-1} 5i$

$$\sum_{j=0}^{n^2-1} 5i = 5\sum_{j=0}^{n^2-1} i = 5n^2 \cdot i$$ 1. Factoring out a constant 2. Summation of a constant

Plugging back into the original summation (0)

$$(1)\quad \sum_{i=0}^{n-1}\left(\frac{i(i-1)}{2} + 5n^2 \cdot i\right) = \sum_{i=0}^{n-1} \frac{i(i-1)}{2} + \sum_{i=0}^{n-1} 5n^2 \cdot i$$ 1. Splitting a sum

Simplifying $\sum_{i=0}^{n-1} \frac{i(i-1)}{2}$

$$\sum_{i=0}^{n-1} \frac{i(i-1)}{2} = \sum_{i=0}^{n-1} \frac{i^2-i}{2} = \frac{1}{2}\sum_{i=0}^{n-1} i^2 - i = \frac{1}{2}\left(\sum_{i=0}^{n-1} i^2 - \sum_{i=0}^{n-1} i\right)$$

$$\frac{1}{2}\left(\sum_{i=0}^{n-1} i^2 - \sum_{i=0}^{n-1} i\right) = \frac{1}{2}\left(\frac{n(n-1)(2n-1)}{6} - \frac{n(n-1)}{2}\right)$$

1. Algebra
2. Factoring out a constant
3. Splitting a sum
4. Sum of squares
5. Gauss's identity

Simplifying $\sum_{i=0}^{n-1} 5n^2 \cdot i$

$$\sum_{i=0}^{n-1} 5n^2 \cdot i = 5n^2 \sum_{i=0}^{n-1} i = 5n^2 \cdot \frac{n(n-1)}{2}$$ 1. Factoring out a constant 2. Gauss's identity

Homework 4

2.a) (Cont'd. from previous page)
Plugging back into summation (1)

$$\boxed{\frac{1}{2}\left(\frac{n(n-1)(2n-1)}{6} - \frac{n(n-1)}{2}\right) + 5n^2 \cdot \frac{n(n-1)}{2}}$$

b) $E(n) = \begin{cases} 4 & \text{when } n \leq 7 \\ E(n-1) + n & \text{Otherwise} \end{cases}$

$E(n-1) + n$ $\quad i=1$
$E(n-2) + n + n$ $\quad i=2$
$E(n-3) + n + n + n$ $\quad i=3$
$\dots$
$E(n-i) + \sum_{j=0}^{i-1} n$ $\quad i = n-i$

When $i = n$..
$E(n-n) + \sum_{j=0}^{n-1} n = 4 + \sum_{j=0}^{n-1} n = 4 + n^2$ (summation of a constant)

3.a) Show that $6n + n \cdot \log(n) \in \Omega(10\log(n))$ is true

$6 \cdot n \geq 1 \cdot 10 \cdot \log(n)$ for all n
$n \cdot \log(n) \geq 1 \cdot 10 \cdot \log(n)$ for all $n \geq 10$
All inequalities are true for $n \geq 10$

$6 \cdot n + n \cdot \log(n) \geq (1+1) \cdot 10\log(n) = 2 \cdot 10\log(n)$ for all $n \geq 10$

Thus, we take $n_0 = 10$ and $c = 2$

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3.b) Show that $\log_3(n) \in O(\log_5(n))$

$$\log_3(n) = \frac{\log_5(n)}{\log_5(3)}$$

$$\frac{1}{\log_5(3)} \cdot \log_3(n) \leq 10 \cdot \log_5(n) \text{ for all } n$$

This inequality is true for all n

$$\frac{1}{\log_5(3)} \cdot \log_3(n) \leq (10) \cdot \log_5(n) \text{ for all } n$$

Thus, we take $n_0 = 10$ and $c = 10$

4a) $\Omega(n^2)$, $\Theta(n^2)$, $O(n^2)$
b) $\Omega(n)$, $\Theta(n)$, $O(n)$
c) $\Omega(1)$, $\Theta(1)$, $O(1)$
d) $\Omega(n)$, $\Theta(n)$, $O(n)$
e) $\Omega(1)$, $O(n)$. Big Θ does not exist for this function because the function does not converge. This means that there exists no $f(n)$ which is in $O(g(n))$ and $\Omega(g(n))$, which means $f(n)$ cannot be in $\Theta(g(n))$.

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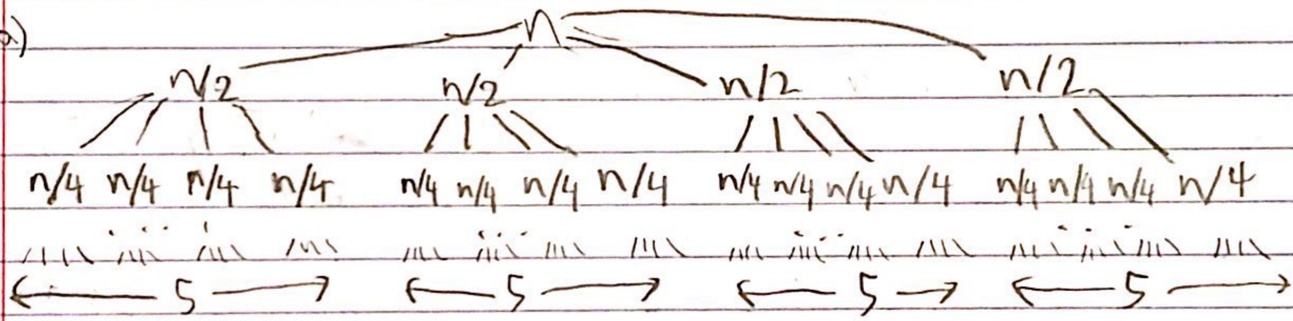
5.a) $T(n) = c_1 + \sum_{i=0}^{n-1}\left(c_2 \sum_{j=0}^{i-1} c_3\right) + c_4$

b) $T(n) = \begin{cases} 1 & n = 0 \\ 1 + T(n/2) & \text{otherwise} \end{cases}$

6.a)



b) $n/2^i$

c) 4^i

d) size of input at level · number of nodes at level = work at ith level

$4^i \cdot \frac{n}{2^i} = 2^i \cdot n$

e) $\log_2(n)$

f) $5 \cdot 4^{\log_2(n)}$

(continued on next page)

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6.g) $\sum_{i=0}^{\log(n)-1} 2^i \cdot n + 5 \cdot 4^{\log_2(n)}$

$\sum_{i=0}^{\log(n)-1} 2^i \cdot n + \sum_{i=0}^{\log(n)-1} 5 \cdot 4^{\log(n)}$ (Splitting a Sum)

1. Factoring Out a Constant
2. Finite Geometric Series

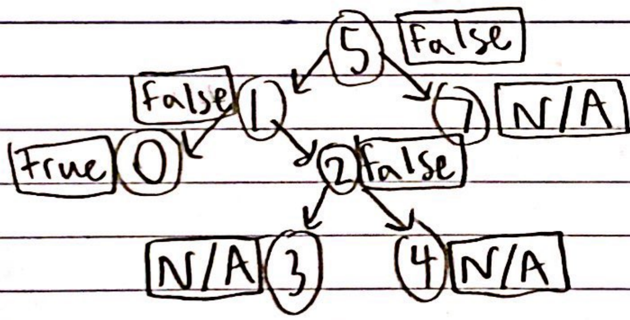
1. Summation of a constant

$n \cdot \frac{2^{\log(n)} - 1}{2 - 1} + 5 \cdot \log(n) \cdot 4^{\log(n)}$

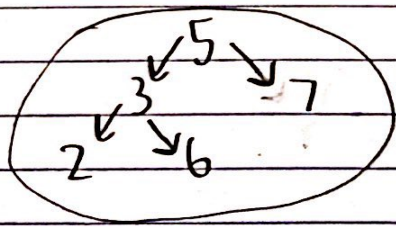
$n^2 - n + 5 \cdot \log(n) \cdot \sqrt{n}$

h) $O(n^2)$

7.a)



b)



6 invalidates the BST but Bob's method returns true.

c) Bob's method does not check if an element in the left subtree is greater than the overall root nor does it check if an element in the right subtree is less than the overall root. Either of these cases invalidates a BST