



Adversarial Search II: Alpha-Beta Pruning

CSE 415: Introduction to Artificial Intelligence
University of Washington
Winter, 2020

Alpha-Beta Pruning

Enhance minimax search with two extra values at each tree node that represent the interval in which the "solution" value must lie.

$$[\alpha, \beta]$$

Initialize the root's to $[-\infty, \infty]$.

Update these at the current node, when possible.

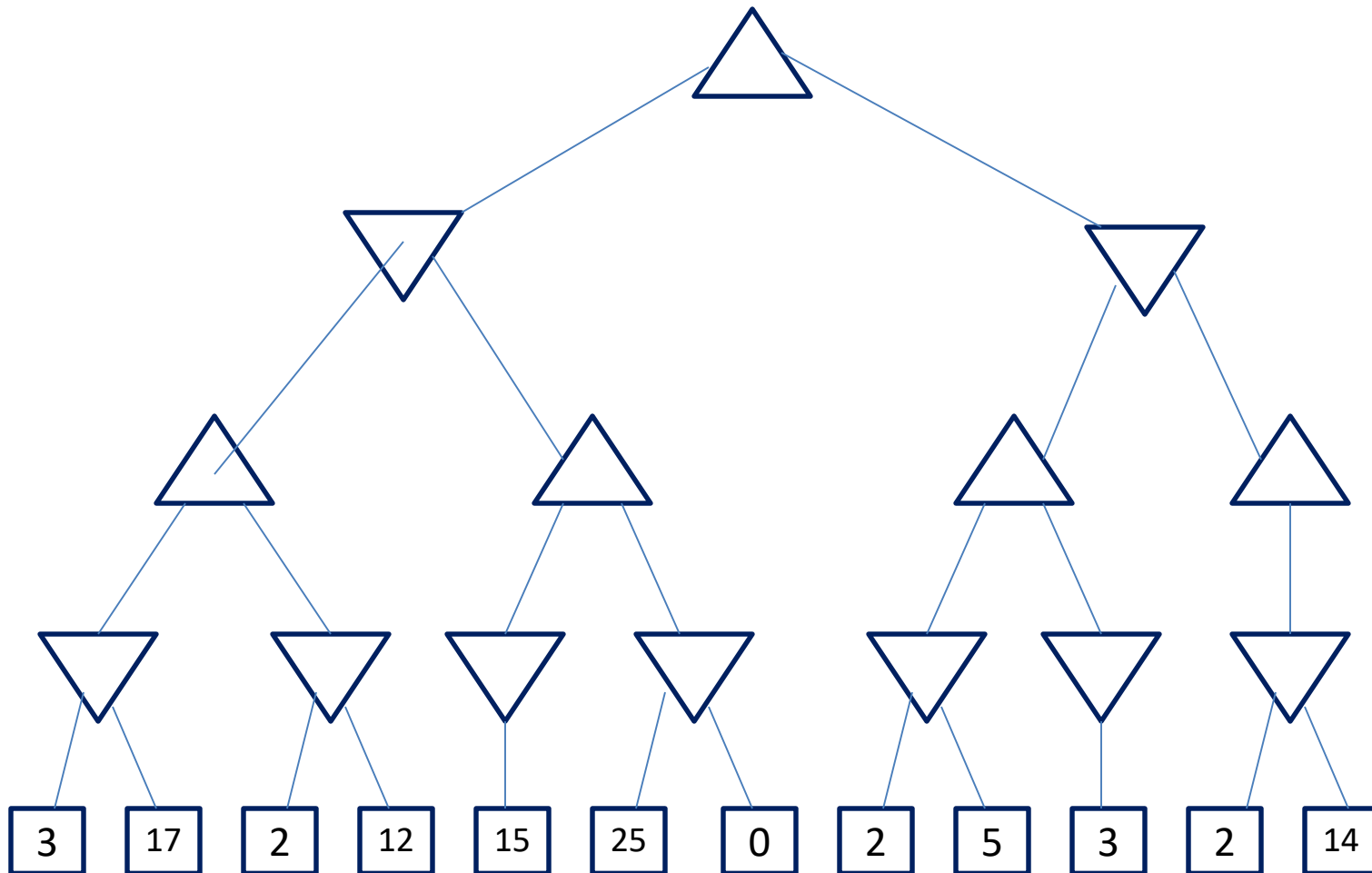
If any node gets $\alpha \geq \beta$, then it is "finished", so "prune off" any of its children that remain.

Alpha-Beta Cutoffs

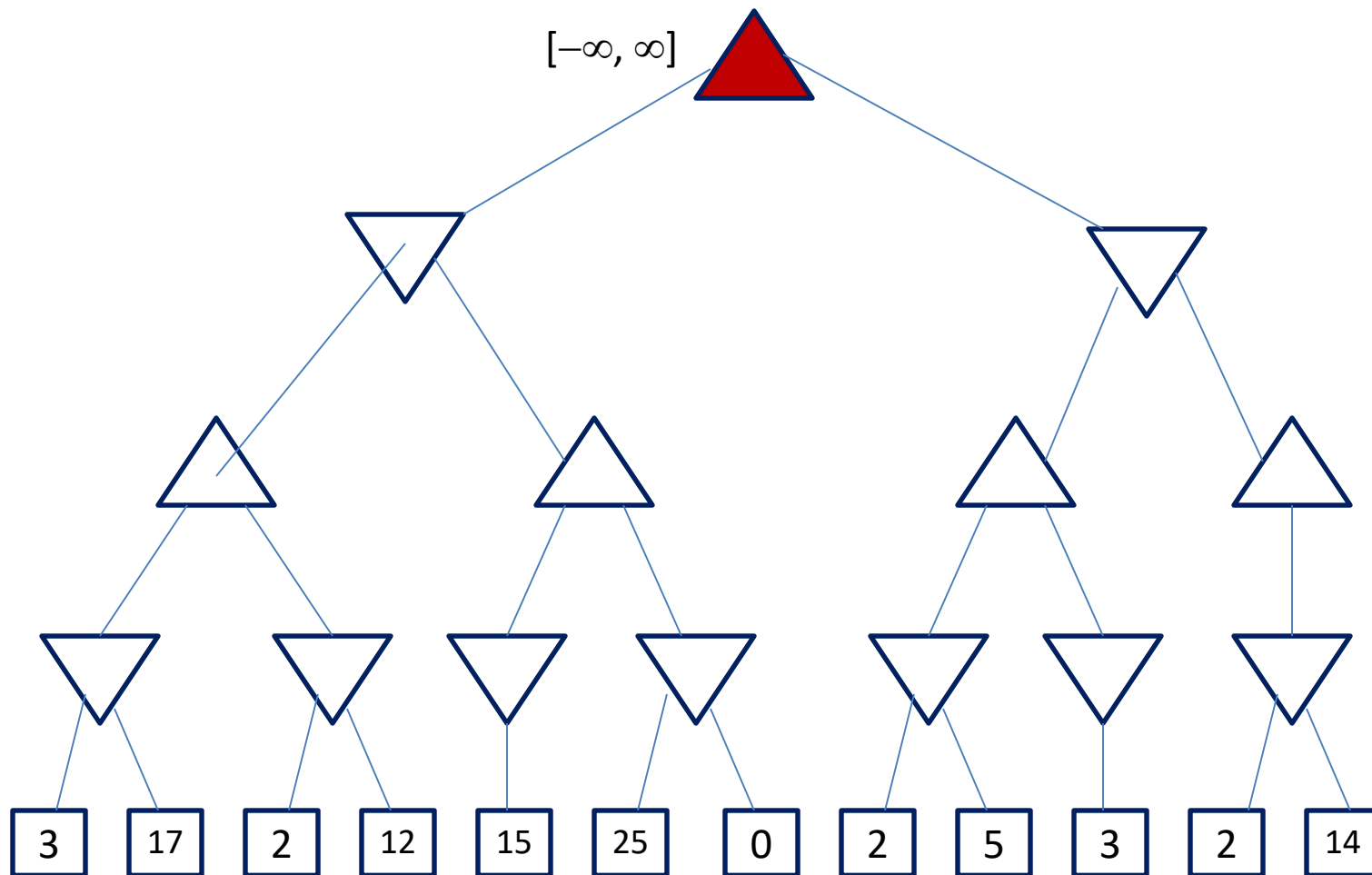
An **alpha** (beta) **cutoff** occurs at a Maximizing (minimizing) node when it is known that the maximizing (minimizing) player has a move that results in a value **alpha** (beta) and, subsequently, when an alternative to that move is explored, it is found that the alternative gives the opponent the option of moving to a lower (higher) valued position.

Any further exploration of the alternative can be canceled.

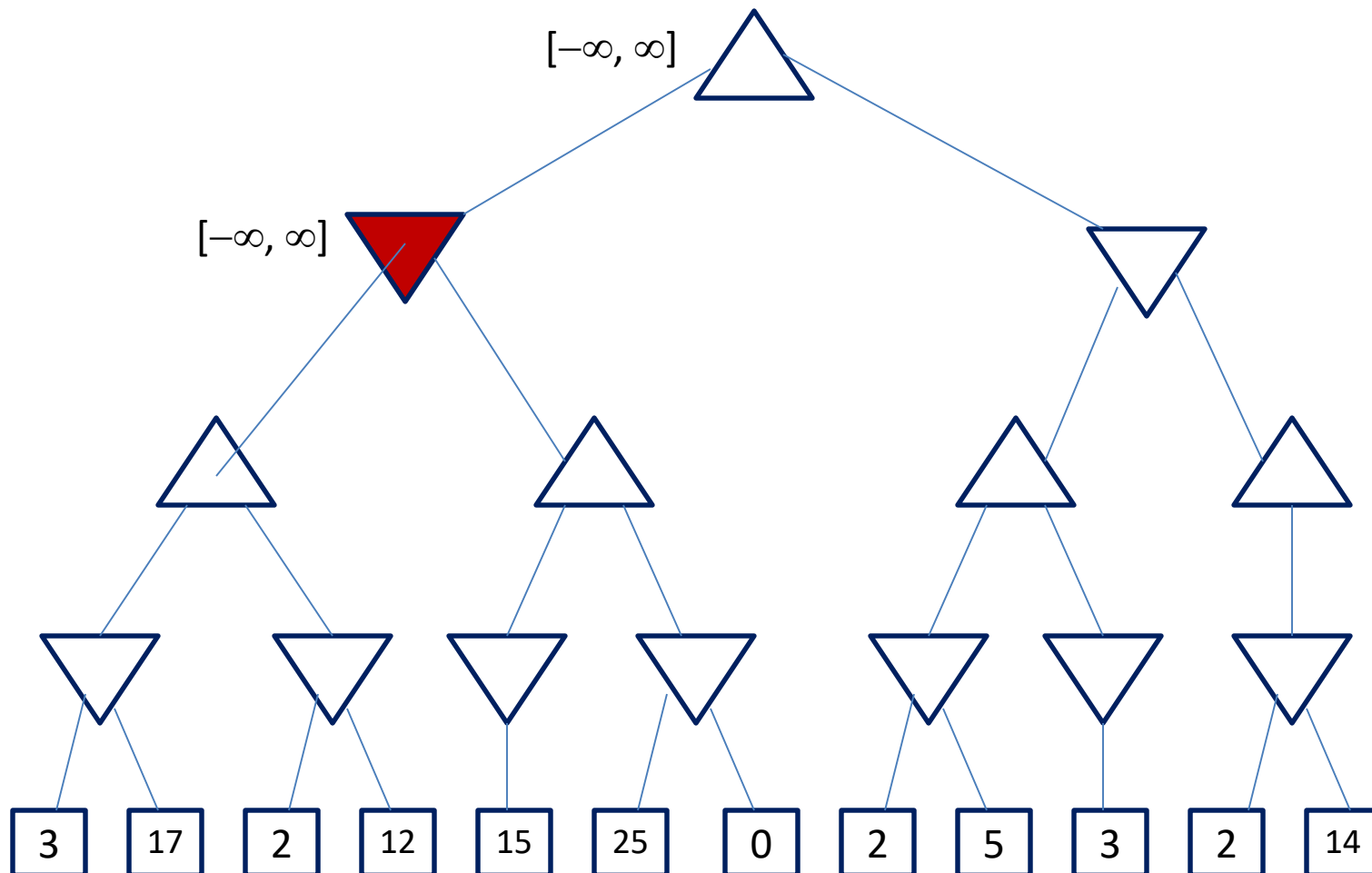
Alpha-Beta Pruning



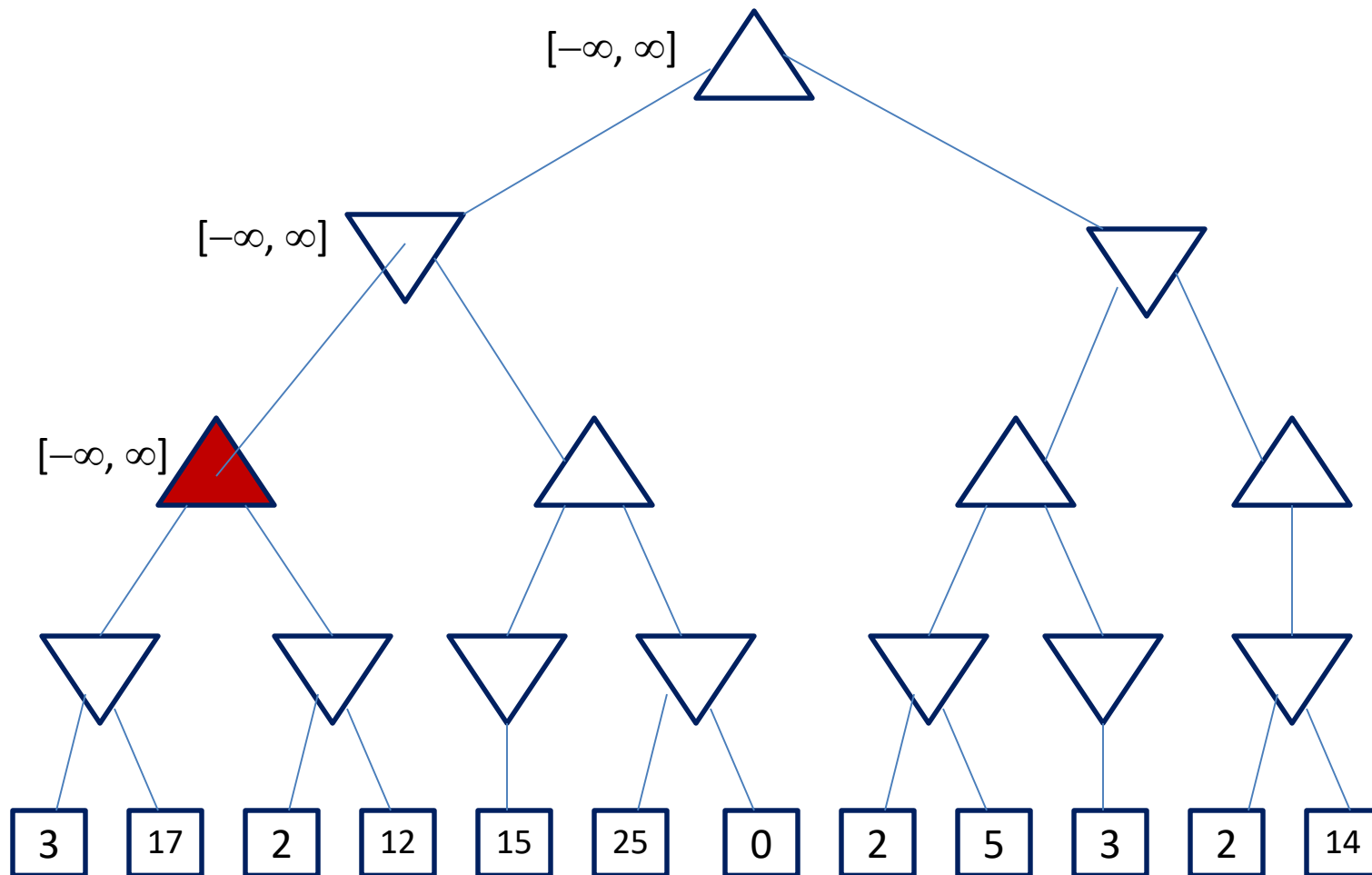
Alpha-Beta Pruning



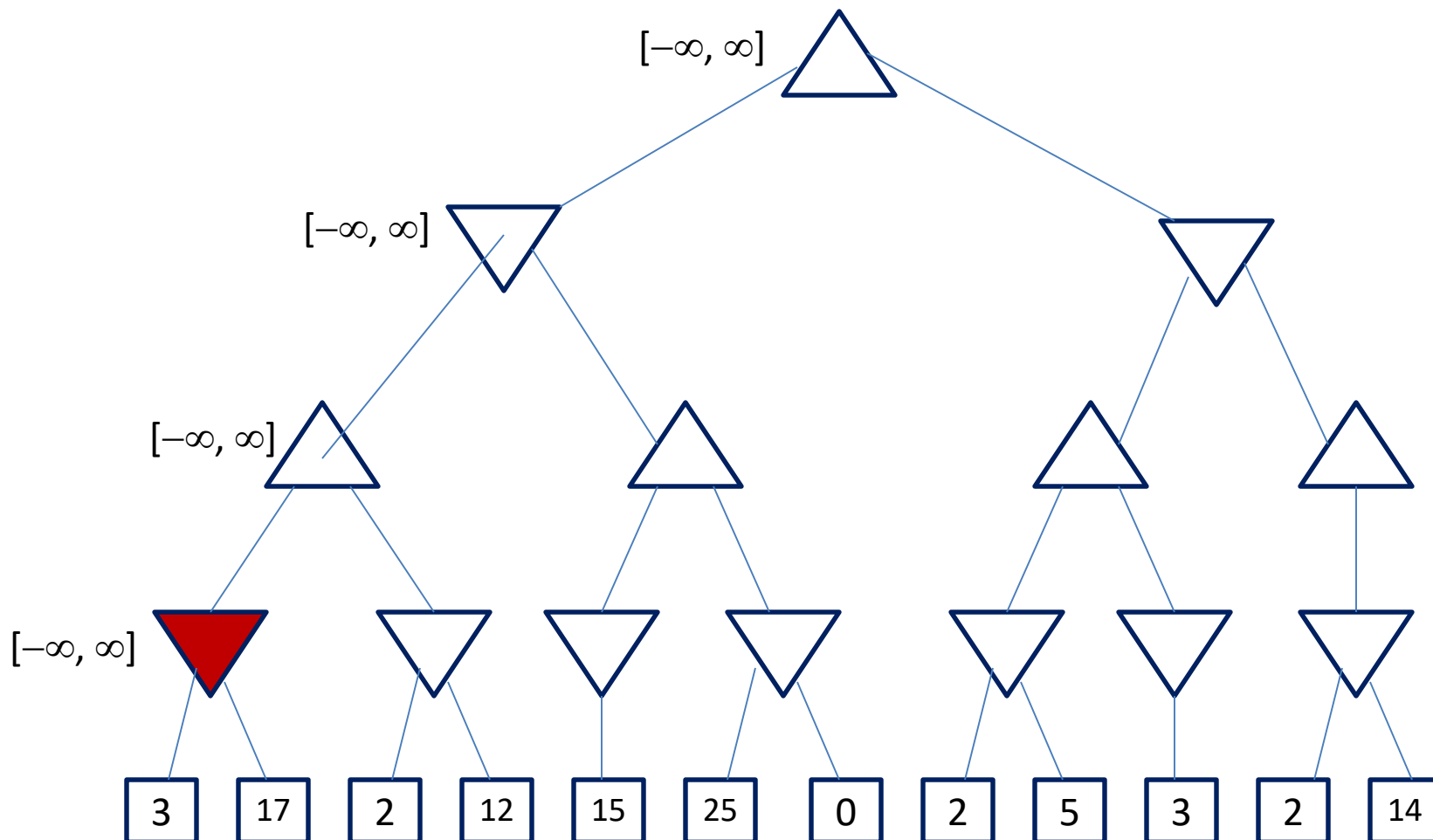
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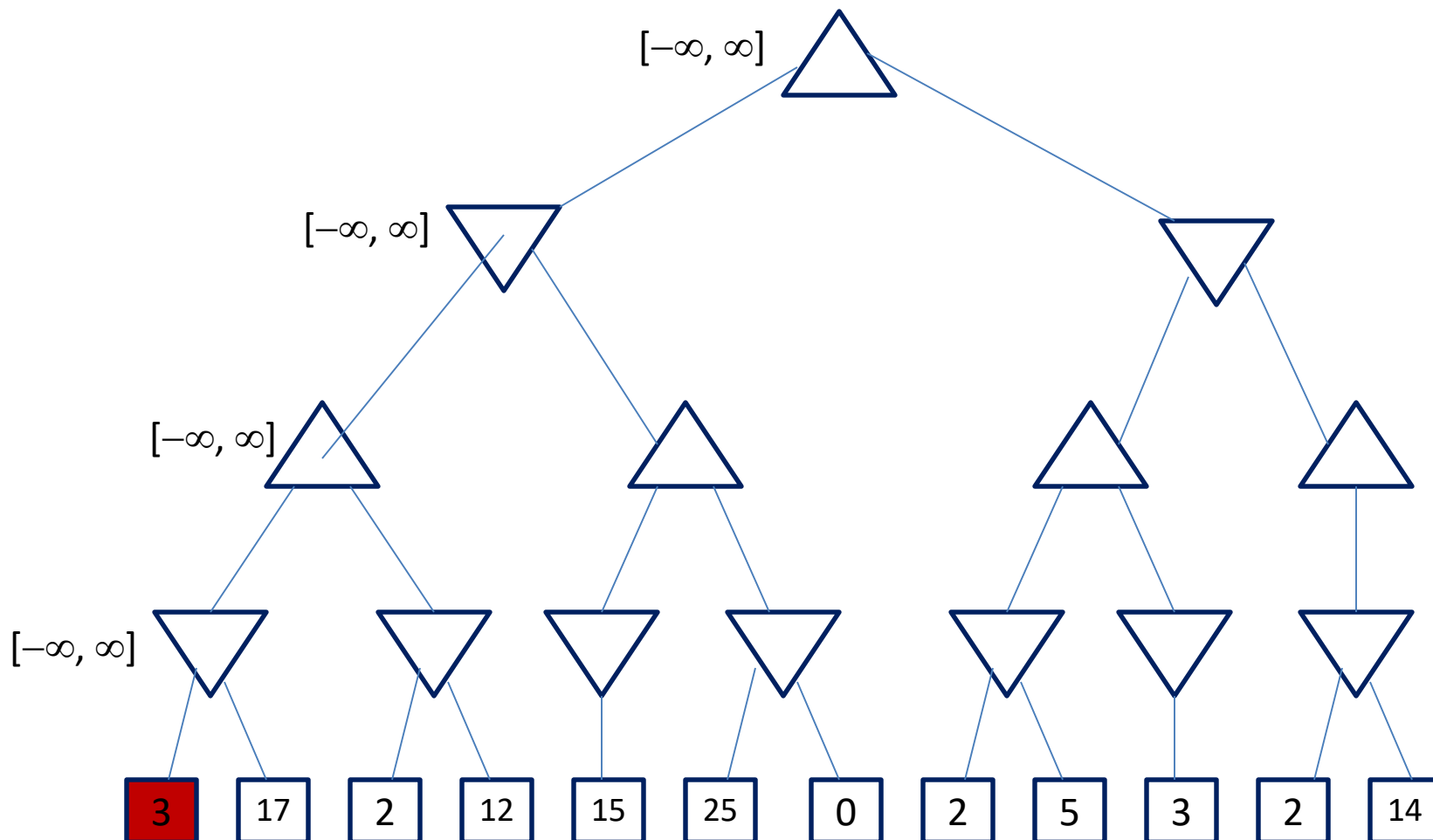
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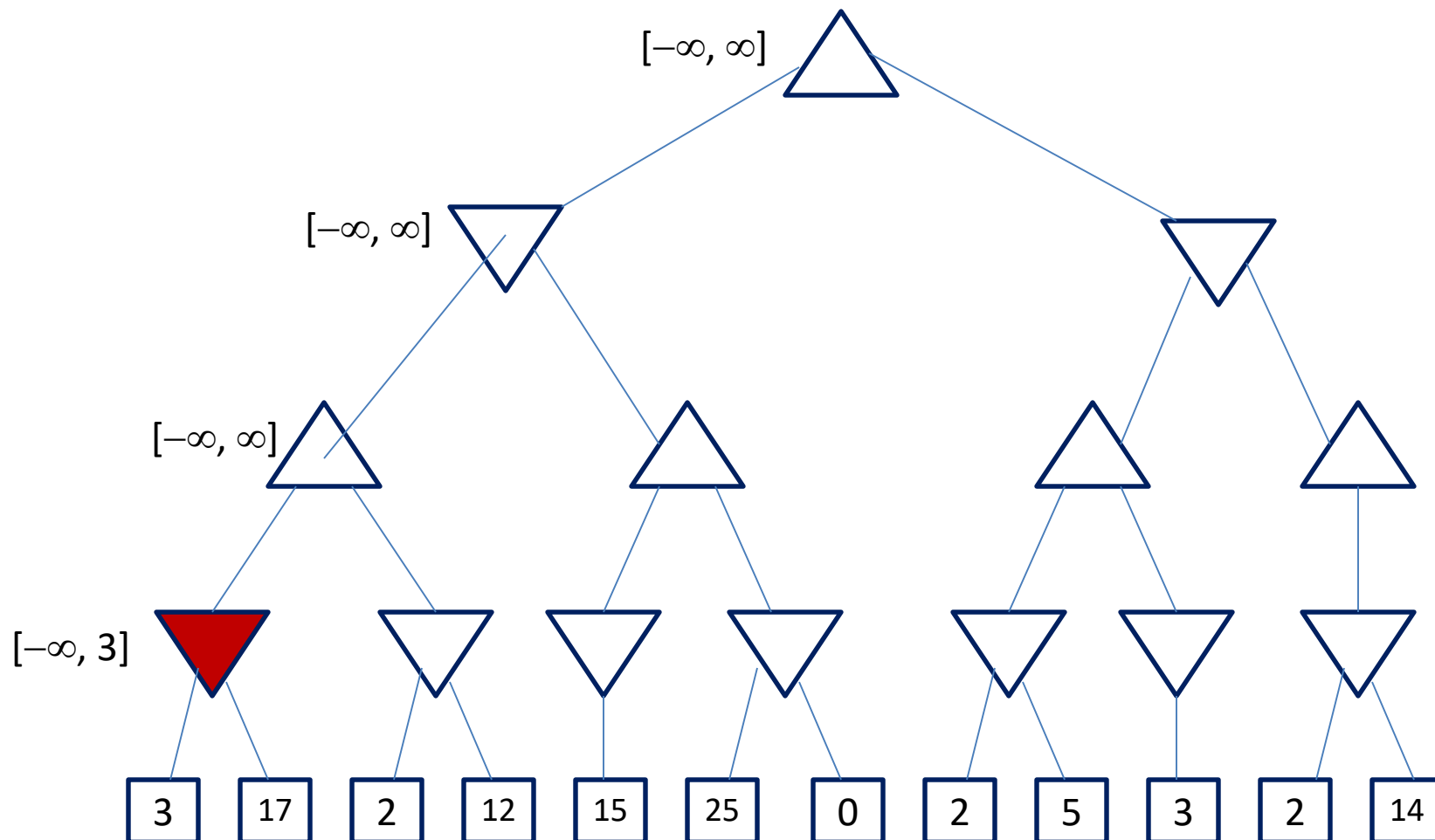
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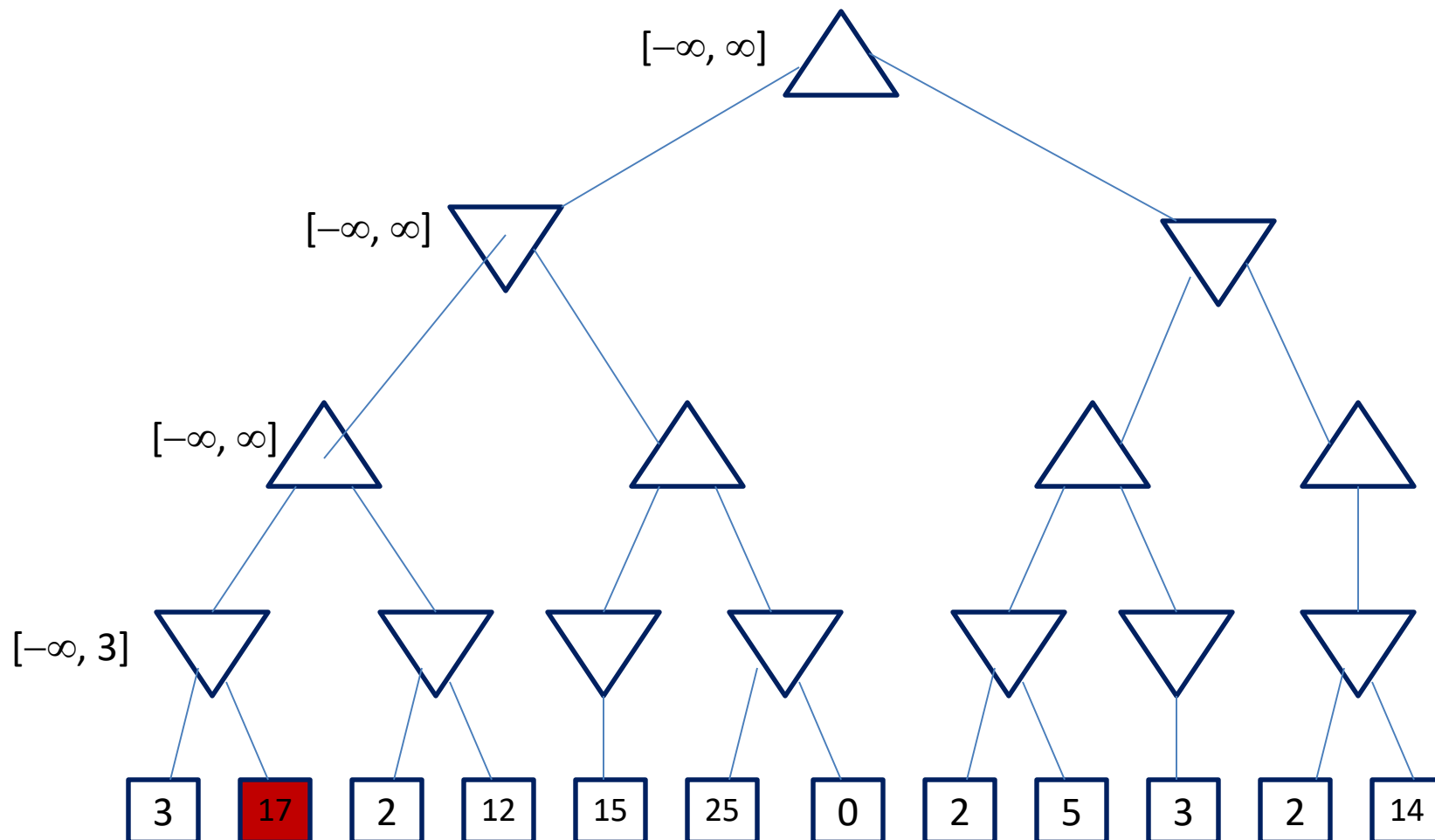
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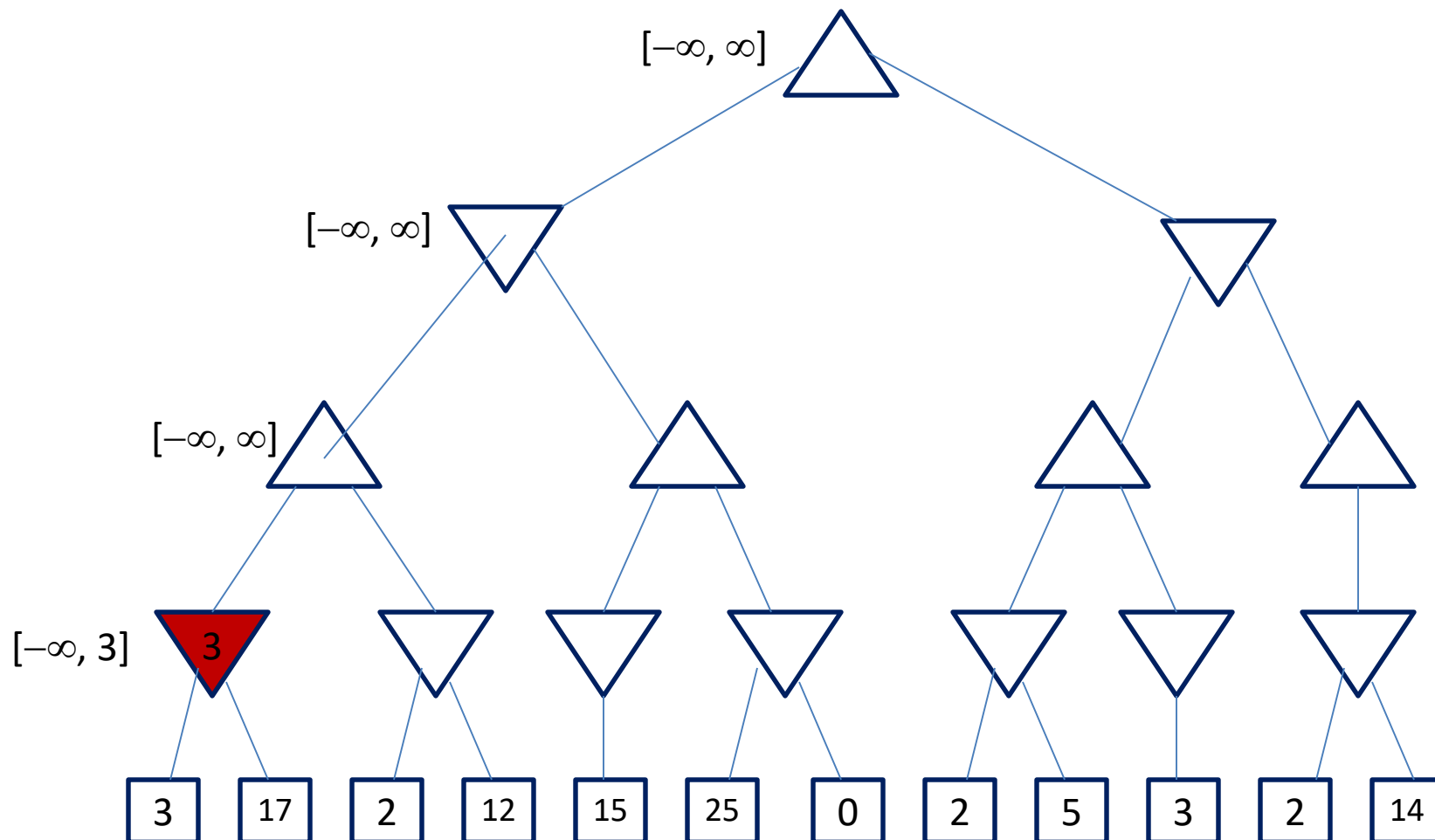
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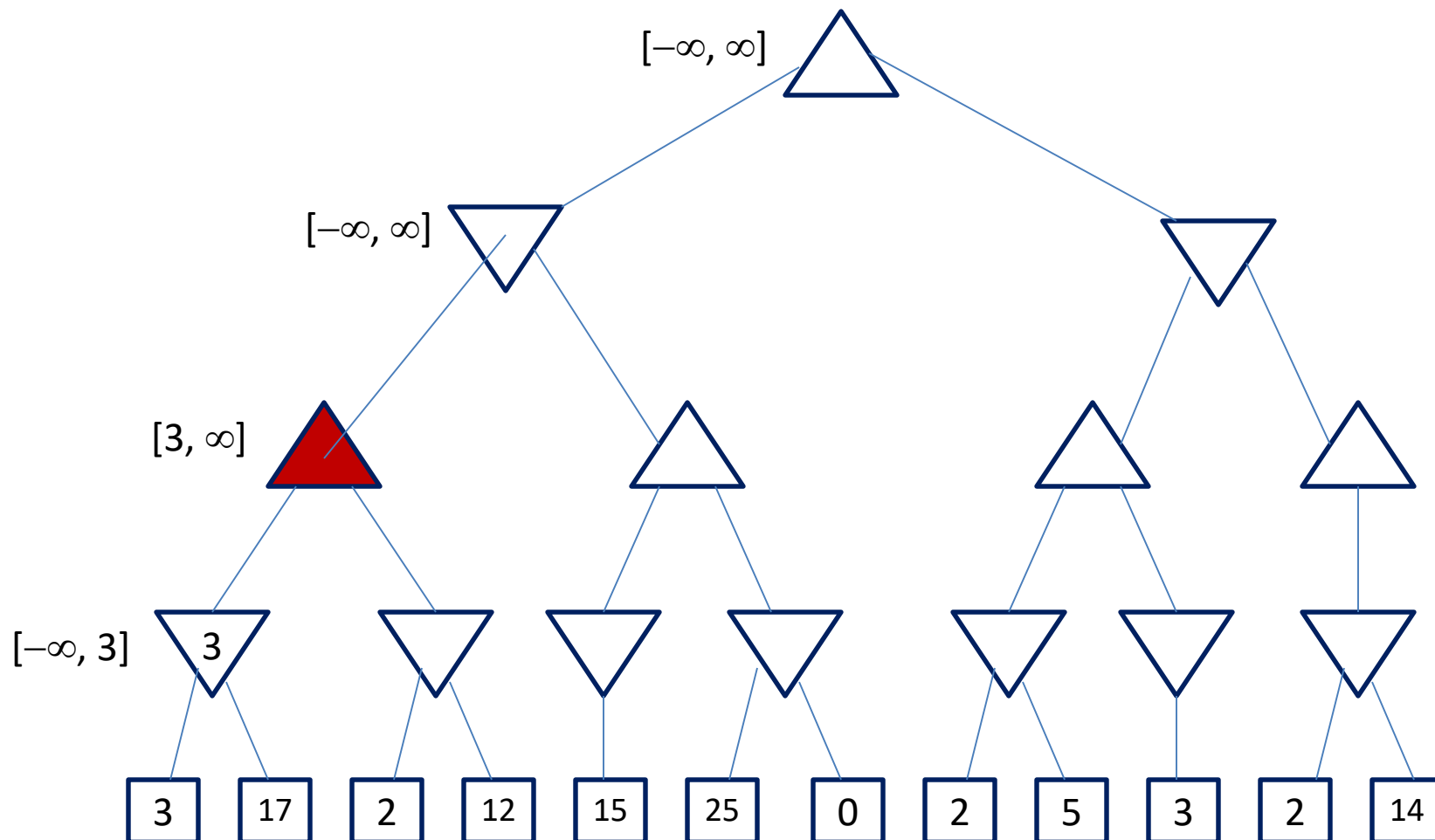
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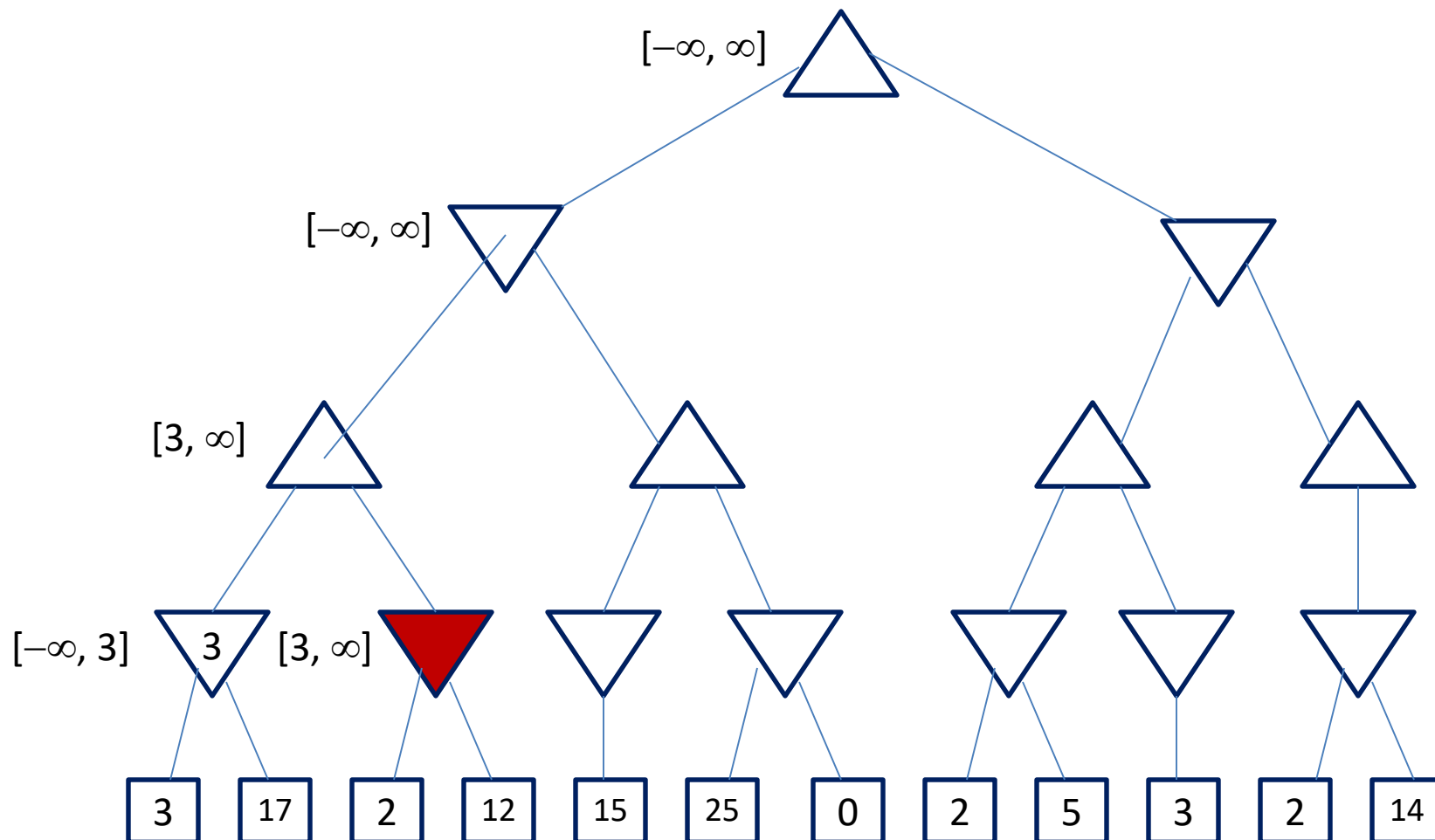
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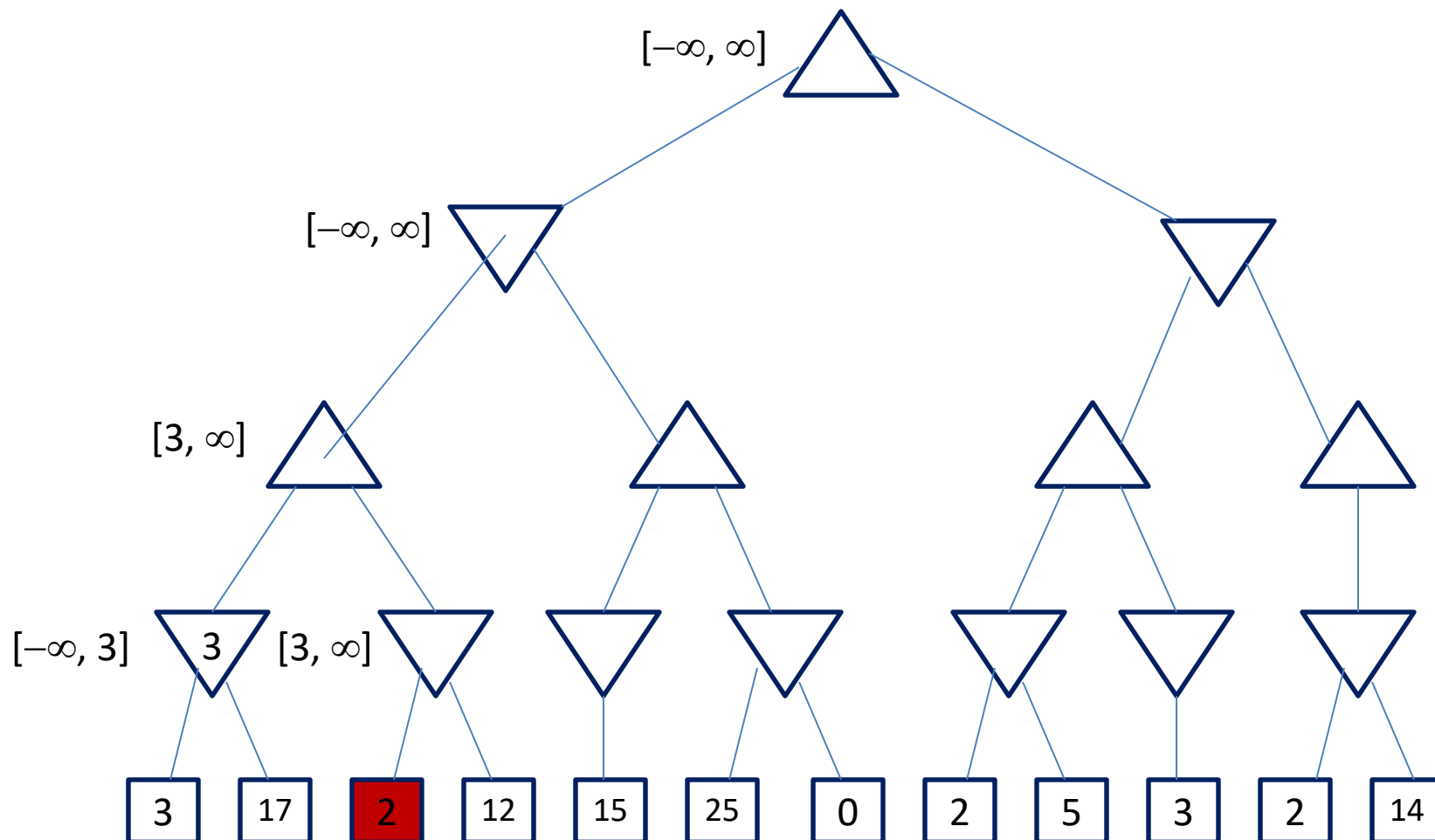
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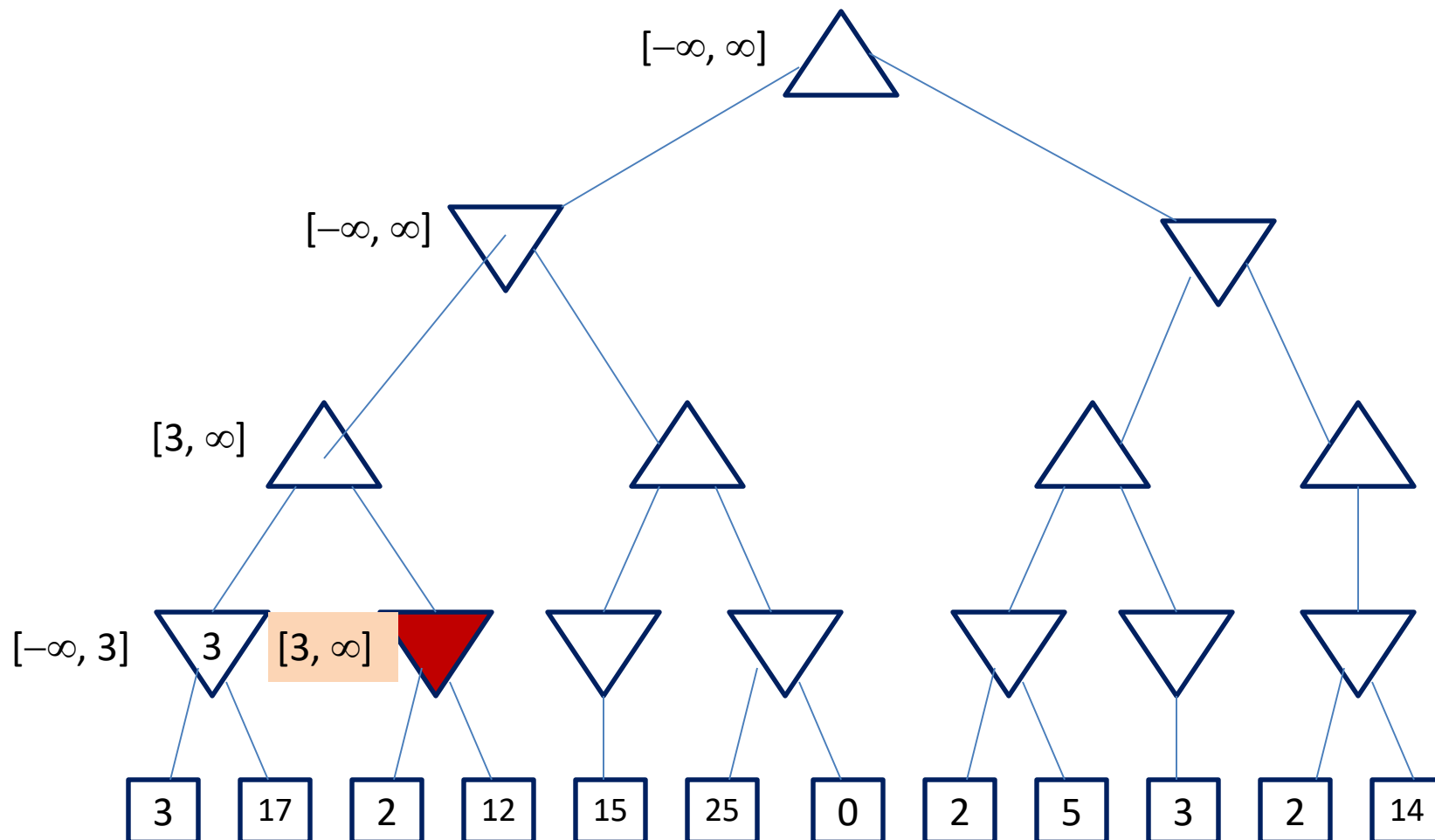
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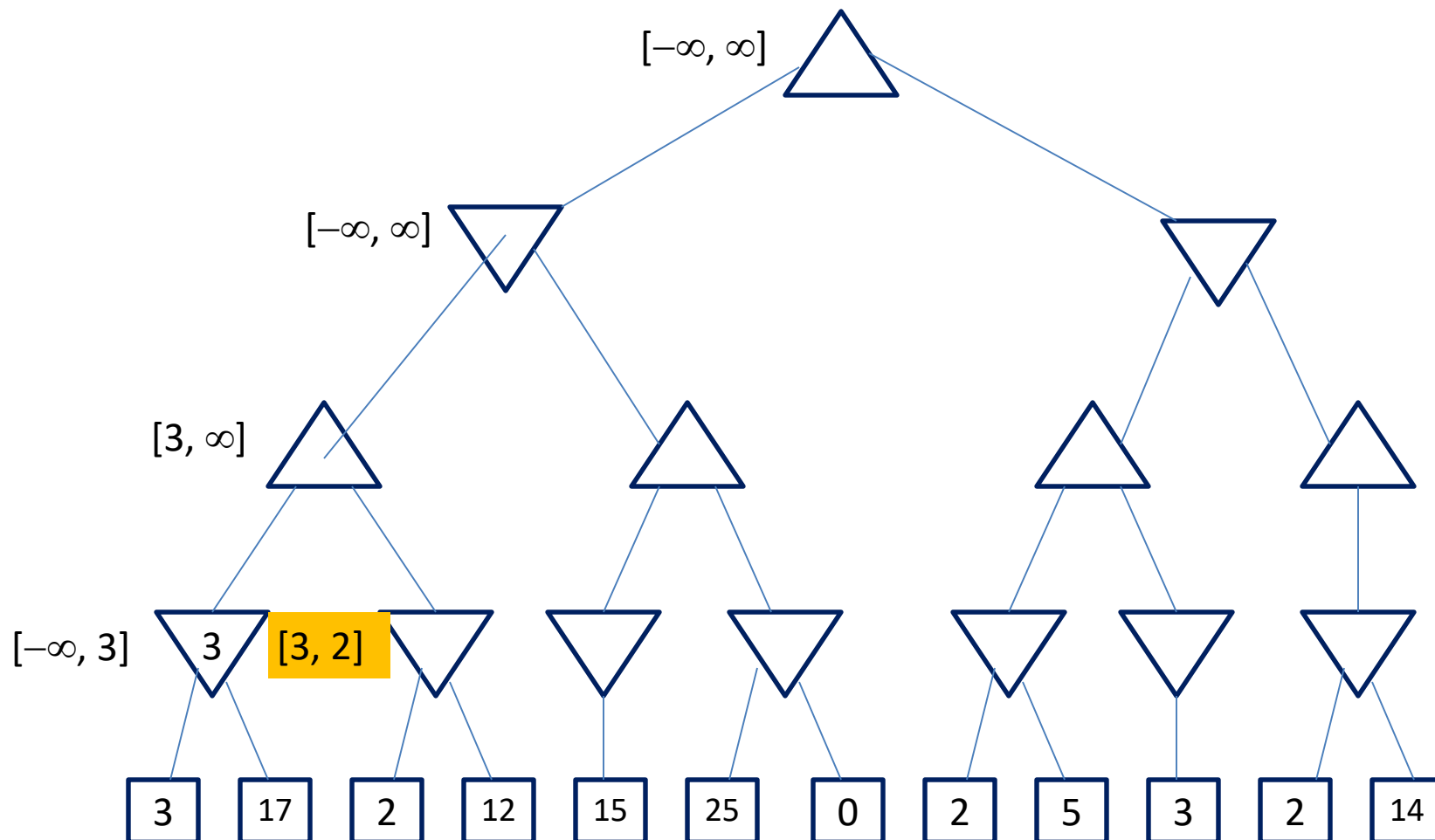


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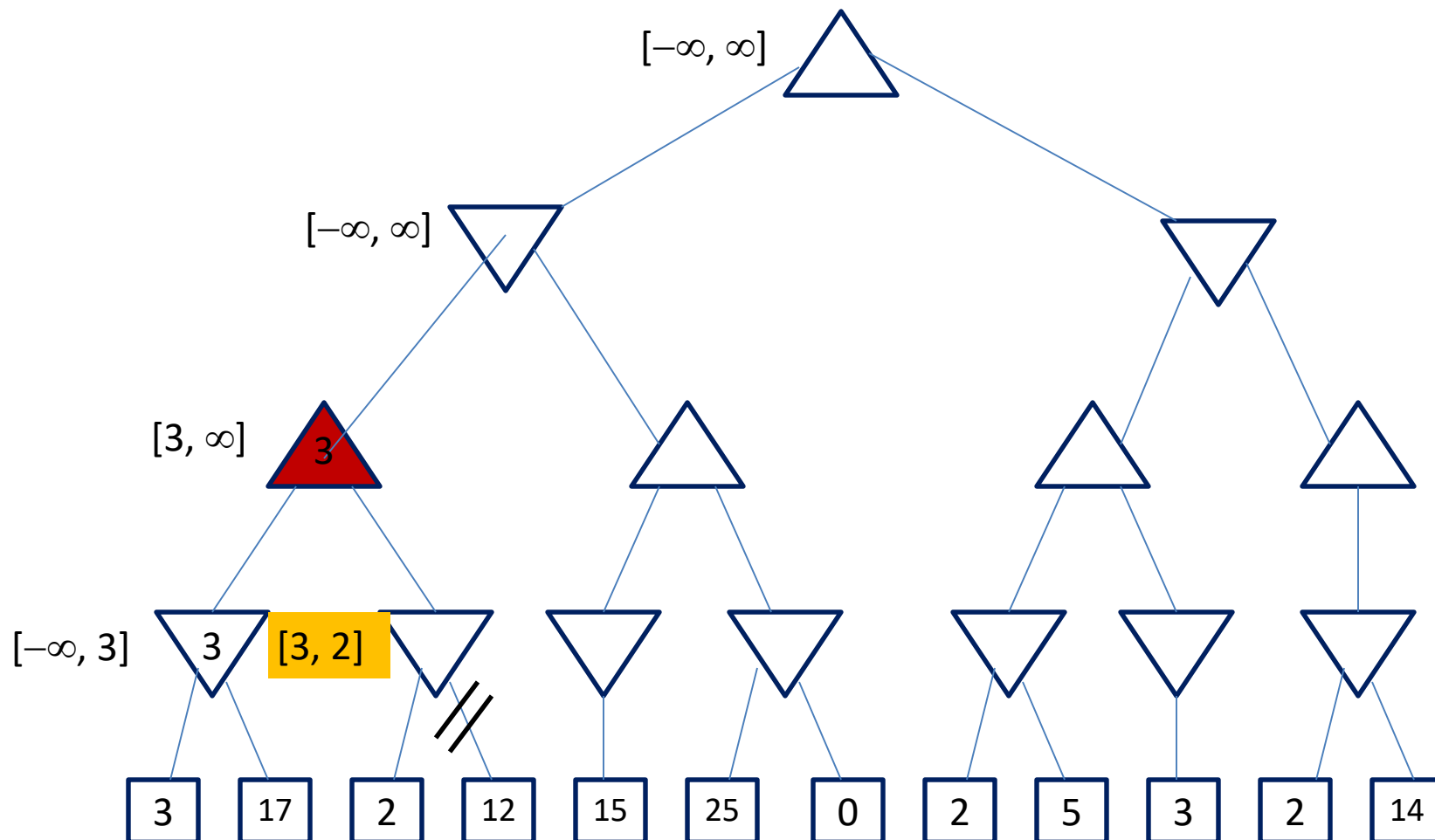


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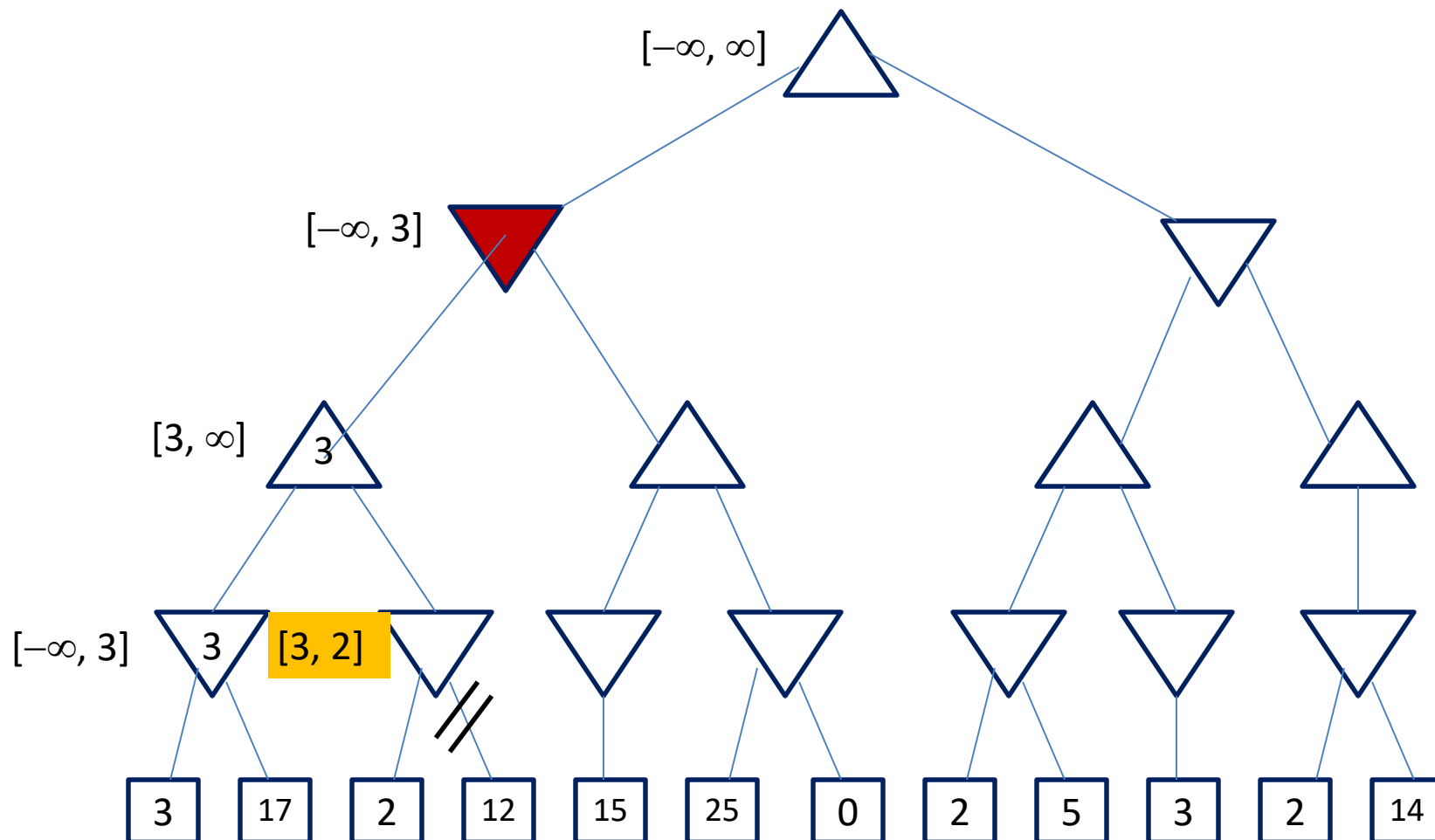




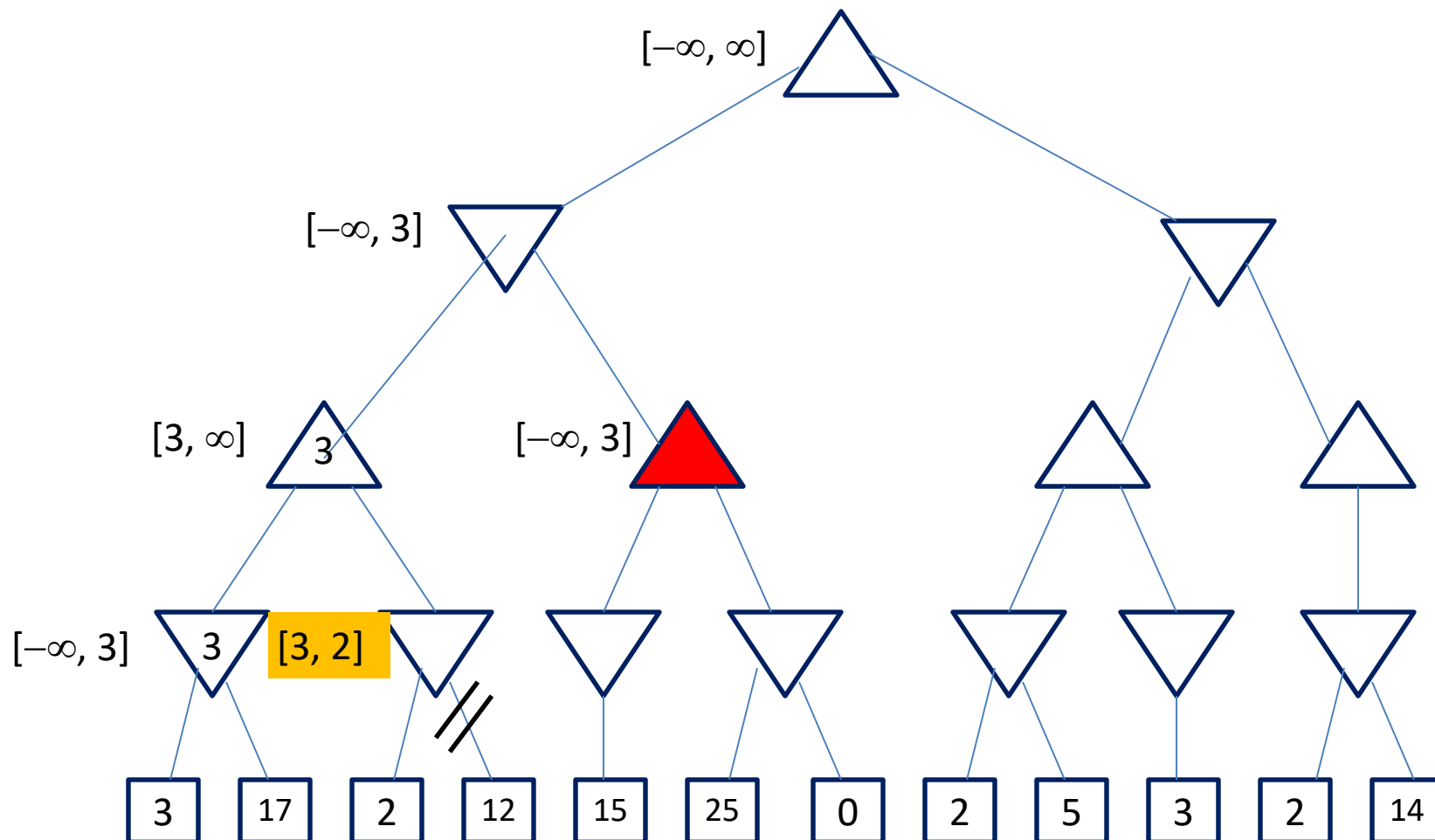
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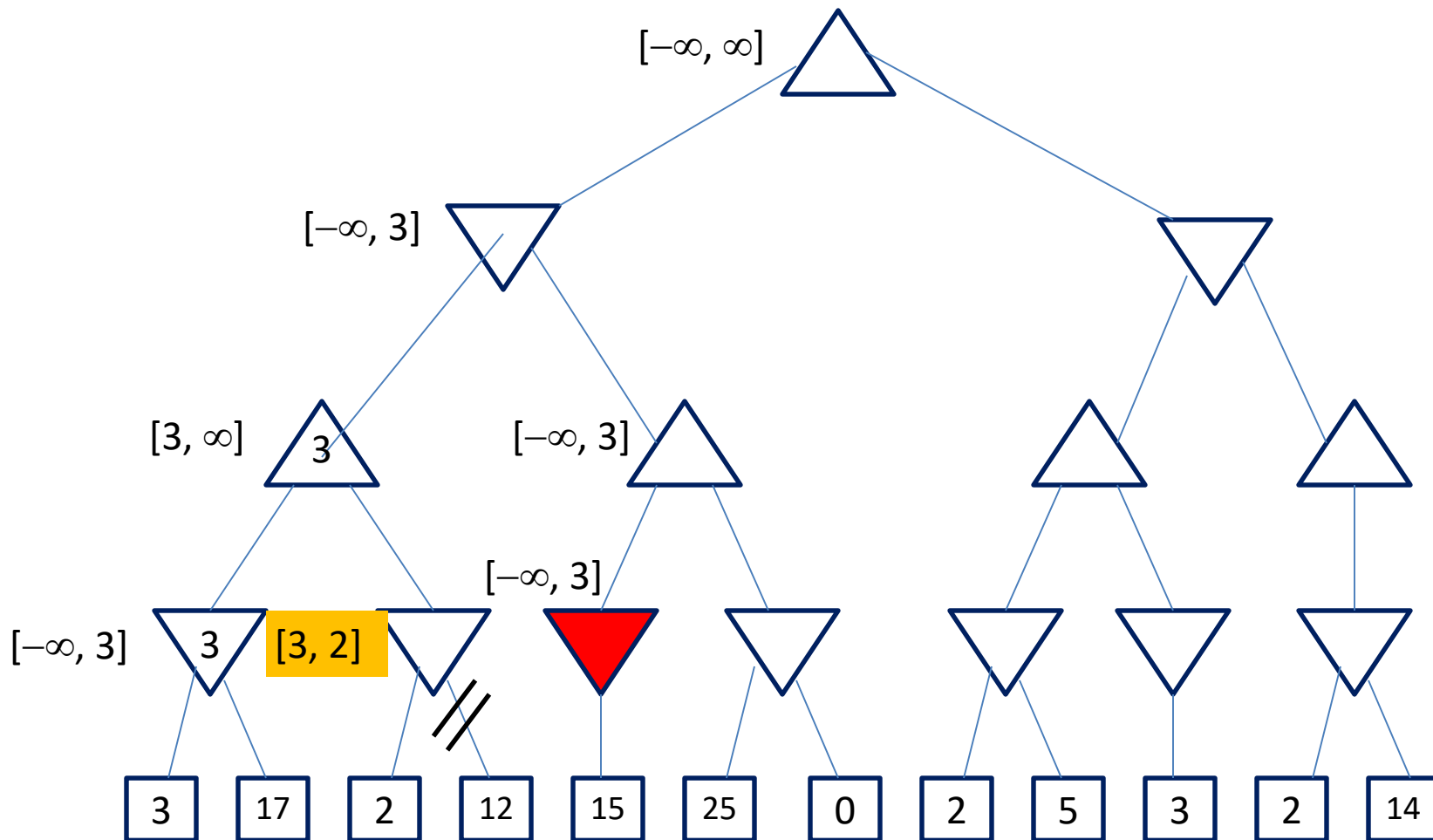
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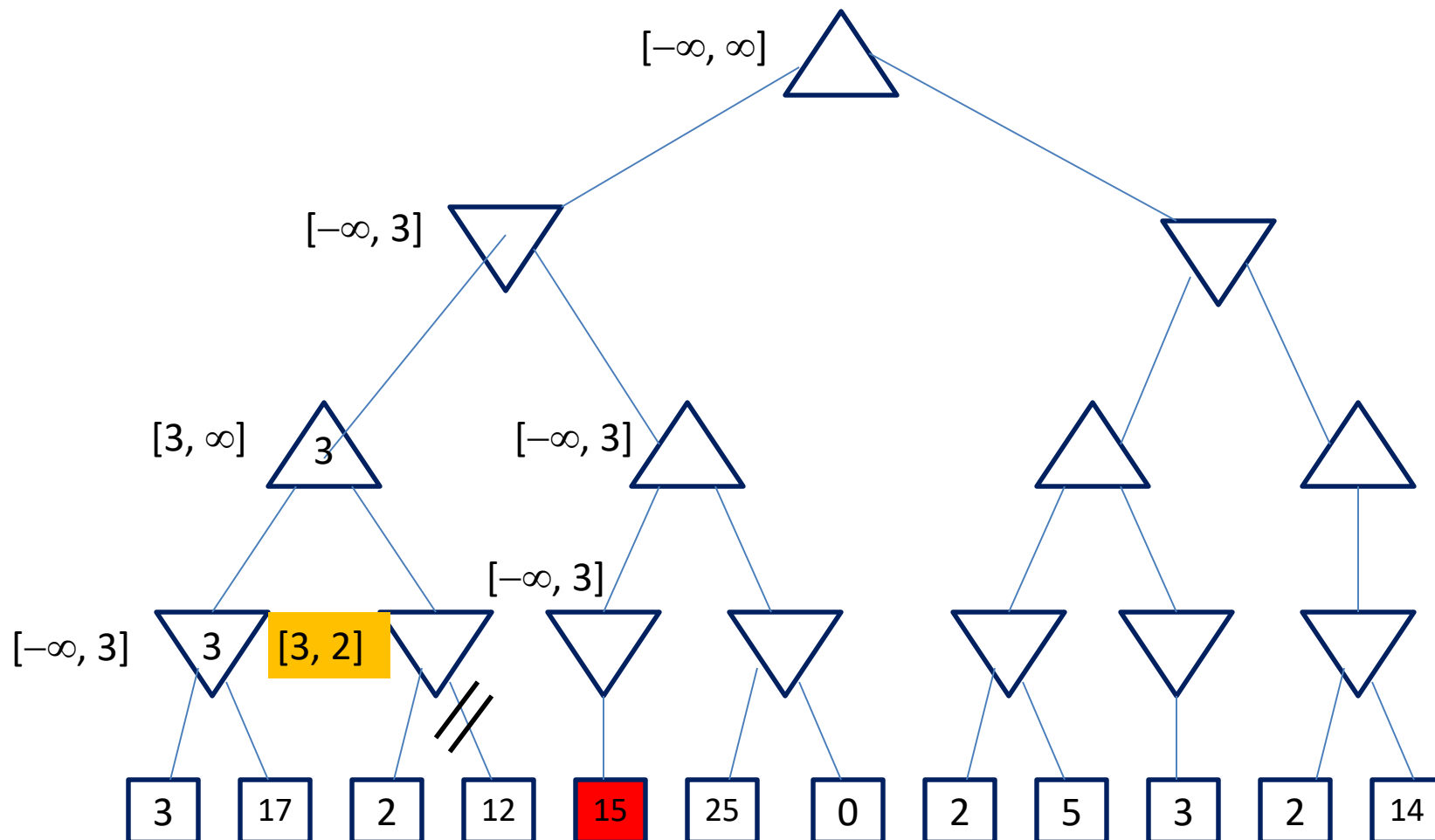
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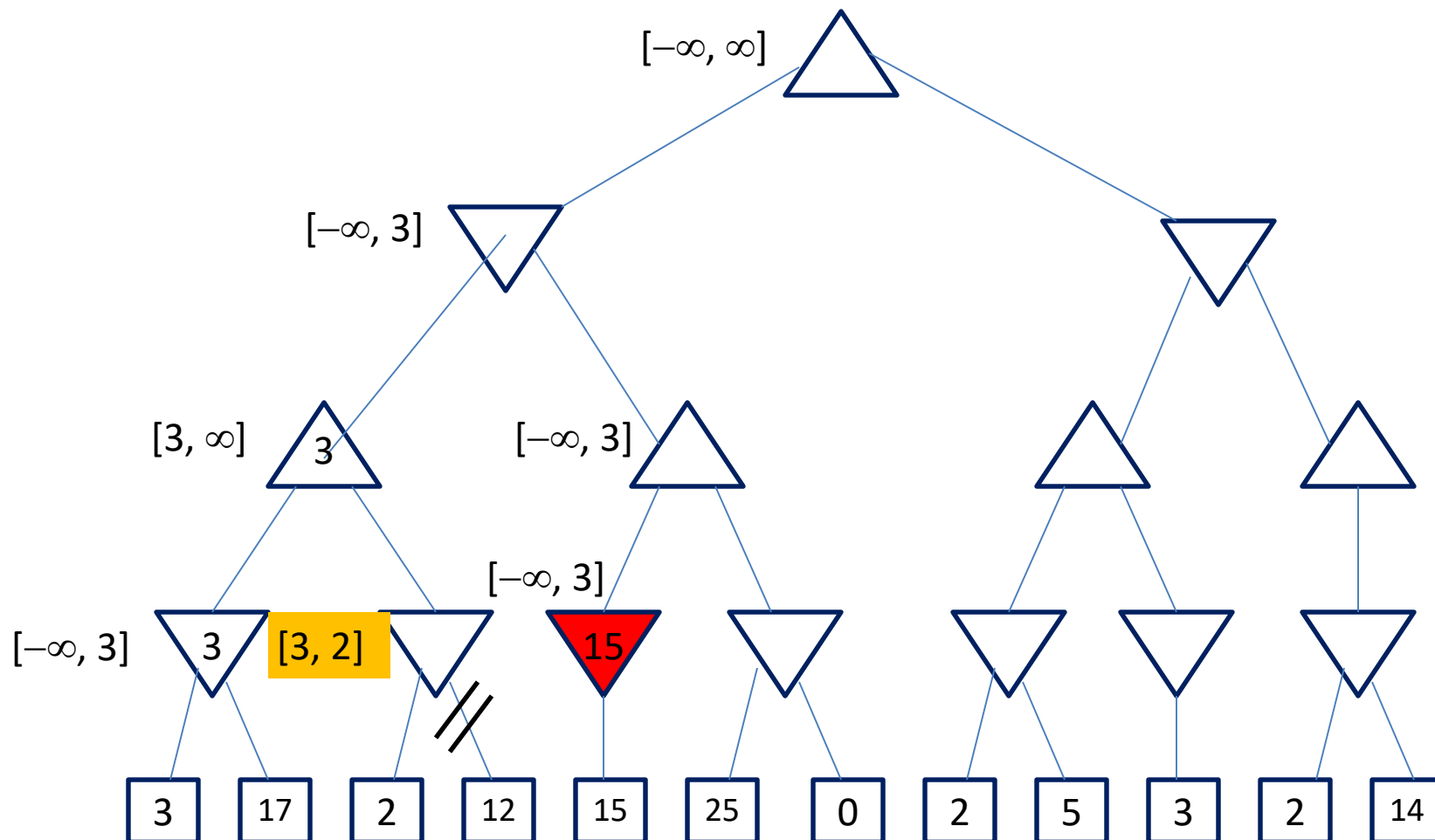
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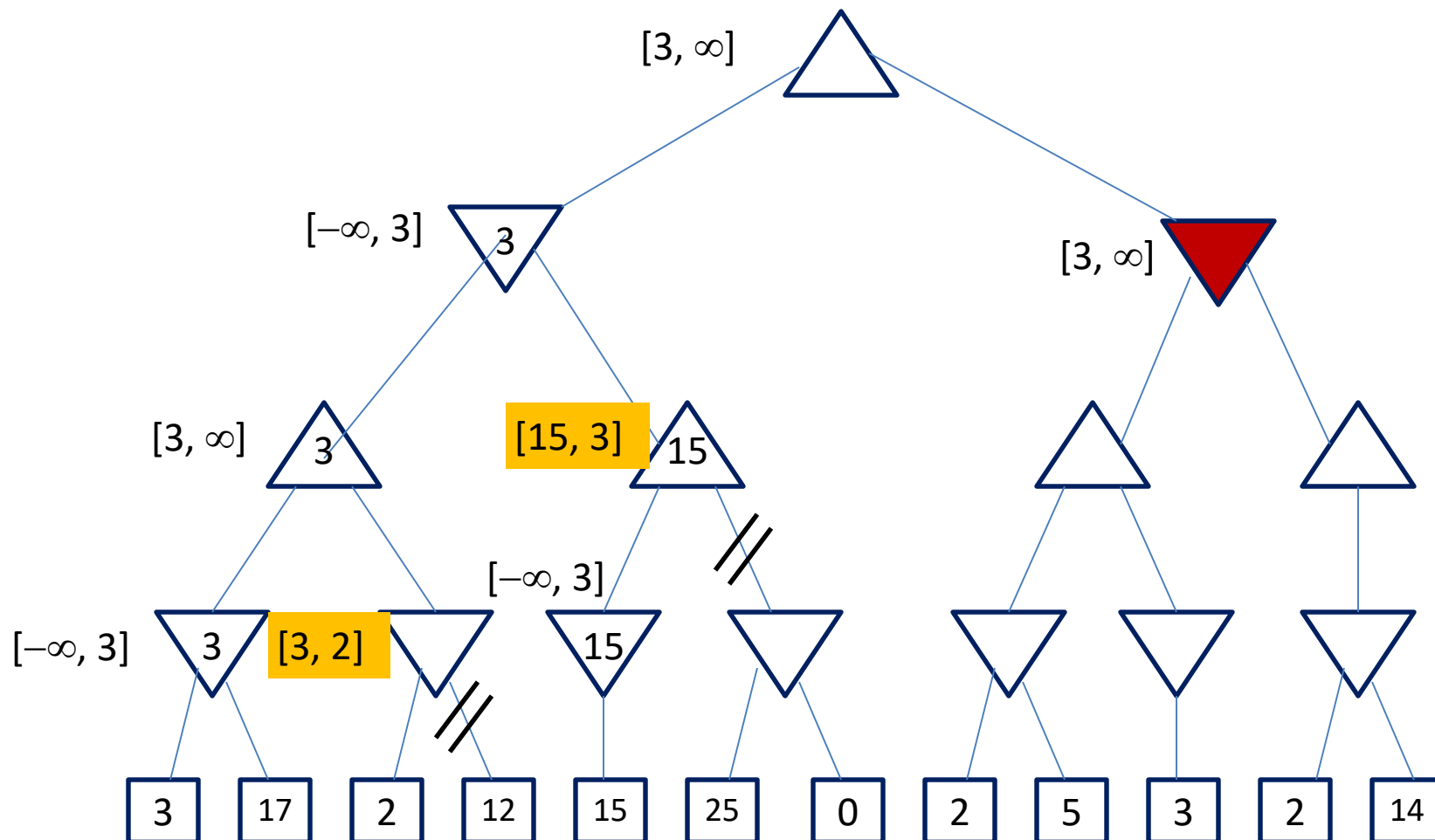




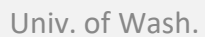


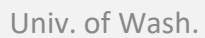


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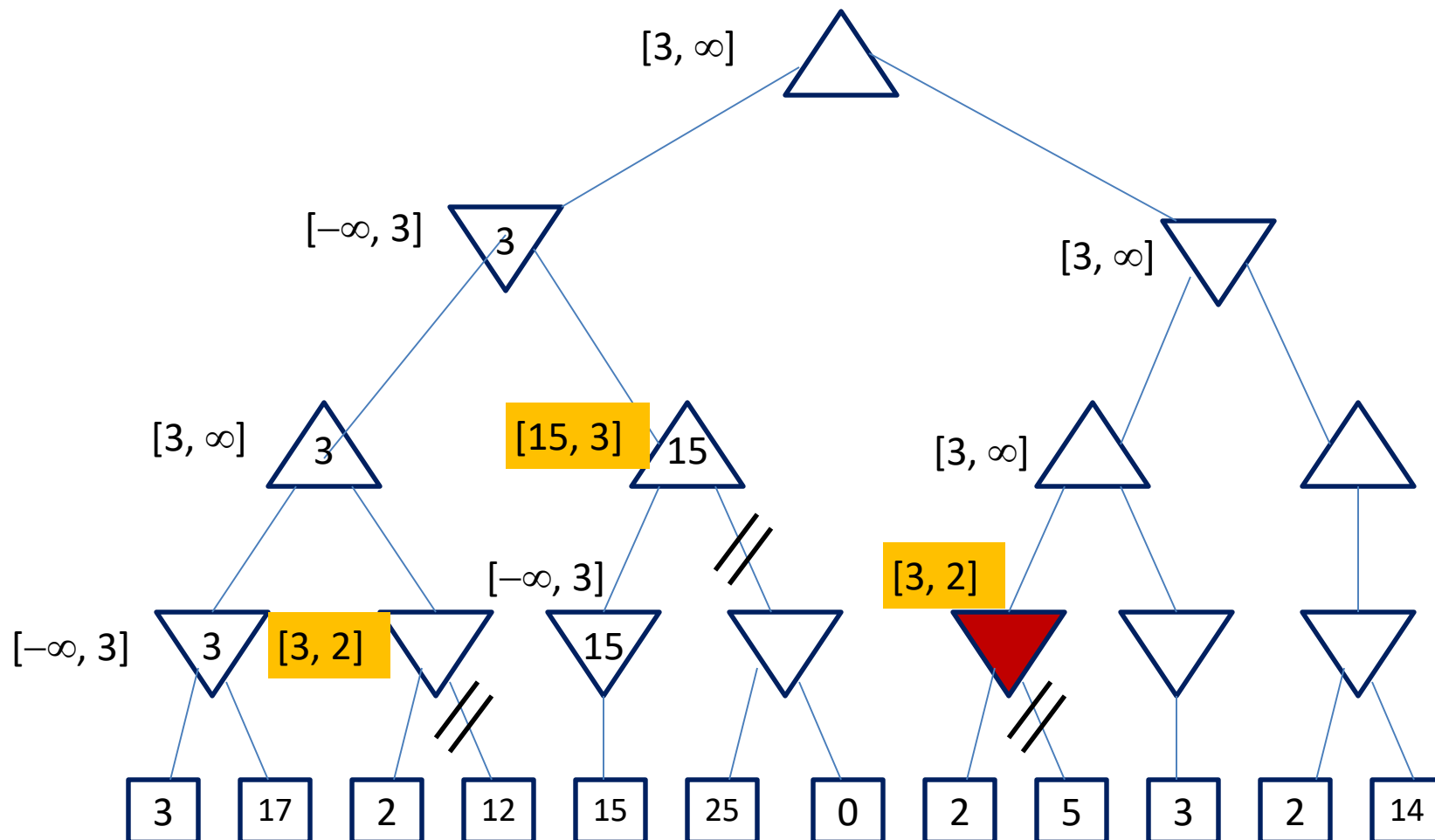


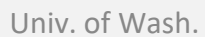


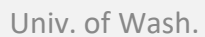




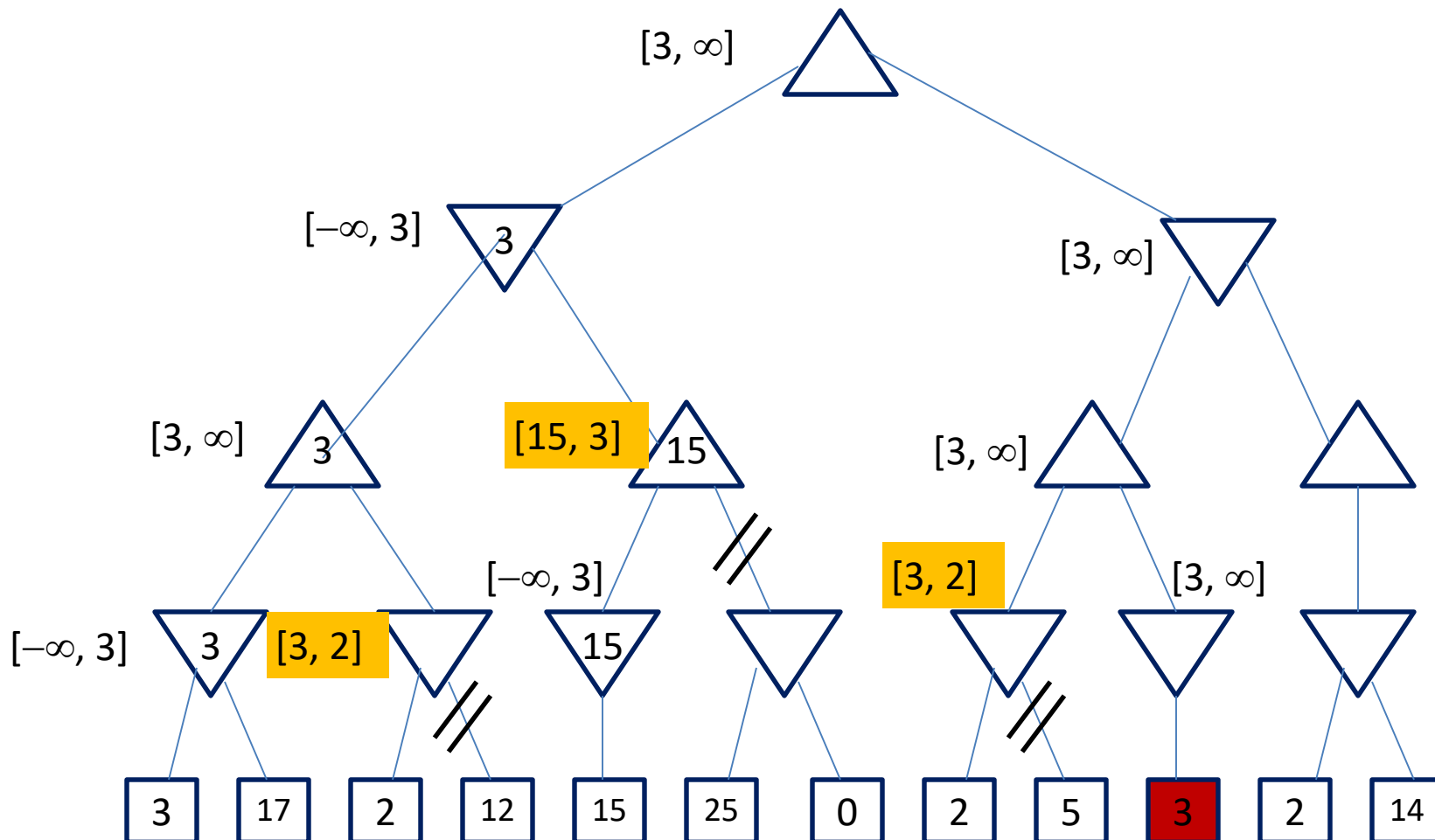
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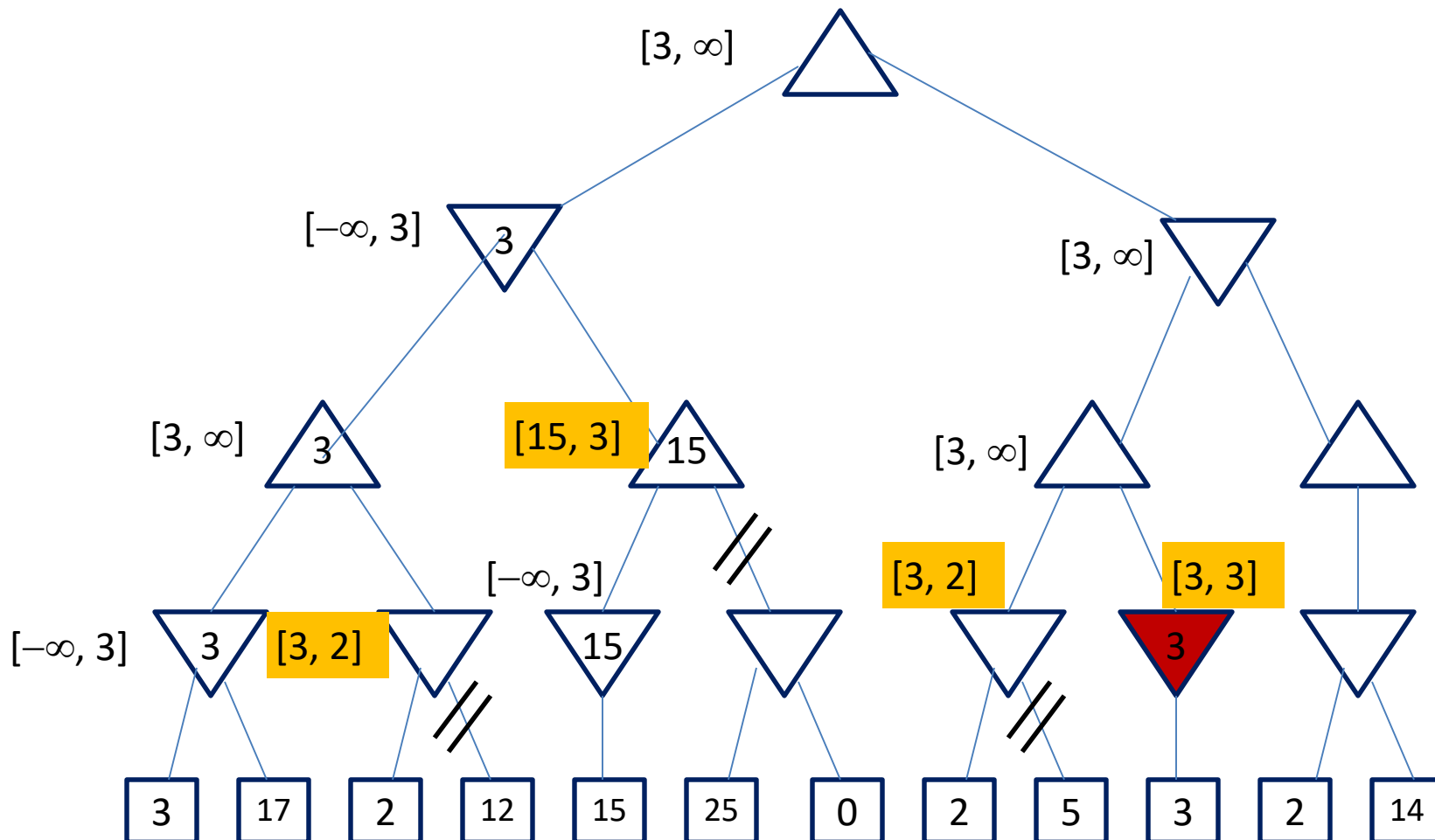




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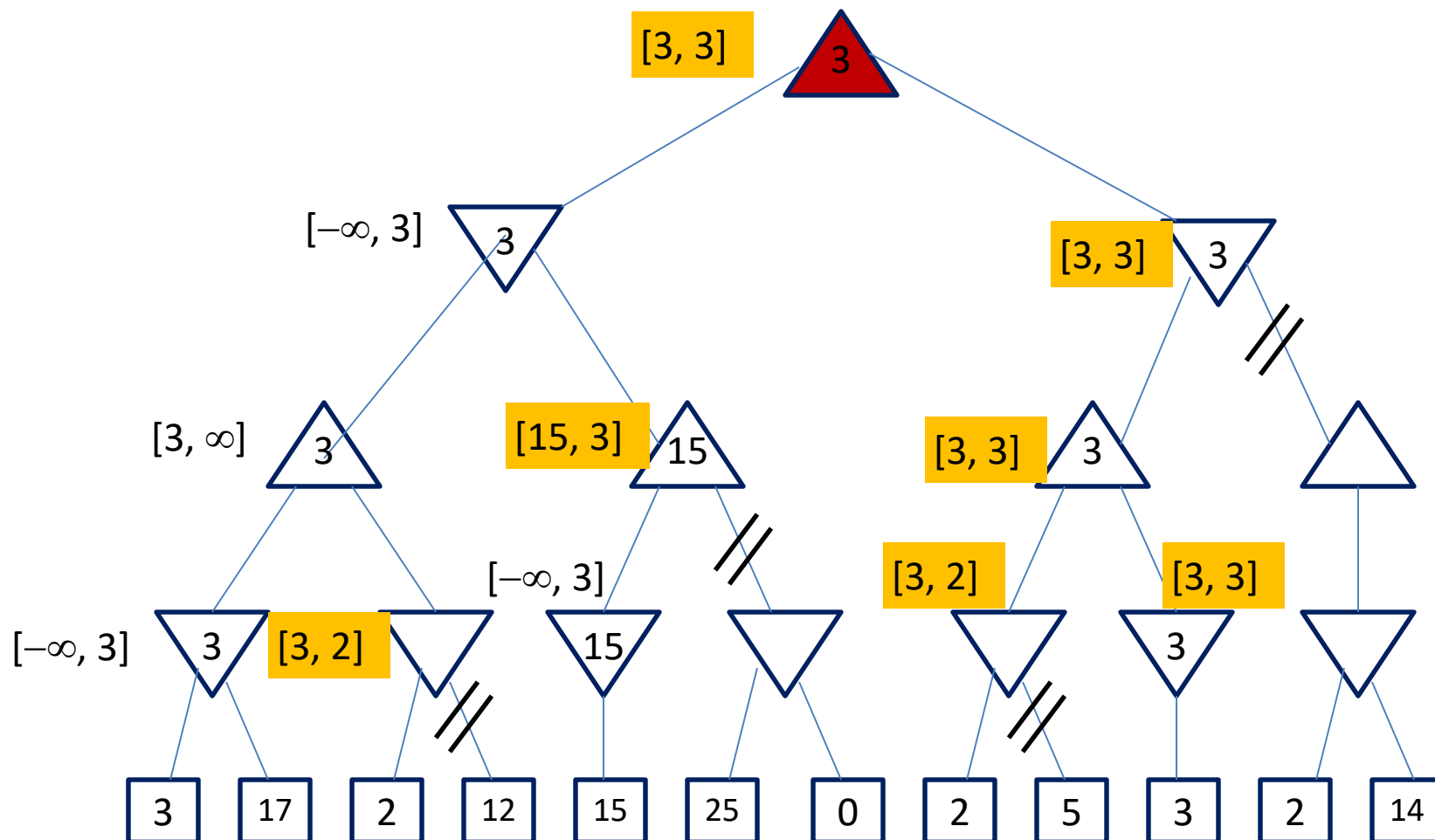
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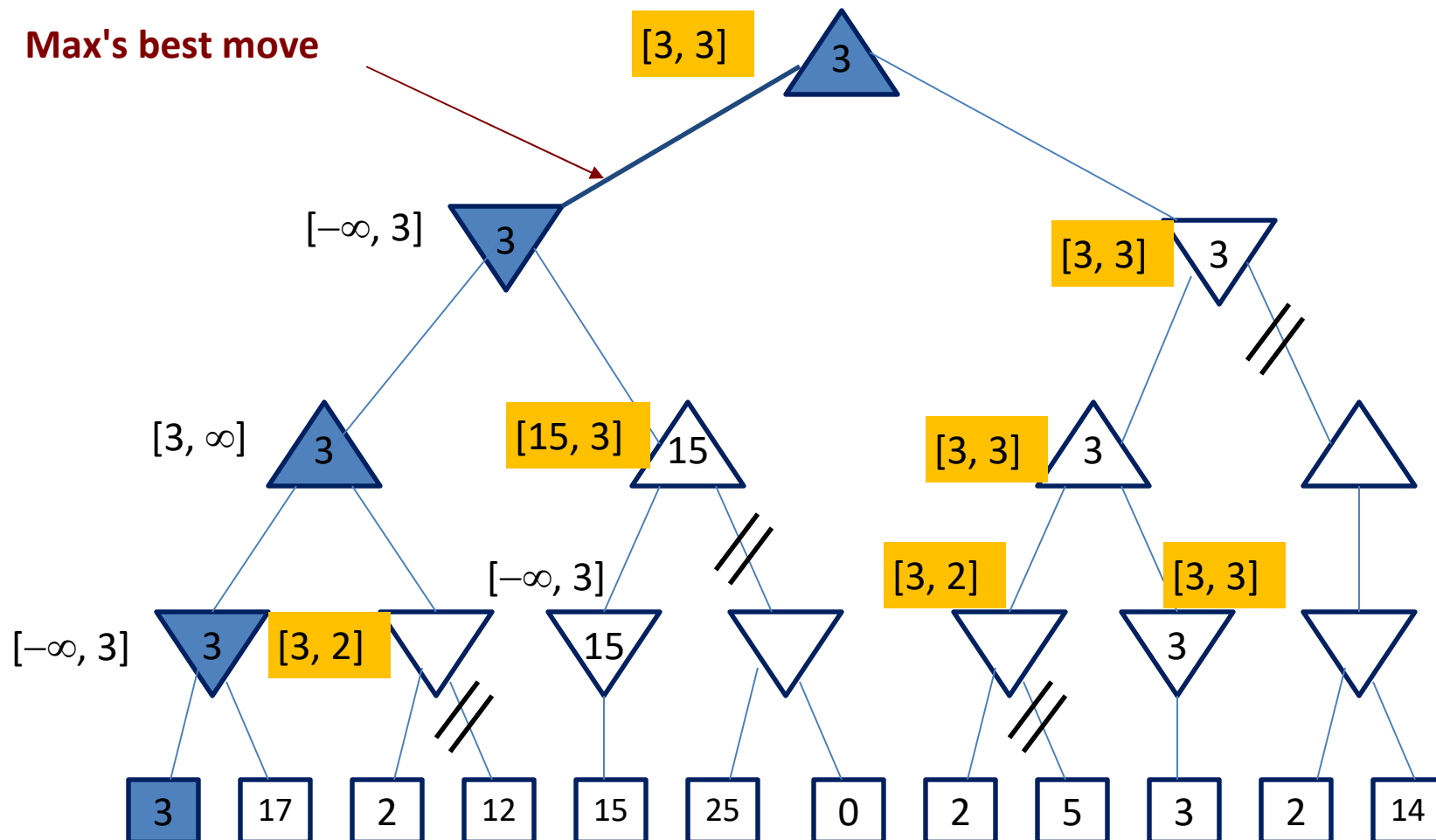


Alpha-Beta Pruning



Alpha-Beta Pruning

Max's best move



Strategy to Increase the Number of Cutoffs

At each non-leaf level, perform a static evaluation of all successors of a node and order them best-first before doing the recursive calls. If the best move was first, the tendency should be to get cutoffs when exploring the remaining ones.

Or, use **Iterative Deepening**, with ply limits increasing from, say 1 to 15. Use results of the last iteration to order moves in the next iteration.

In games like chess, α - β pruning typically allows searching ahead 2 times as deep. It tends to reduce the effective branching factor from d to approx. \sqrt{d} .

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