Hybrid Signal Compression

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1 Motivation

Signal compression is quite an important technology in digital communication. Speech, audio, image, and digital video are all important fields of signal compression, and plenty of compression methods have been put to practical use. Technology advancements and huge data since the arrival of 20th century has turned the table. In the era of ML and AI, it is of utmost need to have the precise calculation and optimum use of the resources such as time and memory.

Based on the papers I mentioned in the reference I model a compression algorithm using two of the most important transforms namely Discrete Cosine Transform and Discrete Wavelet Transform.

2 Discrete Cosine Transform

A discrete cosine transform (DCT) expresses a finite sequence of data points in terms of a sum of cosine functions oscillating at different frequencies. In particular, a DCT is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. The DCTs are generally related to Fourier Series coefficients of a periodically and symmetrically extended sequence whereas DFTs are related to Fourier Series coefficients of a periodically extended sequence. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry. The Transform is given by:

$$X(k) = \sum x(n) cos[\frac{2\pi(n+1)k}{2N}]\ k = 0, 1, 2, ...$$

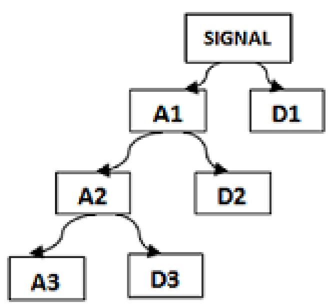
3 Discrete Wavelet Transform

A discrete wavelet transform (DWT) is any wavelet transform for which the wavelets are discretely sampled. As with other wavelet transforms, a key advantage it has over Fourier transforms is temporal resolution: it captures both frequency and location information (location in time). The first DWT was invented by Hungarian mathematician Alfréd Haar. For an input represented by a list of 2^n numbers, the Haar wavelet transform may be considered to pair up input values, storing the difference and passing the sum. This process is repeated recursively, pairing up the sums to prove the next scale, which leads to $2^n - 1$ differences and a final sum.

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable.

$$\psi(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise.} \end{cases}$$

The representation in the form of a tree for a level 3 decomposition can be shown as:



4 Algorithm

4.0.1 compression ratio

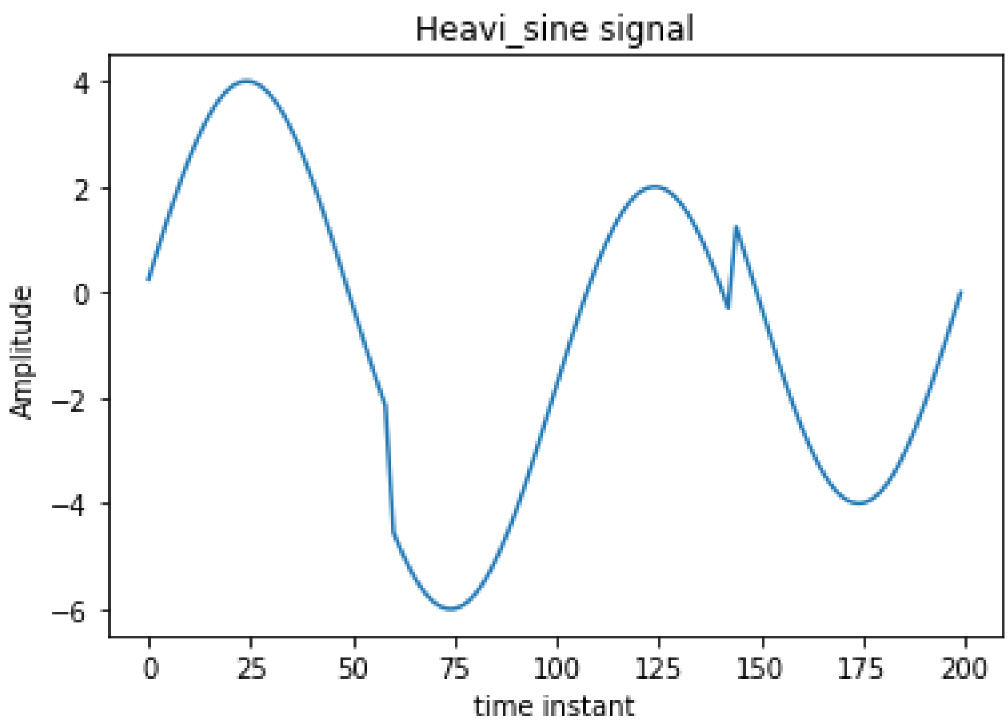
comp ratio = 1-num of zeroed out coefficients/total length of signal

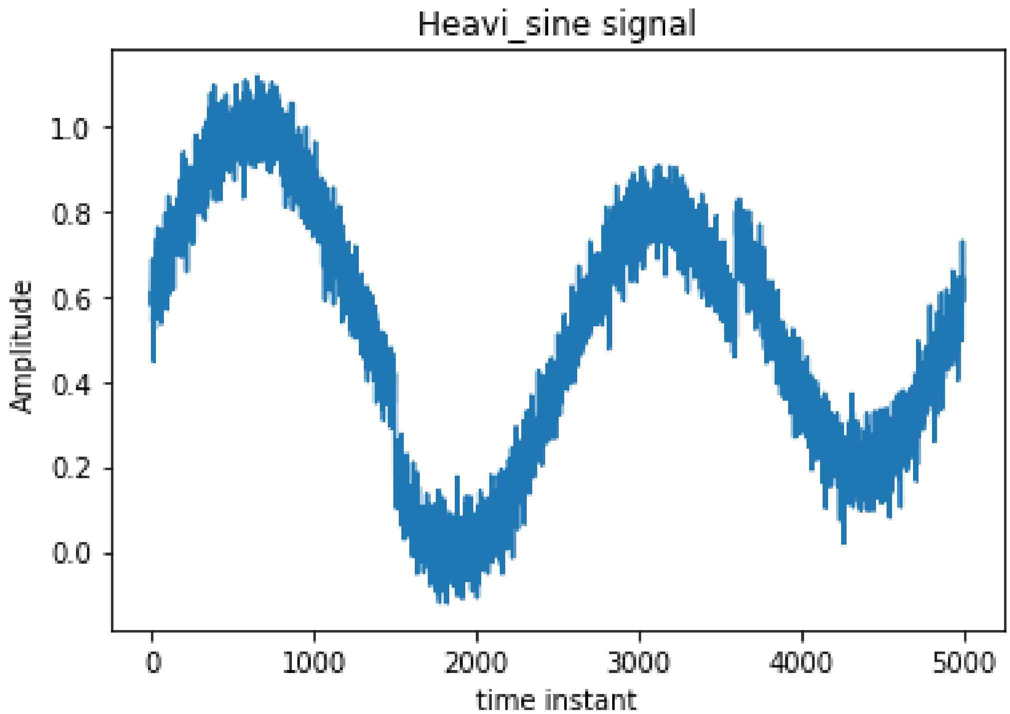
4.0.2 RMSD

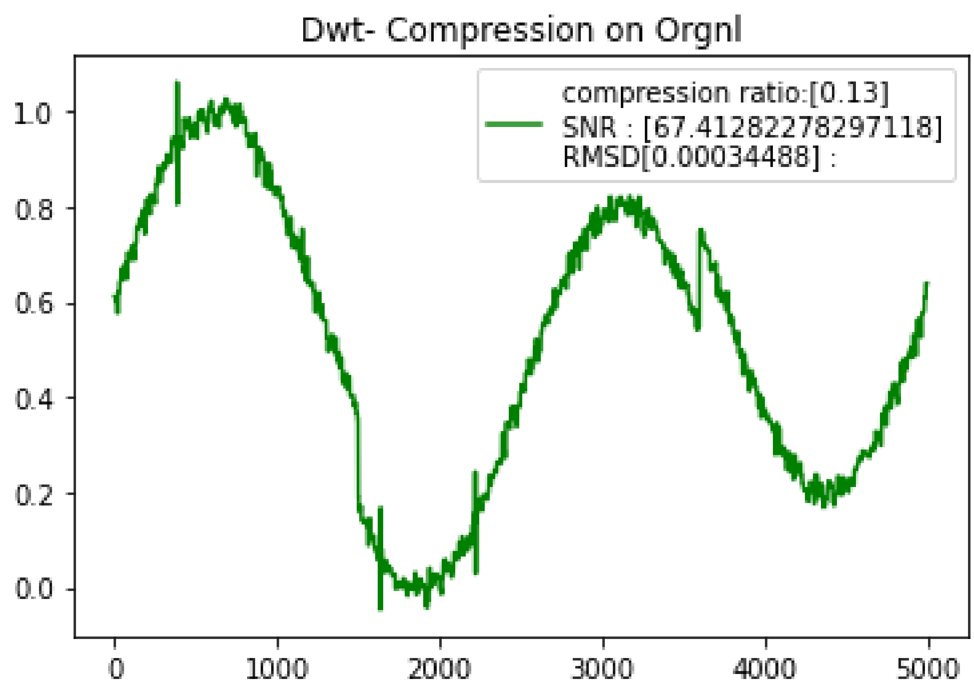
when measuring the average difference between two time series x1(t) and x2(t), the formula becomes RMSD $= \sqrt{\frac{\sum_{t=1}^{T}(x_{1,t}-x_{2,t})^2}{T}}$.

4.0.3 Step 1: DWT based compression and denoising

Step I use the level-3 Haar wavelets decomposition to further reduce the file size. Hence I decompose my dct compressed signal into the wavelet coefficients and further threshold them.
let cA3,cD3,cD2,cD1 be respectively the approximation and the detail coefficients at the label(marked integer). For each of the detail coefficient array These are hard thresholded by choosing only the 5% of the max of given coefficient array.
Doing this not only reduces the high frequency noise component but also reduces the file size.

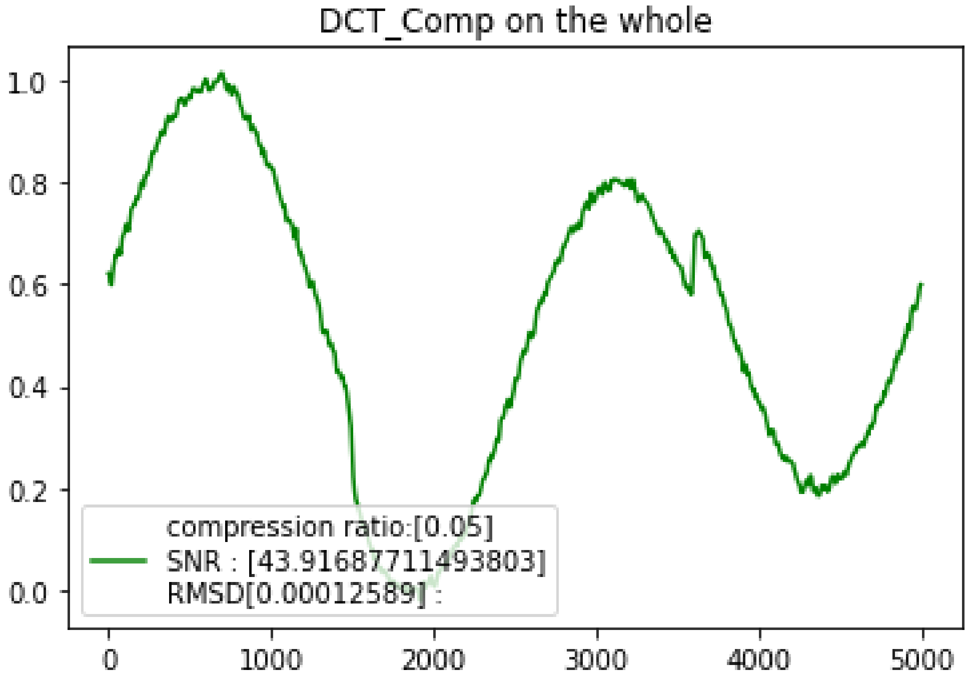



Heavi_sine signal
Amplitude
time instant
0
1000
2000
3000
4000
5000
0.0
0.2
0.4
0.6
0.8
1.0


Dwt- Compression on Orgnl
compression ratio:[0.13]
SNR : [67.41282278297118]
RMSD[0.00034488] :
0
1000
2000
3000
4000
5000
0.0
0.2
0.4
0.6
0.8
1.0

4.0.4 Step 1: DCT based compression

we saw that DCT captures the signal very well by most of them in very few initial coefficients , this is the main idea of compression.
To increase my SNR with the same compression DCT based procedure is carried on to the the signal.



5 Conclusion

With DWT I get the compression upto 0.16 and after applying the output signal to the dct compressor the compressing ratio we get is 0.05.
Hence on the whole product of the two results is 0.008 . I can say that my total reduces by a factor of 120.

if we have only taken the DWT on the original signal, The compression obtained be 0.16 that is by a factor of 6.25 only.

The RMSD also comes pretty low and SNR is also very much acceptable.
The fused algorithm produces a efficient model to compress down the

signal in a very optimised manner.
Please Note more compression can be done still if the quality of the peaks and other minute details is not the need, like in case of cardiogram signal where we just need to study the peaks.

6 References

1.Afonso, V.X., Tompkins, W.J., Nguyen, T.Q., Luo, S.: ECG beat detection using filter banks. IEEE Trans. Biomed. Eng. 46, 192–202 (1999)

2.https://www.researchgate.net/publication/226196019$_{E}CG_{S}ignal_{C}ompression_{U}sing_{D}ifferent_{T}e$

3.$ECGcompressionusingwavelettransformbyRVSShastry, K.Rajgopal$