SYLLABUS

Module 1

Introduction to Engineering Mechanics-statics-basic principles of statics-Parallelogram law, equilibrium law, principles of superposition and transmissibility, law of action and reaction(review) free body diagrams.
Concurrent coplanar forces-composition and resolution of forces-resultant and equilibrium equations – methods of projections – methods of moments – Varignon's Theorem of moments.

Module 2

Friction – sliding friction - Coulomb's laws of friction – analysis of single bodies –wedges, ladder- analysis of connected bodies .
Parallel coplanar forces – couple - resultant of parallel forces – centre of parallel forces – equilibrium of parallel forces – Simple beam subject to concentrated vertical loads. General coplanar force system - resultant and equilibrium equations.

Module 3

Centroid of composite areas- – moment of inertia-parallel axis and perpendicular axis theorems. Polar moment of inertia,radius of gyration,mass moment of inertia-ring,cylinder and disc.
Theorem of Pappus Guldinus(demonstration only)
Forces in space - vectorial representation of forces, moments and couples –resultant and equilibrium equations – concurrent forces in space (simple problems only)

Module 4

Dynamics – rectilinear translation - equations of kinematics(review)
Kinetics – equation of motion – D'Alembert's principle. – motion on horizontal and inclined surfaces, motion of connected bodies. Impulse momentum equation and work energy equation (concepts only).
Curvilinear translation - equations of kinematics –projectile motion(review), kinetics – equation of motion. Moment of momentum and work energy equation (concepts only).

Module 5

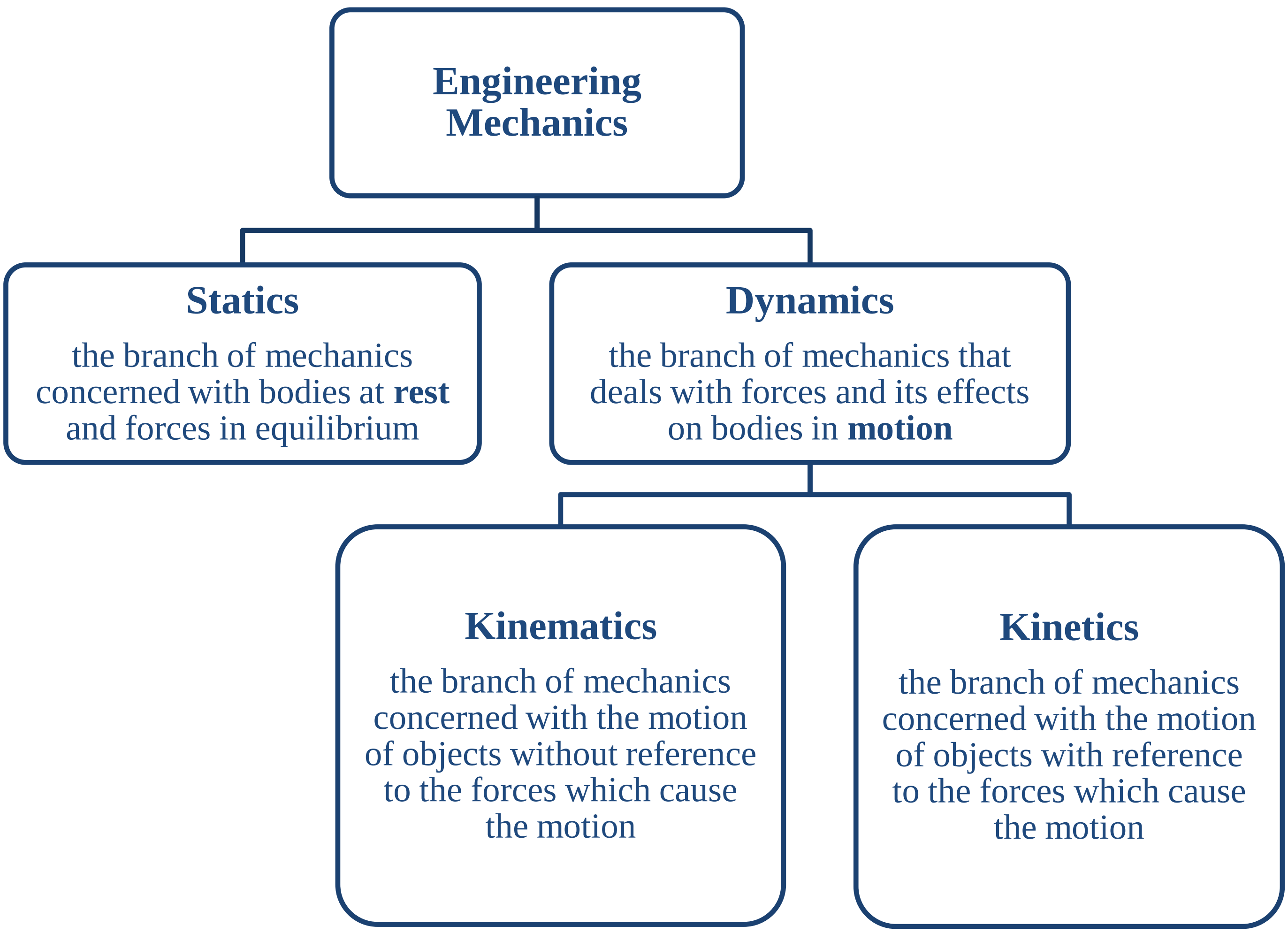
Rotation – kinematics of rotation- equation of motion for a rigid body rotating about a fixed axis – rotation under a constant moment.
Plane motion of rigid body – instantaneous centre of rotation (concept only).
Simple harmonic motion – free vibration –degree of freedom- undamped free vibration of spring mass system-effect of damping(concept only)

FIRST MODULE NOTES

Introduction to Engineering Mechanics-statics-basic principles of statics-Parallelogram law, equilibrium law, principles of superposition and transmissibility, law of action and reaction(review) free body diagrams.

Concurrent coplanar forces-composition and resolution of forces-resultant and equilibrium equations – methods of projections – methods of moments – Varignon's Theorem of moments.

INTRODUCTION TO ENGINEERING MECHANICS



Force

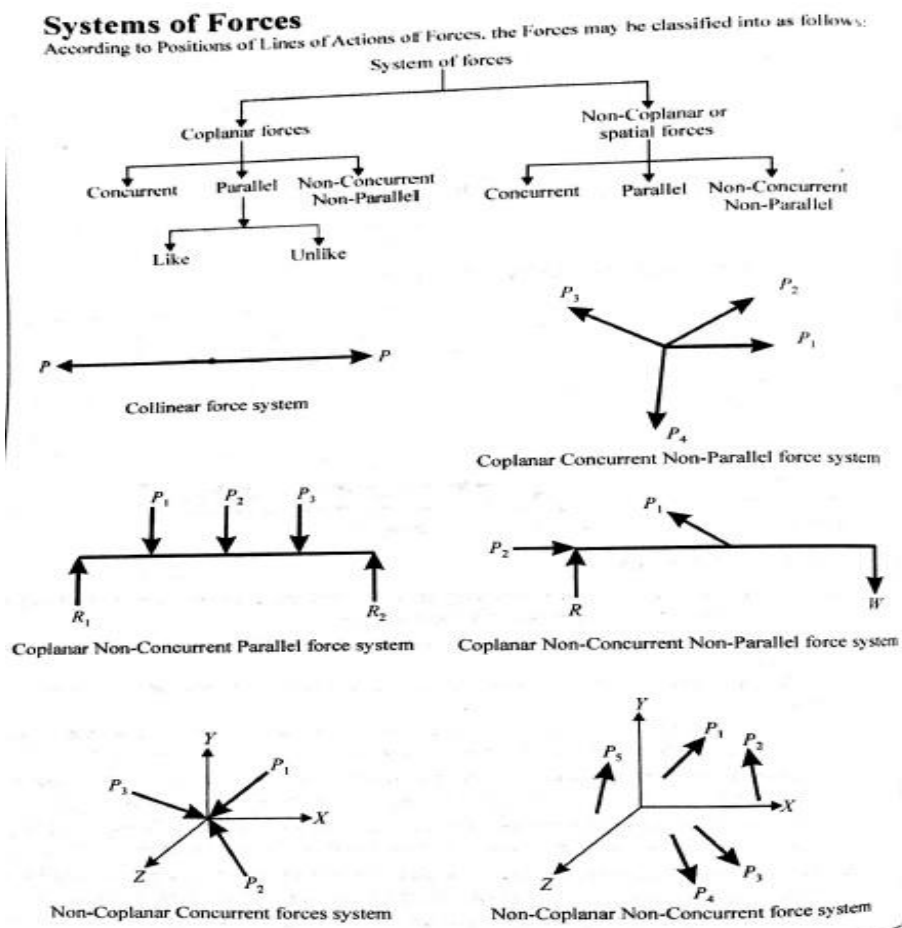
Definition: Force, in mechanics, can be defined as any action that tends to maintain or alter the motion of a body or distort it. A force can cause an object with mass to change its velocity (which includes to begin moving from a state of rest), i.e. to accelerate. Force can also be described intuitively as a push or pull.

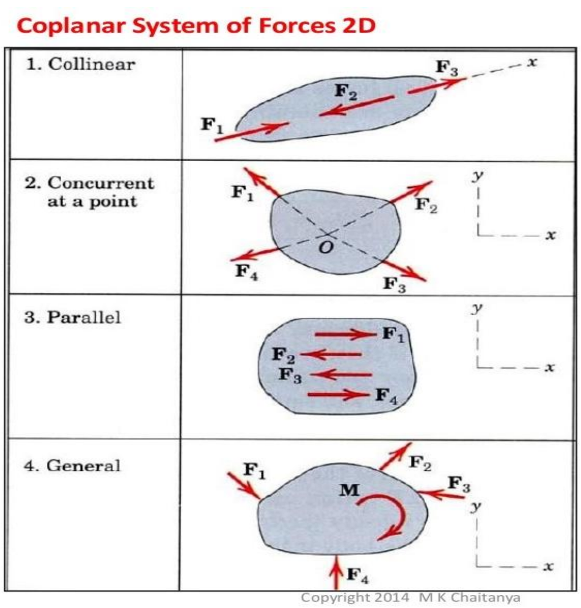
Characteristics of a Force

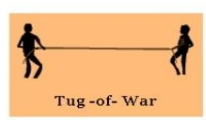
In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

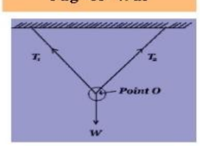
1. Magnitude 2. Direction 3. Point of application 4. Line of action of force

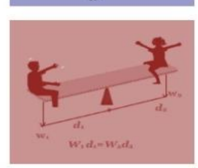
System of Forces

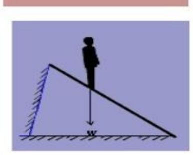


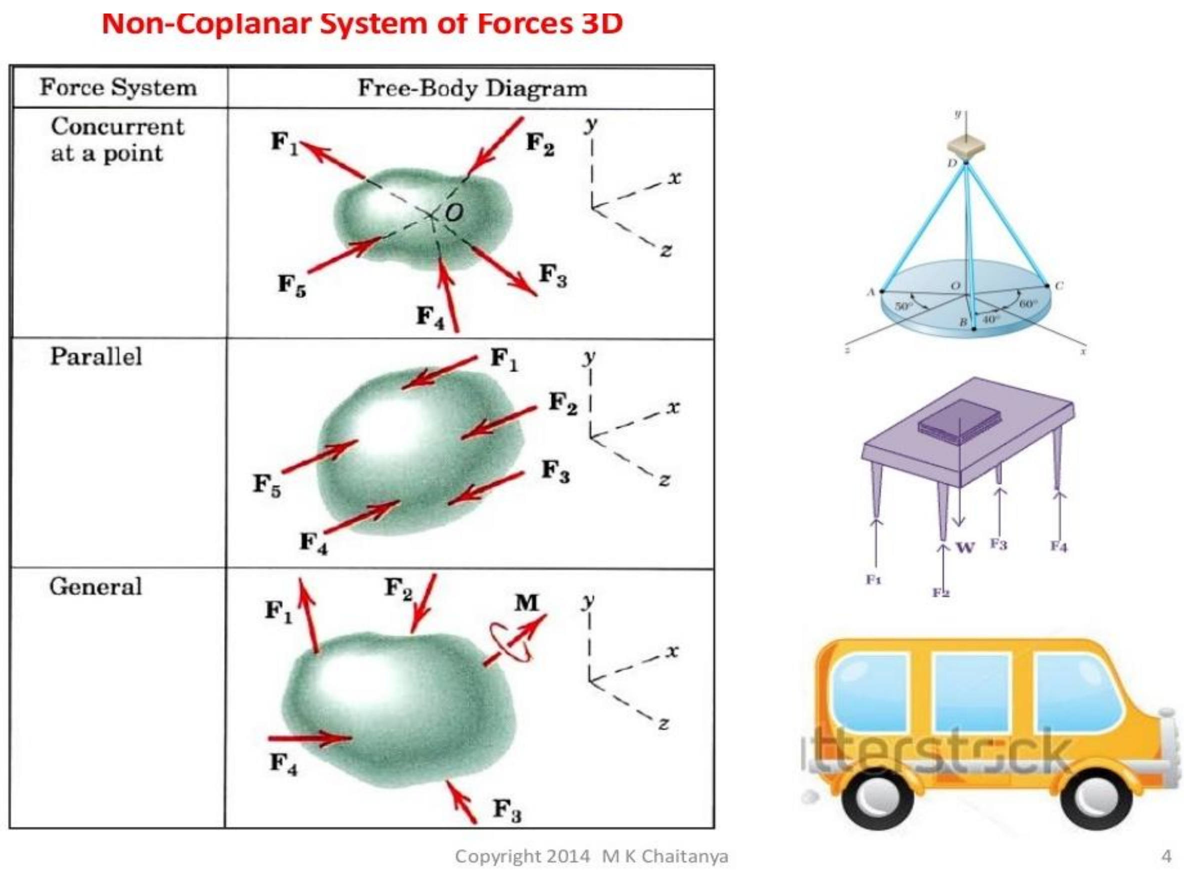












Definitions

1. **Particle:** An object that has infinitely small volume (occupies negligible space) but has a mass which can be considered to be concentrated at a point is known as particle.

2. **Body:** Anything that has a definite shape and consists of number of particles is known as body.

3. **Rigid Body:** Body which does not change its shape and size under the effect of forces acting over it is known as rigid body. It is a combination of a large number of particles which occupy fixed positions with respect to each other both before and after applying a load.

4. **Deformable Body:** Body which undergoes deformation under the effect of forces acting on it is called deformable body.

BASIC PRINCIPLES OF STATICS

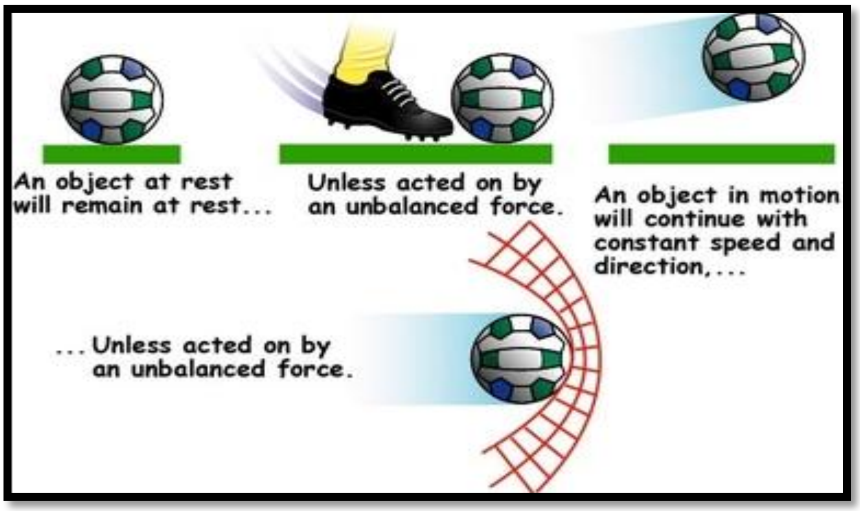
The following are the basic principles of statics:

1. Newton's Laws of Motion
    - First Law
    - Second Law
    - Third Law

2. Newton's Law of Gravitation
3. Law of transmissibility of forces
4. Parallelogram law of forces

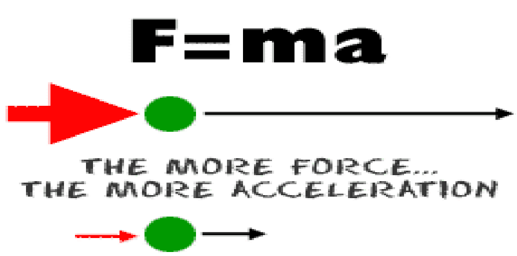
1. Newton's Law of Motion

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

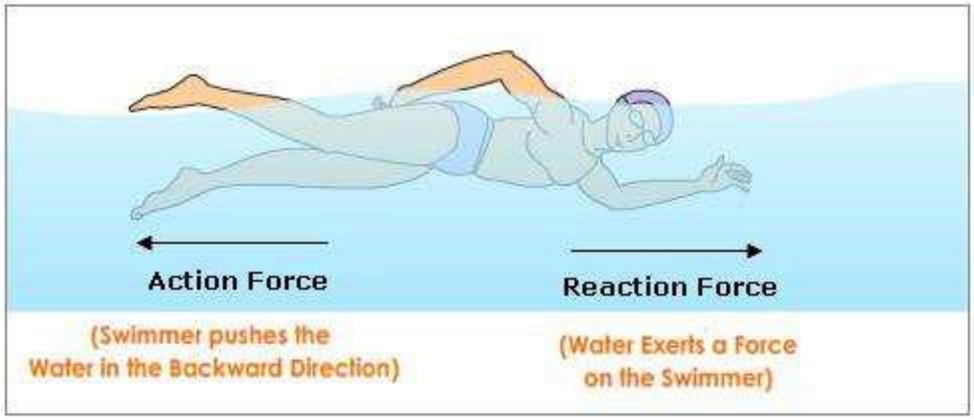


Second Law: The second law states that the rate of change of momentum of a body, is directly proportional to the force applied and this change in momentum takes place in the direction of the applied force.

Newton's second law of motion explains how an object will change velocity if acted by a force. For a body with constant mass, the second law can also be stated in terms of an object's acceleration. **The acceleration of a particle is proportional to the vector sum of forces acting on it and occurs along a straight line in which the force acts.**



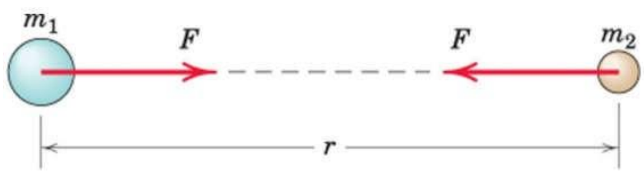
Third Law: To every action there is always an equal reaction: or the mutual interactions of two bodies are always equal but directed in opposite direction



2. **<u>Newton's Law of Gravitation</u>**

Two particles will be attracted towards each other along their connecting line with a force whose magnitude is directly proportional to the product of the masses and inversely proportional to the distance squared between the particles.

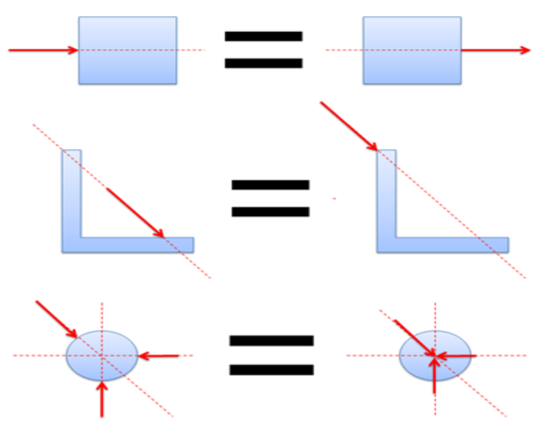
$$F = G\frac{m_1 m_2}{r^2}$$



3. **<u>Law of transmissibility of forces</u>**

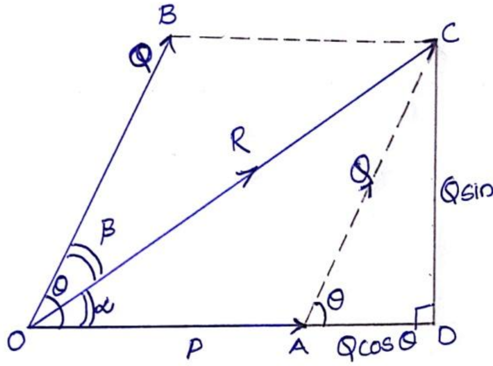
Principle of transmissibility states that the effect of a force on a rigid body remains unchanged if the point of application of the force acting on the rigid body is transferred to any other point along its line of action.

For example, the force F acts on a rigid body at point A. According to the principle of transmissibility of forces, this force has the same effect on the body as a force F applied at point B along its line of action.



<u>PARALLELOGRAM LAW</u>

If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point of intersection of the two forces



Proof

Considering ΔOCD and apply pythagoras theorem

$OC^2 = CD^2 + OD^2$

$R^2 = Q^2\sin^2\theta + (P+Q\cos\theta)^2 = Q^2\sin^2\theta + P^2 + Q^2\cos^2\theta + 2PQ\cos\theta$

rearranging.

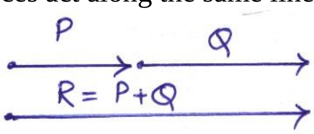
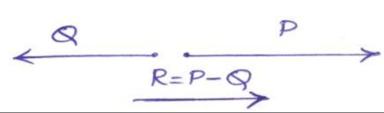
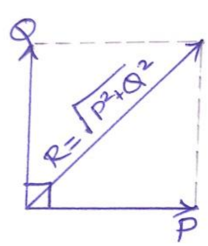
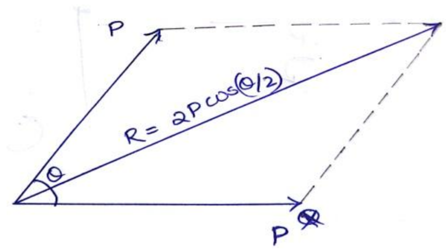
$R^2 = P^2 + Q^2\sin^2\theta + Q^2\cos^2\theta + 2PQ\cos\theta = P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ (since $\sin^2\theta + \cos^2\theta = 1$)

or $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

<table>
<tr><th colspan="2">Special Cases</th></tr>
<tr><td>1. When θ = 0, i.e. when forces act along the same line
R = P + Q
</td><td>2. When θ = 180, i.e. when forces act along the same straight line but opposite in direction
R = P - Q
</td></tr>
<tr><td>3. When θ = 90, i.e. when forces act at right angles
R = $\sqrt{P^2 + Q^2}$
</td><td>4. When P=Q, i.e. two forces are equal
R = 2Pcos(θ/2)
</td></tr>
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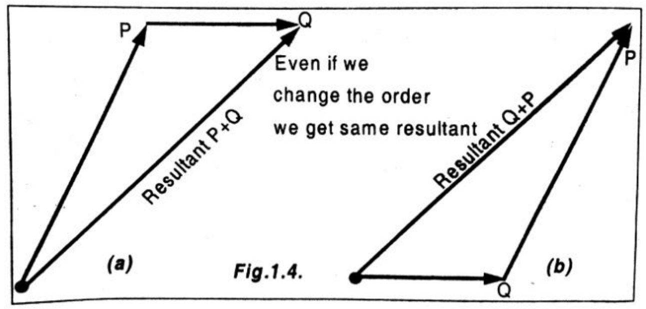
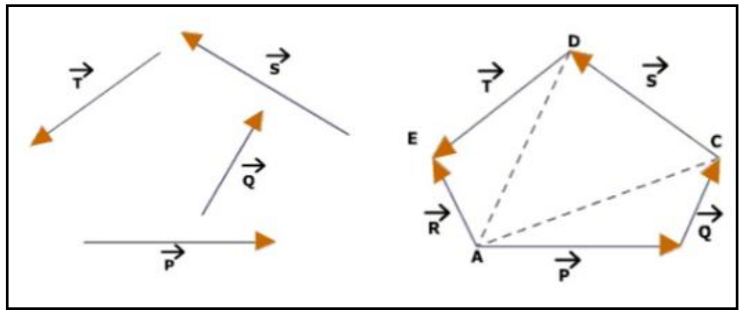
Triangular Law of Forces



If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, then their resultant is represented by the closing side of the triangle taken in the opposite order.

Polygon Law of Forces



If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken in the opposite order.

EQUILIBRIUM LAW (EQULIBRIUM PRINCIPLE)

Two force law/principle: Two forces can be in equilibrium only if they are **equal in magnitude, opposite in direction, and collinear in action**.

Three force law/principle: Three forces can be in equilibrium only if the **resultant of any two forces** is equal, opposite in direction, and collinear in action.

Four force law/principle: Four forces can be in equilibrium only if the **resultant of any two forces** must be equal, opposite in direction and collinear with the **resultant of the other two forces.**

PRINCIPLES OF SUPERPOSITION AND TRANSMISSIBILITY

Principle of superposition: The action of a given system of forces on a RIGID BODY will in no way be changed if we add or subtract from them another system of forces in equilibrium.

Consider a rigid body AB under the action of a force P applied at A and acting along BA as shown in the figure 6(a). From principle of superposition, application of two oppositely directed forces at point B, each equal to and collinear with P, will not change the action of the given force P. That is, the action of the three forces (as in figure 6(b) on the body is same as the action of single force P (as in figure 6(a)).

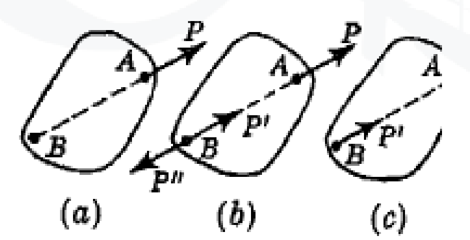


FIG. 6

Similarly, from the system of forces (in figure 6(b)), without changing the action of the three forces on the body, we can remove the equal, opposite and collinear forces P and P'', as the two forces (P and P'') are in equilibrium. Thus we obtain the condition shown in figure 6(c) where, instead of the original force P applied at A, we have an equal force P' applied at B. This proves the principle of transmissibility which is stated below.

Principle of transmissibility of force: The point of application of a force can be transmitted to any other point on the line of action of the force without changing the effect of the force on the RIGID BODY on which the force is applied.

LAW OF ACTION AND REACTION

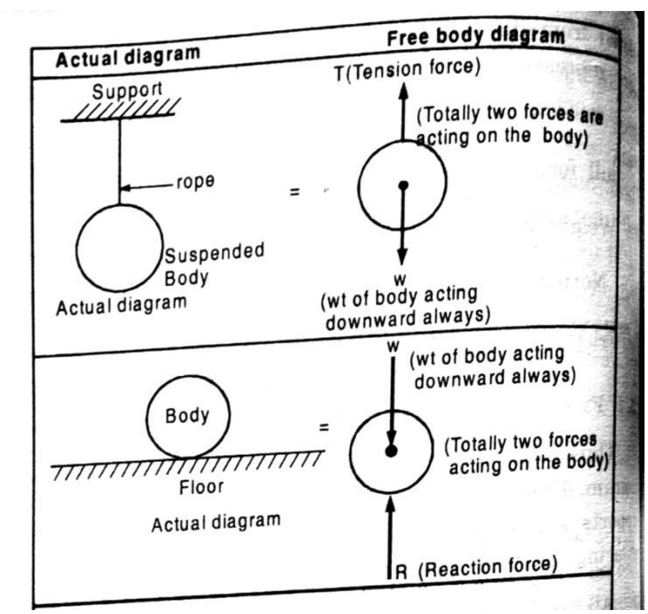
Any force given by a body to its support (action) causes and equal and opposite force from the support to the body (reaction) so that action and reaction are two equal and opposite forces.

This is nothing but Newton's third law of motion stated in a different form that is suitable for solving problems in statics.

FREE BODY DIAGRAM

A Free body diagram is nothing but a sketch of the body obtained by removing the supports and replacing the supports by the reactions the supports exert on the body.

Thus all the external forces and reactions of the supports acting on the body are shown in free body diagram. The self weight of the body acting downwards through the centre of gravity of the body is also considered in free body diagram.



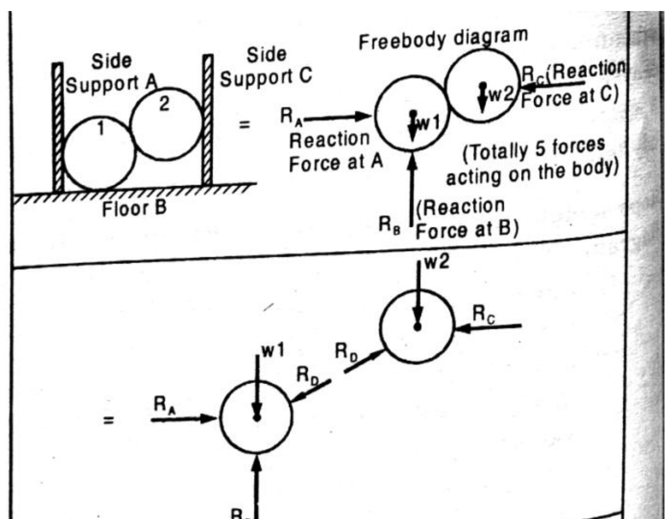


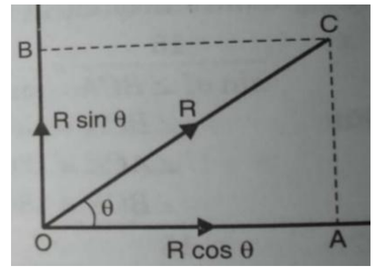
Fig.1.49. Free Body Diagram.

COMPOSITION AND RESOLUTION OF FORCES

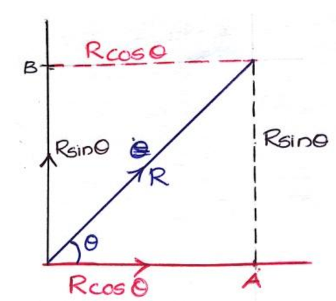
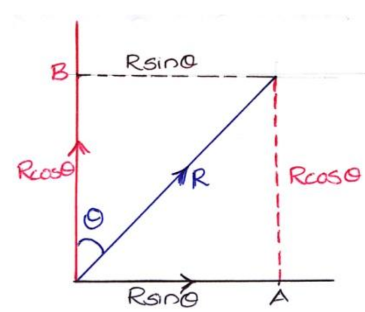
Resolution of forces

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force.

A force is generally resolved along two mutually perpendicular directions, i.e. **horizontal and vertical directions**



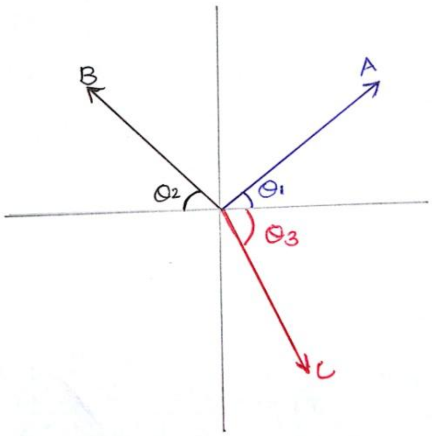
Consider a force R acting at O and making an angle θ with the horizontal. The force P can be resolved into its horizontal component (R cosθ) and vertical component (R sinθ).

Cases	Horizontal Component	Vertical Component	Figure
When Force (R) makes an angle **θ** with **horizontal**	**Rcosθ**	**Rsinθ**	
When Force (R) makes an angle **θ** with **vertical**	**Rsinθ**	**Rcosθ**	

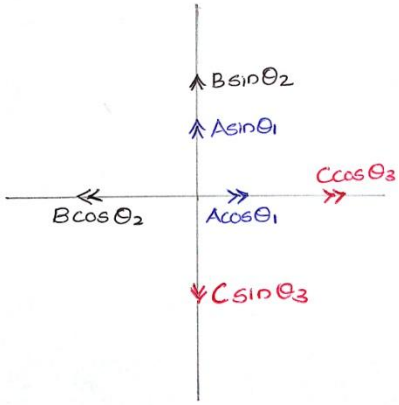
Composition of forces

The reduction of a given system of forces to the simplest system that will be its equivalent is called composition of forces.

Consider the system of concurrent forces A,B and C originating from point O as shown in figure i. In order to find the resultant the forces are resolved into horizontal and vertical components (figure ii). These components can be reduced to a single horizontal and vertical force (figure iii) and finally to a single force R (figure iv). This process of reducing the system of forces (A, B and C) to a single force R is known as composition of forces



Figure(i)



Figure(ii)

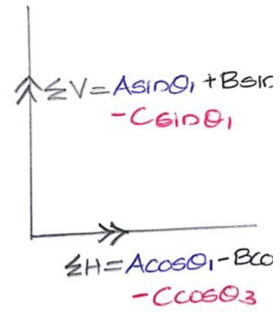
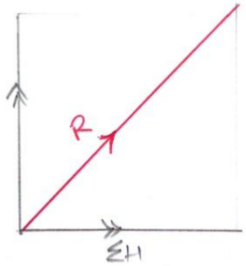


Figure (iii)



Figure(iv)

RESULTANT AND EQUILIBRIUM EQUATIONS

Resultant Force

When a body is acted upon by a number of forces, then the single force which will produce the same effect as produced by all the given forces is known as resultant force.

The resultant force, of a given system of forces, is found out by the method of projection which will be discussed later.

$$\textbf{Resultant Force R} = \sqrt{\Sigma H^2 + \Sigma V^2}$$

Where $\sum$H is the sum of all horizontal forces and $\sum$V is the sum of all vertical forces

The resultant force will be inclined at an angle **θ with the horizontal** such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

Equilibrium Equations

Equilibrium: **A body is said to be in equilibrium when its state, either in rest or in motion along a straight line is not influenced by the external forces acting on it.**

In such a case, the resultant will be zero and the net effect of all the forces acting on the body will be zero and the body will be in equilibrium.

Equilibrium forces: If the resultant of a system of forces is zero, then such a system of forces is known as equilibrium forces.

Equilibrant: The force, which brings the body acted upon by a system of forces, to equilibrium is known as equilibriant. Equilibrant will be always equal to, opposite in direction and collinear to the resultant of the system of forces.

Principles/Laws of Equilibirum discussed in page 8

Conditions/Equations of Equilibrium

For a body to be in equilibrium the resultant should be zero i.e. **R = 0**

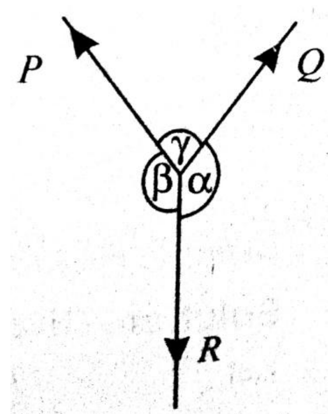
But R = $\sqrt{\Sigma H^2 + \Sigma V^2}$ = **0,** which means ∑H and ∑V and should be equal to zero,

Thus the **condition for equilibrium** is that

$$\boxed{\sum H = 0 \text{ and } \sum V = 0}$$

Lami's Theorem

Lami's theorem states that if three coplanar forces acting at a point is in equilibrium, then each force is proportional to the sin of the angle between the other two



Mathematically,

$$\boxed{\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}}$$

Where P, Q and R are three forces and α, β and γ are the angles as shown in the figure

Proof

Consider three coplanar forces A, B and C acting at point O. Let opposite angles to three forces be α, β and γ as shown in figure

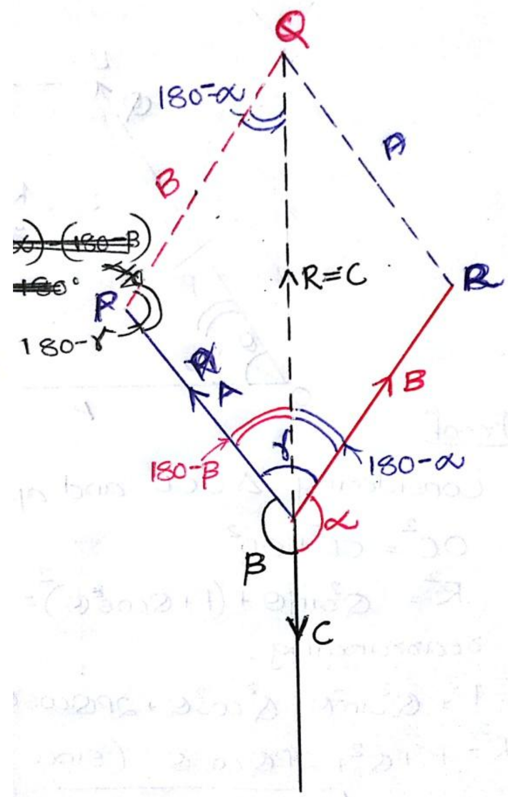
Complete parallelogram OPQR. Resultant of A and B, R will be given by the diagonal of parallelogram OPQR. Since A, B and C are in equilibrium, resultant ~~force~~. ~~R~~ of the forces. A and B, R must be in line with ~~Q~~force C and equal to C. ⇒ R=C.

$\angle POR = 180 - \beta$; $\angle PQO = \angle ROQ = (180 - \alpha)$

$\angle QPO = 180 - \angle POR - \angle PQO$ (sum of angle of $\Delta = 180^\circ$)

$\angle QPO = 180 - (180 - \beta) - (180 - \alpha)$

$\angle QPO = 180 - 180 + \beta - 180 + \alpha = \alpha + \beta - 180^\circ$



Now, $\alpha + \beta + \gamma = 360^\circ$

Subtracting 180 from both sides

$(\alpha + \beta + \gamma) - 180 = 360 - 180$

rearranging

$(\alpha + \beta - 180) + \gamma = 180$

$\angle QPO + \gamma = 180$ or $\angle QPO = 180 - \gamma$ ($\angle QPO = \alpha + \beta - 180^\circ$)

Applying sin rule to ΔPOQ

$$\cancel{\frac{OP}{\sin\angle ACO}} \quad \frac{OP}{\sin\angle PQO} = \frac{PQ}{\sin\angle POQ} = \frac{OQ}{\sin\angle QPO}$$

$$\frac{OP}{\sin(180-\alpha)} = \frac{PQ}{\sin(180-\beta)} = \frac{OQ}{\sin(180-\gamma)}$$

$$\frac{A}{\sin(180-\alpha)} = \frac{B}{\sin(180-\beta)} = \frac{C}{\sin(180-\gamma)}$$

Since $\cancel{180 - \theta =}$ $\sin(180 - \theta) = \sin\theta$

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$