MODULE 3

CENTRE OF GRAVITY

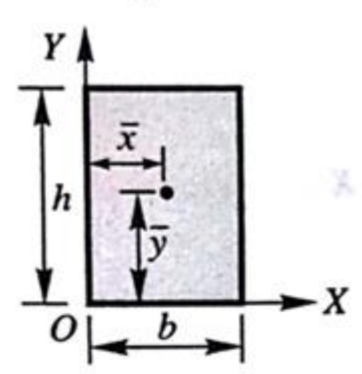
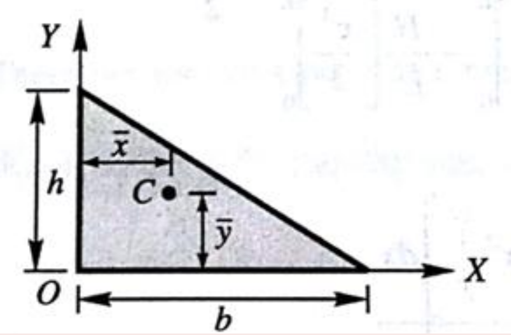
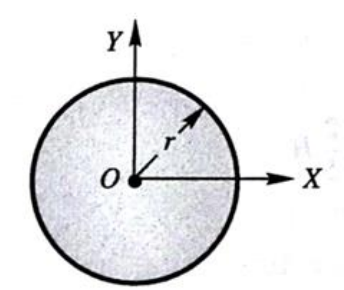
- The point through which the whole weight of the body acts. It is represented by C.G or simply G.
- Applies to bodies with mass and weight

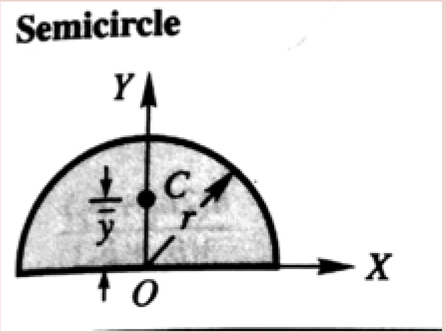
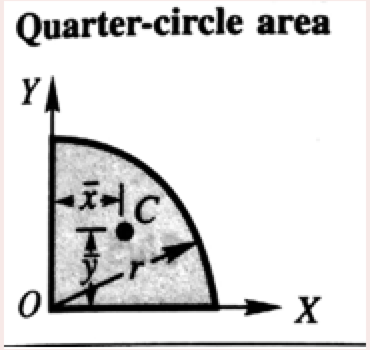
CENTROID

- The point at which the total area of a plane figure is assumed to be concentrated. It is represented by C.G or G.
- Applies to plane areas.

Methods

- By geometrical considerations
- By method of integration
- By method of moments

Sl No.	Shape	Area	Location of centroid x	y
1	Rectangle 	bh	$\frac{b}{2}$	$\frac{h}{2}$
2	Right angle triangle 	$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$
3	Circle 	Πr^2	0	0

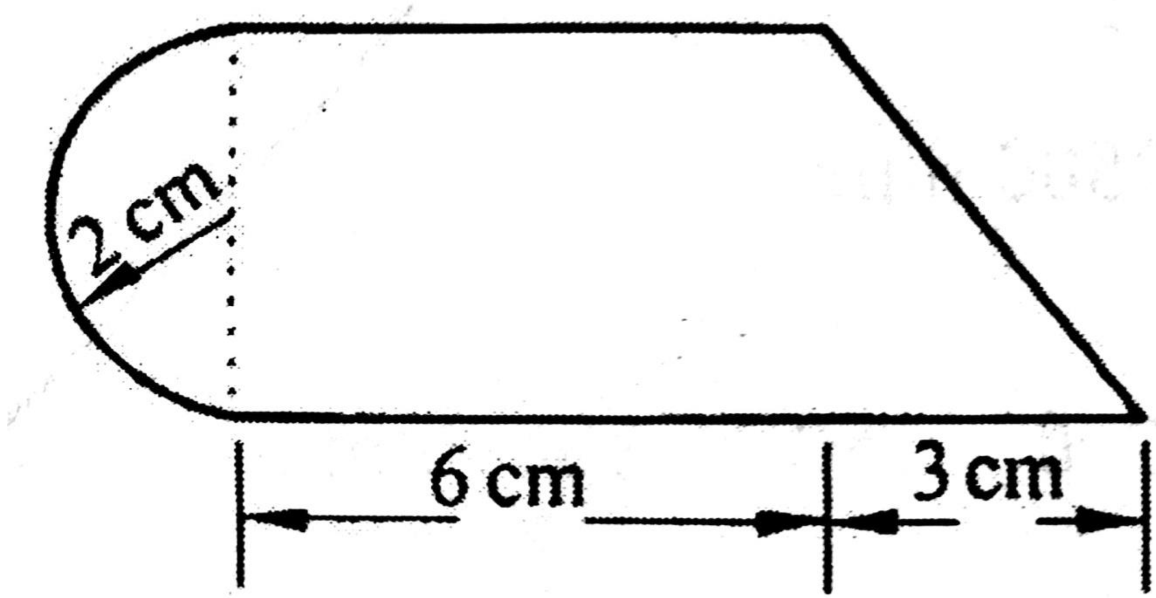
Sl No.	Shape	Area	Location of centroid x	y
4		$\frac{\Pi r^2}{2}$	0	$\frac{4r}{3\Pi}$
5		$\frac{\Pi r^2}{4}$	$\frac{4r}{3\Pi}$	$\frac{4r}{3\Pi}$

IMPORTANT POINTS

- The axis about which the moments of areas are taken is known as axis of reference.
- The lowest line of the figure for calculating y, and left line for calculating x of C.G
- If the given section is symmetrical about XX or YY axis ,the centre of gravity will lie on axis of symmetry
- The C.G of structural sections like T ,I, L sections, etc are obtained by splitting them into rectangular components

Example

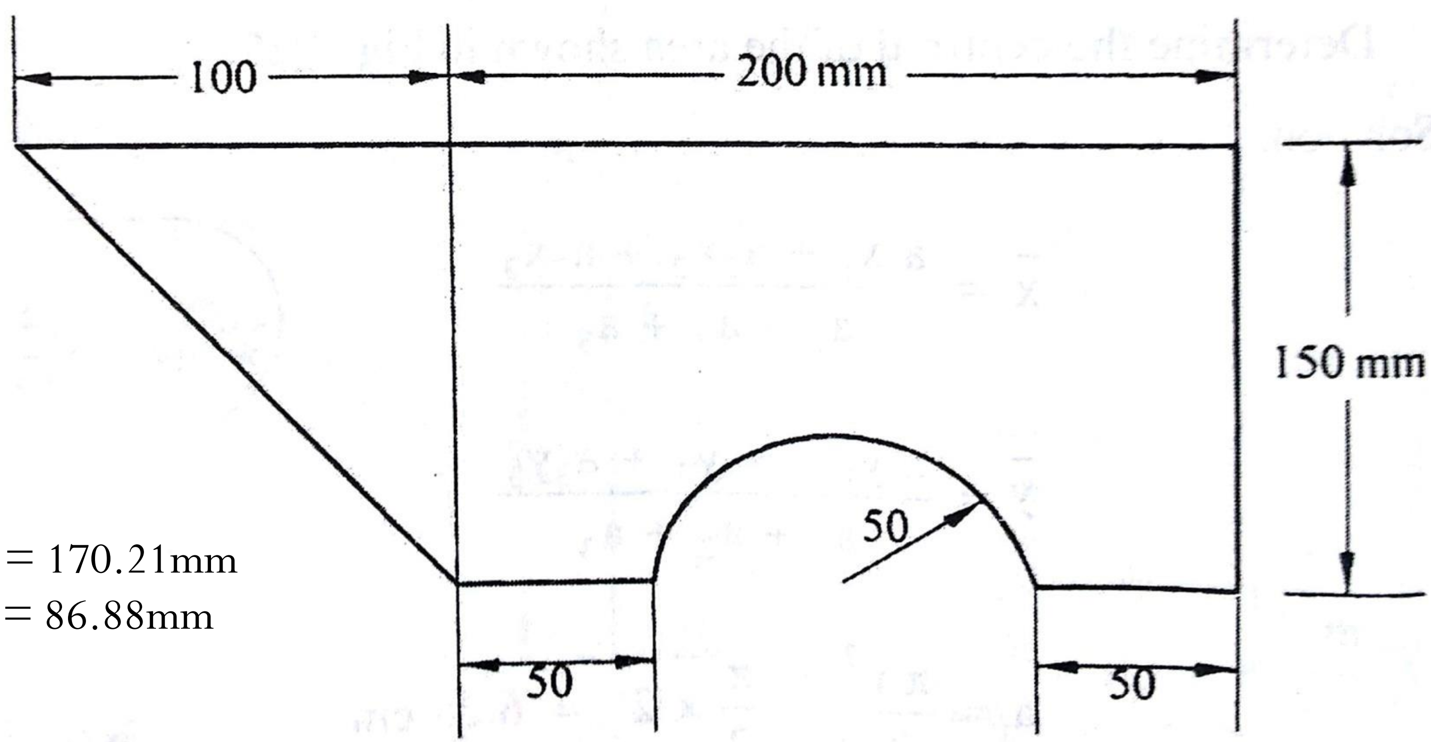
- Locate the centroid of the area shown in the figure.



X= 5cm
Y = 1.89cm

Example

- Locate the centroid of the area shown in the figure.

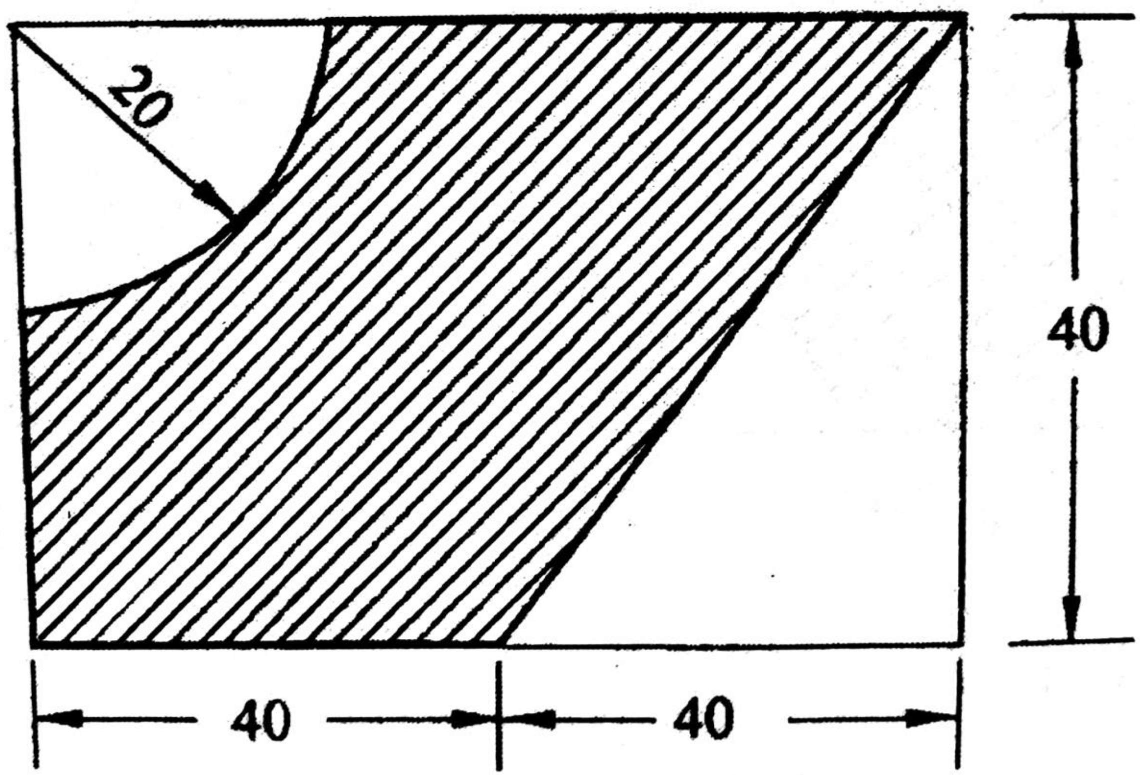


X = 170.21mm

Y = 86.88mm

Example

- Locate the centroid of the shaded area shown in the figure.



X = 34.52mm

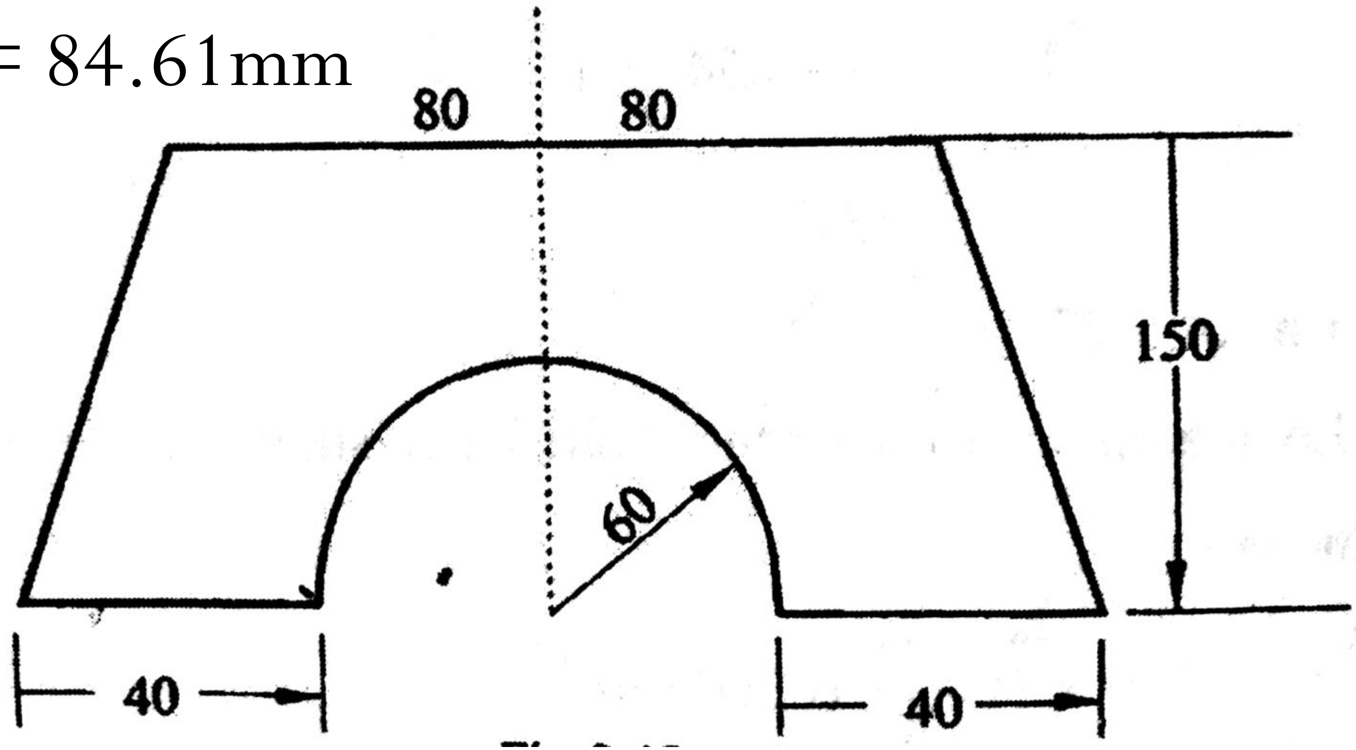
Y = 20.82mm

Example

- Locate the centroid of the area shown in the figure.

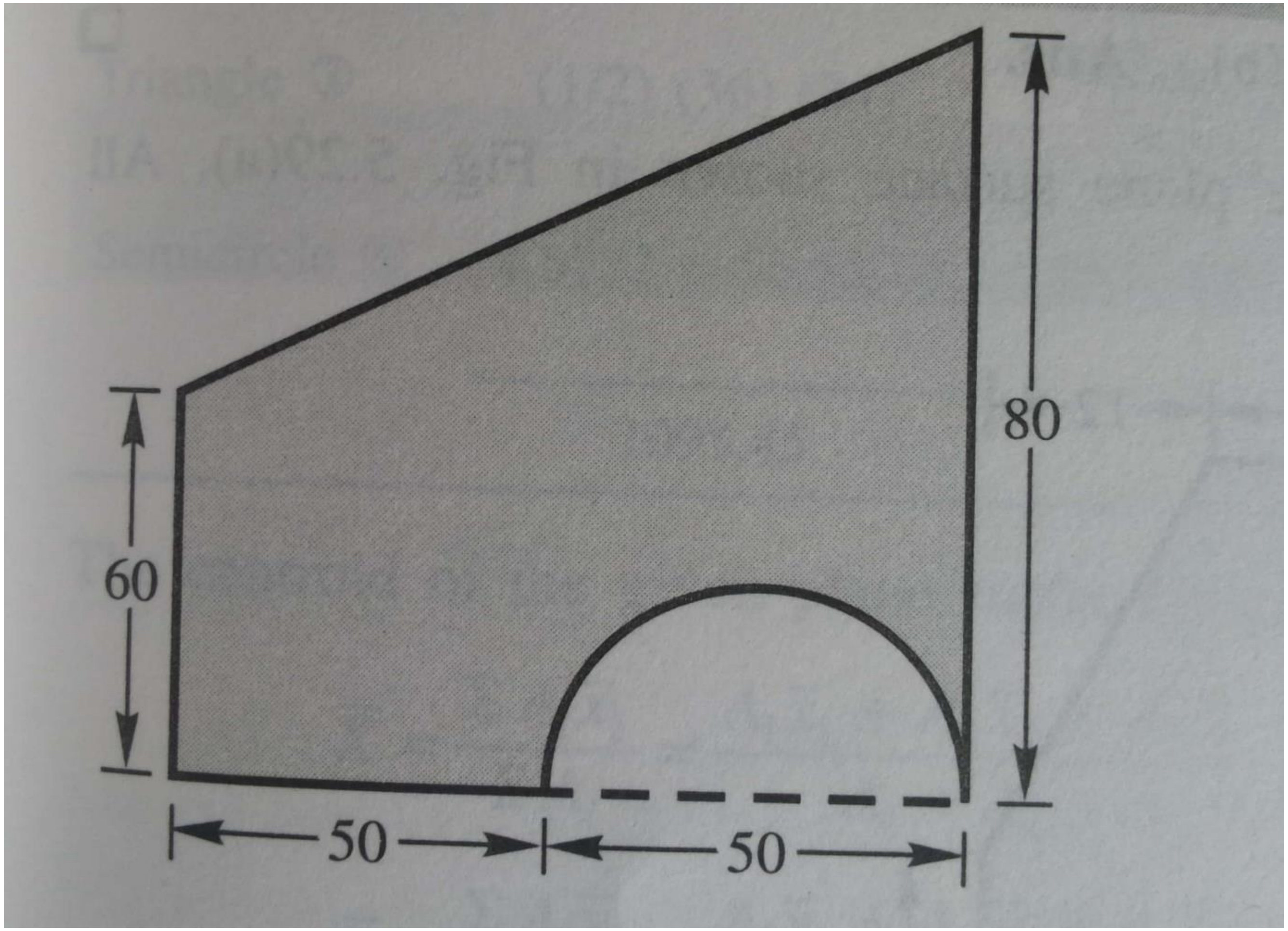
X = 100

Y = 84.61mm

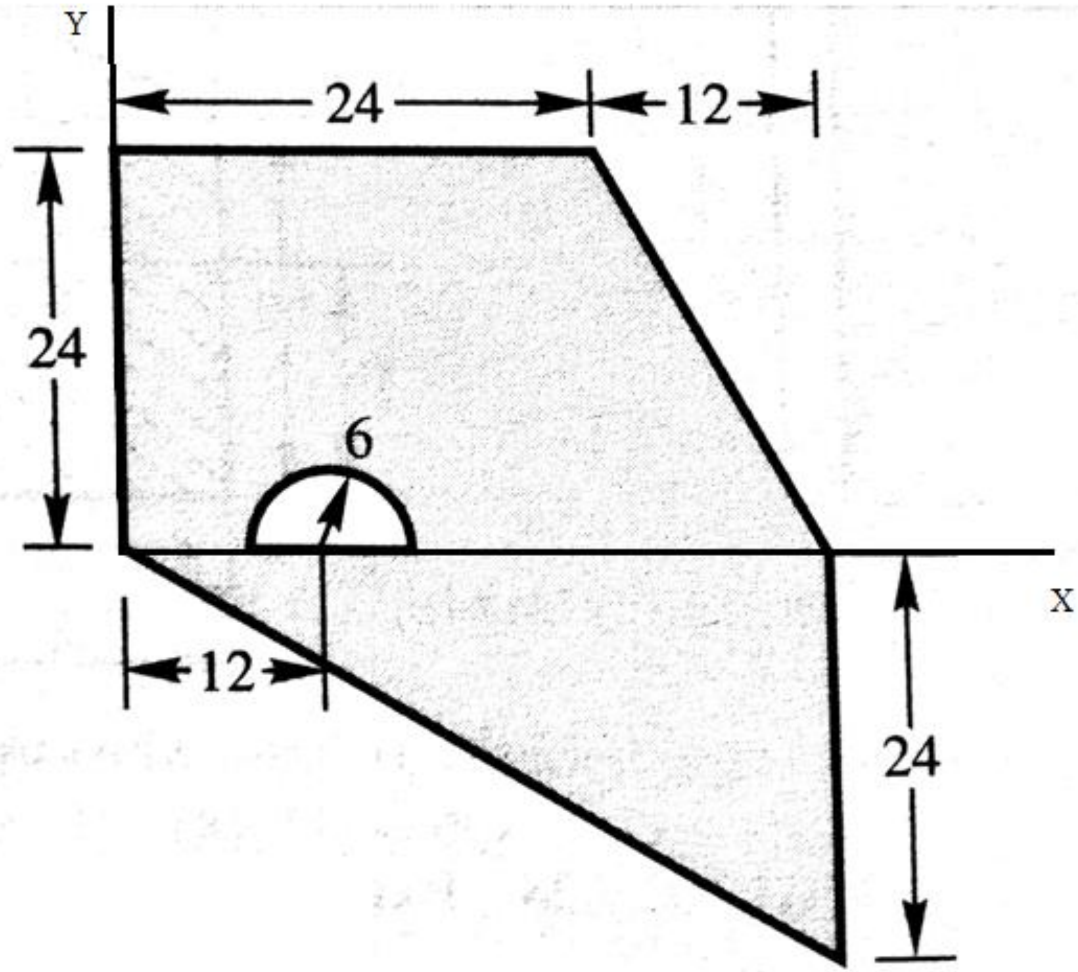


Example

X = 48.69
Y = 39.25



Locate the centroid of the area shown in the figure.



X = 18.84

Y = 4.07

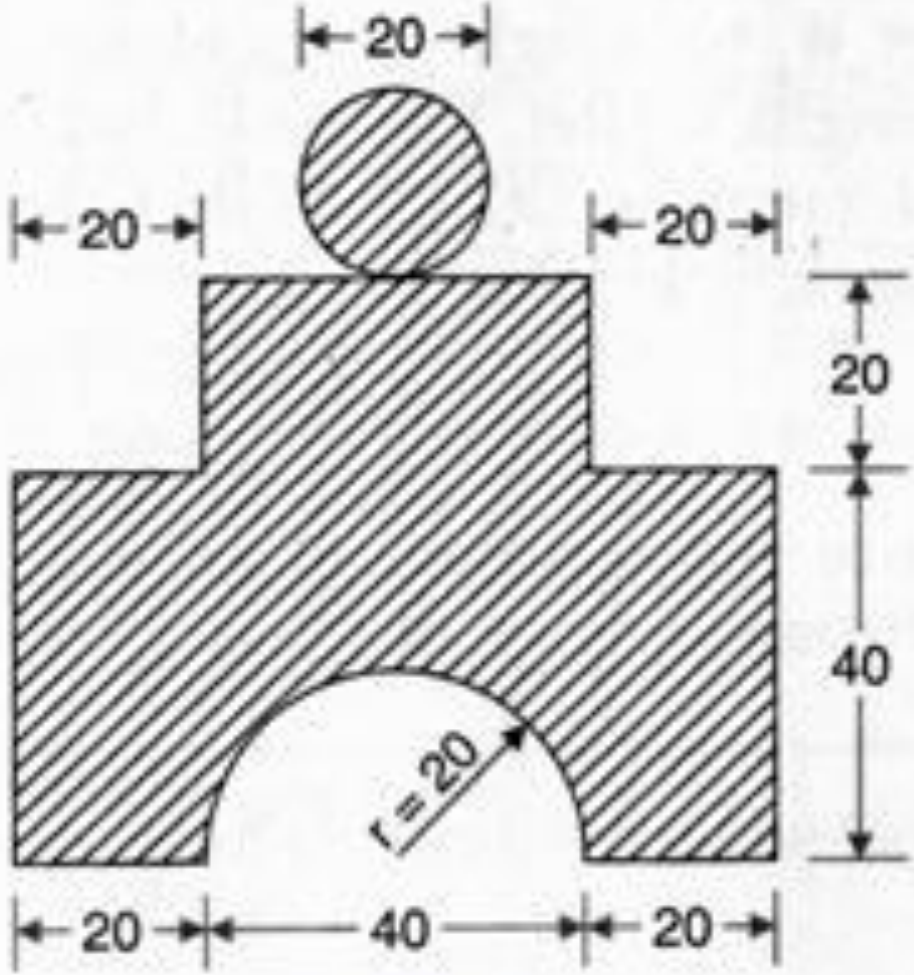
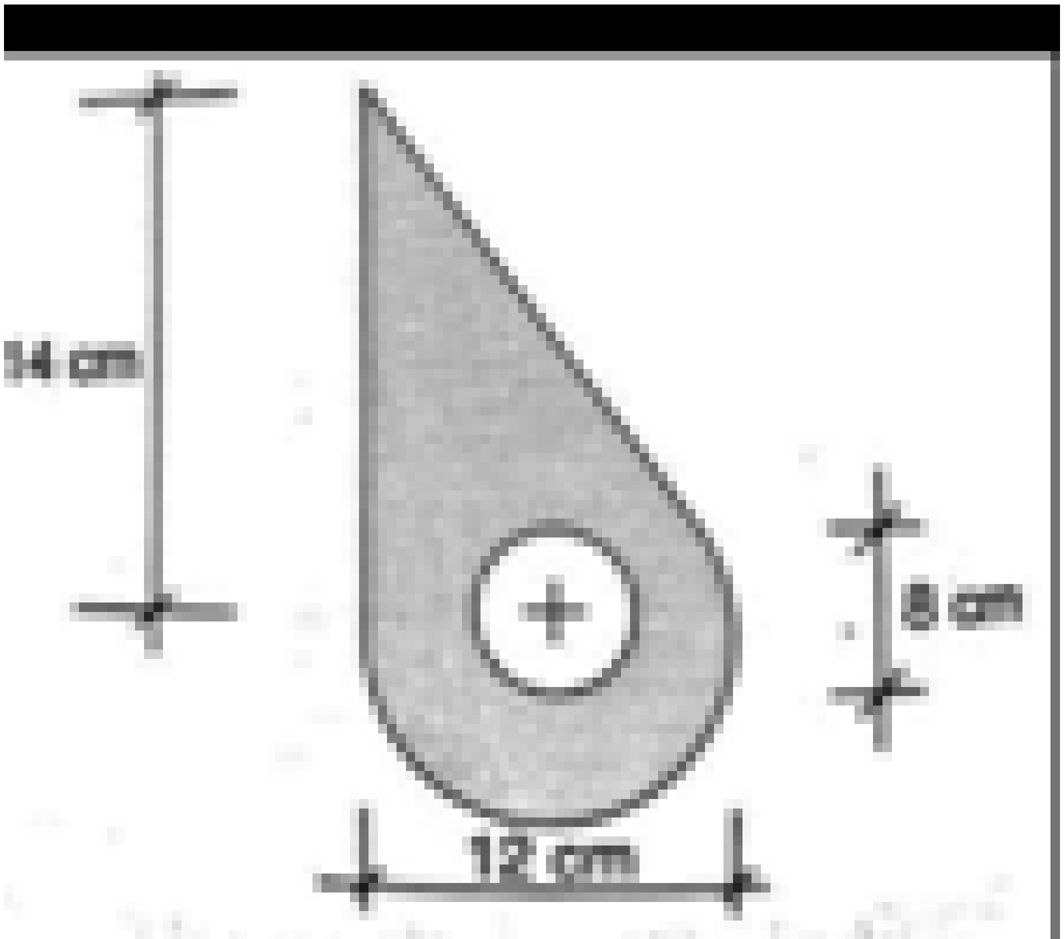


Fig.5

Locate the centroid of the area shown in the figure.



X = 4.13

Y = 8.73

POINTS TO RMEMBER

- The axis about which the moments of areas are taken is known as axis of reference.
- The lowest line of the figure for calculating y, and left line for calculating x of C.G
- If the given section is symmetrical about XX or YY axis ,the centre of gravity will lie on axis of symmetry
- The C.G of structural sections like T ,I, L sections, etc are obtained by splitting them into rectangular components

Moment of Inertia

Area Moment of Inertia

- Second moment of area
- Purely mathematical term
- One of the important properties of area
- Strength of members subject to bending upon the moment of inertia of its cross sectional area

PARALLEL AXIS THEOREM

It states that if I_{GG} is the moment of inertia of a plane lamina of area A, about its centroidal axis of the lamina, then the moment of inertia about any axis AB which is parallel to the centroidal axis and at a distance h from the centroidal axis is given by

$$I_{AB} = I_{GG} + Ah^2$$

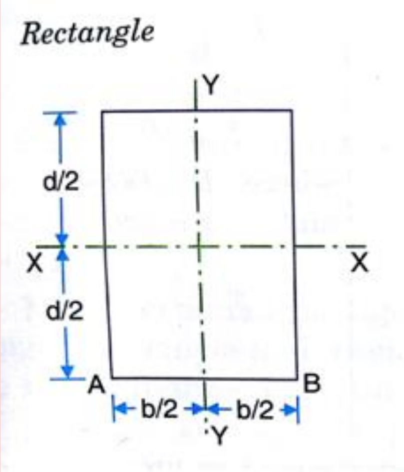
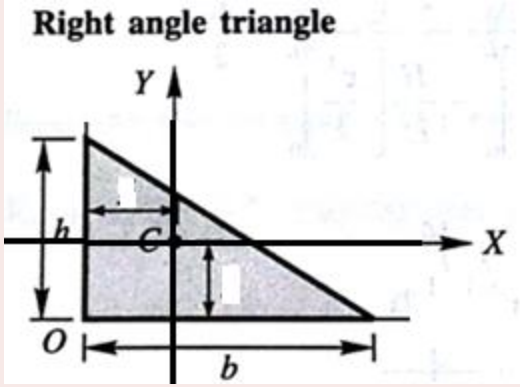
PERPENDICULAR AXIS THEOREM

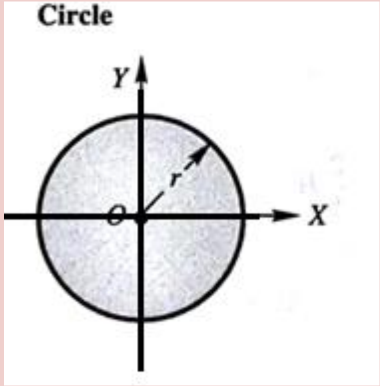
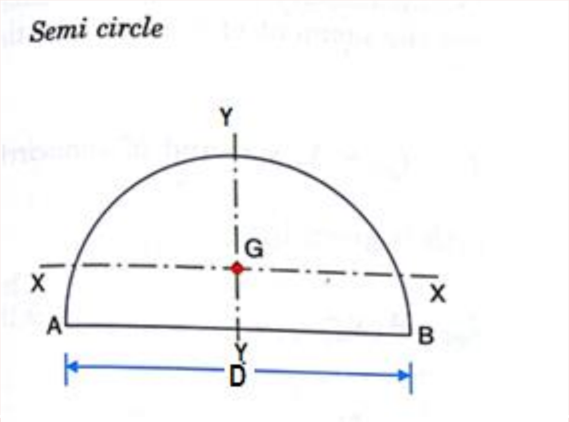
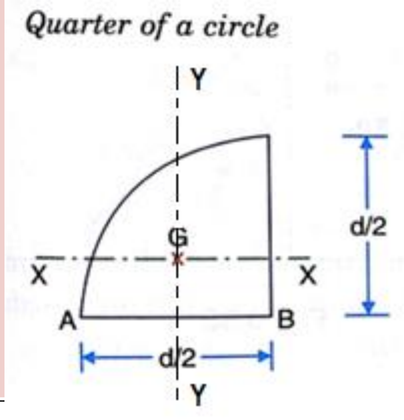
- If I_{XX} and I_{YY} are the moment of inertia of an area A about mutually perpendicular axis XX and YY, in the plane of the area, then the moment of inertia of the area about the ZZ axis which is perpendicular to XX and YY axis and passing through the point of intersection of XX and YY axis is given by

$$Izz = Ixx + Iyy$$

Polar moment of inertia

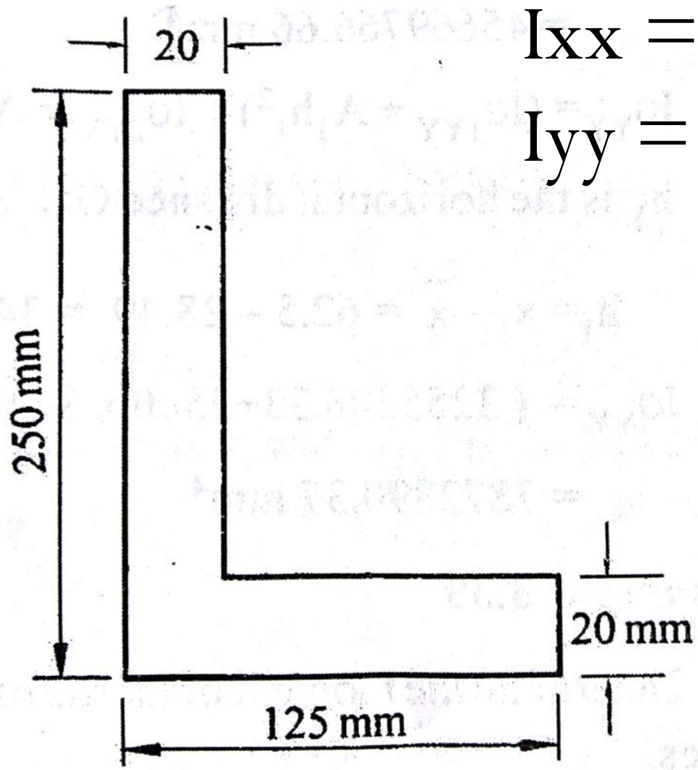
- Moment of inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia. It is denoted as J or Izz.

Shape	I_{Gxx}	I_{Gyy}
	$\frac{bd^3}{12}$	$\frac{db^3}{12}$
	$\frac{bh^3}{36}$	$\frac{hb^3}{36}$

Shape	IGxx	IGyy
	$\dfrac{\pi D^4}{64}$	$\dfrac{\pi D^4}{64}$
	$0.11R^4$	$\dfrac{\pi D^4}{128}$
	$0.055R^4$	$0.055R^4$

Example

Find the moment of inertia of an unequal angle section about its centroidal x axis



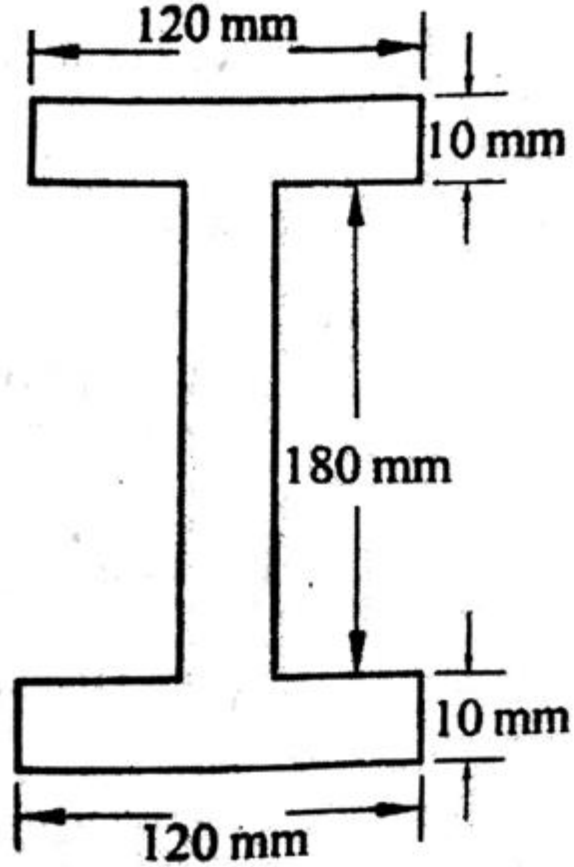
Ixx = 45669766.66

Iyy = 7872890.37

Sl No.	Figure	Area	xi	yi	Aixi	Aiyi
1						
2						

Sl No	Area	Igxx	dy	A(dy)^2	
1					
2					

Determine the moment of inertia about centroidal y axis

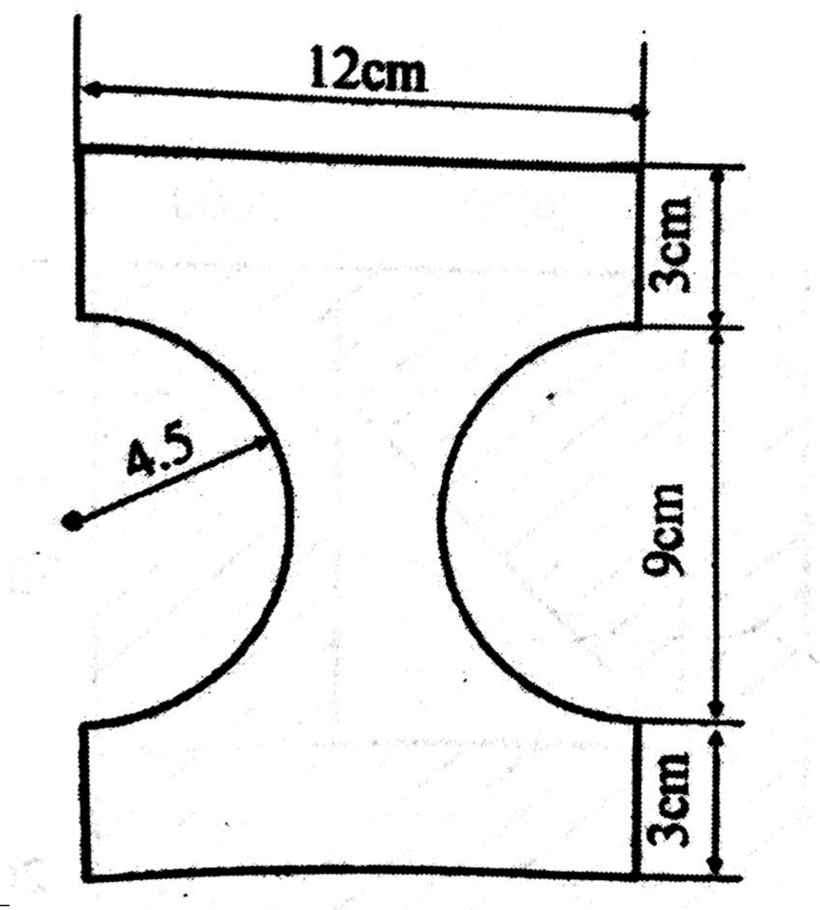


Ixx = 26540000

Iyy = 2895000

Example

Determine the moment of inertia about centroidal axis

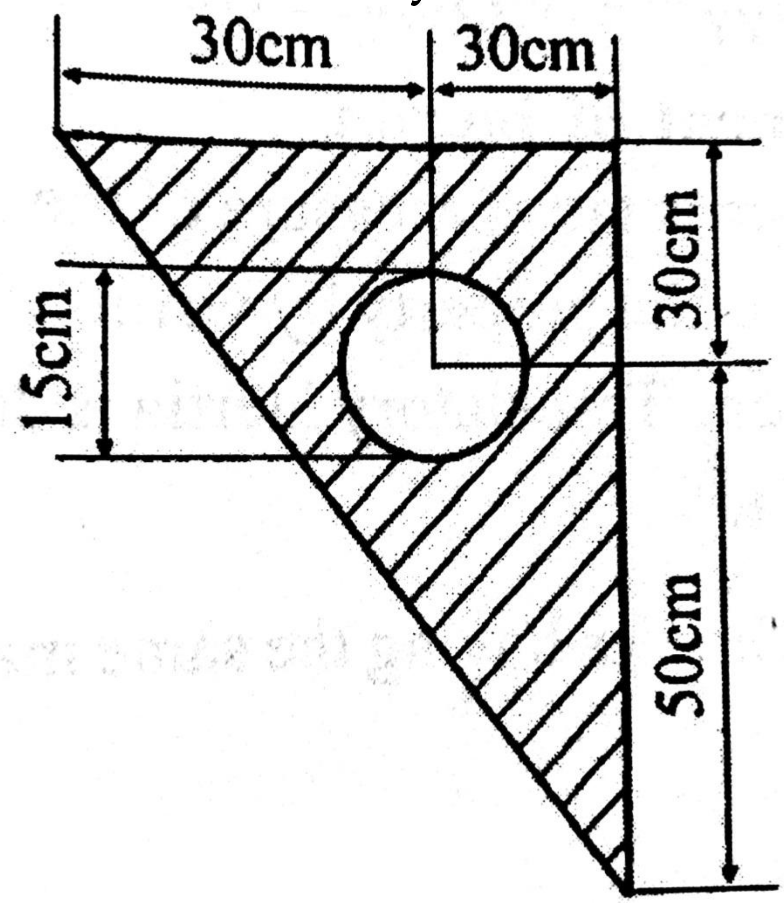


Ixx = 3052.94

Iyy = 1005.52

Example

Calculate the moment of inertia of the shaded area as shown in the figure with respect to centroidal x and y axes

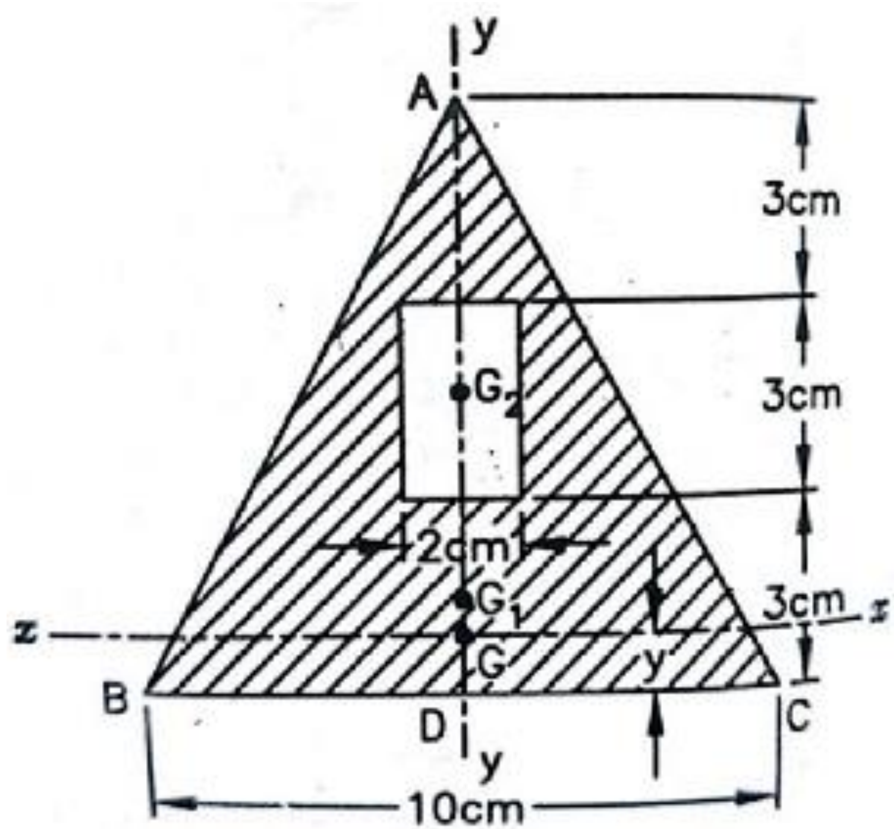


Ixx = 848735.42

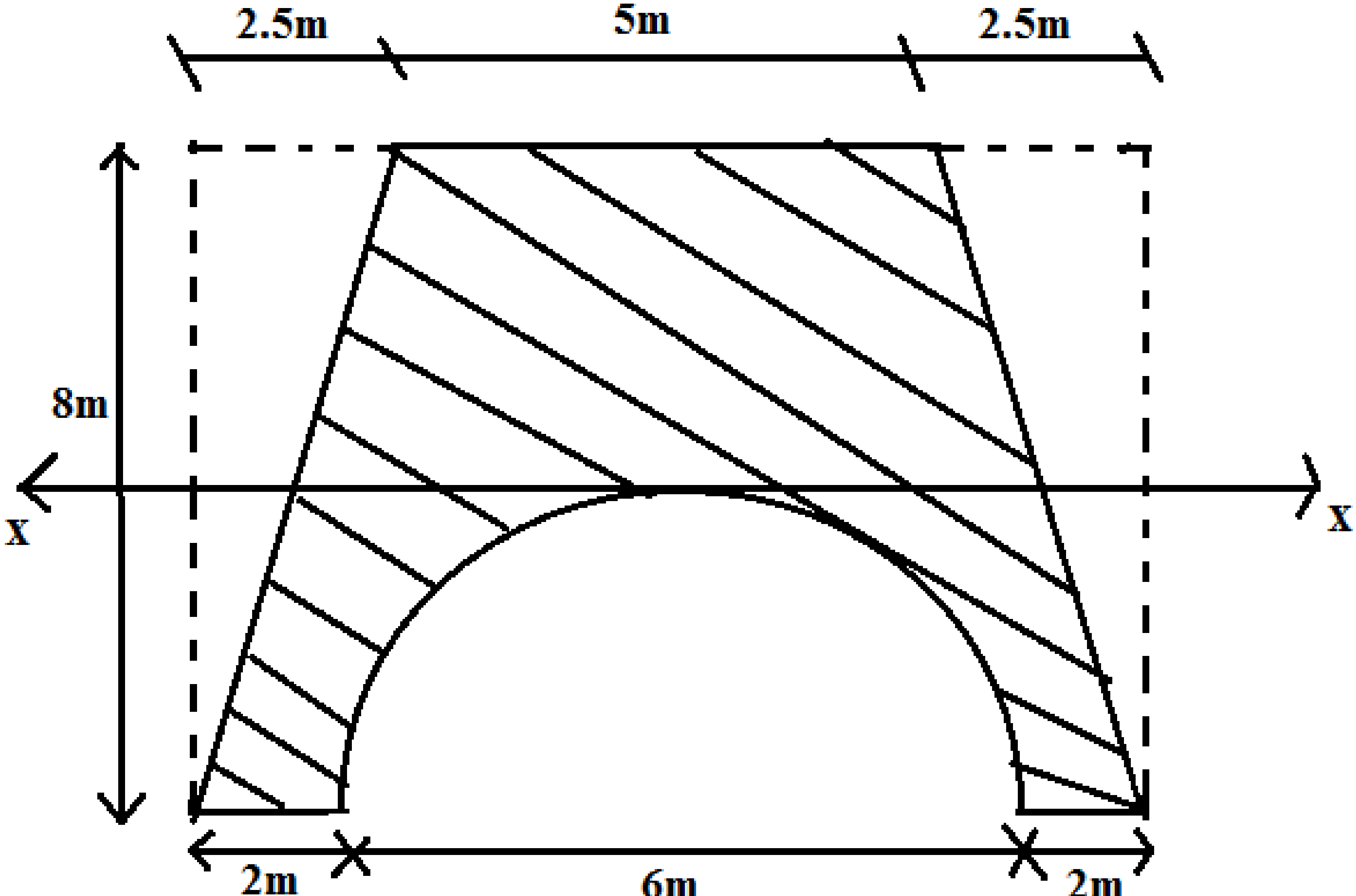
Iyy = 458438.854

Example

- A rectangular hole is made in a triangular section as shown in Fig. Determine the M.I. of the section about x-x axis passing through the CG of the section and parallel to BC. Also find the M.I, with respect to BC.

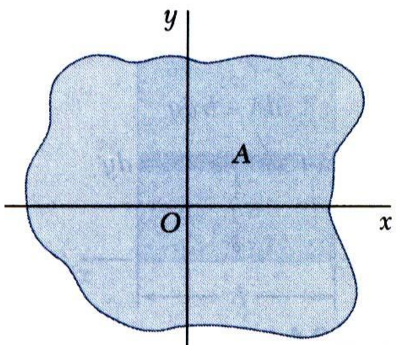


Find the centroid of the cross section of a culvert as shown in figure below. Determine the M. I of horizontal axis XX passing through top of the semi-circle.

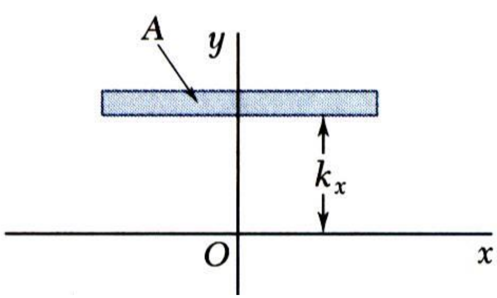


Radius of Gyration of an Area

- Consider area A with moment of inertia I_x , Imagine that the area is concentrated in a thin strip parallel to the x axis with equivalent I_x



$$I_x = k_x^2 A \qquad k_x = \sqrt{\frac{I_x}{A}}$$



$k_x =$ *radius of gyration* with respect to the x axis

Radius of gyration

- **Radius of Gyration of a body (or a given lamina) about an axis is the distance such that its square multiplied by the area gives M.I of the area about the given axis.**

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

$$Kxx = \sqrt{\frac{Ixx}{A}}$$

$$Kyy = \sqrt{\frac{Iyy}{A}}$$

THEOREMS OF PAPPUS AND GULDINUS

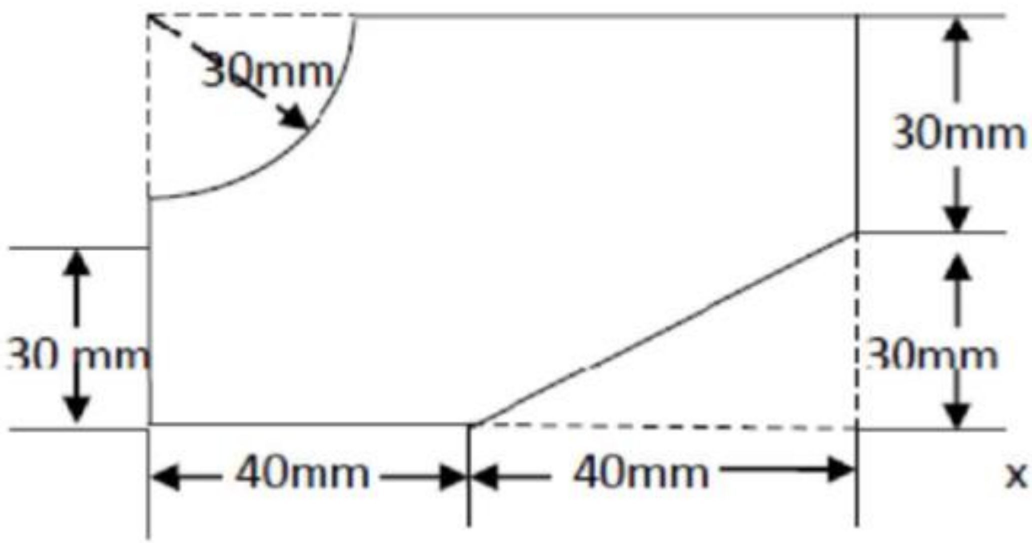
- **THEOREM 1 .**
- **The area of surface generated by revolving a plane curve about a non-intersecting axis in the plane of the curve is equal to the product of length of curve and the distance travelled by the centroid 'G' of the curve during revolution.**

THEOREM 2

- **The volume of solid generated by revolving a plane area about a non-intersecting axis in its plane is equal to the product of area and distance travelled by the centroid 'G' of the area during the rotation about the axis.**

Example

- Calculate the moment of inertia and radius of gyration about X axis for the sectioned area.



Vector operations

- Vector addition
- Vector subtraction
- Cross product of vectors
- Dot product of vectors

Example

- Add the two vectors A = 2i+3j+4k and B= -4i+2j+2k

- -2i+5j+6k

Example

- $F1 = 3i+4j-2k$ and $F2 = 2i+2j-4k$, find

  $F1 - F2$

$i+2j+2k$

Example

- Find the cross product of forces

$F1=2i-4j+5k$ and $F2 = -i+3j+6k$

$-39i-17j+2k$

Example

- Find the angle between the forces

$F1=3i-2j+4k$ and $F2= 5i+3j-4k$

$100.58°$

Example

- A force $F = 8.25i + 12.75j - 18k$ acts through the origin. What is the magnitude of this force and angles it makes with x,y and z axes.

- $F = 23.55$
- 69.49
- 57.22
- 139.84

Example

- A force of magnitude 50kN is acting along the line joining A(2,0,6) and B(3,-2,0). Write the vector form of force.

- F = 7.8i-15.6j-46.85k

Example

- A force F acts at the origin of a coordinate system in a direction defined by the angles $\theta x = 69.3°$, $\theta z = 57.9°$. If the component of force along y direction is -174N, determine

I. The angle θy

II. Other components of force F

III. The magnitude of force

$\theta y = 140.48°$

$Fx = 79.62N$

$Fz = 119.55N$

$F = 225.6N$

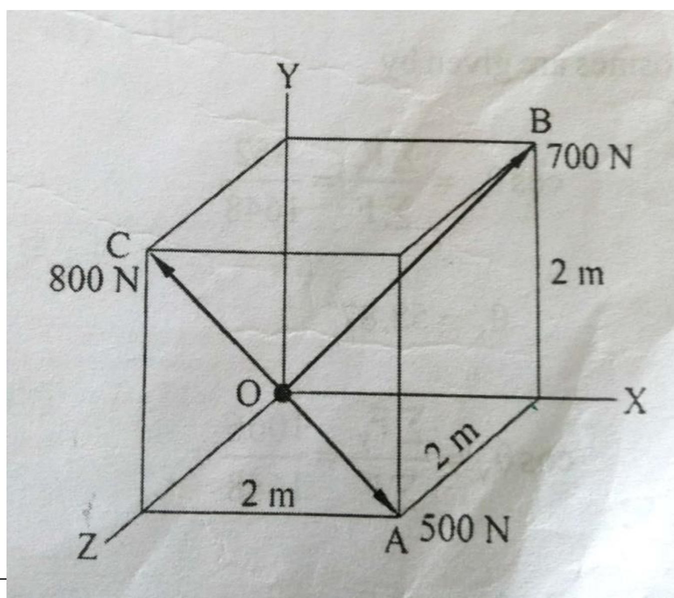
Composition of non coplanar concurrent forces

Example

- Four forces 32kN, 24kN, 24kN and 120kN are concurrent at origin and are respectively directed through the points whose coordinates are A(2,1,6), B(4,-2,5), C(-3,-2,1) and D(5,1,-2). Determine the resultant of the system.
- $R = 114.52i + 6.96j + 10.525k$
- Magnitude of R = 115.21kN
- $\theta x = 6.28, \theta y = 86.54, \theta z = 84.76$

Example

- Three forces 500N, 700N and 800N are acting along the three diagonals of adjacent faces of cube of side 2m as shown in the figure. Determine the resultant of the forces.



R =

Magnitude = 1648N

Θx = 58.87

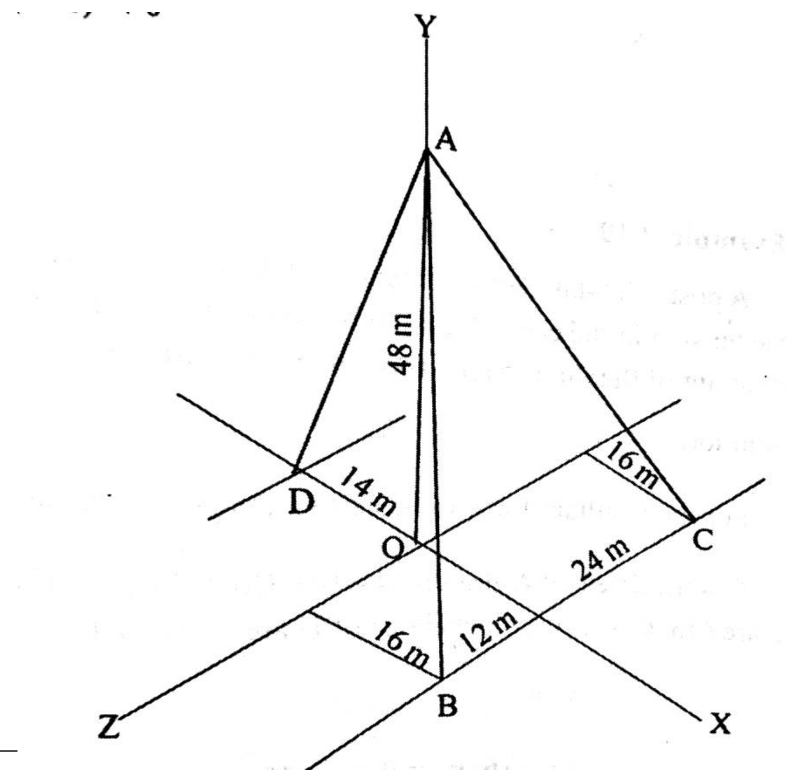
Θy = 49.7

Θz = 55.9

Example

- A post is held in vertical position by three cables AB , AC and AD as shown in the figure. If the tension in the cable AB is 40N, calculate the required tension in AC and AD so that the resultant of three forces applied at A is vertical.

Fac = 21.47N
Fad = 66.20N



Example

- Two forces P and Q are acting at the origin. The force P whose magnitude is 70N is directed towards (3,-6,2). The force Q is inclined at 45°, 60° and 60° respectively with x, y and z axes. Determine the magnitude of Q so that the resultant of P and Q will be in xz plane. Also determine the magnitude and direction cosines of the resultant.
- Q = 120N, Rx = 114.84N, Rz = 80N
- R = 139.96N
- 34.85,90,55.13

Moment of force

Example

- A force $F = 2i+3j-4k$ is applied at the point $B(1,-1,2)$. Find the moment of the force about a point $A(2,-1,3)$.

- $M = 3i-6j-3k$

Example

- A force of magnitude 44N acts through the point A(4,-1,7) in the direction of vector 9i+6j-2k. Find the moment of the force about the point B(1,-3,2).
- M= -136i+204j

Example

- Find the moment about C(-2,3,5) of the force F = 4i+4j-k passing through the points A(1,-2,4) and B(5,2,3).
- M = 9i-j+32k

Example

- A force P is directed from a point A(4,1,4) meters towards a point B (-3,4,1)metres.Determine the moment of force P about x and y axis if it produces a moment of1000Nm about z axis.
- P = 431.22
- Mx = - 789.13 Nm
- My = - 842.5 Nm

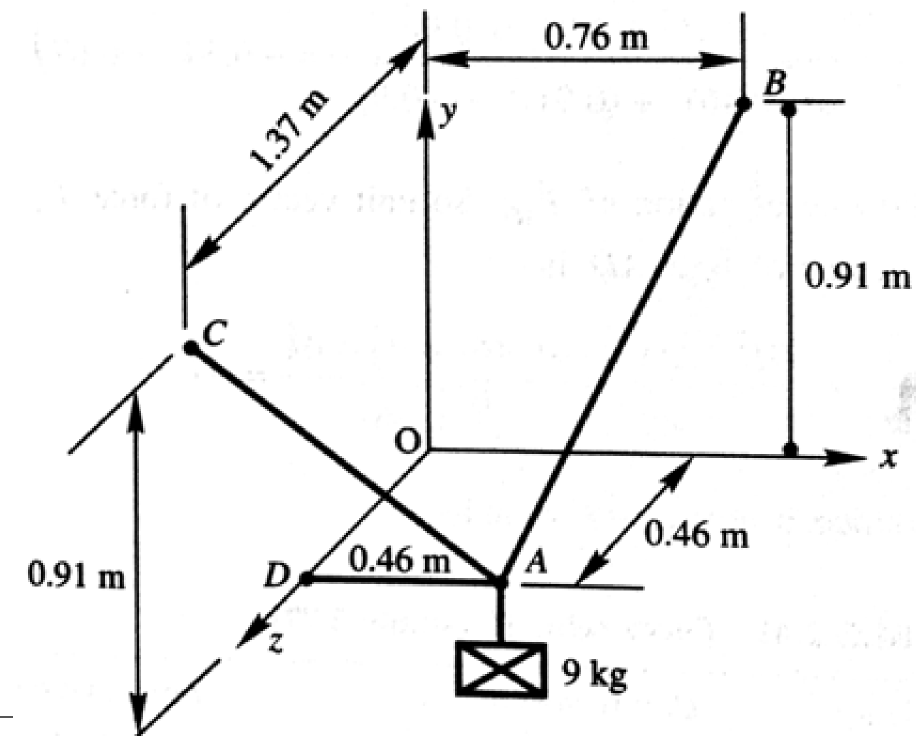
Equilibrium of particles in space

- A particle in space is said to be in equilibrium if the resultant force vector of the system of non-coplanar concurrent forces is zero.
- $\sum Fx = 0$
- $\sum Fy = 0$
- $\sum Fz = 0$

Example

- Determine the magnitude and direction of forces in the cables AB, AC and AD shown in the figure, so that the particle is in equilibrium.

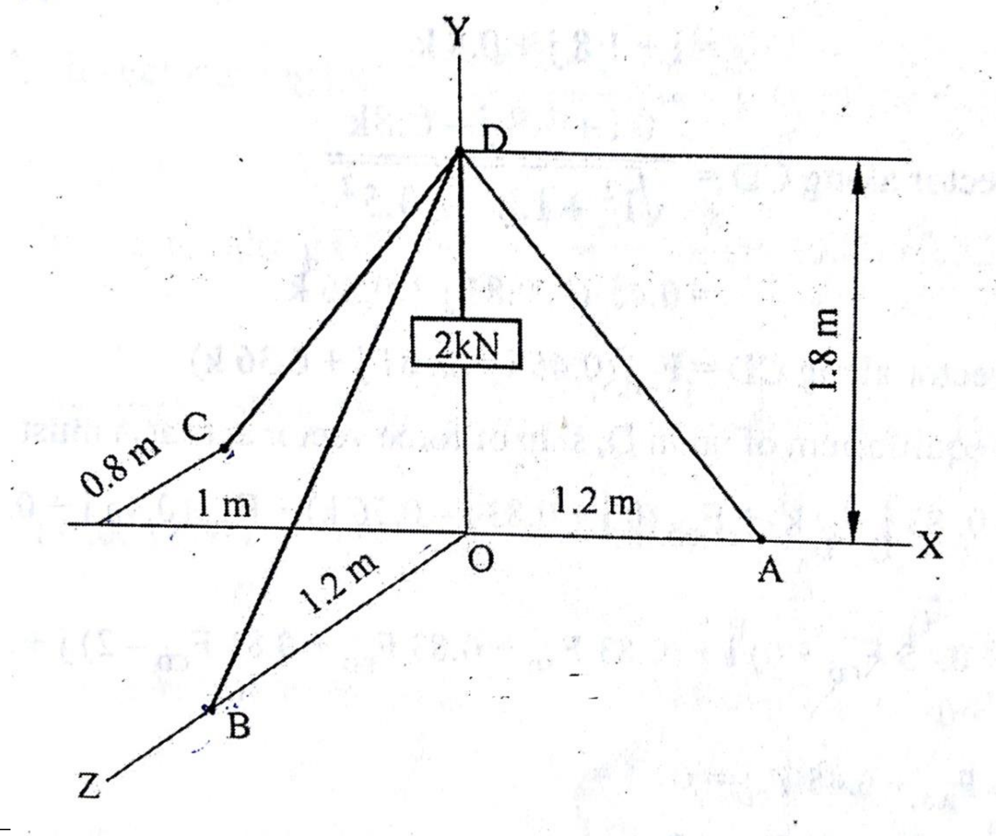
Fab = 68.33N
Fac = 45.09N
Fad = 3.88N



Example

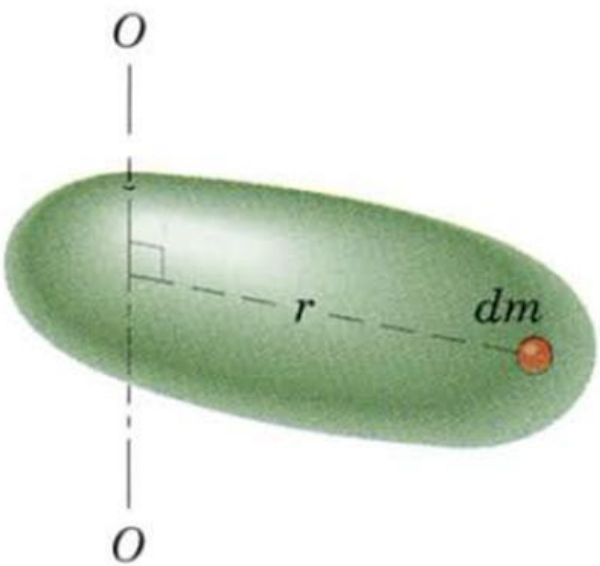
- A tripod supports a load of 2kN as shown in the figure. The ends A,B and C are in the xz plane. Find the force in the three legs of the tripod.

Fad = 802N
Fbd = 642N
Fcd = 990N



Mass moment of inertia

Consider a three dimensional body of mass *m*



$$I = \int r^2 \, dm$$

Mass moment of inertia of this body about axis *O-O*: $I = \int r^2 \, dm$

Integration is over the entire body.

r = *perpendicular distance of the mass element dm* from the axis *O-O*

Mass moment of inertia

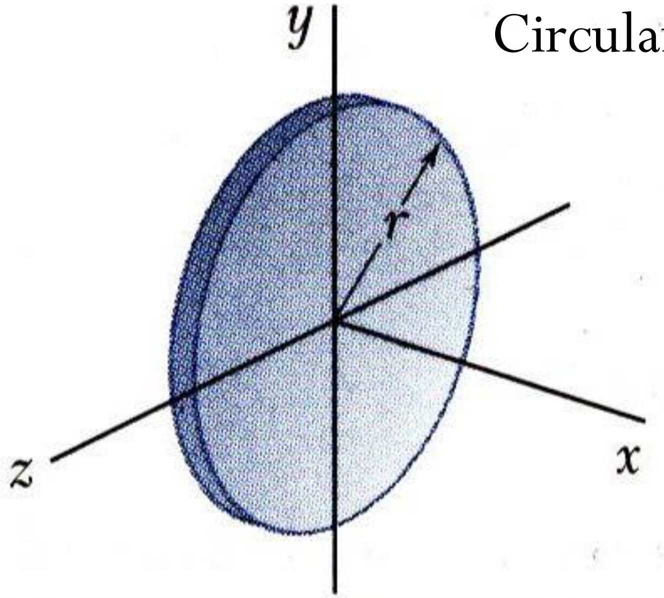
- Mass moment of inertia is an important concept for the study of dynamics of rigid bodies. It is the measure of the resistance of a solid body to angular acceleration, which depends on the distribution of mass of the body about its axis of rotation.
- Units are $(\text{mass})(\text{length})^2 \rightarrow \text{kg.m}^2$

Difference between mass moment of inertia and area moment of inertia

Mass moment of inertia	Area moment of inertia
It is an important concept for the study of dynamics of rigid body	It is an important concept for the study of statics of rigid body
It is a property of 3-D solid objects	It is a property of 2-D areas of planes
It depends on the distribution of mass about some reference axis	It depends on the distribution of area about some reference axis
It is the measure of resistance offered by a solid body to rotate about some reference axis	It is the measure of capacity of a section to resist bending about some reference axis.

Mass moment of inertia of some basic solids

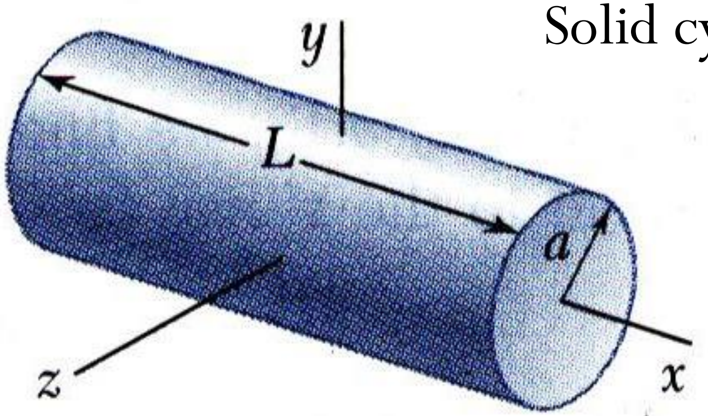
Circular disc



$$I_x = \frac{1}{2} m r^2$$

$$I_y = I_z = \frac{1}{4} m r^2$$

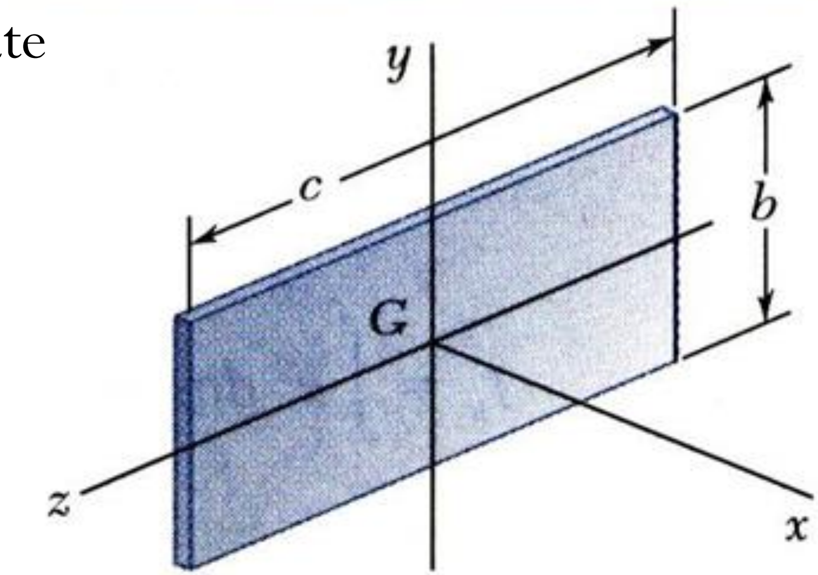
Solid cylinder



$$I_x = \frac{1}{2} m a^2$$

$$I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$$

Rectangular plate

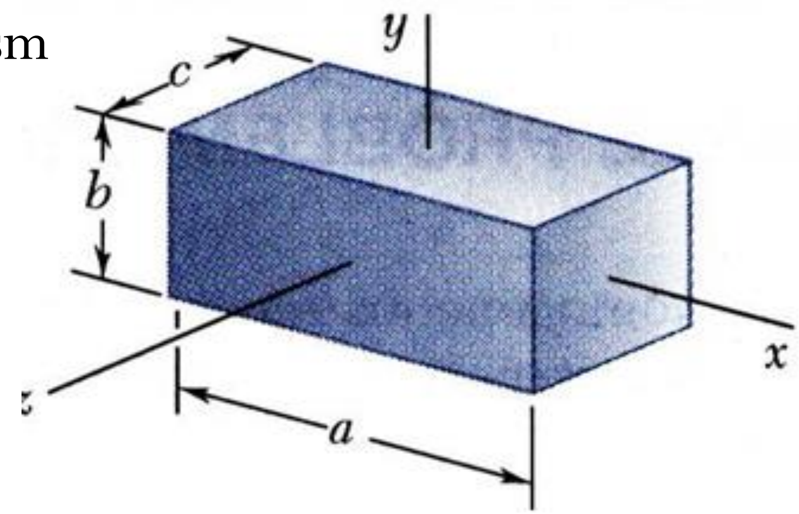


$$I_x = \frac{1}{12} m(b^2 + c^2)$$

$$I_y = \frac{1}{12} mc^2$$

$$I_z = \frac{1}{12} mb^2$$

Rectangular prism



$$I_x = \frac{1}{12} m(b^2 + c^2)$$

$$I_y = \frac{1}{12} m(c^2 + a^2)$$

$$I_z = \frac{1}{12} m(a^2 + b^2)$$