
KTU
NOTES

MODULE - 5

19/11/2020

Two port network parameters

There are 4 variables associated with two port n/w V_1, V_2, I_1 and I_2. Two of these variables can be represented in terms of the other two. There will be two dependent variables and two independent variables.

PARAMETERS	VARIABLES		EQUATION
	DEPENDENT	INDEPENDENT	
Impedence	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$

Open circuited Impedence Parameters (z-parameters)

$$(V_1, V_2) = f(I_1, I_2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$[V] = [Z][I]$$

The individual Z-parameters can be found out by setting each of the port currents to zero.

Case-I : $I_2 = 0$, o/p port open circuited

$$V_{11} = Z_{11} \cdot I_1 \Big|_{I_2=0} \Rightarrow Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

Z_{11} is the driving point impedence with o/p open circuited. It is called open circuit input impedence.

$$V_2 = Z_{21} I_1 \Big|_{I_2=0} \Rightarrow Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

Z_{21} - transfer impedence with o/p open circuited or open circuit forward transfer impedence.

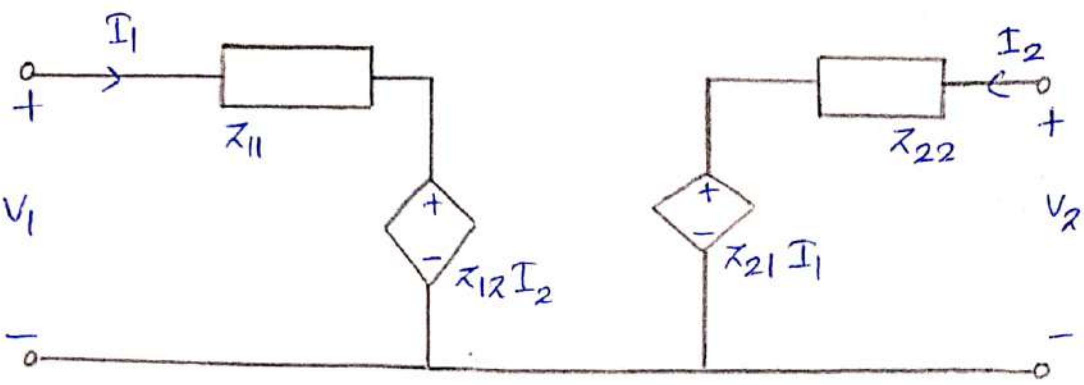
Case-II: $I_1 = 0$, i/p open circuited.

$$V_1 = Z_{12} I_2 \Big|_{I_1=0} \Rightarrow Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

Z_{12} - transfer impedence with Input open circuited or open circuit reverse transfer impedence.

$$\text{IIIrly} \quad Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

Z_{22} - Open circuit driving point Impedence with the input open circuited. Also called open circuit o/p Impedence.



Equivalent of two port n/w in terms of z-parameters.

→ The impedence parameters are measured with either input or o/p port open circuited, hence they are called open circuit impedence parameters.

CONDITION FOR RECIPROCITY

A network is said to be reciprocal if the ratio of excitation at one port to response at the other port is same if excitation and response are interchanged.

(a)

Network for deriving condition for reciprocity.

Voltage V_s is applied at the input port with the o/p short circuited

$$V_1 = V_s, V_2 = 0, I_2 = -I_2'$$

From z parameter eqns,

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$V_s = z_{11} I_1 - z_{12} I_2' \quad \text{——} (1)$$

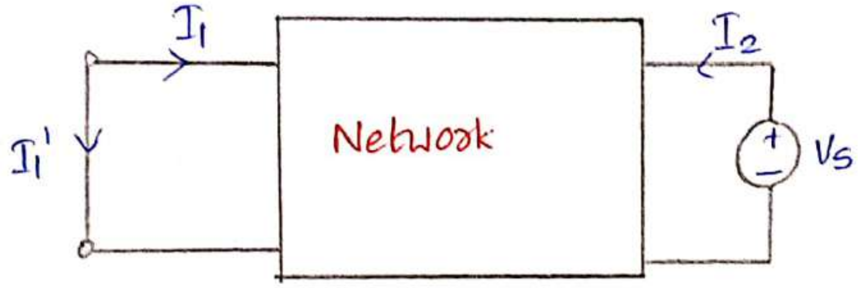
$$0 = z_{21} I_1 - z_{22} I_2' \quad \text{——} (2)$$

$$I_1 = \frac{z_{22}}{z_{21}} I_2' \quad \text{——} (3)$$

Substituting in ①, $V_S = z_{11} \cdot \frac{z_{22}}{z_{21}} I_2' - z_{12} I_2'$

$$\frac{V_S}{I_2'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}$$

ⓑ Interchanging the excitation and response.



$$V_2 = V_S, V_1 = 0, I_1 = -I_1'$$

From the z parameter equations,

$$0 = -z_{11} I_1' + z_{12} I_2 \quad \text{—— ①}$$

$$V_S = -z_{21} I_1' + z_{22} I_2 \quad \text{—— ②}$$

$$I_2 = \frac{z_{11}}{z_{12}} I_1' \quad \text{—— ③}$$

$$V_S = -z_{21} I_1' + z_{22} \cdot \frac{z_{11}}{z_{12}} I_1'$$

$$\frac{V_S}{I_1'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{12}}$$

For the n/w to be reciprocal,

$$\frac{V_S}{I_2'} = \frac{V_S}{I_1'} \Rightarrow \underline{\underline{z_{12} = z_{21}}}$$

23/11/20

CONDITION FOR SYMMETRY

For a network to be symmetrical, the voltage to current ratio at one port should be the same as the voltage to current ratio at the other port with one of the ports open circuited.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

a) When the o/p port is open circuited

$I_2 = 0$

From z-parameter equations,

$$V_S = Z_{11}I_1$$

$$Z_{11} = \frac{V_S}{I_1}$$

b) When the input port is open,

$I_1 = 0$

$$V_S = Z_{22}I_2$$

$$\frac{V_S}{I_2} = Z_{22}$$

Hence for the n/w to be symmetrical,

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

ie $Z_{11} = Z_{22}$

PROBLEMS

1. Test results for a two port n/w are (a) $I_1 = 0.1\angle 0^\circ A$, $V_1 = 5.2\angle 50^\circ V$, $V_2 = 4.1\angle -25^\circ V$ with port 2 open circuited. (b) $I_2 = 0.1\angle 0^\circ A$, $V_1 = 3.1\angle -80^\circ V$, $V_2 = 4.2\angle 60^\circ V$ with port-1 open circuited. Find z-parameters.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{5.2\angle 50}{0.1\angle 0} = 52\angle 50^\circ\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{4.1\angle -25}{0.1\angle 0} = 41\angle -25^\circ\ \Omega$$

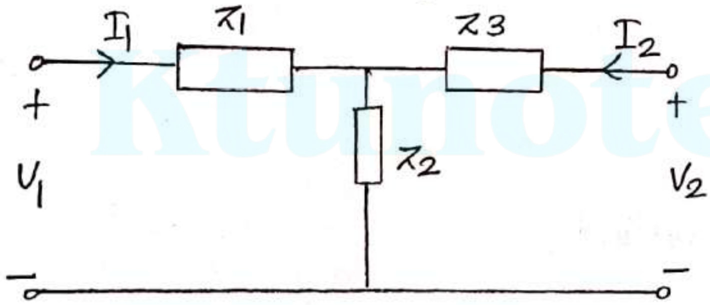
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{3.1\angle -80^\circ}{0.1\angle 0} = 31\angle -80^\circ\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{4.2\angle 60^\circ}{0.1\angle 0^\circ} = 42\angle 60^\circ\ \Omega$$

Hence the z-parameters are

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 52\angle 50^\circ & 31\angle -80^\circ \\ 41\angle -25^\circ & 42\angle 60^\circ \end{bmatrix}$$

2. Find the z-parameters for the n/w.



First method

Case-I: When the o/p is open circuited $I_2=0$.

Applying KVL to mesh-1;

$$V_1 = (z_1+z_2)I_1$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z_1+z_2$$

$$V_2 = z_2 I_1$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z_2$$

Case 2: When i/p is open circuited, $I_1 = 0$

$$V_2 = (z_2 + z_3) I_2$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z_2 + z_3$$

$$V_1 = z_2 I_2$$

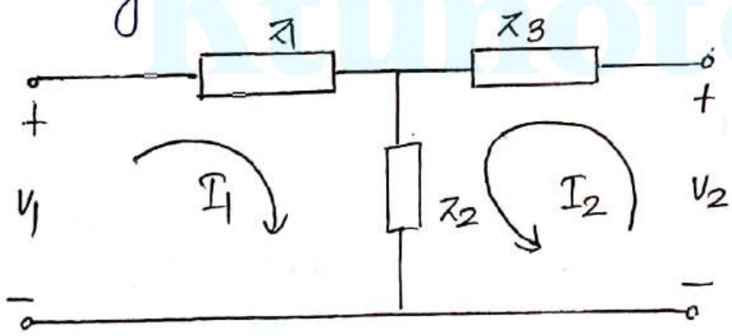
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z_2$$

Hence the z-parameters are,

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_1+z_2 & z_2 \\ z_2 & z_2+z_3 \end{bmatrix}$$

2nd method

Redrawing the n/w,

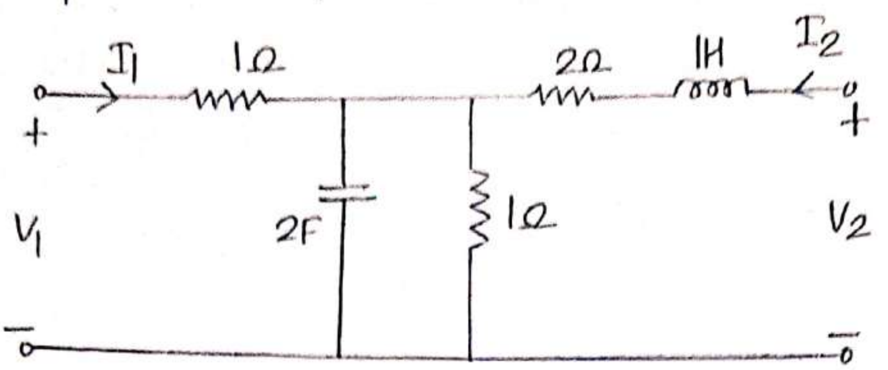


$$V_1 = z_1 I_1 + z_2 (I_1 + I_2)$$
$$= (z_1 + z_2) I_1 + z_2 I_2 \quad \text{——— ①}$$

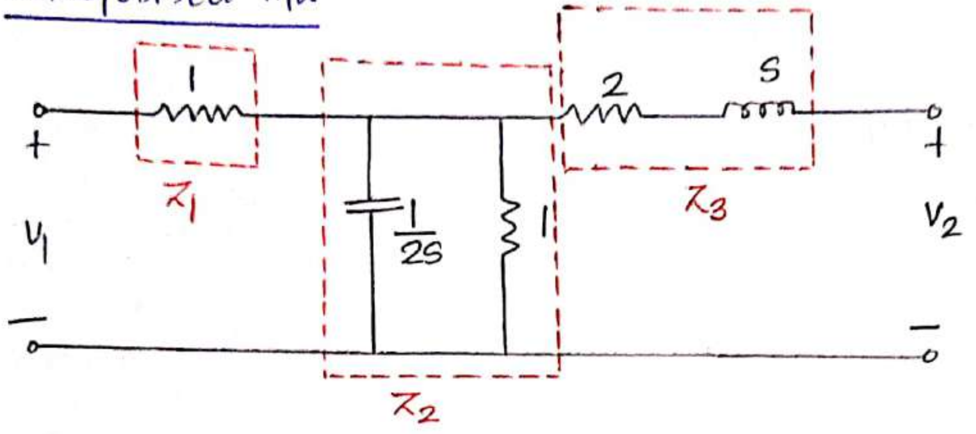
$$V_2 = z_3 I_2 + z_2 (I_1 + I_2)$$
$$= z_2 I_1 + (z_2 + z_3) I_2 \quad \text{——— ②}$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_1+z_2 & z_2 \\ z_2 & z_2+z_3 \end{bmatrix}$$

3. Find z-parameters of the n/w



Transformed n/w



$$z_1 = 1,\ z_2 = \frac{\frac{1}{2s}\cdot 1}{\frac{1}{2s}+1} = \frac{1}{2s+1},\ z_3 = s+2$$

From z-parameter distribution,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z_1 + z_2 = 1 + \frac{1}{2s+1} = \frac{2s+1+1}{2s+1} = \frac{2(s+1)}{2s+1}$$

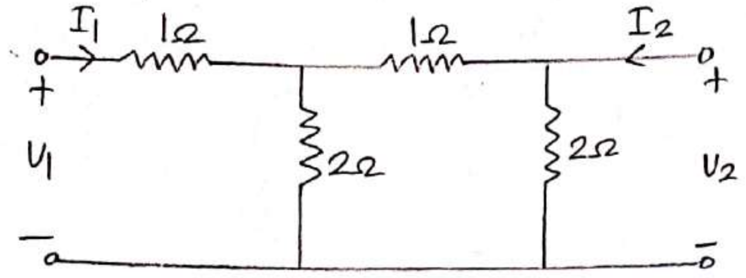
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z_2 = \frac{1}{2s+1}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z_2 = \frac{1}{2s+1}$$

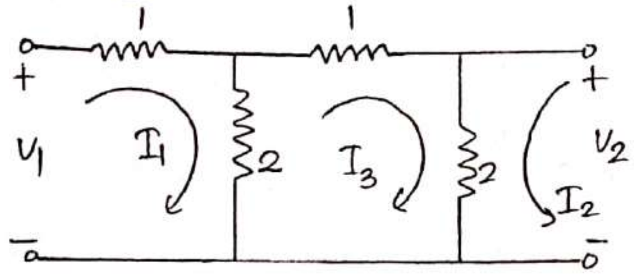
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z_2 + z_3 = \frac{1}{2s+1} + s+2 = \frac{1+(2s+1)(s+2)}{2s+1}$$

$$= \frac{1+2s^2+5s+2}{2s+1} = \frac{2s^2+5s+2}{2s+1}$$

4. Find z- parameters of the n/w.



<u>Redrawing the n/w</u>



<u>Mesh-1</u>:- $-V_1 + 3I_1 - 2I_3 = 0 \Rightarrow V_1 = 3I_1 - 2I_3$ ——— ①

<u>Mesh-3</u>:- $2I_2 + 2I_3 - V_2 = 0 \Rightarrow V_2 = 2I_2 + 2I_3$ ——— ②

<u>Mesh-2</u>:- $-2I_1 + 2I_2 + 5I_3 = 0$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \quad \text{——— ③}$$

Substituting I_3 in ① and ② to eliminate I_3.

$$V_1 = 3I_1 - 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$
$$= 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$$
$$V_1 = \frac{11}{5}I_1 + \frac{4}{5}I_2 \quad \text{——— ④}$$
$$V_2 = 2I_2 + 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$
$$= 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2$$
$$V_2 = \frac{4}{5}I_1 + \frac{6}{5}I_2 \quad \text{——— ⑤}$$

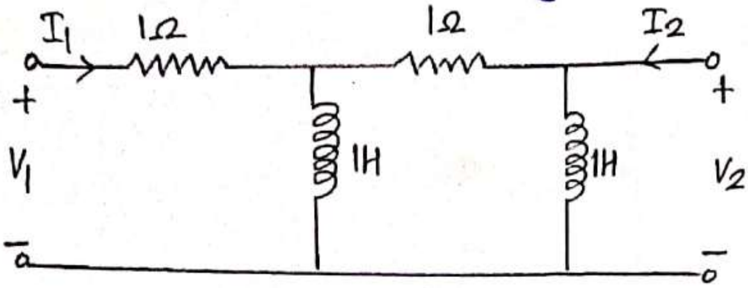
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Comparing ④ and ⑤ with z-parameter eqns.

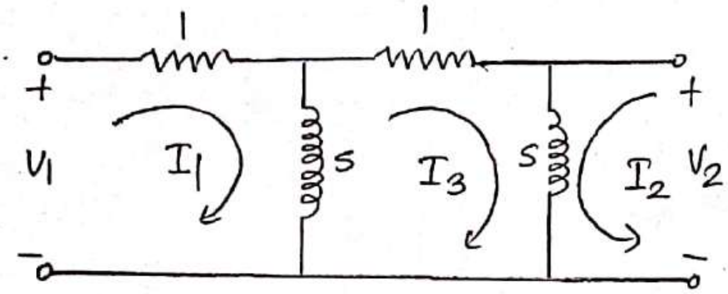
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 11/5 & 4/5 \\ 4/5 & 6/5 \end{bmatrix}$$

5. Find z-parameters of the following n/w.

24/11/20



Transformed n/w



Mesh-1 : $V_1 = (s+1)I_1 - sI_3$ —— ①

Mesh-3 ; $V_2 = sI_2 + sI_3$ —— ②

Mesh-2 : $-sI_1 + sI_2 + (2s+1)I_3 = 0$

$$(2s+1)I_3 = sI_1 - sI_2$$

$$I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2 \text{ —— ③}$$

Substituting in ①;

$$V_1 = (s+1)I_1 - s\left[\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right]$$

$$= \left[(s+1) - \frac{s^2}{2s+1}\right]I_1 + \frac{s^2}{2s+1}I_2$$

$$= \left[\frac{(s+1)(2s+1) - s^2}{(2s+1)}\right]I_1 + \left(\frac{s^2}{2s+1}\right)I_2$$

$$= \left[\frac{2s^2+3s+1-s^2}{2s+1}\right]I_1 + \left(\frac{s^2}{2s+1}\right)I_2$$

$$V_1 = \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I_2 \text{ —— ④}$$

Substituting ③ in ②,

$$V_2 = sI_2 + s\left[\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right]$$

$$= \frac{s^2}{2s+1}I_1 + \frac{2s^2+s-s^2}{2s+1}I_2$$

$$= \frac{s^2}{2s+1}I_1 + \frac{s^2+s}{2s+1}I_2 \quad \text{——— ⑤}$$

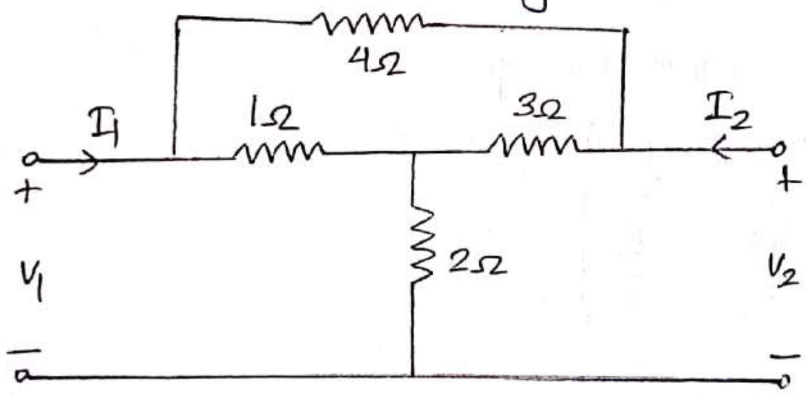
$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

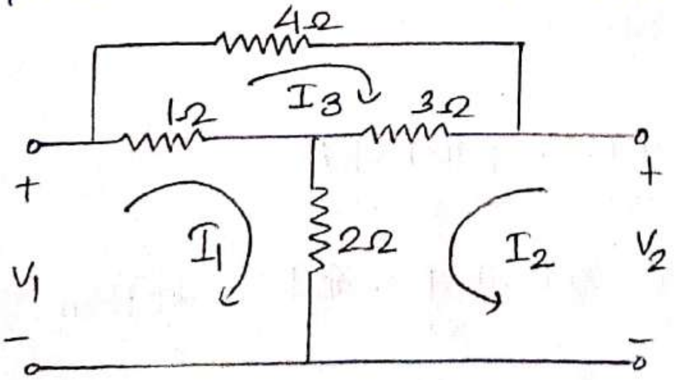
Comparing ④, ⑤ with z-parameter eqns,

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{2s+1} & \frac{s^2}{2s+1} \\ \frac{s^2}{2s+1} & \frac{s^2+s}{2s+1} \end{bmatrix}$$

6. Find the open circuit impedence parameters for the n/w shown below. Determine whether the n/w is symmetrical and reciprocal.



The n/w can be redrawn as shown below,



Applying KVL to mesh 1,

$-V_1 + 3I_1 + 2I_2 - I_3 = 0$

$V_1 = 3I_1 + 2I_2 - I_3$ ——— ①

Mesh-2:

$$5I_2 + 2I_1 + 3I_3 - V_2 = 0$$

$$V_2 = 2I_1 + 5I_2 + 3I_3 \quad \text{——— (2)}$$

Mesh-3:

$$8I_3 + 3I_2 - I_1 = 0$$

$$I_3 = \frac{1}{8}I_1 - \frac{3}{8}I_2 \quad \text{——— (3)}$$

Substituting I_3 in (1);

$$V_1 = 8I_1 + 2I_2 - \left[\frac{1}{8}I_1 - \frac{3}{8}I_2\right]$$

$$= \frac{23}{8}I_1 + \frac{19I_2}{8} \quad \text{——— (4)}$$

Substituting I_3 in (2);

$$V_2 = 2I_1 + 5I_2 + 3\left[\frac{1}{8}I_1 - \frac{3}{8}I_2\right]$$

$$= \frac{19}{8}I_1 + \frac{31}{8}I_2 \quad \text{——— (5)}$$

Comparing (4) and (5) with z-parameter eqns,

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{23}{8} & \frac{19}{8} \\ \frac{19}{8} & \frac{31}{8} \end{bmatrix}$$

$z_{11} \neq z_{22}$, the network is not symmetrical

$z_{12} = z_{21}$; hence the n/w is reciprocal

Short circuit admittance parameters (Y-parameters)

Y-parameters may be defined by expressing I_1 and I_2 in terms of V_1 and V_2

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{——— (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{——— (2)}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ In matrix form

$$[I] = [Y][V]$$

Individual Y-parameters can be defined by setting each of the port voltages to zero.

Case-I

When o/p port is short-circuited, $V_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Y_{11} – driving point admittance with the output port short circuited. It is also called short-circuit input admittance.

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Y_{21} – transfer admittance with o/p short circuited. Also called short-circuit forward transfer admittance.

Case-II

When input port is short circuited, $V_1 = 0$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

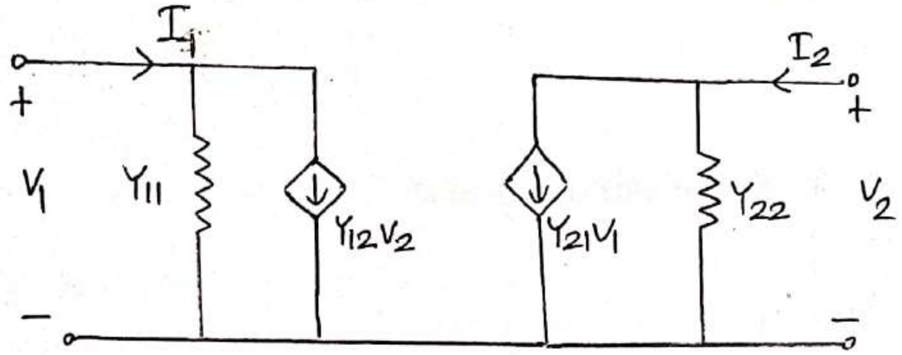
Y_{12} – transfer admittance with input port short circuited. It is also called short circuit reverse transfer admittance.

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Y_{22} – Short circuit driving point admittance with input short circuited. It is also called short-circuit output admittance.

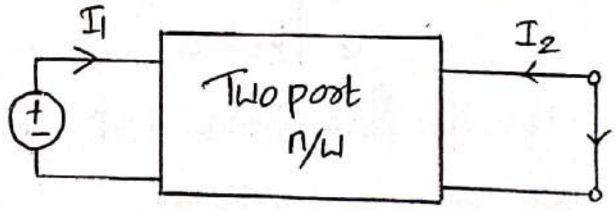
The admittance parameters are measured with either i/p or o/p port short circuited, hence they are called short-circuit admittance parameters.

Equivalent circuit of two port n/w in terms of Y-parameters is shown below



25/11/20 CONDITION FOR RECIPROCITY

ⓐ Vs applied at input port, o/p short circuited.



$V_1 = V_s,\ V_2 = 0,\ I_2 = -I_2'$

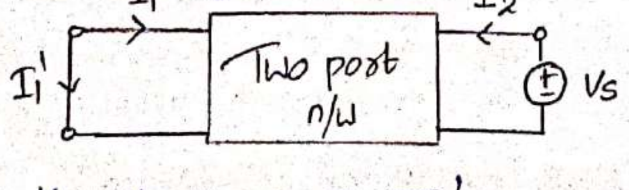
$I_2 = Y_{21}V_1$ From ②

$-I_2' = Y_{21}V_s$

$$\frac{I_2'}{V_s} = -Y_{21}$$

$$\boxed{\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \quad \text{①} \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \quad \text{②} \end{aligned}}$$

ⓑ Interchanging excitation and response



$V_2 = V_s,\ V_1 = 0,\ I_1 = -I_1'$

From ① $I_1 = Y_{12}V_2$

$-I_1' = Y_{12}V_s$

$$\frac{I_1'}{V_s} = -Y_{12}$$

For the n/w to be reciprocal,

$$\frac{I_2'}{V_s} = \frac{I_1'}{V_s}$$

$$Y_{12} = Y_{21}$$

CONDITION FOR SYMMETRY

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{——— (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{——— (2)}$$

(a) When the o/p port is short circuited, $V_2 = 0$

From (1),

$$I_1 = Y_{11}V_s$$

$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

(b) When the i/p port is short circuited, ie $V_1 = 0$

From (2), $I_2 = Y_{22}V_s$

$$\frac{V_s}{I_2} = \frac{1}{Y_{22}}$$

For the n/w to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$Y_{11} = Y_{12}$$

<u>Note</u>: For the n/w to be symmetrical, the voltage current ratio at one port should be the same as the voltage to current ratio of the other port with one of the other ports short circuited.

PROBLEMS

1. Test results for a two port n/w are:

(a) Port-2 short circuited ; $V_1 = 50\angle 0^\circ V$, $I_1 = 2.1\angle 30^\circ A$, $I_2 = -1.1\angle -20^\circ A$

(b) Port-1 short circuited: $V_2 = 50\angle 0^\circ\, V,\ I_2 = 3\angle -15\, A,\ I_1 = -1.1\angle -20^\circ A$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{2.1\angle -30^\circ}{50\angle 0} = 0.042\angle -30^\circ\ S$$

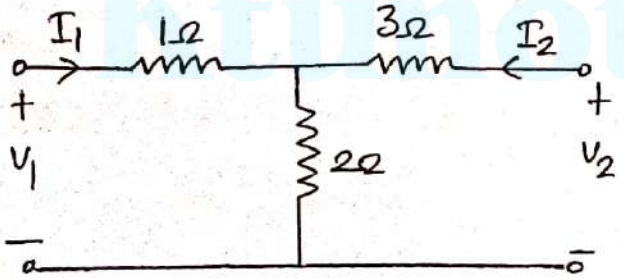
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1.1\angle 20^\circ}{50\angle 0} = -0.022\angle -20^\circ\ S.$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3\angle -15}{50\angle 0} = 0.06\angle -15^\circ\ S.$$

Y-parameters are:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.042\angle -30 & -0.022\angle -20^\circ \\ -0.022\angle -20 & 0.06\angle 15^\circ \end{bmatrix}$$

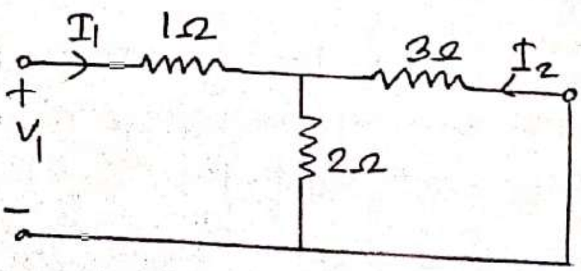
2. Find Y-parameters of the n/w.



First method

Case 1: When the o/p port is short circuited; $V_2 = 0$

$$R_{eq} = 1 + \frac{2\times 3}{2+3} = 1 + \frac{6}{5} = \frac{11}{5}\Omega$$



$$V_1 = \frac{11}{5} I_1$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{5}{11}\ S$$

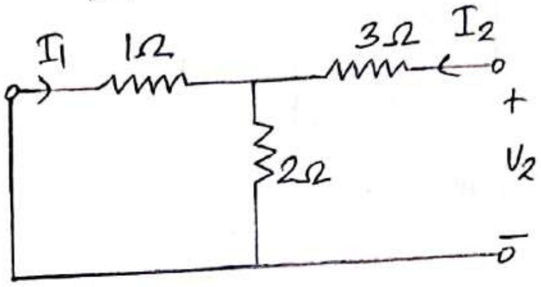
$$I_2 = -I_1 \times \frac{2}{2+3} = -\frac{2}{5}\cdot\frac{5V_1}{11} = \frac{-2}{11} V_1$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{-2}{11}\,S$$

Case2:

When the input port is short circuited, $V_1 = 0$

$$R_{eq} = 3 + \frac{1\times 2}{1+2} = \frac{11}{3}\,\Omega$$



$$V_2 = \frac{11}{3} I_2$$

$$\therefore Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = \frac{3}{4}\,S$$

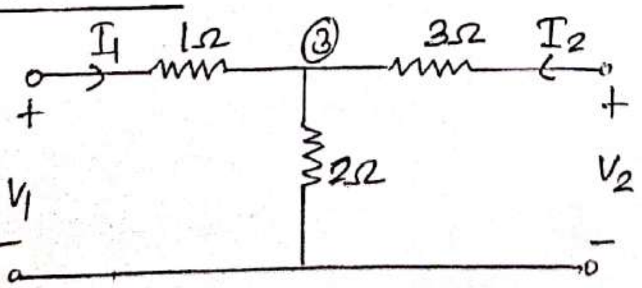
$$I_1 = -I_2 \times \frac{2}{3} = -\frac{2}{3}\cdot\frac{3V_2}{11} = \frac{-2}{11}V_2$$

$$\therefore Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{-2}{11}\,S$$

Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & \frac{-2}{11} \\ \frac{-2}{11} & \frac{3}{11} \end{bmatrix}$$

2nd method



$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \quad \text{---} (1)$$

$$I_2 = \frac{V_2 - V_3}{3} = \frac{V_2}{3} - \frac{V_3}{3} \quad \text{---} (2)$$

Applying KCL at node 3,

$$I_1 + I_2 = \frac{V_3}{2} \quad \text{---} (3)$$

Substituting ① and ② in ③,

$$V_1 - V_3 + \frac{V_2}{3} - \frac{V_3}{3} = \frac{V_3}{2}$$

$$V_3\left[\frac{1}{2} + 1 + \frac{1}{3}\right] = V_1 + \frac{V_2}{3}$$

$$V_3\left[\frac{3+6+2}{6}\right] = V_1 + \frac{V_2}{3}$$

$$V_3 \times \frac{11}{6} = V_1 + \frac{V_2}{3}$$

$$V_3 = \frac{6}{11}V_1 + \frac{2}{11}V_2 \quad \text{——— ④}$$

Substituting ④ in ①,

$$I_1 = V_1 - V_3 = V_1 - \frac{6}{11}V_1 - \frac{2}{11}V_2$$

$$= \frac{5}{11}V_1 - \frac{2}{11}V_2 \quad \text{——— ⑤}$$

$$I_2 = \frac{V_2}{3} - \frac{V_3}{3} = \frac{V_2}{3} - \frac{1}{3}\left[\frac{6}{11}V_1 + \frac{2}{11}V_2\right]$$

$$= \frac{V_2}{3} - \frac{2}{11}V_1 - \frac{2}{33}V_2$$

$$= -\frac{2}{11}V_1 + V_2\left[\frac{1}{3} - \frac{2}{33}\right]$$

$$= -\frac{2}{11}V_1 + V_2\left[\frac{11-2}{33}\right]$$

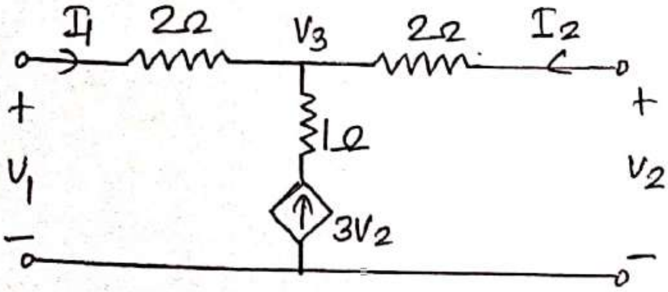
$$= -\frac{2}{11}V_1 + V_2 \times \frac{9}{33}$$

$$= -\frac{2}{11}V_1 + \frac{3}{11}V_2 \quad \text{——— ⑥}$$

Comparing ⑤ and ⑥ with Y-parameter eqns.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

3. Find Y-parameters of the network.



$$I_1 = \frac{V_1 - V_3}{2} = \frac{V_1}{2} - \frac{V_3}{2} \quad \text{——} ①$$

$$I_2 = \frac{V_2 - V_3}{2} = \frac{V_2}{2} - \frac{V_3}{2} \quad \text{——} ②$$

$$① - ② \Rightarrow I_1 - I_2 = \frac{V_1}{2} - \frac{V_2}{2} \quad \text{——} ④$$

Applying KCL at node 3,

$$I_1 + I_2 = -3V_2 \quad \text{——} ③$$

$$I_1 - I_2 = \frac{V_1}{2} - \frac{V_2}{2} \quad \text{——} ④$$

$$③ + ④ \Rightarrow 2I_1 = \frac{V_1}{2} - \frac{7}{2}V_2$$

$$\text{or } I_1 = \frac{V_1}{4} - \frac{7}{4}V_2 \quad \text{——} ⑤$$

From ③, $I_2 = -3V_2 - I_1$

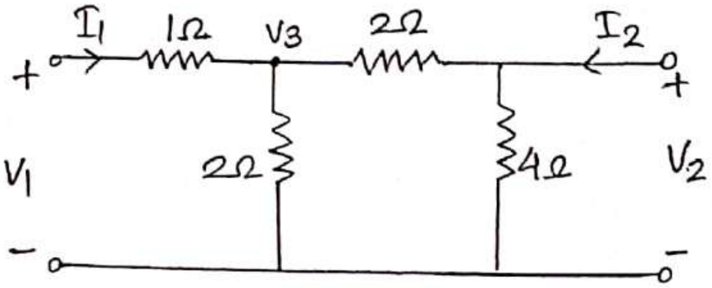
$$= -3V_2 - \left(\frac{V_1}{4} - \frac{7}{4}V_2\right)$$

$$= -\frac{V_1}{4} - \frac{5}{4}V_2 \quad \text{——} ⑥$$

Comparing ⑤ and ⑥ with y-parameter eqns,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{7}{4} \\ -1/4 & -5/4 \end{bmatrix}$$

1/12/2020 4) Determine Y-parameters of the n/w. Determine whether the n/w is symmetrical and reciprocal.



$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \quad \text{——} (1)$$

$$I_2 = \frac{V_2 - V_3}{2} + \frac{V_2}{4}$$

$$= \frac{3}{4}V_2 - \frac{V_3}{2} \quad \text{——} (2)$$

At node 3

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2}$$

$$= V_3 - \frac{V_2}{2} \quad \text{——} (3)$$

From (1) and (3),

$$V_1 - V_3 = V_3 - \frac{V_2}{2}$$

$$2V_3 = V_1 + \frac{V_2}{2}$$

$$V_3 = \frac{V_1}{2} + \frac{V_2}{4} \quad \text{——} (4)$$

Substituting V_3 into (1),

$$I_1 = V_1 - V_3 = V_1 - \left(\frac{V_1}{2} + \frac{V_2}{4}\right) = \frac{V_1}{2} - \frac{V_2}{4} \quad \text{——} (5)$$

Substituting V_3 in (2),

$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}V_3$$

$$= \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right)$$

$$= -\frac{1}{4}V_1 + \frac{5}{8}V_2 \quad \text{——} (6)$$

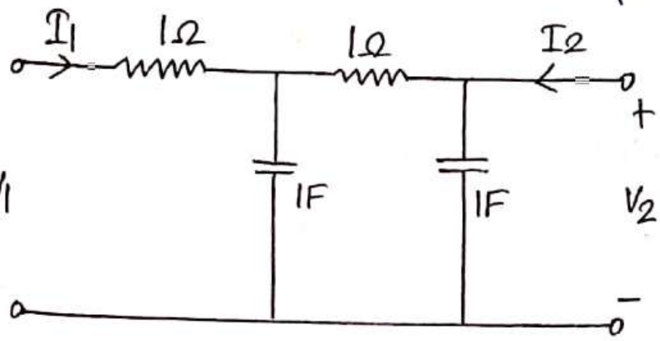
Comparing ⑤ and ⑥ with y-parameter eqns.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{bmatrix}$$

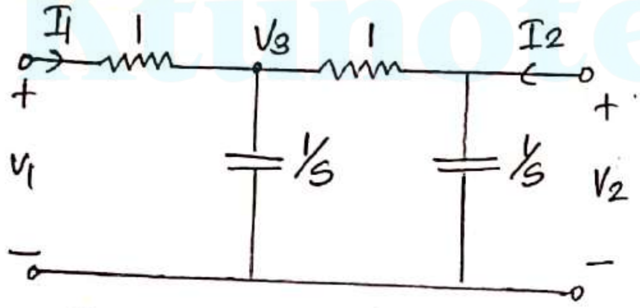
$Y_{11} \neq Y_{22}$ ∴ N/w is not symmetrical.

$Y_{12} \neq Y_{21}$, hence the n/w is reciprocal.

5. Determine the short circuit admittance parameters for the n/w.



Transformed n/w



$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \quad \text{——} ①$$

KCL at node 3,

$$I_1 = \frac{V_3}{1/s} + \frac{V_3 - V_2}{1} = (s+1)V_3 - V_2 \quad \text{——} ②$$

$$I_2 = \frac{V_2}{1/s} + \frac{V_2 - V_3}{1} = (s+1)V_2 - V_3 \quad \text{——} ③$$

From ① and ②,

$$V_1 - V_3 = (s+1)V_3 - V_2$$

$$(s+2)V_3 = V_1 + V_2$$

$$\therefore V_3 = \frac{1}{s+2}V_1 + \frac{1}{s+2}V_2 \quad \text{——} ④$$

Substituting V_3 in ①,

$$I_1 = V_1 - V_3 = V_1 - \frac{1}{s+2}V_1 - \frac{1}{s+2}V_2$$

$$I_1 = \frac{s+1}{s+2}V_1 - \frac{1}{s+2}V_2 \quad \text{——— ⑤}$$

Substituting V_3 in ③,

$$I_2 = (s+1)V_2 - V_3 = (s+1)V_2 - \frac{1}{s+2}V_1 - \frac{1}{s+2}V_2$$

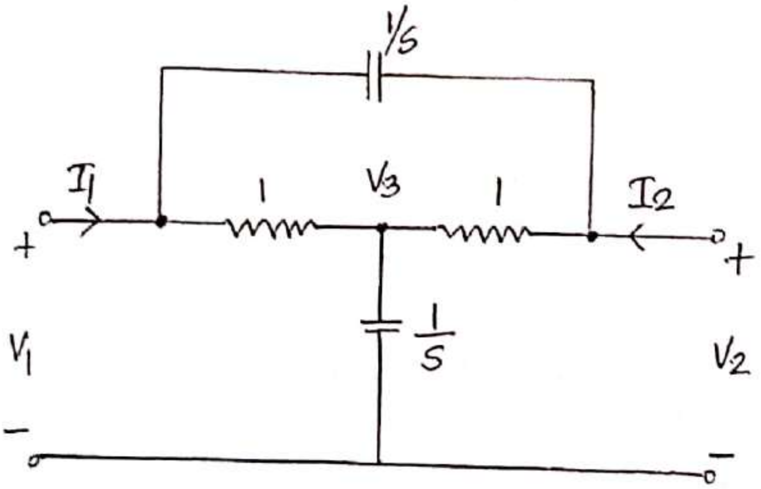
$$= -\frac{1}{s+2}V_1 + V_2\left[\frac{(s+1)(s+2)-1}{s+2}\right]$$

$$= -\frac{1}{s+2}V_1 + \frac{s^2+3s+1}{s+2}V_2 \quad \text{——— ⑥}$$

Comparing ⑤ and ⑥ with Y-parameter eqns.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s+2} & -\frac{1}{s+2} \\ \frac{-1}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}$$

6. Determine Y-parameters of the n/w.



Applying KCL at Node 1.

$$I_1 = \frac{V_1 - V_3}{1} + \frac{V_1 - V_2}{1/s} = (s+1)V_1 - sV_2 - V_3 \quad \text{——— ①}$$

Applying KCL at node-2:

$$I_2 = \frac{V_2 - V_3}{1} + \frac{V_2 - V_1}{1/s}$$

$$= (s+1)V_2 - sV_1 - V_3 \quad \text{——— (2)}$$

Applying KCL at node 3

$$\frac{V_3}{1/s} + \frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1} = 0$$

$$V_3(s+2) - V_1 - V_2 = 0$$

$$V_3 = \frac{1}{s+2}V_1 + \frac{1}{s+2}V_2 \quad \text{——— (3)}$$

Substituting V_3 in (1),

$$I_1 = (s+1)V_1 - sV_2 - \left[\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right]$$

$$= \left[\frac{(s+1)(s+2)-1}{s+2}\right]V_1 - \left[\frac{s(s+2)+1}{s+2}\right]V_2$$

$$= \left[\frac{s^2+3s+1}{s+2}\right]V_1 - \left[\frac{s^2+2s+1}{s+2}\right]V_2 \quad \text{——— (4)}$$

Substituting V_3 in (2),

$$I_2 = (s+1)V_2 - sV_1 - \left[\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right]$$

$$= -\left[\frac{s(s+2)+1}{s+2}\right]V_1 + \left[\frac{(s+1)(s+2)-1}{s+2}\right]V_2$$

$$= -\left[\frac{s^2+2s+1}{s+2}\right]V_1 + \left[\frac{s^2+3s+1}{s+2}\right]V_2 \quad \text{——— (5)}$$

Comparing (4) and (5) with Y-parameter eqns.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{s+2} & -\dfrac{(s^2+2s+1)}{s+2} \\ -\dfrac{(s^2+2s+1)}{s+2} & \dfrac{s^2+3s+1}{s+2} \end{bmatrix}$$

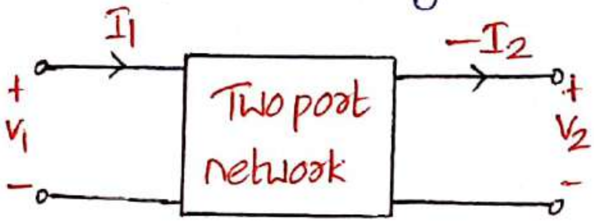
TRANSMISSION PARAMETERS

The transmission parameters or chain parameters or ABCD parameters relate the voltage and current at input port to voltage and current at o/p port.

$(V_1, I_1) = f(V_2, -I_2)$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



The negative sign is used with I_2 and not for parameters B and D. The reason for $-I_2$ is that, in transmission feed, the o/p current is assumed to be coming out of output port and not going into the port.

In matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

CASE 1: When the o/p port is open circuited, $I_2 = 0$.

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ – reverse voltage gain with o/p port open circuited.

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ – transfer admittance with o/p open circuited.

CASE 2: When o/p port is short circuited, $V_2 = 0$.

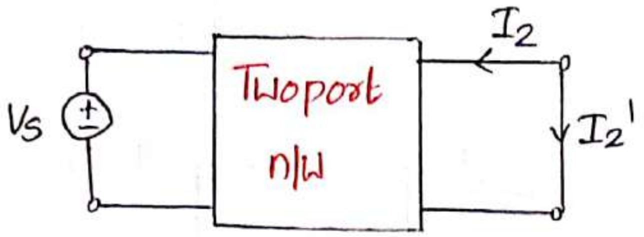
$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$ – transfer impedence with o/p port short circuited.

$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$ – reverse current gain with o/p short circuited.

2|12|2020

CONDITION FOR RECIPROCITY

ⓐ Vs applied at input port o/p short circuited.



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

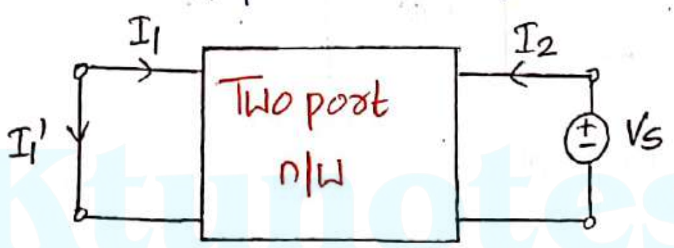
$V_1 = V_s,\ V_2 = 0,\ I_2' = -I_2$

$V_1 = -BI_2$

$V_s = -B \cdot -I_2'$

$\frac{V_s}{I_2'} = B$

ⓑ Vs applied at o/p port with input port short circuited.



$V_2 = V_s$

$V_1 = 0$

$I_1 = -I_1'$

$0 = AV_2 - BI_2$

$0 = AV_s - BI_2$ —— ① $\Rightarrow I_2 = \frac{A}{B} V_s$

$-I_1' = CV_2 - DI_2$

$-I_1' = CV_s - DI_2$

$= CV_s - D\frac{A}{B}V_s$

$= V_s \frac{[BC - AD]}{B}$

$\frac{V_s}{I_1'} = \frac{B}{AD - BC}$

For the n/w to be reciprocal, $\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$

$B = \frac{B}{AD - BC}$

ie $AD - BC = 1$

CONDITION FOR SYMMETRY

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

ⓐ When the o/p port is open circuited,

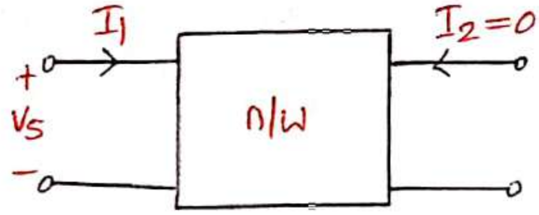
$I_2 = 0,\ V_1 = V_s$

$V_s = AV_2 \Rightarrow V_2 = \frac{V_s}{A}$

$I_1 = CV_2$

$= C\frac{V_s}{A}$

$\frac{V_s}{I_1} = \frac{A}{C}$

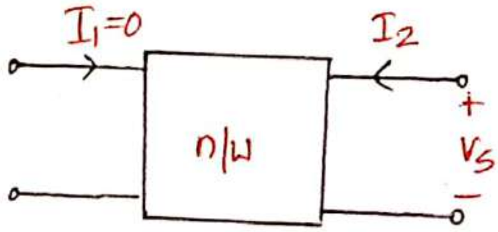


ⓑ When the input port is open circuited, $I_1 = 0;\ V_2 = V_s$.

$0 = CV_s - DI_2$

$CV_s = DI_2$

$\frac{V_s}{I_2} = \frac{D}{C}$



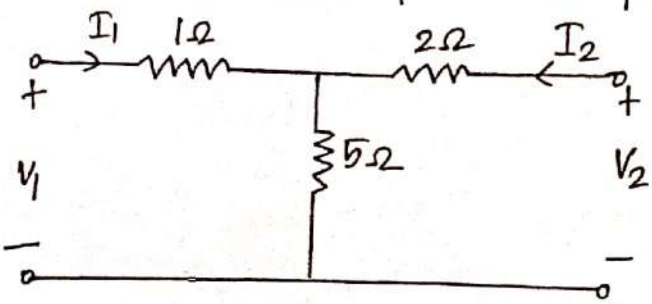
For the n/w to be symmetrical,

$\frac{V_s}{I_1} = \frac{V_s}{I_2}$

$\frac{A}{C} = \frac{D}{C} \Rightarrow \underline{\underline{A=0}}$

PROBLEMS

1. Find the transmission parameters for the n/w shown in fig.



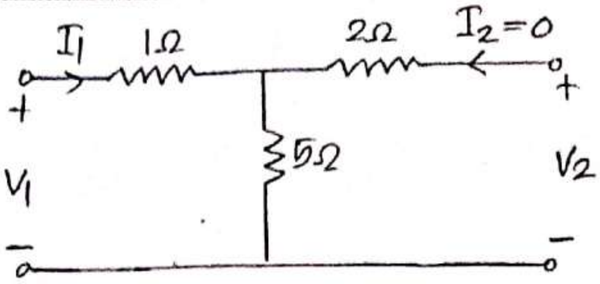
$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

First method

Case-I : o/p port open circuited, $I_2 = 0$

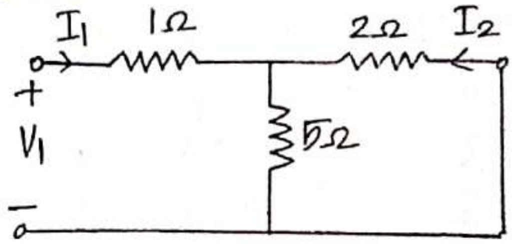
From ckt, $V_1 = 6I_1$, $V_2 = 5I_1$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{6I_1}{5I_1} = \frac{6}{5}$$



$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{5I_1} = \frac{1}{5}\,S$$

Case-2: When o/p port is short circuited. $V_2 = 0$



$$R_{eq} = 1 + (5\|2)$$

$$= 1 + \frac{5\times 2}{5+2} = 1 + \frac{10}{7} = \frac{17}{7}\,\Omega$$

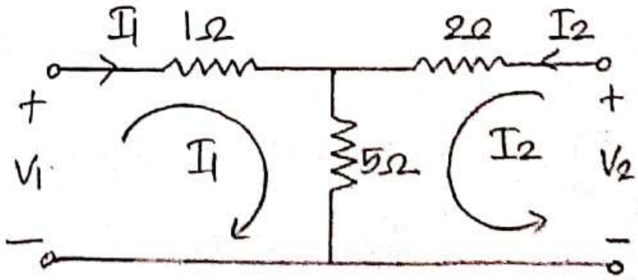
$$V_1 = R_{eq}\cdot I_1 = \frac{17}{7} I_1$$

$$I_2 = -I_1 \times \frac{5}{7} = -\frac{5}{7} I_1$$

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} = \frac{-\frac{17}{7} I_1}{-\frac{5}{7} I_1} = \frac{17}{5}\,\Omega$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} = \frac{-I_1}{-\frac{5}{7} I_1} = \frac{7}{5}$$

2nd method



$$V_1 = 6I_1 + 5I_2 \quad \text{——} (1)$$

$$V_2 = 5I_1 + 7I_2 \quad \text{——} (2)$$

$$5I_1 = V_2 - 7I_2$$

$$I_1 = \frac{1}{5}V_2 - \frac{7I_2}{5} \quad \text{——} (3)$$

Substituting (3) in (1),

$$V_1 = 6I_1 + 5I_2 = 6\left(\frac{1}{5}V_2 - \frac{7}{5}I_2\right) + 5I_2$$

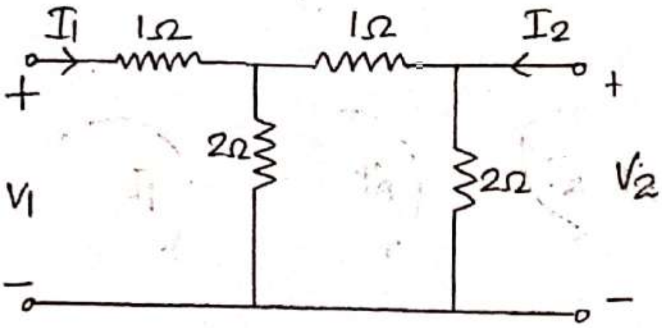
$$= \frac{6}{5}V_2 - \frac{42}{5}I_2 + 5I_2$$

$$= \frac{6}{5}V_2 - \frac{17}{5}I_2 \quad \text{--- (4)}$$

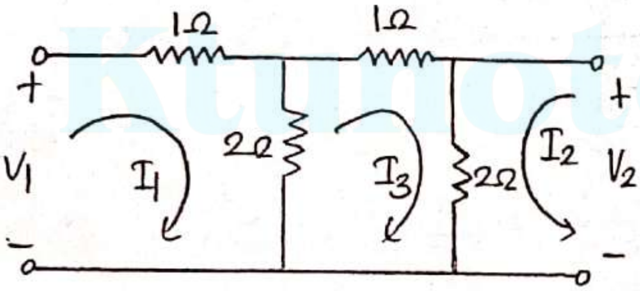
Comparing (3) and (4) with transformission parameter eqns.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

2. Obtain the ABCD parameters for the n/w.



Redrawing the n/w



Mesh-1: $V_1 = 3I_1 - 2I_3$ --- (1)

Mesh-3: $V_2 = 2I_2 + 2I_3$ --- (2)

Mesh-2: $5I_3 - 2I_1 + 2I_2 = 0$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \quad \text{--- (3)}$$

Substituting (3) in (1),

$$V_1 = 3I_1 - 2I_3 = 3I_1 - 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right) = 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2 \quad \text{--- (4)}$$

Substituting ③ in ②,

$$V_2 = 2I_2 + 2I_3 = 2I_2 + 2\left[\frac{2}{5}I_1 - \frac{2}{5}I_2\right]$$

$$= \frac{4}{5}I_1 + \frac{6}{5}I_2$$

$$\frac{4}{5}I_1 = V_2 - \frac{6}{5}I_2$$

$$I_1 = \frac{5}{4}V_2 - \frac{3}{2}I_2 \text{ ——— ⑤}$$

Substituting ⑤ in ④,

$$V_1 = \frac{11}{5}I_1 + \frac{4}{5}I_2 = \frac{11}{5}\left[\frac{5}{4}V_2 - \frac{3}{2}I_2\right] + \frac{4}{5}I_2$$

$$= \frac{11}{4}V_2 - \frac{33}{10}I_2 + \frac{4}{5}I_2$$

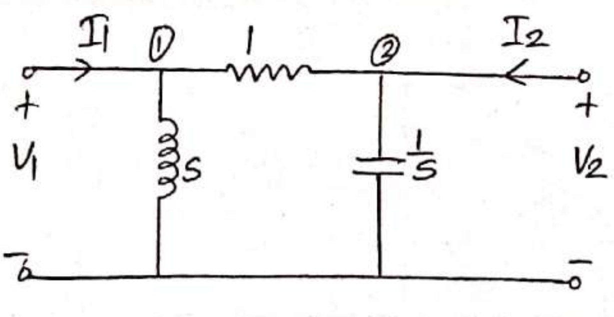
$$= \frac{11}{4}V_2 - \frac{25}{10}I_2$$

$$= \frac{11}{4}V_2 - \frac{5}{2}I_2 \text{ ——— ⑥}$$

Comparing ⑥ and ⑤ with ABCD parameter eqns:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{11}{4} & \frac{5}{2} \\ \frac{5}{4} & \frac{3}{2} \end{bmatrix}$$

3/12/2020 3) Determine transmission parameters for the n/w.



Applying KCL at node-1

$$I_1 = \frac{V_1}{s} + \frac{V_1 - V_2}{1} = V_1\left(\frac{1}{s} + 1\right) - V_2$$

$$= \frac{(s+1)}{s} V_1 - V_2 \quad \text{——— (1)}$$

KCl at node 2

$$I_2 = \frac{V_2}{1/s} + \frac{V_2 - V_1}{1} = sV_2 + V_2 - V_1$$

$$= -V_1 + (s+1)V_2$$

$$V_1 = (s+1)V_2 - I_2 \quad \text{——— (2)}$$

Substituting (2) in (1),

$$I_1 = \frac{s+1}{s}\left[(s+1)V_2 - I_2\right] - V_2$$

$$= \left[\frac{(s+1)^2}{s} - 1\right]V_2 - \frac{s+1}{s} I_2$$

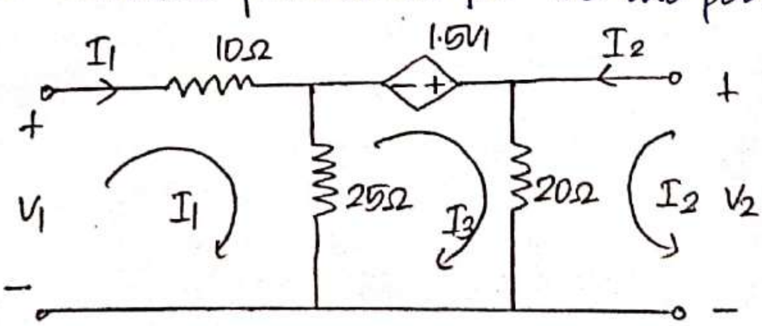
$$= \left(\frac{s^2+2s+1-s}{s}\right)V_2 - \frac{s+1}{s} I_2$$

$$= \left(\frac{s^2+s+1}{s}\right)V_2 - \left(\frac{s+1}{s}\right) I_2 \quad \text{——— (3)}$$

Comparing (2) and (3) with ABCD parameter eqns.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ \frac{s^2+s+1}{s} & \frac{s+1}{s} \end{bmatrix}$$

4. Find transmission parameters for the two port n/w.



Applying KVL to mesh-1

$$V_1 = 35I_1 - 25I_3 \quad \text{——— (1)}$$

Mesh-3

$$V_2 = 20I_2 + 20I_3 \quad \text{--- (2)}$$

Mesh-2

$$45I_3 - 25I_1 + 20I_2 - 1.5V_1 = 0$$

$$45I_3 - 25I_1 + 20I_2 - 1.5(35I_1 - 25I_3) = 0$$

$$82.5I_3 - 77.5I_1 + 20I_2 = 0$$

$$I_3 = 0.94I_1 - 0.24I_2 \quad \text{--- (3)}$$

Subst. (3) in (1),

$$V_1 = 35I_1 - 25I_3 = 35I_1 - 25(0.94I_1 - 0.24I_2)$$

$$= 11.5I_1 + 6I_2 \quad \text{--- (4)}$$

Subst. (3) in (2),

$$V_2 = 20I_2 + 20I_3$$

$$= 20I_2 + 20(0.94I_1 - 0.24I_2)$$

$$= 18.8I_1 + 15.2I_2 \quad \text{--- (5)}$$

$$18.8I_1 = V_2 - 15.2I_2$$

$$I_1 = 0.053V_2 - 0.81I_2 \quad \text{--- (6)}$$

Subst. (6) in (4),

$$V_1 = 11.5I_1 + 6I_2$$

$$= 11.5(0.053V_2 - 0.81I_2) + 6I_2$$

$$= 0.61V_2 - 3.32I_2 \quad \text{--- (7)}$$

Comparing (7) and (6) with ABCD parameters eqns.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.61 & +3.32 \\ 0.053 & +0.81 \end{bmatrix}$$

HYBRID PARAMETERS (h-parameters)

The hybrid parameters of a two port n/w may be defined by expressing V_1 and I_2 in terms of I_1 and V_2.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Individual h-parameters can be defined by setting $I_1 = 0$ and $V_2 = 0$.

Case-1:

When o/p port is short circuited, $V_2 = 0$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$: h_{11} – short circuit Input Impedence.

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$: h_{21} – short circuit forward current gain.

Case-2:

When the input is open circuited, $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

h_{12} – the open circuited reverse voltage gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{22} – the open circuit o/p admittance

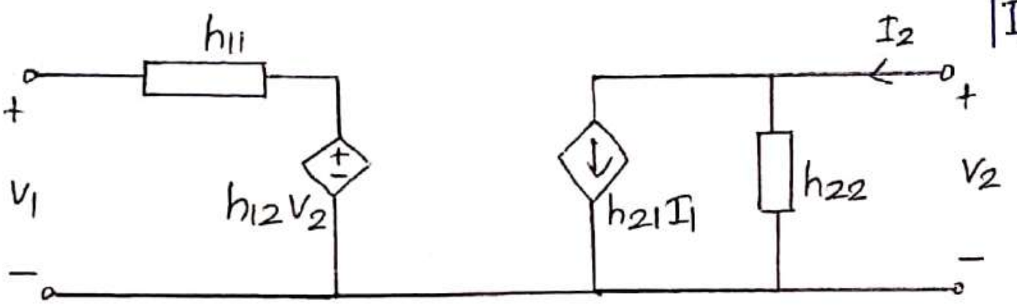
These four parameters are not dimensionally alike, hence they are called

hybrid parameters.

→ The equivalent circuit of a two port n/w in terms of hybrid parameters is shown below.

$V_1 = h_{11} I_1 + h_{12} V_2$

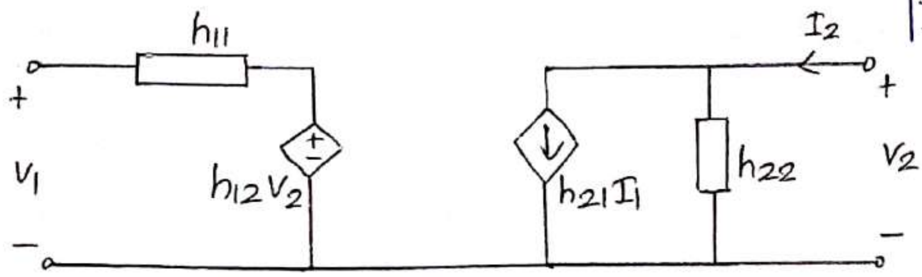
$I_2 = h_{21} I_1 + h_{22} V_2$



hybrid parameters.

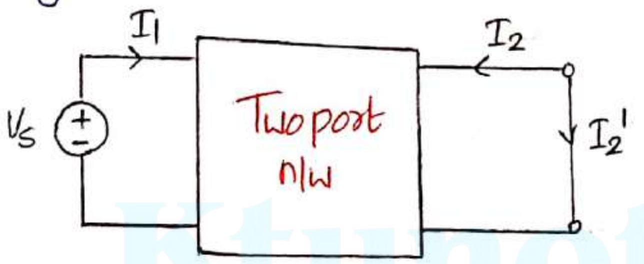
→ The equivalent circuit of a two port n/w in terms of hybrid parameters is shown below.

$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$



14/12/20. CONDITION FOR RECIPROCITY

ⓐ Voltage V_s applied at input port and o/p port short circuited.



$V_1 = V_s$
$V_2 = 0$
$I_2' = -I_2$

From h-parameter eqns.

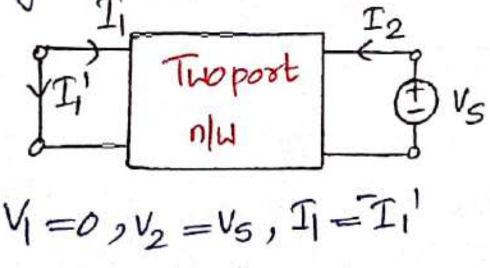
$$V_s = h_{11} I_1$$

$$-I_2' = h_{21} I_1 \Rightarrow I_1 = \frac{-I_2'}{h_{21}}$$

$$\therefore V_s = h_{11} \times \frac{-I_2'}{h_{21}}$$

$$\frac{V_s}{I_2'} = -\frac{h_{11}}{h_{21}}$$

ⓑ Voltage V_s applied at the o/p port with the input port short circuited.



$V_1 = 0$, $V_2 = V_s$, $I_1 = -I_1'$

$0 = h_{11} I_1 + h_{12} V_s$

$$h_{12}V_s = -h_{11}I_1 = h_{11}I_1'$$

$$\frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}}$$

For the n/w to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

$$h_{21} = -h_{12}$$

CONDITION FOR SYMMETRY

Condition for symmetry is obtained from z-parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{h_{11}I_1 + h_{12}V_2}{I_1}$$

$$= h_{11} + h_{12}\frac{V_2}{I_1} \quad \text{——— (1)}$$

$$\therefore I_2 = 0,\; 0 = h_{21}I_1 + h_{22}V_2$$

$$-h_{21}I_1 = h_{22}V_2$$

$$\frac{V_2}{I_1} = \frac{-h_{21}}{h_{22}} \quad \text{——— (2)}$$

Substituting (2) in (1),

$$\therefore z_{11} = h_{11} + h_{12}\cdot\frac{-h_{21}}{h_{22}}$$

$$= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

$$= \frac{\Delta h}{h_{22}} \quad \text{where } \Delta h = h_{11}h_{22} - h_{12}h_{21}$$

Similarly,

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

For a symmetrical n/w, $z_{11} = z_{22}$

$$\frac{\Delta h}{h_{22}} = \frac{1}{h_{22}}$$

$$\Rightarrow \Delta h = 1$$

$$h_{11}h_{22} - h_{12}h_{21} = 1$$

PROBLEMS

1. Find out h-parameters from the following data.

(a) With the o/p port short circuited : $V_1 = 25V$, $I_1 = 1A$, $I_2 = 2A$

(b) With the input port open circuited : $V_1 = 10V$, $V_2 = 50V$, $I_2 = 2A$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{25}{1} = 25\Omega$$

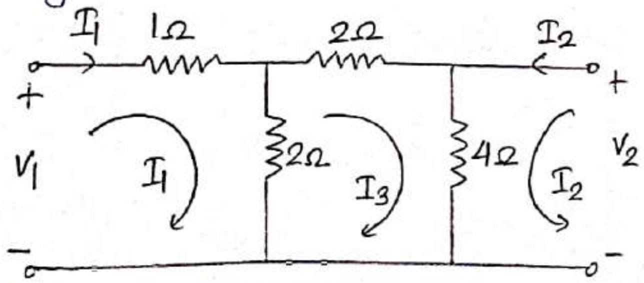
$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{10}{50} = 0.2$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{2}{1} = 2$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2}{50} = 0.04S$$

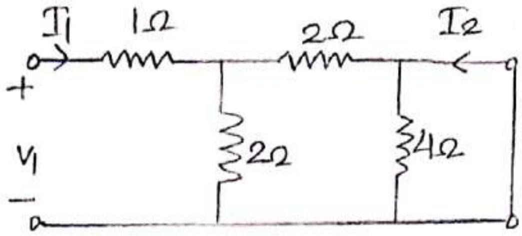
2. Determine hybrid parameters of the n/w. Determine whether the n/w is reciprocal.



First method

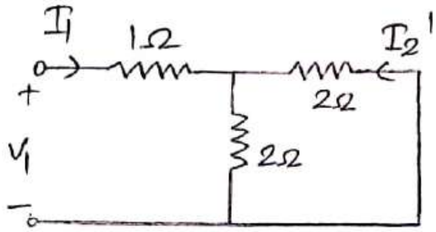
Case I: When port-2 is short circuited, $V_2 = 0$

$R_{eq} = 1 + 2 \| 2 = 2\Omega$



$V_1 = 2I_1$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 2\Omega$$



$$I_2 = -I_1 \times \frac{2}{2+2} = \frac{-I_1}{2}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-1}{2}$$

Case – 2

When port – 2 is open circuited, $I_1 = 0$

$R_{eq} = 4 \| 4 = 2\Omega$

$V_1 = 2I_y \quad I_y = I_x = \frac{I_2}{2}$

$I_y = \frac{I_2}{2}$

$V_2 = 4I_x$

$I_x = \frac{I_2}{2}$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{2I_y}{4I_x} = \frac{1}{2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2I_x}{4I_x} = \frac{1}{2}S$$

Hence h-parameters are,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

2nd method

Mesh -1

$$V_1 = 3I_1 - 2I_3 \quad \text{——} (1)$$

Mesh - 3

$$V_2 = 4I_2 + 4I_3 \quad \text{——} (2)$$

Mesh -2

$$8I_3 - 2I_1 + 4I_2 = 0$$

$$8I_3 = 2I_1 - 4I_2$$

$$I_3 = \frac{I_1}{4} - \frac{I_2}{2} \quad \text{——} (3)$$

Subst. (3) in (1),

$$V_1 = 3I_1 - 2I_3 = 3I_1 - 2\left(\frac{I_1}{4} - \frac{I_2}{2}\right)$$

$$= \frac{5}{2}I_1 + I_2 \quad \text{——} (4)$$

Subs (3) in (2),

$$V_2 = 4I_2 + 4I_3 = 4I_2 + 4\left(\frac{I_1}{4} - \frac{I_2}{2}\right)$$

$$= 4I_2 + I_1 - 2I_2$$

$$= I_1 + 2I_2$$

$$2I_2 = -I_1 + V_2$$

$$\therefore I_2 = -\frac{1}{2}I_1 + \frac{1}{2}V_2 \quad \text{——} (5)$$

Subs. (5) in (4),

$$V_1 = \frac{5I_1}{2} + I_2$$

$$= \frac{5}{2}I_1 + \left(-\frac{I_1}{2} + \frac{1}{2}V_2\right)$$

$$= 2I_1 + \frac{1}{2}V_2 \quad \text{——} (6)$$

Comparing (5) and (6) with h-parameter eqns.

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$

$\because h_{12} = -h_{21}$ the N/W is reciprocal

15/12/20 INTER-RELATIONSHIP AMONG PARAMETERS

When it is required to find out two or more parameters of a particular n/w, then finding each parameter will be tedious. But if we find a particular parameter then the other parameters can be found if the inter-relationship between them is known.

Z-parameters in terms of other parameters

1. Z-parameters in terms of Y-parameters

We know that,

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$V_1 = Z_{11} I_1 + Z\, I_2$$
$$V_2 = Z\, I_1 + Z_{22} I_2$$

By cramer's rule,

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22} I_1 - Y_{12} I_2}{Y_{11} Y_{22} - Y_{12} Y_{21}} = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2 \quad \text{—— (1)}$$

Where $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y} = \frac{Y_{11}}{\Delta Y} I_2 - \frac{Y_{21}}{\Delta Y} I_1 = \frac{-Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2 \quad \text{—— (2)}$$

Comparing (1) and (2) with Z-parameter eqns.

$$Z_{11} = \frac{Y_{22}}{\Delta Y}, \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y}, \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

2. Z-parameters in terms of ABCD parameters.

$$V_1 = AV_2 - BI_2 \quad \text{——} (1)$$

$$I_1 = CV_2 - DI_2 \quad \text{——} (2)$$

From (2), $CV_2 = I_1 + DI_2$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{——} (3)$$

Subs. (3) in (1),

$$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - BI_2$$

$$= \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2$$

$$= \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2 \quad \text{——} (4)$$

Comparing (4) and (3) with Z-parameter eqns.

$$Z_{11} = \frac{A}{C}, \quad Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}$$

3. Z-parameters in terms of h-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——} (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——} (2)$$

From (2), $h_{22} V_2 = I_2 - h_{21} I_1$

$$V_2 = \frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{——} (3)$$

Subst. (3) in (1),

$$V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$= I_1\left[h_{11} - \frac{h_{12}h_{21}}{h_{22}}\right] + \frac{h_{12}}{h_{22}} I_2$$

$$= I_1\left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] + \frac{h_{12}}{h_{22}} I_2 \quad \text{——— (4)}$$

Comparing (4) and (3) with z-parameter eqns.

$$z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}} \quad ; \quad z_{12} = \frac{h_{12}}{h_{22}}$$

$$z_{21} = \frac{-h_{21}}{h_{22}} \quad ; \quad z_{22} = \frac{1}{h_{22}}$$

Y-parameters in terms of other parameters

1. Y-parameters in terms of z-parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

By cramer's rule,

$$I_1 = \frac{\begin{vmatrix} V_1 & z_{12} \\ V_2 & z_{22} \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{22}V_1 - z_{12}V_2}{z_{11}z_{12} - z_{12}z_{21}} = \frac{z_{22}}{\Delta z} V_1 - \frac{z_{12}}{\Delta z} V_2 \quad \text{——— (1)}$$

where $\Delta z = z_{11} z_{22} - z_{12} z_{21}$

$$I_2 = \frac{\begin{vmatrix} z_{11} & V_1 \\ z_{21} & V_2 \end{vmatrix}}{\Delta z} = \frac{z_{11}V_2 - z_{21}V_1}{\Delta z} = \frac{-z_{21}}{\Delta z} V_1 + \frac{z_{11}}{\Delta z} V_2 \quad \text{——— (2)}$$

Comparing ① and ② with Y-parameter eqns.

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

2. Y-parameters in terms of ABCD parameters.

$$V_1 = AV_2 - BI_2 \text{ ——— } ①$$

$$I_1 = CV_2 - DI_2 \text{ ——— } ②$$

From ①, $BI_2 = AV_2 - V_1$

$$I_2 = \frac{-1}{B}V_1 + \frac{A}{B}V_2 \text{ ——— } ③$$

Substituting ③ in ②,

$$I_1 = CV_2 - D\left[\frac{-1}{B}V_1 + \frac{A}{B}V_2\right]$$

$$= \frac{D}{B}V_1 + \left[C - \frac{AD}{B}\right]V_2 = \frac{D}{B}V_1 + \frac{[BC - AD]}{B}V_2 \text{ ——— } ④$$

Comparing ④ and ③ with y-parameter eqns

$$Y_{11} = \frac{D}{B}, \quad Y_{12} = \frac{BC-AD}{B}$$

$$Y_{21} = \frac{-1}{B}, \quad Y_{22} = \frac{A}{B}$$

3. Y-parameters in terms of hybrid-parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \text{ ——— } ①$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \text{ ——— } ②$$

From ①,

$$h_{11}I_1 = V_1 - h_{12}V_2$$

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2 \text{ ——— } ③$$

Subst ③ in ②,

$$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$$

$$= \frac{h_{21}}{h_{11}}V_1 + \left[h_{22} - \frac{h_{12}h_{21}}{h_{11}}\right]V_2$$

$$= \frac{h_{21}}{h_{11}}V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right]V_2 \quad \text{——— ④}$$

Comparing ③ and ④ with y-parameter eqns.

$$Y_{11} = \frac{1}{h_{11}}, \quad Y_{12} = -\frac{h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}}, \quad Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} = \frac{\Delta h}{h_{11}}$$

16/12/20 **ABCD parameters In terms of other parameters**

1. **ABCD parameters in terms of z-parameters**

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad \text{—— ①}$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \quad \text{—— ②}$$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

From ②,

$$z_{21}I_1 = V_2 - z_{22}I_2$$

$$I_1 = \frac{1}{z_{21}}V_2 - \frac{z_{22}}{z_{21}}I_2 \quad \text{——— ③}$$

Subs ③ in ①,

$$V_1 = z_{11}\left[\frac{1}{z_{21}}V_2 - \frac{z_{22}}{z_{21}}I_2\right] + z_{12}I_2$$

$$= \frac{z_{11}}{z_{21}}V_2 - \frac{z_{11}z_{22}}{z_{21}}I_2 + z_{12}I_2$$

$$= \frac{z_{11}}{z_{21}}V_2 - \left[\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}}\right]I_2 \quad \text{——— ④}$$

Comparing (4) and (3) with ABCD parameter eqns.

$$A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}, \quad D = \frac{Z_{22}}{Z_{21}}$$

2. ABCD parameters in terms of Y-parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \text{ ——— (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \text{ ——— (2)}$$

From (2),

$$Y_{21}V_1 = -Y_{22}V_2 + I_2$$

$$V_1 = -\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2 \text{ ——— (3)}$$

Subs. (3) in (1),

$$I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2\right] + Y_{12}V_2$$

$$= \left[Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}\right]V_2 + \frac{Y_{11}}{Y_{21}}I_2$$

$$= \left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right]V_2 + \frac{Y_{11}}{Y_{21}}I_2 \text{ ——— (4)}$$

Comparing (3) and (4) with ABCD parameter eqns.

$$A = \frac{-Y_{22}}{Y_{21}}, \quad B = -\frac{1}{Y_{21}}$$

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} = \frac{-\Delta Y}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

3. ABCD parameters in terms of hybrid parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \text{ ——— } (1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \text{ ——— } (2)$$

From (2),

$$h_{21}I_1 = I_2 - h_{22}V_2$$

$$I_1 = \frac{-h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2 \text{ ——— } (3)$$

Subs. (3) in (1),

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2\right] + h_{12}V_2$$

$$= \left[h_{12} - \frac{h_{11}h_{22}}{h_{21}}\right]V_2 + \frac{h_{11}}{h_{21}}I_2$$

$$= \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right]V_2 + \frac{h_{11}}{h_{21}}I_2 \text{ ——— } (4)$$

Comparing (4) and (3) with ABCD parameter eqns.

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} = \frac{-\Delta h}{h_{21}}, \quad B = -\frac{h_{11}}{h_{21}}$$

$$C = \frac{-h_{22}}{h_{21}}, \quad D = -\frac{1}{h_{21}}$$

Hybrid parameters in terms of other parameters.

1. Hybrid parameters in terms of Z-parameters.

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \text{ ——— } (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \text{ ——— } (2)$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

From (2),

$$Z_{22}I_2 = V_2 - Z_{21}I_1$$

$$I_2 = -\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2 \quad \text{——} (3)$$

Subst. (3) in (1),

$$V_1 = z_{11} I_1 + z_{12}\left[-\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2\right]$$

$$= \left[z_{11} - \frac{z_{12} z_{21}}{z_{22}}\right] I_1 + \frac{z_{12}}{z_{22}} V_2$$

$$= \left[\frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}}\right] I_1 + \frac{z_{12}}{z_{22}} V_2 \quad \text{——} (4)$$

Comparing (4) and (3) with h-parameter eqns.

$$h_{11} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} = \frac{\Delta z}{z_{22}}$$

$$h_{12} = \frac{z_{12}}{z_{22}}$$

$$h_{21} = -\frac{z_{21}}{z_{22}}$$

$$h_{22} = \frac{1}{z_{22}}$$

2. Hybrid parameters in terms of Y-parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{——} (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{——} (2)$$

From (1),

$$Y_{11} V_1 = I_1 - Y_{12} V_2$$

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \quad \text{——} (3)$$

Subs (3) in (2),

$$I_2 = Y_{21}\left[\frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2$$

$$= \left[Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}} \right] V_2 + \frac{Y_{21}}{Y_{11}} I_1$$

$$= \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} \right] V_2 + \frac{Y_{21}}{Y_{11}} I_1 \text{ ——— } (4)$$

Comparing (3) and (4) with h-parameter equations.

$$h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}, \quad h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = \frac{\Delta Y}{Y_{11}}$$

3. Hybrid parameters in terms of ABCD parameters.

$$V_1 = AV_2 - BI_2 \text{ ——— } (1)$$

$$I_1 = CV_2 - DI_2 \text{ ——— } (2)$$

From (2),

$$DI_2 = -I_1 + CV_2$$

$$I_2 = \frac{-1}{D} I_1 + \frac{C}{D} V_2 \text{ ——— } (3)$$

Substituting in (1),

$$V_1 = AV_2 - B\left[\frac{-1}{D} I_1 + \frac{C}{D} V_2\right]$$

$$= \frac{B}{D} I_1 + \left[A - \frac{BC}{D}\right] V_2$$

$$= \frac{B}{D} I_1 + \left[\frac{AD - BC}{D}\right] V_2 \text{ —— } (4)$$

Comparing (4) and (3) with h-parameter eqns.

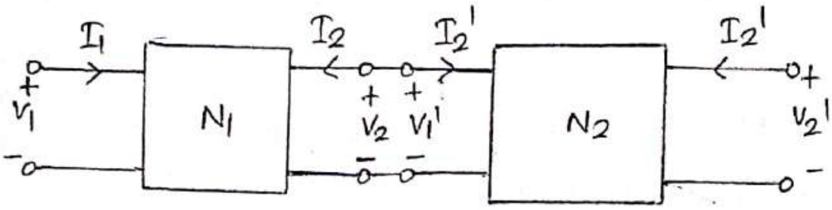
$$h_{11} = \frac{B}{D}, \quad h_{12} = \frac{AD - BC}{D} = \frac{\Delta T}{D}$$

$$h_{21} = \frac{-1}{D}, \quad h_{22} = \frac{C}{D}$$

17/12/20

INTERCONNECTION OF TWO-PORT n/ws

1. Cascade connection



In the cascade connection, the o/p port of the first n/w becomes the input port of 2nd n/w.

Let A_1, B_1, C_1, D_1 be the transmission parameters of n/w N_1 and A_2, B_2, C_2, D_2 that of n/w $-N_2$.

For N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{——— (1)}$$

For $N2$,

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix} \quad \text{——— (2)}$$

$V_2 = V_1'$ and $-I_2 = I_1'$

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \quad \text{——— (3)}$$

From (1) and (3),

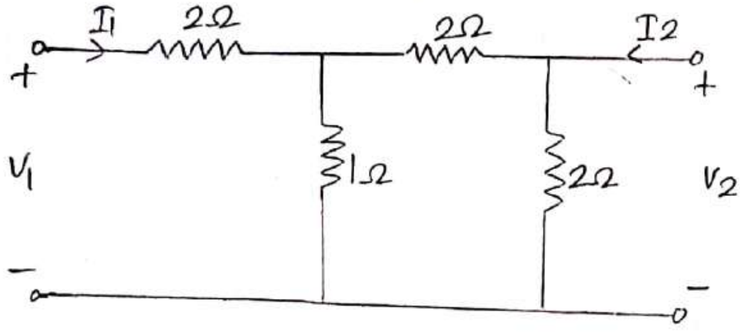
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

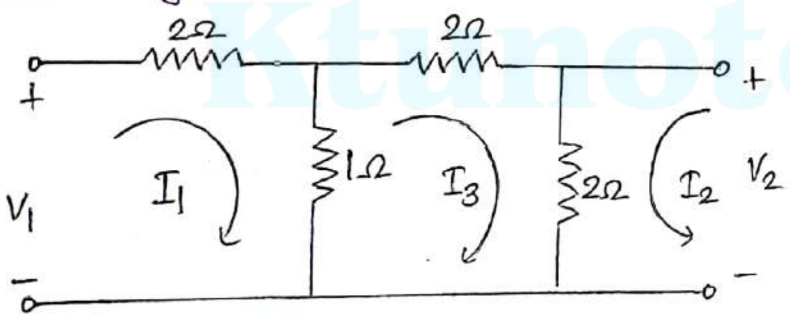
$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

The resultant ABCD matrix of the cascaded connection is the product of individual ABCD matrices.

1). Two identical sections of n/w shown below are connected in cascade. Obtain the transmission parameters of the overall n/w.



Redrawing the n/w



$$V_1 = 3I_1 - I_3 \quad \text{—— (1)}$$

$$V_2 = 2I_2 + 2I_3 \quad \text{—— (2)}$$

$$-I_1 + 2I_2 + 5I_3 = 0$$

$$5I_3 = I_1 - 2I_2$$

$$I_3 = \frac{1}{5}I_1 - \frac{2}{5}I_2 \quad \text{—— (3)}$$

Subst. (3) in (1),

$$V_1 = 3I_1 - I_3$$

$$= 3I_1 - \left(\frac{I_1}{5} - \frac{2}{5}I_2\right)$$

$$= \frac{14}{5}I_1 + \frac{2}{5}I_2 \quad \text{—— (4)}$$

Subst. (3) in (2),

$$V_2 = 2I_2 + 2\left(\frac{I_1}{5} - \frac{2}{5}I_2\right)$$

$$= \frac{2}{5}I_1 + \frac{6}{5}I_2$$

$$\frac{2}{5}I_1 = V_2 - \frac{6}{5}I_2$$

$$I_1 = \frac{5}{2}V_2 - 3I_2 \quad \text{—— (5)}$$

Subst. (5) in (4),

$$V_1 = \frac{14}{5}\left(\frac{5}{2}V_2 - 3I_2\right) + \frac{2}{5}I_2 = 7V_2 - 8I_2 \quad \text{—— (6)}$$

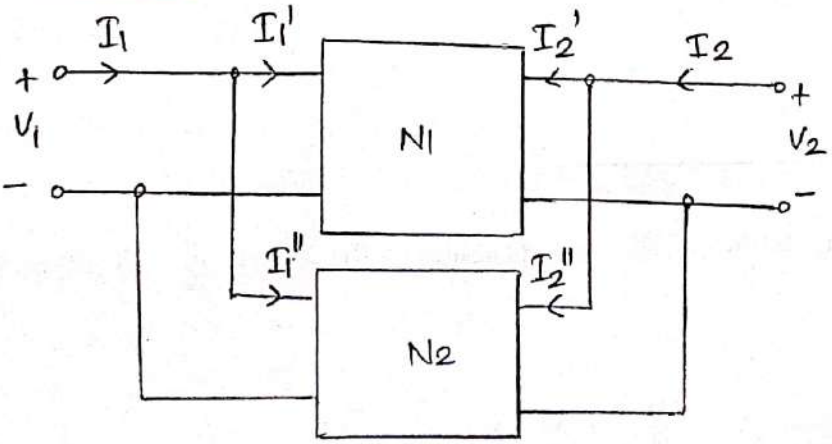
Comparing (6) and (5) with ABCD parameter eqns.

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 2.5 & 3 \end{bmatrix}$$

Hence, transmission parameters of the overall cascaded n/w are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 2.5 & 3 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 2.5 & 3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 69 & 80 \\ 25 & 29 \end{bmatrix}}}$$

Parallel connection



N1 and N2 connected parallel

In parallel connection the two n/ws have the same input and output voltages.

Let Y_{11}', Y_{12}', Y_{21}', Y_{22}' be the Y-parameters of N1 and Y_{11}'', Y_{12}'', Y_{21}'', Y_{22}'' be the Y parameters of N2.

For N1, $$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For n/w N2,

$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the combined n/w,

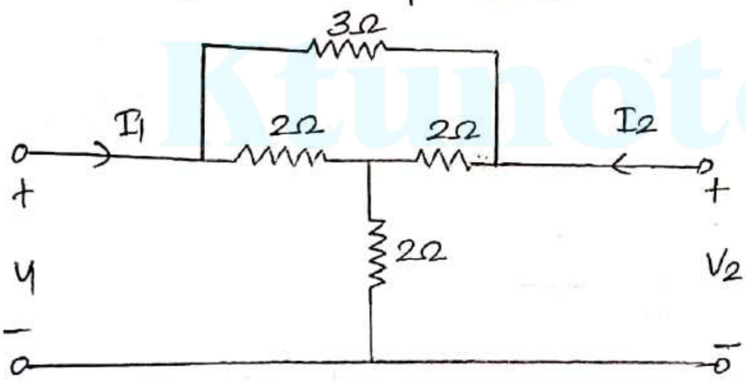
$$I_1 = I_1' + I_1'' \text{ and } I_2 = I_2' + I_2''$$

$$\therefore \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus the resultant Y-parameter matrix for parallel connected n/ws is the sum of Y-matrices of each individual two port n/ws.

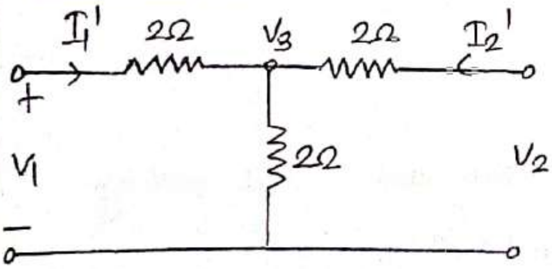
PROBLEMS

1. Determine Y-parameters of the n/w.



The above n/w can be considered as the parallel connection of two n/ws N1 and N2.

Network N1



$$I_1' = \frac{V_1 - V_3}{2} \quad \text{——} ①$$

$$I_2' = \frac{V_2 - V_3}{2} \quad \text{——} ②$$

$$I_1' + I_2' = \frac{V_3}{2} \quad \text{——} ③$$

Subst. ① and ② in ③,

$$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} = \frac{V_3}{2}$$

$$\frac{V_1 + V_2 - 2V_3}{2} = \frac{V_3}{2}$$

$$3V_3 = V_1 + V_2$$

$$V_3 = \frac{V_1}{3} + \frac{V_2}{3} \quad \text{——— (4)}$$

(4) in (1), $I_1' = \frac{V_1}{2} - \frac{V_3}{2} = \frac{V_1}{2} - \frac{1}{2}\left(\frac{V_1}{3} + \frac{V_2}{3}\right)$

$$= \frac{1}{3}V_1 - \frac{1}{6}V_2 \quad \text{——— (5)}$$

Subst (4) in (2),

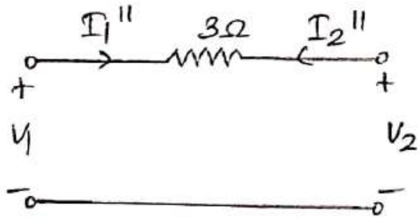
$$I_2' = \frac{V_2}{2} - \frac{1}{2}\left[\frac{V_1}{3} + \frac{V_2}{3}\right]$$

$$= -\frac{1}{6}V_1 + \frac{1}{3}V_2 \quad \text{——— (6)}$$

Comparing (5) and (6) with y-parameter eqns

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} 1/3 & -1/6 \\ -1/6 & 1/3 \end{bmatrix}$$

Network N2 is shown below



$$I_1'' = -I_2'' = \frac{V_1 - V_2}{3} = \frac{1}{3}V_1 - \frac{1}{3}V_2$$

$$I_2' = -\frac{1}{3}V_1 + \frac{1}{3}V_2$$

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

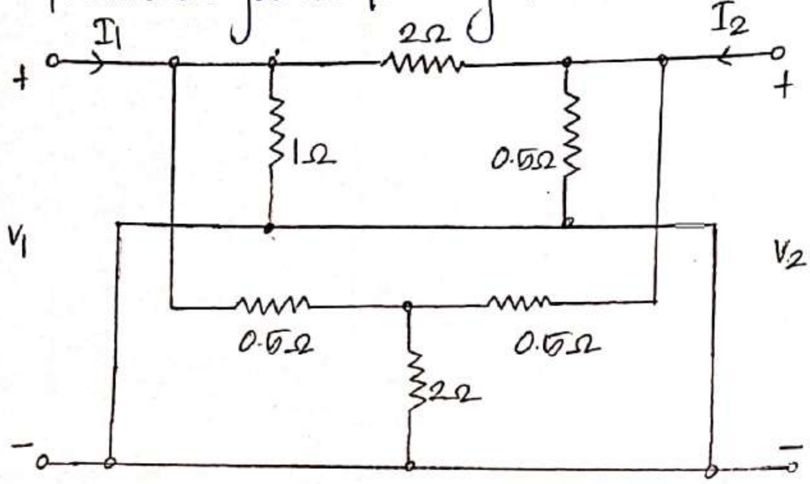
The overall Y-parameters of the n/w are,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix}$$

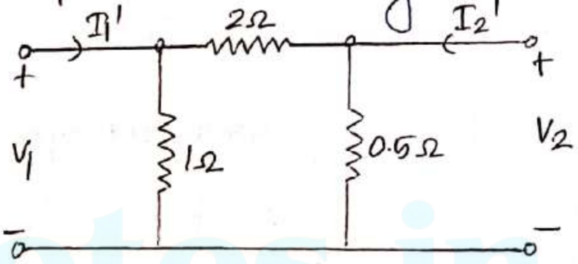
$$= \begin{bmatrix} \frac{1}{3} + \frac{1}{3} & -\frac{1}{6} - \frac{1}{3} \\ -\frac{1}{6} - \frac{1}{3} & \frac{1}{3} + \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{2}{3} \end{bmatrix}$$

28/12/2020

2. Find Y-parameters for the following n/w.



The above n/w can be considered as a parallel combination of two n/ws N_1 and N_2.



Network-N_1

Node-1

$$I_1' = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$

$$= \frac{3}{2}V_1 - \frac{1}{2}V_2 \quad \text{——} (1)$$

Node-2

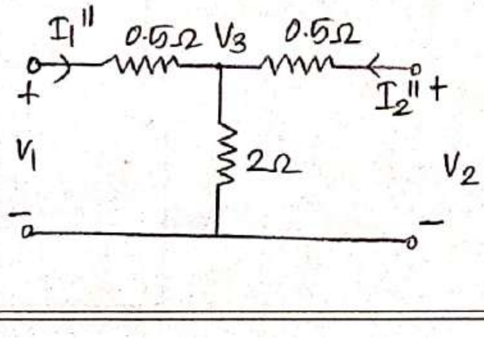
$$I_2' = \frac{V_2}{0.5} + \frac{V_2 - V_1}{2}$$

$$= -\frac{1}{2}V_1 + \frac{5}{2}V_2 \quad \text{——} (2)$$

Comparing (1) and (2) with y-parameter eqns.

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

Network-N_2



$$I_1'' = \frac{V_1 - V_3}{0.5} = 2V_1 - 2V_3 \quad \text{——} (1)$$

$$I_2'' = \frac{V_2 - V_3}{0.5} = 2V_2 - 2V_3 \quad \text{——} (2)$$

$$I_1'' + I_2'' = \frac{V_3}{2} \quad \text{—— (3)}$$

Subst (1) and (2) in (3),

$$2V_1 - 2V_3 + 2V_2 - 2V_3 = \frac{V_3}{2}$$

$$\frac{9}{2}V_3 = 2V_1 + 2V_2$$

$$V_3 = \frac{4}{9}V_1 + \frac{4}{9}V_2 \quad \text{—— (4)}$$

Subst. in (1),

$$I_1'' = 2V_1 - 2\left[\frac{4}{9}V_1 + \frac{4}{9}V_2\right]$$

$$= \frac{10}{9}V_1 - \frac{8}{9}V_2 \quad \text{—— (5)}$$

Substituting V_3 in (2),

$$I_2' = 2V_2 - 2V_3$$

$$= 2V_2 - 2\left(\frac{4}{9}V_1 + \frac{4}{9}V_2\right] = -\frac{8}{9}V_1 + \frac{10}{9}V_2 \quad \text{—— (6)}$$

Comparing (5) and (6) with parameter eqns.

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{10}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{10}{9} \end{bmatrix}$$

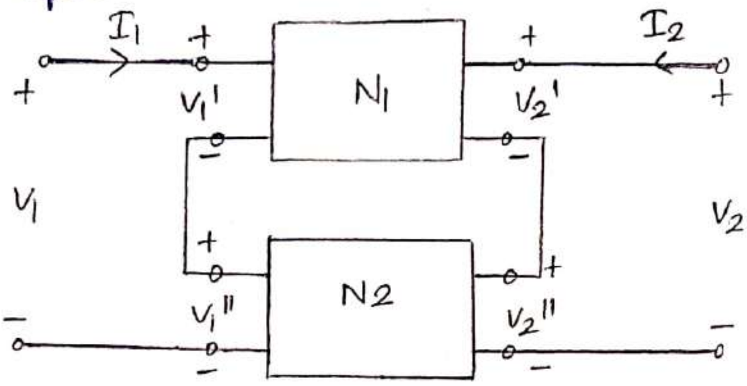
Hence the overall Y-parameters of the n/w are,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} + \frac{10}{9} & \frac{-1}{2} - \frac{8}{9} \\ \frac{-1}{2} - \frac{8}{9} & \frac{5}{2} + \frac{10}{9} \end{bmatrix} = \begin{bmatrix} \frac{47}{18} & \frac{-25}{18} \\ \frac{-25}{18} & \frac{65}{18} \end{bmatrix}$$

SERIES CONNECTION OF n/ws

In series connection, both n/ws carry the same input current. Their o/p currents are also equal.



Let $z_{11}', z_{12}', z_{21}', z_{22}'$ be the z-parameters of N_1 and $z_{11}'', z_{12}'', z_{21}'', z_{22}''$ be that of N_2.

For N_1,

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For N_2,

$$\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} z_{11}'' & z_{12}'' \\ z_{21}'' & z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the combined n/w,

$$V_1 = V_1' + V_1'' \text{ and } V_2 = V_2' + V_2''$$

Hence,

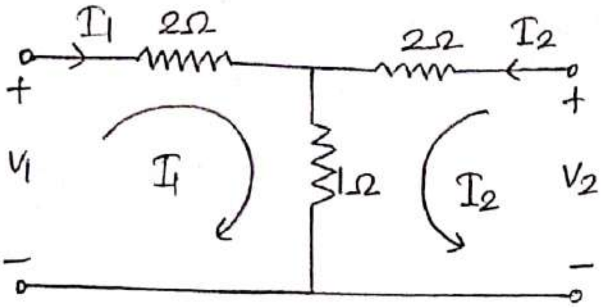
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' + V_1'' \\ V_2' + V_2'' \end{bmatrix} = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where the resultant z-parameter matrix for the series-connected n/ws is the sum of z-matrices of each individual two-port n/ws.

PROBLEMS

1. Two identical sections of the n/w shown below are connected in series. Obtain z-parameters of the overall n/w.



Applying KVL,

$V_1 = 3I_1 + I_2$ —— ①

$V_2 = 3I_2 + I_1$

$= I_1 + 3I_2$ —— ②

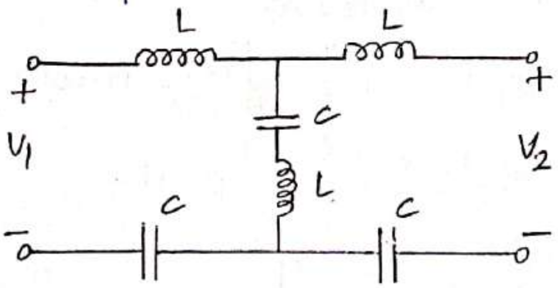
Comparing ① and ② with z-parameter eqns.

$$\begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

z-parameters of the overall n/w are:

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

2. Determine z-parameters for the n/w shown below.

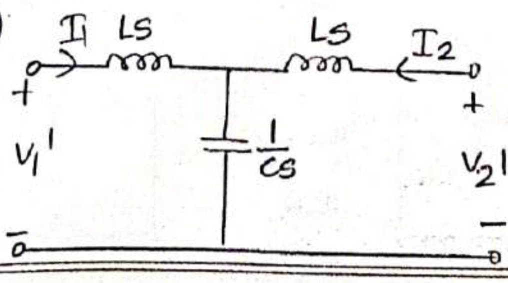


The n/w can be considered as the series connection of two n/ws N_1 and N_2.

$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

Network – N_1 (transformed)



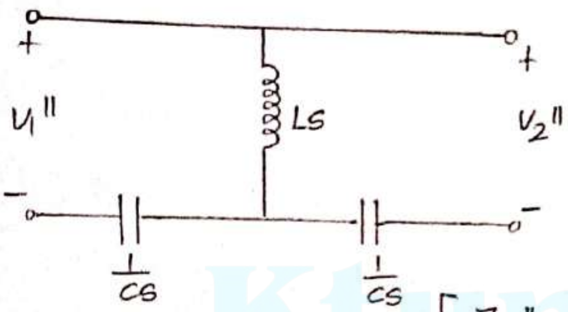
$$V_1' = \left(Ls + \frac{1}{Cs}\right) I_1 + \left(\frac{1}{Cs}\right) I_2 \quad \text{—— (1)}$$

$$V_2' = \left(\frac{1}{Cs}\right) I_1 + \left(Ls + \frac{1}{Cs}\right) I_2 \quad \text{—— (2)}$$

Comparing (1) and (2) with z-parameter eqns,

$$\begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{Cs} & \frac{1}{Cs} \\ \frac{1}{Cs} & Ls + \frac{1}{Cs} \end{bmatrix}$$

<u>Network-N2</u> (transformed)



<u>Mesh-1</u>: $V_1'' = \left(Ls + \frac{1}{Cs}\right) I_1 + (Ls) I_2$ —— (1)

<u>Mesh-2</u>: $V_2'' = (Ls) I_1 + \left(Ls + \frac{1}{Cs}\right) I_2$ —— (2)

Comparing eqns (1) and (2) with z-parameter eqns.

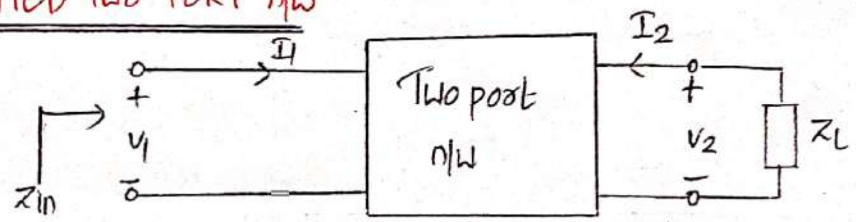
$$\begin{bmatrix} z_{11}'' & z_{12}'' \\ z_{21}'' & z_{22}'' \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{Cs} & Ls \\ Ls & Ls + \frac{1}{Cs} \end{bmatrix}$$

The overall z-parameters of the n/w are,

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{Cs} + Ls + \frac{1}{Cs} & \frac{1}{Cs} + Ls \\ \frac{1}{Cs} + Ls & Ls + \frac{1}{Cs} + Ls + \frac{1}{Cs} \end{bmatrix}$$

$$= \begin{bmatrix} 2Ls + \frac{2}{Cs} & Ls + \frac{1}{Cs} \\ Ls + \frac{1}{Cs} & 2Ls + \frac{2}{Cs} \end{bmatrix} = \left(Ls + \frac{1}{Cs}\right) \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

29/12/20 <u>TERMINATED TWO PORT n/w</u>



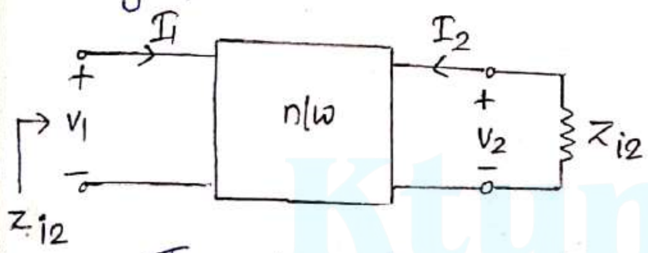
$V_2 = -Z_L I_2$

$$Z_{in} = \frac{V_1}{I_1} = \frac{Z_{11}Z_{22} + Z_{11}Z_L - Z_{12}Z_{21}}{Z_{22} + Z_L}$$

With o/p port open circuited, $Z_L = \infty$. $Z_{in} = Z_{11}$

Image impedences

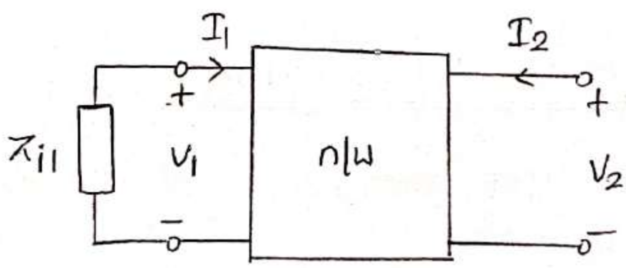
If the driving point impedence at port 1, with impedence Z_{i2} connected across port 2 is Z_{i1} and driving point impedence at port 2, with impedence Z_{i1} connected across port 1 is Z_{i2}, then Z_{i1} and Z_{i2} are known as image impedences of the n/w or image parameters.



Two port n/w terminated at port-2

$$Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad \text{——— (1)}$$

$$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A} \quad \text{——— (2)}$$



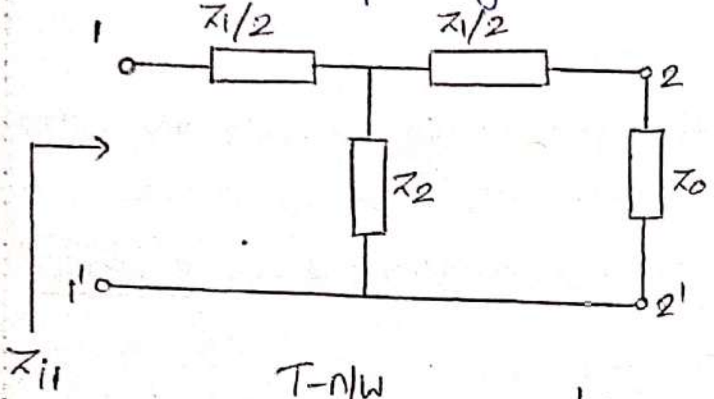
Two port n/w terminated at port-1

Solving (1) and (2),

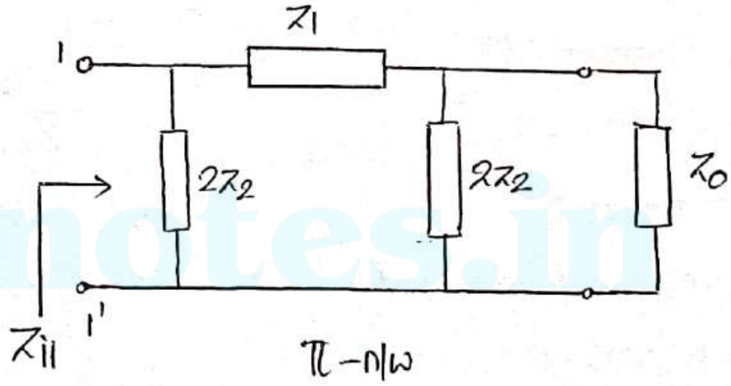
$$Z_{i1} = \sqrt{\frac{AB}{CD}}, \quad Z_{i2} = \sqrt{\frac{BD}{AC}}$$

CHARACTERISTIC IMPEDENCE

For a symmetrical n/w, the image impedences Z_{i1} and Z_{i2} are equal and the image impedence is then called the characteristic or the iterative impedence Z_0. Thus, if a symmetrical T-n/w is ~~determinated~~ terminated in Z_0, its input impedence will also be Z_0. Z_0 for a symmetrical n/w can be found as follows.



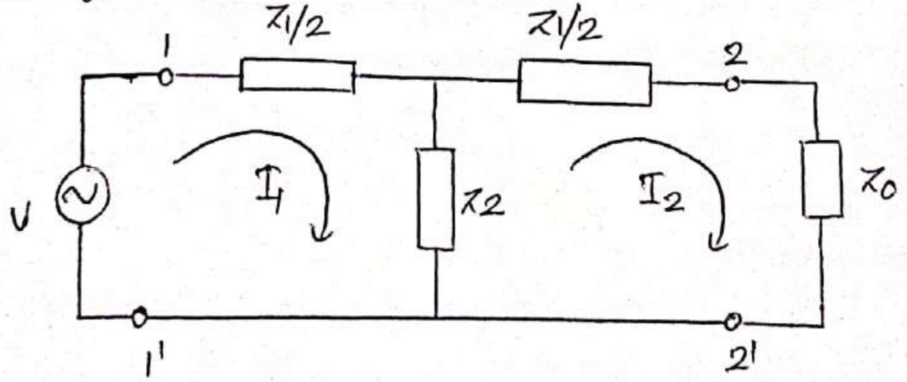
T-n/w



π-n/w

ATTENUATION AND PHASE CONSTANTS

Consider the T-n/w terminated with a load equal to its characteristic impedence Z_0.



$\frac{I_1}{I_2} = e^{\gamma}$ where γ is called propagation constant and is a complex number.

$\gamma = \alpha + j\beta$

α is ~~the~~ known as the attenuation constant, since it determines the magnitude ratio between input and output quantities or the attenuation produced in passing through this n/w. Unit is nepers.

The parameter β is known as the phase constant, since it determines the phase angle between input and output quantities or the shift in phase introduced by the n/w. Its unit is radians.