# MODULE 1

## Introduction to circuit variables and circuit elements

In Electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this we require an interconnection of electrical devices. Such interconnection is referred to an ***electric circuit***, and each component of this circuit is known as an ***element***. ***An electric circuit is an interconnection of electrical elements.***

### Charge:

The most basic quantity in an electric circuit is the electric charge. Charge is an electrical property of an atomic particles of which matter consist, measured in ***coulombs (C).***

***Electric current:*** The directed flow of free electrons (or charge) is called electric current. The conducting wire has a large number of free electrons. When a battery (a source) is connected to conducting wire, then free electrons, being negatively charged, will start moving towards the positive terminal around the circuit. This directed flow of electrons is called electric current.

***Electric current is rate of change of charge.***

$$i = \frac{dQ}{dt}$$

The actual direction of current (i.e. flow of electrons) is from negative terminal to the positive terminal through that part of the circuit external to the cell. However, prior to Electron theory, it was assumed that current flowed from positive terminal to the negative terminal of the cell via the circuit. This convention is so firmly established that it is still in use. This assumed direction of current is now called conventional current.

**Unit of Current**: The strength of electric current I is the rate of flow of electrons i.e. charge flowing per second.

$$\text{Current, } I = \frac{Q}{t}$$

The charge Q is measured in coulombs and time t in seconds. Therefore, the unit of electric current will be ***coulombs/sec or ampere***. If Q = 1 coulomb, t = 1 sec, then I = 1/1 = 1 ampere

*One ampere of current is said to flow through a wire if at any cross-section one coulomb of charge flows in one second.*

The electric current may be classified into three main classes: (i) steady current (ii) varying current and (iii) alternating current.

***(i)*** ***Steady current.*** When the magnitude of current does not change with time, it is called a steady current. Fig. 1.1 (i) shows the graph between steady current and time. Note that value of current remains the same as the time changes. The current provided by a battery is almost a steady current (d.c.).

***(ii)*** Varying current. When the magnitude of current changes with time, it is called a varying current. Fig. 1.1 (ii) shows the graph between varying current and time. Note that value of current varies with time.

***(iii)*** Alternating current. An alternating current is one whose magnitude changes continuously with time and direction changes periodically. Due to technical and economical reasons, we produce alternating currents that have sine waveform (or cosine waveform) as shown in Fig. 1.1 (iii). It is called alternating current because current flows in alternate directions in the circuit, i.e., from 0 to T/2 second (T is the time period of the wave) in one direction and from T/2 to T second in the opposite direction. The current provided by an a.c. generator is alternating current that has sine (or cosine) waveform.

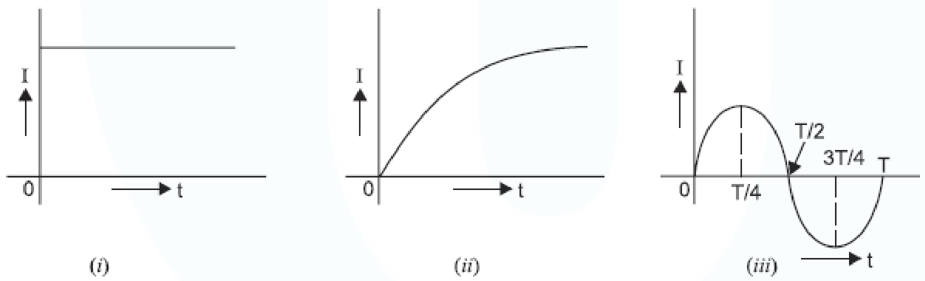


Fig 1.1

## Voltage

To move the electron in a particular direction, it require some work or energy transfer. This work is done by some external electromotive force (emf). This emf is known as ***voltage or potential difference***.

Unit. Since the unit of electric potential is volt, one can expect that unit of potential difference will also be ***volt***. It is defined as under:

*The potential difference between two points is 1 volt if one joule of work is done or released in transferring 1 coulomb of charge from one point to the other.*

Potential Rise and Potential Drop: Fig. 1.2 shows a circuit with a cell and a resistor. The cell provides a potential difference of 1.5 V. Since it is an energy source, there is a rise in potential associated with a cell. The cell's potential difference represents an e.m.f. so that symbol E could be used. The resistor is also associated with a potential difference. Since it is a consumer (converter) of energy, there is a drop in potential across the resistor. We can combine the idea of potential rise or drop with the popular term "voltage". It is customary to refer to the potential difference across the cell as a ***voltage rise*** and to the potential difference across the resistor as a ***voltage drop***.

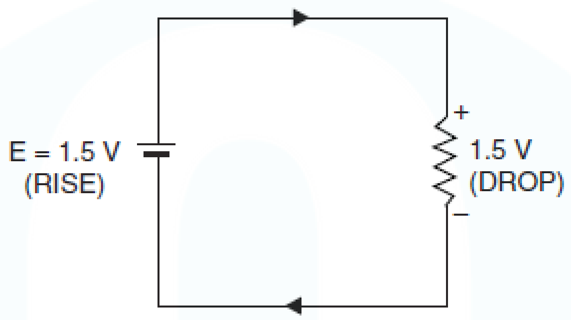



Fig 1.2

## Resistance

The opposition offered by a substance to the flow of electric current is called its ***resistance***.

Unit of resistance: The practical unit of resistance is ***ohm*** and is represented by the symbol **Ω**.

It is defined as under :

A wire is said to have a resistance of 1 ohm if a p.d. of 1 volt across its ends causes 1 ampere to flow through it.

The resistance R of a conductor

(i) is directly proportional to its length i.e. **$R \propto l$**

(ii) is inversely proportional to its area of X-section i.e. **$R \propto 1/a$**

(iii) depends upon the nature of material.

(iv) depends upon temperature.

From the first three points (leaving temperature for the time being), we have,

$R \propto l/a$

Or $$R = \frac{\rho l}{a}$$

where ρ (Greek letter 'Rho') is a constant and is known as resistivity or specific resistance of the material. Its value depends upon the nature of the material.

## Conductance

The reciprocal of resistance of a conductor is called its conductance (G). If a conductor has resistance R, then its conductance G is given by ;

$$\mathbf{G = 1/R}$$

The SI unit of conductance is **mho** (i.e., ohm spelt backward). These days, it is a usual practice to use S***iemen*** as the unit of conductance. It is denoted by the symbol **S**.

## Capacitor

Any two conducting surfaces separated by an insulating material is called a capacitor or condenser. Its purpose is to store charge in a small space.

The ability of a capacitor to store charge is known as its capacitance. It has been found experimentally that charge Q stored in a capacitor is directly proportional to the p.d. V across it

i.e. $Q \propto V$

Or $\mathbf{Q = CV}$

The constant C is called the capacitance of the capacitor.

Hence capacitance of a capacitor can be defined as under :

The ratio of charge on capacitor plates to the p.d. across the plates is called capacitance of the capacitor.

The SI unit of charge is 1 coulomb and that of voltage is 1 volt. Therefore, the SI unit of capacitance is one ***coulomb/volt*** which is also called ***farad*** (Symbol F).

The capacitance of a capacitor depends on the following factors

**(i)** It is directly proportional to the area of plates,A

**(ii)** It is inversely proportional to the distance between two plates,d

**(iii)** It depends upon the absolute permittivity of the medium between the plates, ε(epsilon)

Hence,

$$C \propto \frac{A}{d}$$

$$\mathbf{C = \varepsilon \frac{A}{d}}$$

## Inductance

Inductance is the property of a coil that opposes any change in the amount of current flowing through it.

SI unit of inductance is ***Henry (H).***

## Ohms Law

Ohm's law states that the current I flowing in a circuit is directly proportional to the applied voltageV and inversely proportional to the resistance R, provided the temperature remains constant. ***I = V/R***

| | *Voltage (v)* | *Current (I)* | *Power (P) = VI* | *Energy (E) = Pt* |
|---|---|---|---|---|
| *Resistor (R)* | *V=IR* | *I = V/R* | $P = I^2R = \frac{v^2}{R}$ | $E = I^2Rt = \frac{v^2t}{R}$ |
| *Inductor (L)* | $v = L\frac{di}{dt}$ | $i(t) = \frac{1}{L}\int v\,dt + i(0)$ | $P = Li\frac{di}{dt}$ | $E = \frac{1}{2}L\,I^2$ |
| *Capacitor (C)* | $v(t) = \frac{1}{C}\int i\,dt + v(0)$ | $i = C\frac{dV}{dt}$ | $P = Cv\frac{dv}{dt}$ | $E = \frac{1}{2}C\,V^2$ |

## Problem

A current waveform is shown below is applied to a 2Ω resistor. Draw the voltage waveform

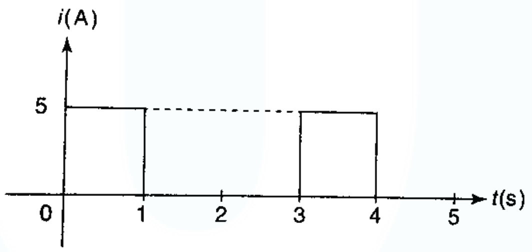
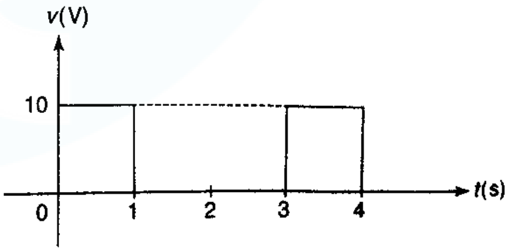



**Fig. 1.1**

**Solution**




For the resistor, $v = Ri$

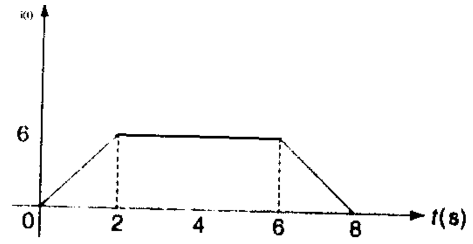
(a) For $0 < t < 1$, $v = Ri = 2 \times 5 = 10$ V

(b) For $1 < t < 3$, $v = Ri = 2 \times 0 = 0$

(c) For $3 < t < 4$, $v = Ri = 2 \times 5 = 10$ V

## Problem

A current waveform is applied to a 2H inductor. Draw the voltage waveform.




**Solution**

$$v = L\frac{di}{dt}$$

**(a)** For $0 < t < 2$,

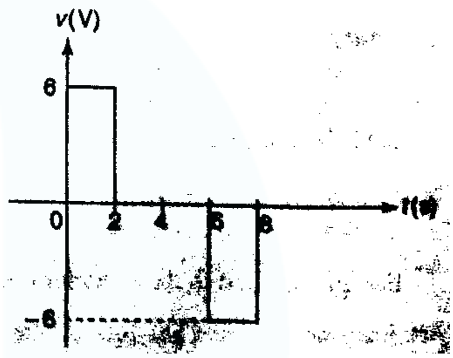
$$v = 2 \times \frac{(6-0)}{(2-0)} = 6 \text{ V}$$

**(b)** For $2 < t < 6$,

$$v = 2 \times \frac{(6-6)}{(6-2)} = 0$$

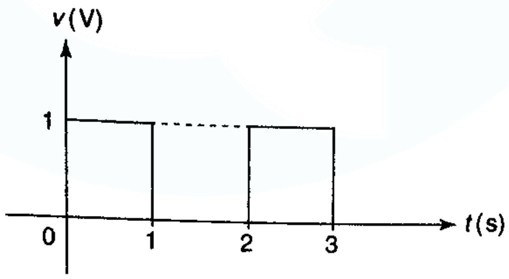
**(c)** For $6 < t < 8$,

$$v = 2 \times \frac{(0-6)}{(8-6)} = -6 \text{ V}$$




## Problem

A voltage waveform is applied across a 1H inductor is shown in figure. Draw the current waveform.



**Solution**

$$i = \frac{1}{L}\int_0^t v\,dt + i(0)$$

(a) For $0 < t < 1$,

$$v = 1$$

$$i = \frac{1}{1}\int_0^t 1\,dt + i(0) = [t]_0^t + 0 = t$$

$$i(1) = 1\text{ A}$$

(b) For $1 < t < 2$,

$$v = 0$$

$$i = \frac{1}{1}\int_1^t 0 \cdot dt + i(1) = 0 + 1 = 1\text{ A}$$

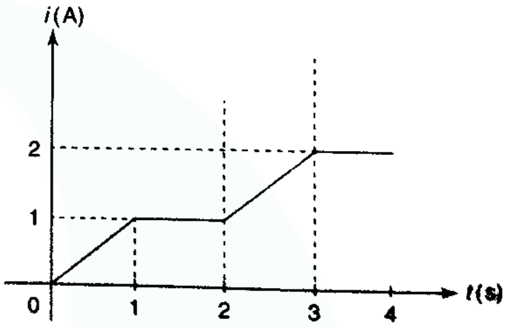
$$i(2) = 1\text{ A}$$

(c) For $2 < t < 3$,

$$v = 1$$

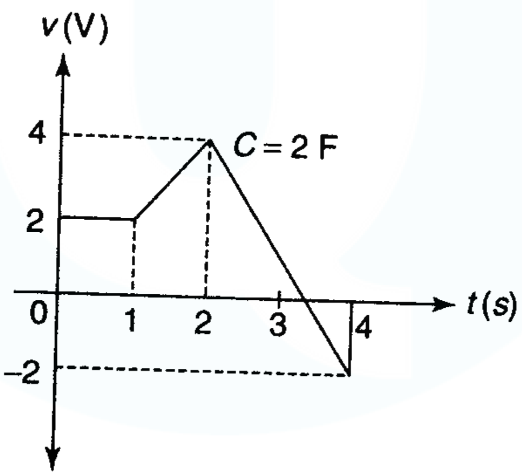
$$i = \frac{1}{1}\int_2^t 1\,dt + i(2) = [t]_2^t + 1 = t - 2 + 1 = t - 1$$

$$i(3) = 3 - 1 = 2\text{ A}$$




## Problem

A current waveform is shown below is applied to a 2F capacitor. Draw the voltage waveform





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**Solution** $i = C\dfrac{dv}{dt}$

(a) $0 < t < 1$

At $t = 0$, $v = 2$ V

At $t = 1$, $v = 2$ V

$$i = C\frac{dv}{dt} = 2\left(\frac{2-2}{1-0}\right) = 0$$

(b) $1 < t < 2$

At $t = 1$, $v = 2$ V

At $t = 2$, $v = 4$ V

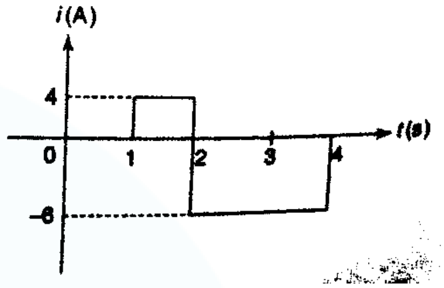
$$i = C\frac{dv}{dt} = 2\left(\frac{4-2}{2-1}\right) = 4 \text{ A}$$

(c) $2 < t < 4$

At $t = 2$, $v = 4$ V

At $t = 4$, $v = -2$ V

$$i = C\frac{dv}{dt} = 2\left(\frac{-2-4}{4-2}\right) = 2\left(\frac{-6}{2}\right) = -6 \text{ A}$$



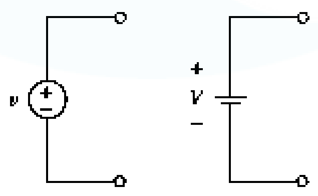

## Sources

Source is a basic network element which supplies energy to the networks. Two types

1. Independent source
2. Dependent Source

## Independent Voltage and Current Sources

The term ***voltage source*** is used to describe a source of energy which establishes a potential difference across its terminals. Most of the sources encountered in everyday life are voltage sources e.g., batteries, d.c. generators, alternators etc.




(a) (b)

The term ***current source*** is used to describe a source of energy that provides a current e.g., collector circuits of transistors. ***Voltage and current sources*** are called ***active elements*** because they provide electrical energy to a circuit.



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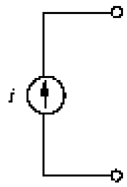
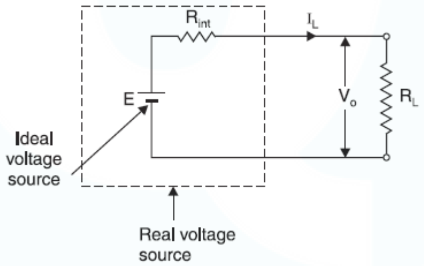



## Ideal Voltage Source or Constant-Voltage Source:

An ***ideal voltage source*** (also called constant-voltage source) is one that maintains a constant terminal voltage, no matter how much current is drawn from it. ***An ideal voltage source has zero internal resistance***. Therefore, it would provide constant terminal voltage regardless of the value of load connected across its terminals.

## Real Voltage Source

A real or non-ideal voltage source has low but finite internal resistance ($R_{int}$) that causes its terminal voltage to decrease when load current is increased and vice-versa. A real voltage source can be represented as an ideal voltage source in series with a resistance equal to its internal resistance ($R_{int}$) as shown in Figure.

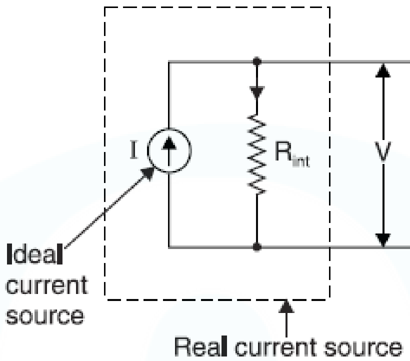



## Ideal Current Source

An ***ideal current source*** or constant current source is one which will supply the same current to any resistance (load) connected across its terminals. ***An ideal current source has infinite internal resistance.*** Therefore, it supplies the same current to any resistance connected across its terminals.

## Real Current Source

A real or non-ideal current source has high but finite internal resistance ($R_{int}$). Therefore, the load current (IL) will change as the value of load resistance ($R_L$) changes. A real current source can be represented by an ideal current source (I) in parallel with its internal resistance ($R_{int}$) as shown in Figure.




## Dependent Voltage and Current Sources

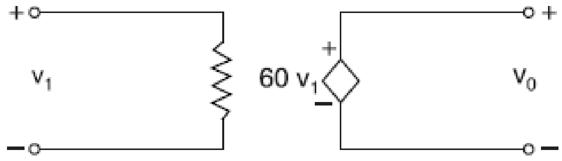
A dependent source provides a voltage or current between its output terminals which depends upon another variable such as voltage or current. For example, a voltage amplifier can be considered to be a dependent voltage source. It is because the output voltage of the amplifier depends upon another voltage i.e. the input voltage to the amplifier. A dependent source is represented by a diamond-shaped symbol as shown in the figures below. There are four possible dependent sources :

(i) Voltage-dependent voltage source (ii) Current-dependent voltage source

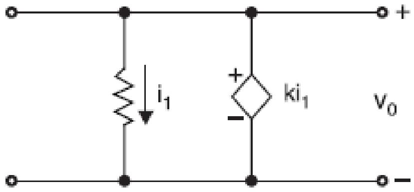
(iii) Voltage-dependent current source (iv) Current-dependent current source

**(i) Voltage-dependent voltage source**. A voltage-dependent voltage source is one whose output voltage ($V_o$) depends upon or is controlled by an input voltage ($V_1$).

**(ii) Current-dependent voltage source.** A current-dependent voltage source is one whose output voltage ($i_0$) depends on or is controlled by an input current ($i_1$).



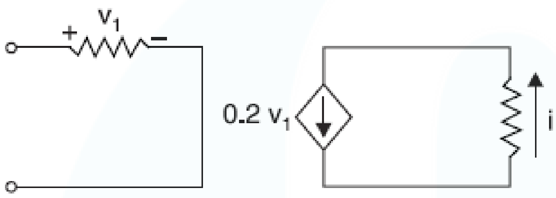
A voltage-dependent voltage source
(i)



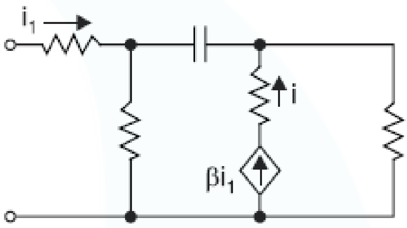
A current-dependent voltage source
(ii)

**(iii) Voltage-dependent current source**. A voltage-dependent current source is one whose output current (i) depends upon or is controlled by an input voltage ($V_1$).

**(iv) Current-dependent current source.** A current-dependent current source is one whose output current (i) depends upon or is controlled by an input current ($i_1$).



A voltage-dependent current source
(i)



A current-dependent current source
(ii)

## Some Definitions

Passive element : An element which receives energy is called a passive element.

Active element : An element that supplies energy to the circuit is called an active element.

Bilateral element : An element which conducts current in both direction is called a bilateral element.

Unilateral elements : An element or a device in which current conduction is possible only in one direction.

Electrical power : Electric power is the rate, per unit time, at which electrical energy is transferred by an electric circuit. The SI unit of power is the watt. Equation $P = VI = I^2.R$. Unit is watts (W)

Electrical energy : Work done by electrical power is called electrical energy. SI unit is Joules.

$E = VI \times t = (V^2/R).t$. Unit is Joules (J).

Resistance in Series : The circuit in which resistances are connected end to end so that there is only one path for current flow, is called a series circuit.

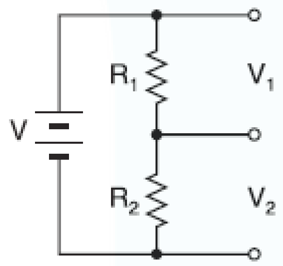
Resistance in parallel : The circuit in which one end of each resistance is joined to a common point and the other the other end of each resistance is joined to another common point so that there are many paths for current flow.

| | Series Connection | Parallel connection |
|---|---|---|
| **Resistor** | $R_{eq} = R_1 + R_2 + R_3$ | $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ |
| **Inductor** | $L_{eq} = L_1 + L_2 + L_3$ | $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$ |
| **Capacitor** | $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ | $C_{eq} = C_1 + C_2 + C_3$ |

**Voltage Division Rule**

$$V_1 = V\frac{R_1}{R_1 + R_2}$$

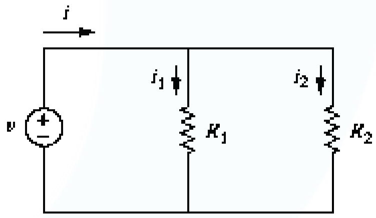
$$V_2 = V\frac{R_2}{R_1 + R_2}$$




**Current Division Rule**

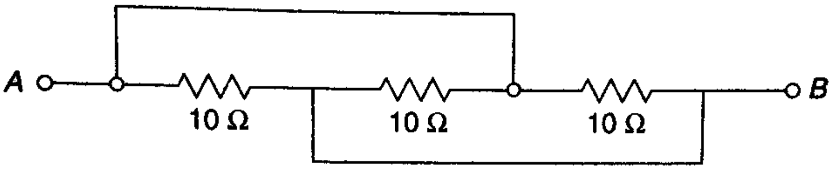
$$I_1 = I\frac{R_2}{R_1 + R_2}$$

$$I_2 = I\frac{R_1}{R_1 + R_2}$$



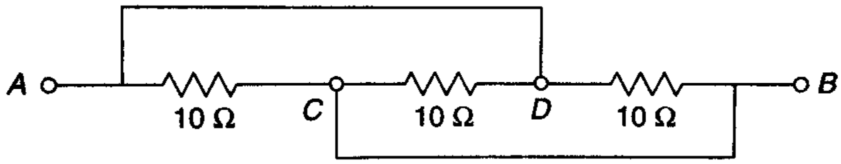

Problem

*Find an equivalent resistance between A and B in the network of Fig. 1.31.*

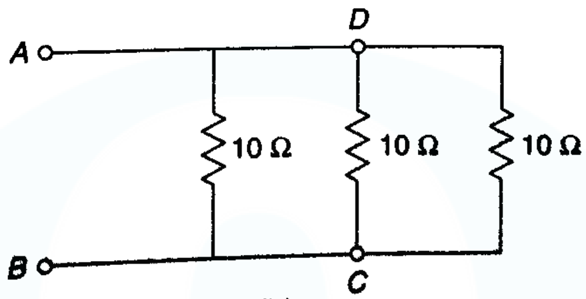


**Solution**

Marking all the junctions and redrawing the network (Fig. 1.32),




(a)

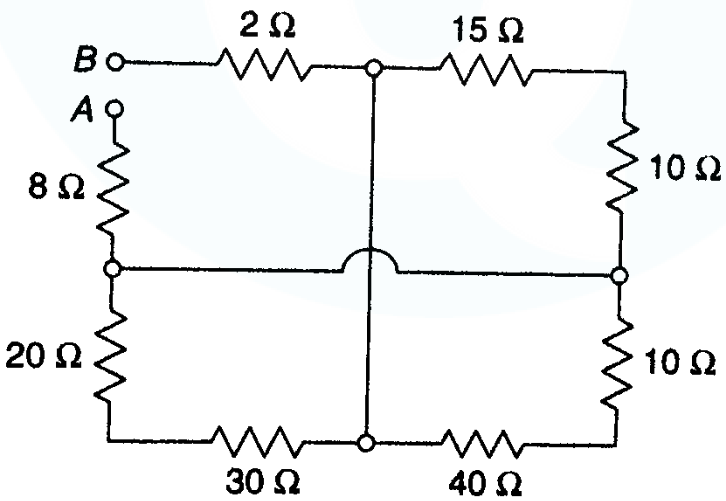



(b)

**Fig. 1.32**

$$R_{AB} = 10 \| 10 \| 10 = 3.33\ \Omega$$

*Find the equivalent resistance between A and B in the network of Fig.*



**Solution** Marking all the junctions and redrawing the network (Fig. 1.36),

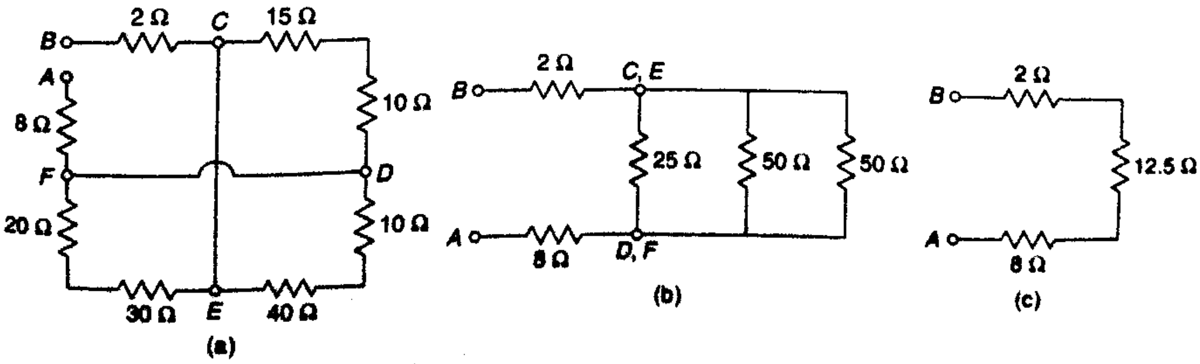
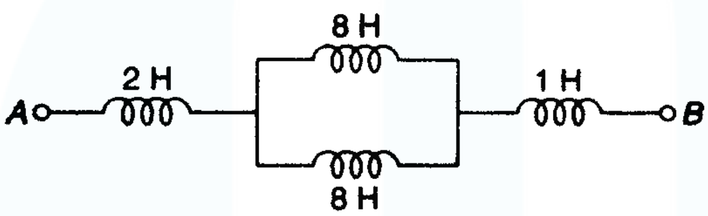



**Fig. 1.36**

$R_{AB} = 22.5\ \Omega$

*Find an equivalent inductance between terminals A and B in the network of Fig.*




**Solution** The network can be simplified by series–parallel reduction technique (Fig. 1.56).

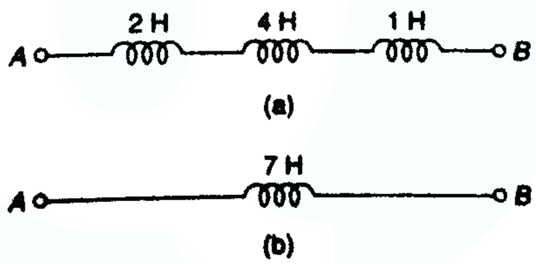



**Fig. 1.56**

$L_{AB} = 7\ \text{H}$

***Find an equivalent inductance of the network shown in Fig 1.57.***

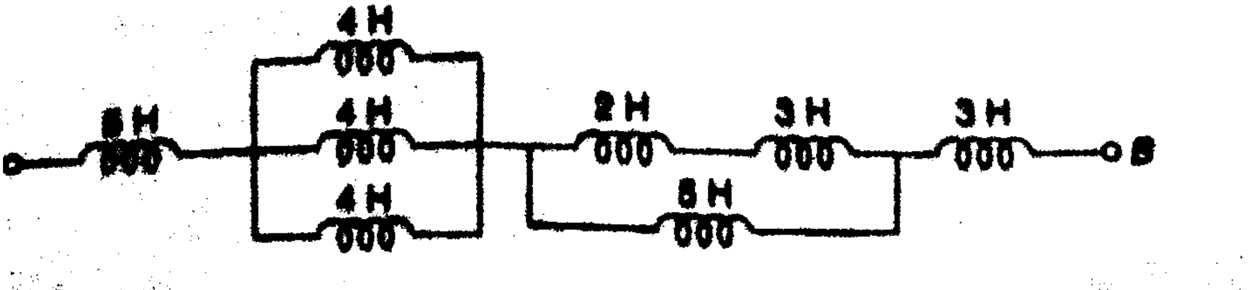



**Fig. 1.57**

The network can be simplified by series–parallel reduction technique (Fig. 1.58).

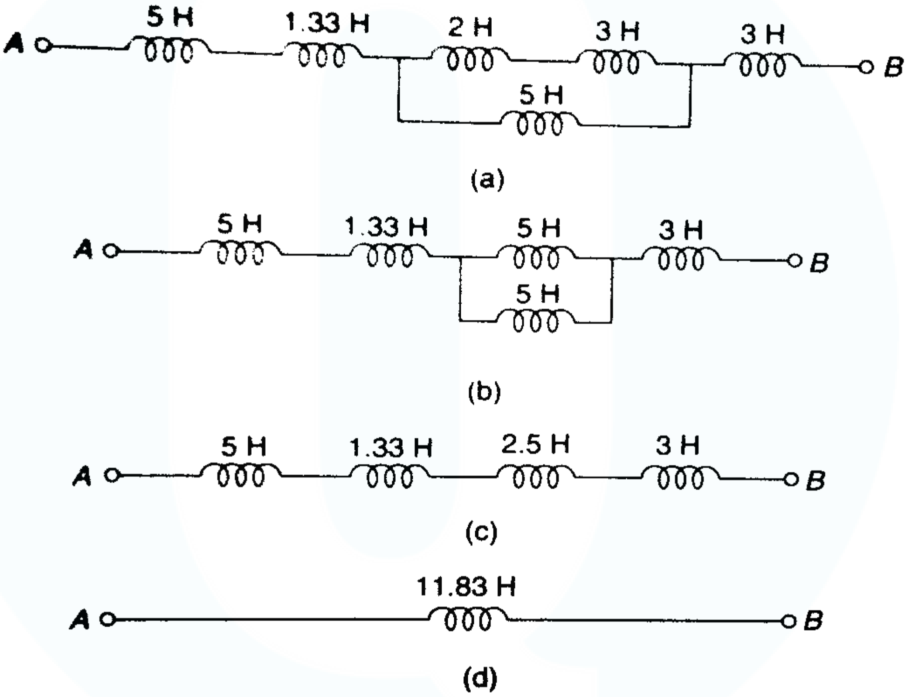
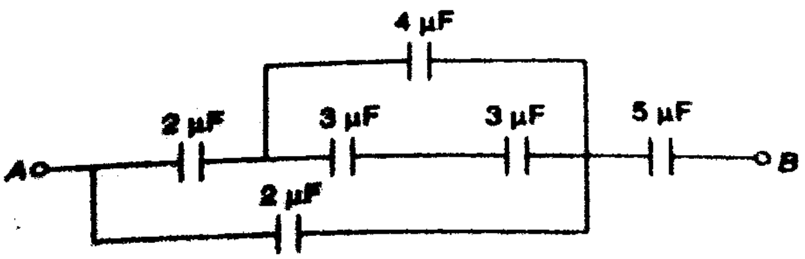



**Fig. 1.58**

$$L_{AB} = 11.83 \text{ H}$$

***Find the equivalent capacitance for the network of Fig. 1.63***



The network can be simplified by series–parallel reduction technique (Fig. 1.64).

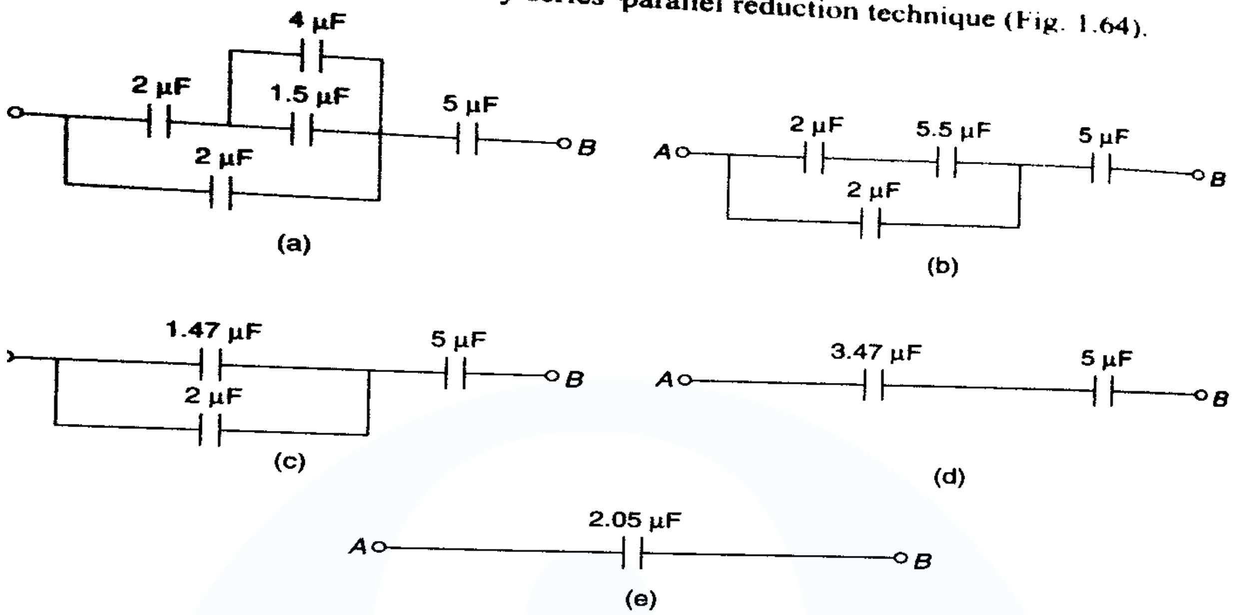



**Fig. 1.64**

$C_{AB} = 2.05\ \mu F$

**Example 1.29** *What is the equivalent capacitance between terminals A and B in the network of Fig. 1.61.*

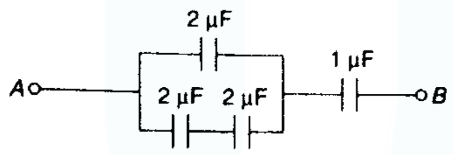



**Fig. 1.61**

**Solution** The network can be simplified by series–parallel reduction technique (Fig. 1.62).

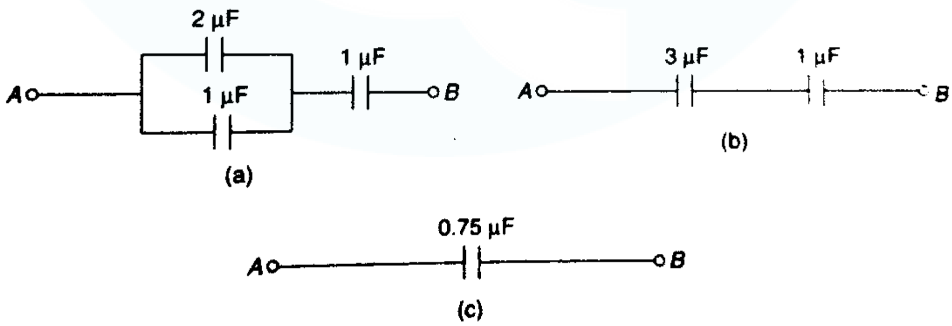



**Fig. 1.62**

$C_{AB} = 0.75\ \mu F$

## Kirchoffs Law

Kirchhoff gave two laws to solve complex circuits, namely ;

1. Kirchhoff's Current Law ( KCL ) 2. Kirchhoff's Voltage Law (KVL)

### Kirchhoff's current law:

Kirchhoff's current law states that at any instant of time, the algebraic sum of current at a node is zero. Mathematically,

$$\sum_{n=1}^{N} i_n = 0.$$

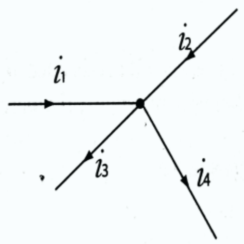
Where N is the number of branches connected to the node and $i_n$ is the $n$th current entering (or leaving) the node.

<u>Illustration of KCL</u>

From the Figure

$$i_1 + i_2 - i_3 - i_4 = 0$$

ie., $$i_1 + i_2 = i_3 - i_4$$




ie., Current entering = Current leaving

### Kirchhoff's voltage law:

Kirchhoff's voltage law states that the algebraic sum of voltages around a closed path at any instant of time is zero. Mathematically,

$$\sum_{m=1}^{M} v_m = 0.$$

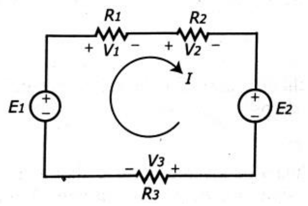
Where m is the number of voltages in the loop (or number of branches in the loop) and $v_m$ is the $m$th voltage.

<u>Illustration of KVL</u>

From the Figure

$$E_1 - V_1 - V_2 - E_2 - V_3 = 0$$

$$E_1 = V_1 + V_2 + E_2 + V_3$$

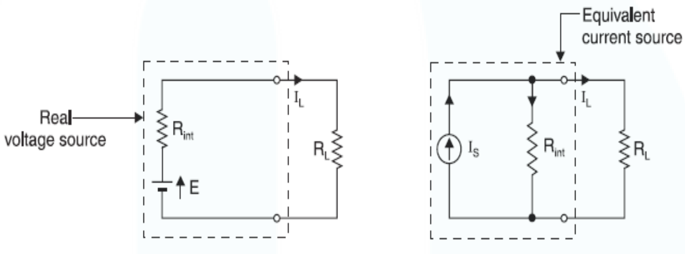


## Source transformation

A real voltage source can be converted to an equivalent real current source and vice-versa. When the conversion is made, the sources are equivalent in every sense of the word; it is impossible to make any measurement or perform any test at the external terminals that would reveal whether the source is a voltage source or its equivalent current source.

## Voltage to current source conversion

Let us see how a real voltage source can be converted to an equivalent current source. We know that a real voltage source can be represented by constant voltage E in series with its internal resistance $R_{int}$ as shown in Figure (i).




It is clear from Fig. (i) that load current $I_L$ is given by

$$I_L = I_s * \frac{R_{int}}{R_{int}+R_L}$$

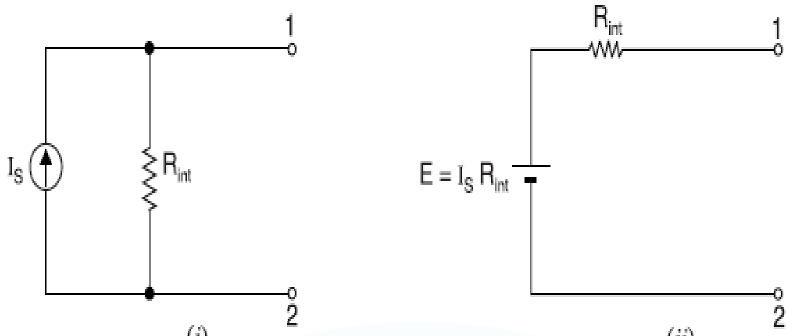
$$I_s = \frac{E}{R_{int}}$$

= Current which would flow in a short circuit across the output terminals of voltage source in Fig (i)

***Thus a real voltage source of constant voltage E and internal resistance $R_{int}$ is equivalent to a current source of current $I_s = \frac{E}{R_{int}}$ and $R_{int}$ in parallel with current source.***

## Current to voltage source conversion.

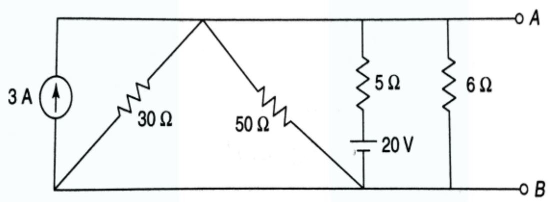
Fig. (i) shows a real current source whereas Fig.(ii) shows its equivalent voltage source. Note that series resistance $R_{int}$ of the voltage source has the same value as the parallel resistance of the original current source. The value of voltage of the equivalent voltage source is E = $I_S$ $R_{int}$ where $I_S$ is the magnitude of current of the current source




(i) (ii)

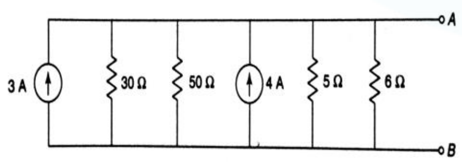
**Problem**

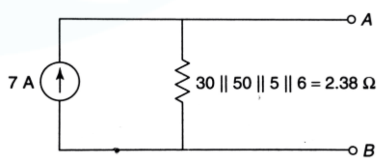
Replace the circuit between A and B in fig. 2.2 with a voltage source in series with a single resistor.

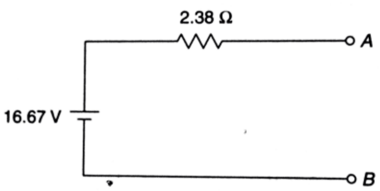



**Solution**

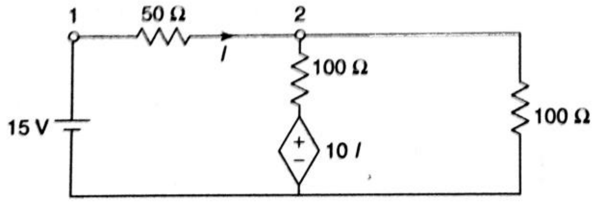
Converting series combination of voltage source of 20 V and the series resistor of 5 Ω into equivalent current source and a parallel resistance as shown in figure. Adding two current sources and simplifying the circuit as shown in figure



1. By source transformation, find the voltage at node 2 of the network shown in figure.




**Solution** We cannot change the network between nodes 1 and 2 since the controlling current $I$, for the controlled source, is in the resistor between these nodes. Applying source transformation to series combination of controlled source and the 100 Ω resistor (Fig. 1.161),

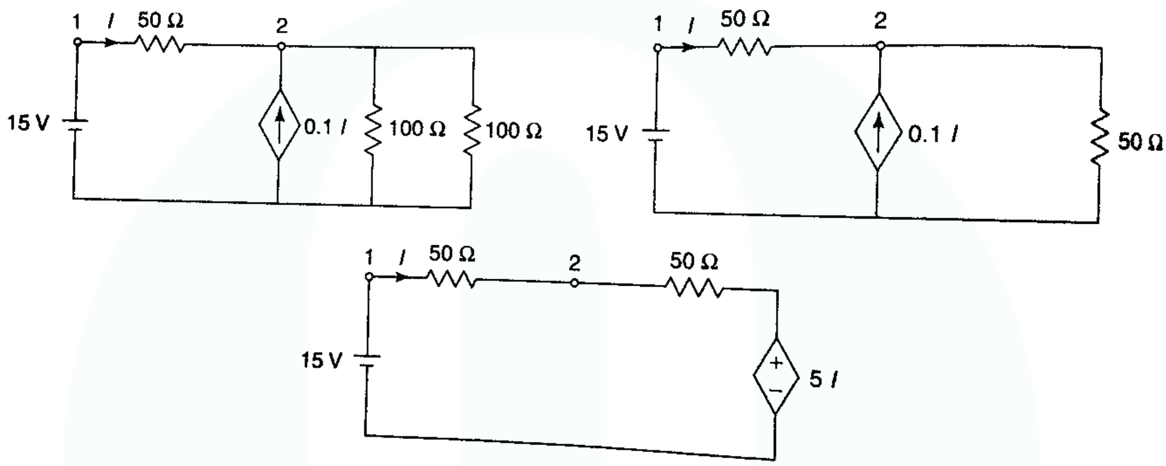



**Fig. 1.161**

Applying KVL to the mesh,

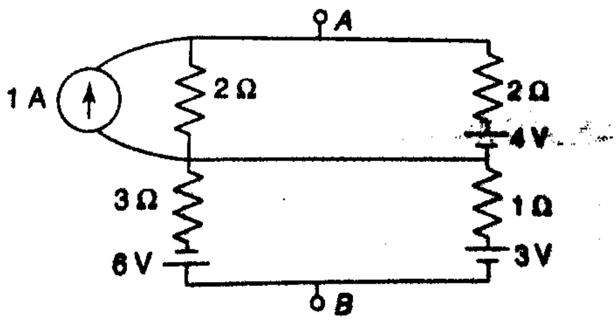
$$15 - 50I - 50I - 5I = 0$$

$$I = \frac{15}{105} = 0.143 \text{ A}$$

Voltage at Node 2 $= 15 - 50I = 15 - 50 \times 0.143 = 7.86$ V

## Problem

Reduce the given network into a single source and single resistor between terminals A and B




**Solution**

**Solution** Converting all voltage sources into equivalent current sources (Fig. 1.1

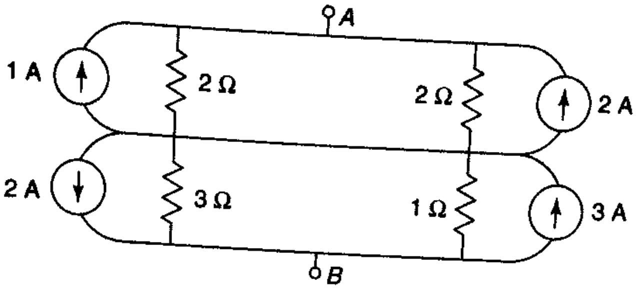



**Fig. 1.140**

Adding the current sources and simplifying the network (Fig. 1.141),

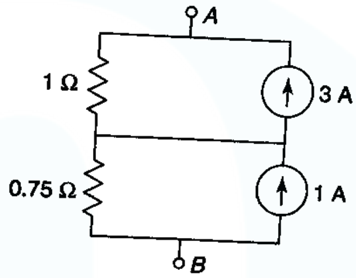
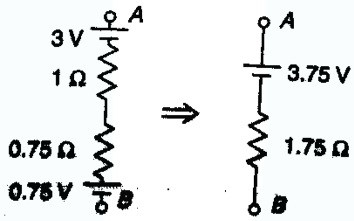



**Fig. 1.141**

Converting the current sources into equivalent voltage sources (Fig. 1.142),




## Network Terminology

I. ***Node.*** A node of a network is an equipotential surface at which two or more circuit elements are joined. Thus in Fig, circuit elements R1 and E1 are joined at A and hence A is the node. Similarly, B, C and D are nodes.

II. ***Junction.*** A junction is that point in a network where three or more circuit elements are joined. In Fig, there are only two junction points viz. B and D. That B is a junction is clear from the fact that three circuit elements R1, R2 and R3 are joined at it. Similarly, point D is a junction because it joins three circuit elements R2, E1 and E2.

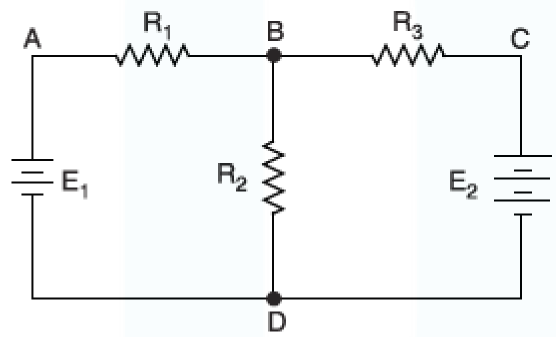
III. ***Branch***. A branch is that part of a network which lies between two junction points. Thus referring to Fig, there are a total of three branches viz. BAD, BCD and BD. The branch

BAD consists of R1 and E1 ; the branch BCD consists of R3 and E2 and branch BD merely consists of R2.

IV. ***Loop.*** A loop is any closed path of a network. Thus in Fig, ABDA, BCDB and ABCDA are the loops.

V. ***Mesh.*** A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig, both loops ABDA and BCDB qualify as meshes because they cannot be further divided into other loops. However, the loop ABCDA cannot be called a mesh because it encloses two loops ABDA and BCDB.

VI. ***Network and circuit.*** Strictly speaking, the term network is used for a circuit containing passive elements only while the term circuit implies the presence of both active and passive elements. However, there is no hard and fast rule for making these distinctions and the terms “network” and “circuit” are often used interchangeably.




## Nodal Analysis

Steps to Determine Node Voltages:

1. Identify the number of nodes
2. Select a node as the reference node.
3. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining (n-1) nodes. The voltages are referenced with respect to the reference node.
4. Apply KCL to each of the (n-1) non reference nodes.
5. Use Ohm's law to express the branch currents in terms of node voltages.
6. Solve the resulting simultaneous equations to obtain the unknown node voltages.

## Example 3.1

Calculate the node voltages in the circuit shown in Fig. 3.3(a).

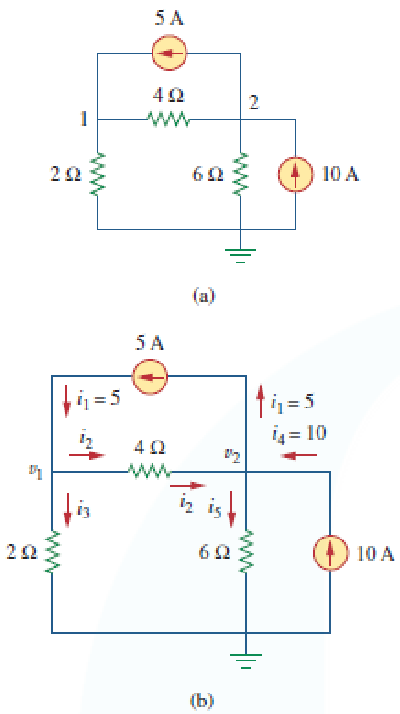



**Figure 3.3**
For Example 3.1: (a) original circuit, (b) circuit for analysis.

**Solution:**
Consider Fig. 3.3(b), where the circuit in Fig. 3.3(a) has been prepared for nodal analysis. Notice how the currents are selected for the application of KCL. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that $i_2$ enters the 4-Ω resistor from the left-hand side, $i_2$ must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages $v_1$ and $v_2$ are now to be determined.

At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4, we obtain

$$20 = v_1 - v_2 + 2v_1$$

or

$$3v_1 - v_2 = 20 \tag{3.1.1}$$

At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

or

$$-3v_1 + 5v_2 = 60 \tag{3.1.2}$$

Now we have two simultaneous Eqs. (3.1.1) and (3.1.2). We can solve the equations using any method and obtain the values of $v_1$ and $v_2$.

**METHOD 1** Using the elimination technique, we add Eqs. (3.1.1) and (3.1.2).

$$4v_2 = 80 \quad \Rightarrow \quad v_2 = 20 \text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \quad \Rightarrow \quad v_1 = \frac{40}{3} = 13.333 \text{ V}$$

**METHOD 2** To use Cramer's rule, we need to put Eqs. (3.1.1) and (3.1.2) in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \tag{3.1.3}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain $v_1$ and $v_2$ as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20 \text{ V}$$

giving us the same result as did the elimination method.

If we need the currents, we can easily calculate them from the values of the nodal voltages.

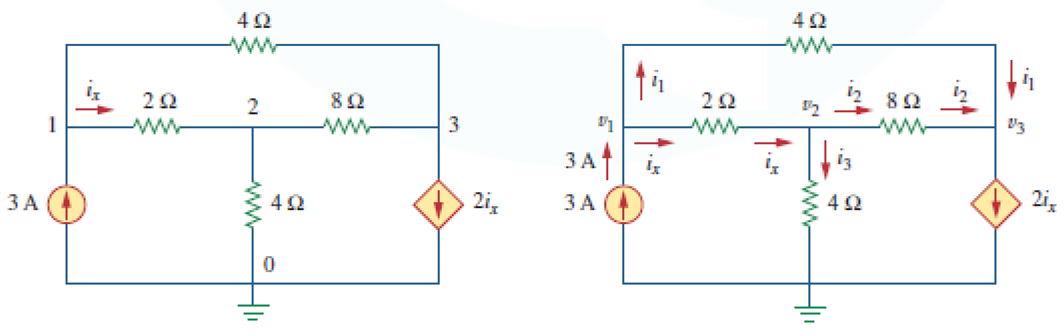
$$i_1 = 5 \text{ A}, \qquad i_2 = \frac{v_1 - v_2}{4} = -1.6668 \text{ A}, \qquad i_3 = \frac{v_1}{2} = 6.666 \text{ A}$$

$$i_4 = 10 \text{ A}, \qquad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

The fact that $i_2$ is negative shows that the current flows in the direction opposite to the one assumed.

## Problem using dependent sourcse

Determine the node voltages shown in Figure (a)




(a) (b)

## Solution

These circuits have four nodes, one node taken as reference node. We assign voltages to other three nodes as shown in Figure (b) and label the currents

At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \tag{3.2.1}$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \tag{3.2.2}$$

At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0 \tag{3.2.3}$$

Using the elimination technique, we add Eqs. (3.2.1) and (3.2.3).

$$5v_1 - 5v_2 = 12$$

or

$$v_1 - v_2 = \frac{12}{5} = 2.4 \tag{3.2.4}$$

Adding Eqs. (3.2.2) and (3.2.3) gives

$$-2v_1 + 4v_2 = 0 \quad \Rightarrow \quad v_1 = 2v_2 \tag{3.2.5}$$

Substituting Eq. (3.2.5) into Eq. (3.2.4) yields

$$2v_2 - v_2 = 2.4 \quad \Rightarrow \quad v_2 = 2.4, \quad v_1 = 2v_2 = 4.8 \text{ V}$$

From Eq. (3.2.3), we get

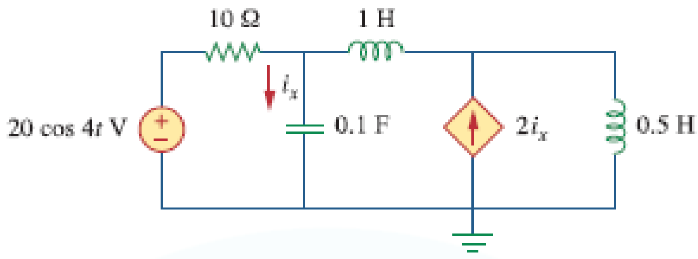
$$v_3 = 3v_2 - 2v_1 = 3v_2 - 4v_2 = -v_2 = -2.4 \text{ V}$$

Thus,

$$v_1 = 4.8 \text{ V}, \quad v_2 = 2.4 \text{ V}, \quad v_3 = -2.4 \text{ V}$$

## Problem with AC circuits

Find $i_x$ in the circuit of Fig. using nodal analysis.




**Solution:**

We first convert the circuit to the frequency domain:

$$20\cos 4t \quad \Rightarrow \quad 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$
$$1 \text{ H} \quad \Rightarrow \quad j\omega L = j4$$
$$0.5 \text{ H} \quad \Rightarrow \quad j\omega L = j2$$
$$0.1 \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$

Thus, the frequency domain equivalent circuit is as shown in Fig. 10.2.

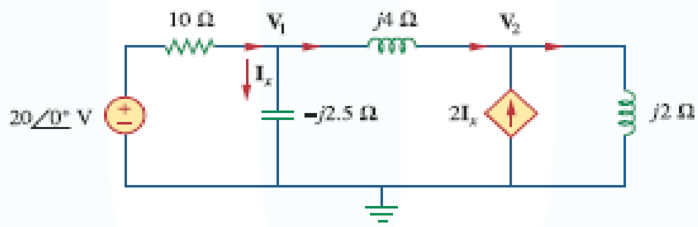



**Figure 10.2**
Frequency domain equivalent of the circuit in Fig. 10.1.

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \qquad \textbf{(10.1.1)}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \qquad \textbf{(10.1.2)}$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

## Nodal Analysis with Voltage Sources

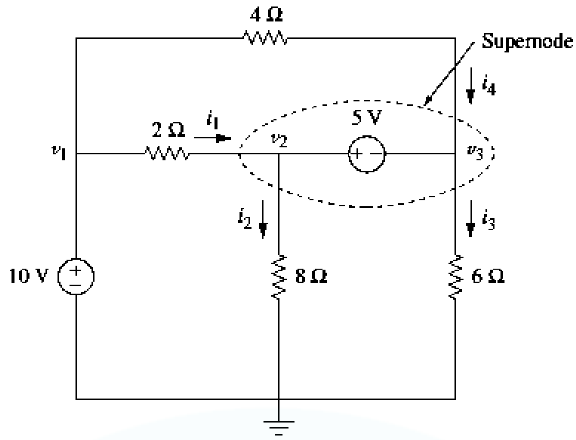
We now consider how voltage sources affect nodal analysis. We use the circuit in Figure for illustration. Consider the following two possibilities.

**CASE 1:** If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source. In Figure, for example, $v_1$ = 10V

Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

**CASE 2**: If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a ***generalized node or super node***; we apply both KCL and KVL to determine the node voltages.

***A super node*** is formed by enclosing a independent or dependent voltage source connected between two non-reference nodes and any element connected parallel with it.




## Problem

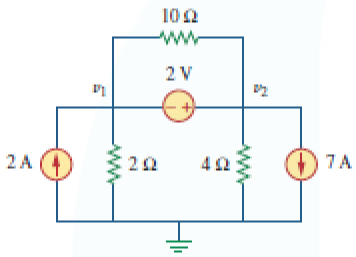



**Figure 3.9**
For Example 3.3.

For the circuit shown in Fig. 3.9, find the node voltages.

**Solution:**
The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying KCL to the supernode as shown in Fig. 3.10(a) gives

$$2 = i_1 + i_2 + 7$$

Expressing $i_1$ and $i_2$ in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow \quad 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \tag{3.3.1}$$

To get the relationship between $v_1$ and $v_2$, we apply KVL to the circuit in Fig. 3.10(b). Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \quad \Rightarrow \quad v_2 = v_1 + 2 \tag{3.3.2}$$

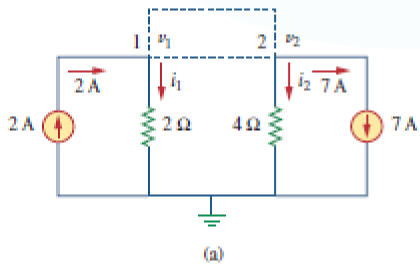
From Eqs. (3.3.1) and (3.3.2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Rightarrow \quad v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.



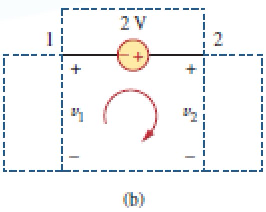




**Figure 3.10**
Applying: (a) KCL to the supernode, (b) KVL to the loop.

Problem

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.

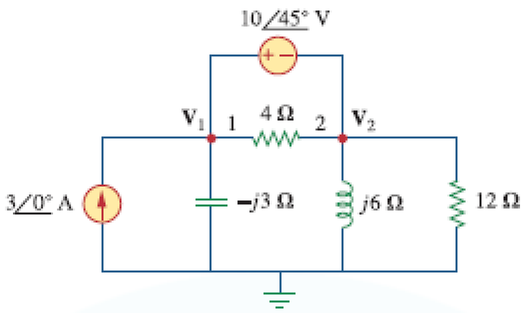



**Figure 10.4**
For Example 10.2.

**Solution:**
Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \qquad \textbf{(10.2.1)}$$

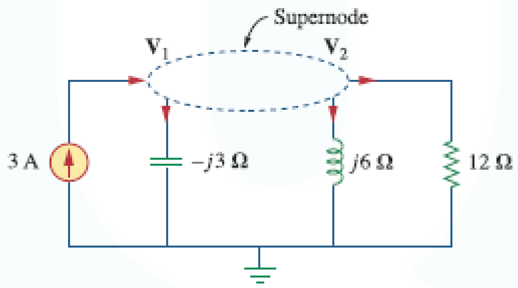



**Figure 10.5**
A supernode in the circuit of Fig. 10.4.

But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} \qquad \textbf{(10.2.2)}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\underline{/135^\circ} = (1 + j2)\mathbf{V}_2 \qquad \Rightarrow \qquad \mathbf{V}_2 = 31.41\underline{/-87.18^\circ}\text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} = 25.78\underline{/-70.48^\circ}\text{ V}$$

## Mesh Analysis

Steps to Determine Mesh Currents:

1. Identify the meshes, n
2. Assign mesh currents $i_1, i_2, \ldots i_n$ in to the n meshes.
3. Apply KVL to each of the n meshes.
4. Use Ohm's law to express the voltages in terms of the mesh currents.
5. Solve the resulting n simultaneous equations to get the mesh currents.

Problem

For the circuit in Fig. 3.18, find the branch currents $I_1$, $I_2$, and $I_3$ using mesh analysis.

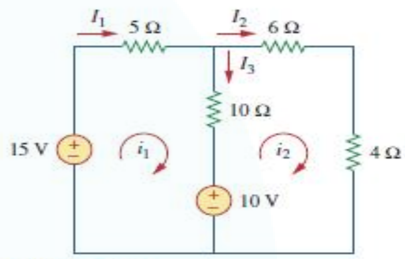



**Figure 3.18**

**Solution:**
We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad (3.5.1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \qquad (3.5.2)$$

**METHOD 1** Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \quad \Rightarrow \quad i_2 = 1 \text{ A}$$

From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

**METHOD 2** To use Cramer's rule, we cast Eqs. (3.5.1) and (3.5.2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

Problem with dependent sources

Use mesh analysis to find the current $I_o$ in the circuit of Fig. 3.20.

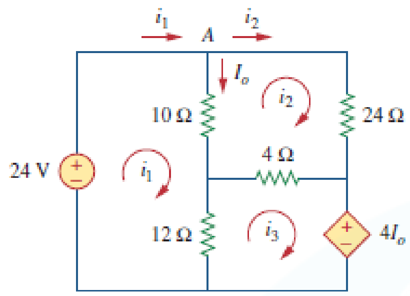



**Figure 3.20**

**Solution:**

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (3.6.1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (3.6.2)$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3.6.3)$$

In matrix form, Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

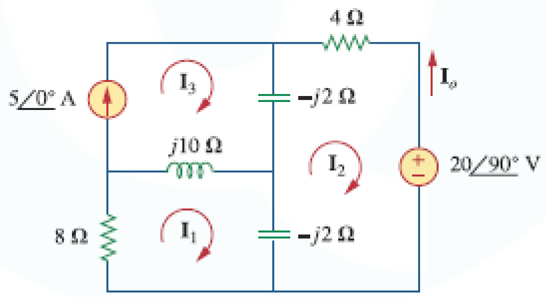
$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

**Problem with AC circuits**

Determine the current Io using mesh analysis




**Solution:**

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad \textbf{(10.3.1)}$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0 \qquad \textbf{(10.3.2)}$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad \textbf{(10.3.3)}$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad \textbf{(10.3.4)}$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22^\circ}}{68} = 6.12\underline{/-35.22^\circ} \text{ A}$$

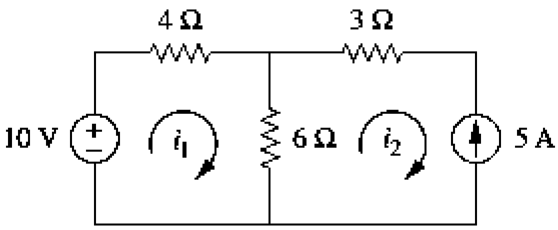
The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78^\circ} \text{ A}$$

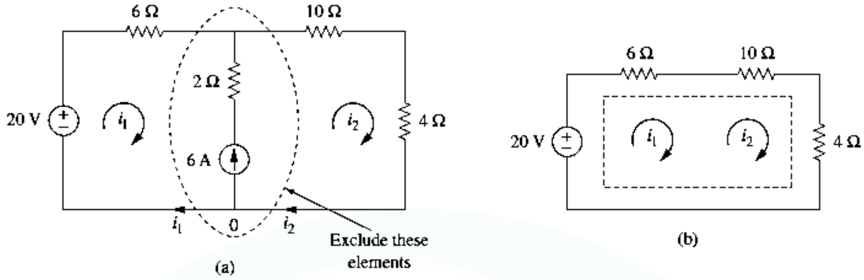
## Mesh Analysis with Current Sources

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

**CASE 1**: When a current source exists only in one mesh: Consider the circuit in Figure, for example. We set $i_2$ = -5 A and write a mesh equation for the other mesh in the usual way; that is,



**CASE 2:** When a current source exists between two meshes: Consider the circuit in Figure (a), for example. We create a ***super mesh*** by excluding the current source and any elements connected in series with it, as shown in Figure (b).




A ***supermesh*** results when two meshes have a (dependent or independent) current source in common.

## Problem

For the circuit in Fig. 3.24, find $i_1$ to $i_4$ using mesh analysis.

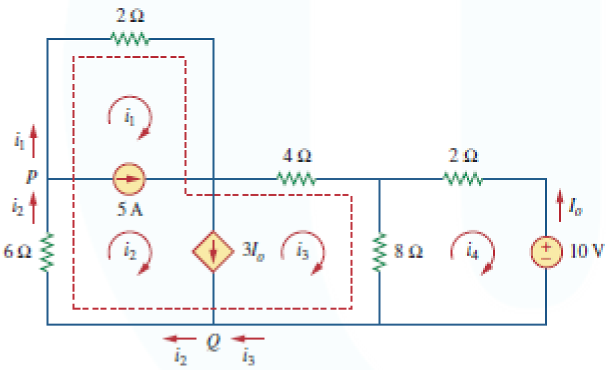



**Figure 3.24**
For Example 3.7.

**Solution:**
Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \qquad (3.7.1)$$

For the independent current source, we apply KCL to node $P$:

$$i_2 = i_1 + 5 \qquad (3.7.2)$$

For the dependent current source, we apply KCL to node $Q$:

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \tag{3.7.3}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

$$5i_4 - 4i_3 = -5 \tag{3.7.4}$$

From Eqs. (3.7.1) to (3.7.4),

$$i_1 = -7.5\text{ A}, \quad i_2 = -2.5\text{ A}, \quad i_3 = 3.93\text{ A}, \quad i_4 = 2.143\text{ A}$$

**Problem with AC circuits**

Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.

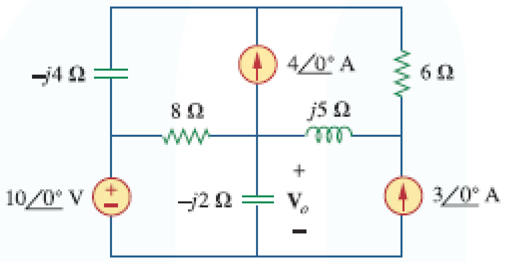



**Figure 10.9**
For Example 10.4.

**Solution:**
As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \tag{10.4.1}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \tag{10.4.2}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \tag{10.4.3}$$

Due to the current source between meshes 3 and 4, at node $A$,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \tag{10.4.4}$$

Instead of solving the above four equations, we reduce them to two by elimination.

Combining Eqs. (10.4.1) and (10.4.2),

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6 \qquad \textbf{(10.4.5)}$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35 \qquad \textbf{(10.4.6)}$$

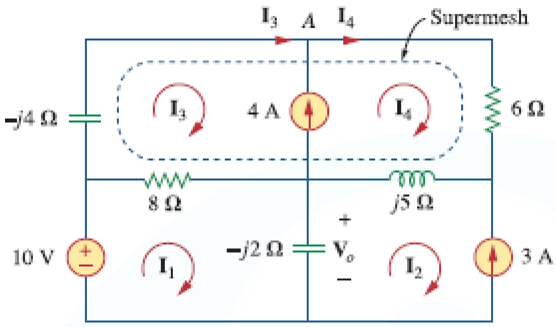



Figure 10.10

From Eqs. (10.4.5) and (10.4.6), we obtain the matrix equation

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_0$ is

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618\underline{/274.5^\circ} + 3)$$

$$= -7.2134 - j6.568 = 9.756\underline{/222.32^\circ} \text{ V}$$

https://www.youtube.com/channel/UClorWwmi7GhAt4W9I2z04gA