EST 130 A
SLOT D1

BASIC ELECTRICAL ENGINEERING

MODULE 1- SYLLABUS

- Elementary Concepts of Electric & Magnetic Circuits
- Elementary concepts of DC electric circuits: Basic Terminology including voltage, current, power, resistance, emf; Resistances in series and parallel; Current and Voltage Division Rules; Capacitors & Inductors: V-I relations and energy stored. Ohms Law and Kirchhoff's laws-Problems; Star-delta conversion (resistive networks only-derivation not required)-problems.
- Analysis of DC electric circuits: Mesh current method - Matrix representation - Solution of network equations. Node voltage methods-matrix representation-solution of network equations by matrix methods. Numerical problems.

Electric Circuit

- The electric circuits are closed path which forms a network of electrical components, where electrons are able to flow.
- Source (battery , generator)
- Conductors
- Load (lamp , heater , motor)

components

- D.C Circuit –closed path followed by direct current
- A.C Circuit - closed path followed by alternating current

- Electric circuit
  - D.C circuit
  - A.C circuit

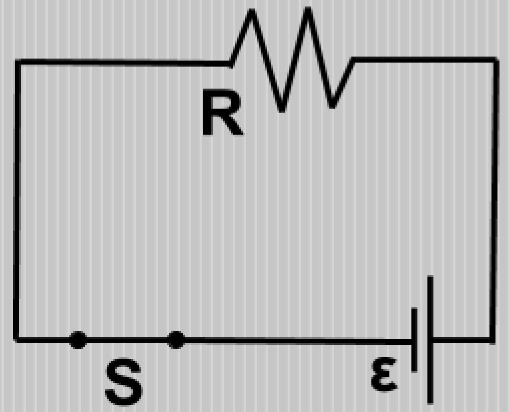
D.C circuit

- Closed path followed by a direct current
- Dc source
- Conductors to carry current
- Load
- It starts from positive terminal of the battery and comes back to the starting point via the load.

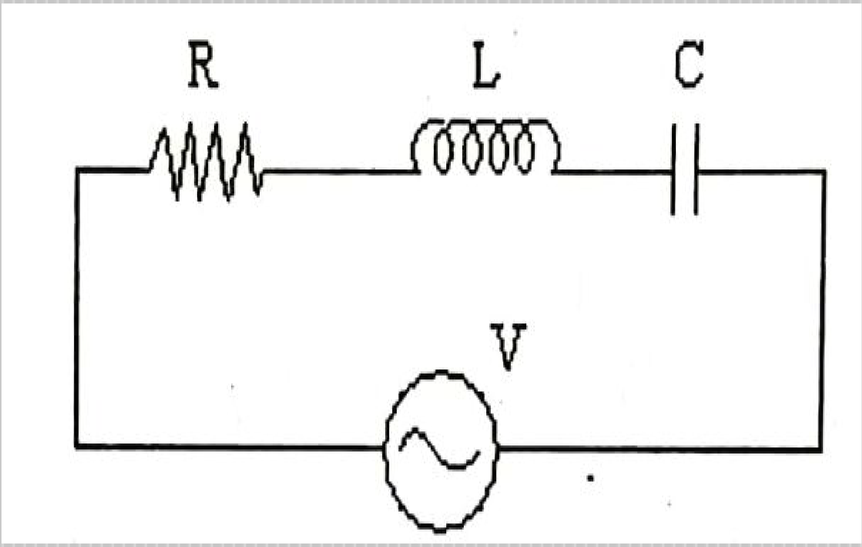
- The current flows in only one direction.
- It is mostly used in low voltage applications.
- The resistor is the main component of the DC circuit.

A.C circuit

- Closed path followed by an alternating current
- Components are resistors, inductors , capacitors.



D.C circuit

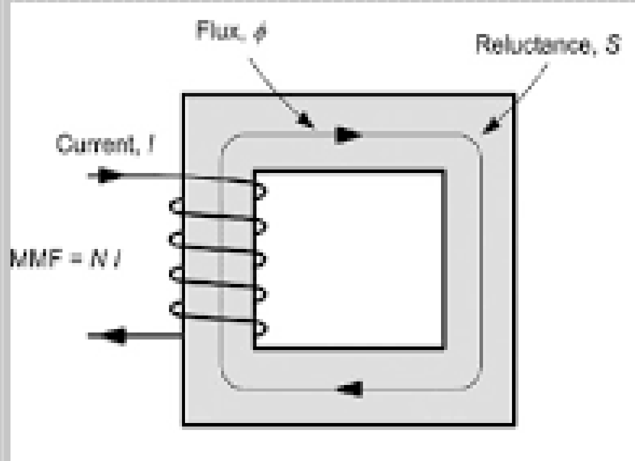


A.C circuit

Magnetic Circuit

- The closed path followed by magnetic flux is called magnetic circuit
- In a magnetic circuit , magnetic flux leaves the N-Pole , passes through the entire circuit and returns to the starting point.
- High permeability materials-iron , soft steel
- Small opposition to flow of flux.
- Magnetic flux is produced by passing an electric current through a wire wound over a magnetic material.

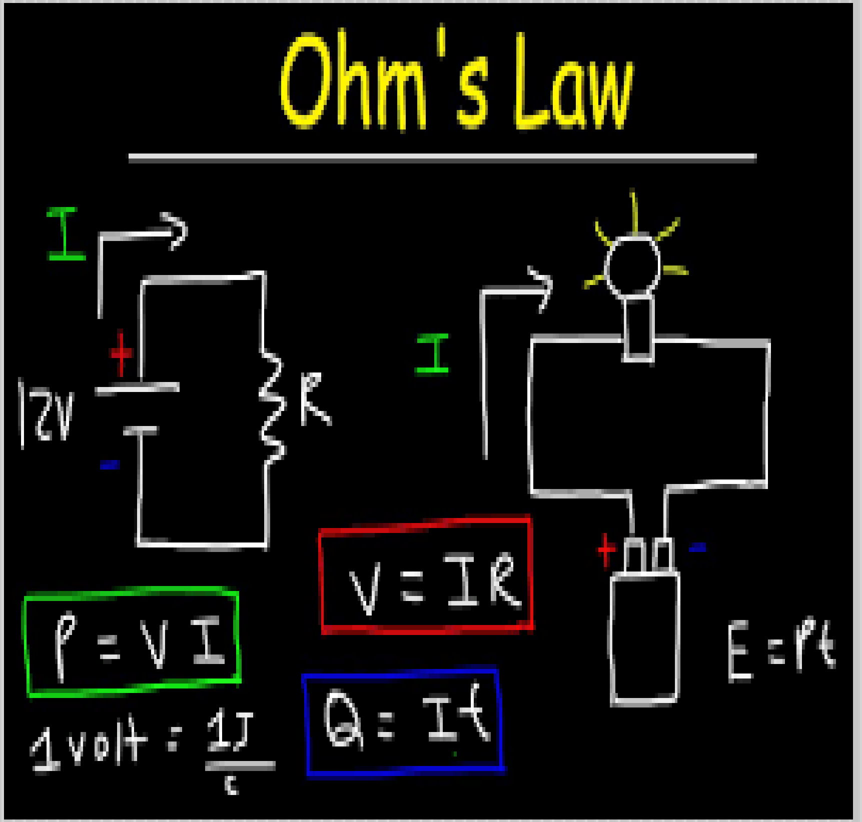
- Press a doorbell, for example, and electric current creates a magnetic field that attracts a ringer which strikes the bell.



A magnetic circuit

Basic Terminologies

- Voltage
- Current
- Emf
- Resistance
- Power
- Energy



ELECTRIC POTENTIAL(V)

- **Voltage** is the pressure from an electrical circuit's power source that pushes charged electrons (current) through a conducting loop, enabling them to do work such as illuminating a light.
- Voltage is symbolized by an uppercase italic letter *V*
- The standard unit is the volt.

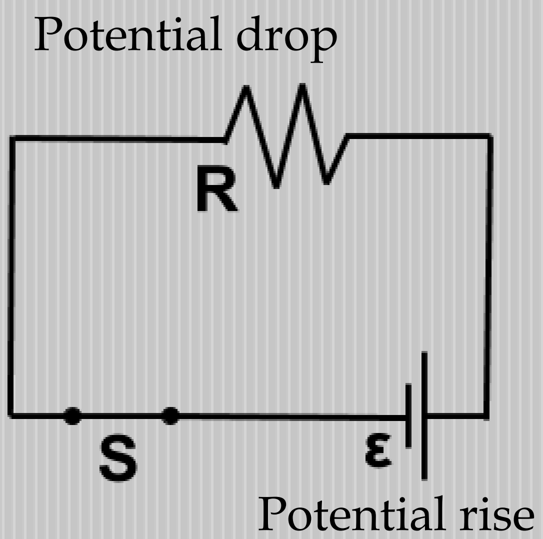
$$\frac{Fd}{q} = \frac{W}{q} = \Delta V$$

$$\frac{\mathrm{N\,m}}{\mathrm{C}} = \frac{\mathrm{Joule}}{\mathrm{C}} = \mathrm{Volts}$$

POTENTIAL DIFFERENCE /EMF

- Potential difference defined as the potential energy difference between two points (charged bodies) in a circuit.
- The amount of difference (expressed in volts) determines how much potential energy exists to move electrons from one specific point to another

- Pd causes current to flow
- Emf maintains the potential difference
- pd maintains the flow of current in a circuit.
- Emf is denoted as E
- Unit is Volt.



- Potential difference across cell is voltage rise
- Potential difference across resistor is voltage drop

CURRENT

- flow of charge (free electrons) within a conductor or how fast charge is moving.
- Direction is from negative to positive terminal

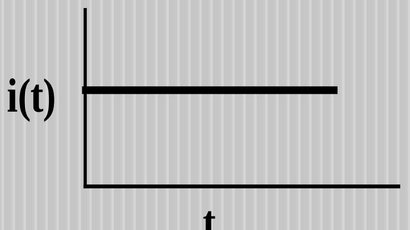
- Charge will only flow if there is a voltage source (potential difference).
- **Symbol for Current** = I
- **Unit for Current** = Amps (A) or Coulombs/sec

- Electricity represents the follow of electric current.
- I = Q/t= ne/t
- n= no.of free electrons
- e- charge of an electron

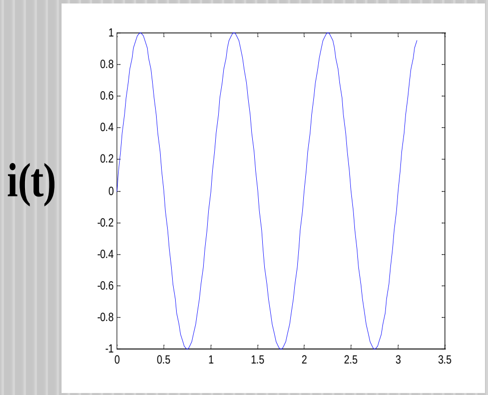
$$I = neAV_d$$

A- area of cross section of wire
V_d- drift velocity of free electrons

- We normally refer to current as being either <u>direct</u> (dc) or
- <u>alternating</u> (ac).
- A closed path is required for the flow of current.



dc current



ac current

- If the voltage in a circuit increases, the current will increase.
- If the voltage in a circuit decreases, the current will decrease.
- This is a ***direct/proportional*** relationship.

Ohm's Law

- Current through a conductor is directly proportional to voltage applied across it's end provided the temperature and all other factors remain constant

$$I \propto V$$

$$I = \frac{A}{\rho}\frac{dV}{dl}$$

$$I = \frac{A}{\rho}\frac{V}{L}$$

$$\frac{V}{I} = R$$

$$\frac{V}{R} = I$$

$$V = IR$$

Voltage, measured in Volts, ***V***

Current, measured in Amps, ***A***

$$V=IR$$

Resistance, measured in Ohms, Ω

Resistance

- **Resistance:** Opposition offered by a substance to the flow of electric current.
- Substance offered high opposition to flow of electrons-insulators-glass , rubber , mica etc.
- Substance offered little opposition to flow of electrons-conductors-silver , copper , aluminium etc.
- **Symbol for Resistance** = R
- **Unit for Resistance** = Ohms (Ω)

- Resistance is the electric friction offered by the substance and causes the production of heat with the flow of electric current.

$$R \propto \frac{L}{A}$$

L- length of conductor in metres (m),
A- area of the conductor in square metres (m^2)

$$R = \rho\left(\frac{L}{A}\right)\Omega$$

the proportional constant ρ (the Greek letter “rho”) is known as **Resistivity**.

- Resistance depends upon
  - Length
  - Area
  - Nature of material
  - Temperature

$$\frac{V}{I} = R$$

- If the resistance in a circuit increases, the current will decrease.
- If the resistance in a circuit decreases, the current will increase.
- This is an ***inversely proportional*** relationship.

Specific resistance / resistivity

- The electrical resistivity of a particular conductor material is a measure of how strongly the material opposes the flow of electric current through it.
- Specific resistance of a material is the resistance offered by 1m length of a wire of material having an area of cross section of $1m^2$

$$\rho = \frac{R \times A}{L} = \frac{\text{ohms} \times \text{meters}^2}{\text{meters}} = \Omega.\text{m}$$

Conductance

- Reciprocal of resistance of a conductor is called conductance

$$G = \frac{1}{R}$$

- Unit – mho or Siemen

$$G = \frac{A}{\rho L}$$

Conductivity

- Reciprocal of resistivity
- Symbol –σ
- Unit- mho/meter
- Siemen/ meter
- **Conductivity**, or specific conductance relates to the ease at which electric current can flow through a material.

ELECTRIC POWER

- The rate at which work is done in an electric circuit is called electric power.
- It is the power consumed by resistor R
- Unit : joules/sec or watt or kilowatt

$$P = \frac{W}{t}$$

P=V*I

$P=V^2/R$

$$W = V \times Q$$

$P=I^2R$

$$Q = It$$

Electrical Energy

- Total work done in an electric circuit is called electrical energy.

Electric Energy = Electrical Power x time

$$VIt = I^2Rt = \frac{V^2}{R}t$$

Series and Parallel Circuits

- In order for electricity to flow we need
  - Power source
  - Closed circuit
- There are two type of circuits we will explore
  - Series circuit
  - Parallel circuit

Series Circuit

- In a series circuit there is only one path for the electrons to flow
  - In other words all the components are in series with each other

- Because there is only one path each charge will go through each resistor

- Since there is only one path for current. Current through all resistors is same.

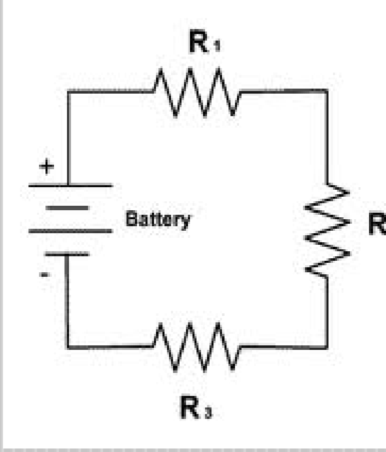
- Each component has resistance that causes a drop in voltage (reduction in voltage).

- Total Voltage = The sum of voltages across each series resistors

$$V_T = V_1 + V_2 + V_3 \ldots$$

- Total Resistance = Sum of all resistors series

$$R_{eq} = R_1 + R_2 + R_3 \ldots$$



- If there are n resistors of same value,

$$R_{eq} = nR$$

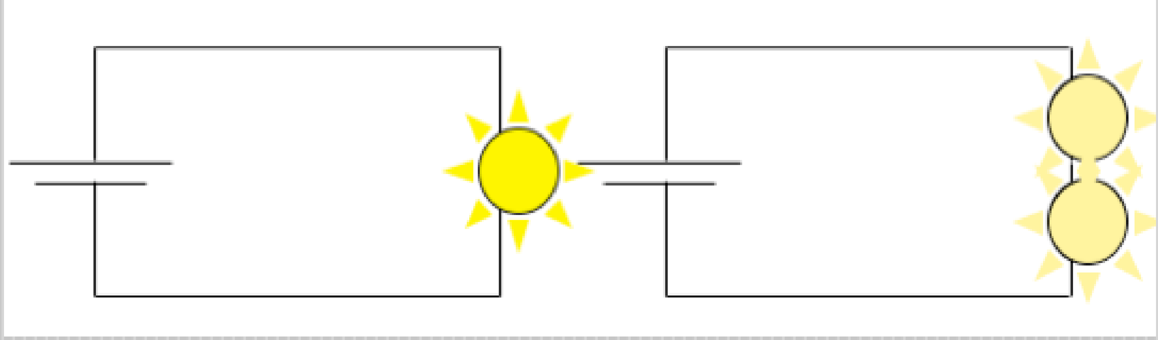
The resistor with the biggest resistance has the greatest voltage.
The equivalent resistance Req, is always more than any resistor in the series

- More components = more resistance
- Increase resistance = decrease current (flow)
- Less current = less bright bulbs

- The current flowing through all components **connected in series is same** while **voltage across each component is different**.
- Series circuit of resistance is also called as **Voltage Divider Circuit.**

Problems with Series:

- The more devices (resistors) in a series circuit, the less current passes through (dimmer bulbs).



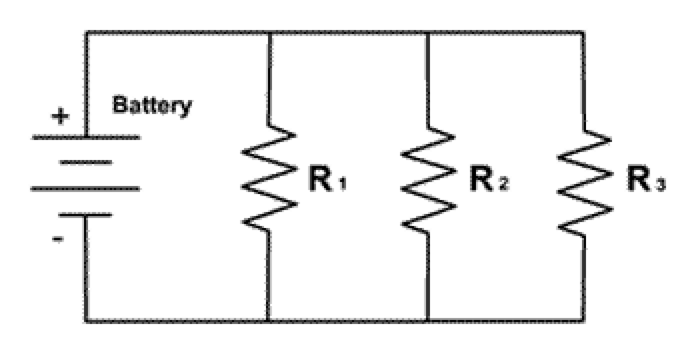
- If one resistor breaks (a bulb goes out) the entire series is turned off.
- Eg: christmas lights- low voltage lamps are connected in series, battery pack in electric car, table lamp

Parallel Circuit

- In a Parallel circuit there are multiple pathways for charge to flow
  - Each device is placed on it’s own separate branch

- Current goes through each of the branches at the same time
- Total current = sum of current in each path

$$\mathbf{I_T = I_1 + I_2 + \ldots}$$



- Voltage drop across the resistor that it *chooses* to pass through must equal the voltage of the battery.
- Total voltage = the voltage across each individual resistor

$$V_T = V_1 = V_2 = \ldots$$

- The smallest resistance gets the most current.
- The equivalent resistance Req is always less than any resistor in the parallel configuration.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

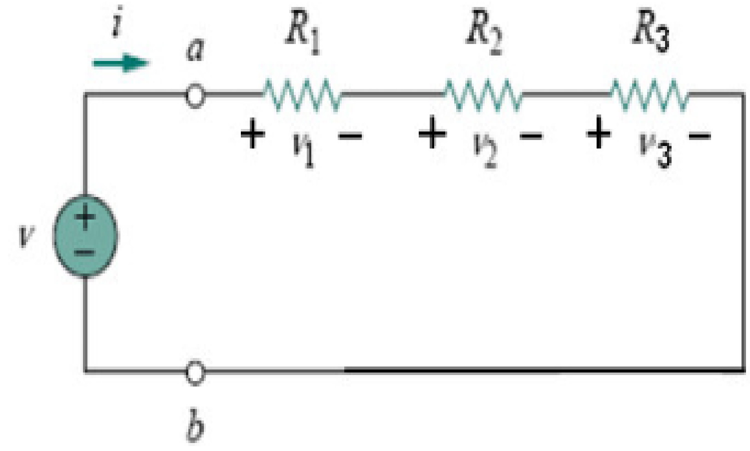
- The more devices (resistors) in a parallel circuit, does not decrease the current (does not dim bulbs).
- If one resistor breaks (a bulb goes out) the rest do not.

Voltage Divider circuits

- Voltage Divider circuits are used to produce different voltage levels from a common voltage source but the current is the same for all components in a series circuit.

- Voltage divider network is to produce a variable voltage output

Voltage Division



For example, we know

$i = V_{TOTAL} / (R_1 + R_2 + R_3)$

so the voltage over the **first resistor** is

$V_1 = i R_1 = R_1 V_{TOTAL} / (R_1 + R_2 + R_3)$

$$\boxed{v_1 = V \frac{R_1}{R_1 + R_2 + R_3}}$$

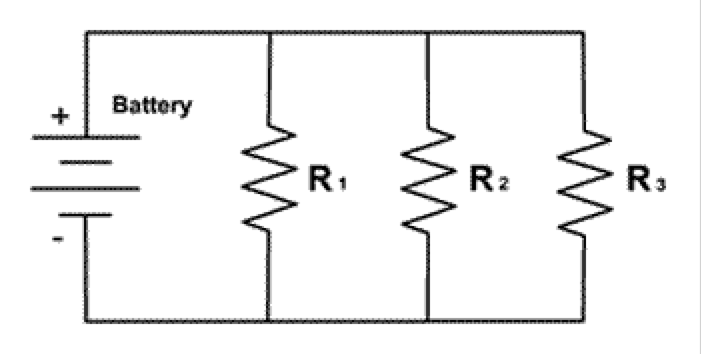
> To find the voltage over an individual resistance in series, take the total series voltage and multiply by the individual resistance over the total resistance.

Current divider circuit

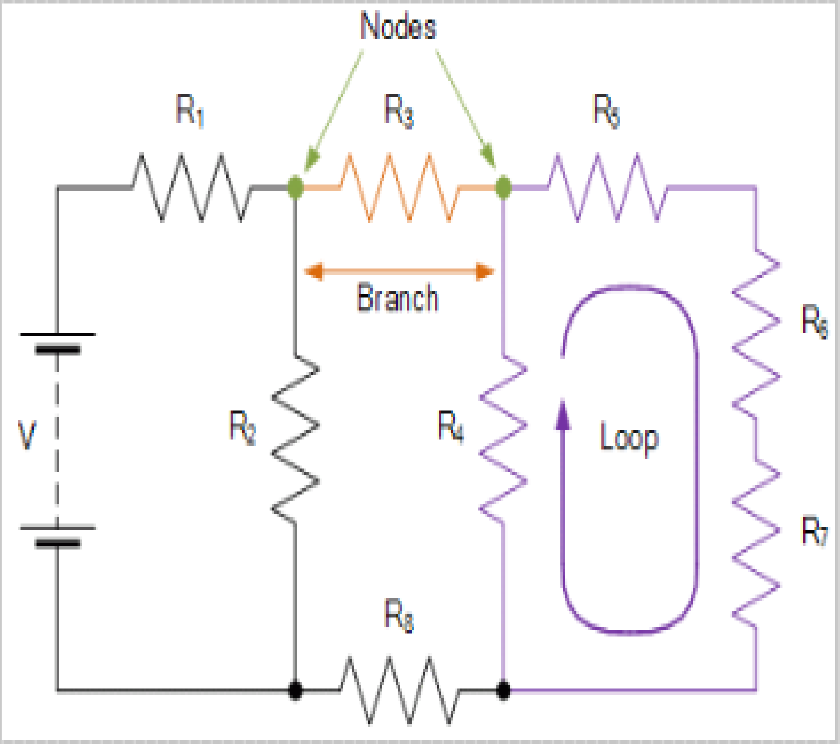
- A parallel circuit acts as a current divider as the current divides in all the branches in a parallel circuit, and the voltage remains the same across them.
- The current division rule determines the current across the circuit impedance.

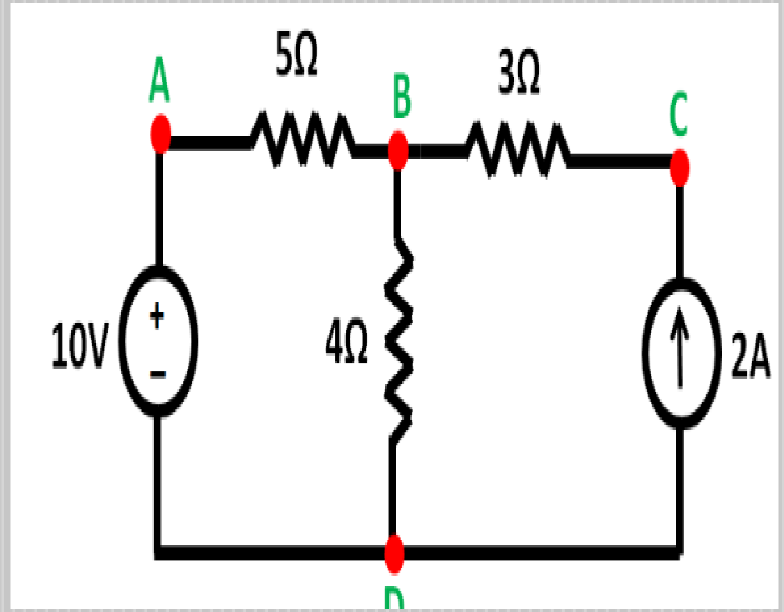
Current division rule

$$I_1 = I\frac{R_2}{R_1 + R_2} \text{ and } I_2 = I\frac{R_1}{R_1 + R_2}$$



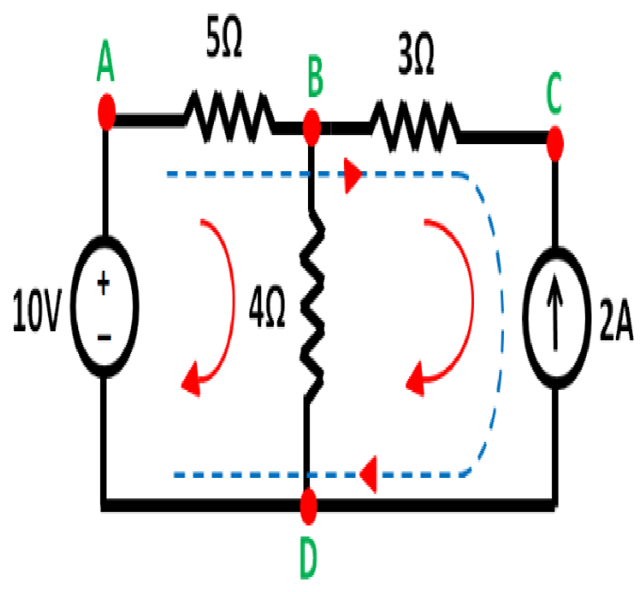
In the current division rule, it is said that the current in any of the parallel branches is equal to the ratio of opposite branch resistance to the total resistance, multiplied by the total current.





A node is a point in the circuit where two or more circuit elements (or branches) are connected.

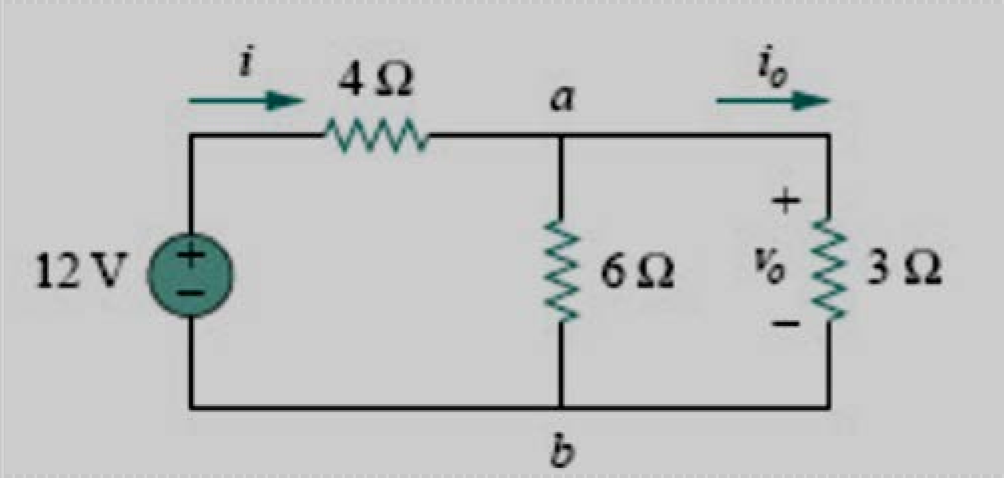
- Branch – a branch is a single or group of components such as resistors or a source which are connected between two nodes.
- Loop – Any closed path in the circuit is called as a loop.
- A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- Mesh – A mesh is a closed path in the circuit, which does not contain any other closed path inside it..



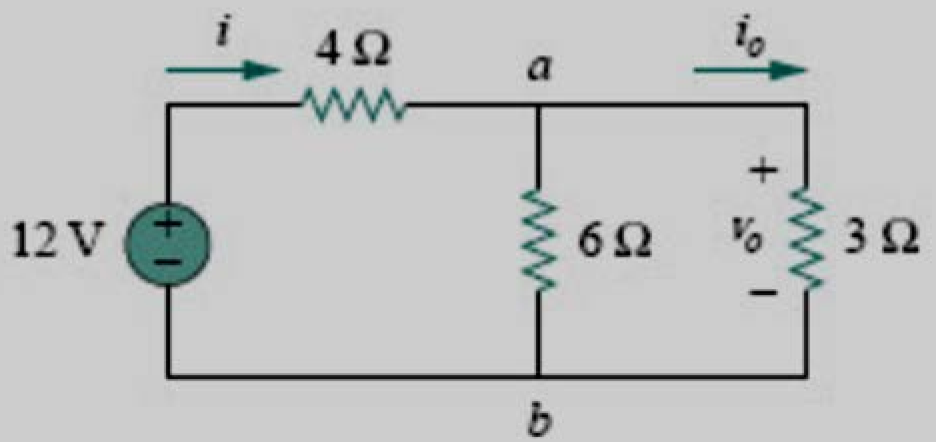
The first is loop A-B-D-A, the second loop is B-C-D-B. And the third loop is A-B-C-D-A

loop 1(A-B-D-A) and loop 2 (B-C-D-B) does not contain any other closed path within them. And they are the example of the Mesh. While loop 3 (A-B-C-D-A) contains loop 1 and loop 2 within it. So, it can't be called as a Mesh.

Use voltage and current division rule to find V_0 and i_0

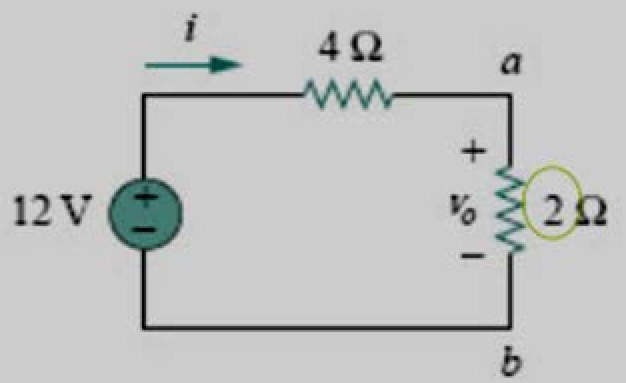


$$\frac{6 \cdot 3}{6+3} = 2$$



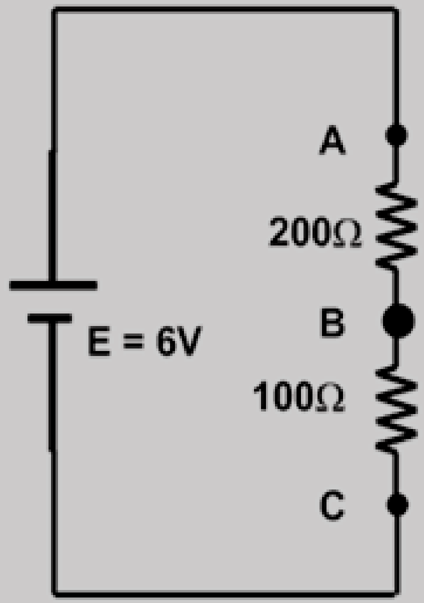
$$i = \frac{12}{4+2} = 2\ \text{A}$$

$$v_o = \frac{2}{2+4}(12\ \text{V}) = 4\ \text{V}$$



$$i_o = \frac{6}{6+3} i = \frac{2}{3}(2\ \text{A}) = \frac{4}{3}\ \text{A}$$

Example of voltage divider rule:



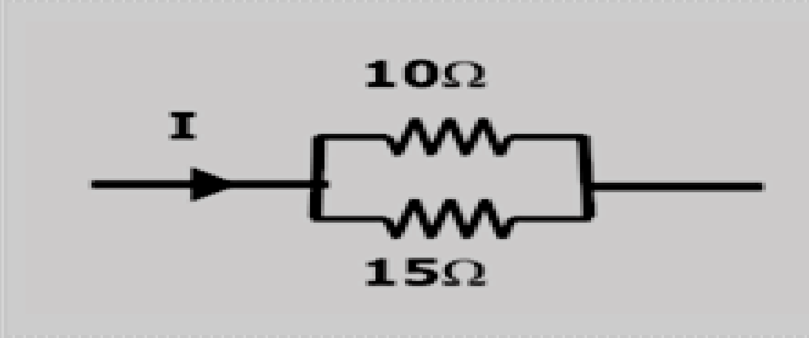
For example of voltage divider rule now we will solve the simple circuit has 6V source and 200 ohm, 100 ohm resistance. We will find voltage drop across each resistance.

Applying formula,

Voltage across 100Ω resistance $V_1 = (100*6)/(200+100) = 2V$

Voltage across 200Ω resistance $V_2 = (200*6)/(200+100) = 4V$

- A circuit carrying I current and divide across two resistors shown in figure, find the current through each resistor.

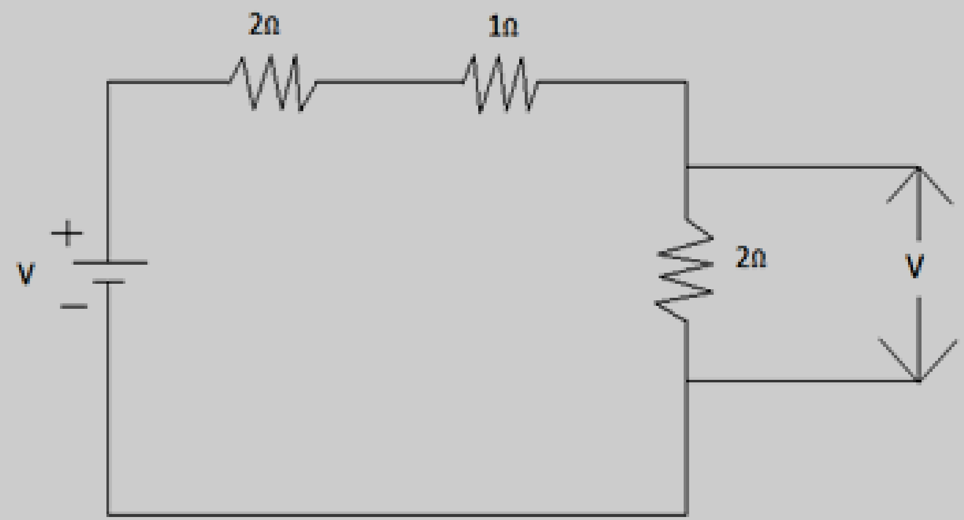


According to current divider rule,

Current for 10Ω resistance, $I_1 = (15*I)/(10+15) = 15I/25 = 0.6I$

Current for 15Ω resistance, $I_2 = (10*I)/(10+15) = 10I/25 = 0.4I$

4.

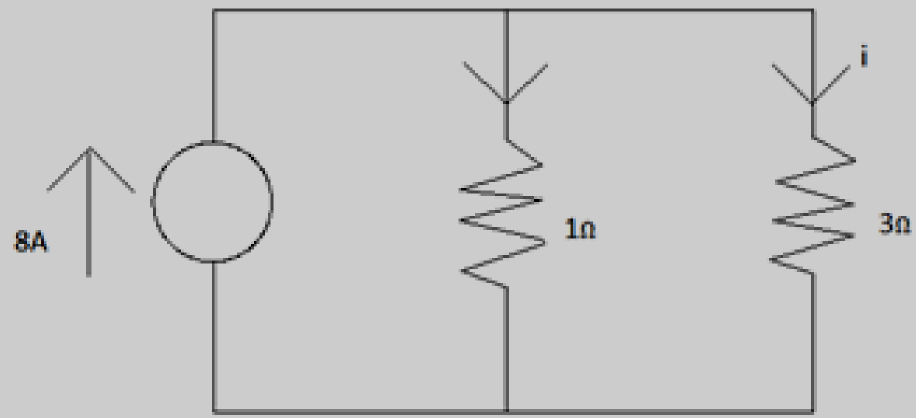


Calculate Voltage across 2Ω Resistor where supply v= 10volts.

- Total V=10V
- R= 5 Ohm
- I= 10/5 = 2 A
- V across 2 ohm resistor = I * R

  =2*2= 4 V

5.

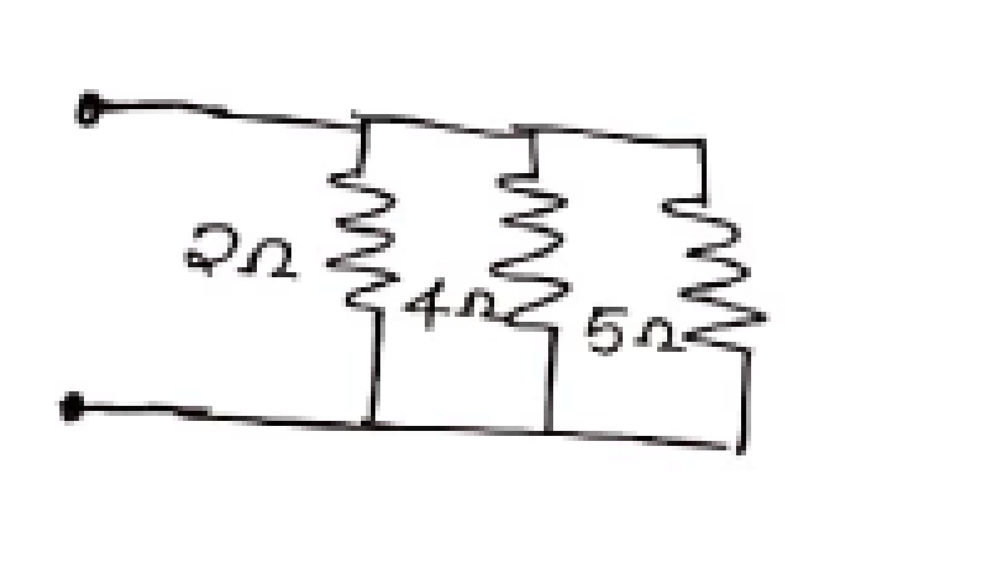


Calculate i =?

- Total resistance= ¾ ohm
- Total voltage = I*R=8*(3/4) = 6 V
- Current through 3 ohm resistor = V/R

  =6/3 = 2 A

- 1.DETERMINE TOTAL RESISTANCE



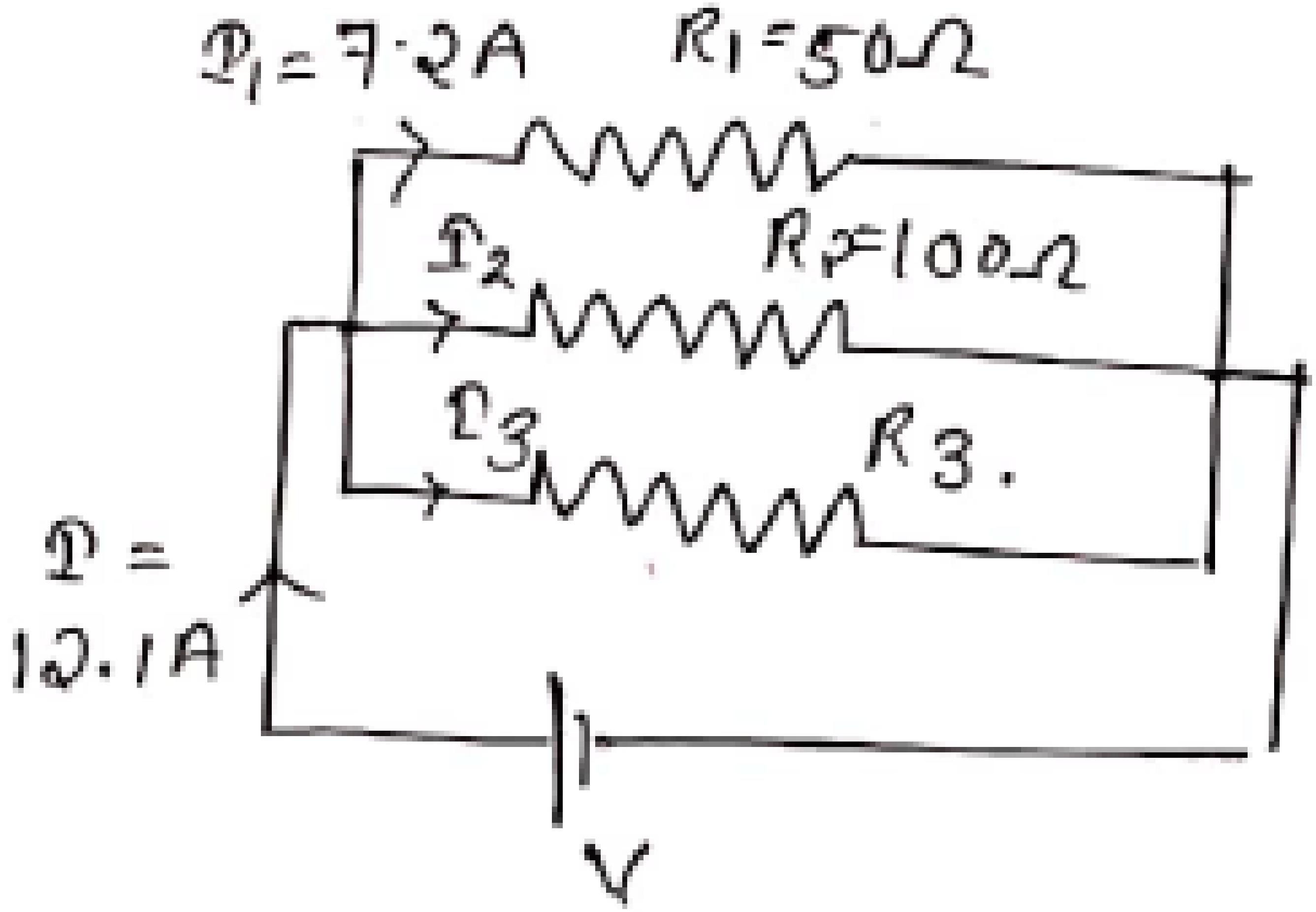
Ans:

$$\frac{1}{R_{total}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6}$$

$$= 0.95\,\Omega$$

$$\therefore R_{total} = \frac{1}{0.95} = \underline{\underline{1.053\,\Omega}}$$

(2) A 50 Ω resistor is in parallel with a 100 Ω resistor. The current in the 50 Ω resistor is 7.2 A. What is the third value of R to be added in parallel to the circuit to make line current as 12.1 A


I1 = 7.2A
R1 = 50Ω
I2
R2 = 100Ω
I3
R3.
I =
12.1A
V

Ans: V will be same across each resistance

$$V_1 = I_1 \cdot R_1$$

$$= 7.2 \times 50 = \underline{\underline{360\,V}}$$

$$V_1 = V_2 = V_3 = V = \underline{\underline{360\,V}}$$

$$I_2 = \frac{V}{R_2} = \frac{V}{100} = \frac{360}{100} = \underline{\underline{3.6\,A}}$$

we know $I = I_1 + I_2 + I_3$

$$12.1 = 7.2 + 3.6 + I_3$$

$$\therefore I_3 = \underline{\underline{1.3\,A}}$$

$$R_3 = \frac{V}{I_3} = \frac{360}{1 \cdot 3} = \underline{\underline{277\ \Omega.}}$$

$\therefore$ Third value of Resistor, $R_3 = \underline{\underline{277\ \Omega}}$

③ For the circuit shown determine the equivalent resistance and current flowing through each branch. Also determine the total power drawn from supply?

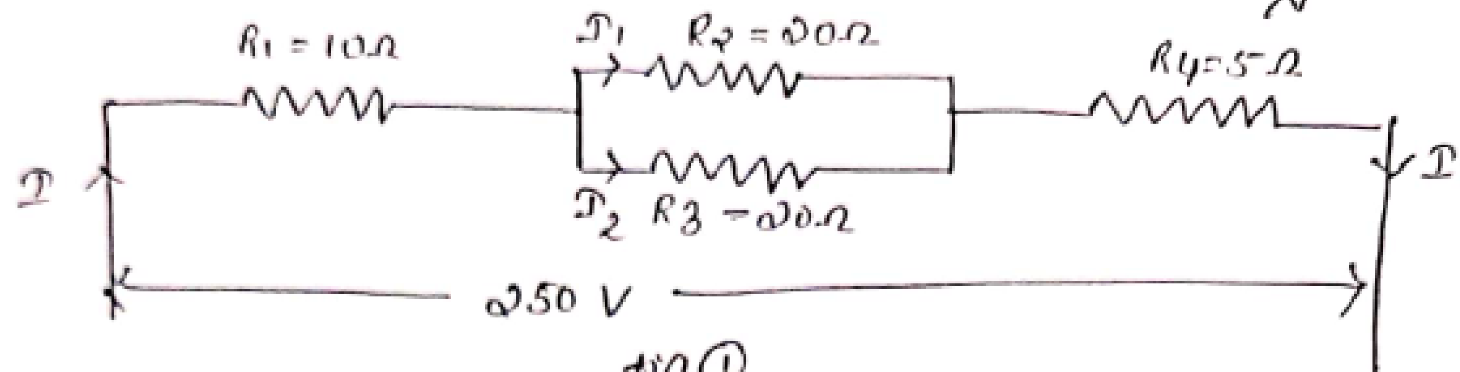
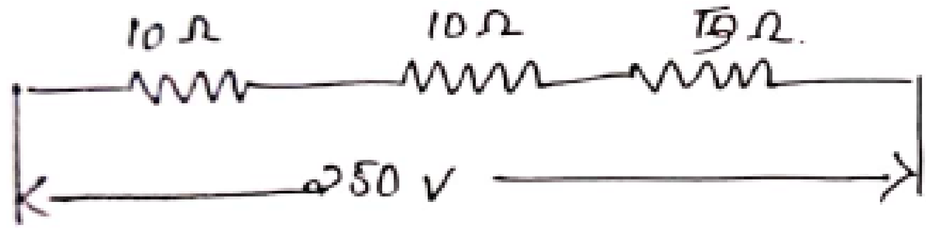


fig ①

Soln: equivalent resistance of 20Ω & 20Ω in parallel

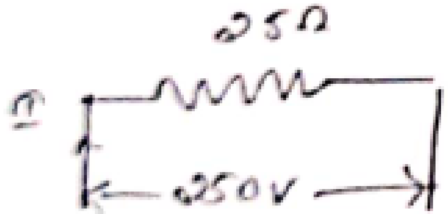
$$\frac{1}{R} = \frac{1}{20} + \frac{1}{20} \qquad \therefore R = \underline{\underline{10\Omega}}$$

$\therefore$



$\therefore$ equivalent R of 10Ω, 10Ω and 5Ω in series

$R = 10+10+5 = 25\,\Omega$



$\therefore \quad I = \frac{250}{25} = \underline{\underline{10\,A}}$

in fig (1) current through $10\,\Omega$ = current through $5\,\Omega = 10\,A$

I_1 = current through 20Ω = $I_1 = \dfrac{I \cdot R_2}{R_2 + R_3} = \dfrac{10 \times 20}{20 + 20}$

$= \underline{\underline{5A}}$

$||^{ly}$ I through 20Ω = $I_2 = \dfrac{I \cdot R_2}{R_2 + R_3} = \underline{\underline{5A}}$

current flowing through $R_1 = I = 10\ A$

" " " $R_2 = I_1 = 5A$

" " " $R_3 = I_2 = 5A$

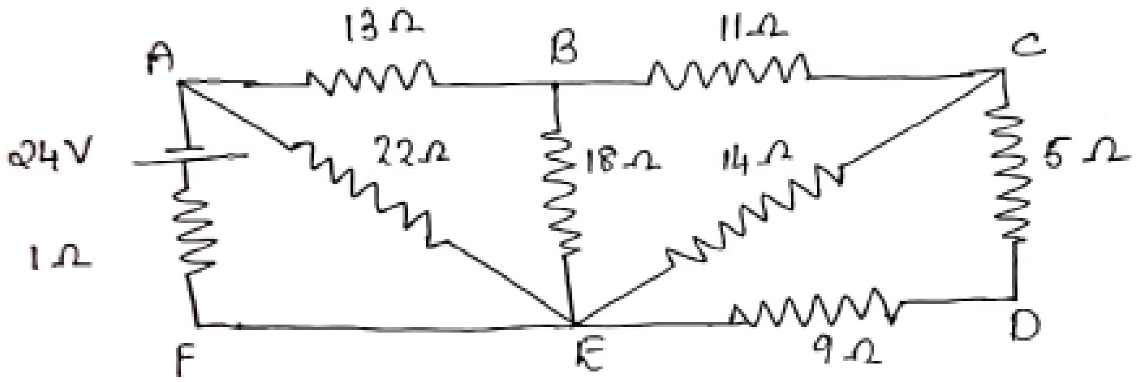
" " " $R_4 = I_1 + I_2 = 5 + 5 = \underline{\underline{10\ A}}$

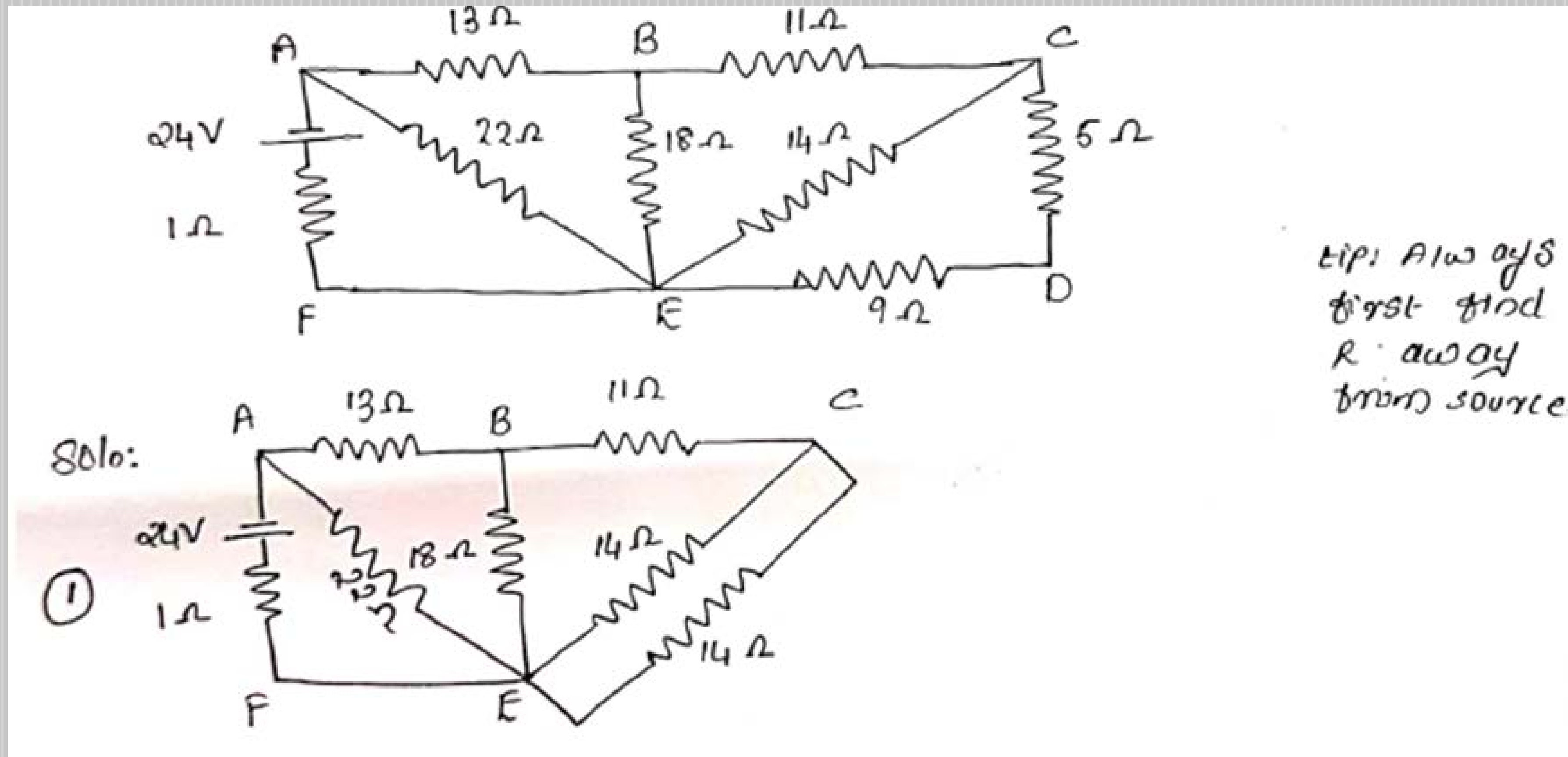
Total power $P = VI = 250 \times 10 = \underline{\underline{2500\ W}}$

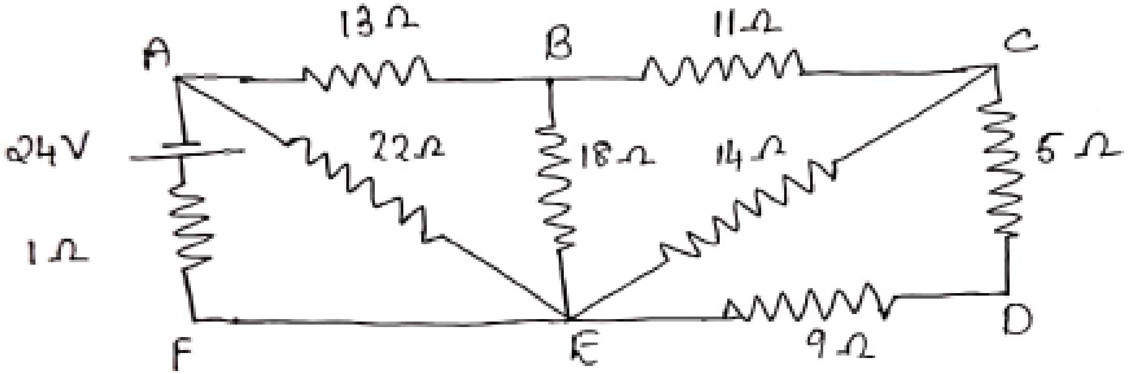
(4) An electric network is arranged as shown in the figure. find ① current in branch AF

② power absorbed in branch BE

③ P.d across branch CD.



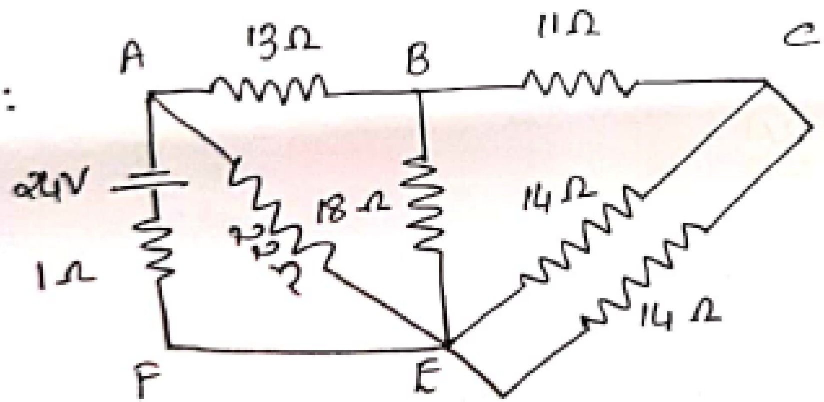

A
13 Ω
B
11 Ω
C
24V
22 Ω
18 Ω
14 Ω
5 Ω
1 Ω
F
E
9 Ω
D
tip: Always first find R away from source
Solo:
1
A
13Ω
B
11Ω
C
24V
18 Ω
14Ω
1Ω
14 Ω
F
E



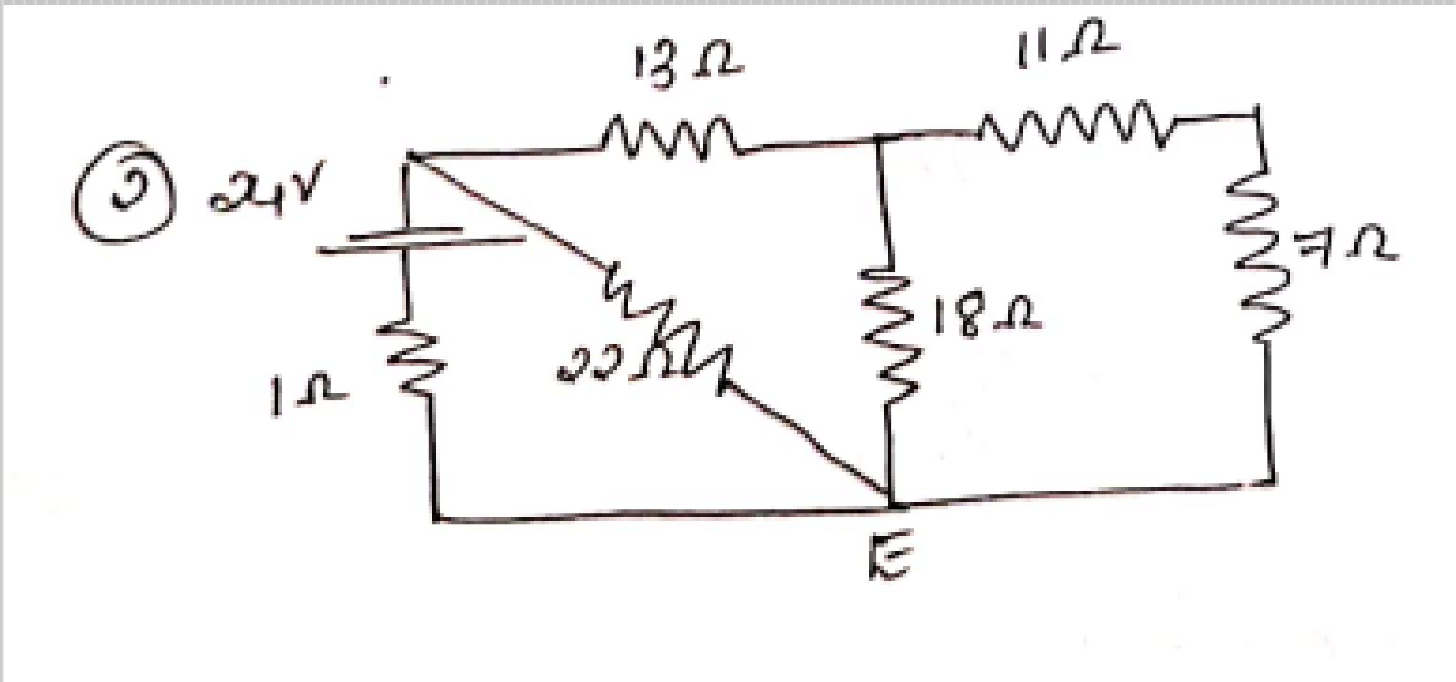
tip: Always first find R away from source

Solo:

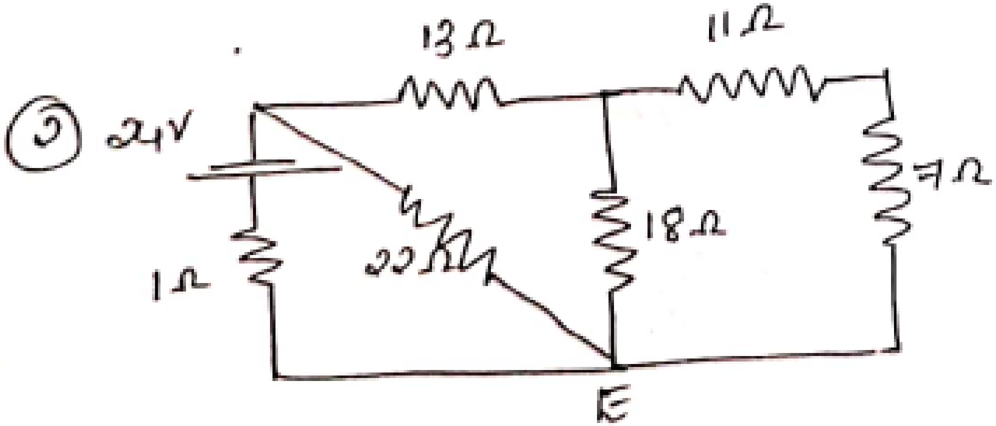
(1)



$$14\,\Omega \,\|\, 14\,\Omega = \frac{14\times 14}{14+14} = \frac{14}{2} = \underline{\underline{7\,\Omega}}$$

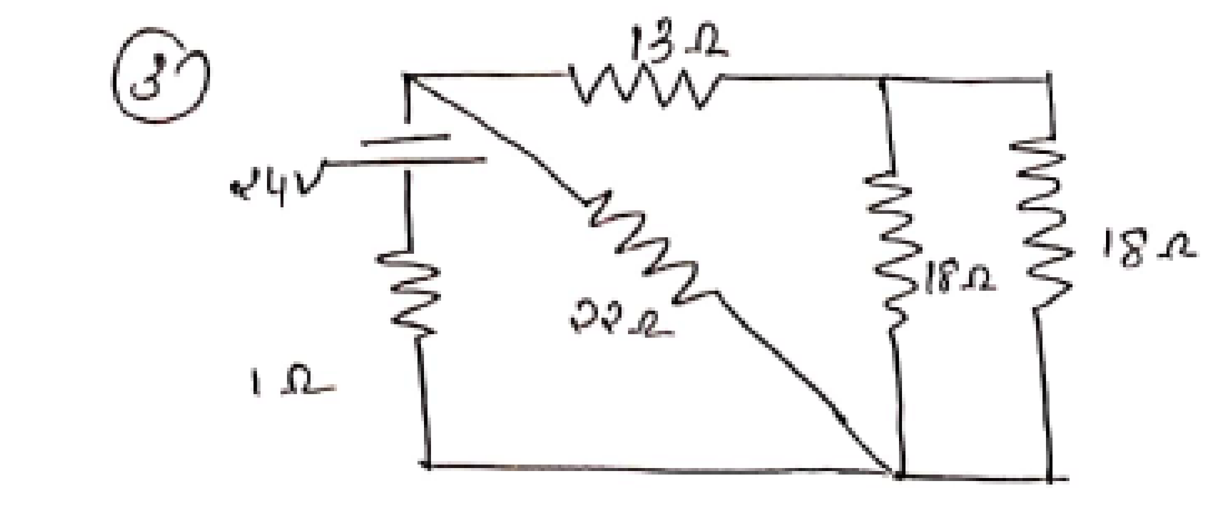


13Ω
11Ω
⑦ 24V
7Ω
18Ω
22Ω
1Ω
E

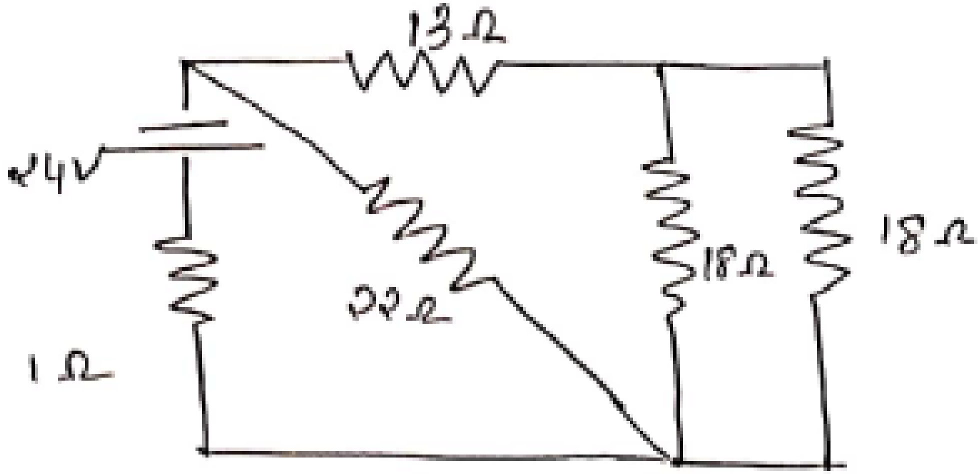


11 Ω + 7 Ω in series

$11+7 = \underline{\underline{18\,\Omega}}$

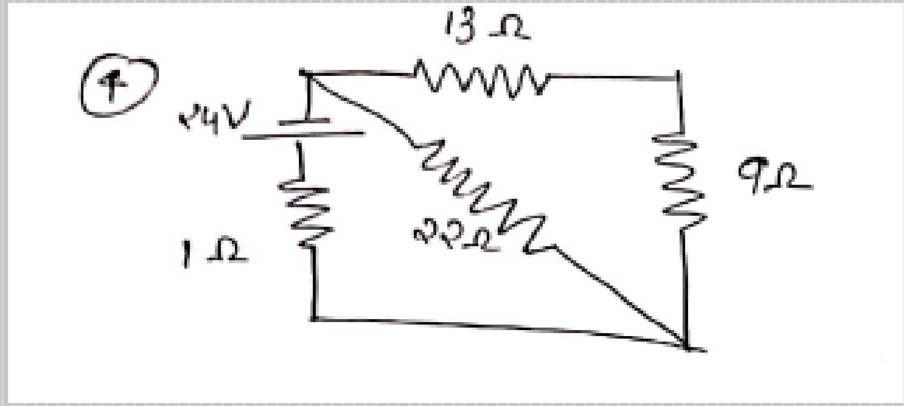

3
13 Ω
24V
22 Ω
18 Ω
18 Ω
1 Ω

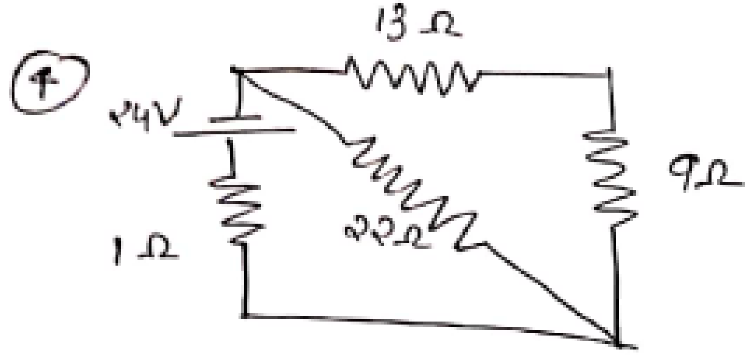
(3)



$18 \parallel 18\,\Omega$

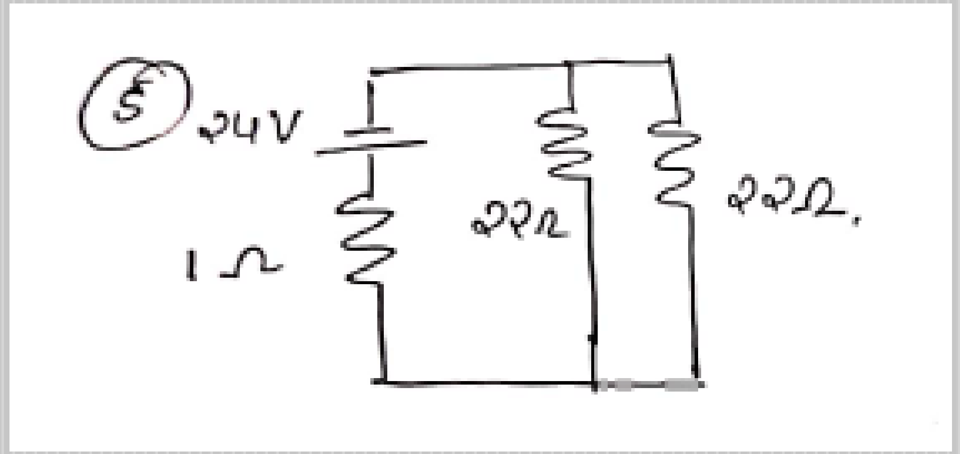
$= \underline{\underline{9\,\Omega}}$


13 Ω
24V
9Ω
22Ω
1 Ω



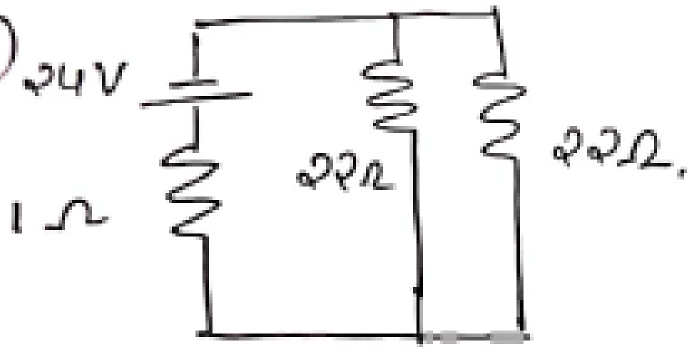
13Ω & 9Ω series connected.

$13\Omega + 9 = \underline{\underline{22\Omega}}$



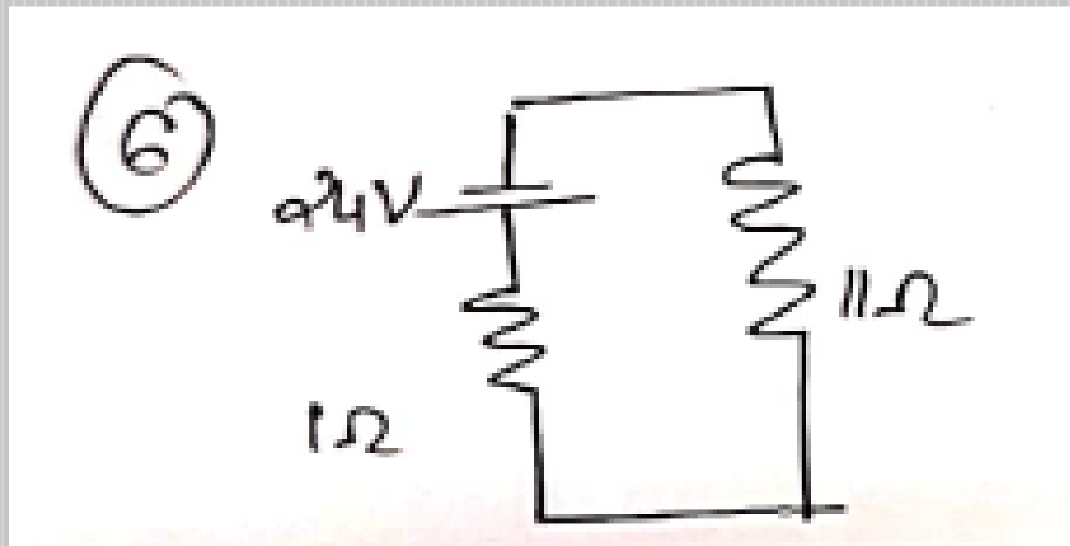
5
24V
1Ω
22Ω
22Ω.

(5)

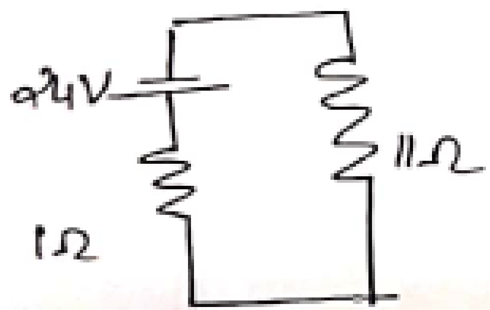


22Ω & 22Ω in parallel

$22 \| 22\Omega = 11\Omega$


6
૨૪V
11Ω
1Ω

6



11Ω in series with 1Ω

$11\Omega + 1\Omega = \underline{\underline{12\Omega}}$

① Total current $= \frac{24}{12} = \underline{\underline{2A}}$

② Power in BE $= I_{BE}^2 \cdot R_{BE}$

from fig ⑤ $I_1 = \frac{2 \times 22}{22+22} = \underline{\underline{1A}}$ $(I = 2)$

$I_2 = \frac{2 \times 22}{44} = \underline{\underline{1A}}$

from fig ③ $I_{BE} = \frac{1 \cdot 18}{18+18} = \underline{\underline{0.5\ A}}$.

$\therefore$ Power in BE $= 0.5^2 \times 18 = \underline{\underline{4.5W}}$

from fig 14

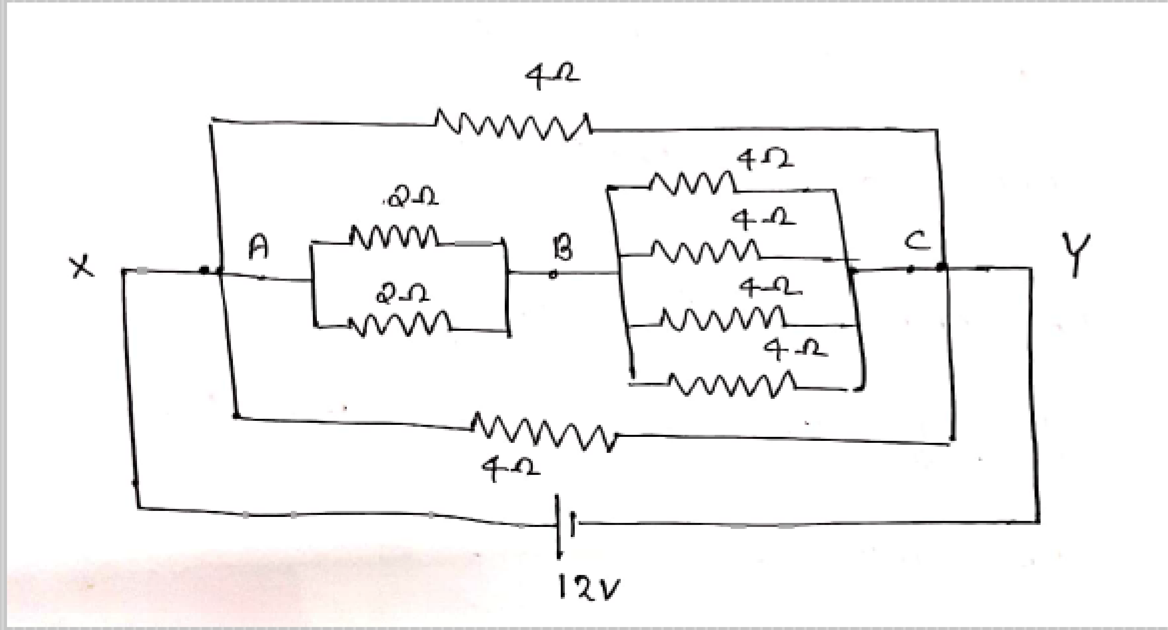
$$I_4 = \frac{I_{BC} \cdot 14}{14+14} = \frac{0.5 \times 14}{28} = \underline{\underline{0.25\,A}}$$

I_4 is current through 9Ω and 5Ω ie CD.

$\therefore$ P.d across CD, $V = I_{CD} \cdot R$

$$= 0.25 \times 5 = \underline{\underline{1.25\,V}}$$

Find equalent resistance between x &y, and also total current.



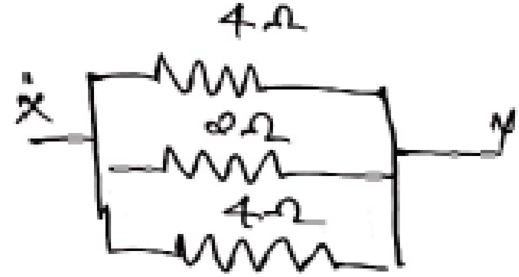
B/w A and B

$$R_{AB} = \frac{2 \times 2}{2+2} = 1\Omega$$

B/w B and C

$$\frac{1}{R_{BC}} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1\Omega. \quad \therefore \quad R_{BC} = 1\Omega$$

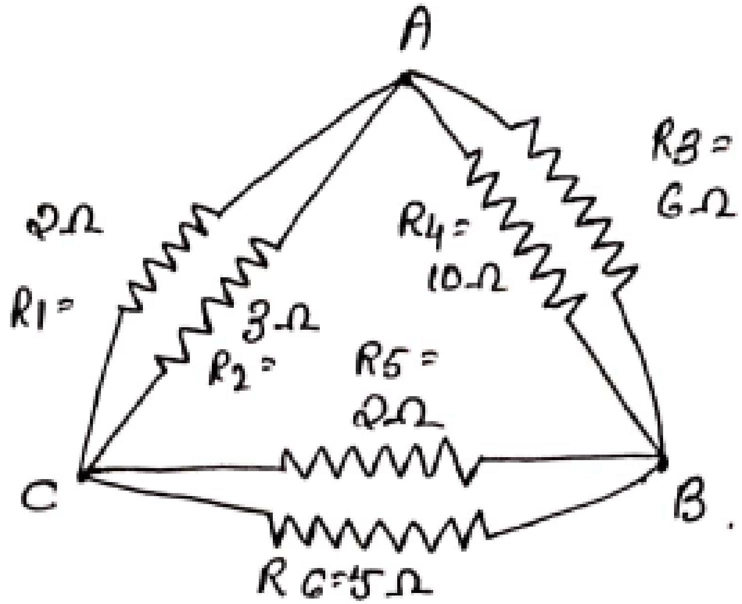
$$\therefore R_{AC} = R_{AB} + R_{BC} = 2\Omega.$$



$$\therefore \frac{1}{R_{xy}} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1\Omega$$

$$R_{xy} = 1\Omega \qquad \therefore \text{Total } I = \frac{V}{R} = \frac{12}{1} = \underline{\underline{12\,A}}$$

(6)



find Req across BC

$$\frac{1}{R_{AC}} = \frac{1}{2} + \frac{1}{3} \quad ; \quad R_{AC} = \underline{\underline{6/5\,\Omega}} = 1.2$$

$$\frac{1}{R_{AB}} = \frac{1}{6} + \frac{1}{10} \quad ; \quad R_{AB} = \frac{15}{4}\,\Omega = 3.75$$

$$R_{AC} + R_{AB} = \frac{6}{5} + \frac{15}{4} = 4.95\,\Omega$$

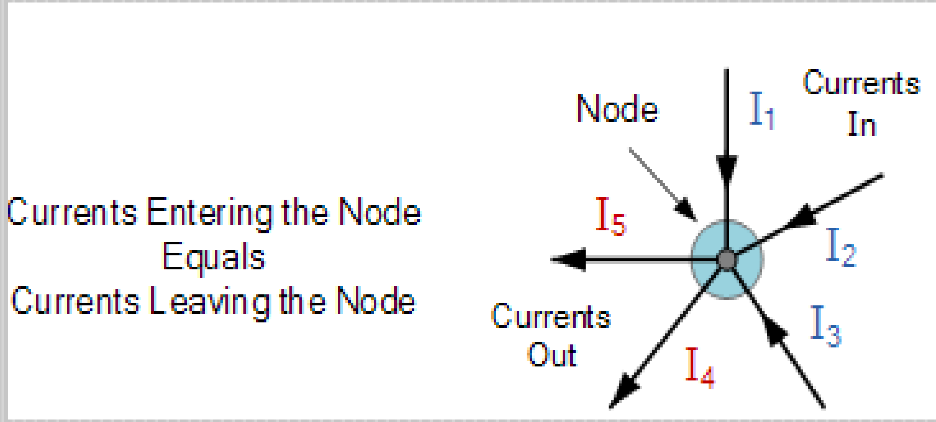
$$\frac{1}{R_{BC}} = \frac{1}{2} + \frac{1}{5} + \left(\frac{1}{4.95}\right) = \cancel{.9765} = 0.9020$$

$$R_{BC} = \underline{\underline{1.1086\,\Omega}}$$

Kirchhoff's law

- Kirchhoff's current law/junction rule
- Kirchhoff's voltage law/loop rule

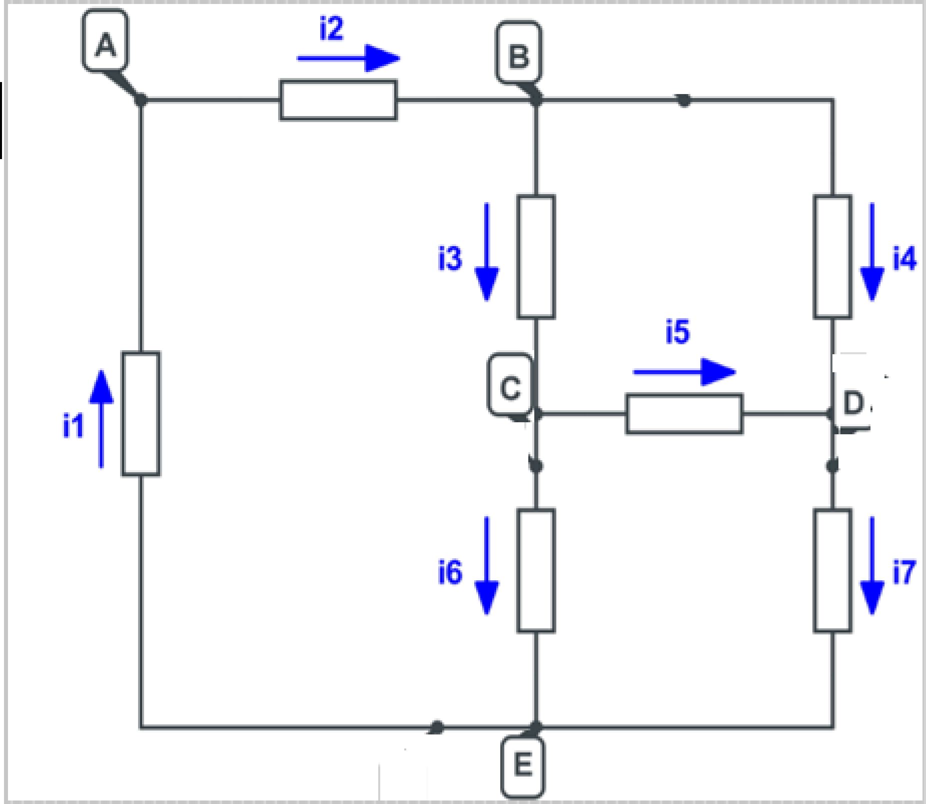
- Kirchhoff's current law/junction rule



The algebraic sum of the currents meeting at a junction in an electric circuit is zero

The sum of currents flowing towards any junction in an electric circuit is equal to the sum of currents flowing away from that junction.

- Here, the three currents entering the node, I_1, I_2, I_3 are all positive in value and the two currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;
- $I_1 + I_2 + I_3 - I_4 - I_5 = 0$

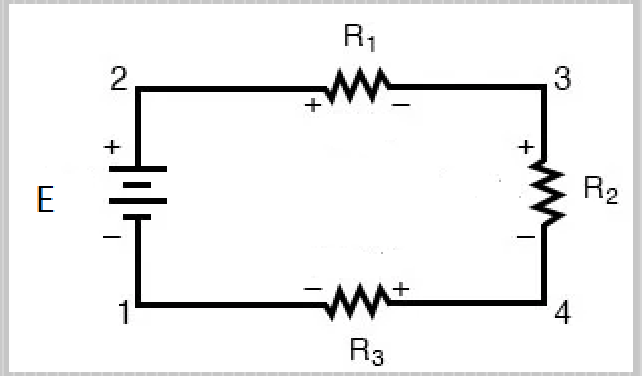


KCL equations at each node

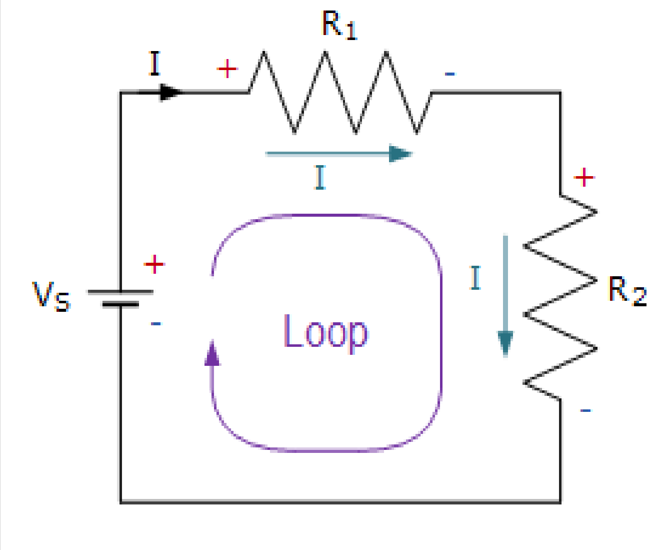
i1−i2=0	Node A
i2−i3−i4=0	Node B
i3−i5−i6=0	Node C
i4+i5−i7=0	Node D
i6+i7−i1=0	Node E

Kirchhoff's voltage law/loop rule

- In any closed electrical circuit or mesh ,the algebraic sum of all the emf's and voltage drops in resistors equal to zero



The sum of the voltage rises around a closed loop must equal the sum of the voltage drops around the loop.



$\Sigma V = 0$.

Since the two resistors, R_1 and R_2 in a series connection, they are both part of the same loop so the same current must flow through each resistor.

Thus the voltage drop across resistor, $R_1 = I*R_1$ and the voltage drop across resistor, $R_2 = I*R_2$ giving by KVL:

$$V_S + (-IR_1) + (-IR_2) = 0$$

$$\therefore V_S = IR_1 + IR_2$$

$$V_S = I(R_1 + R_2)$$

$$V_S = IR_T$$

$$\text{Where: } R_T = R_1 + R_2$$

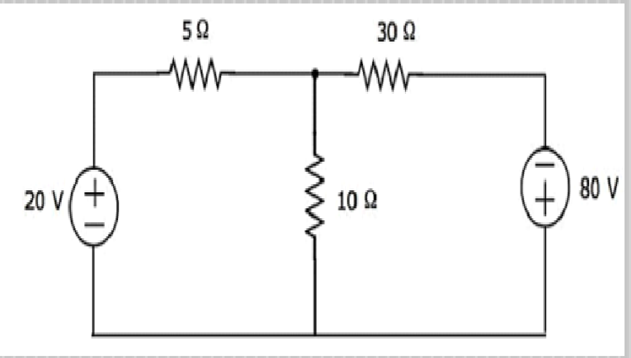
MESH ANALYSIS

- In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as **Mesh-current method**.
- If a branch belongs to only one mesh, then the branch current will be equal to mesh current.
- If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

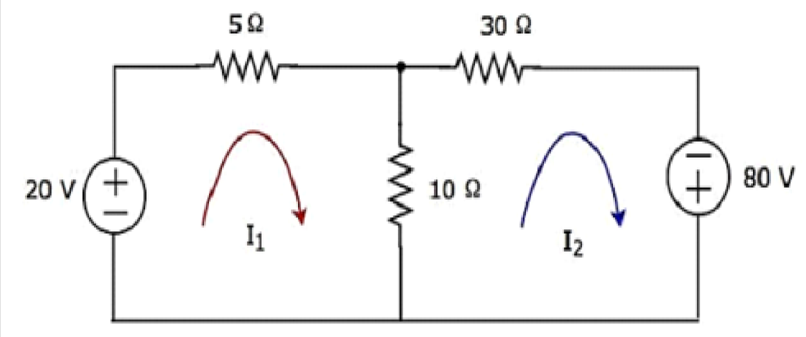
Procedure of Mesh Analysis

- **Step 1** – Identify the **meshes** and label the mesh currents in clockwise direction
- **Step 2** – Observe the amount of current that flows through each element in terms of mesh currents.
- **Step 3** – Write **mesh equations** to all meshes. Mesh equation is obtained by applying KVL first and then Ohm's law.
- **Step 4** – Solve the mesh equations obtained in Step 3 in order to get the **mesh currents**.
- Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using mesh currents

Eg: Find the voltage across 30 Ω resistor using **Mesh analysis**



Step 1 – There are two meshes in the above circuit. The **mesh currents** I_1 and I_2 are considered in clockwise direction. These mesh currents are shown in the following figure.



Step 2 – The mesh current I_1 flows through 20 V voltage source and 5 Ω resistor. Similarly, the mesh current I_2 flows through 30 Ω resistor and -80 V voltage source. But, the difference of two mesh currents, I_1 and I_2, flows through 10 Ω resistor, since it is the common branch of two meshes.

- **Step 3** – In this case, we will get **two mesh equations** since there are two meshes in the given circuit. When we write the mesh equations, assume the mesh current of that particular mesh as greater than all other mesh currents of the circuit.
- The **mesh equation** of first mesh is

$$20 - 5I_1 - 10(I_1 - I_2) = 0$$

$$\Rightarrow 20 - 15I_1 + 10I_2 = 0 \qquad \textbf{Equation 1}$$

$$\Rightarrow 10I_2 = 15I_1 - 20$$

The **mesh equation** of second mesh is

$$-10(I_2 - I_1) - 30I_2 + 80 = 0 \qquad \textbf{Equation 2}$$

Step 4 – Finding mesh currents I_1 and I_2 by solving Equation 1 and Equation 2.

- $I_1 = \frac{16}{5}$ A
- $I_2 = \frac{14}{5}$ A

Step 5 – The current flowing through 30 Ω resistor is nothing but the mesh current I_2

Now, we can find the voltage across 30 Ω resistor by using Ohm's law

$$\boxed{V_{30\Omega} = I_2 R}$$

$$V_{30\Omega} = (\frac{14}{5})30$$

$$\Rightarrow V_{30\Omega} = 84V$$

WE HAVE TO SOLVE 'M' MESH EQUATIONS, IF THE ELECTRIC CIRCUIT IS HAVING 'M' MESHES.

TO USE CALCULATOR TO SOLVE EQNS:

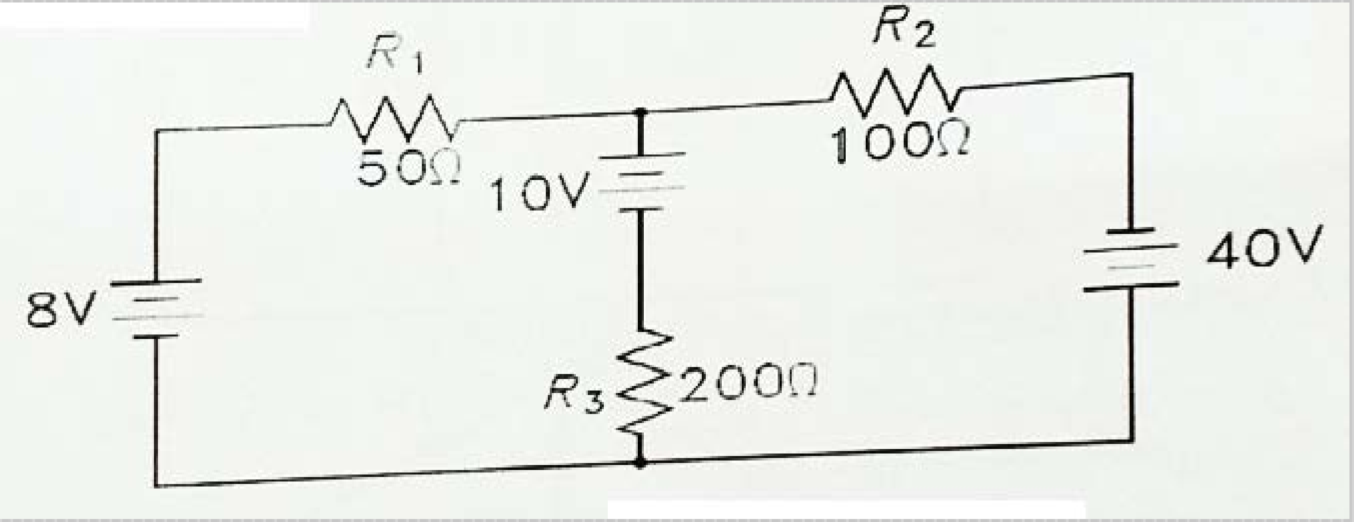
- **CASIO fx -991 ES**

  MODE---EQN(5)---PRESS 1---ENTER THE VALUES

- **CASIO fx-991 MS**

  PRESS MODE 3 TIMES---SELECT EQN BY PRESSING 1---ENTER THE UNKNOWNS---ENTER THE VALUES

1. Determine the power dissipated in all the three resistors using mesh analysis.



mesh: 1

$$-50 I_1 - 10 - 200 (I_1 - I_2) + 8 = 0$$

$$-250 I_1 + 200 I_2 - 2 = 0$$

$$250 I_1 - 200 I_2 = -2 \quad \text{——— } (1)$$

mesh: 2

$$-100 I_2 + 40 - 200 (I_2 - I_1) + 10 = 0.$$

$$-300 I_2 + 200 I_1 + 50 = 0.$$

$$-200 I_1 + 300 I_2 = 50 \quad \text{——— } (2)$$

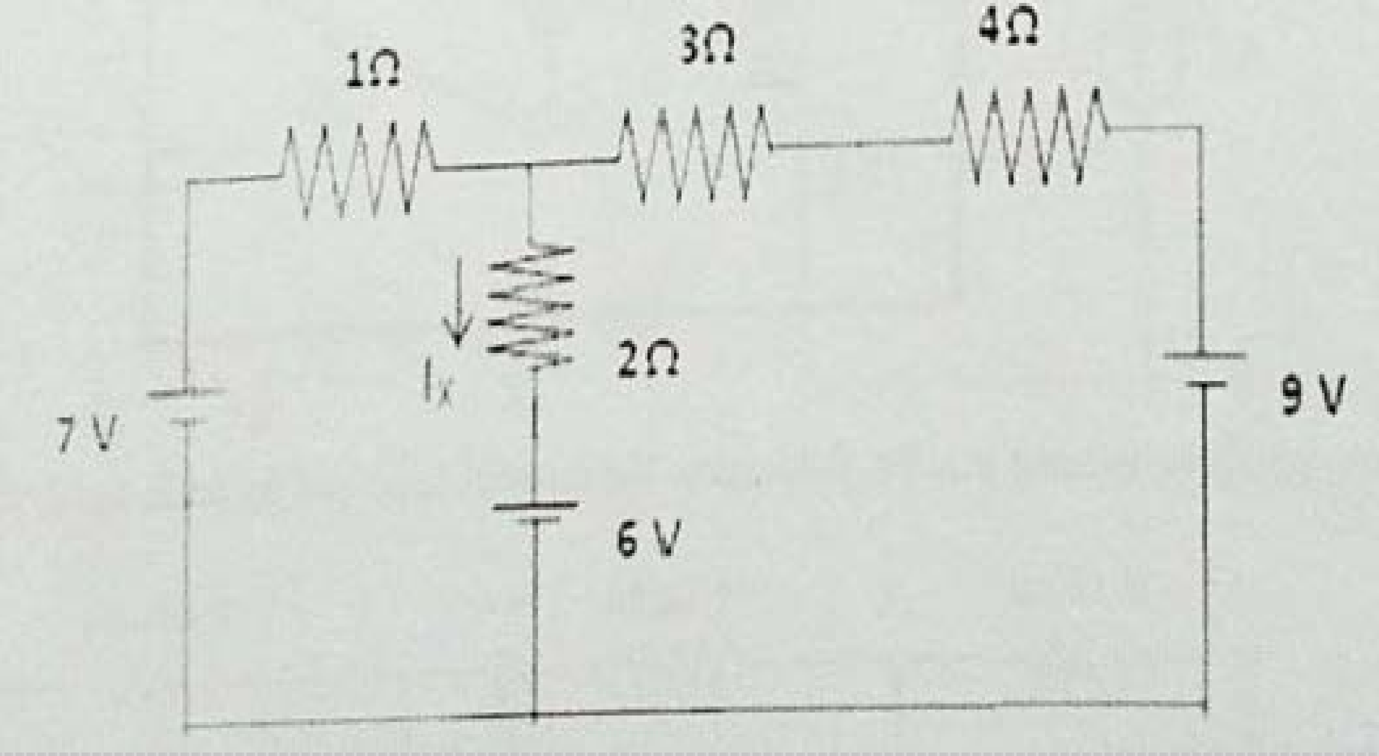
$I_1 = 0.27\,A$, $I_2 = 0.3457\,A$ $\quad I_{R3} = I_2 - I_1 = 0.08\,A$

$$P_{R1} = I_1^2 R_1 = 0.27^2 \times 50 = 3.645\,W$$

$$P_{R2} = I_2^2 R_2 = 0.3457^2 \times 100 = 11.95\,W$$

$$P_{R3} = I_{R3}^2 R_3 = 0.08^2 \times 200 = 1.28\,W$$

2. Solve using mesh analysis and find i_x



$$3I_1 - 2I_2 = 1 \quad \text{— } (1)$$

$$-2I_1 + 9I_2 = -3 \quad \text{— } (2)$$

$I_1 = \underline{0.13\,A}$, $I_2 = -0.\underline{304}\,A$

$i_x = I_1 - I_2 = 0.13 - (-0.304) = \underline{0.434\,A}$

Solution of network equations using matrix method

General expression: by cramer's rule:

$$\pm R_{11} I_1 \pm R_{12} I_2 \pm R_{13} I_3 = V_1$$

$$\pm R_{21} I_1 \pm R_{22} I_2 \pm R_{23} I_3 = V_2$$

$$\pm R_{31} I_1 \pm R_{32} I_2 \pm R_{33} I_3 = V_3$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} \; ; \quad I_2 = \frac{\Delta_2}{\Delta} \; ; \quad I_3 = \frac{\Delta_3}{\Delta}$$

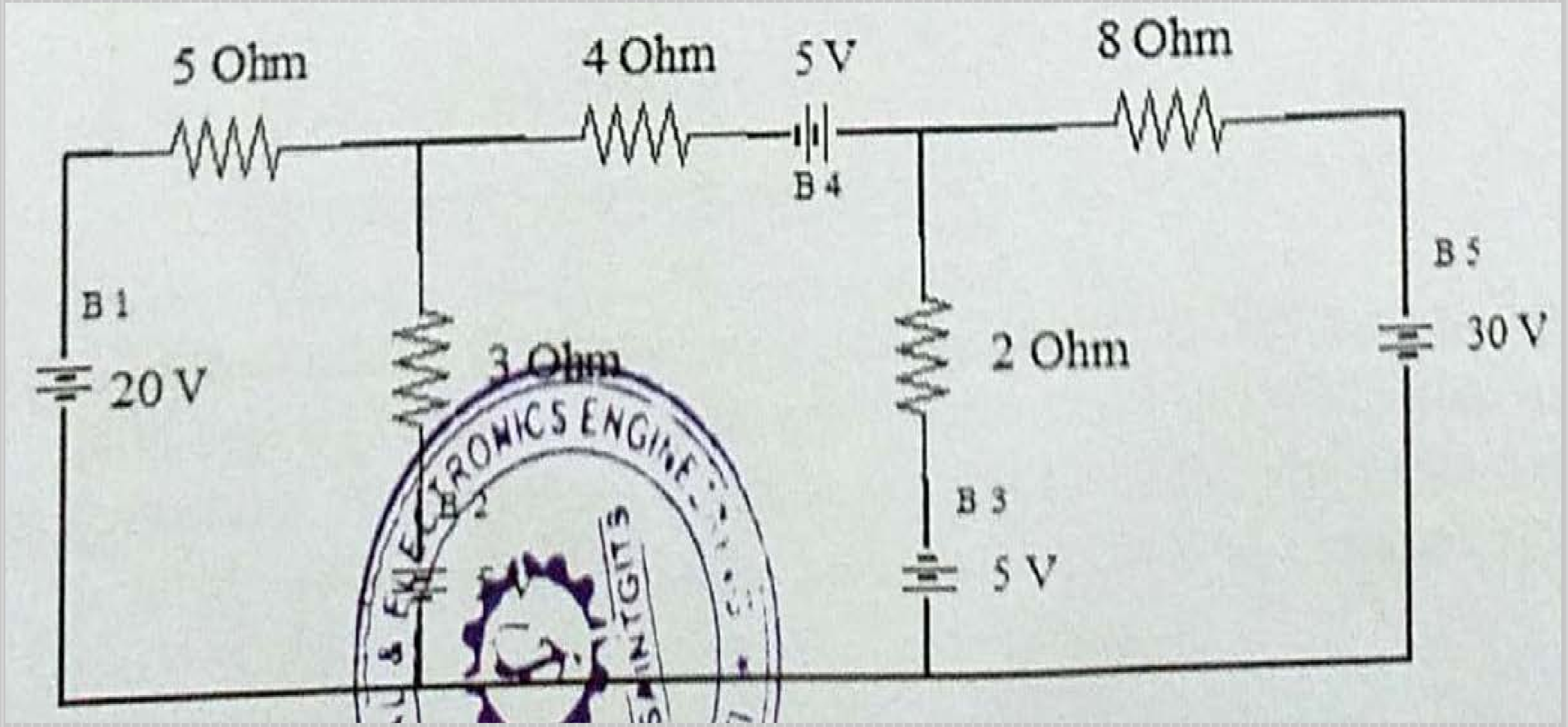
$$\Delta = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix} \longrightarrow \text{Resistance matrix}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & R_{12} & R_{13} \\ V_2 & R_{22} & R_{23} \\ V_3 & R_{32} & R_{33} \end{vmatrix}}{\Delta}$$

$$I_2 = \frac{\begin{vmatrix} R_{11} & V_1 & R_{13} \\ R_{21} & V_2 & R_{23} \\ R_{31} & V_3 & R_{33} \end{vmatrix}}{\Delta}$$

$$I_3 = \frac{\begin{vmatrix} R_{11} & R_{12} & V_1 \\ R_{21} & R_{22} & V_2 \\ R_{31} & R_{32} & V_3 \end{vmatrix}}{\Delta}$$

5. Using mesh resistance matrix calculate the power dissipated in each battery.



$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = \underline{\underline{598}}$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = \underline{\underline{1530}}$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = \underline{\underline{1090}}$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = -\underline{\underline{1875}}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = \underline{\underline{2.56\ A}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = \underline{\underline{1.82\ A}}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598} = \underline{\underline{-3.14\ A}}$$

Current Supplied by $B_1 = I_1 = 2.56\ A$

$B_2 = I_1 - I_2 = 0.74\ A$

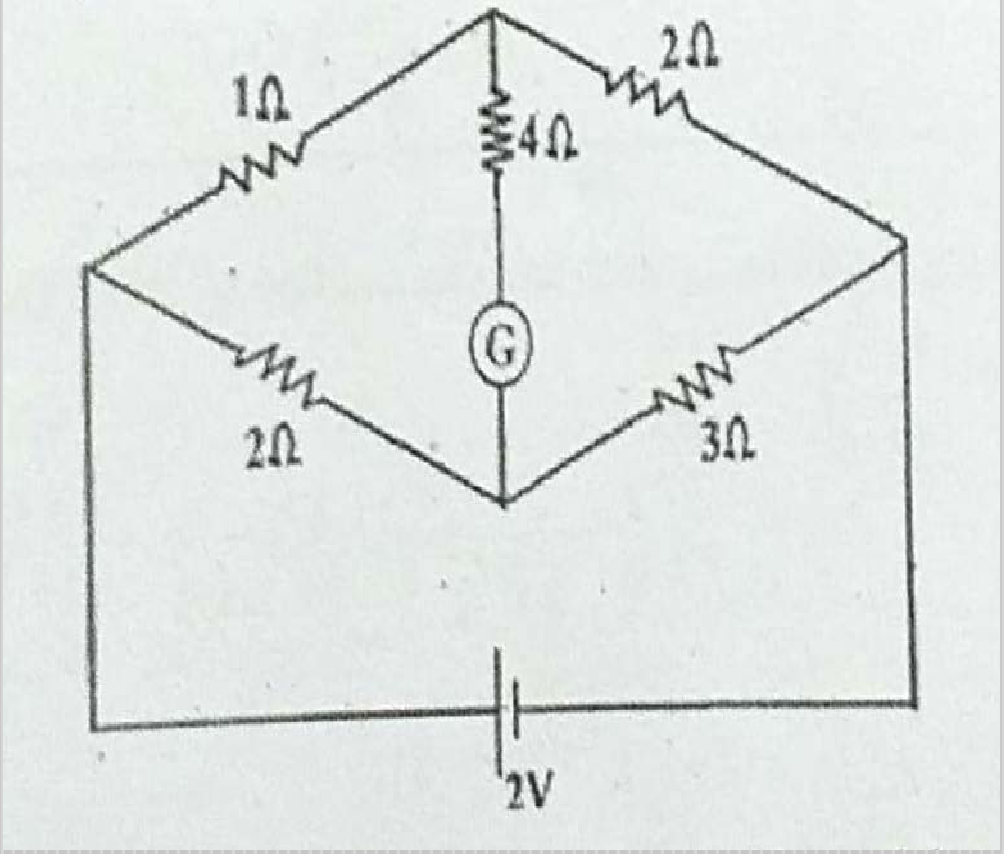
$B_3 = I_2 - I_3 = 1.82 - (-3.13)$

$= 4.95\ A$

$B_4 = I_2 = 1.82\ A$

$B_5 = I_3 = \underline{\underline{3.13\ A}}$

4. Calculate the current through the galvanometer.



$$7I_1 - 4I_2 - 2I_3 = 0 \quad -(1)$$

$$-4I_1 + 9I_2 - 3I_3 = 0 \quad -(2)$$

$$-2I_1 - 3I_2 + 5I_3 = 2 \quad -(3)$$

$I_1 = 0.681$ A

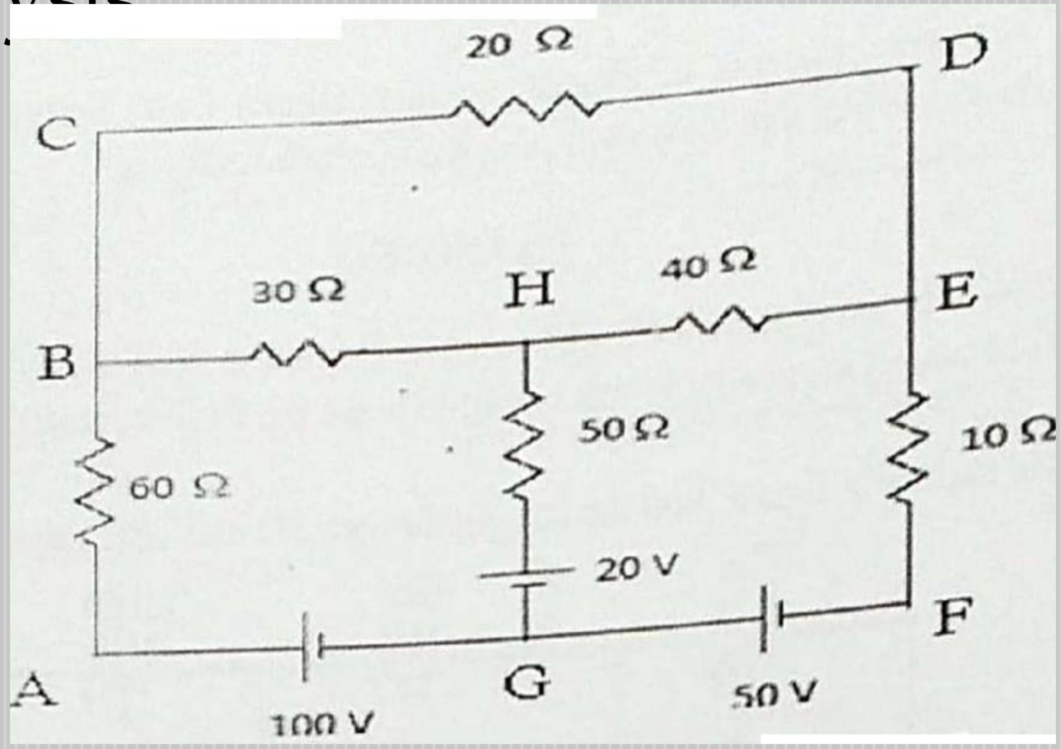
$I_2 = 0.659$ A

$I_3 = 1.068$ A

Galvanometer Current $= I_1 - I_2$

$= 0.681 - 0.659 = \underline{\underline{0.022\text{ A}}}$

6. Calculate the current in each branch using mesh analysis



$$140I_1 - 50I_2 - 30I_3 = 80 \quad - (1)$$

$$-50I_1 + 100I_2 - 40I_3 = 70 \quad - (2)$$

$$-30I_1 - 40I_2 + 90I_3 = 0 \quad - (3)$$

$I_1 = 1.649\,A$, $I_2 = 2.12\,A$, $I_3 = 1.493\,A$

$I_{20\Omega} = I_3 = 1.493\,A$

$I_{30\Omega} = I_1 - I_3 = 0.156\,A$

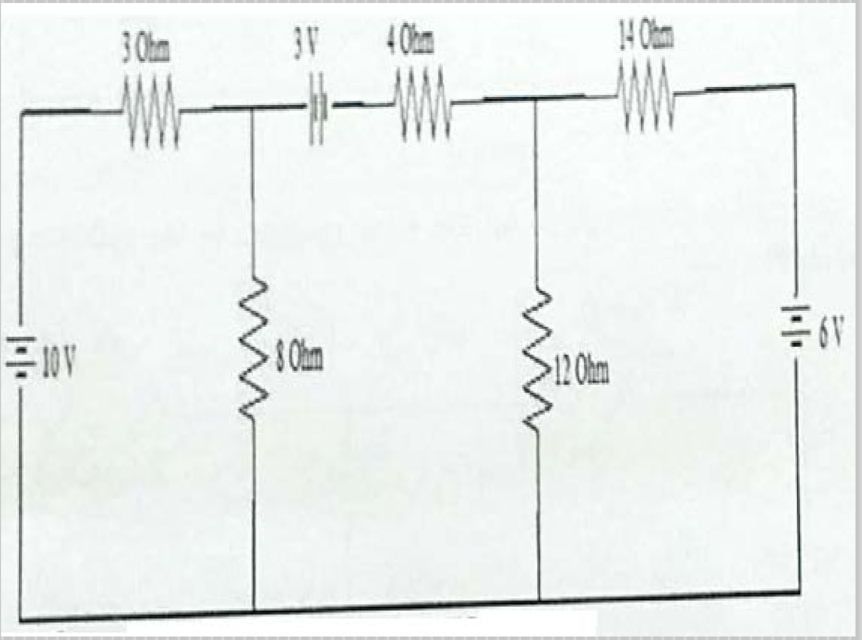
$I_{40\Omega} = I_2 - I_3 = 0.627\,A$

$I_{60\Omega} = I_1 = 1.649\,A$

$I_{50\Omega} = I_2 - I_1 = 0.471\,A$

$I_{10\Omega} = I_2 = 2.12\,A$

3. Find the current through 8ohm and 12ohm resistors using mesh analysis.



$$11 I_1 - 8 I_2 = 10 \quad - (1)$$

$$-8 I_1 + 24 I_2 - 12 I_3 = -3 \quad - (2)$$

$$-12 I_2 + 26 I_3 = -6 \quad - (3)$$

$$I_1 = 0.996\ A$$

$$I_2 = 0.119\ A$$

$$I_3 = -0.176\ A$$

$$I_{8\Omega} = I_1 - I_2 = 0.996 - 0.119 = 0.877\ A$$

$$I_{11\Omega} = I_2 - I_3 = 0.119 + 0.176 = 0.295\ A$$

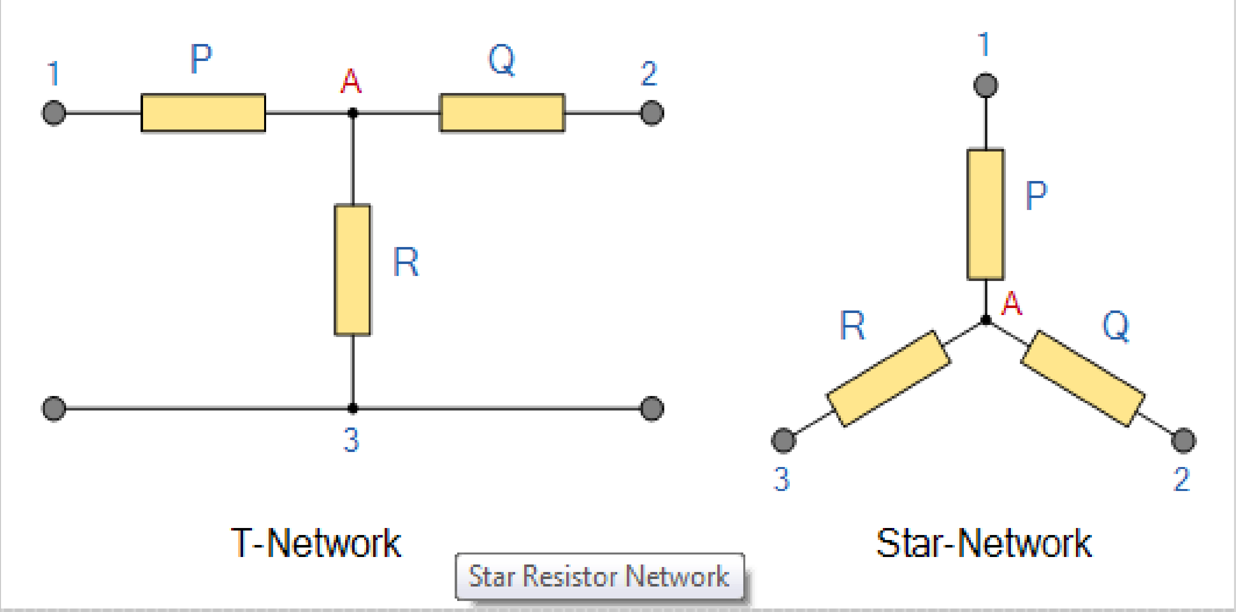
Y-Δ Transformation

- **Star** connected network which has the symbol of the letter, Y (wye)
- **Delta** connected network which has the symbol of a triangle, Δ (delta).

Y- Δ Transformation and Δ- Y Transformation allows us to convert one type of circuit connection into another type in order for us to easily analyse the circuit.

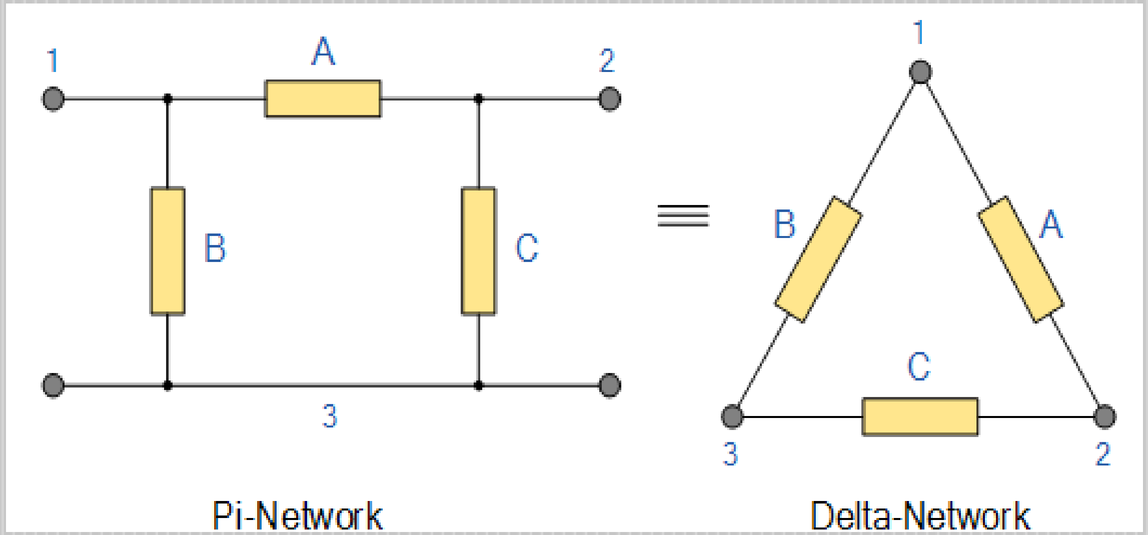
Star or Y type network

- A resistive network consisting of three impedances can be connected together to form a T or “Tee” configuration but the network can also be redrawn to form a **Star** or Y type network as shown below.

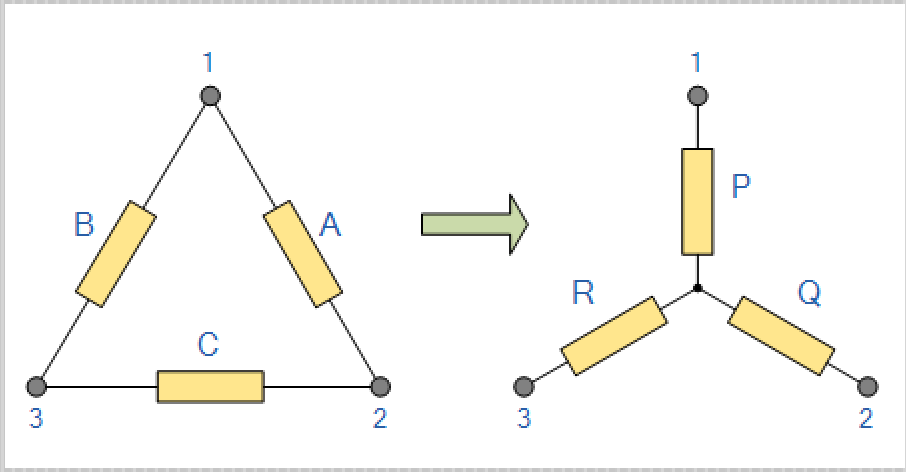


Delta or π type network

- we can also convert a Pi or π type resistor network into an electrically equivalent **Delta** or Δ type network



Delta to star transformation

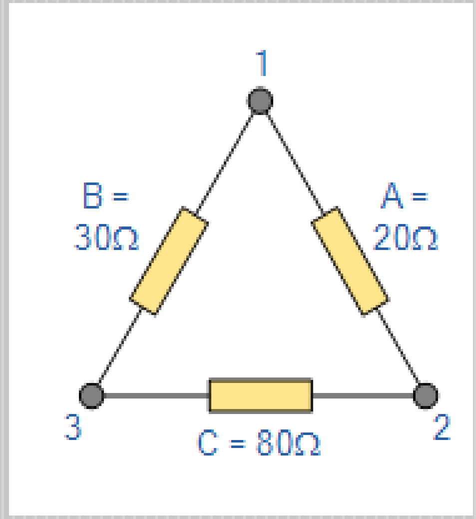


$$P = \frac{AB}{A+B+C}$$

$$Q = \frac{AC}{A+B+C}$$

$$R = \frac{BC}{A+B+C}$$

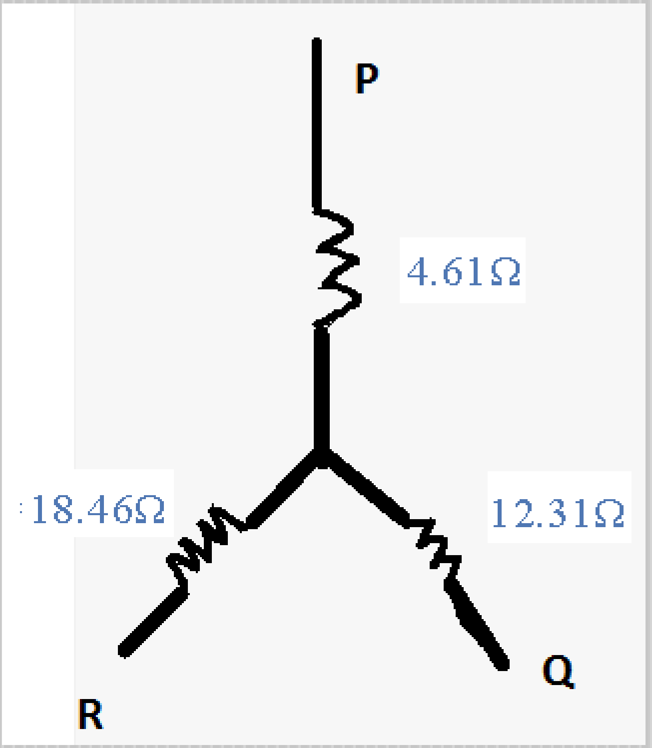
Eg:-



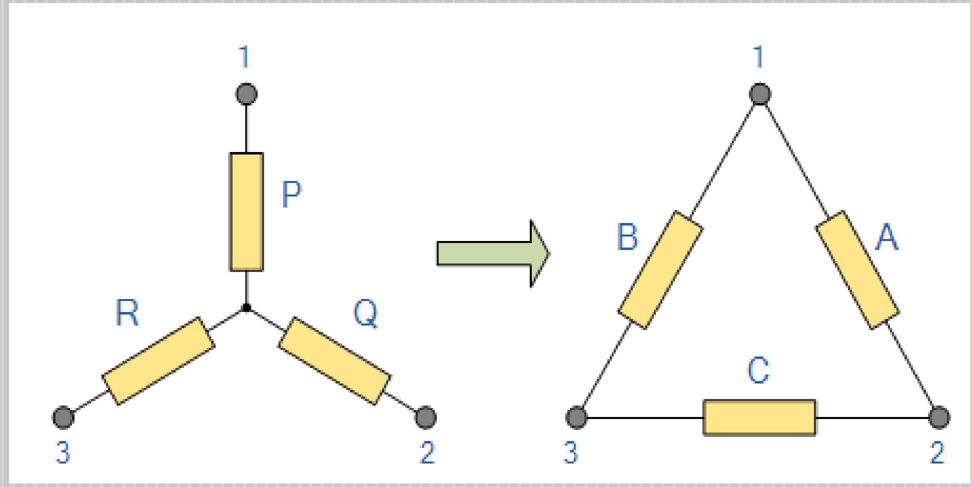
$$P = \frac{AB}{A+B+C}$$

$$Q = \frac{AC}{A+B+C}$$

$$R = \frac{BC}{A+B+C}$$



Star to Delta Transformation

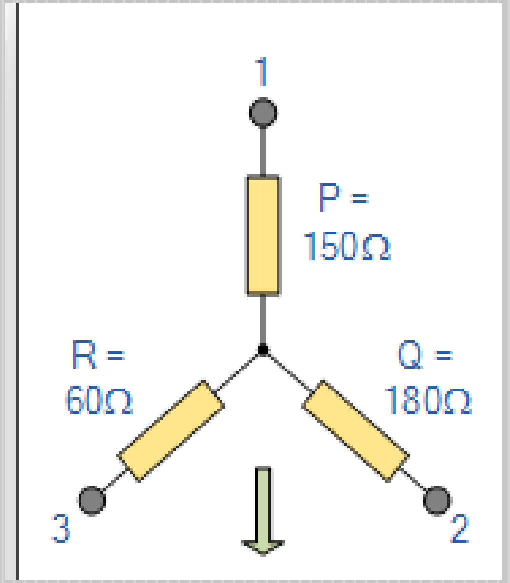


$$A = \frac{PQ + QR + RP}{R}$$

$$B = \frac{PQ + QR + RP}{Q}$$

$$C = \frac{PQ + QR + RP}{P}$$

Eg.



$$A = \frac{PQ + QR + RP}{R}$$

$$B = \frac{PQ + QR + RP}{Q}$$

$$C = \frac{PQ + QR + RP}{P}$$