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10/11/2020

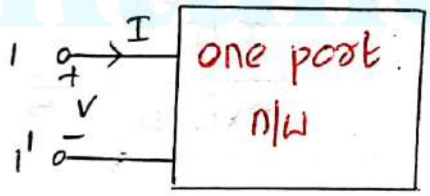
MODULE-4

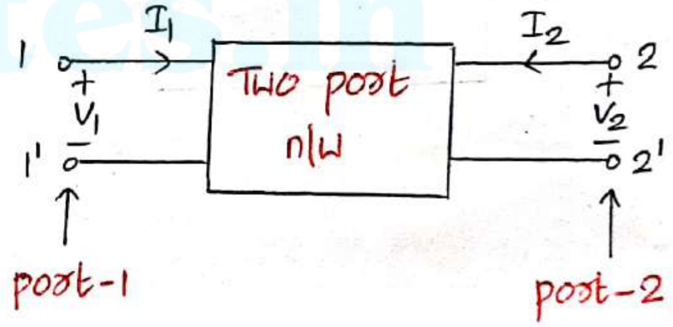
NETWORK FUNCTIONS

A network function gives the relation between currents or voltages at different parts of the network.

Network functions
- → Driving point functions
- → Transfer functions.

→ A network may be schematically represented by a rectangular box. Terminals are used to connect any n/w to any other n/w or for taking measurements. Two such terminals are called a port. Network with one pair of terminals or single port is called a one-port n/w. If there are two ports, it is called a two-port n/w.





→ The port to which energy source (i/p) is connected is called input port and the port to which load is connected is called the o/p port.

→ V_1 and I_1 are the voltage and current assigned to input port and V_2 and I_2 assigned to o/p port. I_1 and I_2 are entering into the n/w at upper terminals 1 and 2 respectively.

DRIVING POINT FUNCTIONS

If excitation and response are measured at the same port, the n/w function is known as the driving point function.

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→ Since for a one port n/w, only a single voltage and current are specified, only one n/w function (and its reciprocal) can be defined.

1. Driving-point impedence function ($Z(s)$)

It is the ratio of voltage transform at one port to the current transform at the same port.

$$Z(s) = \frac{V(s)}{I(s)}$$

2. Driving point admittance function - $Y(s)$

Ratio of $I(s)$ to $V(s)$ at the same port.

$$Y(s) = \frac{I(s)}{V(s)}$$

For a two port n/w, the driving point impedence function and driving point admittance function at port-1 are

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}, \quad Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

Similarly, at port 2

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}, \quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

TRANSFER FUNCTIONS

The transfer function is used to describe networks which have atleast two ports. It relates a voltage or current at one port to the voltage or current at another port. These functions are also defined as the ratio of response transform to an excitation transform. Thus, there are four possible forms of transfer functions.

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1. Voltage transfer function

It is the ratio of voltage transform at one port to the voltage transform at another port. Denoted by $G(s)$.

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)} \; ; \; G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

2. Current transfer function – $\alpha(s)$

Ratio of current transform at one port to the current transform at another port.

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)} \; ; \; \alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

3. Transfer Impedence function – $Z(s)$

Ratio of voltage transform at one port to the current transform at another port.

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)} \; ; \; Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

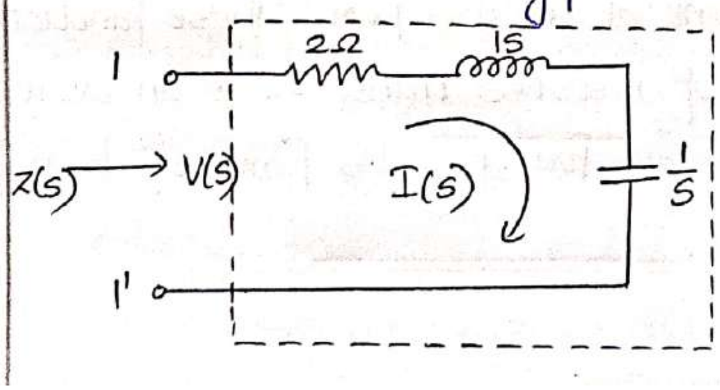
4. Transfer admittance function – $Y(s)$

Ratio of the current transform at one port to the voltage transform at another port.

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)} \; ; \; Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

PROBLEMS

1. For the n/w, obtain driving point impedence.



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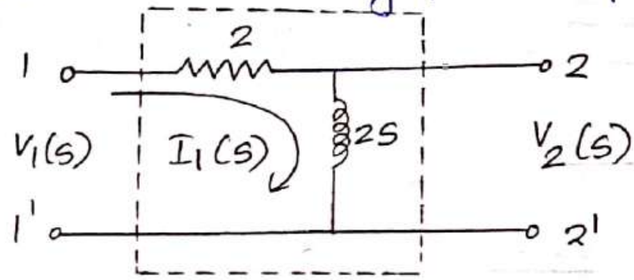
$$Z(s) = \frac{V(s)}{I(s)} = 2+s+\frac{1}{s} = \frac{2s+s^2+1}{s} = \frac{(s+1)^2}{s}$$

2. For the n/w shown below obtain the transfer functions $G_{21}(s)$, $Z_{21}(s)$ and the driving point impedence $Z_{11}(s)$.



$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2s}{2+2s} = \frac{s}{s+1}$$

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = 2s$$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = 2+2s = 2(s+1)$$

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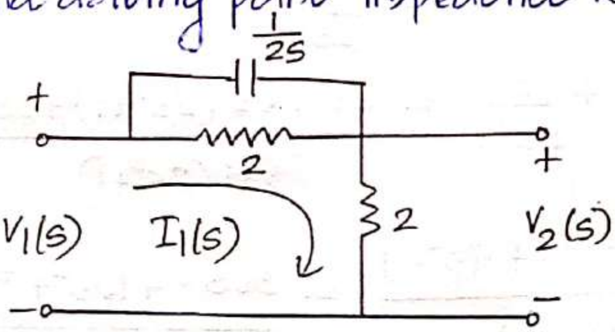
3. For the n/w shown below, obtain the transfer functions $G_{21}(s)$, $Z_{21}(s)$ and driving point impedence $Z_{11}(s)$.



Parallel combination of $\frac{1}{2s}$ and 2, $\frac{2 \times \frac{1}{2s}}{2+\frac{1}{2s}} = \frac{2}{4s+1}$

$$V_1(s) = I_1(s)\left[\frac{2}{4s+1}+2\right] = I_1(s)\left[\frac{2+8s+2}{4s+1}\right] = I_1(s)\left[\frac{8s+4}{4s+1}\right]$$

$$V_2(s) = 2I_1(s)$$

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Transfer functions

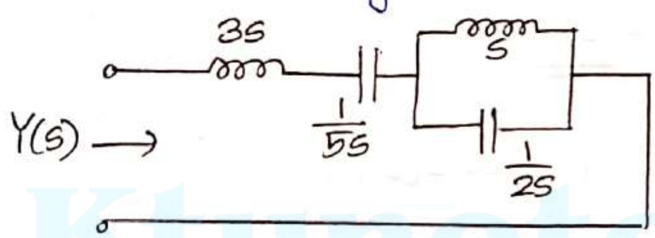
$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2I_1(s)}{I_1(s)\left[\frac{8s+4}{4s+1}\right]} = \frac{4s+1}{4s+2}$$

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = 2$$

Driving point functions

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = \frac{8s+4}{4s+1}$$

4. ~~Re~~ Find the driving point admittance of the following n/w.



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$$Z(s) = 3s + \frac{1}{5s} + \frac{s \times \frac{1}{2s}}{s + \frac{1}{2s}}$$

$$= 3s + \frac{1}{5s} + \frac{s}{2s^2+1} = \frac{3s + 5s(2s^2+1) + 2s^2 + 1 + 5s^2}{5s(2s^2+1)}$$

$$= \frac{15s^2(2s^2+1) + 7s^2 + 1}{5s(2s^2+1)} = \frac{30s^4 + 15s^2 + 7s^2 + 1}{5s(2s^2+1)}$$

$$= \frac{30s^4 + 22s^2 + 1}{5s(2s^2+1)}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{5s(2s^2+1)}{30s^4 + 22s^2 + 1}$$

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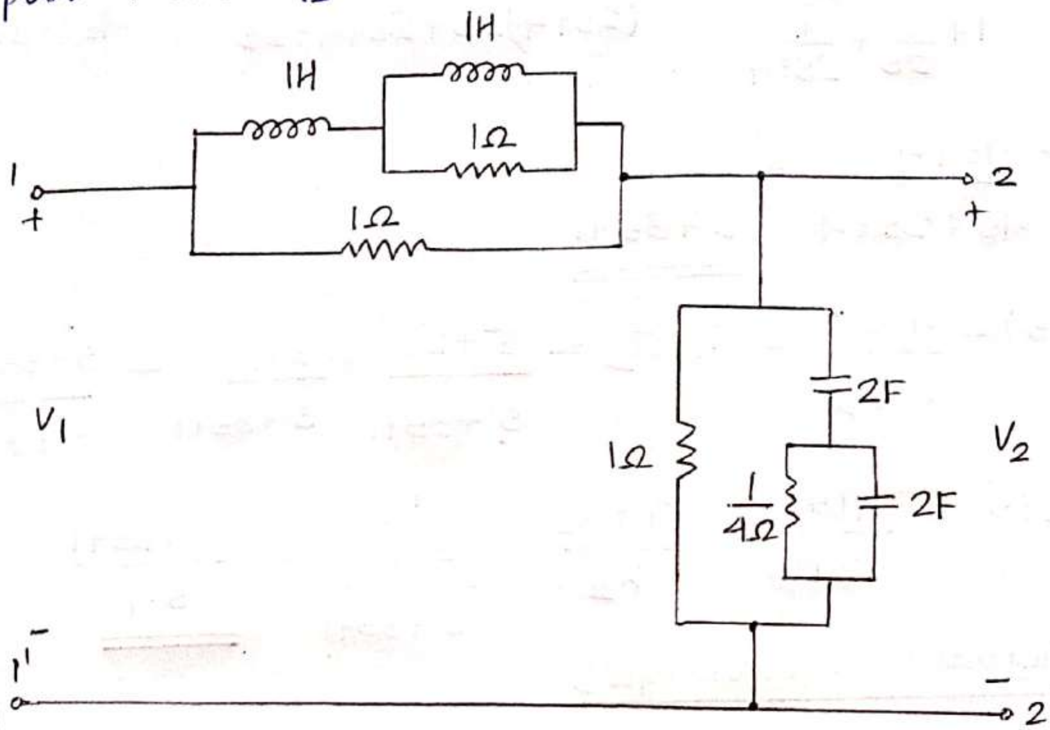
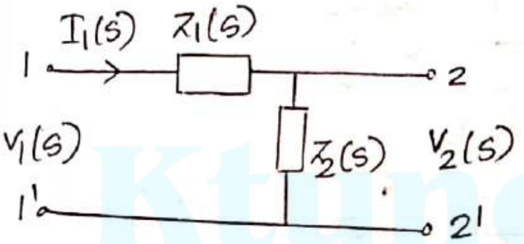
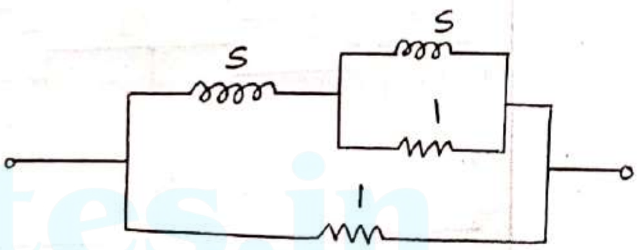
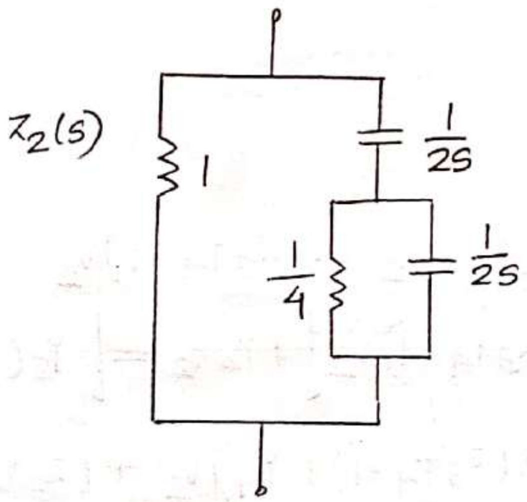
11/11/2020 1. For the following n/w find the driving point impedence at port-1 and G_{12}.







$Z_1(s)$



$$Z_1(s) = \frac{1 \times \left(s + \frac{s}{s+1}\right)}{1 + s + \frac{s}{s+1}}$$

$$= \frac{s^2+s+s}{(s+1)^2+s} = \frac{s^2+2s}{s^2+2s+1+s}$$

$$= \frac{s^2+2s}{s^2+3s+1}$$

$$Z_2(s) = \frac{1 \times \left(\frac{1}{2s} + \frac{\frac{1}{4} \times \frac{1}{2s}}{\frac{1}{4} + \frac{1}{2s}}\right)}{1 + \frac{1}{2s} + \frac{\frac{1}{4} \times \frac{1}{2s}}{\frac{1}{4} + \frac{1}{2s}}} = \frac{\frac{1}{2s} + \frac{\frac{1}{8s}}{\frac{2s+4}{8s}}}{1 + \frac{1}{2s} + \frac{\frac{1}{8s}}{\frac{2s+4}{8s}}}$$

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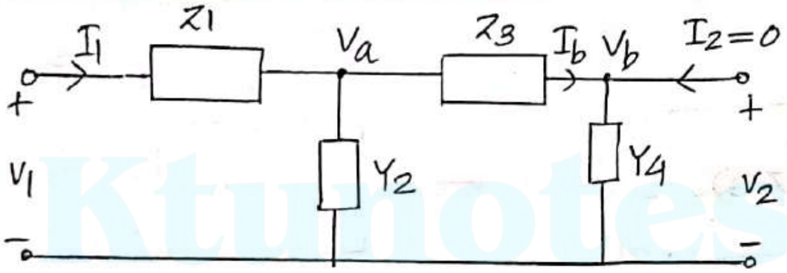
$$= \frac{\frac{1}{2s}+\frac{1}{2s+4}}{1+\frac{1}{2s}+\frac{1}{2s+4}} = \frac{2s+4+2s}{(2s+4)2s+2s+4+2s} = \frac{4s+4}{4s^2+8s+4s+4}$$

$$= \frac{4s+4}{4s^2+12s+4} = \frac{s+1}{s^2+3s+1}$$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = Z_1 + Z_2 = \frac{s^2+2s}{s^2+3s+1} + \frac{s+1}{s^2+3s+1} = \frac{s^2+3s+1}{s^2+3s+1} = 1$$

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)} = \frac{Z_1+Z_2}{Z_2} = \frac{1}{\frac{s+1}{s^2+3s+1}} = \frac{s^2+3s+1}{s+1}$$

ANALYSIS OF LADDER N/W S



Ladder n/w

$V_b = V_2$

$I_b = Y_4 V_2$

$V_a = Z_3 I_b + V_2 = Z_3 Y_4 V_2 + V_2 = (Z_3 Y_4 + 1) V_2$

$I_1 = Y_2 V_a + I_b = [Y_2 (Z_3 Y_4 + 1) V_2] + Y_4 V_2 = [Y_2 (Z_3 Y_4 + 1) + Y_4] V_2$

$V_1 = Z_1 I_1 + V_a = Z_1 \{Y_2 (Z_3 Y_4 + 1) + Y_4\} V_2 + (Z_3 Y_4 + 1) V_2$

$= [Z_1 \{Y_2 (Z_3 Y_4 + 1) + Y_4\} + (Z_3 Y_4 + 1)] V_2$

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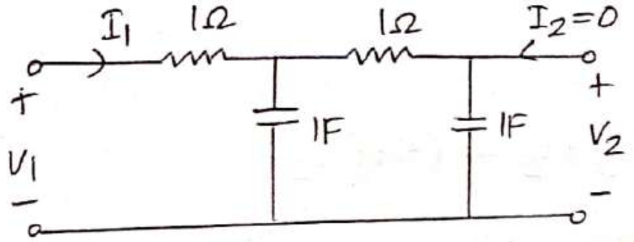
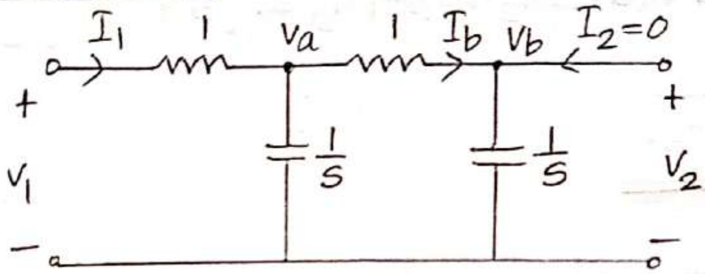
1. For the n/w determine transfer function $\frac{V_2}{V_1}$



Transformed n/w



$$V_b = V_2$$

$$I_b = \frac{V_2}{1/s} = sV_2$$

$$V_a = 1 I_b + V_2 = sV_2 + V_2 = (s+1)V_2$$

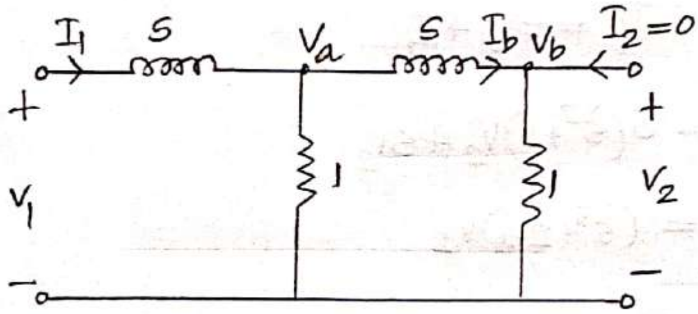
$$I_1 = \frac{V_a}{1/s} + I_b = sV_a + I_b = s(s+1)V_2 + sV_2$$

$$= [s^2+s+s]V_2 = [s^2+2s]V_2$$

$$V_1 = 1 I_1 + V_a = [s^2+2s]V_2 + (s+1)V_2 = (s^2+3s+1)V_2$$

$$\frac{V_2}{V_1} = \frac{1}{s^2+3s+1}$$

2. For the n/w shown below determine the transfer function $\frac{V_2}{V_1}$.



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$V_b = V_2$

$I_b = \frac{V_2}{1} = V_2$

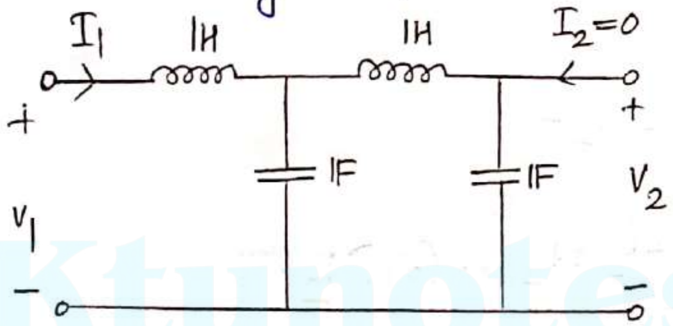
$V_a = sI_b + V_2 = sV_2 + V_2 = (s+1)V_2$

$I_1 = \frac{V_a}{1} + I_b = (s+1)V_2 + V_2 = (s+2)V_2$

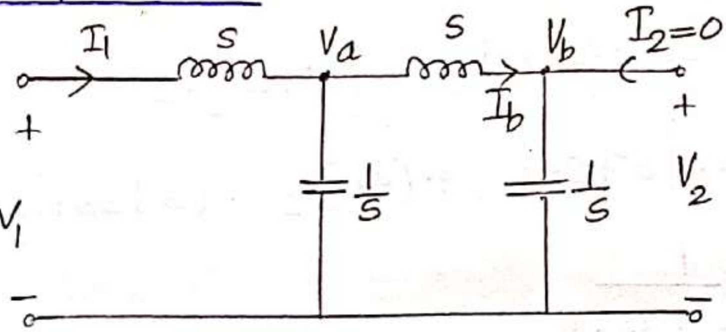
$V_1 = sI_1 + V_a = s(s+2)V_2 + (s+1)V_2 = [s^2+3s+1]V_2$

$$\frac{V_2}{V_1} = \frac{1}{s^2+3s+1}$$

3. For the following n/w find the network functions $\frac{V_1}{I_1}$, $\frac{V_2}{V_1}$ and $\frac{V_2}{I_1}$.



Transformed n/w



$V_b = V_2$

$I_b = \frac{V_2}{1/s} = sV_2$

$V_a = sI_b + V_2$

$= s(sV_2) + V_2$

$= (s^2+1)V_2$

$I_1 = \frac{V_a}{1/s} + I_b = sV_a + I_b$

$= s(s^2+1)V_2 + sV_2$

$= (s^3+2s)V_2$

$V_1 = sI_1 + V_a = s(s^3+2s)V_2 + (s^2+1)V_2$

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$$= (s^4 + 2s^2 + s^2 + 1) V_2$$

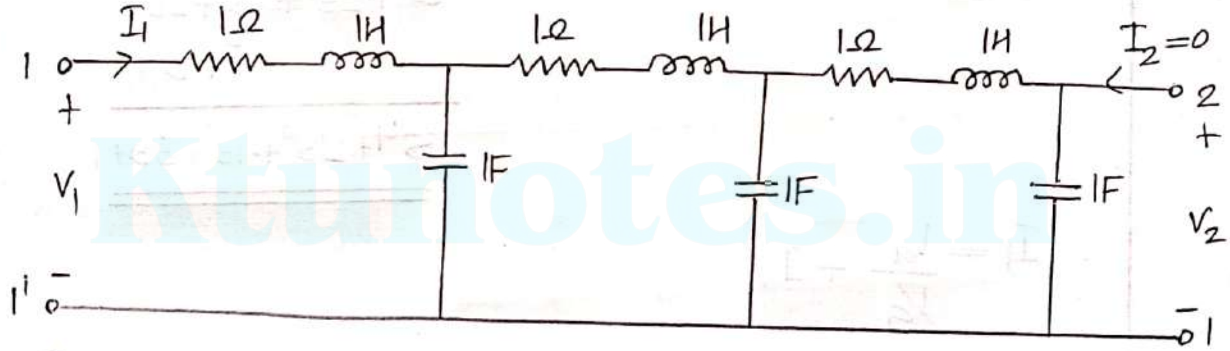
$$= (s^4 + 3s^2 + 1) V_2$$

$$\frac{V_1}{I_1} = \frac{s^4 + 3s^2 + 1}{s^3 + 2s}$$

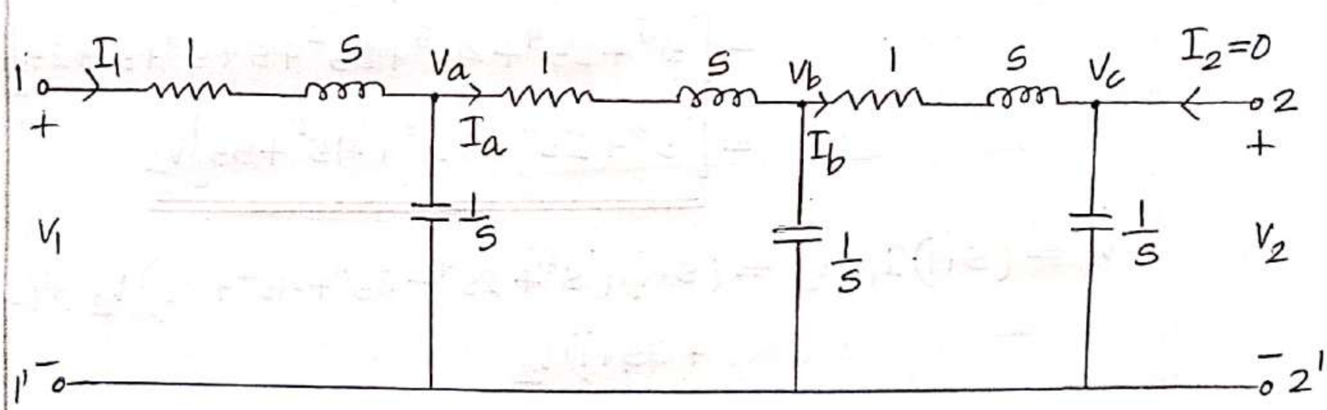
$$\frac{V_2}{V_1} = \frac{1}{s^4 + 3s^2 + 1}$$

$$\frac{V_2}{I_1} = \frac{1}{s^3 + 2s}$$

4. For the following ladder n/w find the driving point impedence at the 1-1' terminal with 2-2' open.



Transformed n/w



$$V_c = V_2$$

$$I_b = \frac{V_2}{1/s} = sV_2$$

$$V_b = (s+1) I_b + V_2 = (s+1) sV_2 + V_2 = (s^2 + s + 1) V_2$$

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$$I_a = \frac{V_b}{1/s} + I_b = sV_b + I_b = s(s^2+s+1)V_2 + sV_2$$

$$= [s^3+s^2+s+s]V_2$$

$$= (s^3+s^2+2s)V_2$$

$$V_a = (s+1)I_a + V_b$$

$$= (s+1)(s^3+s^2+2s)V_2 + (s^2+s+1)V_2$$

$$= (s^4+2s^3+4s^2+3s+1)V_2$$

$$\begin{array}{r} s^3+s^2+2s \\ s+1 \\ \hline s^3+s^2+2s \\ s^4+s^3+2s^2 \\ \hline s^4+2s^3+3s^2+2s+ \\ s^2+s+1 \\ \hline s^4+2s^3+4s^2+3s+1 \\ \hline\hline \end{array}$$

$$I_1 = \frac{V_a}{1/s} + I_a$$

$$= sV_a + I_a = s(s^4+2s^3+4s^2+3s+1)V_2 + (s^3+s^2+2s)V_2$$

$$= [s^5+2s^4+4s^3+3s^2+s+s^3+s^2+2s]V_2$$

$$= \underline{\underline{[s^5+2s^4+5s^3+4s^2+3s]V_2}}$$

$$V_1 = (s+1)I_1 + V_a = (s+1)(s^5+2s^4+5s^3+4s^2+3s)V_2 + (s^4+2s^3+4s^2+3s+1)V_2$$

$$= (s^6+3s^5+8s^4+11s^2+6s+1)V_2$$

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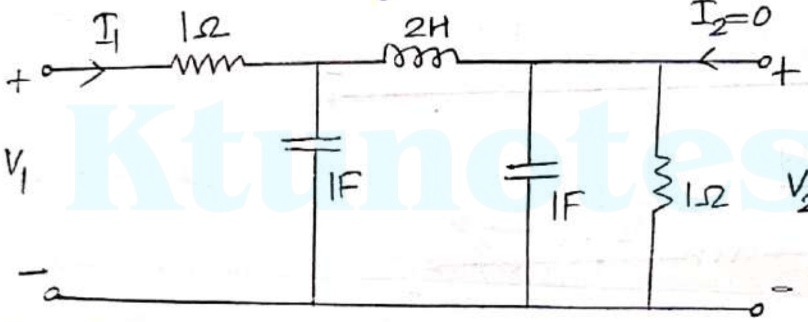
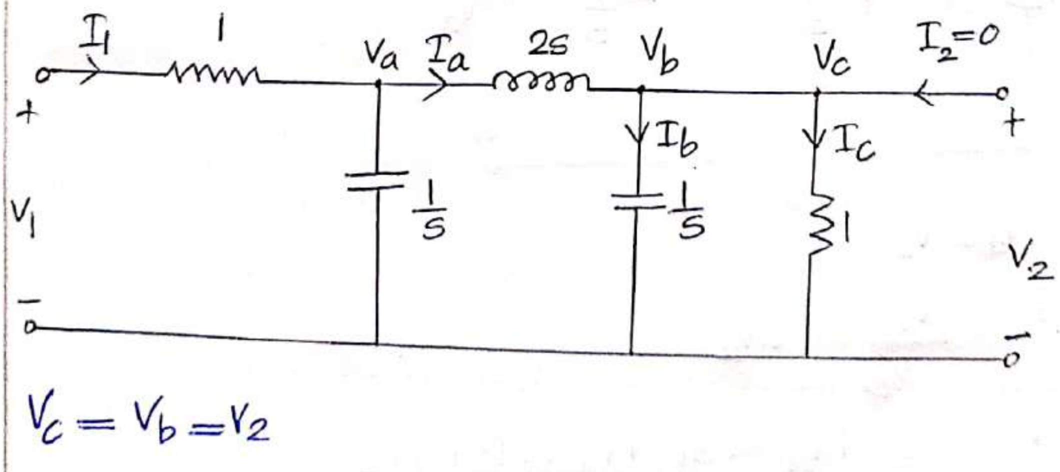
$$\begin{array}{r} s^5+2s^4+5s^3+4s^2+3s \quad \times \\ s+1 \\ \hline s^5+2s^4+5s^3+4s^2+3s \\ s^6+2s^5+5s^4+4s^3+3s^2 \\ \hline s^6+3s^5+7s^4+9s^3+7s^2+3s \quad + \\ s^4+2s^3+4s^2+3s+1 \\ \hline s^6+3s^5+8s^4+11s^3+11s^2+6s+1 \end{array}$$

$$z_{11} = \frac{V_1}{I_1} = \frac{s^6+3s^5+8s^4+11s^3+11s^2+6s+1}{s^5+2s^4+5s^3+4s^2+3s}$$

5. Determine the voltage transfer function $\frac{V_2}{V_1}$ for the N/W shown below.



Transformed n/w



$V_c = V_b = V_2$

$$I_a = I_b + I_c = \frac{V_2}{1/s} + \frac{V_2}{1} = sV_2 + V_2 = (s+1)V_2$$

$$V_a = 2sI_a + V_2 = 2s(s+1)V_2 + V_2 = (2s^2+2s+1)V_2$$

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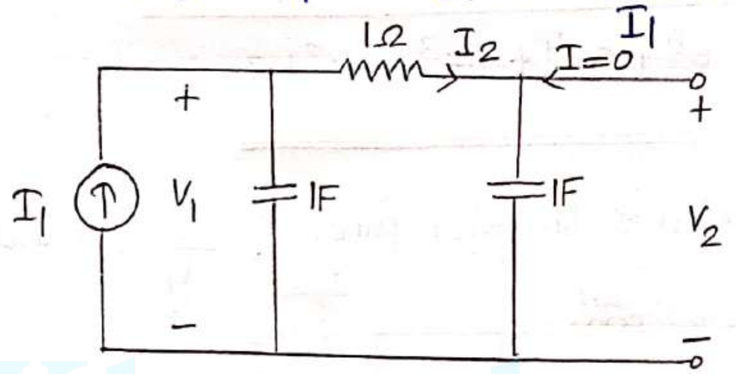
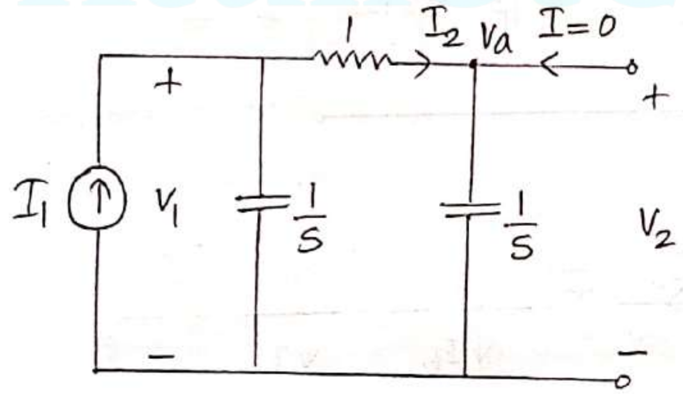
$$I_1 = \frac{V_a}{1/s} + I_a = sV_a + I_a = s(2s^2+2s+1)V_2 + (s+1)V_2$$
$$= (2s^3+2s^2+s+s+1) = (2s^3+2s^2+2s+1)V_2$$
$$V_1 = 1\cdot I_1 + V_a = (2s^3+2s^2+2s+1)V_2 + (2s^2+2s+1)V_2$$
$$= (2s^3+4s^2+4s+2)V_2$$

Hence $$\frac{V_2}{V_1} = \frac{1}{2s^3+4s^2+4s+2}$$

6. For the n/w, compute $\alpha_{21}(s) = \dfrac{I_2}{I_1}$ and $z_{21}(s) = \dfrac{V_2}{I_1}$



Transformed n/w



$$V_a = V_2$$
$$I_2 = \frac{V_2}{1/s} = sV_2$$
$$V_1 = 1\cdot I_2 + V_a = sV_2 + V_2 = (s+1)V_2$$
$$I_1 = \frac{V_1}{1/s} + I_2 = sV_1 + I_2 = s(s+1)V_2 + sV_2 = (s^2+2s)V_2$$
$$\alpha_{21}(s) = \frac{I_2}{I_1} = \frac{sV_2}{(s^2+2s)V_2} = \frac{1}{s+2}$$

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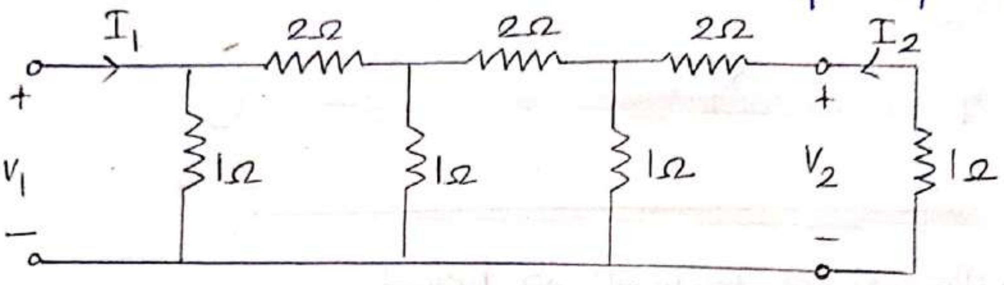
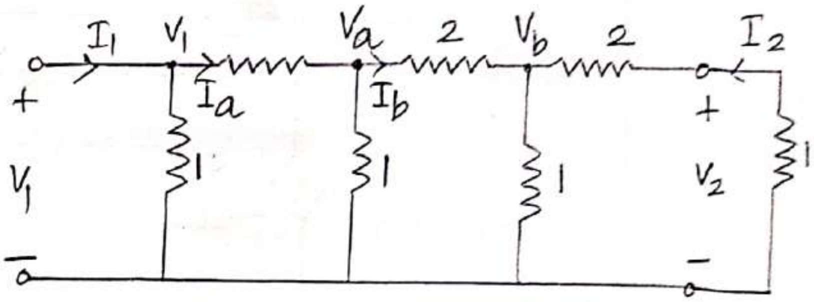
$$Z_{21}(s) = \frac{V_2}{I_1} = \frac{1}{s^2+2s}$$

12/11/2020 7. For the resistive two port n/w, find $\frac{V_2}{V_1}$, $\frac{V_2}{I_1}$, $\frac{I_2}{V_1}$ and $\frac{I_2}{I_1}$.





$$I_2 = \frac{-V_2}{1} = -V_2$$

$$V_b = -3I_2 = 3V_2$$

$$I_b = \frac{V_b}{1} + \frac{V_b}{3} = \frac{4V_b}{3} = 4V_2$$

$$V_a = 2I_b + V_b = 8V_2 + 3V_2 = 11V_2$$

$$I_a = \frac{V_a}{1} + I_b = 11V_2 + 4V_2 = 15V_2$$

$$V_1 = 2I_a + V_a = 2\times 15V_2 + 11V_2 = 41V_2$$

$$I_1 = \frac{V_1}{1} + I_a = 41V_2 + 15V_2 = 56V_2$$

$$\frac{V_2}{V_1} = \frac{1}{41}$$

$$\frac{V_2}{I_1} = \frac{1}{56}\,\Omega$$

$$\frac{I_2}{V_1} = \frac{-1}{41}\,S$$

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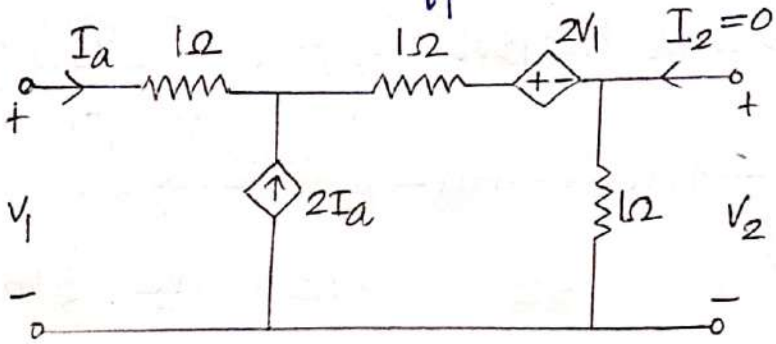
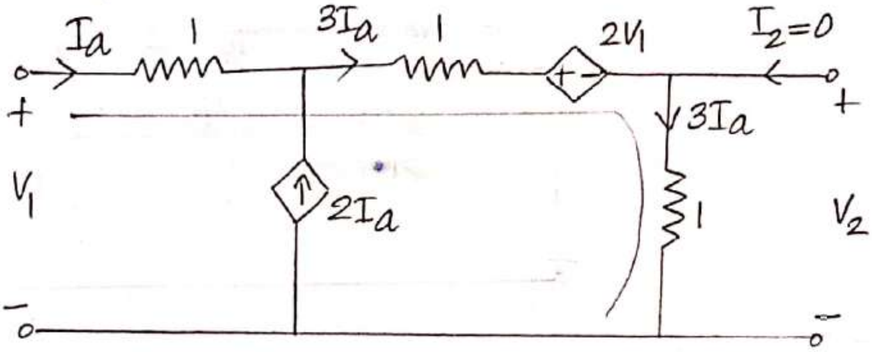
$$\frac{I_2}{I_1} = -\frac{1}{56}$$

8. Find the n/w function $\frac{V_2}{V_1}$ for the n/w.



The n/w can be redrawn as below:



$$V_2 = 1 \cdot 3I_a = 3I_a$$

Applying KVL in the outermost loop,

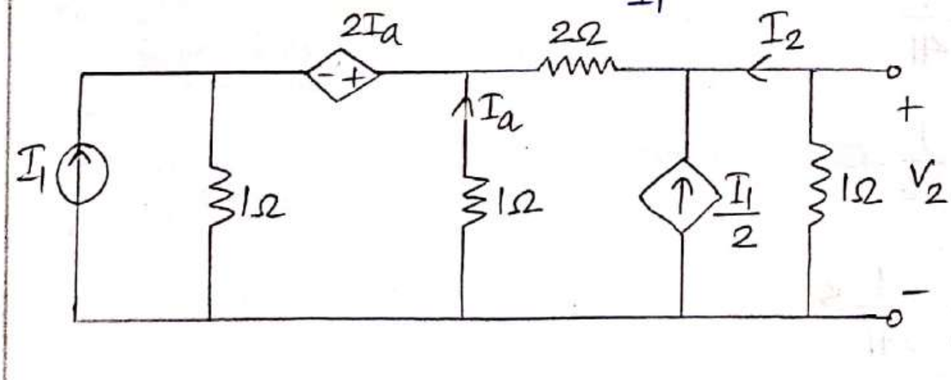
$$-V_1 + 1(I_a) + 1(3I_a) + 2V_1 + 1(3I_a) = 0$$

$$V_1 + 7I_a = 0$$

$$V_1 = -7I_a$$

$$\frac{V_2}{V_1} = \frac{3I_a}{-7I_a} = -\frac{3}{7}$$

9. Find the network function $\frac{I_2}{I_1}$ for the n/w.



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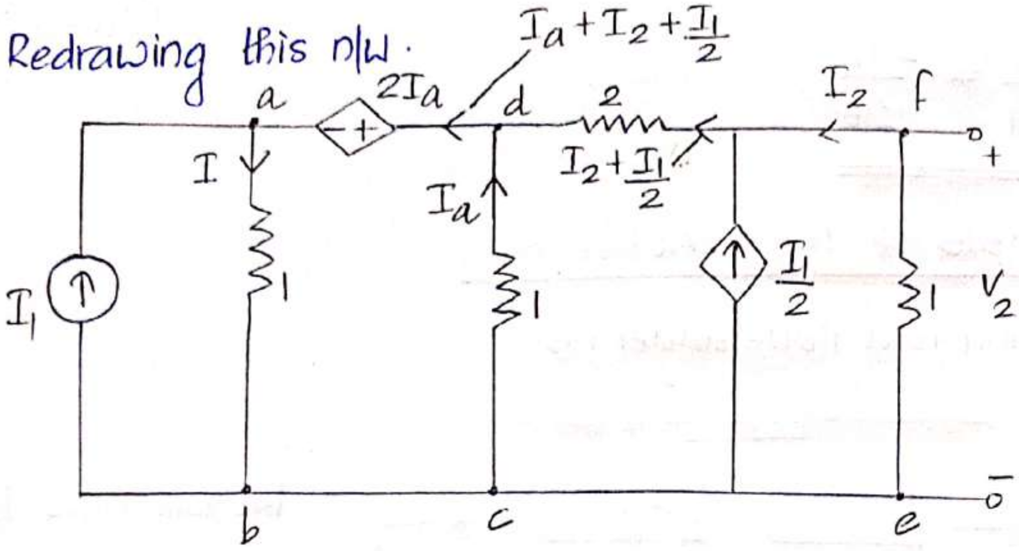
Redrawing this n/w.



$$I = I_1 + I_a + I_2 + \frac{I_1}{2}$$

$$= \frac{3}{2} I_1 + I_a + I_2 \quad \text{---} (1)$$

Applying KVL to loop abcda,

$$-1 \cdot I - 2I_a - I_a = 0$$

$$I + 3I_a = 0 \quad \text{---} (2)$$

Substituting for (2) from (1),

$$\frac{3}{2} I_1 + I_a + I_2 + 3I_a = 0$$

$$\frac{3}{2} I_1 + I_2 + 4I_a = 0 \quad \text{---} (3)$$

Applying KVL to the loop dcefd,

$$1 \cdot I_a - 2\left(I_2 + \frac{I_1}{2}\right) - 1 \cdot I_2 = 0$$

$$I_a - 3I_2 - I_1 = 0$$

$$I_a = I_1 + 3I_2 \quad \text{---} (4)$$

Substituting (4) in (3),

$$\frac{3}{2} I_1 + I_2 + 4(I_1 + 3I_2) = 0$$

$$\frac{3}{2} I_1 + I_2 + 4I_1 + 12I_2 = 0$$

$$\frac{11 I_1}{2} + 13 I_2 = 0$$

$$13 I_2 = \frac{-11 I_1}{2}$$

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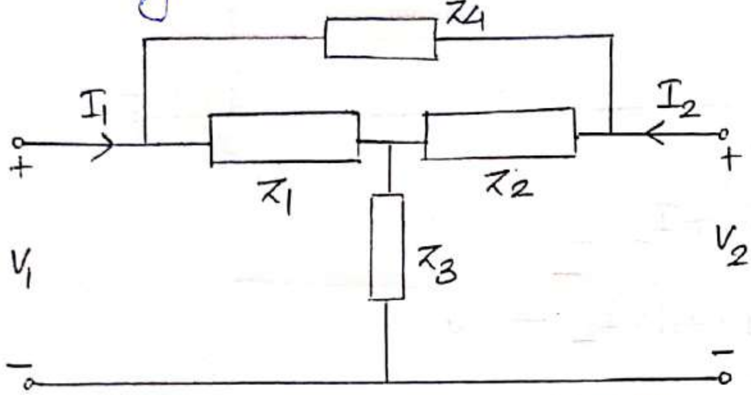
$$\therefore \frac{I_2}{I_1} = \frac{-11}{26}$$

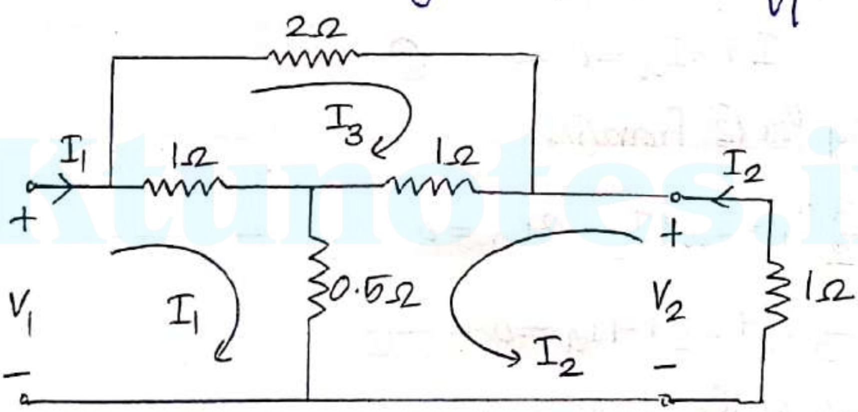
ANALYSIS OF NON-LADDER N/WS

Following is a non-ladder n/w.



We will have to apply KVL or KCL. Then n/w functions are to be represented as a quotient of determinants.

1. For the resistive bridged T-n/w. find $\frac{V_2}{V_1}$, $\frac{V_2}{I_1}$, $\frac{I_2}{V_1}$ and $\frac{I_2}{I_1}$.



Mesh-1 : $V_1 = 1.5I_1 + 0.5I_2 - I_3$ —— ① $\quad 1.5I_1 + 0.5I_2 - I_3 = V_1$

Mesh-2 : $0.5I_1 + 2.5I_2 + I_3 = 0$ —— ②

Mesh-3 : $-I_1 + I_2 + 4I_3 = 0$ —— ③

$$I_1 = \frac{\begin{vmatrix} V_1 & 0.5 & -1 \\ 0 & 2.5 & 1 \\ 0 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 1.5 & 0.5 & -1 \\ 0.5 & 2.5 & 1 \\ -1 & 1 & 4 \end{vmatrix}} = \frac{10V_1 - V_1}{15 - 0.5 - 0.5 - 2.5 - 1.5 - 1} = \frac{9V_1}{9} = V_1 \text{ —— ④}$$

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$$I_2 = \frac{\begin{vmatrix} 1.5 & V_1 & -1 \\ 0.5 & 0 & 1 \\ -1 & 0 & 4 \end{vmatrix}}{9} = \frac{-V_1 - 2V_1}{9} = \frac{-V_1}{3} \quad \text{—— (5)}$$

$$V_2 = 1(-I_2) = -I_2$$

From (5), $V_1 = -3I_2$

From (4), $I_1 = V_1 = -3I_2$

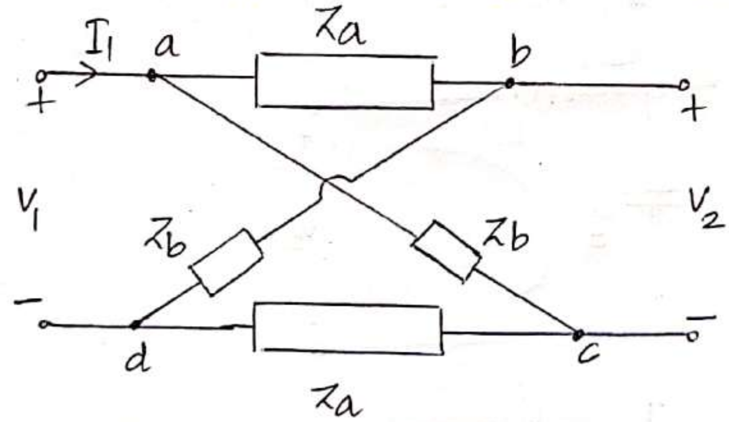
$$\frac{I_2}{V_1} = \frac{-1}{3}\,S$$

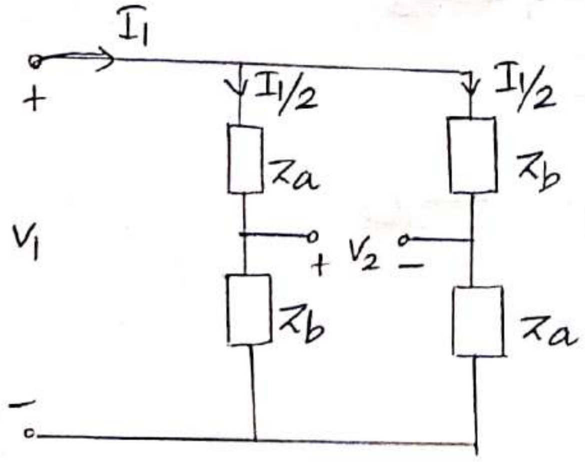
$$\frac{I_2}{I_1} = -\frac{1}{3}$$

$$\frac{V_2}{V_1} = \frac{-I_2}{-3I_2} = \frac{1}{3}$$

$$\frac{V_2}{I_1} = \frac{-I_2}{-3I_2} = \frac{1}{3}\,\Omega$$

2. For the n/w, find z_{11}, z_{21} and G_{21}.





Redrawing,

$$V_1 = (z_a + z_b)\frac{I_1}{2}$$

$$z_{11} = \frac{V_1}{I_1} = \frac{z_a + z_b}{2}$$

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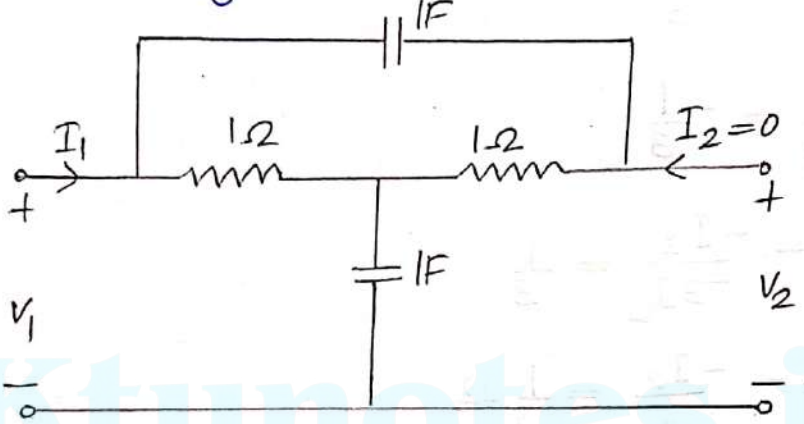
$$V_2 = -Z_a\frac{I_1}{2} + Z_b\frac{I_1}{2} = \frac{(Z_b - Z_a)}{2} I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \underline{\underline{\frac{Z_b - Z_a}{2}}}$$

$$G_{21} = \frac{V_2}{V_1} = \frac{\left(\frac{Z_b - Z_a}{2}\right) I_1}{\frac{(Z_a + Z_b)}{2} I_1} = \underline{\underline{\frac{Z_b - Z_a}{Z_a + Z_b}}}$$

3. For the following n/w, determine $Z_{11}(s)$, $G_{21}(s)$ and $Z_{21}(s)$.



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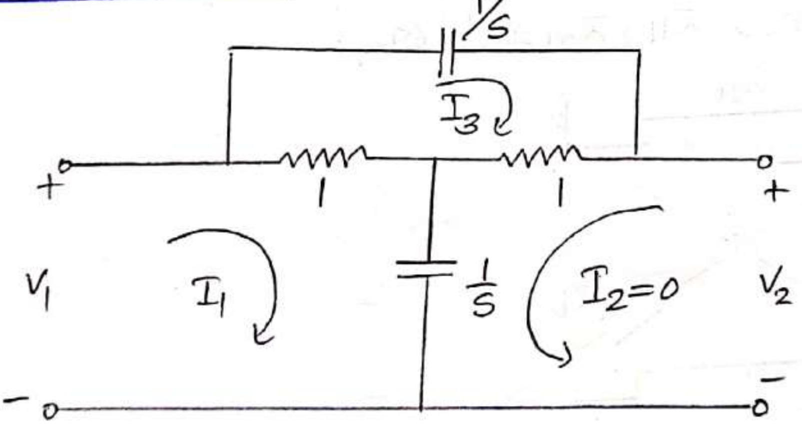
Transformed n/w



Mesh-1 : $\left(1+\frac{1}{s}\right)I_1 - I_3 = V_1$ —— (1)

Mesh-2 : $\frac{1}{s}I_1 + I_3 = V_2$ —— (2)

Mesh-3 : $-I_1 + \left(2+\frac{1}{s}\right)I_3 = 0$ —— (3)

$$I_3 = \left(\frac{s}{2s+1}\right) I_1$$

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Substituting I_3 in (1) and (2),

$$V_1 = \left(1+\frac{1}{s}\right)I_1 - \frac{s}{2s+1}I_1 = I_1\left[\frac{s+1}{s} - \frac{s}{2s+1}\right]$$

$$= I_1\left[\frac{(s+1)(2s+1)-s^2}{s(2s+1)}\right] = \frac{I_1\left[2s^2+3s+1-s^2\right]}{s(2s+1)}$$

$$= \frac{s^2+3s+1}{s(2s+1)} \cdot I_1$$

$$V_2 = \frac{1}{s}I_1 + \frac{s}{2s+1}I_1 = I_1\left[\frac{1}{s}+\frac{s}{2s+1}\right]$$

$$= I_1\left[\frac{2s+1+s^2}{s(2s+1)}\right] = I_1\frac{(s+1)^2}{s(2s+1)}$$

$$Z_{11}(s) = \frac{V_1}{I_1} = \frac{s^2+3s+1}{s(2s+1)}$$

$$G_{21}(s) = \frac{V_2}{V_1} = \frac{(s+1)^2}{s(2s+1)}I_1 \times \frac{s(2s+1)}{(s^2+3s+1)I_1}$$

$$= \frac{(s+1)^2}{(s^2+3s+1)}$$

$$Z_{21}(s) = \frac{V_2}{I_1} = \frac{(s+1)^2}{s(2s+1)}$$

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16/11/2020

POLES AND ZEROS OF NETWORK FUNCTIONS

The n/w function N(s) can be written as ratio of two polynomials.

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

Where $a_0, a_1, \dots a_n$ and $b_0, b_1, \dots b_m$ are the coefficients of polynomials P(s) and Q(s).

These are real and positive for n/ws with positive elements.

Let $P(s) = 0$ have n-roots as $z_1, z_2, \dots z_n$ and $Q(s) = 0$ have m-roots as $p_1, p_2, \dots p_m$.

Then
$$N(s) = \frac{a_n\left[s^n + \frac{a_{n-1}}{a_n} s^{n-1} + \dots + \frac{a_1}{a_n} s + \frac{a_0}{a_n}\right]}{b_m\left[s^m + \frac{b_{m-1}}{b_m} s^{m-1} + \dots + \frac{b_1}{b_m} s + \frac{b_0}{b_m}\right]}$$

$$= H \cdot \frac{(s-z_1)(s-z_2)\dots(s-z_n)}{(s-p_1)(s-p_2)\dots(s-p_m)}$$

Where $H = \frac{a_n}{b_m}$ is a constant called scale factor. At z_1 to z_n, the n/w function becomes zero and are known as zeros of network function. At p_1 to p_m the n/w function becomes infinite and are known as poles.

A network function is completely specified by its poles, zeros and scale factor. If the poles or zeros are not repeated, then the function is said to be having simple poles or simple zeros. If the poles or zeros are repeated, then the function is said to be having multiple poles and multiple zeros.

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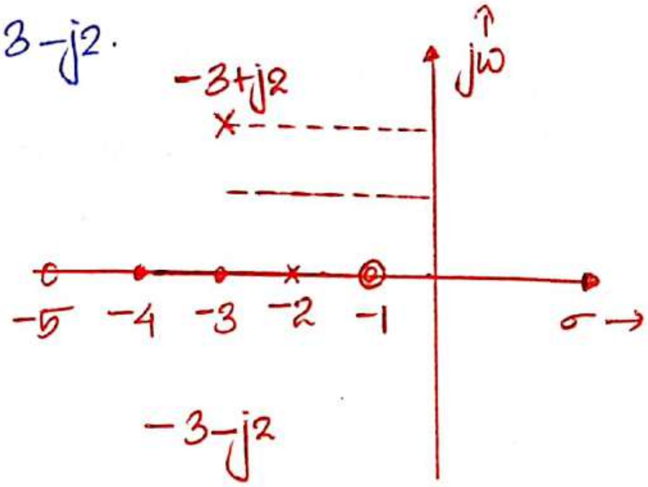
When $n>m$, then $n-m$ poles are at $s=\infty$ and for $m>n$, $(m-n)$ zeros are at $s=\infty$. Zeros are denoted by o and poles are denoted by ∞.

Consider a network function,

$$N(s) = \frac{(s+1)^2(s+5)}{(s+2)(s+3+j2)(s+3-j2)}$$

. It has two zeros at $s=-1$, one zero at $s=-5$. Three poles at $s=-2$, $s=-3+j2$ and $s=-3-j2$.



pole-zero plot

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The network function is said to be stable when the real parts of the poles and zeros are negative. Otherwise, the poles and zeros must be within the left half of the s-plane.

Significance of poles and zeros

Poles and zeros are critical frequencies. The n/w function becomes infinite at poles and zero at zeros. It has a finite non-zero value at other frequencies.

a. **Driving point impedence.**

$$Z(s) = \frac{V(s)}{I(s)}$$

A poles of $Z(s)$ means zero current for a finite voltage which means an open circuit. A zero of $Z(s)$ implies no voltage for a finite current or a short circuit.

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Consider $z(s) = \frac{1}{Cs}$.

The above function has a pole at $s=0$ and zero at $s=\infty$.

∴ The function which represents a capacitor, acts as open circuit at pole frequency and acts as short circuit at zero frequency

b. Driving point Admittance

$$Y(s) = \frac{I(s)}{V(s)}$$

A pole of $Y(s)$ implies zero voltage for a finite value of current which gives a short circuit. A zero of $Y(s)$ implies zero current for a finite value of voltage which gives an open circuit.

c. Voltage Transfer Ratio

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

$$V_2(s) = G_{21}(s) V_1(s)$$

To obtain the o/p voltage, we require the product of input and transfer function. $G_{21}(s) \cdot V_1(s)$ is obtained in the form of a ratio of polynomials in s.

By partial fractions,

$$G_{21}(s) \cdot V_1(s) = \sum_{i=1}^{n} \frac{A}{s-a_i} + \sum_{j=1}^{m} \frac{A}{s-a_j}$$ where n and m are number of poles of $G_{21}(s)$ and $V_1(s)$ respectively.

The frequencies a_i (poles of $G_{21}(s)$) from the natural complex frequencies corresponding to free oscillations and depend on the network function $G_{21}(s)$. While frequencies a_j constitute the complex frequencies corresponding to the forced oscillations and depend on the driving force $V_1(s)$. From the above discussion, we can say that the poles determine the time variation of the response whereas the zeros determine the magnitude response.

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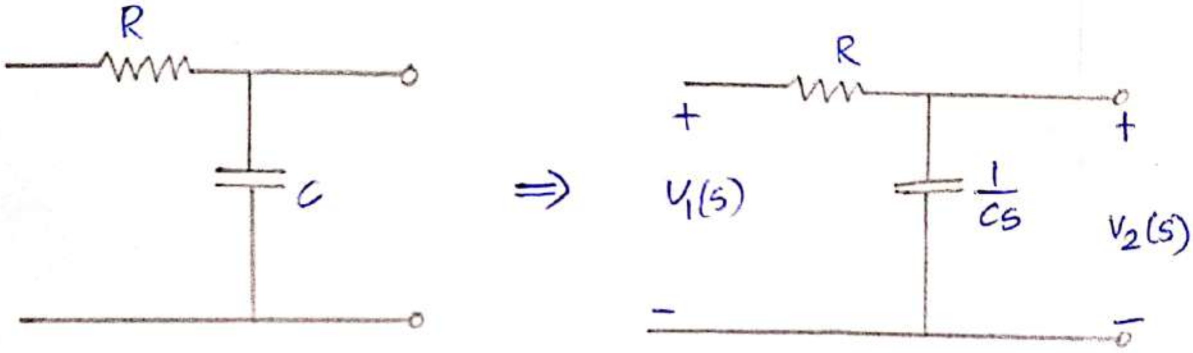
d. Other Network Functions

Significance of poles and zeros in other transfer functions is the same as discussed above. In each of the cases, poles determine the time domain behaviour and zeros determine the magnitude of each of the terms of the response.

Examples:

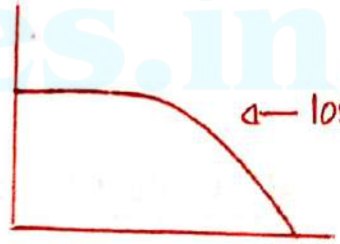
ⓐ



$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{1}{1+s.RC}$$

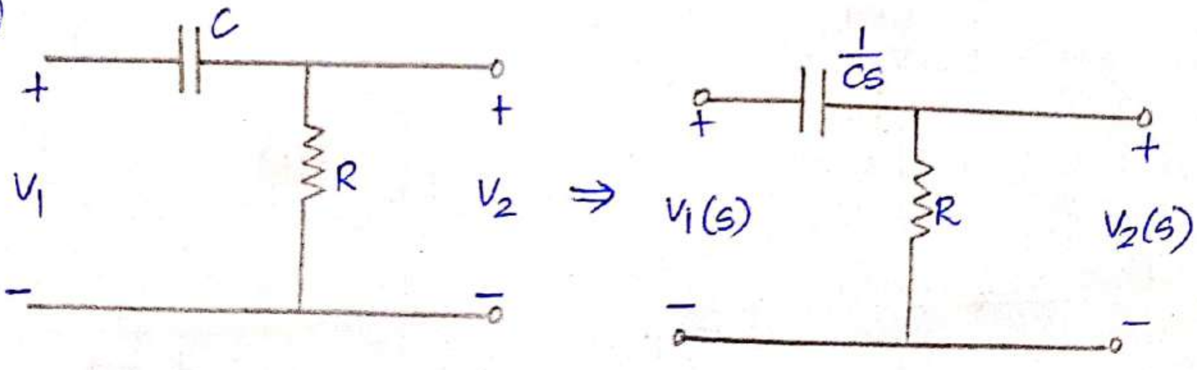
When $s=0$, $G_{21}(s)=1$,

$s=\infty$, $G_{21}(s)=0$



← low pass filter

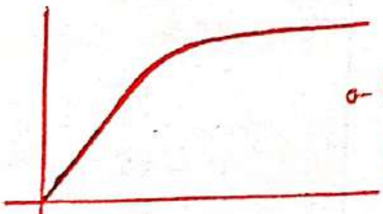
ⓑ



$$G_{21}(s) = \frac{R}{R+\frac{1}{Cs}} = \frac{s.RC}{sRC+1} = \frac{s}{\left(s+\frac{1}{RC}\right)}$$

$s=0 \Rightarrow G_{21}(s)=0$

$s=\infty \Rightarrow G_{21}(s)=1$



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TIME DOMAIN RESPONSE FROM THE POLE-ZERO PLOT

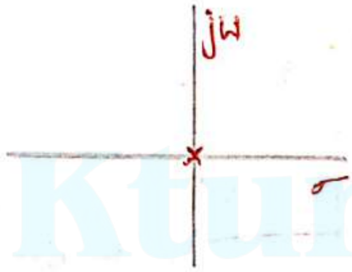
The time domain behaviour of a system can be determined from the pole-zero plot. Consider a n/w function.

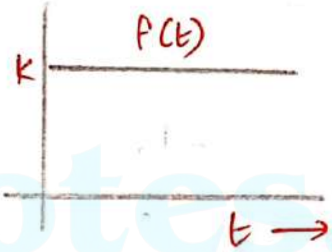
$$F(s) = H \cdot \frac{(s-z_1)(s-z_2)(s-z_3)\cdots\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots\cdots\cdots(s-p_m)}$$

The poles of this function determine the time domain behaviour of f(t). The function f(t) can be determined from the knowledge of the poles, zeros and the scale factor H.

(1) When pole is at origin (at $s=0$)

$$F(s) = \frac{K}{s}, \quad f(t) = Ku(t)$$





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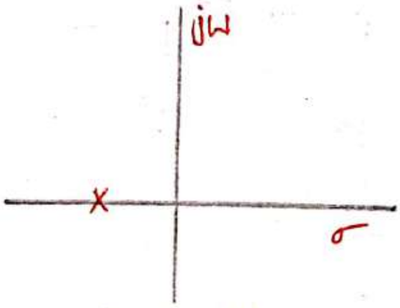
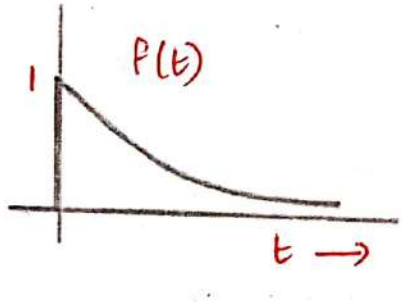
(2) Real and negative pole

$$F(s) = \frac{1}{s+a}, \quad f(t) = e^{-at}u(t)$$





pole location

Real negative pole produces an exponentially decaying time response.

$$F_1(s) = \frac{1}{s+2}, \quad F_2(s) = \frac{1}{s+5} \quad ; \quad f_1(t) = e^{-2t}u(t) \text{ and } f_2(t) = e^{-5t}u(t).$$

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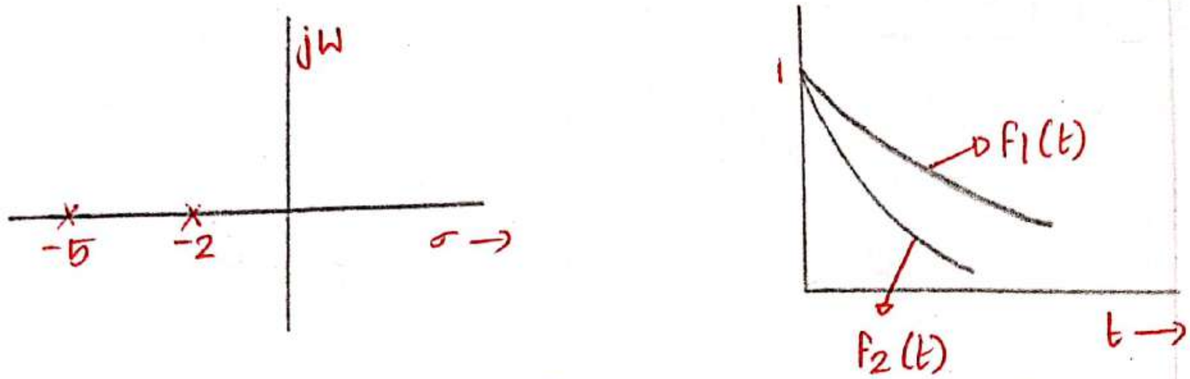
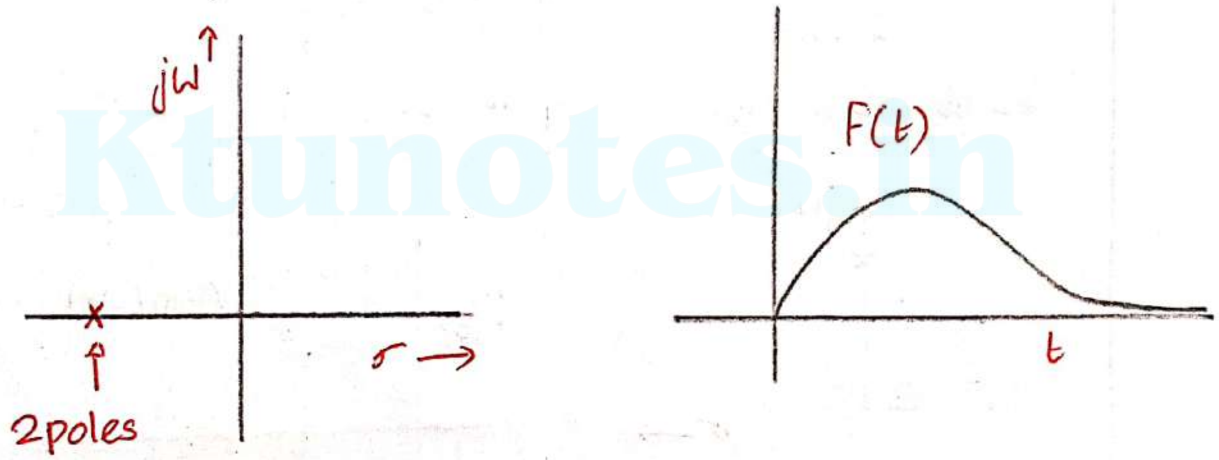


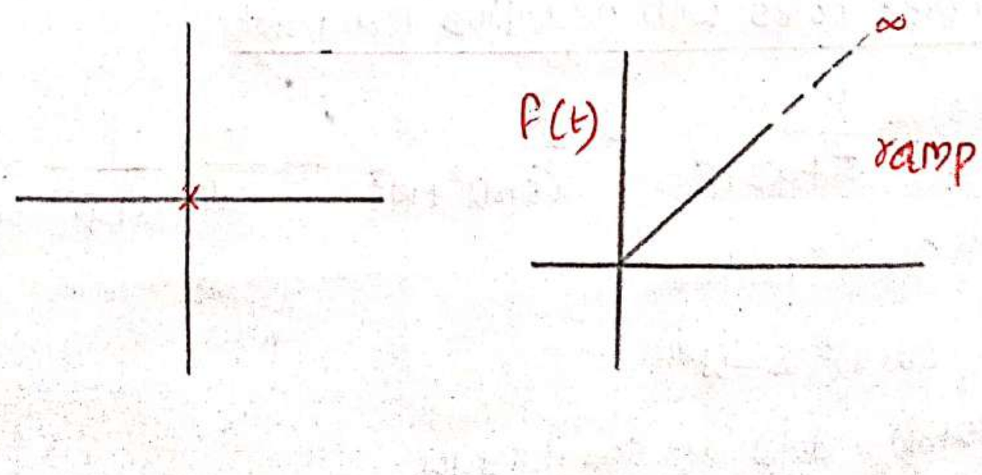
As the pole moves away from origin, or the real axis, in the left half of the s-plane, the corresponding time response decays at a faster rate.

(3) Real, negative repetitive pole

$F(s) = \frac{1}{(s+a)^2}$, $f(t) = te^{-at}u(t)$



If $a=0$, $F(s) = \frac{1}{s^2}$, $f(t) = tu(t) = r(t)$



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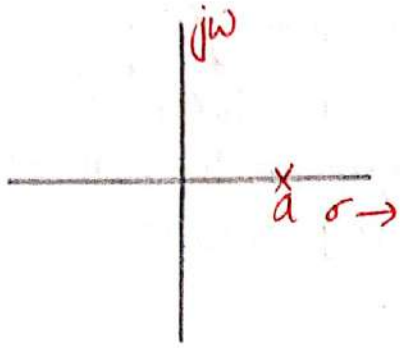
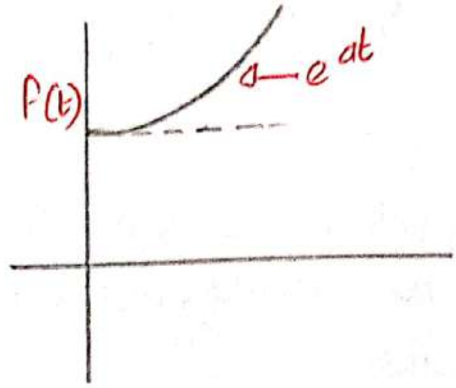
(4) Real +ve pole

$$F(s) = \frac{1}{s-a}, \quad f(t) = e^{at}\,u(t)$$





(5) Complex poles on imaginary axis

$$F(s) = \frac{A\omega}{s^2+\omega^2}, \quad f(t) = A\sin(\omega t)\cdot u(t)$$

Poles at $s = \pm j\omega \quad \because s^2+\omega^2 = 0$

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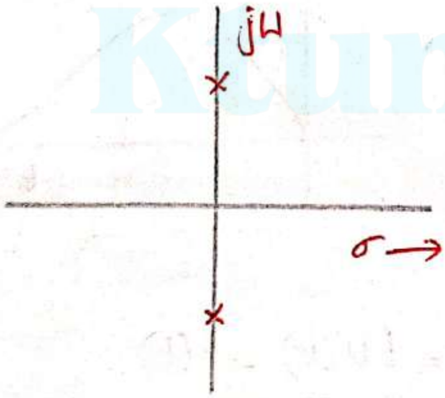

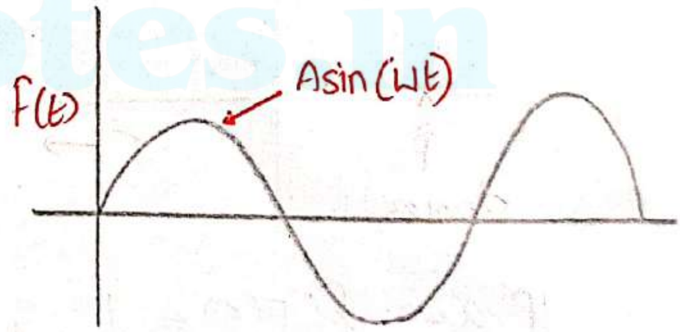

(6) Complex poles with negative real part

$$F(s) = \frac{k}{s^2+\alpha s+\beta} - \frac{k}{(s+a)^2+\omega^2} = \frac{k}{\omega}\cdot\frac{\omega}{(s+a)^2+\omega^2}$$

$$(s+a)^2+\omega^2 = 0$$

$$(s+a)^2 = -\omega^2$$

$$(s+a) = \pm j\omega \text{ or } s = -a \pm j\omega$$

$$f(t) = \frac{k}{\omega}e^{-at}\sin\omega t\cdot u(t)$$

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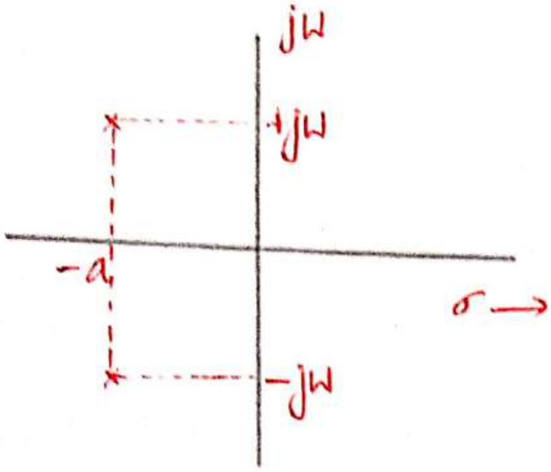

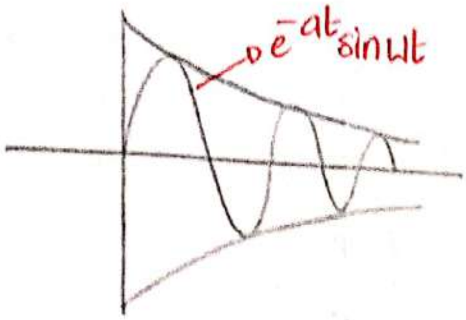

Damped oscillation or oscillations with decreasing amplitude.

(7) Complex poles with positive real part

$$F(s) = \frac{k}{(s-a)^2+w^2} = \frac{k}{w}\cdot\frac{w}{(s-a)^2+w^2}$$

$$f(t) = \frac{k}{w}e^{at}\sin wt\cdot u(t)$$

Poles at : $s = a \pm jw$

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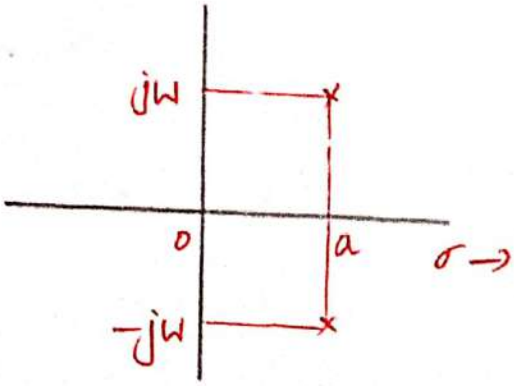
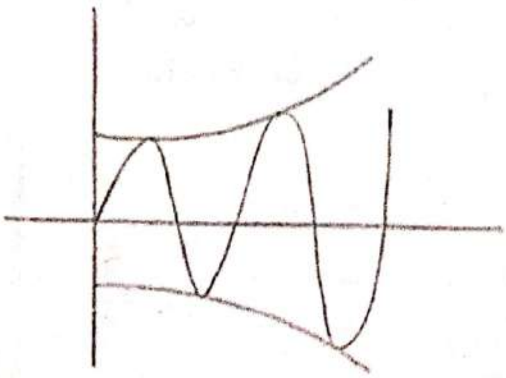
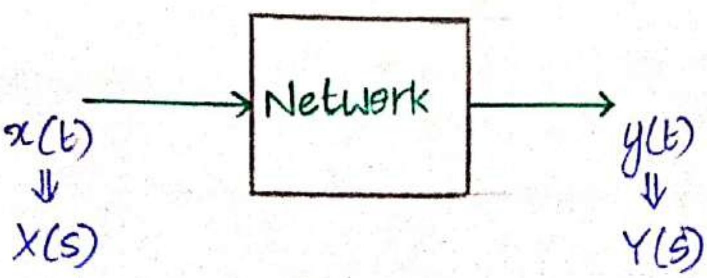




Growing oscillations.

IMPULSE RESPONSE



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Transfer fn, $H(s) = \frac{Y(s)}{X(s)}$

If $x(t) = \delta(t)$, $X(s) = 1$

Then $H(s) = Y(s)$

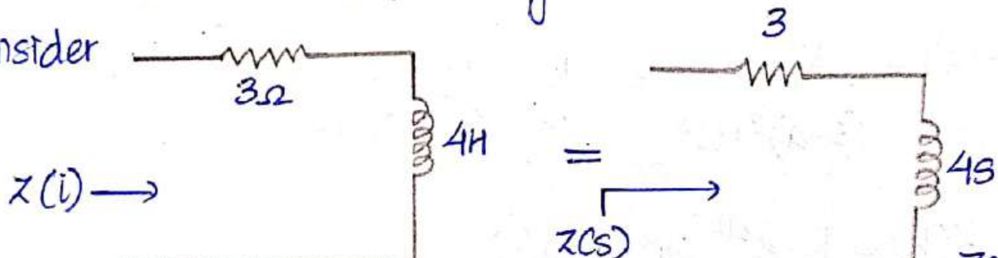
Hence $h(t) = y(t)$

Impulse response of a system is the response or output of the system, when the input is a unit impulse.

COMPLEX ~~COM~~FREQUENCY PLANE

Circuit response as a function of s.

Consider



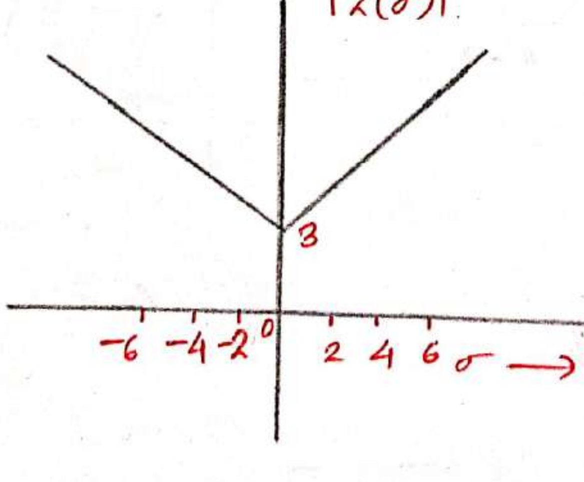
$Z(s) = 3 + 4s$

$s = \sigma + j\omega$

Z variation with σ

Let $s = \sigma + j \cdot 0 = \sigma$

$Z(\sigma) = 3 + 4\sigma$



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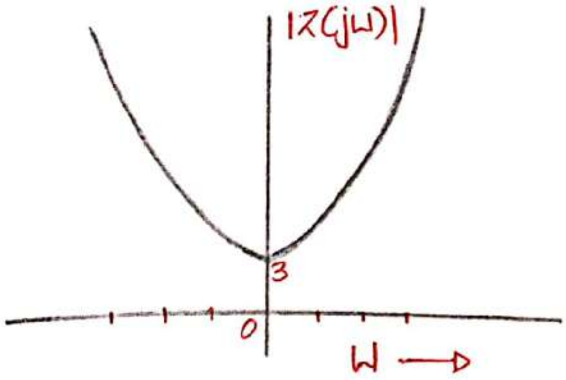
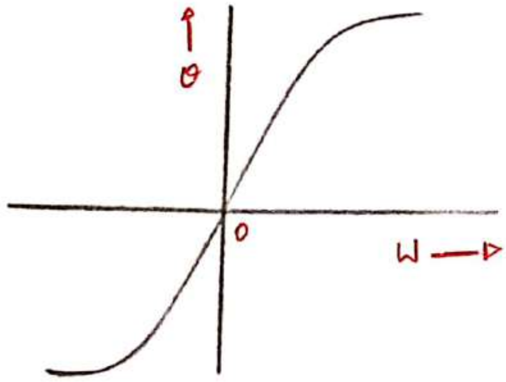
Z(jω) with ω

Let $s = 0 + j\omega$

$Z(j\omega) = 3 + j4\omega$

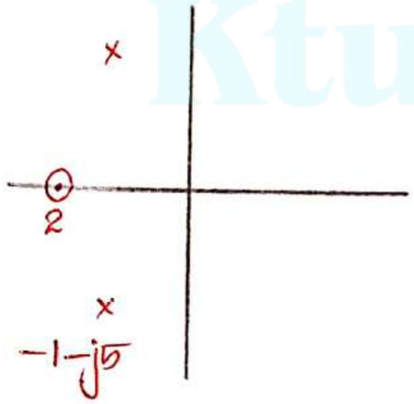
$|Z(j\omega)| = \sqrt{9 + 16\omega^2}$, $\theta = \tan^{-1}\left(\frac{4\omega}{3}\right)$





PROBLEMS

1. From the pole zero plot obtain $Z(s)$. $Z(0) = 1$



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$$Z(s) = K \cdot \frac{s+2}{(s+1-j5)(s+1+j5)}$$

$$= K \cdot \frac{s+2}{(s+1)^2 + 5^2} = K \cdot \frac{s+2}{s^2+2s+1+25}$$

$$= K \cdot \frac{s+2}{s^2+2s+26}$$

$$Z(0) = K \cdot \frac{2}{26} = 1 \Rightarrow K = 13$$

$$\therefore Z(s) = 13 \cdot \frac{s+2}{s^2+2s+26}$$

2. Pole zero plot of $G(s)$ is shown below. Obtain $G(s)$ in s.

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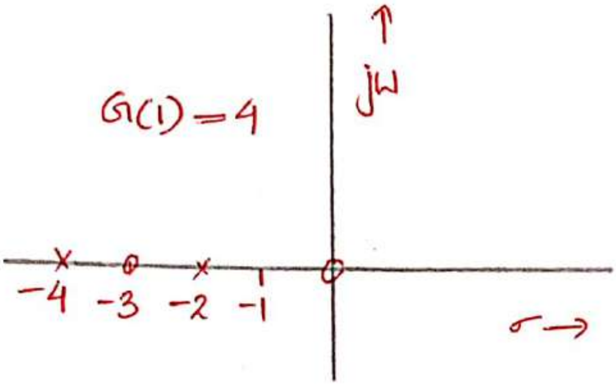


$G(1) = 4$

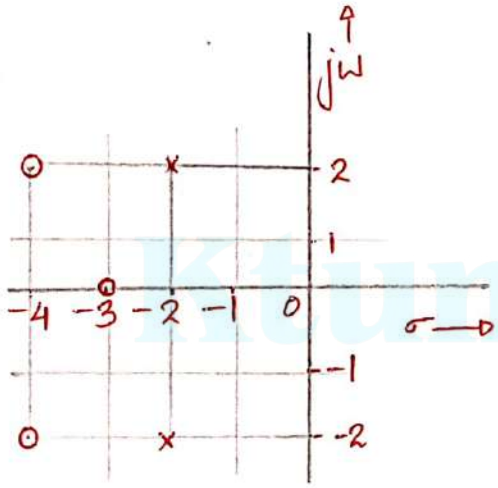
zeros : 0, −3
poles : −2, −4

$$G(s) = k \cdot \frac{s(s+3)}{(s+2)(s+4)}$$

$$G(1) = k \cdot \frac{4}{3 \cdot 5} = 4 \Rightarrow k = 15$$

$$\therefore G(s) = \frac{15 \cdot s(s+3)}{s^2+6s+8} = \frac{15s^2+45s}{s^2+6s+8}$$

18/11/2020 3) Obtain expression for G(s). $G(0) = 15$ | zeros : $-3, -4+j2, -4-j2$
poles : $-2+j2, -2-j2$



$$G(s) = k \cdot \frac{(s+3)(s+4-j2)(s+4+j2)}{(s+2-j2)(s+2+j2)}$$

$$= \frac{k(s+3)\left((s+4)^2+2^2\right)}{\left((s+2)^2+2^2\right)}$$

$$= \frac{k(s+3)(s^2+8s+20)}{(s^2+4s+8)}$$

$$G(0) = \frac{k \cdot 3 \cdot 20}{8} = 15$$

$$\Rightarrow k = \frac{15 \times 8}{3 \times 20} = 2$$

$$G(s) = \frac{2(s+3)(s^2+8s+20)}{s^2+4s+8}$$

$$= \frac{2\left[s^3+11s^2+44s+60\right]}{s^2+4s+8}$$

$$= \frac{2s^3+22s^2+88s+120}{s^2+4s+8}$$

$$\begin{array}{r} s^2+8s+20 \;\times \\ s+3 \\ \hline 3s^2+24s+60 \\ s^3+8s^2+20s \\ \hline s^3+11s^2+44s+60 \end{array}$$

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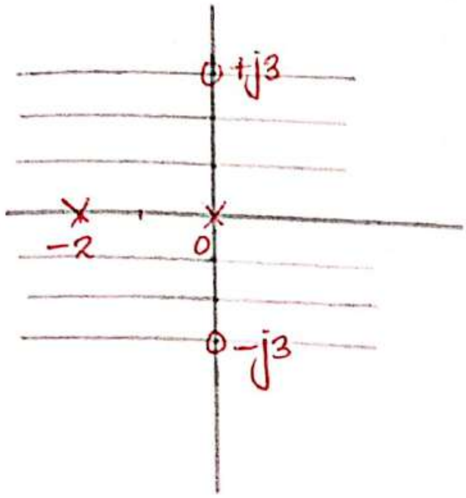
4. obtain G(s). $G(\infty)=3$



zeros : $j3, -j3$

poles : $0, -2$

$$G(s) = k\frac{(s-j3)(s+j3)}{s(s+2)}$$

$$= k\frac{(s^2+9)}{s^2+2s} = k\cdot\frac{\left(1+\frac{9}{s^2}\right)}{1+\frac{2}{s}}$$

$$G(\infty) = 3 = \frac{k\cdot 1}{1} \Rightarrow k=3$$

$$\therefore G(s) = \frac{3(s^2+9)}{s^2+2s} = \frac{3s^2+27}{s^2+2s}$$

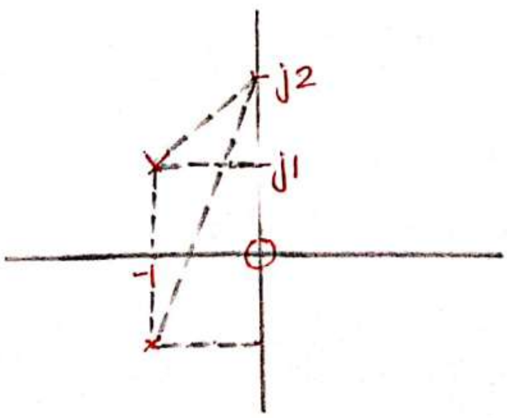
MAGNITUDE AND PHASE RESPONSE

1. Evaluate amplitude and phase of the network function

$$F(s) = \frac{4s}{s^2+2s+2}$$ from the pole-zero plot at $s=j2$

$$F(s) = \frac{4s}{s^2+2s+1+1} = \frac{4s}{(s+1)^2+1^2} = \frac{4s}{(s+1+j1)(s+1-j1)}$$

zero: $s=0$, poles : $-1-j, -1+j$



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$$|F(j2)| = \frac{\text{Product of phasor magnitude from all zeros to } j2}{\text{Product of phasor magnitude from all poles to } j2}$$

$$= \frac{2}{\sqrt{2}\cdot\sqrt{1+9}} = \frac{2}{\sqrt{20}} = \underline{\underline{0.447}}$$

$$\phi(\omega) = \tan^{-1}\left(\frac{2}{0}\right) - \tan^{-1}\left(\frac{3}{1}\right) - \tan^{-1}\left(\frac{1}{1}\right)$$

$$= 90^\circ - 71.56^\circ - 45^\circ$$

$$= \underline{\underline{-26.56^\circ}}$$

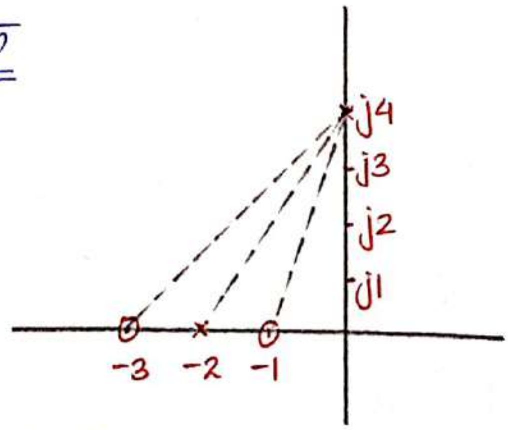
2. Using pole zero plot, find magnitude and phase of the function.

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)} \text{ at } s = j4$$

Zeros : $-1, -3$; poles : $0, -2$

$$|F(j4)| = \frac{\text{Product of phasor magnitude from all zeros to } j4}{\text{Product of phasor magnitude from all poles to } j4}$$

$$= \frac{5\sqrt{17}}{\sqrt{20}\cdot 4} = \underline{\underline{1.15}}$$



$$\phi(\omega) = \tan^{-1}\left(\frac{4}{1}\right) + \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{4}{0}\right) - \tan^{-1}\left(\frac{4}{2}\right)$$

$$= 75.96^\circ + 53.13^\circ - 90^\circ - 63.43^\circ$$

$$= \underline{\underline{-24.34^\circ}}$$

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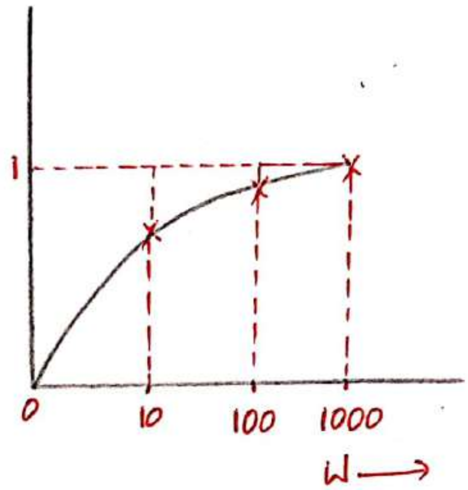
3. Plot the amplitude and plan response for

$$F(s) = \frac{s}{s+10}$$

$$F(j\omega) = \frac{j\omega}{j\omega+10}$$

$$|F(j\omega)| = \frac{\omega}{\sqrt{\omega^2+100}}$$

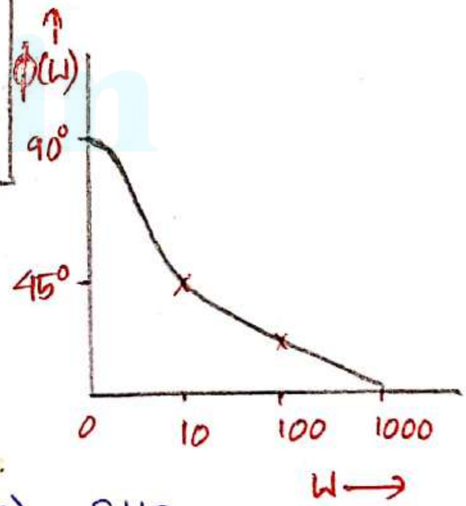
ω	$\lvert F(j\omega)\rvert$
0	0
10	0.707
100	0.995
1000	1



$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$= 90^\circ - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	$\phi(\omega)$
0	90
10	45
100	5.7
1000	0



4. Sketch the amplitude and phase response for $F(s) = \frac{s+10}{s-10}$

$$F(j\omega) = \frac{j\omega+10}{j\omega-10}$$

$$|F(j\omega)| = \frac{\sqrt{100+\omega^2}}{\sqrt{100+\omega^2}} = 1$$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{-\omega}{10}\right)$$

$$= 2\tan^{-1}\left(\frac{\omega}{10}\right)$$

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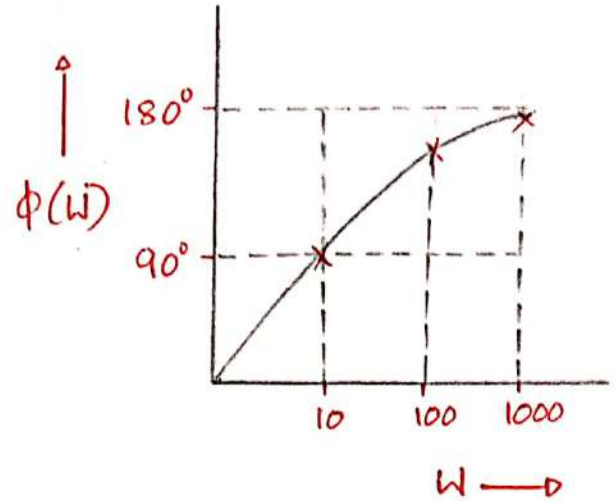
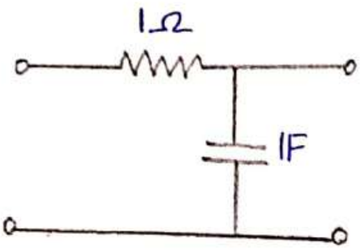
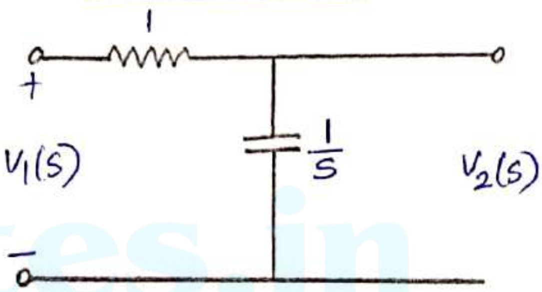
W	$\phi(W)$
0	0
10	90°
100	168.6°
1000	178.90°



5. For the low pass filter (LPF) shown in fig, plot the magnitude and phase response of transfer function for $0 \le W \le 10$.



Transformed n/w



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$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$

$$H(jW) = \frac{1}{jW+1}$$

$$|H(jW)| = \frac{1}{\sqrt{W^2+1}}, \quad \phi(W) = 0 - \tan^{-1}(W) = -\tan^{-1}(W).$$

W	$\lvert H(jW)\rvert$	$\phi(W)$
0	1	0
2	0.45	-63.43
4	0.24	-75.96
6	0.16	-80.53
8	0.12	-82.89
10	0.09	-84.29

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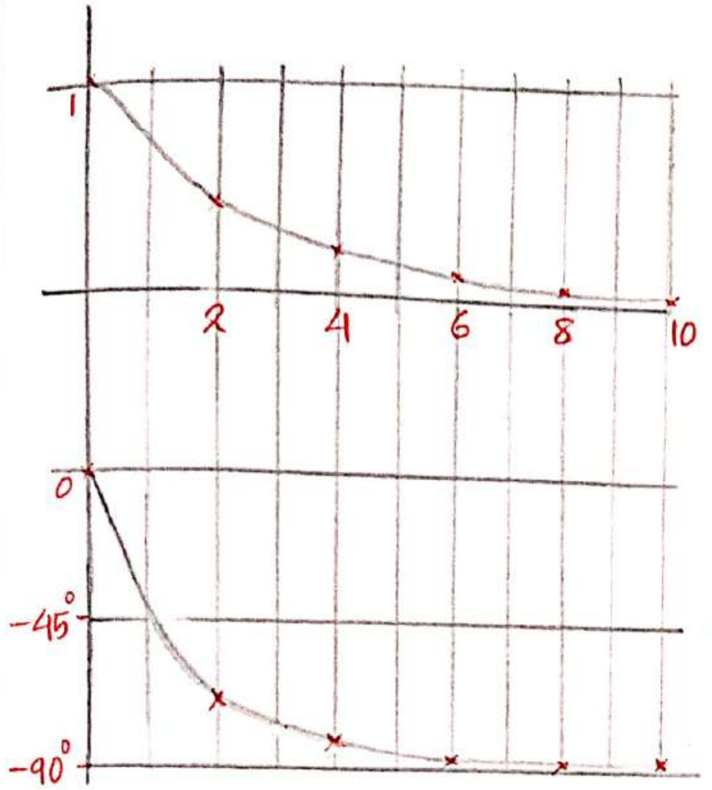
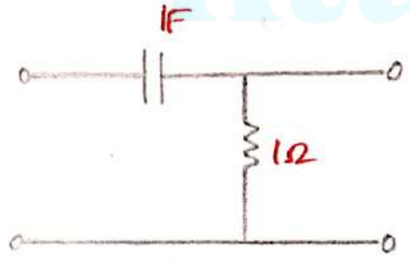
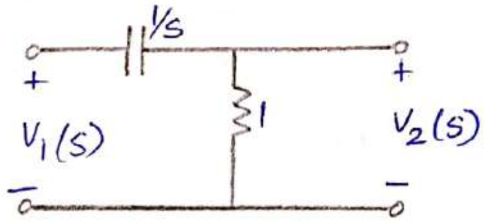


6. For the HPF shown below, plot the magnitude and phase response of the transfer function for $0 \le \omega \le 10$.



$\Rightarrow$

Transformed n/w



$$H(s) = \frac{1}{1 + 1/s} = \frac{s}{s+1}$$

$$H(j\omega) = \frac{j\omega}{j\omega + 1}$$

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2+1}} \quad ; \quad \phi(\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}(\omega)$$

$$= 90 - \tan^{-1}(\omega)$$

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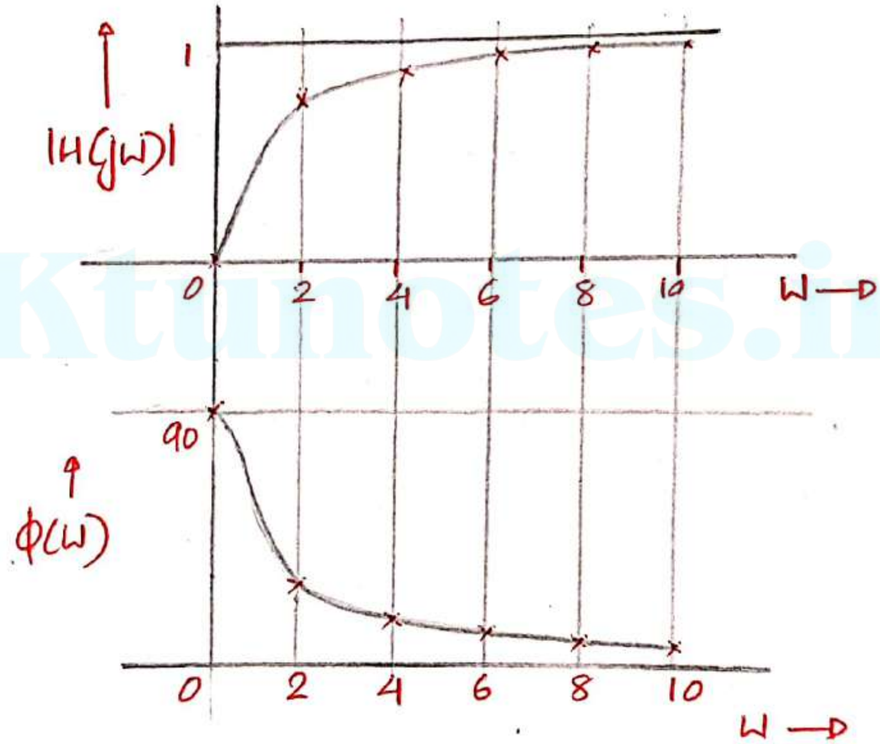
ω	$\lvert H(j\omega)\rvert$	$\phi(\omega)$
0	0	90
2	0.89	26.57
4	0.96	14.04
6	0.96	9.47
8	0.96	7.11
10	0.99	5.71



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