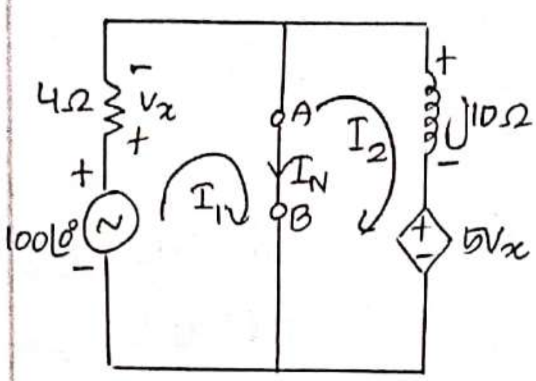

KTU
NOTES

$= 86\angle 3.95^\circ$ V

$I_N$




$V_x = 4I_1$ —— (1)

Mesh 1:

$-100\angle 0 + 4I_1 = 0$

$I_1 = \frac{100\angle 0}{4} = 25A$

Mesh 2:

$(j10)I_2 + 5V_x = 0$

$I_2 = \frac{-5V_x}{(j10)} = \frac{-5(4I_1)}{j10} = \frac{-5(4\times 25)}{j10}$

$= 50\angle 90$ A

$I_N = I_1 - I_2 = 25 - 50\angle 90$

$= 55.9\angle -63.43^\circ$ A

$Z_{Th}$

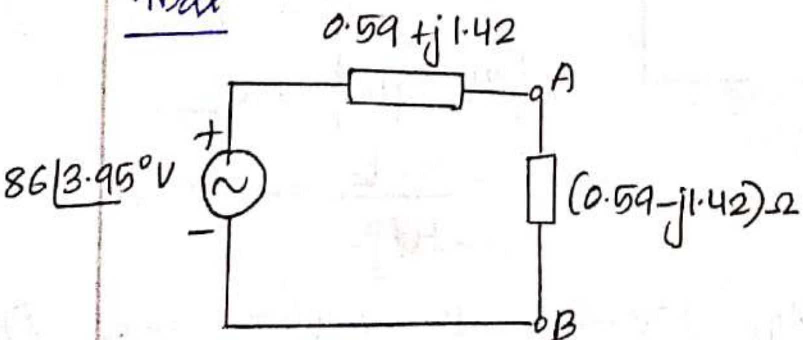
$Z_{Th} = \frac{V_{Th}}{I_N} = \frac{86\angle 3.95^\circ}{55.9\angle -63.43^\circ} = (0.59 + j1.42)\Omega$

$Z_L$

For maximum power transfer,

$Z_L = Z_{Th}^* = (0.59 - j1.42)\Omega$

$P_{max}$




$P_{max} = \frac{|V_{Th}|^2}{4R_L} = \frac{86^2}{4\times 0.59}$

$= 3133.9W$

20/10/2020

# MODULE 3

## APPLICATION OF LAPLACE TRANSFORMS

Time domain analysis is the conventional method of analysing a network (known as classical method). As the order of the network variable equation increases, this method becomes very tedious. For such applications, frequency domain analysis using Laplace transform is very convenient. It is a powerful mathematical technique which enables us to solve linear differential equations by using algebraic methods.

### Laplace transformation

LT of a function $f(t)$ is defined as $F(s) = L\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$ where $s$ is the complex variable. $s = \sigma + j\omega$.

The function $f(t)$ must satisfy the following condition to possess a L.T.

$$\int_0^{\infty} |f(t)| e^{-\sigma t} dt < \infty$$ where $\sigma$ is real and and positive

The inverse Laplace transform, $L^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st} ds$.

### L.T of some important functions

1. Constant k

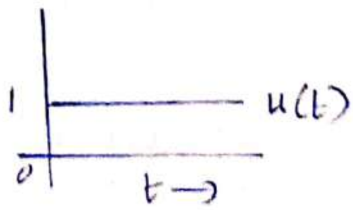
$$L(k) = \int_0^{\infty} k e^{-st} dt = k\left[\frac{e^{-st}}{-s}\right]_0^{\infty} = k\left[0 - \frac{-1}{s}\right] = \frac{k}{s}$$ Eg: $L\{1\} = 1/s$

2. $t^n$

$$L\{t^n\} = \int_0^{\infty} t^n e^{-st} dt$$
$$= \frac{n!}{s^{n+1}}$$

3. Unit step $-u(t)$

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



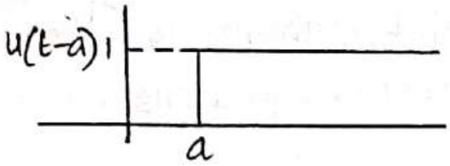

$$L(u(t)) = \frac{1}{s}$$

4. Delayed or shifted unit step

$$u(t-a)=\begin{cases}1 & t>a\\ 0 & t<a\end{cases}$$

or

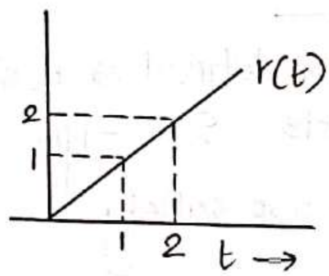
$$u(t-a)=\begin{cases}1 & (t-a)>0 \Rightarrow t>a\\ 0 & t-a<0 \Rightarrow t<a\end{cases}$$




$$L\{u(t-a)\}=e^{-as}\cdot\frac{1}{s}$$

5. Unit ramp – r(t)

$$r(t)=\begin{cases}t & t>0\\ 0 & t<0\end{cases}$$




$$\boxed{L\{t^n\}=\frac{n!}{s^{n+1}}}$$

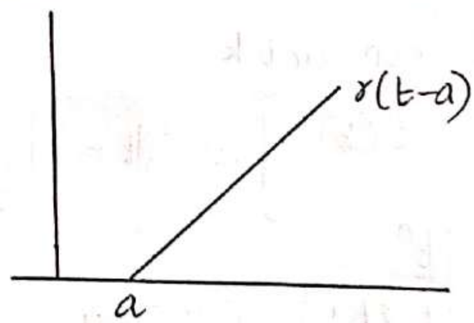
$$L\{r(t)\}=\frac{1}{s^2}$$

6. Delayed unit ramp

$$r(t-a)=\begin{cases}t & t>a\\ 0 & t<0\end{cases}$$

or

$$r(t-a)=\begin{cases}t & t-a>0 \Rightarrow t>a\\ 0 & t-a<0 \Rightarrow t<a\end{cases}$$

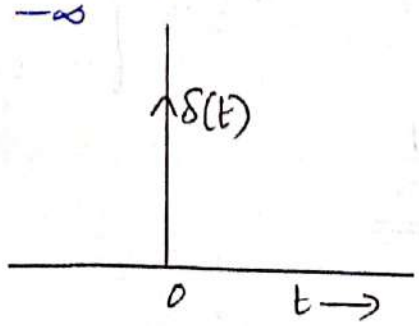



$$L\{r(t-a)\}=e^{-as}\cdot\frac{1}{s^2}$$

7. Unit impulse – δ(t)

$\delta(t)=0$ for $t\neq 0$ and $\int_{-\infty}^{\infty}\delta(t)\,dt=1$

$$L\{\delta(t)\}=1$$



8. Exponential function ($e^{at}$)

$$L\{e^{at}\} = \frac{1}{s-a}$$

9. Sine function

$$L\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$$

10. cosine function

$$L\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$$

11. Hyperbolic sine function

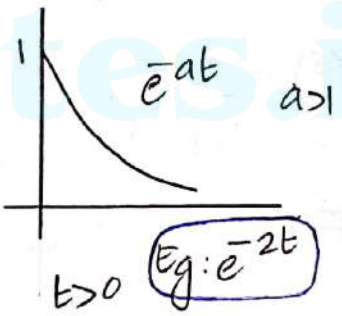
$$L\{\sinh \omega t\} = \frac{\omega}{s^2-\omega^2}$$

12. Hyperbolic cosine function

$$L\{\cosh \omega t\} = \frac{s}{s^2-\omega^2}$$

13. Exponentially damped function

$$L\{e^{-at}f(t)\} = F\{s+a\}$$




Eg: $e^{-2t}$

$f(t) \rightarrow F(s)$

replaces s by s+a.

$$L\{e^{-at}\sin \omega t\} = \frac{\omega}{(s+a)^2+\omega^2}$$

$$L\{e^{-at}\cos \omega t\} = \frac{s+a}{(s+a)^2+\omega^2}$$

$$L\{e^{-at}\sinh \omega t\} = \frac{\omega}{(s+a)^2-\omega^2}$$

$$L\{e^{-at}\cosh \omega t\} = \frac{s+a}{(s+a)^2-\omega^2}$$

## Initial Value Theorem (IVT)

If $L\{f(t)\} = F(s)$, then $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

## Final value Theorem (FVT)

If $L\{f(t)\} = F(s)$, then $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

→ IVT and FVT are together known as Limiting theorems.

## Problems

21/10/2020 1. Verify the initial and final value theorems for $e^{-t}(t+1)^2$

$f(t) = e^{-t}(t+1)^2 = e^{-t}[t^2+2t+1]$

$F(s) = L\{e^{-t}(t^2+2t+1)\}$

$= \frac{2}{(s+1)^3} + 2\cdot\frac{1}{(s+1)^2} + \frac{1}{s+1}$

$sF(s) = \frac{2s}{(s+1)^3} + \frac{2s}{(s+1)^2} + \frac{s}{(s+1)}$

$\lim_{t\to 0} f(t) = \lim_{t\to 0} e^{-t}(t+1)^2 = 1\cdot 1^2 = 1$

$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{2s}{(s+1)^3} + \frac{2s}{(s+1)^2} + \frac{s}{s+1}\right]$

$= \lim_{s\to\infty}\left[\frac{2/s^2}{(1+1/s)^3} + \frac{2/s}{(1+1/s)^2} + \frac{1}{1+1/s}\right]$

$= \underline{\underline{1}} = \lim_{t\to 0} f(t)$. Hence verified

2. Verify the initial and final value theorems for $e^{-t}(t^2+\cos 3t)$

$f(t) = e^{-t}[t^2+\cos 3t)$

$F(s) = \frac{2}{(s+1)^3} + \frac{s+1}{(s+1)^2+9}$

$$sF(s) = \frac{2s}{(s+1)^3} + \frac{s(s+1)}{(s+1)^2+9}$$

$$\lim_{t\to 0} f(t) = \lim_{t\to 0} \left[e^{-t}(t^2+\cos 3t)\right] = 0+1 = 1$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \left[\frac{2s}{(s+1)^3} + \frac{s(s+1)}{(s+1)^2+9}\right]$$

$$= \lim_{s\to\infty} \left[\frac{2/s^2}{(1+1/s)^3} + \frac{(1+1/s)}{(1+1/s)^2+9/s^2}\right]$$

$$= 0 + \frac{1+0}{(1+0)^2+0} = \underline{\underline{1}}$$

Hence initial value theorem is verified.

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty} \left[e^{-t}(t^2+\cos 3t)\right] = 0$$

$$\lim_{t\to\infty} sF(s) = \lim_{s\to 0} \left[\frac{2s}{(s+1)^3} + \frac{s(s+1)}{(s+1)^2+9}\right]$$

$$= 0+0 = 0$$

Hence final value theorem is verified.

3. Find the initial and final values of the function whose Laplace transform is $F(s) = \dfrac{2s+1}{s^3+6s^2+11s+6}$

$$sF(s) = \frac{s(2s+1)}{s^3+6s^2+11s+6}$$

From initial value theorem,

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s(2s+1)}{s^2+6s^2+11s+6}$$

$$= \lim_{s\to\infty} \frac{2/s + 1/s^2}{1+6/s+11/s^2+6/s^2} = \underline{\underline{0}}$$

$$f(\infty) = \underset{s\to 0}{Lt}\, sF(s) = \underset{s\to 0}{Lt} \frac{s(2s+1)}{s^3+6s^2+11s+6} = \underline{\underline{0}}$$

4. Find the final value of the function whose laplace transform is $I(s) = \frac{s+6}{s(s+3)}$.

$$sI(s) = \frac{s+6}{s+3}$$

$$I(\infty) = \underset{s\to 0}{Lt}\, sI(s) = \underset{s\to 0}{Lt} \frac{s+6}{s+3} = \frac{6}{3} = \underline{\underline{2}}$$

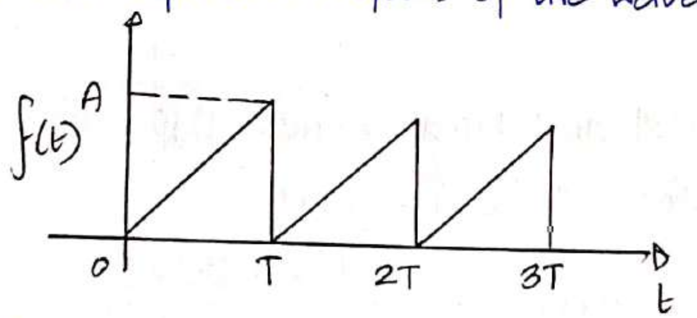
## Laplace transform of periodic functions

A function $f(t)$ is said to be periodic if there exists a constant $T$ $(T>0)$ such that $f(t+T) = f(t)$ for all values of $t$. If $f(t)$ is a piecewise periodic function with period $T$. Then,

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T f(t) e^{-st}\, dt$$

### Problems

1. Find the Laplace transform of the waveform.




$$f(t) = \frac{A}{T} \cdot t, \quad 0<t<T$$

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T f(t) e^{-st}\, dt$$

$$= \frac{1}{1-e^{-Ts}} \int_0^T \frac{At}{T} e^{-st}\, dt.$$



$$= \frac{1}{1-e^{-Ts}} \cdot \frac{A}{T} \int_0^T t e^{-st} dt$$

$$= \frac{A}{T(1-e^{-Ts})} \left[ t \cdot \frac{e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^T$$

$$= \frac{A}{T(1-e^{-Ts})} \left[ \frac{-Te^{-Ts}}{s} - \frac{e^{-Ts}}{s^2} + \frac{1}{s^2} \right]$$

$$= \frac{A}{T(1-e^{-Ts})} \left[ \frac{-Te^{-Ts}}{s} + \frac{1}{s^2}\left[1-e^{-Ts}\right] \right]$$

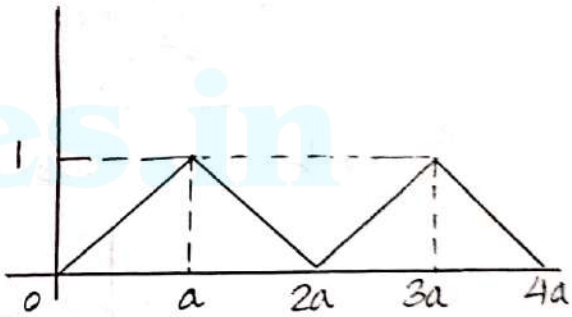
$$= \frac{A}{Ts^2} - \frac{Ae^{-Ts}}{s(1-e^{-Ts})}$$

2. Find the laplace transform of the waveform.

$$f(t) = \frac{1}{a} t \quad 0<t<a$$

$$= \frac{1}{a}(2a-t) \quad a<t<2a$$

$$T = 2a$$




$$L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T f(t) e^{-st} dt$$

$$= \frac{1}{1-e^{-2as}} \int_0^{2a} f(t) e^{-st} dt$$

$$= \frac{1}{1-e^{-2as}} \left[ \int_0^a \frac{t}{a} e^{-st} dt + \int_a^{2a} \frac{1}{a}(2a-t) e^{-st} dt \right]$$

$$= \frac{1}{a(1-e^{-2as})} \left[ \left( t \times \frac{e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^a + \left[ (2a-t)\frac{e^{-st}}{-s} + \frac{e^{-st}}{s^2} \right]_0^{2a} \right]$$

$$= \frac{1}{a(1-e^{-2as})}\left[\frac{-ae^{-as}}{s} - \frac{e^{-as}}{s^2} + \frac{1}{s^2} + \frac{e^{-2as}}{s^2} + \frac{ae^{-as}}{s} - \frac{e^{-as}}{s^2}\right]$$

$$= \frac{-2e^{-as}+1+e^{-2as}}{as^2(1-e^{-2as})}$$

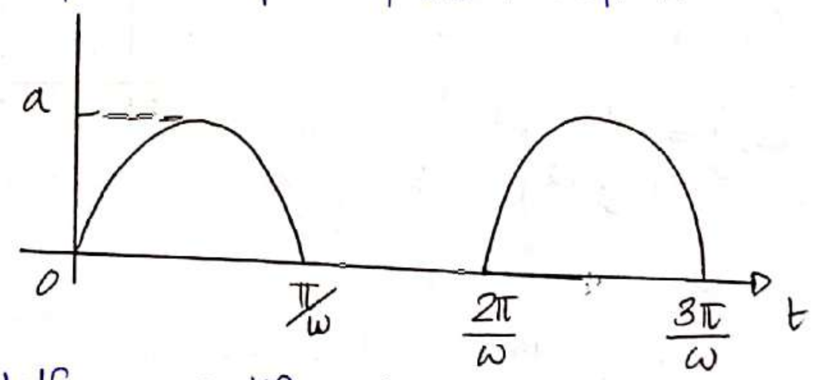
$$= \frac{(1-e^{-as})^2}{as^2(1-e^{-as})(1+e^{-as})}$$

$$= \frac{1-e^{-as}}{as^2(1+e^{-as})}$$

$$= \frac{1}{as^2} \cdot \frac{e^{-as/2}(e^{as/2}-e^{-as/2})}{e^{-as/2}(e^{as/2}+e^{-as/2})}$$

$$= \frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$$

3. Find the Laplace transform of the waveform.




f(t) – Half wave rectifier o/p.

$$f(t) = a\sin\omega t \quad 0<t<\frac{\pi}{\omega}$$

$$= 0 \quad \frac{\pi}{\omega}<t\le\frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\omega}$$

$$L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T f(t)e^{-st}\,dt$$

$$= \frac{1}{1-e^{-s2\pi/\omega}}\left[\int_0^{\pi/\omega} a\sin\omega t \cdot e^{-st}dt + \int_{\pi/\omega}^{2\pi/\omega} 0 \cdot e^{-st}dt\right]$$

$$= \frac{a}{1-e^{-2\pi s/\omega}}\left[\frac{1}{s^2+\omega^2}e^{-st}\left(-s\sin\omega t-\omega\cos\omega t\right)\right]_0^{\pi/\omega}$$

$$= \frac{a}{1-e^{-2\pi s/\omega}}\cdot\frac{1}{s^2+\omega^2}\left[e^{-s\pi/\omega}\left(-s\cdot\sin\pi-\omega\cos\pi\right)+0+\omega\right]$$

$$= \frac{a}{1-e^{-2\pi s/\omega}}\cdot\frac{1}{s^2+\omega^2}\left[e^{-s\pi/\omega}\left(0-\omega(-1)\right)+\omega\right]$$

$$= \frac{a}{1-(e^{-\pi s/\omega})^2}\cdot\frac{1}{s^2+\omega^2}\left[\omega e^{-s\pi/\omega}+\omega\right]$$

$$= \frac{a}{(1+e^{-\pi s/\omega})(1-e^{-\pi s/\omega})}\cdot\frac{1}{s^2+\omega^2}\left[\omega e^{-\frac{s\pi}{\omega}}+\omega\right]$$

$$= \frac{a}{(1+e^{-\pi s/\omega})(1-e^{-\pi s/\omega})}\cdot\frac{1}{s^2+\omega^2}\cdot\omega\left(1+e^{-\pi s/\omega}\right)$$

$$= \frac{a\omega}{1-e^{-\pi s/\omega}}\cdot\frac{1}{s^2+\omega^2}$$

Note: $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

4 Find the Laplace transform of $F(t)=t^2$, $0<t<2$, if $F(t)=F(t+2)$

$F(t)=F(t+2)$ indicates that $f(t)$ is a periodic function with periodis 2.

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}}\int_0^T f(t)e^{-st}\,dt$$

$$= \frac{1}{1-e^{-2s}}\int_0^2 e^{-st}t^2\,dt$$

$$= \frac{1}{1-e^{-2s}}\left[\left[t^2\frac{e^{-st}}{-s}\right]_0^2 - \int_0^2 2t\cdot\frac{e^{-st}}{-s}\,dt\right]$$

$$= \frac{1}{1-e^{-2s}}\left[\left[-\frac{t^2e^{-st}}{s}\right]_0^2 + \frac{2}{s}\left[\left[t\cdot\frac{e^{-st}}{-s}\right]_0^2 - \int_0^2 1\cdot\frac{e^{-st}}{-s}\,dt\right]\right]$$

$$= \frac{1}{1-e^{-2s}}\left[\frac{-t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2}\right]_0^2 + \frac{2}{s}\cdot\frac{1}{s}\int_0^2 e^{-st}\,dt$$

$$= \frac{1}{1-e^{-2s}}\left[-\frac{t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2}{s^3}e^{-st}\right]_0^2$$

$$= \frac{1}{1-e^{-2s}}\left[-e^{-st}\left(\frac{t^2}{s}+\frac{2t}{s^2}+\frac{2}{s^3}\right)\right]_0^2$$

$$= \frac{1}{1-e^{-2s}}\left[-e^{-2s}\left(\frac{4}{s}+\frac{4}{s^2}+\frac{2}{s^3}\right)+\frac{2}{s^3}\right]$$

5. Find the Laplace transform of $f(t)=e^t, 0<t<2\pi$ if $f(t)=f(t+2\pi)$

$T=2\pi$

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}}\int_0^T f(t)e^{-st}\,dt$$

$$= \frac{1}{1-e^{-2\pi s}}\int_0^{2\pi} e^t\cdot e^{-st}\,dt$$

$$= \frac{1}{1-e^{-2\pi s}}\int_0^{2\pi} e^{(1-s)t}\,dt$$

$$= \frac{1}{1-e^{-2\pi s}} \left[ \frac{e^{(1-s)t}}{1-s} \right]_0^{2\pi}$$

$$= \frac{1}{1-e^{-2\pi s}} \left[ \frac{e^{(1-s)2\pi} - 1}{(1-s)} \right]$$

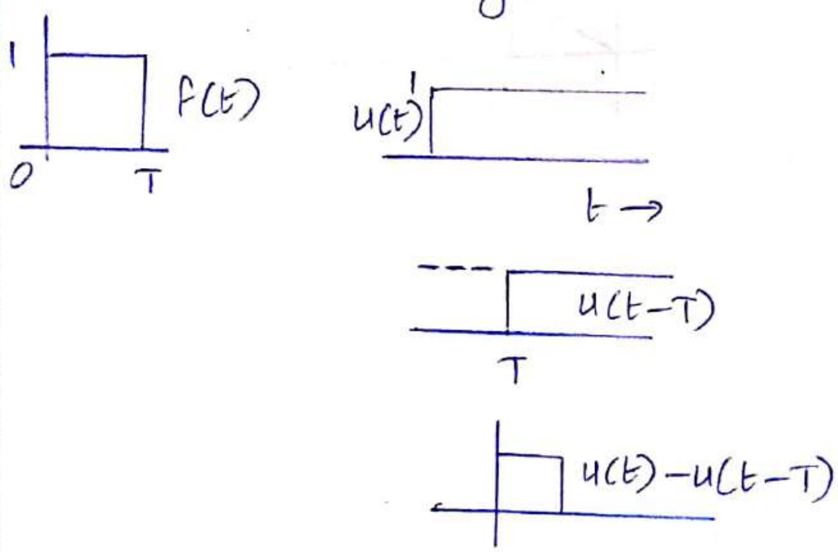
$$= \frac{e^{(1-s)2\pi} - 1}{(1-e^{-2\pi s})(1-s)}$$

## WAVEFORM SYNTHESIS

Synthesis → to combine a number of different pieces into whole.

→ Any waveform can be constructed with unit step, unit ramp and impulse functions. Since we know the LT of these functions, we can find the LT of any function in terms of LT of these functions.

1. Find LT of the rectangular pulse




$f(t) = u(t) - u(t-T)$

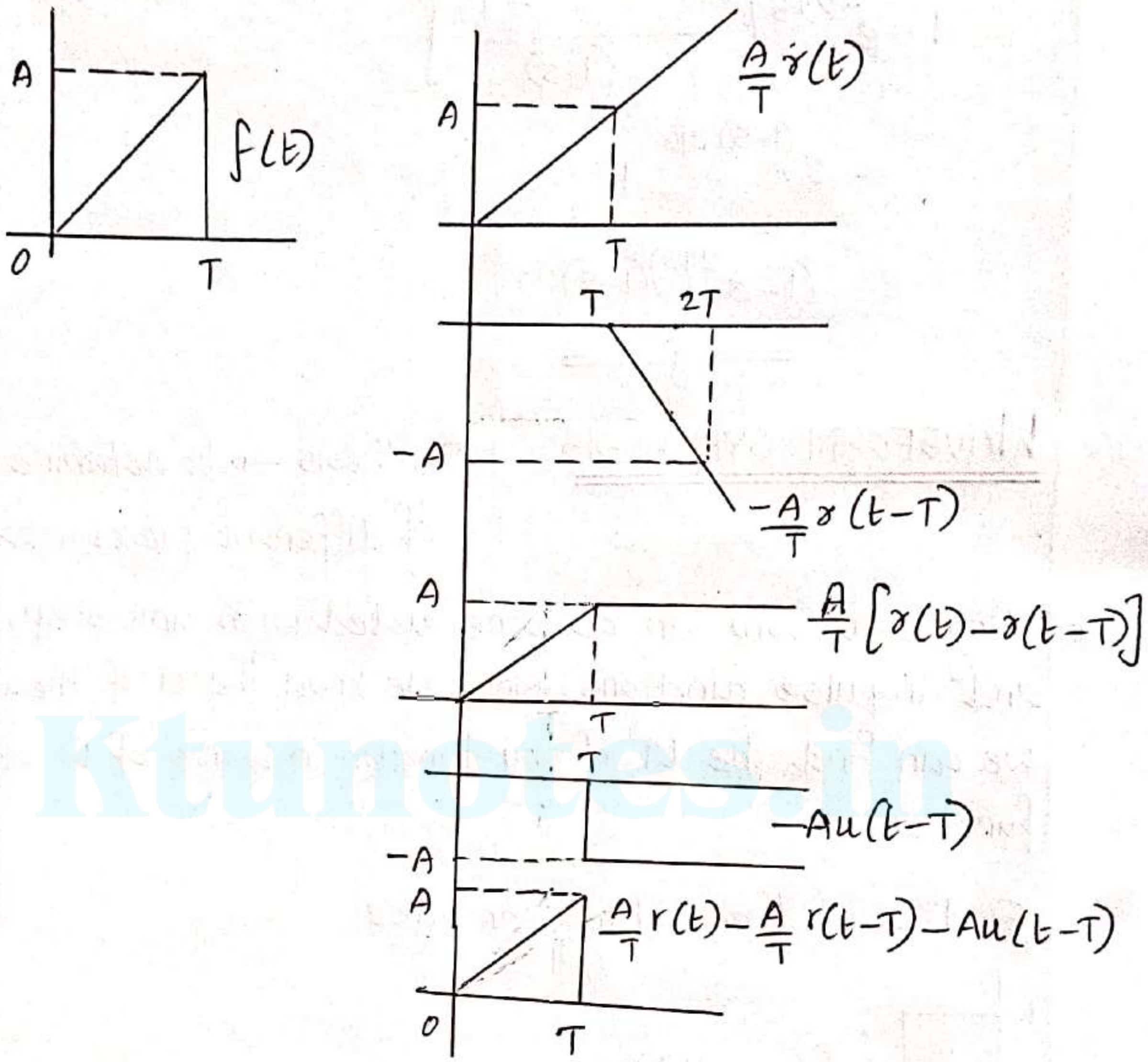
$F(s) = L\{u(t) - u(t-T)\}$

$= 1/s - 1/s\, e^{-Ts}$

$L\{u(t-T)\} = \frac{1}{s} e^{-Ts}$

$= \frac{1}{s}(1-e^{-Ts})$

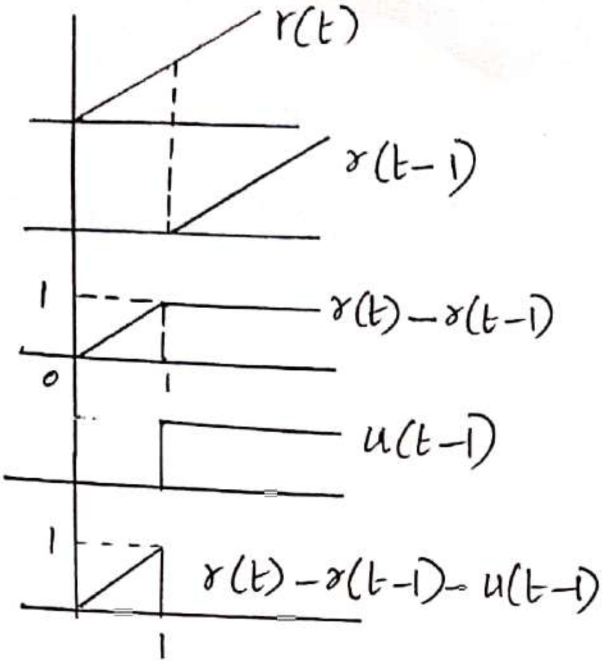
2. Find the LT of the sawtooth waveform shown in fig.



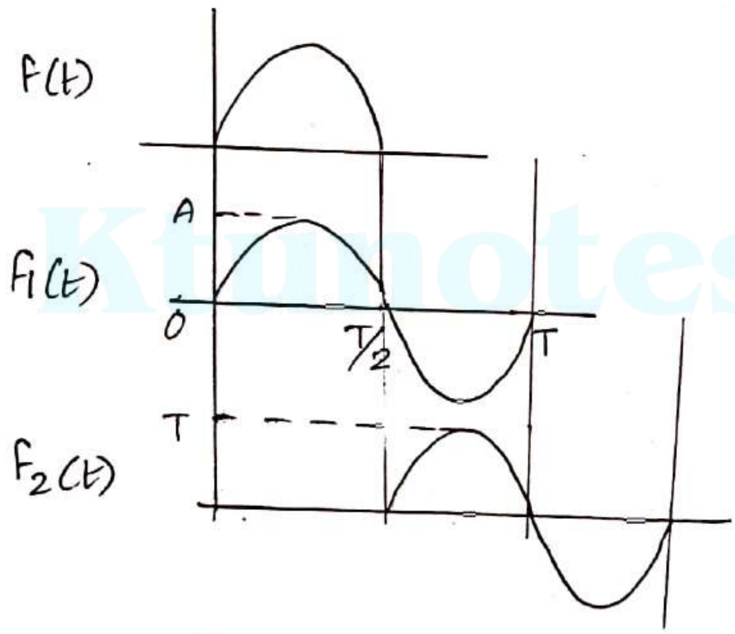

$$f(t) = \frac{A}{T}r(t) - \frac{A}{T}r(t-T) - Au(t-T)$$

$$F(s) = \frac{A}{T}L\{r(t)\} - \frac{A}{T}L\{r(t-T)\} - AL\{u(t-T)\}$$

$$= \frac{A}{T}\frac{1}{s^2} - \frac{A}{T}\frac{1}{s^2}e^{-Ts} - A\cdot\frac{1}{s}e^{-Ts}$$




3. Find the L·T of a sinusoidal waveform shown in figure.




$$F(t) = f_1(t) + f_2(t)$$

$$= A\sin\omega t \cdot u(t) + A\sin\omega\left(t-\frac{T}{2}\right)u\left(t-\frac{T}{2}\right)$$

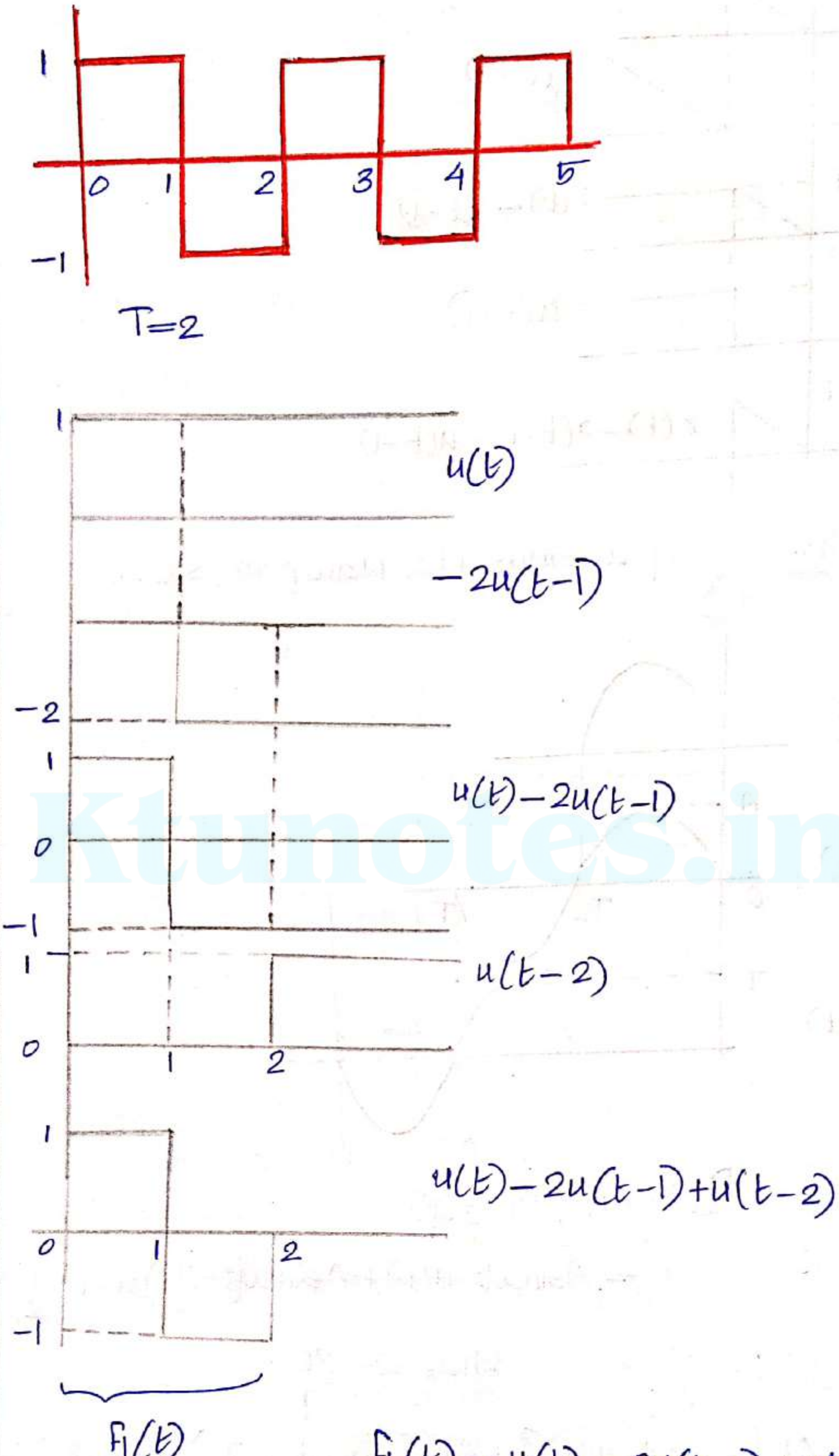
where $\omega = \frac{2\pi}{T}$

$$F(s) = AL\left\{\sin\omega t \cdot u(t)\right\} + AL\left\{\sin\omega\left(t-\frac{T}{2}\right)u\left(t-\frac{T}{2}\right)\right\}$$

$$= \frac{A\omega}{s^2+\omega^2} + \frac{A\omega}{s^2+\omega^2}e^{-Ts/2} = \frac{A\omega}{s^2+\omega^2}\left[1+e^{-Ts/2}\right]$$

27/10/2020 5. Find the L.T of the periodic waveform.




$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$$f_1(s) = L\{u(t)\} - 2L\{u(t-1)\} + L\{u(t-2)\}$$

$$= \frac{1}{s} - \frac{2}{s}e^{-s} + \frac{1}{s}e^{-2s}$$

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}} F_1(s)$$

$$= \frac{1}{1-e^{-2s}}\left[\frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}\right]$$

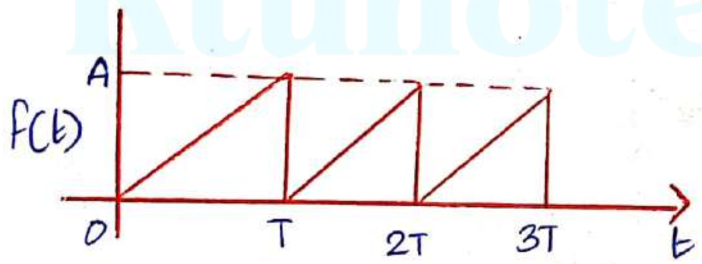
$$= \frac{1-2e^{-s}+e^{-2s}}{s(1+e^{-s})(1-e^{-s})}$$

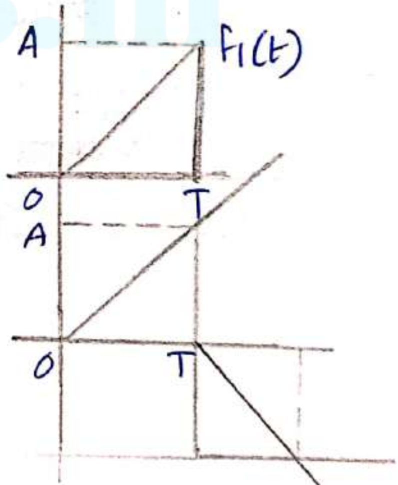
$$= \frac{(1-e^{-s})^2}{s(1+e^{-s})(1-e^{-s})}$$

$$= \frac{1-e^{-s}}{s(1+e^{-s})} = \frac{1}{s}\,\frac{e^{-s/2}(e^{s/2}-e^{-s/2})}{e^{-s/2}(e^{s/2}+e^{-s/2})}$$

$$= \frac{1}{s}\tanh\left(\frac{s}{2}\right)$$

6. Find the L.T of the waveform




r(t) = fu(t)

$$f_1(t) = \frac{A}{T}r(t) - \frac{A}{T}r(t-T) - Au(t-T)$$

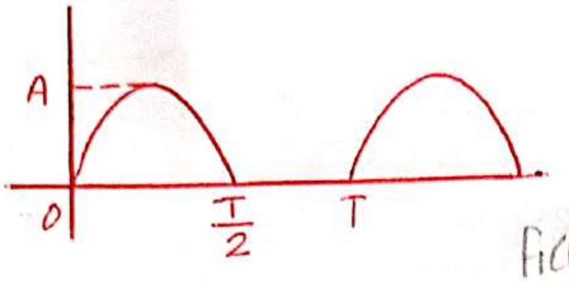
$$F_1(s) = \frac{A}{T}L\{r(t)\} - \frac{A}{T}L\{r(t-T)\} - A.L\{u(t-T)\}$$

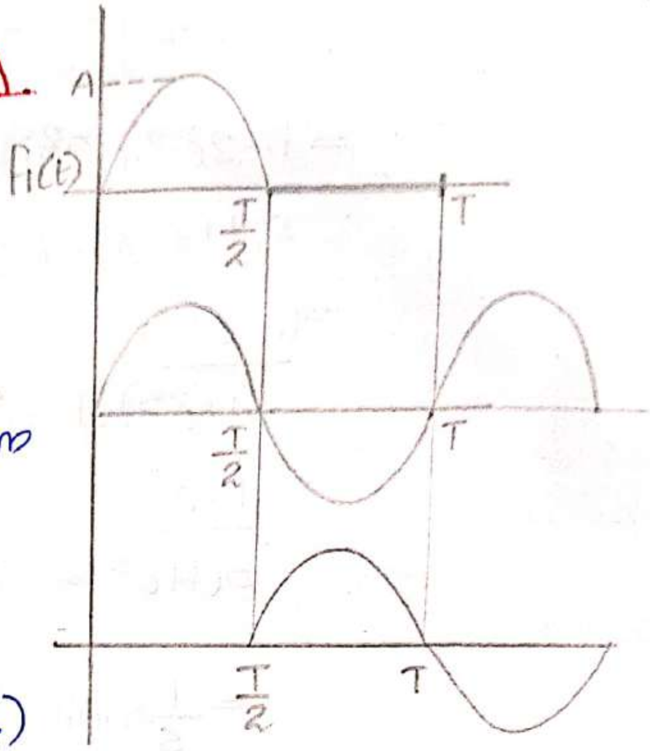
$$= \frac{A}{T}\frac{1}{s^2} - \frac{A}{T}\cdot\frac{1}{s^2}e^{-Ts} - A.\frac{1}{s}e^{-Ts}$$

$$= \frac{A}{s}\left[\frac{1}{Ts} - \frac{1}{Ts}e^{-Ts} - e^{-Ts}\right]$$

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}}F_1(s) = \frac{1}{1-e^{-Ts}}\cdot\frac{A}{s}\left[\frac{1}{Ts} - \frac{1}{Ts}e^{-Ts} - e^{-Ts}\right]$$

7. Find the L.T of the periodic waveform shown below.




F(T) is a periodic function with period T.

$f_1(t)$ can be constructed from two functions by waveform synthesis.

$$f_1(t) = A\sin\omega t \cdot u(t) + A\sin\omega\left(t-\frac{T}{2}\right)u\left(t-\frac{T}{2}\right)$$

$$F_1(s) = A \cdot L\{\sin\omega t \cdot u(t)\} + A \cdot L\left\{\sin\omega\left(t-\frac{T}{2}\right)u\left(t-\frac{T}{2}\right)\right\}$$

$$= A \cdot \frac{\omega}{s^2+\omega^2} + A \cdot \frac{\omega}{s^2+\omega^2} e^{-\frac{T}{2}s}$$

$$= \frac{A\omega}{s^2+\omega^2}\left[1+e^{-\frac{T}{2}s}\right]$$

$$F(s) = L\{f(t)\} = \frac{1}{1-e^{-Ts}} F_1(s)$$

$$= \frac{1}{1-e^{-Ts}} \cdot \frac{A\omega}{s^2+\omega^2}\left(1+e^{-\frac{T}{2}s}\right)$$

$$= \frac{A\omega}{s^2+\omega^2} \frac{\left(1+e^{-\frac{T}{2}s}\right)}{\left(1-e^{-Ts/2}\right)\left(1+e^{-Ts/2}\right)}$$

$$= \frac{A\omega}{s^2+\omega^2} \cdot \frac{1}{1-e^{-Ts/2}}$$

# INVERSE LAPLACE TRANSFORM

If $L\{f(t)\} = F(s)$, then $f(t)$ is called inverse L.T of $F(s)$ and symbolically written as:

$$f(t) = L^{-1}\{F(s)\}$$

$L^{-1}$ – inverse L.T operator

Inverse L.T can be found by the following methods:

(i) Standard results.

(ii) Partial fraction expansion.

(iii) Convolution theorem.

## (i) Standard results:

Inverse L.Ts of some functions can be found by standard results and properties of L.T.

1. Find the inverse L.T of $\dfrac{s^2-3s+4}{s^3}$

$$F(s) = \frac{s^2-3s+4}{s^3} = \frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^3}$$

$$f(t) = L^{-1}\{F(s)\} = 1 - 3t + 2t^2$$

$$\boxed{\begin{aligned} L\{1\} &= \frac{1}{s} \\ L\{t\} &= \frac{1}{t^2} \\ L\{t^2\} &= \frac{1}{t^3} \end{aligned}}$$

2. Find inverse L.T of $\dfrac{3s+4}{s^2+9}$

$\rightarrow L\{\cos 3t\} = \dfrac{s}{s^2+3^2}$

$\rightarrow L\{\sin 3t\} = \dfrac{3}{s^2+3^2}$

$$F(s) = \frac{3s+4}{s^2+9} = \frac{3s+4}{s^2+3^2}$$

$$= \frac{3 \cdot s}{s^2+3^2} + \frac{4}{3}\frac{1 \cdot 3}{s^2+3^2}$$

$$f(t) = 3\cos 3t + \frac{4}{3}\sin 3t$$

3. $F(s) = \frac{4s+15}{16s^2-25}$ Find $f(t)$.

$\rightarrow L\{\cosh \omega t\} = \frac{s}{s^2-\omega^2}$

$\rightarrow L\{\sinh \omega t\} = \frac{\omega}{s^2-\omega^2}$

$F(s) = \frac{4s+15}{16s^2-25}$ $\div$ by 16

$$= \frac{\frac{s}{4} + \frac{15}{16}}{s^2 - \frac{25}{16}} = \frac{\frac{1}{4}s + \frac{15}{16}}{s^2 - \left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{4} \cdot \frac{s}{s^2-(5/4)^2} + \frac{15}{16} \cdot \frac{1}{5/4} \cdot \frac{5/4}{s^2-\left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{4} \cdot \frac{s}{s^2-\left(\frac{5}{4}\right)^2} + \frac{3}{4} \cdot \frac{\frac{5}{4}}{s^2-(5/4)^2}$$

$$\therefore f(t) = \frac{1}{4}\cosh\left(\frac{5}{4}t\right) + \frac{3}{4}\sinh\left(\frac{5}{4}t\right)$$



4. $F(s) = \frac{2s+2}{s^2+2s+10}$. Find $f(t)$

$\rightarrow L\{\cos \omega t\} = \frac{s}{s^2+\omega^2}$

$\rightarrow L\{e^{-at}\cos \omega t\} = \frac{s+a}{(s+a)^2+\omega^2}$

$$F(s) = \frac{2s+2}{s^2+2s+10} = \frac{2s+2}{s^2+2s+1+9}$$

$$= \frac{2(s+1)}{(s+1)^2+3^2}$$

$$f(t) = 2e^{-t}\cos 3t$$

5. $F(s) = \frac{3s+7}{s^2-2s-3}$. Find $f(t)$.

$$F(s) = \frac{3s+7}{s^2-2s-3} = \frac{3s+7}{s^2-2s+1-4}$$

$$= \frac{3s+7}{(s-1)^2-4} = \frac{3(s-1)+10}{(s-1)^2-(2)^2}$$

$$F(t) = 3e^{t}\cosh(2t) + 5e^{t}\sinh(2t)$$

28/10/2020 (ii) Using partial fractions

1. $F(s) = \dfrac{P(s)}{(s+a)(s+b)} = \dfrac{A}{s+a} + \dfrac{B}{s+b}$

2. $F(s) = \dfrac{P(s)}{(s+a)(s+b)^n} = \dfrac{A}{s+a} + \dfrac{B_1}{s+b} + \dfrac{B_2}{(s+b)^2} + \cdots + \dfrac{B_n}{(s+b)^n}$

3. $F(s) = \dfrac{P(s)}{(s^2+as+b)(s^2+cs+d)} = \dfrac{As+B}{s^2+as+b} + \dfrac{Cs+D}{s^2+cs+d}$

4. $F(s) = \dfrac{P(s)}{(s^2+as+b)(s^2+cs+d)^n}$

$$= \frac{As+B}{s^2+as+b} + \frac{C_1s+D_1}{s^2+cs+d} + \frac{C_2s+D_2}{(s^2+cs+d)^2} + \cdots + \frac{C_ns+D_n}{(s^2+cs+d)^n}$$

Problems:

1. $F(s) = \dfrac{s+2}{s(s+1)(s+3)}$. Find $f(t)$

$$F(s) = \frac{s+2}{s(s+1)(s+3)} = \frac{a}{s} + \frac{b}{s+1} + \frac{b}{s+3}$$

$$a = sF(s)\Big|_{s=0} = \frac{s+2}{(s+1)(s+3)}\Big|_{s=0} = \frac{2}{1\cdot 3} = \frac{2}{3}$$

$$b = (s+1)F(s)\Big|_{s=-1} = \frac{s+2}{s(s+3)}\Big|_{s=-1} = \frac{1}{-1\cdot 2} = \frac{-1}{2}$$

$$c = (s+3)F(s)\Big|_{s=-3} = \frac{s+2}{s(s+1)}\Big|_{s=-3} = \frac{-1}{-3(-2)} = \frac{-1}{6}$$

$$F(s) = \frac{2}{3}\cdot\frac{1}{s} - \frac{1}{2}\cdot\frac{1}{s+1} - \frac{1}{6}\cdot\frac{1}{s+3}$$

$$f(t) = \frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}$$

2\. $F(s) = \frac{s+2}{s^2(s+3)}$. Find $f(t)$.

$$F(s) = \frac{s+2}{s^2(s+3)} = \frac{a}{s} + \frac{b}{s^2} + \frac{c}{s+3}$$

$$= as(s+3) + b(s+3) + cs^2$$

$$= as^2 + 3as + bs + 3b + cs^2$$

$a+c=0$ —— (1)

$3a+b=1$ —— (2)

$3b=2$ —— (3) $b=\frac{2}{3}$

$\therefore a = \frac{1-b}{3} = \frac{1-2/3}{3} = \frac{1/3}{3} = \frac{1}{9}$

$c = -a = -1/9$

$$F(s) = \frac{1}{9}\cdot\frac{1}{s} + \frac{2}{3}\cdot\frac{1}{s^2} - \frac{1}{9}\cdot\frac{1}{s+3}$$

$$f(t) = \frac{1}{9} + \frac{2}{3}t - \frac{1}{9}e^{-3t}$$

3\. $F(s) = \frac{5s^2-15s-11}{(s+1)(s-2)^2}$. Find $f(t)$.

$$F(s) = \frac{5s^2-15s-11}{(s+1)(s-2)^2} = \frac{a}{s+1} + \frac{b}{s+2} + \frac{c}{(s+2)^2}$$

$$5s^2 - 15s - 11 = a(s-2)^2 + b(s+1)(s-2) + c(s+1)$$

$$= a(s^2 - 4s + 4) + b(s^2 - s - 2) + c(s+1)$$

$$= s^2(a+b) + s(-4a - b + c) + (4a - 2b + c)$$

$$\left.\begin{array}{l} a+b = 5 \quad \text{---} (1) \\ -4a - b + c = -15 \quad \text{---} (2) \\ 4a - 2b + c = -11 \quad \text{---} (3) \end{array}\right\} \text{On Solving } a = 1, b = 4, c = -7$$

$L\{t\} = \frac{1}{s^2}$

$L\{te^{-at}\} = \frac{1}{(s+a)^2}$

$$F(s) = \frac{1}{s+1} + \frac{4}{s-2} - \frac{7}{(s-2)^2}$$

$$f(t) = e^{-t} + 4e^{2t} - 7te^{2t}$$

4. $F(s) = \dfrac{3s+1}{(s+1)(s^2+2)}$. Find $f(t)$

$$F(s) = \frac{3s+1}{(s+1)(s^2+2)} = \frac{a}{s+1} + \frac{bs+c}{s^2+2}$$

$$3s+1 = a(s^2+2) + (bs+c)(s+1)$$

$$= as^2 + 2a + bs^2 + (b+c)s + c$$

$$= (a+b)s^2 + (b+c)s + (2a+1)$$

$$\left.\begin{array}{l} a+b = 0 \quad \text{---} (1) \\ b+c = 3 \quad \text{---} (2) \\ 2a+c = 1 \quad \text{---} (3) \end{array}\right\} \text{On Solving, } a = -\frac{2}{3}, b = \frac{2}{3}, c = \frac{7}{3}$$

$$F(s) = -\frac{2}{3} \cdot \frac{1}{s+1} + \frac{\frac{2}{3}s + \frac{7}{3}}{s^2+2}$$

$$= -\frac{2}{3} \cdot \frac{1}{s+1} + \frac{2}{3} \cdot \frac{s}{s^2 + (\sqrt{2})^2} + \frac{7}{3} \cdot \frac{1}{s^2 + (\sqrt{2})^2} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$f(t) = \frac{-2}{3} e^{-t} + \frac{2}{3}\cos(\sqrt{2}t) + \frac{7}{3\sqrt{2}}\sin(\sqrt{2}t)$$



# TRANSIENT ANALYSIS

→ Whenever a n/w containing energy storage elements such as inductor or capacitor is switched from one condition to another, either by change in applied source or change in n/w elements, the response current and voltage change from one state to the other. The time taken to change from initial steady state to the final steady state is known as the transient period. This response is known as transient response or transients. The response of the n/w after it attains a final steady state value is independent of time and is called the steady state response. The complete response of the n/w is determined with the help of differential equation.

INITIAL CONDITIONS:

→ Initial conditions are used to determine the arbitrary constant in solving the n/w differential eqns.

→ To differentiate between the time immediately before and after the switching, − and + signs are used.

→ In solving the problems of initial conditions, we divide the time period in the following ways:

1. Just before switching (from $t=-\infty$ to $t=0^-$)
2. Just after switching (at $t=0^+$).
3. After switching (for $t>0$).

If the n/w remains in one conditions for a long time without any switching action, it is said to be under steady-state condition.

## 1. Initial conditions for Resistor:

$v(t) = Ri(t)$. The current changes instantaneously with voltage changes. |||ly voltage will change instantaneously if the current changes instantaneously.

## 2. Initial conditions for Inductor:

$v(t) = L\dfrac{di(t)}{dt}$ - Voltage across the Inductor is proportional to the rate of change of current. Inductor does not allow an abrupt change in current through it.

$i(t) = \dfrac{1}{L}\int^{t} v(t)dt + i(0)$ - $i(0)$ is the initial current through the inductor. If there is no current through the Inductor at $t=0^-$, the inductor will act as open circuit at $t=0^+$.

→ If a current $I_0$ flows through the Inductor at $t=0^-$, the inductor can be regarded as a current source of $I_0$ at $t=0^+$.

## 3. Initial conditions of capacitor:

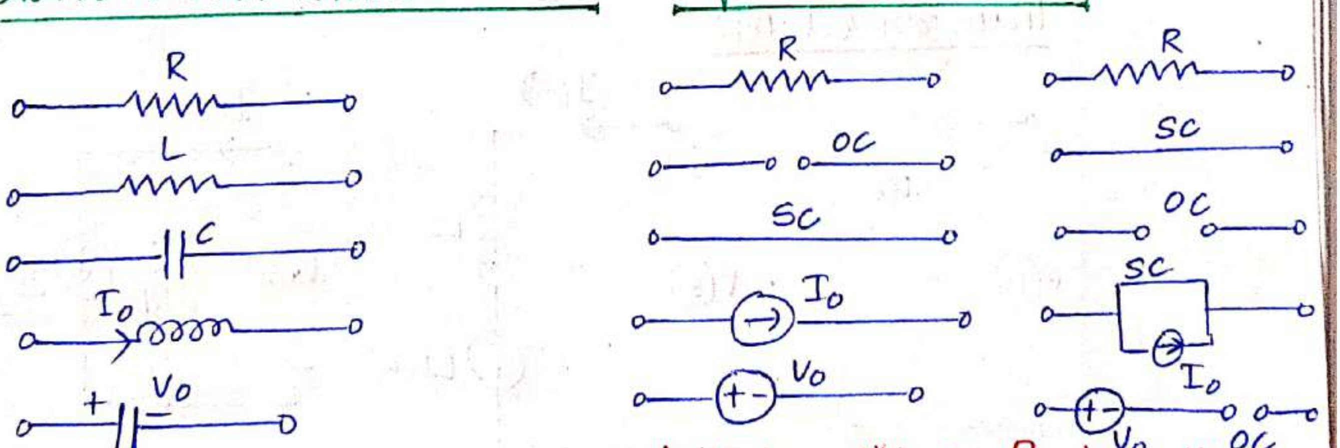
$i(t) = c\cdot\dfrac{dv(t)}{dt}$ - capacitor does not permit an abrupt change in voltage across it.



$v(t) = \dfrac{1}{c}\int_0^t i(t)dt + v(0)$ where $V(0)$ is the initial voltage across the capacitor.

If initial voltage is zero, the capacitor will act as a short circuit at $t=0^+$. If the capacitor is charged to a voltage $v_0$ at $t=0^-$, it can be regarded as a voltage source of $V_0$ at $t=0^+$.

Element with Initial conditions    Equivalent at $t=0^+$




Initial conditions    Final conditions

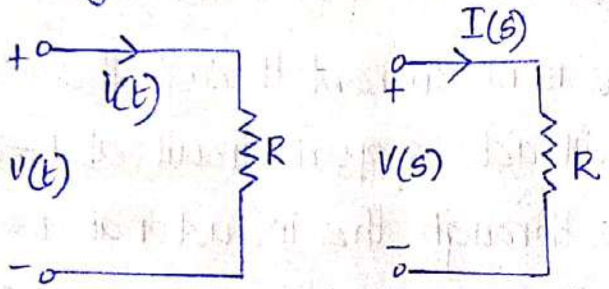
# Transformed Circuit

Voltage–current relationships of network elements can also be represented in frequency domain.

## 1. Resistor

$v(t) = Ri(t)$

Taking L.T, $V(s) = RI(s)$.




## 2. Inductor

For the inductor, the v–i relationships in the time domain are

$$v(t) = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t v(t) + i(0)$$

| $L\{f'(t)\} = sF(s) - f(0)$

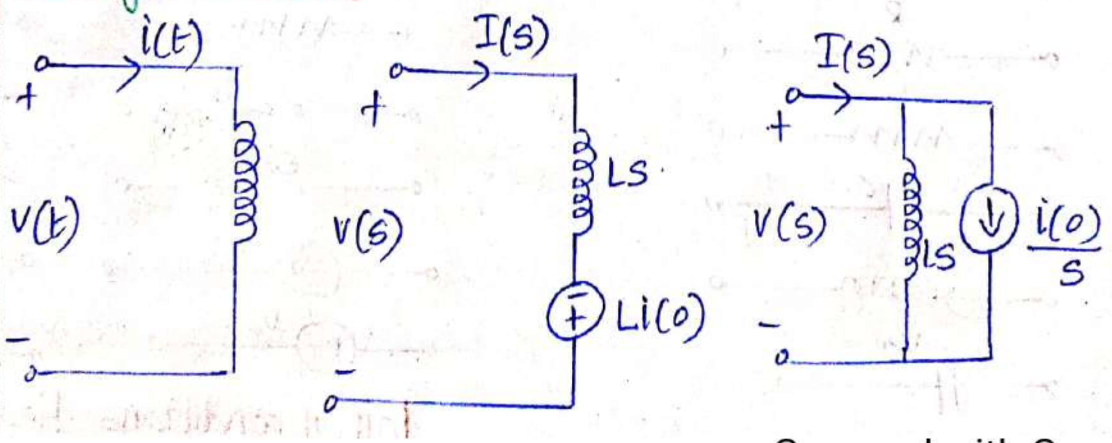
The corresponding frequency are given as,

$$V(s) = L[sI(s) - i(0)]$$

$$V(s) = LsI(s) - Li(0)$$

$$I(s) = \frac{1}{Ls}V(s) + \frac{i(0)}{s}$$

### Transformed n/w



3. Capacitor: V–i relationship in time domain are:

$$v(t) = \frac{1}{c}\int_0^t i(t)\,dt + v(0)$$

$$i(t) = c\frac{dv}{dt}$$

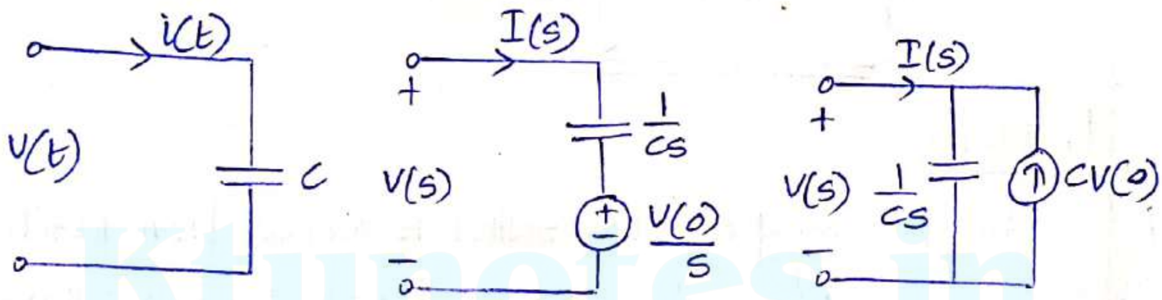
Corresponding frequency domain relations are given as,

$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0)}{s}$$

$$I(s) = C\left[sV(s) - V(0)\right]$$

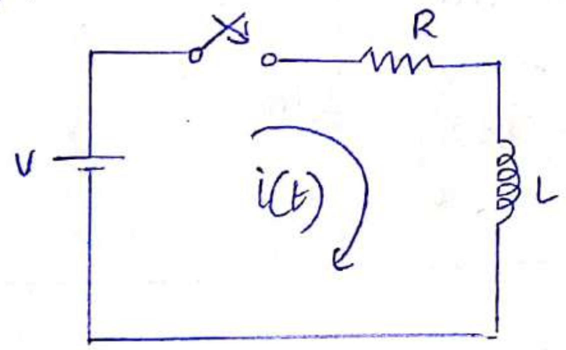
$$I(s) = CsV(s) - CV(0)$$

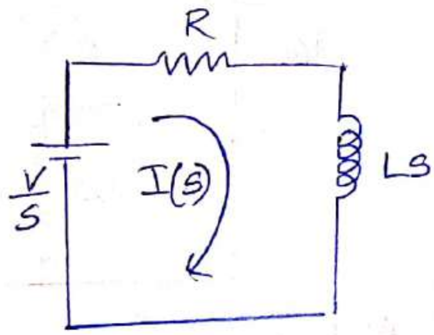
The transformed n/w is shown below.




29/10/2020 Resistor – Inductor circuit (RL circuit)

Consider a series RL circuit as shown. The switch is closed at $t=0$.




Transformed n/w for $t=0$

$$-\frac{V}{s} + RI(s) + LsI(s) = 0$$

$$I(s)\left[R + Ls\right] = \frac{V}{s}$$

$$I(s) = \frac{V}{s(R+Ls)} = \frac{V/L}{s\left(s+\frac{R}{L}\right)}$$

$$= \frac{a}{s} + \frac{b}{s+\frac{R}{L}}$$

$$a = I(s)\cdot s\Big|_{s=0} = \frac{V/L}{s+\frac{R}{L}}\Bigg|_{s=0} = \frac{V}{R}$$

$$b = I(s)\left(s+\frac{R}{L}\right)\Big|_{s=-\frac{R}{L}} = \frac{V/L}{s}\Big|_{s=-\frac{R}{L}} = \frac{-V}{R}$$

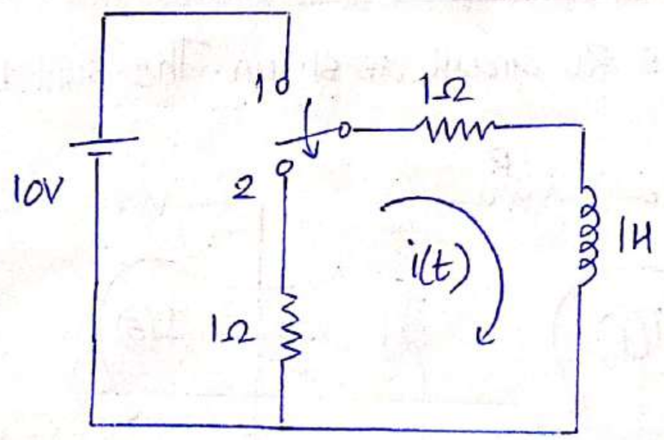
$$I(s) = \frac{V/R}{s} - \frac{V/R}{s+\frac{R}{L}}$$

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-(R/L)t}$$

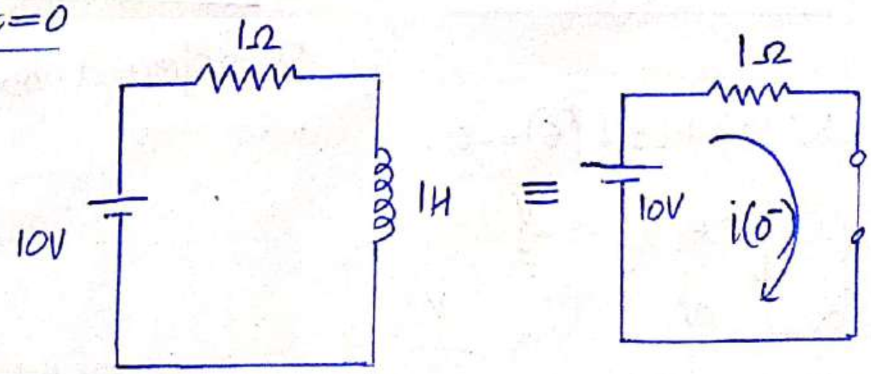
$$= \frac{V}{R}\left[1 - e^{-R/L\,t}\right] \text{ For } t>0$$

## Problems

1. In the following n/w, the switch is moved from position 1 to 2 at $t=0$, steady state condition having been established in position 1. Determine $i(t)$ for $t>0$.




At $t=0$

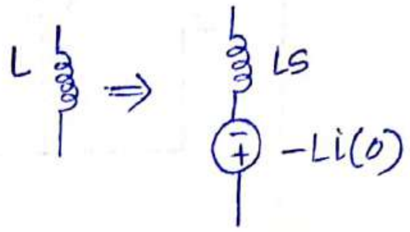


At $t=0^-$, the n/w has attained steady state condition. Hence the inductor acts as short circuit.

$$i(0^-) = \frac{10}{1} = 10A$$

Since the current through the inductor cannot change instantaneously,

$$i(0^+) = 10A$$



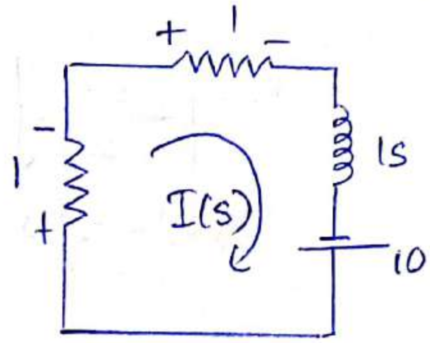

Applying KVL,

$$I(s)+I(s)+sI(s)-10=0$$

$$I(s)(s+2)=10$$

$$I(s) = \frac{10}{s+2}$$

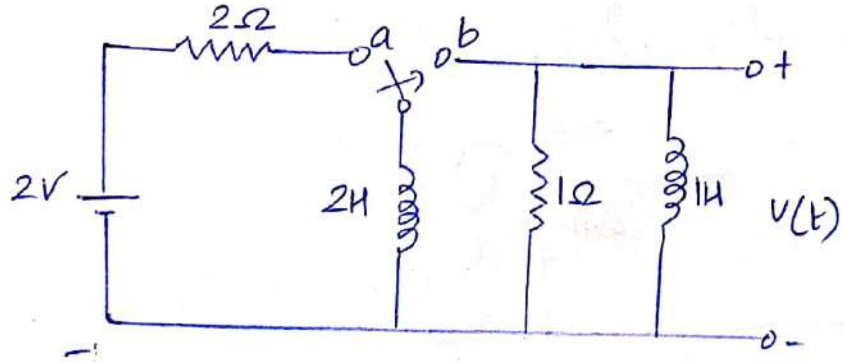
$$\therefore i(t) = 10e^{-2t}, \text{ for } t>0$$



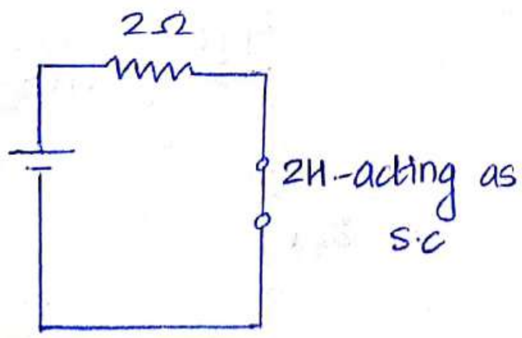

Transformed n/w for $t>0$



2. The n/w was initially in steady state with the switch in the position a. At $t=0$, the switch goes from a to b. Find an expression for $v(t)$ for $t>0$.




$t=0^-$

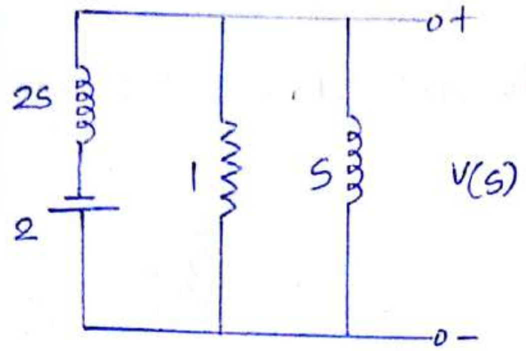



$$i(0^-) = \frac{2}{2} = 1A$$

$\because$ current through inductor cannot change instantaneously

$i(0^+) = 1A$

Transformed n/w for $t>0$




Applying KCL at node,

$$\frac{V(s)+2}{2s} + \frac{V(s)}{1} + \frac{V(s)}{s} = 0$$

$$V(s)\left[\frac{1}{2s} + 1 + \frac{1}{s}\right] = \frac{-1}{s}$$

$$V(s)\left[\frac{1+2s+2}{2s}\right] = -\frac{1}{s}$$

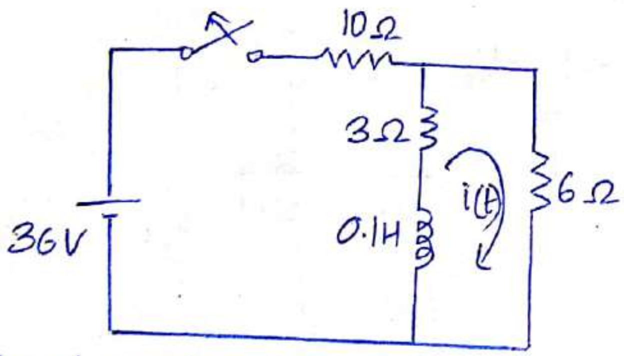
$$V(s)\left[\frac{2s+3}{2s}\right] = \frac{-1}{s}$$

$$V(s) = -\frac{1}{s} \times \frac{2s}{2s+3} = -\frac{2}{2s+3}$$

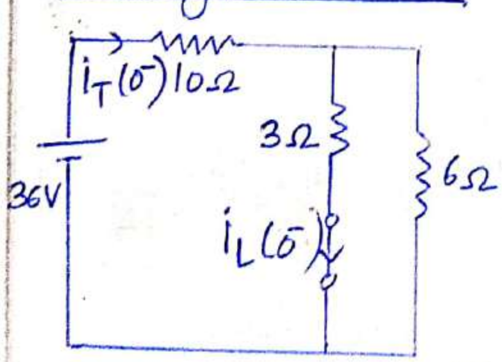
$$= -\frac{1}{s+3/2}$$

$$\therefore v(t) = -e^{-\left(\frac{3}{2}\right)t}, \quad t>0$$

3. In the n/w, the switch is opened at $t=0$. Find $i(t)$.




Steady state, $t=0^-$

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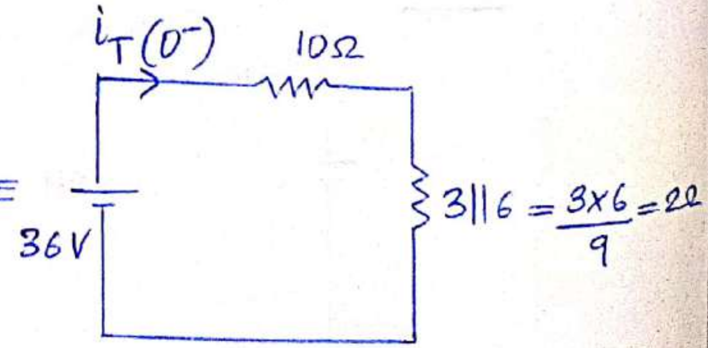
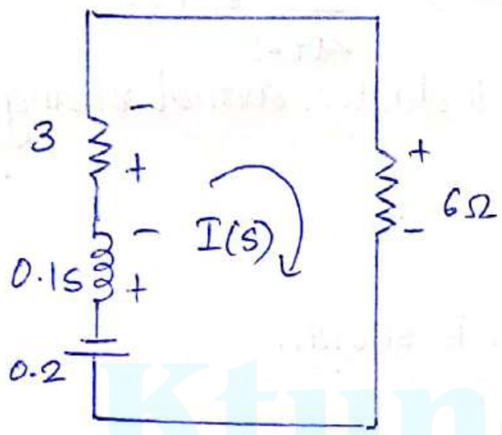


$i_T(0^-) = \frac{36}{10+2} = 3A$

$i_L(0^-) = 3 \times \frac{6}{3+6} = 2A$

$\because$ The current through the inductor cannot change instantaneously $i_L(0^+) = 2A$

Transformed circuit for $t>0$




Applying KVL,

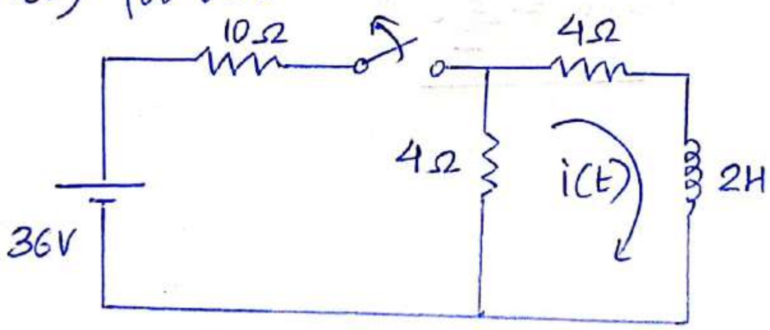
$0.2 + 0.1sI(s) + 3I(s) + 6I(s) = 0$

$I(s)[0.1s + 9] = -0.2$

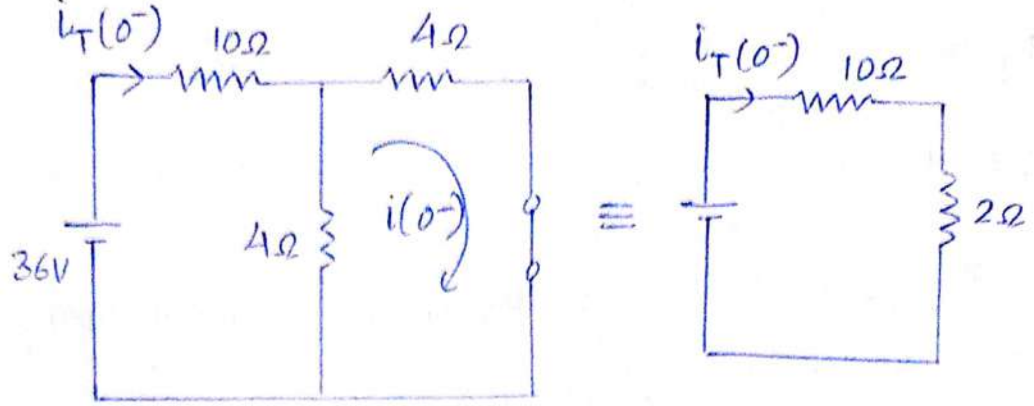
$I(s) = \frac{-0.2}{0.1s+9} = -\frac{2}{s+90}$

$i(t) = -2e^{-90t}$

4. The network shown below has acquired steady state with the switch closed for $t<0$. At $t=0$, the switch is opened. Obtain $i(t)$ for $t>0$.




At $t=0^-$, the network is as shown below, inductor ~~and~~ its acts as short circuit.




$$i_T(0^-) = \frac{36}{10+2} = 3A$$

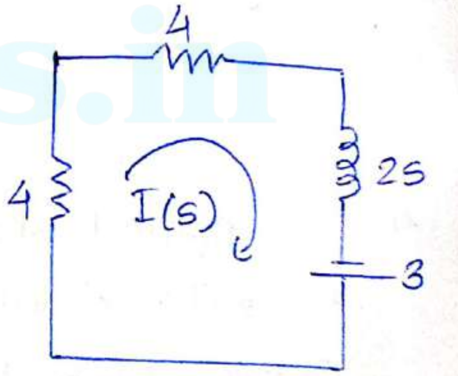
$$i(0^-) = \frac{3 \times 4}{4+4} = 1.5A$$

∵ The current through the inductor cannot change instantaneously,

$$i(0^+) = 1.5A$$

For $t>0$, the transformed n/w is shown,

Applying KVL,

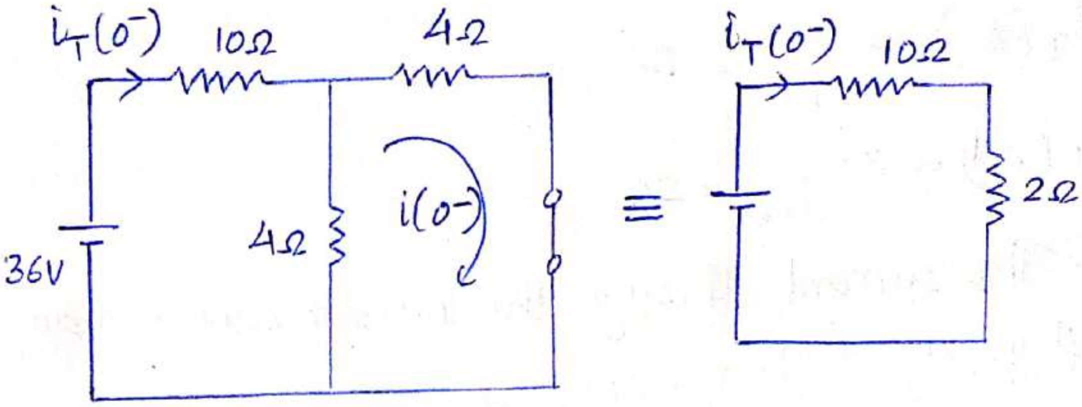



$$4I(s) + 4I(s) + 2sI(s) - 3 = 0$$

$$I_s(2s+8) = 0$$

$$I(s) = \frac{3}{2s+8} = \frac{3/2}{s+4}$$

$$\therefore i(t) = \frac{3}{2}e^{-4t}$$




$$i_T(0^-) = \frac{36}{10+2} = 3A$$

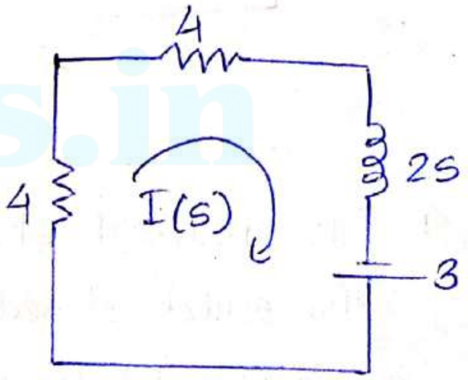
$$i(0^-) = \frac{3 \times 4}{4+4} = 1.5A$$

∵ The current through the inductor cannot change instantaneously,

$$i(0^+) = 1.5A$$

For $t>0$, the transformed n/w is shown,

Applying KVL,



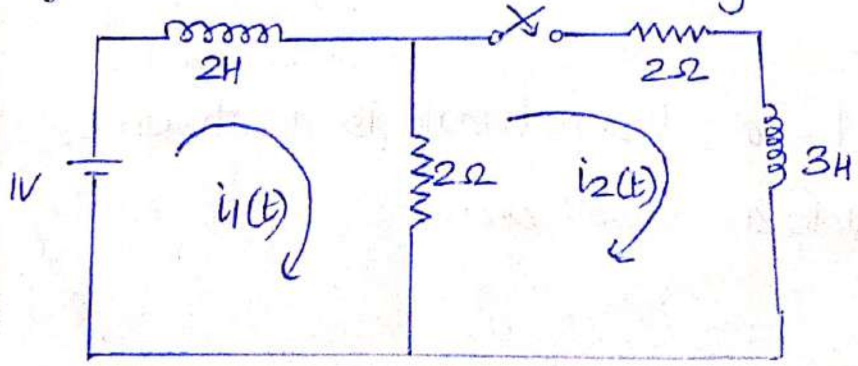

$$4I(s) + 4I(s) + 2sI(s) - 3 = 0$$

$$I_s(2s+8) = 3$$

$$I(s) = \frac{3}{2s+8} = \frac{3/2}{s+4}$$

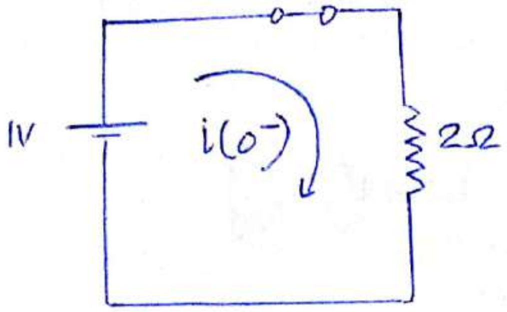
$$\therefore i(t) = \frac{3}{2}e^{-4t}$$

2/11/2020 5. In the n/w, the switch is closed at $t=0$, the steady state being reached before $t=0$. Determine current through 3H.



At $t = 0^-$, the network is shown below;



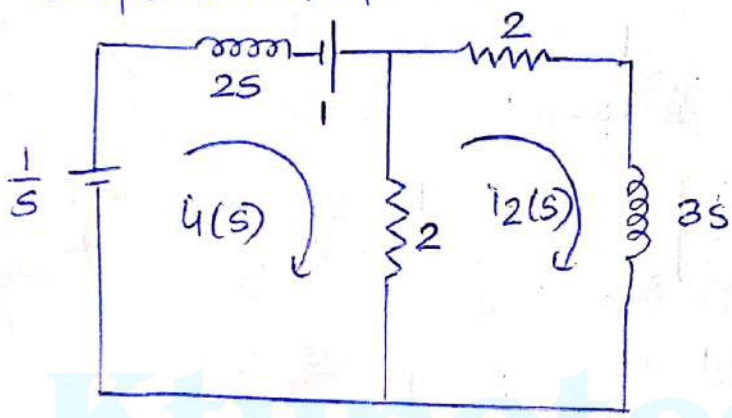

$i_1(0^-) = \frac{1}{2} A$

$i_2(0^-) = 0$

∵ Current through the inductor cannot change instantaneously

$i_1(0^+) = \frac{1}{2} A, \; i_2(0^+) = 0$

Transformed n/w for $t > 0$






Mesh-1: $-\frac{1}{s} + (2s+2) I_1(s) - 2I_2(s) - 1 = 0$

$$(2s+2) I_1(s) - 2I_2(s) = 1 + \frac{1}{s} \quad \text{——} (1)$$

Mesh-2: $(2+2+3s) I_2(s) - 2I_1(s) = 0$

$$-2I_1(s) + (3s+4) I_2(s) = 0 \quad \text{——} (2)$$

$$I_2(s) = \frac{\begin{vmatrix} 2s+2 & 1+\frac{1}{s} \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} 2s+2 & -2 \\ -2 & 3s+4 \end{vmatrix}} = \frac{2(1+1/s)}{(2s+2)(3s+4) - 4}$$

$$= \frac{\frac{2}{s}(s+1)}{2(s+1)(3s+4) - 4} = \frac{\frac{1}{s}(s+1)}{(s+1)(3s+4) - 2}$$

$$= \frac{(s+1)}{s(3s^2+7s+4-2)} = \frac{s+1}{s(3s^2+7s+2)}$$

$$= \frac{s+1}{s(3s^2+6s+s+2)} = \frac{s+1}{s[3s(s+2)+1(s+2)]}$$

$$= \frac{s+1}{s(3s+1)(s+2)} = \frac{1}{3}\cdot\frac{s+1}{s(s+2)(s+1/3)}$$

$$= \frac{a}{s} + \frac{b}{s+2} + \frac{c}{s+1/3}$$

$$a = \frac{1}{3}\cdot\frac{s+1}{(s+2)(s+1/3)}\bigg|_{s=0} = \frac{1}{3}\cdot\frac{1}{2\cdot 1/3} = \frac{1}{2}$$

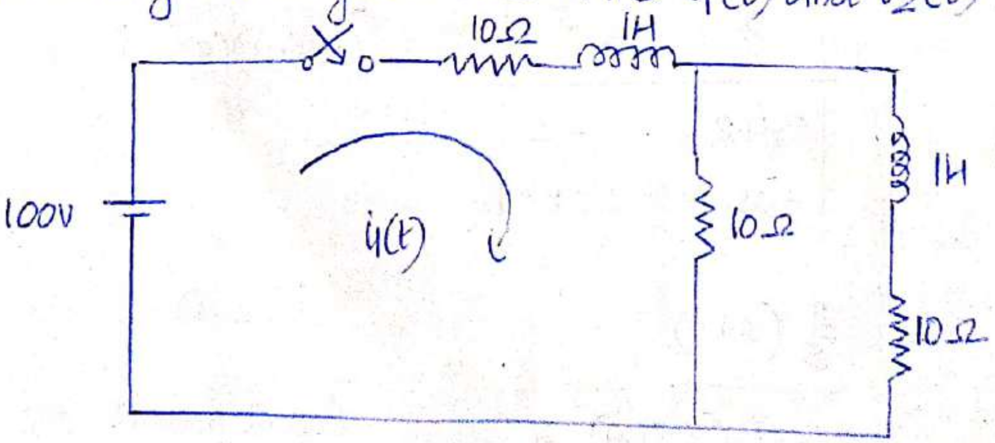
$$b = \frac{1}{3}\cdot\frac{s+1}{s(s+1/3)}\bigg|_{s=-2} = \frac{1}{3}\cdot\frac{-1}{-2\cdot -\frac{5}{3}} = \frac{-1}{10}$$

$$c = \frac{1}{3}\cdot\frac{s+1}{s(s+2)}\bigg|_{s=-1/3} = \frac{1}{3}\cdot\frac{\frac{2}{3}}{-1/3\cdot\frac{5}{3}} = \frac{-2}{5}$$

$$I_2(s) = \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{10}\cdot\frac{1}{s+2} - \frac{2}{5}\cdot\frac{1}{s+\frac{1}{3}}$$

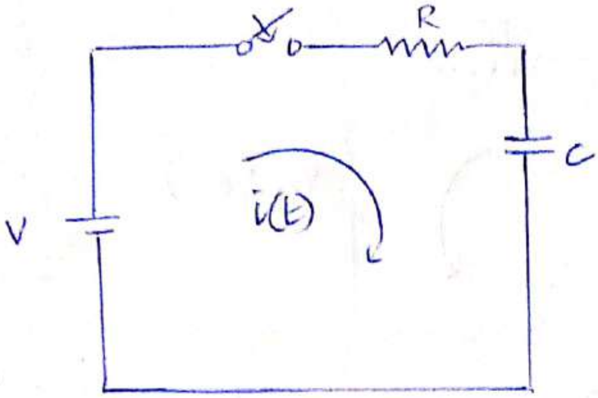
$$i_2(t) = \frac{1}{2} - \frac{1}{10}e^{-2t} - \frac{2}{5}e^{-t/3} \text{ for } t>0$$

6. In the network, the switch is closed at $t=0$ with the network previously unenergised. Determine $i_1(t)$ and $i_2(t)$

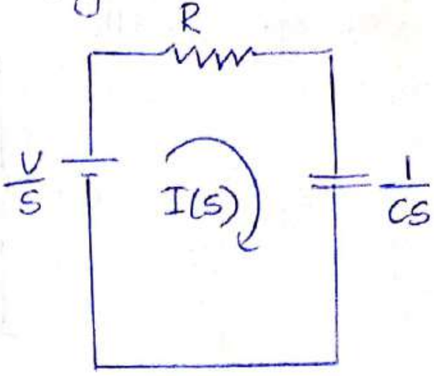


# RESISTOR - CAPACITOR CIRCUIT

Consider a series RC circuit as shown in fig. The switch is closed at $t=0$.




RC circuit




Transformed n/w

$$-\frac{V}{S} + RI(S) + \frac{1}{CS} I(S) = 0$$

$$\left[R + \frac{1}{CS}\right] I(S) = \frac{V}{S}$$

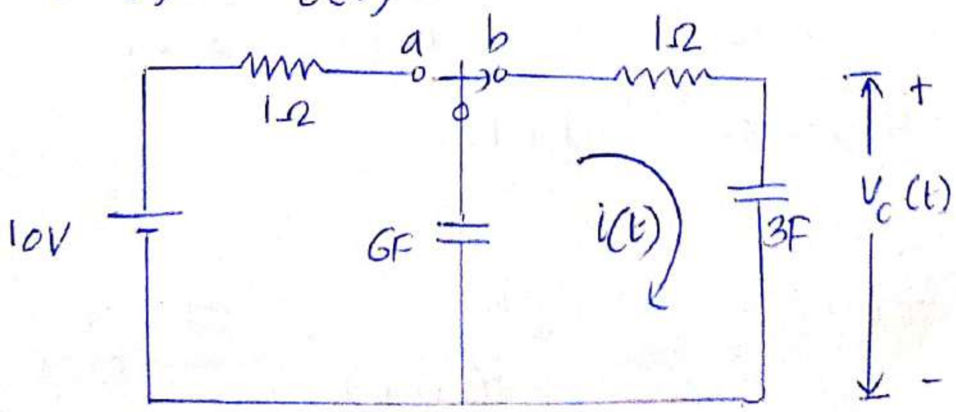
$$I(S) = \frac{V}{S\left(R+\frac{1}{CS}\right)} = \frac{V}{\frac{S(RCS+1)}{CS}} = \frac{V.C}{(S.RC+1)}$$

$$= \frac{V \cdot C}{R\left(S+\frac{1}{RC}\right)} = \frac{V}{R} \cdot \frac{1}{S+\frac{1}{RC}}$$

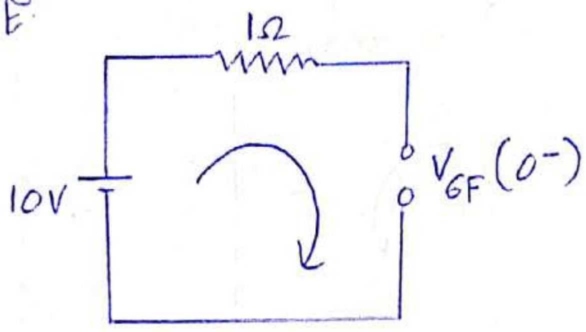
$$i(t) = \frac{V}{R} e^{-\left(\frac{1}{RC}\right)t}$$

## Problems

1. In the network, the switch is moved from a to b at $t=0$. Determine $i(t)$ and $V_c(t)$.



At $t=0^-$, the network is shown below. At $t=0^-$, the n/w has attained steady state condition. Hence, the capacitor of 6F acts as open circuit.

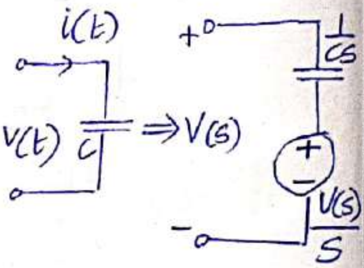


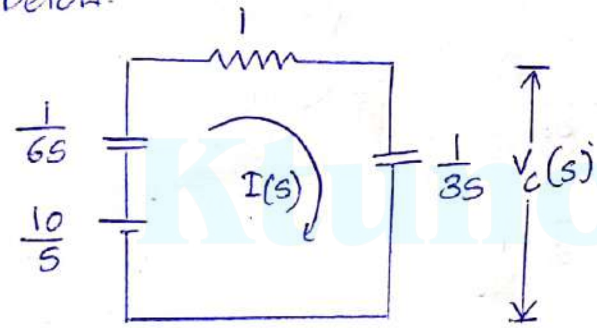

$V_{6F}(0^-) = 10V,\ i(0^-) = 0,\ V_{3F}(0^-) = 0$

Since voltage across the capacitor cannot change instantaneously,

$V_{6F}(0^+) = 10V,\ V_{3F}(0^+) = 0$

For $t>0$, the transformed n/w is shown below:




Applying KVL,

$$-\frac{10}{s} + \frac{1}{6s}I(s) + I(s) + \frac{1}{3s}I(s) = 0$$

$$I(s)\left(1+\frac{1}{6s}+\frac{1}{3s}\right) = \frac{10}{s}$$

$$I(s) = \frac{10}{s\left(1+\frac{1}{6s}+\frac{1}{3s}\right)} = \frac{10}{s+\frac{1}{6}+\frac{1}{3}} = \frac{10\times 6}{6s+1+2}$$

$$= \frac{60}{6s+3} = \frac{10}{s+0.5}$$

$$i(t) = 10e^{-0.5t} \text{ for } t>0$$

Voltage across 3F,

$$V_c(s) = \frac{1}{3s}\cdot I(s) = \frac{10}{3s(s+0.5)} = \frac{a}{s} + \frac{b}{s+0.5}$$

$$a = \left.\frac{10}{3(s+0.5)}\right|_{s=0} = \frac{10}{3\times 0.5} = \frac{20}{3}$$

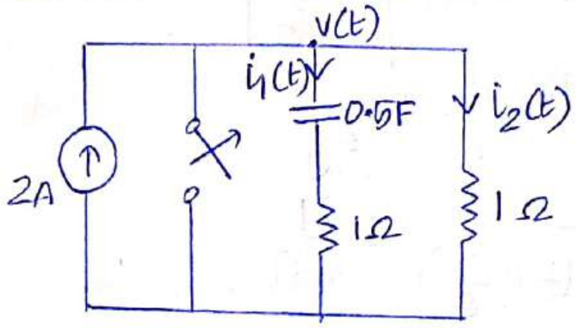
$$b = \left.\frac{10}{3s}\right|_{s=-0.5} = \frac{10}{3\times -0.5} = \frac{-20}{3}$$

$$V_c(s) = \frac{20}{3}\cdot\frac{1}{s} - \frac{20}{3}\cdot\frac{1}{s+0.5}$$

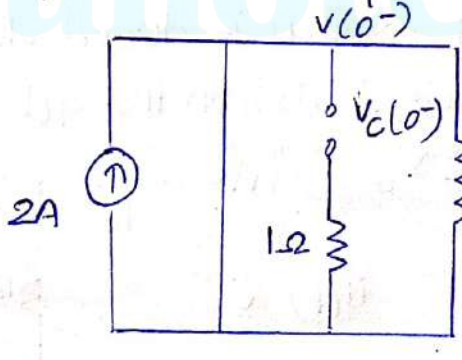
$$V_c(t) = \frac{20}{3} - \frac{20}{3}e^{-0.5t}$$

$$= \frac{20}{3}(1-e^{-0.5t})$$

2. In the n/w, the switch is closed for a long time and at $t=0$, the switch is opened. Determine the current through the capacitor.




At $t=0^-$, the switch is closed, steady state condition is reached, the capacitor acts as open circuit.

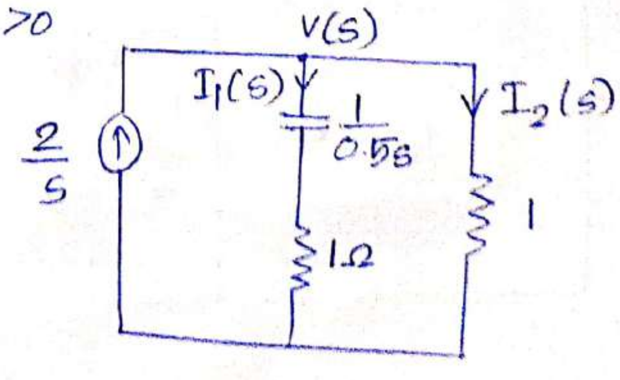



$V_c(0^-) = 0$

∵ The voltage across the capacitor cannot change instantaneously

$$V_c(0^+) = 0$$

Transformed n/w for $t>0$



Applying KVL in the mesh,

$$-I_1(s)(1) - I_1(s)\left(\frac{1}{0.5s}\right) + I_2(s)(1) = 0$$

$$I_2(s) = I_1(s)\left[1 + \frac{2}{s}\right) \quad \text{—— (1)}$$

Applying KCL at node,

$$\frac{2}{s} = I_1(s) + I_2(s)$$

Substituting for $I_2(s)$,

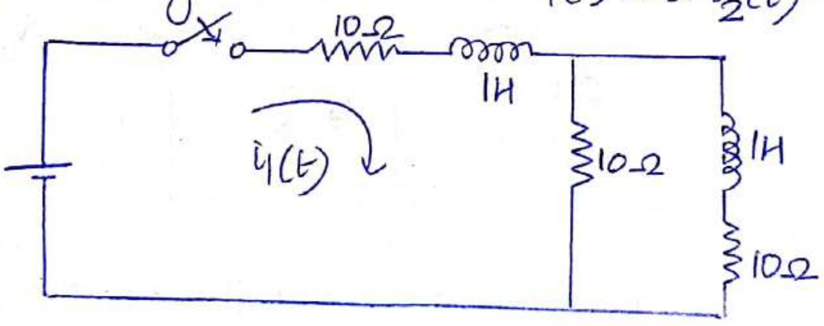
$$\frac{2}{s} = I_1(s) + I_1(s)\left[1 + \frac{2}{s}\right]$$

$$= I_1(s)\left[2 + \frac{2}{s}\right]$$

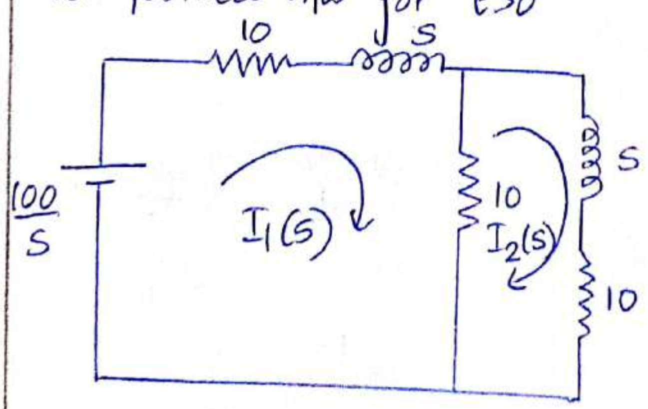
$$\therefore I_1(s) = \frac{2}{s\left(2 + \frac{2}{s}\right)} = \frac{1}{s+1}$$

$$i_1(t) = e^{-t}, t > 0$$

HW: 6. In the network, the switch is closed at $t=0$ with the n/w previously unenergised. Determine $i_1(t)$ and $i_2(t)$.




Transformed n/w for $t>0$



Mesh 1: $-\frac{100}{s} + (10+s+10)I_1(s) - 10I_2(s) = 0$

$$(s+20)I_1(s) - 10I_2(s) = \frac{100}{s} \quad \text{--- (1)}$$

Mesh 2: $(10+s+10)I_2(s) - 10I_1(s) = 0$

$$-10I_1(s) + (s+20)I_2(s) = 0 \quad \text{--- (2)}$$

$$I_1(s) = \frac{\begin{vmatrix} \frac{100}{s} & -10 \\ 0 & s+20 \end{vmatrix}}{\begin{vmatrix} s+20 & -10 \\ -10 & s+20 \end{vmatrix}} = \frac{\frac{100}{s}(s+20)}{(s+20)^2 - 100} = \frac{100(s+20)}{s[s^2+40s+400-100]}$$

$$= \frac{100(s+20)}{s(s^2+40s+300)}$$

$$= \frac{100(s+20)}{s(s+10)(s+30)}$$

$$= \frac{a}{s} + \frac{b}{s+10} + \frac{c}{s+30}$$

$$a = \left.\frac{100(s+20)}{(s+10)(s+30)}\right|_{s=0} = \frac{100\times20}{10\times30} = \frac{20}{3}$$

$$b = \left.\frac{100(s+20)}{s(s+30)}\right|_{s=-10} = \frac{100(10)}{-10\times20} = -5$$

$$c = \left.\frac{100(s+20)}{s(s+10)}\right|_{s=-30} = \frac{100(-10)}{-30\times-20} = \frac{-5}{3}$$

$$I_1(s) = \frac{20}{3}\cdot\frac{1}{s} - 5\cdot\frac{1}{s+10} - \frac{5}{3}\cdot\frac{1}{s+30}$$

$$i_1(t) = \frac{20}{3} - 5\cdot e^{-10t} - \frac{5}{3}e^{-30t}$$

$$I_2(s) = \frac{\begin{vmatrix} s+20 & \frac{100}{s} \\ -10 & 0 \end{vmatrix}}{(s+10)(s+30)} = \frac{\frac{1000}{s}}{(s+10)(s+30)} = \frac{1000}{s(s+10)(s+30)}$$

$$= \frac{a}{s} + \frac{b}{s+10} + \frac{c}{s+30}$$

$$a = \left.\frac{1000}{(s+10)(s+30)}\right|_{s=0} = \frac{1000}{10\times 30} = \frac{10}{3}$$

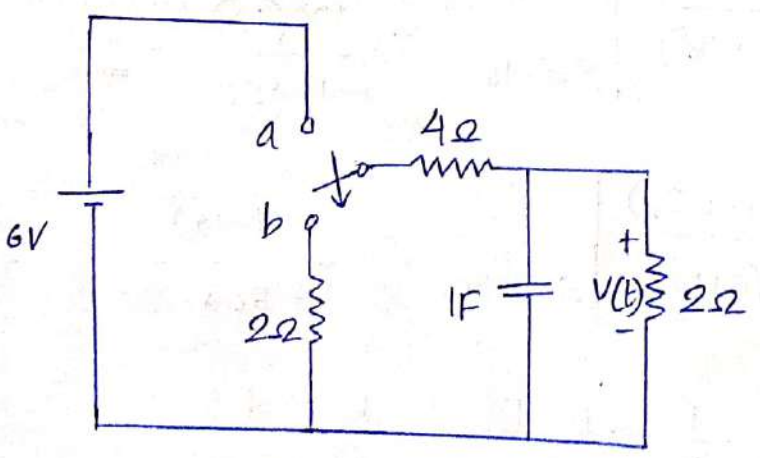
$$b = \left.\frac{1000}{s(s+30)}\right|_{s=-10} = \frac{1000}{-10\times 20} = -5$$

$$c = \left.\frac{1000}{s(s+10)}\right|_{s=-30} = \frac{1000}{-30\times -20} = \frac{5}{3}$$

$$I_2(s) = \frac{10}{3}\cdot\frac{1}{s} - 5\cdot\frac{1}{s+10} + \frac{5}{3}\cdot\frac{1}{s+30}$$

$$i_2(t) = \frac{10}{3} - 5e^{-10t} + \frac{5}{3}e^{-30t}, \quad t>0$$

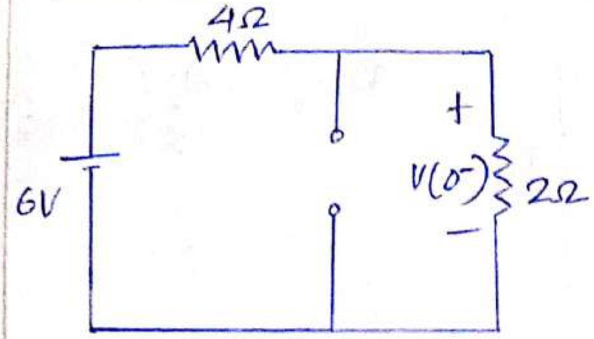
3/11/20 3. In the following n/w, the switch is moved from a to b at $t=0$. Find $v(t)$.





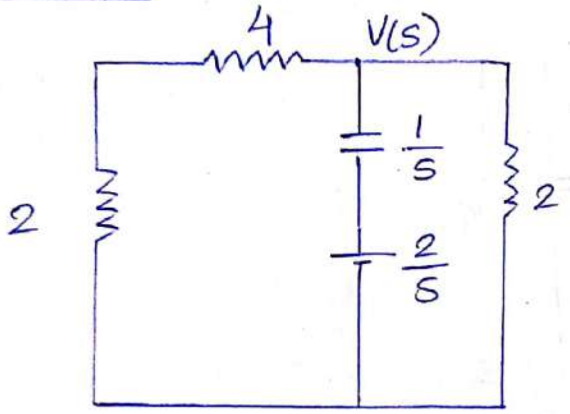
At $t=0^-$




$$V(0^-) = 6 \times \frac{2}{4+2} = 2V$$

$$V(0+) = 2V$$

For $t>0$




KCL at node

$$\frac{V(s)}{6} + \frac{V(s) - 2/s}{1/s} + \frac{V(s)}{2} = 0$$

$$\frac{V(s)}{6} + sV(s) - 2 + \frac{V(s)}{2} = 0$$

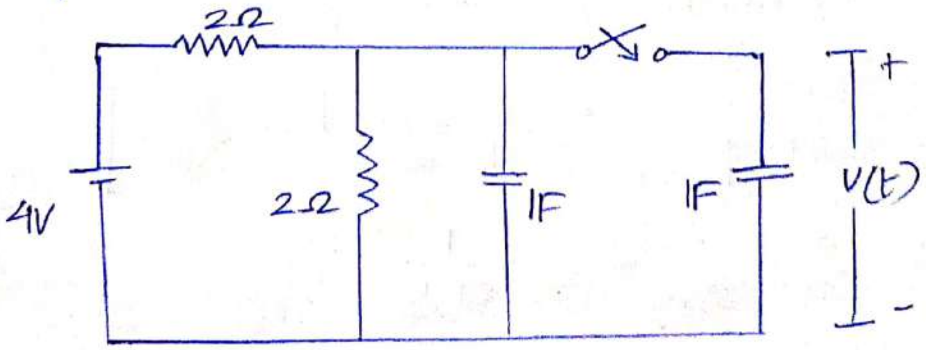
$$V(s)\left[s + \frac{1}{6} + \frac{1}{2}\right] = 2$$

$$V(s)\left[s + \frac{4}{6}\right] = 2$$

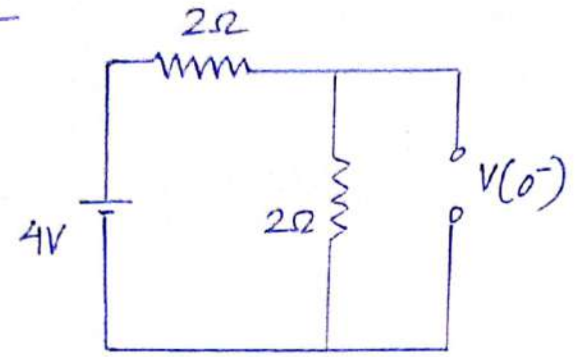
$$V(s) = \frac{2}{s+\frac{4}{6}} = \frac{2}{s+\frac{2}{3}}$$

$$V_c(t) = 2e^{-(2/3)t}$$

4. The n/w shown in fig has acquired steady state at $t<0$ with the switch open. The switch is closed at $t=0$. Determine $V(t)$.



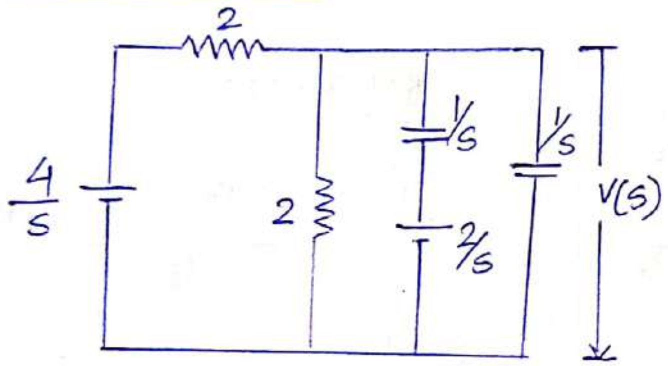
$\underline{t=0^-}$




$$V(0^-) = \frac{4\times 2}{2+2} = \underline{\underline{2V}}$$

$$V(0^+) = 2V$$

Transformed n/w




KCL at node

$$\frac{V(s)-\frac{4}{s}}{2} + \frac{V(s)}{2} + \frac{V(s)-\frac{2}{s}}{1/s} + \frac{V(s)}{1/s} = 0$$

$$\frac{V(s)}{2} - \frac{4}{2s} + \frac{V(s)}{2} + sV(s) - 2 + sV(s) = 0$$

$$V(s)\left[2s+1\right] = 2 + \frac{2}{s}$$

$$= \frac{2s+2}{s}$$

$$V(s) = \frac{2s+2}{s(2s+1)} = \frac{a}{s} + \frac{b}{2s+1}$$

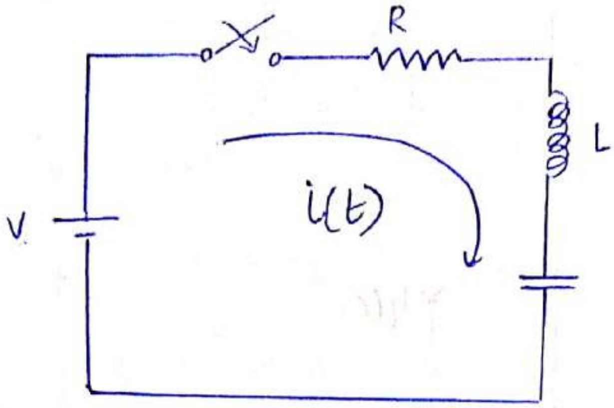
$$a = \left.\frac{2s+2}{2s+1}\right|_{s=0} = \frac{2}{1} = \underline{\underline{2}} \;;\; b = \left.\frac{2s+2}{s}\right|_{s=-1/2} = \frac{1}{-1/2} = \underline{\underline{-2}}$$

$$V(s) = \frac{2}{s} - \frac{2}{2s+1} = \frac{2}{s} - \frac{1}{s+1/2}$$

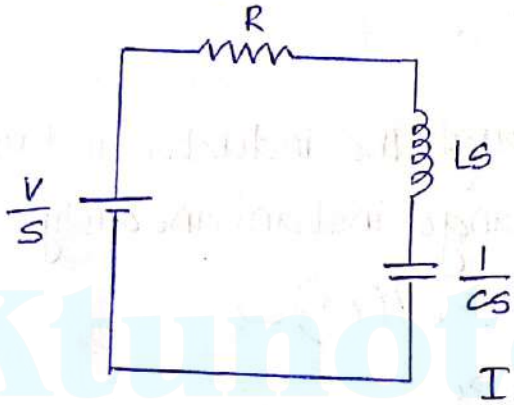
$$\therefore V(t) = 2 - e^{-(1/2)t}$$

## RLC CIRCUIT

Consider a series RLC circuit as shown in fig. The switch is closed at $t=0$




For $t>0$, the transformed n/w is shown below.




$$-\frac{V}{S}+\left(R+LS+\frac{1}{CS}\right)I(s)=0$$

$$I(s)\left(\frac{RCS+LCS^2+1}{CS}\right)=\frac{V}{S}$$

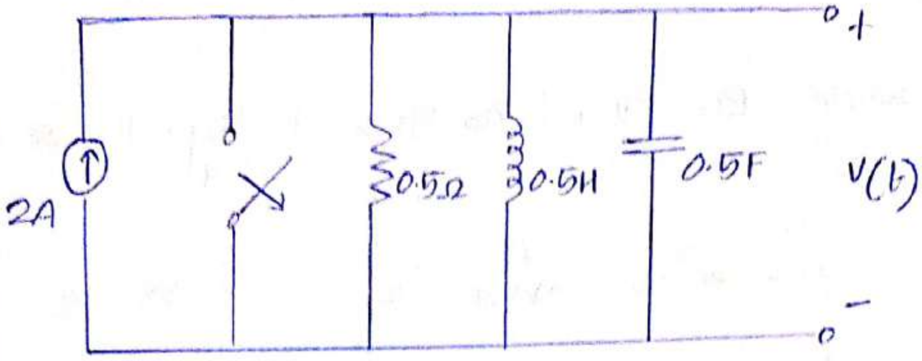
$$I(s)=\frac{\frac{V}{S}}{\frac{LCS^2+RCS+1}{CS}}=\frac{V}{S}\times\frac{CS}{LCS^2+RCS+1}$$

$$=\frac{VC}{LCS^2+RCS+1} \quad \div \text{ by } LC$$

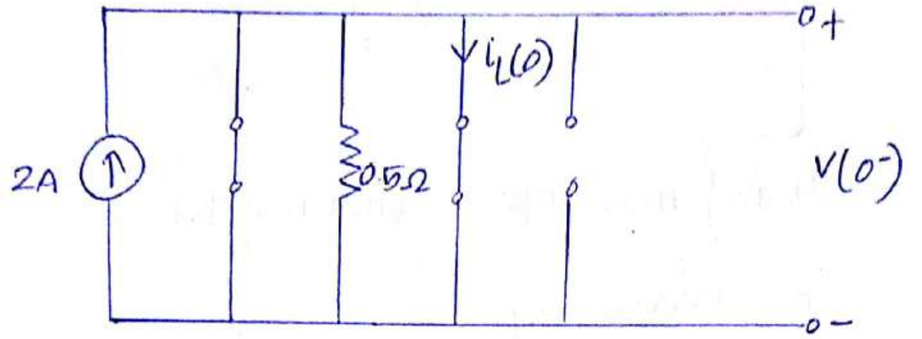
$$=\frac{V/L}{S^2+\frac{R}{L}S+\frac{1}{LC}}=\frac{V/L}{(S-S_1)(S-S_2)} \quad \text{——} ①$$

## PROBLEMS

1. In the n/w shown below, the switch is opened at $t=0$. Determine the voltage $v(t)$ for $t>0$.




$\underline{t=0}$



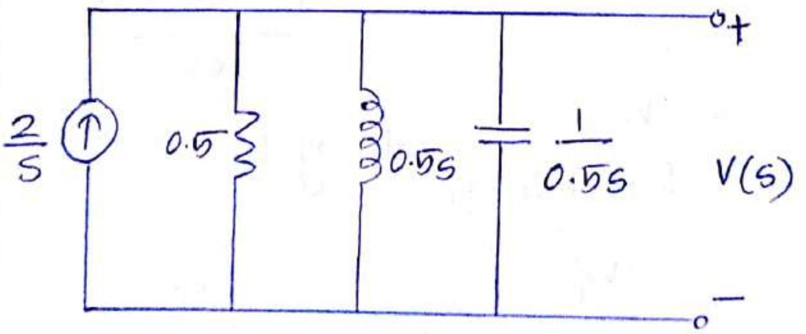

$i_L(0^-) = 0$
$V(0^-) = 0$

$\because$ The current through the inductor and voltage across the capacitor cannot change instantaneously;

$$i_L(0^+) = 0, \; V(0^+) = 0$$



Transformed n/w for $t > 0$




Applying KCL at node,

$$\frac{2}{s} = \frac{V(s)}{0.5} + \frac{V(s)}{0.5s} + \frac{V(s)}{1/0.5s}$$

$$V(s)\left[2 + \frac{2}{s} + 0.5s\right] = \frac{2}{s}$$

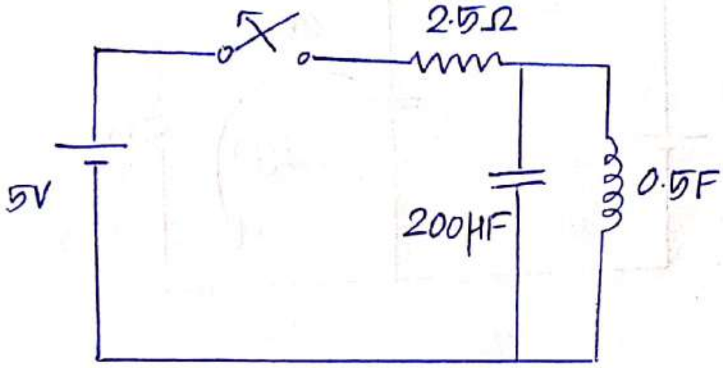
$$V(s) = \frac{\frac{2}{s}}{\frac{2}{s} + 0.5s + 2} = \frac{2}{2 + 0.5s^2 + 2s} = \frac{4}{s^2+4s+4} = \frac{4}{(s+2)^2}$$

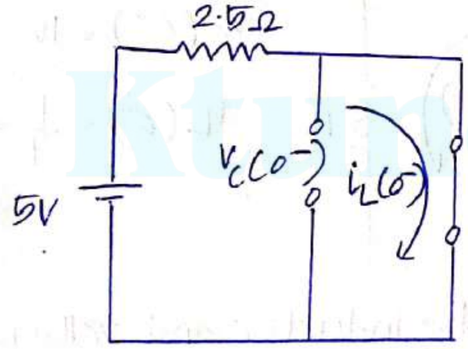
Taking inverse LT,

$$v(t) = 4te^{-2t} \text{ for } t>0$$

2. In the following n/w, the switch is closed and steady state is attained. At t=0, switch is opened. Determine the current through the inductor.




At $t=0^-$, switch is closed, inductor acts as short, capacitor open.




$$i_L(0^-) = \frac{5}{2.5} = 2A$$

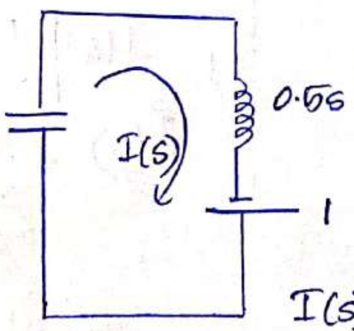
$v_c(0^-) = 0$ ($\because$ c-connected across short)

Since the voltage across the capacitor and current through the inductor cannot change instantaneously

$$i_L(0^+) = 2A, \quad v_c(0^+) = 0$$

For t>0

$$\frac{1}{200\times10^{-6}s} = \frac{5000}{s}$$




Applying KVL,

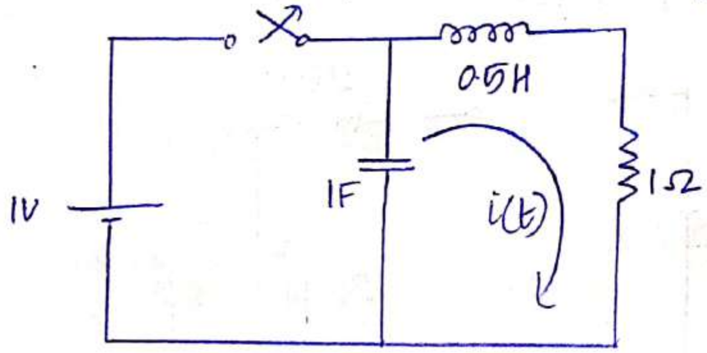
$$\left(\frac{5000}{s} + 0.5s\right)I_s - 1 = 0$$

$$I(s) = \frac{1}{0.5s + \frac{5000}{s}} = \frac{s}{0.5s^2 + 5000}$$

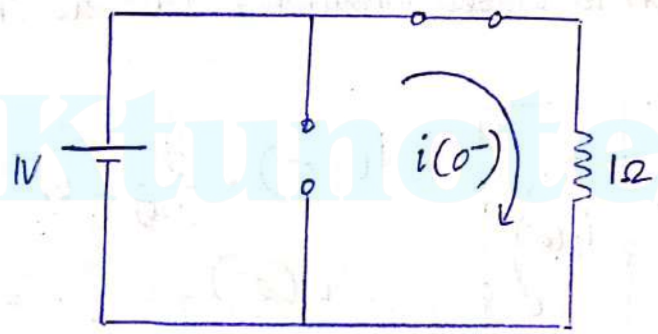
$$= \frac{2s}{s^2+10000} = 2 \cdot \frac{s}{s^2+100^2}$$

$$i(t) = 2\cos(100t) \text{ for } t>0$$

3. In the n/w the switch is opened at $t=0$. Steady state condition is achieved before $t=0$. Find $i(t)$




At $t=0^-$




$V_C(0^-) = 1V$

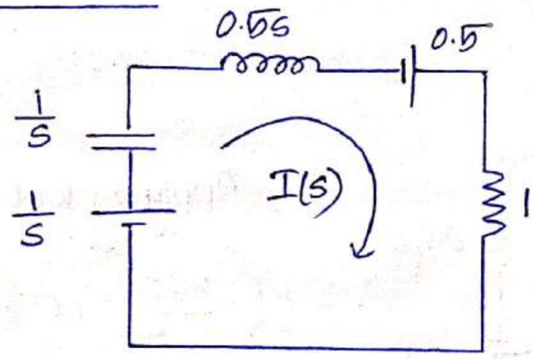
$i_L(0^-) = \frac{1}{1} = 1A$



Since the current through the inductor and voltage across the capacitor cannot change instantaneously,

$$V_C(0^+) = 1V, \quad i(0^+) = 1A$$

For $t>0$




$$-\frac{1}{s} + \frac{1}{s}I(s) + 0.5sI(s) - 0.5 + I(s) = 0$$

$$I(s)\left[\frac{1}{s} + 0.5s + 1\right] = 0.5 + \frac{1}{s}$$

$$I(s) = \frac{0.5 + \frac{1}{s}}{0.5s + 1 + \frac{1}{s}} = \frac{0.5s+1}{0.5s^2+s+1}$$

$$= \frac{s+2}{s^2+2s+2} \quad (\div 0.5)$$

$$= \frac{(s+1)+1}{(s+1)^2+1^2}$$

$$= \frac{s+1}{(s+1)^2+1^2} + \frac{1}{(s+1)^2+1^2}$$

$$i(t) = e^{-t}\cos(t) + e^{-t}\sin(t) \; ; \; t>0$$

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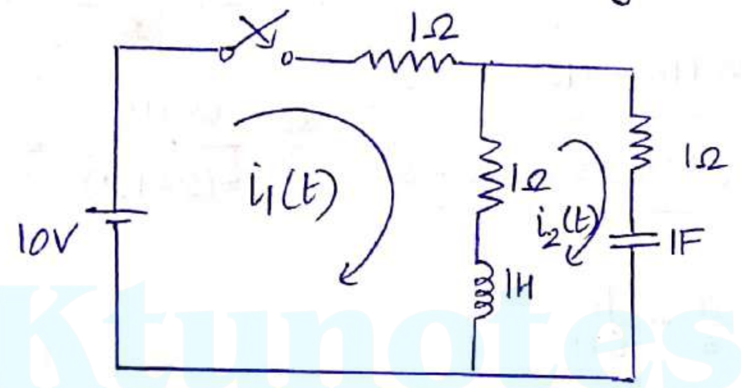
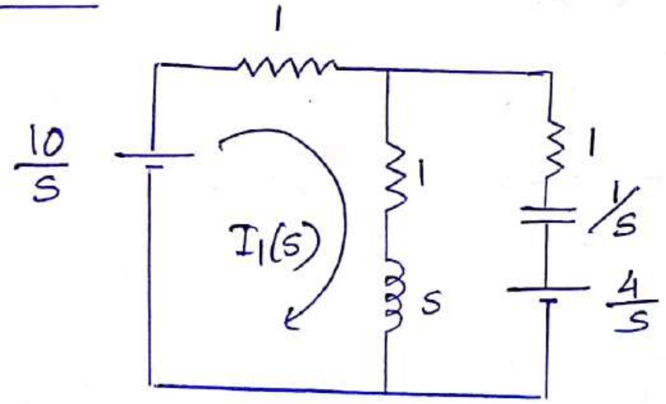
4. In the following n/w, the switch is closed at $t=0$. Find the currents $i_1(t)$ and $i_2(t)$ when initial currents through the inductor is zero and initial voltage on the capacitor is 4V.




For $t>0$




Mesh 1:
$$-\frac{10}{s} + (2+s)I_1(s) - (1+s)I_2(s) = 0$$

$$(s+2)I_1(s) - (s+1)I_2(s) = \frac{10}{s} \quad \text{—} ①$$

Mesh 2:
$$I_2(s)\left[s+2+\frac{1}{s}\right] - (1+s)I_1(s) + \frac{4}{s} = 0$$

$$-(s+1)I_1(s) + \left(s+2+\frac{1}{s}\right)I_2(s) = -\frac{4}{s} \quad \text{—} ②$$

$$I_1(s) = \frac{\begin{vmatrix} \frac{10}{s} & -(s+1) \\ -\frac{4}{s} & (s+2+\frac{1}{s}) \end{vmatrix}}{\begin{vmatrix} s+2 & -(s+1) \\ -(s+1) & s+2+\frac{1}{s} \end{vmatrix}} = \frac{\frac{10}{s}\left(s+2+\frac{1}{s}\right) - \frac{4}{s}(s+1)}{(s+2)\left(s+2+\frac{1}{s}\right) - (s+1)^2}$$

$$= \frac{\frac{10}{s^2}\left[s^2+2s+1\right] - \frac{4}{s}(s+1)}{(s+2)\frac{(s^2+2s+1)}{s} - (s+1)^2}$$

$$= \frac{\frac{10}{s^2}(s+1)^2 - \frac{4}{s}(s+1)}{\frac{(s+2)}{s}(s+1)^2 - (s+1)^2} = \frac{\frac{10}{s^2}(s+1) - \frac{4}{s}}{\frac{(s+2)(s+1)}{s} - (s+1)}$$

$$= \frac{10s+10-4s}{s^2\frac{(s^2+3s+2-s^2-s)}{s}} = \frac{6s+10}{s(2s+2)} = \frac{3s+5}{s(s+1)}$$

$$= \frac{a}{s} + \frac{b}{s+1}$$

$$a = \left.\frac{3s+5}{s+1}\right|_{s=0} = 5$$

$$b = \left.\frac{3s+5}{s}\right|_{s=-1} = \frac{2}{-1} = -2$$

$$I_1(s) = \frac{5}{s} - \frac{2}{s+1}$$

$$i_1(t) = 5 - 2e^{-t}, \; t>0$$

$$I_2(s) = \frac{\begin{vmatrix} s+2 & \frac{10}{s} \\ -(s+1) & -\frac{4}{s} \end{vmatrix}}{\frac{2(s+1)^2}{s}} = \frac{-\frac{4}{s}(s+2) + \frac{10}{s}(s+1)}{\frac{2(s+1)^2}{s}}$$

$$= \frac{\frac{1}{s}\left[-4s-8+10s+10\right]}{\frac{2}{s}\left[s+1\right]^2} = \frac{6s+2}{2(s+1)^2} = \frac{3s+1}{(s+1)^2}$$

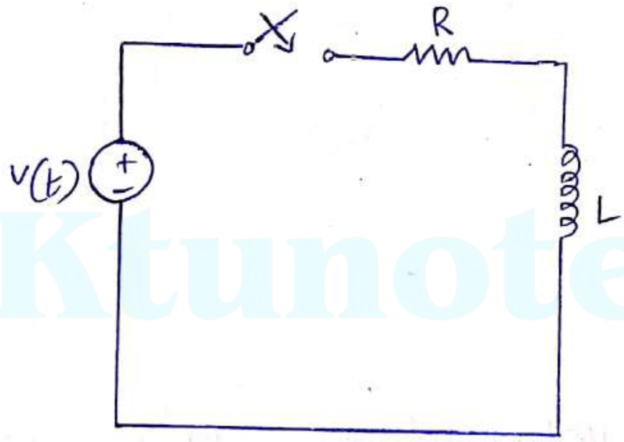
$$= \frac{3(s+1)-2}{(s+1)^2} = \frac{3}{(s+1)} - \frac{2}{(s+1)^2}$$

$$i_2(t) = 3e^{-t} - 2te^{-t}, t>0$$

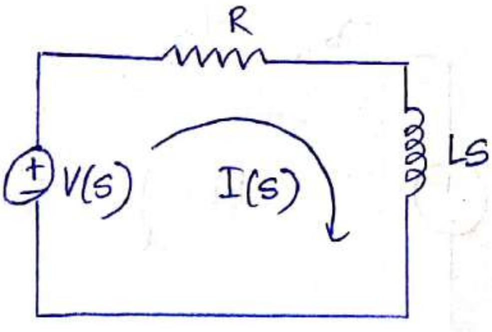
## RESPONSE OF RL CIRCUIT TO VARIOUS FUNCTIONS

Consider the following RL circuit. The switch is closed at $t=0$.

$$i(0^-) = i(0^+) = 0$$




Transformed n/w for $t>0$




Applying KVL,

$$-V(s) + RI(s) + LsI(s) = 0$$

$$I(s) = \frac{V(s)}{R+Ls} = \frac{1}{L} \cdot \frac{V(s)}{s+\frac{R}{L}}$$

Different i/p's

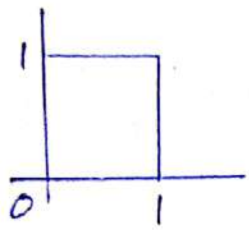
1. Impulse

$$v(t) = \delta(t),\ V(s) = 1$$

2. Step

$u(t) = u(t), V(s) = \frac{1}{s}$

3. Pulse



$u(t) = u(t) - u(t-1)$

$V(s) = \frac{1}{s} - \frac{1}{s} e^{-s}$

$= \frac{1}{s}\left[1 - e^{-s}\right]$

4. Exponential

$u(t) = e^{-t} u(t)$

$V(s) = \frac{1}{s+1}$

5. Sinusoidal i/p

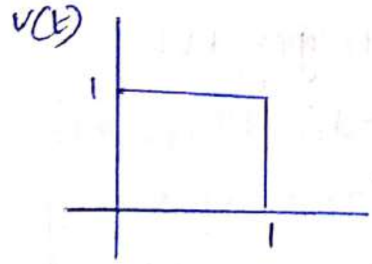
$u(t) = A \sin \omega t$

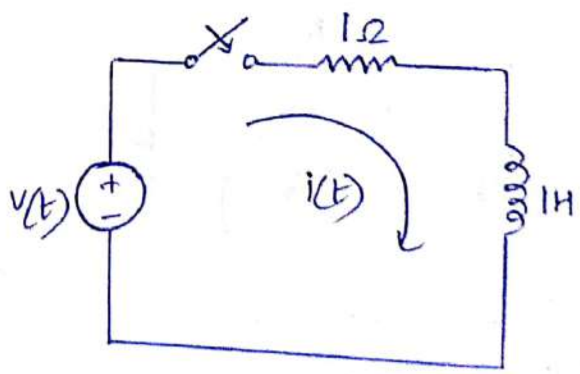
$V(s) = A \cdot \frac{\omega}{s^2 + \omega^2}$



PROBLEMS

1. At $t=0$, unit pulse voltage of unit width is applied to the series RL circuit as shown below. Obtain $i(t)$.

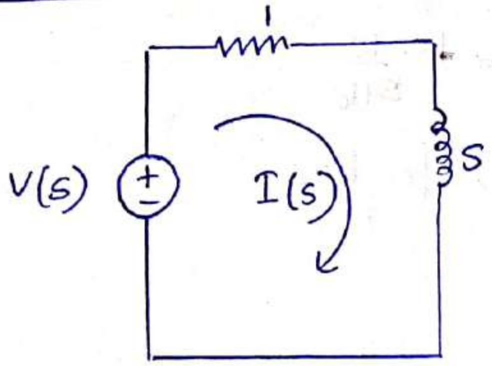





$V(t) = u(t) - u(t-1)$

$V(s) = \frac{1}{s} - \frac{1}{s} e^{-s} = \frac{1}{s}\left[1 - e^{-s}\right]$

## Transformed n/w for $t>0$




Applying KVL,

$$-V(s)+[1+s]I(s)=0$$

$$I(s)=\frac{V(s)}{s+1}=\frac{1-e^{-s}}{s(s+1)}$$

$$I(s)=\frac{1}{s(s+1)}-e^{-s}\cdot\frac{1}{s(s+1)}$$

$$\frac{1}{s(s+1)}=\frac{1}{s}-\frac{1}{s+1}$$

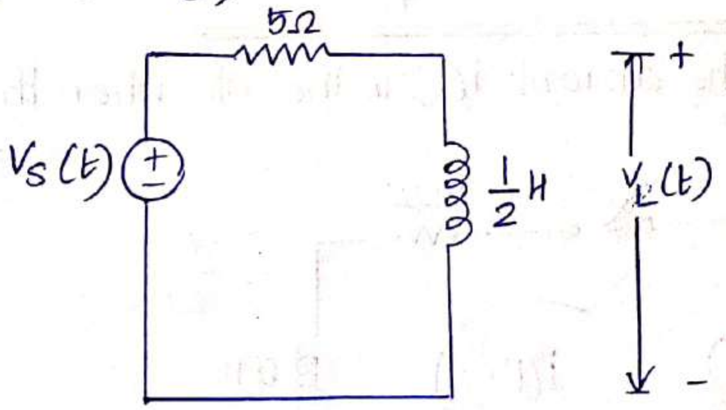
$$=\frac{1}{s}-\frac{1}{s+1}-e^{-s}\left[\frac{1}{s}-\frac{1}{s+1}\right]$$

$$=\frac{1}{s}-\frac{1}{s+1}-\frac{1}{s}e^{-s}+\frac{1}{s+1}e^{-s}$$

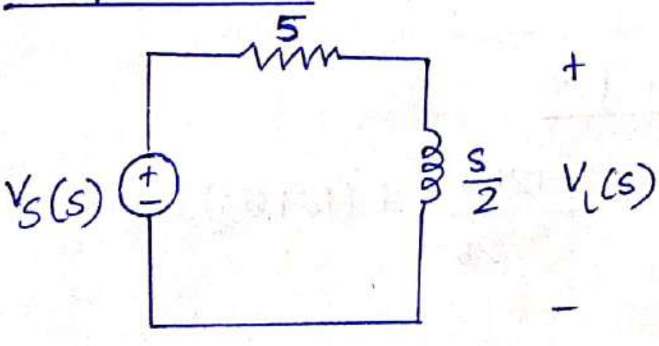
$$i(t)=u(t)-e^{-t}u(t)-u(t-1)+e^{-(t-1)}u(t-1)$$



2. Determine the expression for $V_L(t)$ when (i) $V_s(t)=\delta(t)$ and (ii) $V_s(t)=e^{-t}u(t)$




## Transformed n/w




$$V_L(s)=V_s(s)\times\frac{s/2}{5+s/2}$$

$$=\frac{s}{s+10}\cdot V_s(s)$$

(i) $v_s(t) = \delta(t) \Rightarrow V_s(s) = 1$.

$$V_L(s) = \frac{s}{s+10} = \frac{s+10-10}{s+10} = 1 - \frac{10}{s+10}$$

$$v_L(t) = \delta(t) - 10e^{-10t} \cdot u(t) \text{ for } t>0$$

(ii) $v_s(t) = e^{-t}u(t)$

$$V_s(s) = \frac{1}{s+1}$$

$$V_L(s) = \frac{s}{s+10} \cdot \frac{1}{s+1} = \frac{a}{s+1} + \frac{b}{s+10}$$

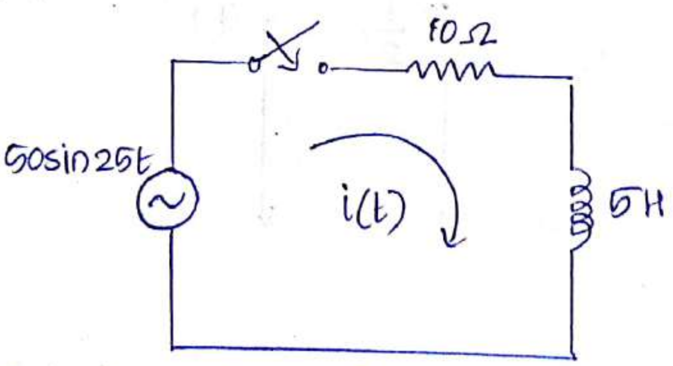
$$a = \left.\frac{s}{s+10}\right|_{s=-1} = \frac{-1}{9}$$

$$b = \left.\frac{s}{s+1}\right|_{s=-10} = \frac{-10}{-9} = \frac{10}{9}$$

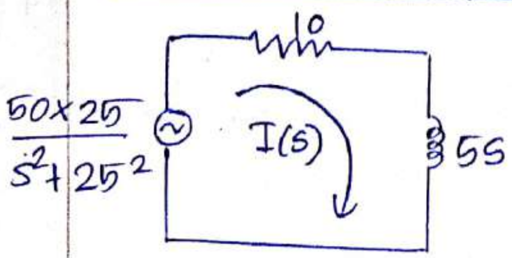
$$V_L(s) = \frac{-1}{9}\frac{1}{s+1} + \frac{10}{9}\cdot\frac{1}{s+10}$$

$$\therefore v_L(t) = \frac{-1}{9}e^{-t}u(t) + \frac{10}{9}\cdot e^{-10t}u(t)$$

5/11/2020 3. Determine the current $i(t)$ in the n/w when the switch is closed at $t=0$.




Transformed n/w for $t>0$




$$\frac{-1250}{s^2+25^2} + (10+5s)I(s) = 0$$

$$I(s) = \frac{1250}{s^2+25^2} \times \frac{1}{5(s+2)} = \frac{250}{(s^2+25)^2(s+2)}$$

$$= \frac{as+b}{s^2+25^2} + \frac{c}{s+2}$$

$$250 = (as+b)(s+2) + c(s^2+25^2)$$

$$a+c=0$$

$$2a+b=0$$

$$2b+625c=0$$

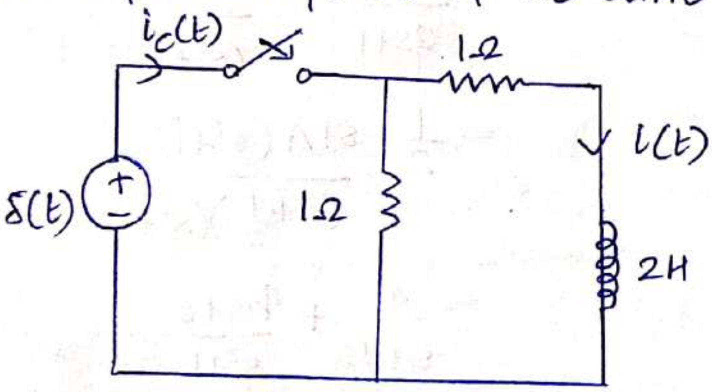
Solving, $a=-0.397$, $b=0.795$, $c=0.397$

$$I(s) = \frac{-0.397s+0.795}{s^2+25^2} + \frac{0.397}{s+2}$$

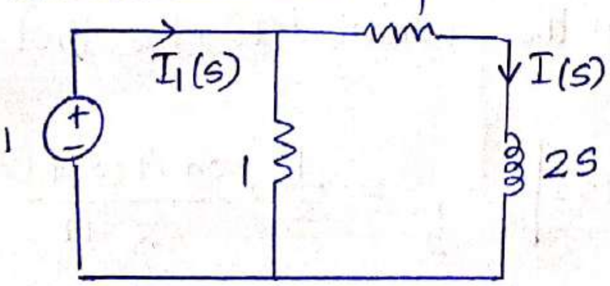
$$= -0.397 \cdot \frac{s}{s^2+25^2} + \frac{0.795}{25} \times \frac{25}{s^2+25^2} + \frac{0.397}{s+2}$$

$$i(t) = -0.397\cos(25t) + 0.032\sin(25t) + 0.397e^{-2t}u(t), \; t>0$$

4. Find impulse response of the current $i(t)$.




Transformed n/w




$$Z(s) = 1 \,||\, (2s+1)$$

$$= \frac{1\cdot(2s+1)}{1+2s+1} = \frac{2s+1}{2s+2}$$

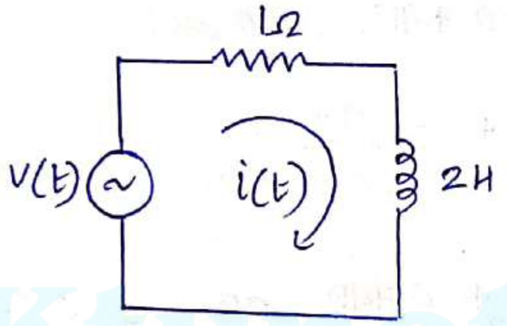
$$I_1(s) = \frac{V(s)}{Z(s)} = \frac{1}{\frac{2s+1}{2s+2}} = \frac{2s+2}{2s+1}$$

$$I(s) = I_1(s) \times \frac{1}{2s+2} = \frac{2s+2}{2s+1} \times \frac{1}{2s+2}$$

$$= \frac{1}{2s+1} = \frac{1}{2} \cdot \frac{1}{s+0.5}$$

$$\therefore i(t) = \frac{1}{2} e^{-0.5t} u(t), \text{ for } t>0$$

5. The following n/w is at rest for $t<0$. If the voltage $v(t) = u(t)\cos t + A\delta(t)$ is applied to the n/w, determine the value of A so that there is no transient term in the current response $i(t)$.

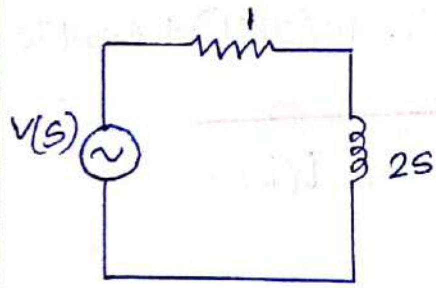



$$v(t) = u(t) = u(t)\cos t + A\delta(t)$$

$$V(s) = \frac{s}{s^2+1} + A = \frac{s + A(s^2+1)}{s^2+1}$$



Transformed n/w at $t>0$




$$V(s) = (2s+1) I(s)$$

$$I(s) = \frac{V(s)}{2s+1} = \frac{s + A(s^2+1)}{(s^2+1)(2s+1)}$$

$$= \frac{1}{2} \cdot \frac{s + A(s^2+1)}{(s+\frac{1}{2})(s^2+1)}$$

$$= \frac{a}{s+\frac{1}{2}} + \frac{bs+c}{s^2+1}$$

The transient part of the response is given by first term. For no transient term in the current $i(t)$, the first term must vanish ie $a=0$.

$$a = \left(s+\frac{1}{2}\right) I(s) \Big|_{s=-\frac{1}{2}} = \frac{1}{2} \cdot \frac{s + A(s^2+1)}{s^2+1} \Big|_{s=-\frac{1}{2}}$$

$$= \frac{1}{2}\frac{\left[-\frac{1}{2} + A\left(\frac{1}{4}+1\right)\right]}{\frac{1}{4}+1} = \frac{1}{2}\frac{\left[-\frac{1}{2} + \frac{5}{4}A\right]}{5/4}$$

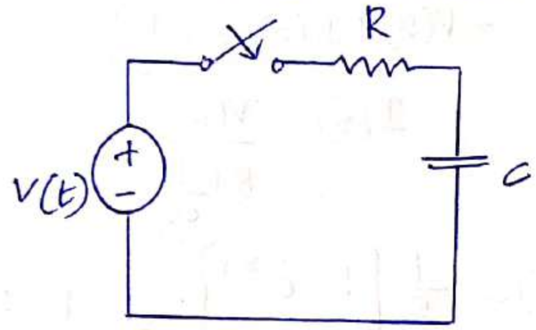
For a to be zero,

$$-\frac{1}{2} + \frac{5}{4}A = 0$$

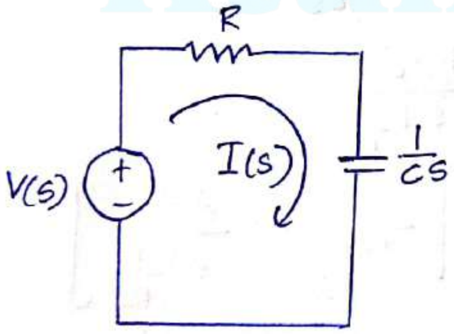
$$\text{or } A = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5} = 0.4$$

## RESPONSE OF RC CIRCUIT TO VARIOUS FUNCTIONS

Consider a series RC circuit as shown below.




Transformed n/w for t>0




Applying KVL,

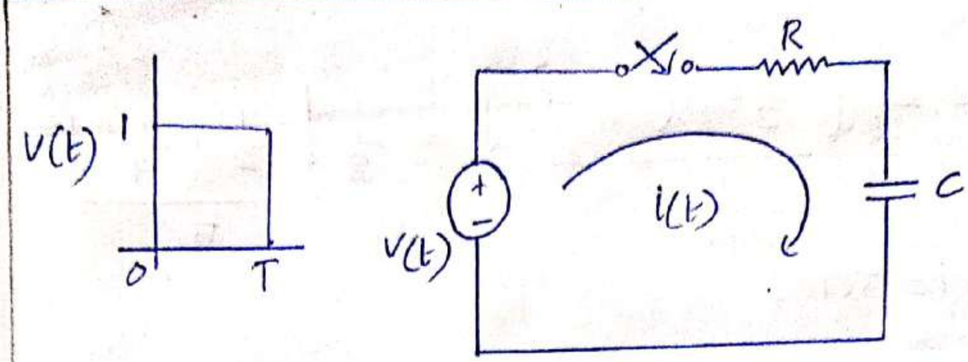
$$-V(s) + I(s)\left[R + \frac{1}{CS}\right] = 0$$

$$I(s) = \frac{V(s)}{R + \frac{1}{CS}} = \frac{S\,V(s)}{SR + \frac{1}{C}}$$

$$= \frac{1}{R} \cdot \frac{S\,V(s)}{S + \frac{1}{RC}}$$

## PROBLEMS

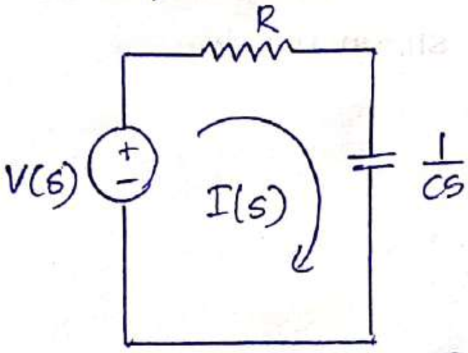
1. A rectangular pulse of unit length and T-seconds duration is applied to a series RC n/w at t=0. Obtain the expression for the current i(t). Assume the capacitor to be initially uncharged.




$$v(t) = u(t) - u(t-T)$$

$$V(s) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1}{s}\left[1 - e^{-sT}\right]$$

For $t>0$, the transformed n/w;




$$-V(s) + I(s)\left(R + \frac{1}{Cs}\right) = 0$$

$$I(s) = \frac{V(s)}{R + \frac{1}{Cs}}$$

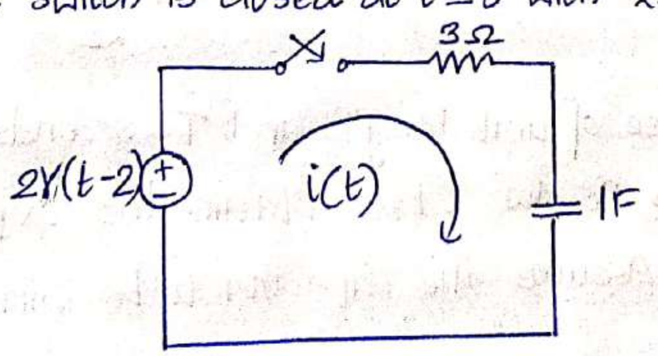
$$I(s) = \frac{\frac{1}{s}\left[1 - e^{-sT}\right]}{R + \frac{1}{Cs}} = \frac{1 - e^{-sT}}{sR + \frac{1}{C}}$$

$$= \frac{1}{R} \cdot \frac{1 - e^{-sT}}{s + \frac{1}{RC}}$$

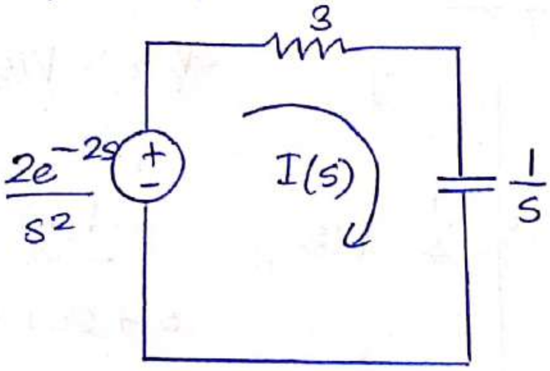
$$= \frac{1}{R}\left[\frac{1}{s + \frac{1}{RC}} - \frac{e^{-st}}{s + \frac{1}{RC}}\right]$$

$$i(t) = \frac{1}{R}\left[e^{-\left(\frac{1}{RC}\right)t} u(t) - e^{-\left[\frac{1}{RC}\right](t-T)} u(t-T)\right] \text{ for } t>0$$

2. For the n/w shown below, determine the current $i(t)$ when the switch is closed at $t=0$ with zero initial conditions.



For $t>0$, transformed n/w




$$-\frac{2e^{-2s}}{s^2}+Is\left(3+\frac{1}{s}\right)=0$$

$$I(s)=\frac{\frac{2e^{-2s}}{s^2}}{3+\frac{1}{s}}=\frac{2e^{-2s}}{s(1+3s)}$$

$$=\frac{2}{3}\cdot\frac{e^{-2s}}{s(s+1/3)}$$

$$=0.67\cdot\frac{e^{-2s}}{s(s+0.33)}$$

By partial fractions,

$$\frac{0.67}{s(s+0.33)}=\frac{a}{s}+\frac{b}{s+0.33}$$

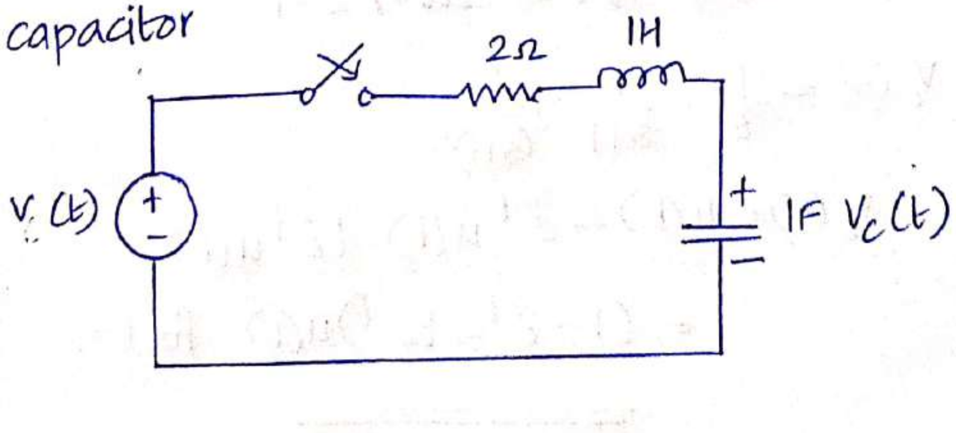
$$a=\left.\frac{0.67}{s+0.33}\right|_{s=0}=2$$

$$b=\left.\frac{0.67}{s}\right|_{s=-0.33}=-2$$

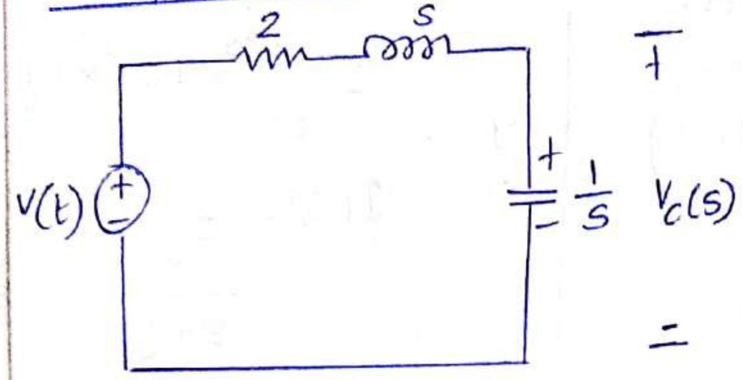
$$I(s)=e^{-2s}\left[\frac{2}{s}-\frac{2}{s+0.33}\right]=2\cdot\frac{1}{s}e^{-2s}-2\cdot\frac{1}{s+0.33}e^{-2s}$$

$$i(t)=2u(t-2)-2\cdot e^{-0.33(t-2)}u(t-2) \text{ for } t>0$$

3. Find the impulse and step response of the voltage across the capacitor



Transformed n/w for t>0




$$V_c(s) = V(s) \times \frac{1/s}{2+s+\frac{1}{s}}$$

$$= \frac{V(s)}{s^2+2s+1} = \frac{V(s)}{(s+1)^2}$$

(a) $v(t) = \delta(t)$

$V(s) = 1$

$V_c(s) = \frac{1}{(s+1)^2}$

$V_c(t) = te^{-t}u(t)$ for $t>0$

(b) $v(t) = u(t)$

$V(s) = \frac{1}{s}$

$V_c(s) = \frac{1}{s(s+1)^2} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{(s+1)^2}$

$1 = a(s+1)^2 + bs(s+1) + cs$

$1 = a(s^2+2s+1) + b(s^2+s) + cs$

$= s^2(a+b) + s(2a+b+c) + a$

$a=1,\ a+b=0 \Rightarrow b=-1$

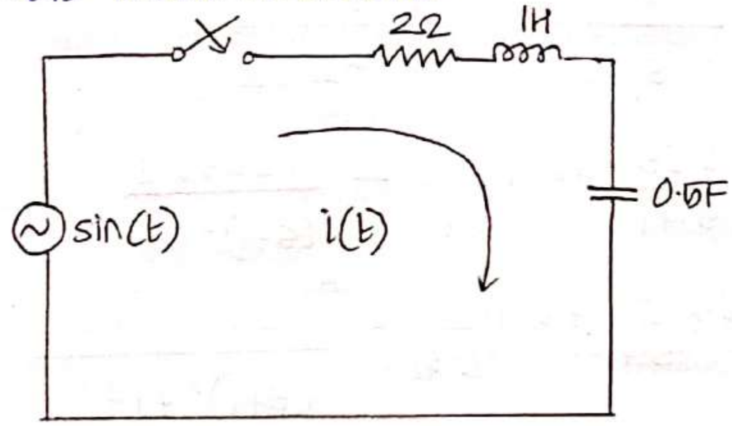
$2a+b+c=0 \Rightarrow c = -2a-b = -1$

$V_c(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$

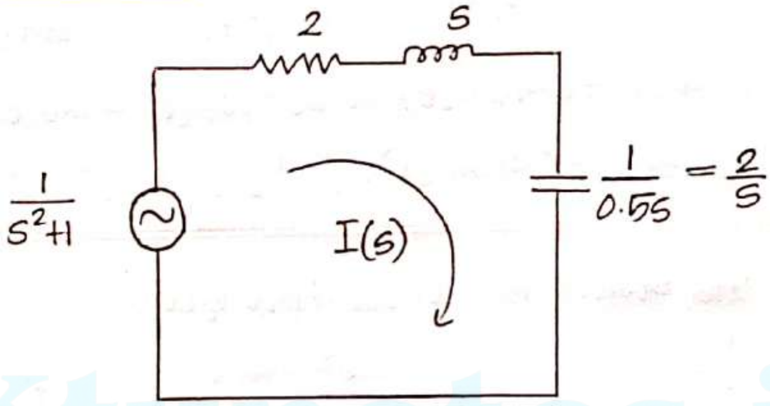
$v_c(t) = u(t) - e^{-t}u(t) - te^{-t}u(t)$

$= (1 - e^{-t} - te^{-t})u(t)$ for $t>0$

9|11|2020 4. In the N/W, the switch is closed at $t=0$. Determine $i(t)$, assuming zero initial conditions.




Transformed n/w






$$-\frac{1}{s^2+1} + I(s)\left[2+s+\frac{2}{s}\right] = 0$$

$$I(s) = \frac{1}{(s^2+1)\left(2+s+\frac{2}{s}\right)}$$

$$I(s) = \frac{as+b}{s^2+1} + \frac{cs+d}{s^2+2s+2}$$

$$s = (as+b)(s^2+2s+2) + (cs+d)(s^2+1)$$

$$= as^3 + 2as^2 + 2as + bs^2 + 2bs + 2b + cs^3 + cs + ds^2 + d$$

$$= (a+c)s^3 + (2a+b+d)s^2 + (2a+2b+c)s + (2b+d)$$

$$\left.\begin{aligned} a+c &= 0 \\ 2a+b+d &= 0 \\ 2a+2b+c &= 1 \\ 2b+d &= 0 \end{aligned}\right\} \quad \begin{aligned} d &= -2b \\ a+c &= 0 \text{ ——— (1)} \\ 2a+b-2b &= 0 \\ 2a-b &= 0 \text{ ——— (2)} \\ 2a+2b+c &= 1 \text{ ——— (3)} \end{aligned}$$

Solving, $a=0.2, b=0.4, c=-0.2, d=-2b=-0.8$

$$\therefore I(s)=\frac{0.2s+0.4}{s^2+1}-\frac{0.2s+0.8}{s^2+2s+2}$$

$$=\frac{0.2s}{s^2+1}+0.4.\frac{1}{s^2+1}-\frac{(0.2s+0.8)}{(s+1)^2+1^2}$$

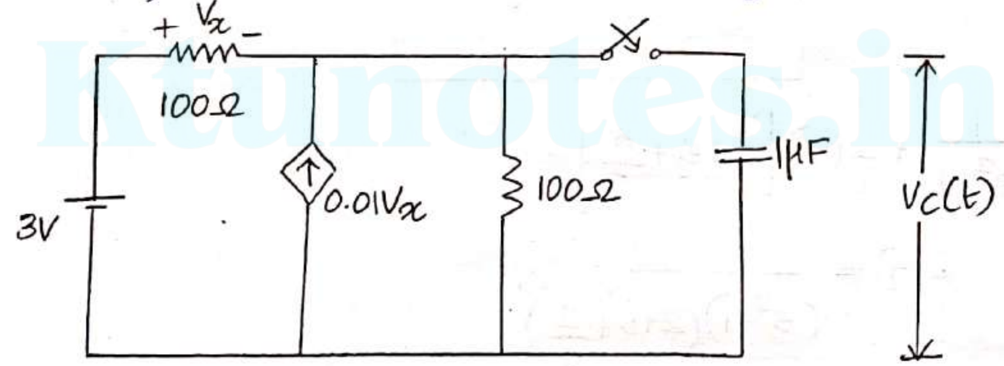
$$=0.2\cdot\frac{s}{s^2+1}+0.4\cdot\frac{1}{s^2+1}-\frac{[0.2s+0.2+0.6]}{(s+1)^2+1^2}$$

$$=0.2\cdot\frac{s}{s^2+1}+0.4\cdot\frac{1}{s^2+1}-0.2\frac{(s+1)}{(s+1)^2+1}-0.6\cdot\frac{1}{(s+1)^2+1^2}$$

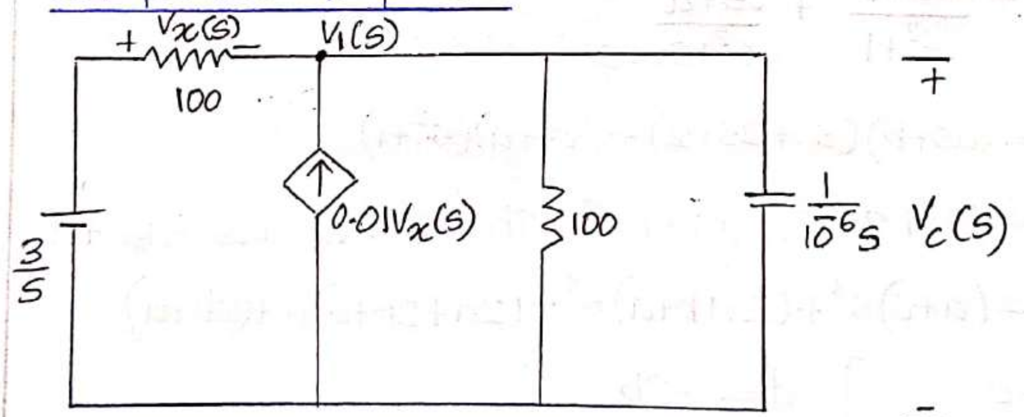
$$i(t)=0.2\cos(t)+0.4\sin(t)-0.2e^{-t}\cos(t)-0.6\cdot e^{-t}\sin(t)$$

$$=0.2\cos(t)+0.4\sin(t)-e^{-t}\left[0.2\cos(t)+0.6\sin(t)\right]$$

Qn: At $t=0$, the switch is closed. Find $v_c(t)$.




Transformed n/w for $t>0$




$$V_x(s)=\frac{3}{s}-V_1(s) \quad —①$$

Applying KCL at node,

$$0.01V_x(s) = \frac{V_1(s) - 3/s}{100} + \frac{V_1(s)}{100} + \frac{V_1(s)}{\left(\frac{1}{s.10^{-6}}\right)}$$

$$0.01\left(\frac{3}{s} - V_1(s)\right) = 0.01V_1(s) - \frac{0.03}{s} + 0.01V_1(s) + s.10^{-6}.V_1(s)$$

$$\frac{0.03}{s} - 0.01V_1(s) = 0.02V_1(s) - \frac{0.03}{s} + s\times10^{-6}V_1(s)$$

$$V_1(s)\left[s.10^{-6} + 0.03\right] = \frac{0.06}{s}$$

$$V_1(s) = \frac{0.06}{s(s\times10^{-6}+0.03)} = \frac{6\times10^4}{s(s+3\times10^4)}$$

$$\boxed{\frac{1}{s(s+a)} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right]}$$

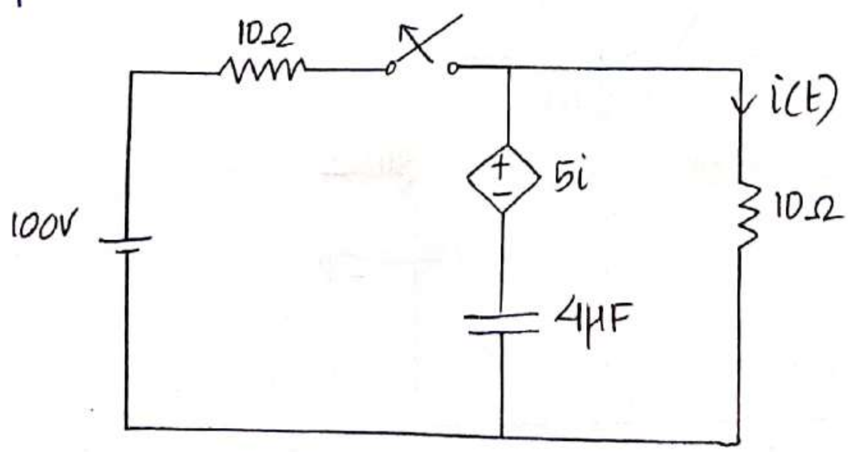
$$= 6\times10^4 . \frac{1}{3\times10^4}\left[\frac{1}{s} - \frac{1}{s+3\times10^4}\right]$$

$$= 2\left[\frac{1}{s} - \frac{1}{s+30000}\right]$$

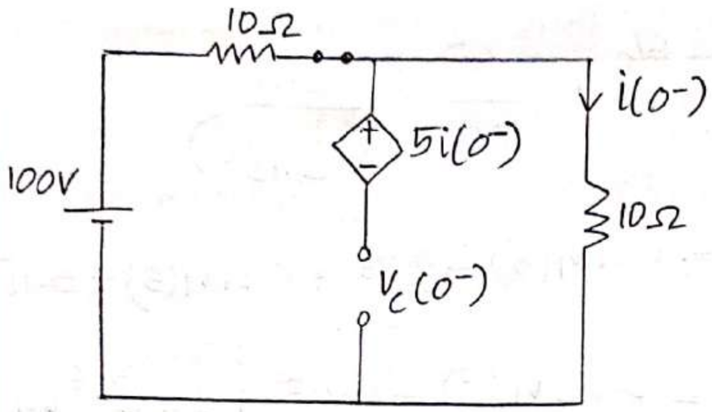
$$V_C(t) = V_1(t) = 2\left[u(t) - e^{-30000t}u(t)\right]$$

$$= 2\left[1 - e^{-30000t}\right]u(t)$$

Qn: For the n/w shown below find i(t) when the switch is opened at t=0.



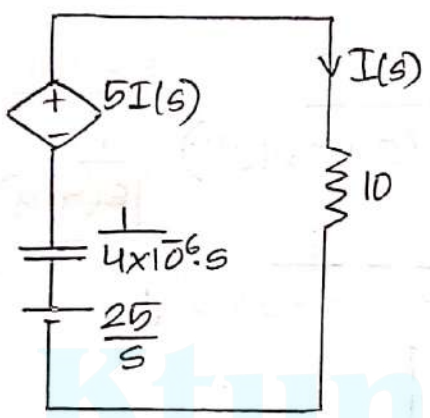
## At $t = 0^-$




$$i(0^-) = \frac{100}{10+10} = 5A$$

$$V_c(0^-) = 100 - 10i(0^-) - 5i(0^-)$$
$$= 100 - 15i(0^-)$$
$$= 100 - 15 \times 5$$
$$= 25V$$

## Transformed n/w at $t=0$




$$-\frac{25}{s} + \frac{I(s)}{4\times10^{-6}\cdot s} - 5I(s) + 10I(s) = 0$$

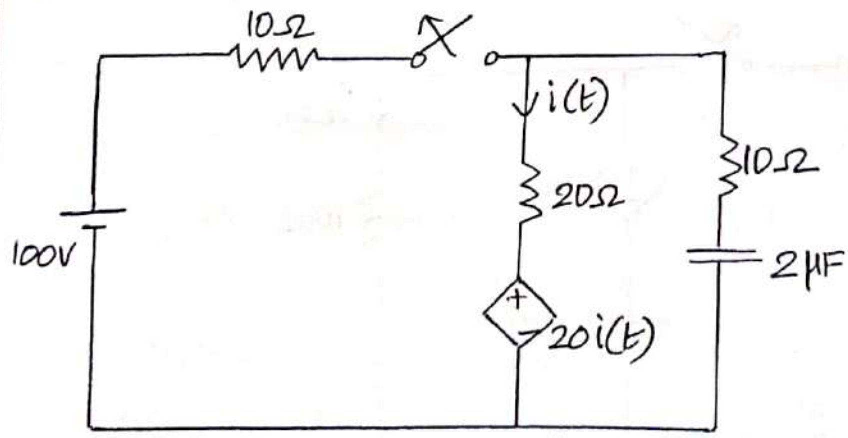
$$I(s)\left[5 + \frac{250000}{s}\right] = \frac{25}{s}$$

$$I(s) = \frac{25}{s\left(5+\frac{25000}{s}\right)} = \frac{25}{5s+25000}$$

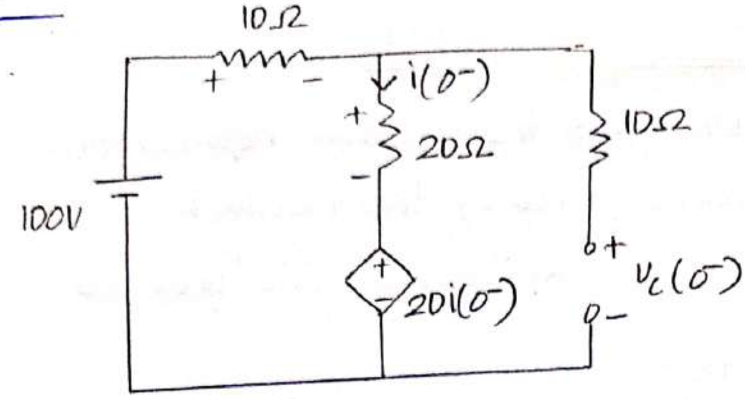
$$I(s) = \frac{5}{s+50000}$$

$$i(t) = 5e^{-50000t}u(t), \text{ for } t=0$$

Qn: For the n/w shown below, find $i(t)$ when the switch is opened at $t=0$.



$t=0^-$




$100 = 10i(0^-) + 20i(0^-) + 20i(0^-)$

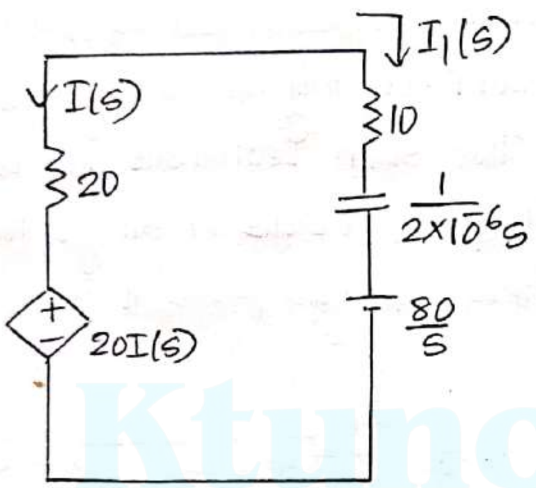
$i(0^-) = 2A$

$v_c(0^-) = 20i(0^-) + 20i(0^-)$

$= 40i(0^-) = 40 \times 2$

$= 80V$

Transformed n/w for $t>0$




Let $I_1(s)$ be the clockwise current through the loop.

$I_1(s) = -I(s)$ —— ①

KVL in loop

$$-20I(s) + 20I(s) + 10I_1(s) + \frac{I_1(s)}{2\times10^{-6}s} + \frac{80}{s} = 0$$

$I_1(s) = -I(s)$

$$\therefore -50I(s) - \frac{5\times10^5}{s}I(s) = -\frac{80}{s}$$

$$I(s)\left[50 + \frac{5\times10^5}{s}\right] = \frac{80}{s}$$

$$I(s) = \frac{80}{s\left[50+\frac{5\times10^5}{s}\right]}$$

$$= \frac{80}{50s + 5\times10^5}$$

$$= \frac{1.6}{s+10^4}$$

$$\therefore i(t) = 1.6e^{-10^4 t}u(t) \text{ for } t>0$$