

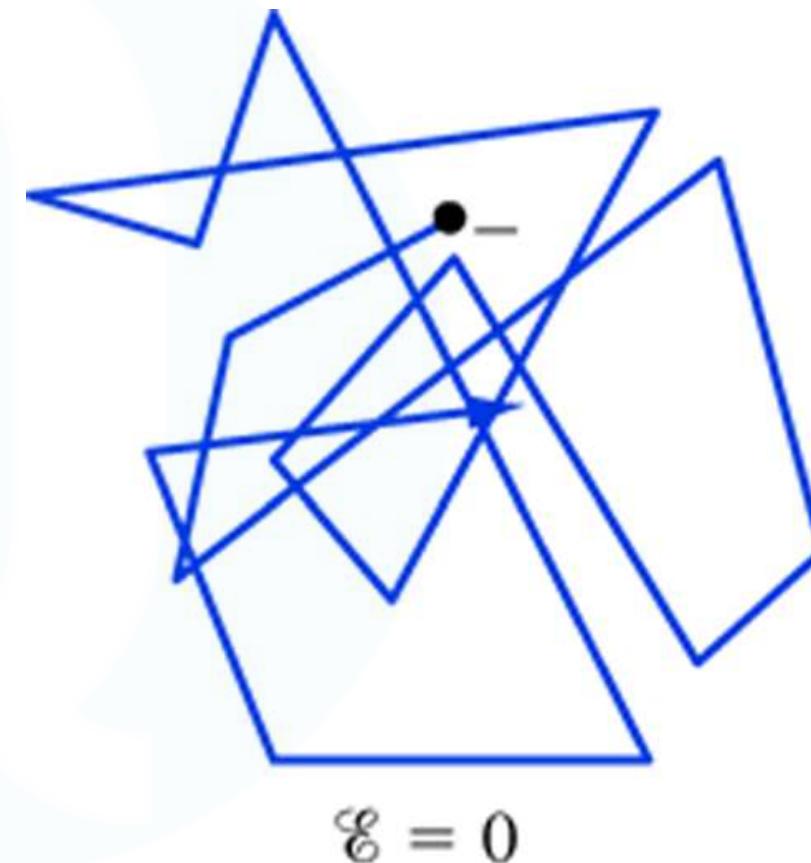
Drift of Carriers in Electric and Magnetic Fields

Conductivity and Mobility

- Knowledge of carrier concentrations in a solid is necessary for calculating current flow in the presence of electric or magnetic fields.
- Also we must be able to take into account the collisions of the charge carriers with the lattice and with the impurities which affect the ease with which electrons and holes can flow through the crystal,
- that is, their *mobility* within the solid.
- The collision and scattering processes depend on temperature, which affects the thermal motion of the lattice atoms and the velocity of the carriers.

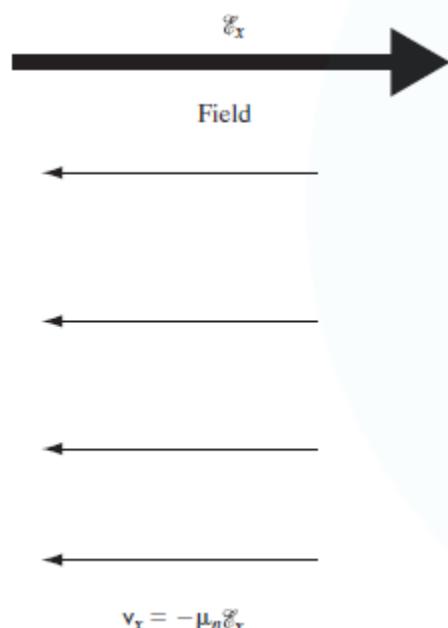
Motion of the electron

- Electrons and holes have thermal energy are moving, even at equilibrium
- After collisions, new direction is random
 - No progress in any particular direction
 - Current is zero



- There is no net motion of the group of n electrons/cm³ over any period of time. If a large number of electrons is considered (e.g., 10^{16} cm⁻³ in an n-type semiconductor), there will be no preferred direction of motion for the group of electrons and no net current flow.
- If an electric field \mathcal{E}_x is applied in the x-direction, each electron experiences
- a net force $-q\mathcal{E}x$ from the field.
- Still random path for an individual electron;
- the effect when averaged over all the electrons, however, is a net motion of the group in the $-x$ direction . If p_x is the x -component of the total momentum of the group,

with an electric field applied along x direction.



- the force of the field on the n electrons/cm³ is
- $-nqE_x = \frac{dp_x}{dt}|_{\text{field}}$ (A)
- Initially, a continuous acceleration of the electrons in the -x-direction. But the net acceleration of Eq. (A) is just balanced in steady state by the decelerations of the collision processes. Thus while the steady field E_x does produce a net momentum
- p_{-x} , the net rate of change of momentum when collisions are included must be zero in the case of steady state current flow.

- To find the total rate of momentum change from collisions,
- consider a group of N_0 electrons at time $t = 0$ and define $N(t)$ as the number of electrons that *have not* undergone a collision by time t .
The rate of decrease in $N(t)$ at any time t is proportional to the number left unscattered at t
- $-\frac{N(t)}{dt} = \frac{1}{\tau} N(t)$ (B) where τ^{-1} is a constant of proportionality.
- The solution to Eq. (B) is an exponential function
- $N(t) = N_0 e^{-t/\tau}$

- and τ represents the mean time between scattering events, called the *mean free time*.
- The probability that any electron has a collision in the time interval dt is dt/τ .
- Thus the differential change in p_x due to collisions in time dt is
- $$\frac{dp_x}{dt} = \frac{-p_x}{\tau} dt$$
- The rate of change of p_x due to the decelerating effect of collisions is
- $$\frac{dp_x}{dt} \Big|_{\text{collisions}} = \frac{-p_x}{\tau} \quad (\text{C})$$

- The sum of acceleration and deceleration effects must be zero for steady state.
- Taking the sum of Eqs. (A) and (C), we have
- $-\frac{p_x}{t} - nq\mathcal{E}_x = 0$
- The average momentum per electron is $\langle p_x \rangle = \frac{p_x}{n} = -q\mathfrak{t}\mathcal{E}_x$ (D)
- where the angular brackets indicate an average over the entire group of electrons.

- Thus for steady state, Eq. (D) indicates that the electrons have *on the average* a constant net velocity in the negative x-direction
- $\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = \frac{-q t \mathcal{E}_x}{m_n^*}$ (E)
- Actually, the individual electrons move in many directions by thermal motion during a given time period, but Eq. (E) tells us the *net drift* of an average electron in response to the electric field. The drift speed described by Eq. (E) is usually much smaller than the random speed due to the thermal motion v_{th} .

- The current density resulting from this net drift is just the number of electrons crossing a unit area per unit time ($n < v_x >$) multiplied by the charge on the electron ($-q$).
$$J_x = -qn < v_x >$$
- $\frac{\text{ampere}}{\text{cm}^2} = \frac{\text{coulomb}}{\text{electron}} \frac{\text{electrons}}{\text{cm}^3} \frac{\text{cm}}{\text{s}} = \frac{\text{coulomb}}{\text{s}-\text{cm}^2}$
- Using Eq. (E) for the average velocity, we obtain $J_x = \frac{n \frac{t}{m} q^2}{m^* n} \mathcal{E}_x$
- from Ohm's law: $J_x = \sigma \mathcal{E}_x$, where $\sigma = \frac{tnq^2}{m^* n}$. The conductivity $\sigma (\Omega\text{-cm})^{-1}$ can be written $\sigma = qn\mu_n$,
where $\mu_n = \frac{tq}{m^* n} (F)$
- The quantity μ_n , called the *electron mobility*, describes the ease with which electrons drift in the material. Mobility is a very important quantity in characterizing semiconductor materials and in device development.
- (Ohm's law: $I=V/R$, $J=I/A = VA/ ApL = \sigma \mathcal{E}_x$, $J = \sigma \mathcal{E}_x$)

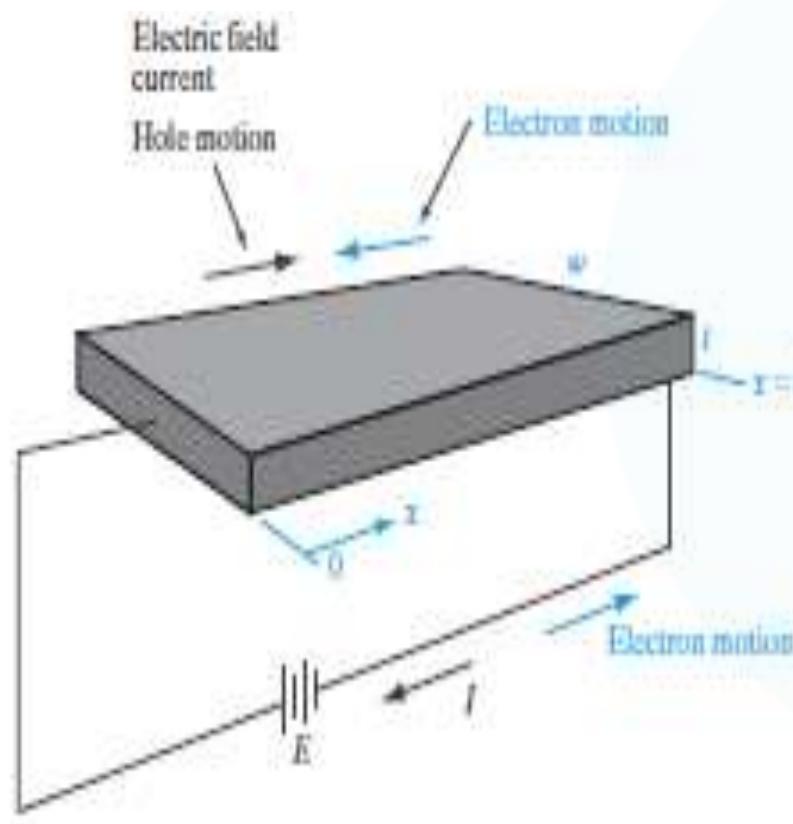
- The mobility expressed as the average particle drift velocity per unit electric field. Comparing Eqs. (E) and (F), we have $\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = \frac{-q t \mathcal{E}_x}{m_n^*}$ (E)
- $\mu_n = \frac{t q}{m_n^*}$ (F)
- Then $\langle v_x \rangle = -\mu_n \mathcal{E}_x$ The unit of mobility is $(\text{cm/s})/(\text{V/cm}) = \text{cm}^2/\text{V-s}$,
- The minus sign in the definition results in a positive value of mobility, since electrons drift opposite to the field. The current density can be written in terms of mobility as $J_x = q n \mu_n \mathcal{E}_x$
- This derivation has been based on the assumption that the current is carried primarily by electrons. For hole conduction we change n to p , $-q$ to $+q$, and μ_n to μ_p , where $\mu_p = + \langle v_x \rangle / \mathcal{E}_x$ is the mobility for holes. If both electrons and holes participate, we must modify the equation to
- $J_x = q(n \mu_n + p \mu_p) \mathcal{E}_x = \sigma \mathcal{E}_x$

Direct & Indirect Semiconductors

Properties of semiconductor materials

	$E_g(\text{eV})$	μ_n	μ_p	ρ	<i>Lattice</i>	\AA
• Si	1.11	1350	480	2.5E5	D	5.43
• Ge	0.67	3900	1900	43	D	5.66
• GaAs	1.43	8500	400	4E8	Z	5.65
• AlAs	2.16	180		0.1	Z	5.66
• GaP	2.26	300	150	1	Z	5.45

Drift and Resistance



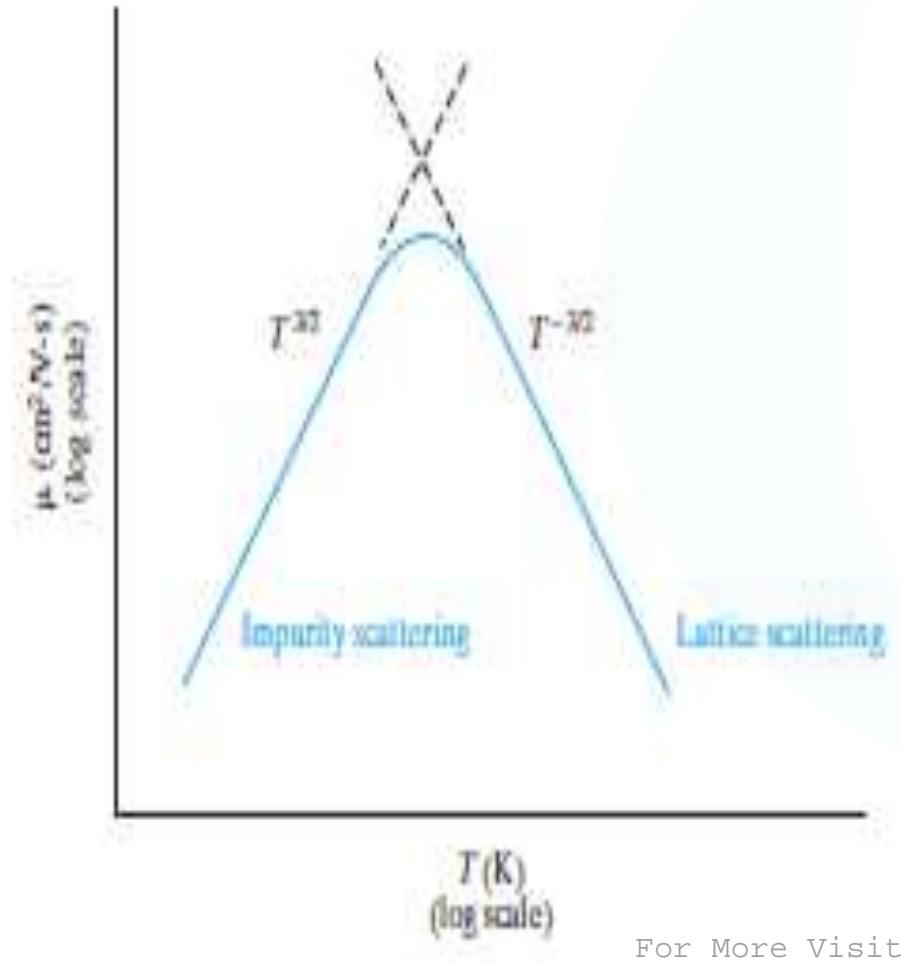
- Let us look more closely at the drift of electrons and holes.
- If the semiconductor bar of Fig. connected to an external circuit & the contacts of the bar is *ohmic*.
- The drift current contains both types of carrier & the conductivity of the material is
- $\sigma = q(n \mu_n + p \mu_p)$.
- The resistance of the bar is then
- $R = \rho L / wt = L / \sigma wt$
- where ρ is the resistivity ($\Omega\text{-cm}$). The physical mechanism of carrier drift requires that the holes in the bar move as a group in the direction of the electric field and that the electrons move as a group in the opposite direction.
- Both current direction along the direction of electric field.

How current is carried around the external circuit by electrons & holes ?

- There is no problem in visualizing electrons flowing into the bar at one end and out at the other (always opposite to I). Thus for every electron leaving the left end ($x = 0$) of the bar in fig. there is a corresponding electron entering at $x = L$, so that the electron concentration in the bar remains constant at n .
- But what happens to the holes at the ohmic contacts?
- As a hole reaches the ohmic contact at $x = L$, it recombines with an electron, which must be supplied through the external circuit.
- As this hole disappears, a corresponding hole must appear at $x = 0$ to maintain space charge neutrality.

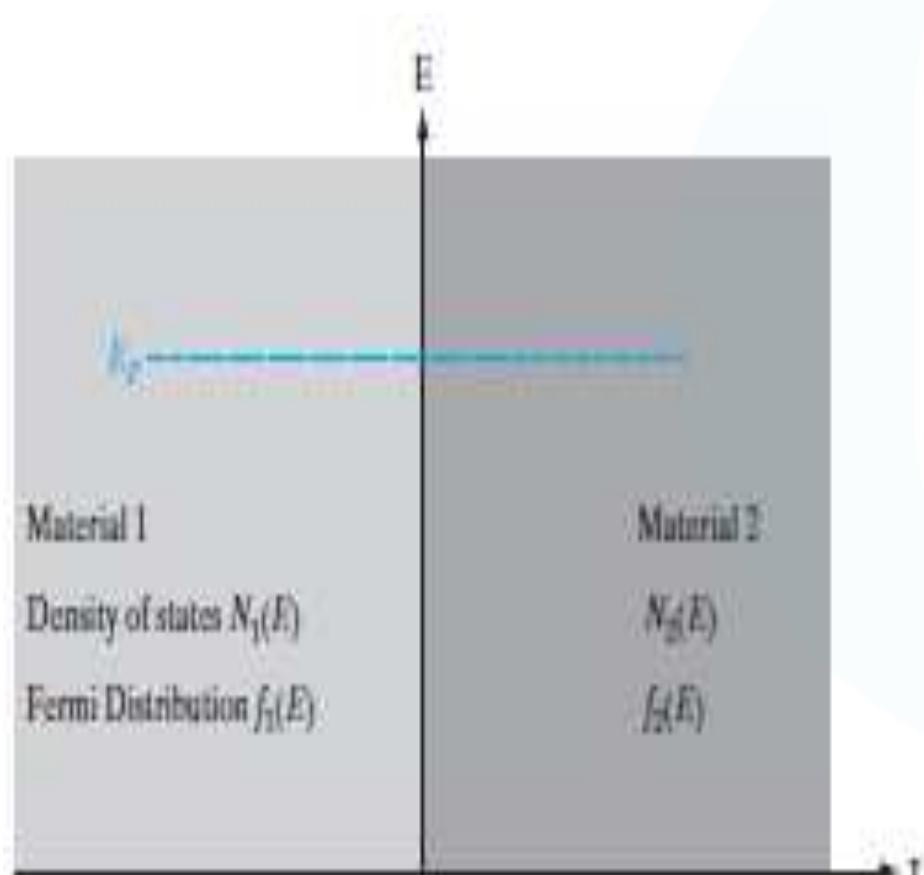
Effects of Temperature and Doping on Mobility

- The two basic types of scattering mechanisms that influence electron and hole mobility are *lattice scattering* and *impurity scattering*.
- In lattice scattering a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature.
- While scattering from crystal defects such as ionized impurities becomes the dominant at low temperatures called *impurity scattering*



- As in fig., mobility proportional with the approximate temperature dependencies are $T^{(-3/2)}$ for lattice scattering and $T^{3/2}$ for impurity scattering.
- $1/\mu = 1/\mu_1 + 1/\mu_2 + \dots$

Invariance of the Fermi Level at Equilibrium



- So far we discussed homogeneous SC with constant doping or no junctions or single material . Suppose if we consider non uniform doping or SC with different materials or junctions we summarize that *no discontinuity or gradient can arise in the equilibrium Fermi level EF* .
- For eg., consider two materials in intimate contact such that electrons can move between the two. This can be dissimilar semiconductors, n- and p-type regions, a metal and a semiconductor, or simply two adjacent regions of a nonuniformly doped semiconductor. Each material is described by a Fermi–Dirac distribution function and some distribution of available energy states that electrons can occupy.

Invariance of the Fermi Level at Equilibrium

- There is no current, and therefore no net charge transport, at thermal equilibrium.
- Therefore any transfer of electrons from material 1 to material 2 must be exactly balanced by the opposite transfer of electrons from 2 to 1.
- Let the density of states at energy E in material 1 be called $N_1(E)$ and in material 2 we will call it $N_2(E)$. At energy E the rate of transfer of electrons from 1 to 2 is proportional to the number of filled states at E in material 1 times the number of empty states at E in material 2:
- rate from 1 to 2 $\propto N_1(E)f_1(E) \times N_2(E)[1 - f_2(E)]$

Invariance of the Fermi Level at Equilibrium

- rate from 2 to 1 $\propto N_2(E)f_2(E) \times N_1(E)[1 - f_1(E)]$
- At equilibrium these must be equal:
- $N_1(E)f_1(E) \times N_2(E)[1 - f_2(E)] = N_2(E)f_2(E) \times N_1(E)[1 - f_1(E)]$
- Rearranging terms, we have, at energy E ,
- $N_1f_1 N_2 - N_1f_1 N_2f_2 = N_2f_2 N_1 - N_2f_2 N_1f_1 \quad (3-56)$
- which results in
- $f_1(E) = f_2(E)$, that is, $[1 + e^{(E-E_{F_1})kT}]^{-1} = [1 + e^{(E-E_{F_2})kT}]^{-1}$
- Thus no gradient exists in the Fermi level at equilibrium: $dE_F/dx = 0$

- Question :
- A Silicon bar of 100 cm long and 1 cm^2 cross sectional area is doped with 10^{17} Arsenic atoms/ cm^3 . Calculate electron and hole concentrations at 300K. Also find the conductivity and the current with 10V applied. Electron mobility at this doping is $700 \text{ cm}^2/\text{V-sec}$.

- $\sigma = q(n \mu_n + p \mu_p)$
- $q = 1.6 \times 10^{-19} C$

Question 2

An n- type silicon sample with $N_d = 10^{15} /cm^3$ is steadily illuminated such that $g_{op} = 10^{20}$ EHP/cm³-sec. If $\tau_n = \tau_p = 1\mu\text{sec}$ for this excitation. Draw the energy band diagram with the quasi Fermi levels at 300K. Intrinsic carrier concentration of silicon is $1.5 \times 10^{10} /cm^3$ (5)

- $n = g_{op} \tau_n$
- $n = n_i \times e^{(Fn - Ei)/kT}$
- $p = n_i \times e^{(Ei - Fp)/kT}$

- A silicon sample doped with 10^{16}cm^{-3} donors at 300K is optically excited such that the optical generation rate is $10^{20} \text{ EHP}/(\text{cm}^3\text{-s})$. Find the separation between quasi- fermi levels if $\tau_p = \tau_n = 2 \mu\text{s}$.

The Hall Effect

The electrical conductivity measurements we've learnt so far are not sufficient for

1) The determination of number of charge carriers

2) Mobility of the charge carriers

3) Whether the conduction is due to ELECTRONS or HOLES

Hence , it was very difficult to differentiate between a P-TYPE semiconductor and a N-TYPE semiconductor.



The Hall effect was discovered in 1879 by Edwin Herbert Hall while he was working on his doctoral degree at Johns Hopkins University in Baltimore, Maryland. His measurements of the tiny effect produced in the apparatus he used were an experimental tour de force, accomplished 18 years before the electron was discovered and published under the name "On a New Action of the Magnet on Electric Currents"

When a piece of conductor (metal or semiconductor) carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction mutually perpendicular to the direction of current flow as well as magnetic field. This phenomenon is known as HALL EFFECT.

- 1) The electric charge moving through a magnetic field will experience a FORCE
- 2) And ELECTRONS that travel through that lies in a MAGNETIC field will also experience a magnetic force. And that force is called as ,

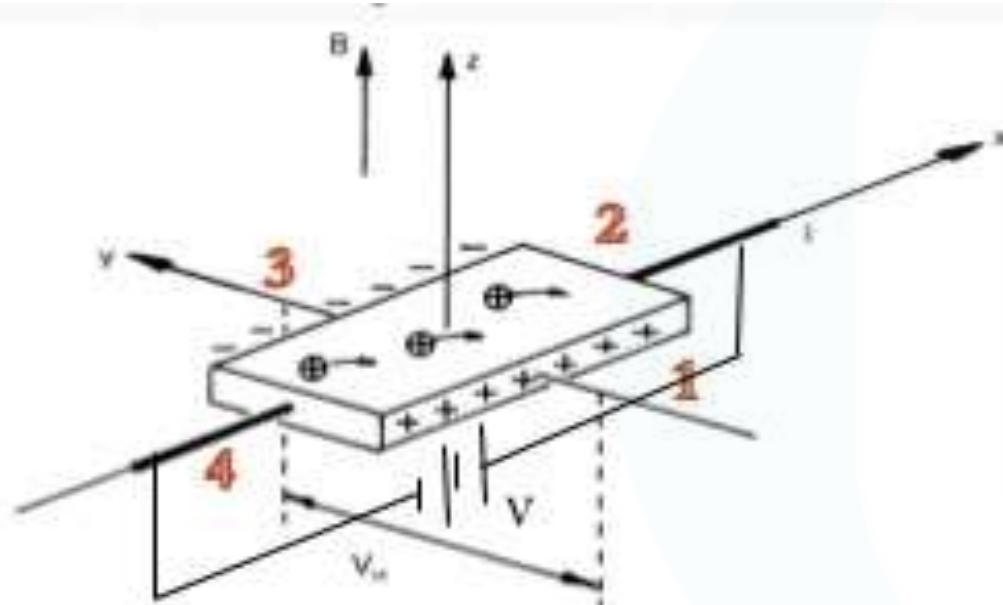
MAGNETIC LORENTZ FORCE

Consider a charged particle of charge 'q' moving with a velocity of 'v' in an uniform magnetic field 'B', then the MAGNETIC LORENTZ force is given by,

$$\mathbf{F} = q(\vec{v} \times \vec{B})$$

Or in scalar form as,

$$F = Bqv \sin\theta$$



Let us consider a P-Type semiconductor Sample placed in a magnetic field as shown in the figure,

When a voltage 'V' is applied to the sample. The HOLES move from contact 2 to contact 4 with a drift velocity

$$V_d = \mu_p E_{app} = \mu_d \frac{V}{l}$$

The magnetic field simultaneously exerts a force called, LORENTZ force = $e (v_d \times B)$ acting on holes and deflecting them towards side 1.

As a consequence, holes completely accumulate on side 1 creating a NET POSITIVE charge, as side 3 is depleted of holes and is negatively charged.

The magnitude of these charges is such that the electric field E_h ($=V_h/d$) created by these charges exactly counterbalances the LORENTZ force on these charges due to the magnetic field and forms an EQUILIBRIUM.

And at this stage NO further accumulation of HOLES takes place on the side1 and HALL FIELD reaches a steady value.

The contacts 1 and 3 are called as HALL contacts and E_h and V_h are called as HALL electric field and HALL voltage. In case of a N-TYPE semiconductor side 1 receives negative charge and side 3 receives positive charge(ie) opp.to P-TYPE semiconductor.

DERIVATION:

Under equilibrium conditions , the force on the charges due to HALL electric field and LORENTZ force counterbalances each other, that is

$$eE_H = e v_d B \quad \text{----- 1}$$

If 'J' is the CURRENT DENSITY then,

$$J = n_c e v_d \quad \text{----- 2}$$

Where n_c = concentration of current carriers

$$E_H = \frac{BJ}{n_c e} \quad \text{----- 3}$$

The HALL effect is generally by means of HALL COEFFICIENT R_h defined in terms of J by,

$$E_H = R_H J B \quad \text{----- 4}$$

From eqns 3 and 4 , we have

$$R_h = \frac{1}{n_c e} \quad \dots \dots \dots \quad 5$$

For N-TYPE, the charge carriers are ELECTRONS, so

$$R_h = \frac{1}{ne} \quad \dots \dots \dots \quad 6$$

For P-TYPE, the charge carriers are HOLES, so

$$R_h = \frac{1}{pe} \quad \dots \dots \dots \quad 7$$

Where n represents the concentration of electrons

and p represents the concentration of holes

More ACCURATE analysis shows that,

$$R_h = \frac{r_H}{pe} \quad \dots \dots \dots \quad 8$$

Where r_H is called the HALL FACTOR that depends on temperature, doping, magnetic field..etc. At room temperature $r_H = \frac{3\pi}{8}$

Thus Hall coefficient for P-TYPE semiconductor is

$$R_h = \frac{3\pi}{8pe} \quad \dots\dots\dots 9$$

While for N-TYPE semiconductor is

$$R_h = \frac{3\pi}{8ne} \quad \dots\dots\dots 10$$

Determination of HALL coefficient:

The Hall Coefficient is determined by measuring hall voltage that generates hall field, then V_h is given by

$$V_h = E_h d \quad \dots\dots\dots 11$$

Using eqn 4,

$$V_h = R_h J B d \quad \dots\dots\dots 12$$

If l is the length of the sample and I_d is its cross sectional area, then the current density J is given by,

$$J = \frac{I}{A} = \frac{I}{ld} \quad \dots \dots \dots \quad 13$$

Therefore,

$$V_H = \frac{R_H I_B}{J} \quad \dots \dots \dots \quad 14$$

And hence,

$$R_H = \frac{V_H l}{IR} \quad \dots \dots \dots \quad 15$$

As the polarity of the V_H will be opposite for n-type and p-type semiconductors, the sign R_H will be different for the two types of the semiconductors.

MOBILITY of charge carriers

We know that the HALL coefficient is ,

$$R_H = -\frac{1}{ne}$$

This expression is correct only if the CHARGE carriers are FREE from any type of ATTRACTIVE forces in ENERGY band and also they are moving with a constant drift velocity v . But this is NOT TRUE in case of semiconductors.

Considering the average speed, it is shown

$$R_H = -\frac{1.18}{ne} \quad \text{for electrons}$$

$$ne = \frac{1.18}{R_H} \quad \text{----- 1}$$

$$R_H = \frac{1.18}{pe} \quad \text{for holes}$$

$$pe = \frac{1.18}{R_H} \quad \text{----- 2}$$

Electrical conductivity of n-type semiconductor ,

$$\sigma_e = ne\mu_e$$

$$\mu_e = \frac{\sigma_e}{ne} \quad \text{----- 3}$$

Where μ_e is the mobility of electrons

Substituting the eqn 1 in eqn 3 , we have

$$\mu_e = \frac{\sigma_e}{-1.18/R_H} = \frac{\sigma_e R_H}{1.18}$$

Electrical conductivity of p-type semiconductor ,

$$\sigma_h = pe\mu_h$$

$$\mu_h = \frac{\sigma_h}{pe} \quad \text{----- 4}$$

Where μ_h is the mobility of holes

Substituting the eqn 2 in eqn 4 , we have

$$\mu_h = \frac{\sigma_h}{1.18/R_H} = \frac{\sigma_h R_H}{1.18}$$

Applications of HALL effect:

1) Determination of semiconductor type:

If HALL coefficient \Rightarrow POSITIVE \rightarrow p-type semiconductor
NEGATIVE \rightarrow n-type semiconductor

2) Calculation of carrier concentration:

By measuring the V_H and R_H the carrier concentration of ELECTRONS in n-type semiconductor and HOLE concentration in p-type semiconductor is measured

3) Determination of mobility of charge carriers:

The mobility of charge carriers is found by 3 and 4 eqn in the previous derivation.

4) Measurement of magnetic flux density

HALL voltage V_H is directly proportional to the magnetic flux density B for a given current I for a semiconducting sample and is used in MAGNETIC FLUX DENSITY METER.

5) Measurement of power in an electromagnetic wave

Effect of Temperature Doping on mobility

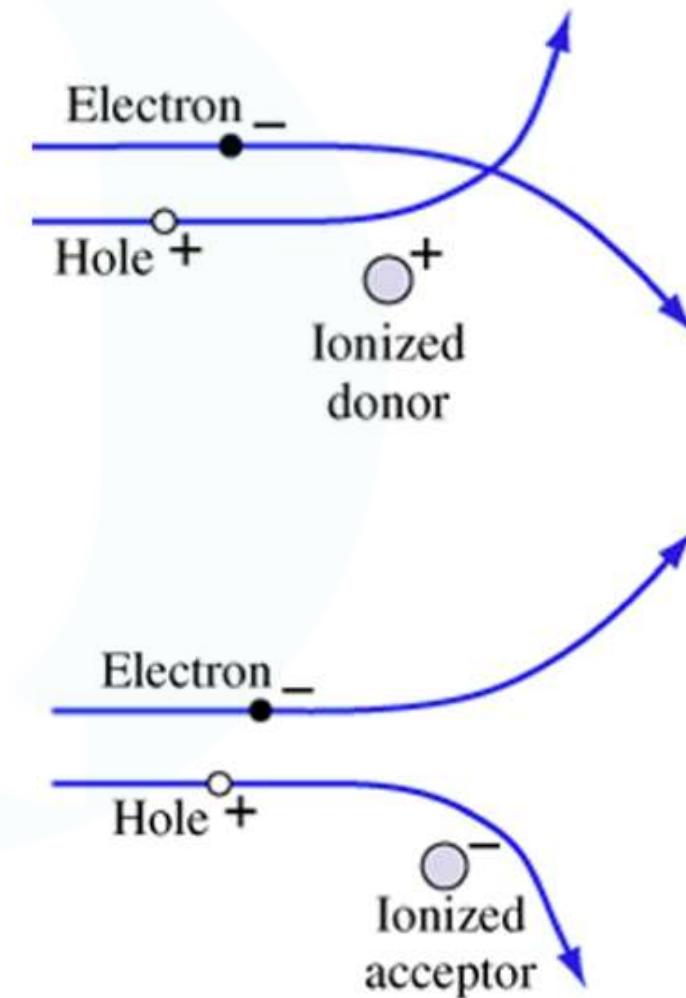
Recall mobility depends on drift velocity

$$m = \left| \frac{v_d}{\mathcal{E}} \right|$$

- Drift influenced by scattering (collisions)
- Particles can collide with
 - Ionized impurities
 - Phonons (parent atoms)
- Drift velocity (and mobility) depends on mean free time between collisions

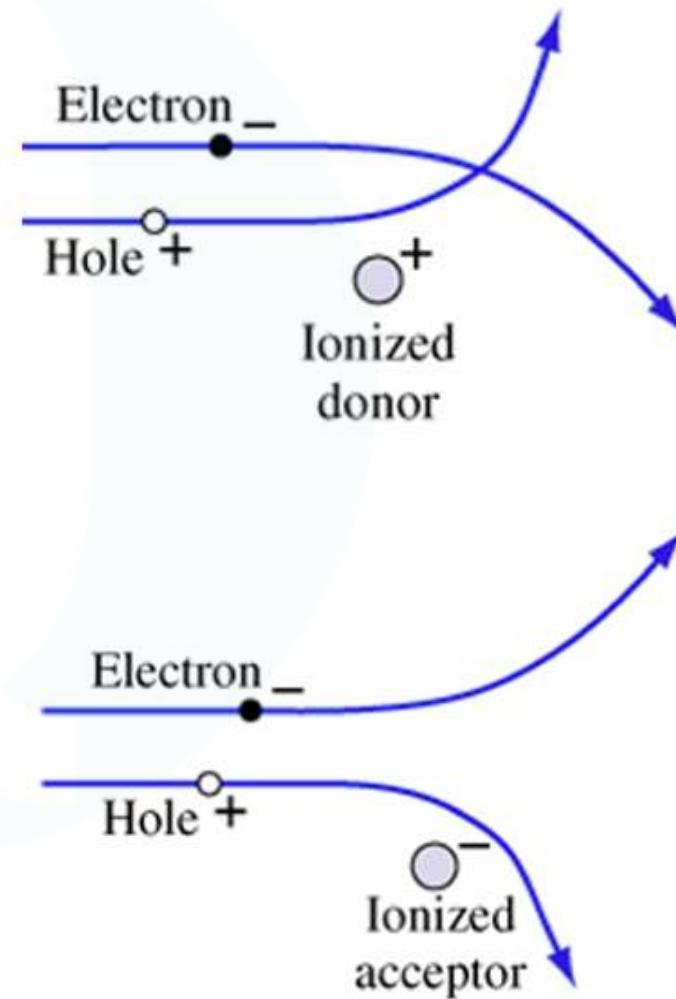
Ionized Impurity Scattering

- Carriers are charged
- Ionized impurities are charged
- Carrier passing near an ionized impurity will get deflected
- The higher the doping, the more often the scattering



Ionized Impurity Scattering, continued

- Note that carriers are deflected by both positively and negatively ionized dopants
- Mobility therefore depends on TOTAL doping $N_D + N_A$
- Minority carrier mobilities can also be estimated because usually $N_D \gg N_A$ or $N_A \gg N_D$



Lattice (phonon) scattering

- Vibrating atoms create acoustic waves in the crystal
 - Acoustic particles are called “phonons”
 - They have energy $E_{phonon} = \hbar W$
 - And wave vector $K = \frac{2p}{l}$
 - They can scatter carriers
 - Phonon energies small, usually less than 0.1 eV
- Vibrations increase with temperature
 - Expect more phonons with increasing temperature
 - Expect more collisions
 - Expect mobility to go down as T goes up

Key points

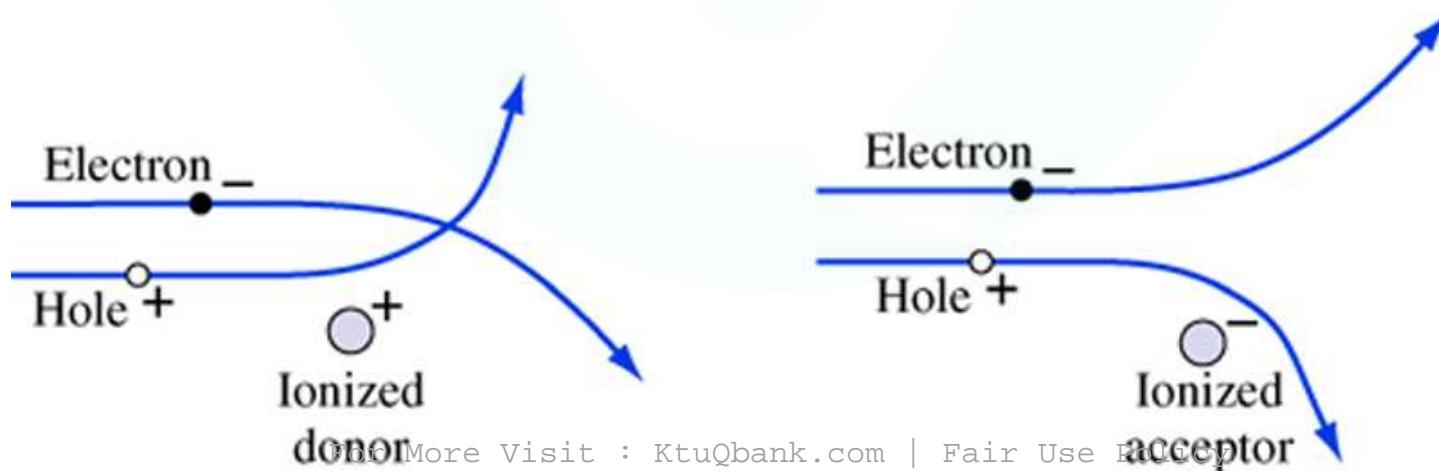
- Carriers can be scattered by
 - Ionized impurities
 - Phonons (lattice vibrations)
 - Both
- Higher doping reduces mobility
- Higher temperature reduces mobility

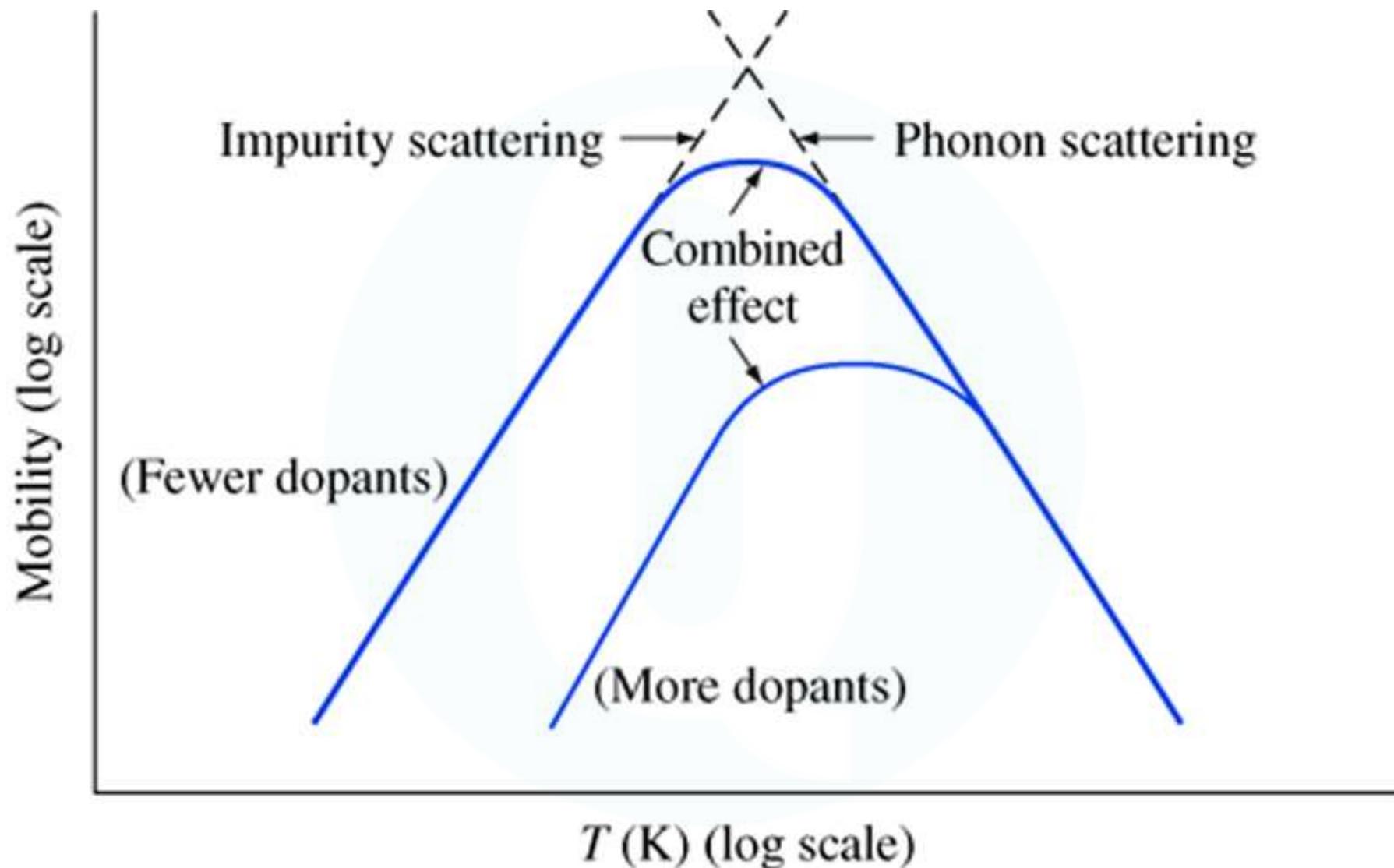
Mobility sensitive to temperature

- We said earlier that phonon scattering increases with temperature
 - Increased T increases lattice vibrations
 - Causes more phonons
 - Lead to more lattice (phonon) scattering

Ionized impurity scattering

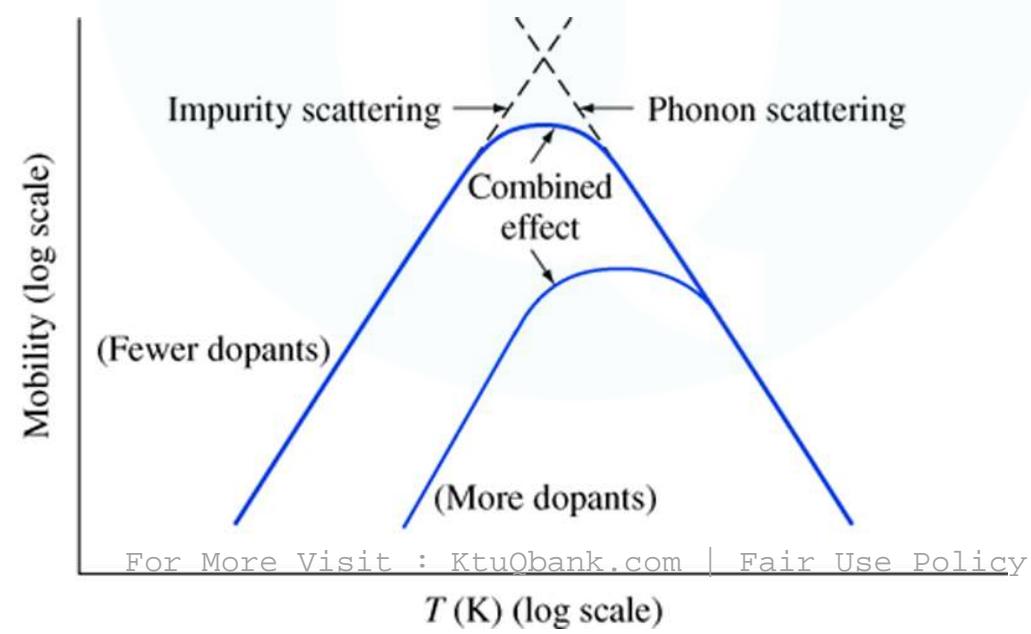
- As temperature increases, average speed of carriers increases
- Carrier moving fast, spend less time near impurity ions
- Don't scatter as much
- Therefore ionized impurity scattering less important at high temperatures





Key points

- At higher temperatures, phonon scattering dominates
- At lower temperatures ionized impurity scattering dominates



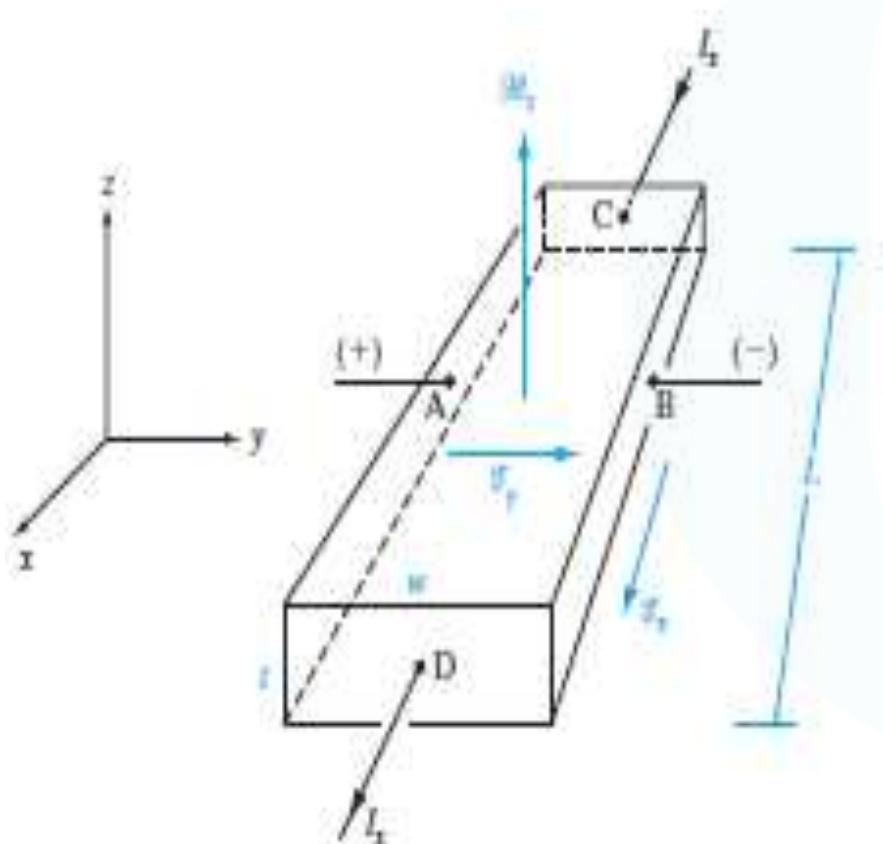
Hall Effect

- So far no method is studied to find experimentally the
 - 1. no. of charge carriers
 - 2. mobility of charge carriers
 - 3. Whether the conduction is due to electrons or holes
 - 4. Also it is very difficult to differentiate n type or p type.
- All these can be determined by Hall experiment method.

Hall Effect

- Discovered by Edwin Herbert Hall in 1879, 18 years before the electrons was discovered .
- When a piece of metal or semiconductor carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction mutually perpendicular to the direction of current flow as well as magnetic field. This phenomenon is called Hall Effect.
-

- If a magnetic field is applied perpendicular to the direction in which holes drift in a p-type bar, the path of the holes tends to be deflected
- Consider a p-type bar placed in a magnetic field. When a voltage is applied holes move with a drift velocity $v_x = \mu_p E_x = \mu_p V/L$ along x direction.
- Magnetic field simultaneously exerts a force acting on holes deflecting towards y direction. As a consequence holes completely accumulate on one side creating a net positive charge as other side depleted of holes creating a net negative charge. As a result an another electric field is developed.
- Using vector notation, the total force on a single hole due to the electric and magnetic fields is
- $\mathbf{F} = q(\mathcal{E} + \mathbf{v} * \mathcal{B})$



- In the y -direction the force is
- $F_y = q(\mathcal{E}_y - v_x B_z) (A)$
- The important result of Eq. (A) is that unless an electric field \mathcal{E}_y is established along the width of the bar, each hole will experience a net force (and therefore an acceleration) in the $-y$ -direction due to the $q v_x B_z$ product.
- Therefore, to maintain a steady state flow of holes down the length of the bar, the electric field \mathcal{E}_y must just balance the product $v_x B_z$:
- $\mathcal{E}_y = v_x B_z$ so that the net force F_y is zero

- Once the electric field \mathcal{E}_y becomes as large as $v_x b_z$, no net lateral force is experienced by the holes as they drift along the bar. The establishment of the electric field \mathcal{E}_y is known as the *Hall effect*, and the resulting voltage $V_{AB} = \mathcal{E}_y w$ is called the *Hall voltage*.
- If we use the expression for the drift velocity (using $+q$ and p_0 for holes), the field \mathcal{E}_y becomes
- $$\mathcal{E}_y = \frac{J_x}{q p_0} B_z \quad (J_x = q p_0 \langle v_x \rangle)$$
- $$\mathcal{E}_y = R_H J_x B_z, R_H = 1/q p_0$$
 called Hall coefficient.

- Thus the Hall field is proportional to the product of the current density and the magnetic flux density.
- the hole concentration $p_0 = 1/R_H$ $q = \frac{J_x}{q\mathcal{E}_y} B_z = \frac{(I_x/wt) B_z}{q(V_{AB}/w)} = \frac{I_x B_z}{qtV_{AB}}$
- Since all of the quantities in the right-hand side of can be measured, the Hall effect can be used to give quite accurate values for carrier concentration.

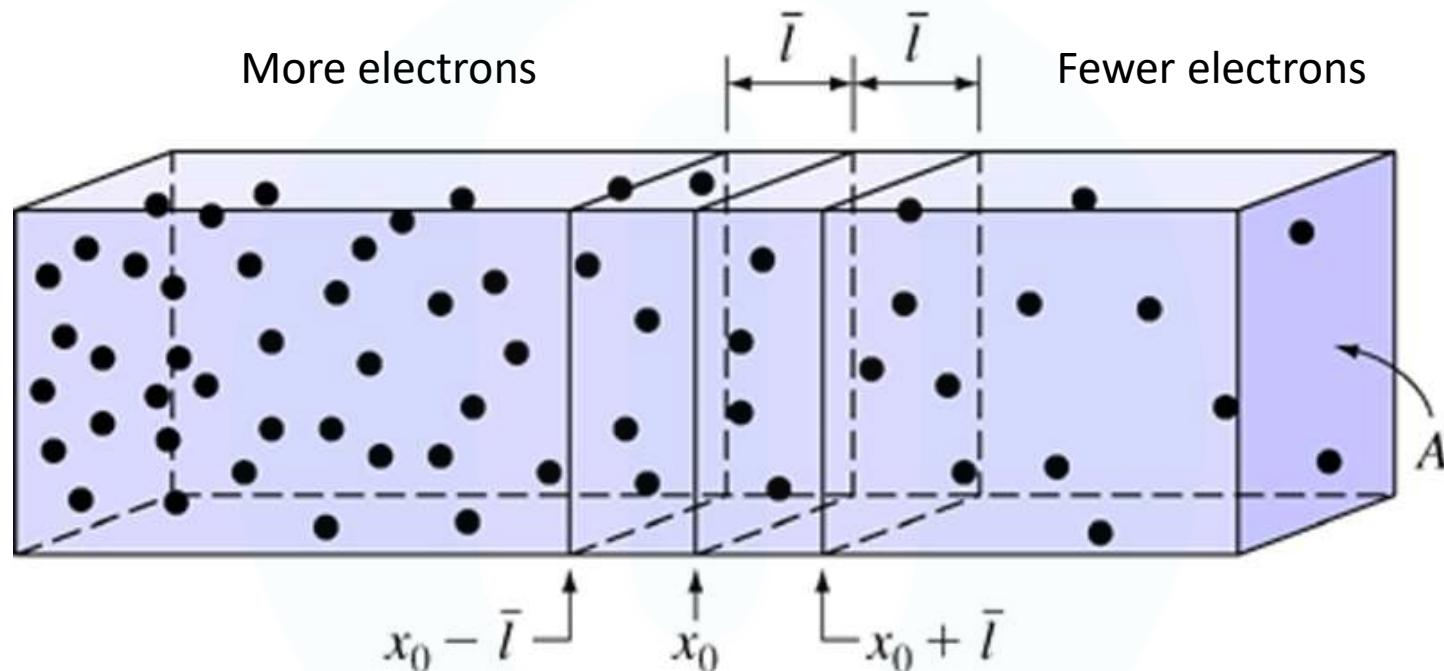
- If a measurement of resistance R is made, the sample resistivity ρ can be calculated: $\rho \text{ } (\Omega - \text{cm}) = R_{\text{wt}}/L = \frac{(V_{CD}/I_x)}{(L/\text{wt})}$
- Since the conductivity $\sigma = 1/\rho$ is given by $q p_0 \mu_p$, the mobility is simply the ratio of the Hall coefficient and the resistivity:
- $\mu_p = \sigma / q p_0 = \frac{(1/\rho)}{q(1/qR_H)} = \frac{R_H}{\rho}$

Diffusion

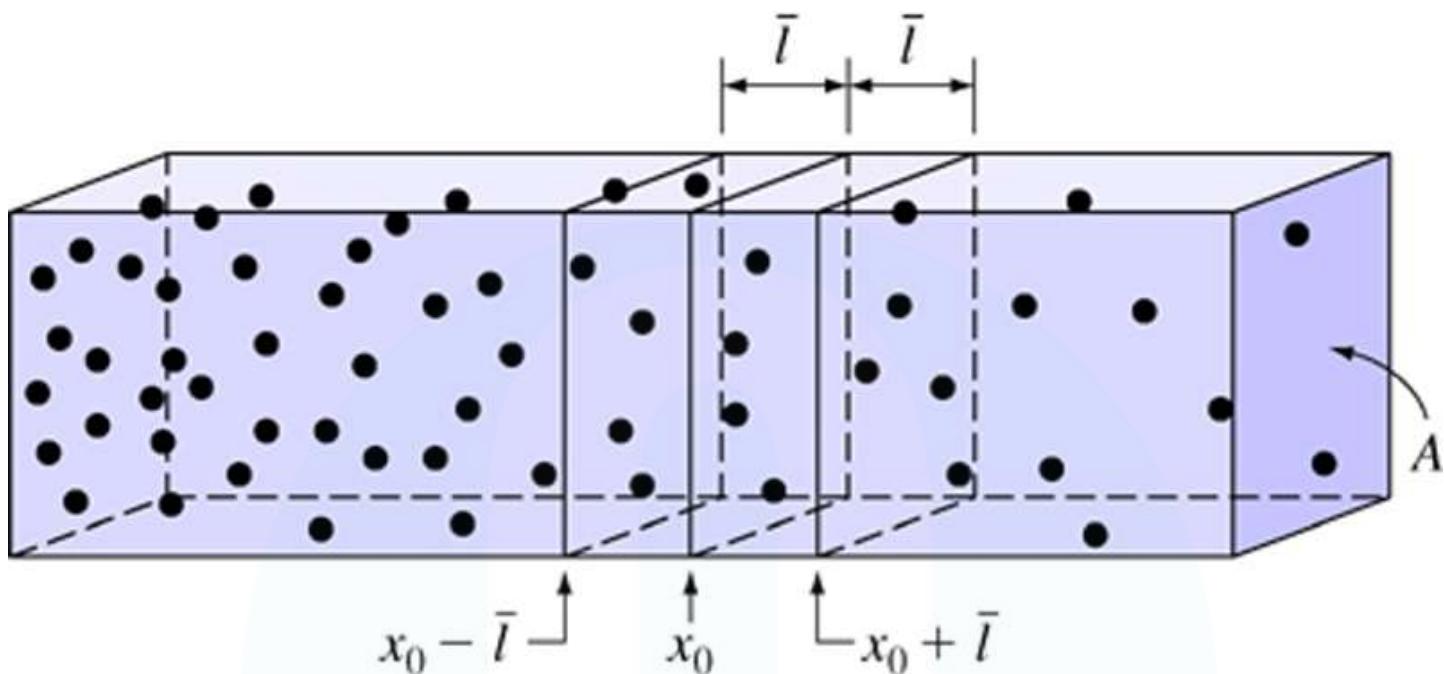
Two kinds of current

- We have discussed drift current
- Now we will look at diffusion current
- Diffusion current is not caused by a force
- Diffusion current arises when there is a gradient in the concentration of carriers

Consider a bar of semiconductor



- All electrons have some thermal energy
- All electrons are moving
- No field is applied
 - Electrons have no particular reason to move in any particular direction



- Consider the plane x_0 . Half of these electrons are moving to the right after each collision.
- Half are moving to the left

(a)

- Here also half of these electrons are moving to the right. Half are moving to the left

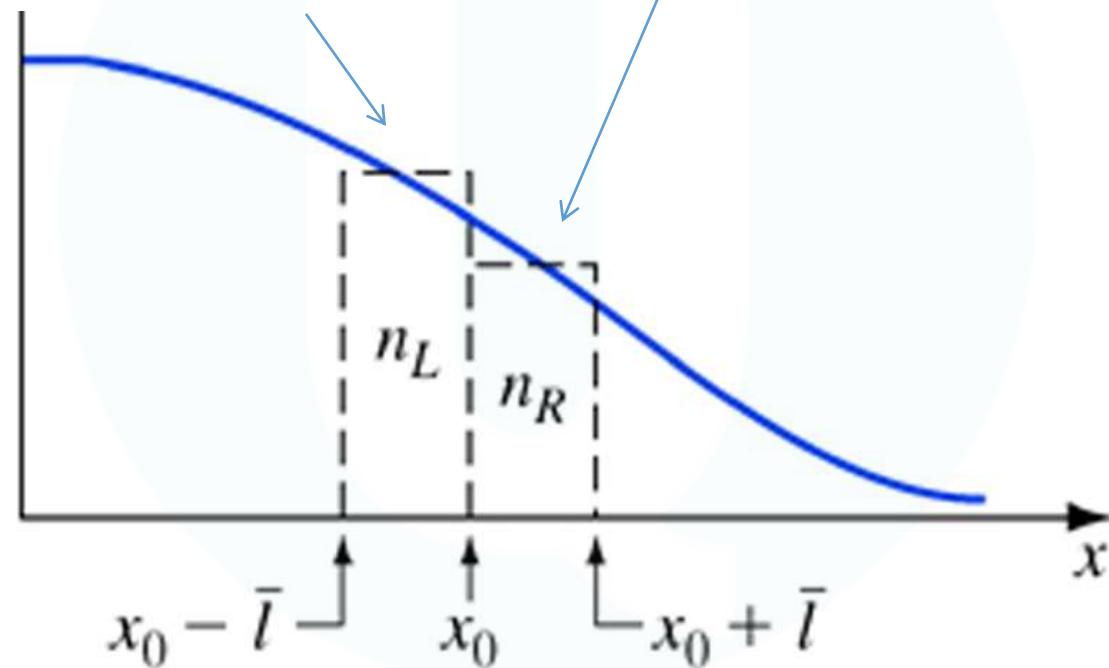
More electrons

Fewer electrons

→ Therefore, more electrons are moving from left to right across the plane x_0 than are crossing from right to left.
Thus, a net current is flowing

Look at another way

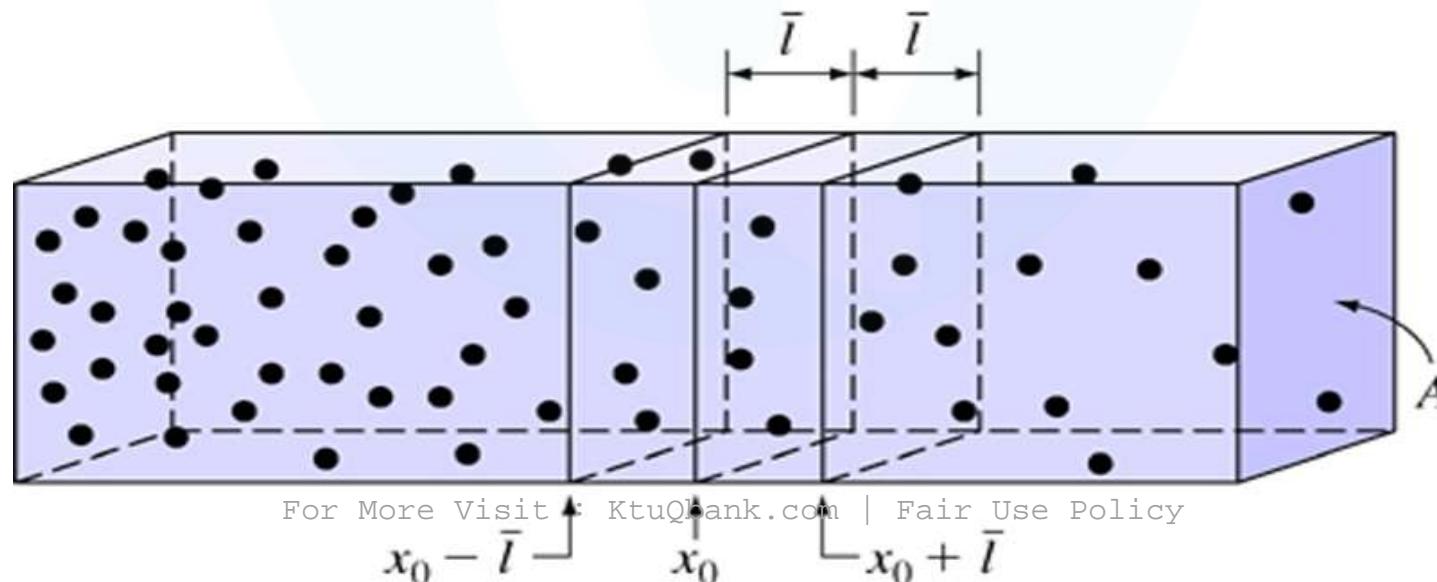
- Half of these electrons are moving to the right
- Half are moving to the left
- Half of these electrons are moving to the right
- Half are moving to the left



→ Therefore, more electrons are moving from left to right across the plane x_0 than are crossing from right to left
Thus, a net current is flowing from left to right

Let ϕ_n be the net flux of electrons

- Flux is net number of electrons crossing the plane
- Let \bar{l} be the mean free path between collisions & is a small incremental distance, we can divide x into segments \bar{l} wide, with $n(x)$ evaluated at the centre of each segment .
- Let \bar{t} be the mean free time between collisions



Diffusion

- The electrons in segment n_L to the left of x_0 in Fig. have equal chances of moving left or right, and in a mean free time \bar{t} one-half of them will move into segment n_R . The same is true of electrons within one mean free path of x_0 to the right; one-half of these electrons will move through x_0 from right to left in a mean free time. Therefore, the *net* number of electrons passing x_0 from left to right in one mean free time is $\frac{1}{2}(n_L \bar{l}A) - \frac{1}{2}(n_R \bar{l}A)$, where the area perpendicular to x_0 is A . The rate of electron flow in the $+x$ -direction per unit area (the electron flux density ϕ_n) is given by $\phi_n(x_0) = \frac{\bar{l}}{2\bar{t}}(n_L - n_R)$

Diffusion

- Since the mean free path \bar{l} is a small differential length,
- The difference in electron concentration ($n_L - n_R$) can be written as
- $(n_L - n_R) = \frac{n(x) - n(x + \Delta x)}{\Delta x} \bar{l}$, If $\text{Limit } \Delta x \rightarrow 0$, then $(n_L - n_R) = \frac{-dn(x)}{dx} \bar{l}$
- $\phi_n(x_0) = \frac{\bar{l}}{2t} (n_L - n_R) = - \frac{\bar{l}^2}{2} \frac{dn(x)}{t dx} \quad (\text{A})$
- $D_n = \frac{\bar{l}^2}{2t}$ where D_n called electron diffusion coefficient with unit $\text{cm}^2/\text{s.}$

Diffusion

- The minus sign in Eq. (A) arises from the definition of the derivative; it simply indicates that the net motion of electrons due to diffusion is in the direction of *decreasing* electron concentration.
- By identical arguments, we can show that holes in a hole concentration gradient move with a diffusion coefficient D_p . Thus
- $\phi_n(x_0) = - D_n \frac{dn(x)}{dx}$
- $\phi_p(x_0) = - D_p \frac{dp(x)}{dx}$

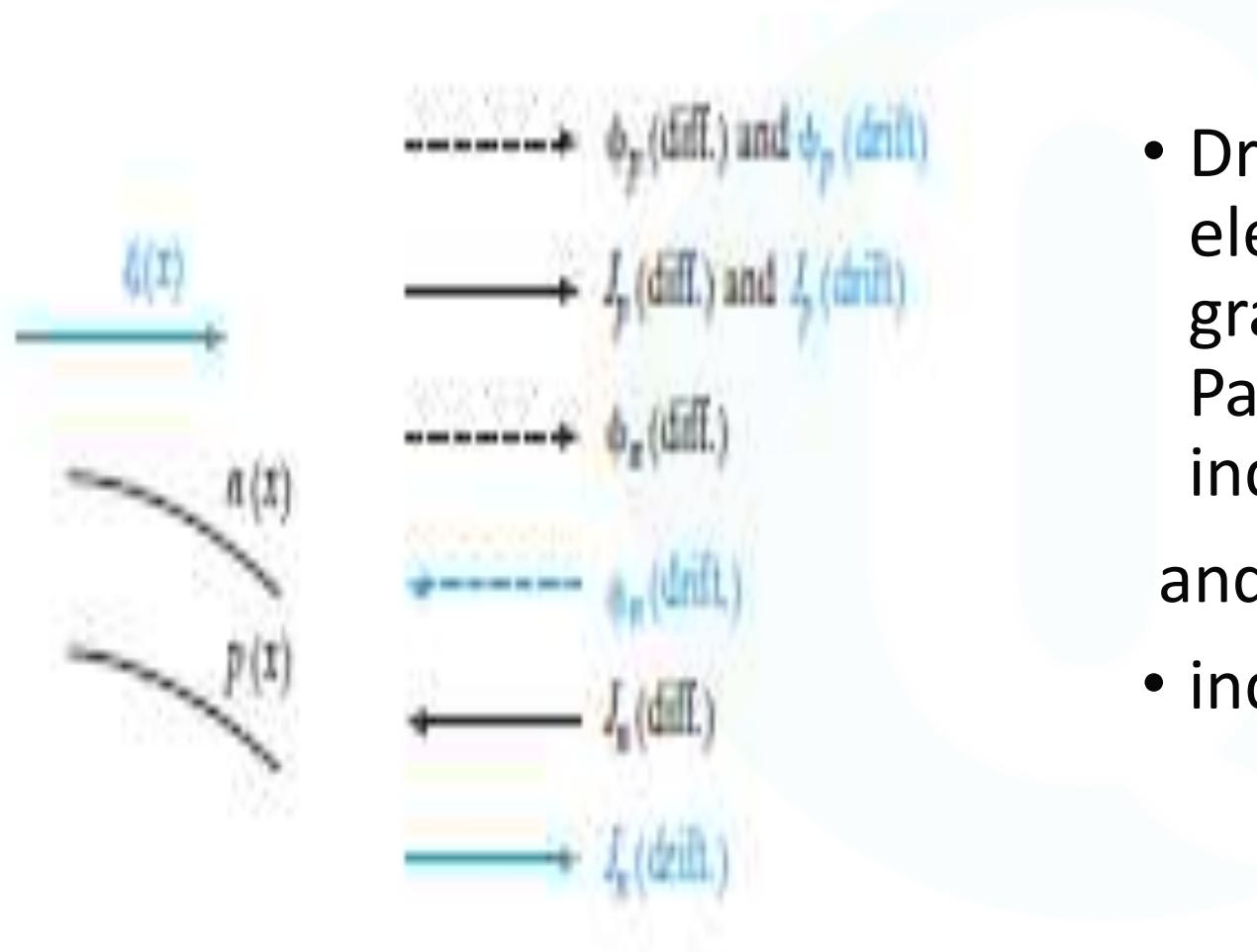
Diffusion

- The diffusion current crossing a unit area (the current density) is the particle flux density multiplied by the charge of the carrier:
- $J_n(\text{diff.}) = -(-q)D_n \frac{dn(x)}{dx} = (q)D_n \frac{dn(x)}{dx}$
- $J_p(\text{diff.}) = -(q)D_p \frac{dp(x)}{dx}$

Diffusion and Drift

- If an electric field is present in addition to the carrier gradient, the current densities will each have a drift component and a diffusion component
- $J_n(x) = qD_n \frac{dn(x)}{dx} + qn(x) \mu_n \mathcal{E}(x)$
- diffusion drift
- *Similarly* $J_p(x) = qp(x) \mu_p \mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$
and the total current density is the sum of the contributions due to electrons and holes: $J(x) = J_n(x) + J_p(x)$

Diffusion and Drift of Carriers; Built-in Fields



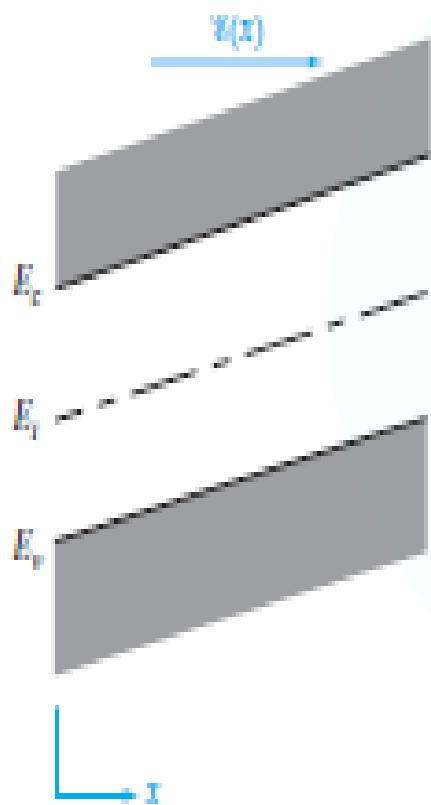
- Drift and diffusion directions for electrons and holes in a carrier gradient and an electric field. Particle flow directions are indicated by dashed arrows,
- and the resulting currents are indicated by solid arrows.

Diffusion and Drift of Carriers; Built-in Fields

- An important result of this equation is that minority carriers can contribute significantly to the current through diffusion. Since the drift terms are proportional to carrier concentration, minority carriers seldom provide much drift current. On the other hand, diffusion current is proportional to the *gradient* of concentration.

For example, in n- type material the minority hole concentration p may be many orders of magnitude smaller than the electron concentration n , but the gradient dp/dx may be significant. As a result, minority carrier currents through diffusion can sometimes be as large as majority carrier currents.

Energy band diagram of a semiconductor in an electric field $\mathcal{E}(x)$.



- Assuming an electric field (x) in the x -direction, we can draw the energy bands as in Fig., to include the change in potential energy of electrons in the field.
- Since electrons drift in a direction opposite to the field, we expect the potential energy for electrons to increase in the direction of the field.
- The electrostatic potential $\mathcal{V}(x)$ varies in the opposite direction, since it is defined in terms of positive charges and is therefore related to the electron potential energy $E(x)$ displayed in the figure
- by $\mathcal{V}(x) = E(x)/(-q)$.

Diffusion and Drift of Carriers; Built-in Fields

- From the definition of electric field, $\mathcal{E}(x) = - d \mathcal{V}(x) / dx$
- we can relate $\mathcal{E}(x)$ to the electron potential energy in the band diagram by choosing some reference in the band(for eg. E_i) for the electrostatic potential.

$$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx} = -\frac{d}{dx} \left[\frac{E_i}{(-q)} \right] = \frac{1}{q} \frac{dE_i}{dx}$$

Since the diagram indicates electron energies, we know the slope in the bands must be such that electrons drift “downhill” in the field. Therefore, $\mathcal{E}(x)$ points “uphill” in the band diagram.

Diffusion and Drift of Carriers; Built-in Fields

- At equilibrium, no net current flows in a semiconductor. Thus any fluctuation which would begin a diffusion current also sets up an electric field which redistributes carriers by drift.
- $J_p(x) = qD_p \frac{dp(x)}{dx} - qp(x) \mu_p \mathcal{E}(x) = 0$ for equilibrium, we have $qD_p \frac{dp(x)}{dx} = qp(x) \mu_p \mathcal{E}(x)$
- $\mathcal{E}(x) = \frac{D_p \frac{dp(x)}{dx}}{\mu_p} \frac{1}{p(x)}$ & for $p(x)$ using the equation $p_0 = n_i e^{\frac{(E_i - E_F)/kT}{}}$
- Differentiating $\frac{dp(x)}{dx} = n_i e^{\frac{(E_i - E_F)/kT}{}} * \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right) = p_0 * \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right)$

$$\frac{dp(x)}{dx} \frac{1}{p_0} = \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right)$$
- $\mathcal{E}(x) = \frac{D_p \frac{dp(x)}{dx}}{\mu_p} \frac{1}{p(x)} \equiv \frac{D_p}{\mu_p} \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right)$

Einstein relation

- But

$$\mathcal{E}(x) = -\frac{dV(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i}{(-q)}\right] = \frac{1}{q} \frac{dE_i}{dx}$$

$$\frac{1}{q} \frac{dE_i}{dx} = \frac{D_p}{\mu_p} \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right) \quad \text{--- where } \left(\frac{dE_F}{dx} = 0 \right)$$

The equilibrium Fermi level does not vary with x , and the derivative of E_i is thus reduces to

$$\frac{kT}{q} = \frac{D_p}{\mu_p} \quad \text{for both carrier type. This important equation is called the *Einstein relation*.}$$

Einstein relation

- Table lists typical values of D and μ for several semiconductors at room temperature.

	D_n [cm ² /s]	D_p [cm ² /s]	μ_n [cm ² /V-s]	μ_p [cm ² /V-s]
Ge	100	50	3900	1900
Si	35	12.5	1350	480
GaAs	220	10	8500	400

- It is clear from these values that $D/\mu = 0.026 \text{ V}$
- An important result of the balance of drift and diffusion at equilibrium is that *built-in* fields accompany gradients in E_i .
- Built-in fields result from doping gradients. For example, a donor distribution $N_d(x)$ causes a gradient in $n_0(x)$, which must be balanced by a built-in electric field $\mathcal{E}(x)$.

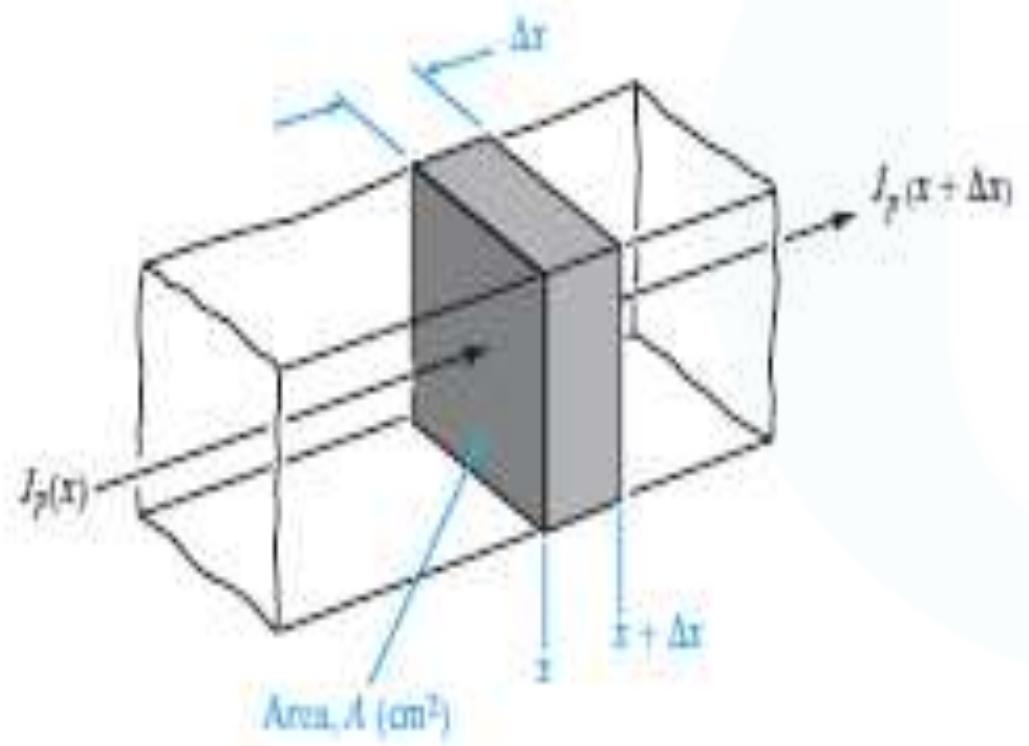
Assignment Questions

- 1. Derive an expression for drift current density in a semiconductor.
- 2. Explain diffusion process in a semiconductor and derive the expression for diffusion current density.
- 3. Define Hall Effect. Derive the expressions for i) majority carrier concentration ii) mobility.
- 4. For the given data, calculate hole and intrinsic carrier concentrations. Also sketch the band diagram. $N_C = 10^{19}\text{cm}^{-3}$, $N_V = 5 \times 10^{18}\text{cm}^{-3}$, $E_g = 2\text{eV}$, $T = 900\text{K}$, $n = 10^{17}\text{cm}^{-3}$

Diffusion and Recombination; The Continuity Equation

- While discussing diffusion of excess carriers, we have neglected the effects of recombination. But this must be included in a description of conduction processes, since recombination can cause a variation in the carrier distribution.
- For example, consider a differential length $\Delta(x)$ of a semiconductor sample with area A in the yz - plane in fig.
- The hole current density leaving the volume, $J_p(x + \Delta x)$, can be larger or smaller than the current density entering, $J_p(x)$, depending on the generation and recombination of carriers taking place within the volume.
- The net increase in hole concentration per unit time, $\frac{\partial p}{\partial t}$, is the difference between the hole flux per unit volume entering and leaving, minus the recombination rate

Continuity equation



- Current entering and leaving a volume $\Delta(x)$ A .
- Converting hole current density to hole particle flux density by dividing J_p by q .
- The current densities are already expressed per unit area; thus dividing $J_p(x)/q$ by $\Delta(x)$ gives the number of carriers per unit volume entering $\Delta x A$ per unit time, and $(1/q)J_p(x + \Delta x)/\Delta x$ is the number leaving per unit volume and time:

Continuity equation

- $$\frac{\partial p}{\partial t} |(x) \rightarrow (x + \Delta x) = \frac{1}{q} \frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} - \frac{\delta_p}{\tau_p}$$

Rate of hole buildup increase of hole concentration in $\Delta x A$ per unit time - recombination

- As Δx approaches zero, we can write the current change in derivative form:
- $$\frac{\partial p(x, t)}{\partial t} = \frac{\partial \delta_p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\delta_p}{\tau_p} \quad (A) \text{ where } p(x, t) = p_0 + \delta_p. (p_0 \text{ thermal equilibrium value constant})$$
- The expression is called the *continuity equation* for holes.
- For electrons we can write $\frac{\partial n(x, t)}{\partial t} = \frac{\partial \delta_n}{\partial t} = +\frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta_n}{\tau_n}$ since the electronic charge is negative.

Continuity equation

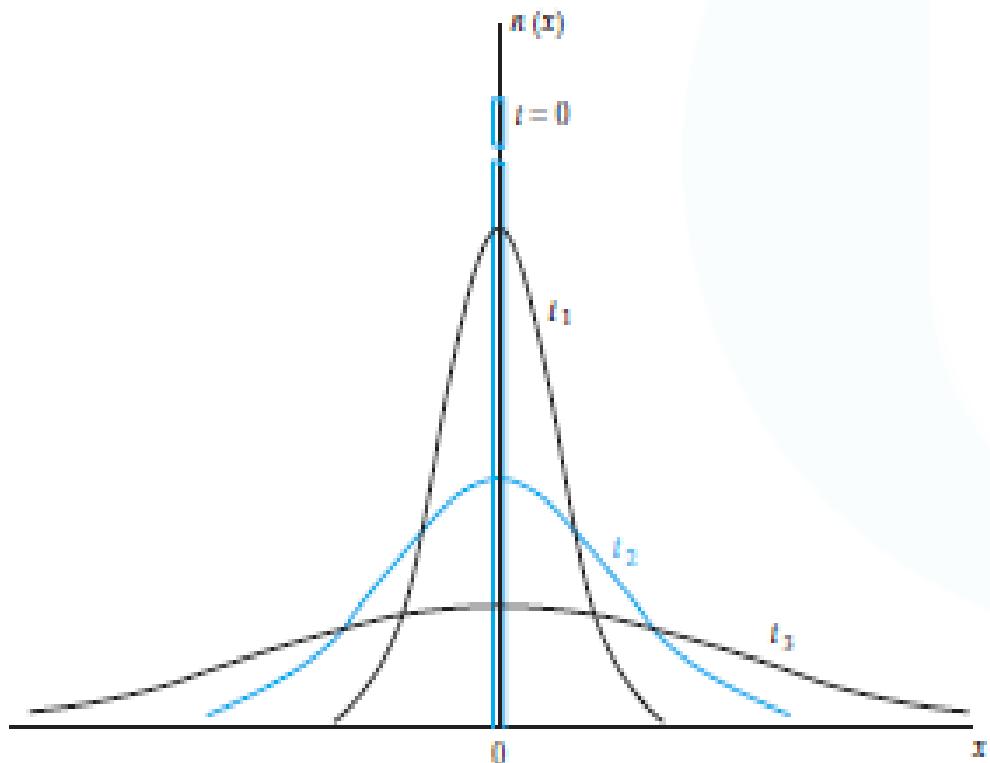
- When the current is carried strictly by diffusion (negligible drift), we can replace the currents in Eq. (A) by the expressions for diffusion current; for example, for electron diffusion we have

- $J_n(\text{diff.}) = qD_n \frac{\partial \delta_n}{\partial x}$
- $\frac{\partial \delta_n}{\partial t} = + \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta_n}{\tau_n} \quad (\text{A})$
- $\frac{\partial \delta_n}{\partial t} = + \frac{1}{q} \frac{\partial q D_n \frac{\partial \delta_n}{\partial x}}{\partial x} - \frac{\delta_n}{\tau_n}$
- $\frac{\partial \delta_n}{\partial t} = + \frac{D_n}{\partial x^2} \frac{\partial^2 \delta_n}{\partial x^2} - \frac{\delta_n}{\tau_n} \quad (\text{B})$

Continuity equation

- and similarly for holes, $\frac{\partial \delta_p}{\partial t} = + \frac{D_p}{\partial x^2} \frac{\partial^2 \delta_p}{\partial x^2} - \frac{\delta_p}{\tau_p}$ (C)
- These equations are useful in solving transient problems of diffusion with recombination. For example, a pulse of electrons in a semiconductor spreads out by diffusion and disappears by recombination as in fig (next slide). To solve for the electron distribution in time, $n(x, t)$, we would begin with the diffusion equation, Eq. (B).

Steady State Carrier Injection; Diffusion Length



- Here shows spreading of a pulse of electrons by diffusion.
- In many problems a steady state distribution of excess carriers is maintained,
- such that the time derivatives in Eqs. (B) or (C) are zero. In the steady state case the diffusion equations become

- $\frac{\partial \delta_n}{\partial t} + \frac{D_n}{\tau_n} \frac{\partial^2 \delta_n}{\partial x^2} = 0 \text{ (steady state)}$
- where $L_n = \sqrt{D_n \tau_n}$ is called the electron diffusion length

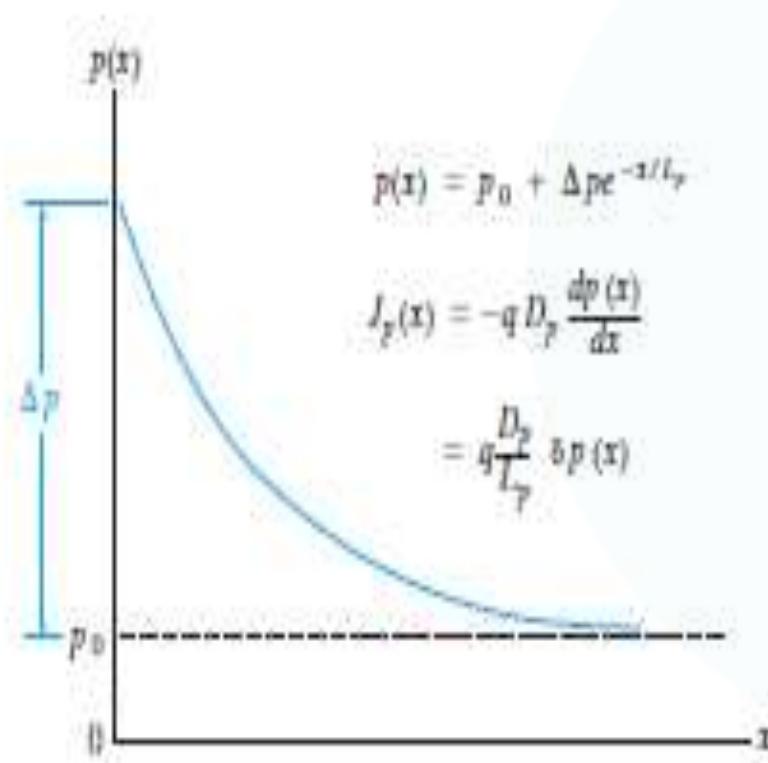
Steady State Carrier Injection; Diffusion Length

- The hole diffusion equation become

- $\frac{\partial \delta_p}{\partial t} + \frac{D_p}{\partial x^2} \frac{\partial^2 \delta_p}{\partial x^2} - \frac{\delta_p}{\tau_p} = 0$ \rightarrow $\frac{\partial^2 \delta_p}{\partial x^2} = \frac{\delta_p}{D_p \tau_p} = \frac{\delta_p}{L_p^2}$

where $L_p = \sqrt{D_p \tau_p}$ is called the hole *diffusion length* and L_p is the diffusion length for holes. We no longer need partial derivatives, since the time variation is zero for steady state.

Significance of diffusion length



- Let us assume that excess holes are somehow injected into a semi-infinite semiconductor bar at $x = 0$, and the steady state hole injection maintains a constant excess hole concentration at the injection
- point $\delta_p(x = 0) = \Delta p$. The injected holes diffuse along the bar, recombining with a characteristic lifetime τ_p . In steady state we expect the distribution of excess holes to decay to zero for large values of x , because of the recombination as in fig.
- For this problem we use the steady state diffusion equation for holes,
- $$\frac{\partial^2 \delta_p}{\partial x^2} = \frac{\delta_p}{L_p^2}$$
- The solution to this equation has the form $\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$

Significance of diffusion length

- We can evaluate C_1 and C_2 from the boundary conditions.
- Since recombination must reduce $\delta_p(x)$ to zero for large values of x , $\delta_p = 0$ at $x = \infty$ Therefore $C_1 = 0$.
- Similarly, the condition $\delta_p = \Delta p$ at $x = 0$ gives $C_2 = \Delta p$,
- and the solution is $\delta p(x) = \Delta p e^{-x/L_p}$
- The injected excess hole concentration dies out exponentially in x
- due to recombination, and the diffusion length L_p represents the distance
- at which the excess hole distribution is reduced to $1/e$ of its value at the
- point of injection. Or L_p is the average distance a hole diffuses before recombining

Gradients in the Quasi-Fermi Levels

- We know under equilibrium implies no gradient in the Fermi level E_F
- In contrast, any combination of drift and diffusion implies a gradient in the steady state quasi- Fermi level.
- Consider the general case of nonequilibrium electron concentration
- with drift and diffusion, the total electron current as
- $$J_n(x) = qD_n \frac{dn(x)}{dx} + qn(x) \mu_n \mathcal{E}(x)$$

Gradients in the Quasi-Fermi Levels

- Where the gradient in electron concentration is
- $\frac{dn(x)}{dx} = \frac{d}{dx} \left[n_i e^{\frac{(F_n - E_i)}{kT}} \right] = \frac{n(x)}{kT} \left(\frac{dF_n}{dx} - \frac{dE_i}{dx} \right)$
- Using the Einstein relation, the total electron current becomes
- $J_n(x) = \mu_n n(x) \left(\frac{dF_n}{dx} - \frac{dE_i}{dx} \right) + qn(x) \mu_n \mathcal{E}(x)$.(But $q\mathcal{E}(x) = \frac{dE_i}{dx}$)
- $J_n(x) = \mu_n n(x) \frac{dF_n}{dx}$

Gradients in the Quasi-Fermi Levels

- Thus, the processes of electron drift and diffusion are summed up by the spatial variation of the quasi- Fermi level. The same derivation can be made for holes, and we can write the current due to drift and diffusion in the form of a *modified Ohm's law*
- $J_n(x) = q\mu_n n(x) \frac{d(F_n/q)}{dx} = \sigma_n(x) \frac{d(F_n/q)}{dx}$
- *Similarly* $J_p(x) = q\mu_p p(x) \frac{d(F_p/q)}{dx} = \sigma_p(x) \frac{d(F_p/q)}{dx}$
- Therefore, any drift, diffusion, or combination of the two in a semiconductor results in currents proportional to the gradients of the two quasi- Fermi levels

