$\boxed{\text{Assignment — 1}}$

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Section : 02

Course : MAT120

1) Use Beta function to evaluate :

$$\int_0^4 x^{3/2}\,(4-x)^{5/2}\,dx$$

$\Rightarrow$ Now, let, $u = \frac{x}{4}$

$\Rightarrow 4u = x$

$\therefore dx = 4du$

$\therefore$ for ~~x0~~ $x = 0$; $u = 0$

and $x = 4$; $u = 1$

$$\therefore \int_0^4 x^{3/2}(4-x)^{5/2}\,dx$$

$$= (4)^{5/2}\int_0^1 (x)^{3/2}\left(1-\frac{x}{4}\right)^{5/2}$$

$$= (4)^{5/2}\int_0^1 (4u)^{3/2}\cdot(1-u)^{5/2}\,4du$$

$$= (4)^{5/2}\cdot(4)^{3/2}\int_0^1 (1-u)^{5/2}\cdot(u)^{3/2}\,du$$

$$= 1024\int_0^1 (u)^{3/2}(1-u)^{5/2}\,du$$

$$= 1024\,\beta\left(\frac{5}{2},\frac{7}{2}\right)$$

$$= 1024\,\frac{\Gamma\frac{5}{2}\,\Gamma\frac{7}{2}}{\Gamma 6}$$ $\left[\because \beta(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\,dt \quad \therefore x=\frac{5}{2},\ y=\frac{7}{2}\right]$

$= 12\pi$

ANS

2) Evaluate the improper integral. See it is convergent or divergent: $\int_0^{+\infty}(1-x)e^{-x}\,dx$

$\Rightarrow$ Here, $\int_0^{+\infty}(1-x)e^{-x}\,dx$

$= \left[(1-x)\cdot\frac{e^{-x}}{-1}\right]_0^{+\infty} - \int_0^{+\infty} e^{-x}\,dx$

$= 0 + 1 + \left[\frac{e^{-x}}{-1}\right]_0^{+\infty}$

$= 0 + 1 + 0 - 1$

$= 0$

$\therefore$ It is convergent. (Ans)

3) Determine the reduction formula for $\int \cos^n x\,dx$ and then find $\int_0^{\pi/2}\cos^6 x\,dx$

$\Rightarrow$ Here, $\int \cos^n x\,dx$

$= \int \cos^{n-1}x\,\cos x\,dx$

$= \cos^{n-1}x\cdot\sin x - \int (n-1)\cos^{n-2}x(-\sin x)\sin x\,dx$

$= \cos^{n-1}x\cdot\sin x + (n-1)\int\cos^{n-2}x\,\sin^2 x\,dx$

$$= \cos^{n-1} x \cdot \sin + (n-1)\int \cos^{n-2} x\,(1-\cos^2 x)\,dx$$

$$= \cos^{n-1} x \cdot \sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$$

$$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,dx$$

$$= \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

Now,

$$\int_0^{\pi/2} \cos^6 x\,dx$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\int_0^{\pi/2} \cos^4 x\,dx$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3}{4}\int_0^{\pi/2} \cos^2 x\,dx\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3}{4}\int_0^{\pi/2} \frac{1+\cos(2x)\,dx}{2}\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3}{4}\times\frac{1}{2}\int_0^{\pi/2} 1+\cos(2x)\,dx\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3}{4}\times\frac{1}{2}\left(\int_0^{\pi/2} 1\,dx + \int_0^{\pi/2} \cos(2x)\,dx\right)\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3}{4}\times\frac{1}{2}\left(\pi/2 + 0\right)\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3}{4} \times \frac{\pi}{4}\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{\sin x \cos^3 x}{4}\right]_0^{\pi/2} + \frac{3\pi}{16}\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[\left[\frac{1}{4} \cdot \sin x \cos^3 x\right]_0^{\pi/2} + \frac{3\pi}{16}\right]$$

$$= \left[\frac{\sin x \cos^5 x}{6}\right]_0^{\pi/2} + \frac{5}{6}\left[0 + \frac{3\pi}{16}\right]$$

$$= \left[\frac{1}{6} \cos^5 x \sin x\right]_0^{\pi/2} + \frac{5\pi}{32}$$

$$= \frac{5\pi}{32} \quad \text{Ans}$$

4) Evaluate the integral by using partial function decomposition :

$$\int \frac{2x^3 - 1}{(4x-1)(x^2+1)}\,dx$$

Hence,

$$\frac{(2x^2-1)}{(4x-1)(x^2+1)} = \frac{A}{(4x-1)} + \frac{Bx+C}{(x^2+1)}$$

Now,

$$(2x^2-1) \equiv A(x^2+1) + (Bx+C)(4x-1)$$

$$= Ax^2 + A + 4Bx^2 + 4Cx - Bx - C$$

When $x = \frac{1}{4}$,

$$2\left(\frac{1}{4}\right)^2 - 1 = A\left\{\left(\frac{1}{4}\right)^2 + 1\right\} + 0$$

$$\Rightarrow 2 \times \frac{1}{16} - 1 = A\left(\frac{1}{16} + 1\right)$$

$$\Rightarrow \frac{1-8}{8} = A\left(\frac{1+16}{16}\right)$$

$$\Rightarrow \frac{-7}{8} = A\left(\frac{17}{16}\right)$$

$$\therefore A = \frac{-14}{17}$$

$\therefore A + 4B = 2$

$\Rightarrow -\frac{14}{17} + 4B = 2$

$\Rightarrow 4B = 2 + \frac{14}{17}$

$\therefore B = \frac{12}{17}$

$\therefore 4C - B = 0$

$\Rightarrow 4C - \frac{12}{17} = 0$

$\Rightarrow 4C = \frac{12}{17}$

$\therefore C = \frac{3}{17}$

Given, $\int \frac{(2x^2-1)}{(4x-1)(x^2+1)}$

$= -\frac{14}{17}\int \frac{1}{(4x-1)} + \frac{12}{17}\int \frac{x}{(x^2+1)}dx + \frac{3}{17}\int \frac{1}{(x^2+1)}dx$

$= -\frac{14}{17}\ln\frac{(4x-1)}{4} + \frac{12}{17}\times\frac{1}{2}\int\left(\frac{1}{z}\right)dz + \frac{3}{17}\cdot\left(\frac{\sec^2\theta}{\sec^2\theta}\right)d\theta$

$= \frac{-14}{17\times 4}\ln(4x-1) + \frac{6}{17}\ln|z| + \frac{3}{17}\int(1)\,d\theta$

let, $(1+x^2) = z$ and $x = \tan\theta$

$\therefore 2x\,dx = dz$ $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta$

$\therefore \int \frac{(2x^2-1)}{(4x-1)(x^2+1)}$

$= \frac{-7}{34}\ln(4x-1) + \frac{6}{17}\ln|z| + \frac{3}{17}d\theta$

$= \frac{1}{34}\left(-7\ln(4x-1) + 12\ln|z| + 6\theta\right)$

$= \frac{1}{34}\left(-7\ln(4x-1) + 12\ln(x^2+1) + 6\tan^{-1}x\right)$

Ans

5) Use Gamma Function to Evaluate $\int_0^{+\infty} e^{-x^2} dx$

$\Rightarrow$ We know,

$$\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$$

Now, let, $y = x^2$

$$\Rightarrow x = \sqrt{y}$$

$$\therefore dx = \frac{1}{2} y^{-1/2} dy$$

$\therefore$ for $x = 0$, $y = 0$

and for $x = \infty$, $y = \infty$

$$\therefore \int_0^{\infty} e^{-y} \frac{1}{2} y^{-1/2} dy = \frac{1}{2} \int_0^{\infty} e^{-y} y^{\frac{1}{2}-1} dy$$

$$= \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} \sqrt{\pi}$$

$$= \frac{\sqrt{\pi}}{2}$$

$$\therefore \int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

Ans

6)

(a)

$f(x) = x^3 - 2x^2$

$g(x) = 2x^2 - 3x$

In[1]:=
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f[x_] = x^3 - 2 x^2;
g[x_] = 2 x^2 - 3 x;
```

In[3]:=
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Plot[{f[x], g[x]}, {x, -2, 4}, PlotLegends → "Expressions"]
```
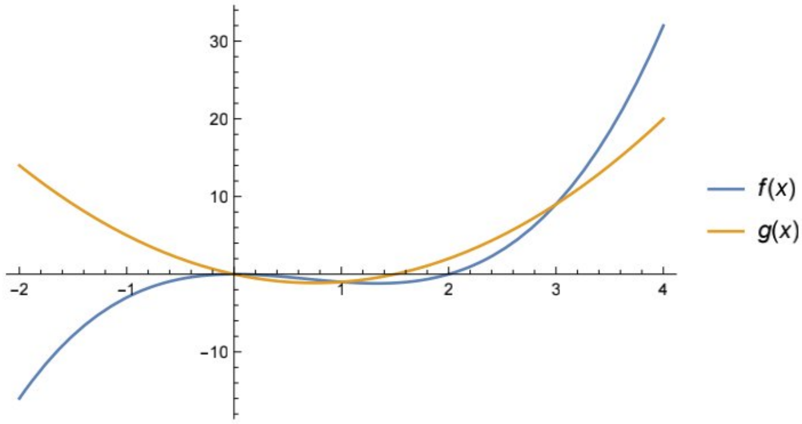

Out[3]=



(b)

In[4]:=
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Solve[f[x] == g[x], x]
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Out[4]= $\{\{x \to 0\}, \{x \to 1\}, \{x \to 3\}\}$

(c)

In[5]:= $\int_0^1 (f[x] - g[x])\, dx + \int_1^3 (g[x] - f[x])\, dx$

Out[5]= $\frac{37}{12}$

So, total area is $\left(\frac{37}{12}\right)$ unit^2.