University of California,
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School of Engineering and Applied Science

Department of Electrical and Computer Engineering

ECE11L Lab Manual
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Experiment 4: Transient Response of the 2nd-Order Circuits

Topics

- Step and Natural Response of Second Order Circuits
- Damping characteristics

Objectives

- To understand and investigate the natural and step response of second order capacitive and inductive circuits
- To analyze different damping effects in second order circuits
- To design a circuit with desired damping characteristics

Background

i. RLC Circuits: The differential equation of the RLC circuit is of the second order. For the series RLC circuit (Figure 1):

$$\frac{d^2 v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = \frac{1}{LC}v_i$$

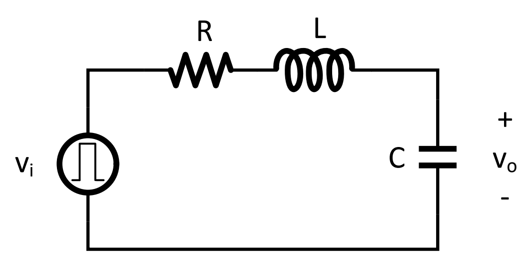


Figure 1: Series RLC circuit

Similarly, the differential equation for the parallel RLC (Figure 2) circuit is:

$$\frac{d^2 v_o}{dt^2} + \frac{1}{RC}\frac{dv_o}{dt} + \frac{1}{LC}v_o = \frac{1}{RC}\frac{dv_i}{dt}$$

The general solution to both the series and parallel RLC circuits can be found through the roots of the characteristic equation:

$$s^2 + 2\alpha s + {\omega_0}^2 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$$

where α is defined as the Neper Frequency, given by $\alpha = \frac{R}{2L}$ for a series RLC, or $\alpha = \frac{1}{2R}$ for a parallel RLC circuit, and $\omega_0 = \frac{1}{\sqrt{LC}}$ is the resonant frequency.

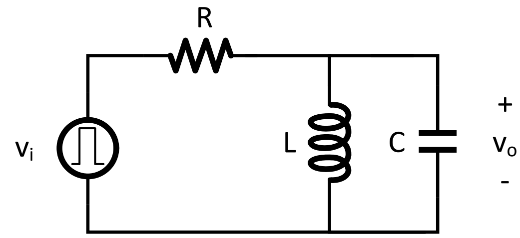


Figure 2: Parallel RLC circuit

ii. Damping: The damping coefficient is defined by: $\zeta = \frac{\alpha}{\omega_0}$.

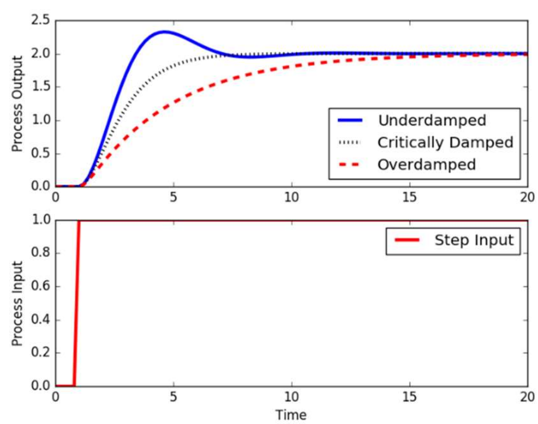
An *overdamped* response occurs if $\zeta > 1$, as characterized by the response reaching its final value without any oscillations. The general solution is of the form: $A_1 e^{s_1 t} + A_2 e^{s_2 t}$.

An *underdamped* response occurs if $\zeta < 1$, as characterized by the response having oscillations about the final value. Underdamped responses oscillate at a reduced frequency of $\omega_d = \sqrt{\omega_0{}^2 - \alpha^2}$, and the general solution is of the form: $e^{-\alpha t}(B_1 cos\omega_d t + B_2 sin\omega_d t)$.

A *critically damped* response occurs if $\zeta = 1$, and is characterized by the response being on the verge of oscillation. The general solution is of the form: $C_1 e^{s_1 t} + C_2 t e^{s_2 t}$. Note that visually, the overdamped and critically damped responses are somewhat similar.

In the case of an underdamped response ($\zeta < 1$), we may define the *overshoot* as the ratio of the difference between the peak voltage and the final voltage:

$$\frac{V_{peak} - V_{final}}{V_{final}}$$



Lab

1. Series RLC Circuit Analysis

Using a 4.7kΩ resistor, 150mH inductor, and 0.1uF capacitor, build the series RLC circuit illustrated in Figure 3. Note that we have considered the resistance of the inductor R_L, which is non-negligible. Write the equation for $v_o(t)$ (including R_L) which describes the voltage across the capacitor as a function of time for your design.

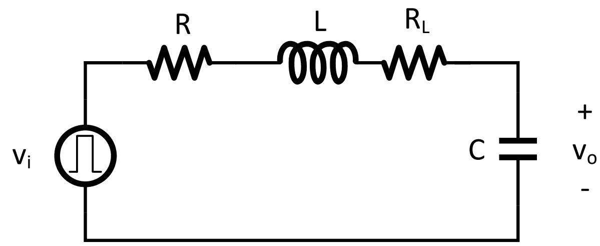


Figure 3: Series RLC circuit (series inductor resistance included)

Set the function generator to 100Hz, and output a square wave with 1V (p-p) from 0 to 1V. Connect one channel of the oscilloscope to view the input waveform, and another to measure the output voltage across the capacitor. Record your observations.

Discussion

- What kind of damping is observed? Verify that this matches with the theoretical expectation.

2. Underdamped RLC Circuit Design

By changing the value of the resistor from part 1 (Figure 3), acquire an underdamped response that overshoots the final voltage value by approximately 0.2. Take into account inductor resistance as well. Record the value of the resistor used, as well as the experimental value of the overshoot now considering the resistance of the inductor and the internal resistance of the generator. Compare the experimental value obtained by measuring the period of the oscillations with the theoretical value. Write the equation for $v_o(t)$ which describes the voltage across the capacitor as a function of time for your design.

Discussion

- How did the experimental damped frequency compare with the theoretical values?
- What happens if you try to make the overshoot smaller?

3. Critically Damped RLC Circuit

Replace the resistor of the circuit of Figure 3 again, but this time by a 10kΩ potentiometer. By using the oscilloscope output as a guide, adjust the resistance until the system is approximately critically damped. Record the output you obtain, remove the potentiometer, and measure the value of the resistance you used.

Note that critically damping is the fastest settling. There may still be a peak (overshoot), but no oscillation or ringing. The way to find the critical damping is to find the point between oscillation and no oscillation.

Discussion

- How close was the value of resistance you ended up with when using the potentiometer to obtain a critically damped response, to the theoretical value you have derived? Consider the effects of inductor resistance as well.
- What did you observe in the output waveform as resistance varied?