23W-EC ENGR-131A-LEC-1 Homework 1

SANJIT SARDA

TOTAL POINTS

86 / 100

QUESTION 1

1 20 pts

1.1 a **5 / 5**

✓ **+ 5 pts** *Correct*

1.2 b **5 / 5**

✓ **+ 5 pts** *Correct*

1.3 c **5 / 5**

✓ **+ 5 pts** *Correct*

1.4 d **5 / 5**

✓ **+ 5 pts** *Correct*

QUESTION 2

2 20 pts

2.1 a **7 / 7**

✓ **+ 7 pts** *Correct*

2.2 b **6 / 6**

✓ **+ 6 pts** *Correct*

2.3 c **7 / 7**

✓ **+ 7 pts** *Correct*

QUESTION 3

3 20 pts

3.1 a **10 / 10**

✓ **+ 10 pts** *Correct*

3.2 b **10 / 10**

✓ **+ 10 pts** *Correct*

QUESTION 4

4 4 **10 / 20**

+ 20 pts Correct

- 10 *Point adjustment*

As long as one number is a multiple of 11,
the product of this number and another
number will be a multiple of 11. One or both
of the numbers can be from {11,22,33,...99}.
It will be 9C2 + (9C1)*(91C1)
Answer: 855 ways to pick 2 numbers such
that their product is a multiple of 11.

QUESTION 5

5 20 pts

5.1 a **6 / 6**

✓ **+ 6 pts** *Correct*

5.2 b **6 / 6**

✓ **+ 6 pts** *Correct*

5.3 c **4 / 8**

+ 8 pts Correct

- 4 *Point adjustment*

Let A be the event ”The absolute value of the difference between your spins is greater than or equal to 8”. Then A = {(1,9),(9,1),(1,10),(10,1),(2,10),(10,2)}.

Thus the answer = 6/100 = 0.06

1. Four schools 1, 2, 3, and 4 are participating in a spelling bee competition. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will face each other for the cup, and the two losers will also play. A possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4).

   (a) List all outcomes in the sample space $\mathcal{S}$.

   (b) Let A denote the event that 1 wins the tournament. List outcomes in A.

   (c) Let B denote the event that 2 gets into the championship game. List outcomes in B.

   (d) What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A^c?

A

1.

a) S= {1324, 1342, 3124, 3142,
1423, 1432, 4123, 4132,
2314, 2341, 3214, 3241, $|S| = 16$
B → 2413, 2431, 4213, 4231}

b) A = {1324, 1342, 1423, 1432}

c) B = {2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}

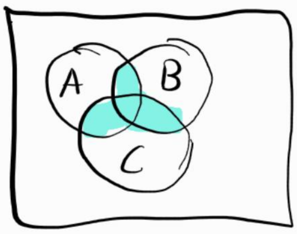
d) A ∪ B = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}

A ∩ B = { } 1 and 2 both cannot be in the championship by the initial condition. $\emptyset$

A^c = {3124, 3142, 4123, 4132, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}

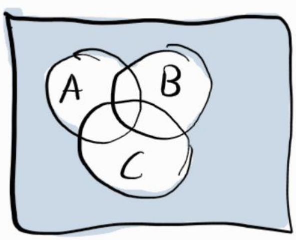
2)

a)



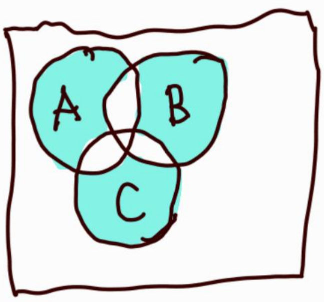
$(A\cap B)\cup(B\cap C)\cup(C\cap A)$

b)



$(A\cup B\cup C)^c$

c)



$(A\cap B^c\cap C^c)\cup(B\cap C^c\cap A^c)\cup(C\cap A^c\cap B^c)$

1.1 a **5 / 5**

✓ **+ 5 pts** *Correct*

1.2 b **5 / 5**

✓ **+ 5 pts** *Correct*

1.3 **C** **5 / 5**

✓ **+ 5 pts** *Correct*

1.4 d **5 / 5**

✓ **+ 5 pts** *Correct*

1. Four schools 1, 2, 3, and 4 are participating in a spelling bee competition. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will face each other for the cup, and the two losers will also play. A possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4).

(a) List all outcomes in the sample space $\mathcal{S}$.

(b) Let A denote the event that 1 wins the tournament. List outcomes in A.

(c) Let B denote the event that 2 gets into the championship game. List outcomes in B.

(d) What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A^c?

A

1.

a) S= {1324, 1342, 3124, 3142,
1423, 1432, 4123, 4132,
B → 2314, 2341, 3214, 3241,
2413, 2431, 4213, 4231}

$|S| = 16$

b) A = {1324, 1342, 1423, 1432}

c) B = {2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}

d) A ∪ B = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}

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Aᶜ = {3124, 3142, 4123, 4132, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}

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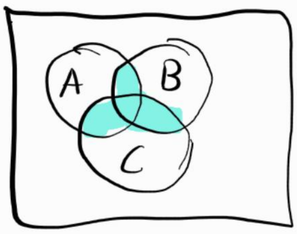
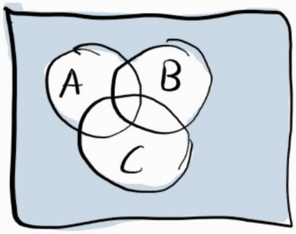
a)



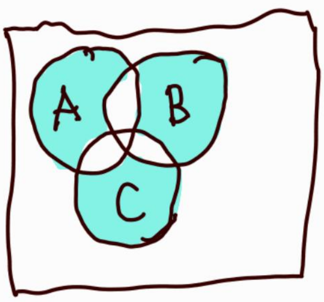
$(A \cap B) \cup (B \cap C) \cup (C \cap A)$

b)



$(A \cup B \cup C)^c$

c)



$(A \cap B^c \cap C^c) \cup (B \cap C^c \cap A^c) \cup (C \cap A^c \cap B^c)$

2.1 a **7 / 7**

✓ **+ 7 pts** *Correct*

2.2 b **6 / 6**

✓ **+ 6 pts** *Correct*

2.3 **C** **7 / 7**

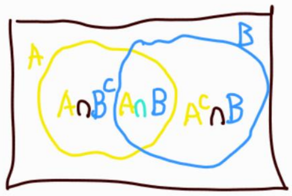
✓ **+ 7 pts** *Correct*

3) a) $P(A\cap B) \geq P(A)+P(B)-1$

$\therefore P(A\cap B)+1 \geq P(A)+P(B)$

$\therefore 1 \geq P(A)+P(B)-P(A\cap B)$ *

From Venn Diagram:



$P(A\cup B) = P(A\cap B^c) + P(A^c\cap B) + P(A\cap B)$

They are Disjoint

$P(A) = P(A\cap B^c) + P(A\cap B) \therefore P(A\cap B^c) = P(A) - P(A\cap B)$

$P(B) = P(A^c\cup B) + P(A\cap B) \therefore P(A^c\cap B) = P(B) - P(A\cap B)$

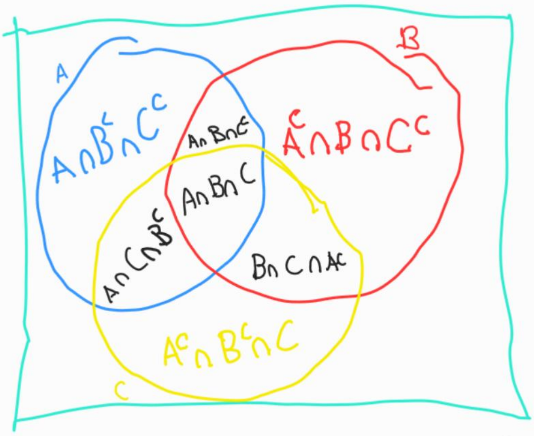
$P(A\cup B) = P(A) - P(A\cap B) + P(B) - P(A\cap B) + P(A\cap B) = P(A)+P(B)-P(A\cap B)$

* $1 \geq P(A)+P(B)-P(A\cap B) \rightarrow \therefore 1 \geq P(A\cup B)$

$\therefore 1 \geq 1 - P((A\cup B)^c) \rightarrow \therefore 1 \geq 1 - P(A^c\cap B^c) \rightarrow \therefore P(A^c\cap B^c) \geq 0$

Probability cannot be negative $\therefore$ Proven

b) $P(A\cup B\cup C) \leq P(A)+P(B)+P(C)$



$P(A\cup B\cup C) = P(A\cap B^c\cap C^c) + P(A^c\cap B\cap C^c) + P(A^c\cap B^c\cap C) + P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C^c) + P(A\cap B\cap C)$

&

$P(A) = P(A\cap B^c\cap C^c) + P(A\cap B\cap C^c) + P(A\cap B^c\cap C) + P(A\cap B\cap C)$

$P(B) = P(A^c\cap B\cap C^c) + P(A^c\cap B\cap C) + P(A\cap B\cap C^c) + P(A\cap B\cap C)$

$P(C) = P(A^c\cap B^c\cap C) + P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C)$

Since they are Disjoint.

We can rewrite the original statement: $P(A\cup B\cup C) \leq P(A) + P(B) + P(C)$

$\therefore P(A\cup B\cup C) \leq P(A\cap B^c\cap C^c) + P(A\cap B\cap C^c) + P(A\cap B^c\cap C) + P(A\cap B\cap C) + P(A^c\cap B\cap C^c) + P(A^c\cap B\cap C) + P(A\cap B\cap C^c) + P(A\cap B\cap C)$
$+ P(A^c\cap B^c\cap C) + P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C)$

$\therefore P(A\cup B\cup C) \leq P(A\cap B^c\cap C^c) + P(A^c\cap B\cap C^c) + P(A^c\cap B^c\cap C) + 2(P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C^c)) + 3P(A\cap B\cap C)$

$\therefore P(A\cap B^c\cap C^c) + P(A^c\cap B\cap C^c) + P(A^c\cap B^c\cap C) + P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C^c) + P(A\cap B\cap C)$
$\leq P(A\cap B^c\cap C^c) + P(A^c\cap B\cap C^c) + P(A^c\cap B^c\cap C) + 2(P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C^c)) + 3P(A\cap B\cap C)$

$\therefore 0 \leq P(A^c\cap B\cap C) + P(A\cap B^c\cap C) + P(A\cap B\cap C^c) + 2P(A\cap B\cap C)$

Since Probability cannot be ≤ 0,
this is true.

3.1 a **10 / 10**

✓ **+ 10 pts** *Correct*

3.2 b **10 / 10**

✓ **+ 10 pts** *Correct*

4) Sample Space = 2 distinct integers = $\frac{100!}{98!} = 9900$

Event = Pick 2 integers such that their product is a multiple of 11.

Observations in event:

$\overbrace{\{11,1\},\{11,2\}\dots\{11,100\}}^{99 \quad \text{exclude } (11,11)}$

$\overbrace{\{22,1\},\{22,2\}\dots\{22,100\}}^{98}$

$\vdots$

$\{99,1\},\{99,2\}\dots\{99,100\}$

$= 99+98+97+96+95+94+93+92+91$

$= 855$

$= 9 \cdot 100 - 9 = 891$

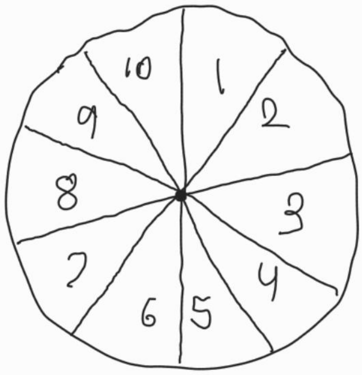
4 4 **10 / 20**

+ 20 pts Correct

- 10 *Point adjustment*

As long as one number is a multiple of 11, the product of this number and another number will be a multiple of 11. One or both of the numbers can be from {11,22,33,...99}. It will be 9C2 + (9C1)*(91C1)

Answer: 855 ways to pick 2 numbers such that their product is a multiple of 11.

5) 

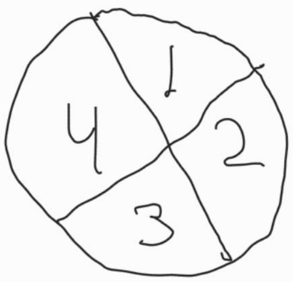
2 spins = 2^{10} = 1024 → Sample Space

a) Sp1 + Sp2 = E: Case 1: Both even: $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$
Case 2: Both odd: $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$ $= \frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

b) Probability at least 1 spin is odd. = 1 − probability both even = $1-\frac{1}{4}=\frac{3}{4}$

c) Probability that $|Sp1 - Sp2| \geq 8$. We know that there are only three

$= \frac{6}{1024} = \frac{3}{512}$



11	12	13	14	15	16	17	18	19	110
21	22	23	24	25	26	27	28	29	210
31	32	33	34	35	36	37	38	39	310
41	42	43	44	45	46	47	48	49	410
51	52	53	54	55	56	57	58	59	
61	62	63	64	65	66	67	68	69	
71	72	73	74	75	76	77	78	79	
81	82	83	84	85	86	87	88	89	
91	92	93	94	95	96	97	98	99	
101	102	103	104	105	106	107	108	109	

5.1 a **6 / 6**

✓ **+ 6 pts** *Correct*

5.2 b **6 / 6**

✓ **+ 6 pts** *Correct*

5.3 **C** **4 / 8**

+ 8 pts Correct

- 4 *Point adjustment*

Let A be the event ”The absolute value of the difference between your spins is greater than or equal to 8”. Then A = {(1,9),(9,1),(1,10),(10,1),(2,10),(10,2)}.

Thus the answer = 6/100 = 0.06