23W-EC ENGR-131A-LEC-1 Homework 3

SANJIT SARDA

TOTAL POINTS

100 / 100

QUESTION 1

1 Question 1 **15 / 15**

✓ **- 0 pts** *All parts correct*

- 2 pts Missing sketch for part a

- 5 pts Inorrect part a

- 5 pts Incorrect part b

- 5 pts Incorrect part c

QUESTION 2

2 Question 2 **20 / 20**

✓ **- 0 pts** *Correct*

- 10 pts Missing part a

- 10 pts Missing part b

QUESTION 3

3 Question 3 **20 / 20**

✓ **- 0 pts** *Correct*

- 10 pts Missing part a

- 10 pts Missing part b

QUESTION 4

4 Question 4 **20 / 20**

✓ **- 0 pts** *Correct*

- 10 pts Missing part a

- 10 pts Missing part b

QUESTION 5

5 Question 5 **25 / 25**

✓ **- 0 pts** *Correct*

- 8 pts Missing part a

- 8 pts Missing part b

- 9 pts Missing part c

ECE 131A HW#3

1) $P(\text{Insured}) = \frac{1}{4}$, $P(\text{Not Insured}) = 1 - \frac{1}{4} = \frac{3}{4}$

4 people @ Random. $X \rightarrow$ # of Insured.

a) $P_X(x<0) = 0$

$P_X(X=0) = P(\text{Insured})^x P(\text{Not Insured})^{4-x} \binom{4}{0} = \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4 \cdot 1 = \frac{81}{256}$

$P_X(X=1) = P(\text{Insured})^x P(\text{Not Insured})^{4-x} \binom{4}{1} = \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^3 \cdot 4 = \frac{108}{256}$

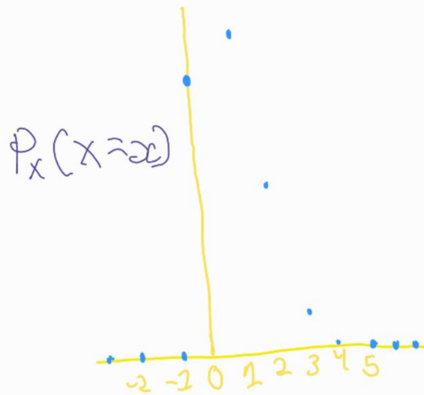
$P_X(X=2) = P(\text{Insured})^x P(\text{Not Insured})^{4-x} \binom{4}{2} = \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2 \cdot 6 = \frac{54}{256}$

$P_X(X=3) = P(\text{Insured})^x P(\text{Not Insured})^{4-x} \binom{4}{3} = \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^1 \cdot 4 = \frac{12}{256}$

$P_X(X=4) = P(\text{Insured})^x P(\text{Not Insured})^{4-x} \binom{4}{4} = \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^0 1 = \frac{1}{256}$

$P_X(X>4) = 0$

ways to arrange.



b) The most likely value for X is that 1 of them has insurance.

c) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) = \frac{54+12+1}{256} = \frac{67}{256}$

1 Question 1 **15 / 15**

✓ **- 0 pts** *All parts correct*

- 2 pts Missing sketch for part a

- 5 pts Inorrect part a

- 5 pts Incorrect part b

- 5 pts Incorrect part c

2) X is a Binomial RV with $n = 4$ & p.

a) PMF: $P(X=x) = \binom{4}{x} p^x (1-p)^{4-x}$

$$\mathbb{E}\left[\sin\left(\frac{\pi X}{2}\right)\right] = \sum_X \sin\left(\frac{\pi}{2}x\right) \cdot P_X(X=x) = \sum_{k=0}^{4} \sin\frac{\pi}{2}k \cdot \binom{4}{k} p^k (1-p)^{4-k}$$

$$= \mathbb{E}\left[\sin\left(\frac{\pi X}{2}\right)\right] = 0 + \sin\left(\frac{\pi}{2}\right) \cdot 4 \cdot p(1-p)^3 + \cancel{\sin(\pi) \cdot 6p^2(1-p)^2} + \sin\left(\frac{3\pi}{2}\right) \cdot 4p^3(1-p) + \cancel{\sin(2\pi) \cdot 1p^4}$$

$$= 4p(1-p)^3 - 4p^3(1-p) = 4p(p-1)(2p-1)$$

b) $$\mathbb{E}\left[\cos\left(\frac{\pi X}{2}\right)\right] = \sum_X \cos\left(\frac{\pi}{2}x\right) \cdot P_X(X=x) = \sum_{k=0}^{4} \cos\left(\frac{\pi}{2}k\right) \binom{4}{k} p^k (1-p)^{4-k}$$

$$= \cos(0)\binom{4}{0} p^0 (1-p)^4 + 0 + \cos(\pi)\binom{4}{2} p^2 (1-p)^2 + 0 + \cos(2\pi)\binom{4}{4} p^4 (1-p)^0$$

$$= (1-p)^4 - 6p^2(1-p)^2 + p^4$$

6

2 Question 2 **20 / 20**

✓ **- 0 pts** *Correct*

- 10 pts Missing part a

- 10 pts Missing part b

3) Two coins : $P(H_1) = p$ & $P(H_2) = q$

X is # rolls till H_1T_2 || H_2T_1. At each toss $P(\text{Success}) = p(1-q) + q(1-p)$

$P(\text{Fail}) = pq + (1-p)(1-q) = 1 - P(\text{success})$

$P_X(x=0) = 0$

$P_X(x=1) = P(\text{Success})P(\text{Fail})^0 = (p(1-q) + q(1-p))$

$P_X(x=2) = P(\text{success})P(\text{Fail})^1 = (p(1-q)+q(1-p))(pq + (1-p)(1-q))$

$P_X(x=3) = P(\text{success})P(\text{Fail})^2 = (p(1-q)+q(1-p))(pq+(1-p)(1-q))^2$

$P_X(x=k) = P(\text{success})P(\text{Fail})^{k-1} = (p(1-q)+q(1-p))(pq+(1-p)(1-q))^{k-1}$

Let $ps = P(\text{Success}) = p(1-q)+q(1-p) = 1 - P(\text{Fail}) = \overset{\text{Let}}{pf}$

$E[x] = \sum_{k=1}^{\infty} k \cdot ps(pf)^{k-1} = ps \sum_{k=1}^{\infty} \frac{d}{dpf}(pf^k) =$

$= ps \frac{d}{dpf} \sum_{k=1}^{\infty} pf^k = ps \frac{d}{dpf} \frac{1}{1-pf} = ps \frac{1}{ps^2} = \frac{1}{ps}$

$\therefore E[x] = \frac{1}{p(1-q)+q(1-p)}$

$p+q-2qp$

Again using ps & pf,

$Var[x] = E[x^2] - (E[x])^2$

$= \sum_{k=1}^{\infty} k^2 ps(pf)^{k-1} - \frac{1}{ps^2}$

$= ps \sum_{k=1}^{\infty} k \frac{d}{dpf}(pf)^{k-1} - \frac{1}{ps^2}$

$= ps \sum_{k=1}^{\infty} \frac{d^2}{dpf^2}(pf^{k+1}) - \frac{d}{dpf} pf^k - \frac{1}{ps^2}$

* $= ps \frac{d^2}{dpf^2} \sum_{k=1}^{\infty} pf^{k+1} - ps \frac{d}{dpf} \sum_{k=1}^{\infty} pf^k - \frac{1}{ps^2}$

$= ps \frac{d^2}{dpf^2}\left(\frac{1}{1-pf} - 1 - pf\right) - ps \frac{d}{dpf}\left(\frac{1}{1-pf} - 1\right) - \frac{1}{ps^2}$

$= ps \cdot \frac{-2}{(-ps)^3} - ps \cdot \frac{1}{(-ps)^2} - \frac{1}{ps^2}$

$= \cancel{ps} \cdot \frac{2}{ps^{\cancel{3}2}} - \frac{ps}{ps^2} - \frac{1}{ps^2}$

$= \frac{2 - ps - 1}{ps^2} = \frac{1-ps}{ps^2} = \frac{pf}{ps^2}$

$= \frac{pq + (1-p)(1-q)}{(p(1-q) + q(1-p))^2} = Var[x]$

* $\frac{d^2}{dpf^2} \sum_{k=1}^{k=\infty} q^{k+1}$, using u-sub, $u = k+1$

$\frac{d^2}{dpf^2} \sum_{u-1=1}^{u-1=\infty} q^u = \frac{d^2}{dpf^2} \sum_{u=2}^{\infty} pf^k$

$= \frac{d^2}{dpf^2}\left(\frac{1}{1-pf} - 1 - pf\right)$

$= \frac{-2}{(pf-1)^3}$

b) $P(H_1T_2 | \text{Success}) = \frac{P(H_1T_2 \cap \text{Success})}{P(\text{Success})}$

$P(H_1T_2 \cap \text{Success}) = \underbrace{P(\text{Success} | H_1T_2)}_{1} P(H_1T_2)$

$\therefore P(H_1T_2|\text{Success}) = \frac{P(H_1T_2)}{P(\text{Success})} = \frac{p(1-q)}{p(1-q)+q(1-p)}$

$= \frac{p(1-q)}{p(1-q)+q(1-p)}$

3 Question 3 **20 / 20**

✓ **- 0 pts** *Correct*

- 10 pts Missing part a

- 10 pts Missing part b

4) PMF

$\therefore P(N=k) = \frac{\alpha^k e^{-\alpha}}{k!}$

Neural Electrodes: 6000 spikes/min

$\alpha =$ average # events in a time interval

$\alpha = \frac{6000}{1m} = \frac{6000}{60s} = \frac{6\cancel{000}}{6\cancel{0000}} = \frac{1}{10}$ events/ms

a) $P(N=0) = \frac{\left(\frac{1}{10}\cdot 100\right)^k e^{-\left(\frac{1}{10}\cdot 100\right)}}{k!} = \frac{10^0 e^{-10}}{0!} = \boxed{e^{-10}}$

b) PMF for between 300 & 400 ms =

$P(N=k) = \frac{\left(\frac{1}{10}\cdot 100\right)^k e^{-\left(1/10 \cdot 100\right)}}{k!}$

$\frac{10^k e^{-10}}{k!}$

$\therefore P(5 \text{ to } 10 \text{ spikes}) = P_{43}(N=5) + P_{43}(N=6) \dots P_{43}(N=10)$

$= \frac{10^5 e^{-10}}{5!} + \frac{10^6 e^{-10}}{6!} + \frac{10^7 e^{-10}}{7!} + \frac{10^8 e^{-10}}{8!} + \frac{10^9 e^{-10}}{9!} + \frac{10^{10} e^{-10}}{10}$

4 Question 4 **20 / 20**

✓ **- 0 pts** *Correct*

- 10 pts Missing part a

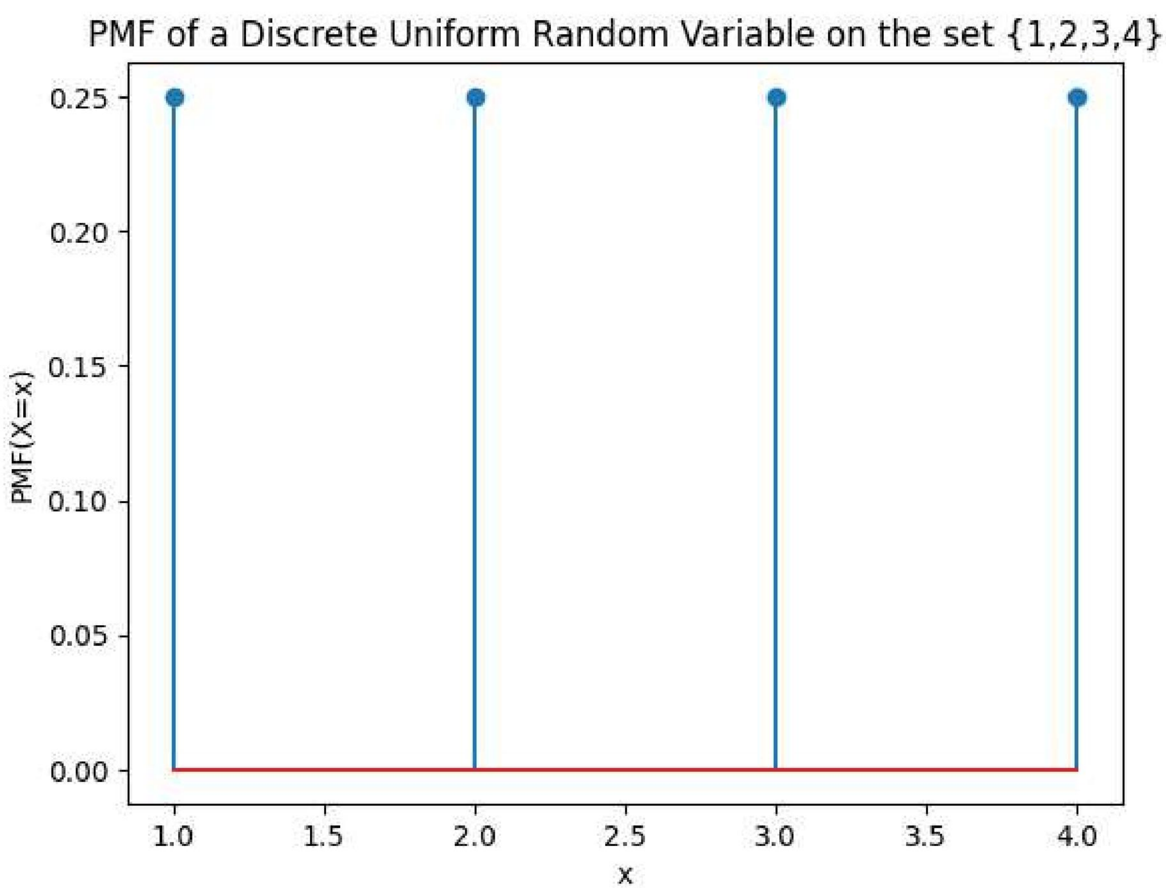
- 10 pts Missing part b

ECE 131A HW 2 Programming Section

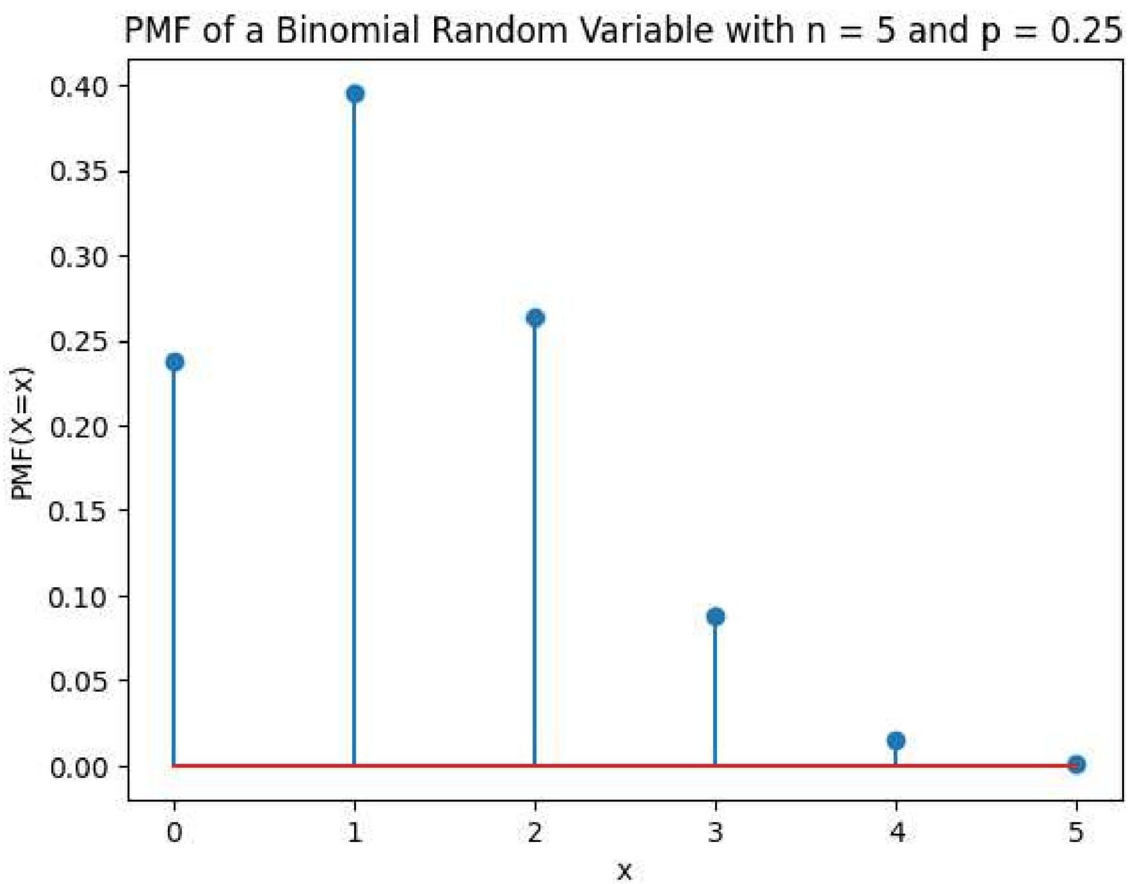
5. Use MATLAB to plot the pmfs of the following random variables. This question is to provide practice with MATLAB which you will use for the future project so do not provide hand-drawn plots.

```
# Imports
import numpy as np
import matplotlib.pyplot as plt
import scipy.io as sio
```

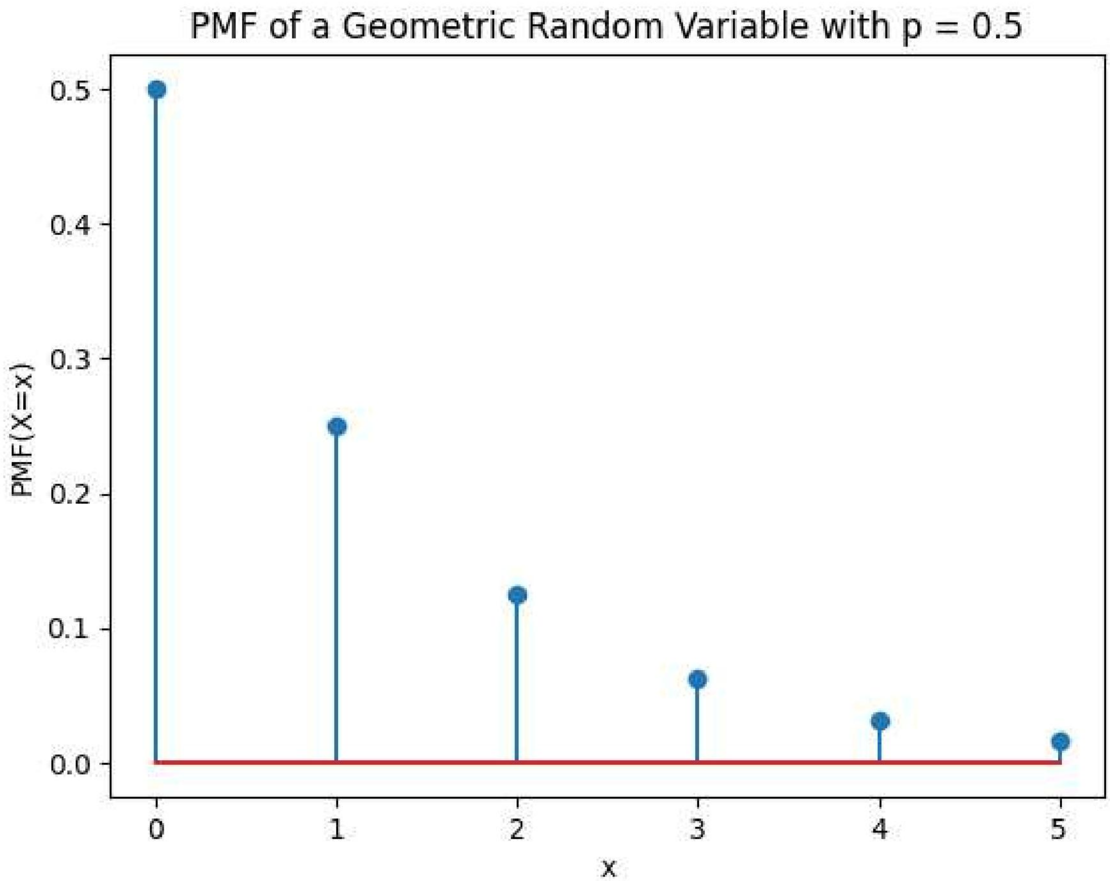
```
# a) Discrete Uniform Random Variable on the set {1,2,3,4}
x = np.array([1,2,3,4])
p = np.array([1/4,1/4,1/4,1/4])
plt.stem(x,p)
plt.title('PMF of a Discrete Uniform Random Variable on the set {1,2,3,4}')
plt.xlabel('x')
plt.ylabel('PMF(X=x)')
plt.show()
```



```
# b) Binomial Random Variable with n = 5 and p = 0.25
n = 5
p = 0.25
x = np.arange(0,6)
pmf = np.zeros(6)
for i in range(0,6):
    pmf[i] = np.math.factorial(n)/(np.math.factorial(i)*np.math.factorial(n-i))*p**i*(1-p)**(
plt.stem(x,pmf)
plt.title('PMF of a Binomial Random Variable with n = 5 and p = 0.25')
plt.xlabel('x')
plt.ylabel('PMF(X=x)')
plt.show()
```




```
# c) Geometric Random Variable with p = 0.5 and plot the PMF for values less than or equal to
p = 0.5
x = np.arange(0,6)
pmf = np.zeros(6)
for i in range(0,6):
    pmf[i] = p*(1-p)**i
plt.stem(x,pmf)
plt.title('PMF of a Geometric Random Variable with p = 0.5')
plt.xlabel('x')
plt.ylabel('PMF(X=x)')
plt.show()
```


PMF of a Geometric Random Variable with p = 0.5
PMF(X=x)
0.0
0.1
0.2
0.3
0.4
0.5
x
0
1
2
3
4
5

5 Question 5 **25 / 25**

✓ **- 0 pts** *Correct*

- 8 pts Missing part a

- 8 pts Missing part b

- 9 pts Missing part c

3) Coin Tossed until heads

$P(H) = p \quad P(T) = q \quad$ also $p+q=1$

$X = \#$ tosses till H

$P_X(X \le 0) = 0$

$P_X(X=1) = p$

$P_X(X=2) = pq$

$P_X(X=3) = pq^2$

$\vdots$

a) $P_X(X=k) = p\,q^{k-1}$

c)

a) $\mathbb{E}[X] = \sum_{k=0}^{\infty} k \cdot P_X(X=k) = \sum_{k=0}^{\infty} k\,p\,q^{k-1} = \sum_{k=0}^{\infty} p \frac{d}{dq} q^k$

$= p \frac{d}{dq} \sum_{k=0}^{\infty} q^k = p \frac{d}{dq} \frac{1}{1-q} = p \frac{1}{(p^2)} = \frac{1}{p} = \frac{1}{1-q}$

$$\mathrm{Var}[X] = \sum_{k=0}^{\infty} k^2 P_X(X=k) - \left(\mathbb{E}[X]\right)^2$$

$$= \sum_{k=0}^{\infty} k^2 p\,q^{k-1} - \frac{1}{p^2}$$

$$= p \sum_{k=0}^{\infty} k \frac{d}{dq} q^k - \frac{1}{p^2}$$

$$= p \sum_{k=0}^{\infty} \left(\frac{d^2}{dq^2} q^{k+1} - \frac{d\,q^k}{dq} \right) - \frac{1}{p^2}$$

$$= p \frac{d^2}{dq^2} \sum_{k=0}^{\infty} q^{k+1} - p \frac{d}{dq} \sum_{k=0}^{\infty} q^k - \frac{1}{p^2}$$

$$= p \frac{d}{dq^2} \left(q^0 + \sum_{k=0}^{\infty} q^k \right) - p \frac{d}{dq} \sum_{k=0}^{\infty} q^k - \frac{1}{p^2}$$

$$= p \frac{d}{dq^2} \left(\frac{q}{1-q} \right) - p \frac{d}{dq} \frac{1}{1-q} - \frac{1}{p^2}$$

$$= p \cdot \frac{-2}{(q-1)^3} - p \cdot \frac{1}{(q-1)^2} - \frac{1}{p^2}$$

$$= p \frac{-2}{(-p)^3} - p \frac{1}{(-p)^2} - \frac{1}{p^2}$$

$$= \frac{2}{p^2} - \frac{p}{p^2} - \frac{1}{p^2}$$

$$= \frac{1-p}{p^2}$$