EE113: DIGITAL SIGNAL PROCESSING

Midterm 1 Practice Problems

Problem 1

Compute the convolution of $x[n]$ and $h[n]$, $y[n] = x[n] * h[n]$.

a) $x[n] = \{\underset{n=0}{\underset{\uparrow}{-3}}, 7, 4\}$ and $h[n] = \{\underset{n=0}{\underset{\uparrow}{4}}, 3, 2, 1\}$

b) $x[n] = u[n] - u[n-7]$ and $h[n] = (0.9)^n u[n]$

Problem 2

a) Consider a discrete-time complex signal $x[n] = A[n]e^{j\phi[n]}$, where $A[n] = |x[n]|$ and $\phi[n]$ is the phase of the signal x[n].
Derive the relationship between $A[n]$ and $A[-n]$, and $\phi[n]$ and $\phi[-n]$ when the signal is

i) conjugate symmetric

ii) conjugate antisymmetric

b) Now consider a discrete-time complex signal $x[n] = a[n] + jb[n]$, where $a[n]$ is the real part and $b[n]$ is the imaginary part of $x[n]$. Is $a[n]$ and $b[n]$ odd or even when the signal x[n] is:

i) conjugate symmetric

ii) conjugate antisymmetric

Problem 3

Assume x[n] has nonzero samples only in the interval $-N_1 \leq n \leq N_2$. Generally, over what interval of time will the following sequence have non-zero samples:

$$y[n] = x[n] * x[n]$$

Problem 4

Prove the distributive property of the periodic convolution:

$$\tilde{x}[n] \otimes (\tilde{y}[n] + \tilde{z}[n]) = \tilde{x}[n] \otimes \tilde{y}[n] + \tilde{x}[n] \otimes \tilde{z}[n]$$

Problem 5

Consider the following system: $y[n] = \sum_{k=0}^{n} \frac{1}{2^k} x[k]$

(a) Is the system linear? Prove your answer.

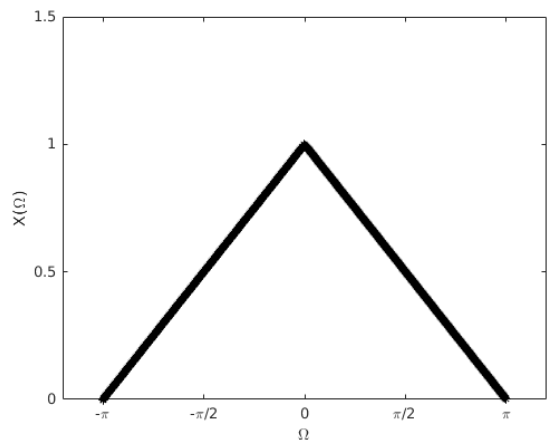
(b) Is the system time-invariant? Prove your answer.

(c) Is the system causal? Prove your answer.

(d) Is the system BIBO stable? Prove your answer. (*Hint: You may need to use triangle inequality:* $|x + y| \leq |x| + |y|$)

Problem 6

Consider a signal $x[n]$ that has a DTFT depicted in the figure below in the range $[-\pi, \pi]$.



Find the expression for the DTFT of the signals below:

(a) $x_1[n] = nx[n-1]$

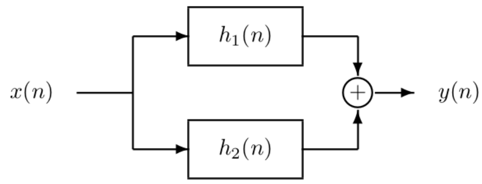
(b) $x_2[n] = e^{j\frac{\pi n}{2}}(x[n] * x[n])$

(b) $x_o[n]$, the odd part of $x[n]$

Problem 7

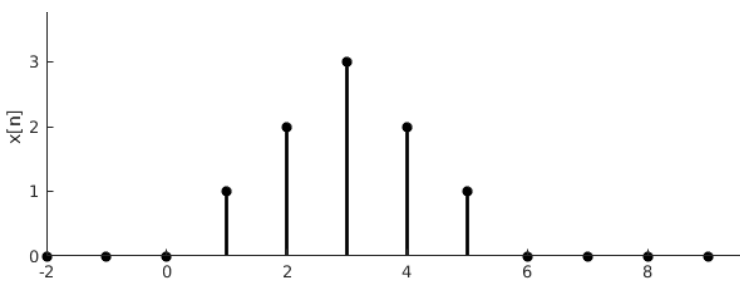
Consider the system composed of parallel connection of two LTI systems.

(a) If unit-step response of the equivalent system (the response when the input is a unit-step function) is $y[n] = r[n+1] - r[n-1]$ and $h_1[n] = u[n] - 2u[n-1] + u[n-2]$, find and sketch $h_2[n]$.



(b) Find the equivalent impulse response of the system $h_{eq}[n]$. The equivalent response is defined by the following relation: $y[n] = x[n] * h_{eq}[n]$.

(c) Find the response of the system $y[n]$ for $x[n]$ shown in the figure below.



Problem 8

Let $\tilde{x}[n]$ be a periodic signal with period N. Its DTFS representation is given by

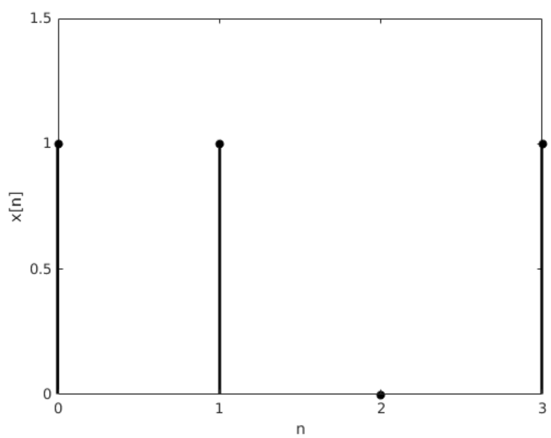
$$\tilde{x}[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn},$$

where $\tilde{c}_k$ are the DTFS coefficients.

Show that if $\tilde{x}[n]$ is a complex signal and conjugate symmetric ($\tilde{x}^*[n] = \tilde{x}[-n]$), then $\mathrm{Im}\{\tilde{c}_k\} = 0$.

Problem 9

Consider a periodic signal $\tilde{x}[n]$ signal with one if its periods shown in the figure below. Calculate its DTFS coefficients $\tilde{c}_k$.


1.5
1
0.5
0
x[n]
0
1
2
3
n