ECE 131A

Axioms: ① $P(A)>0$ ② $P(S)=1$ ③ $A\cap B=\phi \rightarrow P(A\cup B)=P(A)+P(B) \rightarrow P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(\bigcup_K A_K)=\sum_K P(A)$

Choose $\rightarrow {}_nC_K$ $\binom{n}{k}=\frac{n!}{k!(n-k)!}$

① With Replacement & Ordering: Choosing K objects from n choices. $S=n^K$

② W/O Replacement & with ordering: Choosing K objects from n choices. $S=\binom{n}{k}\cdot K!$

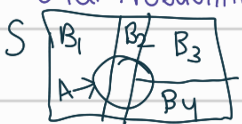
③ W/O Replacement & W/O ordering: Choosing K objects from n choices. $S=\binom{n}{k}$

④ With Replacement & W/O ordering: Choosing . . . . : Figure it out

MCQ : C/K in MCQ w n choices:

Exactly: $\frac{(n-1)^{K-C}}{n^K}\binom{K}{C}$ At least: $\sum_{i=C}^{K}\frac{(n-1)^{K-i}}{n^K}\binom{K}{i}$

Total Probability law



$P(A)=P(A|B_1)+P(A|B_2)+P(A|B_3)+P(A|B_4)$

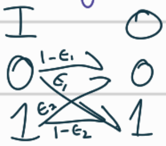
A given B

$P(A|B)=\frac{P(A\cap B)}{P(B)}$

Bayes Rule:

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B\cap A)}{P(A)P(B)}\cdot P(A)=\frac{P(B|A)\cdot P(A)}{P(B)}$$

Binary Comms:



$\rightarrow P(I_0|O_0)=1-\epsilon_1$

Independance: ↑something

$A \perp\!\!\!\perp B$ if $P(A\cap B)=P(A)P(B)$

① Discrete: RV

$E[g(X)]=\sum_{S_X} g(x_i)P(X=x_i)$, E is Linear!

$Var(x)=E[(x-m_x)^2]$ $m_x=E[x]$

ⓐ **Uniform**: $PMF=\begin{cases}\frac{1}{X_E-X_S+1} & X_S<x<X_E\\ 0, & o.w\end{cases}$

$E[X]=\frac{X_E+X_S}{2}$ $Var(x)=E[X^2]-(E[X])^2=\frac{(X_E-X_S+1)^2-1}{12}$

ⓑ **Bernoulli**: 2 outcomes! $PMF=\begin{cases}p, & 1\\ 1-p, & 0\end{cases}$

$E[X]=p$, $Var(x)=p(1-p)$

ⓒ **Geometric**: # trials till first success $PMF=(1-p)^{k-1}p$

$E[X]=p^{-1}$, $Var(x)=\frac{1-p}{p^2}$

ⓓ **Binomial**, $Y=\sum_n X_K$ (Bernoulli), $PMF=\sum_{K=0}^{n}\binom{n}{k}(1-p)^{n-K}p^K$

$E[X]=np$, $Var(x)=np(1-p)$

ⓔ **Fish (Poisson)**: Events/time $PMF=\frac{\lambda^K e^{-\lambda}}{K!}$ λ: Avg rate

$E[X]=\lambda=Var(\lambda)$

② Continuous

CDF: F_x PDF: f_x $f_x=F_x'$, $F_x=\int_{-\infty}^{x}f_x$

$E[g(x)]=\int_{-\infty}^{\infty}g(x)f_x(x)dx$

$Var(x)=E[X^2]-(E[X])^2$

ⓐ **Uniform**: $PDF=\begin{cases}L, & a\le x\le b\\ 0, & ow\end{cases}$ $L=\frac{1}{b-a}$ $CDF=\begin{cases}\frac{x-a}{b-a}, & a\le x\le b\\ 0, & ow\end{cases}$

$E[X]=\frac{a+b}{2}$, $Var(x)=\left(\frac{1}{b-a}\right)\left(\frac{b^3-a^3}{3}\right)$

ⓑ **Exponential**: $PDF=\begin{cases}\lambda e^{-\lambda x}, & x\ge 0\\ 0, & ow\end{cases}$ $CDF=\begin{cases}1-e^{-\lambda x}, & x\ge 0\\ 0, & ow\end{cases}$

$E[X]=1/\lambda$, $Var(x)=1/\lambda^2$

ⓒ **Normal**: (urmom) $PDF=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}=\mathcal{G}(\mu,\sigma^2)$ $CDF=\int_{-\infty}^{x}f_x(x)dx$

$E[X]=\mu$, $Var(x)=\sigma^2$, Def: $Q(x)=P_{\mathcal{G}(0,1)}(x)$, $\Phi=F_x$

ⓓ χ^2: $Y=\sum_K x_i^2$ (x_i is $\mathcal{G}(0,1)$) $f_Y(y)=\frac{y^{\frac{K-2}{2}}e^{-y/2}}{2^{K/2}\Gamma(K/2)}$ $\Gamma(t)=\int_0^{\infty}x^{t-1}e^{-x}dx$

$$\boxed{e^x=\sum_{K=0}^{\infty}\frac{x^K}{K!}}$$

$$\frac{1}{2\pi}\iint_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2}}\,dx\,dy$$

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2+y^2$
$\theta = \arctan y/x$
$dx\,dy = r\,dr\,d\theta$

Stuff

Independance: 2 RV: $\perp\!\!\!\perp$ $P(\{X\in A\}\cap\{X\in B\}) = P(X\in A)P(X\in B)$

CLT: The average behavior of dist RVs converges to $\mathcal{N}(0,1)$, $F_Y\left(\frac{a-\mu}{\sigma}\right)$

Transform Methods: $E[x^K]$ for $K>1$, find $P^{M}_{D}F$ of indep RVs.

$$\Phi_X(\omega) = E[e^{j\omega x}] = \int_{-\infty}^{\infty} e^{j\omega x} f_X(x)\,dx, \quad \text{Exp} \to \frac{\lambda}{\lambda - j\omega}$$

$E[x^n] \to$ nth Moment of x $\leftarrow$ $\Phi_X(\omega) = \sum_{n=0}^{\infty} \frac{(j\omega)^n}{n!} E[x^n]$

$$\left.\frac{d\Phi_X(\omega)}{d\omega}\right|_{\omega=0} = jE[X]$$

$$E[x^n] = \frac{1}{j^n}\frac{d^n \Phi_X(\omega)}{d\omega^n}$$

$$\left.\frac{d^2\Phi_X(\omega)}{d\omega^2}\right|_{\omega=0} = -E[X^2]$$

For $Y = \sum X_i$ RV-Indep, $\prod_i [E\, e^{j\omega X_i}]$

$$\Phi_Y(\omega) = E[e^{j\omega(\sum X_i)}] = E[\prod_i e^{j\omega X_i}]$$

If iid: $\Phi_Y(\omega) = [\Phi_{X_i}(\omega)]^n$

$Z = X+Y$

$$\Phi_Z = E(e^{j\omega z}) = E(e^{j\omega(x+y)})$$
$$= (E(e^{j\omega x}))(E(e^{j\omega y}))$$
$$= \Phi_X(\omega)\Phi_Y(\omega)$$

$$f_Z = f_X \overset{\text{conv.}}{*} f_Y$$

$$PGF = G_X(z)\big|_{z=e^{j\omega}} = \Phi_X(\omega)$$

For normal,

$$\mathcal{N}(\mu,\sigma^2) \to F_X(a) = \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$$Q = 1 - \Phi$$

$$\Phi = CDF = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$$