ECE 102 Homework 7

SANJIT SARDA

TOTAL POINTS

100 / 100

QUESTION 1

1 14 pts

1.1 1a **4 / 4**

✓ **- 0 pts** Correct

- 4 pts Missing

- 2 pts Incorrect, should be 4Hz

1.2 1b **10 / 10**

✓ **- 0 pts** Correct

- 10 pts Missing

- 4 pts Minimum Fs is 1 Hz

- 2 pts Should use a band pass filter

QUESTION 2

2 20 pts

2.1 2a **4 / 4**

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- 4 pts Missing/ Incorrect

2.2 2b **4 / 4**

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- 4 pts Missing/ Incorrect

2.3 2c **4 / 4**

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- 4 pts Missing/ Incorrect

2.4 2d **4 / 4**

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- 4 pts Missing/ Incorrect

2.5 2e **4 / 4**

✓ **- 0 pts** Correct

- 4 pts Missing/ Incorrect

QUESTION 3

3 18 pts

3.1 3a **6 / 6**

✓ **- 0 pts** Correct

- 2 pts Incorrect/missing Xp(w) graph

- 2 pts Incorrect/missing Y(w) graph

- 6 pts Missing

3.2 3b **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Incorrect system (need to multiply by cos before lpf)

- 4 pts Missing

3.3 3c **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Incorrect system (need to multiply by cos before lpf)

- 4 pts Missing

3.4 3d **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Incorrect, should be pi/wm

- 4 pts Missing

QUESTION 4

4 20 pts

4.1 4a **10 / 10**

✓ **- 0 pts** Correct

- 3 pts Incorrect/missing part I

- **3 pts** Incorrect/missing part ii
- **10 pts** Missing

4.2 4b **10 / 10**

✓ **- 0 pts** **Correct**
- **3 pts** Incorrect part I
- **3 pts** Incorrect part ii
- **10 pts** Missing

QUESTION 5

5 12 pts

5.1 5a **6 / 6**

✓ **- 0 pts** **Correct**

5.2 5b **6 / 6**

✓ **- 0 pts** **Correct**
- **3 pts** Incorrect
- **6 pts** Missing

QUESTION 6

LTI System 16 pts

6.1 6a **5 / 5**

✓ **- 0 pts** **Correct**
- **2 pts** Partial Correct
- **5 pts** Missing/ Incorrect

6.2 6b **5 / 5**

✓ **- 0 pts** **Correct**
- **1 pts** Partial Correct
- **5 pts** Missing/ Incorrect

6.3 6c **6 / 6**

✓ **- 0 pts** **Correct**
- **6 pts** Missing Incorrect

102 HW7

1) Bandpass Sampling

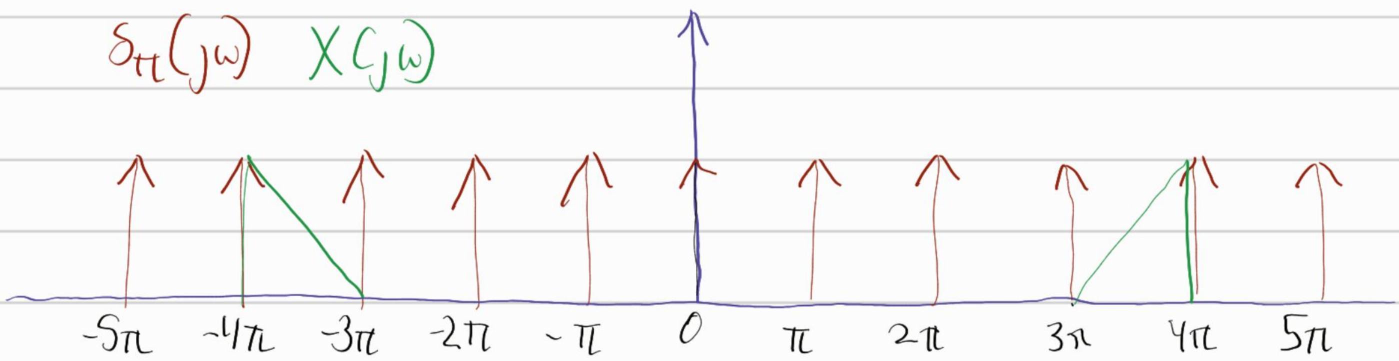
a) $F_s = 2F_{max} = 2 \cdot \frac{\omega_{max}}{2\pi} = 2 \cdot \frac{4\pi}{2\pi} = 4Hz$

$\omega_s = 2\pi \cdot F_s = 8\pi$ rads/s

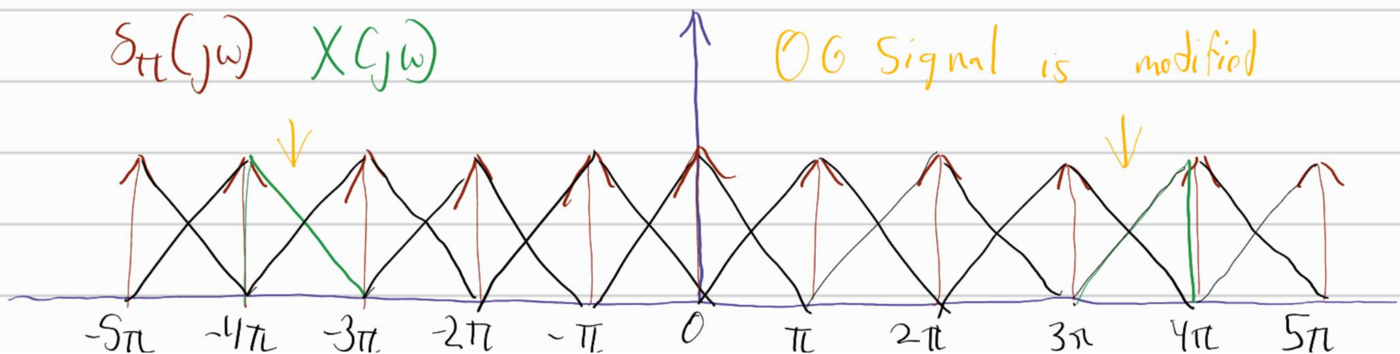
b) Options: 0.5Hz & 1Hz

For 0.5Hz,
$T_s = 1/0.5 = 2s$
$\omega_s = 2\pi \cdot 0.5 = \pi$ rad/s
Using an Impulse train with $T_s = 2$: $\delta_2(t)$

$$x(t) \cdot \delta_2(t) = \frac{1}{2\pi} X(j\omega) * \pi \delta_\pi(\omega)$$
$$= \frac{1}{2} X(j\omega) * \delta_\pi(\omega)$$



Upon Convolution:



1.1 1a **4 / 4**

- ✓ **- 0 pts** Correct
- **- 4 pts** Missing
- **- 2 pts** Incorrect, should be 4Hz

102 HW7

1) Bandpass Sampling

a) $F_s = 2F_{max} = 2 \cdot \frac{\omega_{max}}{2\pi} = 2 \cdot \frac{4\pi}{2\pi} = 4Hz$

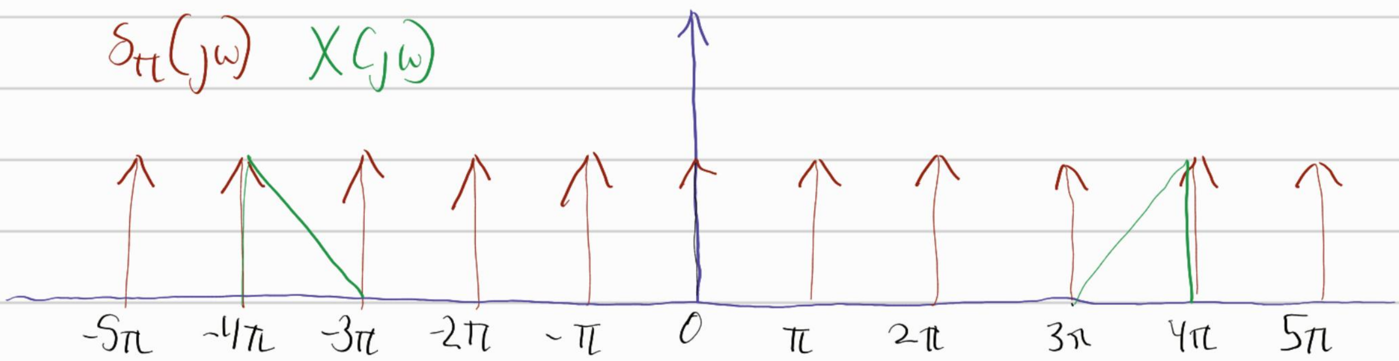
$\omega_s = 2\pi \cdot F_s = 8\pi$ rads/s

b) Options: 0.5Hz & 1Hz

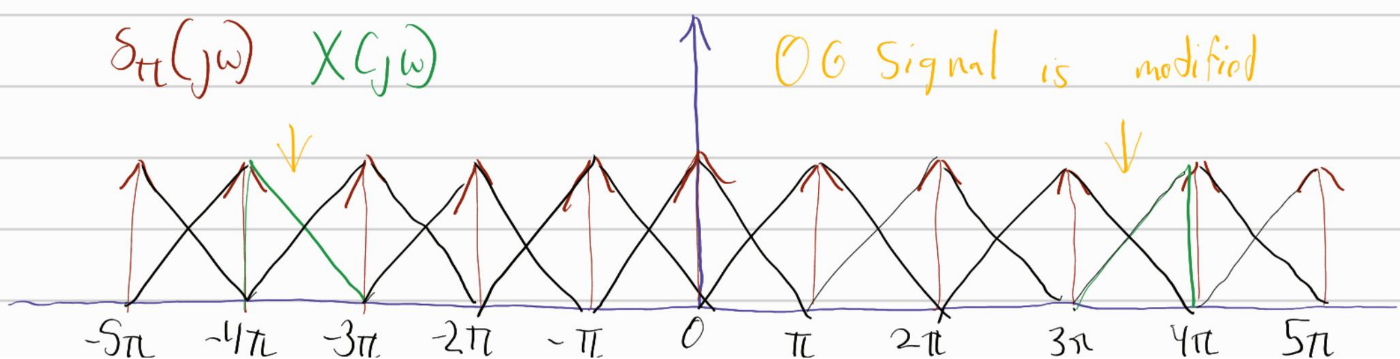
For 0.5Hz,
$T_s = 1/0.5 = 2s$
$\omega_s = 2\pi \cdot 0.5 = \pi$ rad/s
Using an Impulse train with $T_s = 2$: $\delta_2(t)$

$$x(t) \cdot \delta_2(t) = \frac{1}{2\pi} X(j\omega) * \pi \delta_\pi(\omega)$$

$$= \frac{1}{2} X(j\omega) * \delta_\pi(\omega)$$



Upon Convolution:



1.2 1b 10 / 10

- ✓ **- 0 pts** Correct
- **- 10 pts** Missing
- **- 4 pts** Minimum Fs is 1 Hz
- **- 2 pts** Should use a band pass filter

For 1 Hz,

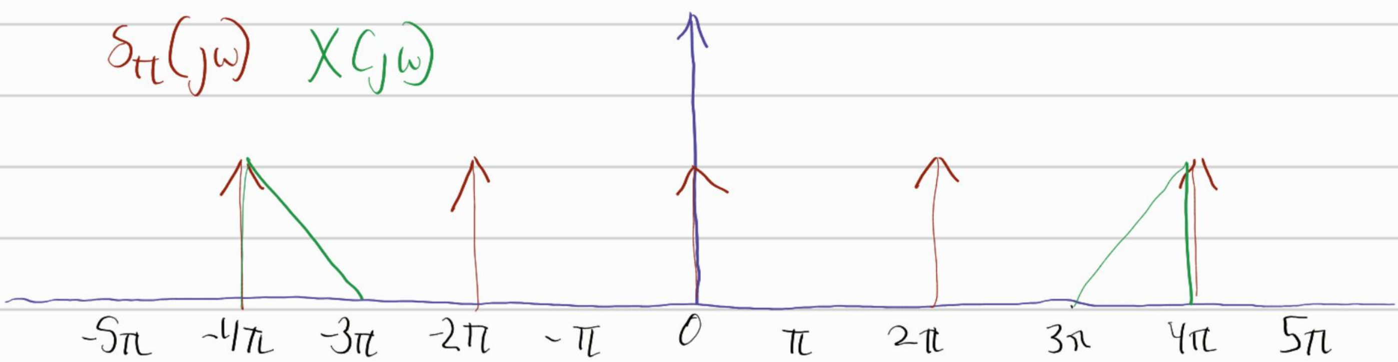
$T_s = 1/1 = 1s$

$\omega_s = 2\pi \cdot 1 = 2\pi$ rad/s

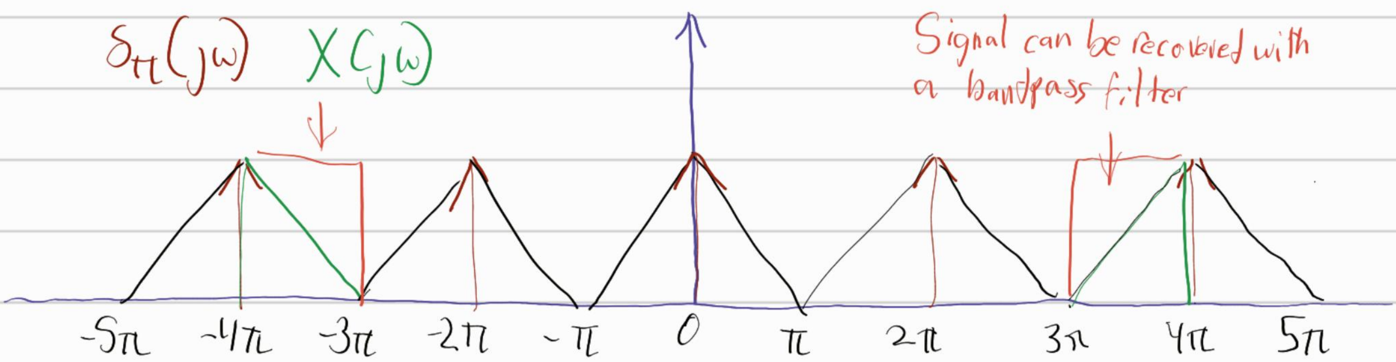
Using an Impulsetrain with $T_s = 1$: $\delta_1(t)$ & $\delta_{2\pi}(\omega)$

$$x(t) \cdot \delta_2(t) = \frac{1}{2\pi} X(j\omega) * 2\pi\, \delta_{2\pi}(\omega)$$

$$= X(j\omega) * \delta_{2\pi}(\omega)$$



Upon Convolution:



$\therefore$ 1 Hz is the minimum to recover

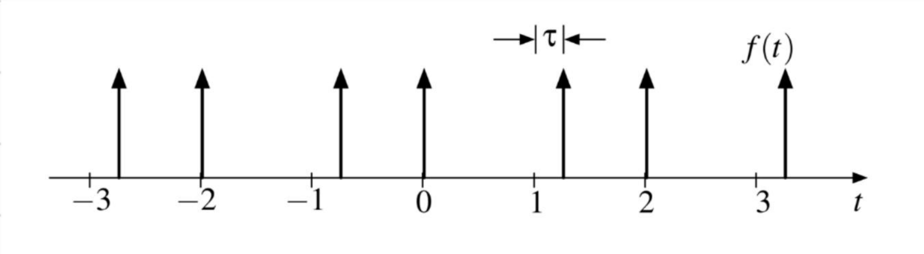
2.1 2a 4 / 4

✓ **- 0 pts** Correct

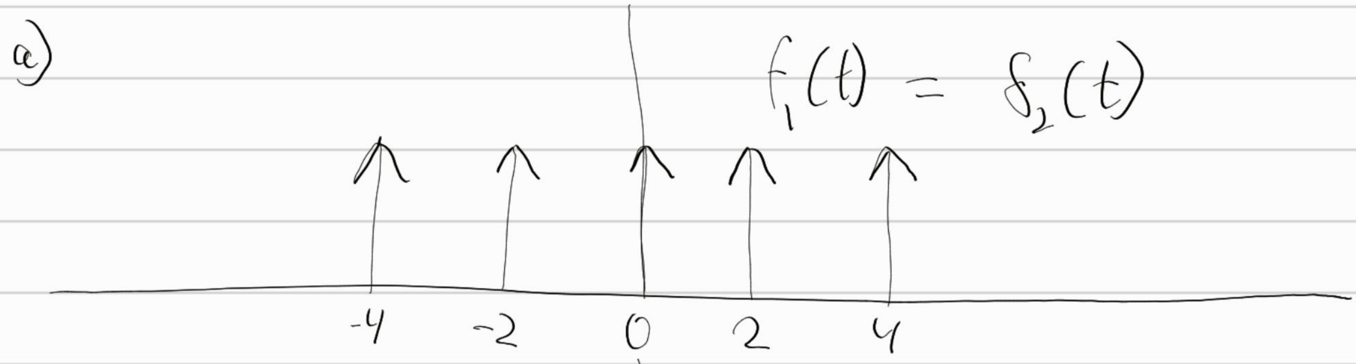
- 1 pts Partial Correct

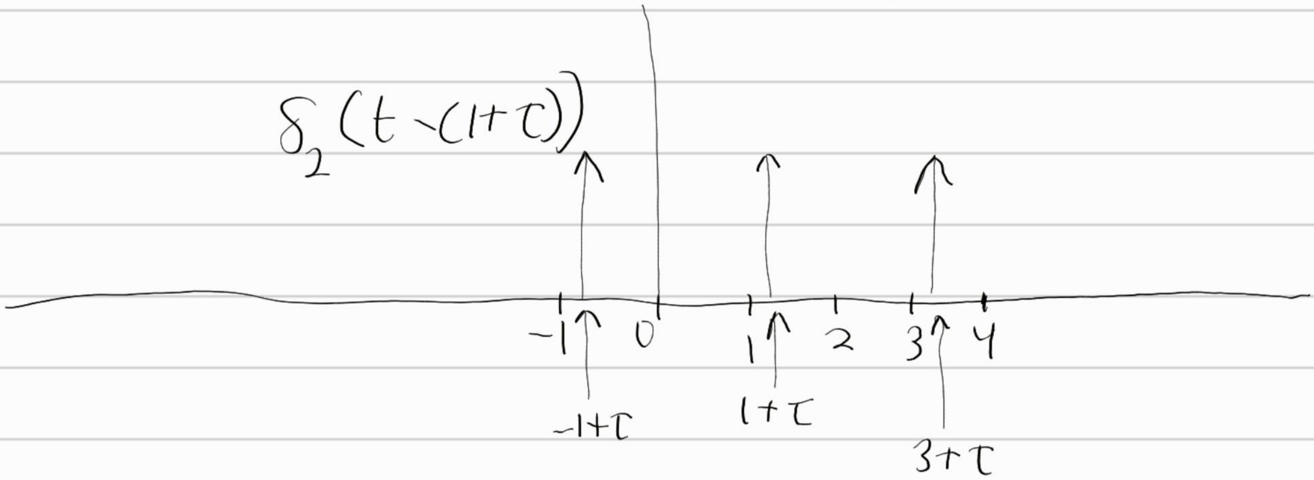
- 4 pts Missing/ Incorrect

2)



a)





$$f(t) = f_1(t) + f_2(t) = \delta_2(t) = \delta_2(t-1-\tau)$$

b) $F(j\omega) = F_1(j\omega) + F_2(j\omega) = \pi\delta_\pi(\omega) + \pi\delta_\pi(\omega)e^{-j\omega(1+\tau)}$

$$F(j\omega) = \pi\left(1+e^{-j\omega(1+\tau)}\right)\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)$$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k) + \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)e^{-jk\pi}e^{-jk\pi\tau}$$

c) Find $F(j\omega)$ for $\tau = 0$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k) + \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)e^{-jk\pi}$$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)\left(1+e^{-jk\pi}\right) = \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)\left(1+(-1)^k\right)$$

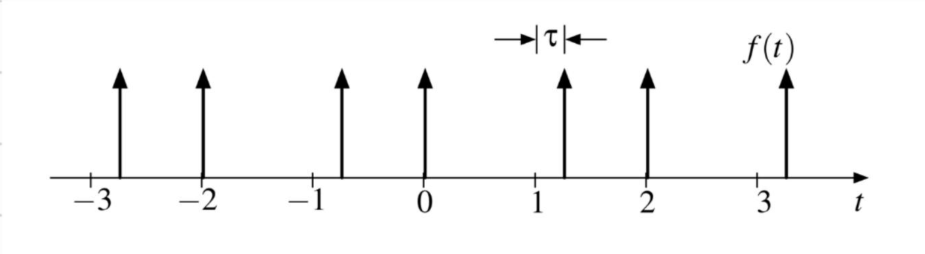
2.2 2b **4 / 4**

✓ **- 0 pts** Correct

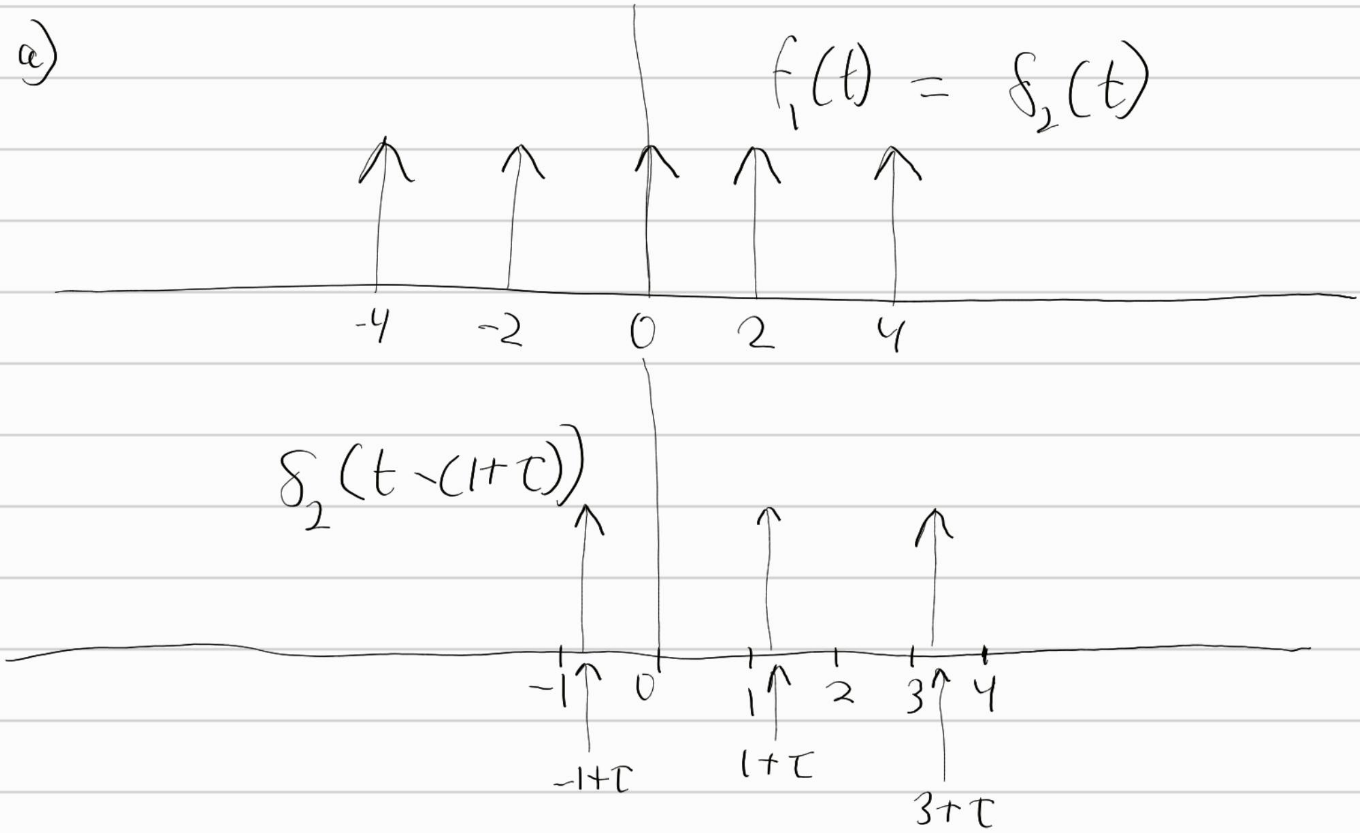
- 1 pts Partial Correct

- 4 pts Missing/ Incorrect

2)



a)



$$f(t) = f_1(t) + f_2(t) = \delta_2(t) = \delta_2(t-1-\tau)$$

b) $$F(j\omega) = F_1(j\omega) + F_2(j\omega) = \pi\delta_\pi(\omega) + \pi\delta_\pi(\omega)e^{-j\omega(1+\tau)}$$

$$F(j\omega) = \pi\left(1+e^{-j\omega(1+\tau)}\right)\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)$$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k) + \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)e^{-jk\pi}e^{-jk\pi\tau}$$

c) Find $F(j\omega)$ for $\tau = 0$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k) + \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)e^{-jk\pi}$$

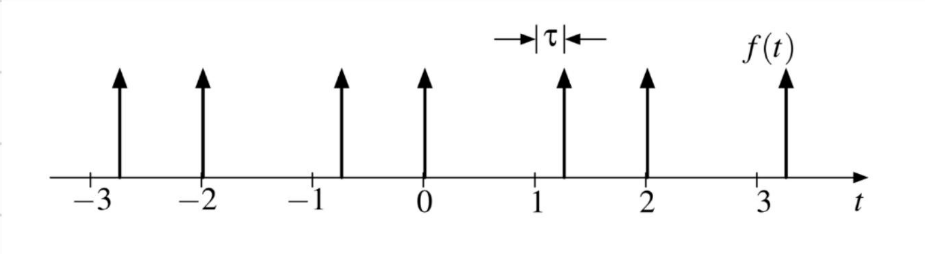
$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)\left(1+e^{-jk\pi}\right) = \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)\left(1+(-1)^k\right)$$

2.3 2c 4 / 4

✓ - 0 pts Correct

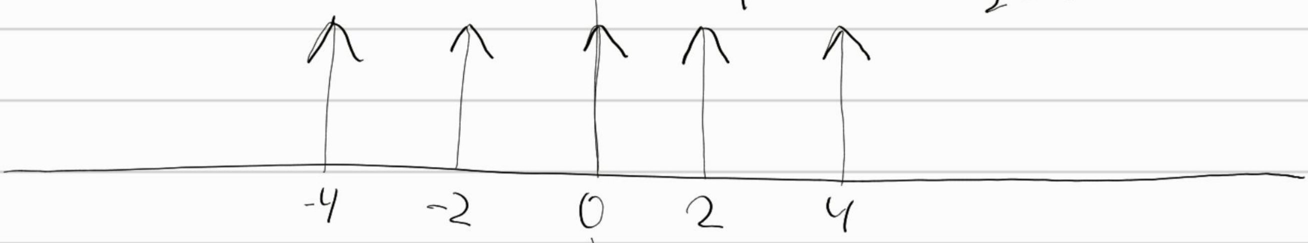
- 1 pts Partial Correct
- 4 pts Missing/ Incorrect

2)

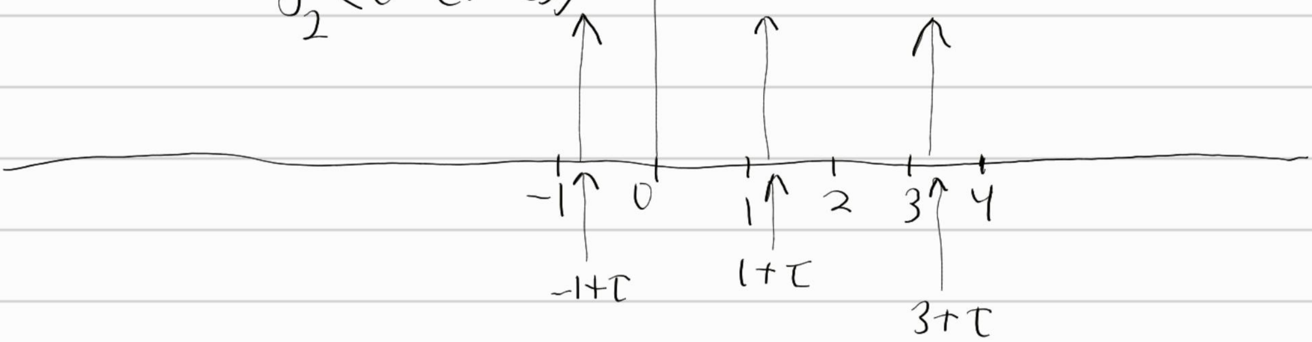


a)

$f_1(t) = \delta_2(t)$



$\delta_2(t-(1+\tau))$



$$f(t) = f_1(t) + f_2(t) = \delta_2(t) = \delta_2(t-1-\tau)$$

b) $F(j\omega) = F_1(j\omega) + F_2(j\omega) = \pi\delta_\pi(\omega) + \pi\delta_\pi(\omega)e^{-j\omega(1+\tau)}$

$$F(j\omega) = \pi\left(1 + e^{-j\omega(1+\tau)}\right)\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)$$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k) + \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)e^{-jk\pi}e^{-jk\pi\tau}$$

c) Find $F(j\omega)$ for $\tau = 0$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k) + \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)e^{-jk\pi}$$

$$= \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)\left(1+e^{-jk\pi}\right) = \pi\sum_{k=-\infty}^{\infty}\delta(\omega-\pi k)\left(1+(-1)^k\right)$$

$n = 2k$

$$= \pi \sum_{n=-\infty}^{\infty} \delta(\omega - \pi n)\cdot\left(1+(-1)^{n-1}\right) + \delta(\omega-\pi n)\left(1+(-1)^{n}\right)$$

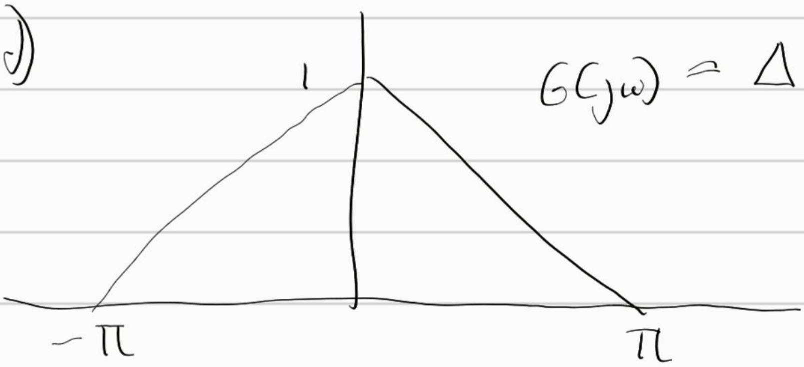
$$= \pi \sum_{n=-\infty}^{\infty} \delta(\omega - \pi n)\cdot 0 + \delta(\omega-\pi n)\cdot 2$$

$n: -2, 0, 2 \ldots\ldots$ so to make it make sense

$$= 2\pi \sum_{n=-\infty}^{\infty} (\omega - \pi n) = 2\pi \sum_{k=-\infty}^{\infty} (\omega - 2\pi k) = 2\pi\, \delta_{2\pi}(\omega)$$

expected because $\tau = 0$ ∴ It samples every unit: $\delta_1(t)$ is the IFT

c)



$$G(j\omega) = \Delta(\omega/\pi)$$

Sample at $\omega = \pm\pi$ & $e^{j\omega\tau} = 1 + j\omega\tau$

$G(j\omega) = F(g(t))$

$f(t)$: sampler =

∴ $g_s(t) = g(t)f(t)$ ∴ $G_s(j\omega) = \frac{1}{2\pi} G(j\omega) F(j\omega)$

g_s → gsampled

$$= \frac{1}{2\pi} \Delta\left(\tfrac{1}{\pi}\omega\right) * \pi \sum_{n=-\infty} \delta(\omega - n\pi)\left(1 + (-1)^n e^{-jn\pi\tau}\right)$$

Computing @ $n = -1, 0, 1$:

$n = -1$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega+\pi)\left(1 - e^{j\pi\tau}\right) = \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega+\pi)\left(1-(1+j\pi\tau)\right)$$

$$= \tfrac{1}{2}\cdot\Delta(\omega/\pi) * \delta(\omega+\pi)(-j\pi\tau) = \frac{1}{2}(-j\pi\tau)\,\Delta\left(\frac{\omega+\pi}{\pi}\right)$$

$n = 0$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega)(1+1) = \tfrac{1}{\cancel{2}}\Delta(\omega/\pi) * \delta(\omega)\cancel{2} = \Delta(\omega/\pi)$$

$n = -1$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega-\pi)\left(1 - e^{-j\pi\tau}\right) = \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega-\pi)\left(1-(1-j\pi\tau)\right)$$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega-\pi)(j\pi t) = \tfrac{1}{2}(j\pi\tau)\,\Delta\left(\frac{\omega-\pi}{\pi}\right)$$

2.4 2d **4 / 4**

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- 4 pts Missing/ Incorrect

$n=2k$

$$= \pi \sum_{n=-\infty}^{\infty} \delta(\omega - \pi n)\cdot(1+(-1)^{n-1}) + \delta(\omega - \pi n)(1+(-1)^{n})$$

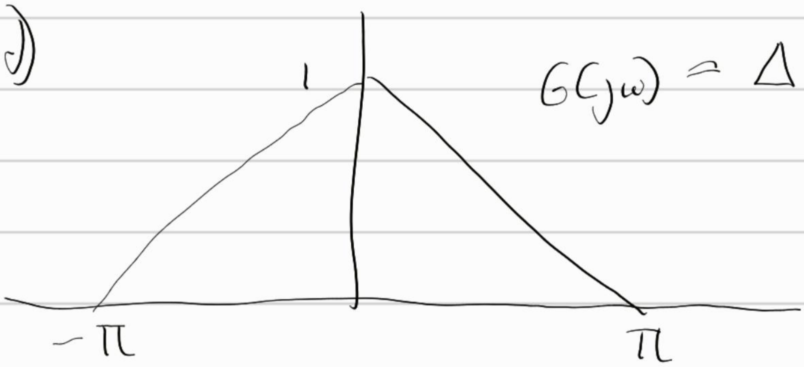
$$= \pi \sum_{n=-\infty}^{\infty} \delta(\omega - \pi n)\cdot 0 + \delta(\omega-\pi n)\cdot 2$$

$n: -2, 0, 2 \ldots\ldots$ so to make it make sense

$$= 2\pi \sum_{n=-\infty}^{\infty} (\omega - \pi n) = 2\pi \sum_{k=-\infty}^{\infty} (\omega - 2\pi k) = 2\pi\, \delta_{2\pi}(\omega)$$

expected because $\tau = 0$ $\therefore$ It samples every unit: $\delta_1(t)$ is the IFT

d)



$$G(j\omega) = \Delta(\omega/\pi)$$

Sample at $\omega = \pm\pi$ & $e^{j\omega\tau} = 1 + j\omega\tau$

$G(j\omega) = F(g(t))$

$f(t)$: sampler $=$

$\therefore g_s(t) = g(t) f(t) \quad \therefore G_s(j\omega) = \frac{1}{2\pi} G(j\omega) F(j\omega)$

g_s ↓ gsampled

$$= \frac{1}{2\pi} \Delta\left(\tfrac{1}{\pi}\omega\right) * \pi \sum_{n=-\infty} \delta(\omega - n\pi)(1+(-1)^n e^{-jn\pi\tau})$$

Computing @ $n = -1, 0, 1$:

$n=-1$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega+\pi)(1 - e^{j\pi\tau}) = \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega+\pi)(1-(1+j\pi\tau))$$

$$= \tfrac{1}{2}\cdot\Delta(\omega/\pi) * \delta(\omega+\pi)(-j\pi\tau) = \frac{1}{2}(-j\pi\tau)\,\Delta\left(\frac{\omega+\pi}{\pi}\right)$$

$n=0$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega)(1+1) = \tfrac{1}{\cancel{2}}\Delta(\omega/\pi) * \delta(\omega)\cancel{2} = \Delta(\omega/\pi)$$

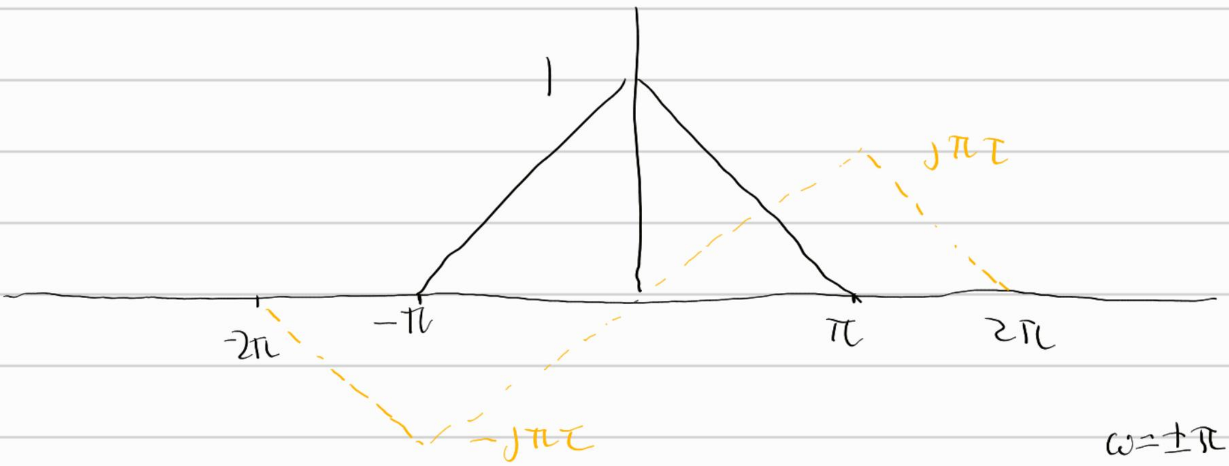
$n=-1$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega-\pi)(1-e^{-j\pi\tau}) = \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega-\pi)(1-(1-j\pi\tau))$$

$$= \tfrac{1}{2}\Delta(\omega/\pi) * \delta(\omega-\pi)(j\pi t) = \tfrac{1}{2}(j\pi\tau)\,\Delta\left(\frac{\omega-\pi}{\pi}\right)$$

$$G_s(\omega) = \Delta(\omega/\pi) + \frac{1}{2}\left(j\pi\tau\,\Delta\left(\frac{\omega-\pi}{\pi}\right) - j\pi\tau\,\Delta\left(\frac{\omega-\pi}{\pi}\right)\right)$$

$\therefore$ The sampled signal is:



$\omega = \pm\pi$
$\uparrow$

e) Knowing $g(t)$ is real and even we can use a low pass filter, it should be able to extract the OG function.

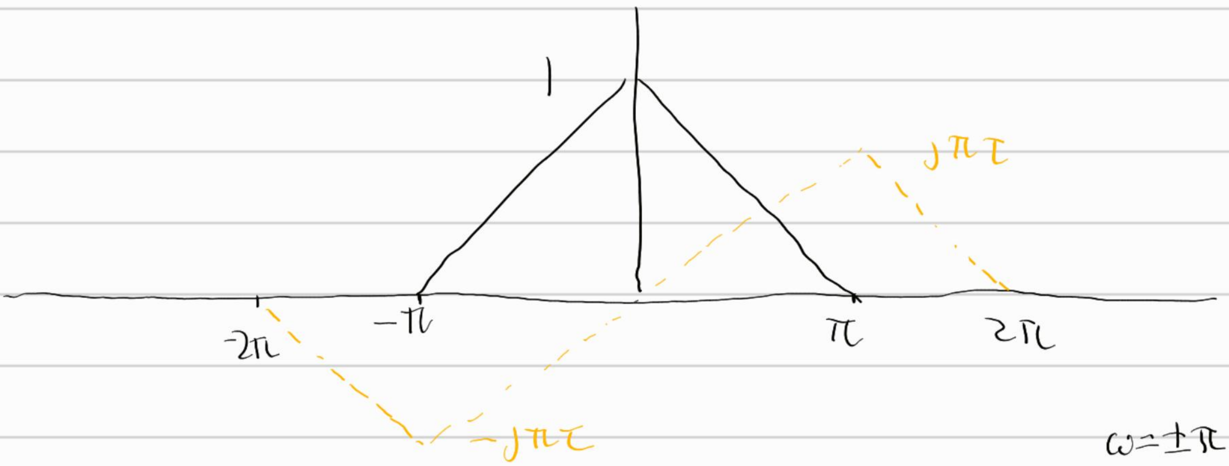
2.5 2e 4 / 4

✓ - 0 pts Correct

- 4 pts Missing/ Incorrect

$$G_s(\omega) = \Delta(\omega/\pi) + \frac{1}{2}\left(j\pi\tau\,\Delta\left(\frac{\omega-\pi}{\pi}\right) - j\pi\tau\,\Delta\left(\frac{\omega-\pi}{\pi}\right)\right)$$

∴ The sampled signal is:



$\omega = \pm\pi$

e) Knowing $g(t)$ is real and even we can use a low pass filter, it should be able to extract the OG function.

3.1 3a **6 / 6**

✓ **- 0 pts** Correct

- 2 pts Incorrect/missing Xp(w) graph

- 2 pts Incorrect/missing Y(w) graph

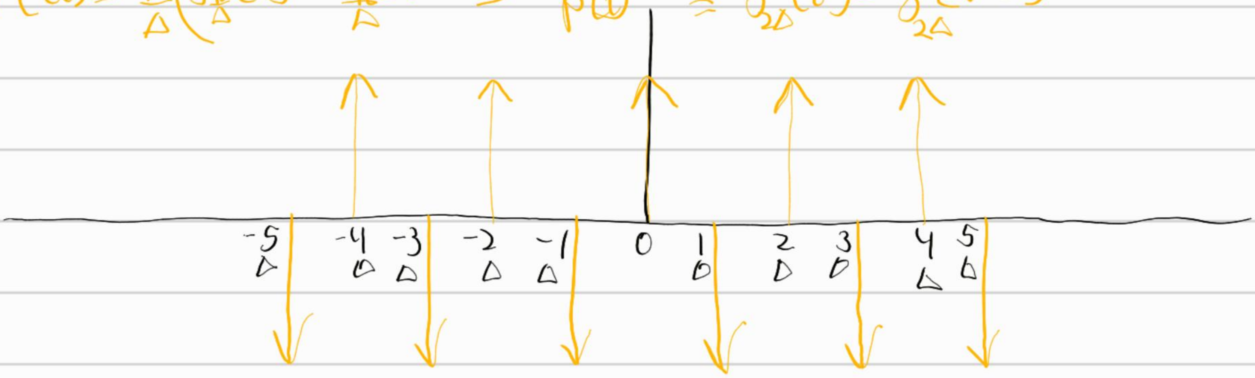
- 6 pts Missing

3) Sampling with alternating impulse train

$$x(t) \longrightarrow \underset{\uparrow\, p(t)}{\otimes} \xrightarrow{x_p(t)} H(\omega) \longrightarrow y(t)$$

$P(\omega) = \frac{\pi}{\Delta}\left(\delta_{\frac{\pi}{\Delta}}(\omega) - \delta_{\frac{\pi}{\Delta}}(\omega)e^{-j\omega\Delta}\right)$

$p(t) = \delta_{2\Delta}(t) - \delta_{2\Delta}(t-\Delta)$

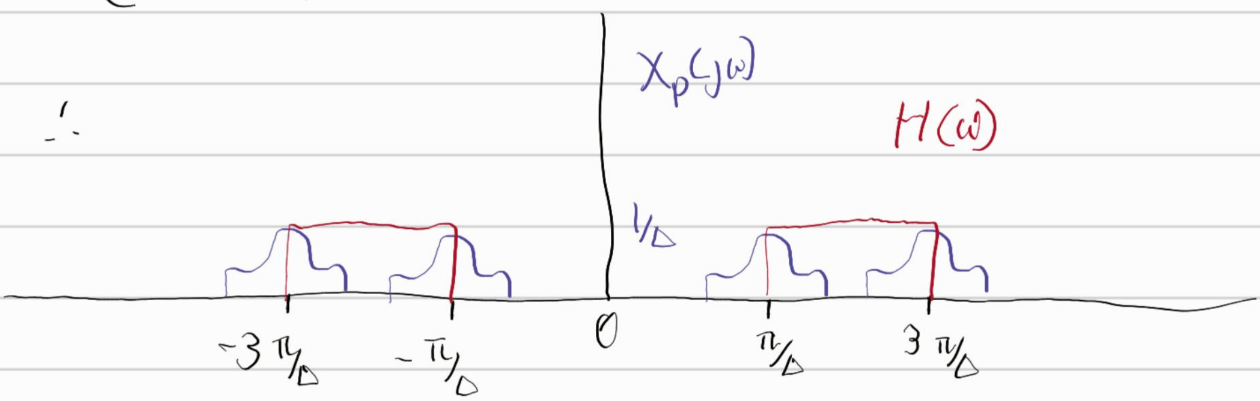


$$x_p(t) = x(t)\,p(t)$$

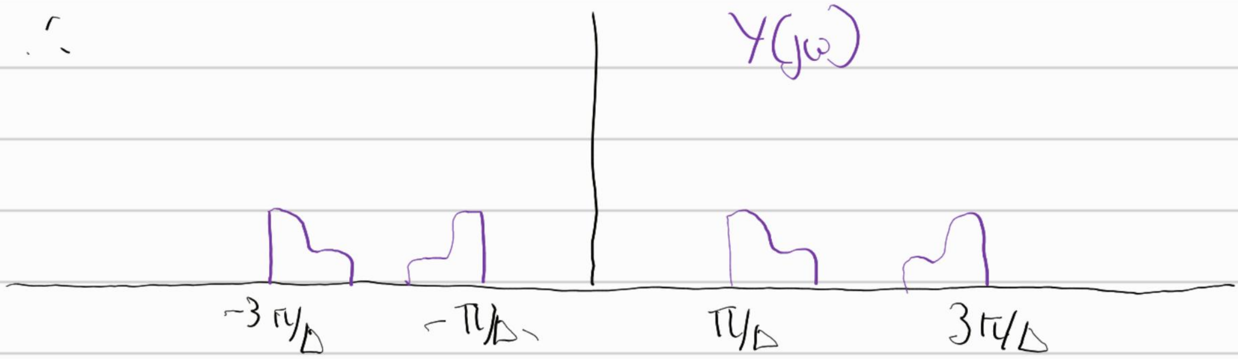
$$X_p(j\omega) = X(j\omega) * P(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * \frac{\pi}{\Delta}\left(\delta_{\pi/\Delta}(\omega) + \delta_{\frac{\pi}{\Delta}}(\omega)e^{-j\omega\Delta}\right)\right)$$

$$= \frac{1}{2\Delta}\left(X(j\omega) * \left(\sum_{k=-\infty}^{\infty} \delta(\omega - k\pi/\Delta) - \delta(\omega - k\pi/\Delta)e^{-j\omega\Delta}\right)\right) = \frac{1}{\Delta}X(j\omega) * \sum_{k=\text{odd}} \delta(\omega - k\pi/\Delta)$$

∴



∴



3.2 3b 4 / 4

✓ **- 0 pts** Correct

- 2 pts Incorrect system (need to multiply by cos before lpf)

- 4 pts Missing

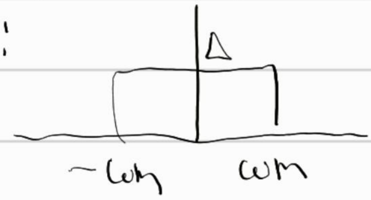
b) We should de cosinify the signal & then use a low pass filter to filter out the extra copies.

$$x_p(t) \rightarrow \underset{\cos(\pi/\Delta\, t)}{\otimes} \longrightarrow \boxed{h_{LP}(t)} \rightarrow x(t)$$

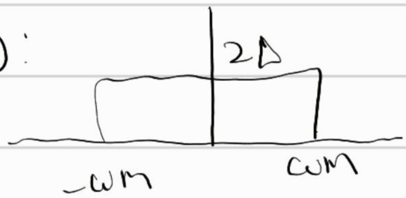
c) The same of systems can be applied to recover the original signal.

$$y(t) \rightarrow \underset{\cos(\pi/\Delta\, t)}{\otimes} \longrightarrow \boxed{h_{LP}(t)} \rightarrow x(t)$$

b&c) The LP Filters from b & c both need to be scaled appropriately. b): $H_{LP}(j\omega)$:



c): $H_{LP}(j\omega)$:



d) Non aliasing Condition:

To prevent overlap, $\frac{3\pi}{\Delta} - \omega m < \frac{\pi}{\Delta} + \omega m$

$\therefore \frac{2\pi}{\Delta} < 2\omega m$

$\therefore \frac{\pi}{\Delta} < \omega m$

$\therefore \Delta > \pi/\omega m$

$\therefore \Delta_{max} = \frac{\pi}{\omega m}$

3.3 3c **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Incorrect system (need to multiply by cos before lpf)

- 4 pts Missing

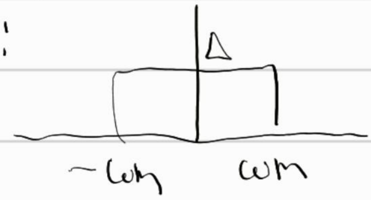
b) We should de cosinify the signal & then use a low pass filter to filter out the extra copies.

$$x_p(t) \rightarrow \otimes \longrightarrow \boxed{h_{LP}(t)} \rightarrow x(t)$$
$$\cos(\pi/_\Delta t)$$

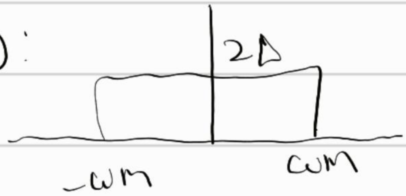
c) The same of systems can be applied to recover the original signal.

$$y(t) \rightarrow \otimes \longrightarrow \boxed{h_{LP}(t)} \rightarrow x(t)$$
$$\cos(\pi/_\Delta t)$$

b&c) The LP Filters from b & c both need to be scaled appropriately. b): $H_{LP}(j\omega)$:



c): $H_{LP}(j\omega)$:



d) Non aliasing Condition:

To prevent overlap, $\frac{3\pi}{\Delta} - \omega m < \frac{\pi}{\Delta} + \omega m$

$\therefore \frac{2\pi}{\Delta} < \cancel{2}\omega m$

$\therefore \frac{\pi}{\Delta} < \omega m$

$\therefore \Delta > \pi/_{\omega m}$

$\therefore \Delta_{max} = \frac{\pi}{\omega m}$

3.4 3d **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Incorrect, should be pi/wm

- 4 pts Missing

4)

a) i) $f(t) = te^{-at}(\sin\omega_0 t)^2 u(t) = \frac{1}{2}\left(te^{-at}u(t) - te^{-at}\cos 2\omega_0 t\right)$

$= \frac{t}{2}\left[\left(e^{-at}u(t) - e^{-at}\cos 2\omega_0 t\right)\right]$

$\therefore L_1(s) = \frac{1}{s+a} - \frac{s+a}{(s+a)^2 + 4\omega_0^2}$

$L_2(s) = -\frac{1}{2}\frac{d L_1(s)}{ds} = \frac{1}{2(s+a)^2} - \frac{s^2+2as-4\omega^2+a^2}{2(s^2+2as+4\omega^2+a^2)^2}$

ROC: $Re\{s\} > -\alpha$

ii) $f(t) = e^{-b|t|}$ where $b \leq 0$.

$\therefore e^{-bt}u(t) \therefore F(s) = \frac{1}{s+b}$ & $Re\{s\} > b$.

b) $X(s) = \frac{1}{s^2+2s+5}$ & $Re\{s\} > -1/2$

i) $x_1(t) = x(t)e^{t/2}$

$\therefore X_1(s) = X(s - 1/2)$

$\therefore X_1(j\omega) = X(j\omega - 1/2) = \frac{1}{(j\omega - 1/2)^2 + 2(j\omega - 1/2) + 5}$

$= \frac{-4}{(2\omega + 4 - j)(2\omega - 4 + j)}$

ii) $y(t) = x(t)e^{2t} \therefore Re\{s\} > 1$

$\downarrow$

$\therefore$ It does not have a $j\omega$ axis

$\therefore$ No fourier transform.

4.1 4a **10 / 10**

✓ **- 0 pts** Correct

- 3 pts Incorrect/missing part I

- 3 pts Incorrect/missing part ii

- 10 pts Missing

4)

a) i) $f(t) = te^{-at}(\sin\omega_0 t)^2 u(t) = \frac{1}{2}\left(te^{-at}u(t) - te^{-at}\cos 2\omega_0 t\right)$

$= \frac{t}{2}\left[\left(e^{-at}u(t) - e^{-at}\cos 2\omega_0 t\right)\right]$

$\therefore L_1(s) = \frac{1}{s+a} - \frac{s+a}{(s+a)^2 + 4\omega_0^2}$

$L_2(s) = -\frac{1}{2}\frac{d\,L_1(s)}{ds} = \frac{1}{2(s+a)^2} - \frac{s^2+2as-4\omega^2+a^2}{2(s^2+2as+4\omega^2+a^2)^2}$

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$= \frac{-4}{(2\omega+4-j)(2\omega-4+j)}$

ii) $y(t) = x(t)e^{2t} \therefore Re\{s\} > 1$

$\downarrow$

$\therefore$ It does not have a $j\omega$ axis

$\therefore$ No fourier transform.

4.2 4b 10 / 10

✓ **- 0 pts** Correct

- 3 pts Incorrect part I

- 3 pts Incorrect part ii

- 10 pts Missing

5) a) $F(s) = \dfrac{e^{-s}(s+1)}{(s-2)^2\ s-3)} = e^{-s} \cdot \left(\dfrac{4}{s-2} + \dfrac{-3}{(s-2)^2} + \dfrac{4}{s-3} \right)$

Inverse laplace $= u(t-1)\left(-4e^{2t-2} - 3(t-1)e^{2t-2} + 4e^{3t-3}\right)$

b) $F(s) = \dfrac{s+4}{s^3+4s} = \dfrac{-s}{s^2+4} + \dfrac{1}{s^2+4} + \dfrac{1}{s}$

$\therefore f(t) = \left(1 + \sin(2t) - \cos(2t)\right)u(t)$

6) a) $y'' + 5y' + 4y = ax(t)$, $y(0) = 0$, $y'(0) = 0$

$\therefore s^2 Y(s) + 5sY(s) + 4Y(s) = aX(s)$

$\dfrac{Y(s)}{X(s)} = \dfrac{a}{s^2 + 5s + 4} = \dfrac{a}{3s+3} - \dfrac{a}{3s+12}$

$x(t) = e^t u(t)$, $y(t) = \dfrac{1}{2} e^t u(t)$

for $e(t)$

$X(s) = \dfrac{1}{s-1}$. $Y(s) = \dfrac{1}{2} \cdot \dfrac{1}{(s-1)}$ $\therefore H_1(s)\ 1/2$

@ $s = 1$

$H_1(s) = \dfrac{1}{2} = \dfrac{a}{10}$ $\therefore a = 5$

$\therefore H_1(s) = \dfrac{5}{(s+1)(s+4)}$

5.1 5a 6 / 6

✓ - 0 pts Correct

5) a) $F(s) = \dfrac{e^{-s}(s+1)}{(s-2)^2 \; s-3)} = e^{-s} \cdot \left(\dfrac{4}{s-2} + \dfrac{-3}{(s-2)^2} + \dfrac{4}{s-3} \right)$

Inverse laplace $= u(t-1)\left(-4e^{2t-2} - 3(t-1)e^{2t-2} + 4e^{3t-3}\right)$

b) $F(s) = \dfrac{s+4}{s^3+4s} = \dfrac{-s}{s^2+4} + \dfrac{1}{s^2+4} + \dfrac{1}{s}$

$\therefore f(t) = \left(1 + \sin(2t) - \cos(2t)\right)u(t)$

6) a) $y'' + 5y' + 4y = ax(t)$, $y(0) = 0$, $y'(0) = 0$

$\therefore s^2 Y(s) + 5sY(s) + 4Y(s) = aX(s)$

$\dfrac{Y(s)}{X(s)} = \dfrac{a}{s^2+5s+4} = \dfrac{a}{3s+3} - \dfrac{a}{3s+12}$

$x(t) = e^t u(t)$, $y(t) = \dfrac{1}{2} e^t u(t)$

for $e(t)$

$X(s) = \dfrac{1}{s-1}$. $Y(s) = \dfrac{1}{2} \cdot \dfrac{1}{(s-1)}$ $\therefore H_1(s)$ 1/2

@ $s=1$

$H_1(s) = \dfrac{1}{2} = \dfrac{a}{10}$ $\therefore a = 5$

$\therefore H_1(s) = \dfrac{5}{(s+1)(s+4)}$

5.2 5b 6 / 6

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect
- **- 6 pts** Missing

5) a) $F(s)=\dfrac{e^{-s}(s+1)}{(s-2)^2(s-3)} = e^{-s}\cdot\left(\dfrac{4}{s-2}+\dfrac{-3}{(s-2)^2}+\dfrac{4}{s-3}\right)$

Inverse laplace $= u(t-1)\left(-4e^{2t-2}-3(t-1)e^{2t-2}+4e^{3t-3}\right)$

b) $F(s)=\dfrac{s+4}{s^3+4s} = \dfrac{-s}{s^2+4}+\dfrac{1}{s^2+4}+\dfrac{1}{s}$

$\therefore f(t)=\left(1+\sin(2t)-\cos(2t)\right)u(t)$

6) a) $y''+5y'+4y=ax(t)$, $y(0)=0$, $y'(0)=0$

$\therefore s^2Y(s)+5sY(s)+4Y(s)=aX(s)$

$\dfrac{Y(s)}{X(s)}=\dfrac{a}{s^2+5s+4}=\dfrac{a}{3s+3}-\dfrac{a}{3s+12}$

$x(t)=e^t u(t)$, $y(t)=\dfrac{1}{2}e^t u(t)$

for $e(t)$

$X(s)=\dfrac{1}{s-1}$. $Y(s)=\dfrac{1}{2}\cdot\dfrac{1}{(s-1)}$ $\therefore H_1(s)$ $1/2$

@ $s=1$

$H_1(s)=\dfrac{1}{2}=\dfrac{a}{10}$ $\therefore a=5$

$\therefore H_1(s)=\dfrac{5}{(s+1)(s+4)}$

6.1 6a **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Partial Correct

- 5 pts Missing/ Incorrect

b) $x(t) = u(t)$

$$\therefore Y(s) = \frac{5}{s(s+1)(s+4)} = \frac{5}{4s} + \frac{-5}{3(s+1)} + \frac{5}{12(s+4)}$$

$$\therefore y(t) = (5/4)u(t) - (5/3)e^{-t}u(t) + (5/12)e^{-4t}u(t)$$

c) $H_2(s) = \frac{Y(s)}{X(s)}$

$$Y(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

$$X(s) = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$H_2(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}\left(\frac{1}{s} - \frac{e^{-s}}{s}\right)^{-1}$$

$$\therefore H_2(s) = \frac{1}{s}(1-e^{-s})$$

$$H(s) = H_1(s)H_2(s) = \left[\frac{3}{s(s+1)(s+4)} - \frac{3e^{-s}}{s(s+1)(s+4)}\right]$$

$$= \left(\frac{3}{4s} + \frac{-1}{s+1} + \frac{1}{4(s+4)}\right)\left(1 - e^{-s}\right)$$

$$\therefore H(s) = u(t)\left(\frac{5}{4} - \frac{5}{3}e^{-t} + \frac{5}{12}e^{-4t}\right) - u(t-1)\left(\frac{5}{4} - \frac{5}{3}e^{-(t-1)} + \frac{5}{12}e^{-4(t-1)}\right)$$

6.2 6b **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** Partial Correct
- **- 5 pts** Missing/ Incorrect

b) $x(t) = u(t)$

$$\therefore Y(s) = \frac{5}{s(s+1)(s+4)} = \frac{5}{4s} + \frac{-5}{3(s+1)} + \frac{5}{12(s+4)}$$

$$\therefore y(t) = (5/4)u(t) - (5/3)e^{-t}u(t) + (5/12)e^{-4t}u(t)$$

c) $H_2(s) = \frac{Y(s)}{X(s)}$ $\qquad Y(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$

$$X(s) = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$H_2(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}\left(\frac{1}{s} - \frac{e^{-s}}{s}\right)^{-1}$$

$$\therefore H_2(s) = \frac{1}{s}(1 - e^{-s})$$

$$H(s) = H_1(s)H_2(s) = \left[\frac{3}{s(s+1)(s+4)} - \frac{3e^{-s}}{s(s+1)(s+4)}\right]$$

$$= \left(\frac{3}{4s} + \frac{-1}{s+1} + \frac{1}{4(s+4)}\right)\left(1 - e^{-s}\right)$$

$$\therefore H(s) = u(t)\left(\frac{5}{4} - \frac{5}{3}e^{-t} + \frac{5}{12}e^{-4t}\right) - u(t-1)\left(\frac{5}{4} - \frac{5}{3}e^{-(t-1)} + \frac{5}{12}e^{-4(t-1)}\right)$$

6.3 6c 6 / 6

✓ - 0 pts Correct

- 6 pts Missing Incorrect