23W-EC ENGR-113-LEC-1 HW1

SANJIT SARDA

TOTAL POINTS

50 / 50

QUESTION 1

1 1 **10 / 10**

- **+ 1 pts** a
- **+ 1 pts** b
- **+ 1 pts** c
- **+ 2 pts** d
- **+ 2 pts** e
- **+ 2 pts** f
- **+ 1 pts** g
- ✓ **+ 10 pts** *All correct*
- **+ 0 pts** Nothing submitted

QUESTION 2

2 2 **10 / 10**

- **+ 2 pts** a
- **+ 2 pts** b
- **+ 2 pts** c
- **+ 2 pts** d
- **+ 1 pts** e
- **+ 1 pts** f
- ✓ **+ 10 pts** *All Correct*
- **+ 0 pts** None Correct

QUESTION 3

3 3 **10 / 10**

- **+ 2 pts** a
- **+ 2 pts** b
- **+ 2 pts** c
- **+ 2 pts** d
- **+ 2 pts** e
- ✓ **+ 10 pts** *All Correct*
- **+ 0 pts** None Correct

QUESTION 4

4 4 **10 / 10**

- **+ 5 pts** i
- **+ 5 pts** ii
- ✓ **+ 10 pts** *All Correct*
- **+ 2 pts** Partial Credit

QUESTION 5

5 5 **10 / 10**

- **+ 4 pts** (i) Can Be Reconstructed
- **+ 3 pts** (ii) Cannot Be Reconstructed
- **+ 3 pts** (iii) Cannot Be Reconstructed
- ✓ **+ 10 pts** *All Correct*
- **+ 3 pts** Partial Credit

1 1 **10 / 10**

- **+ 1 pts** a
- **+ 1 pts** b
- **+ 1 pts** c
- **+ 2 pts** d
- **+ 2 pts** e
- **+ 2 pts** f
- **+ 1 pts** g
- ✓ **+ 10 pts** *All correct*
- **+ 0 pts** Nothing submitted

2 2 **10 / 10**

+ 2 pts a

+ 2 pts b

+ 2 pts c

+ 2 pts d

+ 1 pts e

+ 1 pts f

✓ **+ 10 pts** *All Correct*

+ 0 pts None Correct

3 3 **10 / 10**

+ 2 pts a

+ 2 pts b

+ 2 pts c

+ 2 pts d

+ 2 pts e

✓ **+ 10 pts** *All Correct*

+ 0 pts None Correct

4 4 **10 / 10**

+ 5 pts i

+ 5 pts ii

✓ **+ 10 pts** *All Correct*

+ 2 pts Partial Credit

5 5 **10 / 10**

- **+ 4 pts** (i) Can Be Reconstructed
- **+ 3 pts** (ii) Cannot Be Reconstructed
- **+ 3 pts** (iii) Cannot Be Reconstructed
- ✓ **+ 10 pts** *All Correct*
- **+ 3 pts** Partial Credit

ECE 113 Hw 1

1. Given

$$x[n] = \{2, 0, -1, 6, -3, 2, 0\},\ -3 \le n \le 3$$
$$y[n] = \{8, 2, -7, -2, 0, 1, 1\},\ -5 \le n \le 1$$
$$w[n] = \{3, 6, -1, 2, 6, 6, 1\},\ -2 \le n \le 4$$

Find:

a) $c[n] = x[n+3]$
b) $d[n] = y[n-2]$
c) $e[n] = x[-n]$
d) $u[n] = x[n-3] + y[n+3]$
e) $v[n] = y[n-3] \cdot w[n+2]$
f) $s[n] = y[n+4] - w[n-3]$
g) $r[n] = 3.9w[n]$

Answer

a) $c[n] = \{2, 0, -1, 6, -3, 2, 0\},\ -6 \le n \le 0$
b) $d[n] = \{8, 2, -7, -2, 0, 1, 1\},\ -3 \le n \le 3$
c) $e[n] = \{0, 2, -3, 6, -1, 0, 2\},\ -3 \le n \le 3$
d) $u[n] = \{8, 2, -7, -3, 0, 1, 1, 0, 2, 0, -1, 6, -3, 2, 0\},\ -8 \le n \le 6$
e) $v[n] = \{-8, 4, -42, -18, 0\},\ -2 \le n \le 2$
f) $s[n] = \{8, 2, -9, -3, 0, 1, 1, 0, 0, 0, -3, -6, 1, -2, -6, -6, -1\},\ -9 \le n \le 7$
g) $r[n] = \{11.7, 23.4, -3.9, 7.8, 23.4, 23.4, 3.9\},\ -2 \le n \le 4$

2. Determine

The fundamental period of the sinusoidal sequence $x[n] = A\sin(\omega_0 n)$ for the following values the angular frequency ω_0:

a) 0.3π

b) 0.48π

c) 0.45π

d) 0.525π

e) 0.7π

f) 0.75π

Answer

a) If the sinosuid has a fundametal frequency F_0,
$A\sin(0.3\pi n) = A\sin(2\pi F_0 n) = A\sin(2\pi\frac{0.3}{2}n)$,
$\therefore F_0 = \frac{0.3}{2} = \frac{3}{20}$
$N = \frac{k}{f} = \frac{20}{3}\cdot k = 20$, for $k = 3$, since $k \in \mathbb{R}$
$\therefore N = 20$

b) If the sinosuid has a fundametal frequency F_0,
$A\sin(0.48\pi n) = A\sin(2\pi F_0 n) = A\sin(2\pi\frac{0.48}{2}n)$,
$\therefore F_0 = \frac{0.48}{2} = \frac{6}{25}$
$N = \frac{k}{f} = \frac{25}{6}\cdot k = 20$, for $k = 6$
$\therefore N = 25$

c) If the sinosuid has a fundametal frequency F_0,
$A\sin(0.45\pi n) = A\sin(2\pi F_0 n) = A\sin(2\pi\frac{0.45}{2}n)$,
$\therefore F_0 = \frac{0.45}{2} = \frac{9}{40}$
$N = \frac{k}{f} = \frac{40}{9}\cdot k = 20$, for $k = 9$
$\therefore N = 40$

d) If the sinosuid has a fundametal frequency F_0,
$A\sin(0.525\pi n) = A\sin(2\pi F_0 n) = A\sin(2\pi\frac{0.525}{2}n)$,
$\therefore F_0 = \frac{0.525}{2} = \frac{21}{80}$
$N = \frac{k}{f} = \frac{80}{21}\cdot k = 80$, for $k = 21$
$\therefore N = 80$

e) If the sinosuid has a fundametal frequency F_0,
$A\sin(0.7\pi n) = A\sin(2\pi F_0 n) = A\sin(2\pi\frac{0.7}{2}n)$,
$\therefore F_0 = \frac{0.7}{2} = \frac{7}{20}$
$N = \frac{k}{f} = \frac{20}{7}\cdot k = 20$, for $k = 7$
$\therefore N = 20$

f) If the sinosuid has a fundametal frequency F_0,
$A\sin(0.75\pi n) = A\sin(2\pi F_0 n) = A\sin(2\pi\frac{0.75}{2}n)$,
$\therefore F_0 = \frac{0.75}{2} = \frac{3}{8}$

$N = \frac{k}{f} = \frac{4}{8} \cdot k = 8$, for $k = 3$
$\therefore N = 8$

3. Determine

The fundamental period of the following periodic sequences:
a) $x_a[n] = e^{j0.25\pi n}$
b) $x_b[n] = cos(0.6\pi n + 0.3\pi)$
c) $x_c[n] = \Re(e^{j\pi n/8}) + \Im(e^{j\pi n/5})$
d) $x_d[n] = 6\sin(0.15\pi n) - \cos(0.12\pi n + 0.1\pi)$
e) $x_e[n] = \sin(0.1\pi n + 0.75\pi) - 3\cos(0.8\pi n + 0.2\pi) + \cos(1.3\pi n)$

Answer

a) $0.25\pi = 2\pi f$
$f = \frac{0.25}{2} = \frac{1}{8}$
$N = \frac{k}{f} = \frac{8}{1} \cdot k = 8$, for $k = 1$
$\therefore N = 8$

b) $0.6\pi = 2\pi f$
$f = \frac{0.6}{2} = \frac{3}{10}$
$N = \frac{k}{f} = \frac{10}{3} \cdot k = 10$, for $k = 3$
$\therefore N = 10$

c) $Re(e^{j\pi n/8}) + \Im(e^{j\pi n/5}) = \cos(\pi n/8) + \sin(\pi n/5)$
$\frac{1}{8}\pi = 2\pi f_1$
$f_1 = \frac{1}{8\cdot 2} = \frac{1}{16}$
$N_1 = \frac{k_1}{f_1} = \frac{16}{k}_1 \cdot k = 16$, for $k = 1$
$\frac{1}{5}\pi = 2\pi f_2$
$f_2 = \frac{1}{5\cdot 2} = \frac{1}{10}$
$N_2 = \frac{k_2}{f_2} = \frac{10}{k}_2 \cdot k = 10$, for $k = 1$
$\therefore N = LCM(16, 10) = 80$

d) $6\sin(0.15\pi n) - \cos(0.12\pi n + 0.1\pi) = 6\sin(\frac{3}{20}\pi n) - \cos(\frac{3}{25}\pi n + \frac{3}{40}\pi)$
$\frac{3}{20}\pi = 2\pi f_1$
$f_1 = \frac{3}{20\cdot 2} = \frac{3}{40}$
$N_1 = \frac{k_1}{f_1} = \frac{40}{3}k_1 = 40$, for $k = 3$
$\frac{3}{25}\pi = 2\pi f_2$

$f_2 = \frac{3}{25 \cdot 2} = \frac{3}{50}$
$N_2 = \frac{k_2}{f_2} = \frac{50}{3} k_2 = 50$, for $k = 3$
$\therefore N = LCM(40, 50) = 200$

e) $\sin(0.1\pi n + 0.75\pi) - 3\cos(0.8\pi n + 0.2\pi) + \cos(1.3\pi n) = \sin(\frac{1}{10}\pi n + \frac{3}{4}\pi) - 3\cos(\frac{4}{5}\pi n + \frac{2}{10}\pi) + \cos(\frac{13}{10}\pi n)$
$\frac{1}{10}\pi = 2\pi f_1$
$f_1 = \frac{1}{10 \cdot 2} = \frac{1}{20}$
$N_1 = \frac{k_1}{f_1} = \frac{20}{1} k_1 = 20$, for $k = 1$
$\frac{4}{5}\pi = 2\pi f_2$
$f_2 = \frac{4}{5 \cdot 2} = \frac{2}{5}$
$N_2 = \frac{k_2}{f_2} = \frac{5}{2} k_2 = 5$, for $k = 2$
$\frac{13}{10}\pi = 2\pi f_3$
$f_3 = \frac{13}{10 \cdot 2} = \frac{13}{20}$
$N_3 = \frac{k_3}{f_3} = \frac{20}{13} k_3 = 20$, for $k = 13$
$\therefore N = LCM(20, 5, 20) = 20$

4. Assume

$x[n]$ has a period N. Are the following periodic?
i) $x[1 - 2n]$
ii)$x[n] + (-1)^n x[0]$

Answer
i) If $x[n]$ is periodic with period N, then $x[n] = x[n + N]$
Let $y[n] = x[1 - 2n]$
$\therefore x[2n] = x[2(n + N)]$ $\quad$ $x[-2n] = x[-2(n + N)]$ $\quad$ $x[-2n + 1] = x[1 - 2(n + N)]$ $\quad$ $y[n] = x[1 - 2(n + N)]$ $\quad$ $y[n] = y[n + \frac{1}{2}N]$
$\therefore x[1 - 2n]$ is periodic with a period of $LCM(\frac{1}{2}N, 1) = 1$
The $\frac{1}{2}N$ accounts for the $\frac{1}{2}N$ and the 1 accounts for the discrete nature of the new signal which cannot have a fractional period. In simpler English, x[1-2n] is periodic and if the period N is odd, the period will remain N otherwise it will be $2N$.

ii) If $x[n]$ is periodic with period N, then $x[n] = x[n + N]$
Let $y[n] = (-1)^n x(0)$, and let $z[n] = x[n] + y[n]$
From the problem, we know that $x[n]$ has a period N.
To find the period of $y[n]$ we need to find an N_y such that $y[n] = y[n + N_y]$.

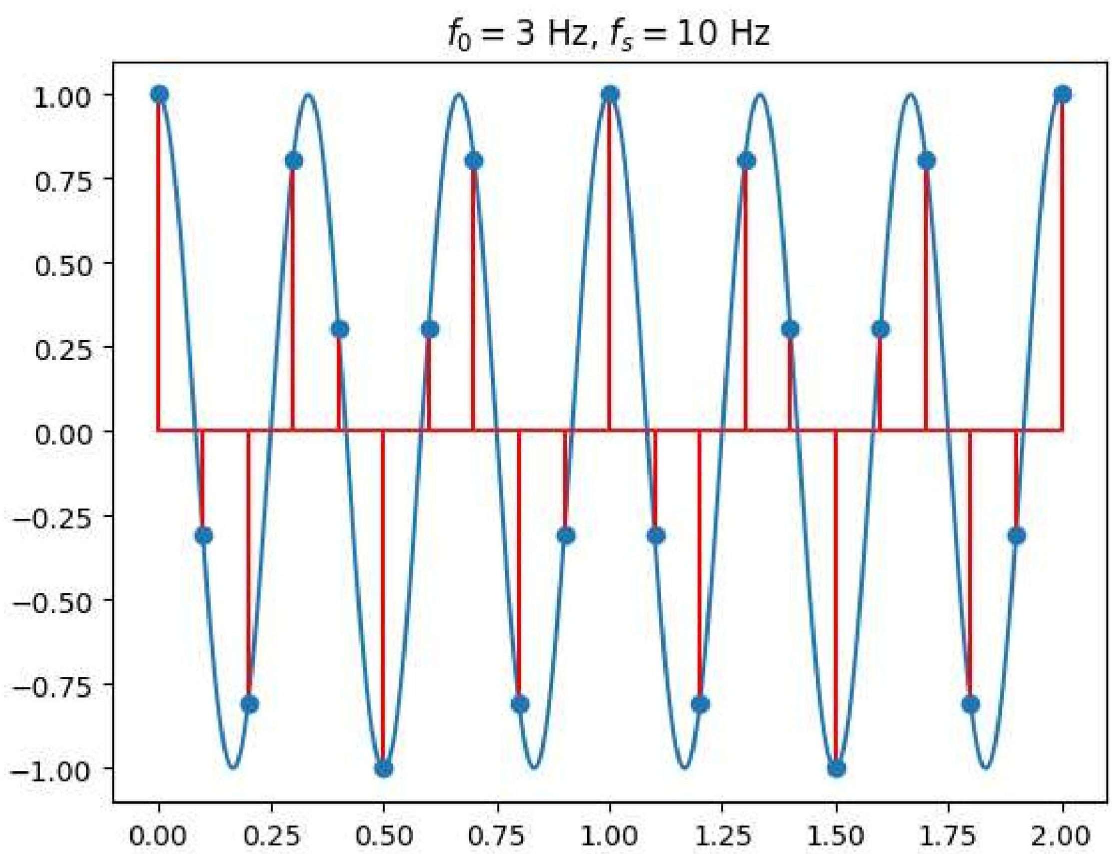
Solving $(-1)^N x[0] = (-1)^{n+N_y} x[0], \quad (-1)^N - (-1)^{n+N_y} = 0$
$\therefore (-1)^n((-1)^{N_y} - 1) = 0, \quad (-1)^{N_y} = 1$
$\therefore N_y = 2$
$\therefore$ The period of $N_z = LCM(N_x, N_y) = LCM(N, 2)$
It is periodic with period $LCM(N, 2)$

5.

In [15]:
```python
# imports
import numpy as np
import matplotlib.pyplot as plt

# Define the function
def x(t, f0):
    return np.cos(2*np.pi*f0*t)
```

In [16]:
```python
# i. $f_0=3$ Hz, $f_s=10$ Hz
t = np.linspace(0, 2, 1000)
f0 = 3
fs = 10
plt.plot(t, x(t, f0))
plt.stem(np.arange(0, 2 + 1/fs, 1/fs), x(np.arange(0, 2 + 1/fs, 1/fs), f0),
'r')
plt.title('$f_0=3$ Hz, $f_s=10$ Hz')
plt.show()
```



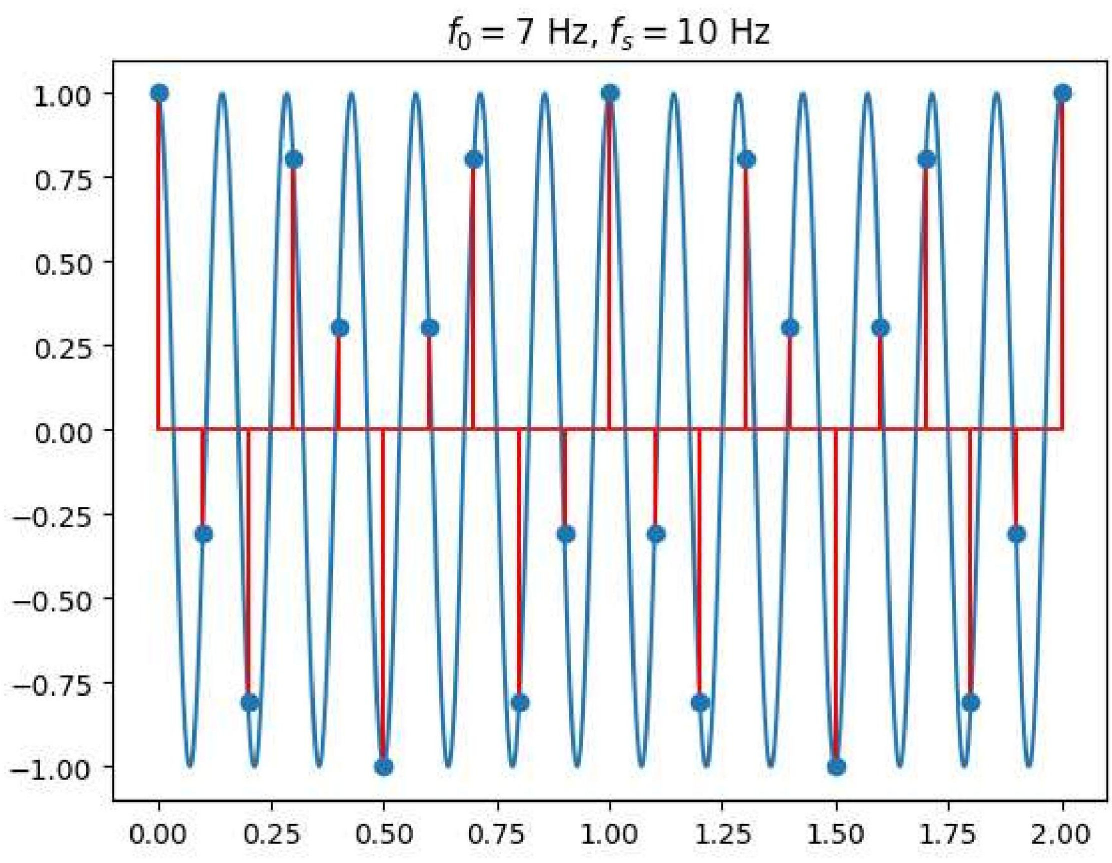
i) F_0= 3Hz

$$2 \cdot F_0 = 6Hz \leq f_s = 10Hz$$

$\therefore$ We should be able to recover the signal since we are above the Nyquist frequency.

$

In [17]:
```
# ii. $f_0=7$ Hz, $f_s=10$ Hz
t = np.linspace(0, 2, 1000)
f0 = 7
fs = 10
plt.plot(t, x(t, f0))
plt.stem(np.arange(0, 2 + 1/fs, 1/fs), x(np.arange(0, 2 + 1/fs, 1/fs), f0),
'r')
plt.title('$f_0=7$ Hz, $f_s=10$ Hz')
plt.show()
```

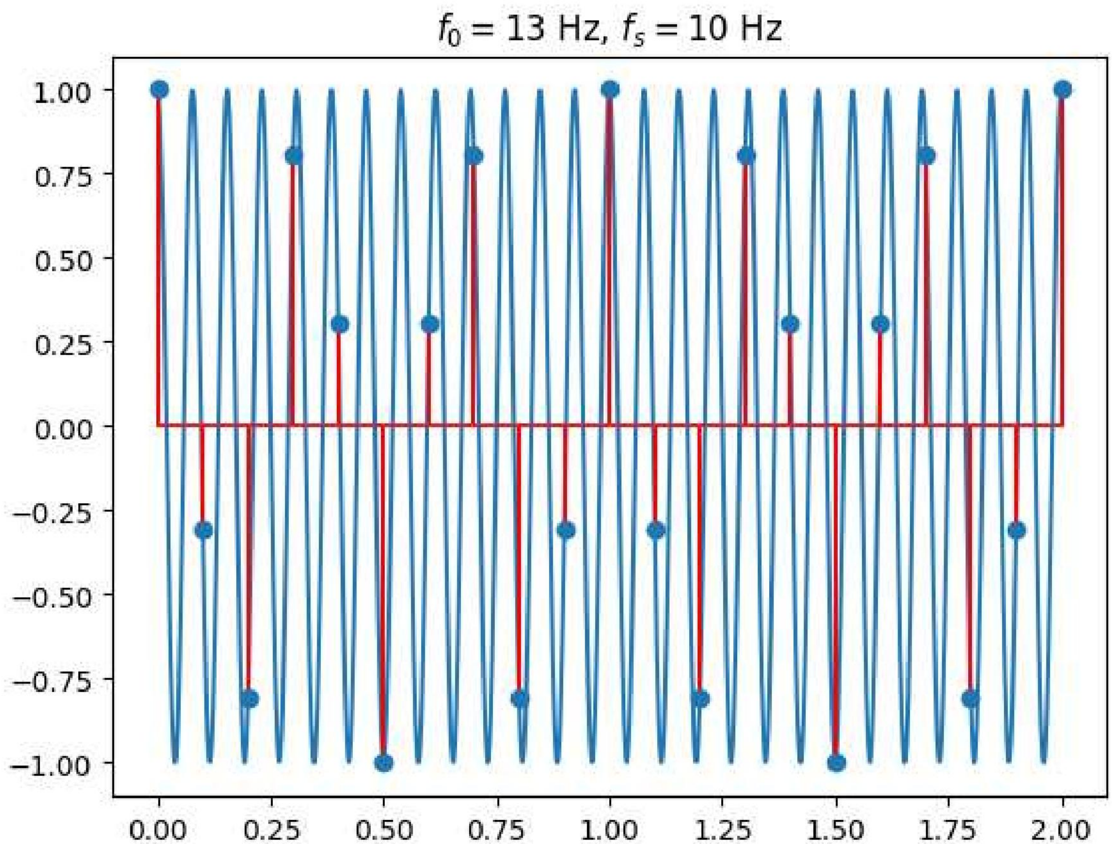


iii) F_0= 7Hz

$$2 \cdot F_0 = 14Hz \geq f_s = 10Hz$$

$\therefore$ We should be not be able to recover the signal since we are below the Nyquist frequency.

```
In [18]: # iii. $f_0=13$ Hz, $f_s=10$ Hz
         t = np.linspace(0, 2, 1000)
         f0 = 13
         fs = 10
         plt.plot(t, x(t, f0))
         plt.stem(np.arange(0, 2 + 1/fs, 1/fs), x(np.arange(0, 2 + 1/fs, 1/fs), f0),
         'r')
         plt.title('$f_0=13$ Hz, $f_s=10$ Hz')
         plt.show()
```



iii) F_0= 13Hz

$$2 \cdot F_0 = 26Hz \geq f_s = 10Hz$$

$\therefore$ We should be not be able to recover the signal since we are below the Nyquist frequency.