23W-EC ENGR-131A-LEC-1 Homework 2

SANJIT SARDA

TOTAL POINTS

100 / 100

QUESTION 1

1 1 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts incorrect answer

- 7 pts incorrect and little/no work shown

- 20 pts missing

QUESTION 2

2 2 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts incorrect answer

- 7 pts incorrect answer with little/no work

- 20 pts Click here to replace this description.

QUESTION 3

3 3 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts incorrect answer

- 7 pts incorrect answer and little/no work

- 20 pts missing

QUESTION 4

4 4 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts partially incorrect

- 6 pts fully incorrect

- 10 pts incorrect and little/no work

- 20 pts missing

QUESTION 5

5 5 **20 / 20**

✓ **- 0 pts** *Correct*

- 2 pts (a)(i) incorrect

- 2 pts (a)(ii) incorrect

- 2 pts (a)(iii) incorrect

- 2 pts (b)(i) incorrect

- 2 pts (b)(ii) incorrect

- 2 pts (b)(iii) incorrect

- 20 pts missing

- 10 pts (a) or (b) missing

ECE 131A HW2

1) 2 Dice.

$A = \{D1_1 \text{ or } D1_2 \text{ or } D1_5\}$

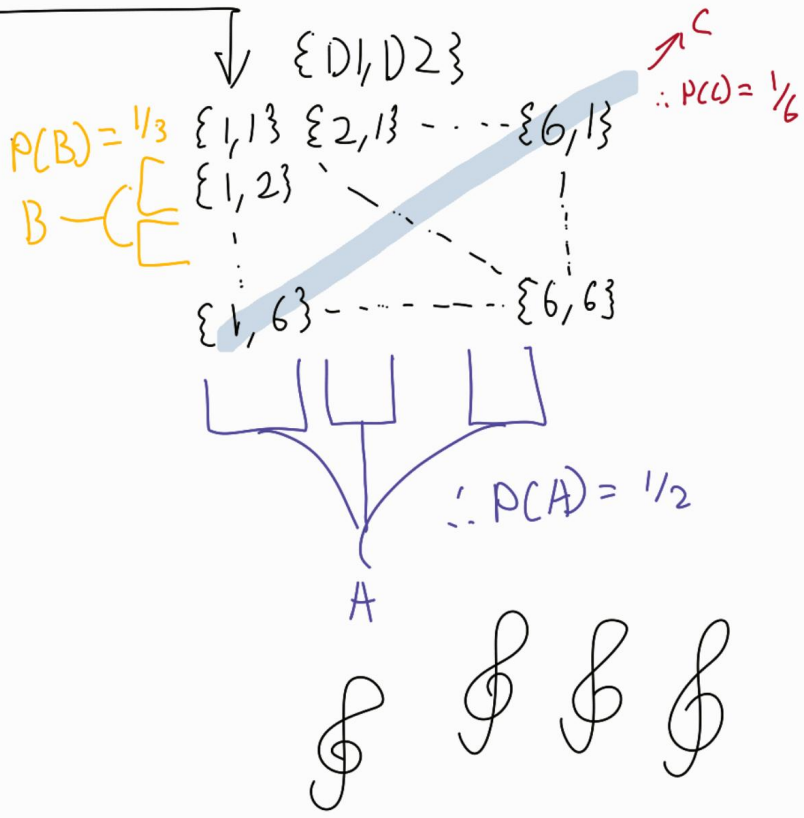
$B = \{D2_2 \text{ or } D2_3\}$

$C = \{(D1+D2)_7\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{2}{6} = \frac{1}{3}$

$P(C) = \frac{6}{36} = \frac{1}{6}$



$P(A \cap B) = P(A)P(B)$

$P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$

$= \frac{1}{3} \cdot \frac{1}{2} \quad = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$P(B)P(A) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} \therefore P(A \cap B) = P(A)P(B)$ ✓

$P(A \cap C) = P(A)P(C)$

$P(A \cap C) = P(C)P(A|C) = P(A)P(C|A)$

$= \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

$P(A)P(C) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \therefore P(A \cap C) = P(A)P(C)$ ✓

$P(B \cap C) = P(B)P(C)$

$P(B \cap C) = P(C)P(B|C) = P(B)P(C|B)$

$= \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

$P(B)P(C) = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18} \therefore P(B \cap C) = P(B)P(C)$ ✓

$P(A \cap B \cap C) = P(A)P(B)P(C)$

$P(A \cap B \cap C) = P((A \cap B) \cap C)$

$= P(C)P((A \cap B)|C)$

$= \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{36}$

$P(A)P(B)P(C) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{36}$

$\therefore P(A \cap B \cap C) = P(A)P(B)P(C)$ ✓

∴ These three events are Independant.

1 1 **20 / 20**

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2)

$P(C \cup D) = .75$, $P(A \cup B) = .25$

$P(\text{Defective} \mid \text{San Diego}) = 5\, P(\text{Defective} \mid \text{Cupertino})$

$\therefore P(B \cup D \mid A \cup B) = 5P(B \cup D \mid C \cup D)$

(Doesn't Happen)

$\therefore P(B \mid A \cup B) = 5P(D \mid C \cup D)$

$\therefore \frac{P((A \cup B) \cap B)}{P(A \cup B)} = \frac{5P((C \cup D) \cap D)}{P(C \cup D)}$

$\therefore \frac{P(B)}{P(A \cup B)} = \frac{5P(D)}{P(C \cup D)}$

$P(C \cup D) = .75$ & $C \cap D = \phi$

$\therefore P(C \cup D) = .75 = P(C) + P(D)$

$\therefore -P(C) = P(D) - .75$

$P(A \cup B) = .25$ & $A \cap B = \phi$

$\therefore P(A \cup B) = .25 = P(A) + P(B)$

$P(B \cup D) = .04$ & $B \cap D = \phi$

$\therefore P(B \cup D) = .04 = P(B) + P(D)$

$P(A \cup B \cup C \cup D) = 1$ & A, B, C, D are disjoint

$\therefore P(A \cup B \cup C \cup D) = 1 = P(A) + P(B) + P(C) + P(D)$

$\therefore P(A) + P(C) = 100 - P(B \cup D) = .96$

Using this,

$P(B) + P(D) = .04$

$P(B) = \frac{5}{3} P(D)$

$\frac{5}{3} P(D) + P(D) = .04$

$\therefore \frac{8}{3} P(D) = .04$

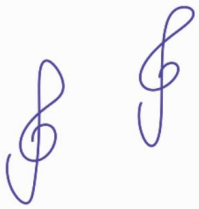
$\therefore P(D) = (.04)\left(\frac{3}{8}\right) = \frac{3}{200} = .015$

$\therefore P(B) = \frac{5}{3} \cdot \frac{3}{200} = \frac{1}{40}$

$\therefore P(A) = P(A \cup B) - P(B) = \frac{10}{40} - \frac{1}{40} = \frac{9}{40}$

$P(B \cup D) = .04 = P(B) + P(D)$

since $B \cap D = \phi$



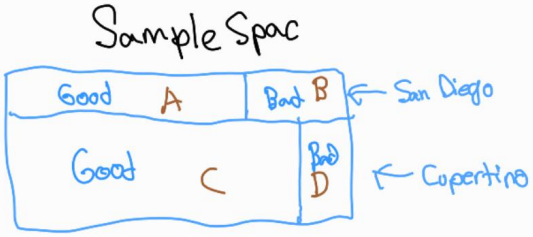


Fig A

$P(\text{San Diego} \mid \text{Working}) =$

$P(A \cup B \mid A \cup C)$

$= \frac{P((A \cup C) \cap A)}{P(A \cup C)}$

$= \frac{P(A)}{1 - P(A \cup C)} = \frac{9}{40} \cdot \frac{1}{1 - \frac{4}{100}} = \frac{9}{40} \cdot \frac{100}{96} = \frac{15}{64}$

2 2 **20 / 20**

✓ **- 0 pts** *Correct*

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3)

Lets find $P(\text{water}|H)$ & $P(\text{water}|T)$

$B_2 = \{W,?\}$ $B_2 = \{W,?,?\}$

$P(H|\text{Water}) = \frac{P(H \cap \text{Water})}{P(\text{Water})}$

$P(\text{Water}|H) = \frac{P(H \cap \text{Water})}{P(H)}$

$B_2|H = \{W,?\}: \{W,W\}$ or $\{W,F\}$

$P = \frac{3}{5}$ $P = \frac{2}{5}$

$P(W|WW) = 1$ $P(W|WF) = 1/2$

$P(W|H) = \frac{3}{5} + \frac{2}{5}\cdot\frac{1}{2} = \frac{4}{5}$

$B_2|T = \{W,?,?\}: \{W,W,?\}$ or $\{W,F,?\}$

$P = \frac{3}{5}$ $P = \frac{2}{5}$

$\{W,W,W\}$ $\{W,W,F\}$ $\{W,W,F\}$ $\{W,F,F\}$

$P = \frac{1}{2}$ $P = \frac{1}{2}$ $P = \frac{3}{4}$ $P = \frac{1}{4}$

$P(W|W,W,W) = 1$ $P(W|WWF) = \frac{2}{3}$ $P(W|W,W,F) = \frac{2}{3}$ $P(W|WFF) = \frac{1}{3}$

$P(W|WW?) = \frac{1}{2} + \frac{1}{2}\cdot\frac{2}{3} = \frac{5}{6}$

$P(W|WF,?) = \frac{2}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{4} = \frac{7}{12}$

$P(W|T) = \frac{5}{6}\cdot\frac{3}{5} + \frac{7}{12}\cdot\frac{2}{5}$

$= \frac{1}{2} + \frac{7}{30} = \frac{11}{15}$

$P(W) = \frac{1}{2}\cdot\frac{4}{5} + \frac{1}{2}\cdot\frac{11}{15}$

$= \frac{23}{30}$

We want to know $P(H|W)$,

$P(H|W) = \frac{P(H \cap W)}{P(W)}$

$\therefore P(H|W) = \frac{P(H)P(W|H)}{P(W)}$

$= \frac{1}{2}\cdot\frac{4}{5}\cdot\frac{30}{23}$

$= \frac{12}{23}$

3 3 **20 / 20**

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4) 2 Dice:

$X_1 = \{1,2,3,4,5,6\}$

$X_2 = \{1,2,3,4,5,6\}$

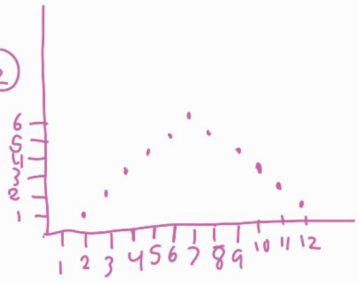
$Z = X_1 + X_2$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a)

z	2	3	4	5	6	7	8	9	10	11	12
$P_Z(Z=z)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$36 \cdot pmf(z)$



b) $Z = 10 = X_1 + X_2$

$\therefore X_1 = 10 - X_2$

$P(X_1 = k \mid Z = 10)$ for $k \in \{1,2,3,4,5,6\}$

$= \frac{P(X_1 = k \cap Z = 10)}{P_Z(Z=10)} = \frac{P(X_1 = k \cap Z = 10)}{3}$.

$\therefore$

X_1	1	2	3	4	5	6
$P(X_1 = k \mid Z=10)$	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

4 4 **20 / 20**

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5) From Problem we have:

$P(O_0|I_0) = 1-\epsilon$

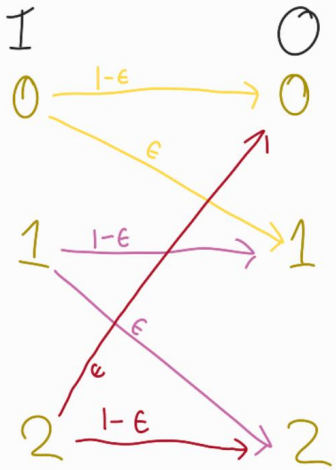
$P(O_1|I_0) = \epsilon$

$P(O_1|I_1) = 1-\epsilon$

$P(O_2|I_1) = \epsilon$

$P(O_2|I_2) = 1-\epsilon$

$P(O_0|I_2) = \epsilon$



a)

$P(O_0) = P(O_0|I_0)\cdot\frac{1}{6} + P(O_0|I_1)\cdot\frac{1}{6} + P(O_0|I_2)\cdot\frac{2}{3} = \frac{1-\epsilon}{6} + \frac{4\epsilon}{6} = \frac{1+3\epsilon}{6}$

$P(O_1) = P(O_1|I_0)\cdot\frac{1}{6} + P(O_1|I_1)\cdot\frac{1}{6} + P(O_1|I_2)\cdot\frac{2}{3} = \frac{\epsilon}{6} + \frac{1-\epsilon}{6} = \frac{1}{6}$

$P(O_2) = P(O_2|I_0)\cdot\frac{1}{6} + P(O_2|I_1)\cdot\frac{1}{6} + P(O_2|I_2)\cdot\frac{2}{3} = \frac{\epsilon}{6} + \frac{4-4\epsilon}{6} = \frac{4-3\epsilon}{6}$

b) $P(I_0|O_0) = \frac{P(I_0 \cap O_0)}{P(O_0)} = \frac{P(I_0)P(O_0|I_0)}{P(O_0)} = \frac{1}{6}\cdot(1-\epsilon)\cdot\frac{6}{1+3\epsilon} = \frac{1-\epsilon}{1+3\epsilon}$

$P(I_1|O_0) = \frac{P(I_1 \cap O_0)}{P(O_0)} = \frac{P(I_1)P(O_0|I_1)}{P(O_0)} = \frac{1}{6}\cdot 0\cdot\frac{6}{1+3\epsilon} = 0$

$P(I_2|O_0) = \frac{P(I_2 \cap O_0)}{P(O_0)} = \frac{P(I_2)P(O_0|I_2)}{P(O_0)} = \frac{4}{6}\cdot\epsilon\cdot\frac{6}{1+3\epsilon} = \frac{4\epsilon}{1+3\epsilon}$

5 5 **20 / 20**

✓ **- 0 pts** *Correct*

- 2 pts (a)(i) incorrect

- 2 pts (a)(ii) incorrect

- 2 pts (a)(iii) incorrect

- 2 pts (b)(i) incorrect

- 2 pts (b)(ii) incorrect

- 2 pts (b)(iii) incorrect

- 20 pts missing

- 10 pts (a) or (b) missing

Sample Spac

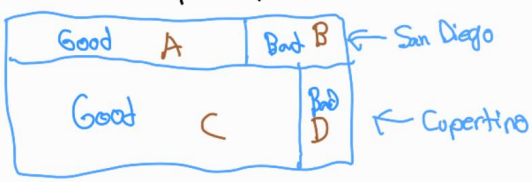


Fig A

$A+B+C+D=100$

$A+B=25$

$A+C=96$

$\frac{B}{A+B}=\frac{5D}{C+D}$

$B=\frac{5}{3}D$

$C+D=75$

$-C=D-75$

$25-B=96-C$

$25-B=96+D-75$

$-B=96+D-100$

$B+D=4$

$B=4-D$

$\frac{5}{3}D=4-D$

$\frac{8}{3}D=4$

$D=\frac{3}{2}$

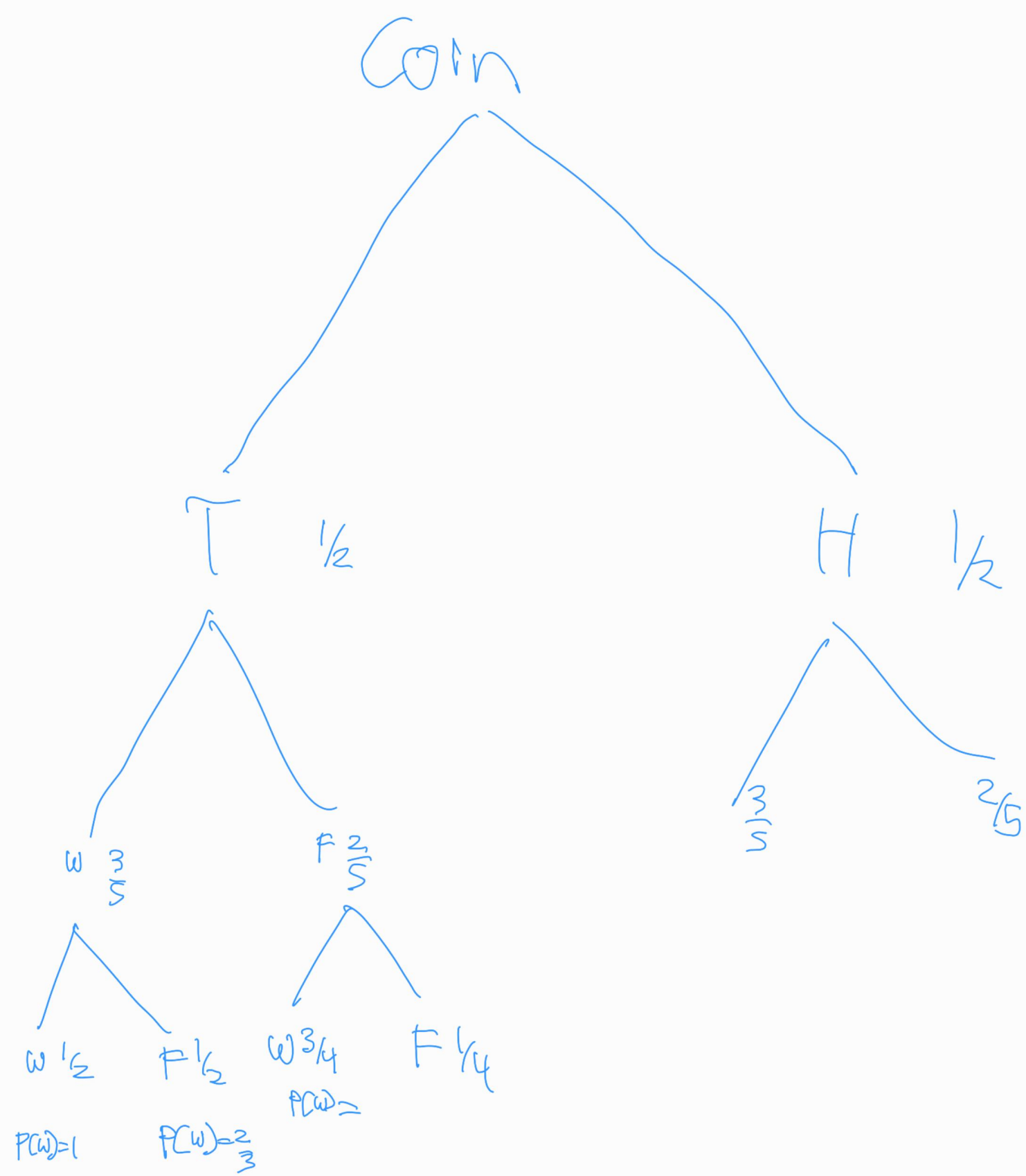
$C=\frac{147}{2}$

$B=\frac{5}{2}$

$A=\frac{45}{2}$

$\frac{45}{2}\cdot\left(\frac{147+45}{2}\right)^{-1}$

$\frac{45}{147+45}=\frac{45}{192}=\frac{15}{}$


Coin
T 1/2
H 1/2
W 3/5
F 2/5
3/5
2/5
W 1/2
F 1/2
W 3/4
F 1/4
P(W)=1
P(W)=2/3
P(W)=