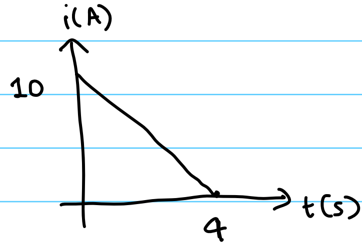
Discussion 1 Solutions

Ans 1.

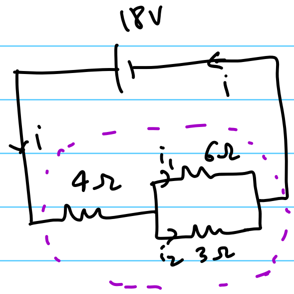


$q = \int i\,dt$; Area under the graph (i versus t) will give us net charge flow.

Hence, $\Delta q =$ Area $\Rightarrow \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 10 = 20C$

Ans. 2

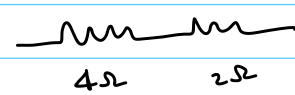
a)



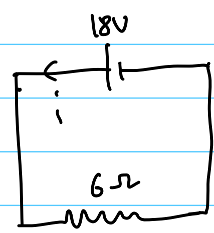
6Ω & 3Ω resistors are in parallel. Their equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{3} \; ; \; R_{eq} = 2\Omega$$

Now this 2Ω and 4Ω resistances are in series and their equivalent resistance is $4+2 = 6\Omega$.



Find equivalent circuit



$$i = \frac{18V}{6\Omega} = 3A$$

b) Current i splits between 3Ω & 6Ω resistors in the inverse ratio of the resistances since they are in parallel.

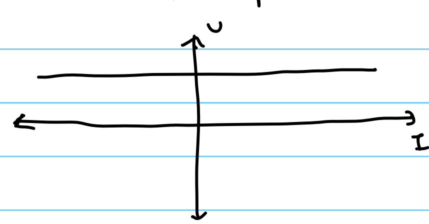
$$i_1 = \frac{3}{3+6} \times 3A \Rightarrow 1A$$

$$i_2 = \frac{6}{3+6} \times 3A \Rightarrow 2A$$

c) Ideal voltage source has zero resistance.
Ideal current source has ∞ resistance

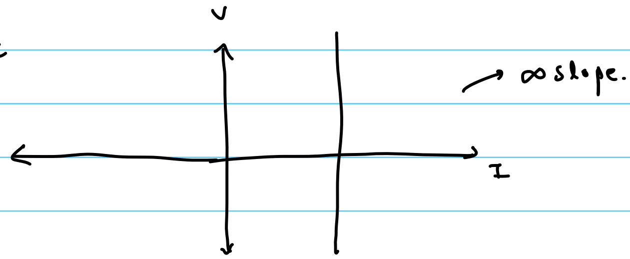
It can also be understood from the slope of (V-I) characteristic.

Ideal Voltage source



→ zero slope.

Ideal current slope



Ans 3.

$$\frac{dy}{dt} = 2(25-y)$$

$$\int \frac{dy}{25-y} = \int 2\,dt \Rightarrow -\ln(y-25) = 2t + c$$

$$y - 25 = e^{-2t-c}$$

$$y = 25 + e^{-c} e^{-2t} \quad ; \text{ Assume } e^{-c} = k$$

$$y(t) = 25 + ke^{-2t} \quad ; \text{ Since } y(0) = 40$$

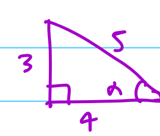
$$y(0) = 25 + k = 40 \Rightarrow (k = 15)$$

$$\boxed{y(t) = 25 + 15e^{-2t}}$$

Ans. 4

$$Y = 3\cos\theta + 4\sin\theta$$

$$Y = \sqrt{3^2+4^2}\cdot\left[\frac{3}{\sqrt{3^2+4^2}}\cos\theta + \frac{4}{\sqrt{3^2+4^2}}\sin\theta\right] \Rightarrow 5\cdot\underbrace{\left[\underset{\sin\alpha}{\frac{3}{5}}\cos\theta + \underset{\cos\alpha}{\frac{4}{5}}\sin\theta\right]}$$



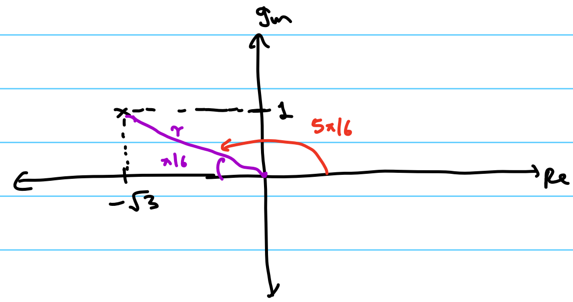
$$\boxed{\text{Identity: } \sin(\theta_1+\theta_2) = \sin\theta_1 \cos\theta_2 + \sin\theta_2 \cos\theta_1}$$

$$5\sin\left(\theta + \tan^{-1}\left(\frac{3}{4}\right)\right)$$

$$\boxed{A = 5, \quad \beta = \tan^{-1}\left(\frac{3}{4}\right)}$$

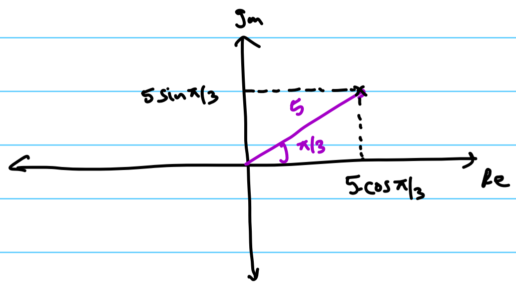
Ans.5

a)



$r = \sqrt{(\sqrt{3})^2 + 1} \Rightarrow 2$; $\theta = 5\pi/6 \Rightarrow$ Polar form $= 2e^{i5\pi/6}$

b)



real component $\Rightarrow 5\cos(\pi/3)$

Imaginary component $\Rightarrow 5\sin(\pi/3)$

$z = 5\cos\left(\frac{\pi}{3}\right) + i\,5\sin(\pi/3)$

$z = \frac{5}{2} + i\frac{5\sqrt{3}}{2}$

Rectangular form

Ans.6

Substituting i as $A_1 e^{S_1 t}$ in the equation, we get

$$A_1 s_1 e^{S_1 t} + \frac{R}{L} A_1 e^{S_1 t} = 0$$

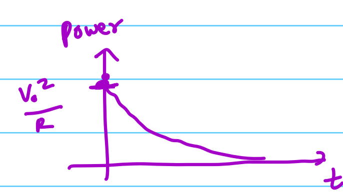
$$S_1 + \frac{R}{L} = 0 \quad ; \quad S_1 = -\frac{R}{L}$$

$$i(t) = A_1 e^{-\frac{R}{L}t} \quad ; \text{since } i(0) = I_0,\ A_1 = I_0$$

$$i(t) = I_0 e^{-\frac{Rt}{L}}$$

Ans.7

$P = VI$, Energy dissipated $= \int P\,dt$



$V(\text{across } R) = V_0 e^{-2t}$, $I(\text{across } R) = \frac{V_0 e^{-2t}}{R}$

$$\text{Energy dissipated} = \int_0^\infty \frac{V_0^2 e^{-4t}}{R} \Rightarrow -\frac{V_0^2}{4R}\Big[e^{-4t}\Big]_0^\infty \Rightarrow \frac{V_0^2}{4R}$$