

Reading: Chapters 2.6, 3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. Some parts of Texas are particularly tornado-prone. Suppose that in one metropolitan area, 25% of all home-owners are insured against tornado damage. Four home-owners are to be selected at random; let X denote the number among the four who have tornado insurance.
 - (a) Find and sketch the pmf of X .
 - (b) What is the most likely value for X ?
 - (c) What is the probability that at least two of the four selected have tornado insurance?
2. Suppose X is a Binomial random variable with parameters $n = 4$, and p .
 - (a) Express $E[\sin(\pi X/2)]$ in terms of p .
 - (b) Express $E[\cos(\pi X/2)]$ in terms of p .
3. Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability p and the second with probability q . All tosses are assumed independent.
 - (a) Find the PMF, the expected value, and the variance of the number of tosses.
 - (b) What is the probability that the last toss of the first coin is a head?
4. Messages passed amongst neurons in the brain can be recorded using an external electrode, and they show up as a series of spikes of equal magnitude. These spikes can be modelled by a Poisson random variable with rate 6000 spikes per minute. The PMF of the Poisson RV is given by:

$$P(N = k) = \frac{\alpha^k e^{-\alpha}}{k!} \quad \text{for } k = 0, 1, 2, \dots \quad (1)$$

- (a) What is the probability of recording no spikes in the first 100 milliseconds?
- (b) What is the probability of recording between 5 to 10 spikes (both included) in the [300ms, 400ms] time range?

Leave your answers as an expression in terms of a sum of 1 or more exponentials.

Hint: The parameter α of the Poisson RV is the average number of event occurrences in a specified time interval, so utilize the given information to find out this parameter.

5. Use MATLAB to plot the pmfs of the following random variables. This question is to provide practice with MATLAB which you will use for the future project so do not provide hand-drawn plots.
- (a) Discrete Uniform Random Variable on the set $\{1, 2, 3, 4\}$.
 - (b) Binomial Random Variable with parameters $n = 5$ and $p = 0.25$.
 - (c) Geometric Random Variable with $p = 0.5$ and plot the pmf for values less than or equal to 5.