ECE113, Winter 2023 **Quiz #6**
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Monday, 30 Jan 2023
10 points total.

Name: _____
UID: _____

1. (10 points) Consider a periodic signal $\tilde{x}[n]$ signal with one of its periods as:

$$\{2, \quad 0, \quad 2, \quad 3\}, 0 \leq n \leq 3,$$

(a) (2 points) How many DTFS coefficients are there in the DTFS representation for $\tilde{x}[n]$?

(b) (8 points) What is the DTFS representation of $\tilde{x}[n]$?

Solution:

(a) There are 4 DTFS coefficients.

(b) We can see that the period $N = 4$.

According to the synthesis equation:

$$\begin{aligned}
\tilde{x}[n] &= \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn} \\
&= \tilde{c}_0 e^{j0} + \tilde{c}_1 e^{j\frac{2\pi}{4}1n} + \tilde{c}_2 e^{j\frac{2\pi}{4}2n} + \tilde{c}_3 e^{j\frac{2\pi}{4}3n} \\
&= \tilde{c}_0 + \tilde{c}_1 e^{j\frac{\pi}{2}n} + \tilde{c}_2 e^{j\pi n} + \tilde{c}_3 e^{j\frac{3\pi}{2}n}
\end{aligned}$$

And now we need to use the analysis equation to find out $\tilde{c}_k$:

$$\begin{aligned}
\tilde{c}_k &= \frac{1}{N} \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn} \\
&= \frac{1}{4} \sum_{n=0}^{3} \tilde{x}[n] e^{-j\frac{\pi}{2}kn} \\
&= \frac{1}{2} + \frac{1}{2} e^{-jk\pi} + \frac{3}{4} e^{-j\frac{3\pi}{2}k}
\end{aligned}$$

Plug in $k = \{0, 1, 2, 3\}$, we can calculate the values as:

$$\begin{aligned}
\tilde{c}_0 &= \frac{7}{4} \\
\tilde{c}_1 &= j\frac{3}{4} \\
\tilde{c}_2 &= \frac{1}{4} \\
\tilde{c}_3 &= -j\frac{3}{4}
\end{aligned}$$

Therefore,

$$\tilde{x}[n] = \frac{7}{4} + j\frac{3}{4}e^{j\frac{\pi}{2}n} + \frac{1}{4}e^{j\pi n} - j\frac{3}{4}e^{j\frac{3\pi}{2}n}$$