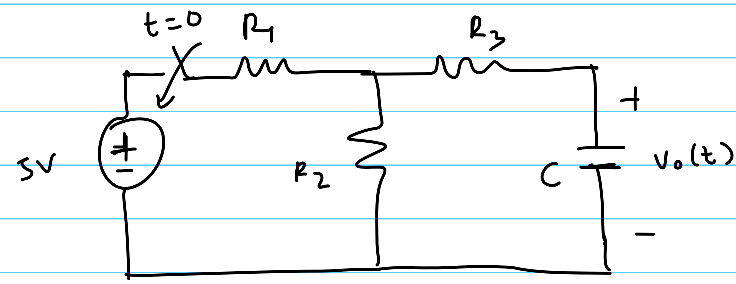
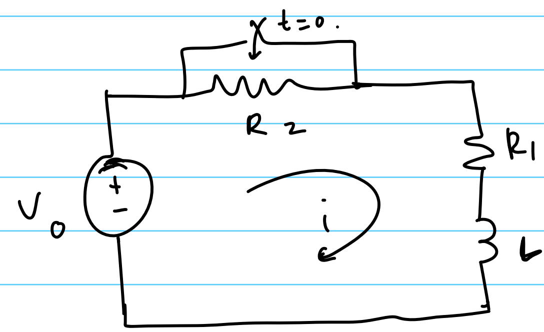
Q1.



$V_o(t=0^-) = 1V$.

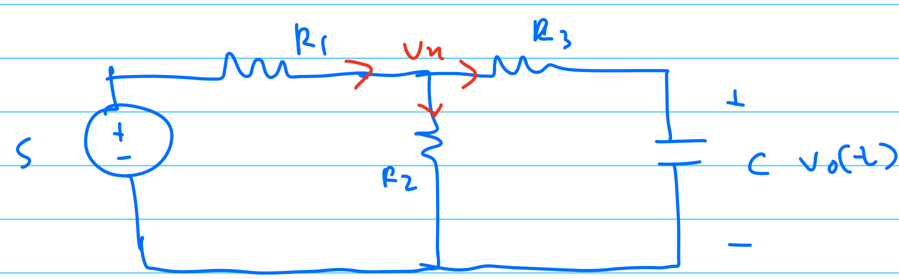
Switch is closed at t=0. Write the differential equation to find $V_o(t)$. What is the time constant of the circuit?

Q2.



Switch is closed at t=0. Assume that the circuit attained steady state previously. Find the current in the circuit as a funcⁿ of time.

Ans.1



$$\frac{5-V_x}{R_1} = \frac{V_x}{R_2} + \frac{V_x - V_o}{R_3}$$

$$\frac{5}{R_1} + \frac{V_o}{R_3} = V_x\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V_x = \left(\frac{5}{R_1} + \frac{v_0}{R_3}\right)\left(R_1 \| R_2 \| R_3\right)$$

$$\frac{V_x - V_0}{R_3} = C\frac{dv_0}{dt}$$

$$\left(\frac{5}{R_1 R_3} + \frac{v_0}{R_3^2}\right)\left(R_1\|R_2\|R_3\right) - \frac{V_0}{R_3} = C\frac{dv_0}{dt}$$

Chracteristic eqn:- (without the source.)

$$\frac{v_0}{R_3^2} \times \left(R_1\|R_2\|R_3\right) - \frac{V_0}{R_3} = C\frac{dv_0}{dt}$$

$$\frac{V_0}{R_3}\left[\frac{\left(R_1\|R_2\|R_3\right)}{R_3} - 1\right] = C\frac{dv_0}{dt}$$

$$\frac{V_0}{R_3}\left[\frac{1}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)R_3} - 1\right] = C\frac{dv_0}{dt}$$

$$\frac{V_0}{R_3}\left[\frac{1}{\left(\frac{R_3}{R_1}+\frac{R_3}{R_2}+1\right)} - 1\right] = C\frac{dv_0}{dt}$$

$$-\frac{V_0}{R_3}\frac{\left[\left(\frac{R_3}{R_1}+\frac{R_3}{R_2}\right)\right]}{\left(\frac{R_3}{R_1}+\frac{R_3}{R_2}+1\right)} = C\frac{dv_0}{dt}$$

$$\frac{-V_0\,(R_1+R_2)}{R_3(R_1+R_2)+R_1R_2} = C\,\frac{dv_0}{dt}$$

Assume $V_0 = k_1 e^{st}$

$$\frac{-\cancel{k_1 e^{st}}\,(R_1+R_2)}{R_3(R_1+R_2)+R_1R_2} = C\,\cancel{k_1} s\,\cancel{e^{st}}$$

$$S = -\frac{1\,(R_1+R_2)}{C\left[R_3(R_1+R_2)+R_1R_2\right]} \Rightarrow -\frac{1}{C\left[R_3+\frac{R_1R_2}{R_1+R_2}\right]}$$

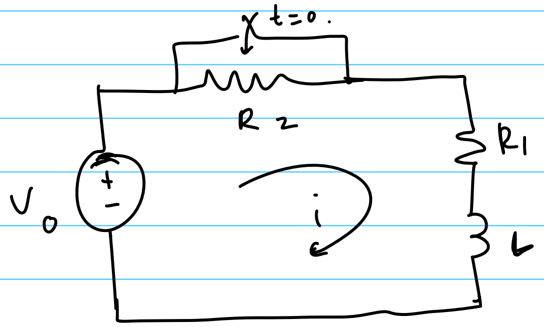
$$\text{time constant } (\tau) = \left(R_3+\frac{R_1R_2}{R_1+R_2}\right).C \Rightarrow \underline{\underline{\left(R_3+(R_1\|R_2)\right)}}.C$$

$V_c(t=0^+) \Rightarrow 1V$

$V_c(t=\infty) \Rightarrow 5V \times \frac{R_2}{R_1+R_2}$

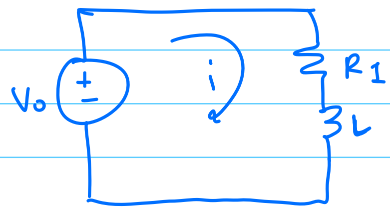
$$\boxed{V_c(t) = \left[V_c(\infty)-V_c(0)\right]\left(1-e^{-t/\tau}\right) + V_c(0)}$$

Ans.2



$$\underline{\underline{i_L(t=0^-) = i_L(t=0^+) = \frac{V_0}{R_1+R_2}}}$$

$$\underline{i_L(t=\infty) = \frac{V_0}{R_1}}$$



$$V_0 = iR_1 + L\frac{di}{dt}$$

To find chracteristic eq^n, set $V_0 = 0$

$$iR_1 + L\frac{di}{dt} \Rightarrow 0$$

$$i(t) = K_1 e^{st}$$

$$\cancel{K_1}\,\cancel{e^{st}}\,R_1 + L\,\cancel{K_1}\,s\,\cancel{e^{st}} = 0$$

$$s = -\frac{R_1}{L} \qquad \boxed{\tau = \frac{L}{R_1}}$$

$$\boxed{i(t) = \left[i_L(\infty) - i_L(0)\right]\left(1-e^{-t/\tau}\right) + i_L(0)}$$