23W-EC ENGR-131A-LEC-1 Homework 4

SANJIT SARDA

TOTAL POINTS

97 / 100

QUESTION 1

1 1 **20 / 20**

✓ **- 0 pts** *Correct*

- 5 pts incorrect answer

- 10 pts incorrect answer and little/no work

- 20 pts missing

QUESTION 2

2 2 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts (a) incorrect

- 3 pts (b) incorrect

- 5 pts incorrect and little/no work on (a)

- 5 pts incorrect and little/no work on (b)

- 20 pts missing

QUESTION 3

3 3 **17 / 20**

- 0 pts Correct

- 1 pts plot (i) wrong in 3(a)

- 1 pts plot (ii) wrong in 3(a)

- 1 pts plot (iii) wrong in 3(a)

- 3 pts (b) incorrect

✓ **- 3 pts** *(c) incorrect*

- 10 pts incorrect answers and little/no work shown

- 20 pts missing

- 5 pts any part missing

- 10 pts two parts missing

QUESTION 4

4 4 **20 / 20**

✓ **- 0 pts** *Correct*

- 5 pts incorrect answer

- 10 pts incorrect answer and little/no work

- 20 pts missing

QUESTION 5

5 5 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts (a) incorrect

- 3 pts (b) incorrect

- 10 pts incorrect answer and little/no work shown

- 20 pts missing

ECE 131A HW4

① Battery of Tesla Model Y is modeled by an exp RV: X

$$\therefore F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$\therefore f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$\mathbb{E}[X] = 9 = \int_{-\infty}^{\infty} x \cdot f_x(x)\,dx = \lambda \int_0^{\infty} x e^{-\lambda x}\,dx = \frac{1}{\lambda} = 9 \quad \therefore \lambda = 1/9$$

$$\therefore F_X(x) = \begin{cases} 1 - e^{-x/9}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$\therefore f_X(x) = \begin{cases} \frac{1}{9} e^{-x/9}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

∴ Probability of battery lasting another 9 hours given it has lasted 9 hours is $F_X(18h \mid 9h) = F_X(18-9) = F_X(9)$

↑ memoryless property.

$$= 1 - \left(1 - e^{-9/9}\right) = e^{-1} = \boxed{1/e}$$

1 1 **20 / 20**

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②

PDF

$$f_x(x) = \begin{cases} 2c, & 0 < x \leq 3 \\ c, & 4 < x \leq 14 \\ 3c, & 16 < x \leq 18 \\ 0, & \text{ow} \end{cases}$$

(a) We know $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$

$$\int_0^3 2c\,dx + \int_4^{14} c\,dx + \int_{16}^{18} 3c\,dx$$

$$= 2cx\Big|_0^3 + cx\Big|_4^{14} + 3cx\Big|_{16}^{18}$$

$$= 6c + 10c + 6c = 1$$

$\therefore 22c = 1 \quad \therefore c = 1/22$

(b) $P(1 < x \leq 5)$

$$P(1 < x \leq 5) = \int_1^5 f_x(x)\,dx = \int_1^3 2(1/22)\,dx + \int_4^5 (1/22)\,dx =$$

$$= 2\cdot(1/22)(3-1) + (1/22)\cdot(5-4)$$

$$= 2/11 + 1/22 = \boxed{5/22}$$

2 2 **20 / 20**

✓ **- 0 pts** *Correct*

- 3 pts (a) incorrect

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- 5 pts incorrect and little/no work on (a)

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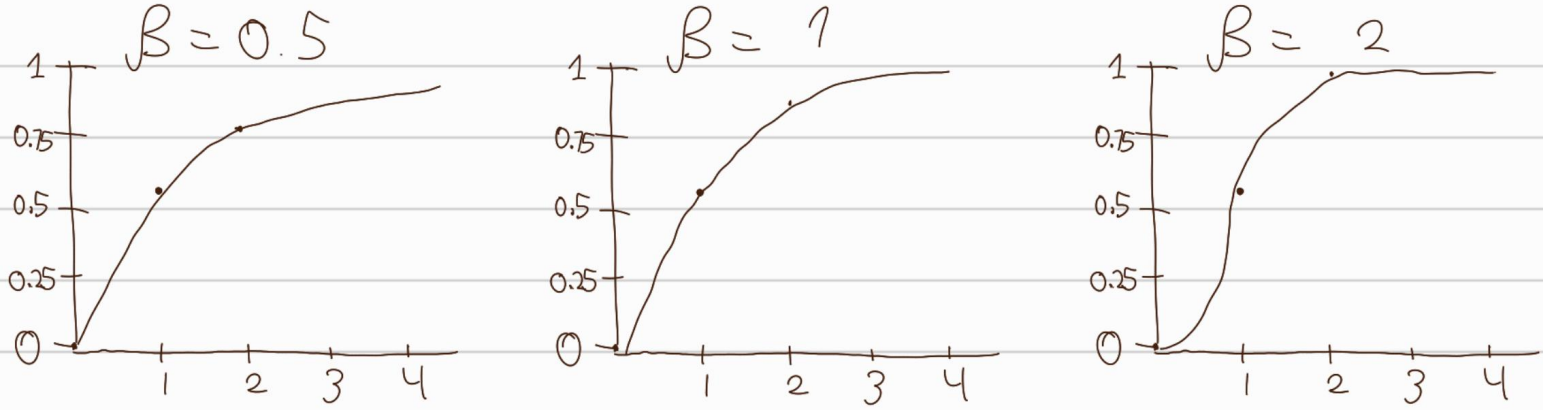
③ $\beta > 0$ $\lambda > 0$ &

CDF: $F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-(x/\lambda)^\beta} & x \geq 0 \end{cases} = \left(1 - e^{-(x/\lambda)^\beta}\right) u(x)$

ⓐ $\lambda = 1$, $\beta = 0.5, 1, 2$

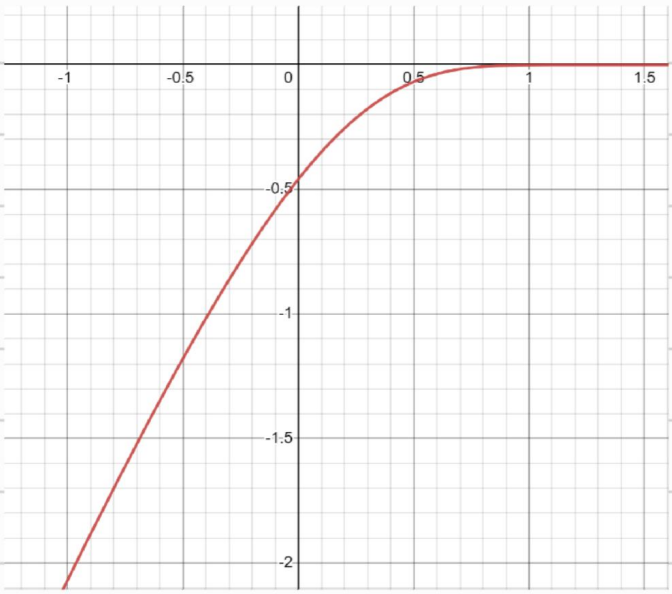


ⓑ

$P[k\lambda < X < (k+1)\lambda] = F_X(x)|_{x=(k+1)\lambda} - F_X(x)|_{x=k\lambda}$

$= 1 - e^{-(k+1)^\beta} - 1 + e^{-k^\beta}$

$= e^{-k^\beta} - e^{-(k+1)^\beta}$



$P(x > k\lambda)$

$= 1 - P(x < k\lambda)$

$= 1 - (1 - e^{-k^\beta})$

$= e^{-k^\beta}$

ⓒ $\ln(P(X > x)) = \ln(F_{\bar{X}}(x)) = \ln\left(\left(1 - e^{-(x/1)^2}\right)\right)$

vs $(\ln(x))$

3 3 **17 / 20**

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- **3 pts** (b) incorrect

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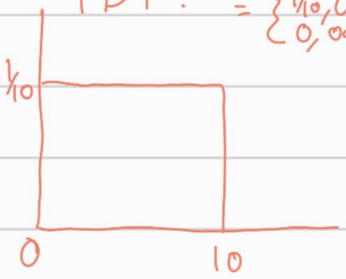
(4)

Let the point assignment be $H(x)$, where x represents the distance from the target, then

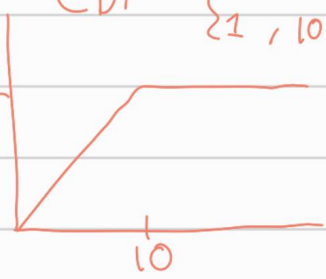
$$H(x) = \begin{cases} 10, & 0 \le x \le 1 \\ 7, & 1 < x \le 3 \\ 5, & 3 < x \le 5 \\ 3, & 5 < x \le 7 \\ 0, & \text{ow} \end{cases}$$

Probability of getting x distance from target = Uniform

PDF: $= \begin{cases} 1/10, & 0 < x \le 10 \\ 0, & \text{ow} \end{cases}$



CDF $= \begin{cases} 0, & x < 0 \\ x/10, & 0 \le x \le 10 \\ 1, & 10 < x \end{cases}$



Expectation of points $= E(H(X)) = \int_0^{10} H(x) \cdot f_X(x)\,dx$

$$= \int_0^{10} (1/10) H(x)\,dx = \int_0^1 (1/10)10\,dx + \int_1^3 (1/10)7\,dx + \int_3^5 (1/10)5\,dx + \int_5^7 (1/10)3\,dx$$

$$= (1-0) + (7/10)(3-1) + (5/10)(5-3) + (3/10)(7-5)$$

$$= 1 + 7/5 + 1 + 3/5 = (5+7+5+3)/5 = \boxed{4}$$

4 4 **20 / 20**

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(5)

(a)

TT	Sched	Where can you go
00	A	–
05	B	B
10		A | 10
15	A	A
20	B	B
25		A | 10
30	A	A
35	B	B
40		A
45	A	A | 10
50	B	B
55		A
00	A	A | 10

$= \frac{40}{60} = 2/3$

(b)

TT	Sched	Where can you go
10		–
15	A	A | 5
20	B	B
25		A | 10
30	A	A
35	B	B
40		A | 10
45	A	A
50	B	B
55		A | 10
00	A	A
05	B	B
10		A | 5

$= \frac{40}{60} = 2/3$

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