DTFS
Synthesis Equation:

$$\tilde{x}[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn}$$

Analysis Equation:

$$\tilde{c}_k = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn}$$

Sum of geometric sequence

$$\sum_{k=1}^{n} aq^{k-1} = \frac{a(1-q^n)}{1-q}$$

DTFT Cheat Sheet (Courtesy of Signals and Systems by Alkin, page 461-462)

Name	Signal	Transform
Discrete-time pulse	$x[n] = \begin{cases} 1\,, & \lvert n\rvert \le L \\ 0 & \text{otherwise} \end{cases}$	$X(\Omega) = \dfrac{\sin\left(\dfrac{(2L+1)\,\Omega}{2}\right)}{\sin\left(\dfrac{\Omega}{2}\right)}$
Unit-impulse signal	$x[n] = \delta[n]$	$X(\Omega) = 1$
Constant-amplitude signal	$x[n] = 1\,,\ \text{all}\ n$	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - 2\pi m)$
Sinc function	$x[n] = \dfrac{\Omega_c}{\pi}\,\text{sinc}\left(\dfrac{\Omega_c n}{\pi}\right)$	$X(\Omega) = \begin{cases} 1\,, & \lvert\Omega\rvert < \Omega_c \\ 0\,, & \text{otherwise} \end{cases}$
Right-sided exponential	$x[n] = \alpha^n\,u[n]\,,\ \lvert\alpha\rvert < 1$	$X(\Omega) = \dfrac{1}{1 - \alpha\,e^{-j\Omega}}$
Complex exponential	$x[n] = e^{j\Omega_0 n}$	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi m)$

Table 5.4 – Some DTFT transform pairs.

Theorem	Signal	Transform	
Linearity	$\alpha\,x_1[n] + \beta\,x_2[n]$	$\alpha\,X_1(\Omega) + \beta\,X_2(\Omega)$	
Periodicity	$x[n]$	$X(\Omega) = X(\Omega + 2\pi r)$	for all integers r
Conjugate symmetry	$x[n]$ real	$X^*(\Omega) = X(-\Omega)$	
		Magnitude:	$\lvert X(-\Omega)\rvert = \lvert X(\Omega)\rvert$
		Phase:	$\Theta(-\Omega) = -\Theta(\Omega)$
		Real part:	$X_r(-\Omega) = X_r(\Omega)$
		Imaginary part:	$X_i(-\Omega) = -X_i(\Omega)$
Conjugate antisymmetry	$x[n]$ imaginary	$X^*(\Omega) = -X(-\Omega)$	
		Magnitude:	$\lvert X(-\Omega)\rvert = \lvert X(\Omega)\rvert$
		Phase:	$\Theta(-\Omega) = -\Theta(\Omega) \mp \pi$
		Real part:	$X_r(-\Omega) = -X_r(\Omega)$
		Imaginary part:	$X_i(-\Omega) = X_i(\Omega)$
Even signal	$x[n] = x[-n]$	$\text{Im}\{X(\Omega)\} = 0$	
Odd signal	$x[n] = -x[-n]$	$\text{Re}\{X(\Omega)\} = 0$	
Time shifting	$x[n-m]$	$X(\Omega)\ e^{-j\Omega m}$	
Time reversal	$x[-n]$	$X(-\Omega)$	
Conjugation	$x^*[n]$	$X^*(-\Omega)$	
Frequency shifting	$x[n]\,e^{j\Omega_0 n}$	$X(\Omega - \Omega_0)$	
Modulation	$x[n]\,\cos(\Omega_0 n)$	$\frac{1}{2}\left[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)\right]$	
	$x[n]\,\sin(\Omega_0 n)$	$\frac{1}{2}\left[X(\Omega - \Omega_0)\ e^{-j\pi/2} + X(\Omega + \Omega_0)\ e^{j\pi/2}\right]$	
Differentiation in frequency	$n^m\,x[n]$	$j^m \dfrac{d^m}{d\Omega^m}\left[X(\Omega)\right]$	
Convolution	$x_1[n] * x_2[n]$	$X_1(\Omega)\ X_2(\Omega)$	
Multiplication	$x_1[n]\,x_2[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X_1(\lambda)\ X_2(\Omega - \lambda)\ d\lambda$	
Parseval's theorem	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(\Omega)\rvert^2\ d\Omega$		

Table 5.3 – DTFT properties.