

Monday, 1 March 2023
 10 points total.

Name: _____
 UID: _____

- (10 points) You are tasked with programming the motor of a robotic arm to perform various motions. Your task is to match a theoretical motion, y , with a motion, x , that closely matches it under the physical constraints of the motor. The constraint is that neighboring elements in x cannot have an absolute difference greater than 5 units (the motor cannot have a change in position greater than 5 over 1 discrete unit of time). The difference matrix is represented by the matrix D . An example of a theoretical and reachable motion plan is shown in Figure 1.

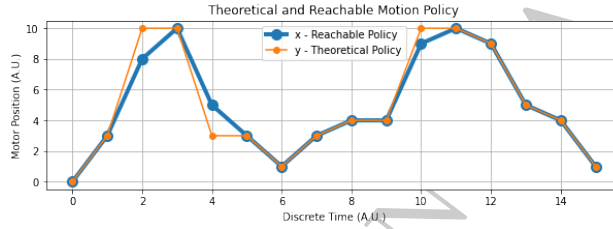


Figure 1: Example of a theoretical and reachable motion plan.

Pick the optimization problem from the multiple choice below that best finds the optimal x^* . Justify why it is the correct choice.

A.

$$x^* = \arg \min_x \|y - x\|_2$$

subject to $Dx \leq 5 \cdot \vec{1}$

B.

$$x^* = \arg \min_x \|y - Dx\|_2$$

subject to $Dx \leq 5 \cdot \vec{1}$

C.

$$x^* = \arg \min_x \|y - x\|_2$$

subject to $Dx \leq 5 \cdot \vec{1},$
 $Dx \geq -5 \cdot \vec{1}$

D.

$$x^* = \arg \min_x \|y - Dx\|_2$$

subject to $Dx \geq 5 \cdot \vec{1},$
 $Dx \leq -5 \cdot \vec{1}$

Solution:

The answer to the multiple choice question is **(C.)**.

(B.) and **(D.)** are incorrect because they are minimizing the distance between y and the derivative of x , Dx (also the shape of Dx would not match the shape of y).

(A.) forms the minimization correctly, but is missing a constraint. **(A.)**'s constraint only forces x to have differences between elements be less than 5, while we require that the 'absolute' difference between elements is less than 5. Therefore, it would not constrain differences that are less than -5 correctly.

(C.) is correct because it poses the minimization correctly. It minimizes the distance between x and y through an $L2$ norm, while making sure that the absolute difference between successive elements in x is less than 5 (or that differences in successive elements are greater than or equal to -5 or less than or equal to 5).

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