ECE113 Practice Final

1. **Problem 1**
   Consider the arbitrary signal $x[n]$ whose DTFT is $X(\Omega)$. Find the DTFT of the following signals as a function of $X(\Omega)$.

   (a) $x_1[n] = nx[n-1]$

   (b) $x_2[n] = e^{j\frac{\pi n}{2}}(x[n] * x[n])$

   (c) $x_o[n]$, the odd part of $x[n]$

2. **Problem 2**

(a) Find an expression for the inverse discrete time Fourier transform (DTFT) of:

$$X(\Omega) = cos^2(\Omega)$$

(b) Find the DTFT of:

$$x[n] = \frac{sinc(n/4)}{4}\frac{sinc(n/2)}{2}$$

3. **Problem 3** (20 points)
Let $x[n]$ be a signal with non-zero values from $n = 0, 1, \cdots, N-1$. Assume that $x[n] = 0$ for $n > N-1$ and for $n < 0$.
Let $y_M[n]$ be an M length finite version of $x[n]$.

$$y_M[n] = \begin{cases} x[n], & 0 \leq n < N \\ 0, & N \leq n < M \end{cases}$$

Show that the M point DFT of y[n] satisfies

$$Y_M[k] = X\left(\frac{2\pi k}{M}\right), \text{ for } k = 0, 1, ..., M-1.$$

where $X(\Omega)$ is the DTFT of x[n].

4. **Problem 4**

Assume $x[n] = cos(2\pi\frac{3}{10}n)$ is a $N = 10$ length signal. Similar to Problem 4, $y[n]$ is the zero-padded $x[n]$ signal to length $M = 20$.

(a) What is the DFT of x[n]? Plot the magnitude and phase.

(b) Describe how the DFT of $y[n]$ compares to the DFT of $x[n]$.

(c) Sketch the magnitude of the DFT of y[n]. (Just a sketch, you do not have to compute the DFT of y[n]).

(d) We now have another signal, $g[n] = cos(2\pi\frac{3.14159}{10}n)$ which is the same length as $x[n]$. Can you easily get the DFT of $g[n]$ by hand as you did in part (a) without a calculator? Justify why you can or cannot.

(e) Sketch the magnitude of the DFT of $g[n]$, $|G[k]|$ and comment on the features that make it look different from the magnitude spectrum of $|X[k]|$.

5. **Problem 5**

Consider the z transform of the signal $h[n]$:

$$H(z) = \frac{(1 - z^{-1})(1 + z^{-1})}{(1 - 0.9e^{\mathrm{j}\frac{5\pi}{6}} z^{-1})(1 - 0.9e^{-\mathrm{j}\frac{5\pi}{6}} z^{-1})(1 - 0.9e^{\mathrm{j}\frac{2\pi}{3}} z^{-1})(1 - 0.9e^{-\mathrm{j}\frac{2\pi}{3}} z^{-1})}$$

Plot the poles and zeros for $H(z)$ on the z-plane. Also, plot the approximate DTFT/DFT spectrum for the sequence sequences $h[n]$.