ECE113, Winter 2023 **Homework #3.5**
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Due day, 13 Feb 2023, by 11:59am to Gradescope. **(Optional & Unscored)**
30 points total.

Note: Unless specified, you are free to use any of the properties of DTFT that were taught in class. Also, all repeated derivations can be referenced with appropriate equation/result numbers.

1. (10 points) Let $x[n] = 1 + e^{j\omega_0 n}$ and $y[n] = 1 + \frac{1}{2}e^{j4\omega_0 n} + \frac{1}{2}e^{j3\omega_0 n}$ be two signals with a fundamental period N, such that $\omega_0 = \frac{2\pi}{N}$.

   Find the DTFS coefficients of their product $z[n] = x[n]y[n]$, assuming $N = 3$.

2. (10 points) Let $x[n] = 3^n$.

   a.) Show that the DTFT does not exist.

   b.) Evaluate whether the DTFT exists for the following modifications to $x[n]$. If they exist, compute DTFT:

   i.) $x[n]\ u[n]$
   ii.) $x[-n]\ u[n]$
   iii.) $x[-|n|\ ]$
   iv.) $x[n]\ u[-n]$

3. (10 points) Let $x[n] = sin(\Omega_o n)$.

   (a) Derive the DTFT of $x[n]$.

   (b) Write the real and imaginary part of $X(\Omega)$ as well as its magnitude and phase.