Wednesday, 15 March 2023

10 points total.

Name: _____

UID: _____

1. (10 points) You are given the following Z-transform pairs and properties:

Mini Z-transform table		
Time-domain: x[n]	Frequency Domain: X(z)	ROC: R_x
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$|z| > |\alpha|$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$|z| < |\alpha|$
$x[n-n_o]$	$z^{-n_o} X(z)$	R_x, except $z = 0$ if $n_o > 0$

Find the inverse Z-transform of the following transform:

$$X(Z) = \frac{2z^{-2}}{1 - \frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2} \tag{1}$$

Solution:

To get the inverse Z-transform using the table, we have to get $X(z)$ into the form of one of the transform pairs.

$$X(z) = 2 \cdot z^{-2} \cdot \frac{1}{1 - \frac{1}{2}z^{-1}} = 2 \cdot z^{-2} \cdot G(z) \tag{2}$$

The last term is clearly the Z-transform for the first pair in the table, given the ROC. Then we can use this information along with the time-shifting property to get the final answer:

$$x[n] = 2 \cdot g[n-2] = 2 \cdot \left(\frac{1}{2}\right)^{n-2} \cdot u[n-2] = \left(\frac{1}{2}\right)^{n-3} u[n-2] \tag{3}$$