ECE113, Winter 2023 **Homework #4**
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Due Friday, 3 March 2023, by 11:59pm to Gradescope.
50 points total.

Submission instructions: Generate a .pdf file for the written problems and .pdf file of the jupyter notebook from the coding portion (the discussion video shows how to do this). Merge the 2 pdf files into one and submit to it to the assignment called HW4 on Gradescope. Additionally, please put your edited "dsp_toolbox.py" and "hw4.ipynb" files into a .zip file and submit to the assignment called HW4_code on Gradescope.
Note: Problems appearing in blue are practice problems and are not graded.

1. (5 points) Determine the DFT of the following sequences:
   a.) $x[n] = sin(2\pi\frac{1}{8}n)$ for n = [0,1,...,7]
   b.) $y_1 = [0, -1, 0, 1]$
   c.) $y_2 = [j, 0, j, 1]$
2. (5 points) Plot the the magnitude and phase of the function y_3 in the frequency domain. Where $y_3 = y_1 \circ y_2$ which is the circular convolution of the functions y_1 and y_2.
3. (40 points) Coding Assignment Instructions:
   (a) Find the hw4_coding.zip file on BruinLearn. Download and unzip it.
   (b) The zip contains three files. The main files you will be editing are "hw4.ipynb" and "dsp_toolbox.py". The last file is there for utility only.
   (c) Begin with the Jupyter notebook and follow through the sections linearly (some sections may depend on previous ones). The Jupyter notebook will ask you to implement empty functions in the "dsp_toolbox.py" file. Make sure you only make changes in the code where specifically asked of you by Python comments.
   (d) A discussion video will be released before Monday's lecture that goes over the coding assignment instructions in more details as well as installation instructions if you do not have the Numpy or Matplotlib Python libraries.

Practice Problems

1. (a) Derive the DFT of a sinusoid with an arbitrary frequency, $x[n] = sin(2\pi\frac{f_o}{N}n)$. ($f_o$ can be any value and is not necessarily a rational number).
   (b) The solution to part (a.) looks familiar, as if it were the Fourier transform of the multiplication of two functions in the time domain. What are these functions? What is the connection between this derivation and the leaky DFT spectrums we saw in Part 3.1 of the "hw4_coding" assignment?

2. Evaluate the following expressions that are a function of the arbitrary sequence $x[n]$. Your final answer should be in a functional form such that $X[k]$ or $x[n]$ is in the expression.

   (a) $DFT(DFT(DFT(DFT(x[n]))))$

   (b) $DFT(x[-n])$