ECE 102, Fall 2022 **Homework #3**

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Due Friday, 21 Oct 2022, by 11:59pm to Gradescope.
Covers material up to Lecture 6.
100 points total.

1. (15 points) **Linear systems**

   Determine whether each of the following systems is linear or not where the input is $x(t)$ and the output is $y(t)$. Explain your answers.

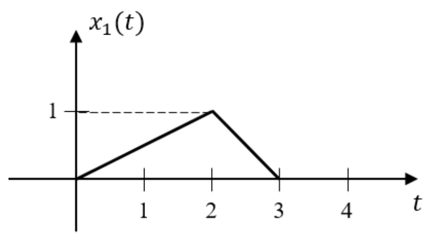
   (a) $y(t) = x(t)e^{-jwt}$ (5 points)

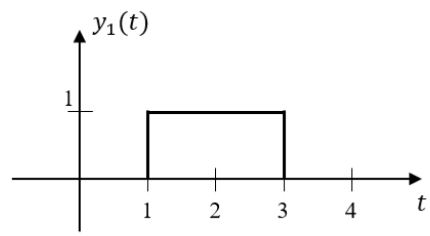
   (b) $y(t) = \int_{-\infty}^{\infty} [x(t)]^2 + x(t)dt$ where $x(t)$ is real (5 points)

   (c) $y(t) = e^{x(t)}$ (5 points)

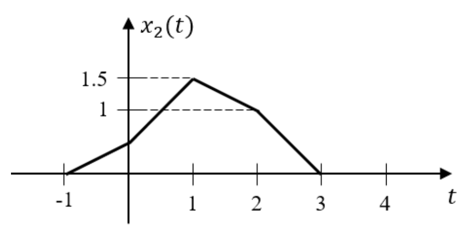
2. (20 points) **LTI (Linear Time-Invariant) systems**

   (a) (10 points) Consider an LTI system whose response to $x_1(t)$ is $y_1(t)$.





   Sketch the response of the system to $x_2(t)$.



(a) $x_2(t)$

   (b) (10 points) Assume we have an LTI system whose output is $a^t \cos(t)$ when the input is $u(t)$. What is the system output when the input is $0.5[\delta(t+1) + \delta(t-1)]$? Is this system causal?

3. (36 points) **Convolution**

   (a) (12 points) Compute the convolution integral for each pair of signals below.

      i. $f(t) = \delta(t+1) + 5\delta(t-2),\ g(t) = e^{-t}u(t)$ (2 points)

      ii. $f(t) = 2\mathrm{rect}(t-\frac{3}{2}),\ g(t) = 2r(t-1)\mathrm{rect}(t-\frac{3}{2})$ (10 points)

   (b) (12 points) Find the impulse response $h(t)$ for a system whose input-output relationship is described as $y(t) = \int_{t-T}^{t}(t-\tau)^2 x(\tau)d\tau$.

   (c) (12 points) Simplify the following expressions:

      i. $e^t * \sum_{k=0}^{\infty} \delta(t-k)$ Where $*$ means convolution (6 points)
      Hint: Geometric progression is of the form $a, ar, ar^2, ar^3, \ldots$ and the sum of these elements is $\frac{a}{1-r}$.

      ii. $\frac{d}{dt}\{[u(t) - u(t-1)] * u(t-2)\}$ (6 points)
      Hint: First show that $u(t) * u(t) = r(t)$ where $r(t)$ is the ramp function.

4. (12 points) **LTI Systems and impulse response**

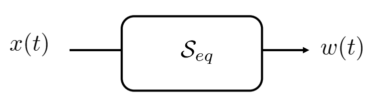
   Consider the following three LTI systems:

   - $\mathcal{S}_1$: $y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)}x(\tau)d\tau$;
   - $\mathcal{S}_2$: $y(t) = \int_{-\infty}^{t-2} x(\tau)d\tau$
   - $\mathcal{S}_3$ is characterized by its impulse response: $h_3(t) = \delta(t-3)$.

   (a) (4 points) Compute the impulse response $h_1(t)$ of $\mathcal{S}_1$.

   (b) (2 points) Define $w(t) = \mathcal{S}_1[x(t)] - \mathcal{S}_3\{\mathcal{S}_2[x(t)]\}$. Represent this relationship using a block diagram where $x(t)$ is the input and $w(t)$ is the output.

   (c) (2 points) Determine the impulse response $h_{eq}(t)$ of the above system.



   (d) (4 points) Determine the response of the overall system to $\delta(t) + 2\delta(t-3)$.

5. (17 points) **Python tasks**

   We provide a helper function `nconv()` as defined below:

```
import numpy as np

def nconv(x, tx, h, th):
    y = np.convolve(x, h) * (th[1] - th[0])
    ty = np.linspace(tx[0] + th[0], tx[-1] + th[-1], len(y))
    return y, ty
```

where the inputs are:

`x` : input signal vector
`tx`: times over which `x` is defined
`h` : impulse response vector
`th`: times over which `h` is defined

and the outputs are:

`y` : output signal vector
`ty`: times over which `y` is defined.

The function is implemented using numpy's `convolve()` function Link.

(a) (10 points) Use `nconv()` to check your result for problem 3(a)(ii) and plot the output. Use the same step size for `tx` and `th` and label the plots.

(b) (7 points) Use `nconv()` to convolve two unit rectangles: $\mathrm{rect}(t) * \mathrm{rect}(t)$. Plot the result and label the axes.