ECE102, Fall 2022 — **Homework #4**

Signals & Systems — Prof. Jonathan C. Kao

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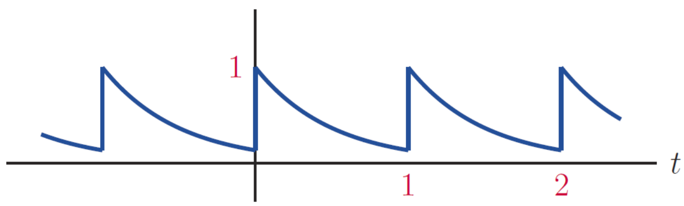
Due Friday, 28 Oct 2022, by 11:59pm to Gradescope.
Covers material up to Lecture 8.
100 points total.

1. (28 points) **Fourier Series**

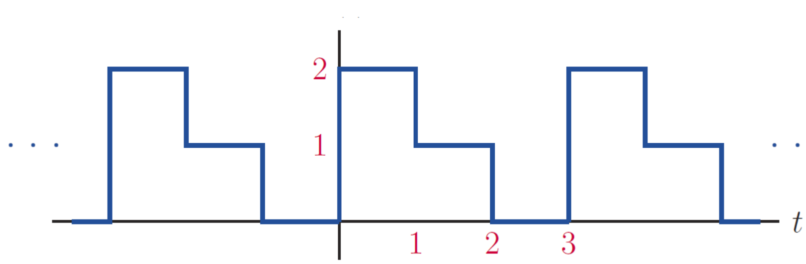
   (a) (18 points) Find the Fourier series coefficients for each of the following periodic signals:

      i. (9 points) $f(t) = \sin(7\pi t) + \frac{1}{3}\cos(4\pi t)$

      ii. (9 points) $f(t)$ is a periodic signal with period $T = 1$ s, where one period of the signal is defined as e^{-t} for $0 < t < 1$ s, as shown below.



      iii. (**Optional**) (0 points) $f(t)$ is the periodic signal shown below:



   (b) (10 points) Suppose you have two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 respectively. Let x_k and y_k be the Fourier series coefficients of $x(t)$ and $y(t)$.

      i. (5 points) If $T_1 = T_2$, express the Fourier series coefficients of $z(t) = 3x(t) + 2y(t)$ in terms of x_k and y_k.

      ii. (5 points) If $T_1 = 2T_2$, express the Fourier series coefficients of $w(t) = x(t) + y(t)$ in terms of x_k and y_k.

2. (20 points) **Fourier series of transformation of signals**

   (a) (15 points) Suppose that $f(t)$ is a periodic signal with period T_0, with the following Fourier series:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Determine the period of each of the following signals, then express its Fourier series in terms of c_k:

i. (5 points) $g(t) = 3f(t)$

ii. (5 points) $g(t) = f(-at)$ where a is a positive real number.

iii. (5 points) $g(t) = f(t - t_0)$

(b) (5 points) Given two periodic signals and their corresponding Fourier series representation as follows:

$$x_1(t) = \sum_{k=-100}^{100} cos(k\pi) e^{jk\frac{2\pi}{50}t}$$

$$x_2(t) = \sum_{k=-100}^{100} jsin(\frac{k\pi}{2}) e^{jk\frac{2\pi}{50}t}$$

Identify whether the signals is/are even.

3. (10 points) **Eigenfunctions and LTI systems**

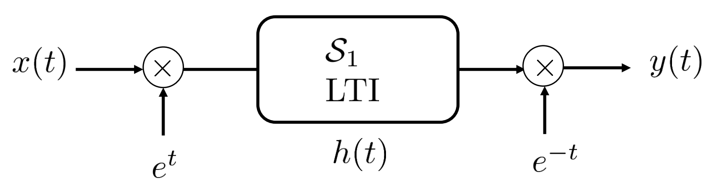
Determine if the following signals are Eigenfunctions of LTI systems

(a) (5 points) $f(t) = t^2$

(b) (5 points) $f(t) = e^{jwt}u(t)$

4. (29 points) **LTI systems**

(a) Consider the following system:



The system takes as input $x(t)$, it first multiplies the input with e^t, then sends it through an LTI system. The output of the LTI system gets multiplied by e^{-t} to form the output $y(t)$.

i. (5 points) Show that we can write $y(t)$ as follows:

$$y(t) = \left[\left(e^t x(t)\right) * h(t)\right] e^{-t} \tag{1}$$

ii. (5 points) Use the definition of convolution to show that (1) can be equivalently written as:

$$y(t) = \int_{-\infty}^{\infty} h'(\tau) x(t - \tau) d\tau \tag{2}$$

where $h'(t)$ is a function to define in terms of $h(t)$.

iii. (5 points) Equation (2) represents a description of the equivalent system that maps $x(t)$ to $y(t)$. Show using (2) that the equivalent system is LTI and determine its impulse response $h_{eq}(t)$ in terms of $h(t)$.

(b) Suppose $x(t)$ is periodic with period T and is specified in the interval $0 < t < T/4$ as shown in figure 1.

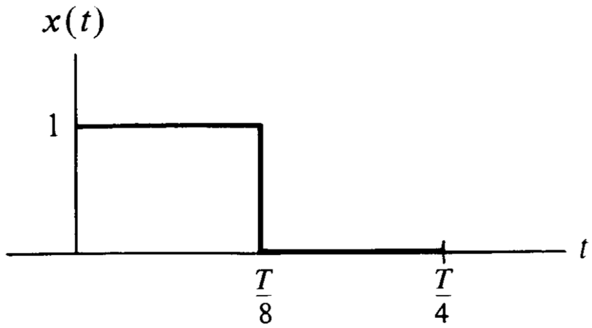


Figure 1: $x(t)$ in the interval $0 < t < T/4$

Sketch $x(t)$ in the interval $0 < t < T$ if

i. (7 points) the Fourier series has only odd harmonics and $x(t)$ is an even function

ii. (7 points) the Fourier series has only odd harmonics and $x(t)$ is an odd function

5. (13 points) **Python**

(a) (6 points) **Task 1**

Write an python function that takes a set of Fourier series coefficients, a fundamental frequency, and a vector of output times, and computes the truncated Fourier series evaluated at these times. The declaration and help for the python file might be:

```
  def myfs(Dn,omega0,t):
'''
  Evaluates the truncated Fourier Series at times t

  Dn     -- vector of Fourier series coefficients
  omega0 -- fundamental frequency
  t      --  vector of times for evaluation
'''
fn = myfs(Dn, omega0, t)
'''
  fn     --  truncated Fourier series evaluated at t}
'''
```

The output of the python function should be

$$f_N(t) = \Sigma_{n=-N}^{N} D_n e^{j\omega_0 nt}$$

The length of the vector Dn should be $2N + 1$. You will need to calculate N from the length of Dn.

(b) (7 points) **Task 2**

Verify the output of your routine by checking the Fourier series coefficients for the sawtooth waveform. The sawtooth signal is given by f(t) = t mod 1 described in the class notes. Try for $N = 10$, $N = 50$. Use the Python subplot command to put multiple plots on a page.

(c) (Optional) (0 points) **Task 3**

Repeat the steps of Task 2 for the case of the signal from Problem 1-a-ii. Also plot for N =100 and observe change in the output with higher N.