23W-EC ENGR-113-LEC-1 HW3

SANJIT SARDA

TOTAL POINTS

50 / 50

QUESTION 1

1 1 **10 / 10**

+ 0 pts No Correct Work

+ 5 pts Some Correct Work

✓ **+ 10 pts** *Fully Correct*

QUESTION 2

2 2 **10 / 10**

+ 0 pts No Correct Work

+ 4 pts a

+ 3 pts b

+ 3 pts c

✓ **+ 10 pts** *Fully Correct*

+ 5 pts Partial

QUESTION 3

3 3 **10 / 10**

+ 0 pts No Correct Work

+ 5 pts Some Correct Work

✓ **+ 10 pts** *Fully Correct*

QUESTION 4

4 4 **10 / 10**

+ 0 pts No Correct Work

+ 5 pts i

+ 5 pts ii

✓ **+ 10 pts** *Fully Correct*

QUESTION 5

5 5 **10 / 10**

+ 0 pts No Correct Work

✓ **+ 10 pts** *Correct*

+ 5 pts Partial Credit

ECE 113 HW#3

(1) $x[n] = \{2, -3, 4, 1\}, -1 \le n \le 2,$ $\quad -2 \le n+1 \le 1$, $\quad -3 \le n \le 0$

$h[n] = \{-3, 5, -6, 4\}, -2 \le n \le 1.$

Testing

$\sum_{k=-4}^{4} x[k]h[n-k]$

$=$

Normalizing,

$x[n] = \{0, 2, -3, 4, 1\}, \; -2 \le n \le 2 \qquad L=4$

$h[n] = \{-3, 5, -6, 4, 0\} \; -2 \le n \le 2 \qquad L=4$

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Find $y[-1]$ = We just need to evalute the summation, where $n=-1$

$$\therefore y[-1] = \sum_{k=-2}^{p} x[k]h[-1-k]$$

p represents where evaluated index $-k >$ lower bound of h
since we are flipping

$\therefore -1 - k > -2$

$\therefore k+1 < 2$

$\therefore k < 1$

$\therefore$ p means $k = 1$

$$= x[-2]h[\overset{1}{-1--2}] + x[-1]h[\overset{0}{-1--1}] + x[0]h[\overset{-1}{-1-0}]$$

$$+ x[1]h[\underset{-2}{-1-1}] = 0 \cdot 4 + 2 \cdot -6 + -3 \cdot 5 + 4 \cdot -3$$

$$= -12 - 15 - 12 = -39$$

1 1 **10 / 10**

+ 0 pts No Correct Work

+ 5 pts Some Correct Work

✓ **+ 10 pts** *Fully Correct*

②

ⓐ $x_1[n] = \{1, -1, 1\}, \quad -1 \leq n \leq 1$

$x_1[n] * x_1[n] = \{1, -2, 3, -2, 1\}, \ -2 \leq n \leq 2$

ⓑ $x_2[n] = \{1, -1, 0, 1, -1\}, 0 \leq n \leq 4$

$\therefore \ x_2[n] * x_2[n] = \{1, -2, 1, 2, -4, 2, 1, -2, 1\}, 0 \leq n \leq 8$

ⓒ $x_3[n] * x_3[n] = \{-1, 2, 0, -2, 1\}, -3 \leq n \leq 1$

$x_3[n] * x_3[n] = \{1, -4, 4, 4, -10, 4, 4, -4, 1\}, -6 \leq n \leq 2$

2 2 **10 / 10**

- **+ 0 pts** No Correct Work
- **+ 4 pts** a
- **+ 3 pts** b
- **+ 3 pts** c
- ✓ **+ 10 pts** *Fully Correct*
- **+ 5 pts** Partial

③ $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$x[n] = 2^n u[-n]$

$$H(x[n]) = h[n] * x[n]$$

$$= \sum_{k=-\infty}^{k=\infty} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k]\, 2^{n-k} u[k-n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} 2^{-k} u[k]\, 2^{n-k} u[n-k]$$

$$= \sum_{k=-\infty}^{\infty} 2^{-k+n-k}\, u[n-k]\, u[k]$$

$n < 0$

$$= 2^n \sum_{k=0}^{\infty} 2^{-2k} = 2^n \sum_{k=0}^{\infty} (2^{-2})^k$$

$$= 2^n \cdot \frac{1}{1-2^{-2}} = \frac{2^{n+2}}{3}$$

$n \geq 0$

$$= \sum_{k=-\infty}^{0} 2^{k-n} = 2^{-n} \sum_{k=0}^{\infty} (2^{-2})^k$$

$$= 2^{-n} \cdot \frac{1}{2-2^{-2}} = \frac{2^{2-n}}{3}$$

$$y[n] = \begin{cases} \frac{2^{n+2}}{3}, & n < 0 \\ \frac{2^{2-n}}{3}, & n \geq 0 \end{cases}$$

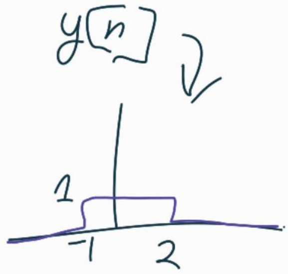
3 3 **10 / 10**

+ 0 pts No Correct Work

+ 5 pts Some Correct Work

✓ **+ 10 pts** *Fully Correct*

④ $x[n]=\{3,2,1,1,3,3,2,1,1\}, -2\le n\le 6$



(i) $y[n]=u[n+1]-u[n-2] = \{1,1,1\}, -1\le n\le 2$

$x[n]*y[n]=\{3,5,6,4,5,7,8,6,4,2,1\}, -3\le n\le 7$

(ii) $h_1[n]=\left(\frac{1}{2}\right)^n h[n]u[n]$, $h[n]=\frac{1}{2}x[n+2]-\frac{3}{2}\delta[n]+u[n-3]$

$$y[n]=h_1[n]*\left(\left(\frac{1}{3}\right)^n u[n]\right)$$

$$=\left(\frac{1}{2^{n+1}}x[n+2]u[n]-\frac{3}{2}\delta[n]u[n]+\frac{1}{2^n}u[n-3]u[n]\right)*\left(\left(\frac{1}{3}\right)^n u[n]\right)$$

$$=\underbrace{\left(\frac{1}{2^{n+1}}x[n+2]u[n]\right)*\left(\left(\frac{1}{3}\right)^n u[n]\right)}_{PT1}-\underbrace{\left(\frac{3}{2}\delta[n]u[n]\right)*\left(\left(\frac{1}{3}\right)^n u[n]\right)}_{PT2}+\underbrace{\left(\frac{1}{2^n}u[n-3]u[n]\right)*\left(\left(\frac{1}{3}\right)^n u[n]\right)}_{PT3}$$

@PT1: $n<0: 0$

else

$$\frac{1}{2^{n+1}}x[n+2]u[n]*\left(\left(\frac{1}{3}\right)^n u[n]\right)=\sum_{k=-\infty}^{\infty}\frac{1}{2^{k+1}}x[k+2]u[k]\left(\left(\frac{1}{3}\right)^{n-k}u[n-k]\right)$$

$$=\left(\frac{1}{3}\right)^n\sum_{k=0}^{n}2^{-k-1}3^k x[k+2]$$

@PT2:

$$\delta[n]*\left(\left(\frac{1}{3}\right)^n u[n]\right)=\left(\frac{1}{3}\right)^n u[n]$$

@PT3: $n<3: 0$

else

$$\left(2^{-n}u[n-3]\right) * \left(3^{-n}u[n]\right) = \sum_{k=-\infty}^{\infty} 2^{-k}u[k-3]\,3^{-k+n}u[n-k]$$

$$= \sum_{k=3}^{n} 2^{-k}3^{k-n} = 3^{-n}\sum_{k=3}^{n}\left(\frac{3}{2}\right)^{k} = -2\left(3^{-n}\right)\left(\left(\frac{3}{2}\right)^{n} - \left(\frac{3}{2}\right)^{2}\right)$$

$$\therefore y[n] = \begin{cases} 0 & , n<0 \\ -\frac{3}{2}\left(\frac{1}{3}\right)^{n} + \left(3^{-n}\right)\sum_{k=0}^{n} 2^{-k-1}3^{k}x[k+2] & , 0\le n<3 \\ -\frac{3}{2}\left(\frac{1}{3}\right)^{n} - 2\left(3^{-n}\right)\left(\left(\frac{3}{2}\right)^{n} - \frac{9}{4}\right) + 3^{-n}\sum_{k=0}^{4} 2^{-k-1}3^{k}x[k+2] & , n\ge 3 \end{cases}$$

4 4 **10 / 10**

+ 0 pts No Correct Work

+ 5 pts i

+ 5 pts ii

✓ **+ 10 pts** *Fully Correct*

⑤ $y[n] = x[n] * h[n]$

$$x[n] = f[n]\left(u[n-n_1] - u[n-n_2]\right)$$

&

$$h[n] = g[n]\left(u[n-n_3] - u[n-n_3]\right)$$

aka $x[n]$ is non neg between n_1 & n_2

$h[n]$ - - \ \ . . . , n_3 & n_4

ⓐ It happens @ $n - n_1 - n_3 = 0$

$\therefore n = n_1 + n_3$

ⓑ It happens @ $n - (n_2 - 1) - n - (n_4 - 1) = 0$

$\therefore n = n_2 + n_4 - 2$

$\therefore$ Length $= n_2 + n_4 - 2 - n_1 - n_3 + 1$

$= n_x + n_h - 1$

5 5 **10 / 10**

+ 0 pts No Correct Work

✓ **+ 10 pts** *Correct*

+ 5 pts Partial Credit