EC ENGR 131A
Probability and Statistics
Instructor: Lara Dolecek

Winter 2023
Thursday, January 26th, 2023
Exam 1 - Version 3

Maximum score is 100 points. You have 110 minutes to complete the exam. Please show your work. Good luck!

Your Name: Sanjit Sarda

Your ID Number: 805964031

Name of person on your left: Kevin Zhou

Name of person on your right: Maya Charathi

Problem	Score	Possible
1	20	20
2	15	15
3	20	20
4	10+15	25
5	20	20
Total	100	100

~~$-100 \quad 0 \quad 100 \quad (100)^2 \quad 2000000$~~

~~$\frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4}$~~

1. Suppose X has the following PMF. **Show all your work.**

$$X = \begin{cases} -2, & \text{with probability } 1/6, \\ -1, & \text{with probability } 2/6, \\ 1, & \text{with probability } 2/6. \\ 2, & \text{with probability } 1/6. \end{cases} \tag{1}$$

(a) Compute $\mathbf{E}[X]$. (10 points)

(b) Compute $Var(X)$. (10 points)

a) $\mathbb{E}[X] = \sum_{S_X} x_i \cdot P(X=x_i) = -2\cdot\frac{1}{6} + -1\cdot\frac{2}{6} + 1\cdot\frac{2}{6} + -2\cdot\frac{1}{6} = \boxed{0}$ ✓

b) $Var(X) = \sum_{S_X} (x_i - m_x^2)\cdot P(X=x_i) = (-2)^2\cdot\frac{1}{6} + (-1)^2\cdot\frac{2}{6} + (1)^2\cdot\frac{2}{6} + (2)^2\cdot\frac{1}{6}$

$= \frac{4}{6} + \frac{2}{6} + \frac{2}{6} + \frac{4}{6} = \boxed{2}$ ✓

20

2. There are two identical boxes B_1 and B_2. B_1 contains 4 red balls and 6 blue balls; and B_2 contains 2 red balls and 8 blue balls. A box is selected at random, and then we pick up a ball from the selected box. What is the probability that the chosen ball is red? (15 points)

$B_1 \to \{RRRRBBBBBB\}$

$B_2 \to \{RRBBBBBBBB\}$

A box is selected at Random. Sum of all Disjoint Pairs in Sample Space

Sample Space

Since there are only 2 Boxes $P(B_1)+P(B_2)=1$

Since Identical $P(B_1)=P(B_2)=\frac{1}{2}$

$P(R|B_1)=\frac{4}{4+6}=\frac{2}{5}$

$P(R|B_2)=\frac{2}{2+8}=\frac{1}{5}$

$\therefore P(R)=P(R|B_2)P(B_2)+P(R|B_1)P(B_1)$

$=\frac{1}{5}\cdot\frac{1}{2}+\frac{2}{5}+\frac{1}{2}=\frac{3}{10}$ ✓

15

325 $\frac{1}{16}$ $\frac{36}{625}$???? $\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{2}{5}\cdot\frac{2}{5}$ $+\frac{3}{5}$

3. A biased coin with probability of tossing a heads being $\frac{3}{5}$, is tossed 4 times.

   (a) List all outcomes in the sample space S. (4 points) 4

   (b) What is the probability of getting exactly 2 heads? (8 points) 8

   (c) What is the probability of getting at least 2 heads? (8 points) 8

You may leave your answer as a fraction/sum of fractions.

$P(H) = \frac{3}{5}$ $\therefore P(T) = 1 - P(H) = 2/5$

ⓐ Tossed 4 Times $\therefore$ Outcomes : ~~{H, H, H, H} {H, H, H, T} {~~

(H, H, H, H); (H, H, H, T); (H, H, T, H); (H, H, T, T);
(H, T, H, H); (H, T, H, T); (H, T, T, H); (H, T, T, T);
(T, H, H, H); (T, H, H, T); (T, H, T, H); (T, H, T, T);
(T, T, H, H); (T, T, H, T); (T, T, T, H); (T, T, T, T);

4 tosses.

$|S| = 2^4 = 16$

H/T

(HHTT); (HTHT); (HTTH)
(THHT); (THTH); (TTHH)

ⓑ From Counting, ~~2 heads~~ the event: 2 heads, happens 6 times

$\therefore P(2\text{heads}) = P(HHTT) + P(HTHT) + P(HTTH) \ldots$

$= \frac{3}{5}\cdot\frac{3}{5}\cdot\frac{2}{5}\cdot\frac{2}{5} + \frac{3}{5}\cdot\frac{2}{5}\cdot\frac{3}{5}\cdot\frac{2}{5} + \frac{3}{5}\cdot\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{3}{5} \ldots$

$= \frac{3^2 2^2}{5^4} \cdot 6\text{ possible ways} = \boxed{\frac{216}{625}} = P(2\text{ heads})$

ⓒ Also from counting, the event at least 2 heads happens 11 times

$\therefore P = \frac{11}{16}$

$P(\text{at least } 2\text{ heads}) = P(HHTT)\cdot 6 + P(HHHT)\cdot 4 + P(HHHH)$

$= \frac{3^2 2^2}{5^4}\cdot 6 + \frac{3^3 2^1}{5^4}\cdot 4 + \frac{3^4 2^0}{5^4}\cdot 1$

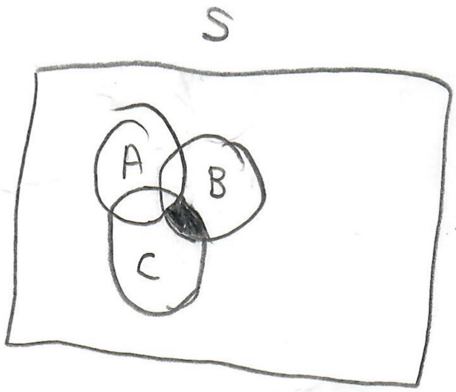
$= \frac{216 + 54 + 81}{5^4} = \boxed{\frac{351}{625}}$

4. Answer the following. **Show all your work.**

(a) For 3 events A, B, and C defined on the sample space S, draw the Venn diagram and find the expression for "B and C occur, but A does not". Use only complements, intersections, and unions. (10 points) 10

(b) Suppose $P(A)$ is 4/10, $P(B)$ is 5/10, and $P(A \cup B) = 6/10$. Compute $P(A \cap B)$, $P(A|B)$, and $P(B|A)$. (15 points)

a)



$= \cancel{B \cap C \cap A^c}$ $B \cap C \cap A^c$

b) $P(A) = \frac{4}{10}$, $P(B) = \frac{5}{10}$

$P(A \cup B) = \frac{6}{10} = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{4}{10} + \frac{5}{10} - \frac{6}{10} = \frac{3}{10}$ ✓

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{10} \cdot \frac{10}{5} = 3/5$ ✓

$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{3}{10} \cdot \frac{10}{4} = 3/4$ ✓

15

$\binom{k}{c} = \frac{k!}{c!(k-c)!}$ 2

$\frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$

$\binom{k}{c} = \leftarrow \boxed{6}$ to account for order

5. Suppose you are taking a test with 12 multiple choice questions where each question can be answered by one of 7 different choices.

reps correct

(a) In how many ways can you answer this test? (4 points) 4

(b) What is the probability that you answer 5 of the questions correctly? (8 points) 8

(c) What is the probability that you answer 10 of the questions correctly? (8 points) 8

You may leave answers as fractions and unexpanded binomial coefficients.

Smaller example samplespace

4mcq = Kmcq
3choices = nchoices
P(2 correct) = c correct

from all nets = $3^4 = n^k$

to answer c/k

$1/n$
$1/n$
$(n-1)/n$
$(n-1)/n$

1 2 3 4 5 6

$\boxed{a}$ 12mcq, 7choices

$\therefore$ # of ways to answer test $= \boxed{7^{12}}$

this means unordered

ways to get first c correct & next k-c wrong

$\frac{1}{n^k} \cdot 1^c \cdot (n-1)^{k-c}$

$= \frac{(n-1)^{k-c}}{n^k}$

Showing possible ways to get diff combinations of c correct

$\boxed{b}$ We need exactly 5/12, with 7 choices.

$P = \frac{\text{\# ways to get exactly 5/12}}{\text{\# ways to answer test}} = \frac{}{7^{12}}$

ways to get exactly 5/12 = (# number of ways to get the first 5 correct & other 7 wrong) · # ways to arrange 5 different questions as correct.

$= (7-1)^{12-5} \cdot \binom{12}{5} = 6^7 \binom{12}{5}$

$\therefore P = \text{\# ways to get 5/12} \cdot \frac{1}{\text{\# ways to answer test}} = \left(6^7 \binom{12}{5}\right) \cdot \left(\frac{1}{7^{12}}\right) = \boxed{\frac{6^7}{7^{12}} \binom{12}{5}}$

$\boxed{c}$ Similarly $P = \frac{(7-1)^{12-10}}{7^{12}} \cdot \binom{12}{10}$

$P = \boxed{\frac{6^2}{7^{12}} \cdot \binom{12}{10}}$