**ECE113, Winter 2023** **Homework #1**

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Due Friday, 27 Jan 2023, by 11:59pm to Gradescope.
50 points total.

1. (10 points) Consider the following sequences:

$$x[n] = \{2, \quad 0, \quad -1, \quad 6, \quad -3, \quad 2, \quad 0\}, -3 \le n \le 3,$$
$$y[n] = \{8, \quad 2, \quad -7, \quad -3, \quad 0, \quad 1, \quad 1\}, -5 \le n \le 1,$$
$$w[n] = \{3, \quad 6, \quad -1, \quad 2, \quad 6, \quad 6, \quad 1\}, -2 \le n \le 4.$$

The sample values of each of the above sequences outside the ranges specified are all zeros. Generate the following sequences:

(a) $c[n] = x[n+3]$,

(b) $d[n] = y[n-2]$,

(c) $e[n] = x[-n]$

(d) $u[n] = x[n-3] + y[n+3]$

(e) $v[n] = y[n-3] \cdot w[n+2]$,

(f) $s[n] = y[n+4] - w[n-3]$, and

(g) $r[n] = 3.9w[n]$

**Solution:**

(a) $c[n] = x[n+3] = \left\{ \begin{array}{ccccccc} 2 & 0 & -1 & 6 & -3 & 2 & 0 \end{array} \right\}, \quad -6 \le n \le 0.$

(b) $d[n] = y[n-2] = \left\{ \begin{array}{ccccccc} 8 & 2 & -7 & -3 & 0 & 1 & 1 \end{array} \right\}, \quad -3 \le n \le 3.$

(c) $e[n] = x[-n] = \left\{ \begin{array}{ccccccc} 0 & 2 & -3 & 6 & -1 & 0 & 2 \end{array} \right\}, \quad -3 \le n \le 3.$

(d) $x[n-3] = \{2, 0, -1, 6, -3, 2, 0\} \quad 0 \le n \le 6$

$y[n+3] = \{8, 2, -7, -3, 0, 1, 1\} \quad -8 \le n \le -2$

$u[n] = x[n-3] + y[n+3] = \{8, 2, -7, -3, 0, 1, 1, 0, 2, 0, -1, 6, -3, 2, 0\} \quad -8 \le n \le 6$

(e) $y[n-3] = \{8, 2, -7, -3, 0, 1, 1\} \quad -2 \le n \le -4$

$w[n+2] = \{3, 6, -1, 2, 6, 6, 1\} \quad -4 \le n \le 2$

$v[n] = y[n-3]w[n+2] = \{-8, 4, -42, -18\} \quad -2 \le n \le 1$

(f) $y[n+4] = \{8, 2, -7, -3, 0, 1, 1\} \quad -9 \le n \le -3$

$w[n-3] = \{3, 6, -2, 2, 6, 6, 1\} \quad 1 \le n \le 7$

$s[n] = y[n+4] - w[n-3] = \{8, 2, , -7, -3, 0, 1, 1, 0, 0, 0, -3, -6, 2, -2, -6, -6, -1\}, -9 \le n \le 7$

(g) $r[n] = 3.9w[n] = \{11.7, 23.4, -3.9, 7.8, 23.4, 23.4, 3.9\} \quad -2 \le n \le 4$

2. (10 points) Determine the fundamental period of the sinusoidal sequence $\tilde{x}[n] = A\sin(\omega_0 n)$ for the following values the angular frequency $\omega_0$:

   (a) $0.3\pi$,

   (b) $0.48\pi$,

   (c) $0.45\pi$,

   (d) $0.525\pi$,

   (e) $0.7\pi$,

   (f) $0.75\pi$.

   **Solution:**

   (a) For this problem, $\omega_0 = 0.3\pi$, so the equation reduces to $0.3\pi N = 2\pi r$, which is satisfied with $N = 20, r = 3$.

   (b) For this problem, $\omega_0 = 0.48\pi$, so the equation reduces to $0.48\pi N = 2\pi r$, which is satisfied with $N = 25, r = 6$.

   (c) For this problem, $\omega_0 = 0.45\pi$, so the equation reduces to $0.45\pi N = 2\pi r$, which is satisfied with $N = 40, r = 9$.

   (d) For this problem, $\omega_0 = 0.525\pi$, so the equation reduces to $0.525\pi N = 2\pi r$, which is satisfied with $N = 80, r = 21$.

   (e) For this problem, $\omega_0 = 0.7\pi$, so the equation reduces to $0.7\pi N = 2\pi r$, which is satisfied with $\mathrm{N} = 20, \mathrm{r} = 7$

   (f) For this problem, $\omega_0 = 0.75\pi$, so the equation reduces to $0.75\pi N = 2\pi r$, which is satisfied with $N = 8, r = 3$.

3. (10 points) Determine the fundamental period of the following periodic sequences:

   (a) $\tilde{x}_a[n] = e^{j0.25\pi n}$,

   (b) $\tilde{x}_b[n] = \cos(0.6\pi n + 0.3\pi)$

   (c) $\tilde{x}_c[n] = \mathrm{Re}\left(e^{j\pi n/8}\right) + \mathrm{Im}\left(e^{j\pi n/5}\right)$,

   (d) $\tilde{x}_d[n] = 6\sin(0.15\pi n) - \cos(0.12\pi n + 0.1\pi)$

   (e) $\tilde{x}_e[n] = \sin(0.1\pi n + 0.75\pi) - 3\cos(0.8\pi n + 0.2\pi) + \cos(1.3\pi n)$

   **Solution:**

   (a) Here, $\omega_0 = 0.25\pi$, so that the equation for fundamental period reduces to $0.25\pi N = 2\pi r$, which is satisfied with $N = 8, r = 1$.

   (b) Here, $\omega_0 = 0.6\pi$, so that the equation for fundamental period reduces to $0.6\pi\mathrm{N} = 2\pi\mathrm{r}$, which is satisfied with $N = 10, r = 2$.

   (c) We first determine the fundamental period $N_1$ of $\mathrm{Re}\left\{e^{j\pi n/8}\right\} = \cos(\pi n/8)$. In this case, the equation reduces to $0.125\pi N_1 = 2\pi r_1$, which is satisfied with $N_1 = 16$ and $r_1 = 1$. Next, we determine the fundamental period $N_2$ of $\mathrm{Im}\left\{e^{j\pi n/5} = j\sin(0.2\pi n)\right.$. In this case, the equation reduces to $0.2\pi N_2 = 2\pi r_2$, which is satisfied with $N_2 = 10$ and $r_2 = 1$. Hence the fundamental period is given by $\mathrm{LCM}(N_1, N_2) = \mathrm{LCM}(10, 16) = 80$.

(d) We first determine the fundamental period $N_1$ of $\sin(0.15\pi n)$ In this case, the equation reduces to $0.15\pi N_1 = 2\pi r_1$, which is satisfied with $N_1 = 40$ N1 = 40 and $r_1 = 3$. Next, we determine the fundamental period $N_2$ of $\cos(0.12\pi n - 0.1\pi)$. In this case, the equation reduces to $0.12\pi N_2 = 2\pi r_2$, which is satisfied with $N_2 = 50$ and $r_2 = 3$. Hence the fundamental period is given by $\mathrm{LCM}(N_1, N_2) = \mathrm{LCM}(50, 40) = 200$.

(e) Again, we start by finding the fundamental period of each sinusoidal component and then find the least common multiple of the three to determine the overall fundamental period. The fundamental period $N_1$ of $\sin(0.1\pi n + 0.75\pi)$ In this case, the equation reduces to $0.1\pi N_1 = 2\pi r_1$, which is satisfied with $N_1 = 20$ and $r_1 = 1$. Next, we determine the fundamental period $N_2$ of $\cos(0.8\pi n + 0.2\pi)$. The equation reduces to $0.8\pi N_2 = 2\pi r_2$, which is satisfied with $N_2 = 5$ and $r_2 = 2$. Lastly, we determine the fundamental period $N_3$ of $\cos(1.3\pi n)$. The equation reduces to $1.3\pi N_3 = 2\pi r_3$, which is satisfied with $N_3 = 20$ and $r_3 = 13$. Hence the fundamental period is given by $\mathrm{LCM}(N_1, N_2, N_3) = \mathrm{LCM}(20, 5, 20) = 20$.

4. (10 points) Assume $x(n)$ has period $N$. Are the following sequences periodic? Please provide your reasoning:

   (i) $x(1 - 2n)$

   (ii) $x(n) + (-1)^n x(0)$

   **Solution:**

   (i) Since $x(n)$ has period $N$, we know $x(n) = x(n - N)$ for all $n$. If we let $y(n) = x(1 - 2n)$, we have

   $$y(n - N) = x(1 - 2(n - N)) = x(1 - 2n) = y(n), \text{ for all } n.$$

   Hence, $y(n) = x(1 - 2n)$ is periodic.

   (ii) The sequence $z(n) = x(n) + (-1)^n x(0)$ can be considered as the summation of two sequences: $x(n)$ and $y(n) = (-1)^n x(0)$. Since $y(n) = y(n - 2)$ for all $n, y(n)$ has period 2. Hence, the period of $z(n)$ (let it be $M$) can be computed as

   $$M = \frac{2N}{GCD(2, N)}$$

   If $N$ is odd, $GCD(2, N) = 1$, and $M = 2N$; if $N$ is even, $GCD(2, N) = 2$, and $M = N$.

5. (10 points) Write a **Python** or **MATLAB** program to plot a continuous-time signal $x(t) = \cos(2\pi f_0 t)$ and its sampled version with the following frequency $f_0$ and sampling frequency $f_s$:

   (i) $f_0 = 3$ Hz, $f_s = 10$ Hz

   (ii) $f_0 = 7$ Hz, $f_s = 10$ Hz

   (iii) $f_0 = 13$ Hz, $f_s = 10$ Hz

   Is it possible to perfectly reconstruct the original continuous-time function from the samples? Why? Please provide your code, plots, and answers in your report.

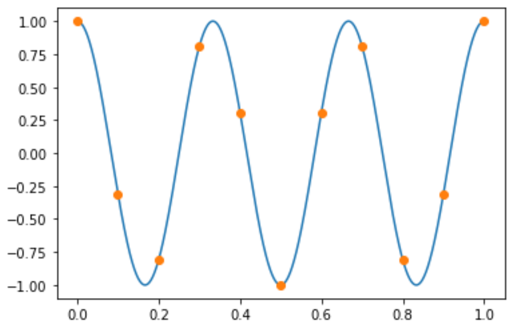
   **Solution:**

   Only (i) can be perfectly reconstructed according to nyquist sampling theorem.

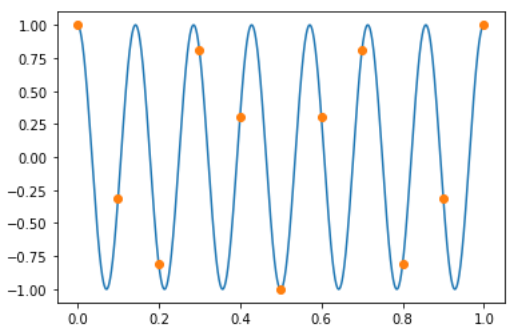
```
import numpy as np
import matplotlib.pyplot as plt
def sampling(f0, sf):
  t = np.arange(0,1,0.00001)
  g1 = np.cos(2* np.pi * f0 * t)
  plt.plot(t,g1)

  n = np.arange(0,sf+1,1)
  sample_points = np.cos(2* np.pi * f0 * n/sf)
  plt.plot(n/sf, sample_points, 'o')
  plt.show()
```

```
sampling(f0=3, sf=10)
```




```
sampling(f0=7, sf=10)
```




```
sampling(f0=13, sf=10)
```