

23W-EC ENGR-113-LEC-1 HW6

SANJIT SARDA

TOTAL POINTS

48 / 50

QUESTION 1

1 10 / 10

✓ + 10 pts *Correct*

+ 9 pts Mostly correct/Does not state if DFT exists

+ 7 pts Incorrect ROC/DFT analysis

QUESTION 2

2 10 / 10

✓ + 10 pts *Correct*

+ 9 pts Slightly incorrect analysis

+ 8 pts Incorrect pole(s) of system

+ 8 pts Missing an ROC

+ 6 pts No analysis after finding poles

+ 5 pts Did not find poles of system

+ 3 pts Incomplete

QUESTION 3

3 8 / 10

✓ + 4 pts *a) Correct*

+ 3 pts *a) Mostly correct*

+ 3 pts *b) Correct*

✓ + 2 pts *b) Mostly correct*

+ 3 pts *c) Correct*

✓ + 2 pts *c) Mostly correct*

+ 1.5 pts *a) Partially correct*

+ 1 pts *b) Partly correct*

+ 1 pts *c) Partly correct*

QUESTION 4

4 10 / 10

✓ + 4 pts *a) Correct*

+ 2 pts *a) Incorrect Application of $G(z)$ /Incorrect Feedback*

✓ + 3 pts *b) Correct*

+ 2 pts *b) Correct result from incorrect a/Mostly correct*

+ 1 pts *Incorrect approach*

✓ + 3 pts *c) Correct*

+ 2 pts *c) Correct result from incorrect a*

+ 0 pts *Incorrect*

QUESTION 5

5 10 / 10

✓ + 10 pts *Correct*

+ 9 pts *Mostly correct/Algebra mistake*

+ 6 pts *Incorrect Decomposition/Technique*

+ 0 pts *Incorrect/No Answer*

ECE 113 HW 6

① Z-transform

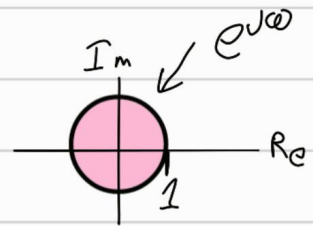
① $x[n] = a u[n]$

$$\therefore X(z) = a \sum \{u[n]\}$$

$$\therefore X(z) = \frac{a}{1-z^{-1}} \text{ with ROC } |z| > 1$$

DTFT @ $z = e^{j\omega}$

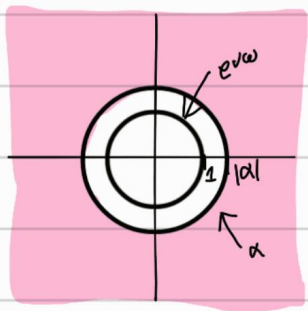
\therefore DTFT does not exist for $|e^{j\omega}| > 1$, since $|e^{j\omega}| = 1$ not > 1



② $x[n] = a^n u[n]$

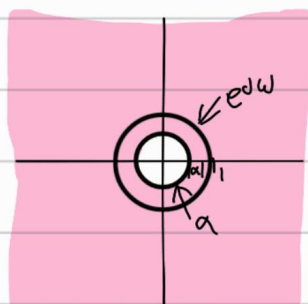
$$\therefore X(z) = \frac{1}{1-\alpha z^{-1}} \text{ with ROC } |z| > |\alpha|$$

\therefore DTFT exists for $|e^{j\omega}| > |\alpha|$, since $|e^{j\omega}| = 1$



DTFT does
not exist

\therefore if $|\alpha| < 1$ then the DTFT exists



DTFT exists

1 10 / 10

✓ + 10 pts *Correct*

+ 9 pts Mostly correct/Does not state if DFT exists

+ 7 pts Incorrect ROC/DFT analysis

② Z transform with multiple ROCs

$$G(z) = \frac{(2z^2 - 0.8z + 0.5)(3z^2 + 2z + 3)}{(z^2 - 0.8z + 0.38)(z^2 + 4z + 5)}$$

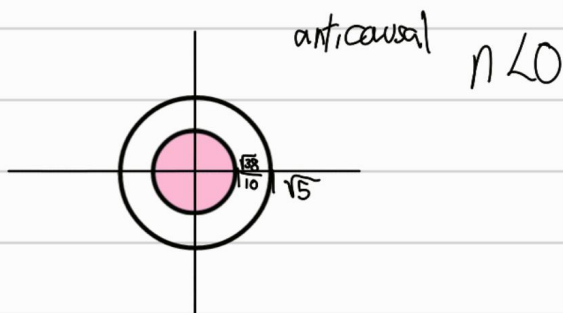
$$G(z) = \frac{6(z - \frac{2+\sqrt{21}}{10})(z - \frac{2-\sqrt{21}}{10})(z - \frac{1-2\sqrt{2}}{3})(z - \frac{1+2\sqrt{2}}{3})}{(z - \frac{4+\sqrt{32}}{10})(z - \frac{4-\sqrt{32}}{10})(z + (2-i))(z + (2+i))}$$

ROCs for each denominator:

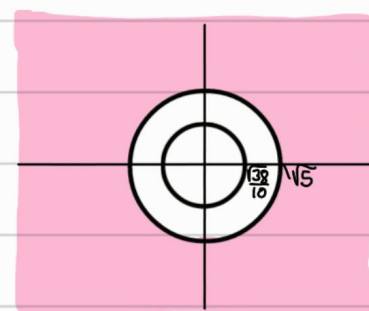
$$z = \frac{\sqrt{38}}{10} \quad \& \sqrt{5}$$

∴ Three ROCs:

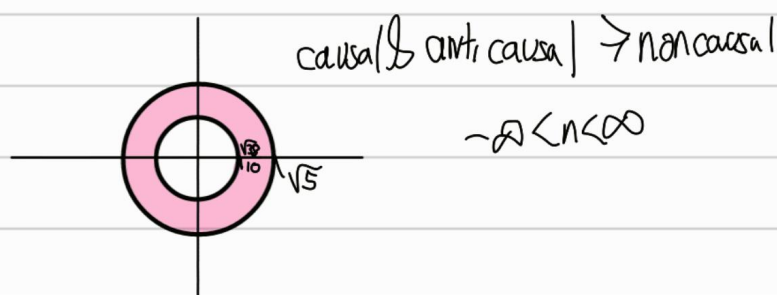
Left sided



Right sided



Two sided



causal
 $n \geq 0$

2 10 / 10

✓ + 10 pts *Correct*

+ 9 pts Slightly incorrect analysis

+ 8 pts Incorrect pole(s) of system

+ 8 pts Missing an ROC

+ 6 pts No analysis after finding poles

+ 5 pts Did not find poles of system

+ 3 pts Incomplete

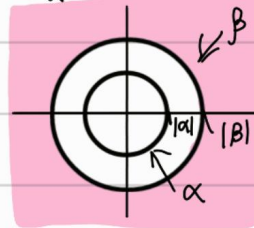
③ Z transform Properties

$|β| > |α| > 0$

$$a) x_1[n] = (α^n + β^n)u[n+2] = α^n u[n+2] + β^n u[n+2] = \frac{α^{n+2} u[n+2]}{α^2} + \frac{β^{n+2} u[n+2]}{β^2}$$

$$Z\{x_1[n]\} = \frac{1}{α^2} Z\{α^{n+2} u[n+2]\} + \frac{1}{β^2} Z\{β^{n+2} u[n+2]\} = \frac{z^2}{α^2} Z\{α^n u[n]\} + \frac{z^2}{β^2} Z\{β^n u[n]\} \quad \text{ROC } z \neq 0$$

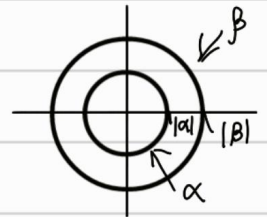
$$= \frac{z^2}{α^2(1-αz^{-1})} + \frac{z^2}{β^2(1-βz^{-1})} \quad \text{ROC: } |z| > |β|$$



$$b) x_2[n] = α^n u[-n-2] + β^n u[n-1] = -(-α^{n+1-1} u[-n-1-1]) + β^{n+1-1} u[n-1] = -α(-α^{n-1} u[-n-1-1]) + ββ^{n-1} u[n-1]$$

$$Z\{x_2[n]\} = -α Z\{-α^{n-1} u[-n-1-1]\} + β Z\{β^{n-1} u[n-1]\} = \frac{-α z^{-1}}{1-α z^{-1}} + \frac{β \cdot z^{-1}}{1-β z^{-1}} \quad \text{ROC: DNE}$$

$$\text{ROC: } |z| < |α| \text{ \& } |z| > |β|$$

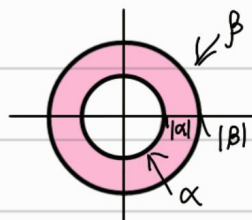


$$c) x_3[n] = α^n u[n+1] + β^n u[-n-2] = \frac{α^{n+1}}{α} u[n+1] - β(β^{n-1} u[-n-1-1]) =$$

$$Z\{x_3[n]\} = \frac{z}{α} \frac{1}{1-α z^{-1}} - \frac{β z^{-1}}{1-β z^{-1}} = \frac{z}{α(1-α z^{-1})} - \frac{β}{z(1-β z^{-1})}$$

$$\text{ROC: } |z| < |β| \text{ \& } |z| > |α|$$

$$\text{ROC} \rightarrow |α| < |z| < |β|$$



3 8 / 10

✓ + 4 pts a) Correct

+ 3 pts a) Mostly correct

+ 3 pts b) Correct

✓ + 2 pts b) Mostly correct

+ 3 pts c) Correct

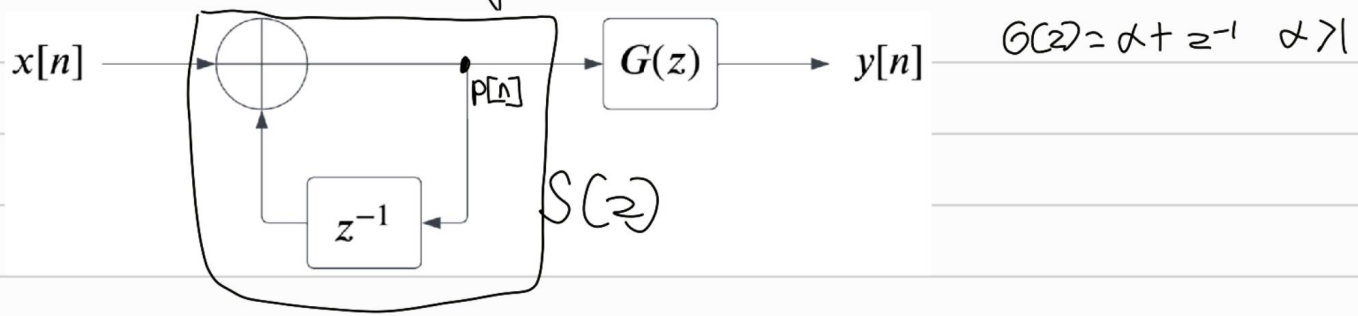
✓ + 2 pts c) Mostly correct

+ 1.5 pts a) Partially correct

+ 1 pts b) Partly correct

+ 1 pts c) Partly correct

④ z transform Block diagram



$$p[n] = x[n] + p[n-1]$$

$$P[z] = X[z] + P[z]z^{-1} \rightarrow X[z] = P[z](1 - z^{-1})$$

$$\therefore S[z] = \frac{P[z]}{X[z]} = \frac{1}{1 - z^{-1}}$$

① $H[z] = \frac{Y[z]}{X[z]} = S[z]G[z] = \frac{\alpha + z^{-1}}{1 - z^{-1}}$

② $H[z] = \frac{\alpha}{1 - z^{-1}} + \frac{z^{-1}}{1 - z^{-1}} \rightarrow h[n] = \alpha u[n] + u[n-1]$

③ ROC: $|z| > 1$

4 10 / 10

✓ + 4 pts a) Correct

+ 2 pts a) Incorrect Application of $G(z)$ /Incorrect Feedback

✓ + 3 pts b) Correct

+ 2 pts b) Correct result from incorrect a/Mostly correct

+ 1 pts Incorrect approach

✓ + 3 pts c) Correct

+ 2 pts c) Correct result from incorrect a

+ 0 pts Incorrect

$$\textcircled{5} \quad H(z) = \frac{1 - 2z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}, \quad |z| > \frac{1}{2}$$

$$H(z) = \frac{(1 - 2z^{-1})}{\frac{1}{6}(z^{-1} - \frac{1}{2})(z^{-1} - \frac{1}{3})}$$

$$\frac{z(z-2)}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{-9z}{z-\frac{1}{2}} + \frac{10z}{z-\frac{1}{3}}$$

$$= \frac{-9}{1-\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{3}z^{-1}}$$

$$\therefore h[n] = -9\left(\frac{1}{2}\right)^n u[n] + 10\left(\frac{1}{3}\right)^n u[n]$$

5 10 / 10

✓ + 10 pts *Correct*

+ 9 pts Mostly correct/Algebra mistake

+ 6 pts Incorrect Decomposition/Technique

+ 0 pts Incorrect/No Answer