

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\sin(A)\sin(B) = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$\cos(A)\cos(B) = \frac{\cos(A+B) + \cos(A-B)}{2}$$

$$\sin(A)\cos(B) = \frac{\sin(A+B) + \sin(A-B)}{2}$$

$$\cos(A)\sin(B) = \frac{\sin(A+B) - \sin(A-B)}{2}$$

$$\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = -2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$j \triangleq \sqrt{-1}$$

$$z \triangleq x + jy \triangleq re^{j\theta}$$

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$r = |z| = \sqrt{x^2 + y^2}$$

$$\theta = \angle z = \tan^{-1}(y/x)$$

$$|z_1 z_2| = |z_1| |z_2|$$

$$|z_1 / z_2| = |z_1| / |z_2|$$

$$|e^{j\theta}| = 1$$

$$\angle r = 0$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$$

$$z_1 z_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

$$\bar{z} \triangleq x - jy = re^{-j\theta}$$

$$z\bar{z} = |z|^2 = x^2 + y^2 = r^2$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

DTFT Cheat Sheet (Courtesy of Signals and Systems by Alkin, page 461-462)

Name	Signal	Transform
Discrete-time pulse	$x[n] = \begin{cases} 1, & n \leq L \\ 0, & \text{otherwise} \end{cases}$	$X(\Omega) = \frac{\sin\left(\frac{(2L+1)\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$
Unit-impulse signal	$x[n] = \delta[n]$	$X(\Omega) = 1$
Constant-amplitude signal	$x[n] = 1, \text{ all } n$	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - 2\pi m)$
Sinc function	$x[n] = \frac{\Omega_c}{\pi} \text{sinc}\left(\frac{\Omega_c n}{\pi}\right)$	$X(\Omega) = \begin{cases} 1, & \Omega < \Omega_c \\ 0, & \text{otherwise} \end{cases}$
Right-sided exponential	$x[n] = \alpha^n u[n], \alpha < 1$	$X(\Omega) = \frac{1}{1 - \alpha e^{-j\Omega}}$
Complex exponential	$x[n] = e^{j\Omega_0 n}$	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi m)$

Table 5.4 – Some DTFT transform pairs.

Theorem	Signal	Transform
Linearity	$\alpha x_1[n] + \beta x_2[n]$	$\alpha X_1(\Omega) + \beta X_2(\Omega)$
Periodicity	$x[n]$	$X(\Omega) = X(\Omega + 2\pi r) \text{ for all integers } r$
Conjugate symmetry	$x[n] \text{ real}$	$X^*(\Omega) = X(-\Omega)$ Magnitude: $ X(-\Omega) = X(\Omega) $ Phase: $\Theta(-\Omega) = -\Theta(\Omega)$ Real part: $X_r(-\Omega) = X_r(\Omega)$ Imaginary part: $X_i(-\Omega) = -X_i(\Omega)$
Conjugate antisymmetry	$x[n] \text{ imaginary}$	$X^*(\Omega) = -X(-\Omega)$ Magnitude: $ X(-\Omega) = X(\Omega) $ Phase: $\Theta(-\Omega) = -\Theta(\Omega) \mp \pi$ Real part: $X_r(-\Omega) = -X_r(\Omega)$ Imaginary part: $X_i(-\Omega) = X_i(\Omega)$
Even signal	$x[n] = x[-n]$	$\text{Im}\{X(\Omega)\} = 0$
Odd signal	$x[n] = -x[-n]$	$\text{Re}\{X(\Omega)\} = 0$
Time shifting	$x[n-m]$	$X(\Omega) e^{-j\Omega m}$
Time reversal	$x[-n]$	$X(-\Omega)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Frequency shifting	$x[n] e^{j\Omega_0 n}$	$X(\Omega - \Omega_0)$
Modulation	$x[n] \cos(\Omega_0 n)$ $x[n] \sin(\Omega_0 n)$	$\frac{1}{2} [X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$ $\frac{j}{2} [X(\Omega - \Omega_0) e^{-j\pi/2} + X(\Omega + \Omega_0) e^{j\pi/2}]$
Differentiation in frequency	$n^m x[n]$	$j^m \frac{d^m}{d\Omega^m} X(\Omega)$
Convolution	$x_1[n] * x_2[n]$	$X_1(\Omega) X_2(\Omega)$
Multiplication	$x_1[n] x_2[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda) X_2(\Omega - \lambda) d\lambda$
Parseval's theorem		$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) ^2 d\Omega$

Table 5.3 – DTFT properties.

Table 8-2 Basic discrete Fourier transform properties.

Table of DFT Properties		
<i>Property Name</i>	<i>Time-Domain: $x[n]$</i>	<i>Frequency-Domain: $X[k]$</i>
Periodic	$x[n] = x[n + N]$	$X[k] = X[k + N]$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
Conjugate Symmetry	$x[n]$ is real	$X[N - k] = X^*[k]$
Conjugation	$x^*[n]$	$X^*[N - k]$
Time-Reversal	$x[((N - n))_N]$	$X[N - k]$
Delay	$x[((n - n_d))_N]$	$e^{-j(2\pi k/N)n_d} X[k]$
Frequency Shift	$x[n]e^{j(2\pi k_0/N)n}$	$X[k - k_0]$
Modulation	$x[n] \cos((2\pi k_0/N)n)$	$\frac{1}{2}X[k - k_0] + \frac{1}{2}X[k + k_0]$
Convolution	$\sum_{m=0}^{N-1} h[m]x[((n - m))_N]$	$H[k]X[k]$
Parseval's Theorem	$\sum_{n=0}^{N-1} x[n] ^2 = \frac{1}{N} \sum_{k=0}^{N-1} X[k] ^2$	

Table of DFT Pairs	
<i>Time-Domain: $x[n]$</i>	<i>Frequency-Domain: $X[k]$</i>
$\delta[n]$	1
$\delta[n - n_d]$	$e^{-j(2\pi k/N)n_d}$
$r_L[n] = u[n] - u[n - L]$	$\underbrace{\frac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))}}_{=D_L(2\pi k/N)} e^{-j(2\pi k/N)(L-1)/2}$
$r_L[n] e^{j(2\pi k_0/N)n}$	$D_L(2\pi(k - k_0)/N) e^{-j(2\pi(k-k_0)/N)(L-1)/2}$

TABLE 10.2 SOME COMMON z-TRANSFORM PAIRS

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
3. $-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
4. $\delta[n - m]$	z^{-m}	All z , except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z > \alpha $
6. $-\alpha^n u[-n - 1]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z < \alpha $
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z > \alpha $
8. $-n\alpha^n u[-n - 1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z < \alpha $
9. $[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r \cos \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r \sin \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z > r$

Property	Time Domain	z- Domain	Region of Convergence
Notation	$h[n]$	$H[z]$	R_h
	$g[n]$	$G[z]$	R_g
Linearity	$\alpha h[n] + b g[n]$	$\alpha H[z] + b G[z]$	At least $R_h \cap R_g$
Time shifting	$h[n - n_0]$	$z^{-n_0} H(z)$	R_h , except $z=0$ if $n_0 > 0$ and $z=\infty$ if $n_0 < 0$
Time reversal	$h[-n]$	$H(z^{-1})$	$1/R_h$
Convolution	$h[n] * g[n]$	$H[z]G[z]$	At least $R_h \cap R_g$