

Monday, 22 Feb 2023

10 points total.

Name: _____

UID: _____

1. (10 points) Determine a N -point **DFT** of the following sequence where $N = 8$.

$$x[n] = \cos\left(2\pi\frac{3}{8}n\right), \quad n = 0, 1, \dots, N-1.$$

Solution:

The long way is by computing the **DFT** in a brute force manner:

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi\frac{k}{N}n} \quad (1)$$

However, after computing a couple k values, you will find that $X[k] = 0$ for most values. If you take the 'basis view' of the **DFT**, it is apparent that $x[n]$ is a linear combination of 2 basis vectors:

$$x[n] = \cos\left(2\pi\frac{3}{8}n\right) = \frac{1}{2}(e^{j2\pi\frac{3}{8}n} + e^{-j2\pi\frac{3}{8}n}) = \frac{1}{2}(e^{j2\pi\frac{3}{8}n} + e^{j2\pi\frac{5}{8}n}) \quad (2)$$

Now, if we plugin Equation 2 into Equation 1, the calculation becomes much simpler. If k equals 3 or 5, then $X[k] = 4$, otherwise $X[k] = 0$. Then your solution is:

$$X[k] = \begin{cases} 4, & k = 3, 5 \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$