ECE113, Winter 2023 **Quiz #12**
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Monday, 27 Feb 2023
10 points total.

Name: _____
UID: _____

1. (10 points) Assume we have a discrete signal $x[n]$ of length N, and we also know the signal's corresponding N-point DFT, $X[k]$. If I apply a shift in the time domain by m such that the signal is now $x[n-m]$, how does the corresponding DFT spectrum change? Does the magnitude, phase, or both change? Please justify.

   Solution:

   According to the DFT shifting property:

   $$x[n-m]_{mod\, N} \xrightarrow{\text{DFT}} X[k]e^{-j\frac{2\pi}{N}km} \tag{1}$$

   For the magnitude, we have

   $$|X[k]e^{-j\frac{2\pi}{N}km}| = |X[k]||e^{-j\frac{2\pi}{N}km}| = |X[k]|\sqrt{\cos(\frac{2\pi}{N}km)^2 + \sin(\frac{2\pi}{N}km)^2} = |X[k]|, \tag{2}$$

   so the magnitude does not change.

   The phase of $X[k]$ for an arbitrary k is $\angle(X[k]) = \angle(e^{j\theta_k}) = \theta_k$.
   Then the phase of the time-shifted DFT signal is

   $$\angle(X[k]e^{-j\frac{2\pi}{N}km}) = \angle(e^{j\theta_k}e^{-j\frac{2\pi}{N}km}) = \theta_k - \frac{2\pi}{N}km \tag{3}$$

   Then the phase of the time shifted DFT spectrum changes only if km is not an integer multiple of N.