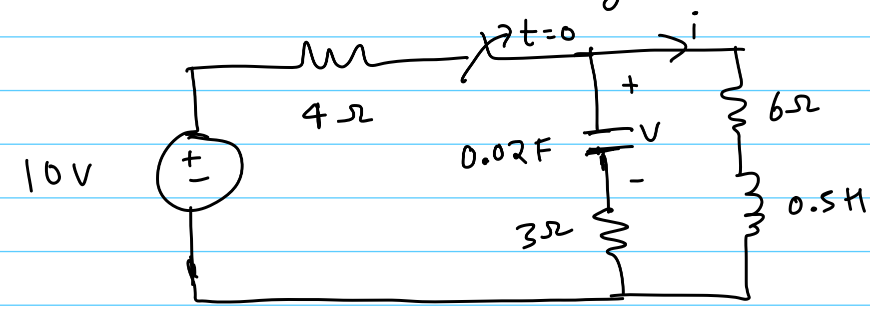
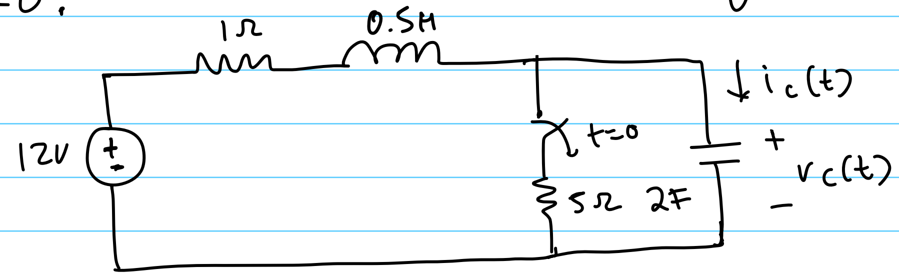
Discussion 7

Q1. Find current $i(t)$ in the circuit shown in the following figure assuming that the circuit reached the steady state at time $t=0^-$.

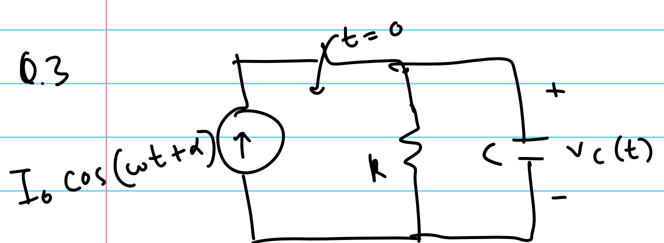


Q2. The switch has been closed for a sufficiently long time and it is opened at t=0.



Find the expression for a) $v_c(t)$ b) $i_c(t)$ for $t>0$.

Q.3



Assuming $v_c(t) = V_o \cos(\omega t + \beta)$ is solution, find V_o & β.

Ans.1

$$\boxed{\zeta \Rightarrow \frac{R}{2\sqrt{\frac{L}{C}}} \Rightarrow \frac{9}{2\times 5} \Rightarrow 0.9}$$ underdamped

$$\sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{50}{2}} \Rightarrow 5 \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}} = 10}$$

$$\boxed{\sigma = \zeta\omega_0 \Rightarrow 9}$$

$$i(0) \Rightarrow \frac{10}{6+4} = 1\,A \qquad v_C(0) = 6V$$

Since the circuit is underdamped

$$i(t) = e^{-\sigma t}(A\cos\omega t + B\sin\omega t) + k$$

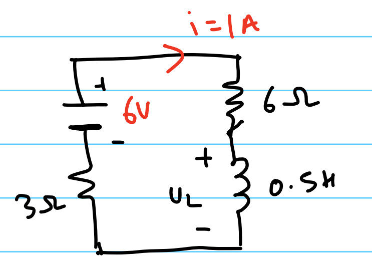
$$i(\infty) = 0 \Rightarrow k = 0$$

$$\omega = \left(\sqrt{1-\zeta^2}\right)\omega_0 \Rightarrow \left(\sqrt{1-0.9^2}\right)\times 10 \Rightarrow 4.358$$

$$i(t) = e^{-6t}(A\cos 4.358t + B\sin 4.358t)$$

$$i(0) = 1A \Rightarrow \underline{A = 1}.$$

@ t = 0



$$v_L(0) = -3$$

$$V_L\Big|_{t=0^+} = L\frac{di}{dt}\Big|_{t=0^+} = -3$$

$$-6 = \frac{di}{dt}\Big|_{t=0^+} \Rightarrow e^{-6t}\left(-A\times 4.358\sin(4.358t) + B(4.358)\cos(4.358t)\right)$$

$$-6e^{-6t}(A\cos 4.358t + B\sin 4.358t)\Big|_{t=0^+}$$

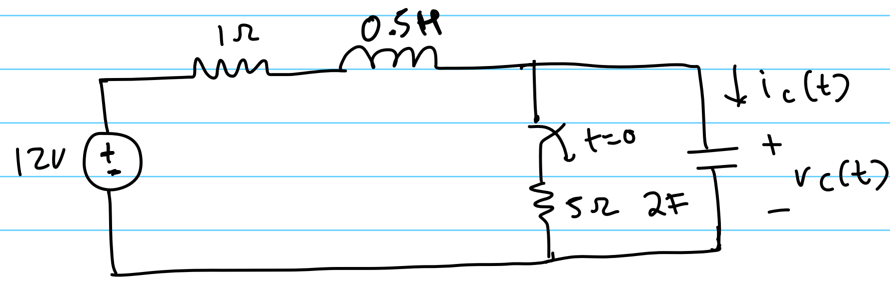
$$-6 = (4.358B) - 6A$$

$$-6 = 4.358B - 9$$

$$3 = 4.358B \; ; \quad \boxed{B = 0.688}$$

$$\boxed{i(t) \Rightarrow e^{-9t}(\cos 4.358t + 0.688 \sin 4.358t)}$$

Ans.2



$$V_c(0) \Rightarrow \frac{5}{5+1} \times 12 \Rightarrow 10V$$

$$i_c(0^+) \Rightarrow 2A \qquad V_c(\infty) = 12$$

$$\sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{0.5}{2}} \Rightarrow \frac{1}{2} \; ; \; \zeta = \frac{R}{2\sqrt{\frac{L}{C}}} \Rightarrow 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{1} \Rightarrow 1$$

Critically damped → 2 roots @ $-\zeta\omega_n \Rightarrow -1$

$$V_c(t) = K_1 e^{-\zeta\omega t} + K_2 t e^{-\zeta\omega_n t} + K_3$$

$v_c(t) = K_1 e^{-t} + K_2 t e^{-t} + K_3$

$v_c(\infty) = \boxed{K_3 = 12}$

$v_c(0) = K_1 + K_3 = 10$; $\boxed{K_1 = -2}$

$i_c = C \frac{dv_c}{dt}\Big|_{t=0^+} \Rightarrow 2$

$i_c(t) = C\left[-K_1 e^{-t} - K_2 t e^{-t} + K_2 e^{-t}\right]$

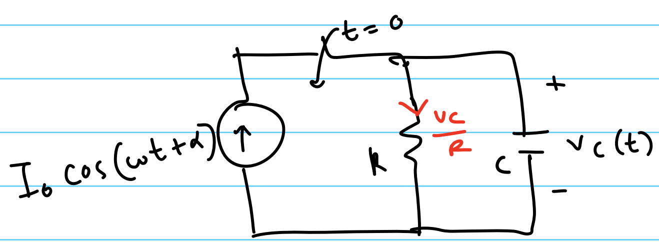
$i_c(0) \Rightarrow C\left[-K_1 + K_2\right] = 2$

$K_2 - K_1 = 1$; $\boxed{K_2 = -1}$

$$\boxed{v_c(t) = -2e^{-t} - t e^{-t} + 12}$$

$$\boxed{i_c(t) = C\frac{dv_c}{dt} \Rightarrow 2 \times \left[(t+2)e^{-t} - e^{-t}\right] = 2(t+1)e^{-t}}$$

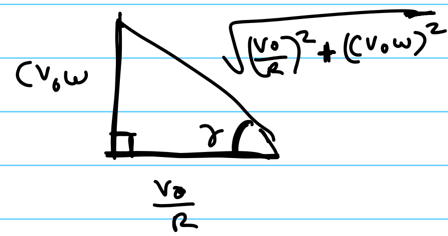
Ans.3



$V_c = V_0 \cos(\omega t + \beta)$

$$I_0 \cos(\omega t + \alpha) = \frac{v_c}{R} + C\frac{dv_c}{dt}$$

$$I_0 \cos(\omega t+\alpha) = \frac{V_0}{R}\cos(\omega t+\beta) - CV_0\omega \sin(\omega t+\beta)$$



$$I_0 \cos(\omega t+\alpha) \Rightarrow \left[\cos\gamma \cos(\omega t+\beta) - \sin\gamma \sin(\omega t+\beta)\right]\sqrt{\left(\frac{V_0}{R}\right)^2+(CV_0\omega)^2}$$

$$I_0 \cos(\omega t+\alpha) \Rightarrow \sqrt{\left(\frac{V_0}{R}\right)^2+(CV_0\omega)^2}\,\cos(\omega t+\beta+\gamma)$$

$$\alpha = \beta+\gamma \quad ; \quad \underline{\underline{\beta = \alpha-\gamma \Rightarrow \alpha - \tan^{-1}(C\omega R)}}$$

$$\left(\frac{V_0}{R}\right)^2 + (CV_0\omega)^2 = I_0^2$$

$$\boxed{V_0 = R\sqrt{I_0^2-(CV_0\omega)^2}}$$