

Part I

Exercise 1

- a) The actual area between these two values is $1 - 2 * 0.1587 = 0.6826$. The empirical rule says that this area should be 0.68.
- b) The actual area between two standard deviations of the mean is $1 - 2 * 0.0228 = 0.9544$ and the empirical rule says that this area should be 0.95. The actual area between three standard deviations of the mean is $1 - 2 * 0.0013 = 0.9974$ and the empirical rule says that this area should be 0.997.
- c) The 25th percentile is approximately 0.671 standard deviations below the mean. The 50th percentile is 0 standard deviations below the mean. The 75th percentile is approximately 0.671 standard deviations above the mean.

Exercise 2

- a) Input: `pnorm(65, mean = 69, sd = 2.8)`
Output: 0.07656373, or 7.66%
- b) Input: `1 - pnorm(75, mean = 69, sd = 2.8)`
Output: 0.01606229, or 1.61%
- c) Input: `pnorm(72, mean = 69, sd = 2.8) - pnorm(66, mean = 69, sd = 2.8)`
Output: 0.7160232 or 71.60%

Exercise 3

- a) Input: `qnorm(0.005, mean = 69, sd = 2.8)`
Output: 61.78768 (inches)
- b) Input: `qnorm(0.9975, mean = 69, sd = 2.8)`
Output: 76.85969 (inches)

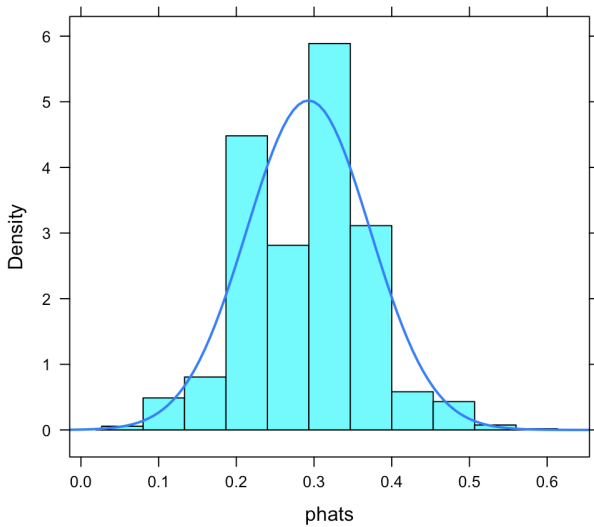
Exercise 4

- a) Input:

```
pawnee <- read.csv("pawnee.csv")
n <- 30
N <- 541
M <- 1000
phats <- numeric(M)
set.seed(123)
for (i in seq_len(M)) {
  index <- sample(N, size = n)
  sample_i <- pawnee[index, ]
  phats[i] <- mean(sample_i$New_hlth_issue == "Y")
}

library(mosaic)
histogram(phats, fit = "normal")
```

Output:



b) Input: `mean(phats)`

`sd(phats)`

Output: 0.2928 (mean)

0.07951963 (standard deviation)

c) The simulated distribution of sample proportions is approximately normal. It is unimodal and largely symmetric.

d) Input: `mean(pawnee$New_hlth_issue == "Y")`

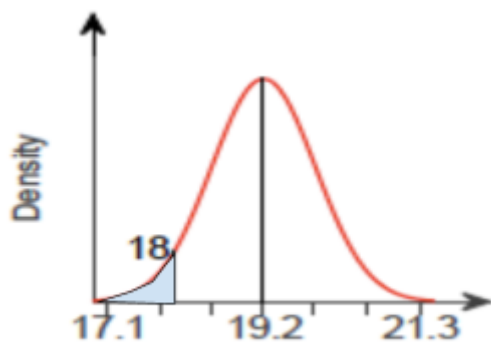
Output: 0.2920518 (mean, theoretical)

Theoretical SD: $\sqrt{(0.2921) \cdot (1 - 0.2921) / 30} = 0.083$

Our empirical and theoretical means (0.2928 and 0.2921 respectively) and standard deviations (0.0795 and 0.083 respectively) are almost exactly the same.

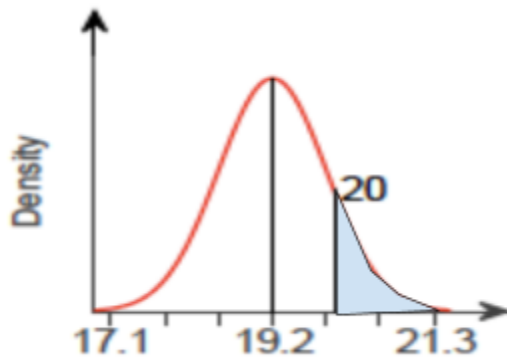
Part II

Exercise 1



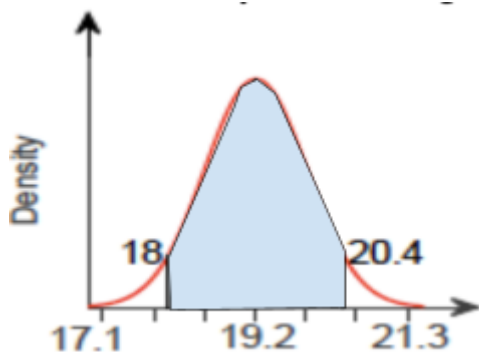
a)

The z-score is $(18 - 19.2) / 0.7 = -1.71$. This would mean that 4.363% of newborns are shorter than 18 inches.



b)

The z-score is $(20-19.2)/0.7 = 1.14$. This would mean that $1 - 0.87286$ or 12.714% of newborns are longer than 20 inches.



c)

From part a) we know that 4.363% of newborns are shorter than 18 inches. The z-score corresponding to a raw value of 20.4 would be $(20.4-19.2) = 1.71$. We note that this is the absolute value of the z-score that corresponds to a raw value of 18. Therefore, $1 - 0.04363 * 2 = 91.274\%$ of newborn babies are between 18 and 20.4 inches.

Exercise 2

The threshold for the top 30% would be the 70th percentile. The z-score that corresponds to a raw value of 428 would be $(428-400)/60 = 0.47$. Unfortunately, that z-score corresponds to the 68th percentile meaning that student won't be admitted.

Exercise 3

- We should expect our sample proportion to be 58% or 0.58.
- $SE = \sqrt{0.58*(1-0.58)/100} = 0.0493$
- We expect 58% of the students in the sample to have their driver's license, give or take 4.93%.
- This would reduce the standard error. The new SE would be $\sqrt{0.58*(1-0.58)/100} = 0.0187$.

Exercise 4

- We expect 58% of the sample to watch television primarily through streaming services.
- Condition 1: Random and independent observations are guaranteed in the problem statement.

Condition 2: $np = 300 \cdot 0.58 = 174 > 10$ and $n(1-p) = 300 \cdot 0.42 = 126 > 10$. The sample is sufficiently large.

Condition 3: $10n = 3000$ and we can assume that there are way more than 3000 people in America who are between the ages of 18 and 29. The population is sufficiently large.

- c) We have a normal distribution denoted by $N(0.58, \sqrt{0.58 \cdot 0.42 / 300}) = N(0.58, 0.028)$. $181/300 = 0.603$, which is less than one standard deviation from the mean. Therefore, finding 181 people in the sample who watch television primarily through streaming services wouldn't be surprising.
- d) A raw value of 0.65 corresponds to a z-score $= (0.65 - 0.58) / 0.028 = 2.5$ which corresponds to the 99.4th percentile. The probability that more than 65% of the sample watched television primarily through streaming services is $1 - 99.4$ or 0.6%.

Exercise 5

- a) Condition 1: Random and independent observations are guaranteed in the problem statement.
- Condition 2: $np = 800 \cdot 0.82 = 656 > 10$, $n(1-p) = 800 \cdot 0.18 = 144 > 10$. The sample is sufficiently large.
- Condition 3: $10n = 8000$ and we can assume that there are more than 8000 adults in that country so the population is sufficiently large.
- b) $SE = \sqrt{0.82 \cdot (1 - 0.82) / 800} = 0.014$. The z-score that corresponds to the 95th percentile is 1.96, so the margin of error $= 0.014 \cdot 1.96 = 0.02744$. Therefore, our confidence interval is $(0.82 - 0.02744, 0.82 + 0.02744) = (0.79256, 0.84744)$. We are 95% confident that the actual proportion of the adults in the country who believe that protecting the rights of those with unpopular views is a very important component of a strong democracy lies within that interval.
- c) The confidence interval would be narrower. The z-score that corresponds to the 90th percentile is lower than the z-score that corresponds to the 95th percentile and since $ME = Z \cdot SE$, a smaller z-score means a smaller margin of error means a narrower interval.