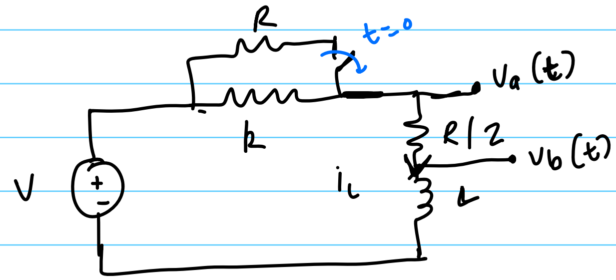
Ans.1



a) $i_L(t=0^-) = i_L(t=0^+) = \frac{V}{R}$

b) $V_a(t=0^+) = V - i_L(t=0^+)R = 0 \quad -(1)$

$V_b(t=0^+) = V - i_L(t=0^+)\frac{3R}{2} = -\frac{V}{R}\times\frac{R}{2} = -\frac{V}{2} \quad -(2)$

$V_a(t=\infty) = \frac{R/2 \times V}{R+R/2} \Rightarrow \frac{V}{3} \quad -(3)$

$V_b(t=\infty) = 0 \quad -(4)$

$\boxed{\tau \Rightarrow \frac{L}{R+R/2} \Rightarrow \frac{2L}{3R}}$

$V_a(t) = K_1 e^{-t/\tau} + K_2 \quad -(A)$

$V_b(t) = K_3 e^{-t/\tau} + K_4 \quad -(B)$

Substituting (1) & (3) in A & (2) & (4) in B

$\boxed{K_2 = \frac{V}{3}}$, $\boxed{K_4 = 0}$

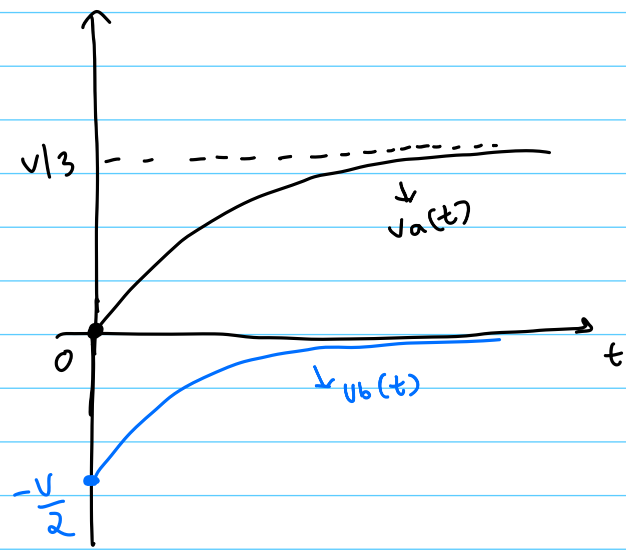
$K_1 + K_2 = 0$, $K_3 + K_4 = -\frac{V}{2}$

$\boxed{K_1 = -\frac{V}{3}}$ $\boxed{K_3 = -\frac{V}{2}}$

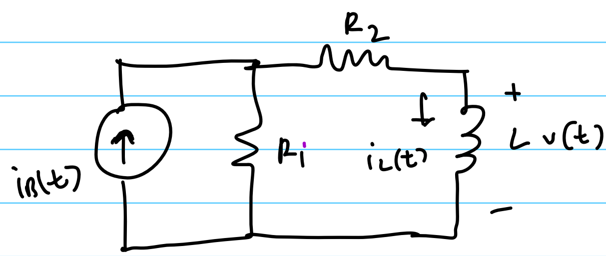
$$\boxed{V_a(t) = -\frac{V}{3} e^{-t/\tau} + \frac{V}{3}}$$

$$\boxed{V_b(t) = -\frac{V}{2} e^{-t/\tau}}$$

c)



Ans. 2



$i_L(0^-) = 1$

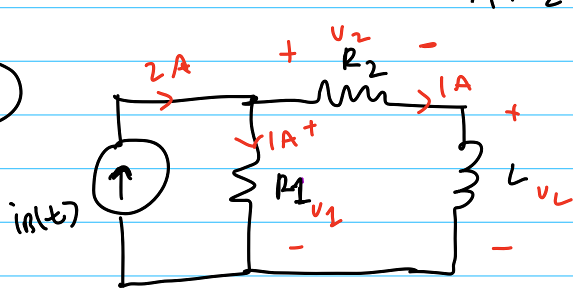
$R_1 = 1\Omega$, $R_2 = 3\Omega$

$L = 1H$

$i_B(t) = 2A$ for $t \geq 0$.

a) time constant $\Rightarrow \frac{L}{R_1+R_2} \Rightarrow \frac{1}{4} \Rightarrow 0.25s$

b)



$V_1 = V_2 + V_L$ (KVL)

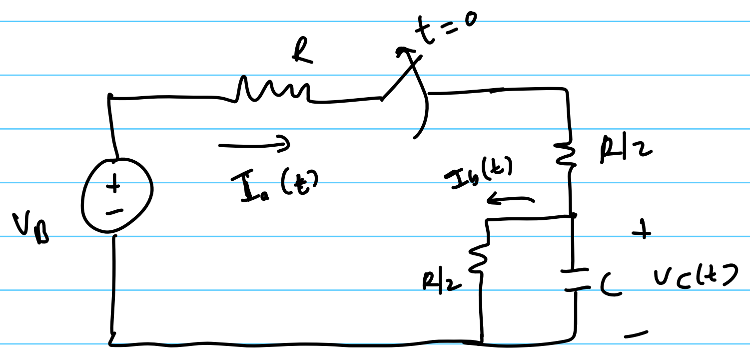
$\downarrow$

$1 \times 1 = 1 \times 3 + V_L \Rightarrow \boxed{V_L(t=0^+) = -2V}$

c) $V_L(\infty) = 0$

d) $V_L(t) = V_L(t=0^+)\, e^{-(R_1+R_2)t/L} \Rightarrow -2e^{-t/0.25} \Rightarrow -2e^{-4t}$

Ans.3



$V_B = 3V$

$R = 20\Omega$

$C = 0.5mF$

a) $V_C(t=0^+) = \dfrac{R/2\ V_B}{R/2 + R/2 + R} \Rightarrow \dfrac{V_B}{4} \Rightarrow \dfrac{3}{4} \Rightarrow \underline{\underline{0.75V}}$

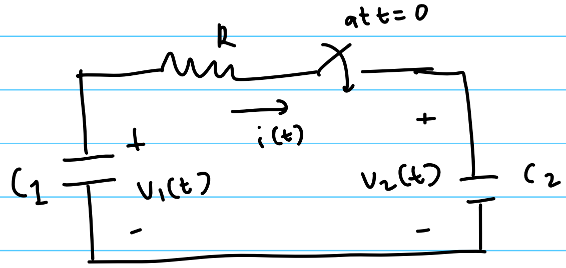
b) $I_a(t=0^+) = 0$

$I_b(t=0^+) = \dfrac{V_c(t=0^+)}{R/2} \Rightarrow \dfrac{0.75}{10} \Rightarrow \underline{\underline{0.075A}}$

c) $0.075 = -C \left.\dfrac{dv_c}{dt}\right|_{(t=0^+)}$

$\left.\dfrac{dv_c}{dt}\right|_{t=0^+} \Rightarrow -\dfrac{0.075}{0.5\times10^{-3}} = -0.15\times10^{3} \Rightarrow \underline{-1.5\times10^{2}\left(\frac{V}{s}\right)}$

Ans.4



$C_1 = 5mF$

$C_2 = 15mF$

$R = 300\Omega$

$Q_1 = -Q_2 = 8$ Coulombs.

$v_1(0^-) = \frac{Q_1}{C_1}$

$v_2(0^-) = \frac{Q_2}{C_2}$

a) $$i(t) = \frac{v_1(t) - v_2(t)}{R} \quad — (1)$$

$$i(t) = -\frac{C_1 dv_1(t)}{dt} = \frac{C_2 dv_2(t)}{dt} \quad -(2)$$

Differentiate eq^n (1)

$$\frac{di(t)}{dt} = \frac{1}{R}\left[\frac{dv_1(t)}{dt} - \frac{dv_2(t)}{dt}\right] \quad \leftarrow (3)$$

Substitute (2) in (3)

$$\frac{di(t)}{dt} = \frac{1}{R}\left[\frac{-i(t)}{C_1} - \frac{i(t)}{C_2}\right] \Rightarrow \boxed{\frac{di(t)}{dt} + \frac{8}{9} i(t) = 0}$$

b) From Part a), we can see that $\tau = R(C_1 \| C_2) \Rightarrow \frac{R C_1 C_2}{C_1 + C_2} \Rightarrow \frac{9}{8} = \underline{1.125 \times 10^9}$ $\underline{ns}$

c) $v_1(0^+) \Rightarrow \frac{8}{C_1} \Rightarrow \underline{1600V}$

$i(0^+) = \frac{v_1 - v_2}{R} \Rightarrow \underline{7.11A}$

$v_2(0^+) = \frac{-8}{C_2} \Rightarrow \underline{-533.33V}$

$i(\infty) = \underline{0}$

$$V_1(\infty) = V_2(\infty) = \frac{Q_1 + Q_2}{C_1 + C_2} = 0V$$

d) $$V_1(t) = [V_1(\infty) - V_1(0)](1 - e^{-t/\tau}) + V_1(0)$$

$$\Rightarrow [0 - 1600](1 - e^{-\frac{8t}{9}}) + 1600 \Rightarrow \underline{\underline{1600e^{-8t/9}}}$$

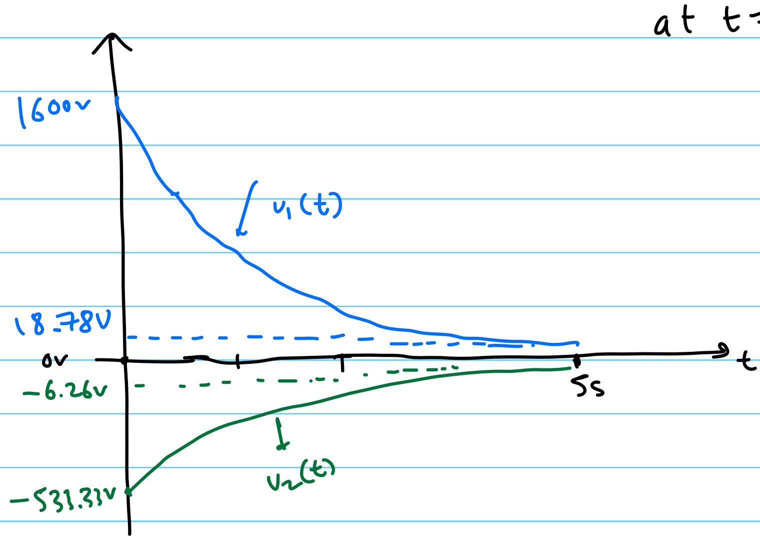
$$V_2(t) = [V_2(\infty) - V_2(0)](1 - e^{-t/\tau}) + V_2(0)$$

$$= [0 + 533.33](1 - e^{-t/\tau}) + (-533.33) \Rightarrow \underline{\underline{-533.33e^{-8t/9}}}$$

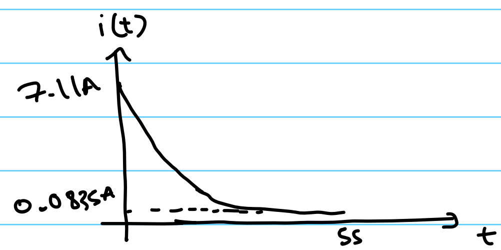
e)

at $t = 5s$, $V_1(t) \Rightarrow 1600e^{-40/9} = 18.78V$

$V_2(t) \Rightarrow -533.33e^{-40/9} = -6.26V$



f)

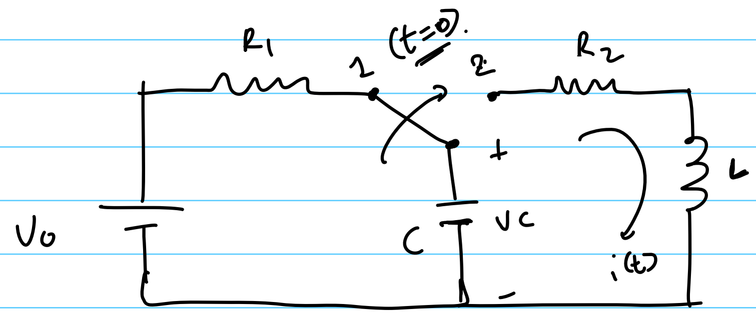


$i(t) = 7.11e^{-8t/9}$

@ $t = 5s$ $i(t) = 7.11e^{-40/9} \Rightarrow 0.0834A$

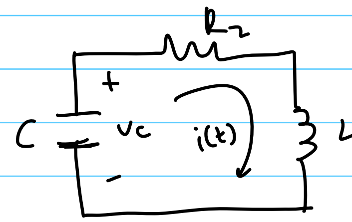
g) As R approaches close to zero, the current would approach ∞ and would exist only for $t=0$. $v_1(t)$ & $v_2(t)$ will then instantaneously go from $1600V \to 0V$ & $-533.33V \to 0V$ (at $t=0$).

Ans.5



$R_1 = 1k\Omega$, $R_2 = 40\Omega$, $L = 10nH$, $C = 10nF$
$V_0 = 2V$

a) for $t \geq 0$



(from KVL) $V_c - i(t) R_2 = V_L$; $V_L = \dfrac{L\,di(t)}{dt}$

$V_c - i(t) R_2 = \dfrac{L\,di(t)}{dt}$; $i(t) = -\dfrac{C\,dv_c}{dt}$

$\downarrow$ differentiate

$$\frac{dv_c}{dt} - \frac{di(t)}{dt} R_2 = \frac{L\,d^2 i(t)}{dt^2}$$

$$-\frac{i(t)}{C} - \frac{di(t)}{dt} R_2 = \frac{L\,d^2 i(t)}{dt^2}$$

$$\boxed{\frac{L\,di^2(t)}{dt^2} + R_2 \frac{di(t)}{dt} + \frac{i(t)}{C} = 0}$$

b) $\underline{i(t=0^+) = 0}$

$V_B = L\frac{di}{dt}$ (V_B is the voltage across capacitor @ $t=0^+$)

$\frac{di}{dt} = \frac{V_B}{L} \Rightarrow \frac{2}{10n} \Rightarrow 0.2\times10^9$ (A/s) $\Rightarrow \underline{2\times10^8 \text{ (A/s)}}$

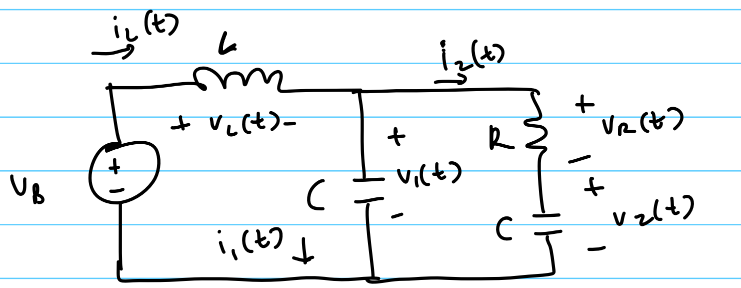
Using results from part a)

$$L\frac{di^2(t)}{dt^2} = -R_2\frac{di(t)}{dt} - \frac{i(t)}{C}$$

$$L\frac{di^2(t)}{dt^2} = -R_2\frac{V_B}{L} \quad (\text{at } t=0^+,\ i(t=0^+)=0)$$

$$\frac{di^2(t)}{dt^2} = -\frac{V_B R_2}{L^2} \Rightarrow \frac{-2\times40}{(10n)^2} \Rightarrow \underline{-80\times10^{16}\ \frac{A}{s^2}}$$

Ans.6



$v_1(0^-) = 0V$

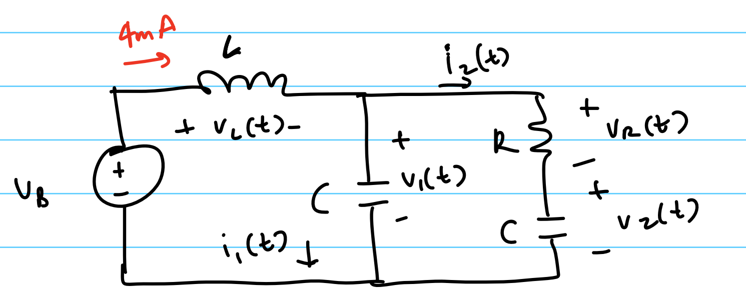
$v_2(0^-) = 2V$

$i_L(0^-) = 4mA$

$R = 1k\Omega$

$V_B = 4V$, $L = 1mH$, $C = 1nF$

a)



$v_1(t) = v_R(t) + v_2(t)$

@ $t = 0^+$

$0 = v_R(t) + 2$

$\boxed{v_R(t) = -2V}$

$$i_2(t=0^+) = \frac{-2}{1k} = -2mA$$

$$\boxed{i_1(t=0^+) = 4+2 = 6mA}$$

b) Applying KVL

$$V_B = V_L + V_1$$

$$\frac{dV_B}{dt} = \frac{dV_L}{dt} + \frac{dV_1}{dt}$$

$$\frac{dV_L}{dt} = -\frac{dV_1}{dt} \quad \left(\text{from part (a)} \quad i_1(t=0^+) = C\frac{dV_1}{dt}\right)$$

$$\frac{dV_L}{dt} = -\frac{6mA}{C} \Rightarrow -\frac{6m}{1n} \Rightarrow \underline{-6M}(V/s)$$

$$\Updownarrow$$

$$\underline{-6\times10^6}(V/s)$$