EE 131A
Probability and Statistics
Instructor: Lara Dolecek

Homework 1
Monday, January 09, 2023
TA: Jayanth Shreekumar, Rushi Bhatt

Reading: Chapters 2.1 - 2.3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. Four schools 1, 2, 3, and 4 are participating in a spelling bee competition. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will face each other for the cup, and the two losers will also play. A possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4).
   (a) List all outcomes in the sample space $\mathcal{S}$.
   (b) Let A denote the event that 1 wins the tournament. List outcomes in A.
   (c) Let B denote the event that 2 gets into the championship game. List outcomes in B.
   (d) What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A^c?
2. Let A, B, and C be three events. Find an expression and draw a Venn diagram for the following events:
   (a) Two or more of the events occur.
   (b) None of the events occur.
   (c) Exactly one of the three events occurs.
3. Suppose A and B are two events. Use the axioms of probability to prove the following:
   (a) $P(A \cap B) \geq P(A) + P(B) - 1$
   (b) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$
4. Suppose we pick 2 distinct integers in the range from 1 to 100. How many ways are there to pick these 2 integers such that their product is a multiple of 11?
5. Consider a spinning wheel that is divided into 10 parts that are numbered from 1 to 10. You spin the wheel 2 times, and each time, you are equally likely to land on any of the 10 numbers.
   (a) What is the probability that the sum of your spins is even?
   (b) What is the probability that at least one of the spins is odd?
   (c) What is the probability that the absolute value of the difference between your spins is greater than or equal to 8?