ECE 102 HW5

SANJIT SARDA

TOTAL POINTS

99 / 100

QUESTION 1

Fourier Series 18 pts

1.1 1a) 5 / 6

- **0 pts** Correct
- **1 pts** Incorrect/missing conjugate property (should be ck* = - c-k)
- ✓ **- 1 pts Incorrect/missing phase property (should be <ck = -<ck* or <ck = -<c-k +/- pi)**
- **1 pts** Incorrect/missing magnitude property (should be |ck| = |c-k|)
- **1 pts** Incorrect/missing real property (should be Re(ck) = -Re(c-k))
- **1 pts** Incorrect/missing imaginary property (should be Im(ck) = Im(c-k))
- **6 pts** Missing

1.2 1b) 12 / 12

- ✓ **- 0 pts Correct**
- **2 pts** Incorrect coefficient for cosine part
- **2 pts** Missing DC component
- **12 pts** Missing
- **4 pts** Incomplete answer (no final x(t) written)

QUESTION 2

Symmetry properties of Fourier Transform 32 pts

2.1 2)a) 16 / 16

- ✓ **- 0 pts Correct**
- **1 pts** Partial correct 1
- **1 pts** Partial correct 2
- **1 pts** Partial correct 3
- **1 pts** Partial correct 4
- **1 pts** Partial correct 5
- **1 pts** Partial correct 6
- **1 pts** Partial correct 7
- **1 pts** Partial correct 8
- **2 pts** Incorrect 5
- **2 pts** Incorrect 6
- **2 pts** Incorrect 7
- **2 pts** Incorrect 8
- **16 pts** Missing/ incorrect

2.2 2)b) 8 / 8

- ✓ **- 0 pts Correct**
- **2 pts** Partial Part 1 (Correct Ans: True)
- **2 pts** Partial Part 2 (Correct Ans: False)
- **4 pts** Part 1 incomplete/ incorrect
- **4 pts** Part 2 incomplete/ incorrect

2.3 2)c) 8 / 8

- ✓ **- 0 pts Correct**
- **2 pts** Partial correct 1
- **2 pts** Partial correct 2
- **4 pts** Incorrect/ Missing 1
- **4 pts** Incorrect Missing 2

QUESTION 3

Halloween Adventures with the Mystery Box 20 pts

3.1 3)a) 3 / 3

- ✓ **- 0 pts Correct**
- **3 pts** Missing

3.2 3)b) 10 / 10

- ✓ **- 0 pts Correct**
- **2 pts** Incorrect/missing part ii
- **2 pts** Incorrect/missing part iii
- **1 pts** Incorrect added component in part v

- **2 pts** Incorrect/missing part I
- **2 pts** Incorrect/missing part iv
- **10 pts** Missing
- **2 pts** Incorrect/missing part v

3.3 3)c) 7 / 7

✓ **- 0 pts** Correct

- 7 pts Missing

QUESTION 4

Fourier Transform and its Inverse 30 pts

4.1 4)a) 21 / 21

✓ **- 0 pts** Correct

- **1 pts** part 2 minor mistake
- **4 pts** part 2 mostly wrong
- **1 pts** part 3 minor mistake
- **4 pts** part 3 wrong
- **5 pts** part 3 incomplete
- **1 pts** part 4 minor mistake
- **2 pts** part 4 major mistake
- **4 pts** part 4 mostly wrong
- **5 pts** part 4 incomplete
- **7 pts** part 2 missing
- **7 pts** part 3 missing
- **7 pts** part 4 missing

4.2 4)b) 9 / 9

✓ **- 0 pts** Correct

- **1 pts** part 1 minor mistake
- **2 pts** part 1 partially correct
- **2 pts** part 2 wrong
- **4 pts** part 1 incomplete
- **3 pts** part 2 missing
- **6 pts** part 1 missing

ECE 102

1)a) $f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

let, $g(t) = f(t)$

If $f(t)$ = imag, $Re(f(t)) = 0$

$Im(f(t)) = f(t)$

$g(t) = \sum_{g=-\infty}^{\infty} c_g e^{jk\omega_0 t}$

where, $jc_g = c_k$

$c_k = \frac{1}{T_0}\int_{T}^{T+T_0} f(t) e^{-jk\omega_0 t}\, dt$

$c_g = \frac{c_k}{j}$

$= \frac{1}{T_0}\int_{T}^{T+T_0} f(t)[\cos(k\omega_0 t) - j\sin(k\omega_0 t)]\, dt$

$= \frac{1}{T_0}\int_{T}^{T+T_0} jg(t)\cos k\omega_0 t - j^2 g(t)\sin k\omega_0 t\, dt$

$= j\frac{1}{T_0}\int_{T}^{T+T_0} g(t)\cos k\omega_0 t\, dt + \frac{1}{T_0}\int_{T}^{T+T_0} g(t)\sin k\omega_0 t\, dt$

$\therefore Re(c_k) = \frac{1}{T_0}\int_{T}^{T+T_0} g(t)\sin k\omega_0 t\, dt$

$Re(c_{-k}) = \frac{1}{T_0}\int_{T}^{T+T_0} g(t)\sin -k\omega_0 t\, dt = \frac{1}{T_0}\int_{T}^{T+T_0} g(t) \cdot -\sin k\omega_0 t\, dt$

$= -R(c_k)$

$Im(c_k) = \frac{1}{T_0}\int_{T}^{T+T_0} g(t)\cos k\omega_0 t\, dt =$

$\therefore Im(c_{-k}) = Im(c_k)$

$Im(c_{-k}) = \frac{1}{T_0}\int_{T}^{T+T_0} g(t)\cos -k\omega_0 t\, dt = \int_{T}^{T+T_0} g(t)\cos k\omega_0 t\, dt$

$$c_k^* = Re(c_k) - j\,Im(c_k)\ ..$$

$$c_{-k} = Re(c_{-k}) + j\,Im(c_{-k}) \therefore -c_{-k} = Re(c_k) - j\,Im(c_k)$$

$$\therefore c_k^* = -c_{-k}$$

$$\angle c_k = \arctan\left(\frac{Im(c_k)}{Re(c_k)}\right)$$

$$\angle c_k^* = \arctan\left(-\frac{Im(c_k)}{Re(c_k)}\right) = -\arctan\left(\frac{Im(c_k)}{Re(c_k)}\right)$$

$$\therefore \angle c_k = \angle c_k^*$$

$$|c_k| = \sqrt{Im^2(c_k) + Re^2(c_k)} = \sqrt{(-Im(c_k))^2 + (-Re(c_k))^2}$$

$$= |-c_k|$$

$$Re(c_k) = -Re(c_{-k})$$

$$Im(c_k) = Im(c_{-k})$$

$$c_k^* = -c_{-k}$$

$$\angle c_k = \angle c_k^*$$

$$|c_k| = |-c_k|$$

1.1 1a) **5 / 6**

- **- 0 pts** Correct
- **- 1 pts** Incorrect/missing conjugate property (should be ck* = - c-k)
- ✓ **- 1 pts Incorrect/missing phase property (should be <ck = -<ck* or <ck = -<c-k +/- pi)**
- **- 1 pts** Incorrect/missing magnitude property (should be |ck| = |c-k|)
- **- 1 pts** Incorrect/missing real property (should be Re(ck) = -Re(c-k))
- **- 1 pts** Incorrect/missing imaginary property (should be Im(ck) = Im(c-k))
- **- 6 pts** Missing

1b)

$$x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

1) $a_k = a_{-k}$
2) $T_0 = 16$
3) $a_k = 0$, for $a_k \neq -1, 0, 1$
4) $\int_0^4 |x(t) - (\sum_{l=0}^{\infty} a_{2l})|^2 dt = 2$
5) $a_0 = 5$

Using #3, $x(t) = a_{-1} e^{-j\omega_0 t} + a_0 e^0 + a_1 e^{j\omega_0 t}$

Using #1, $x(t) = a_1 e^{-j\omega_0 t} + a_1 e^{j\omega_0 t} + a_0$

Using #5, $x(t) = a_1 e^{-j\omega_0 t} + a_1 e^{j\omega_0 t} + 5$

Using #2, $x(t) = 5 + a_1 (e^{-j\frac{2\pi}{16}t} + e^{j\frac{2\pi}{16}t}) = 5 + a_1 (e^{-j\frac{\pi t}{8}} + e^{j\frac{\pi t}{8}}) = 5 + 2a_1 \cos(\pi/8\, t)$

Using #3, $\int_0^4 |x(t) - \sum_{l=0}^{\infty} a_{2l}|^2 dt = 2$, $\int_0^4 |5 + 2a_1 \cos(\frac{\pi}{8}t) - 5|^2 dt = 2$

$\therefore \int_0^4 |2a_1 \cos(\frac{\pi}{8}t)|^2 dt = 8a_1^2 = 2 \quad \therefore a_1 = \pm 1/2$

$\therefore x(t) = 5 \pm \cos(\pi/8\, t)$

1.2 1b) 12 / 12

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect coefficient for cosine part
- **- 2 pts** Missing DC component
- **- 12 pts** Missing
- **- 4 pts** Incomplete answer (no final x(t) written)

2) a)

i) $x(t)$ is even: a, d, e

ii) $x(t)$ is odd: f

iii) $x(t)$ is real: c & e

iv) $x(t)$ is complex: a & b

v) $x(t)$ is real & even: e

vi) $x(t)$ is imaginary & odd: f

vii) $x(t)$ is imaginary & even: d

viii) $e^{j\omega_0 t}x(t)$ is real & even: b.

2b) i) $x_o(t)$ & $x_e(t)$

$$g(t) = (x_o * x_e)(t) = \mathcal{F}^{-1}(\mathcal{F}(x_o(t))\mathcal{F}(x_e(t)))$$

$$G(j\omega) = X_o(j\omega)X_e(j\omega)$$

$\therefore G(j\omega)$ is Imaginary & Odd

$\therefore g(t) = \mathcal{F}^{-1}(G(j\omega))$ is Real & Odd

ii) $x_1(t)$ is Im & Odd, $x_2(t) = x_1(-t)$

$$g(t) = (x_1 * x_2)(t) = \mathcal{F}^{-1}(\mathcal{F}(x_1(t))\mathcal{F}(x_2(t)))$$

$$G(j\omega) = X_1(j\omega)X_2(j\omega)$$

$\therefore G(j\omega)$ is even & real

$\therefore g(t) = \mathcal{F}^{-1}(G(j\omega))$ is even & real.

2.1 2)a) **16 / 16**

- ✓ **- 0 pts** Correct
- **- 1 pts** Partial correct 1
- **- 1 pts** Partial correct 2
- **- 1 pts** Partial correct 3
- **- 1 pts** Partial correct 4
- **- 1 pts** Partial correct 5
- **- 1 pts** Partial correct 6
- **- 1 pts** Partial correct 7
- **- 1 pts** Partial correct 8
- **- 2 pts** Incorrect 5
- **- 2 pts** Incorrect 6
- **- 2 pts** Incorrect 7
- **- 2 pts** Incorrect 8
- **- 16 pts** Missing/ incorrect

2) a)

i) $x(t)$ is even: a, d, e

ii) $x(t)$ is odd: f

iii) $x(t)$ is real: c & e

iv) $x(t)$ is complex: a & b

v) $x(t)$ is real & even: e

vi) $x(t)$ is imaginary & odd: f

vii) $x(t)$ is imaginary & even: d

viii) $e^{j\omega_0 t}x(t)$ is real & even: b.

2b) i) $x_o(t)$ & $x_e(t)$

$$g(t) = (x_o * x_e)(t) = \mathcal{F}^{-1}(\mathcal{F}(x_o(t))\mathcal{F}(x_e(t)))$$

$$G(j\omega) = X_o(j\omega)X_e(j\omega)$$

$\therefore G(j\omega)$ is Imaginary & Odd

$\therefore g(t) = \mathcal{F}^{-1}(G(j\omega))$ is Real & Odd

ii) $x_1(t)$ is Im & Odd, $x_2(t) = x_1(-t)$

$$g(t) = (x_1 * x_2)(t) = \mathcal{F}^{-1}(\mathcal{F}(x_1(t))\mathcal{F}(x_2(t)))$$

$$G(j\omega) = X_1(j\omega)X_2(j\omega)$$

$\therefore G(j\omega)$ is even & real

$\therefore g(t) = \mathcal{F}^{-1}(G(j\omega))$ is even & real.

2.2 2)b) **8 / 8**

✓ **- 0 pts** Correct

- 2 pts Partial Part 1 (Correct Ans: True)

- 2 pts Partial Part 2 (Correct Ans: False)

- 4 pts Part 1 incomplete/ incorrect

- 4 pts Part 2 incomplete/ incorrect

c) i) $x(t) = x^*(t)$

$\therefore \quad X^*(j\omega) = \left(\int_{-\infty}^{\infty} x(t) e^{j\omega t} dt\right)^*$

$= \int_{-\infty}^{\infty} x^*(t) e^{j\omega t} dt$

$= -\int_{-\infty}^{\infty} x^*(-(-t)) e^{-j\omega(-t)} dt$

$= -\int_{-\infty}^{\infty} x^*(-t) e^{j\omega(-t)} dt$

$= X(j\omega) \quad = X^*(j\omega) \therefore X(j\omega)$ is real.

ii) $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} (x_o(t) + x_e(t)) (\cos\omega t - j\sin\omega t) dt$

$= \int_{-\infty}^{\infty} x_o(t)\cos\omega t - jx_o(t)\sin\omega t + x_e(t)\cos\omega t - jx_e(t)\sin\omega t \, dt$

$= \int_{-\infty}^{\infty} x_e(t)\cos\omega t \, dt - j\int_{-\infty}^{\infty} x_o(t)\sin\omega t \, dt$

$\therefore \quad X_e(j\omega) = \int_{-\infty}^{\infty} x_e(t)\cos\omega t \, dt = Re(X(j\omega))$

$\therefore \quad X_o(j\omega) = \int_{-\infty}^{\infty} x_o(t)\sin\omega t \, dt = jIm(X(j\omega))$

2.3 2)c) **8 / 8**

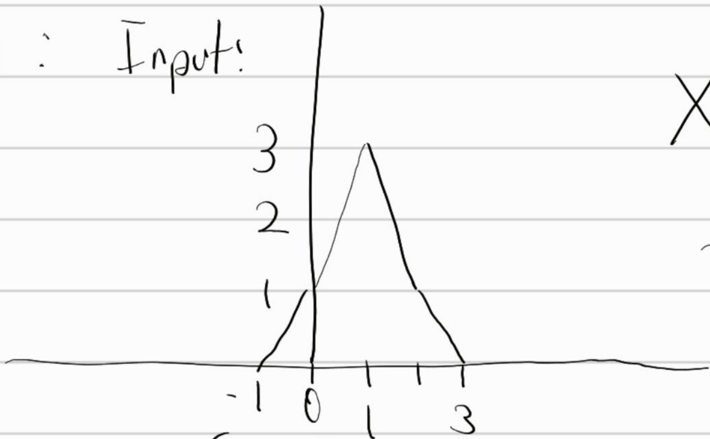
- ✓ **- 0 pts** Correct
- **- 2 pts** Partial correct 1
- **- 2 pts** Partial correct 2
- **- 4 pts** Incorrect/ Missing 1
- **- 4 pts** Incorrect Missing 2

3) a) $\mathcal{F}(\delta(t)) = 1$

$\mathcal{F}(\underbrace{\cancel{\sin(2\pi 5t)}}_{\sin(10\pi t)}) = j\pi \cdot \delta(\omega + 10\pi) - \delta(\omega - 10\pi)$

Since the FTs of the inputs match
∴ The mysterg box is doing foorier transforms

b) $x(t)$: Input:



$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$
$= \int_{-1}^{3} x(t) e^{-j\omega t} dt$

∴ Output: $X(j\omega)$:

i) $X(0) = \int_{-1}^{3} x(t) e^{0t} dt = \int_{1}^{3} x(t)\, dt =$

Area under curve $= \frac{(0--1)(1)}{2} + \frac{(2-0)(2)}{2} + (2-0)(1) + \frac{(3-2)(1)}{2}$

$= \frac{1}{2} + \frac{4}{2} + 2 + \frac{1}{2} = 1 + 2 + 2 = 5$

∴ Frined is wrong.

ii) $\int_{-\infty}^{\infty} X(j\omega)\, d\omega = \int_{-\infty}^{\infty} X(j\omega) e^{j\omega \cdot 0} d\omega$

$= 2\pi \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega \cdot 0} d\omega$

$= 2\pi \cdot x(0) = 2\pi$

3.1 3)a) 3 / 3

✓ **- 0 pts** Correct

- 3 pts Missing

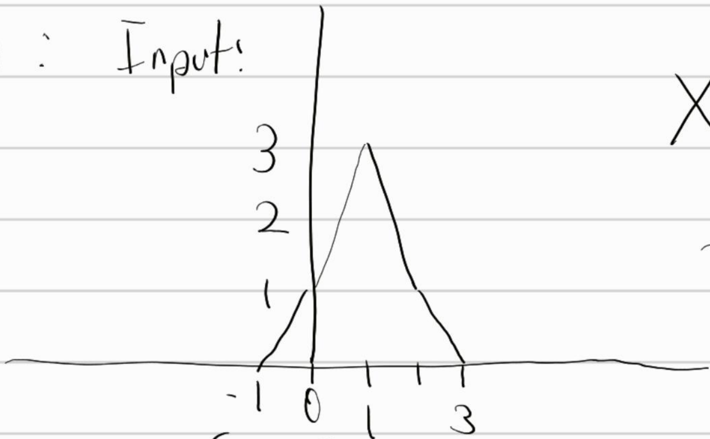
3) a) $\mathcal{F}(\delta(t)) = 1$

$\mathcal{F}(\underbrace{\cancel{\sin(2\pi 5t)}}_{\sin(10\pi t)}) = j\pi \cdot \delta(\omega + 10\pi) - \delta(\omega - 10\pi)$

Since the FTs of the inputs match

∴ The mystery box is doing fourier transforms

b) $x(t)$: Input:



$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$= \int_{-1}^{3} x(t) e^{-j\omega t} dt$

∴ Output: $X(j\omega)$:

i) $X(0) = \int_{-1}^{3} x(t) e^{0t} dt = \int_{1}^{3} x(t)\, dt =$

Area under curve $= \frac{(0--1)(1)}{2} + \frac{(2-0)(2)}{2} + (2-0)(1) + \frac{(3-2)(1)}{2}$

$= \frac{1}{2} + \frac{4}{2} + 2 + \frac{1}{2} = 1 + 2 + 2 = 5$

∴ Friend is wrong.

ii) $\int_{-\infty}^{\infty} X(j\omega)\, d\omega = \int_{-\infty}^{\infty} X(j\omega) e^{j\omega \cdot 0} d\omega$

$= 2\pi \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega \cdot 0} d\omega$

$= 2\pi \cdot x(0) = 2\pi$

iii) $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= 2\pi\left(\int_{-1}^{0}(t+1)^2 dt + \int_{0}^{1}(2t+1)^2 dt + \int_{-1}^{2}(2t+5)^2 dt + \int_{2}^{3}(-t+3)^2 dt\right)$$

$$= 56\pi/3$$

iv) $= Im(X(j\omega))$ for $x(t+1) + x(-t+1)$

$x(t+1)$ is real & even $\therefore x(-t+1)$ is also real & even

$\therefore Im(X(j\omega)) = 0$

v) $x(t) \rightarrow \boxed{\mathcal{F}} \rightarrow X(j\omega)$

$\therefore x_1(t) \rightarrow \boxed{\mathcal{F}} \rightarrow \frac{1}{5}X\left(\frac{j\omega}{5}\right)e^{j2\omega}$

Using shifting & scaling,

$x(5t) = \frac{1}{5}X\left(\frac{j\omega}{5}\right)$

$x(5t+10) = \frac{1}{5}X\left(\frac{j\omega}{5}\right)e^{j2\omega}$

$\therefore x(5t+10)$

c) $x(t) = t^4$

For transform to exist, $\int x(t)dt < \infty$

$\int_{-\infty}^{\infty} t^4 dt = t^5/5 \Big|_{-\infty}^{\infty} = \infty$

3.2 3)b) **10 / 10**

✓ **- 0 pts** Correct

- 2 pts Incorrect/missing part ii

- 2 pts Incorrect/missing part iii

- 1 pts Incorrect added component in part v

- 2 pts Incorrect/missing part I

- 2 pts Incorrect/missing part iv

- 10 pts Missing

- 2 pts Incorrect/missing part v

iii) $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$= 2\pi\left(\int_{-1}^{0} (t+1)^2 \, dt + \int_{0}^{1} (2t+1)^2 \, dt + \int_{-1}^{2} (2t+5)^2 \, dt + \int_{2}^{3} (-t+3)^2 \, dt\right)$

$= 56\pi/3$

iv) $= Im(X(j\omega))$ for $x(t+1) + x(-t+1)$

$x(t+1)$ is real & even $\therefore x(-t+1)$ is also real & even

$\therefore Im(X(j\omega)) = 0$

v) $x(t) \rightarrow \boxed{F} \rightarrow X(j\omega)$

$\therefore x_1(t) \rightarrow \boxed{F} \rightarrow \frac{1}{5} X\left(\frac{j\omega}{5}\right) e^{j2\omega}$

Using shifting & scaling,

$x(5t) = \frac{1}{5} X\left(\frac{j\omega}{5}\right)$

$x(5t+10) = \frac{1}{5} X\left(\frac{j\omega}{5}\right) e^{j2\omega}$

$\therefore x(5t+10)$

c) $x(t) = t^4$

For transform to exist, $\int x(t) dt < \infty$

$\int_{-\infty}^{\infty} t^4 \, dt = t^5/5 \Big|_{-\infty}^{\infty} = \infty$

Since t^4 does not converge, there is no fourier transform

To avoid doing this, dont generate signals that has an integral that converges.

(Also, just aside question, is it even possible to generate t^4 on a signal generator?)

3.3 3)c) **7 / 7**

✓ **- 0 pts** Correct

- 7 pts Missing

4) a) ii) $x_2(t) = e^{(2+3j)t} u(-t+1) = e^{2t} e^{3jt} u(-t+1)$

Starting with $e^{-at} u(t) = \dfrac{1}{a+j\omega}$

$x_3(t) = e^{2t} u(-t+1) = e^{2} e^{-2(t+1)} u(-t+1)$

$e^{2t} u(t) \rightarrow FT \rightarrow \dfrac{1}{2+j\omega}$

Using time shift.

$e^{2} e^{-2(t+1)} u(t+1) \rightarrow FT \rightarrow e^{2} \dfrac{1}{2-j\omega} e^{-j\omega}$

Using Modulation:

$e^{j\omega_0 t} x_3(t) = X_3(j(\omega-\omega_0))$

$\therefore e^{3jt} e^{2t} u(t+1) = e^{2} \dfrac{1}{2-j(\omega-3)} e^{-j(\omega-3)}$

iii) $\begin{cases} 1+\cos(\pi t), & -1<t<1 \\ 0 & \text{else} \end{cases}$

$(1+\cos(\pi t))\, \mathrm{rect}(t/2)$

$= \mathrm{rect}(t/2) + \mathrm{rect}(t/2)\cos(\pi t)$

$\text{rect}(t/2) \rightarrow FT \rightarrow 2\,\text{sinc}\left(\frac{\omega}{\pi}\right)$

Using Modulation:

$\text{rect}(t/2)\cos(\pi t) = \text{sinc}\left(\frac{\omega}{\pi}-1\right) + \text{sinc}\left(\frac{\omega}{\pi}+1\right)$

$\therefore \mathcal{F}(x_3(t)) = 2\,\text{sinc}\left(\frac{\omega}{\pi}\right) + \text{sinc}\left(\frac{\omega}{\pi}-1\right) + \sin\left(\frac{\omega}{\pi}+1\right)$

iv) $te^{-2t}u(t)$

$e^{-2t}u(t) \leftrightarrow \frac{1}{2+j\omega}$

$\therefore te^{-2t}u(t) \rightarrow FT \rightarrow j\frac{d}{d\omega}\frac{1}{2+j\omega}$

$\therefore te^{-2t}u(t) \rightarrow FT \rightarrow j\left(\frac{-j}{(2+j\omega)^2}\right) = \frac{1}{(2+j\omega)^2}$

b) i) $f_1(t) = \text{sinc}(2t)$

$f_2(t) = \text{sinc}(t)\cos(3.1\pi t)$

$f(t) = f_1(t) * f_2(t) = F(j\omega)$

$F_1(j\omega) = \mathcal{F}(\text{sinc}(2t)) = \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$

$F_2(j\omega) = \mathcal{F}(\cos(3.1\pi t)) = \frac{1}{2}\left(\text{rect}\left(\frac{\omega-3.1\pi}{2\pi}\right) + \text{rect}\left(\frac{\omega+3.1\pi}{2\pi}\right)\right)$

$\therefore F(j\omega) = F_1(j\omega)F_2(j\omega) = 0$

ii) $f(t) = \mathcal{F}^{-1}(0) = 0$

4.1 4)a) **21 / 21**

- ✓ **- 0 pts** Correct
- **- 1 pts** part 2 minor mistake
- **- 4 pts** part 2 mostly wrong
- **- 1 pts** part 3 minor mistake
- **- 4 pts** part 3 wrong
- **- 5 pts** part 3 incomplete
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- **- 4 pts** part 4 mostly wrong
- **- 5 pts** part 4 incomplete
- **- 7 pts** part 2 missing
- **- 7 pts** part 3 missing
- **- 7 pts** part 4 missing

$\text{rect}(t/2) \rightarrow FT \rightarrow 2\,\text{sinc}\left(\frac{\omega}{\pi}\right)$

Using Modulation:

$\text{rect}(t/2)\cos(\pi t) = \text{sinc}\left(\frac{\omega}{\pi}-1\right) + \text{sinc}\left(\frac{\omega}{\pi}+1\right)$

$\therefore \mathcal{F}(x_3(t)) = 2\,\text{sinc}\left(\frac{\omega}{\pi}\right) + \text{sinc}\left(\frac{\omega}{\pi}-1\right) + \sin\left(\frac{\omega}{\pi}+1\right)$

iv) $te^{-2t}u(t)$

$e^{-2t}u(t) \leftrightarrow \frac{1}{2+j\omega}$

$\therefore te^{-2t}u(t) \rightarrow FT \rightarrow j\frac{d}{d\omega}\frac{1}{2+j\omega}$

$\therefore te^{-2t}u(t) \rightarrow FT \rightarrow j\left(\frac{-j}{(2+j\omega)^2}\right) = \frac{1}{(2+j\omega)^2}$

b) i) $f_1(t) = \text{sinc}(2t)$

$f_2(t) = \text{sinc}(t)\cos(3.1\pi t)$

$f(t) = f_1(t) * f_2(t) = F(j\omega)$

$F_1(j\omega) = \mathcal{F}(\text{sinc}(2t)) = \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$

$F_2(j\omega) = \mathcal{F}(\cos(3.1\pi t)) = \frac{1}{2}\left(\text{rect}\left(\frac{\omega-3.1\pi}{2\pi}\right) + \text{rect}\left(\frac{\omega+3.1\pi}{2\pi}\right)\right)$

$\therefore F(j\omega) = F_1(j\omega)F_2(j\omega) = 0$

ii) $f(t) = \mathcal{F}^{-1}(0) = 0$

4.2 4)b) **9 / 9**

✓ **- 0 pts** Correct

- 1 pts part 1 minor mistake

- 2 pts part 1 partially correct

- 2 pts part 2 wrong

- 4 pts part 1 incomplete

- 3 pts part 2 missing

- 6 pts part 1 missing