23W-EC ENGR-131A-LEC-1 Homework 6

SANJIT SARDA

TOTAL POINTS

97 / 100

QUESTION 1

1 1 **20 / 20**

✓ **- 0 pts** *Correct*

- **- 3 pts** (a) incorrect
- **- 3 pts** (b) incorrect
- **- 3 pts** (c) incorrect
- **- 5 pts** incorrect and little/no work shown
- **- 20 pts** missing

QUESTION 2

2 2 **20 / 20**

✓ **- 0 pts** *Correct*

- **- 5 pts** incorrect
- **- 10 pts** incorrect and little/no work shown
- **- 20 pts** missing

QUESTION 3

3 3 **17 / 20**

- **- 0 pts** Correct
- **- 3 pts** (a) incorrect
- **- 3 pts** (b) incorrect

✓ **- 3 pts** *(c) incorrect*

- **- 3 pts** (d) incorrect
- **- 2 pts** any part partially incorrect
- **- 20 pts** missing
- **- 1 pts** any part partially incorrect

QUESTION 4

4 4 **20 / 20**

✓ **- 0 pts** *Correct*

- **- 5 pts** (a) incorrect
- **- 5 pts** (b) incorrect
- **- 10 pts** either part incorrect and little/no work shown
- **- 20 pts** missing

QUESTION 5

5 5 **20 / 20**

✓ **- 0 pts** *Correct*

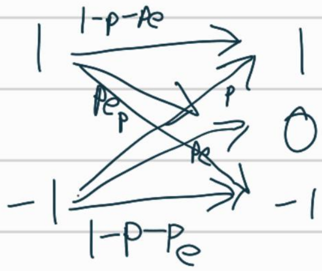
- **- 3 pts** (a) incorrect
- **- 3 pts** (b) incorrect
- **- 3 pts** (c) incorrect
- **- 10 pts** little/no work shown and incorrect
- **- 20 pts** missing

ECE 131A HW6

①

$$PMF_X = \begin{cases} 1/4, & x = -1 \\ 3/4, & x = 1 \\ 0, & ow \end{cases}$$

$$Y = \begin{cases} X, & 1-p-p_e \\ -X, & p \\ 0, & p_e \end{cases} \rightarrow$$



ⓐ S		S_{XY}	ⓑ P
I=1	O=1	(1,1)	$P_{XY}(1,1) = P(X=1)P(Y=1\|X=1) = \frac{3}{4}(1-p-p_e)$
I=1	O=0	(1,0)	$P_{XY}(1,0) = P(X=1)P(Y=0\|X=1) = \frac{3}{4}p_e$
I=1	O=-1	(1,-1)	$P_{XY}(1,-1) = P(X=1)P(Y=-1\|X=1) = \frac{3}{4}p$
I=-1	O=1	(-1,1)	$P_{XY}(-1,1) = P(X=-1)P(Y=1\|X=1) = \frac{1}{4}p$
I=-1	O=0	(-1,0)	$P_{XY}(-1,0) = P(X=-1)P(Y=0\|X=1) = \frac{1}{4}p_e$
I=-1	O=-1	(-1,-1)	$P_{XY}(-1,-1) = P(X=-1)P(Y=-1\|X=1) = \frac{1}{4}(1-p-p_e)$

ⓒ $P(X \neq Y) = 1 - P(X=Y) = 1 - \left(P_{XY}(1,1) + P_{XY}(-1,-1)\right) = 1 - \cancel{\left(\frac{3}{4}+\frac{1}{4}\right)}(1-p-p_e)$

$$= 1-(1-p-p_e) = p + p_e$$

$$P(Y=0) = \sum_{x_i} P_{XY}(x_i, 0) = P_{XY}(-1,0) + P_X(1,0) = \frac{1}{4}p_e + \frac{3}{4}p_e = p_e$$

1 1 20 / 20

✓ **- 0 pts** *Correct*

- 3 pts (a) incorrect

- 3 pts (b) incorrect

- 3 pts (c) incorrect

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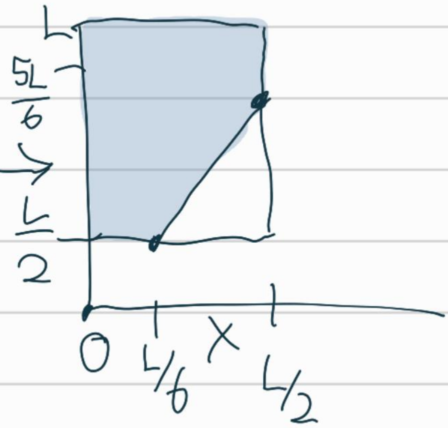
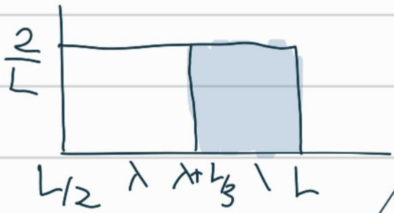
2

$X = U(0, L/2)$ M $Y = U(L/2, L)$

Valid!



$X \to FP1$ $\quad$ $Y \to FP2$



Find $P(Y - X > L/3)$

$$= \int_{L/2}^{L} \int_{x=0}^{y-L/3} \frac{2}{L}\frac{2}{L}\, dA = \quad = \frac{Sq - Tri}{Sq} = 1 - \frac{Tri}{Sq}$$

$$=$$

$$= 1 - \left(\frac{1}{2}\left(\frac{L}{2} - \frac{L}{6}\right)\left(\frac{5L}{6} - \frac{L}{2}\right)\right) \cdot \frac{1}{(L/2)^2}$$

$$= 1 - \frac{\cancel{4}^{2}}{L^2} \cdot \frac{1}{\cancel{2}} \cdot \left(\frac{3L - L}{6}\right)\left(\frac{5L - 3L}{6}\right)$$

$$= 1 - \frac{2}{\cancel{L^2}} \frac{1}{36} \cdot 2L \cdot 2L = 1 - \frac{8}{36} = 1 - \frac{2}{9}$$

$$= 7/9$$

2 2 **20 / 20**

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3 3 **17 / 20**

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- **- 3 pts** (a) incorrect
- **- 3 pts** (b) incorrect
- ✓ **- 3 pts** *(c) incorrect*
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- **- 1 pts** any part partially incorrect

(4)

(a) X is Poisson: λ :

$$PMF = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & x \geq 0 \\ 0, & ow \end{cases}$$

$E[X] = Var[X] = \lambda$

From Markov

$$P(X \geq \alpha) \leq \frac{E[X]}{\alpha}$$

$$\therefore P(X \geq 2\lambda) \leq \frac{1\cancel{\lambda}}{2\cancel{\lambda}} \leftarrow \text{Not Helpful}$$

From Chebyshev,

$$P(|X-\mu| \geq \alpha) \leq \frac{\sigma^2}{\alpha^2} \qquad \therefore P(X-\mu \geq \alpha) + P(X-\mu \leq -\alpha)$$

$$\begin{aligned} P(X \geq 2\lambda) &= P(X-\lambda \geq \lambda) \\ &= P(X-\lambda \geq \lambda) + P(X \leq \lambda - \lambda) \\ &= P(X-\lambda \geq \lambda) + P(X-\lambda \leq -\lambda) \\ &= P(|X-\lambda| \geq \lambda) \end{aligned}$$

0

$$\therefore P(X \geq 2\lambda) = P(|X-\lambda| \geq \lambda) \leq \frac{\lambda}{\lambda^2}$$

$$\therefore P(X > 2\lambda) \leq \frac{1}{\lambda}$$

Markov Proof:

$$E[X] = \int_0^\infty x f_X(x)\,dx = \underbrace{\int_0^\alpha x f_X(x)}_{\geq 0} + \int_\alpha^\infty x f_X(x)$$

$$E[X] \geq \int_\alpha^\infty x f_X(x)\,dx \qquad \therefore P(X \geq \alpha) \leq \frac{E[X]}{\alpha}$$

$$E[X] \geq \int_0^\infty \alpha f_X(x)\,dx$$

$$E[X] \geq \alpha P(X \geq \alpha)$$

(b) X is Std Normal : $N(0,1)$:

$$Q(x) = P(X \geq x)$$

$$P(X \geq x) \leq e^{-sx} \int_{-\infty}^{\infty} e^{st} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$$

$$\leq \frac{e^{-sx}}{\cancel{\sqrt{2\pi}}} \cancel{\sqrt{2\pi}} \cdot e^{s^2/2}$$

$$\leq e^{s^2/2 - sx}$$

$$\therefore Q(x) \leq e^{s^2/2 - sx}$$

$$\left(\frac{s^2}{2} - sx\right)' = 0 \quad \therefore s = x$$

$$\therefore Q(x) \leq e^{\frac{x^2}{2} - x^2}$$

$$\therefore Q(x) \leq e^{-x^2/2}$$

4 4 **20 / 20**

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5

```python
import numpy as np
import matplotlib.pyplot as plt
```

✓ 0.0s

```python
# a)
def X1(u1,u2):
    return np.sqrt(-2*np.log(u1))*np.cos(2*np.pi*u2)

def X2(u1,u2):
    return np.sqrt(-2*np.log(u1))*np.sin(2*np.pi*u2)


sampled_u1 = np.random.uniform(0,1,5000)
sampled_u2 = np.random.uniform(0,1,5000)

sampled_x1 = [X1(sampled_u1[i], sampled_u2[i]) for i in range(5000)]
sampled_x2 = [X2(sampled_u1[i], sampled_u2[i]) for i in range(5000)]
print("First 10 values of X1: ", sampled_x1[:10])
print("First 10 values of X2: ", sampled_x2[:10])
```

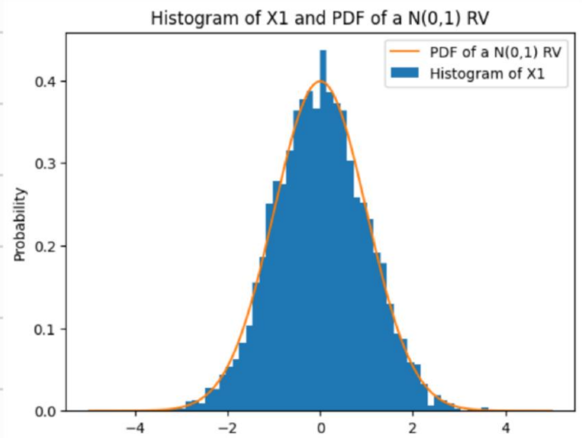
✓ 0.1s Python

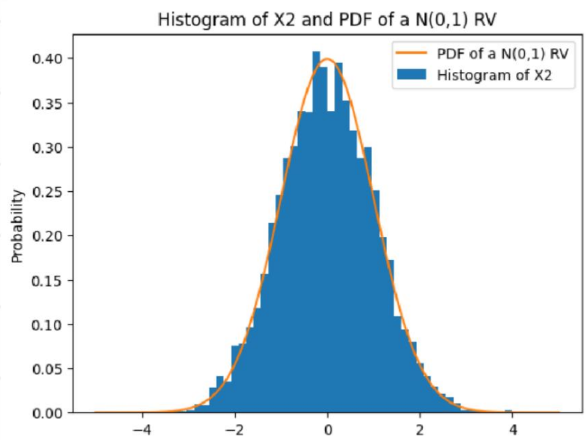
```
First 10 values of X1:  [0.171770360388136, -1.1894795166894625, 1.2844555496429833, -0.4494359613719006, -0.8708973815160764, -1.5339972658327283,
-0.8853944279508194, 0.28576777760962824, -0.4245813101770509, 1.0198734083516727]
First 10 values of X2:  [-0.11471749137628738, -0.23952635076276863, -1.1943194971357385, -2.024873284319121, 1.2539078247394384, -2.2898787070182367,
0.44962245203408, -0.7126611388667015, -0.3099193160396991, -0.5233295630173487]
```

```python
# b)
# Plot the histogram of X1
plt.hist(sampled_x1, bins=50, density=True)
plt.ylabel('Probability')
# Plot the PDF of a N(0,1) RV
x = np.linspace(-5,5,1000)
plt.plot(x, 1/np.sqrt(2*np.pi)*np.exp(-x**2/2))
plt.legend(['PDF of a N(0,1) RV', 'Histogram of X1'])
plt.ylabel('Probability')
plt.title('Histogram of X1 and PDF of a N(0,1) RV')
plt.show()


# Plot the histogram of X2
plt.hist(sampled_x2, bins=50, density=True)
plt.ylabel('Probability')
# Plot the PDF of a N(0,1) RV
x = np.linspace(-5,5,1000)
plt.plot(x, 1/np.sqrt(2*np.pi)*np.exp(-x**2/2))
plt.legend(['PDF of a N(0,1) RV', 'Histogram of X2'])
plt.ylabel('Probability')
plt.title('Histogram of X2 and PDF of a N(0,1) RV')
plt.show()
```

✓ 0.7s

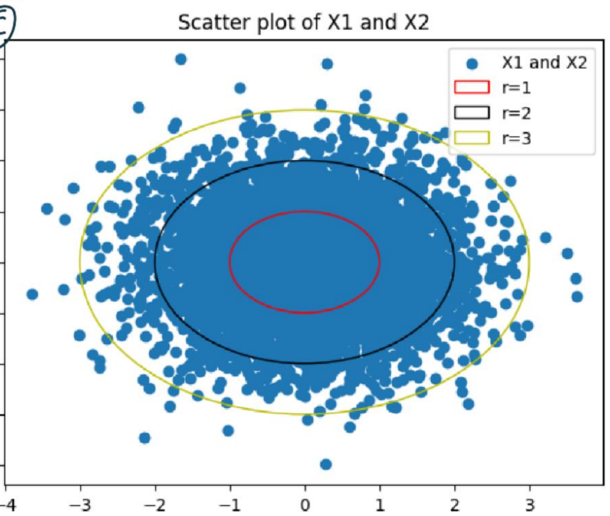





```python
# c)
plt.scatter(sampled_x1, sampled_x2)
plt.title('Scatter plot of X1 and X2')
# Draw circles with radii 1, 2, and 3
circle1 = plt.Circle((0, 0), 1, color='r', fill=False)
circle2 = plt.Circle((0, 0), 2, color='k', fill=False)
circle3 = plt.Circle((0, 0), 3, color='y', fill=False)
ax = plt.gca()
ax.add_artist(circle1)
ax.add_artist(circle2)
ax.add_artist(circle3)
plt.legend(['X1 and X2', 'r=1', 'r=2', 'r=3'])
plt.show()
```

✓ 0.3s

6



As you can see from the above picture, the scatter exhibits circular symmetry. We can also see that the there are more dots in the inner circles than outer.

5 5 **20 / 20**

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