ECE 10, Winter 2023, Homework #1

Problem 1: The current (in Amperes) in a circuit varies in time as

$$i(t) = \begin{cases} \frac{1}{2}e^{-5t}, & t \geq 6 \\ 0, & t < 6 \end{cases}.$$

What is the total charge that passes through the circuit, over all time, in Coulombs? **(5 points)**

Problem 2: A 2.5F capacitor, charged to 10 Volts is connected across a 2 Ohm resistor at time t = 0, by closing a switch, as shown in Figure 1. The current in the circuit is shown to be of the form

$$i(t) = \begin{cases} 5e^{-t/5}, & t \geq 0 \\ 0, & t < 0 \end{cases}.$$

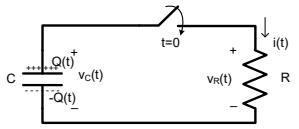

Figure 1

At a certain time, t = t_0, the current has a value 2 Amperes. The switch behaves like a wire of zero resistance when closed. At t = t_0,

(a) at what rate is the voltage across the capacitor, $v_C(t)$ changing? Don't forget the correct units.

(b) what is the voltage across the resistor, $v_R(t)$?

(c) what is the value of the charge on the capacitor, Q(t)?

(3 + 3 + 3 = 9 points)

Problem 3: A parallel plate capacitor is connected to an ideal 5V voltage source. Suppose that the plates of the capacitor are slowly moved in time such that the capacitance decreases linearly from C = 10pF to C = 5pF in 5 seconds.

(a) Sketch the current coming from the positive terminal of the battery and flowing into the capacitor as a function of time while the plates were being moved.

(b) How much energy was stored in the capacitor before the plates started to move?

(c) How much energy is stored in the capacitor after the plates have finished their movement?

(d) How much total energy was delivered to (or removed from) the capacitor during the 5 seconds of movement?

(e) How much energy was delivered by (or delivered to) the ideal voltage source during the 5 seconds of movement?

(f) Do your answers to (e) and (f) indicate that energy is conserved? Explain.

(3 + 3 + 3 + 3 + 3 + 5 = 18 points)

Problem 4: Starting at time t = 0, a battery is applied across an inductor of 3H. The battery voltage increases linearly from 0 to 10V in time 5s after which, it stays at 10V.

(a) Derive an expression for the current through the inductor as a function of time t.

(b) Derive an expression for the energy delivered to the inductor as a function of time t.

(10 points)

Problem 5: Figure 3 shows a common, realistic model for a battery. The resistance, R_S, is often referred to as the source resistance. Derive an expression for the voltage across the battery's terminals, V_{ab} as a function of the current flowing through it, I.
(6 points)
Solution:

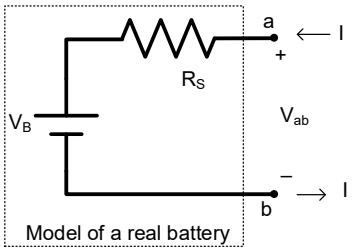

Figure 2