23W-EC ENGR-10-LEC-1 HW4

SANJIT SARDA

TOTAL POINTS

93 / 95

QUESTION 1

20 pts

1.1 a **2 / 2**

✓ **- 0 pts** *Correct*

1.2 b **12 / 12**

✓ **- 0 pts** *Correct*

   - 5 pts wrong expression

   - 2 pts k1 = 2

   - 2 pts k2 = -4

   - 2 pts k3 = 0

1.3 c **6 / 6**

✓ **- 0 pts** *Correct*

   - 3 pts wrong expression

   - 2 pts k1 = 1

   - 2 pts k2 = 2

   - 2 pts k3 = 1

QUESTION 2

20 pts

2.1 a **2 / 2**

✓ **- 0 pts** *Correct*

2.2 b **12 / 12**

✓ **- 0 pts** *Correct*

   - 5 pts wrong expression

   - 2 pts k1 = 0

   - 2 pts k2 = $$-2* 10^{-3}$$

   - 2 pts k3 = 0

   - 12 pts no answer

2.3 c **6 / 6**

✓ **- 0 pts** *Correct*

   - 3 pts wrong expression

   - 6 pts no answer

QUESTION 3

15 pts

3.1 a **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts missing/wrong

3.2 b **2 / 2**

✓ **- 0 pts** *Correct*

   - 0.5 pts answer should be 1/8 or 0.125

   - 2 pts did not answer

3.3 c **7 / 7**

✓ **- 0 pts** *Correct*

   - 1.5 pts A = 1

   - 1.5 pts B = -0.378

   - 7 pts wrong or missing

   - 1.5 pts C = 0

3.4 d **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts wrong or missing

QUESTION 4

15 pts

4.1 a **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts wrong or missing

4.2 b **4 / 4**

✓ **- 0 pts** *Correct*

   - 2 pts wrong

   - 4 pts missing

4.3 c **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts wrong or missing

4.4 d **5 / 5**

✓ **- 0 pts** *Correct*

   - 3 pts wrong expression

   - 1 pts A = 2

   - 1 pts B =1

   - 1 pts k =0

   - 5 pts missing

QUESTION 5

5 **8 / 10**

   - 0 pts Correct

✓ **- 2 pts** *B = 0.4322*

   - 2 pts $\phi = -\tan^{-1}(46/5)$ or $=-1.463$

   - 5 pts wrong

   - 10 pts missing

QUESTION 6

15 pts

6.1 **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts missing or incorrect

6.2 **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts missing or incorrect

6.3 **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts wrong or missing

6.4 **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts missing or incorrect

6.5 **3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts missing or incorrect

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①

at t = 0

1Ω R

N

$i_L(t)$ L $v_c(t)$ 2H

$+$ C $\frac{1}{2}$F $-$

$1V_B$

$v_c(0-)=2V$

$I_L(0-)=2A =$

$I_L(0+)=I_L(0-)=2A$

$V_c(0+)=V_c(0-)=2V$

$V_c'(0+)=\left(\dfrac{V_B-V_c(0+)}{R}-I_L(0+)\right)\cdot\dfrac{1}{C}$

$\therefore V_c'(0+)=(1-2-2)\cdot 2$

$= -6V$

$V=LI_L'$ $\therefore I_L=\dfrac{1}{L}\int V$

@ N $\dfrac{V_B-V_c}{R}=I_L+CV_c'$

$\therefore V_B-V_c=\dfrac{R}{L}\int v+RCv_c'$

$\therefore 0=V_c'+\dfrac{R}{L}v+RCv''$

$\therefore V_c''+\dfrac{V_c'}{RC}+\dfrac{V_c}{LC}=0$

$\therefore V_c''+2V_c'+V_c=0$

$\therefore \omega_0^2=1, \quad 2\mathcal{S}\omega_0=2 \therefore \mathcal{S}=1$

ⓐ $\mathcal{S}=1 \therefore$ Critically damped

© $I_L(t)=\dfrac{V_B-V_c}{R}-CV_c'$

$\therefore I_L(t)=1+(4t-2)e^{-t}+2(6-4t)e^{-t}$

$= (2t+1)e^{-t}+1$

ⓑ $V_c=(C_1t+C_2)e^{-t}$, $V_c(0)=2V$, $V_c'(0)=-6V/s$

$\therefore V_c(0)=C_2=2$

$\therefore V_c'(0)=C_1-C_2=-6 \therefore C_1=-4$

$\therefore V_c=(2-4t)e^{-t}$

1.1 a **2 / 2**

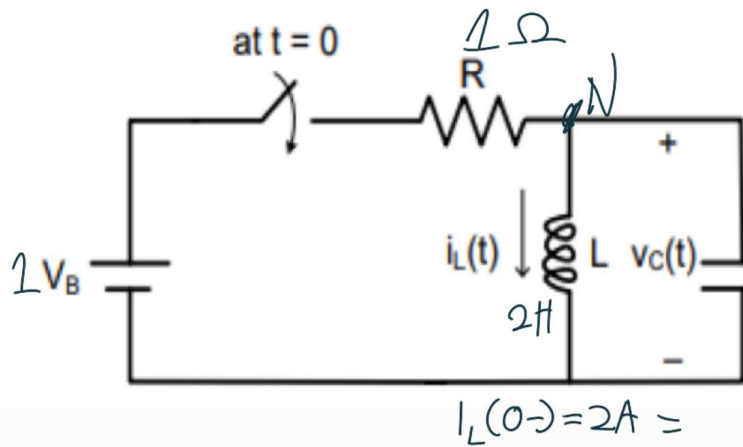✓ **- 0 pts** *Correct*

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ECE 10 HW4

①

at t = 0

$I_L(0+) = I_L(0-) = 2A$

$V_C(0+) = V_C(0-) = 2V$

$V_C(0-) = 2V$ $V_C'(0+) = \left(\dfrac{V_B - V_C(0+)}{R} - I_L(0+)\right) \cdot \dfrac{1}{C}$

$C = \dfrac{1}{2}F$ $\therefore V_C'(0+) = (1 - 2 - 2) \cdot 2$

$= -6V$

$I_L(0-) = 2A =$

$V = L I_L'$ $\therefore I_L = \dfrac{1}{L}\int V$

@ N $\dfrac{V_B - V_C}{R} = I_L + C V_C'$

$\therefore V_B - V_C = \dfrac{R}{L}\int V + R C V_C'$ ⓒ $I_L(t) = \dfrac{V_B - V_C}{R} - C V_C'$

$\therefore 0 = V_C' + \dfrac{R}{L} V + R C V''$ $\therefore I_L(t) = 1 + (4t-2)e^{-t} + 2(6-4t)e^{-t}$

$= (2t+1)e^{-t} + 1$

$\therefore V_C'' + \dfrac{V_C'}{RC} + \dfrac{V_C}{LC} = 0$

$\therefore V_C'' + 2V_C' + V_C = 0$

$\therefore \omega_0^2 = 1, \ 2\zeta\omega_0 = 2 \ \therefore \zeta = 1$

ⓐ $\zeta = 1 \ \therefore$ Critically damped

ⓑ $V_C = (C_1 t + C_2)e^{-t}$, $V_C(0) = 2V$, $V_C'(0) = -6V/s$

$\therefore V_C(0) = C_2 = 2$

$\therefore V_C'(0) = C_1 - C_2 = -6 \ \therefore C_1 = -4$

$\therefore V_C = (2 - 4t)e^{-t}$

1.2 b **12 / 12**

✓ **- 0 pts** *Correct*

    - 5 pts wrong expression
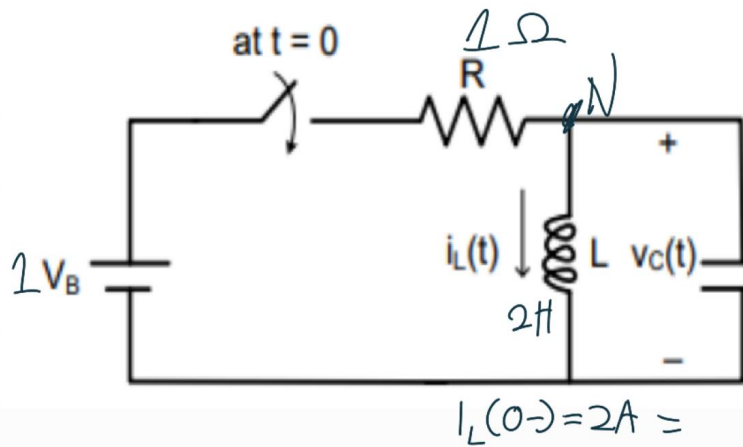
    - 2 pts k1 = 2

    - 2 pts k2 = -4

    - 2 pts k3 = 0

✓ **- 0 pts** *Correct*

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①

at t = 0

$1\,\Omega$ R

$1 V_B$

$i_L(t)$ L $v_c(t)$ 2H

$+$ C $\frac{1}{2}F$ $-$

$V_c(0-)=2V$

$I_L(0-)=2A =$

$I_L(0+)=I_L(0-)=2A$

$V_c(0+)=V_c(0-)=2V$

$V_c'(0+) = \left(\dfrac{V_B-V_c(0+)}{R} - I_L(0+)\right)\Big/ C$

$\therefore V_c'(0+) = (1-2-2)\cdot 2$

$= -6V$

$V = L I_L' \qquad \therefore I_L = \frac{1}{L}\int V$

@N $\qquad \dfrac{V_B-V_c}{R} = I_L + CV_c'$

$\therefore V_B-V_c = \dfrac{R}{L}\int v + RCV_c'$

$\therefore 0 = V_c' + \dfrac{R}{L}v + RCV''$

$\therefore V_c'' + \dfrac{V_c'}{RC} + \dfrac{V_c}{LC} = 0$

$\therefore V_c'' + 2V_c' + V_c = 0$

© $I_L(t) = \dfrac{V_B-V_c}{R} - CV_c'$

$\therefore I_L(t) = 1 + (4t-2)e^{-t} + 2(6-4t)e^{-t}$
$\qquad = (2t+1)e^{-t} + 1$

$\therefore \omega_0^2 = 1, \quad 2\zeta\omega_0 = 2 \therefore \zeta = 1$

ⓐ $\zeta = 1 \therefore$ Critically damped

ⓑ $V_c = (C_1 t + C_2)e^{-t}$, $V_c(0) = 2V$, $V_c'(0) = -6V/s$

$\therefore V_c(0) = C_2 = 2$
$\therefore V_c'(0) = C_1 - C_2 = -6 \therefore C_1 = -4$

$\therefore V_c = (2-4t)e^{-t}$

1.3 C **6 / 6**

✓ **- 0 pts** *Correct*
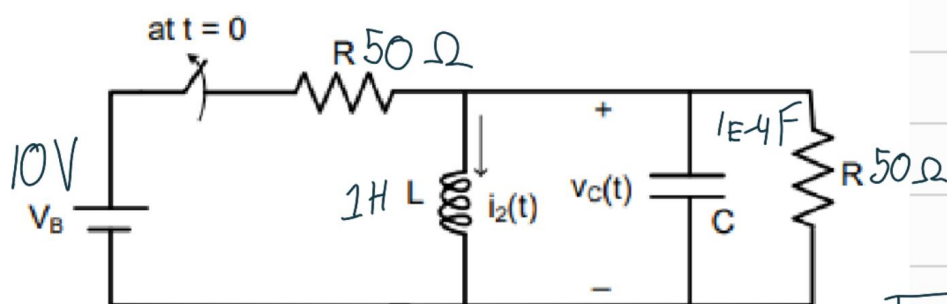
- 3 pts wrong expression

- 2 pts k1 = 1

- 2 pts k2 = 2

- 2 pts k3 = 1

✓ **- 0 pts** *Correct*

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②

at t = 0   R 50 Ω

10V V_B 1H L $i_2(t)$ $v_C(t)$ 1E-4F C   R 50 Ω

@ t=0-,

$I_2 = \dfrac{10}{50} = \boxed{.5 A}$

$V_C = \boxed{0V}$

$\dfrac{V}{L} = I'$

(a) $V_C'' + \dfrac{1}{RC} V_C' + \dfrac{1}{LC} V_C = 0$

$\omega_0 = (LC)^{-1/2}$, $2\zeta\omega_0 = \dfrac{1}{RC} \to \zeta = \dfrac{\sqrt{LC}}{2RC}$

$\therefore \omega_0 = \dfrac{1}{\sqrt{LC}} = 100$ $\zeta = \dfrac{1}{200 \cdot 50 \cdot 1E-4} = 1$

$\zeta = 1 \therefore$ it is critically damped
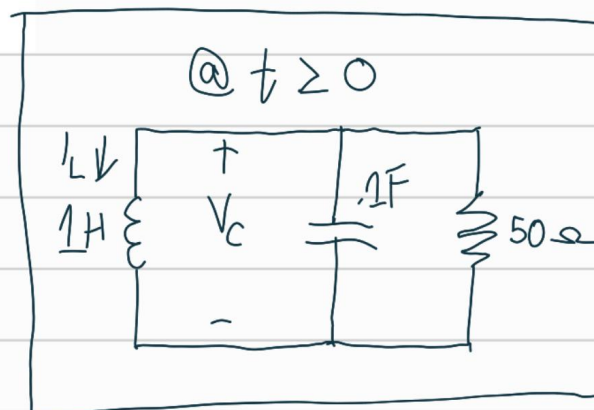
(b) $V_C'' + 200 V_C' + \dfrac{V_C}{10000} = 0$

$V_C(t) = -2000 t e^{-100t}$

@ t ≥ 0

$I_L \downarrow$ +
1H ⧙ V_C .1F   50 Ω
        –
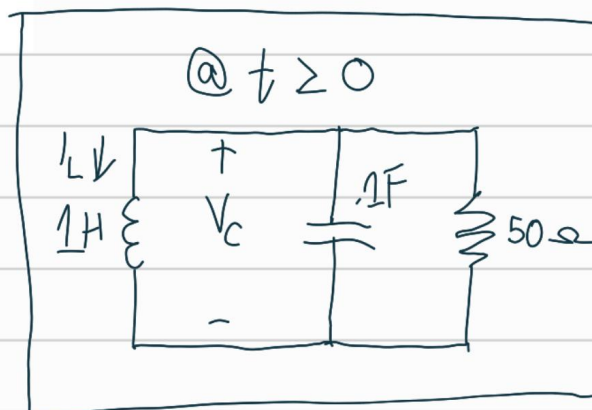
$I = CV'$ $L I' = V$

$\dfrac{1}{L}\int V + CV_C' + \dfrac{V_C}{R} = 0$

$\therefore \dfrac{1}{LC} V + \cancel{C} V_C'' + \dfrac{V_C'}{RC} = 0$
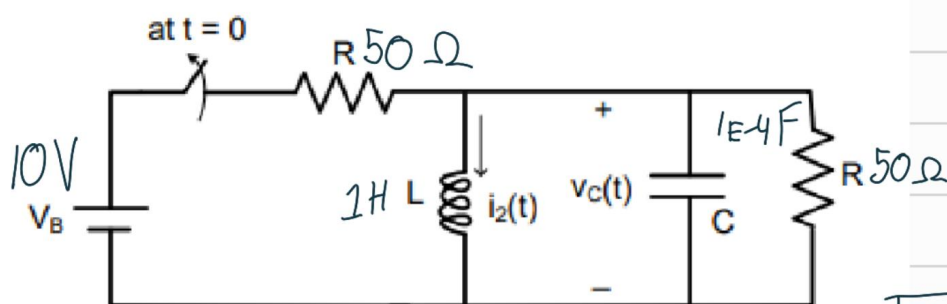
$V_C'' + \dfrac{V_C'}{RC} + \dfrac{V_C}{LC} = 0$

(c) $I_2 = \dfrac{1}{L}\int_0^t V_L + I_L(0-) = \dfrac{1}{5} + -2000\int_0^t t e^{-t} =$

$= -2000\left(\dfrac{-t e^{-100t}}{100} - \dfrac{e^{-100t}}{10000}\Big|_0^t\right) + \dfrac{1}{5} = e^{-100t}\left(\dfrac{100t+1}{5}\right)$
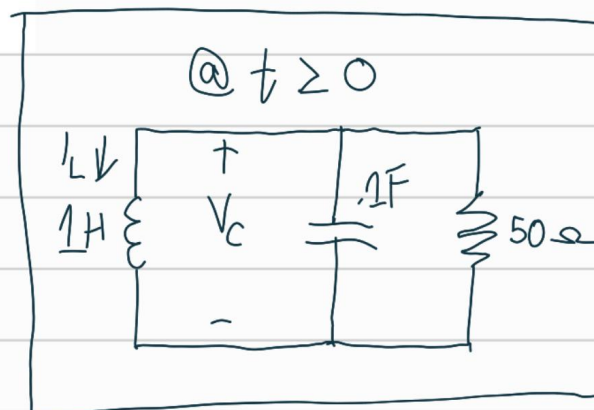
2.1 a **2 / 2**

✓ **- 0 pts** *Correct*

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②

at t = 0   R 50 Ω

10V
V_B

1H L ⌇ $i_2(t)$ $v_C(t)$ 1E4F   C   R 50 Ω

+ —

$\frac{V}{L} = I'$

@ t = 0-,

$I_2 = \frac{10}{50} = 1.5A$ actually $= .2A$

$V_C = 0V$

@ t ≥ 0

$I_L ↓$ +
1H ⌇ V_C .1F   50 Ω
      —

$I = CV'$ $LI' = V$

$\frac{1}{L}\int V + CV_C' + \frac{V_C}{R} = 0$

$\therefore \frac{1}{LC}V + \cancel{C}V_C'' + \frac{V_C'}{RC} = 0$

$V_C'' + \frac{V_C'}{RC} + \frac{V_C}{LC} = 0$

ⓐ $V_C'' + \frac{1}{RC}V_C' + \frac{1}{LC}V_C = 0$

$\omega_0 = (LC)^{-1/2}$, $2\zeta\omega_0 = \frac{1}{RC} \rightarrow \zeta = \frac{\sqrt{LC}}{2RC}$

$\therefore \omega_0 = \frac{1}{\sqrt{LC}} = 100$ $\zeta = \frac{1}{2 \cdot 100 \cdot 50 \cdot 1E-4} = 1$

$\zeta = 1 \therefore$ it is critically damped

ⓑ $V_C'' + 200V_C' + \frac{V_C}{10000} = 0$

$V_C(t) = -2000te^{-100t}$

ⓒ $I_2 = \frac{1}{L}\int_0^t V_L + I_L(0-) = \frac{1}{5} + -2000\int_0^t te^{-t} =$

$= -2000\left(\frac{-te^{-100t}}{100} - \frac{e^{-100t}}{10000}\bigg|_0^t\right) + \frac{1}{5} = e^{-100t}\left(\frac{100t+1}{5}\right)$

2.2 b **12 / 12**

   ✓ **- 0 pts** *Correct*

      - 5 pts wrong expression
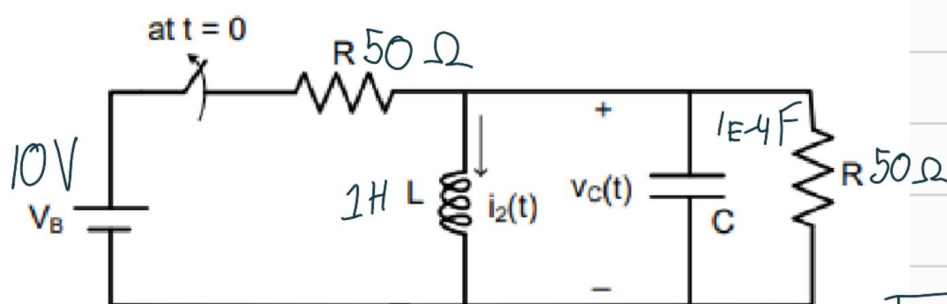
      - 2 pts k1 = 0

      - 2 pts k2 = $-2*10^{-3}$

      - 2 pts k3 = 0

      - 12 pts no answer

   ✓ **- 0 pts** *Correct*

ıll gradescope

②

at t = 0

R 50 Ω

10V
V_B

1H L, $i_2(t)$

$v_C(t)$

1E-4F C

R 50 Ω

@t=0⁻,

$$I_2 = \frac{10}{50} = \boxed{\frac{1}{5} A}$$

$$V_C = \boxed{0V}$$

@ t ≥ 0

$I_L \downarrow$

1H

$+$ V_C $-$

.1F

50 Ω

$$\frac{V}{L} = I'$$

ⓐ $V_C'' + \frac{1}{RC} V_C' + \frac{1}{LC} V_C = 0$

$\omega_0 = (LC)^{-1/2}$, $2\zeta\omega_0 = \frac{1}{RC} \rightarrow \zeta = \frac{\sqrt{LC}}{2RC}$

$\therefore \omega_0 = \frac{1}{\sqrt{LC}} = 100$ $\zeta = \frac{1}{2 \cdot 100 \cdot 50 \cdot 1E-4} = 1$

$\zeta = 1 \therefore$ it is critically damped

ⓑ $V_C'' + 200 V_C' + \frac{V_C}{10000} = 0$

$V_C(t) = -2000 t e^{-100t}$

$I = CV'$ $LI' = V$

$\frac{1}{L}\int V + CV_C' + \frac{V_C}{R} = 0$

$\therefore \frac{1}{LC} V + \cancel{C} V_C'' + \frac{V_C'}{RC} = 0$

$V_C'' + \frac{V_C'}{RC} + \frac{V_C}{LC} = 0$

ⓒ $I_2 = \frac{1}{L}\int_0^t V_L + I_L(0^-) = \frac{1}{5} + -2000 \int_0^t t e^{-t} =$

$= -2000 \left(\frac{-t e^{-100t}}{100} - \frac{e^{-100t}}{10000} \Big|_0^t \right) + \frac{1}{5} = e^{-100t}\left(\frac{100t+1}{5}\right)$
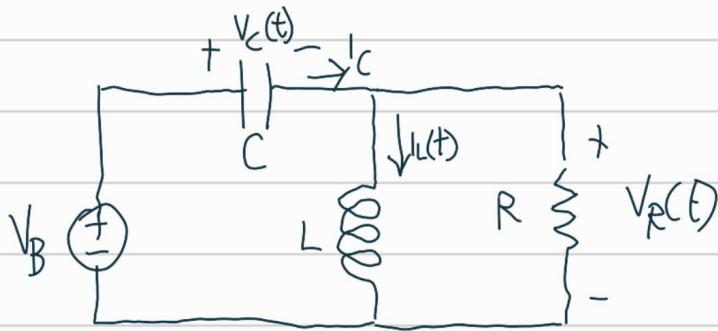
2.3 C **6 / 6**

✓ **- 0 pts** *Correct*

- 3 pts wrong expression

- 6 pts no answer

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③

$V_C(0^+) = 1V$, $V_R(0^+) = V_B - V_C(0^+) = 1V$

$I_L(0^+) = 1mA$, $V_R'(0^+) = 250000$ ←

$V_R' = -V_C' = -\frac{1}{C}I_C = \frac{-1}{C}2mA$

$V_B = V_C + V_R$ $V_C = V_B - V_R$, $V_C' = -V_R'$

$C V_C' = I_L + \frac{V_R}{R}$

$V_C' = \frac{1}{LC}\int V_R + \frac{V_R}{RC}$
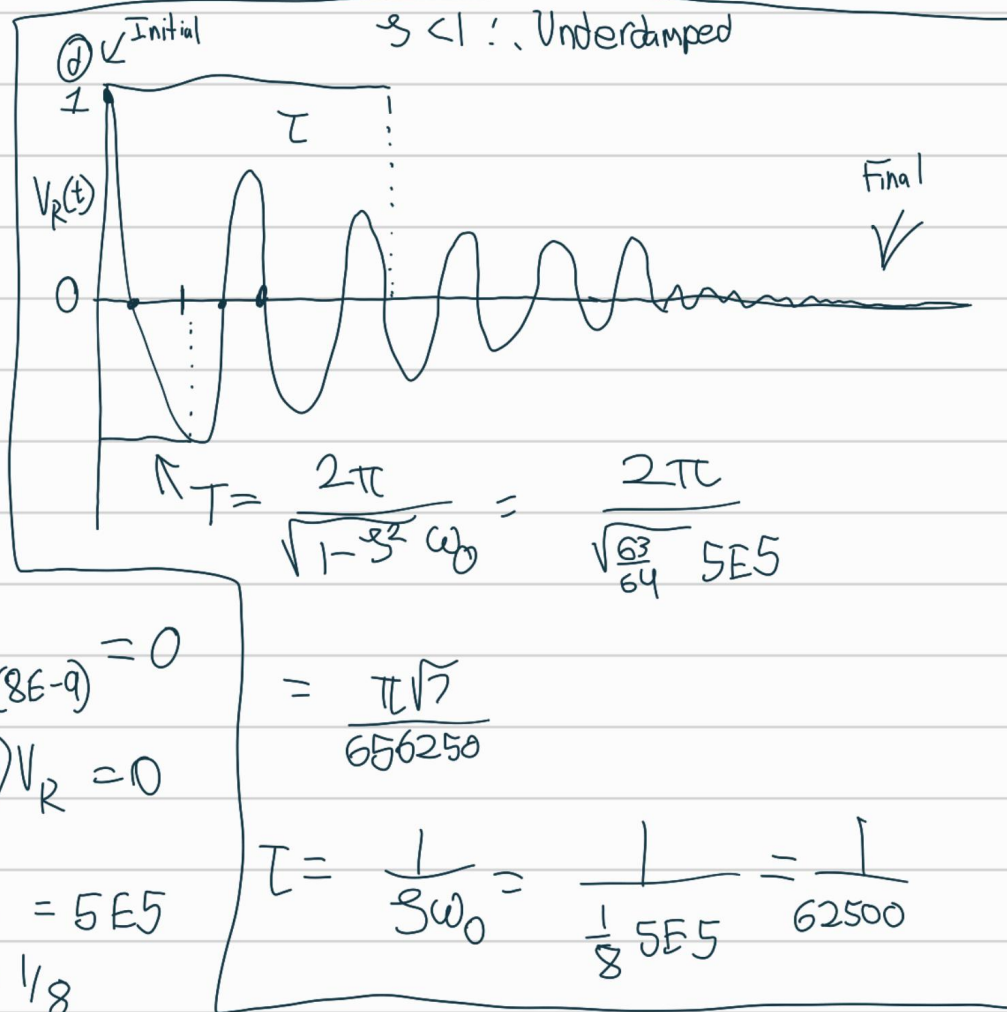
$-V_R' = \frac{1}{LC}\int V_R + \frac{V_R}{RC}$

$\therefore V_R'' + \frac{V_R'}{RC} + \frac{V_R}{LC} = 0$

$V_R'' + \frac{V_R'}{(8E-9)\cdot 1000} + \frac{V_R}{(5E-4)(8E-9)} = 0$

ⓐ $V_R'' + 125000 V_R' + (25E10)V_R = 0$

ⓑ $\omega_0 = \sqrt{2.5E11} = 500000 = 5E5$

$2\zeta\omega_0 = 125000 \rightarrow \zeta = 1/8$

ⓓ

$\zeta < 1 \therefore$ Underdamped

$T = \frac{2\pi}{\sqrt{1-\zeta^2}\,\omega_0} = \frac{2\pi}{\sqrt{\frac{63}{64}}\,5E5}$

$= \frac{\pi\sqrt{7}}{656250}$

$\tau = \frac{1}{\zeta\omega_0} = \frac{1}{\frac{1}{8}5E5} = \frac{1}{62500}$

ⓒ $V_R(t) = C_1 e^{-62500t}\cos(187500 t\sqrt{7}) + C_2 e^{-62500t}\sin(187500 t\sqrt{7})$

$V_R(0) = C_1 = 1$, $V_R'(0) = 250000 = 187500\sqrt{7}\,C_2 - 62500 \rightarrow C_2 = \frac{\sqrt{7}}{7}$

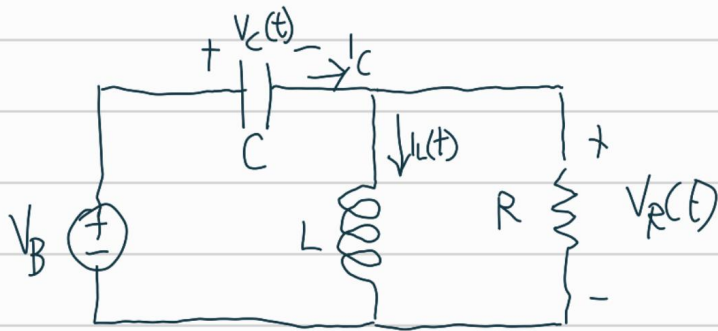$\therefore V_R(t) = e^{-62500t}\left(\cos(187500 t\sqrt{7}) - \frac{\sqrt{7}}{7}\sin(187500 t\sqrt{7})\right)$

3.1 a **3 / 3**

✓ **- 0 pts** *Correct*

- 3 pts missing/wrong

✓ **- 0 pts** *Correct*

- 3 pts missing/wrong

③

$$V_C(0^+)=1V, \quad V_R(0^+)=V_B-V_C(0^+)=1V$$
$$I_L(0^+)=1mA, \quad V_R'(0^+)=250000 \leftarrow$$
$$V_R'=-V_C'=-\frac{1}{C}I_C=\frac{-1}{C}\cdot 2mA$$

$$V_B = V_C + V_R \qquad V_C = V_B - V_R, \quad V_C' = -V_R'$$

$$C V_C' = I_L + \frac{V_R}{R}$$
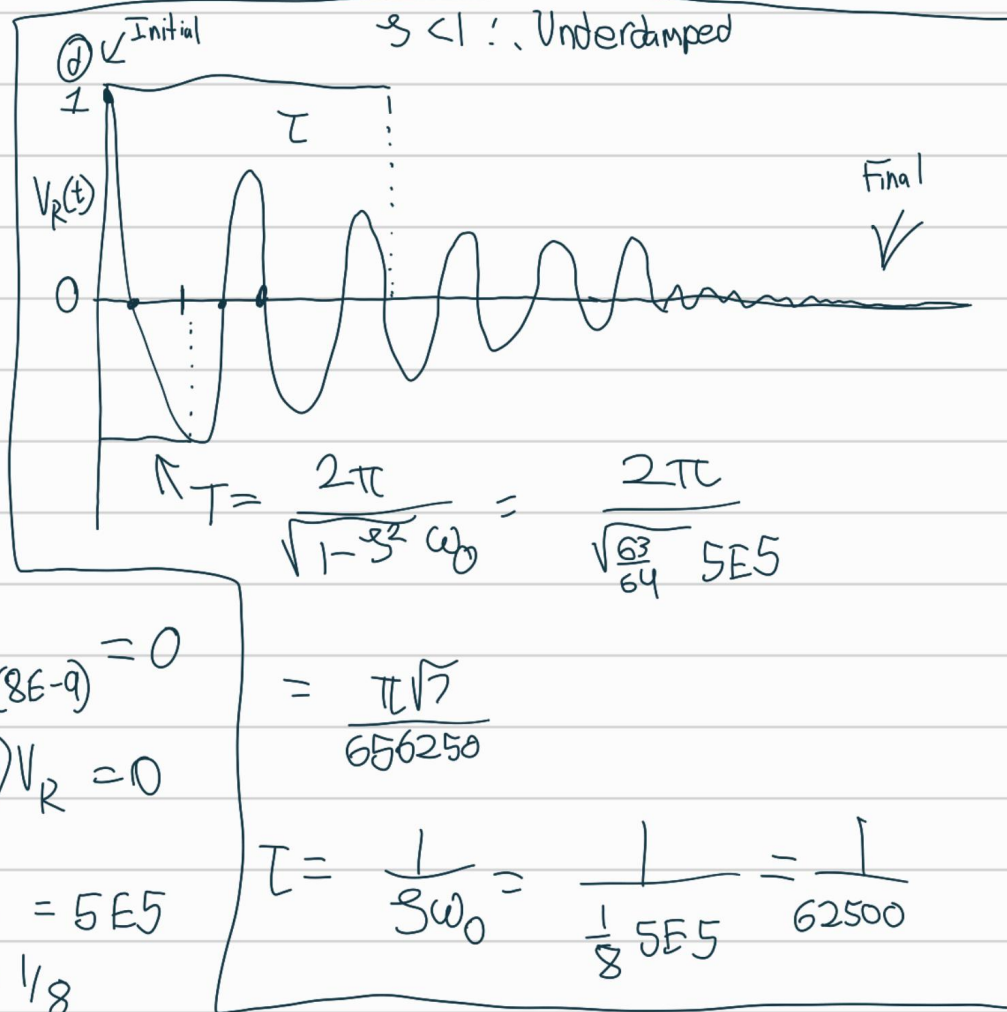
$$V_C' = \frac{1}{LC}\int V_R + \frac{V_R}{RC}$$

$$-V_R' = \frac{1}{LC}\int V_R + \frac{V_R}{RC}$$

$$\therefore V_R'' + \frac{V_R'}{RC} + \frac{V_R}{LC} = 0$$

$$V_R'' + \frac{V_R'}{(8E-9)\cdot 1000} + \frac{V_R}{(5E-4)(8E-9)} = 0$$

ⓐ $\quad V_R'' + 125000V_R' + (25E10)V_R = 0$

ⓑ $\quad \omega_0 = \sqrt{2.5E11} = 500000 = 5E5$
$\quad 2\zeta\omega_0 = 125000 \rightarrow \zeta = 1/8$

$\zeta < 1 \therefore$ Underdamped

Final

$$T = \frac{2\pi}{\sqrt{1-\zeta^2}\,\omega_0} = \frac{2\pi}{\sqrt{\frac{63}{64}}\,5E5}$$

$$= \frac{\pi\sqrt{7}}{656250}$$

$$\tau = \frac{1}{\zeta\omega_0} = \frac{1}{\frac{1}{8}5E5} = \frac{1}{62500}$$

ⓒ $V_R(t) = C_1 e^{-62500t}\cos(187500t\sqrt{7}) + C_2 e^{-62500t}\sin(187500t\sqrt{7})$

$V_R(0) = C_1 = 1, \quad V_R'(0) = 250000 = 187500\sqrt{7}\,C_2 - 62500 \rightarrow C_2 = \frac{\sqrt{7}}{7}$

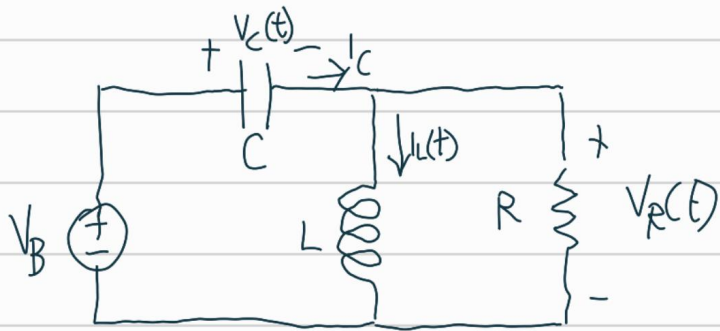$\therefore V_R(t) = e^{-62500t}\left(\cos(187500t\sqrt{7}) - \frac{\sqrt{7}}{7}\sin(187500t\sqrt{7})\right)$

3.2 b **2 / 2**

✓ **- 0 pts** *Correct*

- 0.5 pts answer should be 1/8 or 0.125

- 2 pts did not answer

✓ **- 0 pts** *Correct*

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③

$V_C(0) = 1V$, $V_R(0) = V_B - V_C(0) = 1V$

$I_L(0) = 1mA$, $V_R'(0) = 250000$ ←

$V_R' = -V_C' = -\frac{1}{C} I_C = \frac{-1}{C} 2mA$

$V_B = V_C + V_R$

$V_C = V_B - V_R$, $V_C' = -V_R'$

$C V_C' = I_L + \frac{V_R}{R}$
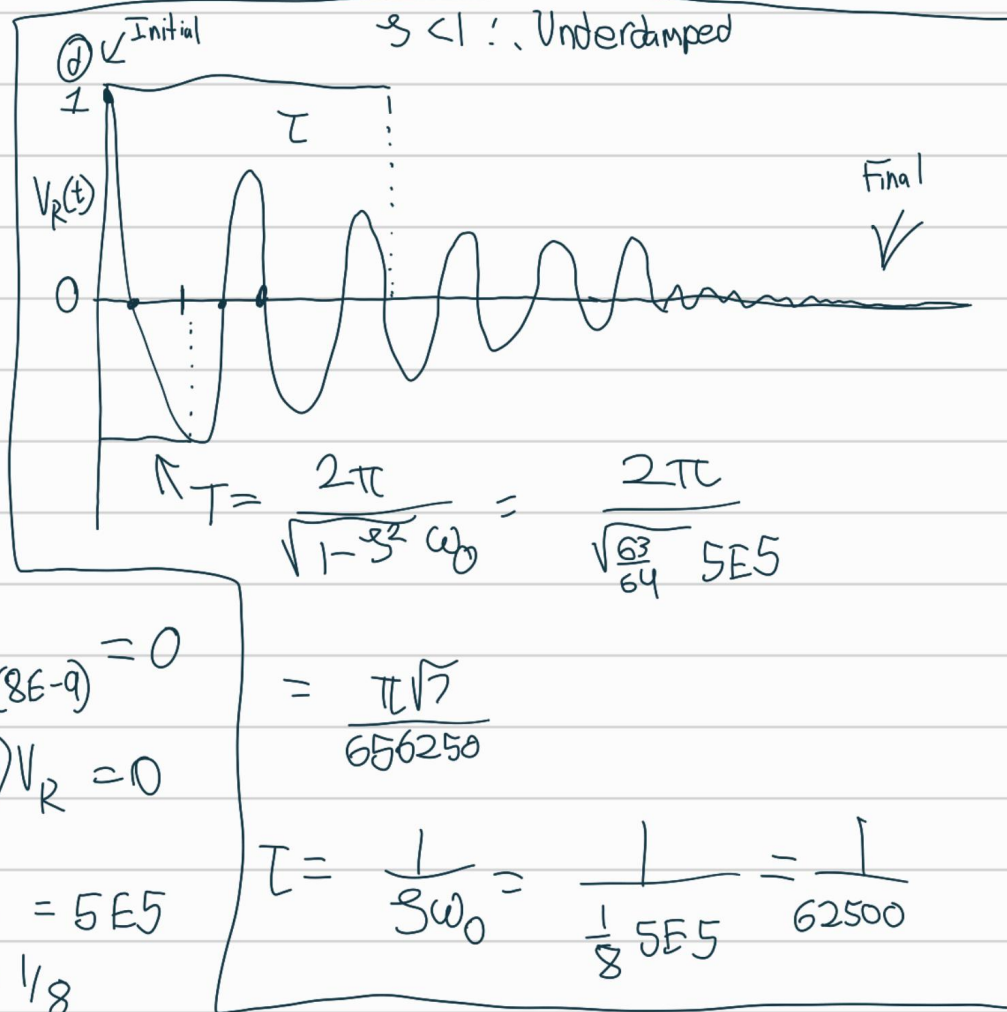
$V_C' = \frac{1}{LC} \int V_R + \frac{V_R}{RC}$

$-V_R' = \frac{1}{LC} \int V_R + \frac{V_R}{RC}$

$\therefore V_R'' + \frac{V_R'}{RC} + \frac{V_R}{LC} = 0$

$V_R'' + \frac{V_R'}{(8E-9) \cdot 1000} + \frac{V_R}{(5E-4)(8E-9)} = 0$

ⓐ $V_R'' + 125000 V_R' + (25 E10) V_R = 0$

ⓑ $\omega_0 = \sqrt{2.5E11} = 500000 = 5E5$

$2 \zeta \omega_0 = 125000 \rightarrow \zeta = 1/8$

$\zeta < 1 \therefore$ Underdamped

$T = \frac{2\pi}{\sqrt{1-\zeta^2} \omega_0} = \frac{2\pi}{\sqrt{\frac{63}{64}} 5E5}$

$= \frac{\pi\sqrt{7}}{656250}$

$\tau = \frac{1}{\zeta\omega_0} = \frac{1}{\frac{1}{8} 5E5} = \frac{1}{62500}$

ⓒ $V_R(t) = C_1 e^{-62500t} \cos(187500t\sqrt{7}) + C_2 e^{-62500t} \sin(187500t\sqrt{7})$

$V_R(0) = C_1 = 1$, $V_R'(0) = 250000 = 187500\sqrt{7} C_2 - 62500 \rightarrow C_2 = \frac{\sqrt{7}}{7}$

$\therefore V_R(t) = e^{-62500t} \left(\cos(187500t\sqrt{7}) - \frac{\sqrt{7}}{7} \sin(187500t\sqrt{7}) \right)$
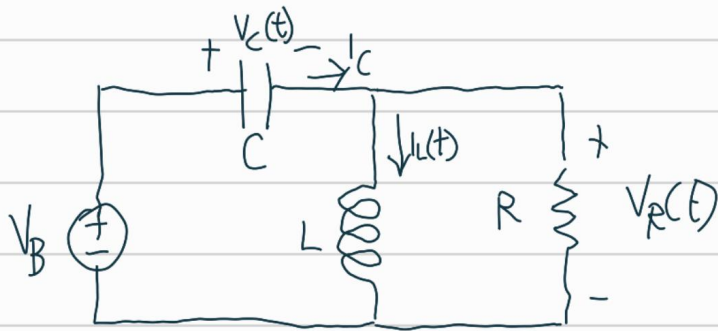
3.3 C **7 / 7**

✓ **- 0 pts** *Correct*

- 1.5 pts A = 1

- 1.5 pts B = -0.378

- 7 pts wrong or missing

- 1.5 pts C = 0

✓ **- 0 pts** *Correct*

gradescope

③

$V_C(0^+)=1V, \quad V_R(0^+)=V_B-V_C(0^+)=1V$

$I_L(0^+)=1mA, \quad V_R'(0^+)=250000 \leftarrow$

$V_R' = -V_C' = -\frac{1}{C}I_C = \frac{-1}{C}2mA$

$V_B = V_C + V_R \qquad V_C = V_B - V_R, \quad V_C' = -V_R'$

$C V_C' = I_L + \frac{V_R}{R}$
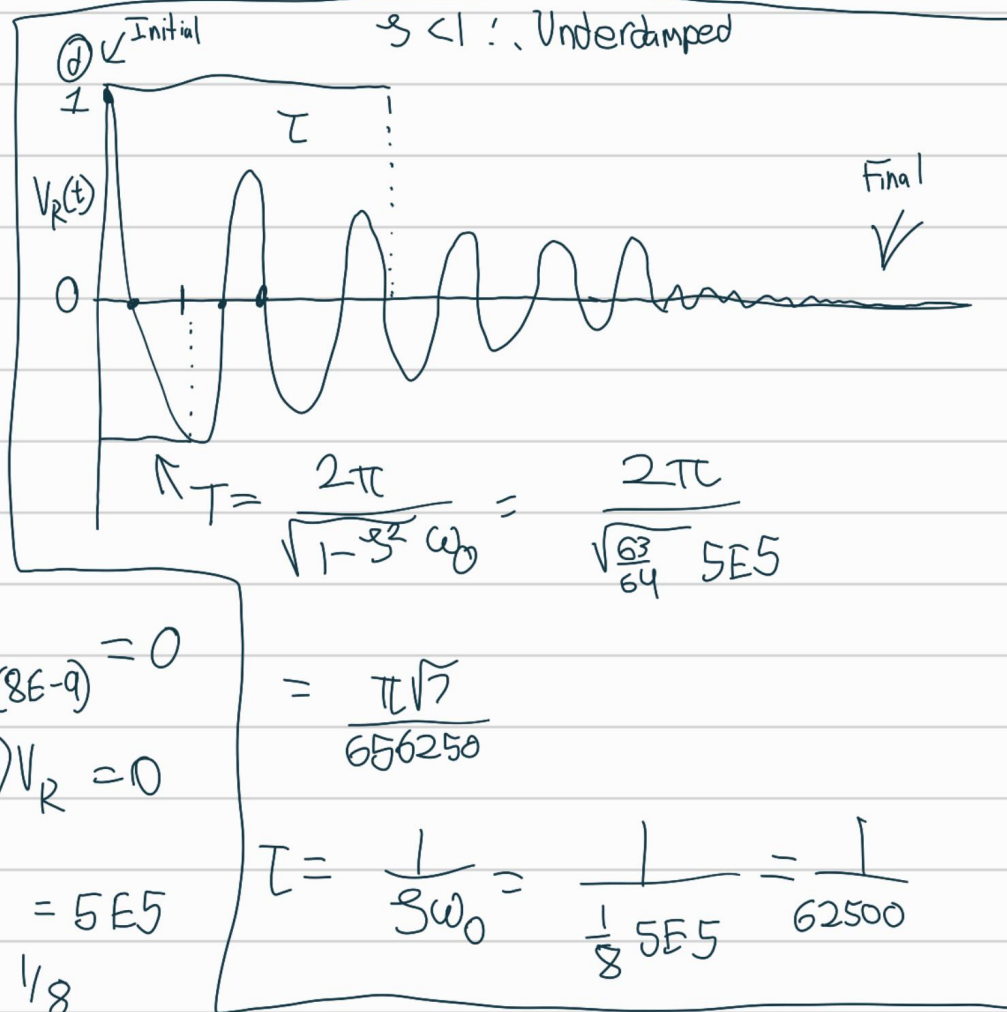
$V_C' = \frac{1}{LC}\int V_R + \frac{V_R}{RC}$

$-V_R' = \frac{1}{LC}\int V_R + \frac{V_R}{RC}$

$\therefore V_R'' + \frac{V_R'}{RC} + \frac{V_R}{LC} = 0$

$V_R'' + \frac{V_R'}{(8E-9)\cdot 1000} + \frac{V_R}{(5E-4)(8E-9)} = 0$

ⓐ $V_R'' + 125000 V_R' + (25E10)V_R = 0$

ⓑ $\omega_0 = \sqrt{2.5E11} = 500000 = 5E5$

$2\zeta\omega_0 = 125000 \rightarrow \zeta = 1/8$

$\zeta < 1 \therefore$ Underdamped

$T = \frac{2\pi}{\sqrt{1-\zeta^2}\,\omega_0} = \frac{2\pi}{\sqrt{\frac{63}{64}}\,5E5}$

$= \frac{\pi\sqrt{7}}{656250}$

$\tau = \frac{1}{\zeta\omega_0} = \frac{1}{\frac{1}{8}5E5} = \frac{1}{62500}$

ⓒ $V_R(t) = C_1 e^{-62500t}\cos\left(187500t\sqrt{7}\right) + C_2 e^{-62500t}\sin\left(187500t\sqrt{7}\right)$

$V_R(0) = C_1 = 1, \quad V_R'(0) = 250000 = 187500\sqrt{7}C_2 - 62500 \rightarrow C_2 = \frac{\sqrt{7}}{7}$

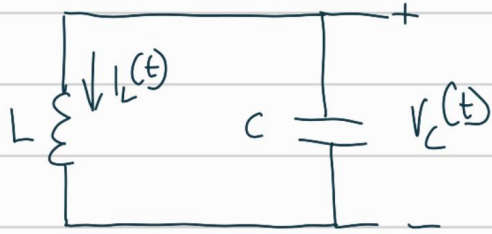$\therefore V_R(t) = e^{-62500t}\left(\cos(187500t\sqrt{7}) - \frac{\sqrt{7}}{7}\sin(187500t\sqrt{7})\right)$

3.4 d **3 / 3**

  ✓ **- 0 pts** *Correct*

    - 3 pts wrong or missing

ıl gradescope

④

$L = 1H$

$C = \frac{1}{4}F$

$V_C(0-) = 2V$

$I_L(0-) = 2A$

$L I_L' = V_C$ $\qquad$ $I_L = -C V_C$

$I_L'' = \frac{-I_L}{LC}$ $\qquad$ $I_L'' + \frac{I_L}{LC} = 0$

$\qquad\qquad\qquad\qquad\qquad I_L'' + 4I_L = 0$

$I_L = C_1 \cos(2t) + C_2 \sin(2t)$

$I_L(0) = C_1 = 2$

$I_L'(0) = 2C_2 = 2$ ∴ $C_2 = 1$

∴ $I_L = 2\cos 2t + \sin 2t$

$V = L I_L' = I_L' = 2\cos 2t - 4\sin 2t$

ⓐ $I_L'' + 4I_L = 0$,

ⓑ $I_L(0+) = I_L(0-) = 2A$

$\quad I_L'(0+) = \frac{V_C(0+)}{L} = 2 As^{-1}$

ⓒ $I_L''(t) + 4I_L(t) = 0$

$\quad$ ∴ $I_L''(0+) = -4I_L(0+) = -8 As^{-2}$

ⓓ $I_L(0+) = 2$

$\quad I_L'(0) = 2$

$\quad I_L'' + 4I_L = 0$

$\quad I_L(t) = C_1 \cos 2t + C_2 \sin 2t$

$\quad I_L(0) = C_1 = 2$

$\quad I_L'(0) = 2C_2 = 2$ ∴ $C_2 = 1$

∴ $I_L(t) = 2\cos 2t + \sin 2t$

4.1 a **3 / 3**

✓ **- 0 pts** *Correct*

- 3 pts wrong or missing

(4)

$$L = 1H$$
$$C = \frac{1}{4}F$$
$$V_C(0-) = 2V$$
$$I_L(0-) = 2A$$

$$LI_L' = V_C \qquad I_L = -CV_C$$

$$I_L'' = \frac{-I_L}{LC} \qquad I_L'' + \frac{I_L}{LC} = 0$$

$$I_L'' + 4I_L = 0$$

$$I_L = C_1 \cos(2t) + C_2 \sin(2t)$$
$$I_L(0) = C_1 = 2$$
$$I_L'(0) = 2C_2 = 2 \quad \therefore C_2 = 1$$

$$\therefore I_L = 2\cos 2t + \sin 2t$$
$$V = LI_L' = I_L' = 2\cos 2t - 4\sin 2t$$

(a) $$I_L'' + 4I_L = 0,$$

(b) $$I_L(0+) = I_L(0-) = 2A$$
$$I_L'(0+) = \frac{V_C(0+)}{L} = 2\,As^{-1}$$

(c) $$I_L''(t) + 4I_L(t) = 0$$
$$\therefore I_L''(0+) = -4I_L(0+) = -8\,As^{-2}$$

(d) $$I_L(0+) = 2$$
$$I_L'(0) = 2$$
$$I_L'' + 4I_L = 0$$
$$I_L(t) = C_1 \cos 2t + C_2 \sin 2t$$
$$I_L(0) = C_1 = 2$$
$$I_L'(0) = 2C_2 = 2 \quad \therefore C_2 = 1$$
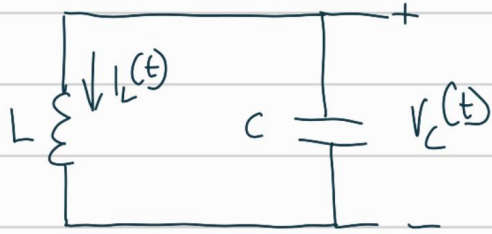
$$\therefore I_L(t) = 2\cos 2t + \sin 2t$$

4.2 b **4 / 4**

   ✓ **- 0 pts** *Correct*

    - 2 pts wrong

    - 4 pts missing

④

$L = 1H$

$C = \frac{1}{4}F$

$V_C(0-) = 2V$

$I_L(0-) = 2A$

$L I_L' = V_C \qquad \qquad I_L = -C V_C$

$I_L'' = -\frac{I_L}{LC} \qquad \qquad I_L'' + \frac{I_L}{LC} = 0$

$L = C_1 \cos(2t) + C_2 \sin(2t)$

$I_L(0) = C_1 = 2$

$I_L'(0) = 2C_2 = 2 \quad \therefore C_2 = 1$

$I_L'' + 4I_L = 0 \qquad \qquad \therefore I_L = 2\cos 2t + \sin 2t$

$V = L I_L' = I_L' = 2\cos 2t - 4\sin 2t$

ⓐ $I_L'' + 4I_L = 0$,

ⓑ $I_L(0+) = I_L(0-) = 2A$

$I_L'(0+) = \frac{V_C(0+)}{L} = 2 As^{-1}$

ⓒ $I_L''(t) + 4I_L(t) = 0$

$\therefore I_L''(0+) = -4I_L(0+) = -8 As^{-2}$

ⓓ $I_L(0+) = 2$

$I_L'(0) = 2$

$I_L'' + 4I_L = 0$

$I_L(t) = C_1 \cos 2t + C_2 \sin 2t$

$I_L(0) = C_1 = 2$

$I_L'(0) = 2C_2 = 2 \quad \therefore C_2 = 1$
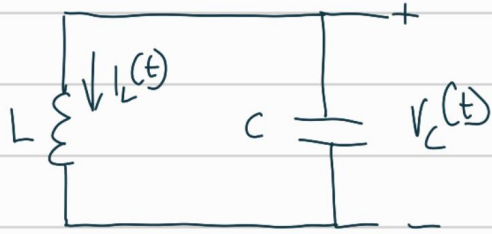
$\therefore I_L(t) = 2\cos 2t + \sin 2t$

4.3 **C 3 / 3**

✓ **- 0 pts** *Correct*

   - 3 pts wrong or missing

ıl gradescope

④

$L = 1H$

$C = \frac{1}{4}F$

$V_C(0-) = 2V$

$I_L(0-) = 2A$

$L I_L' = V_C$ $\qquad$ $I_L = -C V_C$

$I_L'' = -\frac{I_L}{LC}$ $\qquad$ $I_L'' + \frac{I_L}{LC} = 0$

$\qquad\qquad\qquad\qquad$ $I_L'' + 4I_L = 0$

$I_L = C_1 \cos(2t) + C_2 \sin(2t)$

$I_L(0) = C_1 = 2$

$I_L'(0) = 2C_2 = 2 \quad \therefore C_2 = 1$

$\therefore I_L = 2\cos 2t + \sin 2t$

$V = L I_L' = I_L' = 2\cos 2t - 4\sin 2t$

ⓐ $I_L'' + 4I_L = 0$,

ⓑ $I_L(0+) = I_L(0-) = 2A$

$I_L'(0+) = \frac{V_C(0+)}{L} = 2 As^{-1}$

ⓒ $I_L''(t) + 4I_L(t) = 0$

$\therefore I_L''(0+) = -4I_L(0+) = -8 As^{-2}$

ⓓ $I_L(0+) = 2$

$I_L'(0) = 2$

$I_L'' + 4I_L = 0$

$I_L(t) = C_1 \cos 2t + C_2 \sin 2t$

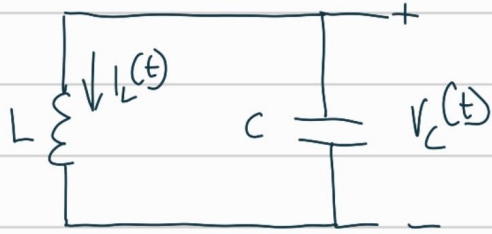$I_L(0) = C_1 = 2$

$I_L'(0) = 2C_2 = 2 \quad \therefore C_2 = 1$

$\therefore I_L(t) = 2\cos 2t + \sin 2t$
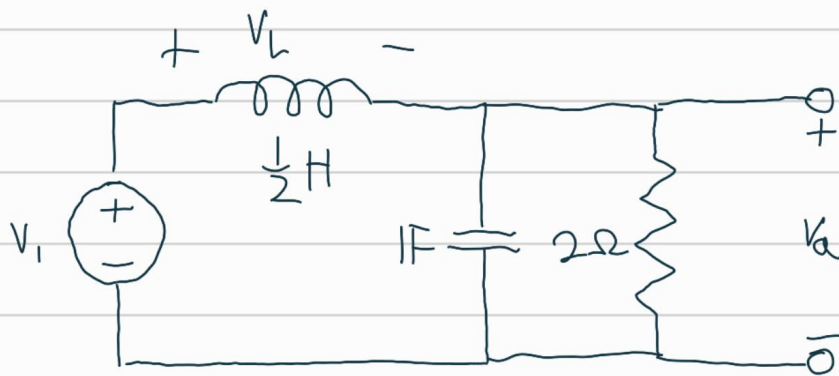
✓ **- 0 pts** *Correct*

- 3 pts wrong expression

- 1 pts A = 2

- 1 pts B =1

- 1 pts k =0

- 5 pts missing

✓ **- 0 pts** *Correct*

⑤

Circuit: V_i source, inductor $\frac{1}{2}$H with $+V_L-$, $1F$ capacitor, 2Ω resistor, output V_a with $+$ on top.

(a) $V_1 = V_L + V_a$ $V_L = V_1 - V_a$

$$\frac{1}{L}\int V_L = CV_a' + \frac{V_a}{R}$$

$$\therefore \frac{1}{L}\int V_1 - \frac{1}{L}\int V_a = CV_a' + \frac{V_a}{R}$$

$$\therefore \frac{1}{L}(V_1 - V_a) = CV_a'' + \frac{V_a'}{R}$$

$$\therefore \frac{1}{LC}V_1 = V_a'' + \frac{V_a'}{RC} + \frac{V_a}{LC}$$

$$\therefore 2V_1 = V_a'' + \frac{1}{2}V_a' + 2V_a$$

$46B\sin(5t+\phi) + 20\cos5t = 5B\cos(5t+\phi)$

$V_a = B\sin(5t+\phi)$

$$\therefore 10\cos(5t) = -25B\sin(5t+\phi) + \frac{5B}{2}\cos(5t+\phi) + 2B\sin(5t+\phi)$$

$$20\cos(5t) = -50B\sin(5t+\phi) + 5B\cos(5t+\phi) + 4B\sin(5t+\phi)$$

$$\omega = 5$$

$$5B\left(\cos(5t+\phi) - 5\frac{46}{25}\sin(5t+\phi)\right) = 20\cos(5t+\dot0)$$

$$\alpha = \tan^{-1}\left(\frac{46}{5}\right) = 1.46 \qquad 5B = \frac{20}{\sqrt{1 + \frac{46^2}{25^2}}} \qquad \therefore B = 1.91$$

    - 0 pts Correct

✓ **- 2 pts** *B = 0.4322*

    - 2 pts $$\phi = -\tan^{-1}(46/5) \text{ or } =-1.463$$

    - 5 pts wrong

    - 10 pts missing

ıl gradescope

(6)

$$i(t) = Ae^{-t/\tau} \sin(\omega t + \phi)$$

$$\omega = \sqrt{1-\zeta^2}\,\omega_0 \qquad \tau = \frac{1}{\zeta\omega_0}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 9.472 - 3.157 = 6.315 = \frac{2\pi}{\omega} \quad \therefore \omega \approx 1,$$

$$\vec{r}\ \phi = 0$$

$$i(0) = 0 = \sin(\omega t + \phi)$$

$$0.8626 = Ae^{-1.429/\tau} \qquad \therefore A \approx 1$$

$$0.4587 = Ae^{-7.794/\tau} \qquad \therefore \tau \approx 10$$

$$\therefore 10 = \frac{1}{\zeta\omega_0}, \qquad 1 = \sqrt{1-\zeta^2}\,\omega_0$$

Solving, $\omega_0 = \dfrac{1}{10\zeta} \quad \therefore 1 = \dfrac{\sqrt{1-\zeta^2}}{10\zeta}$

$$\therefore (10\zeta)^2 = 1 - \zeta^2$$

$$\therefore 101\zeta^2 = 1$$

$$\therefore \zeta = \frac{\sqrt{101}}{101} \qquad \therefore \omega_0 = \frac{\sqrt{101}}{10}$$

(a) $\omega_0 = \dfrac{\sqrt{101}}{10}$ 　　(b) $\zeta = \dfrac{\sqrt{101}}{101}$ 　　(c) $i(t) = e^{-t/10}\sin t$

(d) $LC = \dfrac{1}{\omega_0^2} \quad \therefore C = \dfrac{1}{\omega_0^2} = \dfrac{10^2}{101} = \boxed{\dfrac{100}{101}\,F} = C$

(e) $\zeta = \dfrac{\sqrt{101}}{101} = \dfrac{1}{2}\dfrac{R}{\sqrt{101/100}} \quad \therefore R = \dfrac{2\sqrt{101}\,\sqrt{101/100}}{101}$

$$= .2\,\Omega$$

6.1 **3 / 3**

✓ **- 0 pts** *Correct*

- 3 pts missing or incorrect

⑥

$$i(t) = Ae^{-t/\tau} \sin(\omega t + \phi)$$

$$\omega = \sqrt{1-\zeta^2}\,\omega_0 \qquad \tau = \frac{1}{\zeta\omega_0}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 9.472 - 3.157 = 6.315 = \frac{2\pi}{\omega} \quad \therefore \omega \approx 1,$$

$$\Rightarrow \phi = 0$$
$$i(0) = 0 = \sin(\omega t + 0)$$

$$0.8626 = Ae^{-1.429/\tau} \qquad \therefore A \approx 1$$
$$0.4587 = Ae^{-7.794/\tau} \qquad \therefore \tau \approx 10$$

$$\therefore 10 = \frac{1}{\zeta\omega_0}, \qquad 1 = \sqrt{1-\zeta^2}\,\omega_0$$

Solving, $\omega_0 = \dfrac{1}{10\zeta} \qquad \therefore 1 = \dfrac{\sqrt{1-\zeta^2}}{10\zeta}$

$$\therefore (10\zeta)^2 = 1 - \zeta^2$$

$$\therefore 101\zeta^2 = 1$$
$$\therefore \zeta = \frac{\sqrt{101}}{101} \qquad \therefore \omega_0 = \frac{\sqrt{101}}{10}$$

ⓐ $\omega_0 = \dfrac{\sqrt{101}}{10}$ ⓑ $\zeta = \dfrac{\sqrt{101}}{101}$ ⓒ $i(t) = e^{-t/10}\sin t$

ⓓ $LC = \dfrac{1}{\omega_0^2} \qquad \therefore C = \dfrac{1}{\omega_0^2} = \dfrac{10^2}{101} = \boxed{\dfrac{100}{101}\,F} = C$

ⓔ $\zeta = \dfrac{\sqrt{101}}{101} = \dfrac{1}{2}\dfrac{R}{\sqrt{101/100}} \qquad \therefore R \approx \dfrac{2\sqrt{101}\,\sqrt{101/100}}{101}$

$$= .2\,\Omega$$

✓ **- 0 pts** *Correct*

- 3 pts missing or incorrect

gradescope

⑥

$$i(t) = Ae^{-t/\tau} \sin(\omega t + \phi)$$

$$\omega = \sqrt{1-\varsigma^2}\,\omega_0 \qquad \tau = \frac{1}{\varsigma\omega_0}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 9.472 - 3.157 = 6.315 = \frac{2\pi}{\omega} \quad \therefore \omega \approx 1,$$

$$\because \phi = 0$$
$$i(0) = 0 = \sin(\omega \cdot 0 + 0)$$

$$0.8626 = Ae^{-1.429/\tau} \qquad \therefore A \approx 1$$

$$0.4587 = Ae^{-7.794/\tau} \qquad \therefore \tau \approx 10$$

$$\therefore 10 = \frac{1}{\varsigma\omega_0}, \qquad 1 = \sqrt{1-\varsigma^2}\,\omega_0$$

Solving, $\omega_0 = \dfrac{1}{10\varsigma} \quad \therefore 1 = \dfrac{\sqrt{1-\varsigma^2}}{10\varsigma}$

$$\therefore (10\varsigma)^2 = 1 - \varsigma^2$$

$$\therefore 101\varsigma^2 = 1$$

$$\therefore \varsigma = \frac{\sqrt{101}}{101} \qquad \therefore \omega_0 = \frac{\sqrt{101}}{10}$$

ⓐ $\omega_0 = \dfrac{\sqrt{101}}{10}$ 　　ⓑ $\varsigma = \dfrac{\sqrt{101}}{101}$ 　　ⓒ $i(t) = e^{-t/10}\sin t$

ⓓ $LC = \dfrac{1}{\omega_0^2} \qquad \therefore C = \dfrac{1}{\omega_0^2} = \dfrac{10^2}{101} = \boxed{\dfrac{100}{101}\,F} = C$

ⓔ $\varsigma = \dfrac{\sqrt{101}}{101} = \dfrac{1}{2}\dfrac{R}{\sqrt{101/100}} \qquad \therefore R \approx \dfrac{2\sqrt{101}\sqrt{101/100}}{101}$

$$= .2\,\Omega$$

6.3 **3 / 3**

✓ **- 0 pts** *Correct*

  - 3 pts wrong or missing

✓ **- 0 pts** *Correct*

  - 3 pts wrong or missing

ıll gradescope

⑥

$$i(t) = A e^{-t/\tau} \sin(\omega t + \phi)$$

$$\omega = \sqrt{1-\zeta^2}\,\omega_0 \qquad \tau = \frac{1}{\zeta \omega_0}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 9.472 - 3.157 = 6.315 = \frac{2\pi}{\omega} \quad \therefore \omega \approx 1,$$

$$\phi = 0$$

$$i(0) = 0 = \sin(\omega t + \phi)$$

$$0.8626 = A e^{-1.479/\tau} \qquad \therefore A \approx 1$$
$$0.4587 = A e^{-7.794/\tau} \qquad \therefore \tau \approx 10$$

$$\therefore 10 = \frac{1}{\zeta \omega_0}, \qquad 1 = \sqrt{1-\zeta^2}\,\omega_0$$

Solving, $\quad \omega_0 = \frac{1}{10\zeta} \qquad \therefore 1 = \frac{\sqrt{1-\zeta^2}}{10\zeta}$

$$\therefore (10\zeta)^2 = 1 - \zeta^2$$

$$\therefore 101\zeta^2 = 1$$

$$\therefore \zeta = \frac{\sqrt{101}}{101} \qquad \therefore \omega_0 = \frac{\sqrt{101}}{10}$$

ⓐ $\omega_0 = \dfrac{\sqrt{101}}{10}$ 　　ⓑ $\zeta = \dfrac{\sqrt{101}}{101}$ 　　ⓒ $i(t) = e^{-t/10} \sin t$

ⓓ $LC = \dfrac{1}{\omega_0^2} \qquad \therefore C = \dfrac{1}{\omega_0^2} = \dfrac{10^2}{101} = \boxed{\dfrac{100}{101} F} = C$

ⓔ $\zeta = \dfrac{\sqrt{101}}{101} = \dfrac{1}{2} \dfrac{R}{\sqrt{101/100}} \qquad \therefore R \approx \dfrac{2\sqrt{101}\sqrt{101/100}}{101}$

$$= .2 \,\Omega$$

6.4 **3 / 3**

✓ **- 0 pts** *Correct*

- 3 pts missing or incorrect

✓ **- 0 pts** *Correct*

- 3 pts missing or incorrect

⑥

$$i(t) = Ae^{-t/\tau} \sin(\omega t + \phi)$$

$$\omega = \sqrt{1-\varsigma^2}\,\omega_0 \qquad \tau = \frac{1}{\varsigma\omega_0}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 9.472 - 3.157 = 6.315 = \frac{2\pi}{\omega} \quad \therefore \omega \simeq 1,$$

$$\because \phi = 0$$
$$i(0) = 0 = \sin(\omega t + 0)$$

$$0.8626 = Ae^{-1.429/\tau} \qquad \therefore A \simeq 1$$

$$0.4587 = Ae^{-7.794/\tau} \qquad \therefore \tau \simeq 10$$

$$\therefore 10 = \frac{1}{\varsigma\omega_0}, \qquad 1 = \sqrt{1-\varsigma^2}\,\omega_0$$

$$\text{Solving}, \quad \omega_0 = \frac{1}{10\varsigma} \quad \therefore 1 = \frac{\sqrt{1-\varsigma^2}}{10\varsigma}$$

$$\therefore (10\varsigma)^2 = 1 - \varsigma^2$$

$$\therefore 101\varsigma^2 = 1$$

$$\therefore \varsigma = \frac{\sqrt{101}}{101} \qquad \therefore \omega_0 = \frac{\sqrt{101}}{10}$$

ⓐ $\omega_0 = \dfrac{\sqrt{101}}{10}$ ⓑ $\varsigma = \dfrac{\sqrt{101}}{101}$ ⓒ $i(t) = e^{-t/10}\sin t$

ⓓ $LC = \dfrac{1}{\omega_0^2} \quad \therefore C = \dfrac{1}{\omega_0^2} = \dfrac{10^2}{101} = \boxed{\dfrac{100}{101}\,F} = C$

ⓔ $\varsigma = \dfrac{\sqrt{101}}{101} = \dfrac{1}{2}\dfrac{R}{\sqrt{101/100}} \quad \therefore R \simeq \dfrac{2\sqrt{101}\,\sqrt{101/100}}{101}$

$$= .2\,\Omega$$

6.5 **3 / 3**

✓ **- 0 pts** *Correct*

- 3 pts missing or incorrect

6.5 **3 / 3**

✓ **- 0 pts** *Correct*

- 3 pts missing or incorrect

ılı gradescope