ECE 10, Winter 2023, Midterm #2 – March 2, 2022

Instructions: This exam booklet consists of four problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch…" will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. Put a box around your final answer. Illegible solutions will not be graded.**
7. **Be brief.**
8. **Open book & open notes only. NO homework solutions allowed!**
9. **Regular, scientific, and graphing calculators are allowed.**

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NAMES OF ADJACENT STUDENTS:

LEFT: Inesh Chakrabarti

RIGHT: ———

Problem	Score
#1	7 /8
#2	10 /12
#3	5 /10
#4	10 /20
Total	32 /50

Problem 1: Assume that the circuit shown in Figure 1 has reached steady state i.e. it has existed in this form for a very long time. Given $R_1 = 1$ kOhms, $R_2 = 4$ kOhms, $L = 1$nH, $C_1 = 1$nF, $C_2 = 1$nF, and $V_B = 3$V, calculate the steady state values of $v_{C1}(t)$ and $i_L(t)$.

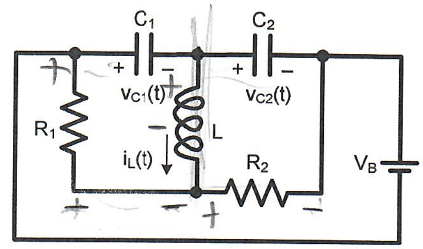


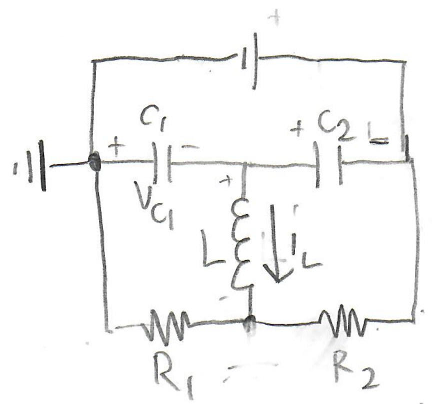
Figure 1.

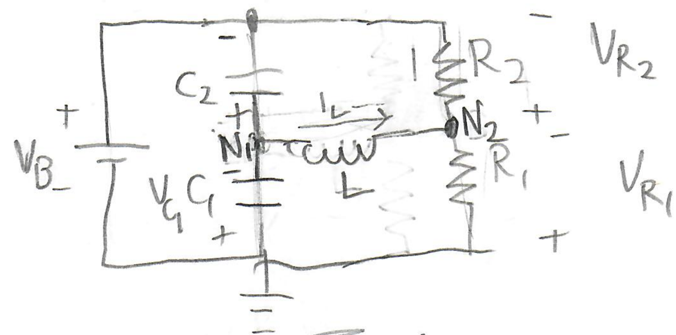
(4 + 4 = 8 points)

Solution:

7

Redrawn





$V_{N_1} - V_{N_2} = L i_L'$

$V_{R_2} = I_{R_2} R_2$

$V_{R_1} = I_{R_1} R_1$

$I_L + I_{R_2} = I_{R_1}$

$V_{C_2} - V_{R_2} - L i_L' = 0$

$V_{C_1} + L i_L' - V_{R_1} = 0$

$V_{C_1} + V_{C_2} - I_{R_2} R_2 - I_{R_1} R_1 = 0$

@ Steady State, $L i_L' = 0$

$\therefore V_{C_1} = V_{R_1}$

@ Steady State

$V_{C_1} = V_{R_1}$ $\quad V_B = -V_{R_2} - V_{R_1}$

$V_{C_2} = V_{R_2}$ $\quad V_B = -V_{C_2} - V_{C_1}$

$I_L + \frac{V_{C_1}}{R_1} = \frac{V_{C_2}}{R_2}$

$\cancel{V_B = -V_{C_1} - V_{C_2}}$

$V_{C_1}(t) = \frac{V_B}{R_1 + R_2} \cdot R_1 = -\frac{3}{5} V$

$i_L(t) = 0$

@ steady State

Problem 2: Consider the circuit shown in Figure 2. Assume that the inductors have no stored energy prior to t = 0. Assume also that I_B = 4mA, R_1 = 2kOhm, R_2 = 6kOhm, L_1 = 4mH, and L_2 = 2mH.
(a) Show that this is a 1st order circuit.
(b) What is the time constant of this circuit?
(c) Derive an expression for $i_1(t)$ valid for $t \geq 0$.

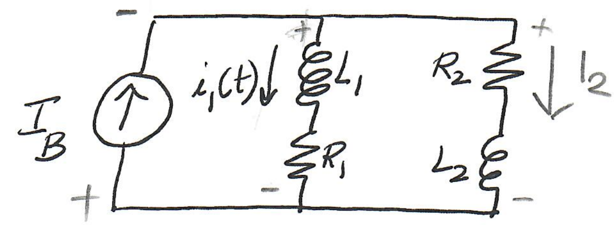


Figure 2.

(4 + 3 + 5 = 12 points)

10

Solution:

$I_B = I_1 + I_2 \quad \therefore I_2 = I_B - I_1$

(a) $L_1 i_1' + R_1 i_1 = L_2 i_2' + R_2 I_2$

$\therefore L_1 I_1' + R_1 I_1 = L_2 (I_B - I_1)' + R_2 (I_B - I_1)$

$\therefore L_1 I_1' + R_1 I_1 = -L_2 I_1' + R_2 I_B - I_1 R_2$

(a) $\therefore (L_1 + L_2) I_1' + (R_1 + R_2) i_1 = R_2 I_B$ $\therefore$ this is a 1st order circuit

$\therefore I_1' + \frac{(R_1+R_2)}{(L_1+L_2)} i_1 = \frac{R_2}{L_1+L_2} I_B$ $\quad \Big| \quad I_1' + \frac{i}{7.5E-7} = 4000$

Homogenized: $I_1' + \frac{R_1+R_2}{L_1+L_2} I_1 = 0$

(b) $\therefore \tau = \frac{L_1+L_2}{R_1+R_2} = \frac{4mH + 2mH}{2k\Omega + 6k\Omega} = \frac{3\,mH}{4\,k\Omega} = \frac{3}{4}E-6\,s$

$= 7.5E-7\,s$

(3)

$\therefore i(t) = C_1 e^{-t/(7.5E-7)} + \frac{3}{1000}$

$I(0+) = I(0-) = 0$ (-3) $\quad 0 = C_1 + \frac{3}{1000} \quad \therefore C_1 = \frac{-3}{1000}$

(c) $\therefore i_1(t) = \frac{3}{1000}\left(1 - e^{-t/(7.5E-7)}\right)$

Problem 3: Refer to the circuit in Figure 3. Assume that V_B=3V, I_B=4mA, R=1kOhm, L_1=1mH, and L_2=2mH.
(a) Calculate the value of $i_1(t)$ at t = 0–.
(b) Calculate the value of $i_1(t)$ at t = 0+.

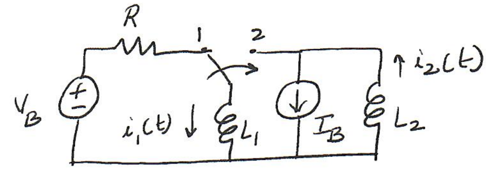


Figure 3.

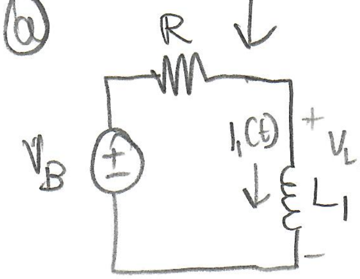
(3 + 7 = 10 points)

5

Solution:

@ t<0

(a)



3

@ steady state: $I_1(0^-) = \frac{V_B}{R} = \boxed{\frac{3}{1000}\text{ A.}}$

$V_{L_1} = L_1 i_1'$

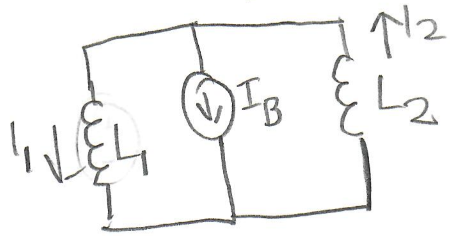
$I_1' = 0 \therefore V_{L_1} = 0 \therefore V_B - I_1 R - V_L = 0$

$\therefore V_B = I_1 R \therefore I_1 = \frac{V_B}{R}$

(b) $I(0^-) = \frac{V_B}{R} = \frac{3}{1000}$ & $I_2(0^-) = I_B$

$I_2(0^+) = I_B$

@ t>0



2

$I_1(0^+) = -\frac{L_2}{L_1} I_2(0^+) + I_1(0^-) = 1.4\,mA + \frac{3}{1000}A$

$I_2 = I_B + I_1$

$L_1 I_1' = -L_2 I_2'$

X $= \boxed{7mA}$

$I_1' = -\frac{L_2}{L_1} I_2' \therefore I_1(t) = \frac{-L_2}{L_1}\int_0^t I_2'(t) + I_1(0^-) = I_1(0^-) - \frac{L_2}{L_1} I_2(t)\Big|_0^t = I_1(0^-) - \frac{L_2}{L_1}$

$I_2(0^+)$

$C V_C' + \frac{V_C}{R} = I_L \quad \therefore \frac{I_L}{C} - \frac{V_C}{RC} = V_C'$

$\therefore V_C'(0) = \frac{4mA}{1nF} - \frac{5V}{(2500)\Omega\, 1nF}$

$V_C'(0) = 2E6 \frac{V}{S}$

Problem 3: Refer to the circuit in Figure 4. Assume that $V_B = 2V$, $L = 16mH$, $C = 1nF$, $R = 2500$ Ohms, $v_C(0+) = 5V$, and $i_L(0+) = 4mA$.
(a) Derive a differential equation in terms of $V_C(t)$ to show that this is a 2nd order circuit.
(b) Is this circuit overdamped or underdamped? Give numerical reasons.
(c) Derive an expression for the total solution of $V_C(t)$.
(d) Should you increase or decrease the value of R to make the circuit critically damped? Explain.

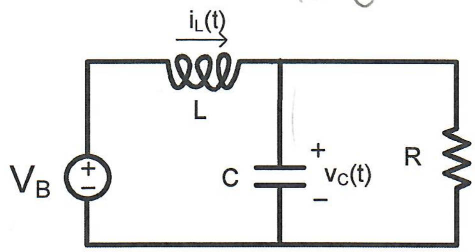


Figure 4.

(4 + 3 + 10 + 3 = 20 points)
Solution:

10

$I_L = CV_C' + \frac{V_C}{R} \quad \therefore I_L' = CV_C'' + \frac{V_C'}{R}$

$V_B - V_C = LI_L'$

$\therefore V_B - V_C = LCV_C'' + \frac{L}{R}V_C'$

4

(a) $\therefore V_C'' + \frac{V_C'}{RC} + \frac{V_C}{LC} = V_B$ ✓ ∴ Second order

$\therefore V_C'' + 400000\, V_C' + (6.25E10)V_C = 2, \quad V_C(0+) = 5V, \; V_C'(0+) = 2E6 \frac{V}{S}$

$\omega_0 = \sqrt{1/LC}$
$= \sqrt{1.6E11}$
$= \sqrt{16E12}$
$= \sqrt{4}\sqrt{1E12}$
$= 2E6$ X!

$2\zeta\omega_0 = 400000$

$\therefore \zeta = 400000 \cdot \frac{1}{2} \cdot \frac{1}{2E6}$

$\therefore \zeta = 0.1$

(b) $\zeta < 1 \therefore$ $\boxed{\text{Underdamped}}$ ✓

1

(c) $V_C(t) = C_1 e^{-200000t}\cos(150000t) + C_2 e^{-200000t}\sin(1500000t) + 3.2E{-11}$

3

$C_1 = 5 - 3.2E{-11} \approx 5$

$C_2 = 20 - 3.2E{-11} \approx 20$

$\therefore V_C(t) = 5e^{-200000t}\cos(150000t) + C_2 e^{-200000t}\sin(1500000t) + 3.2E{-11}$

(d) For Crit damped, $\zeta = 1 \therefore \frac{1}{RC} = 2\zeta\cdot(2E6) \quad \therefore R = \frac{1}{C}\cdot\frac{1}{2\cdot(2E6)}$

3 ✓ $\therefore R = 250 \leftarrow$ Since $250 < 2500$

∴ We need to decrease R

Reference Sheet #1

Trigonometric Identities:

$$\sin A = \cos\left(A - 90^0\right) = \cos\left(A - \pi/2\right)$$

$$\cos A = \sin\left(A + 90^0\right) = \sin\left(A + \pi/2\right)$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$

$$\sin\left(A \pm B\right) = \cos A \cos B \pm \sin A \sin B$$

$$\cos A + \cos B = 2\cos\left(\left(A+B\right)/2\right)\cos\left(\left(A-B\right)/2\right)$$

$$\cos A - \cos B = -2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A + \sin B = 2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A - \sin B = 2\cos\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$a\cos A + b\sin A = \sqrt{a^2 + b^2}\cos\left(A - \tan^{-1}\left(b/a\right)\right)$$

Complex Arithmetic:

$$\mathrm{Re}\{z_1 \pm z_1\} = \mathrm{Re}\{z_1\} \pm \mathrm{Re}\{z_2\}$$

$$\mathrm{Im}\{z_1 \pm z_1\} = \mathrm{Im}\{z_1\} \pm \mathrm{Im}\{z_2\}$$

$$\mathrm{Re}\{z_1 z_2\} = \mathrm{Re}\{z_1\}\mathrm{Re}\{z_2\} - \mathrm{Im}\{z_1\}\mathrm{Im}\{z_2\}$$

$$\mathrm{Im}\{z_1 z_2\} = \mathrm{Re}\{z_1\}\mathrm{Im}\{z_2\} + \mathrm{Im}\{z_1\}\mathrm{Re}\{z_2\}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$x + jy = re^{j\theta} \text{ where } r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\left(y/x\right)$$

$$re^{j\theta} = x + jy \text{ where } x = r\cos\theta, y = r\sin\theta$$

$$|z_1 z_2| = |z_1||z_2|, \quad angle\left(z_1 z_2\right) = angle\left(z_1\right) + angle\left(z_2\right)$$

$$|1/z| = 1/|z|, \qquad angle\left(1/z\right) = -angle\left(z\right)$$

$$\left(x + jy\right)^* = x - jy, \qquad angle\left(z^*\right) = -angle\left(z\right)$$

Quadratic Equations:

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

SCRATCH (Will NOT Be Graded)

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