University of California,
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School of Engineering and Applied Science

Department of Electrical and Computer Engineering

ECE11L Lab Manual
Winter 2023

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Experiment 3: Transient Response of the 1st-Order Circuits

Topics

- Step response of first order circuits
- Initial conditions and transient response

Objectives

- To understand and investigate the natural and step response of first order capacitive and inductive circuits
- To design a first-order circuit with certain characteristics
- To analyze initial conditions within a circuit containing inductors and capacitors

Background

i. Capacitors: Capacitors store energy in the form of an electrostatic field between the two plates as current flows through them, and charge accumulates upon the plates. The charge of a capacitor is given as:

$$q = Cv$$

Which leads to the following $i - v$ relation:

$$i(t) = C\frac{dv(t)}{dt}$$

Or alternatively:

$$v(t) = \frac{q(t)}{C} = \frac{1}{C}\int_{0^-}^{t} i(\theta)d\theta + v(0^-)$$

ii. RC Circuits: We may derive the linear differential equation of the RC circuit in Figure 1:

$$v_o + RC\frac{dv_o}{dt} = v_i$$

The step response ($v_i(t) = Vu(t)$) of the RC circuit for the voltage across the capacitor is:

$$v_o(t) = V(1 - e^{-\frac{t}{RC}})$$

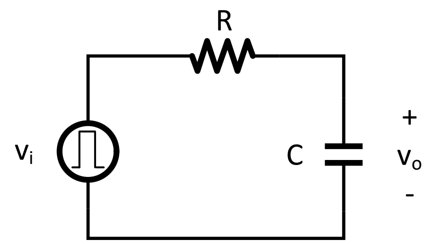


Figure 1: RC Circuit

iii. Inductors: Inductors are a passive circuit element which stores energy within magnetic fields generated by a change in current. The relationship for the voltage across an inductor is a function of the change in current given as:

$$v(t) = L\frac{di(t)}{dt}$$

iv. RL Circuits: The linear differential equation of the RL circuit in Figure 2 is:

$$v_o + \frac{L}{R}\frac{dv_o}{dt} = v_i$$

The step response of interest for the current is given as:

$$v_o(t) = V(1 - e^{-\frac{R}{L}t})$$

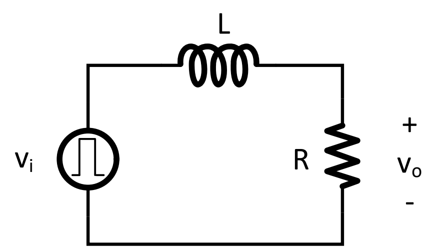


Figure 2: RL Circuit

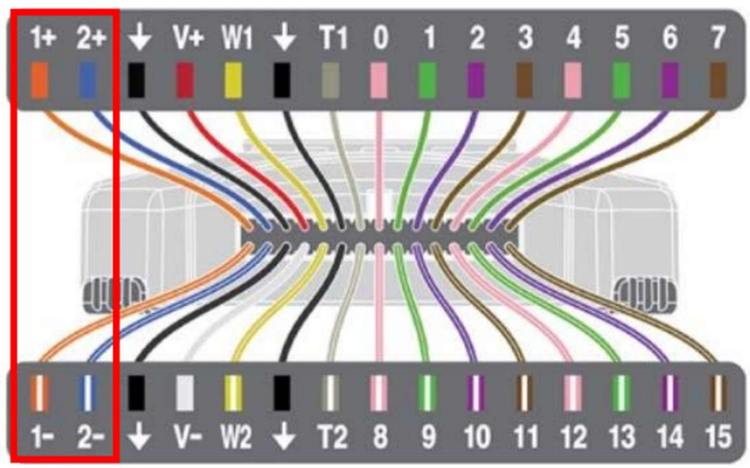
v. Time Constant: The time constant τ is defined as the time it takes the step response to gain a value of $1 - e^{-1}$ (approximately 0.63) of its final value. For the RC circuit, the time constant $\tau = RC$, which is easily seen from the equation above. For the RL circuit, $\tau = \frac{L}{R}$.

vi. Initial Conditions: Looking at a circuit which contains a sudden switch (we can simulate this with step function), the following properties hold:

- At steady state, ideal capacitors act as open circuits (zero current), while ideal inductors act as short circuits (zero voltage).

- At the instant after switching, voltage across the capacitor is equal to voltage before switching, while current across the inductor is equal to the current before switching. In other words, unless an infinite source (like an impulse) is applied, the voltage cannot change instantaneously in a capacitor, and the current cannot change instantaneously in an inductor.

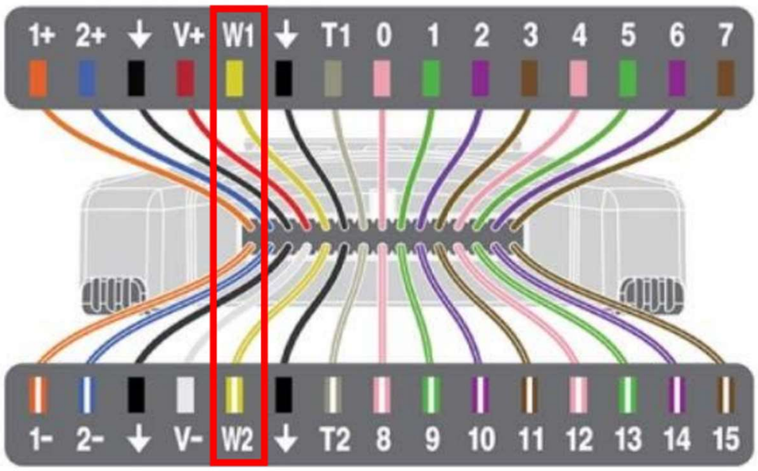
vii. The oscilloscope in Analog Discovery 2: There are two oscilloscope channels on Analog Discovery 2 with pins 1+/1- as channel 1, and pins 2+/2- as channel 2. Jump wires must be used to connect both channels to the breadboard and components.



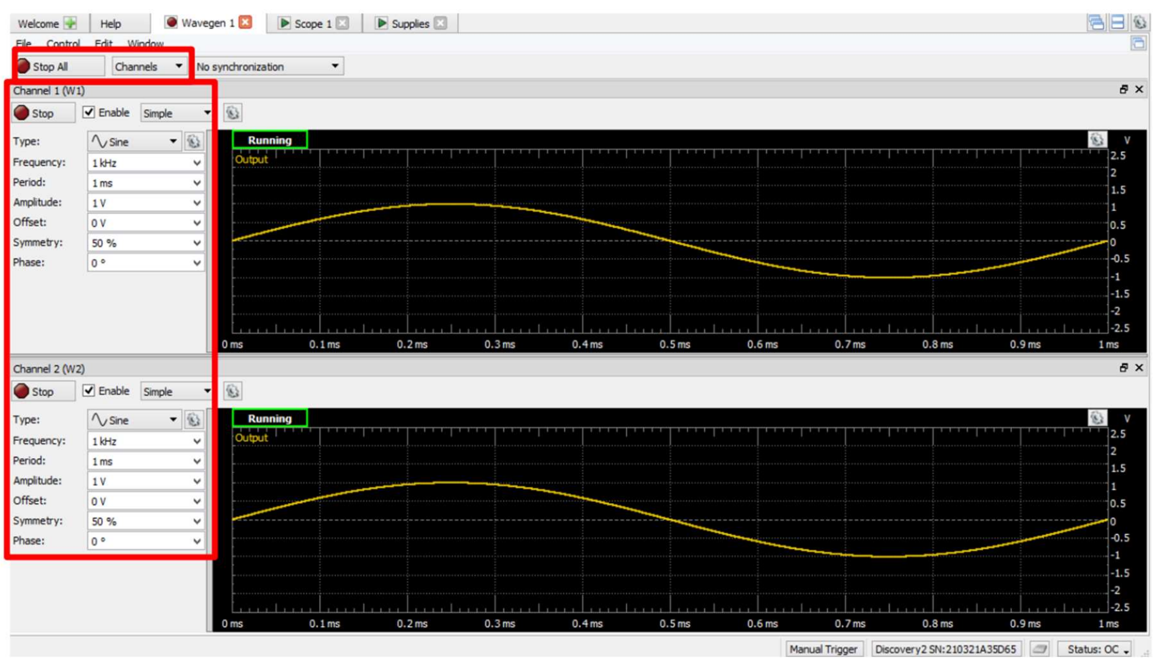
In the WaveForms software, open the Scope applet from the left column and the transient voltages across 1+/1- and 2+/2- will be readily available when you click run.

Two settings regarding the time and amplitude resolutions of the oscilloscope are on the right and are boxed. Time resolution (mS/div) means each horizontal interval captures how many mS of the signal. Amplitude resolution (mV/div) means each vertical interval spans a certain mV of the input signal. It is common practice to adjust the time and amplitude resolutions so that at least one or two periods of the signal spans the entire visible area.

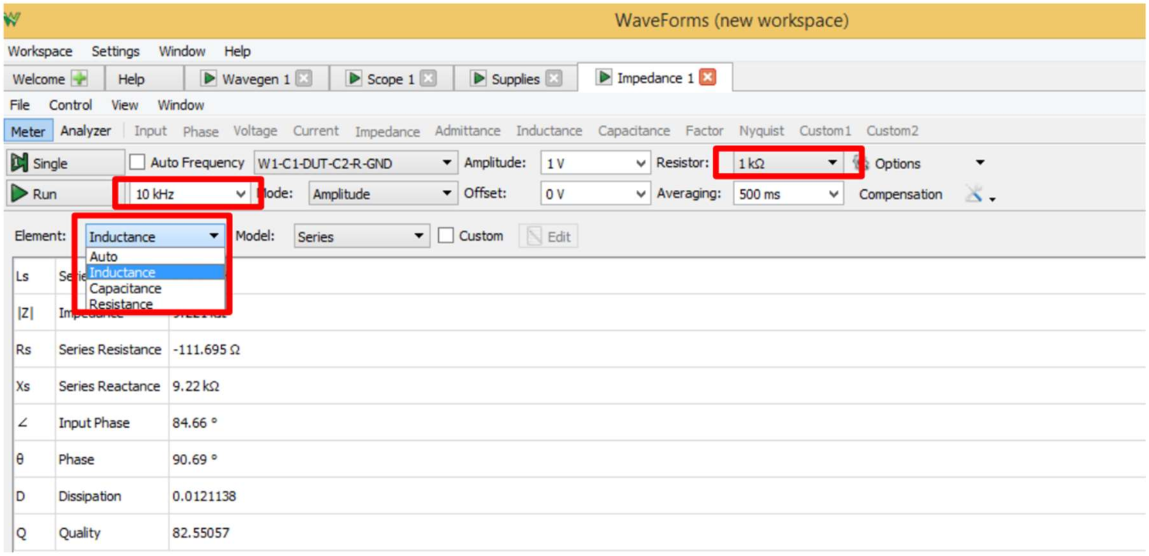
viii. The waveform generator in Analog Discovery 2: There are two independent waveform generator channels on Analog Discovery 2 with pins W1 as channel 1 and W2 as channel 2. Jump wires are used to connect either channels to the breadboard and components. Do not forget to connect ground pins to the circuit to form a loop.



In the WaveForms software, open the Wavegen applet from the left column and the desired signals will be available at W1 and W2 when enabled.



ix. Measuring capacitor and inductor with the impedance analyzer: In the first lab we touched on how to measure resistance with impedance analyzer, and in this section the measurement of C and L with impedance analyzer is shown below.



Select the element type from the pull-down menu as capacitance or inductance and set the testing frequency to 10kHz. The readings should be readily available. It is strongly recommended to measure C and L at a multitude of frequencies, such as 1kHz/10kHz/100kHz to see how the readings change over frequency. It is also recommended that the reference resistor to be swapped with larger ones for smaller capacitances and larger inductances, and vice versa. Don't worry about slight differences in the capacitance/inductance values. The following table outlines recommended pairing table between reference resistor and capacitors and inductors to be measured at 1kHz.

Capacitance	Ref Resistor	Inductance
100 pF	1 MΩ	
1 nF	100 kΩ	
10 nF	10 kΩ	1uH
1 uF	1 kΩ	10 uH
10 uF	100 Ω	100 uH
100 uF	10 Ω	1 mH

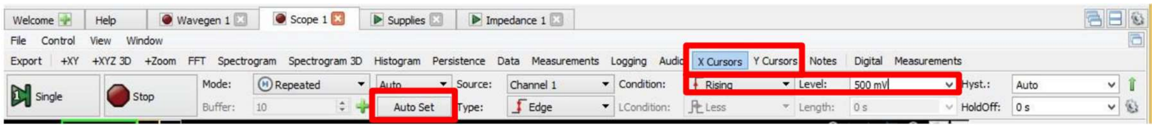
For more information on the oscilloscope, waveform generator and impedance analyzer visit: https://reference.digilentinc.com/learn/instrumentation/tutorials/start

Lab

1. RC Circuit Analysis

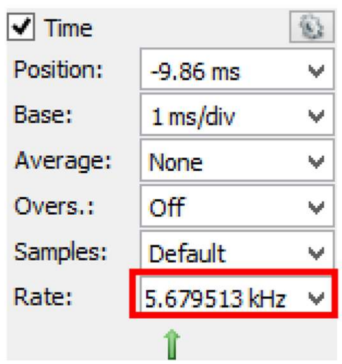
Construct an RC circuit as illustrated in Figure 1. Use a 4.7nF capacitor, and 3.3kΩ resistor. Set the AD2 waveform generator to output a square wave of $1V$ peak-to-peak.

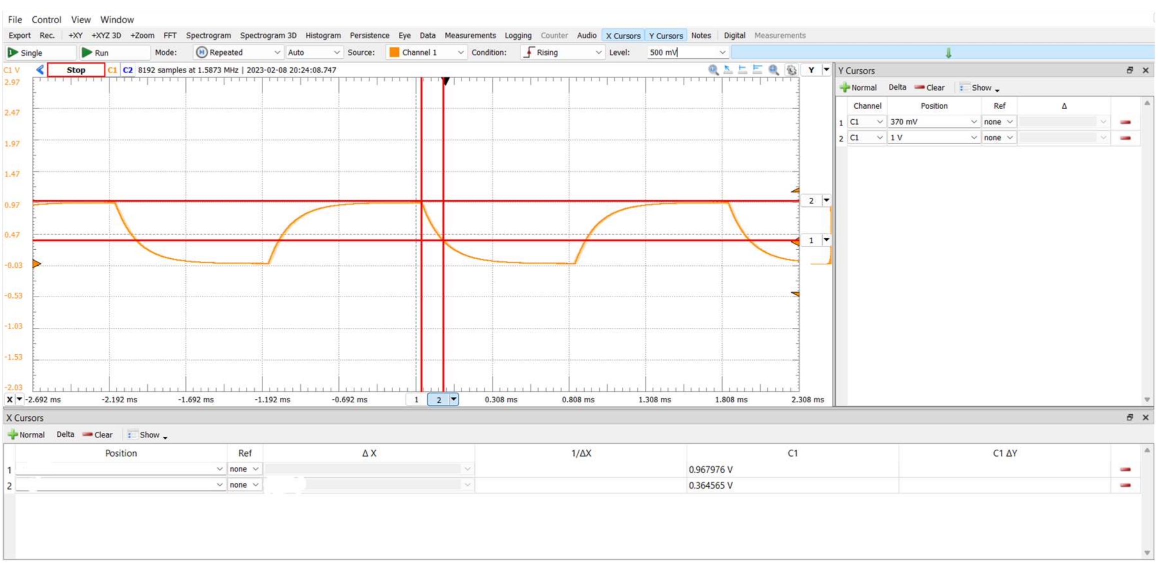
Set the offset such that the input voltage ranges from 0-1V. Finally, choose an appropriate frequency with a period of 5 to 10 times the time constant to allow the output voltage to reach steady state. Connect one channel of the oscilloscope to view the input voltage, and the other channel to read the output voltage of interest. As shown below, set the oscilloscope to be edge triggered based on the input voltage at 0.5V and appropriate resolution, which will enable easier viewing of the waveform. You can also try Auto Set as a starting point.



To use the cursors on the AD2 oscilloscope, click X Cursors and Y Cursors as shown above. Note that you can drag the waveform (time position) to facilitate the measurement. Experimentally measure the time constant of the RC circuit and compare with the theoretical value you have calculated. This can be done by setting one cursor at approximately 63.2% of the final voltage value and the other on the edge at which the input signal changes, and then reading the time difference.

AD2 supports up to 100MHz sampling rate and it can be adjusted in the expandable Time panel as shown below. In Auto Set mode, the sampling rate is tied to the time resolution.





Discussion

- How does the experimental time constant compare with the theoretical values?
- How does the voltage response of the resistor differ from that of the capacitor?
- How does zooming in time resolution (mS/div -> µS/div) change the sampling rate and your measurements?
- Explain what happened when you increased the frequency of the input.

2. RL Circuit Analysis

Maintaining a similar set-up as in the RC circuit, construct an RL circuit with a 150mH inductor and 10kΩ resistor (Figure 2). Measure and record the resistance of the inductor you are using. Note that the inductor resistance is relatively large, around a few hundreds of Ohm.
Obtain an experimental value for the time constant of the RL circuit. Measure the voltage response of the inductor as well with the oscilloscope.

Discussion

- How does the experimental time constant compare with the theoretical value?
- When calculating the theoretical time constant of the RL circuit, should the resistance of the inductor be included?
- How does the response of the RL circuit compare with the response of the RC circuit?

3. DC Switching Analysis

Using what we know of the behavior of capacitors and inductors, we may analyze initial values as well as final values of voltage and current. (i.e. $v(0^-)$, $v(0^+)$, and $v(\infty)$). The source in the given circuit below (Figure 3) switches from 3V to 2V at t=0. Find the theoretical values of all the voltages: $v_{C1}(0^-)$, $v_{C1}(0^+)$, $v_{C1}(\infty)$, $v_{R1}(0^-)$, $v_{R1}(0^+)$, $v_{R1}(\infty)$, and $v_{R2}(0^-)$, $v_{R2}(0^+)$, $v_{R2}(\infty)$.

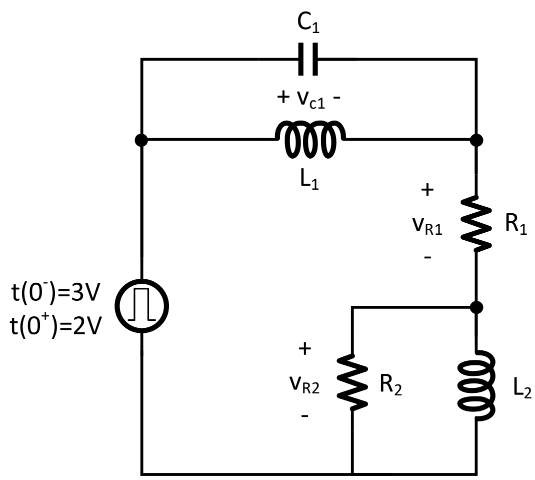


Figure 3: DC Switching Analysis

Construct the circuit on your breadboard. You may set-up the voltage source using a square wave as before with a low enough frequency such that you may observe the steady-state response. Set a 1V p-p wave with an appropriate DC offset to achieve the correct input waveform. Use the following values for components: $R_1 = R_2 = 680\Omega$, $L_1 = L_2 = 150mH$, and $C_1 = 1\mu F$.

Using the oscilloscope, experimentally measure the values. Note the inductors internal resistance may affect the measurements some.

Discussion

- What did you observe of the voltage across the elements? Explain any behavior that strays from the ideal expectations.

4. First Order Circuit Design

Based on the given step response in Figure 4, design a first-order circuit that matches the response.

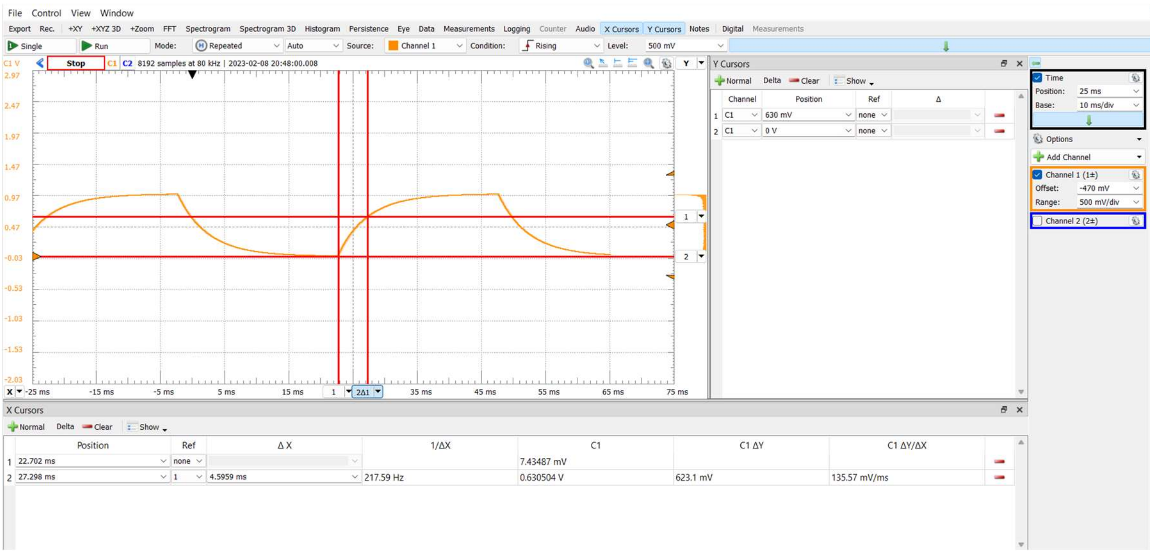


Figure 4: Step response of a 1st-order circuit

Compare the step response of your circuit with the one of Figure 4 and prepare it as part of the demonstration.