ECE 10, Winter 2023, Homework #3

Problem 1: Refer to Figure 1 for this problem. Assume that the circuit achieved steady state before the switch changes positions**.**

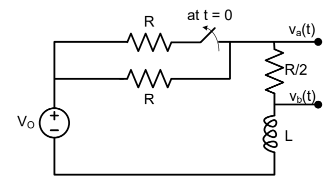

Figure 1.

a. Determine the current in the inductor just before and after the switch changes state i.e. at time t = 0– and t = 0+.

b. Determine the voltages $v_a(t)$ and $v_b(t)$ for all time $t \geq 0$.

c. Sketch the $v_a(t)$ and $v_b(t)$ waveforms clearly marking the values at t = 0+ and as $t \rightarrow \infty$.

(4 + 7 + 4 = 15 points)

Problem 2: Refer to Figure 2 for this problem. Assume that at time t = 0–, the current through the inductor is i(0–) = 1 Ampere and that $R_1 = 1\Omega$, $R_2 = 3\Omega$, L = 1H, and $i_B(t) = 2$ Amperes for time $t \geq 0$.

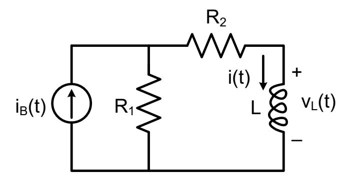

Figure 2.

a. What is the time constant of the circuit?

b. Determine $v_L(0+)$

c. Determine $v_L(\infty)$

d. Determine the voltage, $v_L(t)$, for all time $t \geq 0$.

(5 + 5 + 5 + 5 = 20 points)

Problem 3: Refer to Figure 3 for this problem. Assume that the circuit achieved steady state before the switch changes positions**.** Assume that $V_B = 3V$, R = 20 Ohms, and C = 0.5mF. Assume that the capacitor is charged to $V_B/4 = 0.75V$ just before t = 0.

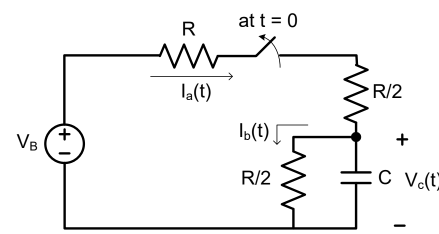

Figure 3

a. Determine the voltage across the capacitor just after the switch changes state i.e. at time t = 0+.

b. Determine the currents $I_a(t)$ and $I_b(t)$ just after the switch changes state i.e. for t = 0+.

c. Determine the value of dVc(t)/dt just after the switch changes state i.e. for t = 0+. **Hint**: Note that the current through the capacitor is C*dVc(t)/dt.

(8 + 4 + 8 = 20 points)

Problem 3: Consider the circuit in Figure 4, for $t \geq 0$. Prior to t = 0, the capacitors have been charged with charges, Q_1 and Q_2 respectively, such that $V_1(0-) = Q_1/C_1$ and $V_2(0-) = Q_2/C_2$. Assume that $C_1 = 5mF$, $C_2 = 15mF$, R = 300 Ohms, and $Q_1 = -Q_2 = 8$ Coulumbs.

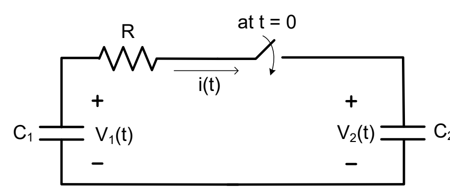

Figure 4

(a) Derive a differential equation in terms of the current through the resistor, i(t), that governs circuit operation for $t \geq 0$.

(b) What is the time constant of the circuit for operation during $t \geq 0$? Express it in terms of nanoseconds.

(c) Determine the boundary values, $V_1(0+)$, $V_2(0+)$, i(0+), and $V_1(\infty)$, $V_2(\infty)$, $i(\infty)$.

(d) Derive an expression for the voltages across the capacitors, $V_1(t)$ and $V_2(t)$, for all time $t \geq 0$.

(e) Draw a sketch of $V_1(t)$ and $V_2(t)$ as a function of time from t = 0 to t = 5s on the same graph.

Clearly mark the initial and final values of $V_1(t)$ and $V_2(t)$, on your sketch.
(f) Draw a sketch of i(t) as a function of time from t = 0 to t = 5s. Clearly mark the initial and final values of i(t) on your sketch.
(g) Briefly describe how the $V_1(t)$, $V_2(t)$, and i(t) waveforms would be for the case where R is very small (close to zero).
(2 + 2 + 6 + 3 + 2 + 2 + 3 = 20 points)

Problem 5: Refer to Figure 5 for this problem. The inductor has no energy stored before t = 0. Suppose that the circuit reached steady state before the switch changed position. For the following assume R_1 = 1 kΩ, R_2 = 40 Ω, L = 10 nH, C = 10 nF, and V_0 = 2 Volts.

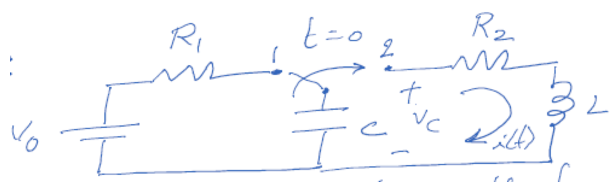

Figure 5

a. Derive the 2nd order differential equation in i(t) that governs circuit operation for $t \geq 0$.
b. Calculate the values of i(t), di(t)/dt, and $d^2i(t)/dt^2$ just after t = 0.
(5 + (3 + 3 + 4) = 15 points)

Problem 6: Refer to the circuit in Figure 6. Assume that $V_1(0-)$ = 0V, $V_2(0-)$ = 2V, $i_L(0-)$ = 4mA, R = 1kOhm, L = 1mH, C = 1nF, and V_B = 4V.
(a) Calculate the value of $i_1(t)$ at t = 0+.
(b) Calculate the value of $dV_L(t)/dt$ at t = 0+.
(4 + 6 = 10 points)

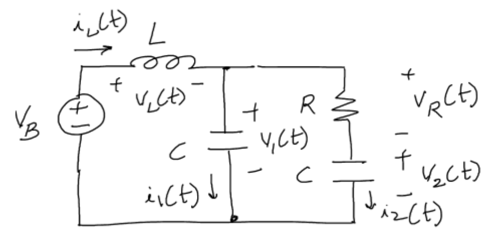

Figure 6.