Chapters 2.4-2.5 & 3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. Roll two fair dice independently. In terms of the possible outcomes, define the events:

$$A = \{\text{First die is 1, 2 or 5}\}$$
$$B = \{\text{Second die is 2, 3}\}$$
$$C = \{\text{Sum of outcomes is 7}\}$$

   Are A, B, and C mutually independent? Hint: Three events A, B, and C are independent if all the four following constraints hold:

$$P(A \cap B) = P(A)P(B),$$
$$P(A \cap C) = P(A)P(C),$$
$$P(B \cap C) = P(B)P(C),$$
$$P(A \cap B \cap C) = P(A)P(B)P(C).$$

2. Apple manufactures their M2 chips only in two locations: San Diego and Cupertino, with the latter producing 75% of their M2 chips. As per the data published by Apple, only 4% of all their manufactured M2 chips turn out to be defective. It is known that;

   P(M2 chip turns out to be defective given that it is manufactured in San Diego) = 5*P(M2 chip turns out to be defective given that it is manufactured in Cupertino)

   An M2 chip is randomly selected, and we find out that it works fine (not defective). Then what is the probability that it was manufactured in San Diego?

3. There are two bags B_1 and B_2 containing a mix of Water-type and Fire-type Pokémon cards. B_1 contains 3 Water-type cards and 2 Fire-type cards, and B_2 contains only 1 Water-type card. A fair coin is tossed ($p = 0.5$), and based on the result of the toss, the following actions are taken:

   (a) If heads appear, then 1 card is randomly drawn from B_1 and put into B_2.
   (b) If tails appear, then 2 cards are randomly drawn from B_1 and put into B_2.

   A card is randomly drawn from B_2; it turns out to be a Water-type Pokémon card. What is the probability that heads appeared on the coin?

4. Throw a pair of six-sided dice. Let X_1 be the number of dots on the resulting face of the first die and let X_2 be the number of dots on the resulting face of the second die. Let $Z = X_1 + X_2$ be the sum of the two dice rolls.

   (a) What is the pmf of Z?

(b) Given that $Z = 10$, what is the probability that $X_1 = k$ for $k \in \{1, 2, 3, 4, 5, 6\}$?

5. A ternary communication channel is shown in the figure. Assume that input symbols $0, 1$, and 2 are chosen for transmission with probabilities $\frac{1}{6}, \frac{1}{6}$, and $\frac{2}{3}$.
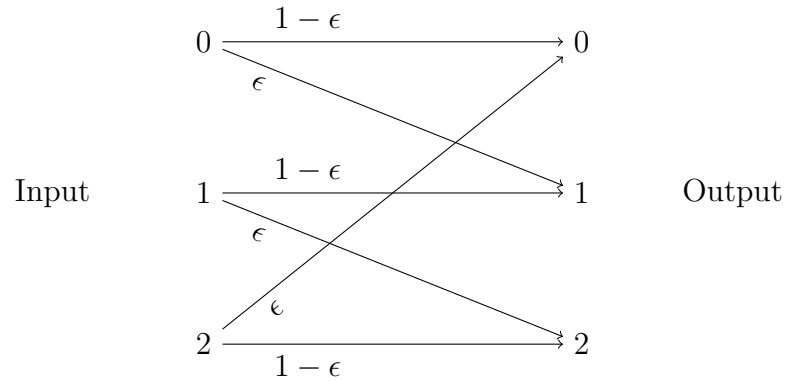
(a) Calculate the probability of each output.

(b) Given that the output was 0, what is the probability that the input was 0? 1? 2?