ECE 102 HW3

SANJIT SARDA

TOTAL POINTS

92 / 100

QUESTION 1

Linear Systems 15 pts

1.1 1 a) 5 / 5

✓ **- 0 pts** Correct

- 2 pts The system is linear

1.2 1 b) 5 / 5

✓ **- 0 pts** Correct

- 3 pts The system is linear

1.3 1 c) 5 / 5

✓ **- 0 pts** Correct

- 3 pts The system is not linear

QUESTION 2

LTI Systems 20 pts

2.1 2 a) 10 / 10

✓ **- 0 pts** Correct

- 4 pts Partial Correct

- 7 pts Missing sketch

- 10 pts Missing/ Incorrect

2.2 2 b) 10 / 10

✓ **- 0 pts** Correct

- 4 pts Correct h(t) but incorrect/ unsimplified y2(t)

- 6 pts Final ans for y2(t) missing

- 10 pts Missing/ Incorrect

QUESTION 3

Convolution 36 pts

3.1 3 a) 9 / 12

- 0 pts Correct

✓ **- 2 pts 2 partially correct**

✓ **- 1 pts 1 partially wrong**

- 6 pts 2 mostly wrong

- 4 pts 2 partially correct (2 cases missing/wrong)

- 10 pts did not attempt/complete part 2

- 2 pts did not attempt/complete part 1

3.2 3 b) 12 / 12

✓ **- 0 pts** Correct

- 4 pts partially correct

- 12 pts did not attempt

3.3 3 c) 12 / 12

✓ **- 0 pts** Correct

- 2 pts part 2 partially correct

- 4 pts part 2 mostly wrong

- 10 pts part 2 missing

- 1 pts 1 partially correct

- 12 pts did not attempt

QUESTION 4

LTI Systems and Impulse response 12 pts

4.1 4 a) 4 / 4

✓ **- 0 pts** Correct

- 2 pts Partial Correct

- 4 pts Incorrect/ Missing

4.2 4 b) 2 / 2

✓ **- 0 pts** Correct

- 1 pts Adder block missing/ incomplete block diagram from x(t) to w(t)

- 2 pts Incorrect

4.3 4 c) 2 / 2

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- **2 pts** Incomplete

4.4 4 d) **4 / 4**

✓ **- 0 pts** **Correct**

- **2 pts** Partial Correct
- **4 pts** Incorrect/ Missing

QUESTION 5

Python Tasks 17 pts

5.1 5 a) **5 / 10**

- **0 pts** Correct

✓ **- 5 pts** **Incorrect convolution (improper inputs to nconv)**

- **2 pts** Please show the y axis tick marks
- **10 pts** Missing

5.2 5 b) **7 / 7**

✓ **- 0 pts** **Correct**

- **4 pts** Incorrect convolution (incorrect inputs to nconv)
- **2 pts** Should be rect(t) * rect(t) (rectangles both centered at the origin)
- **7 pts** Missing

102 HW3

1) Linear Systems

a) $y(t) = x(t)e^{-j\omega t} = -x(t)$

To show linearity: $S[ax_1(t)+bx_2(t)] =$
$aS[x_1(t)] + bS[x_2(t)]$

$\therefore S[ax_1(t)+bx_2(t)] = (ax_1(t)+bx_2(t))\cdot e^{-j\omega t}$

$= (ax_1(t)+bx_2(t))\cdot e^{-j\omega t} = ax_1(t)e^{-j\omega t} + bx_2(t)e^{-j\omega t}$
$= aS[x_1(t)] + bS[x_2(t)]$

$\therefore$ It is linear

b) $y(t) = \int_{-\infty}^{\infty} [x(t)]^2 + x(t)\, dt$

$\therefore S[ax_1(t) + bx_2(t)] = aS[x_1(t)] + bS[x_2(t)]$

$S[ax_1(t) + bx_2(t)] = \int_{-\infty}^{\infty} [ax_1(t) + bx_2(t)]^2 + ax_1(t) + bx_2(t)\, dt$

$\int_{-\infty}^{\infty} [ax_1(t)+bx_2(t)]^2 + ax_1(t)+bx_2(t)\, dt = \int_{-\infty}^{\infty} a^2x_1(t)^2 + 3ax_1(t) + bx_2(t) + b^2x_2(t)^2\, dt$

$\neq$

$aS[x_1(t)] + bS[x_2(t)] = a\int_{-\infty}^{\infty} x_1(t)^2 + x_1(t)\, dt + b\int_{-\infty}^{\infty} x_2(t)^2 + x_2(t)\, dt$

$\therefore$ It is not linear.

c) $y(t) = e^{x(t)}$

$S[ax_1(t)+bx_2(t)] = e^{ax_1(t)+bx_2(t)}$

$\neq$

$aS[x_1(t)] + bS[x_2(t)] = ae^{x_1(t)} + be^{x_2(t)}$

$\therefore$ It is not linear.

1.1 1 a) **5 / 5**

✓ **- 0 pts** Correct

- 2 pts The system is linear

102 HW3

1) Linear Systems

a) $y(t) = x(t)e^{-j\omega t} = -x(t)$

To show linearity: $S[ax_1(t)+bx_2(t)] = aS[x_1(t)] + bS[x_2(t)]$

$\therefore S[ax_1(t)+bx_2(t)] = (ax_1(t)+bx_2(t))\cdot e^{-j\omega t}$

$= (ax_1(t)+bx_2(t))\cdot e^{-j\omega t} = ax_1(t)e^{-j\omega t} + bx_2(t)e^{-j\omega t}$

$= aS[x_1(t)] + bS[x_2(t)]$

$\therefore$ It is linear

b) $y(t) = \int_{-\infty}^{\infty} [x(t)]^2 + x(t)\,dt$

$\therefore S[ax_1(t)+bx_2(t)] = aS[x_1(t)] + bS[x_2(t)]$

$S[ax_1(t)+bx_2(t)] = \int_{-\infty}^{\infty} [ax_1(t)+bx_2(t)]^2 + ax_1(t)+bx_2(t)\,dt$

$\int_{-\infty}^{\infty} [ax_1(t)+bx_2(t)]^2 + ax_1(t)+bx_2(t)\,dt = \int_{-\infty}^{\infty} a^2x_1(t)^2 + 3ax_1(t)+bx_2(t) + b^2x_2(t)^2\,dt$

$\neq$

$aS[x_1(t)] + bS[x_2(t)] = a\int_{-\infty}^{\infty} x_1(t)^2 + x_1(t)\,dt + b\int_{-\infty}^{\infty} x_2(t)^2 + x_2(t)\,dt$

$\therefore$ It is not linear.

c) $y(t) = e^{x(t)}$

$S[ax_1(t)+bx_2(t)] = e^{ax_1(t)+bx_2(t)}$

$\neq$

$aS[x_1(t)] + bS[x_2(t)] = ae^{x_1(t)} + be^{x_2(t)}$

$\therefore$ It is not linear.

1.2 1 b) **5 / 5**

✓ **- 0 pts** Correct

- 3 pts The system is linear

102 HW3

1) Linear Systems

a) $y(t) = x(t)e^{-j\omega t} = -x(t)$

To show linearity: $S[ax_1(t)+bx_2(t)] =$
$a\,S[x_1(t)] + b\,S[x_2(t)]$

$\therefore S[ax_1(t)+bx_2(t)] = (ax_1(t)+bx_2(t))\cdot e^{-j\omega t}$

$= (ax_1(t)+bx_2(t))\cdot e^{-j\omega t} = a x_1(t)e^{-j\omega t} + b x_2(t)e^{-j\omega t}$
$= a\,S[x_1(t)] + b\,S[x_2(t)]$

$\therefore$ It is linear

b) $y(t) = \int_{-\infty}^{\infty} [x(t)]^2 + x(t)\,dt$

$\therefore S[ax_1(t)+bx_2(t)] = a\,S[x_1(t)] + b\,S[x_2(t)]$

$S[ax_1(t)+bx_2(t)] = \int_{-\infty}^{\infty} [ax_1(t)+bx_2(t)]^2 + ax_1(t)+bx_2(t)\,dt$

$\int_{-\infty}^{\infty} [ax_1(t)+bx_2(t)]^2 + ax_1(t)+bx_2(t)\,dt = \int_{-\infty}^{\infty} a^2x_1(t)^2 + 3ax_1(t)+bx_2(t) + b^2x_2(t)^2\,dt$

$\neq$

$a\,S[x_1(t)] + b\,S[x_2(t)] = a\int_{-\infty}^{\infty} x_1(t)^2 + x_1(t)\,dt + b\int_{-\infty}^{\infty} x_2(t)^2 + x_2(t)\,dt$

$\therefore$ It is not linear.

c) $y(t) = e^{x(t)}$

$S[ax_1(t)+bx_2(t)] = e^{ax_1(t)+bx_2(t)}$

$\neq$

$a\,S[x_1(t)] + b\,S[x_2(t)] = ae^{x_1(t)} + be^{x_2(t)}$

$\therefore$ It is not linear.

1.3 1 c) 5 / 5

✓ - **0 pts** Correct

- **3 pts** The system is not linear

2) a) $y(t) = S[x_1(t)]$

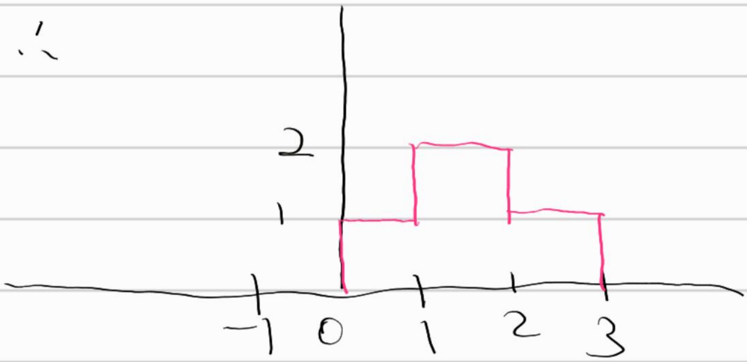
$\therefore \text{rect}(\frac{1}{2}(t-2)) = S[x_1(t)]$

by Observation, $x_2(t) = x_1(t) + x_1(t+1)$

$\therefore y_2(t) = S[x_2(t)] = S[x_1(t) + x_1(t+1)]$ since LTI?

$\therefore y_2(t) = \text{rect}(\frac{1}{2}(t-2)) + \text{rect}(\frac{1}{2}(t-1))$

$\therefore$



b) $y(t) = S[x(t)]$

$a^t \cos(t) = S[u(t)]$

Non causal.
The output of the system uses future values $\therefore$ it is not causal.

$\therefore S[0.5\,\delta(t+1) + \delta(t-1)]$

$= S\left[0.5 \frac{d}{dt}\left(u(t+1) + u(t-1)\right)\right]$

$= 0.5 \frac{d}{dt}\left[S[u(t+1) + u(t-1)]\right) =$

$= 0.5 \frac{d}{dt}\left(a^{t+1}\cos(t+1) + a^{t-1}\cos(t-1)\right) =$

$0.5\left[a^{t+1}\ln(a) \cdot \cos(t+1) - a^{t+1}\sin(t) + a^{t-1}\ln(a)\cos(t-1) - a^{t-1}\sin(t-1)\right)$

2.1 2 a) 10 / 10

- ✓ - **0 pts** Correct
- - **4 pts** Partial Correct
- - **7 pts** Missing sketch
- - **10 pts** Missing/ Incorrect

2) a) $y(t) = S[x_1(t)]$

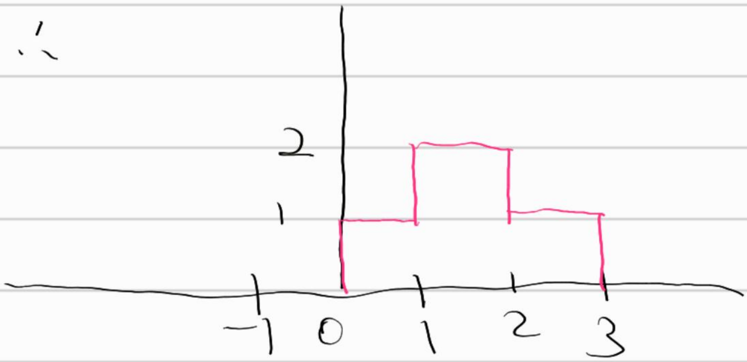
$\therefore \text{rect}(\frac{1}{2}(t-2)) = S[x_1(t)]$

by Observation, $x_2(t) = x_1(t) + x_1(t+1)$

$\therefore y_2(t) = S[x_2(t)] = S[x_1(t) + x_1(t+1)]$ since LTI?

$\therefore y_2(t) = \text{rect}(\frac{1}{2}(t-2)) + \text{rect}(\frac{1}{2}(t-1))$

$\therefore$



b) $y(t) = S[x(t)]$

$a^t\cos(t) = S[u(t)]$

Non causal: The output of the system uses future values $\therefore$ it is not causal.

$\therefore S[0.5\,\delta(t+1) + \delta(t-1)]$

$= S\left[0.5\frac{d}{dt}\left(u(t+1) + u(t-1)\right)\right]$

$= 0.5\frac{d}{dt}\left[S[u(t+1) + u(t-1)]\right] =$

$= 0.5\frac{d}{dt}\left(a^{t+1}\cos(t+1) + a^{t-1}\cos(t-1)\right) =$

$0.5\left[a^{t+1}\ln(a)\cdot\cos(t+1) - a^{t+1}\sin(t) + a^{t-1}\ln(a)\cos(t-1) - a^{t-1}\sin(t-1)\right)$

2.2 2 b) **10 / 10**

✓ **- 0 pts** Correct

- 4 pts Correct h(t) but incorrect/ unsimplified y2(t)

- 6 pts Final ans for y2(t) missing

- 10 pts Missing/ Incorrect

3 a) i) $f(t) = \delta(t+1) + 5\delta(t-2)$, $g(t) = e^{-t}u(t)$

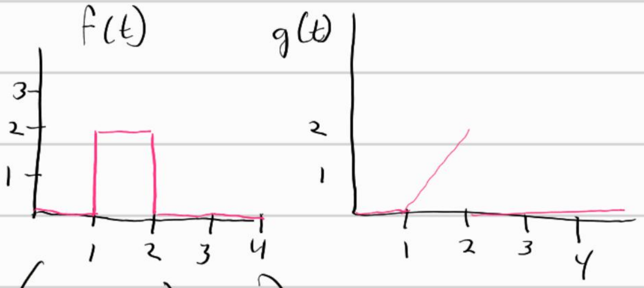
$f(t)*g(t) = \delta(t+1)*g(t) + 5(\delta(t-2)*g(t)) = g(t+1) + g(t-2)$

Distributive property

Delay via convolution

$f(t)*g(t) = e^{-t-1}u(t+1) + e^{2-t}u(t-2)$

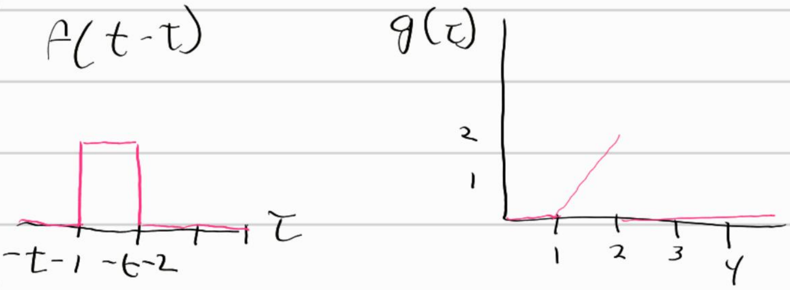
ii) $f(t) = 2\,\mathrm{rect}(t-3/2)$ $g(t) = 2r(t-1)\,\mathrm{rect}(t-3/2)$



$(f*g)(t) =$

$\int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau$

Flip n Drag



Case 1: $t<2$

$\int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau$

$=0$ ∵ No overlap

Case 2: $2 \leq t < 3$

$\int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau$

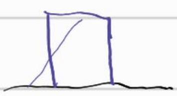
$= \int_1^{t-1} 2\cdot(\tau-1)\cdot 2 d\tau$

$= 4\cdot\int_1^{t-1} \tau - 1 d\tau = 4\left(\frac{\tau^2}{2} - \tau\right)\Big|_1^{t-1}$

$= 2(t-1)^2 - 4(t-1) + 2$

Case 3: $3 \leq t < 4$

$\int_{-\infty}^{\infty} g(\tau)f(t-\tau)dt$:



$= \int_2^{t-2} 2(2\tau-2)d\tau$

$= 4\int_2^{t-2} \tau - 1 d\tau$

$= 4\left(\frac{\tau^2}{2} - \tau\right)\Big|_2^{t-2} = 2(t-4)(t-2)$

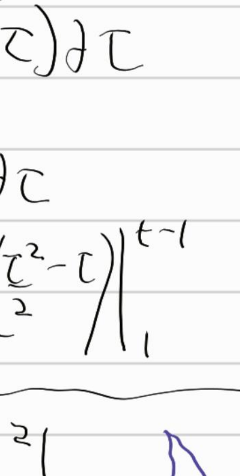
Case 4: $4<t$

$\int_{-\infty}^{\infty} g(\tau)f(t-\tau)dt$:

$= 0,$

Putting it together:

$$(f*g)(t) = \begin{cases} 2(t-1)^2 - 4(t-1) + 2, & 2 \leq t < 3 \\ 2(t-4)(t-2), & 3 \leq t < 4 = \\ 0, & \text{else} \end{cases}$$



3.1 3 a) **9 / 12**

- **- 0 pts** Correct
- ✓ **- 2 pts** **2 partially correct**
- ✓ **- 1 pts** **1 partially wrong**
- **- 6 pts** 2 mostly wrong
- **- 4 pts** 2 partially correct (2 cases missing/wrong)
- **- 10 pts** did not attempt/complete part 2
- **- 2 pts** did not attempt/complete part 1

b) $y(t) = \int_{t-T}^{t} (t-\tau)^2 x(\tau)\, d\tau$

$\therefore$ Impulse Response $= \int_{t-T}^{t} (t-\tau)^2 \delta(\tau) d\tau$

$\therefore \quad x(\tau)\delta(\tau) = x(0)\delta(\tau) = t^2 \delta(\tau)$

$\therefore \quad y(t) = \int_{t-T}^{t} t^2 \delta(\tau) d\tau = \begin{cases} t^2, & 0 < t < T \\ 0, & \text{else} \end{cases}$

$$\therefore = t^2 \begin{cases} 1 & 0 < t < T \\ 0, & \text{else} \end{cases} = \boxed{\underbrace{t^2 \cdot \mathrm{rect}\left(\frac{x}{T} - \frac{1}{2}\right)}_{h(t)}}$$

c) i) $e^t * \sum_{k=0}^{\infty} \delta(t-k)$

$$= \sum_{k=0}^{\infty} e^t * \delta(t-k) = \sum_{k=0}^{\infty} e^{t-k} = e^t \sum_{k=0}^{\infty} \left(\frac{1}{e}\right)^k = \frac{e^t}{1-\frac{1}{e}}$$

$$= e^t \cdot \frac{1}{(e-1)e^{-1}} = \frac{e^t \cdot e}{e-1} = \frac{e^{t+1}}{e-1}$$

ii) $\frac{d}{dt}\left[(u(t) - u(t-1)) * u(t-2)\right]$

$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau) u(t-\tau)\, d\tau$

Case 1:

$x < 0$, $(u * u)(t) = 0 \because$ No overlap

$x \geq 0$, $(u*u)(t) = \int_0^t 1 \cdot 1\, d\tau = t$

$\therefore = \begin{cases} t, & x \geq 0 \\ 0, & \text{else} \end{cases} = \mathrm{relu}(t)$

$(u(t) - u(t-1)) * u(t-2)$

$= u(t) * u(t-2) - u(t-1) * u(t-2)$

$= r(t-2) - r(t-1-2)$

$= r(t-2) - r(t-3)$

$\frac{d}{dx}\left(r(t-2) - r(t-3)\right)$

$= u(t-2) - u(t-3)$

3.2 3 b) **12 / 12**

✓ **- 0 pts** Correct

- 4 pts partially correct

- 12 pts did not attempt

b) $y(t) = \int_{t-T}^{t} (t-\tau)^2 x(\tau)\, d\tau$

$\therefore$ Impulse Response $= \int_{t-T}^{t} (t-\tau)^2 \delta(\tau)\, d\tau$

$\therefore\ x(\tau)\delta(\tau) = x(0)\delta(\tau) = t^2\delta(\tau)$

$\therefore\ y(t) = \int_{t-T}^{t} t^2 \delta(\tau)\, d\tau = \begin{cases} t^2, & 0<t<T \\ 0, & \text{else} \end{cases}$

$h(t)$

$\therefore = t^2 \begin{cases} 1, & 0<t<T \\ 0, & \text{else} \end{cases} = \boxed{t^2 \cdot \text{rect}\left(\frac{x}{T} - \frac{1}{2}\right)}$

c) i) $e^t * \sum_{k=0}^{\infty} \delta(t-k)$

$= \sum_{k=0}^{\infty} e^t * \delta(t-k) = \sum_{k=0}^{\infty} e^{t-k} = e^t \sum_{k=0}^{\infty} \left(\frac{1}{e}\right)^k = \frac{e^t}{1-\frac{1}{e}}$

$= e^t \cdot \frac{1}{(e-1)e^{-1}} = \frac{e^t \cdot e}{e-1} = \frac{e^{t+1}}{e-1}$

ii) $\frac{d}{dt}\left[(u(t) - u(t-1)) * u(t-2)\right]$

$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)\, d\tau$

Case 1:

$x<0,\ (u*u)(t) = 0 \because$ No overlap

$x \geq 0,\ (u*u)(t) = \int_0^t 1 \cdot 1\, d\tau = t$

$\therefore = \begin{cases} t, & x \geq 0 \\ 0, & \text{else} \end{cases} = \text{relu}(t)$

$(u(t) - u(t-1)) * u(t-2)$

$= u(t) * u(t-2) - u(t-1) * u(t-2)$

$= r(t-2) - r(t-1-2)$

$= r(t-2) - r(t-3)$

$\frac{d}{dx}\left(r(t-2) - r(t-3)\right)$

$= u(t-2) - u(t-3)$

3.3 3 c) **12 / 12**

✓ **- 0 pts** Correct

- 2 pts part 2 partially correct

- 4 pts part 2 mostly wrong

- 10 pts part 2 missing

- 1 pts 1 partially correct

- 12 pts did not attempt

$h(t) = H(\delta(t))$

4)

- S_1: $y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} x(\tau) d\tau$;
- S_2: $y(t) = \int_{-\infty}^{t-2} x(\tau) d\tau$
- S_3 is characterized by its impulse response: $h_3(t) = \delta(t-3)$.

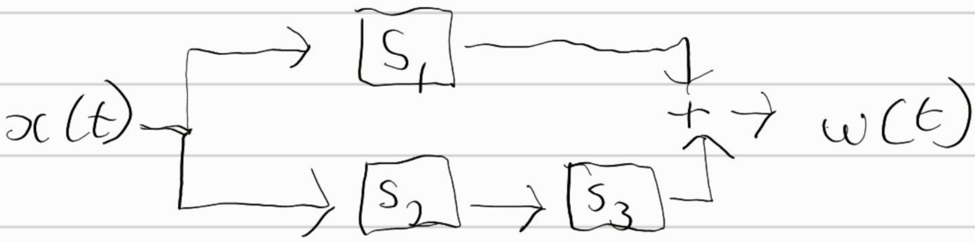
(a) (4 points) Compute the impulse response $h_1(t)$ of S_1.

a) $h_1(t) = S_1(\delta(t))$

$h_1(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} \delta(\tau) d\tau = e^{-3t} \int_{-\infty}^{t} \delta(\tau) d\tau = e^{-3t} u(t)$

b) (b) (2 points) Define $w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$. Represent this relationship using a block diagram where $x(t)$ is the input and $w(t)$ is the output.

$w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$



c) $S_{eq} = S_1[\text{signal}] - S_3[S_2[\text{signal}]]$

$h_1(t) = e^{-3t} u(t)$, $h_2(t) = \int_{-\infty}^{t-2} \delta(\tau) d\tau = u(t-2)$

$\therefore h_{eq} = e^{-3t} u(t) - S_3[S_2[\delta(t)]] = e^{-3t} u(t) - S_3[u(t-2)]$

S_1 - imp

$\therefore h_{eq} = e^{-3t} u(t) - \delta(t-3) * u(t-2) = e^{-3t} u(t) - u(t-3-2)$

$\therefore h_{eq}(t) = e^{-3t} u(t) - u(t-5)$

d) $h_4(t) = S_{eq}[\delta(t) + 2\delta(t-3)]$

$\therefore h_4(t) = \delta(t) * h_{eq}(t) + 2\delta(t-3) h_{eq}(t)$

$= e^{-3t} u(t) - u(t-5) + 2(e^{-3(t-3)} u(t-3) - u(t-8)))$

4.1 4 a) 4 / 4

✓ - **0 pts** Correct

- **2 pts** Partial Correct

- **4 pts** Incorrect/ Missing

$h(t) = H(\delta(t))$

4)

- S_1: $y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} x(\tau) d\tau$;
- S_2: $y(t) = \int_{-\infty}^{t-2} x(\tau) d\tau$
- S_3 is characterized by its impulse response: $h_3(t) = \delta(t-3)$.

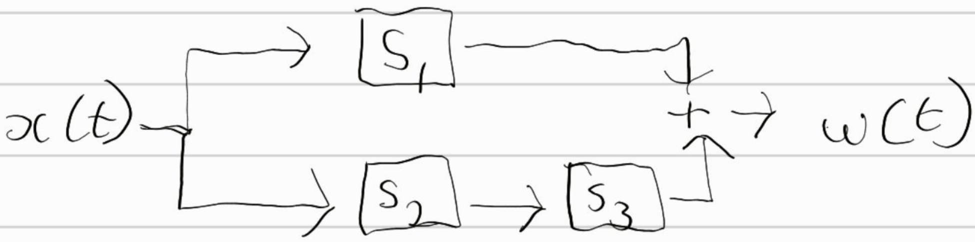
(a) (4 points) Compute the impulse response $h_1(t)$ of S_1.

a) $h_1(t) = S_1(\delta(t))$

$h_1(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} \delta(\tau) d\tau = e^{-3t} \int_{-\infty}^{t} \delta(\tau) d\tau = e^{-3t} u(t)$

b) (b) (2 points) Define $w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$. Represent this relationship using a block diagram where $x(t)$ is the input and $w(t)$ is the output.

$w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$



c) $S_{eq} = S_1[\text{signal}] - S_3[S_2[\text{signal}]]$

$h_1(t) = e^{-3t} u(t)$, $h_2(t) = \int_{-\infty}^{t-2} \delta(\tau) d\tau = u(t-2)$

$\therefore h_{eq} = e^{-3t} u(t) - S_3[S_2[\delta(t)]] = e^{-3t} u(t) - S_3[u(t-2)]$

$\swarrow$ S_1-time

$\therefore h_{eq} = e^{-3t} u(t) - \delta(t-3) * u(t-2) = e^{-3t} u(t) - u(t-3-2)$

$\therefore h_{eq}(t) = e^{-3t} u(t) - u(t-5)$

d) $h_4(t) = S_{eq}[\delta(t) + 2\delta(t-3)]$

$\therefore h_4(t) = \delta(t) * h_{eq}(t) + 2\delta(t-3) h_{eq}(t)$

$= e^{-3t} u(t) - u(t-5) + 2(e^{-3(t-3)} u(t-3) - u(t-8)))$

4.2 4 b) **2 / 2**

✓ **- 0 pts** Correct

- 1 pts Adder block missing/ incomplete block diagram from x(t) to w(t)

- 2 pts Incorrect

$h(t) = H(\delta(t))$

4)

- $\mathcal{S}_1$: $y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} x(\tau) d\tau$;
- $\mathcal{S}_2$: $y(t) = \int_{-\infty}^{t-2} x(\tau) d\tau$
- $\mathcal{S}_3$ is characterized by its impulse response: $h_3(t) = \delta(t-3)$.

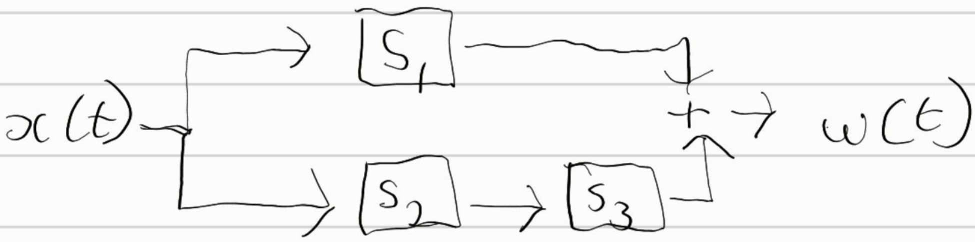
(a) (4 points) Compute the impulse response $h_1(t)$ of $\mathcal{S}_1$.

a) $h_1(t) = S_1(\delta(t))$

$h_1(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} \delta(\tau) d\tau = e^{-3t} \int_{-\infty}^{t} \delta(\tau) d\tau = e^{-3t} u(t)$

b) (b) (2 points) Define $w(t) = \mathcal{S}_1[x(t)] - \mathcal{S}_3\{\mathcal{S}_2[x(t)]\}$. Represent this relationship using a block diagram where $x(t)$ is the input and $w(t)$ is the output.

$w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$



c) $S_{eq} = S_1[\text{signal}] - S_3[S_2[\text{signal}]]$

$h_1(t) = e^{-3t} u(t)$, $h_2(t) = \int_{-\infty}^{t-2} \delta(\tau) d\tau = u(t-2)$

$\therefore h_{eq} = e^{-3t} u(t) - S_3[S_2[\delta(t)]] = e^{-3t} u(t) - S_3[u(t-2)]$

S_1 imp

$\therefore h_{eq} = e^{-3t} u(t) - \delta(t-3) * u(t-2) = e^{-3t} u(t) - u(t-3-2)$

$\therefore h_{eq}(t) = e^{-3t} u(t) - u(t-5)$

d) $h_4(t) = S_{eq}[\delta(t) + 2\delta(t-3)]$

$\therefore h_4(t) = \delta(t) * h_{eq}(t) + 2\delta(t-3) h_{eq}(t)$

$= e^{-3t} u(t) - u(t-5) + 2(e^{-3(t-3)} u(t-3) - u(t-8)))$

4.3 4 c) 2 / 2

✓ **- 0 pts** Correct

- 1 pts Partial Correct

- 2 pts Incomplete

$h(t) = H(\delta(t))$

4)

- S_1: $y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} x(\tau) d\tau$;
- S_2: $y(t) = \int_{-\infty}^{t-2} x(\tau) d\tau$
- S_3 is characterized by its impulse response: $h_3(t) = \delta(t-3)$.

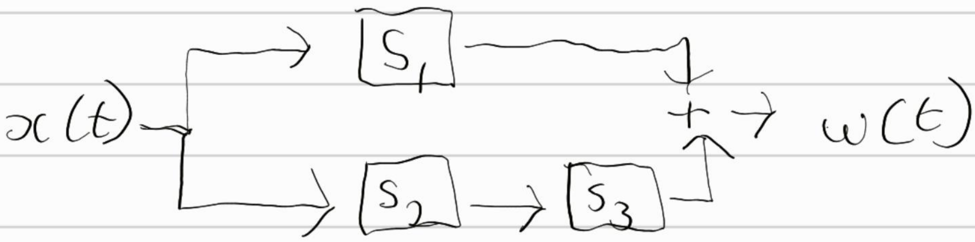
(a) (4 points) Compute the impulse response $h_1(t)$ of S_1.

a) $h_1(t) = S_1(\delta(t))$

$h_1(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} \delta(\tau) d\tau = e^{-3t} \int_{-\infty}^{t} \delta(\tau) d\tau = e^{-3t} u(t)$

b) (b) (2 points) Define $w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$. Represent this relationship using a block diagram where $x(t)$ is the input and $w(t)$ is the output.

$w(t) = S_1[x(t)] - S_3\{S_2[x(t)]\}$



c) $S_{eq} = S_1[\text{signal}] - S_3[S_2[\text{signal}]]$

$h_1(t) = e^{-3t} u(t)$, $h_2(t) = \int_{-\infty}^{t-2} \delta(\tau) d\tau = u(t-2)$

$\therefore h_{eq} = e^{-3t} u(t) - S_3[S_2[\delta(t)]] = e^{-3t} u(t) - S_3[u(t-2)]$

S_1 + imp

$\therefore h_{eq} = e^{-3t} u(t) - \delta(t-3) * u(t-2) = e^{-3t} u(t) - u(t-3-2)$

$\therefore h_{eq}(t) = e^{-3t} u(t) - u(t-5)$

d) $h_4(t) = S_{eq}[\delta(t) + 2\delta(t-3)]$

$\therefore h_4(t) = \delta(t) * h_{eq}(t) + 2\delta(t-3) h_{eq}(t)$

$= e^{-3t} u(t) - u(t-5) + 2(e^{-3(t-3)} u(t-3) - u(t-8)))$

4.4 4 d) **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Partial Correct

- 4 pts Incorrect/ Missing

5.

We start with ncov where

x : input signal vector

tx: times over which x is defined

h : impulse response vector

th: times over which h is defined

and the outputs are:

y : output signal vector

ty: times over which y is defined.

```
# imports
import matplotlib.pyplot as plt
import numpy as np
from scipy.integrate import odeint
import seaborn
import math
```

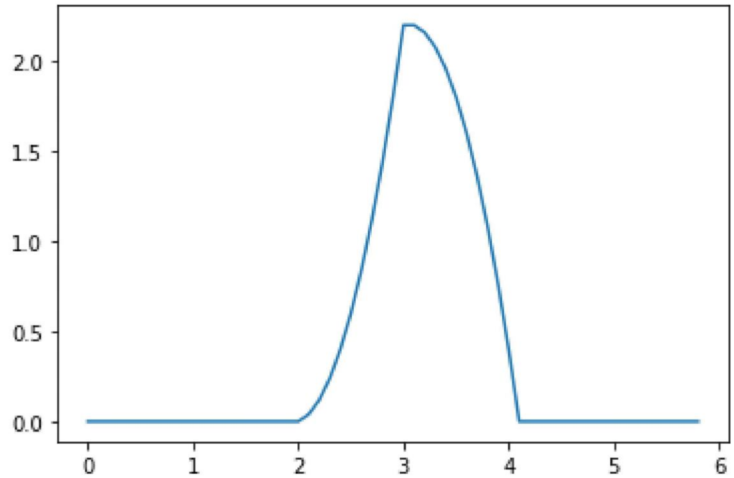
```
import numpy as np
def nconv(x, tx, h, th):
  y = np.convolve(x, h) * (th[1] - th[0])
  ty = np.linspace(tx[0] + th[0], tx[-1] + th[-1], len(y))
  return y, ty
```

```
# Define Rect and Relu
def rect(t):
  return 1 if abs(t)<= 1/2 else 0
def relu(t):
  return t if t > 0 else 0
```

a) Use nconv() to check your result for problem 3(a)(ii) and plot the output. Use the same step size for tx and th and label the plots.

```
def f(t):
  return 2*rect(t - 3/2)
def g(t):
  return 2*relu(t-1)*rect(t-3/2)
```

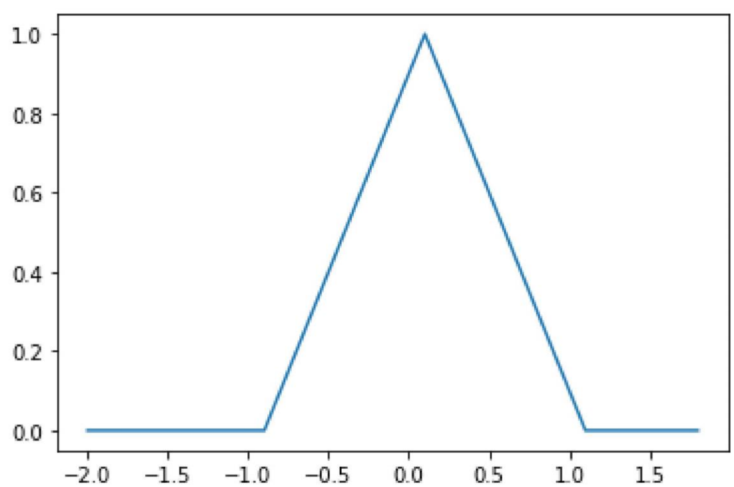
```
t = np.arange(0, 3, 0.1)
ty, y = nconv([f(te) for te in t], t, [g(te) for te in t], t)
plt.plot(y, ty)
plt.show()
```



This looks like the plot of the answer for 3a thus verifying it.

b) Use nconv() to convolve two unit rectangles: rect(t) ∗ rect(t). Plot the result and label the axes.

```
t = np.arange(-1, 1, 0.1)
ty, y = nconv([rect(te) for te in t], t, [rect(te) for te in t], t)
plt.plot(y, ty)
plt.show()
```



5.1 5 a) **5 / 10**

- **- 0 pts** Correct
- ✓ **- 5 pts Incorrect convolution (improper inputs to nconv)**
- **- 2 pts** Please show the y axis tick marks
- **- 10 pts** Missing

5.

We start with ncov where

x : input signal vector

tx: times over which x is defined

h : impulse response vector

th: times over which h is defined

and the outputs are:

y : output signal vector

ty: times over which y is defined.

```
# imports
import matplotlib.pyplot as plt
import numpy as np
from scipy.integrate import odeint
import seaborn
import math
```

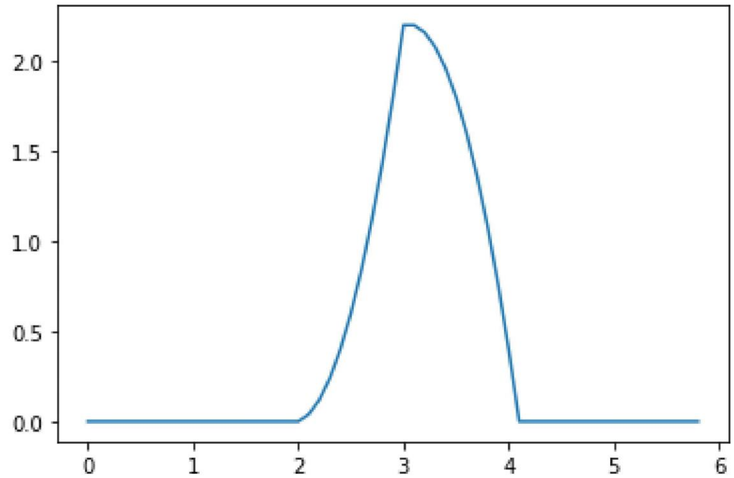
```
import numpy as np
def nconv(x, tx, h, th):
  y = np.convolve(x, h) * (th[1] - th[0])
  ty = np.linspace(tx[0] + th[0], tx[-1] + th[-1], len(y))
  return y, ty
```

```
# Define Rect and Relu
def rect(t):
  return 1 if abs(t)<= 1/2 else 0
def relu(t):
  return t if t > 0 else 0
```

a) Use nconv() to check your result for problem 3(a)(ii) and plot the output. Use the same step size for tx and th and label the plots.

```
def f(t):
  return 2*rect(t - 3/2)
def g(t):
  return 2*relu(t-1)*rect(t-3/2)
```

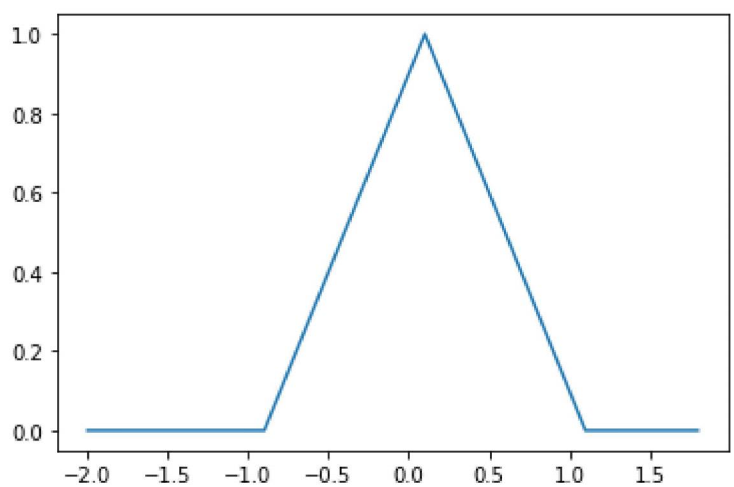
```
t = np.arange(0, 3, 0.1)
ty, y = nconv([f(te) for te in t], t, [g(te) for te in t], t)
plt.plot(y, ty)
plt.show()
```



This looks like the plot of the answer for 3a thus verifying it.

b) Use nconv() to convolve two unit rectangles: rect(t) ∗ rect(t). Plot the result and label the axes.

```
t = np.arange(-1, 1, 0.1)
ty, y = nconv([rect(te) for te in t], t, [rect(te) for te in t], t)
plt.plot(y, ty)
plt.show()
```



5.2 5 b) **7 / 7**

✓ **- 0 pts** Correct

- 4 pts Incorrect convolution (incorrect inputs to nconv)

- 2 pts Should be rect(t) * rect(t) (rectangles both centered at the origin)

- 7 pts Missing

0s completed at 10:11 PM