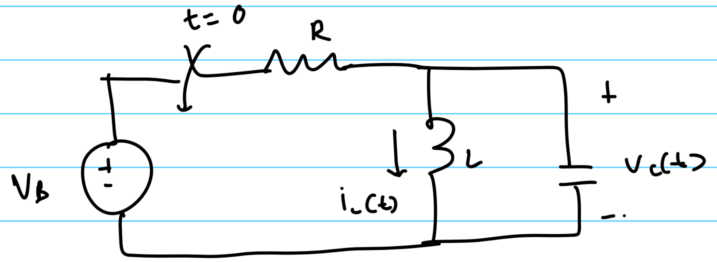
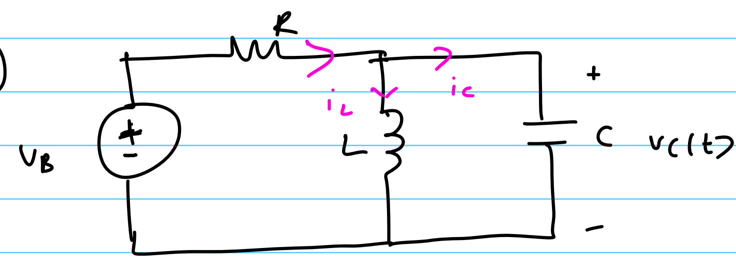
Ans. 1



C = 0.5F
L = 2H
R = 1Ω
V_B = 1V

$i_L(0^-) = 2A$
$v_c(0^-) = 2V$

a)



$$\frac{V_B - v_c}{R} = i_L + i_c$$

$$\frac{V_B - v_c}{R} = \frac{1}{L}\int v_c\,dt + C\frac{dv_c}{dt}$$

differentiate

$$-\frac{dV_c}{dt}\frac{1}{R} = \frac{V_c}{L} + C\frac{d^2V_c}{dt^2}$$

$$\frac{d^2V_c}{dt^2} + \frac{V_c}{LC} + \frac{1}{RC}\frac{dv_c}{dt} = 0$$

$$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} \Rightarrow \frac{1}{2}\sqrt{\frac{2}{0.5}} \Rightarrow 1$$

Critically damped

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1$$

b) $V_c(t) = K_1 e^{-\zeta \omega_0 t} + K_2 t e^{-\zeta \omega_n t} + K_3$

$$\boxed{V_c(t) = K_1 e^{-t} + K_2 t e^{-t} + K_3}$$

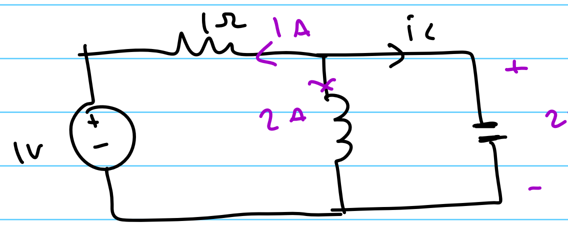
$V_c(0) = 2 \Rightarrow K_1 + K_3 = 2$

$V_c(\infty) = 0 \Rightarrow K_3 = 0$

$$\boxed{K_1 = 2}$$

$i(0^+) = \frac{V_B}{R} \Rightarrow 1A$

@ $t = 0^+$



Using KCL, $i_c = \underline{-3A}$ (@ $t = 0^+$).

$i_c(t) = C\frac{dV_c}{dt} \Rightarrow 0.5 \times \left[-K_1 e^{-t} + K_2 e^{-t} - K_2 t e^{-t}\right]$

$i_c(0^+) = -3 = 0.5 \times \left[-K_1 + K_2\right]$

$K_2 - K_1 = -6$

$$\boxed{K_2 = -4}$$

$$\boxed{V_c(t) \Rightarrow 2e^{-t} - 4te^{-t}}$$

c) Assume $i(t) = K_1 e^{-t} + K_2 t e^{-t} + K_3$

$i(0) = 2 \Rightarrow K_1 + K_3 = 2$

$i(\infty) = 1 \Rightarrow K_3 = 1$

$$\boxed{K_1 = K_3 = 1}$$

$$V_L = L\frac{di}{dt}$$

@ $t = 0^+$, $v_L = 2\,V$

$$\frac{di}{dt} = -K_1 e^{-t} + K_2 e^{-t} - K_2 t e^{-t}$$

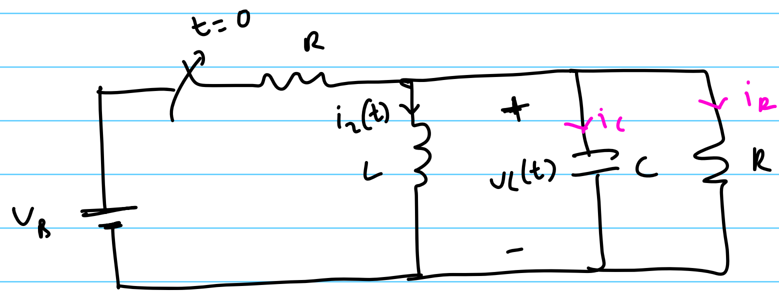
$$\left.\frac{di}{dt}\right|_{t=0^+} \Rightarrow K_2 - K_1 \qquad \boxed{i_L(t) = e^{-t} + 2te^{-t} + 1}$$

$$2(K_2 - K_1) = 2$$

$$K_2 - K_1 = 1$$

$$\boxed{K_2 = 2}$$

Ans. 2



$V_B = 10V$
$C = 100\mu F$
$R = 50\,\Omega$
$L = 1\,H$

a) $\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}} \Rightarrow \frac{1}{2\times 50}\sqrt{\frac{1}{100\mu}} \Rightarrow \frac{100}{100} = 1$ $\qquad \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow 100$

circuit is critically damped.

$\zeta\omega_0 = 100$

b) $i_L + i_C + i_R = 0$

$$\frac{1}{L}\int v_C\,dt + C\frac{dv_C}{dt} + \frac{v_C}{R} = 0$$

differentiate

$$\frac{v_c}{L} + C\frac{d^2 v_c}{dt^2} + \frac{1}{R}\frac{dv_c}{dt} = 0$$

$$\boxed{\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{v_c}{LC} = 0}$$

Since $\xi = 1$

$$V_c(t) = K_1 e^{-\xi\omega_0 t} + K_2 t e^{-\xi\omega_0 t} + K_3 \Rightarrow \underline{K_2 t e^{-100t}}$$

$v_c(0) = 0, \quad v_c(\infty) = 0$

$\downarrow \qquad \downarrow$

$K_1 = 0 \qquad K_3 = 0$

$$\boxed{I_c\big|_{t=0^+} = -I_L \Rightarrow -0.2}$$

$$I_c = C\frac{dv_c}{dt} \Rightarrow \left(K_2 e^{-100t} - K_2 t e^{-100t}\right) 100\mu$$

$$I_c\big|_{t=0^+} \Rightarrow K_2 \times 100\mu = -0.2$$

$$\boxed{K_2 = -2 \times 10^3}$$

$$\boxed{V_c(t) = -2 \times 10^3 t e^{-100t}}$$

c) $V_c(t) = L\frac{di}{dt}$

$$-2\times10^3 t e^{-100t} = L\frac{di}{dt}$$

$$i(t) = -2k \int t e^{-100t} + C$$

$$i(t) = -2k\left[t \frac{e^{-100t}}{-100} - \int 1 \left(\int e^{-100t} dt \right) dt \right] + C$$

$$\Rightarrow -2k\left[\frac{t e^{-100t}}{-100} - \int 1 \frac{e^{-100t}}{-100} dt \right] + C$$

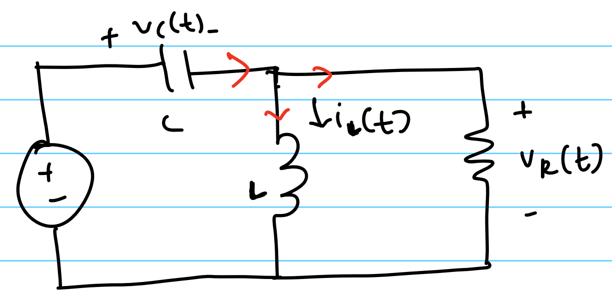
$$\Rightarrow -2k\left[\frac{t e^{-100t}}{-100} - \frac{e^{-100t}}{(100)^2} \right] + C$$

$$i(t) \Rightarrow 20te^{-100t} + 0.2e^{-100t} + C$$

Since $i(\infty) = 0 \Rightarrow C = 0$

$$\boxed{i(t) = 20te^{-100t} + 0.2e^{-100t}}$$

Ans.3



$V_B = 2V$
$R = 1000\Omega$
$C = 8nF$
$L = 0.5mH$
$v_C(0^+) = 1V$
$i_L(0^+) = 1mA$

a) $V_B = V_C + V_R$; $\underline{v_C = V_B - V_R}$

$$\underbrace{C\frac{dv_C}{dt} = i_L + \frac{V_R}{R}}_{\text{KCL}} \quad ; \quad V_R = L\frac{di_L}{dt} \Rightarrow i_L = \int \frac{V_R}{L} dt$$

$$C\frac{d}{dt}\left[V_B - V_R\right] \Rightarrow \int \frac{V_R\,dt}{L} + \frac{V_R}{R}$$

differentiate

$$-C\frac{d^2V_R}{dt^2} = \frac{V_R}{L} + \frac{dV_R}{R\,dt}$$

$$\boxed{\frac{d^2V_R}{dt^2} + \frac{dV_R}{(RC)dt} + \frac{V_R}{LC} = 0}$$

b) $\omega_0 \Rightarrow \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{\sqrt{8n \times 0.5m}} \Rightarrow \frac{1}{2} \times 10^6 \Rightarrow \underline{5 \times 10^5}$ rad/sec

$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}} \Rightarrow \frac{1}{2 \times 1K} \times \sqrt{\frac{0.5 \times 10^{-3}}{8 \times 10^{-9}}} \Rightarrow \frac{1}{2 \times 1\cancel{K}} \times \frac{1}{4} \times 1\cancel{0^3}$

$$\boxed{\zeta \Rightarrow \frac{1}{8}}$$

c) $V_R(t) = e^{-\sigma t}(A\cos\omega t + B\sin\omega t) + K_3$

$\sigma = \zeta\omega_0 \Rightarrow \frac{1}{8} \times 5 \times 10^5 \Rightarrow \underline{6.25 \times 10^4}$

$\omega = \left(\sqrt{1-\zeta^2}\right)\omega_0 \Rightarrow \sqrt{1-\left(\frac{1}{8}\right)^2} \times 5 \times 10^5$

$\Rightarrow \underline{4.96 \times 10^5}$ rad/s

at $t = \infty$, $V_R(\infty) \Rightarrow 0 \Rightarrow \underline{K_3 = 0}$

at $t=0$,

$$V_R(0) = 1V \Rightarrow \underline{A \Rightarrow 1}$$

To find B

$$V_R(t) = e^{-\sigma t}(\cos\omega t + B\sin\omega t)$$

$$C\frac{dV_C}{dt} \Rightarrow i_L + i_R$$

$$C\frac{d(V_B - V_R)}{dt} \Rightarrow 1m + 1m$$

$$-C\frac{dV_R}{dt} \Rightarrow 2m$$

$$-\frac{dV_R}{dt} \Rightarrow \frac{2m}{8n} \Rightarrow \boxed{\frac{dV_R}{dt} = -0.25\times10^6}$$

$$\frac{dV_R}{dt} \Rightarrow -\sigma e^{-\sigma t}(\cos\omega t + B\sin\omega t) + e^{-\sigma t}(-\omega\sin\omega t + B\omega\cos\omega t)$$

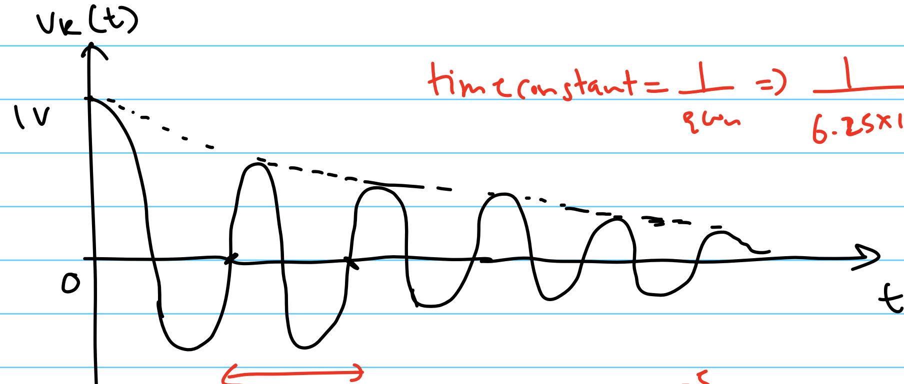
$$\left.\frac{dV_R}{dt}\right|_{t=0^+} \Rightarrow -\sigma + B\omega$$

$$-6.25\times10^4 + B\times4.96\times10^5 = -2.5\times10^5$$

$$\boxed{B = -0.378}$$

$$\boxed{V_R(t) = e^{-(6.25\times10^4)t}\left(\cos(4.96\times10^5 t) - 0.378\sin(4.96\times10^5 t)\right)}$$

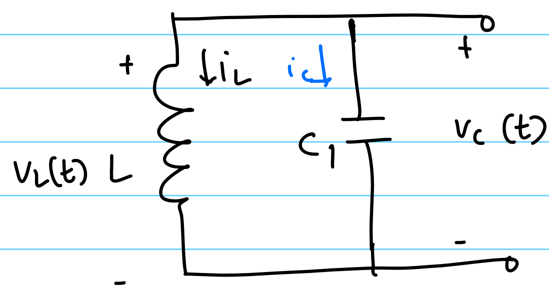
d)



time constant $= \frac{1}{\xi\omega_n} \Rightarrow \frac{1}{6.25\times10^{+4}} \Rightarrow \underline{1.6\times10^{-5}\,s}$

$T = \frac{2\pi}{4.96\times10^{5}} \Rightarrow \underline{1.266\times10^{-5}\,s}$

Ans.4



$i_L(0^-) = 2A$

$v_c(0^-) = 2V$

$L = 1H,\ C1 = 0.25F$

a) $V_C = V_L$

$V_C = \frac{L\,di_L}{dt}$; $i_c \Rightarrow -i_L \Rightarrow \frac{C_1 dv_c}{dt}$

$\int \frac{-i_L}{C_1} dt \Rightarrow \frac{L\,di_L}{dt}$

differentiate

$\frac{-i_L}{C_1} = \frac{L\,d^2 i_L}{dt^2} \Rightarrow \boxed{\frac{d^2 i_L}{dt^2} + \frac{i_L}{LC_1} = 0}$

b) $v_L = \frac{L\,di}{dt}$

$$V_L\Big|_{t=0^+} = L\frac{di}{dt}\Big|_{t=0^+}$$

$$\frac{2}{L} = \frac{di}{dt}\Big|_{t=0^+} \; ; \; \boxed{\frac{di}{dt}\Big|_{t=0^+} \Rightarrow 2\frac{A}{s}}$$

c) from part (a)

$$\frac{d^2 i_L}{dt^2} = \frac{-i_L}{LC_1}$$

$$\boxed{\frac{d^2 i_L}{dt^2}\Big|_{t=0^+} = \frac{-2}{1\times 0.25} \Rightarrow -8\frac{A}{s^2}}$$

d) $\xi = 0$; $i_L(t) = e^{-\sigma t}\left[A\cos\omega t + B\sin\omega t\right] + K$

$$\sigma = \xi\omega_n = 0 \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow 2 \text{ rad/s}$$

$$\omega = \left(\sqrt{1-\xi^2}\right)\omega_0 \Rightarrow 2$$

$$i_L(t) = A\cos\omega t + B\sin\omega t + K$$

$$i_L(0) = 2 \Rightarrow \boxed{A + K = 2}$$

$$2 = \frac{di_L}{dt}\Big|_{t=0^+} = -A\omega\sin\omega t + B\omega\cos\omega t\Big|_{t=0^+} \Rightarrow B\omega$$

$$\boxed{B = \frac{2}{\omega} \Rightarrow 1}$$

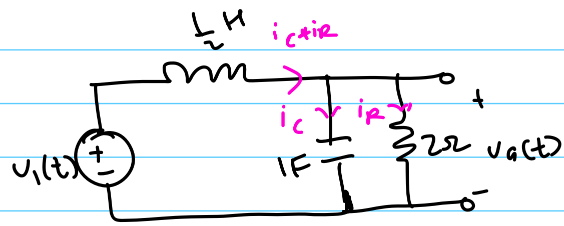
$$\frac{d^2 i_L}{dt^2}\Big|_{t=0^+} \Rightarrow -A\omega^2\cos\omega t - B\omega^2\sin\omega t\Big|_{t=0^+} \Rightarrow -A\omega^2 = -8$$

$$-A \times 4 = -8$$

$$\boxed{A = 2} \qquad \boxed{K = 0}$$

$$\boxed{i_L(t) = 2\cos 2t + \sin 2t}$$

Ans.5



$$i_c = C\frac{dv_a}{dt} = \frac{dv_a}{dt} \qquad \boxed{v_a(t) = B\sin(5t+\varphi)}$$

$$i_R \Rightarrow \frac{v_a}{2}$$

$$v_1(t) - L\frac{d}{dt}(i_c + i_R) \Rightarrow v_a$$

$$v_1(t) - L\frac{d}{dt}\left(\frac{dv_a}{dt} + \frac{v_a}{2}\right) \Rightarrow v_a$$

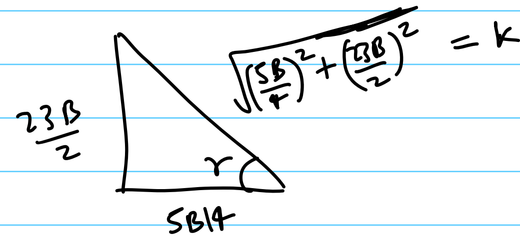
$$v_1(t) - L\left[\frac{d^2 v_a}{dt^2} + \frac{1}{2}\frac{dv_a}{dt}\right] = v_a$$

$$5\cos 5t = \frac{1}{2}\frac{d^2 v_a}{dt^2} + \frac{1}{4}\frac{dv_a}{dt} + v_a$$

$$5\cos 5t = \frac{1}{2}\left[-B(5)^2\sin(5t+\varphi)\right] + \frac{B}{4}(5)\cos(5t+\varphi) + B\sin(5t+\varphi)$$

$$5\cos 5t = \frac{-25B}{2}\sin(5t+\varphi) + \frac{5B}{4}\cos(5t+\varphi) + B\sin(5t+\varphi)$$

$$\Rightarrow \frac{-23B}{2}\sin(5t+\varphi) + \frac{5B}{4}\cos(5t+\varphi)$$



$$5\cos 5t = k\cos(5t+\varphi+r)$$

$$k = 5 \quad ; \quad \varphi + r = 0$$

$$\boxed{\varphi = -r = -\tan^{-1}(46/5)}$$

$$25 = \left(\frac{5B}{4}\right)^2 + \left(\frac{23B}{2}\right)^2$$

$$25 = \left(\frac{25}{16} + \frac{529}{4}\right)B^2$$

$$\boxed{B = 0.4322}$$

$$\boxed{V_a(t) = 0.4322\sin\left(5t - \tan^{-1}(46/5)\right)}$$

Ans.6

a) Time period => $7.794 - 1.479 \Rightarrow 6.315\,s = \frac{2\pi}{\omega}$

$$i(t) = Ae^{-\sigma t}\sin\omega t.$$

$$i(\pi/2) \Rightarrow Ae^{-\sigma\pi/2} = 0.8626 \quad -(1)$$

$$i(5\pi/2) \Rightarrow Ae^{-\sigma 5\pi/2} = 0.4587 \quad -(2)$$

① ÷ ②

$$e^{+\sigma 2\pi} \Rightarrow \frac{0.8626}{0.4587} \Rightarrow 1.8805$$

$$\sigma 2\pi \Rightarrow \ln_e(1.8805)$$

$$\boxed{\sigma \Rightarrow \frac{\ln_e(1.8805)}{2\pi} \Rightarrow 0.1005}$$

$\underline{\zeta\omega_0 = 0.1005}$

$$\frac{2\pi}{\omega} = 6.315 \; ; \quad \boxed{\omega \Rightarrow 0.995}$$

$\underline{\omega = \left(\sqrt{1-\zeta^2}\right)\omega_0}$

$$0.995 = \left(\sqrt{1-\zeta^2}\right)\omega_0$$

$$\Rightarrow 0.9899 = (1-\zeta^2)\omega_0^2$$

$$0.9899 + \zeta^2\omega_0^2 = \omega_0^2$$

$$\boxed{\omega_0 \Rightarrow 1}$$

Resonant freq.

b) $\zeta = 0.1005$

c) $A e^{-0.1 \times \pi/2} \Rightarrow 0.8626$

$A \Rightarrow 1.0093$ $\quad \boxed{i(t) = 1.009 e^{-0.1005t} \sin(0.995t)}$

d) $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C = 1F}$

e) $\zeta = \frac{1}{2}\frac{R}{\sqrt{\frac{L}{C}}} \Rightarrow 0.1005 = \frac{R}{2}$

$$\boxed{R \Rightarrow 0.201\Omega}$$