23W-EC ENGR-113-LEC-1 HW2

SANJIT SARDA

TOTAL POINTS

40 / 50

QUESTION 1

1 Q1 **7 / 10**

- **- 0 pts** Correct
- ✓ **- 1 pts** *1a) Partially incorrect formula/simplified equation(s)*
- ✓ **- 1 pts** *1b) Slightly incorrect formula/simplified equation(s)*
- ✓ **- 1 pts** *1c) Partially incorrect formula/simplified equation(s)*
- **- 1 pts** 1d) Slightly incorrect formula/simplified equation(s)

QUESTION 2

2 Q2 **10 / 10**

- ✓ **- 0 pts** *Correct*
- **- 1.5 pts** 1a) Illogical, Partial, or Incorrect proof/example
- **- 1.5 pts** 1b) Illogical, Partial, or incorrect proof
- **- 2.5 pts** 1b) Incorrect answer
- **- 1.5 pts** 1c) Illogical, Partial, or Incorrect Proof
- **- 1.5 pts** 1d) Illogical, Partial, or Incorrect proof
- **- 2.5 pts** 1d) Incorrect

QUESTION 3

3 Q3 **10 / 10**

- ✓ **- 0 pts** *Correct*
- **- 2.5 pts** Said a is correct
- **- 2.5 pts** Said b is correct
- **- 2.5 pts** Said c is incorrect
- **- 2.5 pts** Said d is correct
- **- 2.5 pts** No answer for time invariance
- **- 2 pts** Incorrect explanation of which system is BIBO stable and which is not

QUESTION 4

4 Q4 **6 / 10**

- ✓ **+ 1.5 pts** *4a) Partially correct*
- **+ 2.5 pts** 4a) All correct
- ✓ **+ 1.5 pts** *4b) Partially correct*
- **+ 2.5 pts** 4b) All correct
- ✓ **+ 1.5 pts** *4c) Partially correct*
- **+ 2.5 pts** 4c) All correct
- ✓ **+ 1.5 pts** *4d) Partially correct*
- **+ 2.5 pts** 4d) All correct

QUESTION 5

5 Q5 **7 / 10**

- **- 0 pts** Correct
- **- 2 pts** Incorrect a
- **- 2 pts** Incorrect b
- **- 1 pts** Not specific as to a needing to be finite
- **- 2 pts** Incorrect c
- **- 2 pts** Incorrect d
- **- 2 pts** Incorrect e
- **- 1 pts** Stated not summable but failed to account for K = 0 in part e

- 1 pts Missing absolute value signs in final

answer(s)

✓ **- 1 pts** *Either a, c, or d include 1 and -1 on bounds*

- 2 *Point adjustment*

No answer for e

ECE 113 Hw 2

1. Determine

the even and odd parts of the following signals:

a) $x_1[n] = u[n-3]$

b) $x_2[n] = \alpha^n u[n-1]$

c) $x_3[n] = n\alpha^n u[n+1]$

d) $x_4[n] = \alpha^{|n|}$

Answer

General Formula:

$$x_e[n] = \frac{x[n]+x[-n]}{2}$$

$$x_o[n] = \frac{x[n]-x[-n]}{2}$$

a) $x_1[n] = u[n-3]$

$$x_e[n] = \frac{u[n-3]+u[(-n)-3]}{2}$$

$$x_e[n] = \frac{u[n-3]+u[-n-3]}{2}$$

$$x_e[n] = \begin{cases} 0 & -3 < n < 3 \\ 0.5 & else \end{cases}$$

$$x_o[n] = \frac{u[n-3]-u[(-n)-3]}{2}$$

$$x_o[n] = \frac{u[n-3]-u[-n-3]}{2}$$

$$x_o[n] = \begin{cases} -0.5 & n < -3 \\ 0 & -3 \leq n \leq 3 \\ 0.5 & n > 3 \end{cases}$$

b) $x_2[n] = \alpha^n u[n-1]$

$$x_e[n] = \frac{\alpha^n u[n-1] + \alpha^{-n} u[(-n)-1]}{2}$$

$$x_e[n] = \frac{\alpha^n u[n-1] + \alpha^{-n} u[-n-1]}{2}$$

$$x_e[n] = \begin{cases} \alpha^{-n} & n < -1 \\ 0 & -1 \leq n \leq 1 \\ \alpha^n & n > 1 \end{cases}$$

$$x_o[n] = \frac{\alpha^n u[n-1] - \alpha^{-n} u[(-n)-1]}{2}$$

$$x_o[n] = \frac{\alpha^n u[n-1] - \alpha^{-n} u[-n-1]}{2}$$

$$x_o[n] = \begin{cases} -\alpha^{-n} & n < -1 \\ 0 & -1 \leq n \leq 1 \\ \alpha^n & n > 1 \end{cases}$$

c) $x_3[n] = n\alpha^n u[n+1]$

$$x_e[n] = \frac{n\alpha^n u[n+1] + (-n)\alpha^{-n} u[(-n)+1]}{2}$$

$$x_e[n] = \frac{n\alpha^n u[n+1] - n\alpha^{-n} u[-n+1]}{2}$$

$$x_e[n] = \begin{cases} -n\alpha^{-n} & n < -1 \\ n\alpha^n - n\alpha^{-n} & -1 \leq n \leq 1 \\ n\alpha^n & n > 1 \end{cases}$$

$$x_o[n] = \frac{n\alpha^n u[n+1] - (-n)\alpha^{-n} u[(-n)+1]}{2}$$

$$x_o[n] = \frac{n\alpha^n u[n+1] + n\alpha^{-n} u[-n+1]}{2}$$

$$x_o[n] = \begin{cases} n\alpha^{-n} & n < -1 \\ n\alpha^n + n\alpha^{-n} & -1 \leq n \leq 1 \\ n\alpha^n & n > 1 \end{cases}$$

d) $x_4[n] = \alpha^{|n|}$

$x_e[n] = \frac{\alpha^{|n|}+\alpha^{|(-n)|}}{2}$

$x_e[n] = \frac{\alpha^{|n|}+\alpha^{|n|}}{2}$

$x_e[n] = \alpha^{|n|}$

$x_o[n] = \frac{\alpha^{|n|}-\alpha^{|(-n)|}}{2}$

$x_o[n] = \frac{\alpha^{|n|}-\alpha^{|n|}}{2}$

$x_o[n] = 0$

2. True or False

a) A power sequence is necessarily an energy sequence.

b) Every energy sequence has zero average power.

c) If $x[n]$ is an energy sequence, then $x[n] \to 0$ as $n \to \infty$

d) There does not exist a sequence with infinite average power.

Answer

a) False. A power sequence cannot be an energy sequence, since the energy of a power sequence is infinite. Ex a simple sinusoid:

$x[n] = sin(\frac{\pi}{2}n)$ is a power sequence, with Power = $\frac{1}{N-1}\sum_{n=0}^{n=N}(\sin(\frac{\pi}{2}n))^2 = \frac{1}{2}$, Energy= $\sum_{n=0}^{n=N}(\sin(\frac{\pi}{2}n))^2 = \infty$.

b) True. Since the energy of the sequence is finite, the power has to be $\frac{1}{T_0}E$ therefore making it zero.

c) True. If a sequence does not converge to zero, then the sum of the squares of the sequence will be infinite, making it a non-energy sequence.

1 Q1 **7 / 10**

- **- 0 pts** Correct
- ✓ **- 1 pts** *1a) Partially incorrect formula/simplified equation(s)*
- ✓ **- 1 pts** *1b) Slightly incorrect formula/simplified equation(s)*
- ✓ **- 1 pts** *1c) Partially incorrect formula/simplified equation(s)*
- **- 1 pts** 1d) Slightly incorrect formula/simplified equation(s)

$x_e[n] = \frac{\alpha^{|n|} + \alpha^{|(-n)|}}{2}$

$x_e[n] = \frac{\alpha^{|n|} + \alpha^{|n|}}{2}$

$x_e[n] = \alpha^{|n|}$

$x_o[n] = \frac{\alpha^{|n|} - \alpha^{|(-n)|}}{2}$

$x_o[n] = \frac{\alpha^{|n|} - \alpha^{|n|}}{2}$

$x_o[n] = 0$

2. True or False

a) A power sequence is necessarily an energy sequence.

b) Every energy sequence has zero average power.

c) If $x[n]$ is an energy sequence, then $x[n] \to 0$ as $n \to \infty$

d) There does not exist a sequence with infinite average power.

Answer

a) False. A power sequence cannot be an energy sequence, since the energy of a power sequence is infinite. Ex a simple sinusoid:

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b) True. Since the energy of the sequence is finite, the power has to be $\frac{1}{T_0}E$ therefore making it zero.

c) True. If a sequence does not converge to zero, then the sum of the squares of the sequence will be infinite, making it a non-energy sequence.

d) False. A sequence with infinite average power would be possible. Ex: ReLU.

$$\lim_{m\to\infty} \frac{1}{2m+1} \sum_{n=-m}^{n=m} (n)^2 = \infty$$

3. System Stability

I is defined by $y[n] = \ln(|x[n-1|)$ and II is defined by $y[n] = e^{x[2n]}$. Which of the following statements is true?

a) Both systems are BIBO stable.

b) Both systems are BIBO unstable.

c) System I is BIBO unstable and system II is BIBO stable.

d) Both systems are time-invariant.

Answer

For I, If $|x[n]| < B$, then $|y[n]| = \ln(B)$ is not bounded, for all B, since at $B \to 0$, $y[n] \to -\infty$. Therefore, I is not BIBO stable.

For II, if $|x[n]| < B$, then $|y[n]| < e^B$ is bounded, for all B. As long as not $B \to \infty$, $y[n]$ is bounded. Therefore, II is BIBO stable.

a) False. I is not BIBO stable.

b) False. II is BIBO stable.

c) True. I is not BIBO stable and II is BIBO stable.

$I.\, y[n] = \ln(|x[n-1|)$

Let input be $z[n]$, therefore output is $y[n] = \ln(|z[n-1|)$.

For Delayed input $z[n-1]$, output is $\ln(|z[n-2|)$.

For Delayed output $y[n-1] = \ln(|z[n-2|)$

Therefore delayed input = delayed output, Therefore it is time-invariant.

2 Q2 **10 / 10**

✓ **- 0 pts** *Correct*

- 1.5 pts 1a) Illogical, Partial, or Incorrect proof/example

- 1.5 pts 1b) Illogical, Partial, or incorrect proof

- 2.5 pts 1b) Incorrect answer

- 1.5 pts 1c) Illogical, Partial, or Incorrect Proof

- 1.5 pts 1d) Illogical, Partial, or Incorrect proof

- 2.5 pts 1d) Incorrect

d) False. A sequence with infinite average power would be possible. Ex: ReLU.

$$\lim_{m \to \infty} \frac{1}{2m+1} \sum_{n=-m}^{n=m} (n)^2 = \infty$$

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Answer

For I, If $|x[n]| < B$, then $|y[n]| = \ln(B)$ is not bounded, for all B, since at $B \to 0$, $y[n] \to -\infty$. Therefore, I is not BIBO stable.

For II, if $|x[n]| < B$, then $|y[n]| < e^B$ is bounded, for all B. As long as not $B \to \infty$, $y[n]$ is bounded. Therefore, II is BIBO stable.

a) False. I is not BIBO stable.

b) False. II is BIBO stable.

c) True. I is not BIBO stable and II is BIBO stable.

$I.\, y[n] = \ln(|x[n-1|)$

Let input be $z[n]$, therefore output is $y[n] = \ln(|z[n-1|)$.

For Delayed input $z[n-1]$, output is $\ln(|z[n-2|)$.

For Delayed output $y[n-1] = \ln(|z[n-2|)$

Therefore delayed input = delayed output, Therefore it is time-invariant.

$II. y[n] = e^{x[2n]}$.

Let input be $z[n]$, therefore output is $y[n] = e^{z[2n]}$.

For Delayed input $z[n-1]$, output is $e^{z[2(n-1)]}$.

For Delayed output $y[n-1] = e^{z[2n-1]}$.

Therefore delayed input $\neq$ delayed output, Therefore it is not time-invariant.

d) False. I is time-invariant and II is not time-invariant.

4. Determine

whether each of the following systems is linear or not, time-invariant or not, and BIBO stable or not, relaxed or not.

a) $y[n] = ln(|x[n]| + 1)$

b) $y[n] = y[n-1] + x[n],\ y[-1] = 0$

c) $y[n] = y[n-1] + x[n],\ y[-1] = 1$

d) $y[n] = 2 + x[n]$

Answer

a)

$y[n] = ln(|x[n]| + 1)$

Linearity:

Applying input $a \cdot x_1[n] + b \cdot x_2[n]$, output is $a \cdot \ln(|x_1[n]| + 1) + b \cdot \ln(|x_2[n]| + 1)$, $= a \cdot y[x_1[n]] + b \cdot y[x_2[n]]$, therefore it is linear.

Time-invariance:

Let input be $z[n]$, therefore output is $y[n] = \ln(|z[n]| + 1)$.

3 Q3 **10 / 10**

✓ **- 0 pts** *Correct*

- 2.5 pts Said a is correct

- 2.5 pts Said b is correct

- 2.5 pts Said c is incorrect

- 2.5 pts Said d is correct

- 2.5 pts No answer for time invariance

- 2 pts Incorrect explanation of which system is BIBO stable and which is not

$II.\ y[n] = e^{x[2n]}$.

Let input be $z[n]$, therefore output is $y[n] = e^{z[2n]}$.

For Delayed input $z[n-1]$, output is $e^{z[2(n-1)]}$.

For Delayed output $y[n-1] = e^{z[2n-1]}$.

Therefore delayed input $\neq$ delayed output, Therefore it is not time-invariant.

d) False. I is time-invariant and II is not time-invariant.

4. Determine

whether each of the following systems is linear or not, time-invariant or not, and BIBO stable or not, relaxed or not.

a) $y[n] = ln(|x[n]| + 1)$

b) $y[n] = y[n-1] + x[n],\ y[-1] = 0$

c) $y[n] = y[n-1] + x[n],\ y[-1] = 1$

d) $y[n] = 2 + x[n]$

Answer

a)

$y[n] = ln(|x[n]| + 1)$

Linearity:

Applying input $a \cdot x_1[n] + b \cdot x_2[n]$, output is $a \cdot \ln(|x_1[n]| + 1) + b \cdot \ln(|x_2[n]| + 1)$ $, = a \cdot y[x_1[n]] + b \cdot y[x_2[n]]$, therefore it is linear.

Time-invariance:

Let input be $z[n]$, therefore output is $y[n] = \ln(|z[n]| + 1)$.

For Delayed input $z[n-1]$, output is $\ln(|z[n-1]|+1)$.

For Delayed output $y[n-1] = \ln(|z[n-1]|+1)$.

Therefore delayed input = delayed output, Therefore it is time-invariant.

Causality:

$y[n] = \ln(|x[n]|+1)$, since it does not depend on future values of $x[n]$, it is causal.

BIBO Stability:

If $|x[n]| < B$, then $|y[n]| < \ln(B+1)$ is bounded, for all B. As long as not $B \to \infty$, $y[n]$ is bounded. Since the +1 term is in the input, it does not $\to \infty$ as $|x[n]| \to 0$. Therefore, it is BIBO stable.

Relaxed:

At t_0, $y[n] = ln(|0|+1) = 0$, therefore it is relaxed system.

a) Linear, Time-invariant, BIBO stable, relaxed.

b)

$$y[n] = y[n-1] + x[n],\ y[-1] = 0$$

Linearity:

$y[n] = y[n-1] + x[n]$, we can rewrite it as $y[n] = \sum_{k=0}^{n} x[k]$, therefore it is linear.

Applying input $a \cdot x_1[n] + b \cdot x_2[n]$, output is $a \cdot y[n-1] + a \cdot x[n] + b \cdot y[n-1] + b \cdot x[n], = a \cdot y[x_1[n]] + b \cdot y[x_2[n]]$, therefore it is linear.

Time-invariance:

Let input be $z[n]$, therefore output is $y[n] = y[n-1] + z[n]$.

For Delayed input $z[n-1]$, output is $y[n-2] + z[n-1]$. This happens because when the the input is delayed, the output which starts at $y[0]$ remains the same until the input is delayed.

For Delayed output $y[n-1] = y[n-2] + z[n-1]$.

Therefore delayed input = delayed output, Therefore it is time-invariant.

Causality:

$y[n] = y[n-1] + x[n]$. Although it depends on past values of y[n], it does not depend on future values of x[n] or y[n], therefore it is causal.

BIBO Stability:

$y[n] = y[n-1] + x[n]$, we can rewrite it as $y[n] = \sum_{k=0}^{n} x[k]$.

$y[n]$ is an accumulator. Even if $|x[n]| < B$, $|y[n]|$ is not bounded. Ex: $\ln(n+1)$ as an input, $y[n]$ will accumulate to ∞. Therefore, it is not BIBO stable.

Relaxed:

At t_0, $y[n]$ is not necessarily 0, since it is an accumulator. Even if $x[t_0]$ is 0, $y[n]$ when $n > t_0$ retains the value of $y[n-1]$, which is not necessarily 0. Therefore, it is not relaxed system.

b) Linear, Time-invariant, Causal, NOT BIBO stable, not relaxed.

c)

$y[n] = y[n-1] + x[n], \ y[-1] = 1$

Linearity:

Applying input $a \cdot x_1[n] + b \cdot x_2[n]$, output is $a \cdot y[n-1] + a \cdot x[n] + b \cdot y[n-1] + b \cdot x[n], = a \cdot y[x_1[n]] + b \cdot y[x_1[n]]$, therefore it is linear.

Time-invariance:

Let input be $z[n]$, therefore output is $y[n] = y[n-1] + z[n]$.

For Delayed input $z[n-1]$, output is $y[n-2] + z[n-1]$. This happens because when the the input is delayed, the output which starts at $y[0]$ remains the same until the input is delayed.

For Delayed output $y[n-1] = y[n-2] + z[n-1]$.

Therefore delayed input = delayed output, Therefore it is time-invariant.

Causality:

$y[n] = y[n-1] + x[n]$. Although it depends on past values of y[n], it does not depend on future values of x[n] or y[n], therefore it is causal.

BIBO Stability:

$y[n] = y[n-1] + x[n]$, we can rewrite it as $y[n] = 1 + \sum_{k=0}^{n} x[k]$.

$y[n]$ is an accumulator. Even if $|x[n]| < B$, $|y[n]|$ is not bounded. Ex: $\ln(n+1)$ as an input, $y[n]$ will accumulate to ∞. Therefore, it is not BIBO stable.

Relaxed:

At t_0, $y[n]$ is not necessarily 0, since it is an accumulator. Even if $x[t_0]$ is 0, $y[n]$ when $n > t_0$ retains the value of $y[n-1]$, which is not necessarily 0. Therefore, it is not relaxed system.

c) Linear, Time-invariant, Causal, not BIBO stable, not relaxed.

d)

$y[n] = 2 + x[n]$

Linearity:

Applying input $a \cdot x_1[n] + b \cdot x_2[n]$, output is $2a + a \cdot x[n] + 2b + b \cdot x[n], = a \cdot y[x_1[n]] + b \cdot y[x_2[n]]$, therefore it is linear.

Time-invariance:

Let input be $z[n]$, therefore output is $y[n] = 2 + z[n]$.

For Delayed input $z[n-1]$, output is $2 + z[n-1]$.

For Delayed output $y[n-1] = 2 + z[n-1]$.

Therefore delayed input = delayed output, Therefore it is time-invariant.

Causality:

$y[n] = 2 + x[n]$. Since it does not depend on past values of x[n], it is causal.

BIBO Stability:

If $|x[n]| < B$, then $|y[n]|$ is bounded, for all B. As long as not $B \to \infty$, $y[n]$ is bounded. Therefore, it is BIBO stable.

Relaxed:

At t_0, $y[n] = 2 + 0 = 2$, therefore it is not relaxed system.

d) Linear, Time-invariant, Causal, BIBO stable, not relaxed.

5. Determine

The conditions on the parameters of the following systems for stability:

a) $h[n] = \alpha^n u[-n]$

b) $h[n] = \alpha^n (u[n] - u[n-100])$

c) $h[n] = r^n \sin[n\omega_0]u[n]$

d) $h[n] = $\alpha^{|n|}$

e) $h[n] = K(-1)^n u[n]$

Answers

a)

$$h[n] = \begin{cases} \alpha^n, & n \leq 0 \\ 0, & n > 0 \end{cases}$$

4 Q4 **6 / 10**

- ✓ **+ 1.5 pts** *4a) Partially correct*
- **+ 2.5 pts** 4a) All correct
- ✓ **+ 1.5 pts** *4b) Partially correct*
- **+ 2.5 pts** 4b) All correct
- ✓ **+ 1.5 pts** *4c) Partially correct*
- **+ 2.5 pts** 4c) All correct
- ✓ **+ 1.5 pts** *4d) Partially correct*
- **+ 2.5 pts** 4d) All correct

For Delayed output $y[n-1] = 2 + z[n-1]$.

Therefore delayed input = delayed output, Therefore it is time-invariant.

Causality:

$y[n] = 2 + x[n]$. Since it does not depend on past values of x[n], it is causal.

BIBO Stability:

If $|x[n]| < B$, then $|y[n]|$ is bounded, for all B. As long as not $B \to \infty$, $y[n]$ is bounded. Therefore, it is BIBO stable.

Relaxed:

At t_0, $y[n] = 2 + 0 = 2$, therefore it is not relaxed system.

d) Linear, Time-invariant, Causal, BIBO stable, not relaxed.

5. Determine

The conditions on the parameters of the following systems for stability:

a) $h[n] = \alpha^n u[-n]$

b) $h[n] = \alpha^n (u[n] - u[n-100])$

c) $h[n] = r^n \sin[n\omega_0] u[n]$

d) $h[n] = $\alpha^{|n|}$

e) $h[n] = K(-1)^n u[n]$

Answers

a)

$$h[n] = \begin{cases} \alpha^n, & n \leq 0 \\ 0, & n > 0 \end{cases}$$

For stability, $|h[n]|$ must be bounded, for all n.

We know that it is bounded for $n \geq 0$, but for it to be bounded for all n, it must be bounded for $n \leq 0$: $\alpha^n < B$, for $n \leq 0$.

Thus condition for stability is that $|\alpha| \geq 1$.

b)

$$h[n] = \begin{cases} \alpha^n, & 0 \leq n \leq 100 \\ 0, & else \end{cases}$$

For stability, $|h[n]|$ must be bounded, for all n.

Since it is bounded for $0 \geq n$ and $n \geq 100$, we need to make sure it is bounded for $0 \leq n \leq 100$: $\alpha^n < B$, for $0 \leq n \leq 100$. If $\alpha \geq 1$, then $|h[n]| = \alpha^{100}$ else $|h[n]| = \alpha^0$, since both are bounded for any α, it is stable.

Thus condition for stability is that α is bounded.

c)

$$h[n] = \begin{cases} r^n \sin[n\omega_0], & n \geq 0 \\ 0, & else \end{cases}$$

For stability, $|h[n]|$ must be bounded, for all n.

Let $h[n] = k[n]l[n]$, such that $k[n] = r^n$ and $l[n] = \sin[n\omega_0]$. Therefore, $|h[n]| = |k[n]| \cdot |l[n]|$.

$|l[n]|$ = 1, since the absolute value of the max/min of a sinusoid is always the amplitude, which is 1.

Therefore $|h[n]| = |k[n]|$, which is bounded for r<1, since the exponent is only active for n>0, thus it needs to be bounded for n>0 which is true for |r|<1.

Thus condition for stability is $|r| < 1$.

d)

$h[n] = \begin{cases} \alpha^n, & n \geq 0 \\ \alpha^{-n}, & n < 0 \end{cases}$ Thus $|h[n]| = |\alpha^n|$ for n>0, since α never takes a negative exponent. Therefore $\alpha < 1$ makes $|h[n]|$ bounded for all n.

Thus condition for stability is $|\alpha| < 1$.

5 Q5 **7 / 10**

- **- 0 pts** Correct
- **- 2 pts** Incorrect a
- **- 2 pts** Incorrect b
- **- 1 pts** Not specific as to a needing to be finite
- **- 2 pts** Incorrect c
- **- 2 pts** Incorrect d
- **- 2 pts** Incorrect e
- **- 1 pts** Stated not summable but failed to account for K = 0 in part e
- **- 1 pts** Missing absolute value signs in final answer(s)

✓ **- 1 pts** *Either a, c, or d include 1 and -1 on bounds*

- 2 *Point adjustment*

No answer for e