ECE113, Winter 2023 **Homework #3**

Digital Signal Processing Prof. A. Kadambi

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Due Friday, 10 Feb 2023, by 11:59pm to Gradescope.
50 points total.

1. (10 points) Let $y[n]$ denote the linear convolution of the two sequences:

   $x[n] = \{2, -3, 4, 1\}, -1 \leq n \leq 2,$

   $h[n] = \{-3, 5, -6, 4\}, -2 \leq n \leq 1.$

   Determine the value of $y[-1]$ without computing the convolution sum.

2. (10 points) Evaluate the linear convolution of each of the following sequences with itself:

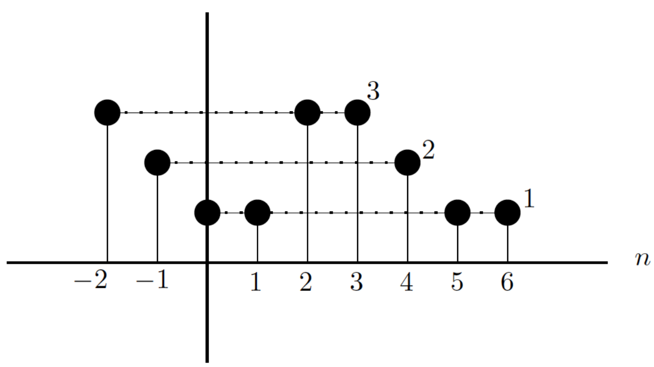
   (a) $x_1[n] = \{1, -1, 1\}, -1 \leq n \leq 1,$

   (b) $x_2[n] = \{1, -1, 0, 1, -1\}, 0 \leq n \leq 4,$

   (c) $x_3[n] = \{-1, 2, 0, -2, 1\}, -3 \leq n \leq 1.$

3. (10 points) Determine the output of a LTI system with impulse response $h[n] = (\frac{1}{2})^n u[n]$ when excited by input $x[n] = 2^n u[-n]$.

4. (10 points) Consider the sequence $x[n]$ that is shown below. It is zero except at the specified time instants. The amplitudes of the non-zero samples are either 1, 2, or 3.



   (i) Define the sequence $y[n] = u[n+1] - u[n-2]$. Compute the convolution $x[n] * y[n]$.

   (ii) Define

$$h_1[n] = (\frac{1}{2})^n h[n] u[n]$$

where

$$h[n] = (\frac{1}{2})x[n+2] - \frac{3}{2}\delta[n] + u[n-3]$$

Take $h_1[n]$ to be the impulse response of an LTI system. What would the response of the system be to the input sequence $(\frac{1}{3})^n u[n]$?

5. (10 points) Let $y[n] = x[n] * h[n]$ with

$$x[n] = f[n]\left(u\left[n-n_1\right] - u\left[n-n_2\right]\right)$$

and

$$h[n] = g[n]\left(u\left[n-n_3\right] - u\left[n-n_4\right]\right)$$

where $f[n]$ and $g[n]$ are arbitrary functions, and $n_1 < n_2$ and $n_3 < n_4$. Therefore, $x[n]$ and $h[n]$ are pulse-like signals of finite duration $n_x = n_2 - n_1$ and $n_h = n_4 - n_3$, respectively.

(a) For what value of the index n does the first non-zero output element $y[n]$ occur?

(b) For what value of the index n does the last non-zero output element $y[n]$ occur?

(c) What is the duration n_y of the output sequence $y[n]$ in terms of n_x and n_h?