ECE 102 Homework 6

SANJIT SARDA

TOTAL POINTS

96 / 100

QUESTION 1

Frequency Response 32 pts

1.1 (a) **18 / 18**

✓ - **0 pts** Correct

- **2 pts** part 3 partially correct
- **2 pts** part2 partially correct
- **2 pts** part 1 partially correct

1.2 (b) **7 / 8**

- **0 pts** Correct

✓ - **1 pts** minor mistakes

- **2 pts** major mistakes
- **4 pts** incomlpete

1.3 (c) **6 / 6**

✓ - **0 pts** Correct

- **1 pts** part 2 partially correct
- **2 pts** part 2 missing

QUESTION 2

Filters 18 pts

2.1 (a) **4 / 6**

- **0 pts** Correct
- **1 pts** wrong alpha

✓ - **2 pts** wrong phase

2.2 (b) **2 / 3**

- **0 pts** Correct

✓ - **1 pts** infinite not mentioned

- **1 pts** non causal not mentioned
- **3 pts** missing

2.3 (c) **5 / 5**

✓ - **0 pts** Correct

- **1 pts** not rejected -2pi
- **2 pts** wrong final answer
- **2 pts** incomplete

2.4 (d) **4 / 4**

✓ - **0 pts** Correct

- **1 pts** frequency response partially correct
- **2 pts** frquency response missing
- **1 pts** high pass not mentioned

QUESTION 3

Moving Average Filter 25 pts

3.1 (a) **6 / 6**

✓ - **0 pts** Correct

- **3 pts** Partial Correct
- **1 pts** Incorrect power of exp(.)
- **1 pts** Incorrect sinc(.) term
- **6 pts** Missing/ Fully incorrect

3.2 (b) **6 / 6**

✓ - **0 pts** Correct

- **2 pts** Incorrect Graph
- **1 pts** Incorrect response at w-> infinity
- **4 pts** Missing Graph
- **1 pts** Missing w->infinity response

3.3 (c) **6 / 6**

✓ - **0 pts** Correct

- **2 pts** Partial Correct
- **6 pts** Missing/ incorrect

3.4 (d) **7 / 7**

✓ - **0 pts** Correct

- **3 pts** Partial Correct
- **7 pts** Incorrect/ Missing

QUESTION 4

Modulation and Demodulation 25 pts

4.1 (a) 10 / 10

✓ **- 0 pts** Correct

- 1 pts part 3 partially correct

- 1 pts systen recovers signal

- 3 pts part 1 missing

- 3 pts part 2 missing

- 3 pts part 3 missing

4.2 (b) 15 / 15

✓ **- 0 pts** Correct

- 2 pts partially correct

- 15 pts missing

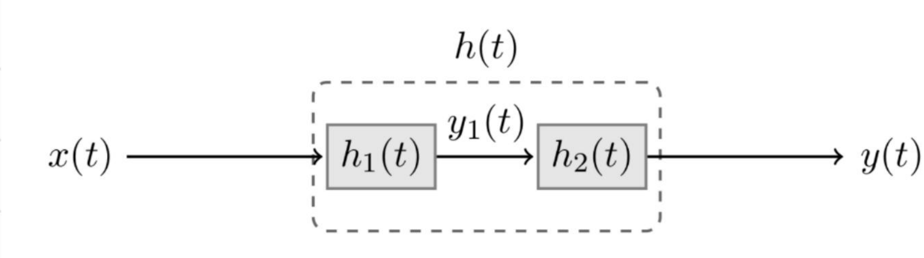
- 10 pts no plots

- 6 pts 2 plots missing

102

HW#6

1) a)



$y(t) = (4e^{-t} - 4e^{-4t})u(t)$

$y_1(t) = 2e^{-t}u(t)$

$y'' + 6y' + 8y = 3x$

Applying a fourier transform

i) $\therefore (j\omega)^2 Y(j\omega) + 6(j\omega)Y(j\omega) + 8Y(j\omega) = 3X(j\omega)$

$\therefore Y(j\omega)((j\omega)^2 + 6(j\omega)Y(j\omega)) + 8Y(j\omega) = 3X(j\omega)$

$\therefore Y(j\omega)(j\omega+2)(j\omega+4) = 3X(j\omega)$

$H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{3}{(j\omega+2)(j\omega+4)}$

ii) $H(j\omega) = H_1(j\omega)H_2(j\omega)$

$y_1(t) = x(t) * h_1(t)$

$\therefore Y_1(j\omega) = X(j\omega)H_1(j\omega)$

$\therefore H_1(j\omega) = \dfrac{Y_1(j\omega)}{X(j\omega)}$

We have:

$x(t) * h_1(t) = y_1(t) \therefore X(j\omega)H_1(j\omega) = Y_1(j\omega)$

$y_1(t) * h_2(t) = y(t) \therefore Y_1(j\omega)H_2(j\omega) = Y(j\omega)$

$\therefore Y(j\omega) = X(j\omega)H_1(j\omega)H_2(j\omega) = X(j\omega)H(j\omega)$

We know $H(j\omega) = \dfrac{3}{(j\omega+2)(j\omega+4)}$

$y_1(t) = 2e^{-t}u(t) \therefore Y_1(j\omega) = \dfrac{2}{(j\omega+1)}$

$y(t) = 4e^{-t}u(t) - 4e^{-4t}u(t) \therefore Y(j\omega) = \dfrac{4(j\omega+4) - 4(j\omega+1)}{(j\omega+4)(j\omega+1)}$

$= \dfrac{12}{(j\omega+4)(j\omega+1)}$

$\therefore$ From $Y(j\omega) = X(j\omega)H(j\omega)$, $X(j\omega) = \dfrac{Y(j\omega)}{H(j\omega)}$

From $X(j\omega)H_1(j\omega) = Y_1(j\omega) \therefore H_1(j\omega) = \dfrac{Y_1(j\omega)}{X(j\omega)} = \dfrac{Y_1(j\omega)H(j\omega)}{Y(j\omega)}$

$$\therefore H_1(j\omega) = \frac{2}{\cancel{(j\omega+1)}} \cdot \frac{3}{(j\omega+2)\cancel{(j\omega+4)}} \cdot \frac{\cancel{(j\omega+4)}\cancel{(j\omega+1)}}{12} = \frac{1}{2(j\omega+2)}$$

$$H_1(j\omega)H_2(j\omega) = H(j\omega) \therefore H_2(j\omega) = \frac{H(j\omega)}{H_1(j\omega)} = \frac{3\cdot 2\cancel{(j\omega+2)}}{\cancel{(j\omega+2)}(j\omega+4)} = \frac{6}{j\omega+4}$$

iii) Find the impulse responses $h(t), h_1(t), h_2(t)$

We have,

$H(j\omega) = \dfrac{3}{(j\omega+2)(j\omega+4)}$ $\therefore h(t) = \mathcal{F}^{-1}(H(j\omega)) = \dfrac{3}{2}\left(e^{-2t}u(t) - e^{-4t}u(t)\right)$

$H_1(j\omega) = \dfrac{1}{2(j\omega+2)}$ $\therefore h_1(t) = \mathcal{F}^{-1}(H_1(j\omega)) = \dfrac{1}{2}e^{-2t}u(t)$

$H_2(j\omega) = \dfrac{6}{j\omega+4}$ $\therefore h_2(t) = \mathcal{F}^{-1}(H_2(j\omega)) = 6e^{-4t}u(t)$

b) Consider

$x_1(t) = \sin(\omega_1 t + \pi/4)$ $\quad \omega_1 = \pi$

$x_2(t) = 2\cos(\omega_2 t - \pi/3)$ $\quad \omega_2 = 2\pi$

$y(t) = (x_1(t) + x_2(t)) * h(t) \Rightarrow$

$\therefore Y(j\omega) = X_1(j\omega)H(j\omega) + X_2(j\omega)H(j\omega)$

Computing the Impulse Responses:

$h_1(t) = |H(j\omega_1)|\sin(\omega_1 t + \pi/4 + \angle H(j\omega_1)) = \frac{1}{4}\sin(\pi t + 3\pi/8)$

$h_2(t) = |H(j\omega_2)|2\cos(\omega_2 t - (\pi/3 + \angle H(j\omega_2))) = \cos(2\pi t - \frac{7}{12}\pi)$

$y(t) = h_1(t) + h_2(t) = \frac{1}{4}\sin(\pi t + 3\pi/8) + \cos(2\pi t - \frac{7}{12}\pi)$

1.1 (a) **18 / 18**

✓ **- 0 pts** Correct

- 2 pts part 3 partially correct

- 2 pts part2 partially correct

- 2 pts part 1 partially correct

$$\therefore \text{From } Y(j\omega) = X(j\omega)H(j\omega), \quad X(j\omega) = \frac{Y(j\omega)}{H(j\omega)}$$

$$\text{From } X(j\omega)H_1(j\omega) = Y_1(j\omega) \therefore H_1(j\omega) = \frac{Y_1(j\omega)}{X(j\omega)} = \frac{Y_1(j\omega)H(j\omega)}{Y(j\omega)}$$

$$\therefore H_1(j\omega) = \frac{2}{\cancel{(j\omega+1)}} \cdot \frac{3}{(j\omega+2)\cancel{(j\omega+4)}} \cdot \frac{\cancel{(j\omega+4)}\cancel{(j\omega+1)}}{12} = \frac{1}{2(j\omega+2)}$$

$$H_1(j\omega)H_2(j\omega) = H(j\omega) \therefore H_2(j\omega) = \frac{H(j\omega)}{H_1(j\omega)} = \frac{3 \cdot 2\cancel{(j\omega+2)}}{\cancel{(j\omega+2)}(j\omega+4)} = \frac{6}{j\omega+4}$$

iii) Find the impulse responses $h(t), h_1(t), h_2(t)$

We have,

$$H(j\omega) = \frac{3}{(j\omega+2)(j\omega+4)} \quad \therefore h(t) = \mathcal{F}^{-1}(H(j\omega)) = \frac{3}{2}\left(e^{-2t}u(t) - e^{-4t}u(t)\right)$$

$$H_1(j\omega) = \frac{1}{2(j\omega+2)} \quad \therefore h_1(t) = \mathcal{F}^{-1}(H_1(j\omega)) = \frac{1}{2}e^{-2t}u(t)$$

$$H_2(j\omega) = \frac{6}{j\omega+4} \quad \therefore h_2(t) = \mathcal{F}^{-1}(H_2(j\omega)) = 6e^{-4t}u(t)$$

b) Consider

$x_1(t) = \sin(\omega_1 t + \pi/4)$ $\quad \omega_1 = \pi$

$x_2(t) = 2\cos(\omega_2 t - \pi/3)$ $\quad \omega_2 = 2\pi$

$y(t) = (x_1(t) + x_2(t)) * h(t) =$

$\therefore Y(j\omega) = X_1(j\omega)H(j\omega) + X_2(j\omega)H(j\omega)$

Computing the Impulse Responses:

$h_1(t) = |H(j\omega_1)|\sin(\omega_1 t + \pi/4 + \angle H(j\omega_1)) = \frac{1}{4}\sin(\pi t + 3\pi/8)$

$h_2(t) = |H(j\omega_2)| 2\cos(\omega_2 t - (\pi/3 + \angle H(j\omega_2))) = \cos(2\pi t - \frac{7}{12}\pi)$

$y(t) = h_1(t) + h_2(t) = \frac{1}{4}\sin(\pi t + 3\pi/8) + \cos(2\pi t - 7/12\,\pi)$

1.2 (b) **7 / 8**

- **- 0 pts** Correct
- ✓ **- 1 pts** minor mistakes
- **- 2 pts** major mistakes
- **- 4 pts** incomlpete

c) $h(t) \to H(j\omega) = \frac{1}{1+j\omega}$ $\therefore h(t) = e^{-t}u(t)$

i) $H(j\omega) = \frac{1}{1+j\omega}$ $\therefore$ we need to find where $|H(j\omega)| \geq 0.99$

$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$ $\therefore 1+\omega^2 = 0.99^{-2}$

$\therefore \omega = \pm\sqrt{0.99^{-2}-1} = \pm 0.142 \text{ rads/s} = \pm 0.0226 \text{Hz}$

If $|H(j\omega_c)| = \frac{1}{\sqrt{2}}|H(j0)|$, $\frac{1}{\sqrt{1+\omega_c^2}} = \frac{1}{\sqrt{2}}$ $\therefore \omega_c = 2-1$

$\therefore \omega_c = \pm 1 \text{ rad/s} = (2\pi)^{-1} \text{Hz}$

ii) For cutoff $= 5\omega_c$,

Since $\Rightarrow f(at) \Rightarrow \frac{1}{a}F(j\frac{\omega}{a})$

$\mathcal{F}\left(\frac{1}{5}H(j\frac{\omega}{5})\right) = 5h(5t)$

cutoff frequency

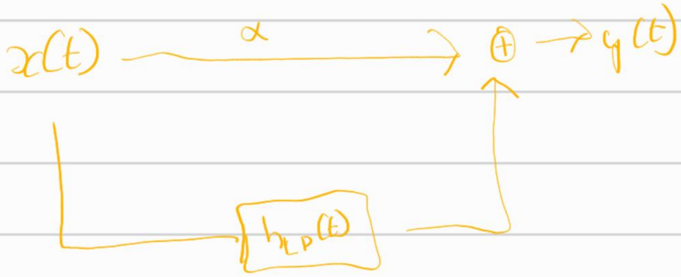
1.3 (c) **6 / 6**

✓ **- 0 pts** Correct

- 1 pts part 2 partially correct

- 2 pts part 2 missing

2) a)



$\therefore y(t) = \alpha x(t) + x(t) * h_{LP}(t) = x(t) * (\alpha \delta(t) + x(t) * h_{LP}(t)$

$\therefore$ For $y(t) = x(t) * h(t)$, $h(t) = \alpha \delta(t) + h_{LP}(t)$

$\therefore H(j\omega) = \alpha + H_{LP}(j\omega)$

For $\alpha = -1$, $H(j\omega) = H_{LP}(j\omega) - 1$

$H(j\omega)$:

which is a lowpass filter

$\therefore \alpha = -1$

b) Ideal filters are non realizable systems because they are noncausal $\therefore$ they depend on future values and thus cannot be implemented $\therefore$ Non realizable.

c) $H_{LP2}(j\omega) = \dfrac{K}{\beta + j\omega}$

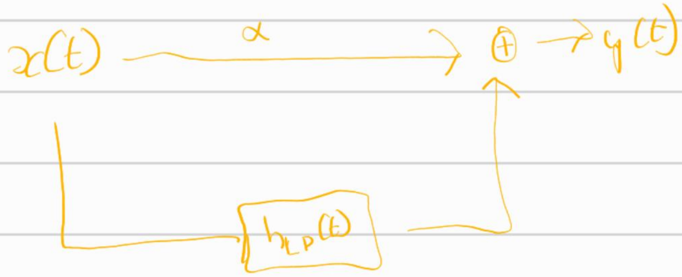
$H_{LP2}(j0) = 1$ & $|H_{LP2}(j2\pi)| = 1/\sqrt{2}$

$\therefore \dfrac{K}{\beta} = 1 \therefore K = \beta$, $\dfrac{|K|}{\sqrt{\beta^2 + 4\pi^2}} = \dfrac{1}{\sqrt{2}} \therefore \beta = K = \pm 2\pi$

2.1 (a) **4 / 6**

- **- 0 pts** Correct
- **- 1 pts** wrong alpha
- ✓ **- 2 pts wrong phase**

2) a)



$\therefore y(t) = \alpha x(t) + x(t) * h_{LP}(t) = x(t) * (\alpha \delta(t) + x(t) * h_{LP}(t)$

$\therefore$ For $y(t) = x(t) * h(t)$, $h(t) = \alpha \delta(t) + h_{LP}(t)$

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$\therefore \dfrac{K}{\beta} = 1 \therefore K = \beta$, $\quad \dfrac{|K|}{\sqrt{\beta^2 + 4\pi^2}} = \dfrac{1}{\sqrt{2}} \therefore \beta = K = \pm 2\pi$

2.2 (b) **2 / 3**

- **- 0 pts** Correct
- ✓ **- 1 pts** **infinite not mentioned**
- **- 1 pts** non causal not mentioned
- **- 3 pts** missing

2) a)

$\therefore y(t) = \alpha x(t) + x(t) * h_{LP}(t) = x(t) * (\alpha\delta(t) + x(t) * h_{LP}(t)$

$\therefore$ For $y(t) = x(t) * h(t)$, $h(t) = \alpha\delta(t) + h_{LP}(t)$

$\therefore H(j\omega) = \alpha + H_{LP}(j\omega)$

For $\alpha = -1$, $H(j\omega) = H_{LP}(j\omega) - 1$

$H(j\omega)$:

which is a lowpass filter

$\therefore \alpha = -1$

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c) $H_{LP2}(j\omega) = \dfrac{K}{\beta + j\omega}$

$H_{LP2}(j0) = 1$ & $|H_{LP2}(j2\pi)| = 1/\sqrt{2}$

$\therefore \dfrac{K}{\beta} = 1 \therefore K = \beta$, $\dfrac{|K|}{\sqrt{\beta^2 + 4\pi^2}} = \dfrac{1}{\sqrt{2}} \therefore \beta = K = \pm 2\pi$

For $\beta = k = -2\pi$, $H_{LP2}(j\omega) = \dfrac{2\pi}{2\pi - j\omega}$ $\therefore$ $h_{LP2}(t) = 2\pi e^{2\pi t} u(-t)$

since this is noncausal, $\beta = k = -2\pi$ is not a valid solution.

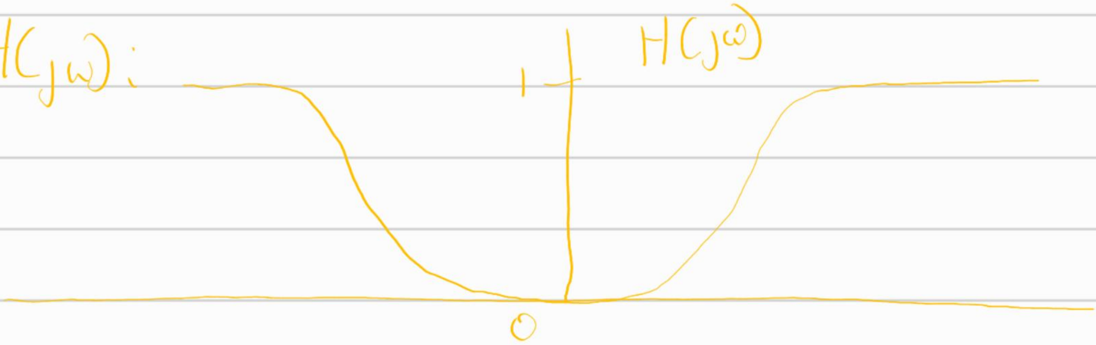
For $\beta = k = 2\pi$, $H_{LP2}(j\omega) = \dfrac{2\pi}{2\pi + j\omega}$ $\therefore$ $h_{LP2}(t) = 2\pi e^{-2\pi t} u(t)$

since this is noncausal, $\beta = k = 2\pi$ is valid.

2d) Using $\alpha = -1$, $H(j\omega) = -1 + H_{LP2}(j\omega) = \dfrac{-j\omega}{2\pi + j\omega}$

$$H(j\omega) = \frac{|\omega|}{\sqrt{4\pi^2 + \omega^2}} = \sqrt{\frac{\omega^2}{\omega^2 + 4\pi^2}}$$

$\therefore H(j\omega)$:



It still cuts low frequencies $\therefore$ is a HPF.

2.3 (c) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** not rejected -2pi
- **- 2 pts** wrong final answer
- **- 2 pts** incomplete

For $\beta = k = -2\pi$, $H_{LP2}(j\omega) = \frac{2\pi}{2\pi - j\omega}$ $\therefore h_{LP2}(t) = 2\pi e^{2\pi t} u(-t)$

since this is noncausal, $\beta = k = -2\pi$ is not a valid solution.

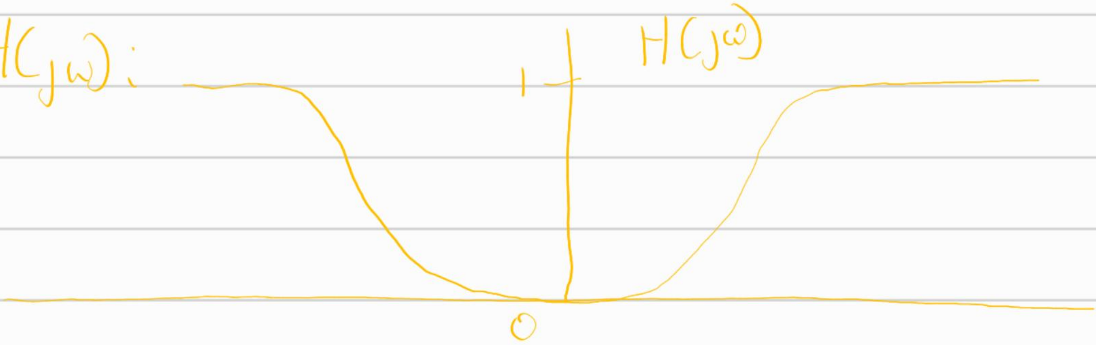
For $\beta = k = 2\pi$, $H_{LP2}(j\omega) = \frac{2\pi}{2\pi + j\omega}$ $\therefore h_{LP2}(t) = 2\pi e^{-2\pi t} u(t)$

since this is noncausal, $\beta = k = 2\pi$ is valid.

2d) Using $\alpha = -1$, $H(j\omega) = -1 + H_{LP2}(j\omega) = \frac{-j\omega}{2\pi + j\omega}$

$$|H(j\omega)| = \frac{|\omega|}{\sqrt{4\pi^2 + \omega^2}} = \sqrt{\frac{\omega^2}{\omega^2 + 4\pi^2}}$$

$\therefore H(j\omega)$:



It still cuts low frequencies $\therefore$ is a HPF.

2.4 (d) **4 / 4**

✓ **- 0 pts** Correct

- 1 pts frequency response partially correct

- 2 pts frquency response missing

- 1 pts high pass not mentioned

3) a) $h(t) = \begin{cases} 1/T, & 0 \le t \le T \\ 0, & \text{else} \end{cases} = \frac{1}{T}\text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) = \frac{1}{T}\text{rect}\left(\frac{t - T/2}{T}\right)$

$\mathcal{F}(h(t)) = H(j\omega) = e^{-j\omega(T/2)}\,\text{sinc}(\omega T/2\pi)$

b) $|H(j\omega)| = |\text{sinc}(\omega T/2\pi)\cdot 1|$

$\lim_{\omega \to \infty} |H(j\omega)| = 0$

c) For $x(t)$ is periodic, $y(t) = x(t) * h(t) = C$

$\therefore \mathcal{F}(C) = X(j\omega)H(j\omega) = 2C\pi\delta(\omega)$

For $x(t)$, Period: T_0 & $T_0 = 2\pi\omega_0$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

$\therefore X(j\omega) = \mathcal{F}\left(\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}\right) = \sum_{k=-\infty}^{\infty} c_k \mathcal{F}(e^{jk\omega_0 t})$

$= \sum_{k=-\infty}^{\infty} |2\pi c_k \delta(\omega - k\omega_0)| \therefore$

$\therefore \sum_{k=-\infty}^{\infty} |2\pi c_k \delta(\omega - k\omega_0)|\,\text{sin}(\omega T/2\pi) = 2C\pi\delta(\omega)$

$\therefore \omega_0 = k\,2\pi/T \quad \therefore T_0 = 2\pi \cdot \frac{T}{2\pi k} \quad \therefore T_0 = T/k$

This will allow the impulses to align

3.1 (a) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 3 pts** Partial Correct
- **- 1 pts** Incorrect power of exp(.)
- **- 1 pts** Incorrect sinc(.) term
- **- 6 pts** Missing/ Fully incorrect

3) a) $h(t) = \begin{cases} 1/T, & 0 \le t \le T \\ 0, & \text{else} \end{cases} = \frac{1}{T}\text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) = \frac{1}{T}\text{rect}\left(\frac{t - T/2}{T}\right)$

$\mathcal{F}(h(t)) = H(j\omega) = e^{-j\omega(T/2)}\ \text{sinc}(\omega T/2\pi)$

b) $|H(j\omega)| = |\text{sinc}(\omega T/2\pi) \cdot 1|$

$\lim_{\omega \to \infty} |H(j\omega)| = 0$

c) For $x(t)$ is periodic, $y(t) = x(t) * h(t) = C$

$\therefore \mathcal{F}(C) = X(j\omega) H(j\omega) = 2C\pi\delta(\omega)$

For $x(t)$, Period: T_0 & $T_0 = 2\pi\omega_0$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

$\therefore X(j\omega) = \mathcal{F}\left(\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}\right) = \sum_{k=-\infty}^{\infty} c_k \mathcal{F}(e^{jk\omega_0 t})$

$= \sum_{k=-\infty}^{\infty} |2\pi c_k \delta(\omega - k\omega_0)| \therefore$

$\therefore \sum_{k=-\infty}^{\infty} |2\pi c_k \delta(\omega - k\omega_0)|\ \text{sin}(\omega T/2\pi) = 2C\pi\delta(\omega)$

$\therefore \omega_0 = k\, 2\pi/T \quad \therefore T_0 = 2\pi \cdot \frac{T}{2\pi k} \quad \therefore T_0 = T/k$

This will allow the impulses to align

3.2 (b) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect Graph
- **- 1 pts** Incorrect response at w-> infinity
- **- 4 pts** Missing Graph
- **- 1 pts** Missing w->infinity response

3) a) $h(t) = \begin{cases} 1/T, & 0 \le t \le T \\ 0, & \text{else} \end{cases} = \frac{1}{T}\text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) = \frac{1}{T}\text{rect}\left(\frac{t - T/2}{T}\right)$

$$\mathcal{F}(h(t)) = H(j\omega) = e^{-j\omega(T/2)} \text{sinc}(\omega T/2\pi)$$

b) $|H(j\omega)| = |\text{sinc}(\omega T/2\pi)\cdot 1|$

$\lim_{\omega \to \infty} |H(j\omega)| = 0$

c) For $x(t)$ is periodic, $y(t) = x(t) * h(t) = C$

$\therefore \mathcal{F}(C) = X(j\omega)H(j\omega) = 2C\pi\delta(\omega)$

For $x(t)$, Period: T_0 & $T_0 = 2\pi\omega_0$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$\therefore X(j\omega) = \mathcal{F}\left(\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}\right) = \sum_{k=-\infty}^{\infty} c_k \mathcal{F}(e^{jk\omega_0 t})$$

$$= \sum_{k=-\infty}^{\infty} |2\pi c_k \delta(\omega - k\omega_0)| \therefore$$

$$\therefore \sum_{k=-\infty}^{\infty} |2\pi c_k \delta(\omega - k\omega_0)| \sin(\omega T/2\pi) = 2C\pi\delta(\omega)$$

$\therefore \omega_0 = k\,2\pi/T \quad \therefore T_0 = \frac{2\pi \cdot T}{2\pi k} \quad \therefore T_0 = T/k$

This will allow the impulses to align

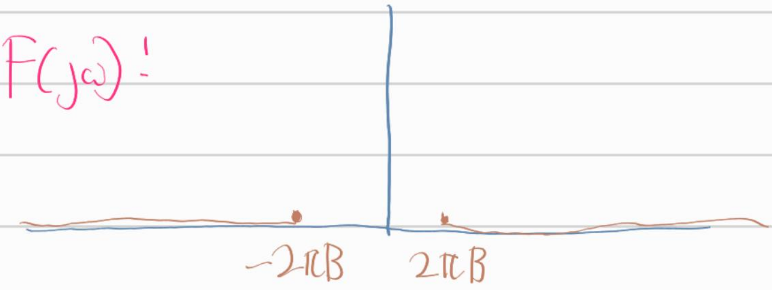
3.3 (c) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 2 pts** Partial Correct
- **- 6 pts** Missing/ incorrect

d) $f(t)$ is baseband,

$$F(j\omega) = \begin{cases} 0, & |\omega| > 2\pi B \\ \not 0, & -2\pi B < \omega < 2\pi B \end{cases}$$

$F(j\omega)$:



$$y(t) = f(t) * h(t)$$

$$Y(j\omega) = F(j\omega) H(j\omega)$$

$$|Y(j\omega)| = |F(j\omega)||H(j\omega)|$$ $H(j\omega)$ touches 0 @ $\pm\frac{2\pi}{T}$

For $\cancel{2\pi} B \le \cancel{2\pi} \frac{1}{T}$ $\quad B < \frac{1}{T}$

$\therefore H(j\omega) = \text{sinc}\left(\frac{\omega T}{2\pi}\right) e^{-j\omega T/2}$ & $\angle H(j\omega) = \cancel{\angle \sin\left(\frac{\omega T}{2\pi}\right)} - \frac{\omega T}{2}$

$\therefore t_g(\omega) = -\frac{d}{d\omega}\angle H(j\omega) = -\frac{d}{d\omega}\frac{T}{2}(-\omega) = T/2$

3.4 (d) **7 / 7**

- ✓ **- 0 pts** Correct
- **- 3 pts** Partial Correct
- **- 7 pts** Incorrect/ Missing

4)

$$m(t) \rightarrow \underset{\cos(\omega_c t)}{\otimes} \xrightarrow{x(t)} \underset{\text{Bandpass}}{\boxed{H(j\omega)}} \rightarrow y(t) \longrightarrow y(t) \rightarrow \underset{\cos \omega_c t}{\otimes} \xrightarrow{z(t)} \underset{\text{Lowpass}}{\boxed{H_{LP}(j\omega)}} \rightarrow m(t)$$

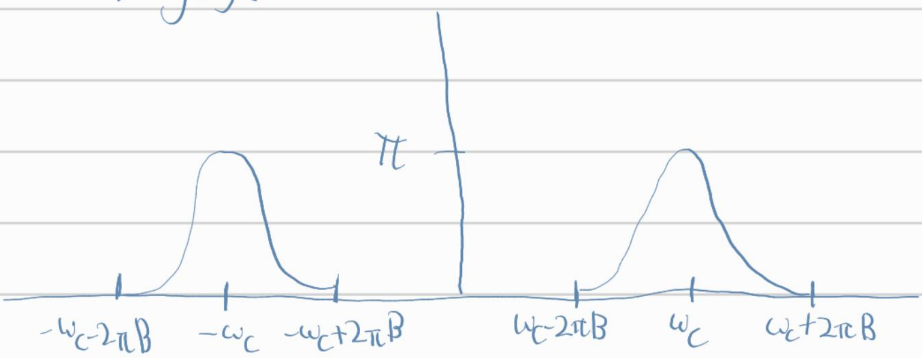
i) Sketch $X(j\omega)$

$$X(j\omega) = F(m(t)\cos\omega_c t) = \frac{1}{2\pi}\left(M(j\omega) * \pi\left(\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right)\right)$$

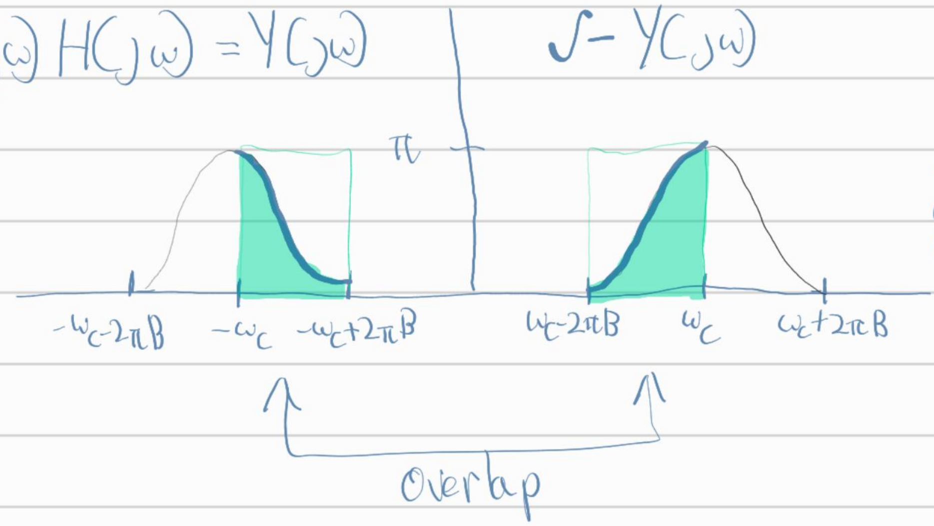
Using sifting & shifting,

i

$X(j\omega)$:



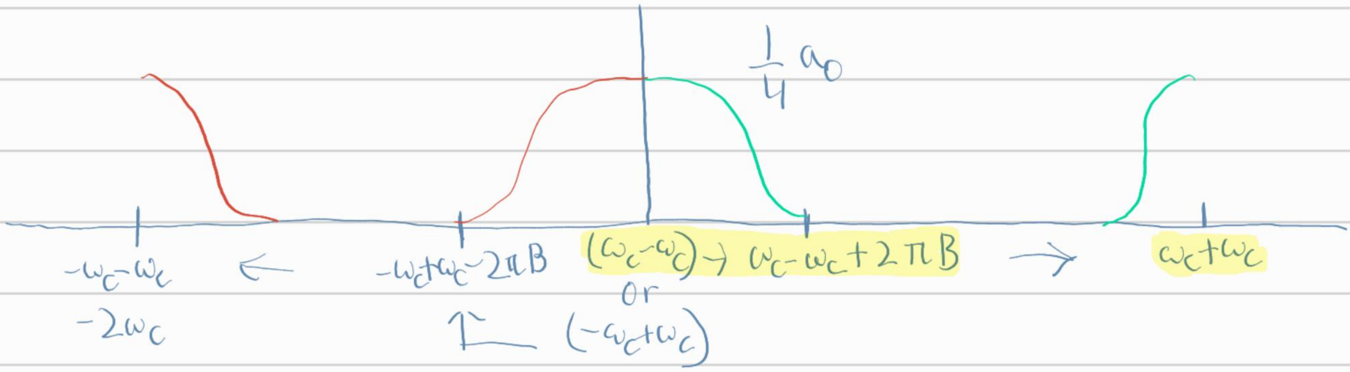
ii $X(j\omega)H(j\omega) = Y(j\omega)$ $\quad$ ✓ – $Y(j\omega)$



* the filter does not have an amplitude of π but if you multiplied the filter by a_0, the overlap would equal the product.

iii $Z(j\omega) = Y(j\omega) * \frac{1}{2\pi}\left(M(j\omega) * \pi\left(\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right)\right)$

$\therefore$ Using sifting and sampling,

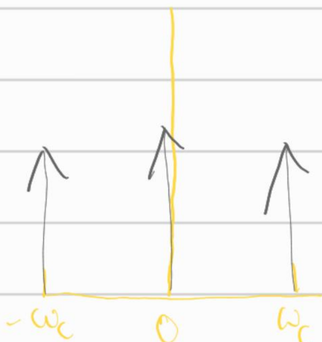


Does this system recover $m(t)$: Sort of.
After Applying the ideal Lowpass filter, you have 1/2 of $m(t)$. The cosines each half the signal & th LP doubles it $\frac{1}{2}\cdot\frac{1}{\cancel{2}}\cdot\cancel{2} = \frac{1}{2}$. So you end up with the signal halved.

b)

$t = nT \to$ periodic impulses with $T_0 = T = \frac{2\pi}{\omega_c}$

$$H_s(j\omega) = \sum_{K=-\infty}^{\infty} \delta(\omega - \omega_c)$$



$$\therefore \delta_T(t) \to \omega_c \delta_{\omega_c}(\omega)$$

$$y(t)h_s(t) = \frac{1}{2\pi}\left(Y(j\omega)\,\omega_c\delta_{\omega_c}(\omega)\right)$$

$$= \frac{1}{2\pi}\,\omega_c\,\frac{1}{2}\,M(j\omega) = \frac{1}{2\pi}\,M(j\omega)$$

4.1 (a) **10 / 10**

✓ **- 0 pts** Correct

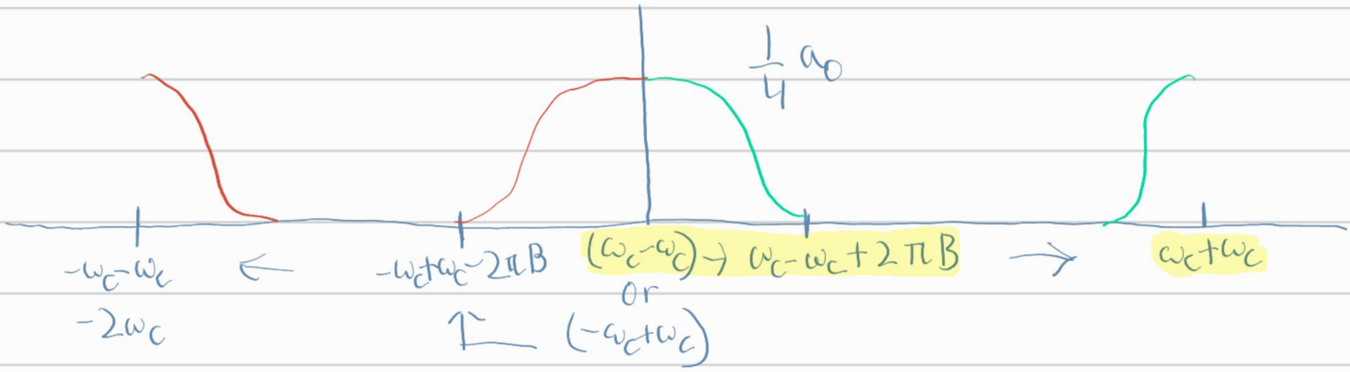
- 1 pts part 3 partially correct

- 1 pts systen recovers signal

- 3 pts part 1 missing

- 3 pts part 2 missing

- 3 pts part 3 missing



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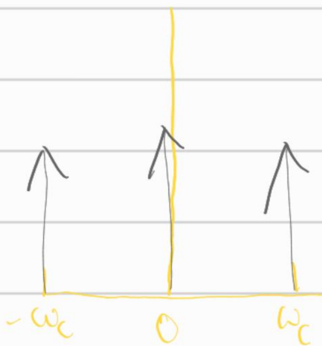

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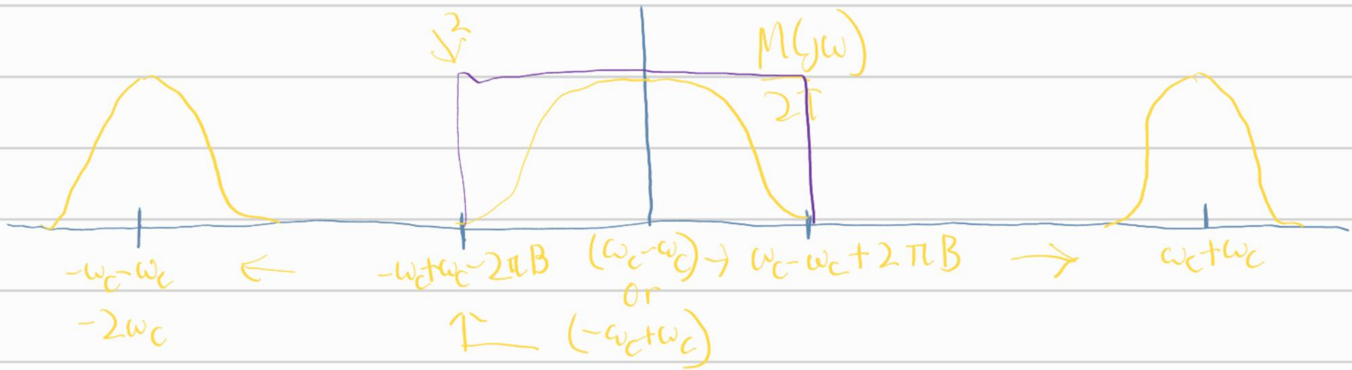
$$= \frac{1}{2\pi}\,\omega_c \frac{1}{2} M(j\omega) = \frac{1}{2\pi} M(j\omega)$$



After applying the LP filter, we get just $\frac{1}{T} M(j\omega)$

4.2 (b) **15 / 15**

- ✓ **- 0 pts** Correct
- **- 2 pts** partially correct
- **- 15 pts** missing
- **- 10 pts** no plots
- **- 6 pts** 2 plots missing