ECE 113 CS

$*113$: $\text{sinc}(x) = \frac{\sin x}{x}$

Signals: 113 → Discrete: $x[n] = \cos(2\pi n)$ || $x[n] = \{1,2,3,4,5\}$ $-2 \le n \le 2$

Signal Manipulation: ① Arithmetic: $\oplus, \otimes$ ② Time scaling & scaling: $x[\alpha n + \beta]$

Basic Bitch Signals: $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & ow \end{cases}$, $u[n] = \begin{cases} 1, & n \ge 0 \\ 0, & ow \end{cases}$, $r[n] = \begin{cases} n, & n \ge 0 \\ 0, & ow \end{cases}$ | Sampling Holds in $\delta[n]$ | $r' = u$, $r'' = u' = \delta$ | Any Signals can be rep by δ: $x[n] = \{1,2,0\}$, $x[n] = \delta[n] + 2\delta[n-1]$ ← Canonical Basis

$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$

$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j}$

~ denotes periodicity

Sinusoid: $\tilde{x}[n] = A\cos(\Omega_0 n + \Theta)$: $\Omega_0 \in [-\pi, \pi]$ $\Omega = 2\pi F_0$; $F_0 \to$ Frequency

Normalized $= \frac{\Omega: \text{angular Freq}}{F_s: \text{sampling Freq}}$

Complex: $x[n] = Ae^{j(2\pi F_0 n + \theta)}$, $e^{j\phi} = \cos\phi + j\sin\phi$

Fundamental Period: Extra constraint: N must be a ratio of rational #s

For $\tilde{x}[n]$, smallest N, such: $\tilde{x}[n+N] = x[n]$

Even & odd signals: E: $x[n] = x[-n]$; O: $x[n] = -x[-n]$

$x_e[n] = \frac{x[n] + x[-n]}{2}$

$x_o[n] = \frac{x[n] - x[-n]}{2}$

E.S.: $x^*[n] = x[-n]$

O.S.: $x^*[n] = -x[-n]$

Complex: $e^{j\pi} - 1 = 0$ Sym $= x^*[n] = x[-n]$, $= x^*[n] = -x[-n]$

$x^*[n] = |x[n]| e^{-j\angle x[n]}$

$x[n] = Re(x[n]) + jIm(x[n]) = |x[n]| e^{j\angle x[n]}$

$z = re^{j\phi}$ r: Mag ϕ = Angle

$re^{j\phi} = r\cos\phi + rj\sin\phi$

$r = \sqrt{Re^2 + Im^2}$, $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$Re(x) = r\cos\phi$

$Im(x) = r\sin\phi$

$x^* = Re(x) - jIm(x)$

$x^2 = xx^*$ $j = -\frac{1}{j}$

$j = e^{j\pi/2}$, $-j = e^{-j\pi/2}$, $1 = e^{j\cdot 0}$, $-1 = e^{j\pi}$ ✱

Constant Coeff Difference

$x \to \boxed{S} \to y$

$y = \sum_{k=0}^{N-1} b_k x[n-k]$

Feed Back → Needs init condit

$\sum_{k=0}^{M-1} a_k y[n-k] = \sum_{k=0}^{N-1} b_k x[n-k]$

Energy: $E_x = \sum_n |x[n]|^2$: Energy Signal if $0 < E_x < \infty$ → similar applies

Power: If $\tilde{x} \to P_x = \frac{1}{N}\sum_{n=0}^{N} |x[n]|^2$ else $P_x = \lim_{m\to\infty} \frac{1}{2m+1} \sum_{n=-m}^{m} |x[n]|^2$

Systems: $x \to \boxed{H} \to y$

Linearity: $H[ax + bx_0] = aH[x] + bH[x_0]$

Homogeneity: $H[ax] = aH[x]$

Superpos: $H[x + x_0] = H[x] + H[x_0]$

Time Invariance: $S[x[n-k]]$ Delay Inp $= y[n-k]$ Delay Out

Causality: No dependance on past values

Stability: BiBO stable

BiBO: $|x[n]| < B$ & $|y[n]| < C$

System Blocks: Unit Delay: $\boxed{D}$, Additional Delay: $\boxed{z^{-n}}$, $\otimes$ation: $\triangleright^{k}$, $\oplus$tion: $\oplus$

Convolution: $x * h = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

1) $x_1 * x_2 = x_2 * x_1$

2) $x * (h_0 + h_1) = x * h_0 + x * h_1$

3) $h[n] * \delta[n-k] = h[n-k]$

$x_1[n]$: $n_1 \le n \le n_1 + L_1$ | $L = L_1 + L_2 + 1$

$x_2[n]$: $n_2 \le n \le n_2 + L_2$

$x_1 * x_2$: $n_1 + n_2 \le n \le n_1 + n_2 + L_1 + L_2$

Thinking of signals with basis vectors.

$x[n] = \sum_k c_k \psi_k[n]$ | c_k: Basis coeff, ψ_k: Basis vector

Syn: Basis + coeff = vector

Anal: Basis + Signal = coeff?

Fourier Series $\Omega_k = k\Omega_0 = k\frac{2\pi}{N}$ ✱

$\tilde{x}[n] = \sum_k c_k e^{jk\Omega_0 n}$ ← Synthesis

Anal: $c_k = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{x}[n] e^{-jk\Omega_0 n}$

$c_k^* = c_{-k}$

for $x \circledast y$, $e_k = LCM(N_x, N_y) \cdot c_k d_k$

Periodic Convolution: $\tilde{x}[n]$ w/ period N_x: c_k, $\tilde{y}[n]$ w/ period N_y: d_k

DT Fourier Transform: $X(\Omega) = \lim_{m\to\infty} \sum_{n=-m}^{m} x[n] e^{-jk\Omega_0 n}$ $\Omega_0 k \in \{-\pi, \pi\}$

Existance: $x[n]$ needs to be absolutely Summable $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$ or sq. Summable $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$

Inverse DT Fourier Transform: $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega) e^{j\Omega n} d\Omega$

DTFT & DTFS bad. replace $\Omega_0 k$ with Ω

$\sum_{0}^{N-1} \tilde{x}[k]h[n-k]$

Dont accidentally count 0 twice

Random Shit

$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, $a < 1$

$\int u\,dv = uv - \int v\,du$

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$

$\sum_{a=a_1}^{a_2} a = \frac{(a_1 + a_2)(a_1 - a_2)}{2}$

$\sum_{n=N_0}^{N_1} r^n = \frac{r^{N_0} - r^{(N_1+1)}}{1-r}$

$(\{1111111\}) * (.9^n u[n])$ 2

For $0 \le n < 7$, $u[n-7-k]^{y[n]} = \sum_{k=0}^{n} 0.9^k = \frac{1-0.9^{n+1}}{1-0.9}$

For $n \ge 7$: $y[n] = \sum_{r=0}^{6} 0.9^{n-r} = 0.9^n \frac{1-0.9^{-7}}{1-0.9^{-1}}$

$|X(\Omega)| = \pi \sum_{l=-\infty}^{\infty} \delta(\Omega - \Omega_o - 2\pi l) + \delta(\Omega + \Omega_o - 2\pi l)$

$\angle X(\Omega) = \begin{cases} -\frac{\pi}{2}, & \text{where } \Omega = +\Omega_o + 2\pi l, \ \forall\, l \in \mathbf{Z} \\ +\frac{\pi}{2}, & \text{where } \Omega = -\Omega_o + 2\pi l, \ \forall\, l \in \mathbf{Z} \end{cases}$

DTFT

$\frac{\pi}{j}\sum_{-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi l) - \delta(\Omega + \Omega_0 - 2\pi l)$

$e^{-j\Omega_0 n}$

$\sum_{-\infty}^{\infty} 2\pi\delta(\Omega + \Omega_0 - 2\pi l)$

$\frac{d}{d\Omega}(X(\Omega)e^{-j\Omega})$ | $n \cdot x[n-1]$

FT Pairs (102)

Series	
$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$ | $a_1 = \{1\}$
$\cos(\omega_0 t)$	$\pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$ | $a_1 = a_{-1} = \frac{1}{2}$
$\sin(\omega_0 t)$	$\frac{\pi}{j}(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$ | $a_1 = \frac{1}{2j}$, $a_{-1} = -\frac{1}{2j}$
1	$2\pi\delta(\omega)$ | $a_0 = 1$
$rect(t/T)$	$T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right)$
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t)$	$\frac{1}{(a + j\omega)^n}$