ECE102, Fall 2022 **Homework #2**
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Due Friday, 14 Oct 2022, by 11:59pm to Gradescope.
Covers material up to Lecture 5.
100 points total.

1. (29 points) **Elementary signals.**

   (a) (9 points) Consider the signal $x(t)$ shown below. Sketch the following:

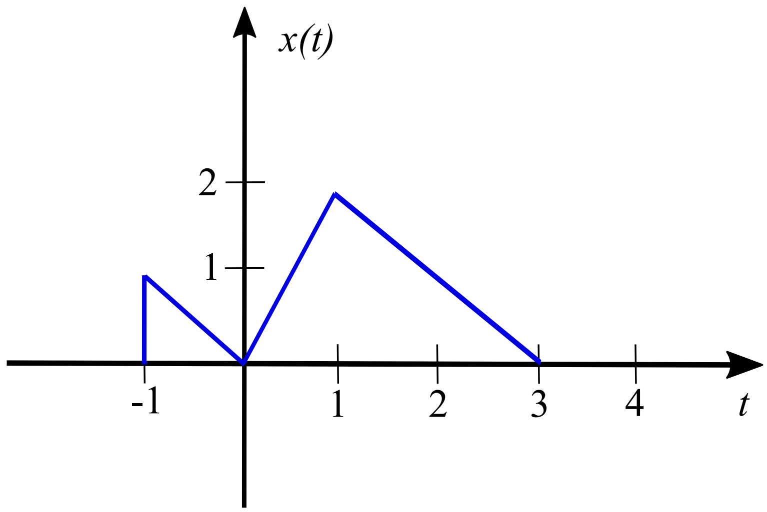


Figure 1: (2 b i)

   i. $y(t) = x(t)\,[1 - u(t-1) + u(t-2)]$
   ii. $y(t) = \int_{-\infty}^{t} [\delta(\tau+1) - \delta(\tau-1) + \delta(\tau-2)]x(\tau)d\tau$
   iii. $y(t) = x(t) + r(t+1) - u(t) - 3r(t) + 3r(t-1) - r(t-3)$

   (b) (12 points) Evaluate these integrals:

   i. $\int_{-\infty}^{\infty} f(t+1)\delta(t+1)dt$
   ii. $\int_{t}^{\infty} e^{-2\tau}u(\tau-1)d\tau$
   iii. $\int_{0}^{\infty} f(t)(\delta(t-1)+\delta(t+1))dt$
   iv. $\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

   (c) (4 points) Evaluate the derivative i.e, $\frac{d}{dt}y(t)$ for $y(t) = \Delta(t)u(t) + r(t-2)$.
   Hint: Use the product rule of derivative $\frac{d}{dx}f(x)g(x) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ *to evaluate the derivative of product of signals*

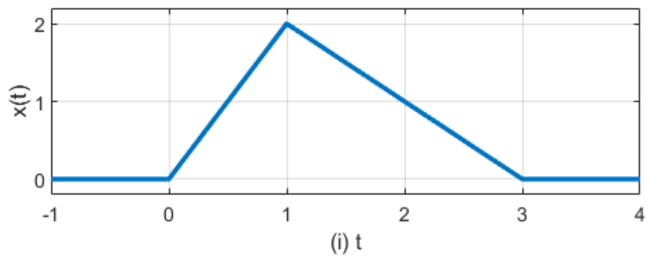
   (d) (4 points) Let b be a positive constant. Show the following property for the delta function:

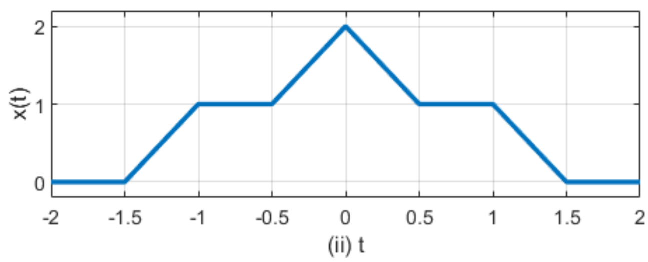
$$\delta(bt) = \frac{1}{b}\delta(t)$$

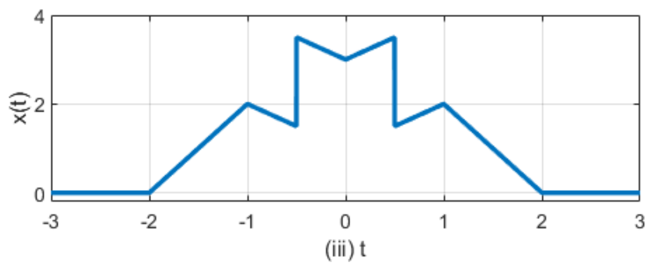
Hint: what function is "delta-like"?

2. (23 points) **Expression for signals.**

   (a) (15 points) Write the following signals as a combination (sums or products) of unit triangles $\Delta(t)$ and unit rectangles $\text{rect}(t)$.







   (b) (8 points) Express each of the signals shown below as sums of scaled and time shifted unit-step functions.

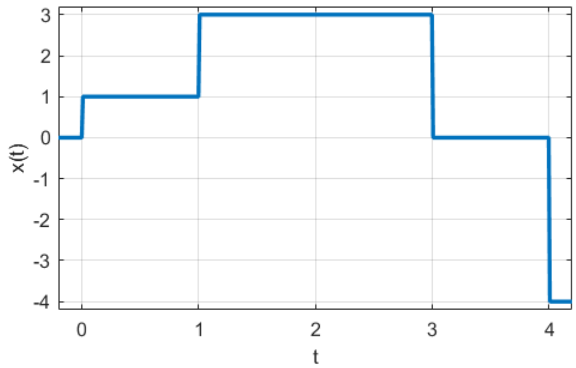
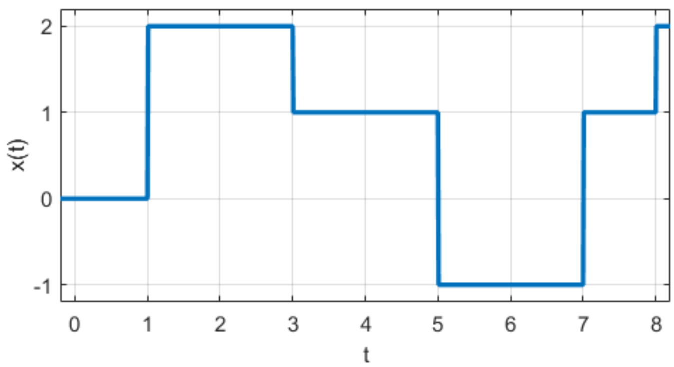


Figure 2: (2 b i)



3. (20 points) **System properties.** A system with input $x(t)$ and output $y(t)$ can be time-invariant, causal or stable. Determine which of these properties hold for each of the following systems. Explain your answer.

   (a) $y(t) = |x(t)| + x(2t)$

   (b) $y(t) = \int_{t-T}^{t+T} x(\lambda)d\lambda$, where T is positive and constant.

   (c) $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)d\lambda$

   (d) $y(t) = 1 + e^{x(t)}$

   (e) $y(t) = \frac{1}{1+x^2(t)}$

4. (13 points) **Power and energy of complex signals**

   (a) (6 points) Let

$$x(t) = Ae^{j\omega t} + Be^{-j\omega t}$$

   where A and B are complex numbers expressed in polar form

$$A = r_1 e^{j\phi}$$
$$B = r_2 e^{j\phi}$$

Is $x(t)$ a power or energy signal? If it is an energy signal, compute its energy. If it is a power signal, compute its power. (*Hint: Use the fact that the square magnitude of a complex number v is: $|v|^2 = v^*v$, where v^* is the complex conjugate of the complex number v.*)

(b) (5 points) Is $y(t) = e^{-(2+j\omega_1+j\omega_2)t}u(t+3)$ an energy signal or power signal? Again, if it is an energy signal, compute its energy. If it is a power signal, compute its power.

(c) (2 points) Given $z(t)$ is an energy signal, Comment on the energy of the signal $z(t)+u(t)$.

5. (15 points) **Python**

(a) (3 points) **Task 1**

A complex sinusoid is denoted:

$$y(t) = e^{(\sigma+j\omega)t}$$

First compute a vector representing time from 0 to 10 seconds in about 500 steps (You can use **np.linspace**). Use this vector to compute a complex sinusoid with a period of 2 seconds, and a decay rate that reduces the signal level at 10 seconds to 1/3 its original value. What σ and ω did you choose? Evaluate ($y(t)$ as shown above i.e.

$$y(t) = e^{(\sigma+j\omega)t}$$

Hint: to define complex number $e^{(5+6j)}$

```
y = np.exp(5 + 1j*6)
```

(b) (7 points) **Task 2**

Use the `np.real(y)` and `np.imag(y)` Python functions to extract the real and imaginary parts of the complex exponential.

i. (5 points) Plot them as a function of time (plot them separately, you can use `subplot` for this task). This should look more reasonable. Label your axes, and check that your signal has the required period and decay rate.

ii. (2 points) Plot the imaginary component of $y(t)$ as a function of real component of $y(t)$. What can be inferred from the plot ? *Hint: Comment on the shape of the plot and what we can infer about the envelope of $y(t)$ from the shape?*

(c) (5 points) **Task 3**

Use the `np.abs()` and `np.angle()` functions to plot the magnitude and phase angle of the complex exponential (plot them in the same figure). Scale the `np.angle()` plot by dividing it by `2*pi` so that it fits well on the same plot as the `np.abs()` plot (i.e. plot the angle in cycles, instead of radians, the function `np.angle(x)` returns the angle in radians).

Feel free to also explore and visualize the change in the wave-forms for different sigma and omega values.