

Due Friday, 17 March 2023, by 11:59pm to Gradescope.

50 points total.

1. (10 points) **Z-transform**

Find the Z-transform of the following sequences and indicate their region of convergence. Additionally, check if the DTFT exists. For these problems, a is an arbitrary constant.

(a) $x[n] = au[n]$

(b) $x[n] = a^n u[n]$

2. (10 points) **Z-transform with multiple ROCs**

Consider the z transform of, $g[n]$:

$$G(z) = \frac{(2z^2 - 0.8z + 0.5)(3z^2 + 2z + 3)}{(z^2 - 0.8z + 0.38)(z^2 + 4z + 5)}$$

There are 3 possible nonoverlapping regions of convergence (ROCs) of this z-transform. Discuss the type of inverse z-transform (left-sided, right-sided, or two-sided sequences) associated with each of the 3 ROCs. It is not necessary to compute the exact inverse transform).

3. (10 points) **Z-transform Properties**

Determine the z-transform and the corresponding ROC of the following sequences. Assume $|\beta| > |\alpha| > 0$.

Show a plot of the z-plane where the ROC is clearly indicated.

(a) $x_1[n] = (\alpha^n + \beta^n)u[n + 2]$

(b) $x_2[n] = \alpha^n u[-n - 2] + \beta^n u[n - 1]$

(c) $x_3[n] = \alpha^n u[n + 1] + \beta^n u[-n - 2]$

4. (10 points) **Z-transform Block Diagram** The following causal system, in Figure 1, has the input $x[n]$ and the output $y[n]$. Where $G(z)$ is defined as:

$$G(z) = \alpha + z^{-1}, \quad \alpha > 1$$

(a) What is the transfer function for this system?

(b) What is the impulse response?

(c) What is the region of convergence corresponding to the impulse response?

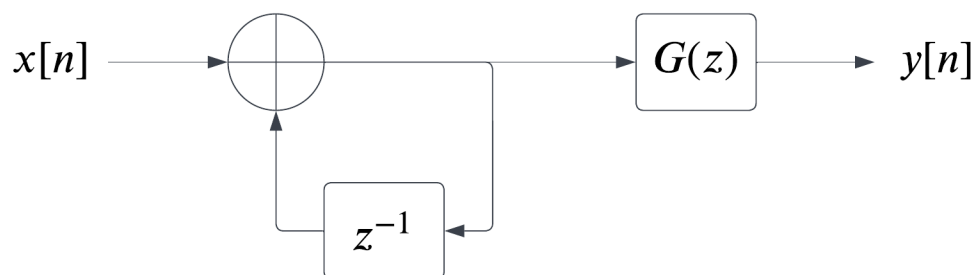


Figure 1: System Block Diagram

5. (10 points) **Inverse Z-transform** Find the impulse response, $h[n]$ of the following transfer function $H(z)$ and its ROC:

$$H(z) = \frac{1 - 2z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}, \quad |z| > \frac{1}{2}$$