ECE102, Fall 2022 — **Homework #1**

Signals & Systems — Prof. Jonathan C. Kao

University of California, Los Angeles; Department of ECE — TAs: Rakshith, Kalai, Yang

Due Friday, 7 Oct 2022, by 11:59pm to Gradescope.
Covers material up to Lecture 3.
100 points total.

1. (10 points) **Even and odd parts.**

   (a) (3 points) Show that the product of two odd signals is even.

   (b) (3 points) Show that the product of an even signal and an odd signal is odd.

   (c) (4 points) Use the properties derived in the previous parts to find the even and odd component of:
   (Hint : Sum of odd signals is odd and sum of even signals is even).

$$x(t) = 5 + t\sin^2(t) + t^3\left(\frac{e^t + e^{-t}}{2}\right) + t^5\sin(t)$$

2. (15 points) **Time scaling and shifting.**

   (a) (10 points) For $x(t)$ indicated in the figure below, sketch the following:

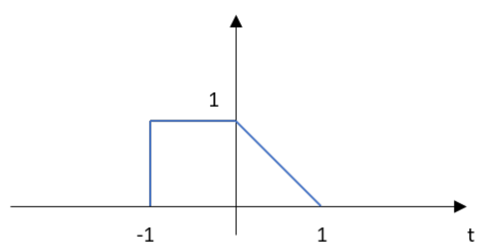
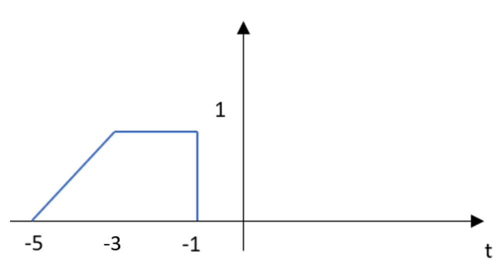


Figure 1: $x(t)$

   i. $\frac{1}{2}x(2t-6)$

   ii. $x(\frac{1}{10} - \frac{1}{5}t)$

   (b) (5 points) The figure below shows another signal: $y(t)$. Could you express $y(t)$ in terms of $x(t)$?



3. (22 points) **Periodic signals.**

   (a) (12 points) For each of the following signals, determine whether it is periodic or not. If the signal is periodic, determine the fundamental period and frequency.

      i. $x(t) = 24 + 50\sin(60\pi t)$

      ii. $x(t) = 10\cos^2(\frac{\pi}{2}t)$

      iii. $x(t) = \sin(5\pi t) + \cos(16\pi^2 t)$

      iv.
$$x(t) = \begin{cases} 24 + 50\sin(60\pi t) & t < 0 \\ 10\cos^2(10\pi t) & t \geq 0 \end{cases} \tag{1}$$

   (b) (5 points) Assume that the signal $x(t)$ is periodic with period T_0, and that $x(t)$ is odd (i.e. $x(t) = -x(-t)$). What is the value of $x(T_0)$?

   (c) (4 points) If $x(t)$ is periodic, is the signal $x(5t + 2)$ periodic ?

   (d) (1 point)What is the effect of **time shifting** on the fundamental period of signal (Think Intuitively) ?

4. (21 points) **Energy and power signals.**

   (a) (15 points) Determine whether the following signals are energy or power signals. If the signal is an energy signal, determine its energy. If the signal is a power signal, determine its power.

      i. $x(t) = e^{-|t|}$

      ii. $x(t) = \begin{cases} \frac{1}{\sqrt{t}}, & \text{if } t \geq 1 \\ 0, & \text{otherwise} \end{cases}$

      iii. $x(t) = \begin{cases} 1 + e^{-t}, & \text{if } t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

   (b) (6 points)If $z(t)$ is an odd signal, then for any $\tau > 0$ show that:

      - $$\int_{-\tau}^{\tau} z(t)dt = 0$$

      .

   We know from question 1 b that the product of an even signal and an odd signal is odd. Use this fact and the property derived above ,to show that the energy of $x(t)$ is the sum of the energy of its even component $x_e(t)$ and the energy of its odd component $x_o(t)$, i.e.,

$$E_x = E_{x_e} + E_{x_o}$$

   Assume $x(t)$ is a real signal.

5. (16 points) **Euler's identity and complex numbers.**

   (a) (8 points) Use Euler's formula to prove the following identities:

      i. $\cos^2(\theta) + \sin^2(\theta) = 1$

      ii. $\cos(\theta + \psi) + \cos(\theta - \psi) = 2\cos(\theta)\cos(\psi)$

   (b) (8 points) $x(t) = (5 + \sqrt{2}j)e^{j(t+2)}$ and $y(t) = 1/(2 - j)$.

      i. Compute the real and imaginary parts of $x(t)$ and $y(t)$.

      ii. Compute the magnitude and phase of $x(t)$ and $y(t)$.

6. (16 points) **Python tasks**

   For this question, please include all relevant code in text format. For plots, please include axis labels and preferably include a grid.

   (a) (5 points) **Task 1**

   Plot the waveform

   $$x(t) = e^{-t}\cos(2\pi t)$$

   for $-10 \leq t \leq 10$, with a step size of 0.2.

   (b) (5 points) **Task 2**

   Create a function `relu(t)` that implements the *ReLU* function:

   $$x(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases} \tag{2}$$

   You will need to create a function called relu

```python
def relu(t):
    # Function body, which is what you will have to modify
    #to get the appropriate implementation of relu function
```

   Then plot the function for $-5 \leq t \leq 5$, with a step size of 0.1.

   (c) (6 points) **Task 3**

   Create functions `even(t, f)` and `odd(t, f)` that take inputs time `t` and function (handle) `f` that compute the respective even and odd parts of `f(t)` at points t.
   For example, the square of a function could be implemented as :
   Here t is the time series array and f is the function

```python
def square(t, f):
    return (f(t) * f(t))
```

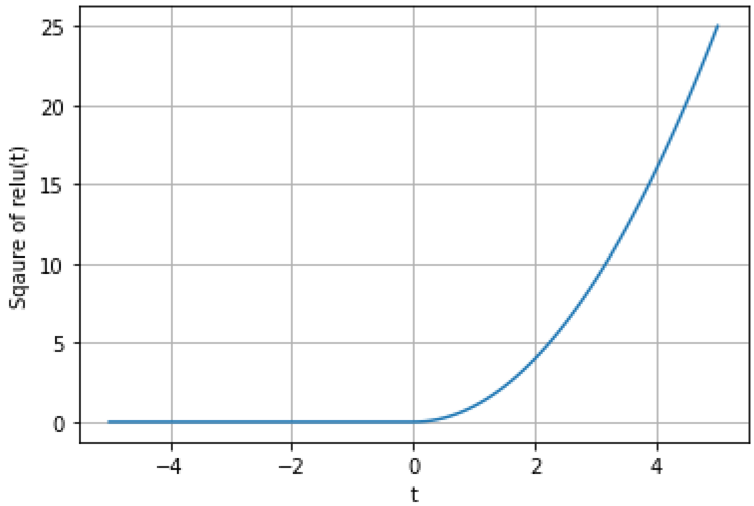
   and run as:

```python
t= np.arange(-5,5+0.1, 0.1)
y= [square(tt, relu) for tt in t]
```

   where `relu` is called a function handle of the function `relu`.

```
plt.plot(t,y)
plt.grid()
plt.ylabel("Sqaure of relu(t)")
plt.xlabel("t")
```

yields the result:



For this question, plot the even and odd components of $relu(t)$ for $-5 \leq t \leq 5$, with a step size of 0.1 using the functions `even(t, f)` and `odd(t, f)`. Feel free to also define and play around with arbitrary functions to look at their even and odd components.