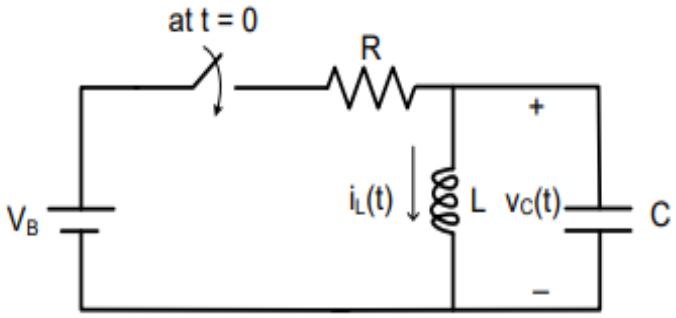
ECE 10, Winter 2022, Homework #4, Due February 28, 2023

Problem 1: Consider the circuit shown below. Assume that C = 0.5F, L = 2H, R = 1 Ohm, VB = 1V, $v_C(0–)$ = 2V, and $i_L(0–)$ = 2A.

(a) Is the circuit under-, over-, or critically damped? Why?

(b) Derive an expression for $v_C(t)$ for time $t \geq 0$.

(c) Derive an expression for $i_L(t)$ for time $t \geq 0$.



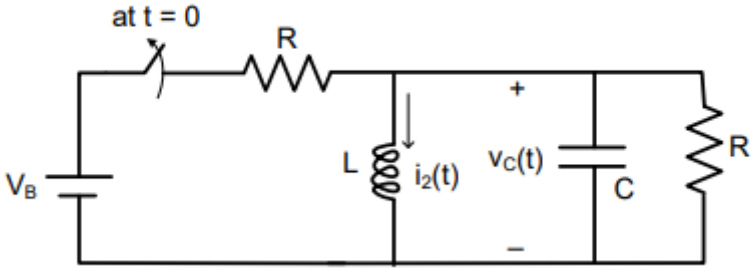
(2 + 12 + 6 = 20 points)

Problem 2: Consider the circuit shown below. Assume that VB = 10V, C = 100µF, R = 50 Ohms, and L = 1H. In the network, the switch is closed and a steady state is reached before t = 0. Subsequently, at time t = 0, the switch is opened.

(a) Is the circuit under-, over-, or critically damped? Why?

(b) Derive an expression for $v_C(t)$ for time $t \geq 0$.

(c) Derive an expression for $i_2(t)$ for time $t \geq 0$.



(2 + 12 + 6 = 20 points)

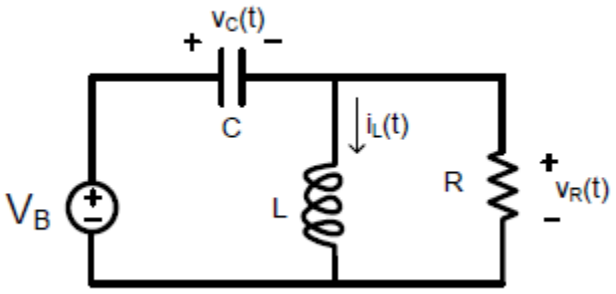
Problem 3: Refer to the figure below for this problem. Use V_B = 2V, R = 1000Ω, C = 8nF, L = 0.5mH, $v_C(0+)$ = 1V, and $i_L(0+)$ = 1mA.

a. Derive a differential equation in terms of $v_R(t)$ that describes circuit operation for t ≥ 0.

b. Determine the resonant frequency and the damping factor.

c. Solve the differential equation and obtain an expression for $v_R(t)$ for t ≥ 0 using the initial conditions provided.

d. Sketch $v_R(t)$ and label important features such as initial and final values, and if the solution is underdamped, calculate and label the oscillation period, and the time constant of the envelope decay.



(3 + 2 + 7 + 3 = 15 points)

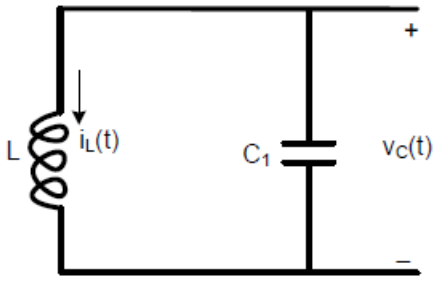
Problem 4: Refer to the figure below for this problem. The circuit came into existence at t = 0, prior to which, $i_L(0–)$ = 2A and $v_C(0–)$ = 2V. Assume that L = 1H, C1 = 0.25F.

a. Derive a differential equation in terms of $i_L(t)$ that characterizes circuit behavior for t ≥ 0.

b. Determine the values of $i_L(0+)$ and $di_L(t)/dt|_{t=0+}$

c. Determine the value of $d^2i_L(t)/dt^2|_{t=0+}$

d. Derive an expression for the complete solution of $i_L(t)$ in this circuit.



(3 + 4 + 3 + 5 = 15 points)

Problem 5: Please refer to Fig. P12-11 (page 367) from the textbook. Calculate the steady state $v_a(t)$ given that $v_1(t) = 5\cos(5t)$. **Note**: Please use the method described in the class which assumes a parametric form i.e. assume $v_a(t) = B\sin(5t+\varphi)$ in steady state and solve for B and φ.

(10 points)

Problem 6: Refer to the below figure for this problem. It plots the current, i(t), flowing through a series RLC circuit along with measurements at various times; the x-axis is in seconds while they-axis is in Amperes. The series RLC circuit has L = 1H, and is connected to a battery, VB= 1V, at t = 0. Note that i(0) = 0 A in the graph.

(a)Calculate resonant frequency, in rad/s

(b)Calculate the damping factor

(c)Calculate an expression for i(t)

(d)Calculate the value of C potentially used.

(e)Calculate the value of R potentially used.

(3+ 3 + 3 + 3 + 3= 15 points)