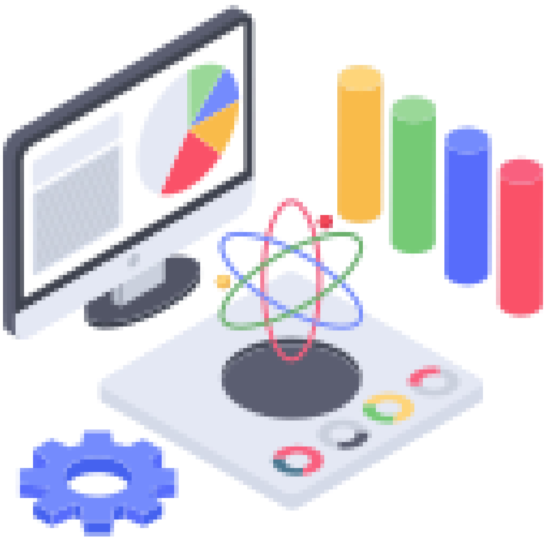


Computer Graphics

(UNIT 2)

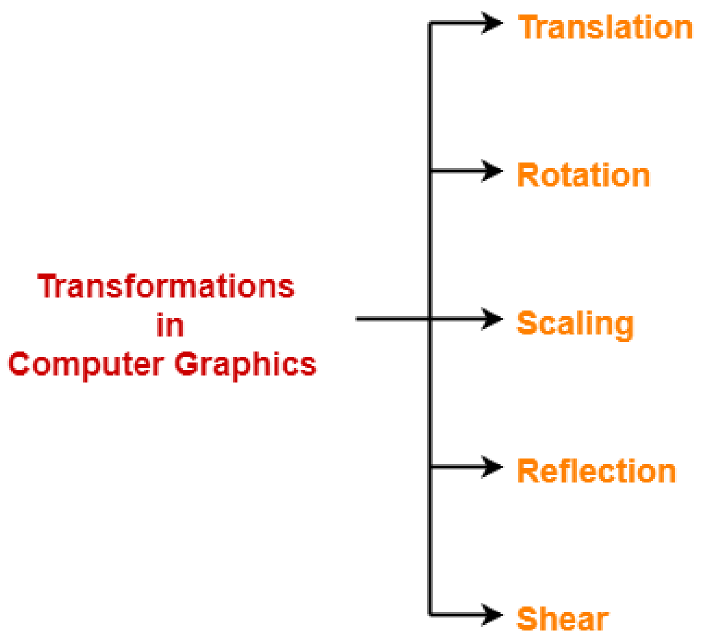
2D Transformations

What is 2d Transformation

- Transformation means changing the graphics in some other form by applying rules.
- We can have various types of transformations such as translation, scaling, rotation, shearing, etc.
- When a transformation takes place on a 2D plane (x and y axis), it is called 2D transformation.

Types of Transformations

- Translation
- Scaling
- Rotating
- Reflection
- Shearing



Types of Transformations:

Translation

- Moving an object from one position to another without changing its orientation/alignment or size.
- Here we add or subtract the values of x and y axis with some constant values to change the position of the object

Rotation:

- Rotating an object around a fixed point by a certain angle is called as rotation.

Scaling

- Scaling is to resize the object by changing its dimensions (size)
- Here the object gets enlarge (Bigger) or it shrinks (Smaller)

Shearing

- In shearing only some points of the object are moved form its current position to different position
- Shearing changes the shape of the Object

Reflection

- Flipping and object means to create a mirror image of the object across the line (axis or a line)
- This mirror image looks like the reflection of the original image

Translation

- Translation refers to Moving an object from one position to another without changing its orientation/alignment or size.

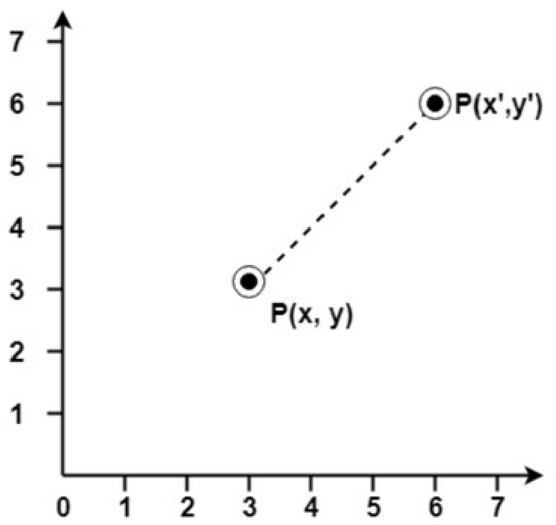
- Here we add or subtract the values of x and y axis with some constant values to change the position of the object

- Translation can be represented as:

x1 = x + Tx
y1 = y + Ty

- Here Tx and Ty are the amounts of horizontal and vertical displacement, respectively.

- Let P is a point with coordinates (x, y). It will be translated as (x1, y1).



Translation

- Here the point P is moved from one position to another

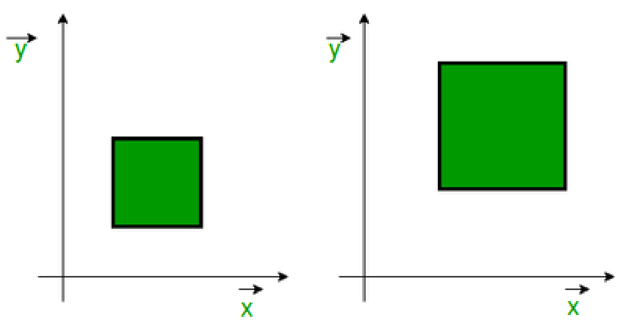
- i.e. from (x, y) to (x1, y1)

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Scaling

- Scaling is to resize the object by changing its dimensions (size)
- Here the object gets enlarge (Bigger) or it shrinks (Smaller)
- The change is done using scaling factors. There are two scaling factors, i.e. Sx in x direction Sy in y-direction.
- the new values of the points can be found by multiplying the Scaling factors with the coordinates
- If Sx and Sy are equal it is also called as Uniform Scaling. If not equal then called as Differential Scaling
- If scaling factors with values less than one will move the object closer to origin, while a value higher than one will move far from origin.

x1 = x * Sx
y1 = y * Sy



$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

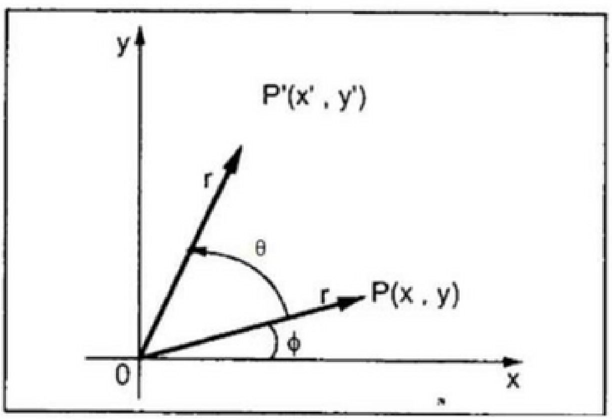
Rotation

- Rotating an object around a fixed point by a certain angle is called as rotation.
- Rotation can be clockwise or anticlockwise.
- For rotation, we have to specify the angle of rotation and rotation point.

$x1 = x * \cos(\theta) - y * \sin(\theta)$
$y1 = x * \sin(\theta) + y * \cos(\theta)$

...For anticlockwise Rotation

- Here, (x1, y1) are the coordinates of the rotated point
- (x, y) are the coordinates of the original point, and θ is the angle of rotation in radians



- Here we rotate the point (x, y) into (x1, y1) by angle A

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

...For anticlockwise Rotation

x1 = x * cos(θ) + y * sin(θ)

y1 = - x * sin(θ) + y * cos(θ) ...For Clockwise Rotation

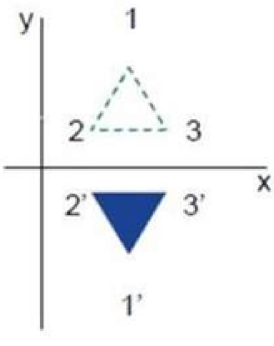
$$\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$ ...For Clockwise Rotation

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

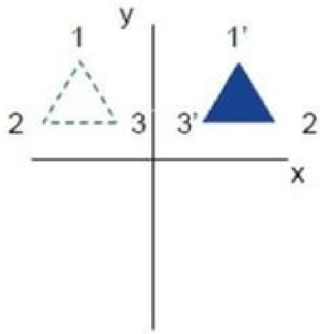
Reflection

- Reflection is to Flipping an object to create a mirror image of the object across the line (axis or a line)
- This mirror image looks like the reflection of the original image
- rotation operation with 180°
- Types of Reflection:

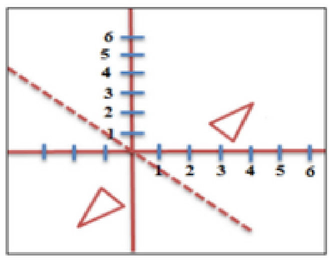
  a) Reflection about the x-axis
  b) Reflection about the y-axis
  c) Reflection about an axis perpendicular to xy plane and passing through the origin
  d) Reflection about line y = x
  e) Reflection about line y = -x



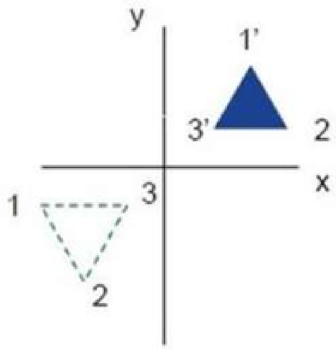
(a)



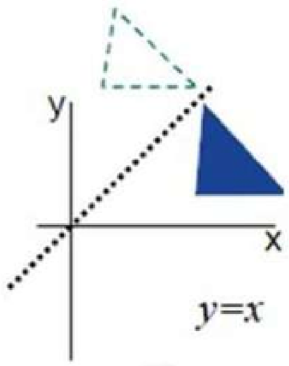
(b)



(e)

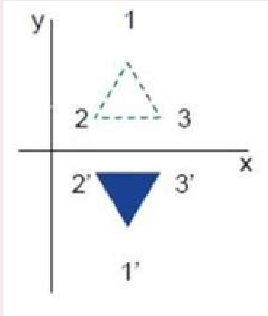


(c)



(d)

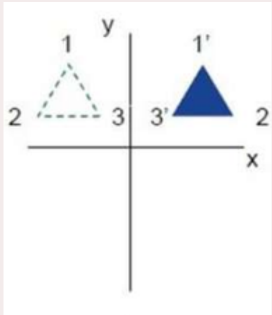
- **Reflection about x axis** (value of x will remain same whereas the value of y will become negative)



x = x

y = -y

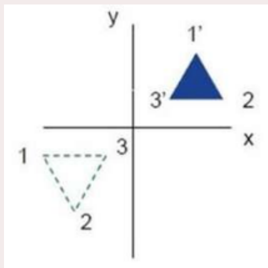
- **Reflection about y axis** (values of x will be reversed, whereas the value of y will remain the same)



x = -x

y = y

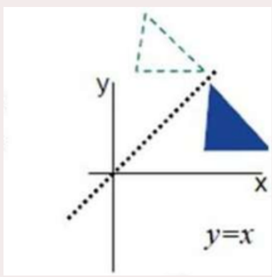
- **Reflection about an axis perpendicular to xy plane and passing through the origin** (value of x and y both will be reversed)



x = -x

y = -y

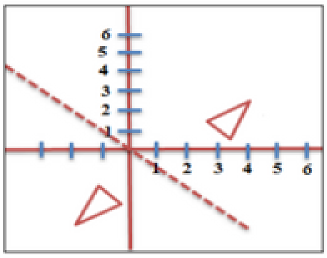
- **Reflection about line y = x** (values of x and y will be interchanged)



x = y

y = x

- **Reflection on Y = -X** (values of x and y will be interchanged and has a negetion)



x = -y

y = -x

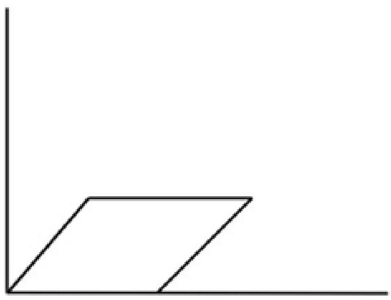
Shearing

- In shearing only some points of the object are moved form its current position to different position
- Shearing changes the shape of the Object
- The change is done using Shearing factors. There are two scaling factors, i.e. Shx in x direction Shy in y-direction.

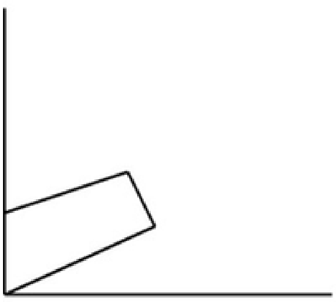
y1 = y + Shy * x (Shearing in X axis)

x1 = x + Shx * y (Shearing in Y axis)

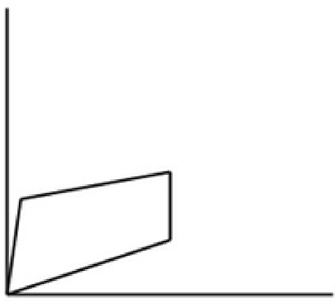
Original Object



Shear in X direction



Shear in Y direction



Shear in both directions

Matrix Representation for 2d Transformation

1. Scaling $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

2. Rotation (clockwise) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

3. Rotation (anti-clock) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

4. Translation $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_x & t_y \end{bmatrix}$

5. Reflection $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(about x axis)

6. Reflection $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(about y axis)

7. Reflection $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(about origin)

8. Reflection about Y=X $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9. Reflection about Y= –X $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

10. Shearing in X direction $\begin{bmatrix} 1 & 0 \\ Sh_x & 1 \end{bmatrix}$

11. Shearing in Y direction $\begin{bmatrix} 1 & Sh_y \\ 0 & 1 \end{bmatrix}$

12. Shearing in both x and y direction $\begin{bmatrix} 1 & Sh_y \\ Sh_x & 1 \end{bmatrix}$

Homogeneous Coordinates

- Homogeneous means a **uniform representation** of rotation, translation, scaling and other transformations.
- Homogeneous coordinates are widely used in computer graphics because they enable effective, simple manipulations of transformations in a specific way.
- **The rotation of a point, straight line or an entire image on the screen, about a point other than origin, is achieved by first moving the image until the point of rotation occupies the origin, then performing rotation, then finally moving the image to its original position.**
- Translation of point by the change of coordinate cannot be combined with other transformation by using simple matrix application. Such a combination is essential if we wish to rotate an image about a point other than origin by translation, rotation again translation.
- To combine these three transformations into a single transformation, homogeneous coordinates are used. In homogeneous coordinate system, two-dimensional coordinate positions (x, y) are represented by triplecoordinates.

Why Homogeneous?

- We have to use 3×3 transformation matrix instead of 2×2 transformation matrix. To convert a 2×2 matrix to 3×3 matrix, we have to add an extra dummy coordinate W.
- In this way, we can represent the point by 3 numbers instead of 2 numbers, which is called Homogenous Coordinate system.
- In this system, we can represent all the transformation equations in matrix multiplication. Any Cartesian point P(X, Y) can be converted to homogenous coordinates by P' (Xn, Yn, h).
- For our convenience take it as one. Each two-dimensional position is then represented with homogeneous coordinates (x, y, 1)

Translation

$$T_v = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \quad P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1*x+0*y+tx*1 \\ 0*x+1*y+ty*1 \\ 0*x+0*y+1*1 \end{bmatrix} = \begin{bmatrix} x+tx \\ y+ty \\ 1 \end{bmatrix}$$

on comparing

$$x' = x + tx$$

$$y' = y + ty$$

If there is a requirement to show homogenies coordinates we can demonstrate it by the example which is mentioned above, we can use other transformations as a example such as rotation, shearing, scaling

Rigid body transformations

- Rigid body transformations are the ones which preserve the shape and size of the object i.e. magnitude and the angle also.
- Pure rotations and pure reflections are rigid body transformation.
- Uniform scaling is not a rigid body transformation as it changes the magnitude.

Combining transformations

- Combining transformations is nothing but doing more than one transformation on a geometric shape.
- That is, one transformation followed by another transformation. The resulting transformation can frequently be described by an equivalent single transformation.

Rotation About an Arbitrary Point

- If we want to rotate an object or point about an arbitrary point, first of all, we translate the point about which we want to rotate to the origin.
- Then rotate point or object about the origin, and at the end, we again translate it to the original place.
- We get rotation about an arbitrary point

3D Transformations

What is 2d Transformation

- In Computer graphics, Transformation is a process of modifying and re-positioning the existing graphics.
- 3D Transformations take place in a three dimensional plane.
- 3D Transformations are a bit more complex than 2D Transformations.
- Properties of 3-D Transformation:
    - Lines are preserved
    - Parallelism is preserved
    - Proportional distances are preserved.

Types of 3D Transformations

- Translation
- Scaling
- Rotating
- Reflection
- Shearing

3D Translation

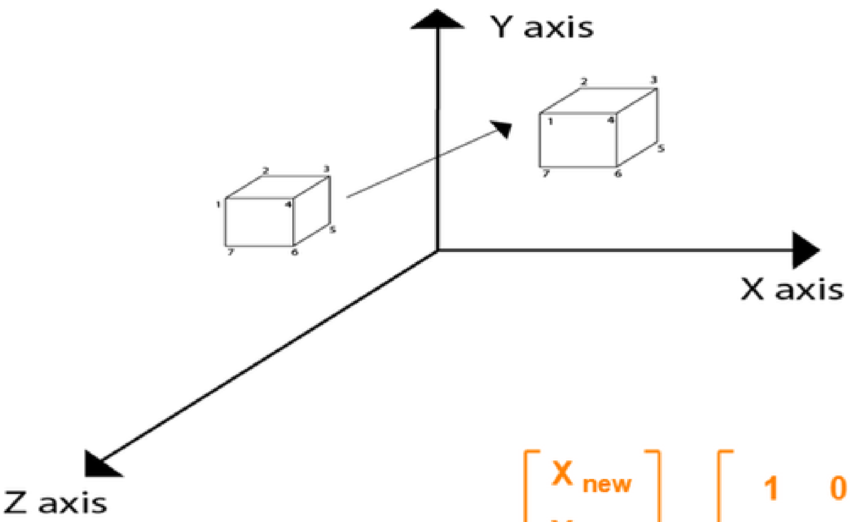
- Translation refers to Moving an object from one position to another without changing its orientation/alignment or size.
- Here we add or subtract the values of x, y and z axis with some constant values to change the position of the object
- 3D Translation can be represented as:

x1= x + Tx

y1= y + Ty

z1= z + Tz

- Translation in the x-direction is represented using Tx. The translation is y-direction is represented using Ty. The translation in the z-direction is represented using Tz.



$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling

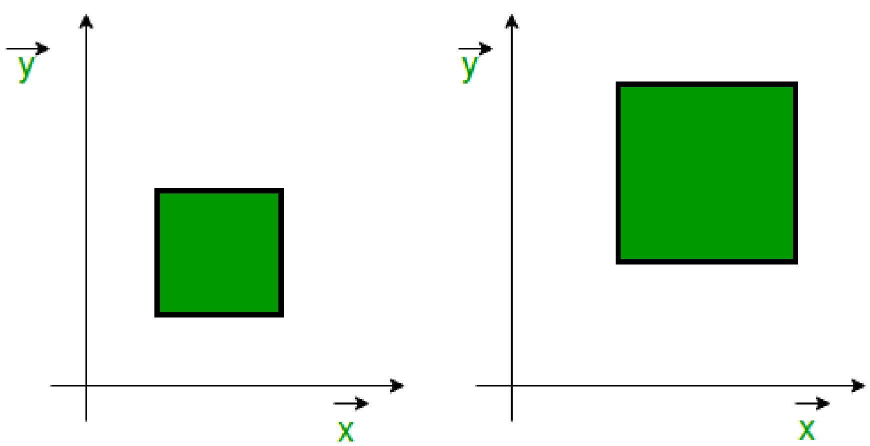
- Scaling is used to change the size of an object.
- The size can be increased or decreased.
- The scaling three factors are required Sx Sy and Sz.
    - Sx=Scaling factor in x- direction
    - Sy=Scaling factor in y-direction
    - Sz=Scaling factor in z-direction

x1= x * Sx

y1= y * Sy

z1= z * Sz

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$



3D Reflection

- Reflection is to Flipping an object to create a mirror image of the object across the line (axis or a line)

- This mirror image looks like the reflection of the original image

- Types of 3D Reflection: xy plane, xz plane, yz plane

  - **Reflection about xy plane** (value of x & y will remain same whereas the value of z will become negative)

x = x

y = y

z = -z

  - **Reflection about xz plane** (value of x and z will remain same whereas the value of y will become negative)

x = x

y = -y

z = z

  - **Reflection about yz plane** (value of y and z will remain same whereas the value of x will become negative)

x = -x

y = y

z = z

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XY plane)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XZ plane)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to YZ plane)

3D Shearing

- In shearing only some points of the object are moved form its current position to different position
- Shearing changes the shape of the Object
- The change is done using Shearing factors. There are Three Shering factors in 3D Shearing , i.e. Shx in x direction, Shy in y-direction, Shz in z direction

Shearing in X axis

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Sh_y & 1 & 0 & 0 \\ Sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In X axis)

Xnew = Xold

Ynew = Yold + Shy x Xold

Znew = Zold + Shz x Xold

Shearing in Y axis

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Y axis)

Xnew = Xold + Shx x Yold

Ynew = Yold

Znew = Zold + Shz x Yold

Shearing in Z axis

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Z axis)

Xnew = Xold + Shx x Zold

Ynew = Yold + Shy x Zold

Znew = Zold

3D Rotation

- 3D Rotation is a process of rotating an object with respect to an angle in a three dimensional plane.

Rotation in X axis

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

$X = X$

$Y = Y * \cos\theta - Z * \sin\theta$

$Z = Y * \sin\theta + Z * \cos\theta$

Rotation in Y axis

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

$X = Z * \sin\theta + X * \cos\theta$

$Y = Y$

$Z = Y * \cos\theta - X * \sin\theta$

Rotation in Z axis

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

$X = X * \cos\theta - Y * \sin\theta$

$Y = X * \sin\theta + Y * \cos\theta$

$Z = Z$

Composition of 3D Transformations

- A number of transformations or sequence of transformations can be combined into single one called as composition.
- The resulting matrix is called as composite matrix.
- The process of combining is called as concatenation.
- Suppose we want to perform rotation about an arbitrary point, then we can perform it by the sequence of two transformations
    - Translation
    - Rotation

Affine Transformation

- Affine transformation is a mapping method that preserves points, straight lines and planes.

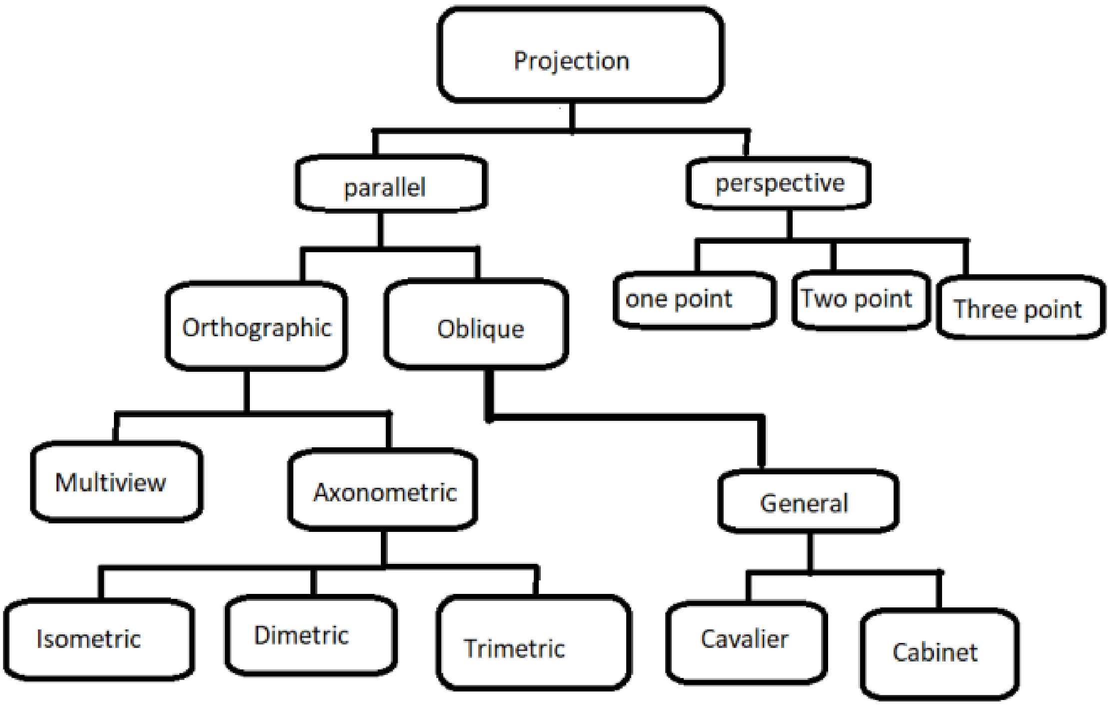
- Affine Transformation preserves parallel lines (Parallel lines remains parallel)

- It is used to fix the distortions or deformations images like satellite images.

- Basically, it helps make sure everything stays straight and in the right place, so pictures look better and it's easier to work with them.

Projections

- Representing an n-dimensional object into an n-1 dimension is known as projection.
- It is process of converting a 3D object into 2D object, we represent a 3D object on a 2D plane {(x,y,z)->(x,y)}.
- It is also defined as mapping or transforming of the object in projection plane or view plane.
- When geometric objects are formed by the intersection of lines with a plane, the plane is called the projection plane and the lines are called projections.
- Types of Projections:
    - **Parallel projections**
    - **Perspective projections**

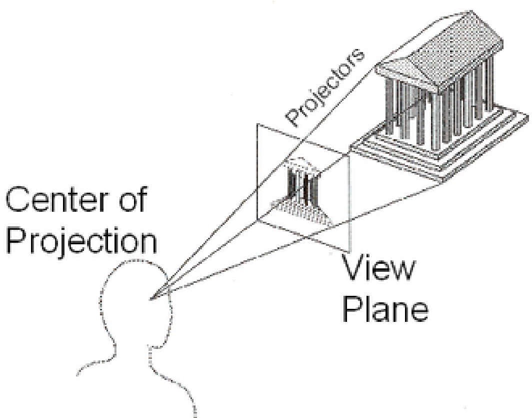


What is Centre of Projection

- It is an arbitrary point from where the lines are drawn on each point of an object.
- Also known as perspective reference point.
- If COP is located at a finite point in 3D space , Perspective projection is the result
- If the COP is located at infinity, all the lines are parallel and the result is a parallel projection

Perspective Projections

- Perspective projection is a way to represent three-dimensional objects on a two-dimensional surface, like a piece of paper or a screen.
- It is similar to what a person sees
- In perspective projection farther away object from the viewer, small it appears. This property of projection gives an idea about depth and distance
- **The distance and angles are not preserved and parallel lines do not remain parallel**
- Two characteristic of perspective are **vanishing point and fore shortening.**
- Due to fore shortening objects and lengths appear smaller from the center of projections.
- Vanishing point is the point where all lines will appear to meet.
- There can be one point, two point, and three point perspectives.



- **View plane:** It is an area of world coordinate system which is projected into viewing plane.
- **Center of Projection**: It's where the eyes are located
- **Projectors:** these are rays starting from objects in the scene, used to form an image on the viewing plane.
- **View Volume**: This is the space that's actually captured in the image. Anything inside this volume appears in the image, while things outside of it are clipped and not visible.

Types Perspective Projections

One-point perspective:

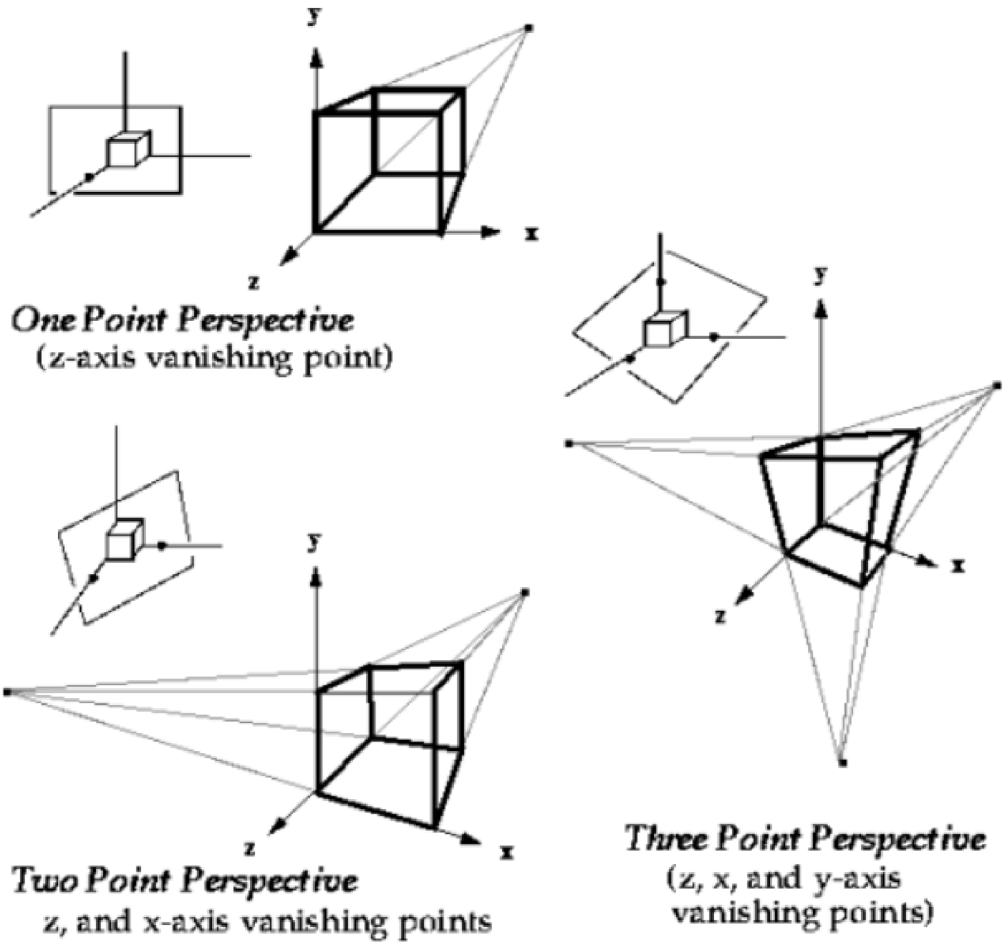
- Principal axis has a single finite vanishing point.
- Perspective projection is simple to draw.

Two-point perspective:

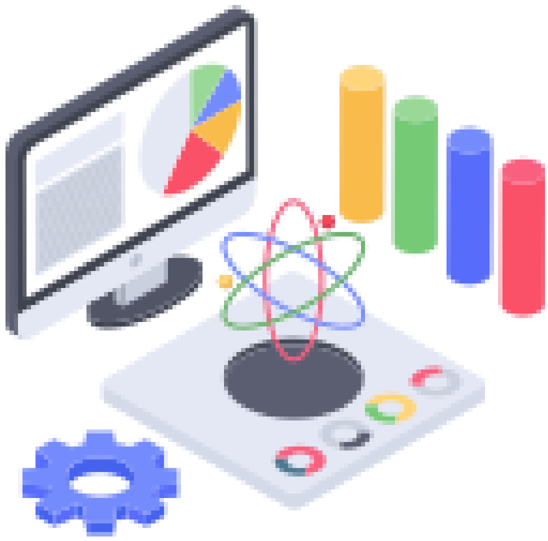
- Two principal axes have vanishing points.
- Perspective projection provides a better impression of depth

Three-point perspective:

- All three principal axes have finite vanishing points.
- Perspective projection is the most challenging to draw.



THE END



-By Dipesh Adelkar